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Emplacement Guidance for Criticality Safety in Low-Level-Waste Disposal

Prepared by
K. R. Elam, C. M. Hopper, C. V. Parks, ORNL
T. E. Harris, NRC

Oak Ridge National Laboratory

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Prepared by

K. R. Elam, C. M. Hopper, C. V. Parks, ORNL

T. E. Harris, NRC

Oak Ridge National Laboratory

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T. E. Harris, Technical Monitor

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ABSTRACT

The disposal of low-level radioactive waste (LLW) containing special nuclear material (SNM) presents some unusual challenges for LLW disposal site operators and regulators. Radiological concerns associated with the radioactive decay of the SNM are combined with concerns associated with the avoidance of a nuclear criticality both during handling and after disposal of the waste. Currently, there are three operating LLW disposal facilities: Envirocare, Barnwell, and Richland. All these facilities are located in Nuclear Regulatory Commission (NRC) Agreement States and are regulated by their respective state: Utah, South Carolina, and Washington. As such, the amount of SNM that can be possessed by each of these facilities is limited to the 10 CFR Part 150 limits (i.e., 350 g of uranium-235, 200 g of uranium-233, and 200 g of Pu, with the sum-of-fractions rule applying), unless an exemption is issued. NRC has applied these SNM possession limits to above-ground possession. The purpose of this guidance is to provide LLW disposal facility licensees which could demonstrate that SNM waste at emplacement will not cause a nuclear criticality accident. In addition, the guidance can be used by regulators in Agreement States that license LLW disposal facilities.

Five different SNM isotopic compositions were studied: 100 wt % enriched uranium, 10 wt % enriched uranium, uranium-233, plutonium-239, and an isotopic mixture of plutonium (76 wt % plutonium-239, 12 wt % plutonium-240, and 12 wt % plutonium-241). Three different graded-approach methods are presented. The first graded-approach method is the most conservative and will be easy to use for facilities that dispose of very low areal densities of SNM, or dispose of material with a low average enrichment. It relies on the calculation of average areal density or on the average enrichment of SNM. The area over which averaging may be performed is also specified, but the emplacement depth is not limited. The second method relies on limiting the average concentration by weight of SNM in the waste, and on limiting the depth of the emplacement. This method may be useful for facilities that emplace somewhat higher areal densities of SNM but do not use vaults or segmentation in the disposal emplacement. The third method relies on limiting the average concentration by weight of SNM in the waste, and on the presence of segmenting barriers, such as vaults, which will mitigate interaction between units of SNM. This method may be useful for facilities that use concrete vaults in their disposal areas, and allows even higher areal density of SNM in the disposal site.

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1 INTRODUCTION

The disposal of low-level radioactive waste (LLW) containing special nuclear material (SNM) presents some unusual challenges for LLW disposal site operators and regulators. Radiological concerns associated with the radioactive decay of the SNM are combined with concerns associated with the avoidance of a nuclear criticality accident both during handling and after disposal of the waste. A criticality accident during emplacement of LLW in a disposal site could result in a radiation dose to people who are relatively close to the incident (i.e., workers). It would also slightly increase the radioactive content of the disposal site, possibly resulting in an increase in dose to the public. Therefore, it is important that attention be paid to how SNM is disposed of in an LLW disposal facility.

Currently, there are three operating LLW disposal facilities: Envirocare, Barnwell, and Richland. All these facilities are located in U.S. Nuclear Regulatory Commission (NRC) Agreement States and are regulated by their respective state: Utah, South Carolina, and Washington. As such, the amount of SNM that can be possessed by each of these facilities is limited to the 10 CFR Part 150 limits (i.e., 350 g of uranium-235, 200 g of uranium-233, and 200 g of Pu, with the sum-of-fractions rule^a applying), unless an exemption is issued. NRC has applied these SNM possession limits to above-ground possession.

The purpose of this guidance is to provide a way that LLW disposal facility licensees could demonstrate that SNM waste at emplacement will not cause a nuclear criticality accident. In addition, the guidance can be used by regulators in Agreement States that license LLW disposal facilities. The guidance presents a graded approach to allow flexibility and is based on several assumptions. Users of the guidance should review the compatibility of these assumptions with the characteristics of the waste and disposal site. In addition, site-specific criticality safety analyses based on other assumptions could be provided by a licensee to demonstrate that SNM waste at emplacement would not cause a criticality accident. Moreover, this guidance is not intended to be applied retrospectively to past disposals.

Previous studies provide the basis for much of the data presented in this guidance. One such study¹ determined areal density limits for 100 wt % enriched uranium, 10 wt %-enriched uranium, plutonium-239, and an isotopic mixture of plutonium (76 wt % plutonium-239, 12 wt % plutonium-240, and 12 wt % plutonium-241) that would ensure subcriticality following emplacement of the waste. However, this study used very conservative models that may be overly restrictive for some disposal sites. Two other studies^{2,3} looked at the result of long-term hydrogeological processes that might mobilize and subsequently increase the concentration of uranium-235 in disposal sites.

In this guidance, five different SNM isotopic compositions were studied: 100 wt %-enriched uranium, 10 wt % enriched uranium, uranium-233, plutonium-239, and the isotopic mixture of plutonium described above. Three different graded-approach methods are presented. The first graded-approach method is the most conservative and will be easy to use for facilities that dispose of very low areal densities^b of SNM, or dispose of material with a low average enrichment. This approach relies on the calculation of average areal density or on the average enrichment of SNM. The area over which averaging may be performed is also specified, but the emplacement depth is not

^aThe quantity of each SNM isotope present is divided by the limit for that isotope. All of these ratios are then added together, and the sum must not exceed 1.0.

^bAreal density is expressed in terms of mass of SNM per area at the base of the disposal unit. Limiting the areal density of SNM in an array of units is an established method of nuclear criticality control.¹

limited. The second method relies on limiting the average concentration by weight of SNM in the waste, and on limiting the depth of the emplacement. This method may be useful for facilities that emplace somewhat higher areal densities of SNM, but which do not use vaults or segmentation in the disposal emplacement. The third method relies on limiting the average concentration by weight of SNM in the waste, and on the presence of segmenting barriers, such as vaults, that will mitigate interaction between units of SNM. This method may be useful for facilities that use concrete vaults in their disposal areas, and it allows even higher areal density of SNM in the disposal site.

2 BACKGROUND

2.1 SNM and LLW Regulations

This section summarizes NRC regulations that are applicable to the disposal of SNM waste. In some cases, equivalent regulations in Agreement States may be slightly different and may vary from state to state. This section also provides a discussion of existing and proposed LLW disposal facilities.

2.1.1 10 CFR Part 61, “Licensing Requirements for Land Disposal of Radioactive Waste”

10 CFR Part 61 sets forth licensing requirements for land disposal of radioactive waste (i.e., low-level waste). Subpart C provides requirements for performance objectives which a LLW facility shall meet to ensure the protection of the public, health, and environment. In addition to operational safety, long-term safety after closure of a facility is evaluated through the use of a performance assessment. This performance assessment estimates the dose to persons resulting from the transport of radionuclides from the disposal site, and limits are set in order to maintain this dose below certain levels. Limits on individual radionuclides based on performance assessment of a disposal site are required, and may be much lower than that required for criticality safety.

Part 61 defines three classes of waste (Class A, B, or C) that have different stability and intrusion protection requirements. This classification system (Part 61.55) is based on concentration values of several key nuclides. Classification of wastes containing only uranium isotopes is not covered specifically. Classification limits are provided for plutonium-241 and “alpha-emitting transuranic nuclides with half-life greater than 5 years,” which does encompass the other plutonium isotopes.

In addition, Part 61.16(b) requires license applicants to describe procedures for avoiding nuclear criticality accidents, which address both storage of SNM prior to disposal and waste emplacement for disposal. It is envisioned that this guidance would provide a basis for demonstrating emplacement criticality safety. Compliance with this requirement could also be based on a site-specific analysis.

2.1.2 10 CFR Part 70, “Domestic Licensing of Special Nuclear Material”

10 CFR Part 70 sets forth licensing requirements for persons who receive, own, acquire, deliver, possess, use and transfer SNM. Although Part 70 is primarily intended for fuel-cycle facilities (enrichment and fuel fabrication facilities), Part 70 does apply to LLW disposal facilities where the SNM above-ground possession quantities are greater than the 10 CFR Part 150 limits.

2.1.3 10 CFR Part 71, “Packaging and Transport of Radioactive Material”

10 CFR Part 71 sets forth requirements for packaging, preparation for shipment, and transportation of license material. In general, the type of package required to ship radioactive material is a function of the quantity (mass and activity) and the form (solid, liquid, or gas) of the material. For instance, strong tight containers, such as cardboard boxes, are acceptable to ship small quantities of medical isotopes, whereas heavy steel casks, which are tested to meet hypothetical accident conditions, are required to ship nuclear fuel. Part 71 provides several general licenses and exemptions for shipping SNM that depend on several factors, including the mass of SNM in the packages, the mass of SNM in the shipment, the concentration of SNM, and the presence of moderating materials. Part 71 also references applicable Department of Transportation (DOT) regulations.

2.1.4 10 CFR Part 150, “Exemptions and Continued Regulatory Authority in Agreement States and in Offshore Waters Under Section 274”

10 CFR Part 150 sets forth provisions where licensees in Agreement States are exempt from NRC licensing requirements and where licensees remain under the regulatory authority of the NRC. As it relates to SNM and LLW disposal, licensees in Agreement States are exempt from NRC regulations for possession of “special nuclear material in quantities not sufficient to form a critical mass.” This term is defined in 150.11 as quantities not exceeding 350 g of uranium-235, 200 g of uranium-233, 200 g of plutonium, or combinations not exceeding the sum-of-fractions rule. However, disposal of SNM in an Agreement State may require an NRC license if the Commission determines that it should, based on “hazards or potential hazards thereof.” Relative to LLW disposal facilities, NRC has applied the possession limits in Part 150 to above-ground possession prior to disposal. SNM that has been emplaced would no longer be subject to these possession limits.

2.2 Current LLW Sites and Licensed Limits for SNM

Currently, there are three LLW disposal facilities (Envirocare, Barnwell, and Richland). All these facilities are located in NRC Agreement States and are regulated by their respective state (Utah, South Carolina, and Washington). As such, the amount of SNM that can be possessed above ground by each of these facilities is limited to the 10 CFR Part 150 limits (i.e., 350 g of uranium-235, 200 g of uranium-233, and 200 g of plutonium, with the sum-of-fractions rule applying), unless an exemption is issued. NRC has applied these SNM possession limits to above-ground possession.

The disposal site designs currently in use vary widely from highly engineered systems with concrete vaults to landfill-style embankments. Emplacement depths range from around 20 to 45 ft. The graded-approach limits given in Sect. 7 are designed to provide guidance for this range of designs and emplacement depths. Even though current license limits for disposal of SNM vary somewhat, this variation is expected due to the differences in site design and emplacement methods. Also, each Agreement State has decided on its preferred methods for setting limits. In some cases, the license limits for SNM isotopes are based upon radiological concerns and are therefore much lower than that needed for criticality safety concerns. For example, the plutonium-239 concentration limit at Envirocare is significantly lower than the limit suggested in this guidance.

2.2.1 Envirocare, Clive, Utah

Waste is received either uncontainerized (i.e., gondola rail cars) or containerized in drums, boxes, or intermodals. The containerized waste is typically removed from the container prior to disposal. Bulk waste materials and waste removed from the containers are placed in lifts with uncompacted thickness not exceeding 12 in. and are then compacted in a landfill-style above-ground embankment. Debris (nonsoil waste material, such as concrete) is coemplaced with waste or clean soil, but is restricted as to its volume fraction and placement. The embankment height is limited to 37 ft of compacted waste with a 10.9-ft-thick multilayer cover.

Unlike other LLW disposal facilities, the Envirocare license sets a limit on the maximum average concentration for specific isotopes, including SNM isotopes, with a sum-of-fractions rule.

2.2.2 Barnwell, South Carolina

Since 1996, the Barnwell facility disposes of waste containers in concrete overpacks or vaults within below-ground trenches. These vaults are approximately cubical with 10-ft outer dimensions (130 in. long, 108 in. wide, 128 in. high) and are stacked three vaults high in the trench. These new trenches are typically 1000 ft long, 200 ft wide, and 25 ft deep. Before 1996, waste containers were placed directly into below-ground earthen trenches. These older trenches are 15 to 22 ft deep. Trenches are backfilled with sand prior to placement of a 6-ft cap. SNM waste must be received in containers that are at least 55 gals in volume (license condition 54).

From 1979 to 1997, the site was licensed by the NRC under 10 CFR Part 70. Under this license, the license allowed 4500 g of uranium-235 and 200 g of uranium-233 in undisposed waste. Plutonium was limited by concentration. This license also had disposal requirements. No single package could exceed 350 g of uranium-235, and had to have a minimum projected surface area of 2 ft². The average areal density could then not exceed 200 g uranium-235/ft², assuming a 95% confidence in the shipping values. If this 95% confidence was not met, the limit was decreased to 100 g uranium-235/ft². This areal density limit is not in the South Carolina license but has been retained in the facility operating procedures. The current license allows 350 g uranium-235, 200 g of uranium-233, or 200 g of plutonium in undisposed waste, with a sum-of-fractions rule applied to mixtures. In addition, transuranic isotopes are limited in license condition 40 to less than 1% of the total activity of the waste. Typically, Barnwell receives only insignificant quantities of plutonium-contaminated waste.

2.2.3 Richland, Washington

The Richland facility disposes of waste in below-ground trenches, similar to the Barnwell facility. Trenches are 20 to 45 ft deep with a 5-ft cap. Trenches may be no more than 150 ft wide, 45 ft deep, and 1000 ft long. Waste is at least 8 ft below grade. Waste is received and buried in “closed containers,” unless specifically approved. No cardboard, corrugated paper, wood, or fiberboard containers are allowed. Metal containers are accepted.

Like the Barnwell facility, the Richland facility was licensed by the NRC under 10 CFR Part 70 prior to 1997. As such, the Richland license includes several criticality-related conditions. No package can have more than 100 g of uranium-235, 60 g of uranium-233, or 60 g of plutonium, with the sum-of-fractions rule applied for mixtures. Also, no package can have more than 15 g per cubic ft of the above three materials, and the SNM is supposed to be “essentially uniformly distributed” in the package (license condition 27). During disposal, there are requirements for package placement for SNM. An “accumulation” of packages is defined as a group containing no more than 350 g uranium-235, 200 g of uranium-233, or 200 g of plutonium (with sum of fractions rule for mixtures). Each accumulation of packages are disposed with a minimum of 8 in. of soil or 4 ft of non-SNM-bearing waste in all directions from other SNM waste accumulations (license condition 52).

2.3 Proposed LLW Disposal Sites

Several LLW disposal facilities have been proposed for construction. However, recently the state regulators for the proposed Nebraska and Texas facilities have denied operating licenses. The California site remains in limbo, pending land transfer from the Federal Government. Even though these facilities may not be constructed, it is believed that the design of future LLW facilities would be similar to these proposed facilities. Therefore, the method of disposal is discussed for each of these facilities.

2.3.1 Proposed Nebraska Site

The Nebraska site proposed disposal of waste containers in above-grade concrete vaults that are backfilled with grout before capping. The vault roof is 3-ft-thick concrete. A multilayer cover, consisting of a waterproof membrane, sand, clay, more concrete, and soil, totaling a thickness of 17 ft, was proposed. The vaults were to be constructed of low-permeability concrete with close-spaced reinforcing steel. Waste in drums would be placed 4 to a pallet, and stacked no more than 4 pallets high. Boxes were proposed to be stacked 3 to 4 high within a vault. Vault heights were estimated to be in the range of 28 ft.

2.3.2 Proposed Texas Site

The Texas site proposed disposal of waste containers in cylindrical concrete vaults that are preplaced into below-ground trenches. The vaults were proposed to be 9 ft in height and 8 ft 4 in. in diameter, with 10-in.-thick walls and 13.5-in.-thick tops and bottoms. The vaults were to be placed a minimum of 18 in. apart in a triangular-pitched array, and stacked two deep. Canisters would have been filled with waste packages and then backfilled with grout. The space between the canisters was to be filled with sand. The trench depth was in the range of 33 ft.

2.3.3 Proposed California Site

The California site proposes disposal of waste containers directly into earthen trenches. This method of disposal is similar to that formerly used by the Barnwell site and currently employed by the Richland site. Unique to the California design, the drums will be placed on their sides. A trench depth of 60 ft is proposed, with 40 ft for the waste and a 20-ft soil cap.

3 CRITICALITY SAFETY CONCERNS RELEVANT TO LLW DISPOSAL FACILITIES

Criticality safety is an important consideration in the disposal of LLW containing SNM. A nuclear criticality accident during emplacement of LLW in a disposal site could result in a radiation dose to people who are relatively close to the incident (i.e., workers). It would also slightly increase the radioactive content of the disposal site, possibly resulting in an increase in doses to the public. Therefore, it is important that attention be paid to how SNM is disposed of in a LLW disposal facility.

Spontaneous nuclear fission occurs naturally in a very small percentage of radioactive decays of SNM atoms. When fission occurs, neutrons are emitted along with the nuclear fragments (e.g., cesium and strontium atoms). These neutrons may be absorbed by a non-SNM nucleus, may be absorbed by a SNM nucleus and cause fission, or may be lost from the system through leakage or some other mechanism. In natural uranium (enrichment of 0.71 wt % uranium-235) that is present in most soil, the neutrons produced during spontaneous fission are typically absorbed by a non-SNM nucleus.

