Westinghouse Technology Systems Manual

Section 2.1

Reactor Physics Review

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2.1 REACTOR PHYSICS REVIEW

Learning Objectives:

1. Define the following terms:

    a. K_{eff}
    b. Reactivity
    c. Reactivity coefficient
    d. Power defect
    e. Poison
    f. Critical
    g. Supercritical
    h. Subcritical
    i. Startup rate

2. Describe the following reactivity coefficients and explain how their values change with core life and reactor power level:

    a. Moderator temperature
    b. Doppler-only power
    c. Void
    d. Power

3. Explain the relative effects of the following poisons in plant operations:

    a. Xenon
    b. Samarium

4. Explain how the following controllable poisons affect core reactivity:

    a. Control rods
    b. Chemical shim

5. Explain the inherent response of the reactor to the following transients:

    a. Secondary load changes
    b. Reactivity additions from control rod motion or boron concentration changes

6. Explain how the neutron population of a subcritical reactor changes in response to reactivity changes.

2.1.1 Introduction

This section represents a summary of basic nuclear physics and nuclear reactor design principles and terminology. The material presented is broader in scope than can be conveniently covered in the classroom time allotted; therefore, all the written material is not covered in detail. Basic explanations and definitions of concepts are

given in the classroom. It is emphasized that the written material is only a summary of the subject.

2.1.2 Fission Process

Nuclear fission is the splitting of the nucleus of an atom into two or more separate nuclei accompanied by the release of a large amount of energy. The reaction can be induced by a nucleus absorbing a neutron, or it can occur spontaneously, because of the unstable nature of some of the heavy isotopes. Very few isotopes have been excited to the state where the fission reaction occurs. These have been in the heavy elements, generally uranium and above on the chemical scale.

Nearly all of the fissions in a reactor are generated in the fuel by neutron absorption, which can result in the splitting of the fissionable atoms that make up the fuel. Only a few of the heavy isotopes are available in quantities large enough or present a sufficient probability of fission to be used as reactor fuel. These are uranium-233 (U-233), uranium-235, (U-235), uranium-238 (U-238) for fast or high-energy fission only, plutonium-239 (Pu-239), and plutonium-241 (Pu-241). Several other isotopes undergo some fission but their contribution is always extremely small. U-235 and U-238 are naturally occurring isotopes with very long half-lives; they are generally the fuels used for reactors. Artificially produced fuels include U-233, which is produced by the irradiation of thorium-232 (Th-232) in a reactor, and Pu-239 (produced by irradiation of U-238 in a reactor). Th-232 and U-238 are called fertile materials and are generally placed in the core or in a blanket surrounding the reactor for the express purpose of producing fuel (fissionable material) as the original fuel is used up in fission. The ratio of the amount of fuel that is produced in a reactor to the amount that is used during any period of time is called the conversion ratio of the reactor. If the amount of fuel produced is greater than the amount consumed, then the excess fuel produced is called a breeding gain. The fissile nucleus absorbs a neutron, and almost immediately, fission occurs. In the case of U-235, the reaction is represented by the following:

U-235 + n --> (U-236)*

(U-236)*--> FP_1 + FP_2 + 2.43 n + Energy

where FP = fission product,
n = neutron, and
"*" indicates the isotope is unstable.

In atomic studies it has become the practice to express energies in "electron volt" units, abbreviated "eV." It has been determined that gamma energies are frequently on the order of a million electron volts (MeV); the MeV has thus become a convenient unit for stating these (and related) energies.

The fission of any of the fissionable isotopes produces gammas, neutrons, betas, and other particles. The total energy released per fission is about 207 MeV for U-235; this energy is distributed as shown in Table 2.1-1.

The energy of the neutrinos, which accompany the radioactivity, is not available for producing power because these particles do not interact appreciably with matter; thus, the net energy available is 197 MeV, or roughly 200 MeV per fission. Table 2.1-1 lists the particles and the energy each particle produces per fission event.

Neutron production (neutrons per fission) varies with the different fissionable isotopes and with the energy at which the fission reaction is caused to take place. Table 2.1-2 shows some relative values for neutrons per fission for some of the common fuels that are now being used in reactors.

More than 99.3% of the neutrons produced are produced within 10^{-14} seconds; these are called prompt neutrons. It should be noted that each fission event does not produce the same number of neutrons. The number of neutrons per fission given in the referenced table represents an average number produced per fission.

The neutrons released from fission are not monoenergetic neutrons (neutrons having a single energy); they vary in energy from essentially thermal energy up to about 15 MeV. The energy distribution of these prompt neutrons is shown in Figure 2.1-1. The horizontal axis shows the range of prompt neutron energy distribution in MeV and the vertical axis shows the fractional neutron distribution in an incremental band (delta) around a selected energy level. The units for the vertical axis are fractional distribution per MeV. It can be seen that the area under the curve in an incremental band around 0.65 MeV yields the highest fraction of neutrons. Using the area under the curve it can also be seen that approximately 98% of all prompt neutrons are born at an energy level less than 8 MeV, and the average prompt neutron energy is approximately 2 MeV.

2.1.3 Moderation

In an actual operating reactor the probability of fission for typical reactor fuels is dependent on the energy of the incident neutrons. Fission neutrons are born fast (at high energies), and the probability that they will cause a fission in U-235 at that energy is very small. It is necessary to reduce this kinetic energy in order to increase the chance that they will cause fission. This is accomplished by interposing relatively non-absorbing nuclei as collision media to absorb the kinetic energy of fission neutrons through the process of elastic scattering. This medium is called the “moderator.” It acts to slow down or “thermalize” the fission neutrons.

Typical moderators are hydrogen, beryllium, and carbon. It should be clear that fewer collisions are necessary in a hydrogen medium to cause complete moderation than in carbon, since the nuclear mass of hydrogen is smaller and therefore more likely to absorb the kinetic energy of the neutron by the elastic scattering process. The amount of moderator in a multiplying system (a reactor, for instance) greatly influences the degree of slowing down that occurs. If there is too little moderator, the neutrons are not adequately slowed down. Therefore, the probability of fission is small when compared with optimum. If there is too much moderator, then the probability that a thermal neutron will be captured by the moderator (or some other nonfissionable material) is greatly increased. Figure 2.1-2 shows a snapshot of the neutron energy spectrum for a light water moderated reactor. The horizontal axis is

neutron energies in electron volts (eV) on a logarithmic scale, while the vertical axis is the neutron flux per unit energy. Neutron flux, ϕ, is the product of the neutron density (typical unit of measurement: neutrons/cm^3) and the neutron velocity (typical unit: cm/sec), and is usually expressed in terms of neutrons/cm^2•sec. As shown in this figure two peaks occur. The first peak is at fast neutron energies due to the prompt neutron production (within 10^{-14} sec) of the fission events. The second peak occurs at thermal neutron energy levels. This is caused by the diffusion of the thermal neutrons until they are absorbed by either a poison or the U-235 fuel. The time it takes a neutron to slow down from fast to thermal energy is relatively short, 5 microseconds, as compared to the thermal diffusion time of 210 microseconds. This results in a relatively low number of intermediate energy neutrons and the peak at the thermal energy level.

2.1.4 Nuclear Cross Section

The previous discussions of neutron reactions alluded to the fact that they have different probabilities of occurrence. A measure of the relative probability that a given reaction will occur is defined as the cross section of the nucleus for that specified reaction. More precisely, this measure is referred to as the microscopic cross section. In an approximate sense, the microscopic cross section may be considered as the effective area for interaction that the nucleus presents to the neutron.

The microscopic cross section has units of area (cm^2), and it is often expressed in units of barns (1 barn = 10^{-24} cm^2) for ease of manipulation. The microscopic cross section is represented by the symbol σ. It is made up of several component parts. The total cross section, σ_T is a combination of σ_c (capture), σ_s (scattering), and sometimes σ_f (fission), given by:

$$\sigma = \sigma_T = \sigma_c + \sigma_s + \sigma_f$$

where:

σ_c is the area presented for neutron capture; a neutron approaching an atom is exposed to an area of this apparent size.

σ_s is the area of the nucleus that will scatter or deflect the neutron.

σ_f, present in only a few of the many hundred nuclei, is the area that is presented for a neutron to strike the nucleus and cause a fission to occur.

These component cross sections are a function of the target nucleus (U-235, boron-10, etc.) and the incident neutron's energy (thermal, epithermal, or fast). In general, the probability of a given reaction is determined by the neutron's energy. The probability of absorption of neutrons for fission tends to follow an inverse velocity trend. Figure 2.1-3 shows this general trend in U-235. The horizontal axis is neutron energy in electron volts (eV) on a logarithmic scale, while the vertical axis is total neutron cross section (fission, capture, and scattering) in barns. For energy

levels above 100 eV, the predominant cross section is scattering. The neutrons collide with and bounce off the nucleus, but no further interaction takes place.

The peaks in the middle of the curve are due to resonance capture. A qualitative explanation of resonance capture relates to the fact that nuclei have discrete excited states. If the energy of the incident neutron is such that the energy of the resultant compound nucleus is equal to one of these states, then the neutron has a high probability of being captured with no resultant fission. These discrete energies determine where these isotopes will have abrupt increases in their capture cross sections. These spikes in the cross section curve are referred to as resonance peaks.

Light element resonances are not as observable, since their peaks are very broad and give the effect of a smooth curve. Below 1 eV the predominant cross section is the fission cross section. The probability of fission increases as the energy level of the neutron decreases. This is referred to as the 1/v region and is approximately a linear function.

The total effective cross section presented by all of the nuclei of a given isotope in a cubic centimeter is referred to as the macroscopic cross section (Σ):

$$\Sigma = N\sigma$$

where N = the number of individual atoms (or nuclei) per cubic centimeter.

The macroscopic cross section has dimensions of reciprocal length (cm^{-1}). It is sometimes convenient to consider the macroscopic cross section as the probability of neutron interaction per unit track length. The reciprocal of Σ is called the neutron mean free path (denoted by λ); it is a measure of the average distance a neutron will travel in the substance before it experiences the nuclear reaction under consideration. If a material contains several isotopes (i.e., a compound or a naturally occurring material with several isotopes), then the effective macroscopic cross section is:

$$\Sigma_T = \Sigma_1 + \Sigma_2 + \bullet \bullet \bullet + \Sigma_i$$

$$= N_1\sigma_1 + N_2\sigma_2 + \bullet \bullet \bullet + N_i\sigma_i$$

2.1.5 Neutron Multiplication

This chapter has thus far discussed the fission process, moderation, and cross sections. In this section, these three topics are combined into the quantity called the "multiplication factor," which describes all the possible events in the life of a neutron and effectively describes the state of a nuclear reactor.

The multiplication factor in a nuclear system is a measure of the change in the fission neutron population from one neutron generation to the subsequent generation. If the multiplication factor for a reactor core (or any nuclear assembly) is less than 1.0, a condition known as subcritical, then the system is decaying or dying

out and will never be self-sustaining. With a multiplication factor greater than 1.0, a condition referred to as supercritical, a nuclear system is producing more neutrons than are needed to be self-sustaining and is subjected to an increasing chain reaction that must be controlled by some exterior factor. The stable or critical condition of a nuclear system occurs when the multiplication factor is equal to 1.0 and there is no change in neutron population from one generation to the next. The effective multiplication factor is defined as:

$$K_{eff} = \varepsilon L_f p L_t f \eta$$

These symbols are defined and considered in detail in the following paragraphs.

In reactor operation, K_{eff} is the most significant property with regard to reactor control. At any specific power level or condition of the reactor, K_{eff} is kept as near to the value of 1.0 as possible. At this point in operation, the neutron balance is kept to exactly one neutron completing the life cycle for each original neutron absorbed in the fuel. An example of this balance is shown in Table 2.1-3.

The operational factors that affect reactor control are all-important because of the ways that they change the factors that make up K_{eff}. If any one of the contributing factors to K_{eff} (Table 2.1-3) changes, the ratio of 1.0 is not maintained. This resultant change in K_{eff} makes the reactor either subcritical or supercritical.

2.1.5.1 Fast Fission Factor

The fast fission factor, ε, is the contribution to neutron multiplication from the fissions that occur at higher-than-thermal energies. This contribution is from the fast fission in U-235 and U-238. The probability for a fission reaction in U-238 is relatively low, but there is so much of this isotope in the reactor core that there is a contribution to the multiplication factor. The fast fission factor is defined as the ratio of the neutrons produced by fissions at all energies to the number of neutrons produced in thermal fission. As core temperature is increased, the value of ε is increased because more fast neutrons are present to cause fission the the U-238 due to poorer moderating properties of the water. There is only a slight, almost insignificant, change over the core lifetime due to loss of U-238 by conversion to Pu-239.

