Fundamentals of Nuclear Engineering

Module 8: *Low Power Reactor Dynamics*

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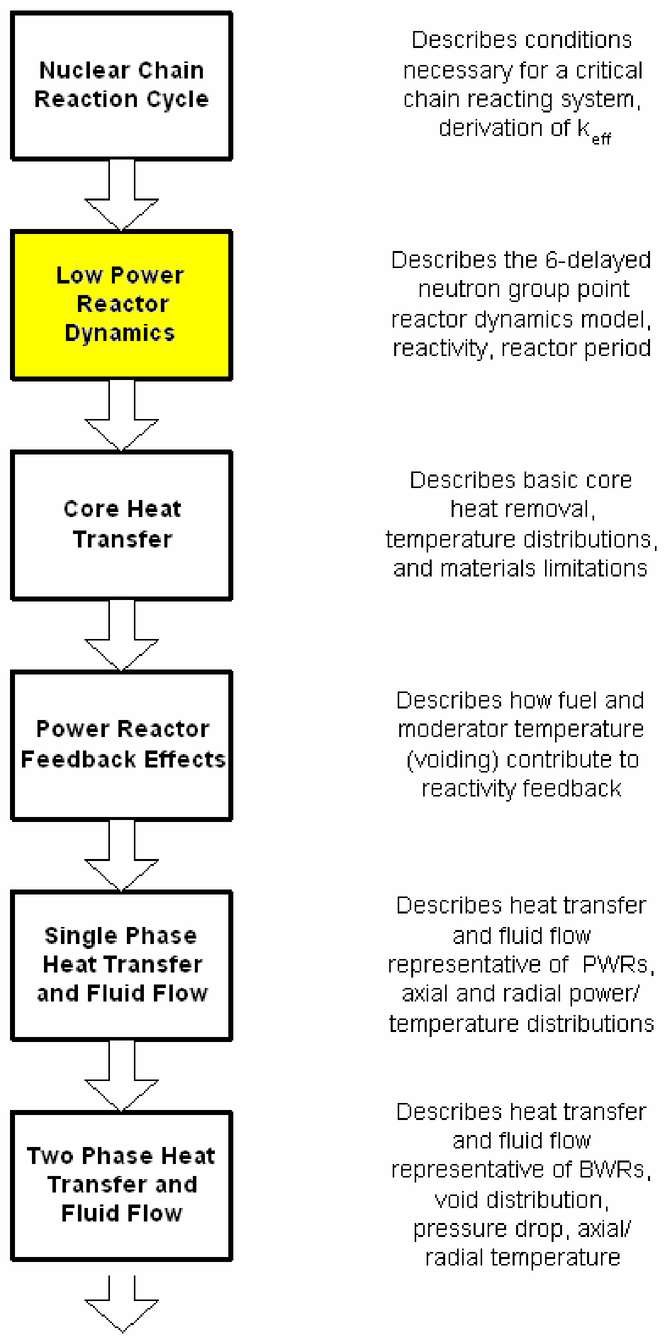

Nuclear Chain Reaction Cycle
Describes conditions necessary for a critical chain reacting system, derivation of k_{eff}
Low Power Reactor Dynamics
Describes the 6-delayed neutron group point reactor dynamics model, reactivity, reactor period
Core Heat Transfer
Describes basic core heat removal, temperature distributions, and materials limitations
Power Reactor Feedback Effects
Describes how fuel and moderator temperature (voiding) contribute to reactivity feedback
Single Phase Heat Transfer and Fluid Flow
Describes heat transfer and fluid flow representative of PWRs, axial and radial power/ temperature distributions
Two Phase Heat Transfer and Fluid Flow
Describes heat transfer and fluid flow representative of BWRs, void distribution, pressure drop, axial/ radial temperature

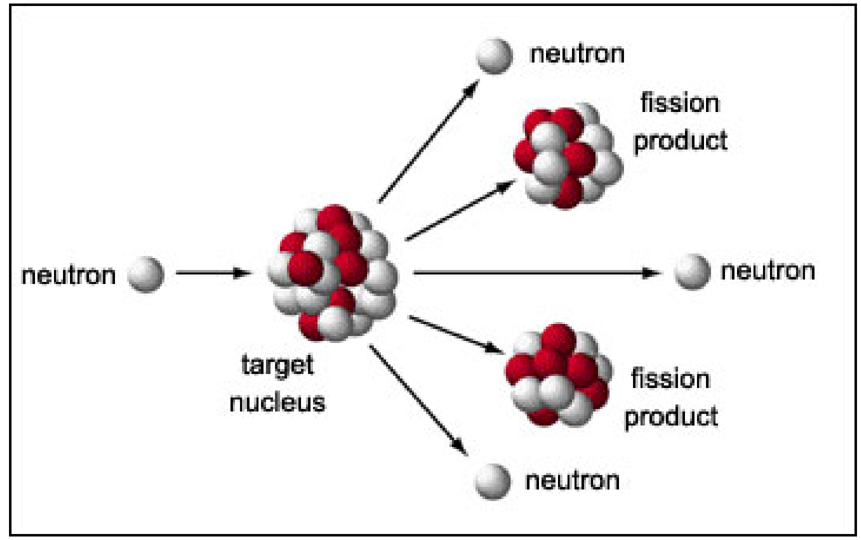
Objectives:

Previous lectures described origins of neutron diffusion equation and balance required for reactor criticality. This lecture will:

1. *Describe time dependent fission neutron source via 6-Delayed Neutron Group Model*
2. *Develop Point Reactor Dynamics neutron density model*
3. *Define: reactivity, delayed neutron fraction, neutron lifetime*
4. *Describe low power (Zero Feedback) reactor dynamics response to step and ramp changes in reactivity*
5. *Demonstrate simulated startup and low power operation*

Time Dependent Neutron Sources

Each Fission produces multiple neutrons:



- Fission yields on average: "*v*" total neutrons
- Fission yield increases *slightly* with neutron energy
- For U^{235}: $v(E) \approx 2.44$
- For U^{233}: $v(E) \approx 2.50$
- For Pu^{239}: $v(E) \approx 2.90$
- In discussions of steady state criticality: *timing of neutron emission* was not necessary to describe

Physics of Neutron Emission

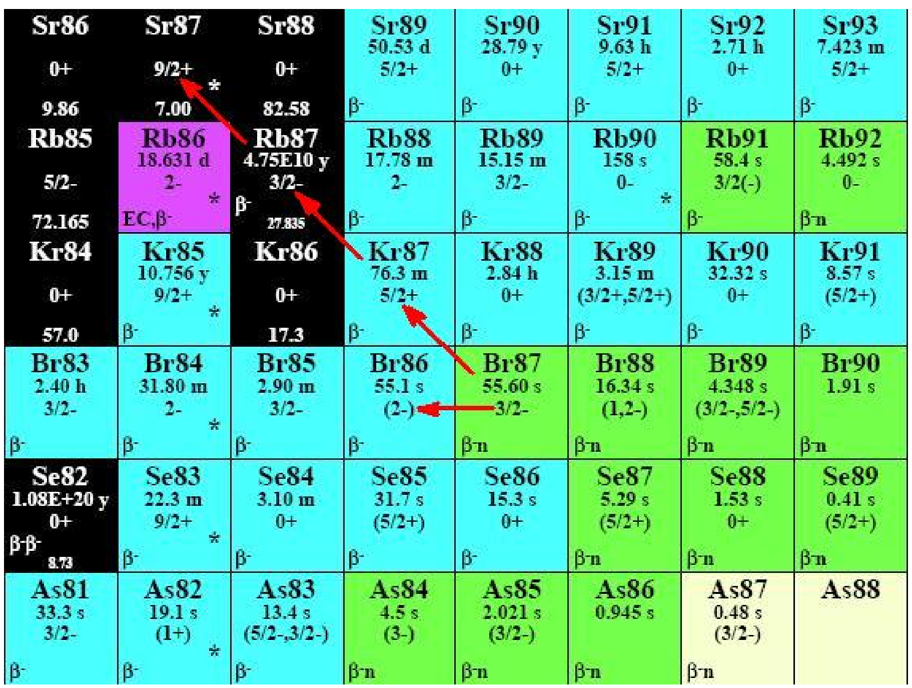
- Neutron flux *promptly emitted* at fission: $\nu\Sigma_f(1-\beta)\phi(t)$
- Delayed neutron flux, characterized by *β:* $\nu\Sigma_f\beta\chi(t)$
- Overall fission neutron source can be described as:

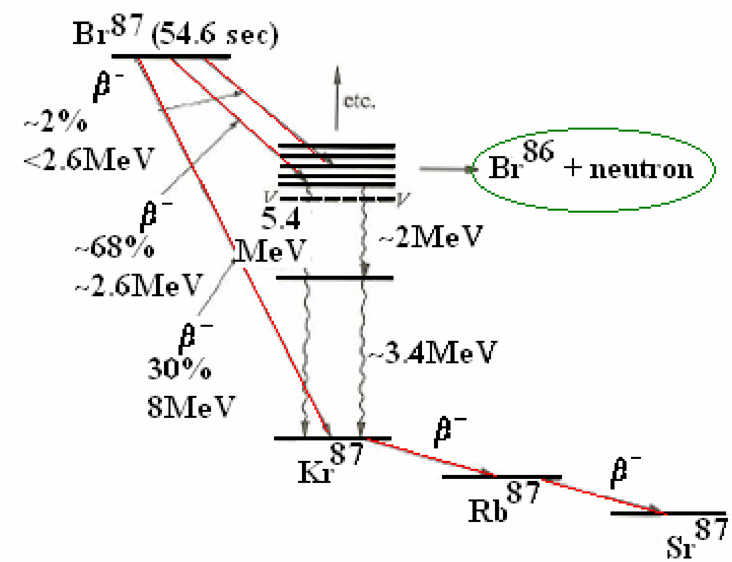
$$S(t) = \nu\Sigma_f[(1-\beta)\phi(t) + \beta\chi(t)]$$

- Delayed neutron emission: combination of:
  - physical insight (known Isotope decay half-lives)
  - experimental observation
- *β*-decay of Br^{87} and I^{137} are known to be sources of longest delayed neutrons
- Other *β*-decay reactions have been *lumped together* in groups with *roughly equivalent decay constants*

Origin of ~55 sec. Delayed Neutron

- ${}_{0}n^{1} + {}_{92}U^{235} \rightarrow$ fission ${}_{35}Br^{87}$ is a fission product
- ${}_{35}Br^{87} \rightarrow {}_{36}Kr^{87} + {}_{0}\beta^{-1} + v$ β-decay (neutron decays to proton)
- ${}_{35}Br^{87} \rightarrow {}_{35}Br^{86} + {}_{0}n^{1}$ neutron emission





Taken from J. Lamarsh, "Nuclear Reactor Theory", p. 98

Delayed Neutrons Grouped into 6-Groups

Delayed-Neutron Precursors. Uncertain Quantities are Indicated by Parentheses.*

Precursor	Precursor half-life (sec) and group assignment	
Br^{87}	54.5	Group 1
I^{137}	24.4	Group 2
Br^{88}	16.3	
I^{138}	6.3	Group 3
$Br^{(89)}$	4.4	
$Rb^{(93,\ 94)}$	~6	
I^{139}	2.0	Group 4
(Cs, Sb or Te)	(1.6–2.4)	
$Br^{(90,\ 92)}$	1.6	
$Kr^{(93)}$	~1.5	
(I^{140} + Kr?)	0.5	Group 5
(Br, Rb, As + ?)	0.2	Group 6

* From G. R. Keepin, *Physics of Nuclear Kinetics*, Reading, Mass.: Addison-Wesley, 1965.