Unlike spontaneous nuclear fission, criticality is a chain reaction of fission events where large numbers of neutrons are produced. Criticality occurs when, on average, one neutron produced during a fission goes on to produce one other fission. Since more than one neutron may be produced during each fission event, some neutrons can be lost to absorption or leakage while still possibly leaving enough neutrons to produce additional fissions. In other words, criticality requires a balance between neutron production and neutron loss. Criticality safety relies on minimizing neutron production and maximizing neutron losses.

The probability of a particular reaction between a neutron and a nucleus, for example absorption that causes fission, is given the term “cross section,” and is a property of the particular isotope and the energy of the neutron. In other words, if a given material is exposed to neutrons, the rate at which any particular nuclear reaction occurs depends upon the number of neutrons, their energy, and the number of nuclei of the particular material. In general, SNM isotopes tend to be more likely to undergo fission when interacting with relatively low-energy neutrons. Therefore, their cross section for fission is higher for slow neutrons than for fast neutrons.⁴

Even though a criticality can intentionally be produced with a relatively small mass of SNM under ideal conditions, it is more difficult for a criticality to occur in more diffuse material, such as LLW. However, it is not inconceivable for a criticality accident to occur at an LLW disposal facility. For example, if containers with SNM of sufficient density, which are not critical individually, were stacked in a large array, such as in a disposal cell, a criticality could occur. Specific considerations that affect criticality safety are discussed in the following sections. In general, the most important factors that affect the criticality safety of an LLW disposal site are the following:

1. the quantity, isotope, enrichment, and distribution of the SNM;
2. the presence of moderating materials, and their distribution; and
3. the presence of neutron-absorbing materials, and their distribution.

The quantity of SNM present in a disposal site can be described using different measures. One common measure is concentration in terms of grams of SNM per cubic foot. Even though this value is generally easy to determine in a waste package, it is heavily dependent on void space. If the package is compacted, or the contents emptied, this concentration could change dramatically. Another way to measure SNM is areal density, which is the mass of SNM per unit area of a disposal site as if it were projected downward onto a horizontal surface. In other words, a disposal unit that contains 5 g per ft³, and is 10 ft deep, would have an areal density of 50 g per ft². The advantage of using areal density as a measure of the SNM in a disposal site is that vertical settling or compaction

will not change the areal density value. This measure is used to describe the limits in the first graded approach. The second and third graded approaches use concentration limits in terms of grams of SNM per gram of waste.

The isotopic composition and enrichment of the SNM depends on the waste stream. To be used as fuel in reactors, natural uranium is enriched in uranium-235. Enrichment is a ratio of the weight of uranium-235 to total uranium, and is typically expressed as a percent. Most nuclear fuel is enriched to <5 wt %; therefore, waste streams from fuel fabrication facilities contain low-enriched uranium. However, nuclear fuel for naval submarines is enriched to >90 wt %. The criticality concern increases with enrichment. To bound these two types of waste streams, the guidance provides limits for 10 wt %- and 100 wt %-enriched uranium. Significant quantities of plutonium and uranium-233 are not common in commercial or industrial waste streams. These waste materials are predominately generated from the cleanup of Department of Energy (DOE) facilities and in high-level waste (spent nuclear fuel).

The distribution of SNM in the disposal site is also an issue. Regions of higher SNM concentration could pose a criticality concern, even if the average concentration is quite low. Because of this concern, Sect. 6 of this report gives limits on the mass of waste or the area over which the SNM distribution may be averaged.

When SNM is in solution, or present as finely divided particles, such as in LLW, the presence of a “neutron moderator,” such as water or hydrocarbons (e.g., plastics), can significantly reduce the amount of SNM required for criticality.⁵ Slow neutrons have a far greater probability of causing fission in uranium-235 or plutonium-239. However, most neutrons produced by a fission are “fast” neutrons. A moderator slows neutrons as the neutrons collide with the moderator atoms. Elements with light nuclei, such as hydrogen, deuterium, and carbon, are particularly good neutron moderators. An optimum degree of moderation exists because if the ratio of hydrogen to uranium becomes too large, the probability that the hydrogen will absorb the neutron becomes larger. Materials that capture neutrons are termed absorbers, and most materials are both moderators and absorbers to varying degrees. The effect of moderators with very low neutron-absorption characteristics, such as beryllium, is discussed in Sect. 3.3. (See also Ref. 6.)

Many materials found in LLW or in the soil surrounding the emplaced waste are very effective neutron absorbers. Section 5.2 discusses the calculational method used to derive the limits in Sect. 6, and why silicon dioxide was chosen as a surrogate material for waste. Because most other elements found in soil or waste are better neutron absorbers than silicon, this calculational method produced conservative results. However, these other elements (e.g., calcium, iron, copper) are likely to be present in waste and soil, and their neutron-absorbing properties reduce the likelihood of an inadvertent nuclear criticality. Some materials, cadmium and boron in particular, are extremely effective neutron absorbers. Such neutron absorbers may be used to provide criticality control in waste packages. However, in a disposal site environment, it is difficult to predict if the neutron absorber will stay with the SNM, and not leach away over time. Therefore, it is not recommended that neutron absorbers in soil be relied upon for long-term criticality control in an LLW disposal site.

3.1 Individual Package Limits

The requirements of 10 CFR Part 71 ensure criticality safety for transport of packages containing SNM. For disposal facilities licensed by NRC Agreement States, the amount of SNM in packages is further constrained by the mass limits in 10 CFR Part 150. Part 150 defines the amount of SNM which can be licensed by an Agreement State as 350 g of uranium-235, 200 g of uranium-233, and 200 g of plutonium. If mixtures of SNM isotopes are present in the waste, the “sum of fractions” rule applies. These package limit quantities of SNM have been shown to be subcritical.

3.2 SNM Migration and Concentration

One assumption that could be made regarding emplacement criticality is that the SNM does not move from its original “as disposed” position. Following disposal, however, it is reasonable to assume that the container (i.e., metal drum) will degrade in tens of years. Therefore, another assumption that could be made is that the containers are no longer able to physically contain the waste. When the containers fail, the waste could either settle and fill the bottom of a concrete vault or settle within the trench. In these situations, the waste could become slightly denser, but the areal density, or grams of SNM per square ft, and the concentration of the SNM within the waste would be essentially unchanged. This guidance considers that this physical movement could occur.

Another situation would be that water could infiltrate the disposal unit and that the SNM, particularly uranium, may change chemically and may move from its original disposal position. In this case the concentration of the SNM in the waste would change. One could further assume that the SNM in solution with the infiltration water could move horizontally, vertically, or both, and be redeposited in another location. This process is called “reconcentration.” Physical barriers to movement, such as vaults, can reduce the amount of SNM that is available for migration and reconcentration. However, such barriers can also act as collection points for mobile SNM. The larger the volume over which SNM collection is considered, the larger the potential increase in the local concentration of SNM. If this situation occurs, the distribution of SNM over the entire disposal site would need to be considered to evaluate the concern that large masses of SNM could be reconcentrated, thereby posing the potential for a nuclear criticality accident.

To evaluate this reconcentration concern, two previous studies^{2,3} have been performed. The scope and results of these studies are discussed in Sects. 4.3 and 4.4. Based on these studies, NRC concluded that the reconcentration process is unlikely and would take tens of thousands of years to reconfigure the uranium to pose a criticality concern. Note that simplifying assumptions were used to reach these conclusions and that large uncertainties remain. The Commission directed NRC staff to discontinue research in this area and consider the recommendations from these studies and the need to limit unusual moderators in this guidance document. Unusual moderators in LLW are discussed in Sect. 3.3.

The above referenced studies^{2,3} assumed that the dominant uranium compound was uraninite, or uranium dioxide (UO₂). Uranium dioxide is a common uranium compound that is relatively insoluble in water. Other compounds of uranium are highly soluble. These compounds include uranyl fluoride, uranyl nitrate, and uranyl potassium carbonate. If soluble uranium compounds were present in LLW, the disposal facility operator may not be able to determine their presence from available documents. NRC Forms 541, “Uniform Low-Level Radioactive Waste Manifest,” and 741, “Nuclear Material Transaction Report,” require that only prevalent chemical forms of the waste be reported. This information may or may not include the SNM chemical form. Therefore, some uncertainty exists on the presence of soluble uranium.

Even though these compounds were not considered in the above-referenced studies, two processes need to happen for reconcentration to occur: The first process is that the uranium needs to be mobilized in water. Highly soluble forms of uranium would be mobilized much faster than UO₂. The second process is that the uranium would then need to be immobilized. (If the uranium is not immobilized, its concentration does not increase and there is no increased likelihood of a criticality accident.) Therefore, the conclusions of the studies relative to the processes and rates of immobilization would not be affected if the uranium was highly soluble. Moreover, if the void space between containers is filled with soil material containing silica (sand), it is likely that the uranium would not

migrate. The good practice segment in Sect. 7 recommends backfilling the void space between SNM containers with sand or grout and minimizing the quantity of soluble uranium to the extent practicable.

In addition, substantial quantities of soluble uranium are not anticipated to be present in LLW. Because uranium is expensive to enrich and soluble uranium is easily recoverable, it is likely that generators would reprocess this type of waste and recover the majority of the soluble uranium.

3.3 Coemplacement of SNM with Unusual Moderating Materials

Because water is both a good moderator (a material that slows neutrons) and a poor absorber (a material that does not capture neutrons), it is common in criticality safety evaluations to assume that water may be present with the SNM. *The guidance presented in this report makes this assumption.* However, there are other materials, such as beryllium, graphite, and heavy water (D₂O), that are less efficient neutron moderators than water, but have lower neutron-absorption characteristics than water. Therefore, a system of SNM with such moderators can become critical at a lower concentration than it would with only water present.⁶ These moderating materials are used at many nuclear facilities, and may be present with the SNM in the LLW. Note that the guidance presented in this document is not applicable if the LLW contains more than trace amounts (0.1 wt %) of these materials. Furthermore, it is a good practice to segregate SNM LLW from LLW containing beryllium, graphite, or heavy water.

4 SUMMARY OF PREVIOUS ANALYSIS RELEVANT TO EMPLACEMENT GUIDANCE

4.1 General Screening Criteria

NUREG/CR-6284,¹ which was prepared to provide screening criteria for NRC licensing of LLW facilities, presents very conservative areal density limits. In a license application, if a licensee proposed values below these limits, then NRC would not need to perform any additional criticality review to ensure safety. Operational limits were determined from surface-density spacing criteria that were developed in the 1960s⁷ and 1970s⁸ and applied by the NRC.⁹ The surface-density spacing criteria developed in this report are based upon SCALE¹⁰ calculations for establishing operational limits by taking into consideration the following:

1. type and isotopic compositions of SNM (100 wt %-enriched uranium, 10 wt %-enriched uranium, plutonium-239, and an isotopic mixture of plutonium containing 76 wt % plutonium-239, 12 wt % plutonium-240, and 12 wt % plutonium-241);
2. single-package mass limits based upon the fraction critical (i.e., ratio of the mass of a single unit to the bare critical mass of the same SNM in a similar shape);
3. optimization of SNM density or degree of moderation;
4. optimization of cylindrical geometry height-to-diameter ratios;
5. use of realistic maximum reflector materials (i.e., silicon dioxide as opposed to unquantifiable damp soil or water);
6. reflector spacing;
7. interspersed moderation and container materials;
8. array lattice patterns; and
9. calculational uncertainties.

The surface-density spacing criteria that evolved from these optimization and maximization studies provided conservative license review screening criteria for which no other assumptions must be made except that significant horizontal migration of SNM will not occur. As augmented with limiting enrichment values to ensure subcriticality, other SCALE computations,⁶ and SCALE computations for uranium-233 performed for this report, NUREG/CR-6284 provides the bases for the first graded approach that can be applied uniformly to all license applications.

4.2 Transportation Exemptions and General-License Conditions

NUREG/CR-5342⁶ was prepared to support possible changes to 10 CFR Part 71. The report reviews the current transportation regulations, assesses their technical bases, and provides recommendations on changes to Part 71 relative to fissile material exemptions and general licenses. Among other information, minimum subcritical masses and limiting concentrations of the three primary SNM isotopes (uranium-235, uranium-233, and plutonium-239) in

various moderators of interest (water, polyethylene, silicon dioxide, carbon, beryllium, and heavy water) were provided. The SCALE computational results in NUREG/CR-5342⁶ augment information in NUREG/CR-6284¹ to provide the basis for the first graded approach.

4.3 Criticality Potential for LLW Blended with Soil

NUREG/CR-6505, Vol. 1² was prepared to aid the NRC in evaluating the possibility that SNM in waste might reconcentrate after disposal. It was developed specifically to examine the potential for uranium migration and deposition into a potential critical configuration within an LLW disposal facility like Envirocare, where the waste is not generally disposed within containers. The report considered the initial uniform distribution and temporal/environmental redistribution of 100 wt %-enriched uranium into infinite slabs, infinite cylinders and spheres having variable uranium-235 densities and variable degrees of water moderation in a surrogate waste matrix (1.6 g SiO₂/cm³), reflected by SiO₂. Critical areal densities and mass values were derived. NUREG/CR-6505, Vol.1, taken together with Ref. 3 and augmented with SCALE calculations for uranium-233, plutonium-239, and the isotopic mixture of plutonium, provides the basis for the second graded approach.

4.4 Criticality Potential for Containerized LLW

NUREG/CR-6505, Vol. 2,³ with containerized waste, was prepared as a companion study to Ref. 2 and explored the reconcentration process at humid sites with containerized waste. It was developed specifically to examine potential migration of 10 wt %-enriched uranium and deposition into a potential critical configuration in a LLW disposal facility like Barnwell, South Carolina, where the waste is disposed of in containers and within vaults. Critical areal densities and mass values were derived. The report considered the initial uniform distribution and temporal/environmental redistribution of 10 wt %-enriched uranium into infinite slabs, infinite cylinders and spheres having variable uranium-235 densities and variable degrees of water moderation in a surrogate waste matrix (i.e., 1.6 g SiO₂/cm³) reflected by SiO₂. NUREG/CR-6505, Vol. 2, taken together with Ref. 2 and augmented with SCALE calculations for uranium-233, plutonium-239, and the isotopic mixture of plutonium, provides the basis for the second graded approach.

5 CALCULATIONAL METHODS AND ASSUMPTIONS

5.1 Calculational Methods

Criticality calculations used in developing the limits in this guidance were performed using the SCALE computer code system, Version 4.4, which includes XSDRNPM and KENO V.a.¹¹ To improve efficiency, many of the preliminary calculations were performed with the 27-group ENDF/B-IV neutron cross-section library. Calculations used to define guidance in this document were performed with the more recently evaluated 238-group ENDF/B-V neutron cross-section library.^c

As discussed in Sect. 4, much of the data were derived from previously published studies.^{1-3,6} The data were augmented by using methods outlined in these studies, and calculating data for SNM isotopes that were not previously evaluated, such as uranium-233. This accounts for much of the variation seen in the calculational methods used for the different graded approaches.

The physical dimensions of the limit specifications (areal density in g/ft², and other dimensions of height in ft) were chosen to be compatible with information available to LLW disposal facility personnel. The gram was selected because grams are the units provided on NRC Form 541 for the SNM content in a container. Square feet was selected because most personnel working at an LLW disposal facility are familiar with their building, trench, bunker, etc., dimensions in terms of square feet. If needed, the conversion of g/ft² to kg/m² may be accomplished by multiplying by the constant 0.010763. Other dimensions of height are given in ft to be compatible with engineering drawings and other LLW disposal facility records. The conversion of ft to m may be accomplished by multiplying by the constant 0.3048.

The basis for comparison between criticality calculations for different cases was the neutron multiplication factor k , which is a measure of the potential of the modeled system to support a self-sustaining fission chain reaction.⁴ In an infinite system with no neutron leakage, k_{inf} is defined as the ratio of the rate of neutron production to the rate of neutron absorption. In a finite system, k_{eff} is defined as the ratio of the rate of neutron production to the sum of the rates of neutron absorption and leakage. In a critical system, k is equal to 1. However, to account for some of the uncertainties in the calculational methods and cross sections, a calculated $k < 1$ may be used as a “critical” value.

5.1.1 Calculational Uncertainties

Even though the SCALE computer code system and the cross-section libraries used have been extensively validated against critical benchmark experiments,¹² many of the systems modeled in this report are not bounded by the available experimental data. Therefore, it is not possible to determine the calculational bias. This is particularly true of the systems modeled for the second and third graded approaches, many of which are dry, silicon-moderated systems. No critical experiments are available for validation of these models. Subcritical and operational margins in this guidance are therefore based on experience and engineering judgement. However, the uncertainties involved in the calculations are considered small compared with the variation found in disposal environments. Also, the methods used to calculate subcritical values from the calculated critical values contain sufficient conservatism so that computational uncertainty is not a major factor. Numerous calculations were performed to determine

^cThe computations were executed on workstations CA37 and CA38 at the Oak Ridge National Laboratory, Computational Physics and Engineering Division, Nuclear Engineering Applications Section. The modules and cross-section data set creation dates were the following: BONAMI—1/12/99; NITAWL—9/18/98; XSDRNPM—5/6/99; KENO V.a—7/31/98; and scale.rev07.xn238—6/22/98.