2.1.5.2 Fast Nonleakage Factor

The fast nonleakage factor, L_f, is the fraction of neutrons that is not lost due to leakage from the core system during the slowing down process from fission energies to thermal energies. It is also the probability that a neutron will remain in the core and become a thermal neutron without being lost by fast leakage. It is represented by:

$$L_f = e^{\tau B^2}$$

for the continuous slowing down model, or by:

$$L_f = \frac{1}{1 + \tau B^2}$$

for the two-group model. "Two-group" indicates that the core can be described as consisting of only one group of thermal and one representative group of fast, or epithermal, neutrons. Fermi age, τ, is a measure of how far fast neutrons travel before being thermalized. B^2 is called buckling and depends on the shape and size of the core. Small cores have larger buckling than large cores.

As the temperature of the core increases, L_f decreases because of the increase in the numerical value of τ from the decreasing density of the water. The change in L_f over core life is almost insignificant; a change in L_f would be primarily due to a change in the core's metal-to-water ratio, which is relatively constant as the core ages.

2.1.5.3 Resonance Escape Probability

The resonance escape probability, symbolized by p, is the probability that a neutron will be slowed to thermal energy and will escape resonance capture. It is also the fraction of neutrons that escape capture during the slowing-down process. It is always less than 1.0 when there is any amount of U-238 or Pu-240 present in the core, which means that high-energy capture by these isotopes always removes some of the neutrons from the neutron life cycle.

As the reactor temperature increases, the resonance escape probability decreases in value because of the decrease in the ratio of the water-moderating atoms to fuel atoms and the broadening of the resonance capture cross sections. The resonance escape probability increases with core lifetime due to the decrease in fuel temperature. These changes in the resonance escape probability will be discussed further in section 2.1.6.1, "Fuel Temperature Coefficient."

2.1.5.4 Thermal Nonleakage Factor

The thermal nonleakage factor, L_t, is the fraction of the thermal neutrons that do not leak out of the core during thermal diffusion but remain to contribute to the chain reaction. L_t is also the probability that a thermal neutron will remain and be utilized in the core. It is a calculated value for each condition of the core and is represented by the equation:

$$L_t = \frac{1}{1 + L^2 B^2}$$

where L^2 is the thermal diffusion length squared and B^2 is the geometric buckling of the system. The value of L_t decreases as the temperature of the core increases; the effect can be seen from the value of L^2, which is a measure of how far thermal neutrons travel before absorption. When temperature is increased, the values of all absorption cross sections decrease. This increases L^2, which in turn decreases L_t. Buckling (B^2) does not change over the range of interest. As the core is operated and fuel is consumed, the value of L_t decreases as a result of fuel burnup.

2.1.5.5 Thermal Utilization Factor

The thermal utilization factor, f, is the ratio of the probability that a neutron will be absorbed in the fuel (fissile nuclides) to the probability that the neutron will be absorbed in all the material that makes up the core. This factor is the one that the plant operator has the greatest control over. It is described by the following equation:

$$f = \frac{\Sigma_a(\text{fuel})}{\Sigma_a(\text{fuel}) + \Sigma_a(\text{other})}$$

where Σ_a= macroscopic absorption cross section, which is the sum of the capture cross section, Σ_c, and the fission cross section, Σ_f.

$$\Sigma_a = \Sigma_c + \Sigma_f$$

An examination of the thermal utilization factor shows that the Σ_a (fuel) comprises only the absorption by the U-235 at the beginning of core life. As the amount of Pu-239 increases because of the irradiation of U-238 in the core, it is necessary to consider the change of fuel concentration in determining the value of f at different times in the core lifetime. The reactor operator can change Σ_a(other) by positioning of the control rods and by addition or removal of boric acid from the moderator.

2.1.5.6 Neutron Production Factor

The neutron production factor, η, is the average number of neutrons produced per thermal neutron absorbed in the fuel (fissile nuclides). It is based on physical measurement for each type of fuel used in a reactor.

The numerical value of η does not change with core temperature over the range considered for most reactors. There is essentially no change in η over the lifetime of the reactor core because the values for U-235 and Pu-239 are very close. As the reactor operates, and Pu-239 begins to contribute to the neutron economy of the core, the average effect on η is expressed by:

$$\eta = \frac{(\nu\Sigma_f)^{235} + (\nu\Sigma_f)^{239}}{\Sigma_a^{235} + \Sigma_a^{239}}$$

where ν is the number of neutrons per fission.

2.1.6 Reactivity and Reactivity Coefficients

In reactor physics, it is more convenient to use a term called reactivity rather than K_{eff} to describe the state of the reactor core. Reactivity (ρ or $\Delta K/K$) is defined in terms of K_{eff} by the following equation:

$$\rho = \frac{K_{eff} - 1}{K_{eff}}$$

Based on this equation, when K_{eff} is equal to 1.0, the reactivity of the core is zero, and the reactor is said to be critical. If K_{eff} is less than 1.0, reactivity of the core is negative, and the reactor is subcritical. Values of K_{eff} greater than 1.0 make the reactivity positive, and the reactor would be supercritical.

Mathematically, reactivity is a dimensionless quantity, but units are commonly applied. The most common units are %ΔK/K and pcm. A reactivity of 0.01 could be expressed as 1 %ΔK/K or 1000 pcm.

As any operating condition of the moderator or fuel (temperature, pressure, or void fraction) changes, the reactivity of the core changes accordingly. It is difficult to change any operating parameter and not affect every other property of the core. Once a change has been made to the core, it is necessary to make some compensating change to maintain criticality at the same power. For instance, assume that the reactor is critical at 50% power and that the reactor operator wants to increase power to 75%. The operator must first bring the reactor supercritical by making a positive reactivity change (control rod withdrawal or boron dilution) to initiate the power increase. As the power increases, the values of core parameters such as moderator temperature and fuel temperature change, causing a negative reactivity effect. If the added negative reactivity offsets the original positive reactivity change, then the reactor returns to the critical condition, and power stabilizes at the new value. The inherent properties of a reactor system that result in positive or negative reactivity additions upon changes in certain parameter values are generally described by reactivity coefficients.

A reactivity coefficient is defined as the change of reactivity per unit change in some operating parameter of the reactor. The coefficients that are significant in the reactor are the fuel temperature (Doppler) coefficient, moderator temperature coefficient, void coefficient, and pressure coefficient. In addition to these coefficients, one other coefficient is considered. It is called the power coefficient. The power coefficient is the combination of the first three coefficients. Most of the reactivity coefficients for the core are important to the safety of reactor operation because they act in a negative manner to oppose any power change in the nuclear system.

The response of the reactor core to plant conditions or operator adjustments during normal operation, as well as the response to abnormal or accidental transients, is evaluated by means of a detailed plant simulation. In these calculations, reactivity coefficients are required to couple the response of the core neutron multiplication to the variables which are set by conditions external to the core. Since the reactivity coefficients change during the life of the core, a range of coefficient values are established to ensure the correct response of the plant throughout its life.

2.1.6.1 Fuel Temperature Coefficient

The fuel temperature coefficient, also called the Doppler coefficient of reactivity, is the reactivity change per degree change in fuel temperature; it exists because the resonance cross sections of the fuel change with a change in fuel temperature. It is related to the resonance cross section broadening of some fuel materials, principally U-238 and Pu-240. Figure 2.1-4 is a plot of the total cross section vs. energy levels for the resonance region of U-238. The solid line represents the cross section of the fuel at cold temperatures (547°F). The dashed line represents the cross section of the fuel at hot temperatures (1500°F). As the temperature of the fuel increases, the heights of the resonance peaks decrease and the bases of the peaks broaden. The broadened peaks result in a larger percentage of neutrons having energies that are susceptible to capture in the fuel pellets. With colder fuel, only neutrons very close to the resonance-peak energy are absorbed. With hotter fuel, neutrons that are only somewhat close to the resonance-peak energy are also absorbed. Although the probability of resonance absorption by any particular fuel nucleus is unchanged at the hotter temperature, a neutron near a resonance energy must "run the gauntlet" of concentrated resonance absorbers (nuclei with fairly large absorption cross sections) in a fuel pin and thus will not avoid capture. This phenomenon leaves fewer neutrons per generation to cause fissions and is thus a negative reactivity effect.

Figure 2.1-5 is a plot of Doppler coefficient vs. effective fuel temperature. The effective fuel temperature is a weighted average temperature. The temperature profile across the fuel pellet makes this temperature lower than the average temperature and closer to the temperature the neutrons will be exposed to when they interact with the fuel. The terms BOL and EOL represent beginning of life and end of life. For this discussion and all others that follow, this is equivalent to new unirradiated fuel (BOL), and the fuel at the end of the first core cycle (EOL). Note that the EOL case is more negative than the BOL case for cooler fuel temperatures. This is caused by the buildup of plutonium-240, which has a higher cross section at lower temperatures. The difference at higher temperatures is minimal due to the broadening of the resonance peaks. Note that the Doppler coefficient is always negative.

In the event of an addition of a positive reactivity to the reactor core, the Doppler coefficient of reactivity would be the first and most important effect in controlling that addition. It is effective almost instantaneously, because an addition of positive reactivity causes the fission rate to increase and produce more heat in the fuel, which in turn causes an addition of negative reactivity. As quickly as the heating rate is increased, the Doppler coefficient becomes effective.

Since the effective fuel temperature can not be measured, the Doppler-only power coefficient used for reactor transients is a function of power. The Doppler coefficient used in the remainder of this text is defined as the change in core reactivity as the result of a change in the reactor power level. It is in units of $\Delta\rho/\Delta$%power, or change in reactivity per percent change in power. Figure 2.1-6 shows the Doppler-only power coefficient as a function of power. The results presented do not include any moderator coefficient even though the moderator temperature changes with power level.

The numerical value of the Doppler-only power coefficient changes with both reactor temperature and time in core life. As fuel temperature is increased, the U-238 resonance peaks broaden. However, the rate of broadening diminishes at higher temperatures, yielding a Doppler-only power coefficient with a smaller negative value at higher reactor power levels. Three important factors influence the Doppler-only power coefficient during core lifetime; these are: thermal conductivity of the gases in the gap between the fuel and the cladding, plutonium production, and fuel-clad gap reduction. Dealing with these factors one at a time will lead to the conclusion that the Doppler-only power coefficient is more negative at BOL than it is at EOL.

1. Initially the fuel rods are pressurized with helium gas, which yields a given fuel-clad gap thermal conductivity coefficient, but as fission gases such as xenon and krypton are produced, they tend to pollute the helium gas causing a reduction in the fuel-clad gap thermal conductivity coefficient. The result of this is an increased fuel temperature for any given power level, and the Doppler-only power coefficient becomes more negative as the core ages.

2. Plutonium buildup is the result of the conversion of U-238 to the plutonium isotopes. Pu-240 has a large thermal resonance for parasitic capture of neutrons. As Pu-240 builds in, the Doppler-only power coefficient becomes more negative as the core ages.

3. The fuel-clad gap reduction is the most dominant effect, outweighing the other two factors combined. The reduction of the gap between the fuel pellets and the cladding is a result of swelling of the fuel pellets and clad creep. Fuel pellet swelling occurs because fission gases cause the pellet to swell and crack. The pellet occupies a larger volume. At the same time, the cladding is distorted by outside pressure. These two effects result in direct fuel-clad contact. With direct contact, the overall thermal conductivity increases due to conductive heat transfer, which results in a lower effective fuel temperature for the same power level over core life. This causes the Doppler-only power coefficient to become less negative over core life. In the beginning of life case, the fuel temperature rises approximately 1000°F for a 0-100% power change, or 10°F/% power. For the end of life case with direct fuel-clad contact, the temperature rise is approximately 800°F, or 8°F/%power.

As a result of the three combined effects, the Doppler-only power coefficient will be more negative early in core life, and become less negative throughout the rest of core life, resulting in the EOL value being slightly less negative than the BOL value. Consequently, it is less effective for controlling power increases at EOL.

Figure 2.1-7 is a plot of the Doppler-only power defect vs. power. For this discussion and all others that follow, "defect" is defined as the total amount reactivity added to the core due to a change in power. The differences in the BOL and EOL values have been explained in the preceding paragraph. An operator would use this graph to determine the total amount of reactivity added to the core solely by the change in fuel temperature associated with a desired load change. For example, changing load from 80% to 100% at EOL would result in a Doppler-only reactivity defect of -225 pcm. This value is calculated as follows:

Final reactivity - Initial reactivity = Defect
-1025pcm - (-800pcm) = -225pcm

2.1.6.2 Moderator Temperature Coefficient

The next important reactivity coefficient is the moderator temperature coefficient (MTC). MTC is defined as the change in reactivity per degree change in moderator temperature; it is expressed in pcm/°F.

