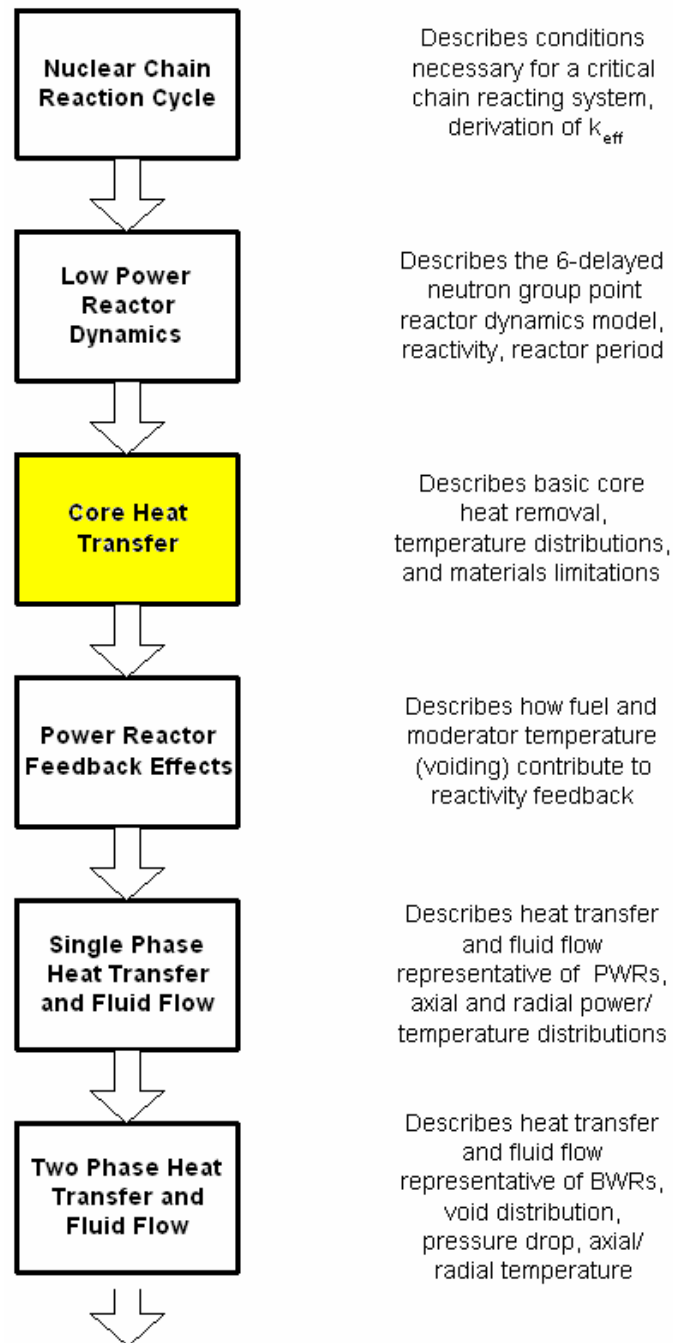


Fundamentals of Nuclear Engineering

Module 9: *Core Heat Transfer*

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Objectives

This lecture will:

1. Review basic laws of heat transfer
2. Describe fuel/cladding thermal and mechanical limitations
3. Describe basic heat sources considered
4. Describe temperature dependent fuel/cladding material properties
5. Describe simplified steady state core heat transfer from fuel to coolant and fuel/clad temperature distributions
6. Describe lumped parameter transient heat transfer model for dynamic behavior of fuel temperature, heat flux

Unit Systems:

- Historically all Nuclear Engineering texts used English units
- USNRC Technical Specifications are in English Units
- A few obvious “*problem children*”: power density in kW/ft
- Reactor Physics historically used CGS units because:
 - macroscopic cross sections Σ_f naturally arise in cm^{-1}
 - fuel pellet diameters are on order of cm.
- Expressing parameters consistently in SI units is cumbersome - particularly when describing length.
- For remainder of this course CGS units will be used for describing power and heat transfer within fuel pellets
- A unit conversion tool (free software) will be distributed with this course

Unit Systems:

	English	CGS	SI
Mass	lb _m	gram	kilogram
Density	lb _m /ft ³	gr/cm ³	Kg/m ³
Power	BTU/hr.	Watts	Watts
Linear Power Density	kW/ft	Watt/cm	Watt/m
Heat Flux	BTU/hr ft ²	Watt/cm ²	Watt/m ²
Pressure	psi (lb _f /in ²)		Pascals
Temperature	°F	°C	°K
Length	ft	cm.	m.

Basic Laws of Heat Transfer

Basic Laws of Heat Transfer

- Energy balance equation describing heat input, temperature, heat transfer is similar to neutron transport equation:
- $$\frac{\partial}{\partial t}(\rho C_p T(r, t)) = P(r, t) + \nabla \cdot k \nabla T(r, t) \cong P(r, t) + k \nabla^2 T(r, t)$$
- $P(r, t)$ is volumetric heat input source in Watts/cm³
- $k = k(T)$ is thermal conductivity in Watts/cm.°K
- $\rho = \rho(T)$ is mass density in grams/cm³
- $C_p = C_p(T)$ is Heat Capacity in Joules/gram°K or alternately: Watt-sec/gram°K
- Units of this overall Heat Transfer equation work out to:
- (grams/cm³)(Watt-sec/gram°K)(°K/sec) = Watts/cm³

Basic Laws of Heat Transfer

- Flow of heat is governed by Fourier's law of thermal conduction:

$$Q = -k A dT/dx = -k A \text{ Grad } T$$

- where: Q is heat expressed in Watts

- Frequently equation is rearranged to units of $Q/A = q$

$$q = -k dT/dx = -k \text{ Grad } T$$

- where: q is expressed in Watts/cm²

$k = k(T)$ is thermal conductivity in Watts/cm.°K

Basic Laws of Heat Transfer

- Heat transfer between two adjacent surfaces is governed by Newton's Law of cooling:

$$Q/A = q = h \Delta T$$

- where: h is conductance in units of Watts/cm².°K

- Newton's law of cooling is particularly useful when evaluating temperature drops across:
- Fuel pellet – clad Characterized via " h_{gap} "
- Clad – Coolant film Characterized via " h_{film} "
- Using these three basic relationships with appropriate property values all temperature, heat transfer effects can be evaluated.

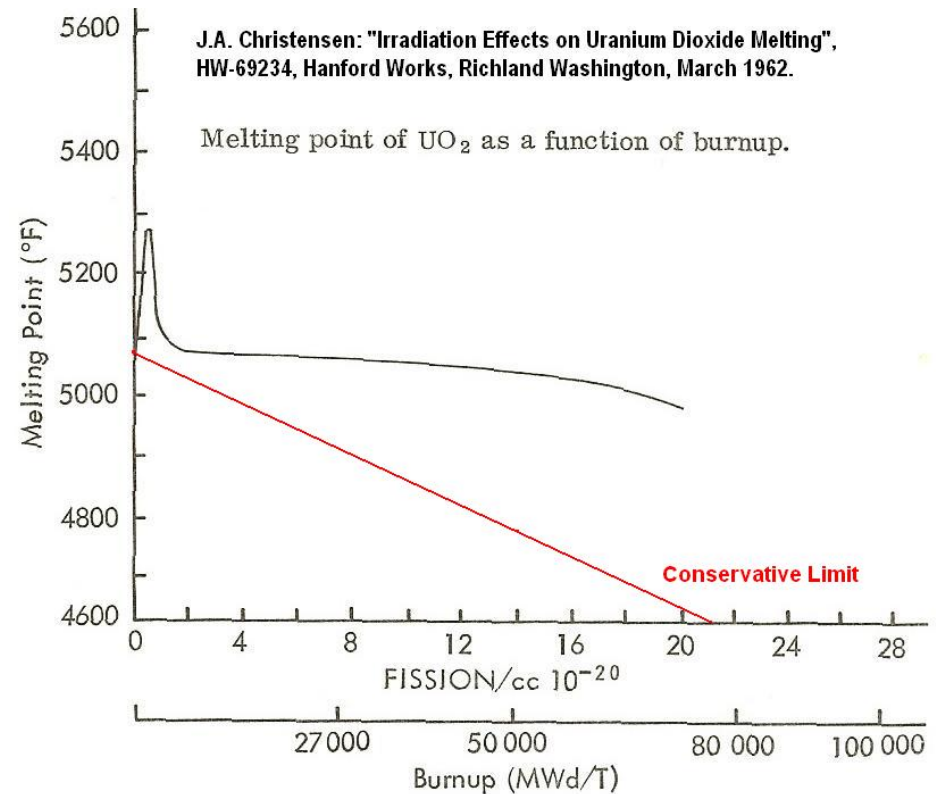
Fuel, Cladding Thermal and Mechanical Limitations

Ceramic UO_2 is Preferred Fuel Material

- Pure Uranium metal has $T_{melt} \sim 2070^\circ F$ ($\sim 1405^\circ K$)
- Pure Uranium metal undergoes 3 separate crystalline phase changes before reaching $2070^\circ F$
- Ceramic UO_2 does not have such phase changes
- Ceramic UO_2 has significantly higher melting point implying higher operating temperature capability

UO_2 Temperature Limits

- Observed UO_2 melting temperature depends on extent of burnup
- Epstein (1967) measured $T_{melt} = 5144 \pm 68 \text{ } ^\circ F$
- NRC has used Peak Fuel Centerline Temperature (PFCT) limit of $5080 \text{ } ^\circ F$ ($3077 \text{ } ^\circ K$)
- Decreasing $58 \text{ } ^\circ F$ ($288 \text{ } ^\circ K$) every 10,000 MWd/MTU as appropriate



UO₂ Enthalpy Rise Limitations

- Rapid enthalpy rise in fuel causes fracturing of ceramic UO₂
- Experimental studies (SPERT-IV, PBF) indicate:
- $\Delta H < 260 \text{ Cal/gram}$ (1.088 Joule/kg): no apparent effects
- ΔH between 260-300 Cal/gram (1.088 - 1.25 Joule/kg) fuel element fractures into large chunks
- $\Delta H \sim 350 \text{ Cal/gram}$ ($\sim 1.46 \text{ Joule/kg}$) in PBF reactivity insertion tests: local fuel melt, 0.3% conversion to mechanical energy destructive pressure pulse from: 870 $\rightarrow$ 5076 psia
- $\Delta H > 600 \text{ Cal/gram}$ (2.51 Joule/kg): fracturing of fuel to less than 0.15mm particles, significantly larger mechanical energy conversion efficiencies
- 1986 Chernobyl-4 reactivity accident exceeded ΔH limitations
- Worst US BWR Rod Drop analyses: $\Delta H < 110 \text{ Cal/gram}$

Zircaloy Replaced Stainless Steel Clad

- Stainless Steel $T_{melt} \sim 2550\text{-}2600^{\circ}\text{F}$ ($< 1700^{\circ}\text{K}$)
- Zircaloy $T_{melt} \sim 3360^{\circ}\text{F}$ ($\sim 2122^{\circ}\text{K}$)
- Zircaloy-2 in BWRs
- Zircaloy-4 used in PWRs
- Key thermal limitation is related to *metal-water reaction*
- *Zr-Water exothermic reaction* accelerates rapidly at surface temperatures $T_c > 2200^{\circ}\text{F}$ and *generates H_2 gas*
- NRC currently uses Peak Clad Temperature limit of $PCT < 2200^{\circ}\text{F}$ (1200°C) as safety limit for LWRs

Zircaloy Clad Strain Limitations

- Because of metallurgical differences between UO_2 and Zircaloy – expansion/contraction vs. temperature is different
- UO_2 thermal expansion rate is $\sim 2\times$ that of Zircaloy
- Rapid local power increase in fuel pellet can cause high local strain on Zircaloy cladding leading to local failure
- *Limit of $< 1\%$ Plastic Strain* is conservative limit below which fuel-clad failure due to excessive strain not expected
- Strain limit can be translated back to specific power density limits.

Heat Sources Considered in Core Thermal Analysis

Three Core Heat Sources are Considered

- *Steady state power operation*, characterized by: $\Phi(r,z)$
 $P(r,z) \text{ (Watts/cm}^3) \approx$
 $E_f \text{ (Watts/fission)} \Phi(r,z) \text{ (neutrons/cm}^2) \Sigma_f \text{ (fission/cm)}$
- *Transient power operation*, described by neutron kinetics models (with feedback), characterized by: $\Phi(r,z,t)$
- *Long term fission product decay heat*, best characterized by ANS 5.1-2005 Decay Heat Standard
- If Clad temperature > 2200°F (1200°C) Zircaloy-Water chemical reaction heat would need consideration.
- Distribution of heat throughout reactor is roughly:
 - 88% within fuel pellets (fission product recoil, α, β)
 - 2.5% in moderator (neutrons slowing down)
 - 9.5% in metals, structural materials (γ -ray absorption)

Core Heat Sources

- Scenarios involving reactor trip from high power operation must consider when fission/activation product *decay heat* exceeds *neutron flux power*.
- Decay heat sources include: α, β, γ -decay modes of fission products and activation products such as: Pu^{239} , Np^{239}
- Example problem
- CE System 80 NPP generating 3800 MWt trips from full power after running for more than one year
- Compare heat generated by post-trip neutron fission power vs. NRC Branch Technical Position ASB-9 (Rev2) Decay Heat Model
- Modern ANSI ANS 5.1-2005 Decay Heat Standard considers equivalent heat sources

CE System 80 NPP Trips from 3800 MWt

Comparison of ASB 9-2 and Simplified Core Decay Heat Models for Long Term Cooling Analysis

The following fission product and activation product decay energy correlations are taken from NRC Branch Technical Position ASB 9-2 Rev. 02 (July 1981). This decay heat model is used for evaluation of long term core cooling capability.