“optimum” conditions for parameters, such as water content and SNM concentration. Soil impurities, such as iron and calcium, that would lower the k_{eff} have not been included.

5.1.2 First Graded Approach

To calculate the areal densities used in the first graded approach, the method described in NUREG/CR-6284¹ was used. Hypothetical cylinders filled with SNM solution were placed in an infinite planar array. The array was reflected with 240 cm (7.9 ft) of silicon dioxide on top and bottom. Each cylinder was limited to a subcritical SNM mass, which was calculated for each type of SNM by (1) determining the minimum critical mass for hydrogenous-reflected and moderated spheres, and (2) applying a safety margin of 2.3 to account for accidental double-batching and uncertainties. These mass limits were 350 g for uranium-235, 250 g for uranium-233, and 225 g for plutonium-239. For a given cylinder height/diameter (H/D) ratio and variable SNM solution concentrations, the height and diameter that would contain this subcritical mass were determined. SCALE critical pitch (i.e., center-to-center spacing) searches utilizing CSAS4 and KENO V.a were used to find the array conditions that would produce a k_{eff} of 1.000 ± 0.005 . This critical pitch was then used to calculate an average critical areal density for each configuration. The configuration producing the minimum critical areal density served as the basis for the limits in the first graded approach. The critical areal densities were reduced by 20% to produce subcritical limits, and then by another 10% to produce operational limits. (The basis for this methodology is given in Ref. 1.) Reference 1 provided the data for 100 wt %-enriched uranium, 10 wt %-enriched uranium, plutonium-239, and the heretofore specified isotopic mixture of plutonium. Additional calculations were performed to provide similar limits for uranium-233 in this guidance. Results are tabulated for the various SNMs in Sect. 7.1.1. The complete data set is given in Appendix A. An example input file is shown in Appendix B.

5.1.3 Second Graded Approach

To calculate the SNM concentrations used in the second graded approach, the method described in Ref. 2 was used. SCALE calculations utilizing CSAS1X and XSDRNPM were used to determine combinations of silicon dioxide, water and the SNM that are critical in an infinite system. In this case, critical was defined as having a k_{inf} greater than 0.95. (A higher level of uncertainty is associated with these calculations compared with those in Sect. 5.1.2, due to the lack of critical experiments involving silicon-moderated systems.)

These combinations were then modeled as infinite slabs. SCALE critical dimension searches utilizing CSAS1X were performed to determine the slab thickness that yielded a k_{eff} of 0.950 ± 0.001 . (An example input file is shown in Appendix B.) Four meters of reflector were placed on each face of the slabs. The composition of the reflector region matched that of the waste region, except that there was no SNM in the reflector. The density of the silicon dioxide was 1.6 g/cm^3 , and the maximum void fraction available for SNM and water together was 0.40. For a given SNM concentration, the water concentration that provided the minimum critical slab thickness was determined. Reference 2 provided this data for 100 wt %-enriched uranium (Ref. 2, Table C-2), and Ref. 3 provided this data for 10 wt % enriched uranium (Ref. 3, Table A.1). Additional calculations were performed to provide similar data for uranium-233, plutonium-239, and the isotopic mixture of plutonium in this guidance. The complete data set for the additional calculations is given in Appendix C.

The SNM concentration required to produce a critical slab of a given thickness was determined from this data. These concentrations were calculated in units of g SNM/cm^3 waste and were based on the assumed waste density of 1.6 g/cm^3 (about 100 lb/ft^3). Because some waste will exceed this density, and in some cases will average about 2.4 g/cm^3 (about 150 lb/ft^3), the concentrations were adjusted to accommodate this increase. Using the

concentration in g/cm^3 and dividing by 2.4 g waste/cm^3 yields an SNM concentration in g SNM/g waste that is a conservative estimate of the critical concentration in a given slab thickness.

In order to account for calculational uncertainty beyond that included in performing critical thickness searches at a k_{eff} of 0.95, and to bound operational uncertainties related to determining actual SNM concentrations in the waste, these critical slab concentrations were reduced by a factor of 0.70 to produce operational limits. For the chosen emplacement depths (up to 10 ft, up to 20 ft, and greater than 20 ft), the operational limits were interpolated from the available data, and are tabulated in Sect. 7.1.2. These operational limits at the chosen emplacement depths were checked using SCALE calculations of k_{eff} using an infinite, silicon dioxide reflected slab, and all of the tabulated systems yield a k_{eff} of less than 0.84.

5.1.4 Third Graded Approach

To calculate the SNM concentrations used in the third graded approach, the method described in Sect. 5.1.3 was altered to include a concrete layer between separate waste regions, and to determine whether segmenting a given emplacement depth into two or three layers would allow for higher SNM concentration limits. Such a comparison was made using a model with a single 30-ft-deep emplacement, and comparing results with a model using three 10-ft-deep vaults separated by 16 in. of concrete. This vault dimension was chosen as being representative of vault sizes currently in use, or proposed for future LLW disposal sites (see Sects. 2.2 and 2.3).

Each vault was modeled as an infinite slab having 20.32 cm (8 in.) of concrete in both the floor and in the ceiling, and having a 3.048 m (10-ft) inner height. The vaults were stacked, and 4 m of silicon dioxide reflector was added to the top and bottom of the stack. The same SNM waste matrix was used for this approach, and the data described in Sect. 5.1.3 were used to determine the water concentration that would yield the minimum critical SNM concentration. In the case of 10 wt %-enriched uranium, the minimum critical concentration at around 10-ft slab thicknesses resulted from using 0.03 g/cm^3 water. All of the other SNM combinations were modeled dry. The SiO_2 density was 2.4 g/cm^3 . SCALE critical concentration searches utilizing CSAS4 and KENO V.a were performed to determine the SNM concentration in the vaults that produced a k_{eff} of 0.950 ± 0.005 . (An example input file is shown in Appendix B.) In order to account for calculational uncertainty beyond that included in performing critical concentration searches at a k_{eff} of 0.95, and to bound operational uncertainties related to determining actual SNM concentrations in the waste, these critical SNM concentrations were reduced by a factor of 0.70 to produce operational limits. The operational limits are tabulated in Sect. 7.1.3. These operational limits were checked using SCALE calculations of k_{eff} using an infinite, silicon dioxide-reflected vault stacks, and all tabulated systems yielded a k_{eff} of less than 0.54.

5.1.5 Uniformity Criteria

To calculate the area over which the areal density may be averaged, and the mass of waste over which the enrichment or concentration of SNM may be averaged, critical sphere radii were calculated using the method described in Ref. 2. Starting with the combinations of silicon dioxide, water and the SNM that were critical in an infinite system as described in Sect. 5.1.3, SCALE critical dimension searches utilizing CSAS1X were performed to determine the spherical radius that yielded a k_{eff} of 0.950 ± 0.001 . Four meters of reflector were placed on the sphere surface. The composition of the reflector region matched that of the waste region, except that there was no SNM in the reflector. The density of the silicon dioxide was 1.6 g/cm^3 , and the maximum void fraction available for SNM and water together was 0.40. The critical sphere radius was used to calculate the critical mass of SNM within the sphere. Reference 2 provided these data for 100 wt %-enriched uranium (Ref. 2, Table C-2), and Ref. 3

provided these data for 10 wt % enriched uranium (Ref. 3, Table A.1). Additional calculations were performed to provide similar data for uranium-233, plutonium-239, and the isotopic mixture of plutonium in this guidance. The complete data set for the additional calculations is given in Appendix C.

For each type of SNM, the minimum critical spherical mass for this combination of materials (silicon dioxide, water, and SNM) was determined (See Table 5.1). Note that these masses are somewhat higher than the minimum critical spherical mass for optimally moderated and reflected SNM metal and water spheres, but such optimum conditions are judged to be highly unlikely in an LLW disposal site. Also, note that these masses were calculated using a silicon dioxide density of 1.6 g/cm^3 , because much of the data were taken from previous studies.^{2,3} Performing similar calculations using a higher silicon dioxide density of 2.4 g/cm^3 would produce higher minimum critical spherical masses. These higher masses would result in higher uniformity criteria values that would not be appropriate for the lower-density-waste materials.

Using the operational limits for areal density under the first graded approach (See Sect. 7.1.1), the area required to obtain this critical mass for each type of SNM was calculated. The operational areal density limit already provides a subcritical margin, so no additional margin was added during this calculation. It was determined that a value of 12 ft^2 will bound all SNM types described in this guidance. Therefore, if the areal density for SNM is averaged over no more than 12 ft^2 , it is extremely unlikely that a critical mass of SNM can be concentrated in a small area of the disposal site, while still meeting the areal density limit once this mass is averaged over a larger area.

The mass of waste over which enrichment and concentration may be averaged was calculated in a similar way. The concentration limits given in Table 7.2 for an up to 10-ft-deep emplacement were adjusted back to a waste density of 1.6 g/cm^3 to be compatible with the minimum critical spherical masses described above. These values were determined by multiplying the limits by the ratio $(2.4/1.6)$; the results are shown in Table 5.1. The mass of waste required to obtain the critical spherical mass of SNM was calculated. (The concentration limits for a 10-ft emplacement were used to produce a conservative value, because deeper emplacement limits would have yielded a higher mass allowable for averaging.) These data are shown in Table 5.1. It was determined that a value of 1500 kg of waste will bound all SNM types described in this guidance. Therefore, if the concentration of SNM is averaged over no more than 1500 kg of waste, it is extremely unlikely that a critical mass of SNM can be concentrated in a small part of the disposal site, while still meeting the operational limit once this mass of SNM is averaged over a larger mass of waste.

5.2 Assumptions and Limitations

Silicon dioxide (SiO_2), or sand, is used in this study as a surrogate for LLW in order to simplify the calculations and provide for a conservative estimate of the critically safe concentration of SNM in LLW. Silicon has a very low-neutron-capture cross section, and silicon dioxide (sand) is often a major constituent in the soil or backfill materials used at LLW facilities. In actual waste disposal environments, neutron absorbers, such as iron, calcium, and sodium, would be expected to be present in the waste, thus making the SNM waste less likely to cause a criticality accident. To ensure that the use of silicon dioxide as a surrogate for LLW would give conservative results, other elements were substituted for the silicon on an atom-for-atom basis in a series of computer calculations. The baseline model was of an infinite dry system containing uranium-235 at a density of 0.00141 g/cm^3 and silicon dioxide at a density of 1.6 g/cm^3 , and which had a calculated k_{inf} of 0.95. SCALE calculations using CSAS1X and cross sections from the 238-group ENDF-B/V cross-section library were performed, and a comparison was made based on the calculated k_{inf} of each system. All other elements resulted in a lower value of k_{inf} , with the following exceptions: helium, beryllium, carbon (graphite), fluorine, magnesium, and

bismuth (see Appendix D). Restrictions concerning beryllium and graphite are described in Sect. 3.3. Helium, fluorine, magnesium, and bismuth are not expected to be dominant materials in disposal sites, relative to silicon. In particular, fluorine is generally found in chemical compounds with elements that are stronger absorbers of neutrons, such as calcium.

Table 5.1 Uniformity criteria

SNM	Minimum spherical critical mass	Areal density operational limit (Sect. 7.1.1)	Area needed to contain minimum spherical critical mass	Adjusted concentration for 10-ft emplacement	Mass of waste needed to contain minimum spherical critical mass
Uranium-235	1569 g ²³⁵ U	94 g ²³⁵ U/ft ²	16.7 ft ²	8.395E-4 g ²³⁵ U/g	1869 kg
10 wt % Uranium-235	2428 g ²³⁵ U	174 g ²³⁵ U/ft ²	14.0 ft ²	1.209E-3 g ²³⁵ U/g	2008 kg
Uranium-233	1140 g ²³³ U	82 g ²³³ U/ft ²	13.9 ft ²	6.277 E-4 g ²³³ U/g	1816 kg
Plutonium-239	920 g ²³⁹ Pu	52 g ²³⁹ Pu/ft ²	17.7 ft ²	3.838E-4 g ²³⁹ Pu/g	2397 kg
Plutonium isotopic mixture	988 g ²³⁹ Pu	51 g ²³⁹ Pu/ft ²	19.4 ft ²	4.324E-4 g ²³⁹ Pu/g	2285 kg

Areal densities (grams SNM per unit area, as projected through a volume onto a flat surface) presented under the first graded approach were derived for optimized parameters of SNM concentration, cylinder H/D, and array unit pitch. This approach is extremely conservative, because SNM in an LLW disposal facility will not be in such an optimum configuration. The calculation of areal density limits also assumed that no single waste package would exceed the limits in 10 CFR Part 150, which are 350 g of uranium-235, 200 g of uranium-233, 200 g of plutonium, or combinations not exceeding the sum of the fractions rule.

Subcritical enrichment levels presented under the first graded approach were derived using homogeneous mixtures of SNM and water. This approach is also extremely conservative, since pure SNM solutions are not expected to be present in the disposal site.

The limits in the second and third graded approaches do not take into account the possibility for significant migration and reconcentration of SNM at a disposal site. As discussed in Sect. 3.2, earlier studies^{2,3} have shown that migration and reconcentration into a configuration posing a potential criticality concern are unlikely occurrences, and may take tens of thousands of years. However, these previous studies did assume that the SNM was not in a highly soluble chemical form (e.g., uranyl nitrate, uranyl fluoride). To account for this assumption, Sect. 7 includes a recommendation that LLW disposal sites minimize the quantity of highly soluble SNM compounds in their waste.

6 GRADED APPROACH TO EMPLACEMENT GUIDANCE

Three graded methods of demonstrating subcriticality in emplacement of LLW containing SNM are described below. Even though the emplacement guidance limits are higher for the second and third graded approaches, there are assumptions associated with these approaches with which the disposal facility must show compliance.

This guidance is applicable to SNM-contaminated LLW that meets the following general conditions:

1. The waste does not contain more than 0.1 wt % of beryllium, graphite, or deuterium.
2. The SNM is homogeneously distributed in the waste or meets the uniformity criteria as described in Sect. 5.1.5.
3. The mass of plutonium-241 does not exceed the mass of plutonium-240.

Although this guidance is applicable to a range of disposal configurations, there may be conditions at a particular disposal site that warrant a site-specific evaluation. Geological or geochemical conditions at a site may make it unique enough to make such a specific evaluation worthwhile. Highly engineered design features that provide long-term protection of the waste from environmental degradation may allow for higher allowable SNM concentrations in the waste, as may specific waste forms that contain sufficient neutron absorbing materials to make nuclear criticality extremely unlikely. For SNM waste that does not meet the above criteria, then site-specific analysis or reliance on not exceeding the mass limits in 10 CFR Part 150 for a particular vault or disposal unit would be required.

6.1 First Graded Approach – Areal Density and Enrichment Limits

The first graded approach method is the most conservative, and will be easy to use for facilities that dispose of very low levels of SNM, or dispose of material with a low average enrichment. It relies on the calculation of average areal density (grams of SNM per square foot), or on the average enrichment of SNM. The area over which averaging may be performed is also specified, but the emplacement depth is not limited.

Waste that contains SNM isotopes at very low enrichments may be disposed of without regard for concentration or areal density. For example, waste contaminated with uranium that has not been enriched in uranium-235 above the natural abundance of approximately 0.71 wt % may be placed in a disposal site with no concentration limits. Under conditions normally found in a disposal environment, uranium that is below 0.96 wt % uranium-235, with the rest being uranium-238, cannot be made critical.¹³ Uranium-233 may also be combined with uranium-238 to form a material that cannot be made critical in a disposal environment.¹⁴ Normal environmental transport mechanisms will not separate fissile from nonfissile isotopes of uranium, so reconcentration of very low-enriched uranium is not an issue. Even though waste that already contains such low enrichments of SNM may be able to use this criteria for safe emplacement, the use of isotopic dilution (addition of uranium-238 to lower the average enrichment of uranium in the waste) that results in a substantial increase in waste volume is not considered a good practice.

6.2 Second Graded Approach – SNM Concentration at Limited Emplacement Depth

The second graded approach relies on limiting the average concentration by weight of SNM in the waste, and on limiting the depth of the emplacement. This method may be useful for facilities that emplace somewhat higher concentrations of SNM, but do not use vaults or segmentation in the disposal emplacement. Results are tabulated for emplacements that are up to 10 ft, up to 20 ft, and greater than 20 ft deep. Emplacements greater than 20 ft deep are approaching an “infinite” system from a nuclear criticality perspective.

6.3 Third Graded Approach – SNM Concentration at Limited Vault Depth

The third graded approach relies on limiting the average concentration by weight of SNM in the waste, and on the presence of segmenting barriers, such as vaults, that will prevent movement of SNM through the barrier. This method may be useful for facilities that use concrete vaults in their disposal areas. Results are tabulated for a configuration with 10-ft-tall vaults, with 8-in.-thick concrete floors and ceilings, and stacked three vaults deep. These results are applicable for vaults that are stacked less than three deep, that are less than 10 ft tall, or that have thicker floors or ceilings.

7 RECOMMENDATIONS AND OPERATIONAL LIMITS

7.1 Selection of Graded Approach for a Given Disposal Site

The graded approach presented in Sect. 6 is applicable to a wide range of existing and potential disposal practices and site conditions. LLW disposal facilities that place discrete waste packages into trenches, and can therefore easily calculate an areal density of the SNM in the trench (such as the Richland site), or that dispose of waste containing depleted uranium or other SNM at low enrichment, may be able to demonstrate compliance with the first graded approach, given in Sect. 7.1.1. LLW disposal facilities that randomly emplace relatively low concentrations of SNM-contaminated LLW (such as Envirocare of Utah, Inc.) may be able to demonstrate compliance with the second graded approach, given in Sect. 7.1.2. Facilities that use engineered design features (e.g., concrete vaults such as those used at the Barnwell site) may be able to justify higher disposal concentration limits using the third graded approach. In any case, disposal site designs and operating procedures will be the key to determining which approaches are appropriate for a given site.