When the moderator temperature increases, its density decreases, so fewer moderator molecules are available to slow fast and epithermal neutrons and bring them to thermal energy. Decreasing the density of the moderator has two effects. First, if the density of the moderator is less, it increases the leakage probability of the neutrons. Second, if the neutrons are not slowing, they stay at a higher energy for a longer period, which increases the probability of non-fission capture of these neutrons. Both effects together cause a net decrease in the neutron population that adds negative reactivity to the core.

Moderator density changes are not linear. At high temperatures an increase in the moderator temperature causes a larger reduction in density than an identical increase at low moderator temperatures. This is illustrated by the zero (0) ppm curve (bottom curve) in Figure 2.1-8. Notice how the slope of this curve becomes more negative as the temperature of the moderator increases. This is a graphical representation of the non-linear density change of the moderator. This curve also displays the reactivity effect due to a 1°F change in the temperature of the moderator. As shown, if the moderator temperature changes from its present value (horizontal axis), a certain amount of reactivity, in pcm (vertical axis), is added to the core. As an example, using the 0 ppm curve, if the moderator temperature is initially at 500°F and its temperature is increased by 1°F, -17 pcm of reactivity would be added to the core.

Historically, commercial reactors sold by Westinghouse have been designed with a water volume to fuel volume ratio such that the value of the MTC is negative. The negative MTC and the negative fuel temperature coefficient ensure stable power operation of the reactor core. As explained in the previous paragraph, if the temperature of the moderator is increased, negative reactivity is added to the core. This negative reactivity causes reactor power to decrease, which acts against any further increase in temperature or power. Therefore, a condition that could cause an increase in reactor power is limited by the negative reactivity added by the increase in the moderator temperature.

Operating with a negative MTC is desired due to the negative feedback mechanism discussed above and to its favorable operational characteristics during power changes. When the operator increases the load on the turbine, governor valves supplying steam to the high pressure turbine open. This action increases the steam demand, causing steam pressure, the saturation temperature in the steam generators, and the moderator temperature to decrease. Reducing the temperature of the moderator adds positive reactivity to the core, causing reactor power to increase.

If the secondary load is reduced by the operator, the governor valves on the turbine close, reducing the steam demand of the turbine. The reduced steam demand causes the following to increase: steam pressure, saturation temperature, and the moderator temperature. Increasing the temperature of the moderator adds negative reactivity, which reduces reactor power. Therefore, the two above examples display how reactor power follows secondary steam demand due to the negative reactivity feedback caused by the MTC.

When the reactor is designed, excess reactivity in the form of fuel is added to the core so that it can operate at 100% power for an extended period. With excess fuel added to the reactor, the reactor could be taken critical and power could be escalated with the rods greatly inserted into the core. However, various operating limits require the rods to be fully withdrawn from the core (the all rods out [ARO] condition), so an additional poison must be added to the core. This poison addition is accomplished by adding soluble boron to the reactor coolant. By adding boron to the reactor coolant, negative reactivity is inserted uniformly throughout the core. Since boric acid is dissolved in the coolant, the excess reactivity in the core can be controlled by boron changes without creating flux distribution problems associated with the control rods.

Consider what happens to the MTC when the moderator contains both light water and boron. Boron is a poison. When it is added to the reactor coolant (moderator), it adds negative reactivity to the core in a similar fashion as the control rods. By adding a certain amount of boron to the reactor coolant, the power of the reactor can be maintained at its desired values of power and temperature with the control rods fully withdrawn from the core.

Although it is true that adding boron to the moderator has a negative reactivity effect on the core's reactivity, it has the opposite effect on MTC. As previously discussed, increasing the temperature of the moderator (0 ppm curve) has a negative reactivity effect. Now consider what happens when the temperature of the boron solution is increased.

During a temperature increase boric acid is expanded out of the core along with the moderator. Since boric acid is a poison, and it is expanding out of the core, positive reactivity is added. (The positive reactivity addition due to the expansion of boron out of the core offsets the negative reactivity addition due to the expansion of the moderator out of the core). As an example, using the 500 ppm curve on Figure 2.1-8, it is shown that for an initial temperature of 500°F, a 1°F increase adds -8 pcm of reactivity.

As explained earlier, -17 pcm was added to the core due to the density decrease of the moderator. For the conditions in the above example the same -17 pcm of reactivity was added to the core via the moderator expansion out of the core, but due to the concurrent expansion of boron out of the core, the net effect was -8 pcm.

The amount of positive reactivity added to the core depends upon the initial concentration of boron in the moderator. With high concentrations of boric acid, the positive effect could be large enough to overcome the negative effect of the moderator loss. In other words, a net positive reactivity addition could result from a

moderator temperature increase. At boric acid concentrations greater than approximately 1400 ppm, the MTC is positive.

Once, all Westinghouse-designed cores were required by technical specifications to operate with a negative MTC. This effectively limited the boric acid concentration of the reactor coolant. To reduce the dissolved poison requirement for control of excess reactivity, burnable poisons were incorporated in the core design, as discussed in Chapter 3.1. However, today, utilities are demanding the vendors to design cores that can operate from 18 to 24 months at full power. To operate this long, more fuel and burnable poisons must be added to the core. In addition, the boron concentration also has to increase. The increase in boron concentration is large enough in magnitude that the MTC has become positive. This has resulted in technical specification changes that allow operation of the plant with a positive MTC for certain conditions.

Operating with a positive MTC makes the operating characteristics of the core less stable. Ignoring the effect of fuel temperature changes, consider what would happen if MTC is positive. With a positive MTC, a 1°F increase in moderator temperature adds positive reactivity to the core, causing reactor power to increase. As reactor power increases, more energy is added to the moderator. As the moderator temperature increases, more positive reactivity is added to the core, further increasing power and raising the moderator temperature, etc. Reactor power is thus self escalating. Fortunately, the fuel temperature coefficient, which is always negative and stronger than the MTC, cannot be ignored. Therefore, the scenario described above (self-escalating reactor power) does not occur.

2.1.6.3 Void Coefficient

The next coefficient of reactivity that acts in response to a reactivity insertion is the void coefficient. It is defined as the change in reactivity as the result of boiling of the moderator in the core region and has the units of $\Delta\rho/\Delta$%void.

The formation of voids in the core has the same effect as the temperature increase of the moderator (decreasing the density of the moderator.) Therefore, the moderating ability of the core is reduced. If the moderator is borated, the decrease in moderator density also reduces the boron concentration. In general, void formation could have a positive or negative effect on reactivity depending on both boron concentration and the departure from the optimum fuel-to-moderator ratio. The void content of the core is about one-half of one percent. The void coefficient varies from -30 pcm/%void at BOL and low temperatures to -250 pcm/%void at EOL and operating temperatures.

2.1.6.4 Pressure Coefficient

The pressure coefficient is the change in reactivity caused by a change in pressure of the primary system. It is affected by the same mechanism that changes the moderator temperature and void coefficients. As pressure increases, the only change that takes place is in the density of the cooling and moderating water. The pressure coefficient is given in units of $\Delta\rho/\Delta P$. The pressure coefficient of reactivity has a slight positive effect on reactivity as the pressure of the system is increased if

boron concentration is very low or absent. At high boron concentrations, an increase in pressure produces a slightly negative effect on reactivity.

2.1.6.5 Power Coefficient and Power Defect

The power coefficient combines the Doppler, moderator temperature, and void coefficients. It is expressed as a change in reactivity per change in percent power, $\Delta\rho/\Delta\%$ power. It is negative at all times in core life but is more negative at the end of life primarily due to the change in the moderator temperature coefficient. Figure 2.1-9 shows the values of the power coefficient vs. percent power at BOL and EOL. This graph can be used to calculate the reactivity change associated with 1% incremental changes in power. Figure 2.1-10 is the integrated power coefficient, or power defect, vs percent power at BOL and EOL. This graph is more useful to the operator for calculating the total reactivity change for any power change.

Operators need to be concerned about the effect of the power defect. As power is increased the power defect has a negative reactivity effect to the core. Therefore an equal amount of positive reactivity must be added to keep the reactor critical or near critical. This positive reactivity will be in the form of rod withdrawal or boron dilution. When power is decreased quickly, as it will after a trip, power defect is a positive reactivity effect to the core, and the resulting rod insertion must be sufficient to make the reactor subcritical. An alternate method, boron addition, is not rapid enough to overcome this positive reactivity addition.

To ensure that the rods can shut down the reactor, they must be maintained above a minimum rod height. Since the magnitude of the power defect is dependent on total power, the minimum rod height is also increased with increasing power.

2.1.7 Poisons

In a reactor system, poisons absorb neutrons without causing fission. These neutrons are no longer in the neutron life cycle. Control rods and soluble boron are examples of poisons which are controllable. Xenon and samarium are examples of poisons which are uncontrollable.

2.1.7.1 Uncontrollable Poisons

In the fission process the nucleus absorbs a neutron and the resulting nucleus breaks into two or more parts called fission fragments. A detailed investigation of the thermal neutron fission of U-235 has shown that the resultant nucleus splits in many different ways, yielding more than 350 primary fission products (or fission fragments). In Figure 2.1-11, the percent fission yields of U-235 are plotted against the mass numbers. Note that each mass number represents several isotopes.

During power operation, the isotope xenon-135 (Xe-135) is formed as one of fission products. This isotope has an extremely large capture cross section (2.6 million barns). Xe-135 is formed in two ways; directly as a fission product (0.3% of the total fission products) and indirectly from the radioactive decay of tellurium-135 (5.9% of

the total fission products). The radioactive decay chain of tellurium-135 is as follows:

$$\text{Te-135} \xrightarrow[19.2\ \text{sec}]{\beta^-} \text{I-135} \xrightarrow[6.6\ \text{hr}]{\beta^-} \text{Xe-135} \xrightarrow[9.1\ \text{hr}]{\beta^-} \text{Cs135 (stable)}$$

Xe-135 is lost in two ways; it can capture a thermal neutron, forming Xe-136, which is stable and has a very small capture cross section, or it can decay to Cs-135. When the reactor is first brought to power, the concentration of Xe-135 (atoms/cm^3) is slowly built up to an equilibrium value. This is due primarily to the relatively long half-lives of iodine-135 (6.6 hours) and Xe-135 (9.1 hours). Because of the high thermal neutron cross section of Xe-135, as the concentration of the isotope increases, the macroscopic absorption cross section of the core increases.

Operationally, as Xe-135 builds up, other poisons in the core (control material such as control rods or boric acid) must be removed to maintain criticality. Provided there is enough control material to remove during this xenon buildup, equilibrium will be reached after approximately 48 hours of power operation. After this 48-hour time period a condition is reached where the production of Xe-135 is equal to the removal of Xe-135 by neutron capture plus the loss of Xe-135 through radioactive decay.

The equilibrium value for the amount of xenon in the core at any time is a function of the reactor's neutron flux (power) (see Figure 2.1-12). Since a neutron absorber (poison) is added to the reactor when xenon is built into the system, its effect on the chain reaction can be described in terms of reactivity. The multiplication factor is lowered primarily through the decrease in thermal utilization with a secondary effect on the thermal nonleakage probability caused by decreasing the thermal diffusion length. The reactivity effect can be shown to be approximately equal to the ratio of the macroscopic absorption cross section of the xenon to that of the fuel.

A change in power will cause a transient in xenon concentration. At the end of the transient, which will take about 2 days, the xenon concentration will reach its new equilibrium, assuming that power is left constant after the change. Three cases will be discussed: a power increase, a power decrease, and a shutdown.

Referring to Figure 2.1-13, following a xenon-free startup, a reactor has reached an equilibrium xenon concentration (time = 48 hr), and then power is increased. There is only a small increase in production of xenon because the production as a direct fission product is small (about 0.3%), and the iodine concentration cannot change very quickly, so the production by decay of iodine remains practically constant. Removal of xenon is increased significantly due to increased burnout (xenon absorbs neutrons). The decay of xenon to cesium remains almost constant initially.

These are only the initial effects, but it can be seen that the removal of xenon has increased more than the production, so xenon concentration decreases. This will continue for a few hours until the iodine concentration has increased enough that xenon production by iodine decay is greater than xenon removal. Eventually, the xenon concentration will reach a higher equilibrium value than that existing before

the transient. This will happen when production and removal rates are again equal (time=96 hr).