USER Input: Reactor Operating Thermal Power in MWt

$$P_o := 3800$$

USER Input: Core Operating time in seconds:
(16000 hours considered to be equilibrium value)

$$t_o := 5.76 \cdot 10^7$$

Fission Product Decay Term:

$$k := \begin{pmatrix} 0.5980 \\ 1.6500 \\ 3.1000 \\ 3.8700 \\ 2.3300 \\ 1.2900 \\ 0.4620 \\ 0.3280 \\ 0.1700 \\ 0.0865 \\ 0.1140 \end{pmatrix}$$

$$\lambda := \begin{pmatrix} 1.77 \cdot 10^0 \\ 5.774 \cdot 10^{-1} \\ 6.743 \cdot 10^{-2} \\ 6.214 \cdot 10^{-3} \\ 4.739 \cdot 10^{-4} \\ 4.810 \cdot 10^{-5} \\ 5.344 \cdot 10^{-6} \\ 5.716 \cdot 10^{-7} \\ 1.036 \cdot 10^{-7} \\ 2.959 \cdot 10^{-8} \\ 7.585 \cdot 10^{-10} \end{pmatrix}$$

$$P(ts) := \frac{P_o}{200} \sum_{i=0}^{10} \left(k_i \cdot e^{-\lambda_i \cdot ts} \right)$$

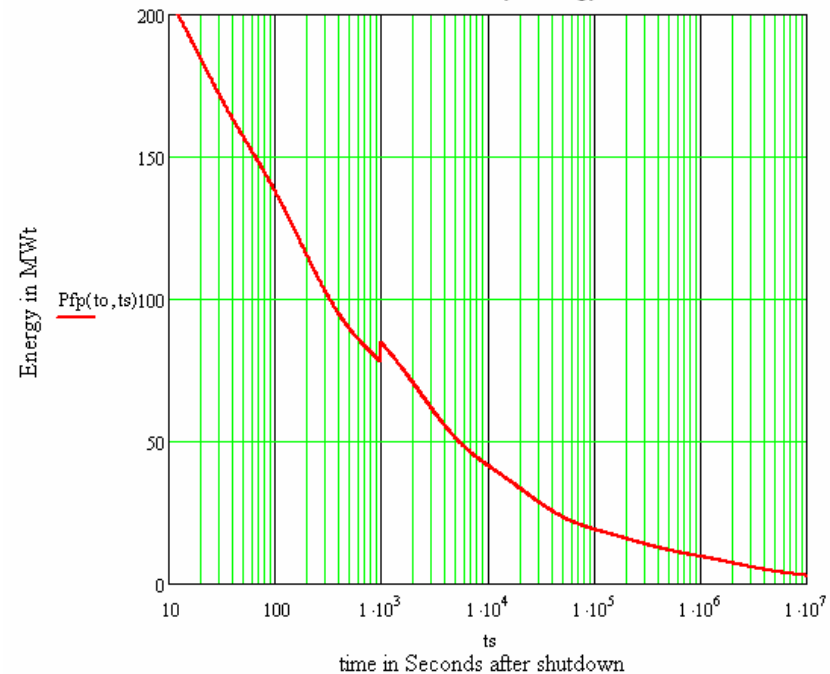
$$K(ts) := \begin{cases} 0.1 & \text{if } 0 \leq ts < 10^3 \\ 0.2 & \text{if } 10^3 \leq ts \leq 10^7 \\ 0.0 & \text{otherwise} \end{cases}$$

Cooling time uncertainty factor
adjustment per NRC Branch Technical
Position ASB 9-2 Rev. 02 (July 1981)

$$Pfp(to, ts) := (1 + K(ts)) \cdot P(ts) - P(to + ts)$$

Adjusted fission product energy source
which accounts for uncertainties.

Fission Product Decay Energy in MWt



Heavy Metal Decay Heat:

Uranium-239 term:

Conversion ratio, atoms of Pu-239 produced per
atom U-235 consumed multiplied by ratio: $\Sigma_a U-235 / \Sigma_f U-235$
Value suggested in NRC Branch Technical Position ASB 9-2
Rev. 02 (July 1981)

$$C := 0.7$$

$$PU239(to, ts) := P_o \left[2.28 \cdot 10^{-3} \cdot C \cdot \left(1 - e^{-4.91 \cdot 10^{-4} \cdot t_o} \right) \cdot \left(e^{-4.91 \cdot 10^{-4} \cdot ts} \right) \right]$$

CE System 80 NPP Trips from 3800 MWt

Neptunium-239 term:

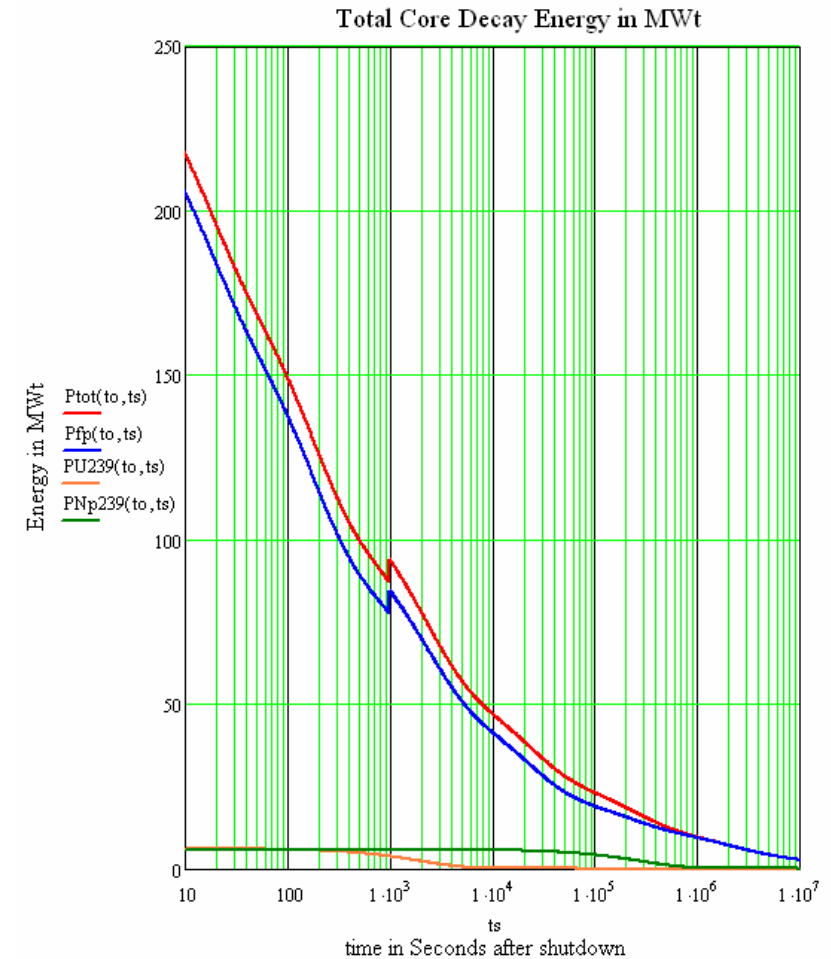
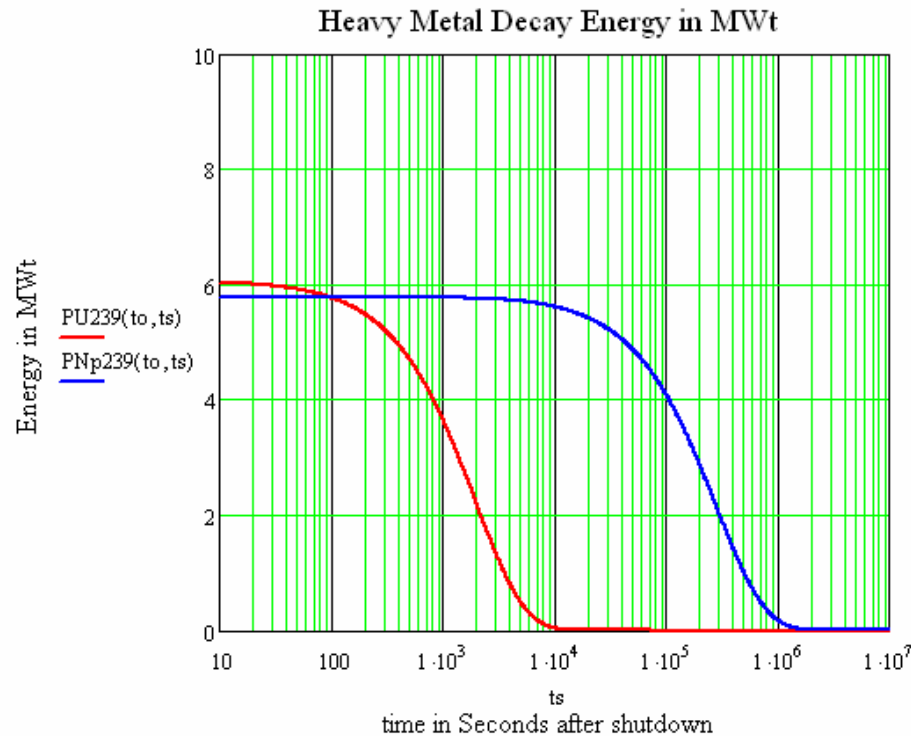
$$F(t_o, t_s) := 1.007 \cdot \left(1 - e^{-3.41 \cdot 10^{-6} \cdot t_o}\right) \cdot \left(e^{-3.41 \cdot 10^{-6} \cdot t_s}\right)$$

$$G(t_o, t_s) := 0.007 \cdot \left(1 - e^{-4.91 \cdot 10^{-4} \cdot t_o}\right) \cdot \left(e^{-4.91 \cdot 10^{-4} \cdot t_s}\right)$$

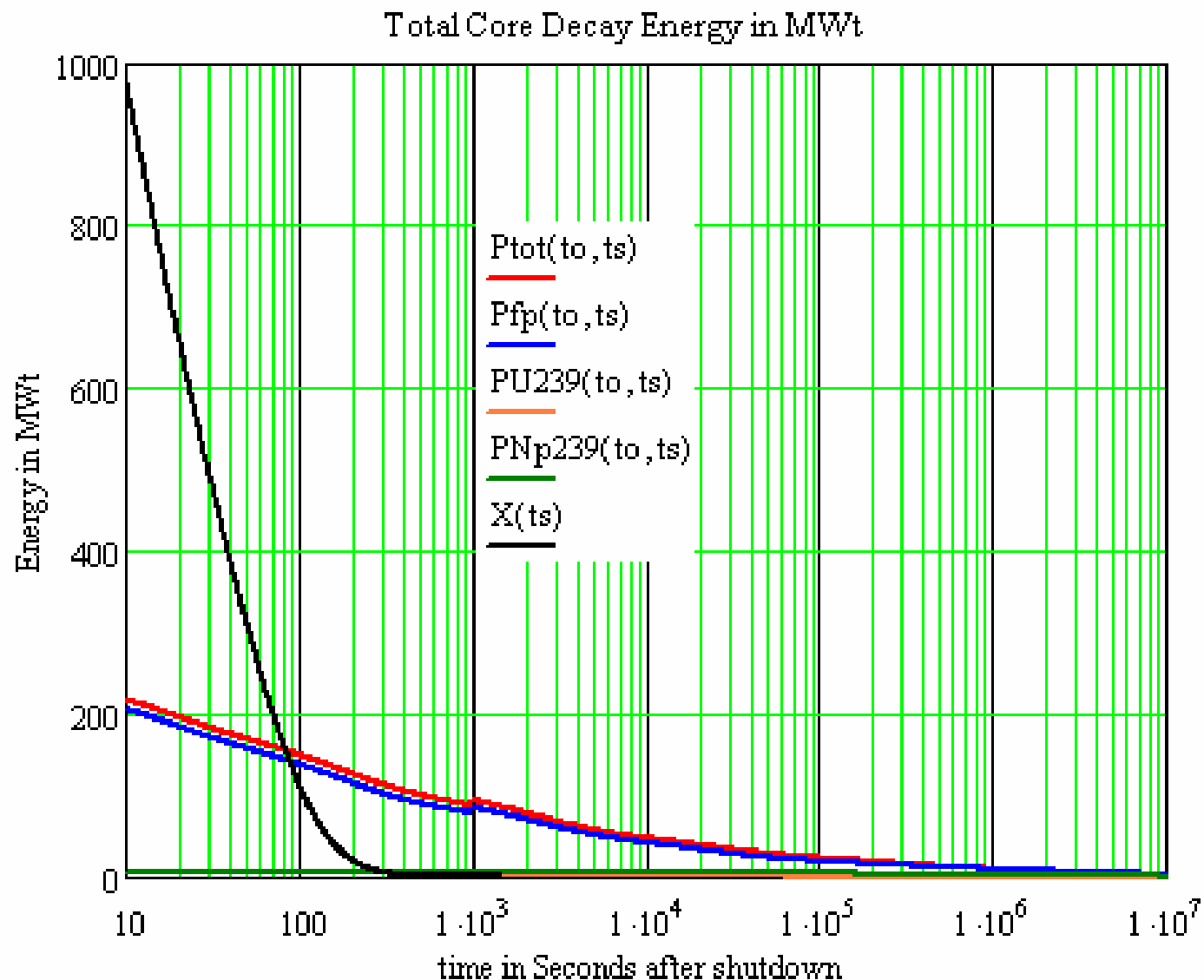
$$PNp239(t_o, t_s) := Po \cdot \left(2.17 \cdot 10^{-3} \cdot C\right) \cdot (F(t_o, t_s) - G(t_o, t_s))$$