The first step in selecting an approach is to verify that the general conditions presented in Sect. 6 are met. If the waste was shipped as fissile exempt waste under 10 CFR Part 71, it will meet the first condition of containing no more than 0.1 wt % of beryllium, graphite, or deuterium. If the waste is not fissile exempt waste, the presence of these materials will be shown on the manifest (NRC Form 541). The disposal site operator should contact the waste generator for further information if needed to determine compliance with this condition.

Demonstrating compliance with the uniformity criteria, as described in Sect. 5.1.5, may be done by container, or for waste that is not disposed of in a container, emplacement records may be used. Instructions for calculating the SNM enrichment, concentration, and areal density, given the information on NRC Form 541, are given in Sect. 7.1. To average the SNM concentration or enrichment over 1500 kg of waste, contiguous packages not exceeding a total of 1500 kg net weight in an emplacement may be averaged. For noncontainerized waste, the SNM concentration or enrichment in any contiguous 1500 kg of waste in the emplacement should not exceed the given limit. Similarly, to average the SNM areal density over 12 ft², the SNM areal density in any contiguous 12 ft² area of the emplacement should not exceed the given limit. Each disposal site should develop procedures detailing the method used for averaging and how the records of such averaging are maintained.

Information is provided on NRC Form 541, Item 15, if plutonium-240 or plutonium-241 are present in the waste. The activity in MBq may be converted to a mass of the isotope in grams by dividing by the specific activity (i.e., MBq per grams - See Appendix E). This value will show if the mass of plutonium-241 exceeds that of plutonium-240.

Guidance is given for some pure SNM isotopes and for some isotopic mixtures. In general, the limits for 10 wt % uranium-235 may be used if the material is less than or equal to 10 wt % uranium-235. For material that contains greater than 10 wt % uranium-235, the 100 wt % uranium-235 limits should be used. For isotopic mixtures of plutonium, if the material contains less than or equal to 76 wt % plutonium-239, the isotopic plutonium mixture limits may be used. If the material contains greater than 76 wt % plutonium-239, the limits for 100 wt % plutonium-239 should be used. In both cases, the plutonium-241 content must not exceed the plutonium-240 content. The next step is to verify that any conditions specific to an approach are met. It is envisioned that the first and second graded approaches could be used at most sites. Also, it is possible to use more than a single approach at a site or even within a disposal trench, depending on the characteristics of the waste. After selecting the appropriate graded approach, the next step will be to determine the isotopic composition and enrichment of the waste. Section 7.1.1.2 provides procedures for calculating the enrichment.

In cases where the waste contains a mixture of SNM isotopes, other than 10 wt %-enriched uranium and the isotopic mixture of plutonium included below, a “sum-of-fractions” rule may be used. The value for each SNM type present (i.e., grams of SNM per gram of waste, grams of SNM per square foot) is divided by the limit for that type. All of these ratios are then added together, and the sum must not exceed 1. For example, if the waste contains a mixture of uranium-233, uranium-235 and plutonium-239, the following condition must be satisfied:

$$\frac{{}^{233}\text{U value}}{{}^{233}\text{U limit}} + \frac{{}^{235}\text{U value}}{{}^{235}\text{U limit}} + \frac{{}^{239}\text{Pu value}}{{}^{239}\text{Pu limit}} \leq 1$$

The limits in the equation above will depend upon the chosen graded approach and upon the isotopic composition of the uranium-235 and plutonium-239. The following sections describe in detail how to verify that the limits have been met.

7.1.1 First Graded Approach

7.1.1.1 Areal Density Limits

Areal density is expressed in terms of mass of SNM per area at the base of the disposal unit. This areal density can also be calculated for a single container using the mass of SNM on the manifest (NRC Form 541, Item 1), dividing by the base area of the container. For example, a 55-gal drum with 104 g of uranium-235 would have an areal density of 36 g/ft² of uranium-235. If the uranium was enriched to 10 wt % uranium-235, then no more than four drums could be stacked on top of each other to comply with the limit below (174 g ²³⁵U/ft²). The first graded approach does not limit concentration; therefore, waste with concentrations higher than allowed in graded approaches 2 or 3 should be placed in accordance with this graded approach.

As described in Sect. 7.1, the areal density of the waste may be averaged over 12 ft² when determining compliance with the areal density limit in Table 7.1. For example, a single 55-gal drum has a base area of approximately 2.9 ft². If this drum contains 350 g uranium-235, its areal density is 350 g/2.9 ft² = 121 g/ft², which exceeds the limit in Table 7.1 of 94 g/ft². However, in a closely packed triangular-pitched array of drums, 12 ft² of base area can contain three 55-gal drums. Therefore, if one drum contains 350 g uranium-235, and the remaining two drums contain 200 g uranium-235 each, the average areal density over the 12 ft² area is 62.5 g/ft² (((350 + 200 + 200)/12 ft²) = 62.5 g/ft²), which meets the limit in Table 7.1.

To demonstrate the sum-of-fractions rule, suppose one of these three drums also contains 200 g of uranium-233. The areal density of uranium-233 averaged over the 12 ft² area is 17 g/ft². The sum-of-fractions rule would be the following:

$$\frac{\text{uranium-233 value}}{\text{uranium-233 limit}} + \frac{\text{uranium-235 value}}{\text{uranium-235 limit}} = \frac{17}{82} + \frac{62.5}{94} = 0.87 .$$

The sum is less than 1, so this combination is in compliance with the areal density limits in Table 7.1.

If the first graded approach is used, LLW facilities should develop operating procedures to track the location of SNM waste within the disposal unit, to verify the areal density limits have been met, and to prevent additional placement of SNM waste above areas where the areal density limits have been reached.

Table 7.1 Areal density limits

SNM	Operational limit ^a
100 wt % enriched uranium	94 g ²³⁵ U/ft ²
10 wt % enriched uranium	174 g ²³⁵ U/ft ²
Uranium-233	82 g ²³³ U/ft ²
Plutonium-239	52 g ²³⁹ Pu/ft ²
Isotopic mixture of plutonium (76 wt % plutonium-239, 12 wt % plutonium-240, and 12 wt % plutonium-241)	51 g ²³⁹ Pu/ft ²

^a The areal density in kg/m² can be obtained by multiplying the g/ft² values by 0.010763.

7.1.1.2 Subcritical Enrichment Levels for Different SNM Isotopes

Enrichment is a ratio of the weight of uranium-235 or uranium-233 to total uranium, and is typically expressed as a percent. This enrichment (wt %) can be calculated using information on the manifest. The activity of uranium-238 in MBq, if present, would be recorded on the manifest (NRC Form 541, Item 15). The activity should be converted to a mass of the isotope by dividing by the specific activity (i.e., MBq per grams - See Appendix E). The mass of SNM isotopes can be obtained directly from the manifest (NRC Form 541, Item 1). The enrichment can then be calculated using the masses of the required isotopes. Waste containing SNM at or below the enrichments listed below may be placed in a disposal site without concern for a nuclear criticality accident.

$$\text{Uranium-235} = 0.96 \text{ wt } \%$$

$$\text{Uranium-233} = 0.66 \text{ wt } \%$$

The uranium enrichment of waste may be averaged over no more than 1500 kg of waste in order to meet these criteria. For example, consider a 55-gal drum containing 350 kg (772 lb) of waste contaminated with 100 g of uranium-233 at an enrichment of 1.5 wt %. This drum would then contain over 6.6 kg of uranium-238. (The quantities of uranium-233 and uranium-238 can be taken from the NRC Form 541 information, as described above.) If three adjacent drums in the emplacement each contain 350 kg of waste contaminated with 100 g of uranium-233 at an enrichment of 0.4 wt %, each of these drums would therefore contain 25 kg of uranium-235. The total mass of uranium-233 in these four drums would be 400 g, and the total mass of uranium-238 would be 81.6 kg, making the average uranium-233 enrichment 0.49 wt %.

$$\frac{400 \text{ g uranium-233}}{400 \text{ g uranium-233} + 81,600 \text{ g uranium-238}} \times 100\% = 0.49 \text{ wt } \%$$

The total mass of waste in these four drums would be 1400 kg. Therefore, these four drums averaged together meet the subcritical enrichment criteria for the first graded approach.

7.1.2 Second Graded Approach

Concentration in this approach is expressed in terms of mass of SNM per mass of waste. This value can be calculated using information on the manifest. The mass of SNM isotopes is recorded on NRC Form 541, Item 1, and the mass of the waste is recorded on NRC Form 541, Item 8. The concentration is calculated by dividing the mass of the SNM by the mass of the waste. As discussed in Sect. 5.1, the concentration should be averaged over no more than 1500 kg of waste.

For example, consider a 25-ft waste emplacement with a 55-gal drum, containing 350 kg (772 lb) of waste contaminated with 200 g of uranium-233. The concentration of uranium-233 in this drum is 5.74E-4 g uranium-233/g waste, which exceeds the limit in Table 7.2. If three adjacent drums in the emplacement each contain 350 kg of waste contaminated with 75 g of uranium-233, the average concentration over these four drums would be 3.036E-4 g uranium-233/g waste, which meets the limit in Table 7.2. Therefore, these four drums averaged together meet the criteria for the second graded approach.

If the second graded approach is used, LLW facilities should develop operating procedures to document that the concentration of SNM waste does not exceed the limits. The appropriate concentration limit should be selected from the following table based in the depth of the disposal unit.

Table 7.2 Second graded-approach limits

SNM	Limit for up-to-10-ft emplacement	Limit for up-to-20-ft emplacement	Limit for greater than 20-ft emplacement
100 wt % enriched uranium	5.597E-4 g ²³⁵ U/g	4.706E-4 g ²³⁵ U/g	4.592E-4 g ²³⁵ U/g
10 wt % enriched uranium	8.060E-4 g ²³⁵ U/g	7.107E-2 g ²³⁵ U/g	6.933E-2 g ²³⁵ U/g
Uranium-233	4.185E-4 g ²³³ U/g	3.573E-4 g ²³³ U/g	3.444E-4 g ²³³ U/g
Plutonium-239	2.559E-4 g ²³⁹ Pu/g	2.035E-4 g ²³⁹ Pu/g	1.961E-4 g ²³⁹ Pu/g
Isotopic mixture of plutonium (76 wt % plutonium-239, 12 wt % plutonium-240, and 12 wt % plutonium-241)	2.883E-4 g ²³⁹ Pu/g	2.256E-4 g ²³⁹ Pu/g	2.102E-4 g ²³⁹ Pu/g

7.1.3 Third Graded Approach

As in Sect. 7.1.2, the concentration in this approach is expressed in terms of mass of SNM per mass of waste. This value can be calculated using information on the manifest. The mass of SNM isotopes is recorded on NRC Form 541, Item 1, and the mass of the waste is recorded on NRC Form 541, Item 8. The concentration is calculated by dividing the mass of the SNM by the mass of the waste. As discussed in Sect. 5.1, the concentration should be averaged over no more than 1500 kg of waste. An example of such averaging is given in Sect. 7.1.2.

If the third graded approach is used, LLW facilities should develop operating procedures to document that the concentration of SNM waste does not exceed the limits.

Table 7.3 Third graded-approach limits

SNM	Limit for 3 × 10 ft vaults as described
100 wt % enriched uranium	7.407E-4 g ²³⁵ U/g
10 wt % enriched uranium	1.168E-3 g ²³⁵ U/g
Uranium-233	5.516E-4 g ²³³ U/g
Plutonium-239	3.304E-4 g ²³⁹ Pu/g
Isotopic mixture of plutonium (76 wt % plutonium-239, 12 wt % plutonium-240, and 12 wt % plutonium-241)	3.748E-4 g ²³⁹ Pu/g

7.2 LLW Emplacement Good Practices

As discussed in Sect. 3.2, SNM migration and reconcentration is an issue that is not addressed by the guidance provided herein. Earlier studies^{2,3} provide data that indicate that SNM migration and reconcentration into a potentially critical configuration is unlikely and would take tens of thousands of years. Engineered design features can be used to mitigate such movement and to detect an increase in SNM within nearby groundwater before significant migration occurs.

Several factors affect SNM migration or concentration. These include the amount of water entering the disposal unit, the chemistry of the waste and water, and the presence of preferential pathways, such as drains, sumps, etc. To minimize the potential for migration, and to maximize the probability of detecting migration before it becomes significant, the following good practices should be incorporated:

- Infiltration of rain or groundwater should be minimized to lengthen the life of the disposal containers and to limit the volume of water available to move the uranium from its disposal location. Unsaturated conditions greatly reduce opportunities for migration and concentration.
- Highly soluble chemical forms of SNM should be minimized in a disposal site. Examples of soluble compounds include uranyl nitrate and uranyl fluoride, which are common chemical forms of uranium in waste from fuel processing facilities.
- Facility designs should minimize preferential pathways that would tend to concentrate or focus effluents. The use of sumps or other means of sampling the water below the LLW emplacement will provide a means of detecting migration before it becomes significant. However, the geometry and size of such collection points should be restricted to avoid potential critical geometries.

- LLW emplacements should minimize zones with strong reducing potential. Uranium compounds tend to become more soluble in oxidizing conditions, and precipitate under reducing conditions.¹⁵ Rainwater tends to be oxidizing, thus is capable of dissolving uranium and transporting it through a disposal site. Organic materials and iron tend to be reducing, so uranium in solution may precipitate when it encounters these materials, forming a reconcentration zone that could pose the potential for a nuclear criticality accident. Limiting rainwater infiltration will also help in keeping the SNM from oxidizing and dissolving.
- Where operationally feasible, the areal density of SNM should be as low as is practical, that is, SNM waste containers should not be stacked and should be placed as far from other SNM containers as is practical.
- Where operationally feasible, the enrichment of SNM should be reduced, that is, depleted uranium, natural uranium, or natural thorium should be placed adjacent to SNM waste.
- Void space between SNM containers should be backfilled with sand or grout to minimize post disposal settling.

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Appendix A

Data for First Graded-Approach Method

Appendix A

Data for First Graded-Approach Method

**Table A.1 100 wt % enriched uranium hydrogenous systems, SiO₂ (ρ = 1.9)-reflected (Z-axis)
350 g uranium-235 per unit, infinite planar array**

H/D	g ²³⁵ U/L	Pitch (cm)	k_{eff}		[g ²³⁵ U/ft ²] ^a
0.5	100	41.570	0.9976	0.0017	188
	75	44.478	1.0008	0.0017	164
	50	47.292	1.0035	0.0017	145
	35	46.044	1.0004	0.0014	153
	25	41.012	0.9997	0.0012	193
1.0	100	43.066	1.0034	0.0027	175
	75	45.826	1.0024	0.0026	155
	50	49.716	0.9960	0.0023	132
	35	48.946	0.9995	0.0021	136
	25	43.324	1.0018	0.0018	173
1.5	100	43.288	0.9966	0.0026	174
	75	45.636	1.0033	0.0026	156
	50	48.898	1.0040	0.0024	136
	35	48.704	0.9968	0.0021	137
	25	43.288	1.0037	0.0017	174
2.0	100	42.110	1.0036	0.0029	183
	75	45.738	0.9969	0.0025	155
	50	48.404	1.0028	0.0023	139
	35	47.890	1.0000	0.0022	142
	25	43.142	1.0030	0.0018	175
2.5	100	42.526	0.9965	0.0026	180
	75	45.222	0.9984	0.0024	159
	50	48.192	0.9972	0.0022	140
	35	46.916	1.0045	0.0020	148
	25	42.870	1.0012	0.0019	177

^a The areal density in kg/m² can be obtained by multiplying the g/ft² values by 0.010763.

**Table A.2 10 wt % enriched uranium systems, SiO₂ (ρ = 1.9)-reflected (Z-axis)
35 g uranium-235 per unit, infinite planar array**

H/D	g ²³⁵ U/L	Pitch (cm)	<i>k</i> _{eff}		[g ²³⁵ U/ft ²] ^a
1.0	100	10.508	0.9969	0.0023	294
	75	11.057	0.9994	0.0021	266
	50	11.470	1.0000	0.0020	247
	35	11.520	0.9956	0.0019	245
	25		not critical		
2.0	100	10.482	0.9991	0.0023	295
	75	11.172	0.9962	0.0020	260
	50	11.585	1.0008	0.0018	242
	35	11.469	0.9955	0.0018	247
	25	10.697	0.9971	0.0016	284
4.0	100	10.487	0.9991	0.0023	295
	75	11.042	1.0043	0.0021	266
	50	11.559	0.9999	0.0020	243
	35	11.401	1.0007	0.0017	250
	25	10.578	1.0005	0.0016	290
6.0	100	10.421	1.0044	0.0021	299
	75	11.063	0.0079	0.0021	265
	50	11.574	0.9976	0.0019	242
	35	11.420	1.0006	0.0017	249
	25	10.535	0.9986	0.0015	293

^a The areal density in kg/m² can be obtained by multiplying the g/ft² values by 0.010763.