Delayed Neutron Groups show slight differences for U^{233}, U^{235}, Pu^{239}

Delayed Neutron Data for Thermal Fission in U^{233}, U^{235}, and Pu^{239}*

U^{233}

Group	Half-life (sec)	Decay constant λ_i (sec^{-1})	Yield (neutrons per fission)	Fraction β_i
1	55.00	0.0126	0.00057	0.000224
2	20.57	0.0337	0.00197	0.000777
3	5.00	0.139	0.00166	0.000655
4	2.13	0.325	0.00184	0.000723
5	0.615	1.13	0.00034	0.000133
6	0.277	2.50	0.00022	0.000088

Total yield: 0.0066
Total delayed fraction (β): 0.0026

U^{235}

Group	Half-life (sec)	Decay constant λ_i (sec^{-1})	Yield (neutrons per fission)	Fraction β_i
1	55.72	0.0124	0.00052	0.000215
2	22.72	0.0305	0.00346	0.001424
3	6.22	0.111	0.00310	0.001274
4	2.30	0.301	0.00624	0.002568
5	0.610	1.14	0.00182	0.000748
6	0.230	3.01	0.00066	0.000273

Total yield: 0.0158
Total delayed fraction (β): 0.0065

Pu^{239}

Group	Half-life (sec)	Decay constant λ_i (sec^{-1})	Yield (neutrons per fission)	Fraction β_i
1	54.28	0.0128	0.00021	0.000073
2	23.04	0.0301	0.00182	0.000626
3	5.60	0.124	0.00129	0.000443
4	2.13	0.325	0.00199	0.000685
5	0.618	1.12	0.00052	0.000181
6	0.257	2.69	0.00027	0.000092

Total yield: 0.0061
Total delayed fraction (β): 0.0021

* Based on G. R. Keepin, *Physics of Nuclear Kinetics*, Reading, Mass.: Addison-Wesley, 1965.

6-Delayed Neutron Groups Model:

- Each delayed neutron precursor group *"C_i"* is modeled via buildup (proportional to: β_i) and decay (with rate: λ_i):

$$\frac{\partial C_i(r,t)}{\partial t} = \beta_i \nu \Sigma_f \phi(r,t) - \lambda_i C_i(r,t)$$

- Overall fission neutron source is expressed as:

$$S(r,t) = (1-\beta)\nu\Sigma_f \phi(r,t) + \sum_{i=1}^{6} \lambda_i C_i(r,t)$$

$$-where: \beta = \sum_{i=1}^{6} \beta_i$$

Substituting Neutron Source Term into Time-Dependent Diffusion Equation:

- Recall:

$$\frac{\partial N(r,t)}{\partial t} = \frac{1}{V}\frac{\partial \phi(r,t)}{\partial t} = S(r,t) - \Sigma_a \phi(r,t) + D\nabla^2 \phi(r,t)$$

- Substituting 6-Delayed Neutron Group Model yields following system of 7 equations:

$$\frac{1}{V}\frac{\partial \phi(r,t)}{\partial t} = (1-\beta)\nu\Sigma_f \phi(r,t) + \sum_{i=1}^{6} \lambda_i C_i(r,t) - \Sigma_a \phi(r,t) + D\nabla^2 \phi(r,t)$$

$$\frac{\partial C_i(r,t)}{\partial t} = \beta_i \nu\Sigma_f \phi(r,t) - \lambda_i C_i(r,t)$$

$$-\,where: i = 1...6$$

For Simplification: Separation of Variables

- Assume: $\Phi(r,t) = \varphi(r)\, VN(t)$ and: $C_i(r,t) = \varphi(r)\, c_i(t)$

$$\varphi(r)\frac{dN(t)}{dt} = \varphi(r)[(1-\beta)\nu\Sigma_f V - \Sigma_a V + \frac{D\nabla^2\varphi(r)}{\varphi(r)}]N(t) + \varphi(r)\sum_{i=1}^{6}\lambda_i c_i$$

$$\varphi(r)\frac{dc_i(t)}{dt} = \varphi(r)\beta_i\nu\Sigma_f N(t) - \varphi(r)\lambda_i c_i(t)$$

- *Dividing out the spatial flux distribution from all equations, and substitution of the Geometrical Buckling coefficient:* B^2 *yields:*

$$\frac{dN(t)}{dt} = [\frac{(1-\beta)\nu\Sigma_f}{\Sigma_a} - 1 - L^2B^2]\Sigma_a VN(t) + \sum_{i=1}^{6}\lambda_i c_i(t)$$

$$\frac{dc_i(t)}{dt} = \beta_i\nu\Sigma_f N(t) - \lambda_i c_i(t)$$

Further Simplifications:

- Define average neutron lifetime as:

$$l = [V\Sigma_a(1 + L^2 B^2)]^{-1}$$

- Recognize full multiplication factor corrected for leakage:

$$k = \frac{\nu\Sigma_f / \Sigma_a}{(1 + L^2 B^2)}$$

- System of equations becomes:

$$\frac{dN(t)}{dt} = \frac{(1-\beta)k - 1}{l} N(t) + \sum_{i=1}^{6} \lambda_i c_i(t)$$

$$\frac{dc_i(t)}{dt} = \frac{\beta_i k}{l} N(t) - \lambda_i c_i(t)$$

Limitations in Point Reactor Dynamics Model

- 6-Delayed Neutron Group Model was derived assuming fission product *β-decay* as the source
- Delayed neutron production via 2.2MeV Deuterium photo-nuclear *(n,γ)* reactions would be significant in any D_2O moderated reactor such as CANDU. Overall dynamics would be slower than in PWR/BWR.
- 6-Delayed Neutron Group Model is function of assumed fissionable isotopes
- Buildup of Pu^{239} decreases β from 0.0065 – but never reaches pure Pu^{239} β value of: 0.0021
- Neutron lifetime is for *thermal reactors* and is typically on order of 10^{-4} - 10^{-5} sec. Neutron lifetime in fast reactor is on order of: 10^{-6} - 10^{-7} sec.

Low Power Reactor Dynamics

- Following discussions pertain to scenarios typical of *very low power reactor operation*
- *Non-linear Feedback Effects* on multiplication factor become significant when usable power (heat) is being generated
- Feedback effects will be discussed in subsequent lecture
- Previously calculation showed:

$(1W_t) / (2.0x10^8 eV/fission)(1.6x10^{-19} W_t\text{-}sec/eV) = 3.1x10^{10} fissions/sec.$

- 4000MW$_t$ reactor with core loading of: 1.2x10^{5}kg 3.5% enriched Uranium would require an average neutron flux of ~ *10^{13} - 10^{14} neutrons/cm2-sec.*
- *THUS:* following discussion of low power reactor dynamics will relate to $\Phi \leq 10^{10}$ *neutrons/cm2-sec.*
- In start-up range all reactors (PWR, BWR) behave same.

Steady State Solution

- Steady state solution is obtained by setting:

$$\frac{dN}{dt} = \frac{dc_i}{dt} = 0$$

- Solving for precursor concentrations yields:

$$c_i(t) = \frac{\beta_i k N(t)}{l\lambda_i}$$

$$\frac{dN}{dt} = 0 = \frac{(1-\beta)k - 1}{l} N(t) + \sum_{i=1}^{6} \lambda_i \frac{\beta_i k N(t)}{l\lambda_i}$$

$$0 = \frac{(1-\beta)k - 1}{l} + \frac{\beta k}{l}$$

- Which is simply: $k = 1$ - or in a state of *criticality*

Point Reactor Dynamics Solutions

- Most applications of Point Reactor Dynamics involve *time dependent changes* to multiplication factor: $k(t)$
- This generally implies solution of a messy system of *non-linear differential equations*.

$$\frac{dN(t)}{dt} = \frac{(1-\beta)k(t)-1}{l} N(t) + \sum_{i=1}^{6} \lambda_i c_i(t)$$

$$\frac{dc_i(t)}{dt} = \frac{\beta_i k(t)}{l} N(t) - \lambda_i c_i(t)$$

- Several "simplified" cases exist which allow hand solution – when $k(t)$ is a step or ramp
- However: Objective is not solving differential equations – but understanding reactor dynamics
- Thus: use *MATHCAD*

Transition from Critical to Supercritical

- Consider situation where system is initially critical: $k = 1.0$
- Adjustment made at 10 seconds and system becomes slightly supercritical: $k = 1.002$
- Initial conditions:

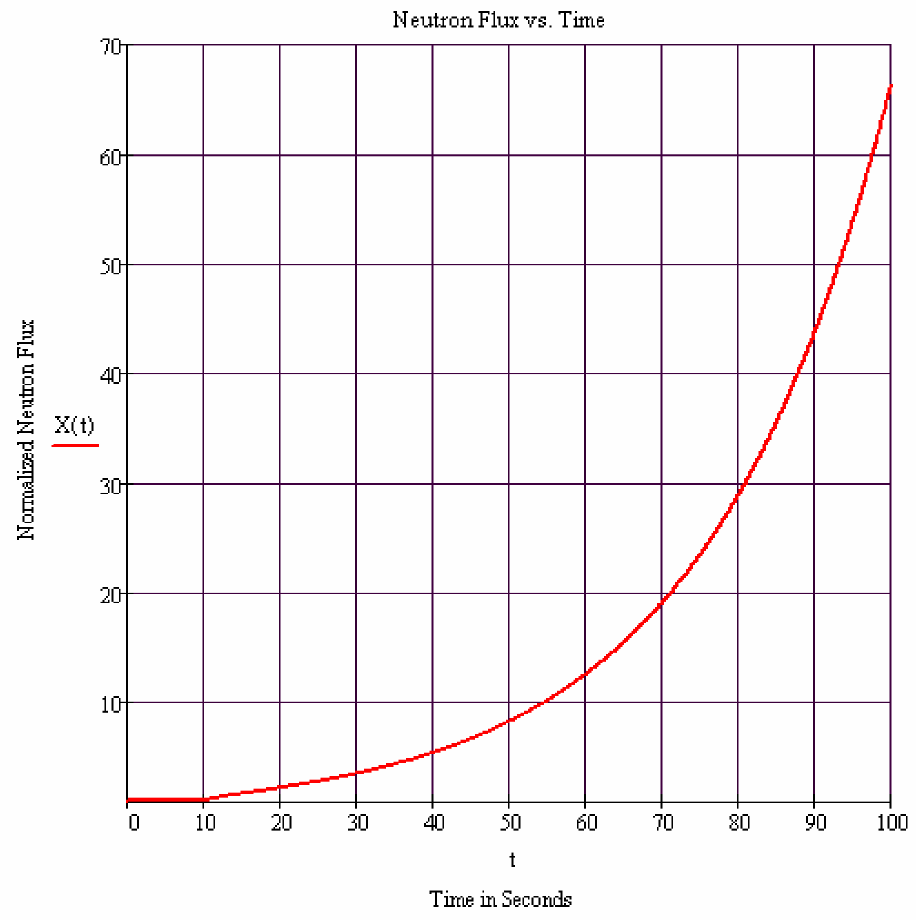
$$\frac{dN}{dt} = \frac{dc_i}{dt} = 0$$

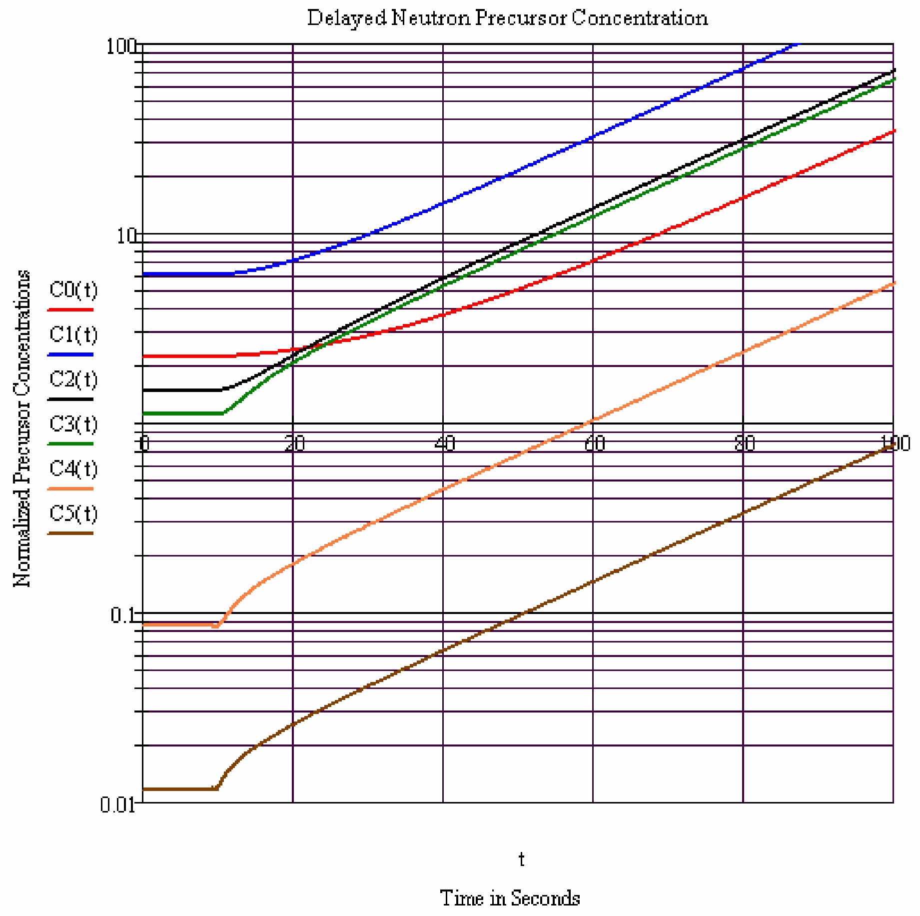
$$c_i(0) = \frac{\beta_i k N(0)}{l \lambda_i}$$

$$N(0) = N_0$$

- Numerical simulation of this scenario yields following

Transition to Supercritical with k = 1.002



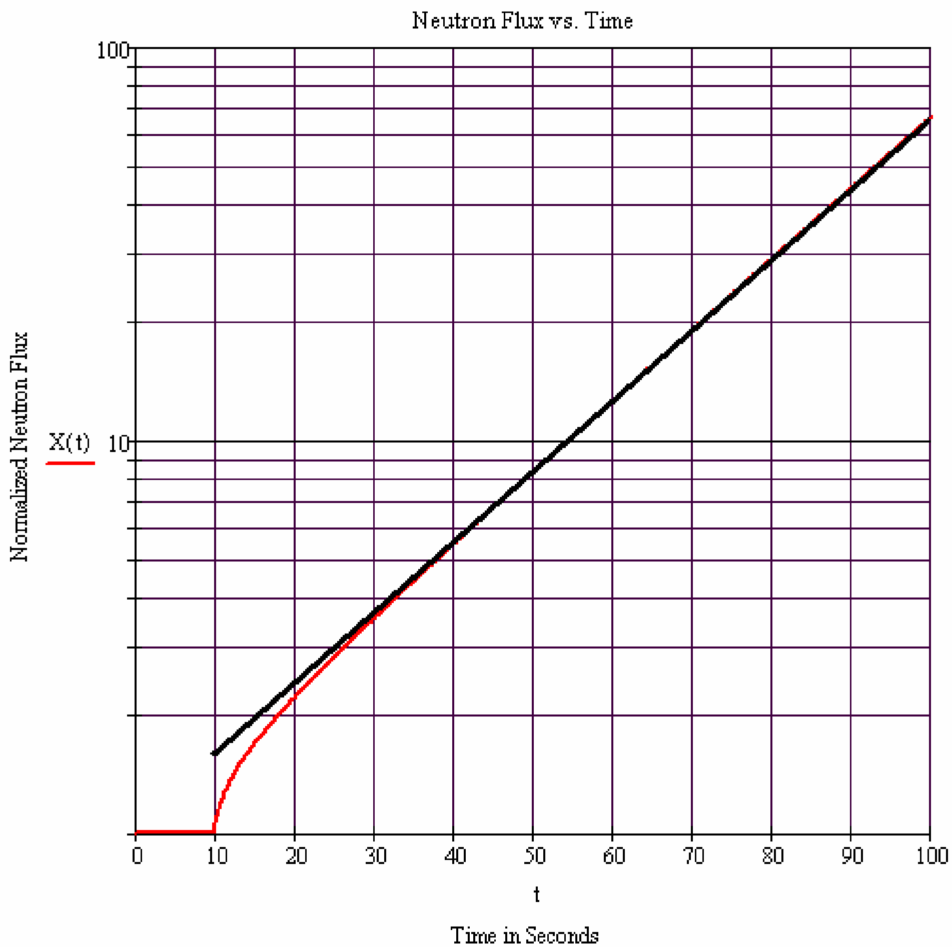


Transition to Supercritical with k = 1.002

- *Log N(t)* gives different perspective
- Note "***prompt jump***" with "***exponential tail***"
- This is related to physics of prompt vs. delayed neutrons
- After prompt neutron transients die out, *N(t)* can be modeled as:

$$N(t) = A_o exp(\omega t)$$

- Reactor period: $T = 1 / \omega$ depends on magnitude of change in *k*

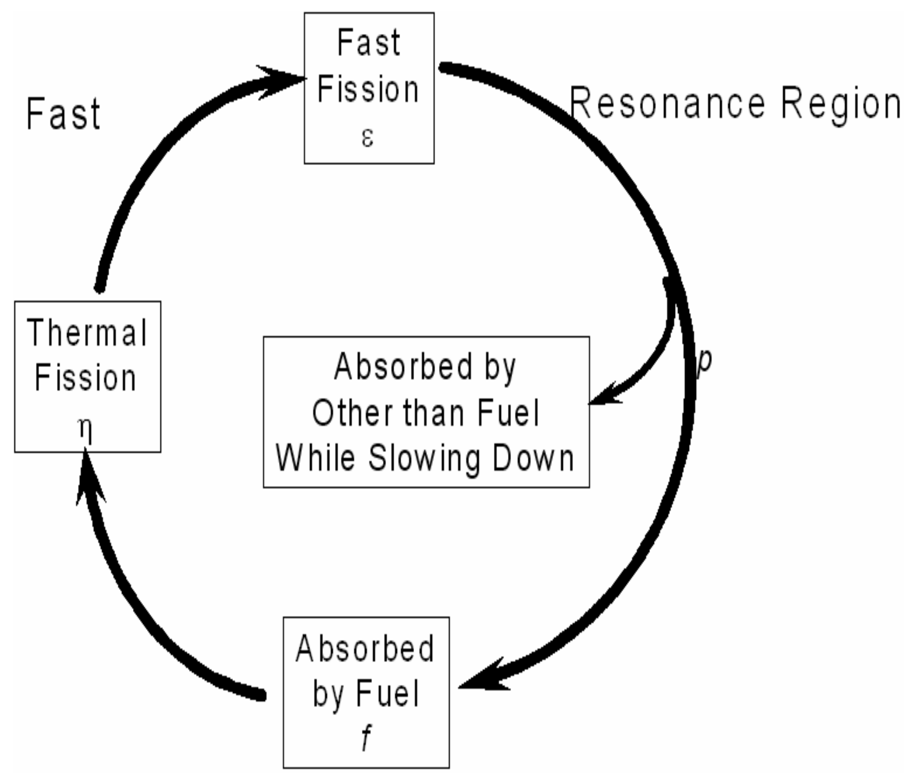


Transition to Supercritical with k = 1.002 at 10 sec.