Calculation of the total energy sums the fission product and heavy metal decay energy terms as follows:

$$P_{tot}(t_o, t_s) := P_{fp}(t_o, t_s) + P_{U239}(t_o, t_s) + P_{Np239}(t_o, t_s)$$



Comparison of Neutron Flux Power to NRC BTP ASB-9 Rev2 Decay Heat



Material Properties Which Impact Core Heat Transfer

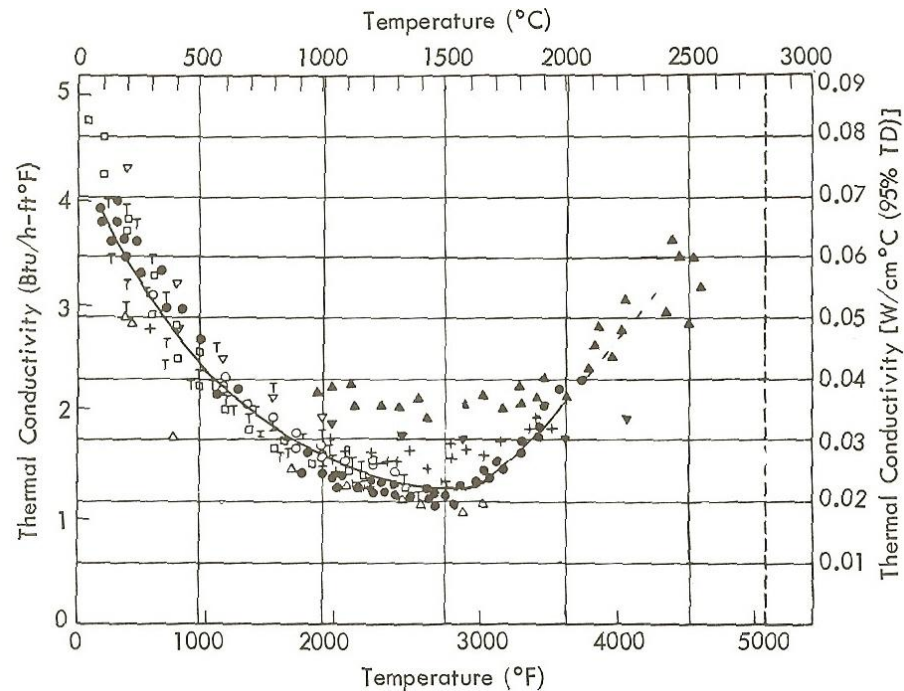
Heat Transfer from Fuel to Coolant

- General equation for heat transfer:

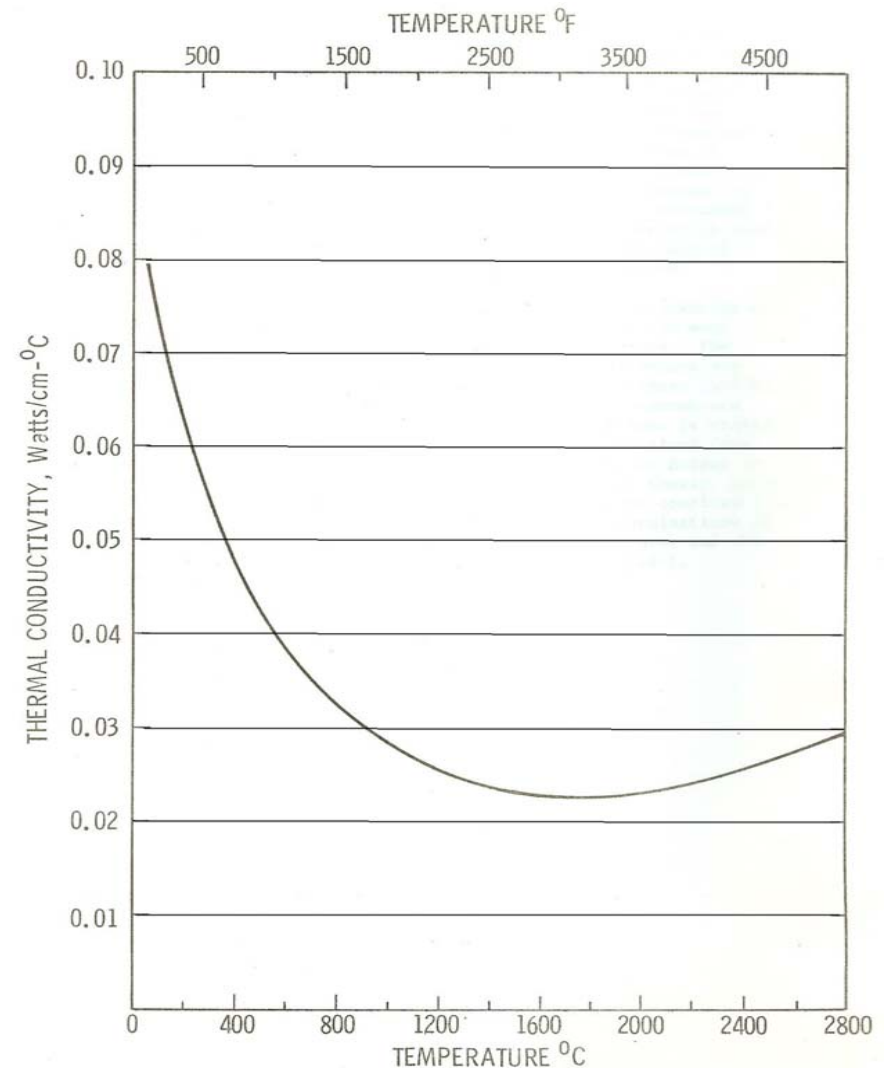
$$\rho(T_f(r,t))c_p(T_f(r,t))\frac{\partial T_f(r,t)}{\partial t} = P(r,t) + \nabla(k_f(T_f(r,t))\nabla T_f(r,t))$$

- Equation is written this way because material parameters are *strongly dependent on temperature*
- Simplified calculations can be performed in narrow temperature ranges assuming relatively constant, temperature averaged $k(T)$, $C_p(T)$ parameters
- Doing accident analysis generally requires more sophisticated analysis

Data for UO_2 Thermal Conductivity

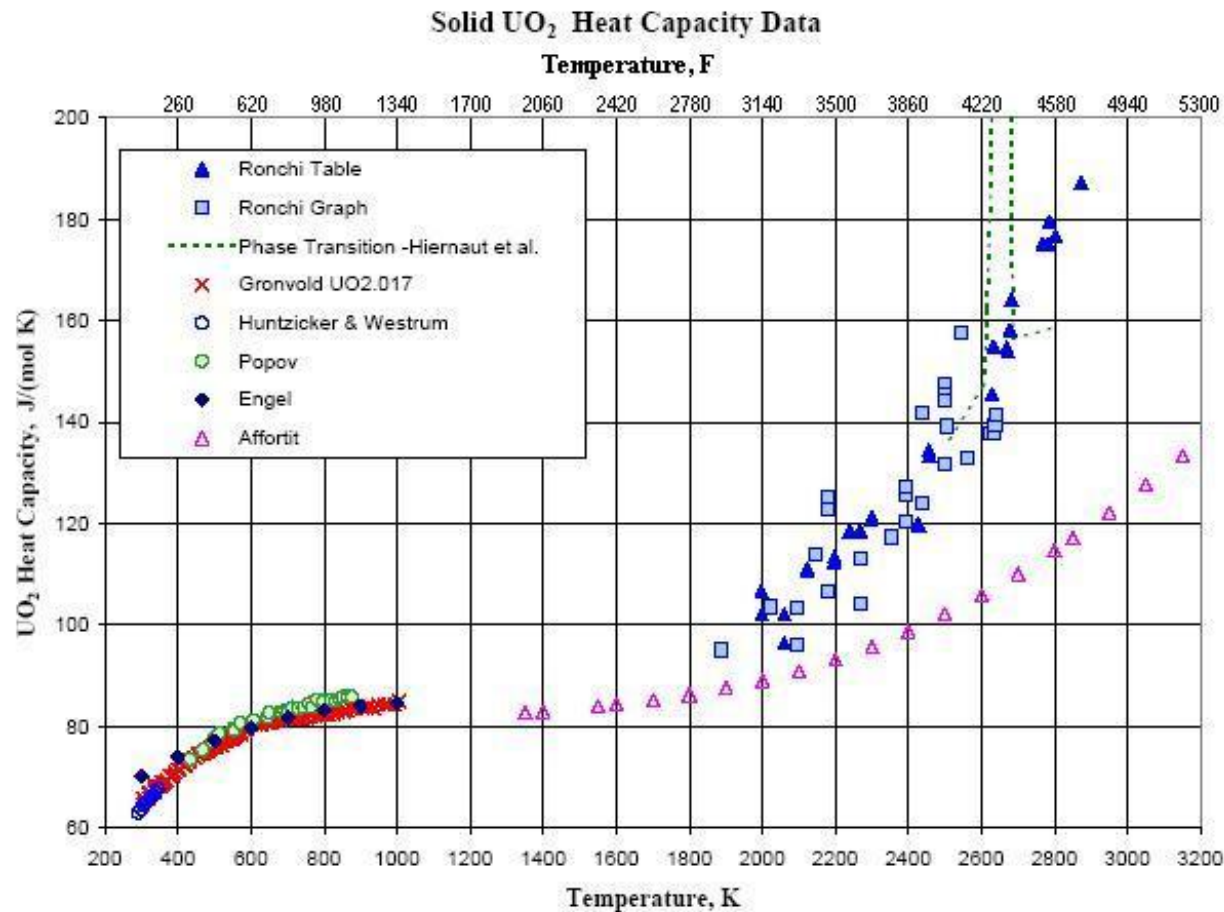


Taken from L.S. Tong & J. Weisman, "Thermal Analysis of Pressurized Water Reactors", p. 60



Taken from "CE Fuel Evaluation Model Topical Report", CENPD-139, July 1974, p.5-21