**Table A.3 Uranium-233 hydrogenous systems, SiO₂ ($\rho = 1.9$)-reflected (Z-axis)
250 g uranium-233 per unit, infinite planar array**

H/D	g ²³³ U/L	Pitch (cm)	k_{eff}		[g ²³³ U/ft ²] ^a
0.5	100	39.102	1.0007	0.0022	152
	75	41.762	1.0018	0.0020	133
	50	43.530	0.9978	0.0017	123
	35	42.656	0.9956	0.0016	128
	25	38.096	0.9978	0.0013	160
1.0	100	40.000	1.0008	0.0019	145
	75	42.804	1.0029	0.0021	127
	60	44.268	1.0004	0.0019	119
	50	45.056	0.9978	0.0018	114
	45	45.212	1.0000	0.0017	114
	40	44.838	0.9975	0.0017	116
	35	44.634	1.0026	0.0016	117
	25	40.000	1.0042	0.0013	145
1.5	100	40.000	0.9972	0.0020	145
	75	42.484	1.0005	0.0020	129
	50	44.956	1.0034	0.0018	115
	35	44.082	0.9993	0.0017	120
	25	40.000	1.0045	0.0013	145
2.0	100	39.924	0.9986	0.0019	146
	75	41.950	1.0000	0.0020	132
	50	44.282	1.0002	0.0018	118
	35	43.536	0.9992	0.0016	123
	25	40.000	1.0005	0.0012	145
2.5	100	38.986	1.0009	0.0019	153
	75	41.396	1.0003	0.0020	136
	50	43.404	0.9971	0.0019	123
	35	43.494	1.0032	0.0016	123
	25	40.000	0.9952	0.0014	145

^a The areal density in kg/m² can be obtained by multiplying the g/ft² values by 0.010763.

**Table A.4 Plutonium-239 hydrogenous systems, SiO₂ ($\rho = 1.9$)-reflected (Z-axis)
225 g plutonium-239 per unit, infinite planar array**

H/D	g ²³⁹ Pu /L	Pitch (cm)	k_{eff}		[g ²³⁹ Pu/ft ²] ^a
1.0	100	37.822	0.9973	0.0028	146
	75	41.856	1.0041	0.0028	119
	50	47.206	1.0030	0.0026	93
	35	51.490	1.0037	0.0026	79
	25	53.390	0.9996	0.0021	73
	20	52.302	0.9977	0.0019	76
	15	46.748	1.0038	0.0020	96
2.0	100	37.366	1.0038	0.0026	150
	75	41.334	1.0011	0.0028	122
	50	46.592	1.0040	0.0027	96
	35	50.652	0.9980	0.0022	81
	25	51.932	1.0039	0.0022	78
	20	51.356	0.9976	0.0019	79
	15	47.326	0.9968	0.0019	93
2.5	10	33.528	1.0017	0.0013	186
	50	46.224	1.0014	0.0026	98
	35	49.852	1.0019	0.0024	84
	25	51.478	0.9987	0.0021	79
	20	50.298	0.9995	0.0021	83
	15	46.428	1.0001	0.0019	97
	10	33.508	1.0013	0.0014	186

^a The areal density in kg/m² can be obtained by multiplying the g/ft² values by 0.010763.

**Table A.5 Plutonium isotopic mixture^a hydrogenous systems,
SiO₂ (ρ = 1.9)-reflected (Z-axis), 225 g plutonium-239 per unit, infinite planar array**

H/D	g ²³⁹ Pu/L	Pitch (cm)	<i>k</i> _{eff}		[g ²³⁹ Pu/ft ²] ^a	[g(²³⁹ Pu + ²⁴¹ Pu) /ft ²] ^b
1.0	100	33.870	0.9968	0.0029	182	211
	75	38.318	0.9988	0.0026	142	164
	50	44.568	1.0005	0.0023	105	122
	35	49.422	1.0043	0.0025	86	100
	25	52.532	1.0050	0.0022	76	88
	20	53.912	0.9990	0.0021	72	83
	15	51.888	0.9981	0.0018	78	90
2.0	100	33.934	0.9979	0.0026	182	211
	75	38.376	0.9955	0.0026	142	164
	50	44.240	1.0046	0.0024	107	124
	35	49.346	0.9999	0.0022	86	100
	25	52.334	1.0016	0.0022	76	88
	20	52.000	1.0036	0.0021	77	89
	15	50.450	1.0026	0.0017	82	95
	10	40.680	1.0024	0.0014	126	146
2.5	50	43.750	1.0018	0.0023	109	126
	35	48.472	1.0013	0.0022	89	103
	25	51.794	0.9965	0.0021	78	90
	20	51.784	1.0005	0.0020	78	90
	15	49.866	1.0039	0.0019	84	97
	10	40.274	1.0018	0.0015	129	149

^a 76 wt % plutonium-239, 12 wt % plutonium-240, and 12 wt % plutonium-241.

^b The areal density in kg/m² can be obtained by multiplying the g/ft² values by 0.010763.

Appendix B

Example Input Files

Appendix B

Example Input Files

Example input for the first graded approach: Critical pitch search for solution-filled cylinders

```
#CSAS4      PARM=SIZE=1000000
PITCH SEARCH U233
238GROUPNDF5          INFHOMMEDIUM
U-233          1 DEN=0.050 1.0 293 END
H2O           1 1.0 293 END
ARBMSIO2 1.9 2 0 1 1 14000 1 8016 2 2 1.0 293 END
END COMP
50 G/L H/D=1.5
READ PARM RUN=YES PLT=NO NUB=YES END PARM
READ GEOM
UNIT 1
CYLINDER 1 1 8.0953 24.2859 0
CUBOID 0 1 20 -20 20 -20 24.2859 0
GLOBAL UNIT 2
ARRAY 2 0 0 0
REPLICATE 2 1 4*0.0 2*240.0 1
END GEOM
READ ARRAY
ARA=2 NUX=100 NUY=100 NUZ=1
FILL F1 END FILL
END ARRAY
READ BNDS +XB=REFLECT -XB=REFLECT +YB=REFLECT -YB=REFLECT +ZB=VACUUM
-ZB=VACUUM END BNDS
END DATA
READ SEARCH CRITICAL PITCH MAXPITCH=70 MINPITCH=20 END SEARCH
END
```

Example input for the second graded approach: Critical slab thickness search

```
#CSAS1X   PARM=SIZE=1000000
ZONE WIDTH SEARCH CRITICAL SLAB  U233=0.0065 G/CC H2O=0.12 G/CC
238GROUPNDF5                      MULTIREGION
U          1  DEN=0.0065 1.0 293 92233 100 END
ARBMSIO2 1.6 2  0 1 1  14000 1 8016 2 1  1.0 293 END
H2O 1  DEN=0.12  1.0 293  END
ARBMSREFL 1.6 2  0 1 1  14000 1 8016 2 2  1.0 293 END
H2O 2  DEN=0.12  1.0 293  END
END COMP
SLAB VACUUM VACUUM 0  END
2 400 1 440 2 840  END ZONE
MORE DATA
ZMD(2)=0.8  KFM=-0.2 KEF=0.95 ICM=100 END
END
```

Example input for the third graded approach: Critical concentration search in vaulted system

```
#CSAS4     PARM=SIZE=1000000
CRITICAL CONCENTRATION SEARCH U233 - 3 X 10 ft vaults
238GROUPNDF5                      INFHOMMEDIUM
U          1  DEN=0.0014 1.0 293 92233 100 END
MGCONCRETE 2  1.0 293  END
ARBMSIO2 2.4 2  0 1 1  14000 1 8016 2 1  1.0 293 END
ARBMSIO2 2.4 2  0 1 1  14000 1 8016 2 3  1.0 293 END
END COMP
CRITICAL CONCENTRATION SEARCH
READ PARM RUN=YES PLT=NO NUB=YES END PARM
READ GEOM
UNIT 1
COM=!DISPOSAL VAULT!
CUBOID 2 1 500 -500 500 -500 20.32 0
CUBOID 1 1 500 -500 500 -500 325.12 0
CUBOID 2 1 500 -500 500 -500 345.44 0
GLOBAL UNIT 2
COM=!VAULTS STACKED THREE HIGH!
ARRAY 2 0 0 0
REPLICATE 3 1 0 0 0 0 400 400 1
END GEOM
READ ARRAY
ARA=2 NUX=1 NUY=1 NUZ=3
COM=!VAULTS STACKED 3 HIGH!
FILL F1 END FILL
END ARRAY
READ BNDS +XB=REFLECT -XB=REFLECT +YB=REFLECT -YB=REFLECT +ZB=VACUUM
-ZB=VACUUM  END BNDS
END DATA
READ SEARCH CRITICAL CONCENTRATION KEF=0.95 MORE
ALTER MIX=1 SCNAME=U FACTOR=1 +CON=0.20 -CON=-0.27
END SEARCH
END
```

Appendix C

Data for Second and Third Graded-Approach Methods

Table C.1 Uranium-233 systems

g ²³³ U /cm ³	g ²³³ U /g SiO ₂	g H ₂ O /cm ³	g H ₂ O /g SiO ₂	<i>k</i> _{inf}	Critical slab		Critical sphere	
					Thickness (cm)	Areal density (kg ²³³ U/ m ²)	Radius (cm)	kg ²³³ U
0.00110	0.00069	0.00	0.00	0.987	1473.70	16.211	773.06	2128.73
0.00125	0.00000	0.00	5.44	1.033	435.23	5.440	423.52	397.76
0.00150	0.00094	0.00	0.00	1.140	258.73	3.881	307.94	183.48
0.00165	0.00103	0.00	0.00	1.196	216.43	3.571	266.10	130.23
0.00165	0.00103	0.03	0.02	1.006	332.07	5.479	387.93	403.49
0.00180	0.00113	0.00	0.00	1.247	189.24	3.406	238.75	102.61
0.00180	0.00113	0.03	0.02	1.056	231.96	4.175	274.32	155.64
0.00200	0.00125	0.00	0.00	1.307	163.91	3.278	213.88	81.97
0.00200	0.00125	0.03	0.02	1.116	174.88	3.498	213.02	80.98
0.00200	0.00125	0.06	0.04	1.000	284.35	5.687	323.58	283.83
0.00220	0.00138	0.00	0.00	1.361	148.00	3.256	196.43	69.84
0.00220	0.00138	0.03	0.02	1.171	153.37	3.374	181.06	54.70
0.00220	0.00138	0.06	0.04	1.054	186.46	4.102	217.98	95.45
0.00220	0.00138	0.09	0.06	0.962	496.86	10.931	596.17	1952.64
0.00250	0.00156	0.00	0.00	1.432	125.45	3.136	178.00	59.06
0.00250	0.00156	0.03	0.02	1.244	121.14	3.029	153.19	37.65
0.00250	0.00156	0.06	0.04	1.127	132.88	3.322	161.95	44.48
0.00250	0.00156	0.09	0.06	1.034	179.38	4.485	206.74	92.53
0.00250	0.00156	0.12	0.08	0.956	596.09	14.902	777.74	4926.43
0.00280	0.00175	0.00	0.00	1.492	108.90	3.049	164.97	52.66
0.00280	0.00175	0.03	0.02	1.307	103.70	2.904	136.11	29.57
0.00280	0.00175	0.06	0.04	1.192	107.22	3.002	135.38	29.10
0.00280	0.00175	0.09	0.06	1.099	125.97	3.527	150.56	40.03
0.00280	0.00175	0.12	0.08	1.020	175.23	4.906	198.76	92.09
0.00280	0.00175	0.15	0.09	0.952	863.78	24.186	147.66	37.76

Table C.1 (continued)

g ²³³ U/cm ³	g ²³³ U /g SiO ₂	g H ₂ O/cm ³	g H ₂ O /g SiO ₂	<i>k</i> _{inf}	Critical slab		Critical sphere	
					Thickness (cm)	Areal density (kg ²³³ U /m ²)	Radius (cm)	kg ²³³ U
0.00320	0.00200	0.00	0.00	1.559	96.46	3.087	152.40	47.45
0.00320	0.00200	0.03	0.02	1.381	89.26	2.856	121.19	23.86
0.00320	0.00200	0.06	0.04	1.268	91.52	2.929	115.21	20.50
0.00320	0.00200	0.09	0.06	1.175	95.87	3.068	118.81	22.48
0.00320	0.00200	0.12	0.08	1.096	111.19	3.558	132.52	31.19
0.00320	0.00200	0.15	0.09	1.027	147.31	4.714	167.80	63.33
0.00320	0.00200	0.18	0.11	0.966	184.58	5.907	350.73	578.31
0.00360	0.00225	0.00	0.00	1.615	87.90	3.164	143.21	44.29
0.00360	0.00225	0.03	0.02	1.444	76.64	2.759	111.05	20.65
0.00360	0.00225	0.06	0.04	1.333	75.81	2.729	102.73	16.35
0.00360	0.00225	0.09	0.06	1.242	79.10	2.848	101.93	15.97
0.00360	0.00225	0.12	0.08	1.163	86.56	3.116	106.71	18.32
0.00360	0.00225	0.15	0.09	1.094	100.46	3.617	118.65	25.19
0.00360	0.00225	0.18	0.11	1.033	127.50	4.590	145.65	46.59
0.00360	0.00225	0.21	0.13	0.978	215.93	7.773	238.76	205.25
0.00450	0.00281	0.00	0.00	1.710	73.12	3.290	129.18	40.63
0.00450	0.00281	0.03	0.02	1.557	64.25	2.891	96.69	17.04
0.00450	0.00281	0.06	0.04	1.454	61.454	2.765	86.39	12.15
0.00450	0.00281	0.09	0.06	1.366	61.716	2.777	82.10	10.43
0.00450	0.00281	0.12	0.08	1.290	63.873	2.874	81.01	10.02
0.00450	0.00281	0.15	0.09	1.221	64.53	2.904	82.39	10.54
0.00450	0.00281	0.18	0.11	1.160	70.14	3.156	86.39	12.15
0.00450	0.00281	0.21	0.13	1.105	78.54	3.534	94.00	15.66
0.00450	0.00281	0.25	0.16	1.039	100.51	4.523	115.01	28.68
0.00450	0.00281	0.29	0.18	0.980	170.86	7.689	188.21	125.67
0.00550	0.00344	0.00	0.00	1.784	62.66	3.446	119.26	39.08
0.00550	0.00344	0.03	0.02	1.650	52.22	2.872	87.23	15.29
0.00550	0.00344	0.06	0.04	1.555	50.744	2.791	76.34	10.25
0.00550	0.00344	0.09	0.06	1.473	49.793	2.739	70.86	8.20
0.00550	0.12000	0.12	2.75	1.400	50.072	2.754	67.97	7.23
0.00550	0.00344	0.15	0.09	1.334	51.248	2.819	66.77	6.86

Table C.1 (continued)

g ²³³ U/cm ³	g ²³³ U /g SiO ₂	g H ₂ O/cm ³	g H ₂ O /g SiO ₂	<i>k</i> _{inf}	Critical slab		Critical sphere	
					Thickness (cm)	Areal density (kg ²³³ U /m ²)	Radius (cm)	kg ²³³ U
0.00550	0.00344	0.18	0.11	1.274	53.272	2.930	66.87	6.89
0.00550	0.00344	0.21	0.13	1.219	53.44	2.939	68.23	7.32
0.00550	0.00344	0.25	0.16	1.153	58.79	3.233	72.21	8.67
0.00550	0.00344	0.29	0.18	1.094	67.50	3.713	79.95	11.77
0.00550	0.00344	0.32	0.20	1.053	78.18	4.300	90.18	16.90
0.00550	0.00344	0.36	0.23	1.004	107.05	5.888	119.13	38.95
0.00550	0.00344	0.40	0.25	0.959	272.32	14.978	305.15	654.62
0.00650	0.00406	0.00	0.00	1.836	55.39	3.600	112.38	38.64
0.00650	0.00406	0.03	0.02	1.720	45.74	2.973	81.00	14.47
0.00650	0.00406	0.06	0.04	1.634	43.31	2.815	69.97	9.33
0.00650	0.00406	0.09	0.06	1.557	41.82	2.718	64.05	7.15
0.00650	0.00406	0.12	0.08	1.488	41.35	2.688	60.52	6.04
0.00650	0.00406	0.15	0.09	1.425	41.38	2.690	58.40	5.42
0.00650	0.00406	0.18	0.11	1.367	42.05	2.733	57.27	5.11
0.00650	0.00406	0.21	0.13	1.313	43.05	2.798	56.95	5.03
0.00650	0.00406	0.25	0.16	1.248	44.41	2.887	57.64	5.21
0.00650	0.00406	0.29	0.18	1.189	47.55	3.091	59.66	5.78
0.00650	0.00406	0.32	0.20	1.149	50.81	3.303	62.30	6.58
0.00650	0.00406	0.36	0.23	1.099	57.11	3.712	67.89	8.52
0.00650	0.00406	0.40	0.25	1.053	67.29	4.374	77.62	12.73
0.01000	0.00625	0.00	0.0000	1.935	40.70	40.700	98.55	40.09
0.01000	0.01	0.03	0.0188	1.867	33.42	3.342	68.11	13.23
0.01000	0.00625	0.06	0.0375	1.805	30.57	3.057	57.56	7.99
0.01000	0.00625	0.09	0.0563	1.746	29.05	2.905	51.53	5.73
0.01000	0.00625	0.12	0.0750	1.690	28.13	2.813	47.53	4.50
0.01000	0.00625	0.15	0.0938	1.638	27.56	2.756	44.69	3.74
0.01000	0.00625	0.18	0.1125	1.588	27.21	2.721	42.59	3.24
0.01000	0.00625	0.21	0.1313	1.542	27.04	2.704	41.02	2.89
0.01000	0.00625	0.25	0.1563	1.483	27.01	2.701	39.52	2.59
0.01000	0.00625	0.29	0.1813	1.429	27.20	2.720	38.54	2.40
0.01000	0.00625	0.32	0.2000	1.391	27.47	2.747	38.08	2.31
0.01000	0.00625	0.36	0.2250	1.343	28.00	2.800	37.78	2.26