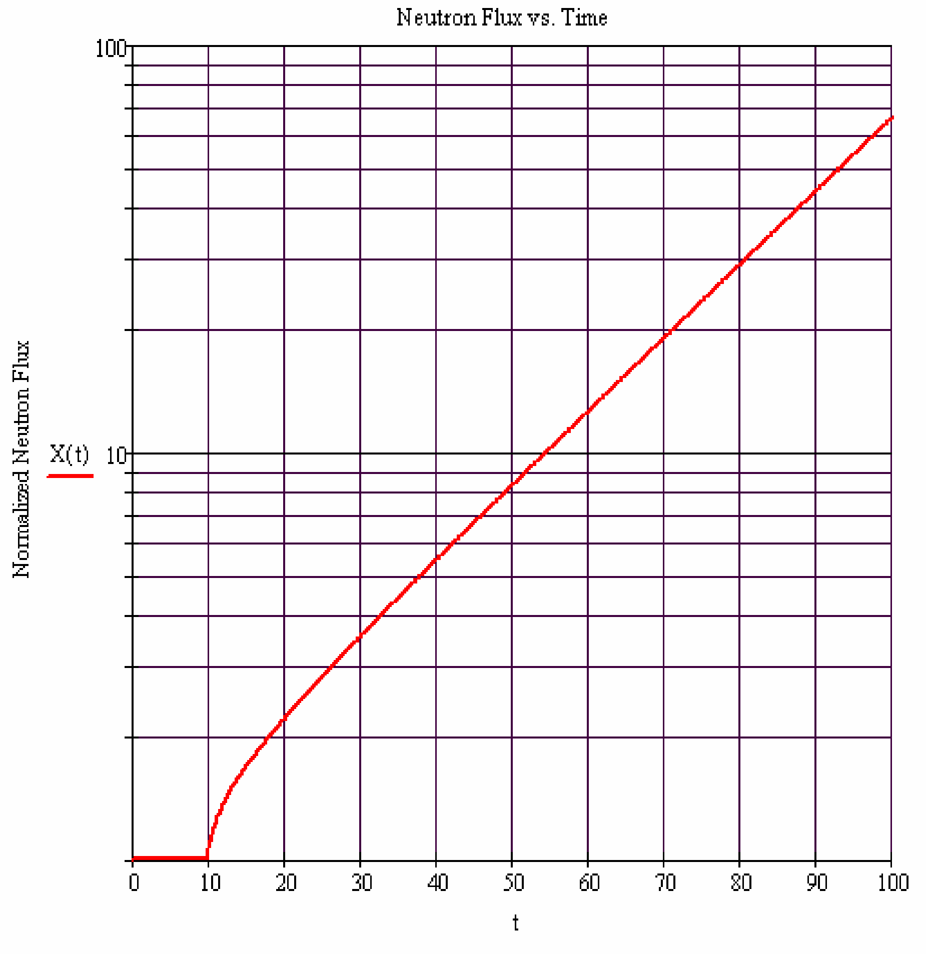
Delayed Neutrons: Key to Reactor Control

- Neutron life-cycle was previously described as →
- Time constant of one cycle:

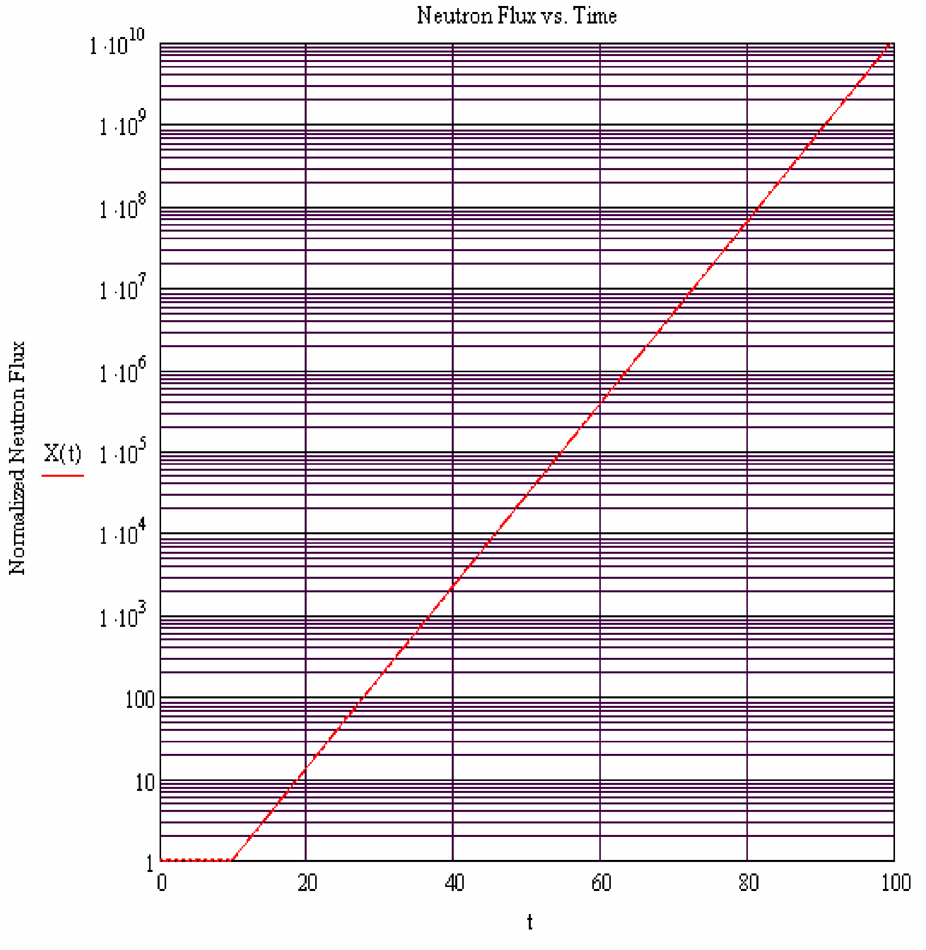
  $\ell = 10^{-4} - 10^{-5}$ *sec.*
- No mechanical device known could operate to intervene in chain reaction growing this fast
- Removing between 0.0021 – 0.0065 neutrons in each $10^{-4} - 10^{-5}$ sec. cycle dramatically cuts back on neutron in growth of chain reaction.



Reactor Dynamics With vs. Without Delayed Neutrons



Transition to Supercritical with k = 1.002 at 10 sec.



Transition to Supercritical with k = 1.002 at 10 sec.
- No Delayed Neutrons Assumed -

Transition to Subcritical

- Consider situation where system is initially critical: $k = 1.0$
- Adjustment made at 10 seconds and system becomes subcritical: $k = 0.99$
- Initial conditions:

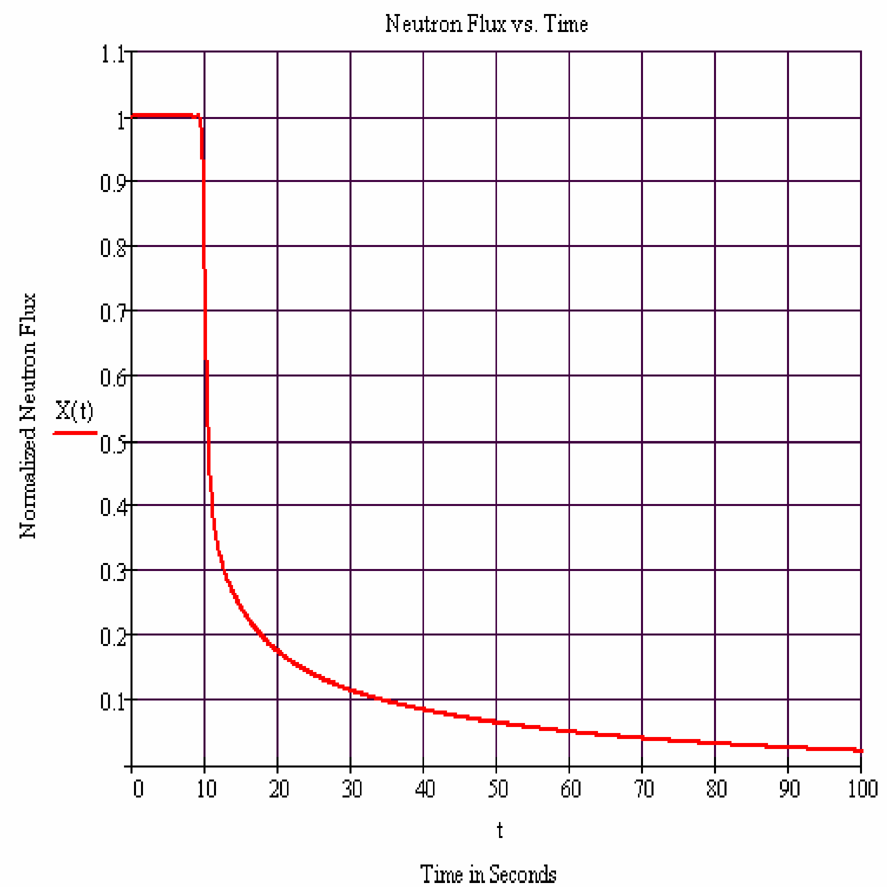
$$\frac{dN}{dt} = \frac{dc_i}{dt} = 0$$

$$c_i(0) = \frac{\beta_i k N(0)}{l\lambda_i}$$

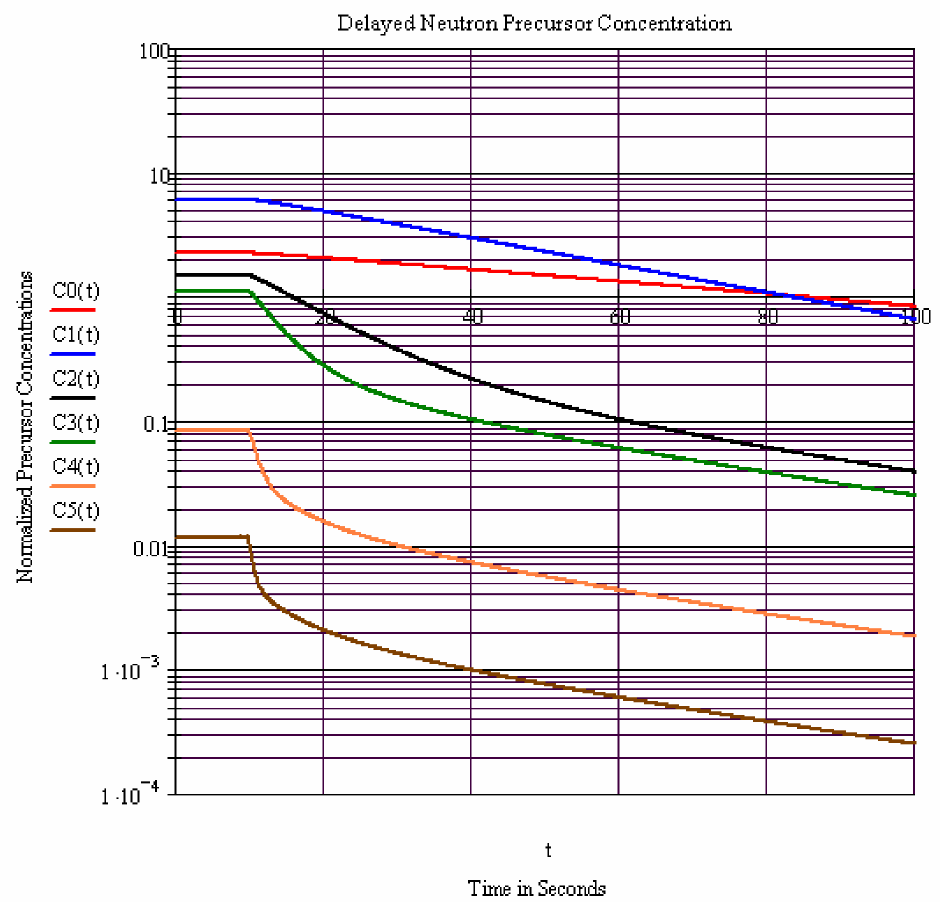
$$N(0) = N_0$$

- Numerical simulation yields following

Transition to Subcritical Simulation

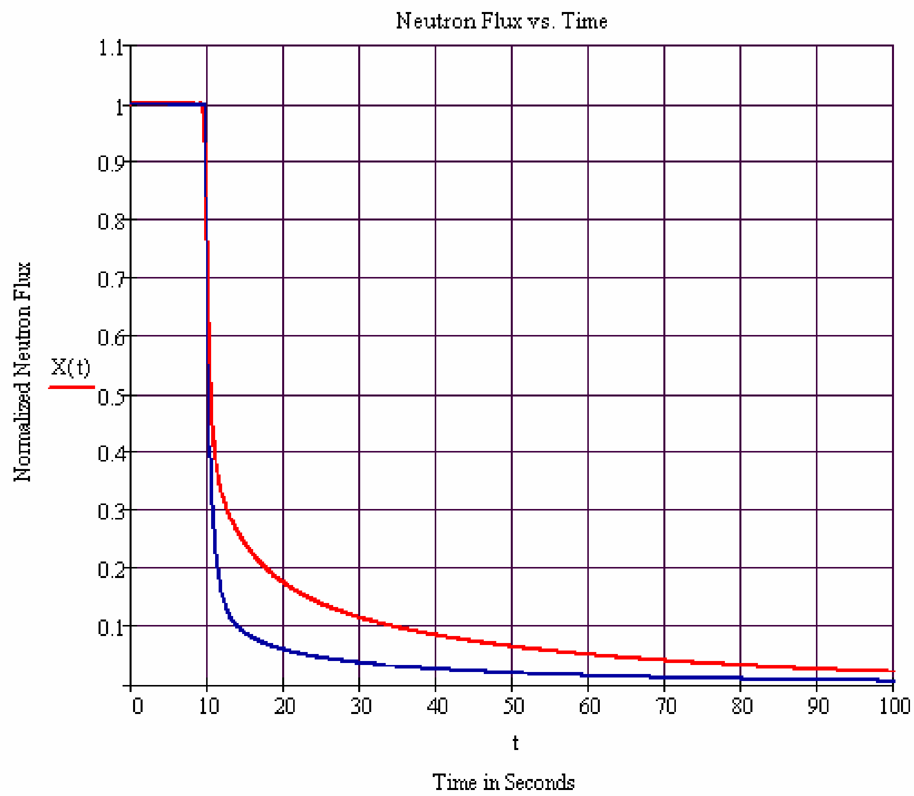


Response to Transition from k = 1.000 to k = 0.99 at 10 sec.



Transition to Subcritical Simulation

- Previous calculation was:
  $k = 1.00$ to $k = 0.99$
- Suppose reduction was 3x
- Change: $k = 1.00$ to $k = 0.97$
- Observe shape combination of "*prompt drop*" and "*exponential tail*"
- Again this is caused by differences between prompt vs. delayed neutrons



Comparison with:
Response to Transition from k = 1.0 to k = 0.97 at 10 sec.

Concept of Reactivity

Reactivity is Fractional "k" Deviation from 1.0

- Reactivity is defined: $\rho(t) = (k(t) - 1) / k$
- Neutron Lifetime is slightly redefined: $\Lambda = \ell / k$
- This formalism works well in vicinity of critical system conditions – where studying deviations of: ~ +/- 0.03
- Substituting these changes into Point Reactor Dynamics equations yield following system of equations:

$$\frac{dN(t)}{dt} = \frac{(\rho(t) - \beta)}{\Lambda} N(t) + \sum_{i=1}^{6} \lambda_i c_i(t)$$

$$\frac{dc_i(t)}{dt} = \frac{\beta_i}{\Lambda} N(t) - \lambda_i c_i(t)$$

Comparison of k_{eff} vs. ρ

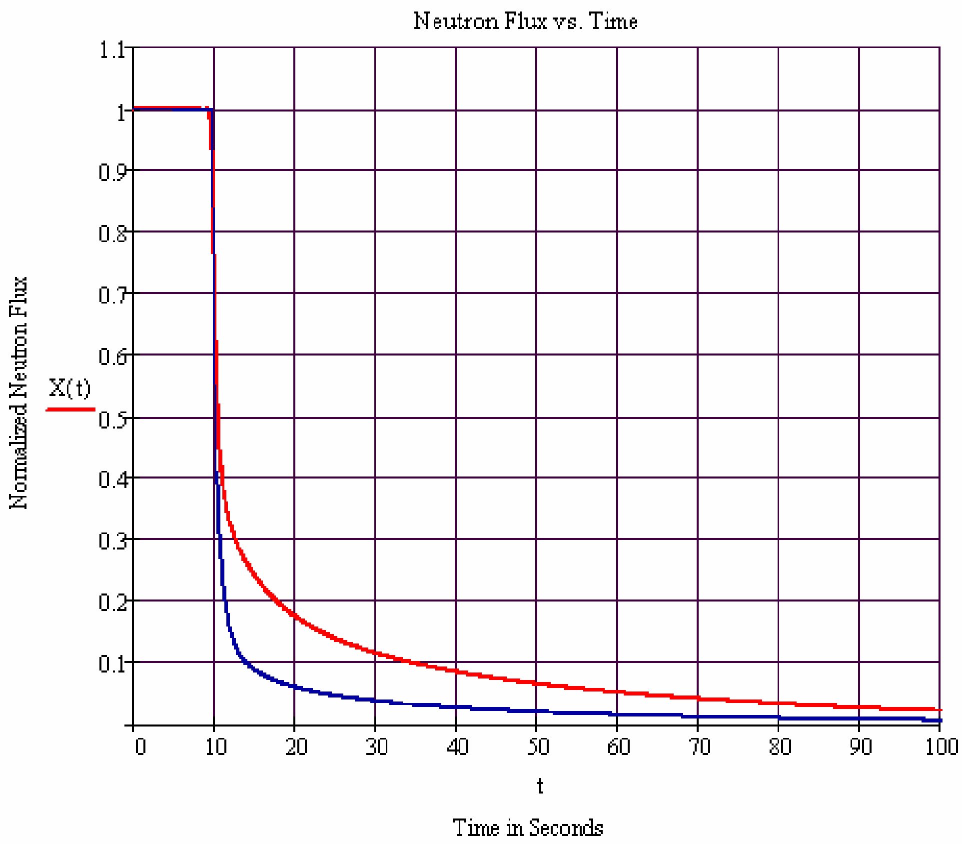
Parameter:	*Multiplication Factor:* k_{eff}	*Reactivity:* ρ
Subcritical:	< 1.0	< 0.0
Critical:	$= 1.0$	$= 0.0$
Supercritical:	> 1.0	> 0.0

Expression of Reactivity Units

- Reactivity can be expressed directly as: $\Delta k/k$ or, as comparison to: β
- Old texts such as Glasstone & Sesonske: "*Nuclear Reactor Engineering*" (1967) used units of: *\$, ¢*
- $\rho = 1\$$ is reactivity change to/from critical conditions equivalent to $\rho = \beta$, or $\rho = 0.0065$
- $\rho = 1¢$ is 1/100th of this, or: $\rho = \sim 6.5x10^{-5}$
- 80's SARs use: $\Delta k/k$, or $\%\Delta k/k$
- 90's SARs use: "pcm" (per cent milli-rho) $1pcm = 1x10^{-5}$
- In Europe, or former Soviet Countries reactivity is expressed directly in units of β, *example:* $\rho = .12\beta$
- Problem with using units of β: *it is not constant*
- Recall that with: U^{235} burnup/ Pu^{239} buildup, β *decreases*

Prompt Drop From Control Rod Insertion

- Sudden change in reactivity results in "Prompt Drop"
- Followed by exponential decay
- Magnitude of initial drop can be directly related to reactivity change



Comparison with:
Response to Transition from k = 1.0 to k = 0.97 at 10 sec.