Data for UO_2 Specific Heat



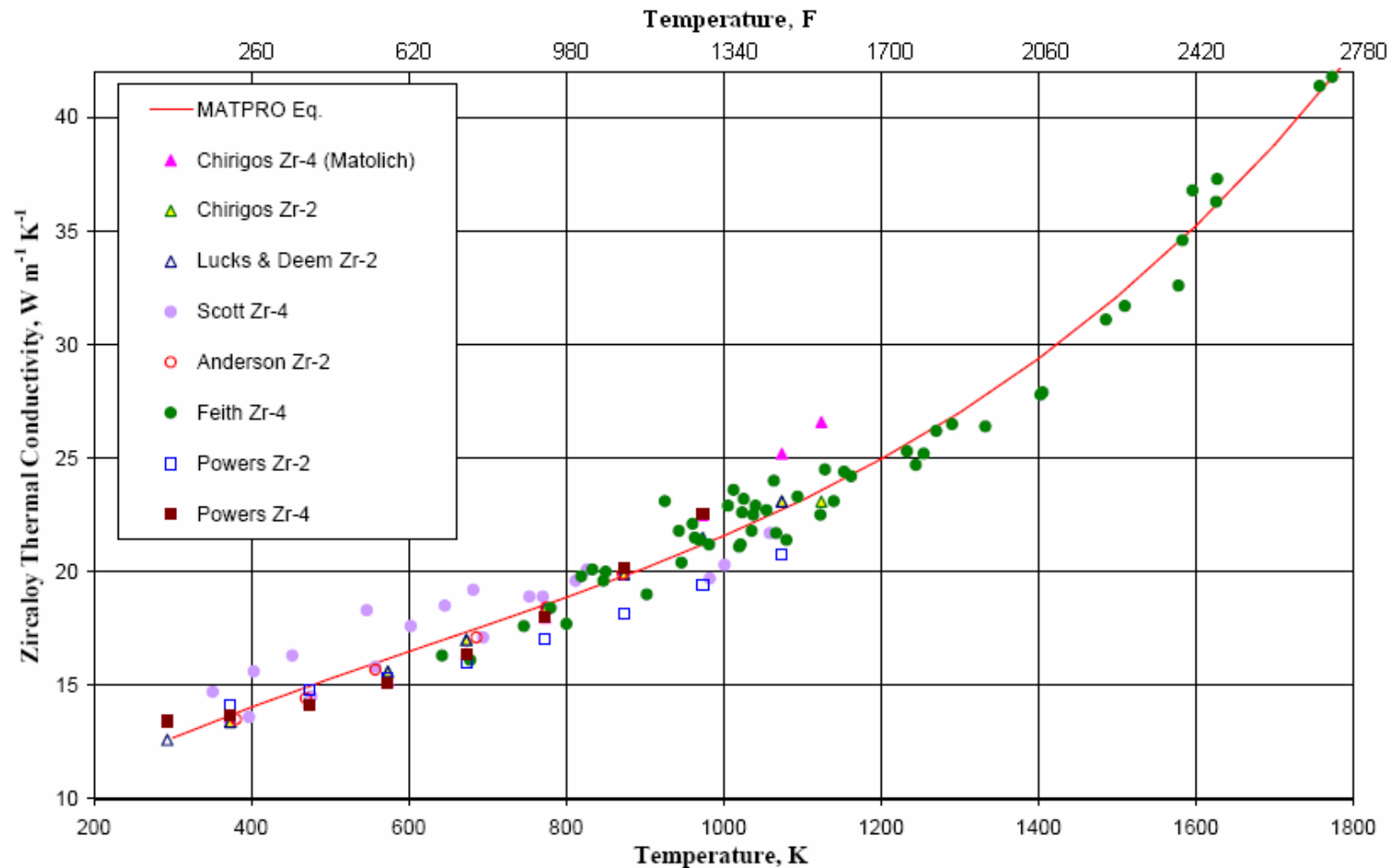
for $298.15 \text{ K} \leq T \leq 3120 \text{ K}$

$$C_p(T) = + 52.1743 + 87.951\tau - 84.2411\tau^2 + 31.542\tau^3 - 2.6334\tau^4 - 0.71391\tau^{-2}$$

where $\tau = T/1000$,

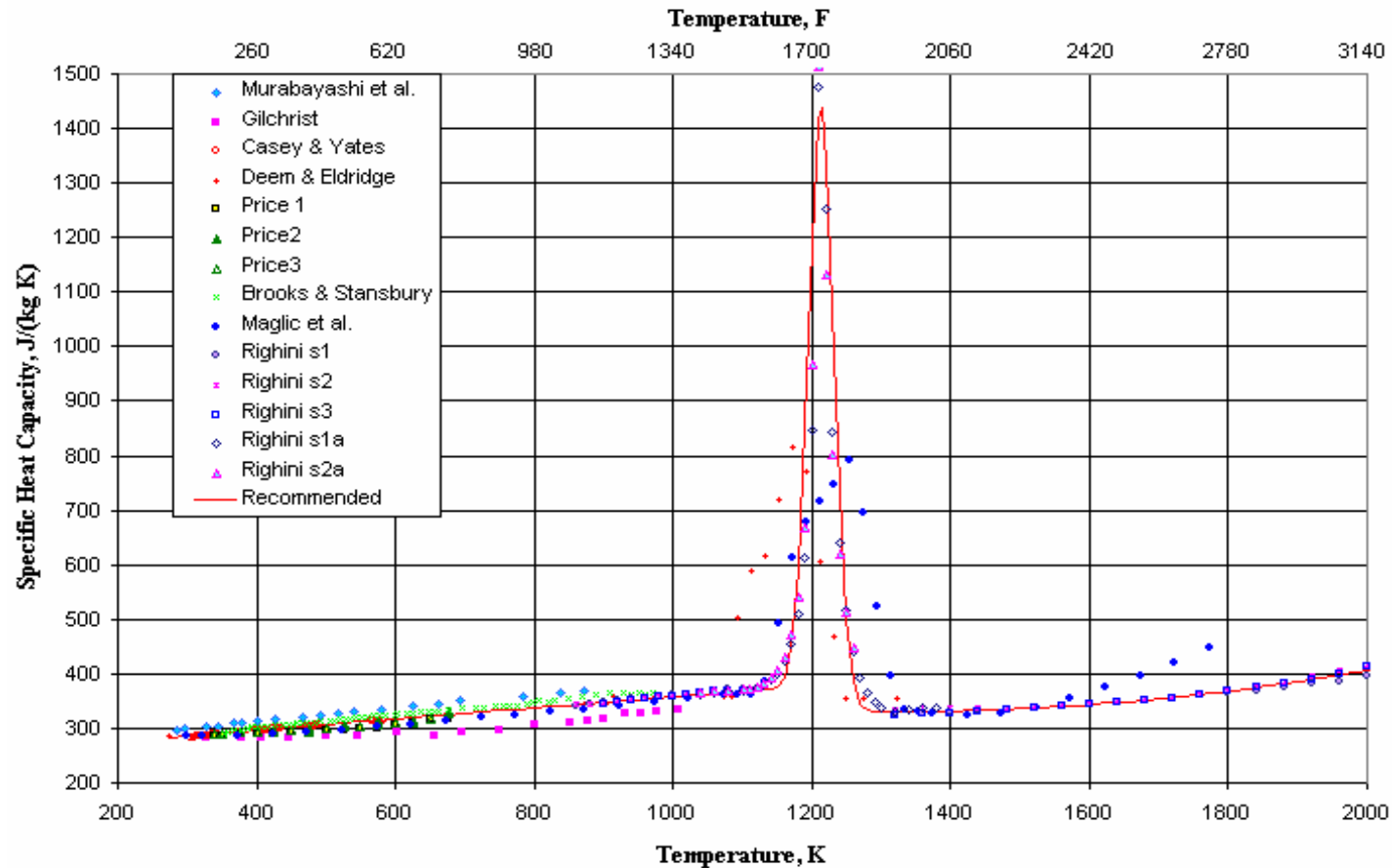
T is the temperature in K,
and the heat capacity, C_p , is in $\text{J} \cdot \text{mol}^{-1} \cdot \text{K}^{-1}$.

Data for Zircaloy Thermal Conductivity



- $k_{Zr}(T)$ data in *Watts/m.°K* vs. $T^{\circ}K$ in fitted as:
- $k_{Zr}(T) = 12.767 - 5.4348 \times 10^{-4}T + 8.9818 \times 10^{-6}T^2$

Data for Zircaloy Specific Heat



- Zircaloy-2 undergoes phase change $>1200^{\circ}\text{K}$
- $C_p(T) = 255.66 + 0.1024T$ for: $T < 1100^{\circ}\text{K}$
- $C_p(T) = 1058.4 \exp[(T-1213.8)^2/719.61]$ $1100^{\circ}\text{K} < T < 1214^{\circ}\text{K}$
- $C_p(T) = 597.1 - 0.4088T + 1.565 \times 10^{-4}T^2$ $1214^{\circ}\text{K} < T < 2000^{\circ}\text{K}$ ²⁷

Steady State Core Heat Transfer

Assumptions:

- Steady state: $dT_f/dt = 0$, $dT_c/dt = 0$
- Power derived from fission is uniformly deposited within fuel pin (no flux depression, no power within clad)
- Fuel pin radius is: R_o in cm.
- Clad outer radius is: R_c in cm.
- Gap is of negligible dimension
- Assume volumetric power density: P in units of Watts/cm³
- Linear power density is: $q = \pi R_o^2 P$ in units of Watts/cm
- Alternately: $P = q / \pi R_o^2$
- Thermal conductivity values: k , in units of Watts/cm.°K
- Conductance values: h , in units of Watts/cm².°K

Heat Transfer Assuming Constant k -values

- Assume averaged heat transfer coefficients – heat conduction equation in fuel pellet becomes:

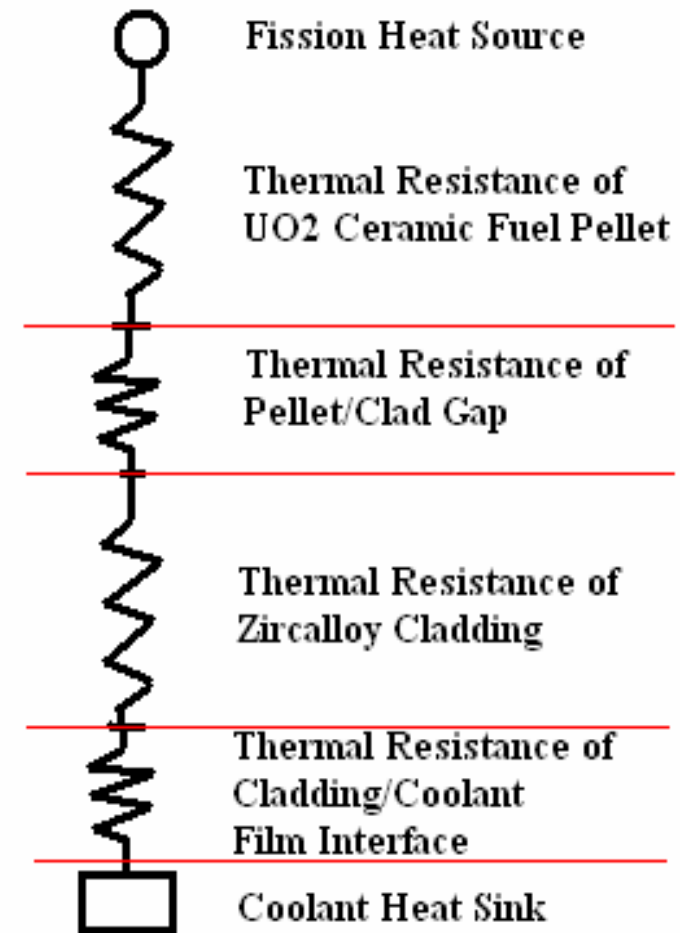
$$\frac{1}{r} \frac{d}{dr} \left(r \frac{dT_f}{dr} \right) + \frac{P}{k_f} = 0$$

$$\text{with: } \left. \frac{dT_f}{dr} \right|_{r=0} = 0;$$

$$T_f(r) \Big|_{r=R_o} = T_f(R_o)$$

- A series of temperature drops are calculated proportional to “ q ”
- Temperature drop across gap is:

$$T_f(R_o) - T_c(R_o) = q / 2\pi R_o h_{gap}$$



Thermal Resistances

Using thermal resistance analogy:

$$\Delta T = q \cdot \text{Resistance}$$

Resistance across gap:

$$1/2\pi R_o h_{gap}$$

Resistance across cladding:

$$\ln(R_c/R_o)/4\pi k_c$$

Resistance across cladding/coolant film:

$$1/2\pi R_c h_{film}$$

*Heat Transfer across fuel pellet
assuming constant k-values*

$$\frac{1}{r} \frac{d}{dr} \left(r \frac{dT_f}{dr} \right) + \frac{P}{k_f} = 0$$

$$\int \frac{d}{dr} \left(r \frac{dT_f}{dr} \right) dr = - \int \frac{rP}{k_f} dr + A$$

$$r \frac{dT_f}{dr} = - \frac{P}{2k_f} r^2 + A$$

$$\frac{dT_f}{dr} = - \frac{P}{2k_f} r + \frac{A}{r}$$

$$\text{Applying : } \left. \frac{dT_f}{dr} \right|_{r=0} = 0$$

$$\text{yields : } A = 0$$

Heat Transfer across fuel pellet assuming constant k-values

$$\int \frac{dT_f}{dr} dr = - \int \frac{P}{2k_f} r dr$$

$$T_f(r) = -\frac{P}{4k_f} r^2 + B$$

$$\text{If : } T_f(r) \Big|_{r=R_o} = T_f(R_o)$$

$$T_f(R_o) = -\frac{P}{4k_f} R_o^2 + B$$

$$\text{Thus : } B = T_f(R_o) + \frac{P}{4k_f} R_o^2$$

$$\text{Substituting : } P = \frac{q}{\pi R_o^2}$$

$$T_f(r) = T_f(R_o) + \frac{P}{4k_f} (R_o^2 - r^2) = T_f(R_o) + \frac{q}{4\pi k_f} \left(1 - \frac{r^2}{R_o^2}\right)$$

Heat Transfer across fuel pellet assuming constant k-values

- Substituting in temperature drop across gap using thermal resistance model yields:

$$T_f(R_o) - T_c(R_o) = q \times \text{Resistance}$$

$$\text{Resistance} = \frac{1}{2\pi R_o h_{gap}}$$

$$T_f(R_o) - T_c(R_o) = \frac{q}{2\pi R_o h_{gap}}$$

$$T_f(r) = T_c(R_o) + \frac{q}{2\pi} \left(\frac{\left(1 - \frac{r^2}{R_o^2}\right)}{2k_f} + \frac{1}{R_o h_{gap}} \right)$$