Later when power is decreased (time=150 hr), the production is again changed very little initially, but removal by burnout is reduced and xenon concentration increases initially. As iodine concentration decreases, production of xenon will decrease until removal is greater than production. Then xenon concentration will decrease until a new lower equilibrium is reached (time= 200 hr).

A shutdown is similar to a power decrease, but more severe (refer to Figure 2.1-14). Production of xenon directly from fission and removal by burnout almost cease. What is left is radioactive decay of iodine to produce xenon, and radioactive decay of xenon to remove it. The xenon concentration increases initially, just as in the power decrease case. However, the increase continues longer and reaches much higher levels before decreasing. The peak occurs about 8 or 9 hours after shutdown. Since iodine is no longer being produced, its concentration continues to decrease as it decays to xenon. The xenon, also decaying away, is almost gone in about 3 days.

In some cases Xenon-135 presents moderately sensitive reactor operating conditions. Consider a reactor that has operated at power long enough to establish a high xenon concentration. Assume that the reactor trips and that a rapid recovery to full power is achieved simultaneously with the point in time at which “peak” xenon occurs. In returning the reactor to power, the negative reactivity associated with the xenon is overcome, as indicated above, by withdrawing the control rods to a higher position than usual. With the high flux of full power operation acting to “burnout” the accumulated Xe-135 and the decay of Xe-135 due to the shutdown, the concentration quickly falls off, and the result is a positive reactivity insertion. It should be pointed out that properly designed power reactors will overcome the xenon burnout quite adequately with proper control system operation.

Figure 2.1-15 shows reactor trips and returns to power at various power levels. Note the drop-off in xenon concentration during the restart. At EOL there is a possibility that there is not enough positive reactivity remaining in the fuel to overcome the negative reactivity added by xenon. In such a case the reactor restart is delayed until the xenon has decayed to lower values.

Another important fission product poison encountered in reactor operation is samarium-149 (Sm-149). This stable isotope has a capture cross section of about 4 x 10^4 barns. It enters the system as the end product of the following decay chain:

$$\text{Nd-149} \xrightarrow[1.7\ \text{hr}]{\beta^-} \text{Pm-149} \xrightarrow[53\ \text{hr}]{\beta^-} \text{Sm-149 (stable)}$$

(Nd: neodymium, Pm: promethium)

These products occur in about 2.1% of U-235 fissions. Because Sm-149 is stable, its equilibrium concentration in any reactor produces about -0.65% $\Delta K/K$ or -650 pcm of reactivity, independent of the neutron flux (power level) as shown in Figures 2.1-16 and 2.1-17. The rate at which the Sm-149 concentration approaches this

equilibrium is related to the Pm-149 (53-hour) half-life. Upon reactor shutdown (Figures 2.1-18 and 2.1-19), the Sm-149 concentration builds up to an asymptotic value dependent on the equilibrium concentration of promethium, which is power-level dependent.

2.1.7.2 Controllable Poisons

Two controllable conditions existing in a nuclear reactor greatly affect the reactivity balance within the system. These are control rod position and soluble poison concentration.

Control rods are made of a material or materials with an extremely high capture cross section for neutrons. Consider a just-critical reactor in which a control rod is partially inserted. If the rod is withdrawn slightly, K_{eff} will increase to a value slightly greater than 1.0. By withdrawing the control rod, neutron absorbing material is removed from the core. The effect is an increase in the thermal utilization factor, and (as seen in the six-factor formula discussed earlier) K_{eff} is increased. Control rod reactivity effects vary with their location in the core. If a control rod is located at the midplane of the core, its worth per step of movement is greater than if it were almost totally withdrawn or inserted.

A typical differential control rod worth ($\Delta\rho/\Delta$ rod position) curve is shown in Figure 2.1-20(a). Note that the maximum differential worth is found when the rod is positioned at about 40% withdrawn. This curve can be integrated to provide a determination of total control rod worth, which is defined as that amount of reactivity change associated with a given total movement of the control rod. Figure 2.1-20(b) shows the integrated control rod worth curve for the given differential rod worth.

Reactivity control is also maintained by using soluble boron poison in the moderator or coolant that passes through the reactor core. Addition of boron reduces the thermal utilization factor and requires that the control rods be further removed from the core to sustain the just-critical system. In most large pressurized water reactors today, large amounts of positive reactivity are held down by a soluble poison such as boric acid. Boron-10 is the principal absorber in this solution. When used as gross control, the plant can operate with almost all control rods fully removed. This works to optimize the power distribution and heat transfer through the core and at the same time enhances the available shutdown safety margin.

Soluble poisons are injected through the makeup portion of the chemical and volume control system. As reactor fuel is consumed and the installed excess reactivity drops, the soluble poison is removed through the chemical and volume control system, compensating for the reactivity loss due to fuel depletion.

2.1.8 Reactor Response to Reactivity Changes

The preceding sections have described the reactivity effects associated with reactor parameters and fission product poisons. This section discusses how reactivity effects interact to control the reactor's response to transient conditions, with a

particular emphasis on the reactor's inherent response when actions to control plant parameters at their desired values are not taken.

If the moderator temperature coefficient is negative and reactor power is greater than the point of adding heat, reactor power follows secondary power. If the turbine load changes from one steady-state value to another, reactor power will undergo the same change, even if no actions (manual or automatic) are taken to control reactor power. Also, if plant operators add reactivity (positive or negative) by changing control rod positions or the coolant boron concentration without changing the turbine load, the final reactor power will be unchanged; the net result of the reactivity addition will be a new steady-state coolant temperature.

Consequently, during normal operations at power, PWR plant operators control reactor power by adjusting the output of the secondary plant (usually the turbine load). Operators adjust control rod positions and coolant boron concentration to maintain the coolant temperature at the desired value for a given power level (i.e., to conform to the sliding T_{avg} program described in Section 1.2).

The terms "secondary power" and "turbine load" are often used interchangeably, because during normal operation, secondary power *is* the turbine load. However, steam exiting the steam generators can also leak from steam piping, or it can travel directly to the condenser via the steam dump valves. Under these circumstances, the total secondary power equals the turbine load PLUS the equivalent power associated with the other steam flows. Therefore, secondary power is a more inclusive term.

At equilibrium, two conditions are always satisfied: reactor power equals secondary power, AND net reactivity equals zero (K_{eff} = 1; the reactor is critical). Following an upset that affects power or coolant temperature, the plant will inherently (without control action) find a new equilibrium state in which those conditions are again satisfied, unless a reactor trip setpoint is reached while the plant "tries" to get there.

Consider an increase in secondary power. The inherent plant response is that reactor power increases to the new higher power level, and the coolant T_{avg} drops to a new lower steady-state value. Increasing the secondary power increases the rate of energy removal from the reactor coolant in the steam generators, thereby reducing the coolant temperature. With a negative MTC, the drop in temperature adds positive reactivity:

Reactivity (moderator) = (MTC)(ΔT);
$\Delta T = T_{avg(FINAL)} - T_{avg(INITIAL)}$

(In the above expression, MTC and ΔT are both negative.) The positive reactivity addition causes reactor power, and thus fuel temperature, to increase. The fuel temperature increase adds negative reactivity:

Reactivity (fuel) = (DOPC) (Δ%power);
DOPC = Doppler-only power coefficient, and
Δ%power = $power_{FINAL}$ - $power_{INITIAL}$

(In the above expression, the DOPC is negative and Δ%power is positive.) Reactor power increases until it again equals secondary power. At that point, the rate of energy transfer to the coolant from the reactor equals the rate of energy removal from the coolant in the steam generators, and the coolant temperature stops decreasing. The coolant T_{avg} decreases to a value for which the positive reactivity from the coolant temperature decrease completely offsets the negative reactivity from the reactor power (fuel temperature) increase. At the new equilibrium, reactor power again equals secondary power (at a higher power level) and net reactivity again equals zero (with different constituent reactivities).

Contrast the inherent response to the "controlled" response. For an increase in secondary power, the sliding T_{avg} program calls for T_{avg} to increase. If the MTC is negative, increasing the coolant temperature over the power change will add negative reactivity. Fuel temperature will also increase and, because of the negative DOPC, also add negative reactivity. To offset both of these negative reactivity additions (combined, they constitute the power defect described in Section 2.1.6.5), the operators must add positive reactivity by withdrawing control rods (or allowing them to automatically withdraw) or by diluting the reactor coolant. These operator-controlled actions are NOT required to make the reactor power follow the secondary power; they are required to maintain T_{avg} at its programmed value.

As a second example, consider a reduction in turbine load from 60% to 50%. This 10% reduction causes an imbalance between reactor power and secondary power; the reactor is adding energy to the coolant at a faster rate than the steam generators (ultimately, the turbine) are removing energy from the coolant. As a result, T_{avg} rises. The increase in T_{avg} adds negative reactivity ($-\rho$) to the reactor (recall that MTC is negative). This $-\rho$ makes the reactor subcritical, and reactor power begins to drop. The decrease in reactor power adds positive reactivity ($+\rho$) via the DOPC (the fuel temperature is decreasing). This $+\rho$ from the power decrease offsets the $-\rho$ from the T_{avg} increase.

For illustrative purposes, consider that the above changes in power and temperature take place in small sequential increments. Assume that the value of the MTC is -20 pcm/°F, and that the value of the DOPC is -10 pcm/%. Each time T_{avg} rises 1°F, the reactor becomes subcritical by 20 pcm, causing reactor power to decrease. When power has decreased by 2%, +20 pcm will have been added by the reactor power change, returning the reactor to an exactly critical condition ($\rho = 0$). Thus, it can be said that each increase in T_{avg} results in a corresponding drop in reactor power that returns the reactor to an exactly critical condition. In this example, five such incremental steps will lower reactor power by 10%. T_{avg} will eventually rise 5°F (adding -100 pcm of reactivity), and reactor power will lower 10% (adding +100 pcm).

As described above, when the operator initially decreases turbine load, the power mismatch between the primary and the secondary causes T_{avg} to rise. Each incremental rise in T_{avg} adds $-\rho$, which causes reactor power to decrease. Each time power decreases, the decrease in fuel temperature adds $+\rho$ to offset the $-\rho$ from the change in moderator temperature. These changes (T_{avg} increasing, reactor power decreasing) continue until reactor power reaches the new value of turbine

load. At the new equilibrium state, reactor power equals secondary power, and T_{avg} stops changing. Net reactivity is zero; the $-\rho$ from the increase in T_{avg} has been offset by the $+\rho$ from the drop in reactor power.

To summarize, the inherent effect of lowering turbine load with no primary control actions is that reactor power drops to equal secondary power, and T_{avg} rises.

In a final example, the operators withdraw control rods without changing turbine load. The rod withdrawal adds $+\rho$, causing reactor power to rise. As the power increases, the fuel temperature increase adds $-\rho$ via the DOPC, which offsets the $+\rho$ from the rod withdrawal. Now the reactor is again exactly critical (net $\rho = 0$), but reactor power exceeds secondary power, so T_{avg} rises. The rise in T_{avg} adds $-\rho$, which causes reactor power to drop. When power drops, the decrease in fuel temperature adds $+\rho$, which offsets the $-\rho$ from the T_{avg} increase and returns the reactor to an exactly critical condition. As in the previous example, T_{avg} continues to rise until reactor power equals secondary power. At the end of the transient, T_{avg} is higher than its starting value, and reactor power has returned to its original value (which equals the unchanged turbine load). In the new equilibrium reactivity balance, $+\rho$ from the rod withdrawal has effectively been exchanged with $-\rho$ from the moderator temperature increase.

The above discussion is slightly oversimplified. Actually, a change in reactor coolant temperature changes the conditions for heat transfer in the steam generators; the resultant change in steam pressure can affect the power level. (Recall the expression introduced in Section 1.2 for primary-to-secondary heat transfer: $Q = UA[T_{avg} - T_{ref}]$.) Some turbine control schemes can respond to steam pressure changes in order to maintain the desired turbine load. However, the predominant effect is that reactor power follows turbine load, and that the reactor coolant temperature assumes whatever value is necessary to return net reactivity to zero. In addition, a change in reactor coolant temperature also changes the conditions for heat transfer from the fuel to the coolant, and thus affects the fuel temperature. (The temperature differential between the fuel and the coolant must remain about the same for heat transfer from the fuel to the coolant to remain constant.) However, any resultant change in fuel temperature is insignificant when compared to the fuel temperature change associated with a power change. It is therefore useful to consider fuel temperature to be a function of reactor power only.