Prompt Drop From Control Rod Insertion

- Assume *control rod reactivity change:* $-\rho_{CR}$ is made faster than shortest delayed neutron precursor response time
- Initially precursor populations would be given by:

$$c_i(t) \approx \frac{\beta_i N(0)}{\lambda_i \Lambda}$$

- Upon substitutions, summing precursor contributions, point reactor dynamics equation becomes:

$$\frac{dN(t)}{dt} = \frac{(-\rho_{CR} - \beta)}{\Lambda} N(t) + \frac{\beta}{\Lambda} N(0)$$

- Expression is linear differential equation solvable as:

$$N(t) = \frac{\beta}{\rho_{CR} + \beta} N(0) + \frac{\rho_{CR}}{\rho_{CR} + \beta} N(0) \exp[-\frac{(\rho_{CR} + \beta)t}{\Lambda}] \approx \frac{\beta}{\rho_{CR} + \beta} N(0)$$

Prompt Drop From Control Rod Insertion

- Doing a little rearranging, ratio of before/after flux immediately after control rod drop would be:

$$\frac{N_0}{N_1} \approx \frac{\rho_{CR} + \beta}{\beta}$$

$$\rho_{CR} \approx \left(\frac{N_0}{N_1} - 1 \right) \beta$$

- This is historic method of checking individual control rod reactivity worth during low power startup testing.
- Example: $\rho_{CR} = 100pcm = 10^{-3}\Delta k/k = 0.154\beta = 0.154\$$
- Dropping control rod would result in immediate drop to:

$$N_0/N_1 = (\rho_{CR} + \beta)/ \beta = (1.154\ \beta)/\beta = 1.154$$

$$N_1 = \quad N_0/1.154 = 0.866\ N_0$$

Reactivity Excursions from Low Power

- Normal process of reactor startup involves slow, controlled evolution to increase k_{eff} to point of criticality

- Prior to reaching criticality *flux increases linearly* as reactivity increased

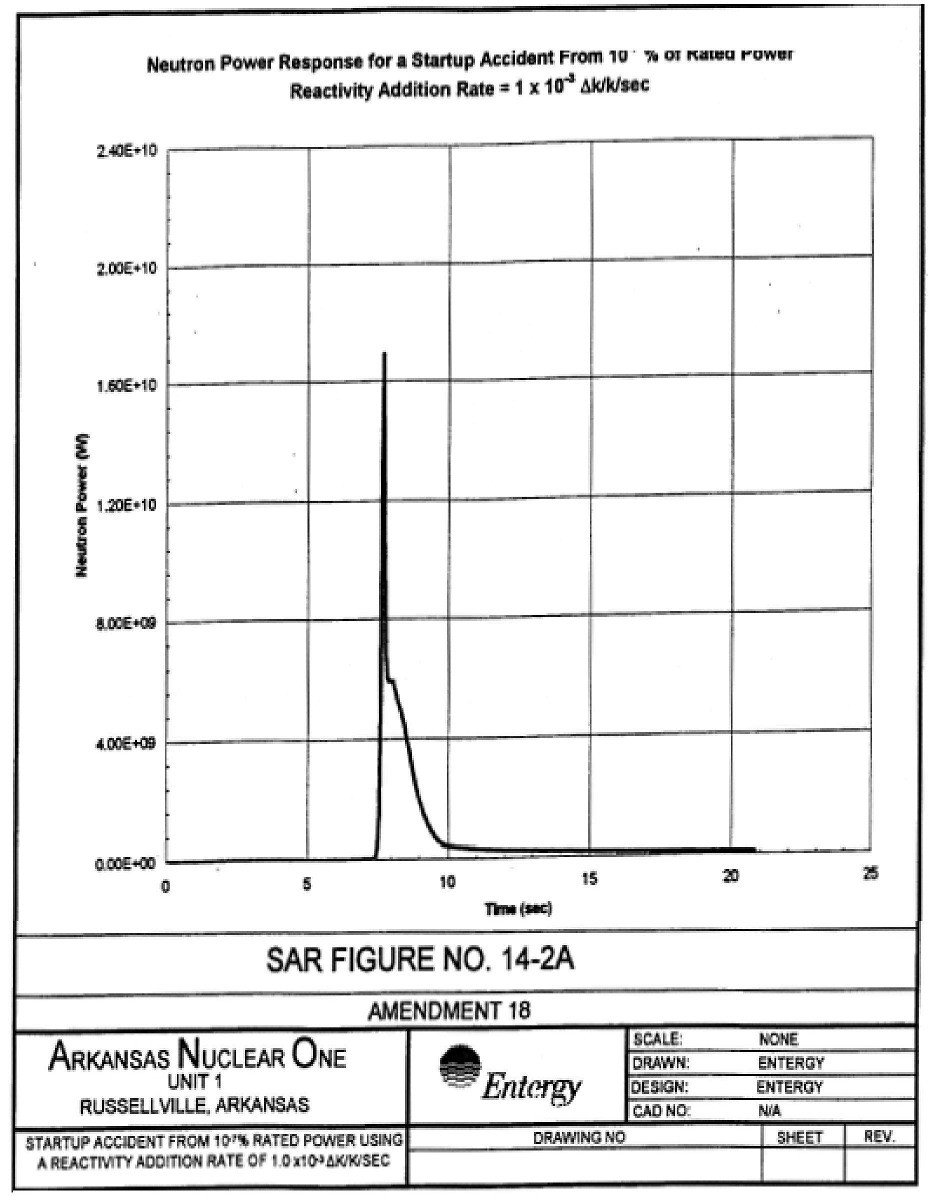
- When criticality reached, *flux increases exponentially* up to point of power/heat generation

- Heat production results in non-linear feedback that will slow down and halt further power increase until reactivity added

- Sudden spike in neutron flux, with corresponding spike in fuel/coolant temperatures *obviously needs to be avoided*

Reactivity Excursions from Low Power

- Example taken from ANO-1 FSAR
- Assumed initial flux: 10^{-7}%
- Assumed reactivity insertion rate: $d\rho/dt = 1x10^{-3}\Delta k/k/sec.$
  $= 100pcm/sec.$
  $= 0.154\beta/sec$
- Note: *prompt drop* followed by *exponential decay* tail
- To avoid startup power excursions, automatic trips provided on: hi flux, hi log power.
- Better to avoid hi $d\rho/dt$ additions !



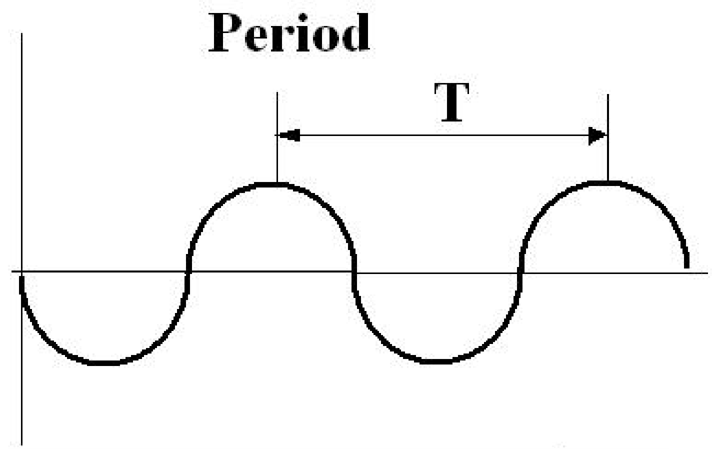
Limiting Rates of Reactivity Addition

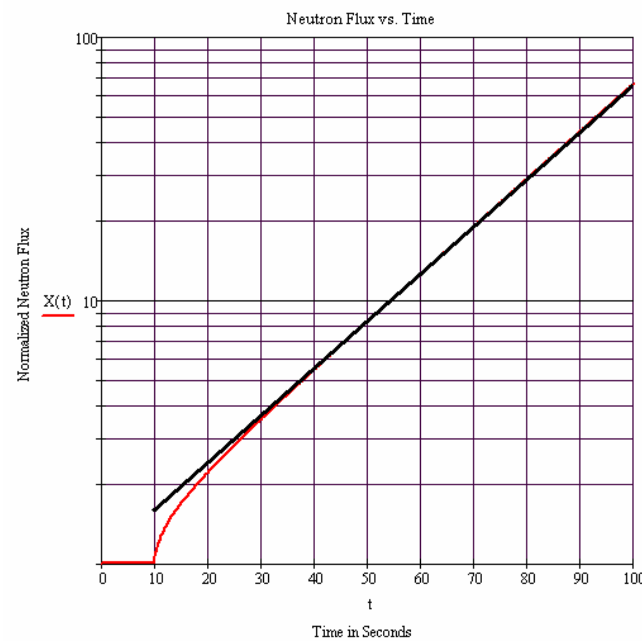
- Given that operators bring reactor to criticality using control rods (BWRs/PWRs) or dilution of soluble Boron (PWRs)
- Features should exist to:
- Alarm to operator if too much reactivity is being added
- Terminate adding further reactivity
- Initiate automatic shutdown if addition rate is excessive
- Measuring reactivity is *difficult*
- Measuring reactor period is actually *straight forward* given ability to measure $\log N(t)$
- Desire is to limit/control reactivity addition rates based upon *reactor period*

"Reactor Period" is ***NOT*** *about periodic or cyclic type phenomenon*

- Many mechanical and electrical systems involve simple harmonic systems
- Period: $T = 1/\omega$
- *Reactor Period* is inverse of exponential rate constant
- *Reactor Period*: $T = 1/\omega$
- In reactor physics "period" is inverse rate of exponential growth:

$$N(t) \sim N_0 \exp(t / T)$$

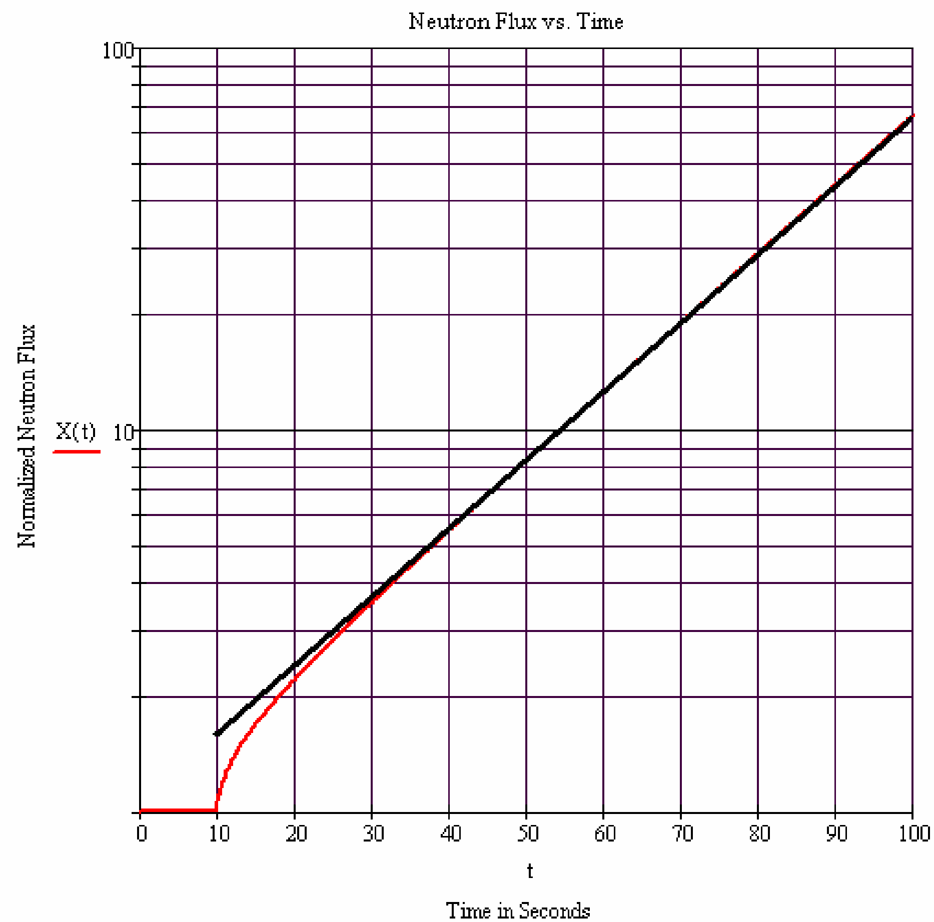




Transition to Supercritical with k = 1.002 at 10 sec.