Heat Transfer across cladding assuming constant k-values

- Additional heat transfer equation used for clad
- *No internal heat* assumed

$$\frac{1}{r} \frac{d}{dr} \left(r \frac{dT_c}{dr} \right) = 0$$

$$\text{with : } T_c(r) \Big|_{r=R_o} = T_c(R_o)$$

$$\text{and : } T_c(r) \Big|_{r=R_c} = T_c(R_c)$$

$$\int \frac{d}{dr} \left(r \frac{dT_c}{dr} \right) dr = C$$

$$\int \frac{dT_c}{dr} dr = \int \frac{C}{r} dr + D$$

$$T_c(r) = C \ln(r) + D$$

$$T_c(R_o) = C \ln(R_o) + D$$

$$T_c(R_c) = C \ln(R_c) + D$$

Heat Transfer across cladding assuming constant k-values

- To solve for coefficients substitute thermal resistance across cladding:

$$T_c(R_o) - T_c(R_c) = q \times \text{Resistance}$$

$$\text{Resistance} = \ln \left[\frac{R_c}{R_o} \right] / 4\pi k_c$$

$$T_c(R_o) - T_c(R_c) = C \ln \left[\frac{R_c}{R_o} \right]$$

$$C = \ln \left[\frac{R_c}{R_o} \right] / 4\pi k_c$$

$$T_c(R_o) = T_c(R_c) + \frac{q R_o^2}{4\pi k_c} \ln \left[\frac{R_c}{R_o} \right]$$

Heat Transfer Assuming Constant k -values

- Substituting in yields following for $T_f(r)$:

$$T_f(r) = T_c(R_c) + \frac{q}{2\pi} \left(\frac{\left(1 - \frac{r^2}{R_o^2}\right)}{2k_f} + \frac{1}{R_o h_{gap}} + \frac{\ln\left(\frac{R_c}{R_o}\right)}{2k_c} \right)$$

- A final substitution (similar to expression for gap) is temperature drop across film layer from clad to coolant:

$$T_f(R_c) - T_{coolant}(R_c) = q/2\pi R_c h_{film}$$

- Film conductance depends on coolant flow, geometry, temperature, pressure, voids, etc. (*more to come later!*)

Heat Transfer Assuming Constant k -values

- Overall solution for fuel pellet temperature becomes:

$$T_f(r) = T_{coolant}(R_c) + \frac{q}{2\pi} \left(\frac{\left(1 - \frac{r^2}{R_o^2}\right)}{2k_f} + \frac{1}{R_o h_{gap}} + \frac{\ln\left(\frac{R_c}{R_o}\right)}{2k_c} + \frac{1}{R_c h_{film}} \right)$$

- Overall solution for clad temperature becomes:

$$T_c(r) = T_{coolant}(R_c) + \frac{q}{2\pi} \left(\frac{\ln\left(\frac{R_c}{r}\right)}{2k_c} + \frac{1}{R_c h_{film}} \right)$$

Example Applications

- Westinghouse 17 x 17 fuel bundle design at 14 kW/ft
- GE 8 x 8 fuel bundle design at 13.4 kW/ft
- Effective (constant) thermal conductivity assumed
- All thermal energy assumed to originate in fuel pin
- No thermal neutron flux depression considered
- Constant $h_{film} = 4.5 \text{ Watts/cm}^{\circ}\text{K}$ in both cases
- Objective is to calculate:
- Peak centerline temperature (compare to melting point)
- Fuel temperature distribution
- Clad temperature distribution

Example: W 17x17 Fuel at 14kW/ft.

Fuel Pin Temperature Distribution Assuming Constant: q, k_{fuel}, k_c

Fuel pin radius in cm:	$R_o := \frac{0.819}{2}$	Source: Westinghouse Duderstadt/Hamilton p.634
Clad radius in cm.:	$R_c := \frac{0.94}{2} \quad R_c = 0.47$	Source: Westinghouse Duderstadt/Hamilton p.634
Maximum linear Power Density in Watts/cm.:	$q := 459$	Source: Westinghouse WCAP-8720, p.A-17 14kW/ft = 459.32 Watts/cm
Effective fuel thermal conductivity in Watts/cm.K:	$k_{fuel} := 0.03$	Source: MATPRO
Effective clad thermal conductivity in Watts/cm.K:	$k_c := 0.17$	Source: MATPRO
Gap conductance in Watts/cm.K:	$h_{gap} := 1.7$	Source: Westinghouse WCAP-8720, p.2-7 notes: 3000 BTU / hr.ft2 degF
Film conductance in Watts/cm.K	$h_{film} := 4.5$	Source: Westinghouse Duderstadt/Hamilton p.481 notes: 4.5 Watts/cm K
Local Coolant temperature in K:	$T_{cool} := 588.15$	Source: Westinghouse Duderstadt/Hamilton p.635 notes: 300-332 C range. 315 C is assumed.

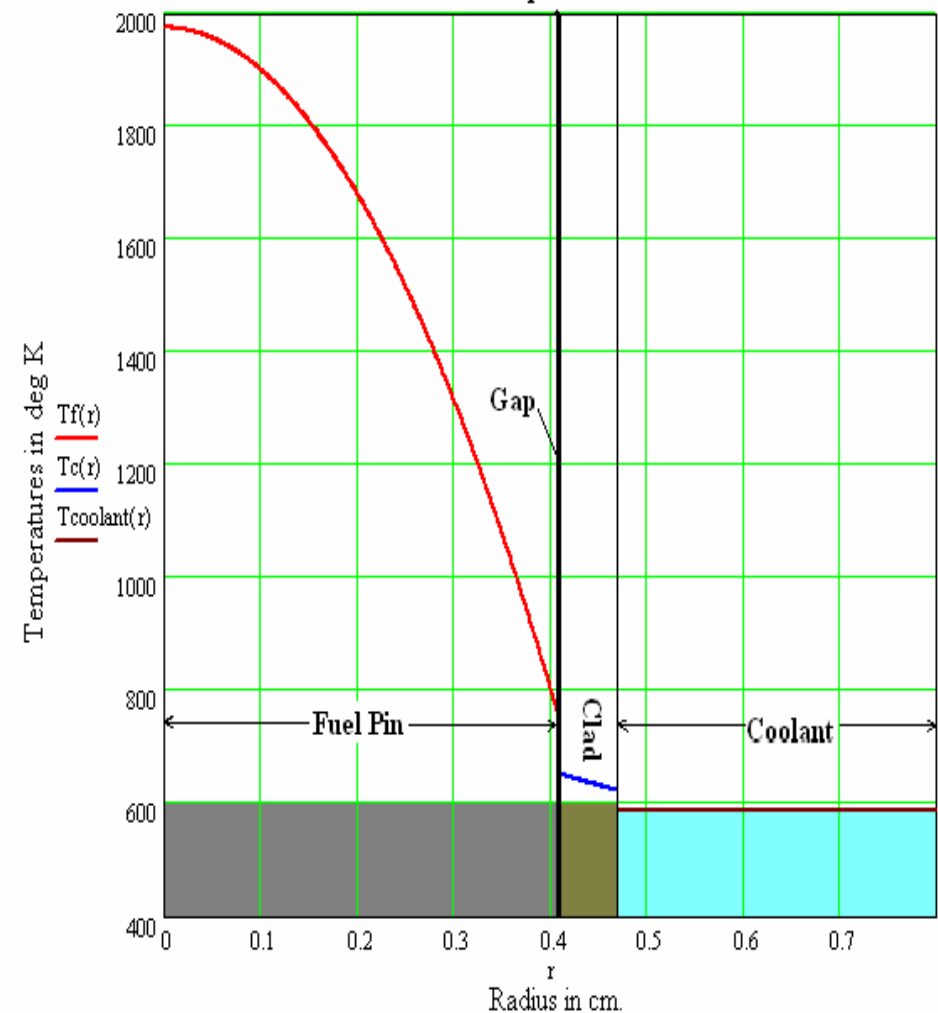
Predicted steady state fuel pin and clad temperature distributions:

$$T_f(r) := \begin{cases} T_{cool} + \frac{q}{2\pi} \left(\frac{1 - \frac{r^2}{R_o^2}}{2 \cdot k_{fuel}} + \frac{1}{R_o \cdot h_{gap}} + \frac{\ln\left(\frac{R_c}{R_o}\right)}{2 \cdot k_c} + \frac{1}{R_c \cdot h_{film}} \right) & \text{if } r \leq R_o \\ \text{break} & \text{otherwise} \end{cases}$$

$$T_c(r) := \begin{cases} T_{cool} + \frac{q}{2\pi} \left(\frac{\ln\left(\frac{R_c}{r}\right)}{2 \cdot k_c} + \frac{1}{R_c \cdot h_{film}} \right) & \text{if } R_o < r \leq R_c \\ \text{break} & \text{otherwise} \end{cases}$$

$$T_{coolant}(r) := \begin{cases} T_{cool} & \text{if } R_c < r \\ \text{break} & \text{otherwise} \end{cases}$$

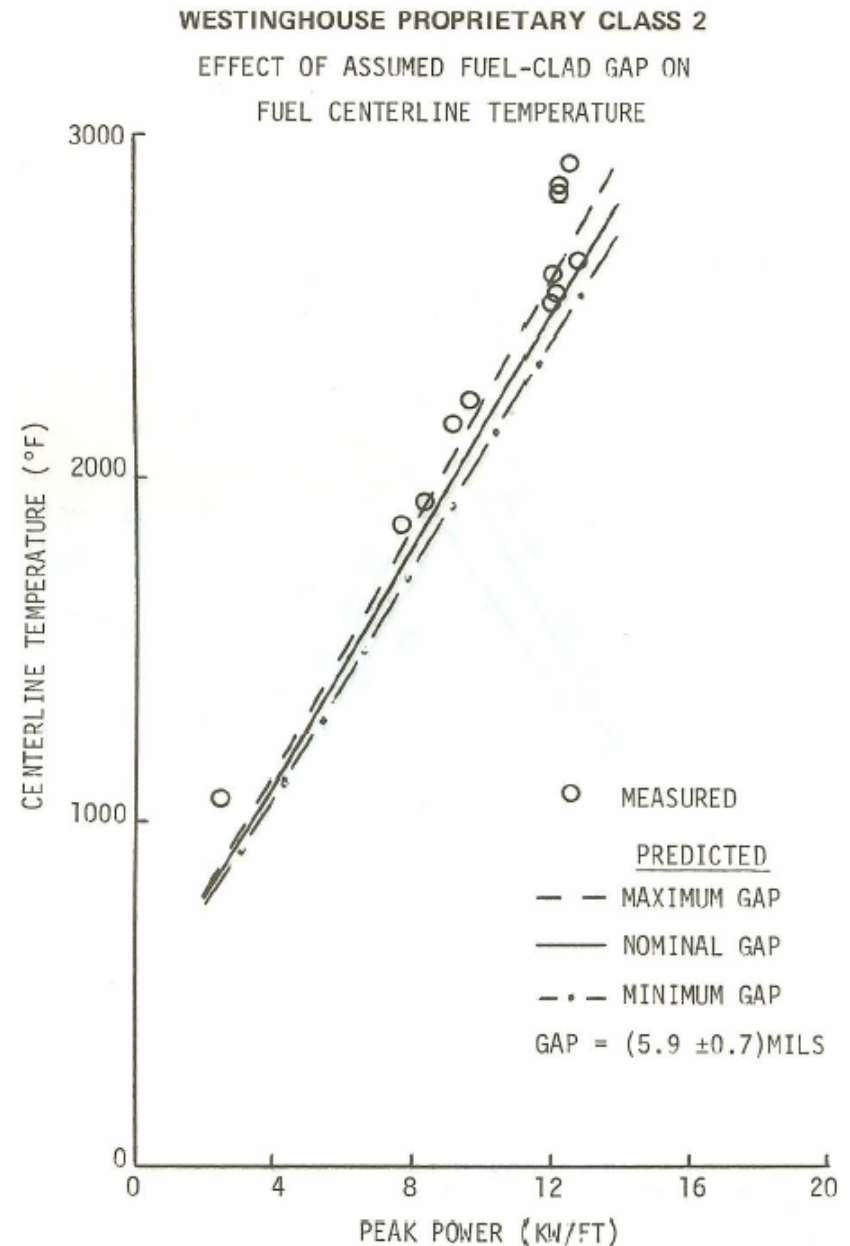
W 17x17 Fuel and Clad Temperature Distributions vs.radius



Peak Centerline Temperature in deg K: $T_f(0) = 1.975 \times 10^3$

Example: W 17x17 fuel at 14kW/ft.