2.1.9 Reactor Kinetics

Consider a reactor in which K_{eff} is exactly 1.0. With equilibrium established, the neutrons entering into the self-sustaining reaction are a mixture of prompt and delayed neutrons. The fraction of this mixture that are delayed neutrons at steady state is known as β. Each fuel has a characteristic β due to its yield of delayed neutron producers. A mixture of fuels, such as what would be present in a commercial reactor after it has operated, has a β which is a weighted average of the βs for the fuels present. For a reactor with only U-235, β is about .007 or 0.7%. Even though this is a small fraction of the neutrons, delayed neutrons are essential to reactor control. Delayed neutrons do not appear until about 13 seconds after the

fission that results in a delayed neutron producer. The long time delay is because the delayed neutron producer must decay before a neutron is released.

After the release of the delayed neutron, it will go through the same life cycle with the prompt neutrons, which were released immediately after the fission. The lifetime for a prompt neutron is about 10^{-4} seconds, but the lifetime for a delayed neutron is about 13 seconds ($13 + 10^{-4} \sim 13$). The life cycle time or generation time is a weighted average of the lifetimes for prompt and delayed neutrons. Since the delayed neutron fraction is β, then the prompt neutron fraction is (1-β). These are used as the weighting factors. The generation time is about 0.1 seconds. The reactor would be uncontrollable with a generation time of 10^{-4} seconds but is controllable with 0.1 seconds, as can be seen in the following discussion.

As previously stated, if the fuel is U-235 and the reactor is critical, 0.7% of the fission neutrons are delayed. That means that 99.3% are prompt. Assume now that an adjustment has been made so that K_{eff} is increased to 1.001, or 0.1% supercritical. The system is now multiplying at a given rate proportional to the number of neutrons present within the system. After the transient effects decay out, the rise in the neutron level can be described by a single exponential equation:

$$n(t) = n_0 e^{t/T}$$

where n_0 = the neutron level at time zero
t = the time in seconds
T = the stable reactor period (the time it takes the neutron level to increase by a factor of e)

At the instant the reactor is made supercritical, the neutrons in the core are a mixture of 0.7% delayed and, in this instance, about 99.4% prompt neutrons (99.3 + 0.1%).

If K_{eff} were increased to 1.007, then the reactor would be critical on prompt neutrons alone, no waiting for delayed neutrons would be necessary, and it would be uncontrollable by normal means. The effect of the delayed neutrons on generation time is lost. To put it another way, the power would be increasing at such a rapid rate that it would be impossible to control the reactor short of spontaneous shutdown on its own inherent mechanisms.

The relationship between reactivity and the reactor period can be expressed mathematically. The period of the reactor is useful in equations for reactor analysis. The more useful term for reactor operation is the related startup rate (SUR). The startup rate is expressed in terms of the reactor period in accordance with the following equation:

$$SUR = 26/T$$

where SUR = startup rate in decades per minute
T = period in seconds.

Therefore, if the reactor power level increases by a factor of 10 every minute, then the reactor SUR is 1 decade per minute, which is equal to a period of 26 seconds. Figure 2.1-21 is a plot of reactivity addition versus reactor startup rate. If a reactor operator wished to establish a 1 dpm startup rate from a critical condition it would require an addition of 175 pcm of positive reactivity. The startup rate can also be used to predict power changes if the time the SUR will be in effect and the starting power level are known.

The power of the reactor can be expressed by:

$$p = p_0 10^{SUR \cdot t}$$

where: p_0 = power at time zero
SUR = startup rate in decades/min
t = time in minutes

2.1.10 Subcritical Multiplication

There is a third category of neutrons, in addition to prompt and delayed neutrons, that are not fission neutrons, but are very important during startup and shutdown operations when the reactor is subcritical. These are known as "source neutrons," and they come from sources other than neutron-induced fission. These sources include spontaneous fission of certain nuclides and source assemblies specifically loaded into the core to emit neutrons. (Source assemblies are described in Section 3.1.) Without source neutrons, the neutron population in the core would gradually dwindle toward 0; each neutron generation would have fewer neutrons than the previous one because K_{eff} is less than 1.0. With source neutrons, the population remains at levels that can be measured by the source range nuclear instruments, so that operators can always ascertain how fast the neutron population is changing.

When the reactor is made subcritical after operating at a critical state, the neutron population at first undergoes nearly a step decrease with the rapid reduction in prompt neutrons, and after a short time begins to decrease exponentially with a startup rate of -1/3 decade per minute. This rate of decrease is the result of the delay of the longest-lived delayed neutron producers. Without source neutrons, the decrease would continue until the core neutron population is negligible. However, the neutron population eventually levels off, because at equilibrium the addition of source neutrons just makes up for the losses from the neutron life cycle. The source neutrons enter the life cycle and experience the same environment that fission neutrons experience. For a defined source strength of S neutrons per neutron generation, the core neutron population contains:

S	neutrons from the current generation,
$S \cdot K_{eff}$	neutrons from the immediately previous generation,
$S \cdot (K_{eff})^2$	neutrons from two generations ago, and…
$S \cdot (K_{eff})^n$	neutrons from n generations ago.

The total core neutron population, N, is thus:

$$N = S(1 + K_{eff} + [K_{eff}]^2 + \ldots + [K_{eff}]^n)$$

The above expression converges to:

$$N = \frac{S}{1 - K_{eff}} \text{ for } K_{eff} < 1$$

Hence, the neutron population of a subcritical reactor does not decrease to 0; it reaches an equilibrium value which depends on the source neutron strength and the value of K_{eff}. The indication of the source range neutron detectors (a measured count rate; see Section 9.1) would be a value proportional to N in the above expression ($CR \propto N$).

The above expression explains what happens to the core neutron population when the value of K_{eff} is changed. During a startup, for example, an initially subcritical reactor is driven to criticality via a series of discrete rod withdrawals. After each withdrawal, the operators pause to allow the reactor to attain equilibrium. On the way to criticality, the value of K_{eff} is made closer and closer to 1.0, and the equilibrium neutron population becomes larger and larger. Put another way, after each rod withdrawal, the core neutron population increases and then levels off at a higher value determined by the new larger (but still subcritical) K_{eff} value. Eventually, of course, enough positive reactivity is added to make the reactor critical, at which time $K_{eff} = 1.0$, and the source neutrons become inconsequential.

Also, the amount of time it takes to reach equilibrium after a reactivity change increases as K_{eff} approaches 1.0. In other words, as criticality is approached, it takes longer for the count rate measured by the source range instruments to level off. The reason is evident in the series expression above; as K_{eff} increases, more terms in the series are significant, meaning that more previous generations of source neutrons contribute to the present population.

A decrease in K_{eff} for a subcritical reactor would cause similar changes in the opposite direction. If K_{eff} is changed from one subcritical value to a smaller one, the neutron population decreases until it levels off at a new lower equilibrium value determined by the neutron population equations defined above.

One use of the count rate is to predict the point of criticality as fuel assemblies are added or reactivity is increased. However, because the source range CR gets infinitely large as K_{eff} approaches 1.0 (recall that the CR is proportional to N), the inverse of CR is plotted. As criticality is approached, 1/CR approaches zero; the value of 1/CR thus provides operators with an effective tool for monitoring the approach to criticality.

Table 2.1-1
Particles and Energy Produced per Fission Event

Instantaneous Energy Release	MeV/fission
Kinetic energy of fission fragments	165
Prompt γ energy	5
Capture γ energy	5
Kinetic energy of prompt neutrons	7
Total	182
Delayed Energy Release	
β Energy from fission products	7
γ Energy from fission products	8
Neutrons	10
Total	25

Table 2.1-2
Neutrons per Fission

Isotope	Neutrons
U-233	2.51
U-235	2.43
U-238 (fast fission)	2.47
Pu-239	2.90
Pu-241	3.06

Table 2.1-3
Typical Neutron Balance for U-235

Neutron Generation Stage	Number	Factor	Gain/Loss
Beginning of generation: fast neutrons from thermal fission	1000		
Fast neutrons from thermal fission and fast fission	1175	ε = 1.175	+175
Fast neutrons remaining after fast leakage	1151	L_f = 0.98	-20
Neutrons that escaped resonance capture while slowing down to thermal energies	1001	p = 0.87	-150
Thermal neutrons remaining after thermal leakage, to be absorbed by core materials	981	L_t = 0.98	-20
Thermal neutrons absorbed by fuel (fissile nuclides), not absorbed by non-fuel core materials	488	f = 0.497	-493
Fast neutrons resulting from thermal fission in fuel (end of generation and beginning of next generation)	1000	η = 2.05	+512