Reactor Period and Reactivity

- Previous simulations of *supercritical* show long term exponential growth
- Exponential growth is expected because of chain multiplication, $k > 1.0$
- Rate of exponential growth or "inverse of period" is directly related to *Δρ*
- Larger changes from critical (*Δρ*) result in shorter periods.



Transition to Supercritical with k = 1.002 at 10 sec.

Reactor Period and Reactivity

- Assume overall solution of form: $N(t) = \sum A_i \exp(\omega_i t)$
- Assume unique long term relationship between reactivity change: "*$\Delta\rho$*" and reactor period: "*T*"
- With: $\omega = 1/T$, assume after short term transients die out, that: $N(t) \sim A_o \exp(\omega t)$ *- all higher order terms gone*
- After initial transients, precursor concentrations can be expressed: $C_i(t) = A_o \exp(\omega t)\beta_i / \Lambda \lambda_i$
- Substituting into point reactor dynamics equation yields following:

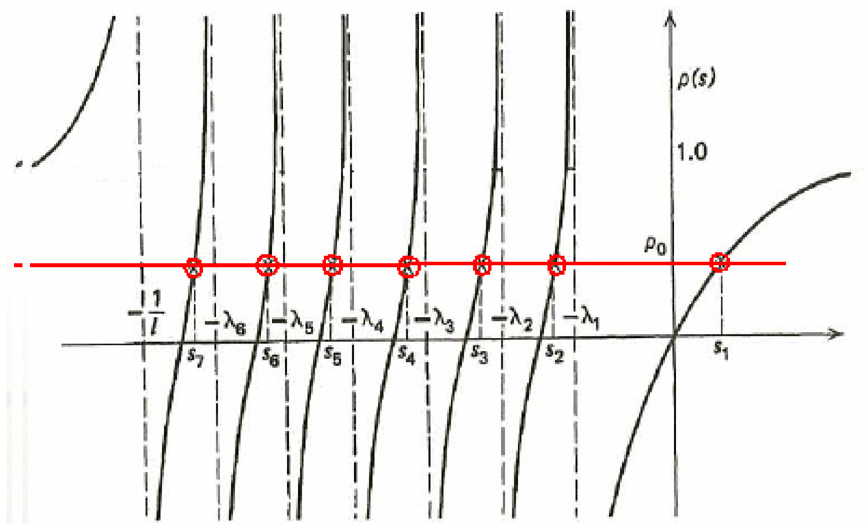
$$\omega = \left(\frac{\rho(\omega) - \beta}{\Lambda}\right) + \sum_{i=1}^{6} \frac{\lambda_i \beta_i}{(\omega + \lambda_i)}$$

$$\Lambda\omega = \rho(\omega) - \Lambda\sum_{i=1}^{6} \frac{\beta_i(\omega + \lambda_i)}{(\omega + \lambda_i)} - \frac{\lambda_i \beta_i}{(\omega + \lambda_i)} = \rho(\omega) - \Lambda\sum_{i=1}^{6} \frac{\beta_i \omega}{(\omega + \lambda_i)}$$

$$\rho(\omega) = \Lambda\omega + \Lambda\sum_{i=1}^{6} \frac{\beta_i \omega}{(\omega + \lambda_i)}$$

Reactor Period and Reactivity Graphical Solution

- Specific reactivity value ρ chosen
- Horizontal line drawn to find intersection with roots
- Roots identified: 6 always negative, 1 root dependent on whether ρ is positive/negative

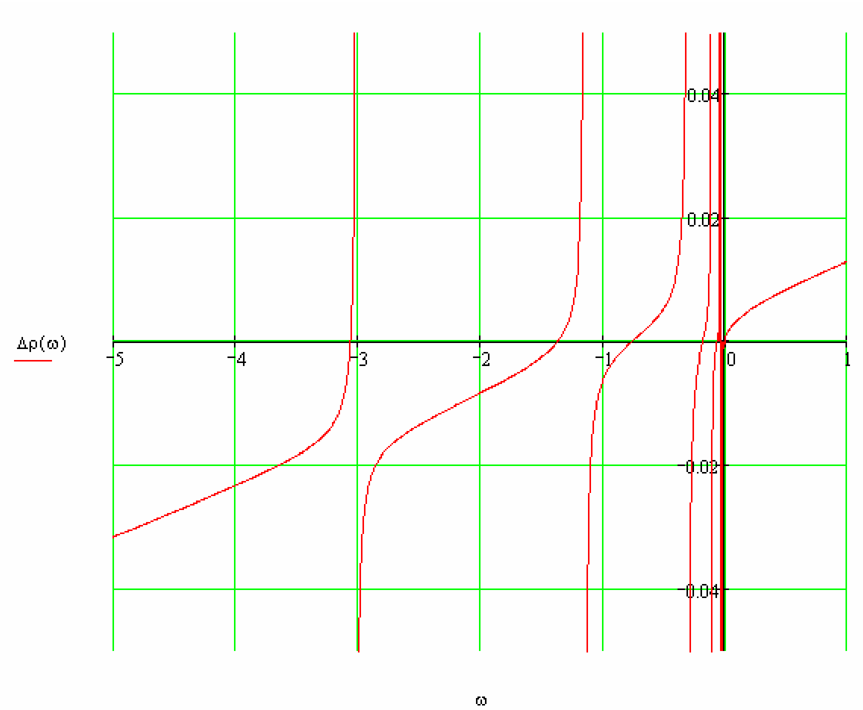


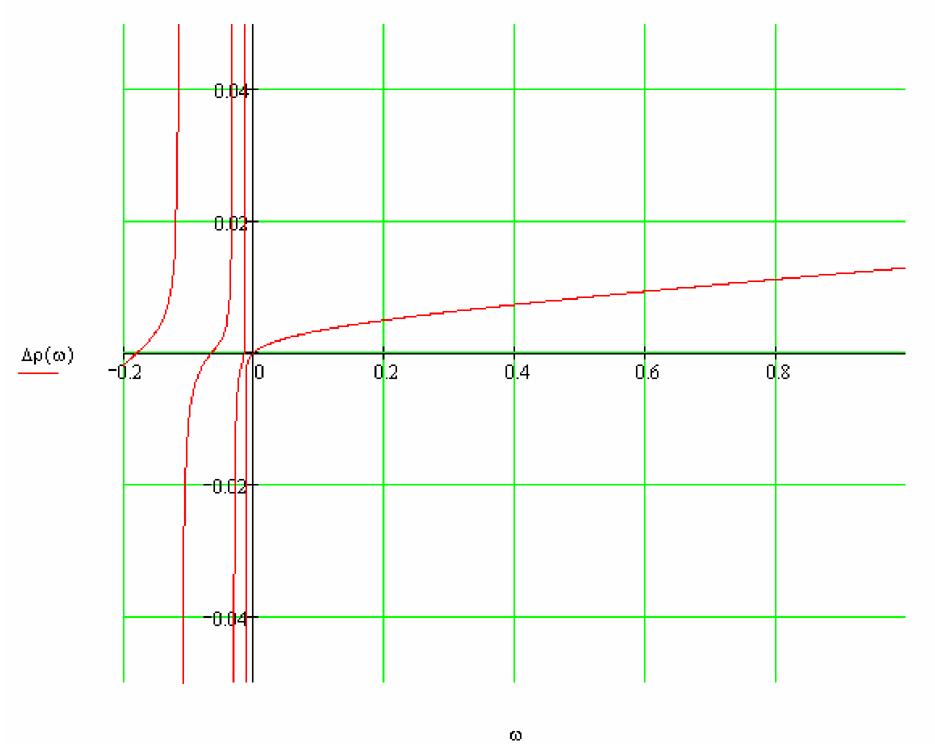
A graphical determination of the roots to the inhour equation

Taken from J. Duderstadt & L. Hamilton, "Nuclear Reactor Analysis", p. 245

MATHCAD Plot of Negative/Positive Roots

- Six negative valued roots are associated with delayed neutron precursor group decay processes (ω_i is always negative)
- Most right-hand root can be positive/negative depending on whether $\Delta\rho$ is positive or negative
- General solution is of form: $N(t) = \sum A_i \exp(\omega_i t)$