- Predicted Peak Centerline Temperature $\sim 1975^{\circ}\text{K}$
- This would be $\sim 3095^{\circ}\text{F}$
- This is comparable to data described in WCAP-8720
- Actual reactor vendor fuel temperature evaluation models consider: burnup and temperature effects on density, heat transfer processes



Example: GE BWR-6 8x8 Fuel at 13.4kW/ft

Fuel Pin Temperature Distribution Assuming Constant: q, k_{fuel}, k_c

Fuel pin radius in cm:	$R_o := \frac{1.056}{2}$	$R_o = 0.528$	Source: GE BWR-6 Duderstadt/Hamilton p.634
Clad radius in cm.:	$R_c := \frac{1.25}{2}$	$R_c = 0.625$	Source: GE BWR-6 Duderstadt/Hamilton p.634
Maximum linear Power Density in Watts/cm.:	$q := 440$		Source: GE BWR-6 Duderstadt/Hamilton p.634
Effective fuel thermal conductivity in Watts/cm.K:	$k_{fuel} := 0.025$		Source: MATPRO
Effective clad thermal conductivity in Watts/cm.K:	$k_c := 0.17$		Source: MATPRO
Gap conductance in Watts/cm.K:	$h_{gap} := 0.6$		Source: Duderstadt/Hamilton p.480 notes: 0.5 - 1.1 Watts/cm ² K
Film conductance in Watts/cm.K	$h_{film} := 4.5$		Source: Duderstadt/Hamilton p.481 notes: 4.5 Watts/cm ² K
Local Coolant temperature in K:	$T_{cool} := 550.65$		Source: BWR-6 Duderstadt/Hamilton p.635 notes: 269-286 C range. 277.5 C is assumed.

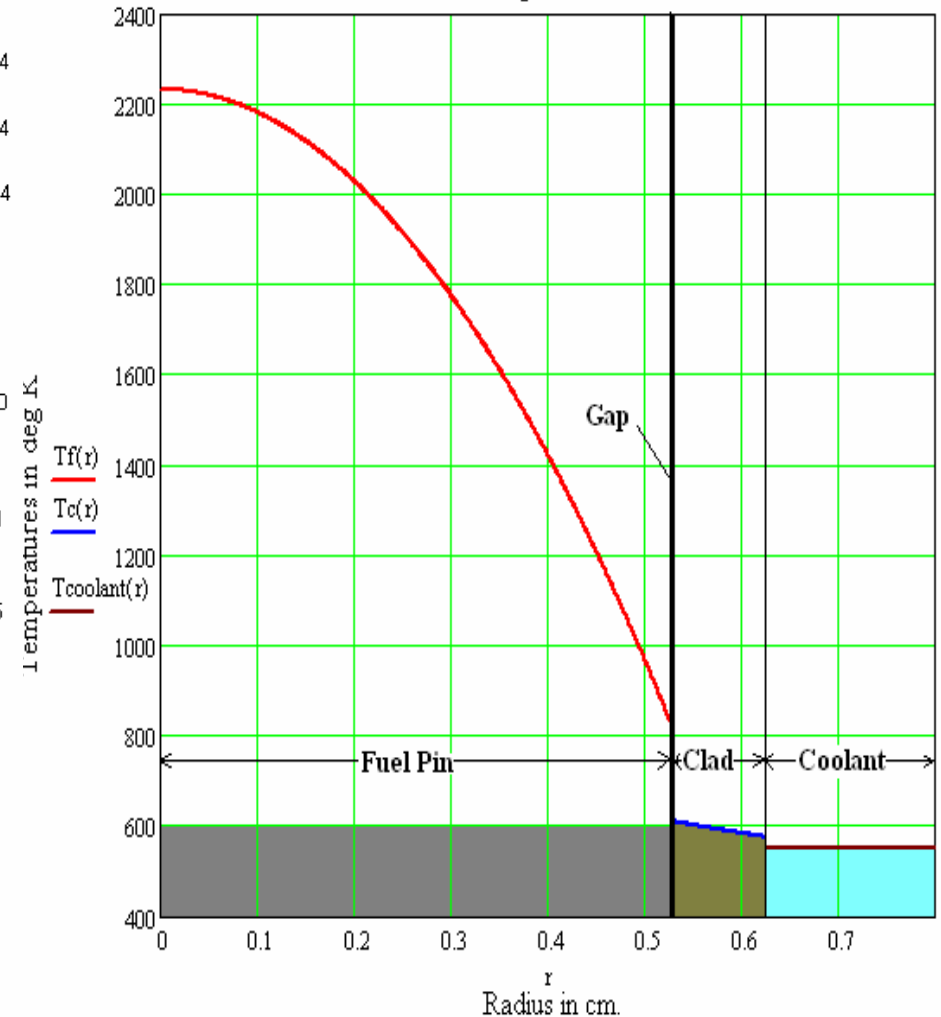
Predicted steady state fuel pin and clad temperature distributions:

$$T_f(r) := \begin{cases} T_{cool} + \frac{q}{2\pi} \left(\frac{1 - \frac{r^2}{R_o^2}}{2 \cdot k_{fuel}} + \frac{1}{R_o \cdot h_{gap}} + \frac{\ln\left(\frac{R_c}{R_o}\right)}{2 \cdot k_c} + \frac{1}{R_c \cdot h_{film}} \right) & \text{if } r \leq R_o \\ \text{break otherwise} \end{cases}$$

$$T_c(r) := \begin{cases} T_{cool} + \frac{q}{2\pi} \left(\frac{\ln\left(\frac{R_c}{r}\right)}{2 \cdot k_c} + \frac{1}{R_c \cdot h_{film}} \right) & \text{if } R_o < r \leq R_c \\ \text{break otherwise} \end{cases}$$

$$T_{coolant}(r) := \begin{cases} T_{cool} & \text{if } R_c < r \\ \text{break otherwise} \end{cases}$$

BWR-6 Fuel and Clad Temperature Distributions vs. radius



Peak Centerline Temperature in deg. K: $T_f(0) = 2.232 \times 10^3$

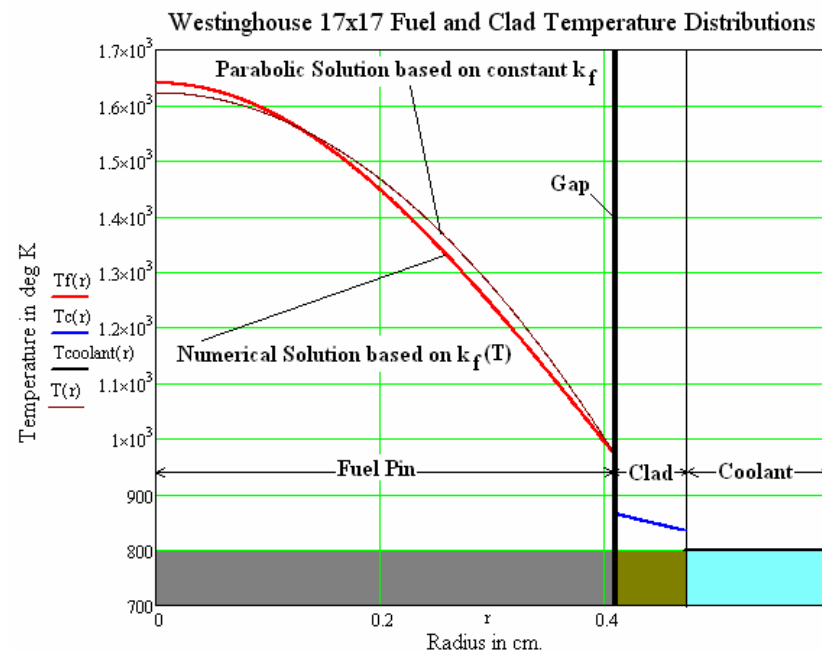
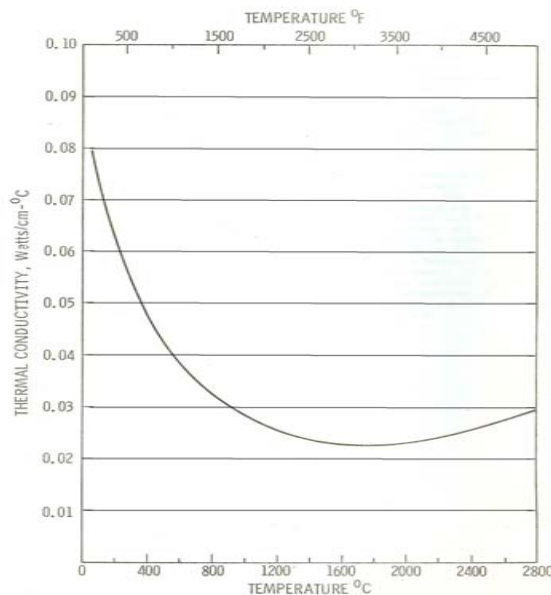
How Significant are Nonlinearities in $k_f(T)$?

- Recall that: $k_f(T)$ temperature is parabolic in range of interest
- Previous calculations used constant $k_f \sim 0.025 \text{ Watts / cm } ^\circ\text{K}$
- Actual equation is:

$$q + \nabla(k_f(T_f(r))\nabla T_f(r)) = 0$$

$$q + k_f(T_f(r))\frac{d^2T_f(r)}{dr^2} + \frac{dk_f(T_f(r))}{dT_f(r)}\left(\frac{dT_f(r)}{dr}\right)^2 + \frac{k_f(T_f(r))}{r}\frac{dT_f(r)}{dr} = 0$$

- Fitting k_f to “detailed calculation” yields: $k_f \sim 0.056 \text{ Watts / cm } ^\circ\text{K}$

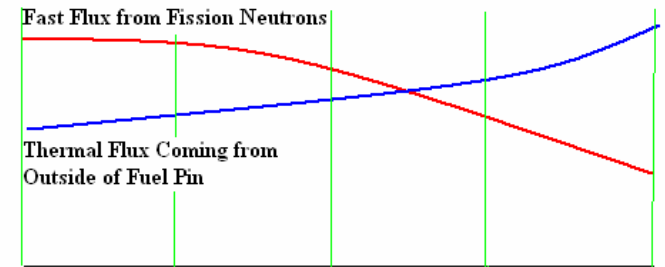


Other Nonlinearities:

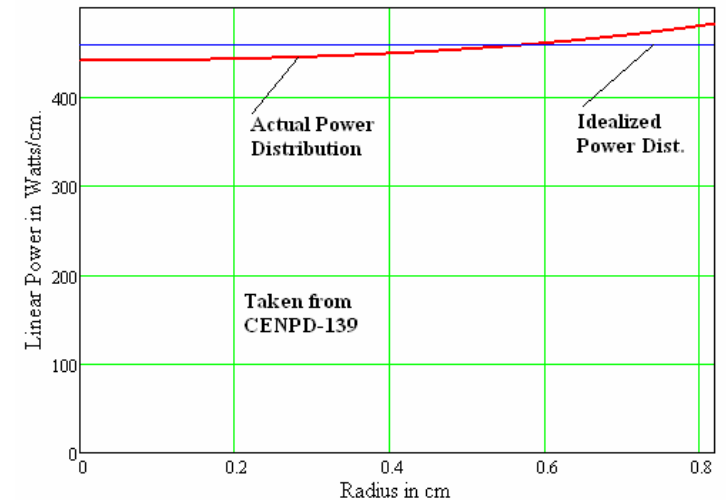
- Thermal/fast neutrons *distribute* differently within fuel pin
- Fast neutrons peak in center
- Thermal neutrons peak in water
- Actual power distribution is *depressed* in center of pin
- Steady state diffusion equation reflecting flux depression:

$$P(r) + \nabla(k_f(T_f(r))\nabla T_f(r)) = 0$$

$$P(r) + k_f(T_f(r))\frac{d^2T_f(r)}{dr^2} + \frac{dk_f(T_f(r))}{dT_f(r)}\left(\frac{dT_f(r)}{dr}\right)^2 + \frac{k_f(T_f(r))}{r}\frac{dT_f(r)}{dr} = 0$$



Radial Power Distribution Considering Flux Depression



Transient Core Heat Transfer

Transient Heat Transfer via Lumped Parameter Method

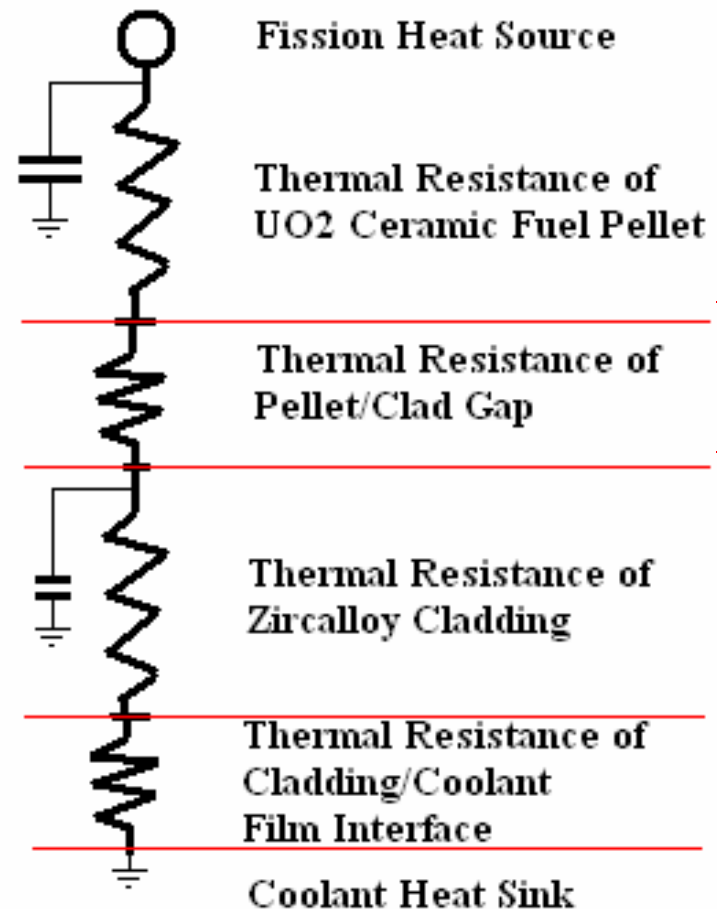
- Heat transfer equation for fuel previously noted as:

$$\rho(T_f(r,t))c_p(T_f(r,t))\frac{\partial T_f(r,t)}{\partial t} = P(r,t) + \nabla(k_f(T_f(r,t))\nabla T_f(r,t))$$