Table C.1 (continued)

g ²³³ U/cm ³	g ²³³ U /g SiO ₂	g H ₂ O/cm ³	g H ₂ O /g SiO ₂	<i>k</i> _{inf}	Critical slab		Critical sphere	
					Thickness (cm)	Areal density (kg ²³³ U /m ²)	Radius (cm)	kg ²³³ U
0.01000	0.00625	0.40	0.2500	1.298	28.73	2.873	37.82	2.27
0.01600	0.01	0.00	0.0000	2.000	29.12	29.120	85.06	41.25
0.01600	0.01	0.03	0.0188	1.975	24.31	3.890	59.57	14.17
0.01600	0.01	0.06	0.0375	1.940	22.22	3.555	49.68	8.22
0.01600	0.01	0.09	0.0563	1.902	21.01	3.362	43.84	5.65
0.01600	0.01	0.12	0.0750	1.863	20.19	3.230	39.86	4.24
0.01600	0.01	0.15	0.0938	1.825	19.60	3.136	36.92	3.37
0.01600	0.01	0.18	0.1125	1.787	19.10	3.056	34.64	2.79
0.01600	0.01	0.21	0.1313	1.751	18.74	2.998	32.83	2.37
0.01600	0.01	0.25	0.1563	1.704	18.37	2.939	30.93	1.98
0.01600	0.01	0.29	0.1813	1.660	18.07	2.891	29.43	1.71
0.01600	0.01	0.32	0.2000	1.628	17.92	2.867	28.52	1.55
0.01600	0.01	0.36	0.2250	1.587	17.78	2.845	27.52	1.40
0.01600	0.01	0.40	0.2500	1.548	17.68	2.829	26.72	1.28
0.02250	0.01406	0.00	0.00	2.031	22.74	22.740	78.24	45.14
0.02250	0.01406	0.03	0.02	2.025	19.40	4.365	54.95	15.64
0.02250	0.01406	0.06	0.04	2.006	17.80	4.005	45.60	8.94
0.02250	0.01406	0.09	0.06	1.982	16.84	3.789	39.99	6.03
0.02250	0.01406	0.12	0.08	1.955	16.16	3.636	36.11	4.44
0.02250	0.01406	0.15	0.09	1.927	15.62	3.515	33.22	3.46
0.02250	0.01406	0.18	0.11	1.899	15.19	3.418	30.96	2.80
0.02250	0.01406	0.21	0.13	1.871	14.84	3.339	29.14	2.33
0.02250	0.01406	0.25	0.16	1.834	14.43	3.247	27.18	1.89
0.02250	0.01406	0.29	0.18	1.798	14.11	3.175	25.62	1.58
0.02250	0.01406	0.32	0.20	1.772	13.91	3.130	24.64	1.41
0.02250	0.01406	0.36	0.23	1.738	13.67	3.076	23.53	1.23
0.02250	0.02250	0.40	0.40	1.705	13.48	3.033	22.93	1.14
0.08	0.05000	0.00	0.00	2.130	8.54	8.540	57.55	63.87
0.08	0.05000	0.03	0.02	2.108	8.20	6.560	42.96	26.57
0.08	0.05000	0.06	0.04	2.108	7.89	6.312	35.87	15.47
0.08	0.05000	0.09	0.06	2.110	7.69	6.152	31.36	10.33

Table C.1 (continued)

g ²³³ U/cm ³	g ²³³ U /g SiO ₂	g H ₂ O/cm ³	g H ₂ O /g SiO ₂	<i>k</i> _{inf}	Critical slab		Critical sphere	
					Thickness (cm)	Areal density (kg ²³³ U /m ²)	Radius (cm)	kg ²³³ U
0.08	0.05000	0.12	0.08	2.109	7.53	6.024	28.11	7.44
0.08	0.05000	0.15	0.09	2.107	7.37	5.896	25.62	5.64
0.08	0.05000	0.18	0.11	2.104	7.22	5.776	23.63	4.42
0.08	0.05000	0.21	0.13	2.100	7.07	5.656	21.99	3.56
0.08	0.05000	0.25	0.16	2.092	6.88	5.504	20.19	2.76
0.08	0.05000	0.29	0.18	2.083	6.70	5.360	18.73	2.20
0.08	0.05000	0.32	0.20	2.077	6.57	5.256	17.79	1.89
0.08	0.05000	0.36	0.23	2.067	6.40	5.120	16.70	1.56
0.08	0.05000	0.396	0.25	2.058	6.27	5.016	15.85	1.33
0.25	0.15625	0.00	0.00	2.191	3.20	3.200	41.75	76.21
0.25	0.15625	0.03	0.02	2.174	3.42	8.550	34.06	41.38
0.25	0.15625	0.06	0.04	2.161	3.52	8.800	29.36	26.50
0.25	0.15625	0.09	0.06	2.154	3.62	9.050	26.08	18.58
0.25	0.15625	0.12	0.08	2.150	3.69	9.225	23.60	13.76
0.25	0.15625	0.15	0.09	2.148	3.72	9.300	21.63	10.60
0.25	0.15625	0.18	0.11	2.147	3.74	9.350	20.00	8.38
0.25	0.15625	0.21	0.13	2.147	3.74	9.350	18.65	6.79
0.25	0.15625	0.25	0.16	2.146	3.73	9.325	17.13	5.26
0.25	0.15625	0.29	0.18	2.146	3.69	9.225	15.87	4.19
0.25	0.15625	0.32	0.20	2.145	3.65	9.125	15.05	3.57
0.25	0.15625	0.36	0.23	2.145	3.60	9.000	14.10	2.94
0.25	0.15625	0.385	0.24	2.144	3.56	8.900	13.57	2.62
1	0.62500	0.00	0.00	2.301	0.87	0.870	25.01	65.53
1	0.62500	0.03	0.02	2.279	1.01	10.060	22.66	48.74
1	0.62500	0.06	0.04	2.263	1.10	11.030	20.64	36.83
1	0.62500	0.09	0.06	2.250	1.20	12.000	19.00	28.73
1	0.62500	0.12	0.08	2.239	1.29	12.900	17.63	22.95
1	0.62500	0.15	0.09	2.230	1.37	13.710	16.48	18.75
1	0.62500	0.18	0.11	2.223	1.44	14.410	15.48	15.54
1	0.62500	0.21	0.13	2.216	1.50	14.990	14.61	13.06
1	0.62500	0.25	0.16	2.209	1.56	15.610	13.61	10.56
1	0.62500	0.29	0.18	2.203	1.61	16.070	12.74	8.66

Table C.1 (continued)

$\text{g } ^{233}\text{U}/\text{cm}^3$	$\text{g } ^{233}\text{U}$ /g SiO_2	$\text{g H}_2\text{O}/\text{cm}^3$	$\text{g H}_2\text{O}$ /g SiO_2	k_{inf}	Critical slab		Critical sphere	
					Thickness (cm)	Areal density ($\text{kg } ^{233}\text{U}$ / m^2)	Radius (cm)	$\text{kg } ^{233}\text{U}$
1	0.62500	0.32	0.20	2.199	1.63	16.330	12.16	7.53
1	0.62500	0.35	0.22	2.196	1.65	16.520	11.64	6.61

Table C.2 Plutonium-239 systems

g ²³⁹ Pu/cm ³	g ²³⁹ Pu/ g SiO ₂	g H ₂ O/cm ³	g H ₂ O /g SiO ₂	<i>k</i> _{inf}	Critical slab		Critical sphere	
					Thickness (cm)	Areal density (kg ²³⁹ Pu/m ²)	Radius (cm)	kg ²³⁹ Pu
0.00064	0.00040	0.00	0.00	0.951	8476.25	54.248	1447.81	8135.86
0.00065	0.00040	0.00	0.00	0.954	2356.15	15.197	1237.47	5119.79
0.00065	0.00041	0.00	0.00	0.958	1628.93	10.588	1098.92	3613.27
0.00068	0.00042	0.00	0.00	0.977	831.79	5.615	766.60	1273.80
0.00070	0.00044	0.00	0.00	0.995	602.33	4.216	623.59	711.01
0.00075	0.00047	0.00	0.00	1.028	447.06	3.353	498.57	389.34
0.00090	0.00056	0.00	0.00	1.115	279.39	2.515	332.20	138.21
0.00100	0.00063	0.00	0.00	1.163	234.39	2.344	289.20	101.32
0.00100	0.00063	0.03	0.02	0.998	338.91	3.389	371.82	215.32
0.00110	0.00069	0.00	0.00	1.205	205.56	2.261	261.55	82.44
0.00110	0.00069	0.03	0.02	1.046	235.60	2.592	268.97	89.66
0.00125	0.00078	0.00	0.00	1.258	186.89	2.336	234.37	67.41
0.00125	0.00078	0.03	0.02	1.110	180.32	2.254	207.15	46.54
0.00125	0.00078	0.06	0.04	1.004	255.54	3.194	281.54	116.84
0.00150	0.00094	0.00	0.00	1.326	151.71	2.276	206.94	55.68
0.00150	0.00094	0.03	0.02	1.197	132.31	1.985	162.56	26.99
0.00150	0.00094	0.06	0.04	1.096	145.22	2.178	171.48	31.68
0.00150	0.00094	0.09	0.06	1.011	202.57	3.039	224.47	71.07
0.00165	0.00103	0.00	0.00	1.359	140.40	2.317	196.10	52.12
0.00165	0.00103	0.03	0.02	1.241	116.75	1.926	147.73	22.28
0.00165	0.00103	0.06	0.04	1.143	120.74	1.992	147.21	22.05
0.00165	0.00103	0.09	0.06	1.060	145.95	2.408	168.03	32.79
0.00165	0.00103	0.12	0.08	0.988	226.02	3.729	245.46	102.21

Table C.2 (continued)

g ²³⁹ Pu/cm ³	g ²³⁹ Pu /g SiO ₂	g H ₂ O/cm ³	g H ₂ O /g SiO ₂	<i>k_{inf}</i>	Critical slab		Critical sphere	
					Thickness (cm)	Areal density (kg ²³⁹ Pu/m ²)	Radius (cm)	kg ²³⁹ Pu
0.0018	0.00113	0.00	0.00	1.386	131.88	2.374	187.64	49.81
0.0018	0.00113	0.03	0.02	1.280	105.31	1.896	137.09	19.43
0.0018	0.00113	0.06	0.04	1.185	105.36	1.896	131.63	17.20
0.0018	0.00113	0.09	0.06	1.104	118.75	2.138	140.61	20.96
0.0018	0.00113	0.12	0.08	1.032	151.06	2.719	169.83	36.93
0.0018	0.00113	0.15	0.09	0.970	272.23	4.900	290.56	184.96
0.0020	0.00125	0.00	0.00	1.417	122.52	2.450	178.85	47.93
0.0020	0.00125	0.03	0.03	1.325	92.60	1.852	126.63	17.01
0.0020	0.00125	0.06	0.04	1.235	91.23	1.825	117.73	13.67
0.0020	0.00125	0.09	0.06	1.156	97.52	1.950	119.56	14.32
0.0020	0.00125	0.12	0.08	1.086	111.97	2.239	131.08	18.87
0.0020	0.00125	0.15	0.09	1.023	153.62	3.072	169.74	40.97
0.0020	0.00125	0.18	0.11	0.968	258.17	5.163	273.31	171.04
0.0022	0.00138	0.00	0.00	1.441	115.16	2.534	172.05	46.93
0.0022	0.00138	0.03	0.03	1.364	84.65	1.862	118.93	15.50
0.0022	0.00138	0.06	0.04	1.279	81.36	1.790	108.04	11.62
0.0022	0.00138	0.09	0.06	1.202	84.23	1.853	106.50	11.13
0.0022	0.00138	0.12	0.08	1.134	91.89	2.022	111.15	12.65
0.0022	0.00138	0.15	0.09	1.072	106.58	2.345	123.56	17.39
0.0022	0.00138	0.18	0.11	1.017	137.08	3.016	152.70	32.81
0.0022	0.00138	0.21	0.13	0.967	242.88	5.343	258.29	158.78
0.0025	0.00250	0.00	0.00	1.471	106.51	2.663	164.21	46.37
0.0025	0.00156	0.03	0.03	1.412	75.81	1.895	110.47	14.12
0.0025	0.00156	0.06	0.04	1.335	71.11	1.778	97.99	9.85
0.0025	0.00156	0.09	0.06	1.263	71.42	1.785	93.81	8.64
0.0025	0.00156	0.12	0.08	1.196	74.68	1.867	94.04	8.71
0.0025	0.00156	0.15	0.09	1.136	81.01	2.025	98.14	9.90
0.0025	0.00156	0.18	0.11	1.082	91.82	2.296	107.27	12.93
0.0025	0.00156	0.21	0.13	1.032	111.67	2.792	125.84	20.87
0.0025	0.00156	0.25	0.16	0.972	195.18	4.880	207.60	93.70
0.0028	0.00175	0.00	0.00	1.493	99.96	2.799	158.23	46.47
0.0028	0.00175	0.03	0.02	1.452	69.38	1.943	104.32	13.31
0.0028	0.00175	0.06	0.04	1.382	64.86	1.816	90.98	8.83

Table C.2 (continued)

g ²³⁹ Pu/cm ³	g ²³⁹ Pu /g SiO ₂	g H ₂ O/cm ³	g H ₂ O /g SiO ₂	<i>k</i> _{inf}	Critical slab		Critical sphere	
					Thickness (cm)	Areal density (kg ²³⁹ Pu/m ²)	Radius (cm)	kg ²³⁹ Pu
0.0028	0.00175	0.09	0.06	1.314	64.04	1.793	85.45	7.32
0.0028	0.00175	0.12	0.08	1.251	66.20	1.854	83.67	6.87
0.0028	0.00175	0.15	0.09	1.192	68.11	1.907	84.55	7.09
0.0028	0.00175	0.18	0.11	1.139	73.21	2.050	88.08	8.01
0.0028	0.00175	0.21	0.13	1.090	81.74	2.289	95.07	10.08
0.0028	0.00175	0.25	0.16	1.030	102.57	2.872	114.40	17.56
0.0028	0.00175	0.29	0.18	0.977	163.68	4.583	175.17	63.04
0.0032	0.00200	0.00	0.00	1.516	93.14	2.980	152.19	47.25
0.0032	0.00200	0.03	0.02	1.494	62.97	2.015	98.29	12.73
0.0032	0.00200	0.06	0.04	1.434	57.79	1.849	84.31	8.03
0.0032	0.00200	0.09	0.06	1.371	56.04	1.793	77.83	6.32
0.0032	0.00200	0.12	0.08	1.312	56.11	1.796	74.70	5.59
0.0032	0.00200	0.15	0.09	1.256	57.46	1.839	73.64	5.35
0.0032	0.00200	0.18	0.11	1.205	58.66	1.877	74.23	5.48
0.0032	0.00200	0.21	0.13	1.157	62.30	1.994	76.52	6.00
0.0032	0.00200	0.25	0.16	1.098	70.05	2.242	82.81	7.61
0.0032	0.00200	0.29	0.18	1.045	83.86	2.684	95.47	11.66
0.0032	0.00200	0.32	0.20	1.008	103.62	3.316	114.43	20.09
0.0032	0.00200	0.36	0.23	0.963	189.77	6.073	201.31	109.35
0.0036	0.00225	0.00	0.00	1.532	87.85	3.163	147.50	48.40
0.0036	0.00225	0.03	0.02	1.528	58.19	2.095	93.79	12.44
0.0036	0.00225	0.06	0.04	1.476	52.02	1.873	79.49	7.57
0.0036	0.00225	0.09	0.06	1.419	50.43	1.815	72.51	5.75
0.0036	0.00225	0.12	0.08	1.363	49.78	1.792	68.67	4.88
0.0036	0.00225	0.15	0.09	1.311	50.13	1.805	66.65	4.46
0.0036	0.00225	0.18	0.11	1.261	50.31	1.811	65.94	4.32
0.0036	0.00225	0.21	0.13	1.215	52.12	1.876	66.36	4.41
0.0036	0.00225	0.25	0.16	1.157	55.90	2.012	68.67	4.88
0.0036	0.00225	0.29	0.18	1.105	61.90	2.229	73.49	5.99
0.0036	0.00225	0.32	0.20	1.069	68.77	2.476	79.63	7.61
0.0036	0.00225	0.36	0.23	1.024	90.00	3.240	99.40	14.81
0.0036	0.00225	0.40	0.25	0.982	120.90	4.352	131.47	34.27
0.0045	0.00281	0.00	0.00	1.557	79.67	3.585	139.93	51.64
0.0045	0.00281	0.03	0.02	1.581	51.07	2.298	86.96	12.39