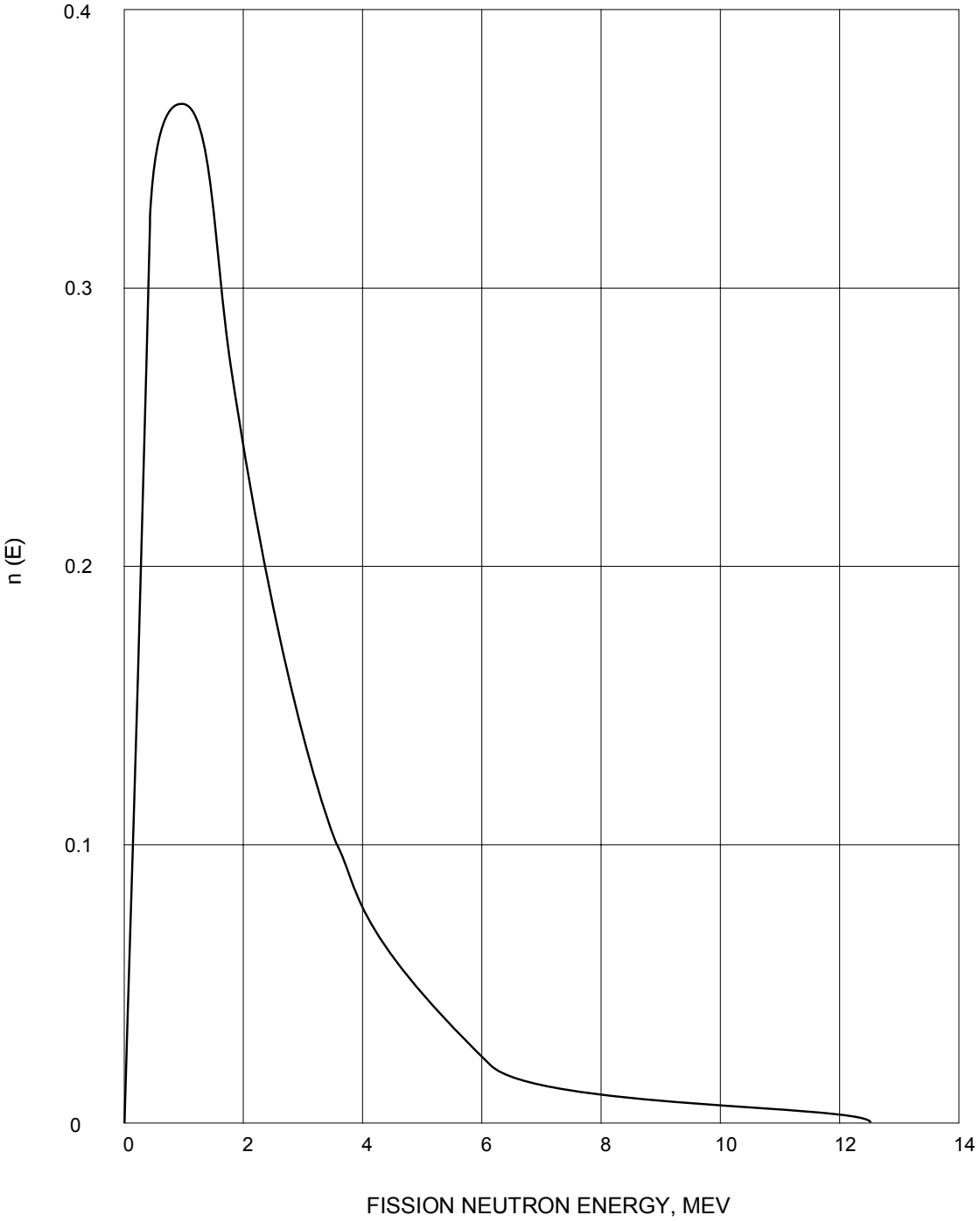


Figure 2.1-1 Fission Neutron Energy

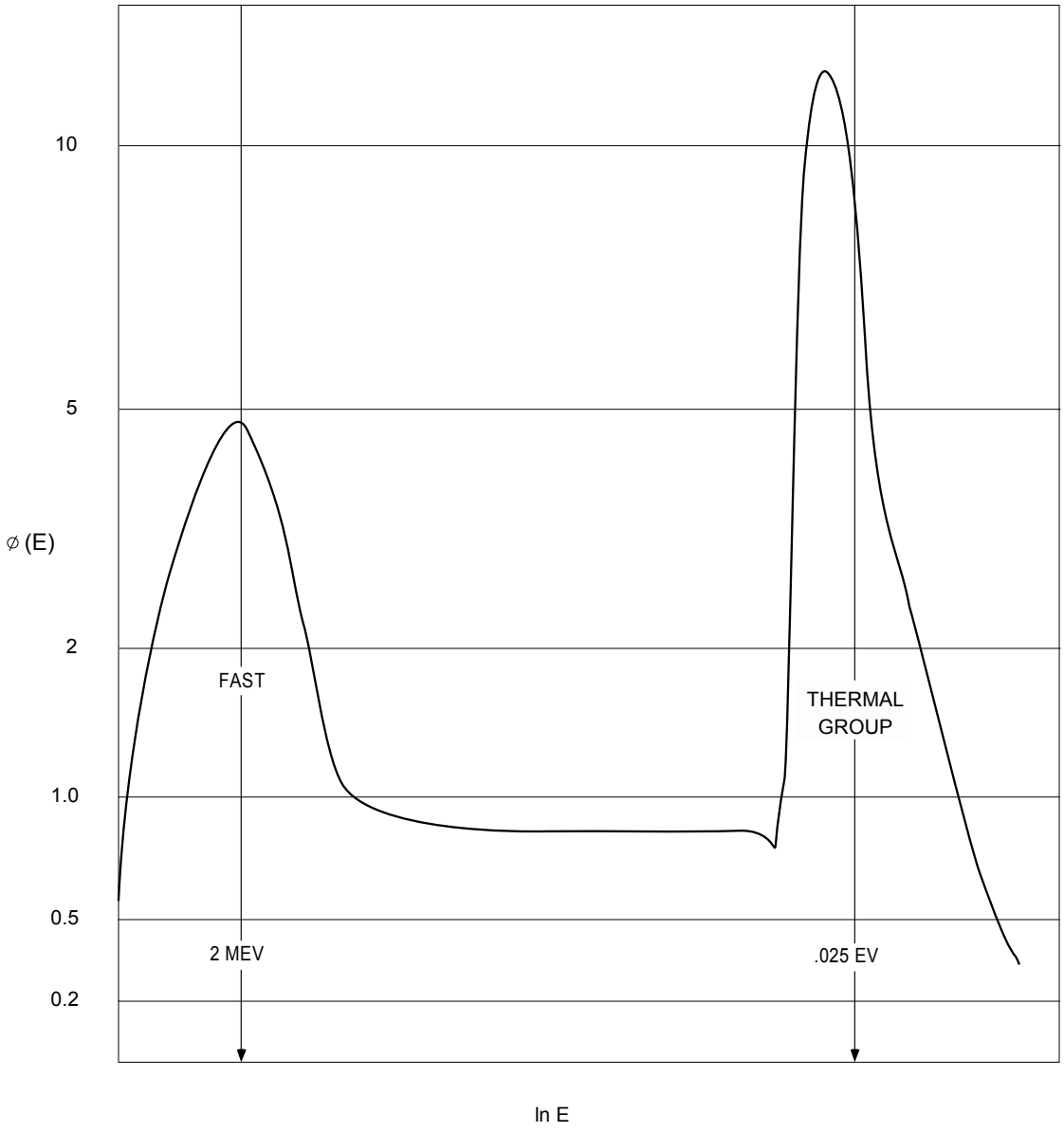


Figure 2.1-2 Flux Distribution

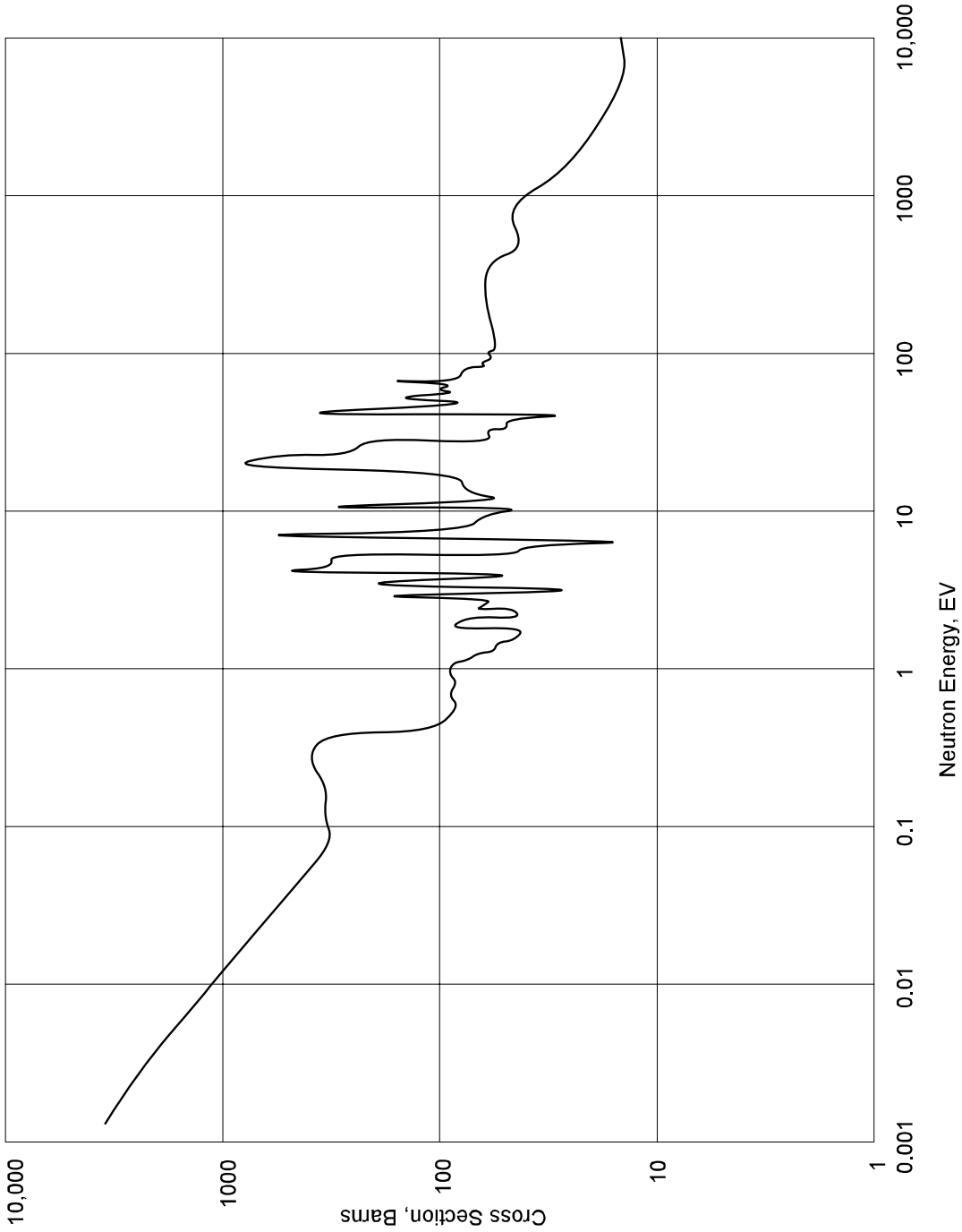


Figure 2.1-3 Total Cross Section for U-235

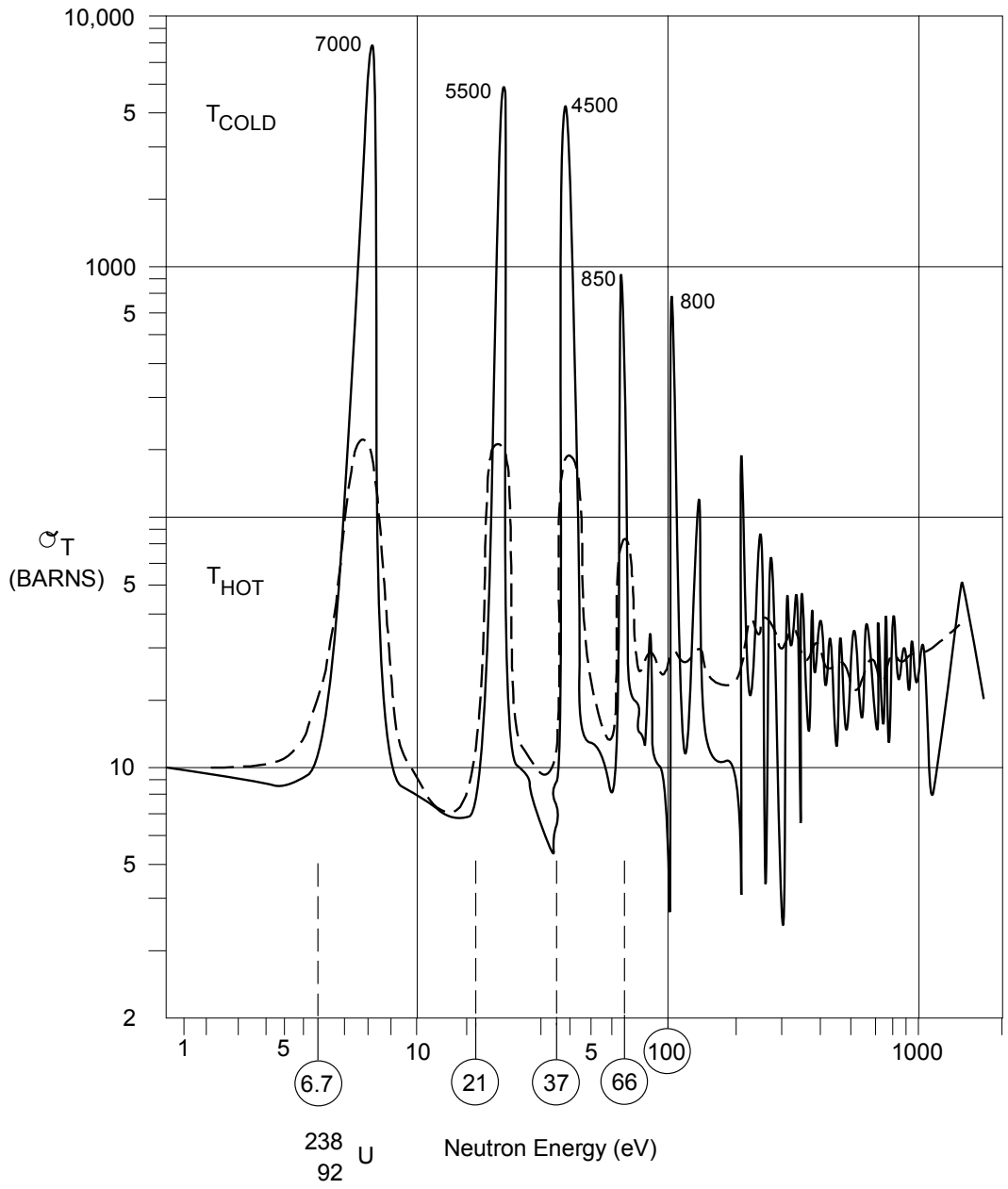


Figure 2.1-4 Cross Section Curve