- Licensing calculations require considerable detail
- Majority of dynamic core heat transfer can be understood based on *Lumped Parameter Heat Transfer Model*
- Lumped Parameter approach recognizes that fuel and clad behave differently due to different “time constants”
- Method involves computing “effective” temperatures and rates of heat transfer between regions.
- Lumped Parameter approach can be directly used with fuel temperature (Doppler) reactivity feedback simulations

Transient Heat Transfer via Lumped Parameter Method

- Steady state heat transfer network modeled as resistance network
- Transient heat transfer network modeled as having “heat storage”
- During steady state heat stored in fuel, clad
- During transients temperature response is delayed via thermal capacitance



Transient Heat Transfer via Lumped Parameter Method

- Using RC electrical circuit analogy:
- Temperature in fuel pellet is like *voltage source*
- Heat flux is like *electrical current*
- Temperature drops are equivalent to *voltage drops*
- There is *thermal resistance* in fuel, gap, clad, film
- Thermal resistance of fuel pellet is: $R_f = 1/(4\pi k_f)$
- Thermal resistance of gap is: $R_{gap} = 1/(2\pi R_o h_{gap})$
- Thermal capacitance in fuel is: $C_f = \pi R_o^2 C_{pf} \rho_f$
- C_{pf} is Heat Capacity in Watt-sec/gram°K
($C_{pf} = 0.377 \text{ Watt-sec/gram}^\circ\text{K}$)
- ρ_f is fuel pellet density in gram/cm³ ($\rho_f = 10.97 \text{ gram/cm}^3$)

Lumped Parameter Fuel Heat Transfer

- Dynamics of (spatially averaged) Fuel temperature:

$$C_f \frac{d\langle T_f \rangle}{dt} = \pi R_o \langle P(t) \rangle - \frac{\langle T_f \rangle - \langle T_c \rangle}{R_{fc}}$$

$$C_f = \pi R_o^2 C_{pf} \rho_f$$

- R_{fc} is effective thermal resistance between $\langle T_f \rangle$, $\langle T_c \rangle$
- $\langle T_f \rangle$, $\langle T_c \rangle$ are respectively spatial averaged fuel and clad temperatures in °K
- $\langle P(t) \rangle$ is volumetric heat source, $\pi R_o \langle P(t) \rangle = \langle q(t) \rangle$ is linear power density in Watts/cm.

Lumped Parameter Clad Heat Transfer

- Dynamics of effective (spatially averaged) clad temperature are similarly given by:

$$C_c \frac{d\langle T_c \rangle}{dt} = \frac{\langle T_f \rangle - \langle T_c \rangle}{R_{fc}} - \frac{\langle T_c \rangle - \langle T_{AVG} \rangle}{R_{cc}}$$

$$C_c = \pi(R_c^2 - R_o^2)C_{pc}\rho_c$$

- R_{cc} is effective thermal resistance between: $\langle T_c(t) \rangle, \langle T_{AVG}(t) \rangle$
- Where: $\langle T_{AVG}(t) \rangle$ is average coolant temperature in °K
- C_{pc} is Heat Capacity in Watt-sec/gram°K
($C_{pc} = 0.331$ Watt-sec/gram°K)
- ρ_c is clad density in gram/cm³ ($\rho_c = 6.57$ gram/cm³)

Lumped Parameter Heat Transfer Model

- Effective fuel and clad temperatures are related to previously calculated temperatures via following averages:

$$\langle T_f(t) \rangle = \frac{1}{\pi R_o^2} \int_0^{R_o} T_f(r, t) 2\pi r dr$$

$$\langle T_c(t) \rangle = \frac{1}{\pi(R_c^2 - R_o^2)} \int_{R_o}^{R_c} T_c(r, t) 2\pi r dr$$

- Heat flux from clad to coolant, which lags power, can be expressed:

$$q_{c-c}(t) = [\langle T_c(t) \rangle - T_{avg}(t)] / R_{cc}$$

Effective Temperatures

Computation of Effective Fuel Temperature $\langle T_f \rangle$:

$$\langle T_f \rangle = \frac{1}{\pi \cdot R_o^2} \int_0^{R_o} \left[T_{cool} + \frac{q}{2 \cdot \pi} \left(\frac{1 - \frac{r^2}{R_o^2}}{2 \cdot k_{fuel}} + \frac{1}{R_o \cdot h_{gap}} + \frac{\ln\left(\frac{R_c}{R_o}\right)}{2 \cdot k_c} + \frac{1}{R_c \cdot h_{film}} \right) \right] \cdot 2 \cdot \pi \cdot r \, dr$$

$$\langle T_f \rangle = T_{cool} + \frac{q}{2 \cdot \pi} \left(\frac{1}{4 \cdot k_{fuel}} + \frac{1}{R_o \cdot h_{gap}} + \frac{\ln\left(\frac{R_c}{R_o}\right)}{2 \cdot k_c} + \frac{1}{R_c \cdot h_{film}} \right) = 1.366 \times 10^3 \text{ } ^\circ\text{K}$$

Computation of Effective Clad Temperature $\langle T_c \rangle$:

$$\langle T_c \rangle = \frac{1}{\pi \cdot (R_c^2 - R_o^2)} \int_{R_o}^{R_c} \left[T_{cool} + \frac{q}{2 \cdot \pi} \left(\frac{\ln\left(\frac{R_c}{r}\right)}{2 \cdot k_c} + \frac{1}{R_c \cdot h_{film}} \right) \right] \cdot 2 \cdot \pi \cdot r \, dr$$

$$\langle T_c \rangle = T_{cool} + \frac{q}{2 \cdot \pi} \left[\frac{1}{R_c \cdot h_{film}} + \frac{1}{4 \cdot k_c} + \frac{R_o^2 \cdot \ln\left(\frac{R_o}{R_c}\right)}{2 \cdot (R_c^2 - R_o^2) \cdot k_c} \right] = 636.814 \text{ } ^\circ\text{K}$$

Lumped Parameter Time Constants

Time constant for fuel:

Fuel Heat Capacity in Watt-sec/gram K:

$$C_{pf} := 0.377$$

Based upon 0.09 BTU/lb.F
Tong & Weissman, p.64

Fuel Density in gr/cm³:

$$\rho_f := 10.97$$

Based upon
Tong & Weissman, p.64

$$C_f := \pi \cdot R_o^2 \cdot C_{pf} \cdot \rho_f$$

$$R_{fc} := \frac{1}{2 \cdot \pi} \left[\frac{1}{4 \cdot k_{fuel}} + \frac{1}{R_o \cdot h_{gap}} - \frac{1}{4 \cdot k_c} + \frac{\ln\left(\frac{R_c}{R_o}\right)}{2 \cdot k_c} - \frac{R_o^2 \cdot \ln\left(\frac{R_o}{R_c}\right)}{2(R_c^2 - R_o^2) \cdot k_c} \right]$$

$\tau_f := C_f \cdot R_{fc}$ Time constant in seconds:

$$\tau_f = 3.461$$

Time constant for clad:

Zircaloy Heat Capacity in Watt-sec/gram K:

$$C_{pc} := 0.3307$$

Based upon 0.0789 BTU/lb.F
Tong & Weissman, p.67

Zircaloy Density in gr/cm³:

$$\rho_c := 6.57$$

Based upon
Tong & Weissman, p.67

$$R_{cc} := \frac{1}{2 \cdot \pi} \left(\frac{1}{4 \cdot k_{fuel}} + \frac{1}{R_o \cdot h_{gap}} + \frac{\ln\left(\frac{R_c}{R_o}\right)}{2 \cdot k_c} + \frac{1}{R_c \cdot h_{film}} \right)$$

$$C_c := \pi \cdot (R_c^2 - R_o^2) \cdot C_{pc} \cdot \rho_c$$

$\tau_c := C_c \cdot R_{cc}$ Time constant in seconds:

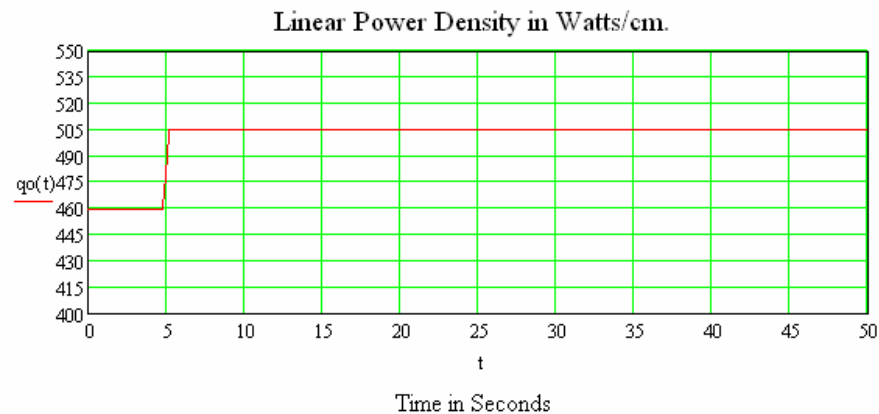
$$\tau_c = 0.615$$

$$R_{eq} := \frac{R_{fc} \cdot R_{cc}}{R_{fc} + R_{cc}}$$

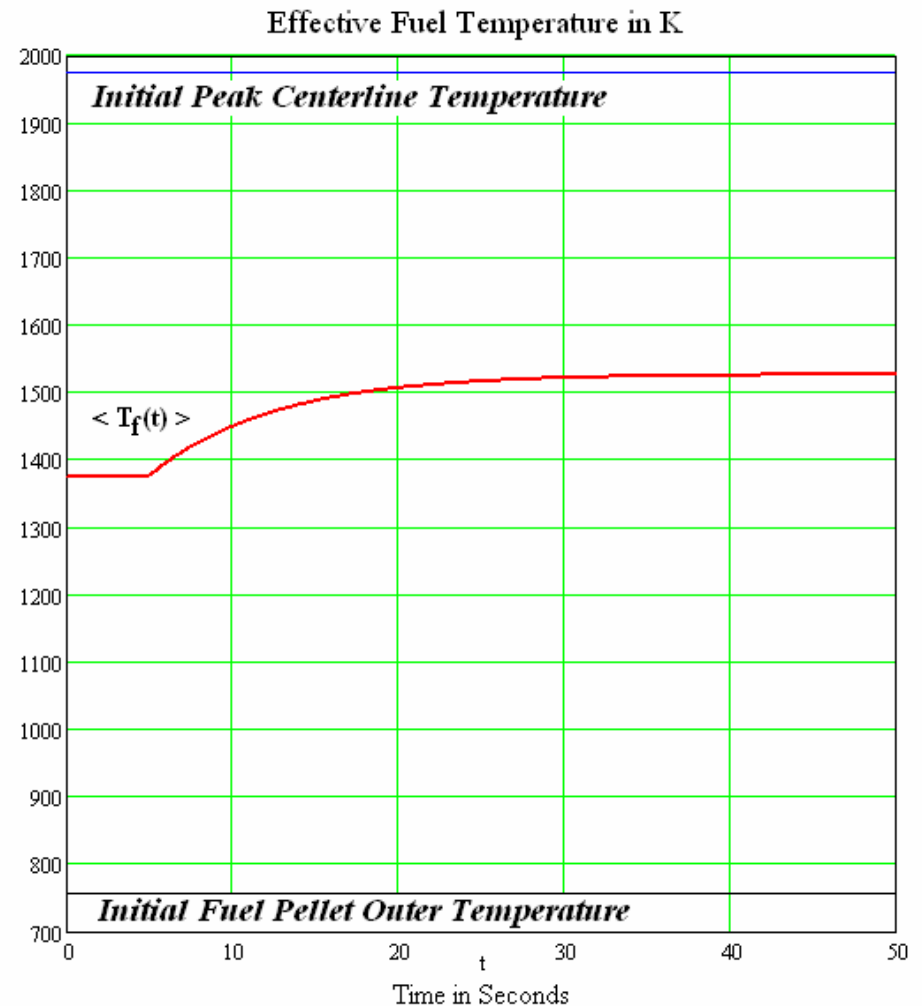
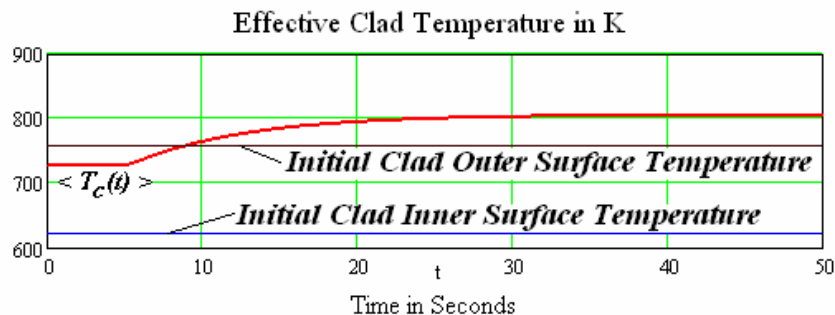
$$\tau_{eq} := C_c \cdot R_{eq}$$

$$\tau_{eq} = 0.298$$

MATHCAD Simulation of +10% Rise in q



Linear Power Density from 459 Watts/cm. by +10% at $t = 5$ seconds
Coolant Temperature Assumed to Remain constant