Table C.2 (continued)

g ²³⁹ Pu/cm ³	g ²³⁹ Pu /g SiO ₂	g H ₂ O/cm ³	g H ₂ O /g SiO ₂	<i>k</i> _{inf}	Critical slab		Critical sphere	
					Thickness (cm)	Areal density (kg ²³⁹ Pu/m ²)	Radius (cm)	kg ²³⁹ Pu
0.0045	0.00281	0.06	0.04	1.545	44.68	2.011	72.32	7.13
0.0045	0.00281	0.09	0.06	1.500	41.98	1.889	64.81	5.13
0.0045	0.00281	0.12	0.08	1.453	40.55	1.825	60.24	4.12
0.0045	0.00281	0.15	0.09	1.406	39.89	1.795	57.28	3.54
0.0045	0.00281	0.18	0.11	1.362	39.80	1.791	55.39	3.20
0.0045	0.00281	0.21	0.13	1.319	40.06	1.803	54.25	3.01
0.0045	0.00281	0.25	0.16	1.266	41.03	1.846	53.69	2.92
0.0045	0.00281	0.29	0.18	1.216	42.61	1.918	54.11	2.99
0.0045	0.00281	0.32	0.20	1.181	44.30	1.993	55.06	3.15
0.0045	0.00281	0.36	0.23	1.137	47.43	2.134	57.34	3.55
0.0045	0.00281	0.40	0.25	1.097	51.90	2.336	61.09	4.30
0.0055	0.00344	0.00	0.00	1.574	72.95	4.012	134.20	55.68
0.0055	0.00344	0.03	0.02	1.619	45.53	2.504	82.18	12.79
0.0055	0.00550	0.06	0.04	1.597	39.36	2.165	67.45	7.07
0.0055	0.00344	0.09	0.09	1.562	36.46	2.005	59.69	4.90
0.0055	0.00344	0.12	0.08	1.523	34.83	1.915	54.81	3.79
0.0055	0.00344	0.15	0.09	1.483	33.88	1.863	51.46	3.14
0.0055	0.00344	0.18	0.11	1.444	33.31	1.832	49.08	2.72
0.0055	0.00344	0.21	0.13	1.406	33.06	1.818	47.37	2.45
0.0055	0.00344	0.25	0.16	1.357	33.06	1.819	45.86	2.22
0.0055	0.00344	0.29	0.18	1.311	33.42	1.838	45.01	2.10
0.0055	0.00344	0.32	0.20	1.278	33.93	1.866	44.75	2.07
0.0055	0.00344	0.36	0.23	1.237	34.88	1.918	44.86	2.08
0.0055	0.00344	0.40	0.25	1.197	36.22	1.992	45.50	2.17
0.0065	0.00406	0.00	0.00	1.585	67.84	4.410	129.88	59.66
0.0065	0.00406	0.03	0.03	1.644	41.76	2.714	78.90	13.37
0.0065	0.00406	0.06	0.04	1.632	35.68	2.319	64.18	7.20
0.0065	0.00406	0.09	0.06	1.606	32.80	2.132	56.33	4.87
0.0065	0.00406	0.12	0.08	1.574	31.11	2.022	51.30	3.68
0.0065	0.004063	0.15	0.0938	1.540	30.00	1.950	47.78	2.97
0.0065	0.004063	0.18	0.1125	1.506	29.28	1.903	45.19	2.51
0.0065	0.004063	0.21	0.1313	1.472	28.79	1.871	43.23	2.20
0.0065	0.004063	0.25	0.1563	1.427	28.43	1.848	41.32	1.92
0.0065	0.004063	0.29	0.1813	1.385	28.34	1.842	40.00	1.74
0.0065	0.004063	0.32	0.2000	1.354	28.41	1.846	39.30	1.65

Table C.2 (continued)

g ²³⁹ Pu/cm ³	g ²³⁹ Pu /g SiO ₂	g H ₂ O/cm ³	g H ₂ O /g SiO ₂	<i>k</i> _{inf}	Critical slab		Critical sphere	
					Thickness (cm)	Areal density (kg ²³⁹ Pu/m ²)	Radius (cm)	kg ²³⁹ Pu
0.0065	0.004063	0.36	0.2250	1.315	28.68	1.864	38.71	1.58
0.0065	0.004063	0.40	0.2500	1.278	29.16	1.895	38.46	1.55
0.0100	0.00625	0.00	0.0000	1.607	56.99	5.699	119.78	71.98
0.0100	0.00625	0.03	0.0300	1.685	34.32	3.432	72.64	16.05
0.0100	0.00625	0.06	0.0375	1.695	28.69	2.869	58.07	8.20
0.0100	0.00625	0.09	0.0563	1.688	25.98	2.598	50.17	5.29
0.0100	0.006250	0.12	0.0750	1.673	24.30	2.430	44.99	3.82
0.0100	0.00625	0.15	0.0938	1.654	23.08	2.308	41.26	2.94
0.0100	0.00625	0.18	0.1125	1.632	22.20	2.220	38.44	2.38
0.0100	0.00625	0.21	0.1313	1.610	21.53	2.153	36.22	1.99
0.0100	0.00625	0.25	0.1563	1.579	20.81	2.081	33.89	1.63
0.0100	0.00625	0.29	0.1813	1.548	20.30	2.030	32.09	1.38
0.0100	0.00625	0.32	0.2000	1.524	19.98	1.998	30.99	1.25
0.0100	0.00625	0.36	0.2250	1.494	19.68	1.968	29.78	1.11
0.0100	0.00625	0.40	0.2500	1.464	19.45	1.945	29.15	1.04
0.0160	0.01	0.00	0.0000	1.625	46.42	7.427	110.03	89.27
0.0160	0.01	0.03	0.0188	1.707	28.20	4.512	67.69	20.79
0.0160	0.01	0.06	0.0375	1.728	23.34	3.734	53.72	10.39
0.0160	0.01	0.09	0.0563	1.735	20.90	3.343	45.94	6.50
0.0160	0.01000	0.12	0.08	1.734	19.33	3.093	40.78	4.54
0.0160	0.01000	0.15	0.09	1.728	18.21	2.914	37.02	3.40
0.0160	0.01000	0.18	0.11	1.719	17.34	2.774	34.13	2.66
0.0160	0.01000	0.21	0.13	1.708	16.65	2.664	31.83	2.16
0.0160	0.01000	0.25	0.16	1.692	15.91	2.545	29.38	1.70
0.0160	0.01000	0.29	0.18	1.674	15.30	2.448	27.44	1.38
0.0160	0.01000	0.32	0.20	1.659	14.93	2.388	26.22	1.21
0.0160	0.01000	0.36	0.23	1.640	14.49	2.319	24.85	1.03
0.0160	0.01000	0.40	0.25	1.619	14.13	2.261	23.91	0.92
0.0225	0.01406	0.00	0.00	1.636	39.64	8.919	103.10	103.30
0.0225	0.01406	0.03	0.02	1.716	24.79	5.577	64.79	25.63
0.0225	0.01406	0.06	0.04	1.740	20.48	4.607	51.42	12.81
0.0225	0.01406	0.09	0.06	1.751	18.28	4.112	43.83	7.94
0.0225	0.01406	0.12	0.08	1.756	16.86	3.792	38.73	5.48
0.0225	0.01406	0.15	0.09	1.757	15.82	3.560	35.00	4.04

Table C.2 (continued)

g ²³⁹ Pu/cm ³	g ²³⁹ Pu /g SiO ₂	g H ₂ O/cm ³	g H ₂ O /g SiO ₂	<i>k_{inf}</i>	Critical slab		Critical sphere	
					Thickness (cm)	Areal density (kg ²³⁹ Pu/m ²)	Radius (cm)	kg ²³⁹ Pu
0.0225	0.01406	0.18	0.11	1.755	15.02	3.379	32.11	3.12
0.0225	0.01406	0.21	0.13	1.750	14.36	3.231	29.79	2.49
0.0225	0.01406	0.25	0.16	1.742	13.65	3.071	27.32	1.92
0.0225	0.01406	0.29	0.18	1.732	13.01	2.927	25.35	1.54
0.0225	0.02250	0.32	0.20	1.724	12.63	2.842	24.11	1.32
0.0225	0.01406	0.36	0.23	1.711	12.20	2.744	22.70	1.10
0.0225	0.01406	0.40	0.25	1.698	11.82	2.660	21.65	0.96
0.08	0.05000	0.00	0.00	1.691	20.38	16.302	78.70	163.32
0.08	0.05000	0.03	0.02	1.756	14.84	11.868	54.42	54.00
0.08	0.05000	0.06	0.04	1.769	12.91	10.329	44.49	29.52
0.08	0.05000	0.09	0.06	1.775	11.78	9.426	38.33	18.87
0.08	0.05000	0.12	0.08	1.780	10.97	8.779	33.98	13.14
0.08	0.05000	0.15	0.09	1.784	10.50	8.398	30.96	9.95
0.08	0.05000	0.18	0.11	1.788	9.79	7.835	28.03	7.38
0.08	0.05000	0.21	0.13	1.791	9.33	7.464	25.87	5.80
0.08	0.05000	0.25	0.16	1.795	8.80	7.037	23.52	4.36
0.08	0.05000	0.29	0.18	1.798	8.34	6.669	21.61	3.38
0.08	0.05000	0.32	0.20	1.800	8.03	6.424	20.40	2.84
0.08	0.05000	0.36	0.23	1.801	7.66	6.131	18.99	2.30
0.08	0.05000	0.40	0.25	1.803	7.50	5.999	18.05	1.97
0.25	0.15625	0.00	0.00	1.861	8.61	21.535	54.10	165.83
0.25	0.15625	0.03	0.02	1.853	7.85	19.635	42.60	80.97
0.25	0.15625	0.06	0.04	1.845	7.62	19.058	36.65	51.55
0.25	0.15625	0.09	0.06	1.838	7.45	18.615	32.51	35.98
0.25	0.15625	0.12	0.08	1.832	7.27	18.165	29.37	26.54
0.25	0.15625	0.15	0.09	1.827	7.20	17.998	27.14	20.94
0.25	0.15625	0.18	0.11	1.823	6.87	17.185	24.83	16.02
0.25	0.15625	0.21	0.13	1.820	6.67	16.683	23.10	12.90
0.25	0.15625	0.25	0.16	1.818	6.41	16.020	21.17	9.93
0.25	0.15625	0.29	0.18	1.816	6.15	15.380	19.56	7.83
0.25	0.15625	0.32	0.20	1.815	5.97	14.923	18.51	6.64
0.25	0.15625	0.36	0.23	1.814	5.74	14.340	17.29	5.41
0.25	0.15625	0.39	0.24	1.814	5.70	14.240	16.78	4.95
1	0.62500	0.00	0.00	2.311	2.42	24.220	27.89	90.88

Table C.2 (continued)

g ²³⁹ Pu/cm ³	g ²³⁹ Pu /g SiO ₂	g H ₂ O/cm ³	g H ₂ O /g SiO ₂	<i>k_{inf}</i>	Critical slab		Critical sphere	
					Thickness (cm)	Areal density (kg ²³⁹ Pu/m ²)	Radius (cm)	kg ²³⁹ Pu
1	0.62500	0.03	0.03	2.157	2.65	26.460	25.61	70.40
1	0.62500	0.06	0.04	2.091	2.89	28.870	23.80	56.48
1	0.62500	0.09	0.06	2.051	3.10	30.960	22.21	45.89
1	0.62500	0.12	0.08	2.022	3.26	32.620	20.82	37.82
1	0.62500	0.15	0.09	2.000	3.42	34.220	19.74	32.20
1	0.62500	0.18	0.11	1.981	3.47	34.690	18.53	26.67
1	0.62500	0.21	0.13	1.966	3.52	35.230	17.58	22.76
1	0.62500	0.25	0.16	1.949	3.56	35.570	16.46	18.67
1	0.62500	0.29	0.18	1.936	3.56	35.600	15.48	15.52
1	0.62500	0.32	0.20	1.927	3.55	35.470	14.82	13.63
1	0.62500	0.35	0.22	1.919	3.57	35.650	14.34	12.35

Table C.3 Plutonium isotopic mixture systems

g Pu/cm ³	g Pu/g SiO ₂	g H ₂ O/cm ³	g H ₂ O /g SiO ₂	k_{inf}	Critical slab		Critical sphere	
					Thickness (cm)	Areal density (kg Pu/m ²)	Radius (cm)	kg Pu
0.00089	0.00056	0.00	0.00	0.952	5273.13	46.931	2671.32	71065.10
0.00090	0.00056	0.00	0.00	0.956	2138.41	19.246	1782.73	21359.34
0.00095	0.00059	0.00	0.00	0.974	869.90	8.264	896.33	2865.64
0.00100	0.00063	0.00	0.00	0.990	641.04	6.410	693.17	1395.09
0.00110	0.00069	0.00	0.00	1.019	463.07	5.094	518.05	640.63
0.00110	0.00069	0.03	0.02	0.953	861.37	9.475	930.44	3711.50
0.00125	0.00078	0.00	0.00	1.053	363.71	4.546	422.04	393.60
0.00125	0.00078	0.03	0.02	1.008	332.26	4.153	344.07	213.27
0.00150	0.00094	0.00	0.00	1.094	288.10	4.321	347.26	263.11
0.00150	0.00094	0.03	0.02	1.084	196.68	2.950	224.73	71.31
0.00150	0.00094	0.06	0.04	1.008	247.54	3.713	274.09	129.38
0.00165	0.00103	0.00	0.00	1.110	263.60	4.349	323.65	234.31
0.00165	0.00103	0.03	0.02	1.122	165.57	2.732	196.11	52.12
0.00165	0.00103	0.06	0.04	1.050	181.80	3.000	208.13	62.31
0.00165	0.00103	0.09	0.06	0.980	294.49	4.859	315.65	217.37
0.0018	0.00113	0.00	0.00	1.123	246.11	4.430	307.44	219.09
0.0018	0.00113	0.03	0.02	1.154	146.48	2.637	177.57	42.22
0.0018	0.00113	0.06	0.04	1.088	148.77	2.678	175.25	40.58
0.0018	0.00113	0.09	0.06	1.020	189.63	3.413	211.57	71.41
0.0018	0.00113	0.12	0.08	0.957	450.70	8.113	476.95	818.03
0.0020	0.00125	0.00	0.00	1.134	229.91	4.598	291.99	208.56
0.0020	0.00125	0.03	0.02	1.191	129.25	2.585	160.71	34.77
0.0020	0.00125	0.06	0.04	1.132	126.69	2.534	150.12	28.34
0.0020	0.00125	0.09	0.06	1.067	139.31	2.786	161.47	35.27
0.0020	0.00125	0.12	0.08	1.006	186.55	3.731	205.39	72.58
0.0020	0.00125	0.15	0.09	0.950	599.80	11.996	631.19	2106.69

Table C.3 (continued)

g Pu/cm ³	g Pu/g SiO ₂	g H ₂ O/cm ³	g H ₂ O /g SiO ₂	k_{inf}	Critical slab		Critical sphere	
					Thickness (cm)	Areal density (kg Pu/m ²)	Radius (cm)	kg Pu
0.0022	0.00138	0.00	0.00	1.142	218.08	4.798	281.13	204.76
0.0022	0.00138	0.03	0.02	1.222	113.69	2.501	148.97	30.46
0.0022	0.00138	0.06	0.04	1.171	109.81	2.416	134.41	22.38
0.0022	0.00138	0.09	0.06	1.109	114.15	2.511	136.67	23.53
0.0022	0.00138	0.12	0.08	1.050	134.02	2.949	153.30	33.20
0.0022	0.00138	0.15	0.09	0.995	186.83	4.110	203.71	77.90
0.0025	0.00156	0.00	0.00	1.148	205.87	5.147	270.62	207.54
0.0025	0.00156	0.03	0.02	1.260	100.95	2.524	136.83	26.83
0.0025	0.00156	0.06	0.04	1.219	91.75	2.294	119.19	17.73
0.0025	0.00156	0.09	0.06	1.164	92.90	2.323	115.62	16.18
0.0025	0.00156	0.12	0.08	1.108	100.35	2.509	119.95	18.07
0.0025	0.00156	0.15	0.09	1.055	121.96	3.049	138.04	27.55
0.0025	0.00156	0.18	0.11	1.006	151.63	3.791	167.10	48.86
0.0025	0.00156	0.21	0.13	0.960	299.25	7.481	324.99	359.46
0.0028	0.00175	0.00	0.00	1.149	197.46	5.529	263.48	214.54
0.0028	0.00175	0.03	0.02	1.289	92.09	2.579	128.39	24.82
0.0028	0.00175	0.06	0.04	1.259	81.54	2.283	109.25	15.29
0.0028	0.00175	0.09	0.06	1.209	80.14	2.244	103.06	12.84
0.0028	0.00175	0.12	0.08	1.157	83.05	2.325	102.82	12.75
0.0028	0.00175	0.15	0.09	1.106	90.05	2.521	107.48	14.56
0.0028	0.00175	0.18	0.11	1.058	103.03	2.885	118.63	19.58
0.0028	0.00175	0.21	0.13	1.014	128.55	3.599	142.65	34.05
0.0028	0.00175	0.25	0.16	0.959	279.76	7.833	297.00	307.26
0.0032	0.00200	0.00	0.00	1.147	189.72	6.071	257.43	228.68
0.0032	0.00200	0.03	0.02	1.319	83.61	2.675	120.49	23.45
0.0032	0.00200	0.06	0.04	1.302	72.20	2.310	100.27	13.51
0.0032	0.00200	0.09	0.06	1.259	69.82	2.234	92.32	10.55
0.0032	0.00200	0.12	0.08	1.212	70.22	2.247	89.39	9.57
0.0032	0.00200	0.15	0.09	1.165	72.69	2.326	89.68	9.67
0.0032	0.00200	0.18	0.11	1.119	77.91	2.493	92.98	10.78
0.0032	0.00200	0.21	0.13	1.076	86.34	2.763	100.08	13.43
0.0032	0.00200	0.25	0.16	1.023	108.47	3.471	120.39	23.39
0.0032	0.00200	0.29	0.18	0.974	176.58	5.650	186.86	87.45