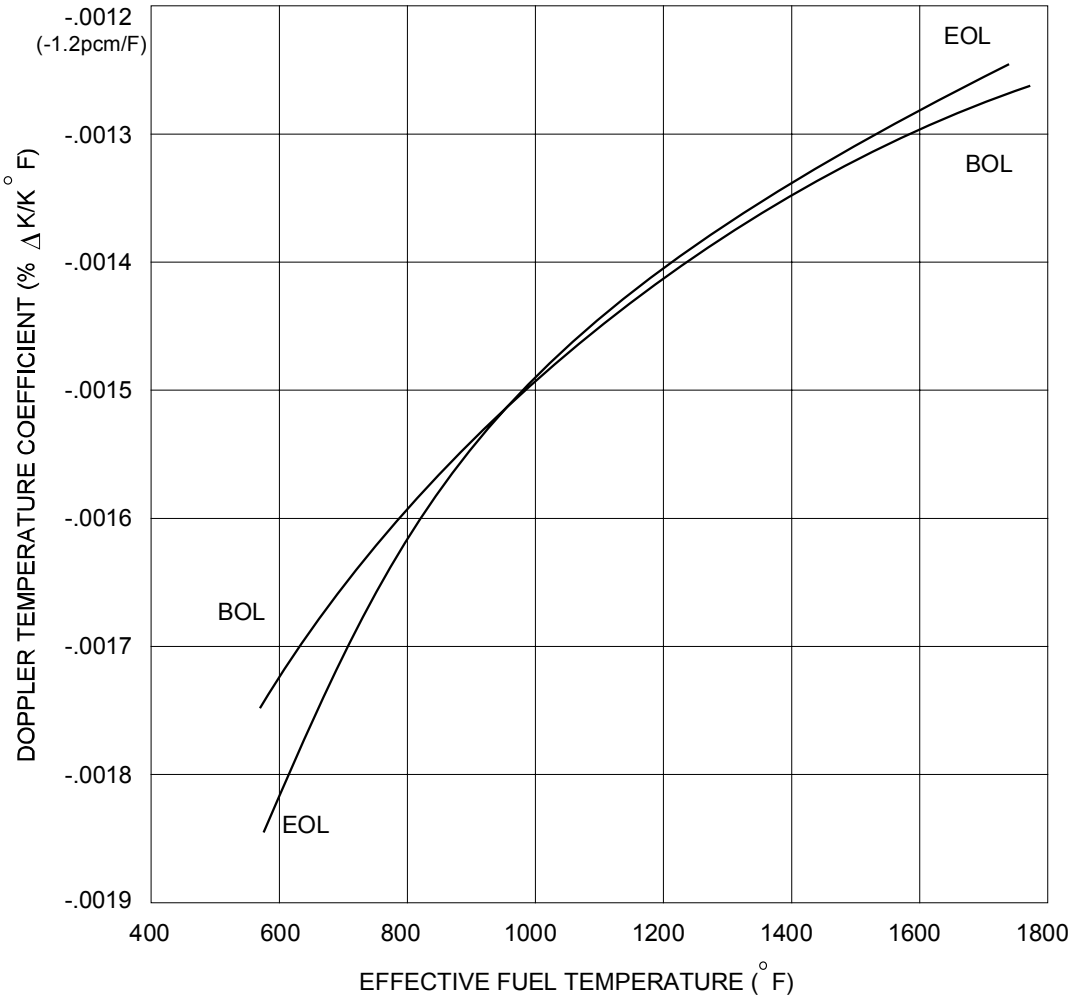


Figure 2.1-5 Doppler Temperature Coefficient, BOL and EOL

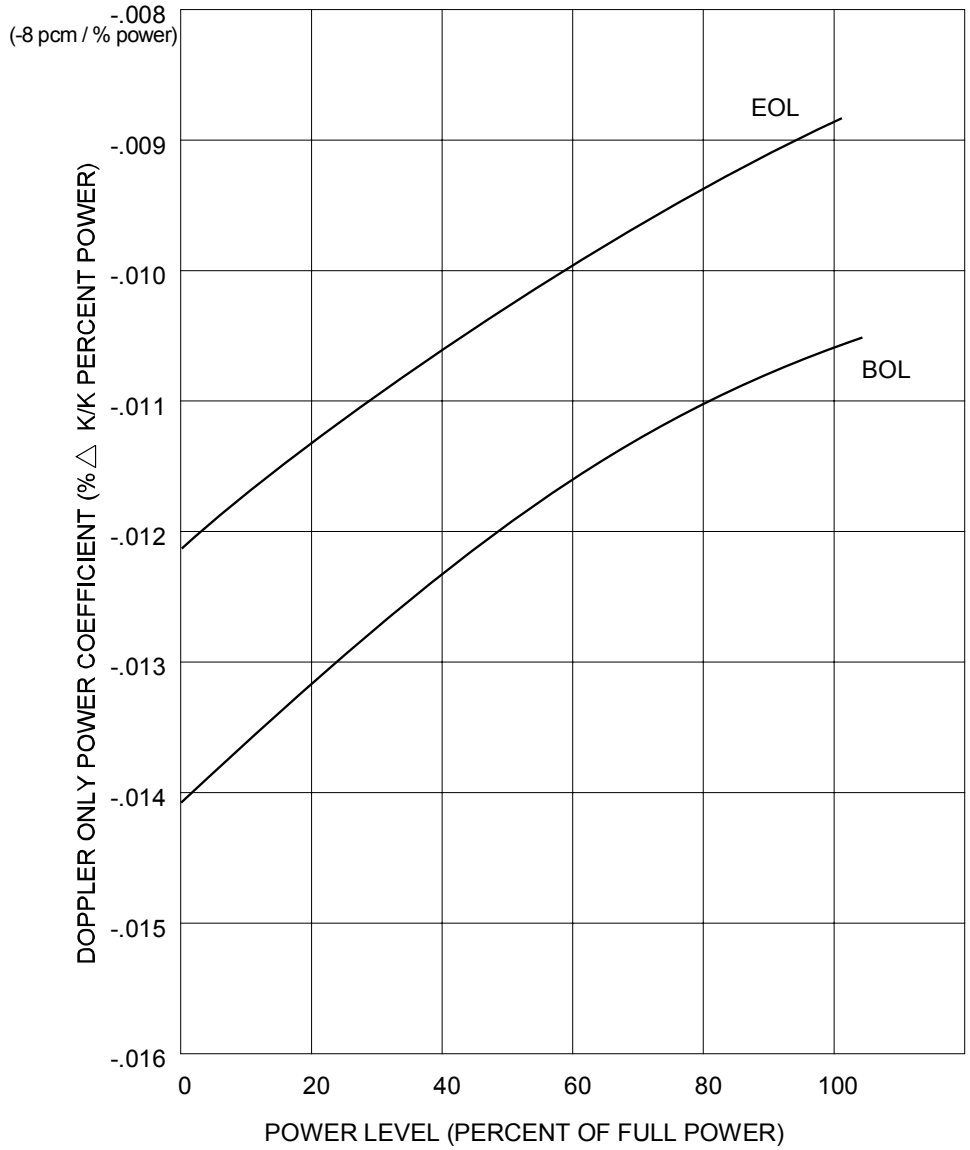


Figure 2.1-6 Doppler Only Power Coefficient, BOL and EOL

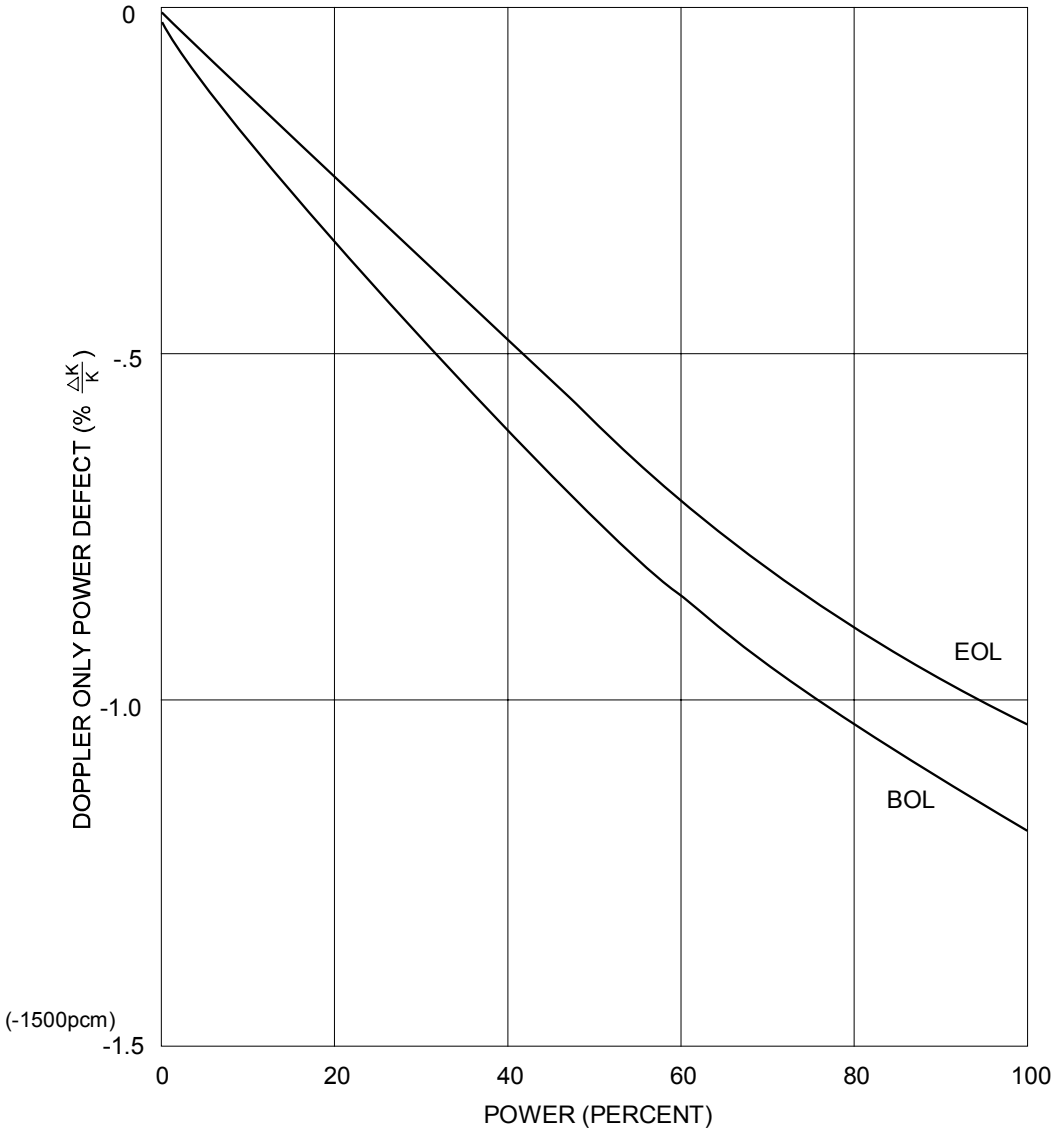


Figure 2.1-7 Doppler Only Power Defect, BOL and EOL

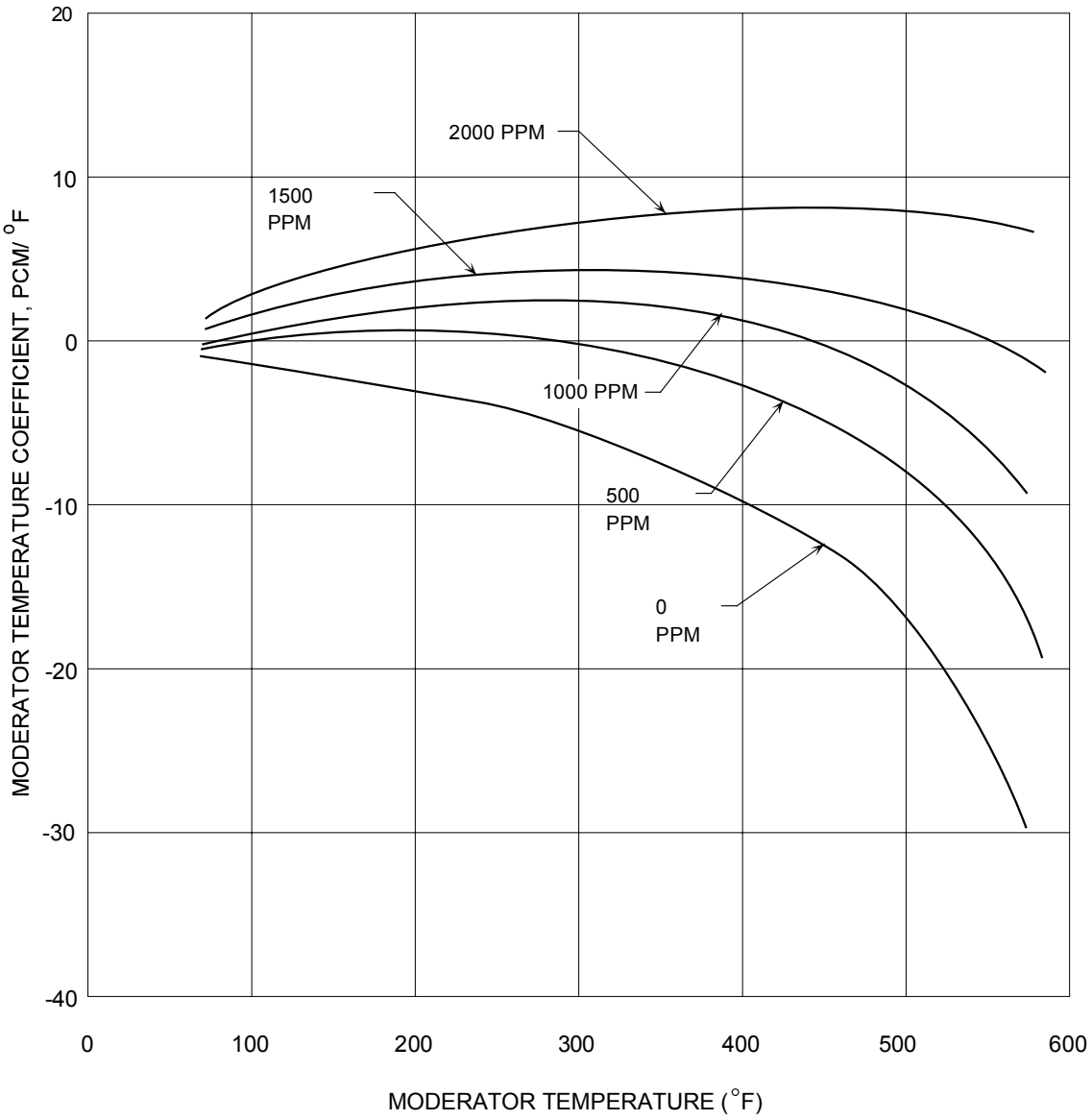