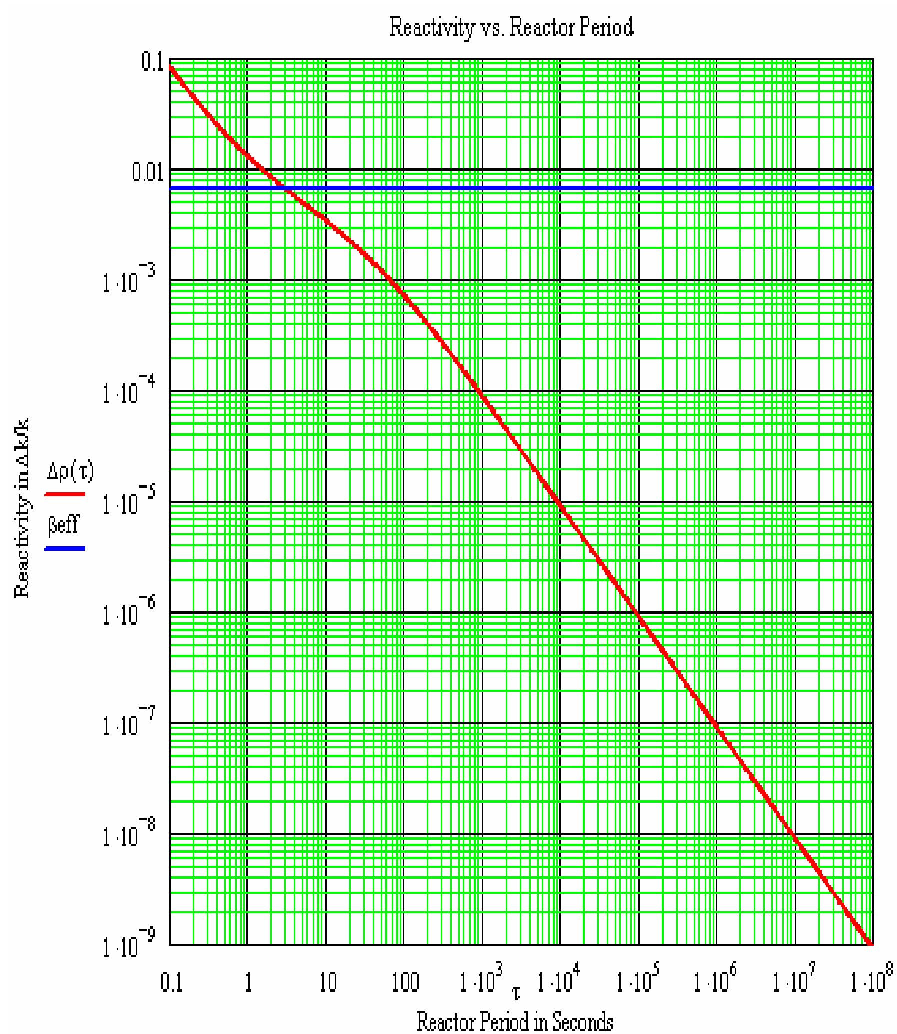


How Reactor Period and Reactivity Used to Control Reactor Startup

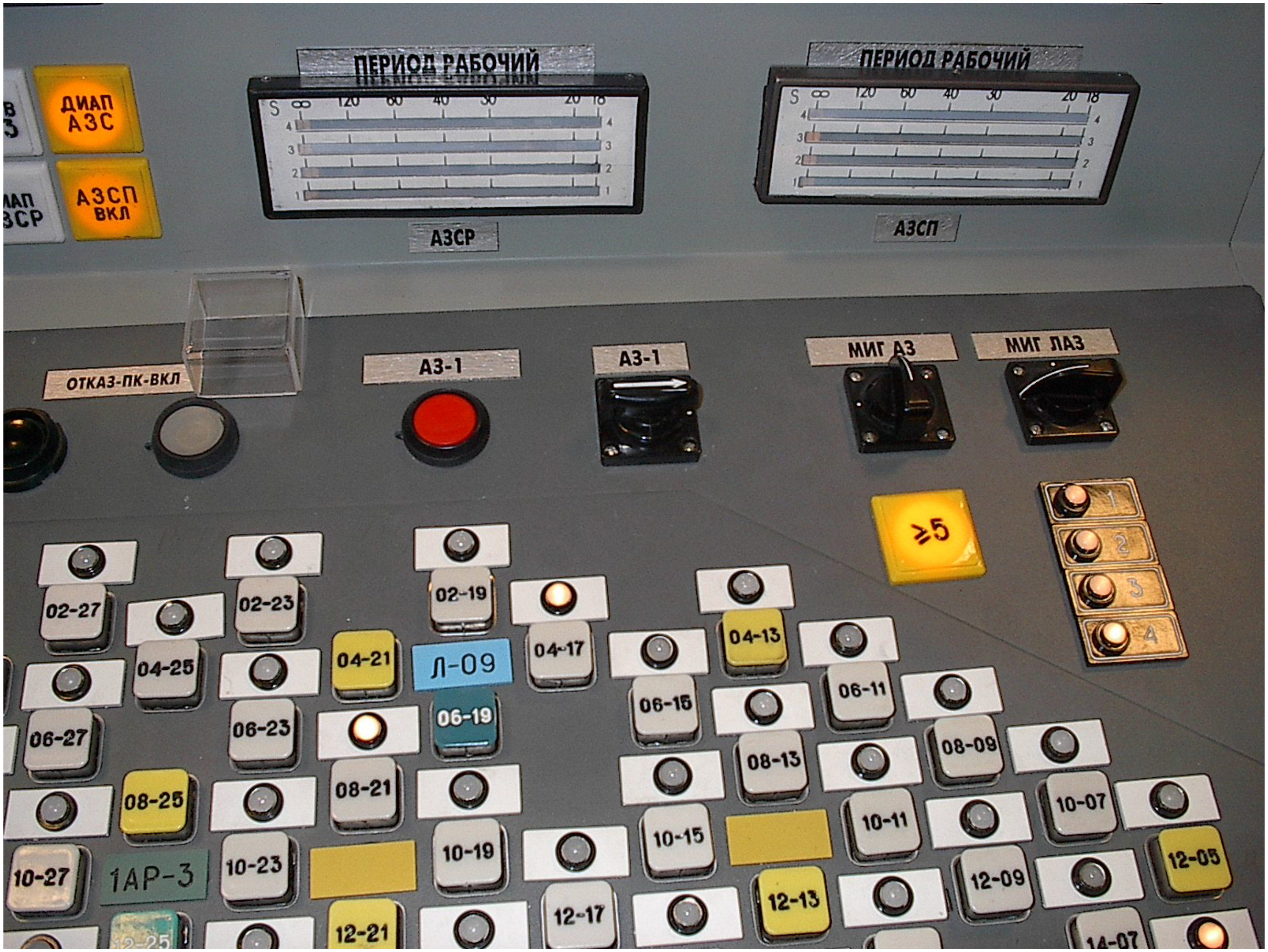
- Reactivity not measurable
- Log power rate is measurable
- Log power rate can be converted to Reactor Period: T
- Reactivity can be computed from:

$$\Delta\rho(\tau) := \frac{\Lambda}{\tau} + \sum_{i=0}^{5} \frac{\beta_i}{1 + \lambda_i \cdot \tau}$$

- *Prompt Critical Period* ~ 2.993 sec. (for assumed: Λ, β values)
- Operator displays and Control Rod Withdrawal Prohibit features are quite common

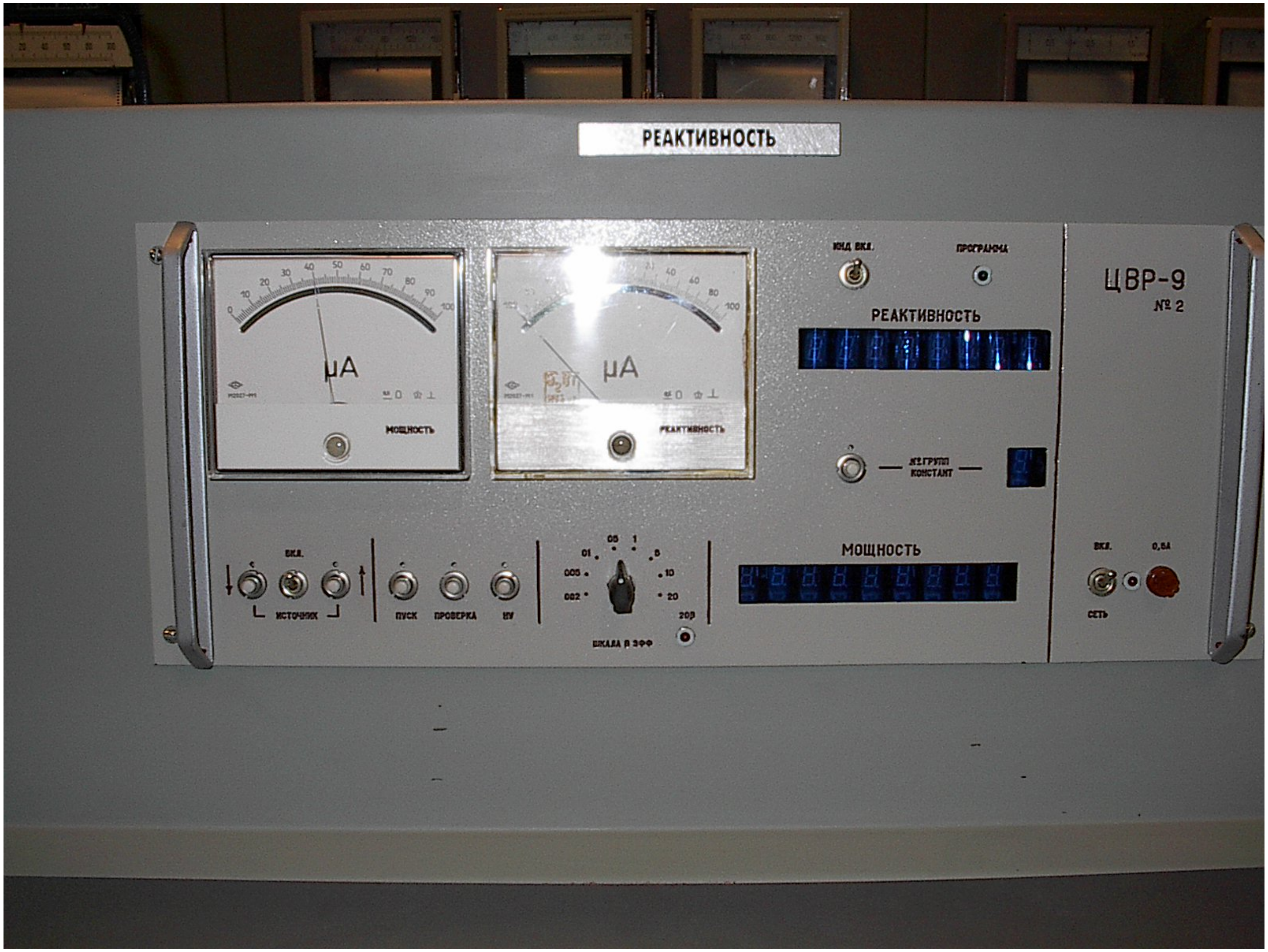


Period Meters on Russian RBMK-1500



Redundant Reactor Period Meters (∞,10sec) shown on Control Rod Panel

Reactimeter Panel on Russian RBMK-1500



Direct Indication of Startup Reactivity (like shown above) was added on all Russian Reactors following April 1986 Accident at Chornobyl Unit 4

Summary: Low Power Reactor Dynamics

- Delayed neutron fraction: β - plays key role in ability to control dynamics of nuclear reactors
- Point reactor dynamics model is commonly used as basis for all safety analysis work – subject to assumed limitations
- Low power reactor dynamics not subject to feedback effects found at power operation
- Subcritical: $k_{eff} < 1.000,\ \rho < 0.0,\ T \sim \infty sec.$
- Critical: $k_{eff} = 1.000,\ \rho = 0.0,\ T \sim \infty sec.$
- Supercritical: $k_{eff} > 1.000,\ \rho > 0.0,\ 10\ sec. < T < \infty\ sec.$
- *Prompt Supercritical:* $k_{eff} > \beta + 1.000,\ \rho \geq \beta,\ T < 2.993\ sec.$
- Reactor startup involves slow controlled evolution from subcritical to critical operation followed by controlled exponential rise to point where heat is being generated.