Table C.3 (continued)

g Pu/cm ³	g Pu/g SiO ₂	g H ₂ O/cm ³	g H ₂ O /g SiO ₂	k_{inf}	Critical slab		Critical sphere	
					Thickness (cm)	Areal density (kg Pu/m ²)	Radius (cm)	kg Pu
0.0036	0.00225	0.00	0.00	1.143	184.19	6.631	253.48	245.59
0.0036	0.00225	0.03	0.02	1.341	77.51	2.790	114.91	22.88
0.0036	0.00225	0.06	0.04	1.335	65.68	2.364	93.98	12.52
0.0036	0.00225	0.09	0.06	1.300	64.48	2.321	85.15	9.31
0.0036	0.00225	0.12	0.08	1.258	63.81	2.297	80.92	7.99
0.0036	0.00225	0.15	0.09	1.214	64.98	2.339	79.34	7.53
0.0036	0.00225	0.18	0.11	1.171	67.77	2.440	79.81	7.67
0.0036	0.00225	0.21	0.13	1.129	67.80	2.441	82.23	8.38
0.0036	0.00225	0.25	0.16	1.078	76.48	2.753	89.38	10.77
0.0036	0.00225	0.29	0.18	1.030	92.76	3.339	104.51	17.21
0.0036	0.00225	0.32	0.20	0.996	117.83	4.242	129.31	32.61
0.0036	0.00225	0.36	0.23	0.955	277.25	9.981	295.38	388.63
0.0045	0.00281	0.00	0.00	1.132	175.46	7.896	247.36	285.30
0.0045	0.00281	0.03	0.02	1.371	69.06	3.108	106.79	22.96
0.0045	0.00281	0.06	0.04	1.388	56.55	2.545	85.12	11.62
0.0045	0.00281	0.09	0.06	1.368	51.95	2.338	75.30	8.05
0.0045	0.00281	0.12	0.08	1.335	51.47	2.316	69.80	6.41
0.0045	0.00281	0.15	0.09	1.299	50.80	2.286	66.54	5.55
0.0045	0.00281	0.18	0.11	1.262	51.02	2.296	64.73	5.11
0.0045	0.00281	0.21	0.13	1.225	52.01	2.341	63.99	4.94
0.0045	0.00281	0.25	0.16	1.178	51.67	2.325	64.47	5.05
0.0045	0.00281	0.29	0.18	1.133	55.04	2.477	66.62	5.57
0.0045	0.00281	0.32	0.20	1.101	58.75	2.644	69.59	6.35
0.0045	0.00281	0.36	0.23	1.061	66.27	2.982	76.23	8.35
0.0045	0.00281	0.40	0.25	1.023	83.05	3.737	91.93	14.65
0.0055	0.00344	0.00	0.00	1.122	167.51	9.213	241.53	324.60
0.0055	0.00344	0.03	0.02	1.387	62.46	3.436	101.55	24.13
0.0055	0.00344	0.06	0.04	1.423	50.14	2.758	79.36	11.52
0.0055	0.00344	0.09	0.06	1.416	45.14	2.483	69.10	7.60
0.0055	0.00344	0.12	0.08	1.394	42.55	2.340	63.03	5.77
0.0055	0.00344	0.15	0.09	1.366	41.15	2.263	59.10	4.76
0.0055	0.00344	0.18	0.11	1.335	41.92	2.306	56.46	4.15
0.0055	0.00344	0.21	0.13	1.303	41.78	2.298	54.70	3.77
0.0055	0.00344	0.25	0.16	1.261	42.21	2.322	53.38	3.51
0.0055	0.00344	0.29	0.18	1.220	41.24	2.268	53.00	3.43

Table C.3 (continued)

g Pu/cm ³	g Pu/g SiO ₂	g H ₂ O/cm ³	g H ₂ O /g SiO ₂	<i>k_{inf}</i>	Critical slab		Critical sphere	
					Thickness (cm)	Areal density (kg Pu/m ²)	Radius (cm)	kg Pu
0.0055	0.00344	0.32	0.20	1.191	42.31	2.327	53.28	3.48
0.0055	0.00344	0.36	0.23	1.153	44.31	2.437	54.41	3.71
0.0055	0.00344	0.40	0.25	1.117	47.20	2.596	56.57	4.17
0.0065	0.00406	0.00	0.00	1.117	159.56	10.371	234.62	351.64
0.0065	0.00406	0.03	0.02	1.393	58.41	3.797	98.18	25.77
0.0065	0.00406	0.06	0.04	1.444	46.07	2.994	75.72	11.82
0.0065	0.00406	0.09	0.06	1.448	40.91	2.659	65.18	7.54
0.0065	0.00406	0.12	0.08	1.435	38.20	2.483	58.84	5.55
0.0065	0.00406	0.15	0.09	1.413	36.47	2.370	54.61	4.43
0.0065	0.00406	0.18	0.11	1.388	35.43	2.303	51.60	3.74
0.0065	0.00406	0.21	0.13	1.361	34.81	2.263	49.43	3.29
0.0065	0.00406	0.25	0.16	1.324	34.39	2.235	47.42	2.90
0.0065	0.00406	0.29	0.18	1.287	34.39	2.235	46.18	2.68
0.0065	0.00406	0.32	0.20	1.260	34.66	2.253	45.66	2.59
0.0065	0.00406	0.36	0.23	1.225	35.29	2.294	45.42	2.55
0.0065	0.00406	0.40	0.25	1.192	36.32	2.361	45.69	2.60
0.0100	0.00625	0.00	0.0000	1.130	130.56	13.056	205.73	364.72
0.0100	0.00625	0.03	0.0188	1.389	50.53	5.053	92.29	32.93
0.0100	0.00625	0.06	0.0375	1.468	38.14	3.814	69.40	14.00
0.0100	0.00625	0.09	0.0563	1.497	33.10	3.310	58.44	8.36
0.0100	0.00625	0.12	0.0750	1.504	30.24	3.024	51.72	5.79
0.0100	0.00625	0.15	0.0938	1.500	28.27	2.827	47.08	4.37
0.0100	0.00625	0.18	0.1125	1.490	26.94	2.694	43.65	3.48
0.0100	0.00625	0.21	0.1313	1.477	25.94	2.594	41.01	2.89
0.0100	0.00625	0.25	0.1563	1.455	24.92	2.492	38.32	2.36
0.0100	0.00625	0.29	0.1813	1.431	24.23	2.423	36.28	2.00
0.0100	0.00625	0.32	0.2000	1.412	23.82	2.382	35.06	1.81
0.0100	0.00625	0.36	0.2250	1.387	23.46	2.346	33.76	1.61
0.0100	0.00625	0.40	0.2500	1.361	23.23	2.323	32.74	1.47
0.0160	0.01	0.00	0.0000	1.203	94.20	15.071	165.02	301.15
0.0160	0.01	0.03	0.0188	1.372	43.76	7.001	87.98	45.64
0.0160	0.01	0.06	0.0375	1.460	32.71	5.233	65.56	18.88
0.0160	0.01	0.09	0.0563	1.505	27.83	4.452	54.42	10.80
0.0160	0.01	0.12	0.0750	1.529	25.00	4.000	47.48	7.17
0.0160	0.01	0.15	0.0938	1.541	23.10	3.696	42.63	5.19

Table C.3 (continued)

g Pu/cm ³	g Pu/g SiO ₂	g H ₂ O/cm ³	g H ₂ O /g SiO ₂	k_{inf}	Critical slab		Critical sphere	
					Thickness (cm)	Areal density (kg Pu/m ²)	Radius (cm)	kg Pu
0.0160	0.01	0.18	0.1125	1.546	21.58	3.452	39.01	3.98
0.0160	0.01	0.21	0.1313	1.546	20.50	3.280	36.18	3.17
0.0160	0.01	0.25	0.1563	1.541	19.39	3.103	33.24	2.46
0.0160	0.01	0.29	0.1813	1.531	18.53	2.965	30.94	1.98
0.0160	0.01	0.32	0.2000	1.523	17.95	2.871	29.52	1.72
0.0160	0.01	0.36	0.2250	1.509	17.36	2.778	27.93	1.46
0.0160	0.01000	0.40	0.25	1.495	16.88	2.701	26.84	1.30
0.0225	0.01406	0.00	0.00	1.292	72.32	16.272	139.16	254.00
0.0225	0.01406	0.03	0.02	1.366	39.74	8.940	84.68	57.23
0.0225	0.01406	0.06	0.04	1.446	29.69	6.680	63.66	24.32
0.0225	0.01406	0.09	0.06	1.494	24.97	5.619	52.67	13.77
0.0225	0.01406	0.12	0.08	1.524	22.25	5.006	45.69	8.99
0.0225	0.01406	0.15	0.09	1.544	20.39	4.588	40.77	6.39
0.0225	0.01406	0.18	0.11	1.557	19.01	4.277	37.07	4.80
0.0225	0.01406	0.21	0.13	1.564	17.90	4.026	34.16	3.76
0.0225	0.01406	0.25	0.16	1.569	16.77	3.772	31.12	2.84
0.0225	0.01406	0.29	0.18	1.569	16.02	3.605	28.73	2.24
0.0225	0.01406	0.32	0.20	1.567	15.47	3.481	27.25	1.91
0.0225	0.01406	0.36	0.23	1.562	14.84	3.340	25.57	1.58
0.0225	0.01406	0.40	0.25	1.555	14.32	3.221	24.37	1.36
0.0800	0.05000	0.00	0.00	1.632	30.62	24.494	86.31	215.49
0.0800	0.05000	0.03	0.02	1.486	22.98	18.385	64.98	91.95
0.0800	0.05000	0.06	0.04	1.465	19.90	15.921	54.44	54.07
0.0800	0.05000	0.09	0.06	1.470	17.81	14.246	47.07	34.96
0.0800	0.05000	0.12	0.08	1.482	16.24	12.993	41.59	24.10
0.0800	0.05000	0.15	0.09	1.495	15.61	12.485	38.27	18.78
0.0800	0.05000	0.18	0.11	1.508	13.94	11.153	33.92	13.08
0.08	0.05000	0.21	0.13	1.520	13.07	10.458	31.12	10.10
0.08	0.05000	0.25	0.16	1.535	12.11	9.684	28.09	7.43
0.08	0.05000	0.29	0.18	1.548	11.30	9.038	25.64	5.65
0.08	0.05000	0.32	0.20	1.556	10.76	8.606	24.09	4.68
0.08	0.05000	0.36	0.23	1.567	10.15	8.118	22.31	3.72
0.08	0.05000	0.40	0.25	1.575	9.94	7.950	21.24	3.21
0.25	0.15625	0.00	0.00	1.848	11.56	28.908	56.71	190.97
0.25	0.15625	0.03	0.02	1.744	10.79	26.978	45.71	100.04

Table C.3 (continued)

g Pu/cm ³	g Pu/g SiO ₂	g H ₂ O/cm ³	g H ₂ O /g SiO ₂	k_{inf}	Critical slab		Critical sphere	
					Thickness (cm)	Areal density (kg Pu/m ²)	Radius (cm)	kg Pu
0.25	0.15625	0.06	0.04	1.669	10.83	27.075	40.60	70.10
0.25	0.15625	0.09	0.06	1.609	10.74	26.858	36.91	52.66
0.25	0.15625	0.12	0.08	1.587	10.56	26.408	33.99	41.11
0.25	0.15625	0.15	0.09	1.566	10.90	27.245	31.84	33.82
0.25	0.15625	0.18	0.11	1.551	10.03	25.085	29.41	26.63
0.25	0.15625	0.21	0.13	1.542	9.73	24.320	27.56	21.92
0.25	0.15625	0.25	0.16	1.535	9.31	23.278	25.42	17.19
0.25	0.15625	0.29	0.18	1.531	8.90	22.245	23.57	13.71
0.25	0.15625	0.32	0.20	1.531	8.60	21.498	22.35	11.69
0.25	0.15625	0.36	0.23	1.531	8.22	20.543	20.89	9.55
0.25	0.15625	0.38	0.24	1.532	8.33	20.835	20.46	8.96
1	0.62500	0.00	0.00	2.279	3.03	30.290	28.93	101.37
1	0.62500	0.03	0.02	2.125	3.25	32.460	26.42	77.26
1	0.62500	0.06	0.04	2.043	3.64	36.360	24.65	62.72
1	0.62500	0.09	0.06	1.983	3.96	39.560	23.15	51.94
1	0.62500	0.12	0.08	1.934	4.21	42.110	21.86	43.77
1	0.62500	0.15	0.09	1.892	4.67	46.730	21.15	39.62
1	0.62500	0.18	0.11	1.855	4.55	45.490	19.79	32.45
1	0.62500	0.21	0.13	1.824	4.65	46.510	18.92	28.39
1	0.62500	0.25	0.16	1.788	4.74	47.350	17.91	24.06
1	0.62500	0.29	0.18	1.757	4.77	47.730	17.01	20.63
1	0.62500	0.32	0.20	1.737	4.78	47.790	16.41	18.50
1	0.62500	0.35	0.22	1.719	5.00	50.000	16.23	17.92

Appendix D

Comparisons of Other Elements to Silicon

Appendix D

Comparisons of Other Elements to Silicon

Table D.1 Substitution of element X into SiO₂, forming XO₂

Atomic No.	Element	k_{inf}
1	H	0.640
2	He	2.020
3	Li	0.014
4	Be	1.985
5	B	0.002
6	C	1.975
7	N	0.132
8	O	1.996
9	F	1.871
10	Ne	N/A ^a
11	Na	0.437
12	Mg	1.408
13	Al	0.768
14	Si	0.949
15	P	0.804
16	S	0.366
17	Cl	0.037
18	Ar	N/A
19	K	0.017
20	Ca	0.440
21	Sc	N/A
22	Ti	0.088
23	V	0.076
24	Cr	0.102
25	Mn	0.028
26	Fe	0.121

Table D.1 (continued)

Atomic No.	Element	k_{inf}
27	Co	0.019
28	Ni	0.073
29	Cu	0.058
30	Zn	0.177
31	Ga	0.010
32	Ge	0.035
33	As	0.004
34	Se	0.019
35	Br	0.003
36	Kr	0.017
37	Rb	0.148
38	Sr	0.077
39	Y	0.179
40	Zr	0.633
41	Nb	0.018
42	Mo	0.009
43	Tc	0.002
44	Ru	0.003
45	Rh	0.002
46	Pd	0.003
47	Ag	0.002
48	Cd	0.004
49	In	0.003
50	Sn	0.039
51	Sb	0.003
52	Te	0.015

Table D.1 (continued)

Atomic No.	Element	k_{inf}
53	I	0.002
54	Xe	0.007
55	Cs	0.003
56	Ba	0.044
57	La	0.046
58	Ce	0.336
59	Pr	0.012
60	Nd	0.009
61	Pm	0.002
62	Sm	0.002
63	Eu	0.001
64	Gd	0.002
65	Tb	0.001
66	Dy	0.002
67	Ho	0.001
68	Er	0.002
69	Tm	N/A
70	Yb	N/A
71	Lu	0.001
72	Hf	0.002
73	Ta	0.002
74	W	0.002
75	Re	0.002
76	Os	N/A
77	Ir	N/A

Table D.1 (continued)

Atomic No.	Element	k_{inf}
78	Pt	N/A
79	Au	0.003
80	Hg	N/A
81	Tl	N/A
82	Pb	0.899
83	Bi	1.430
84	Po	N/A
85	At	N/A
86	Rn	N/A
87	Fr	N/A
88	Ra	N/A
89	Ac	N/A
90	Th	0.065
91	Pa	0.792
92	U (natural)	0.391

^aN/A = Cross sections not available in SCALE libraries.

Table D.2 Substitution of other substances for SiO₂

Substance	k_{inf}
Heavy Water (D ₂ O)	2.032
Be (elemental)	1.572
C (graphite)	1.856
Mg (elemental)	0.970
MgF ₂	1.297
CaF ₂	0.528
Nominal soil ^a	0.672
Limestone ^b	0.648

^aNominal soil composition in weight percent (wt %):

Ca = 1.37%
 C = 4.29%
 O = 49.00%
 Si = 33.00%
 Al = 7.1%
 K = 1.36%
 Na = 0.68%
 Mg = 0.60%
 Fe = 2.60%

^bLimestone composition in weight percent (wt %):

Ca = 34.04%
 C = 9.98%
 O = 47.91%
 Si = 5.51%
 Al = 1.45%
 K = 0.70%
 Na = 0.41%

Appendix E

Specific Activity Values for Selected Isotopes

Appendix E

Table E.1 Specific activity values for selected isotopes

Isotope	Specific activity (Ci/g)	Specific activity (MBq/g)
Uranium-233	9.650E-3	3.571E+2
Uranium-235	2.164E-6	8.007E-2
Uranium-238	3.366E-7	1.245E-2
Plutonium-239	6.212E-2	2.298E+3
Plutonium-240	2.273E-1	8.410E+3
Plutonium-241	1.035E+2	3.830E+6