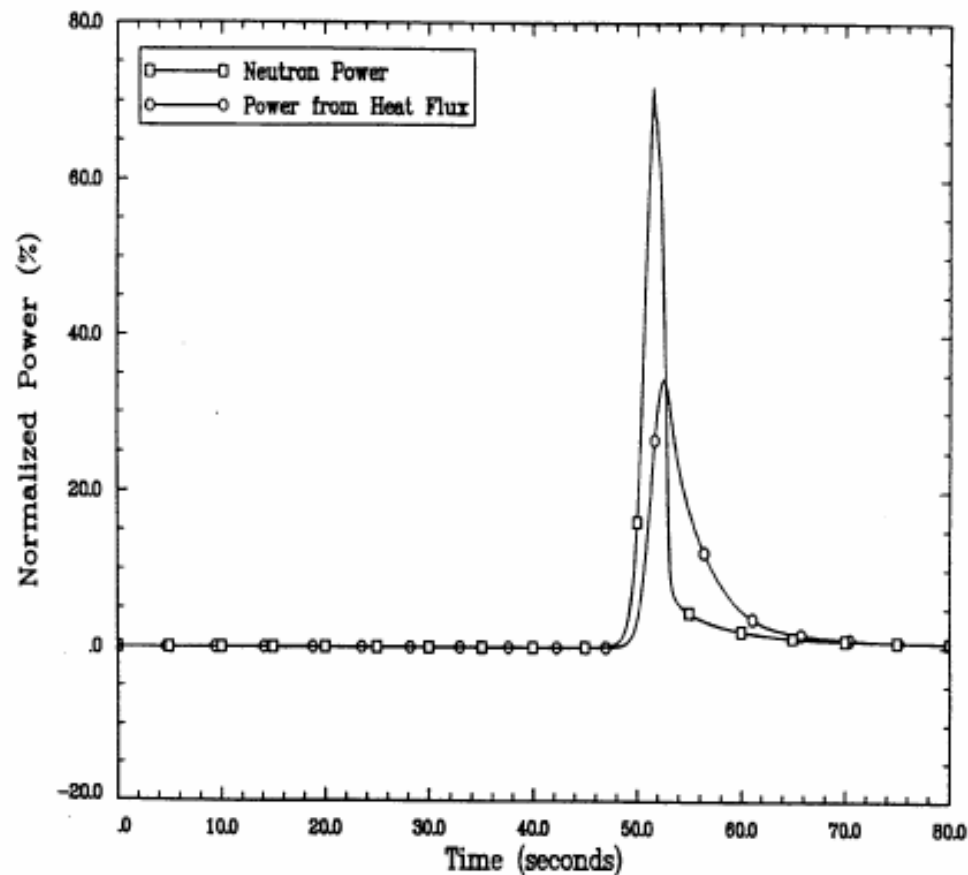


Example: PWR Surface Heat Flux vs Power Response During Startup Transient

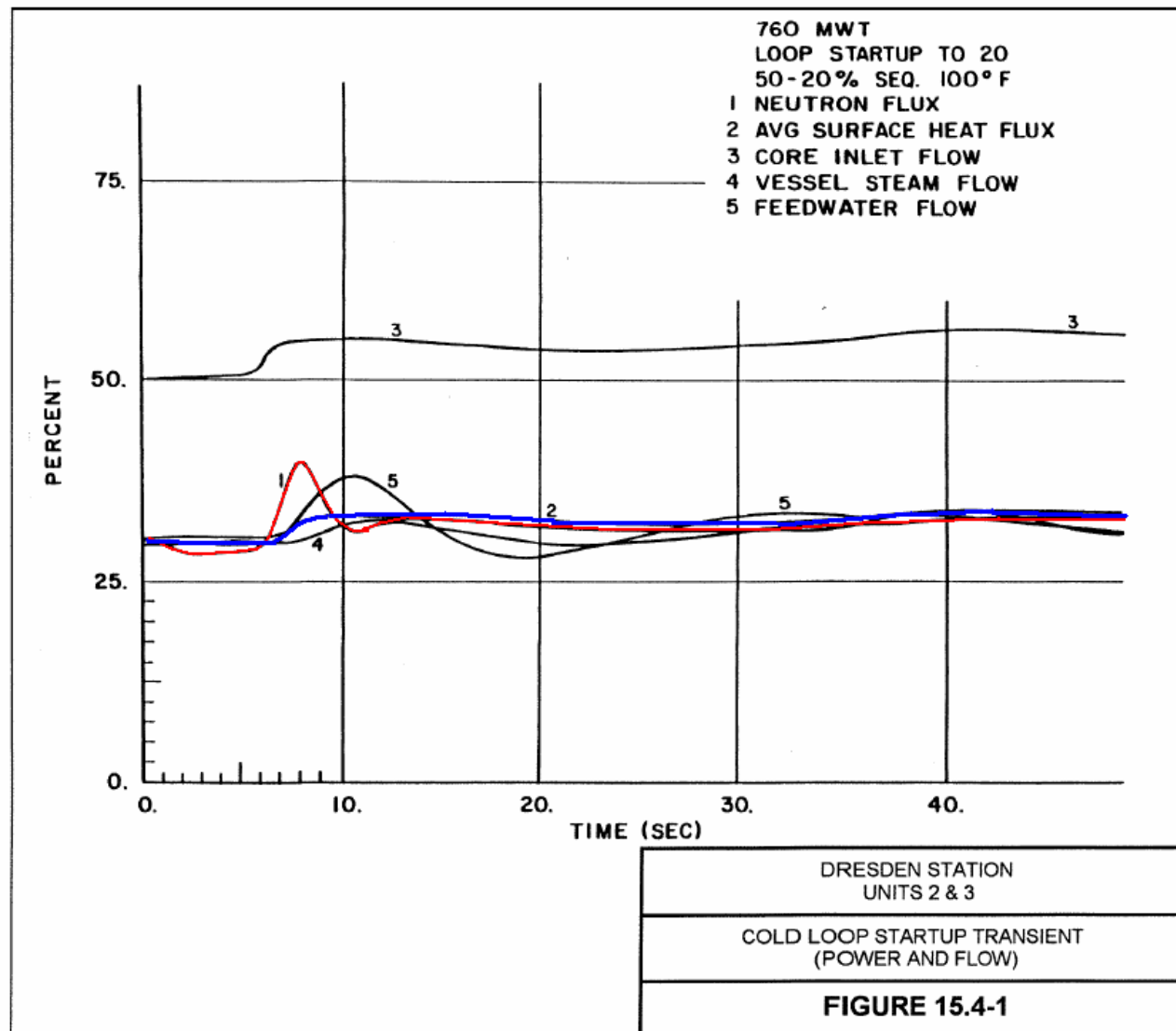
FSAR CHAPTER 14 - SAFETY ANALYSIS

FIGURE 14.2.1-4
Revision 21

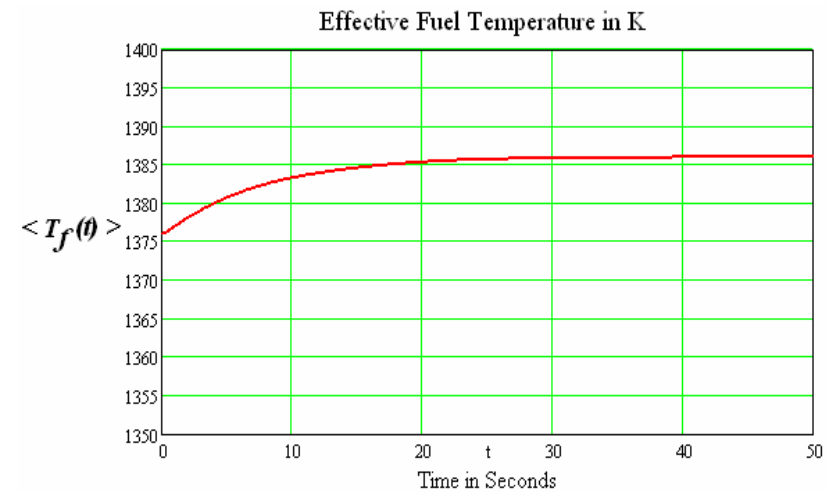
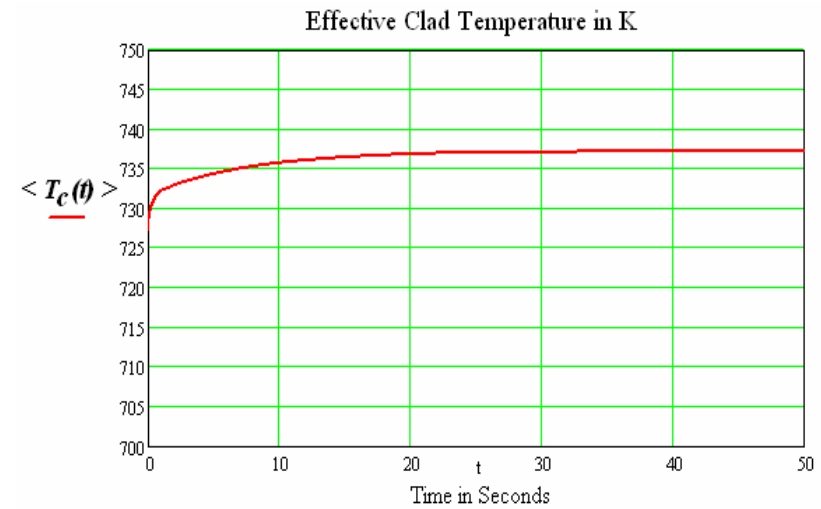
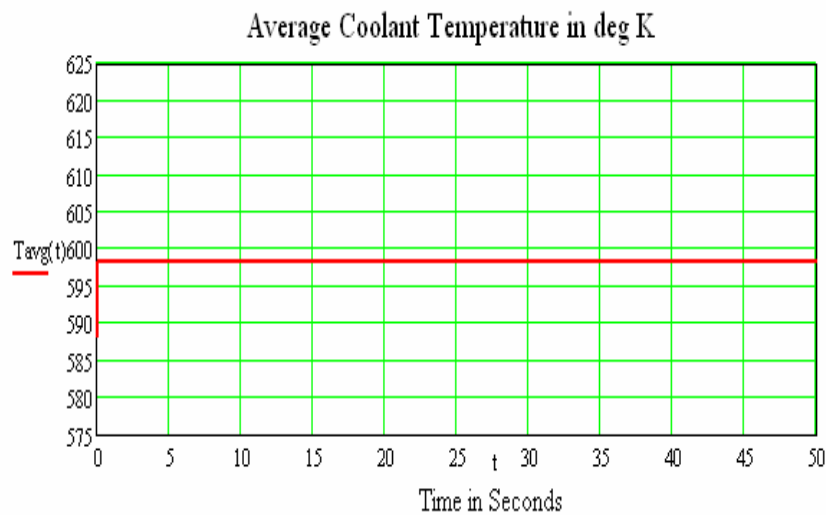
CONTROL ROD WITHDRAWAL INCIDENT HZP POWER AND HEAT FLUX



Example: BWR Surface Heat Flux vs Power Response During Loop Startup Transient



MATHCAD Simulation of +10°K Rise in T_{avg}



Summary

- Laws governing heat transfer can be used to define temperature and heat flow rates in fuel rods
- Key material fuel/clad properties (thermal conductivity, conductance, heat capacity) are *temperature dependent*
- *Simple heat transfer calculation* assuming “flat neutron power distribution” and constant material properties shows:
 - Peak centerline temperature is linear function of *coolant temperature* and *linear power density*
 - Fuel temperature distribution is *roughly parabolic*
- *Detailed fuel models* consider:
 - Thermal neutron flux depression in fuel pin
 - Temperature dependencies of material properties, densification..
 - Gap distance, fill gas composition.....