Figure 2.1-8 Moderator Temperature Coefficient

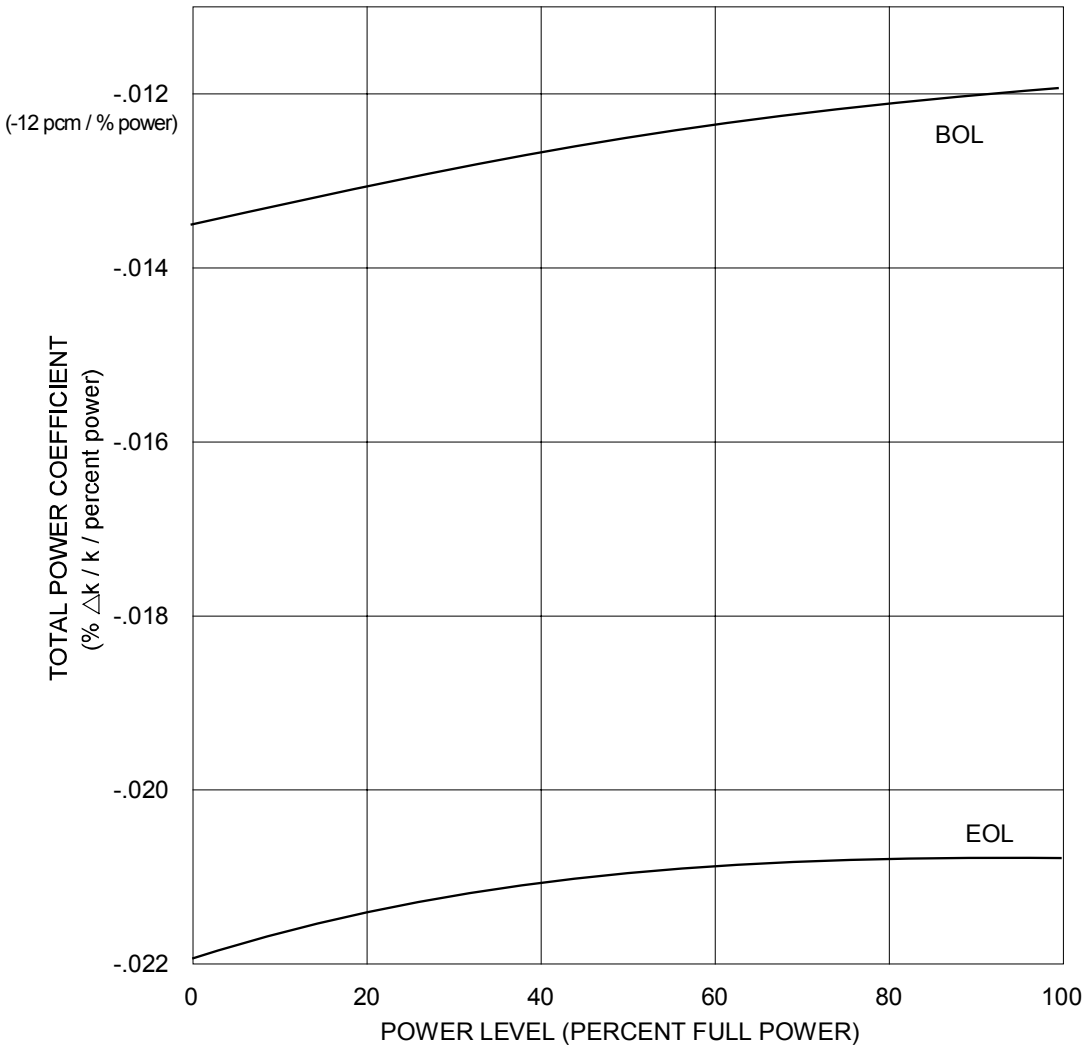


Figure 2.1-9 Total Power Coefficient, BOL and EOL

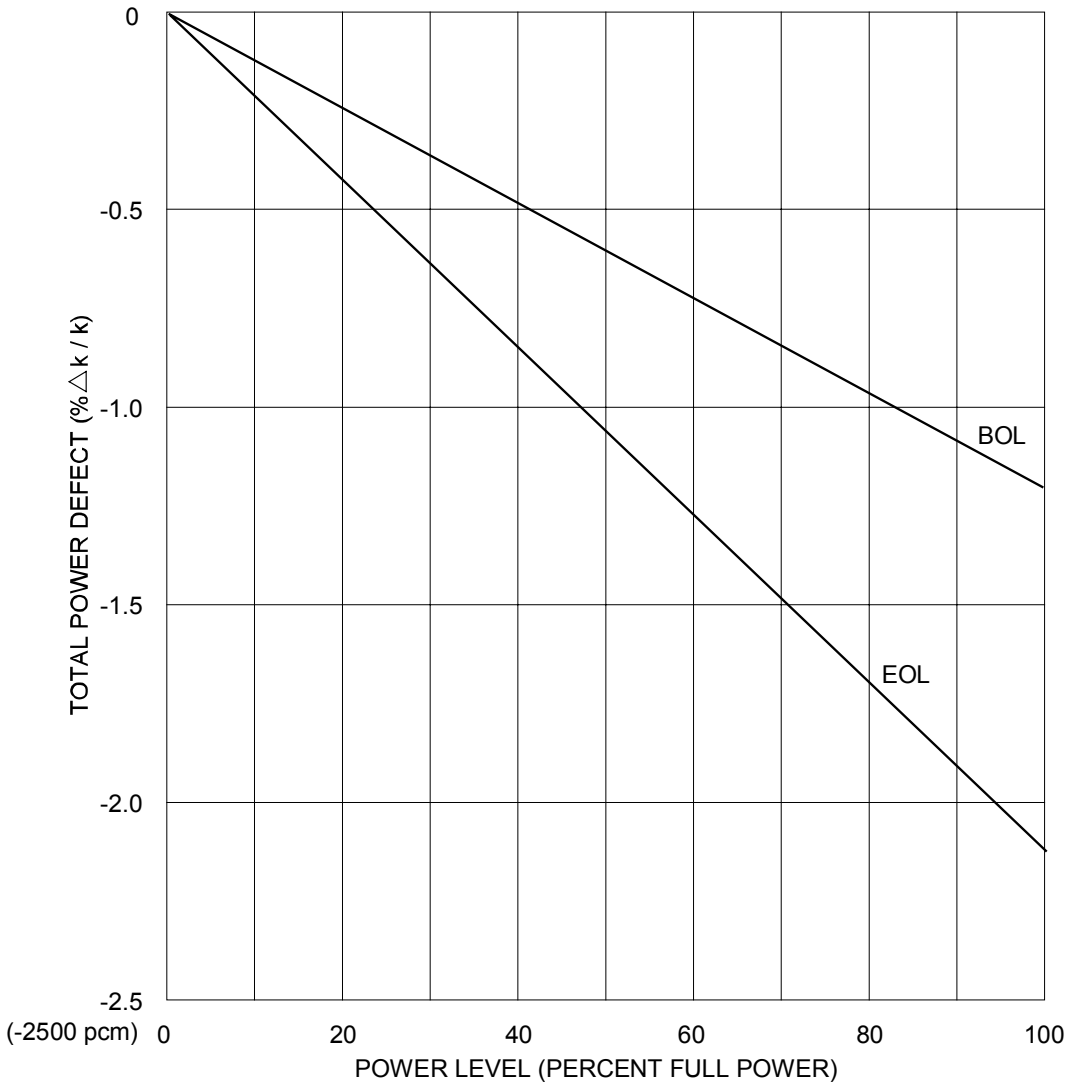


Figure 2.1-10 Total Power Defect, BOL and EOL

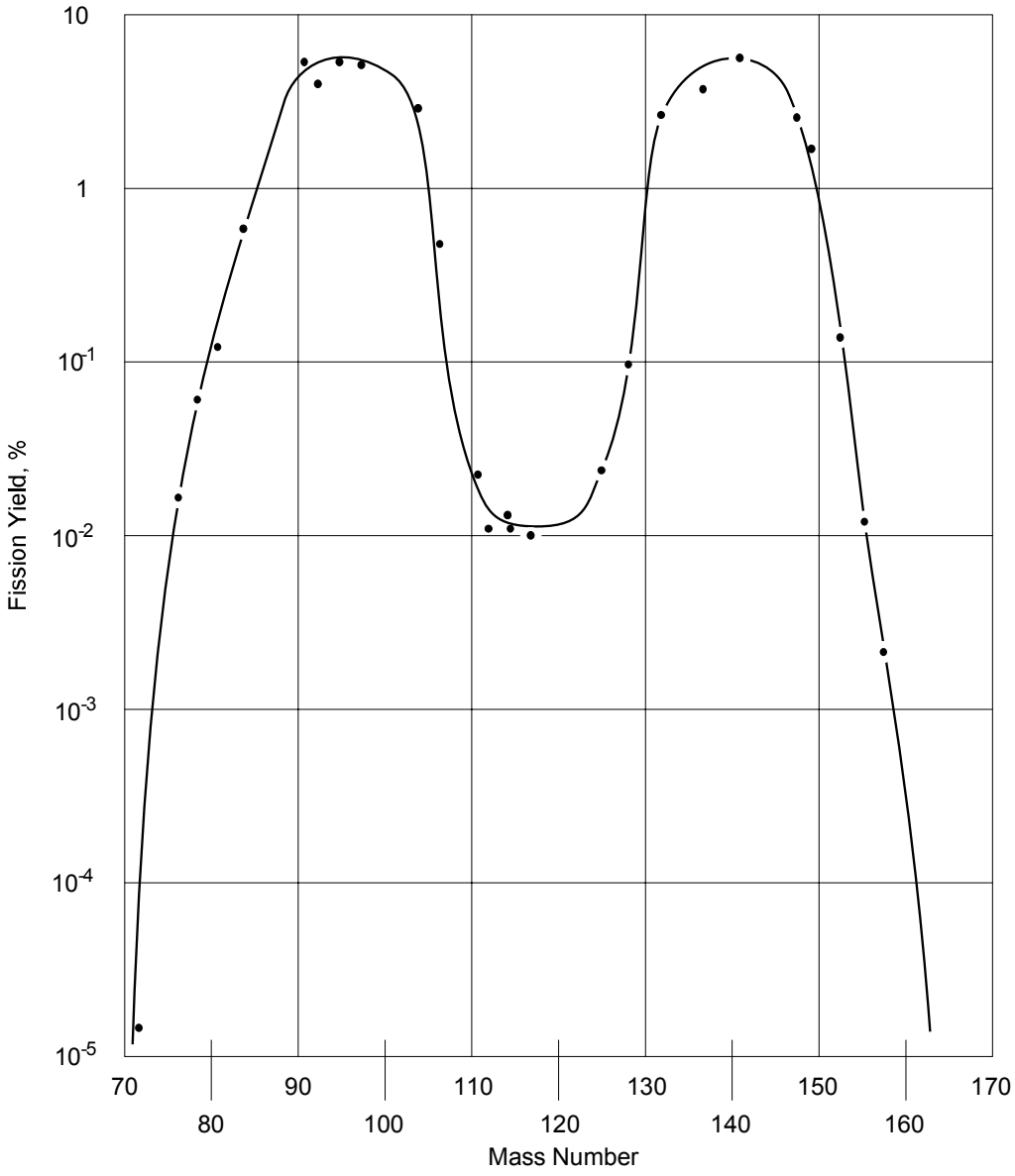


Figure 2.1-11 Fission Yield versus Mass Number

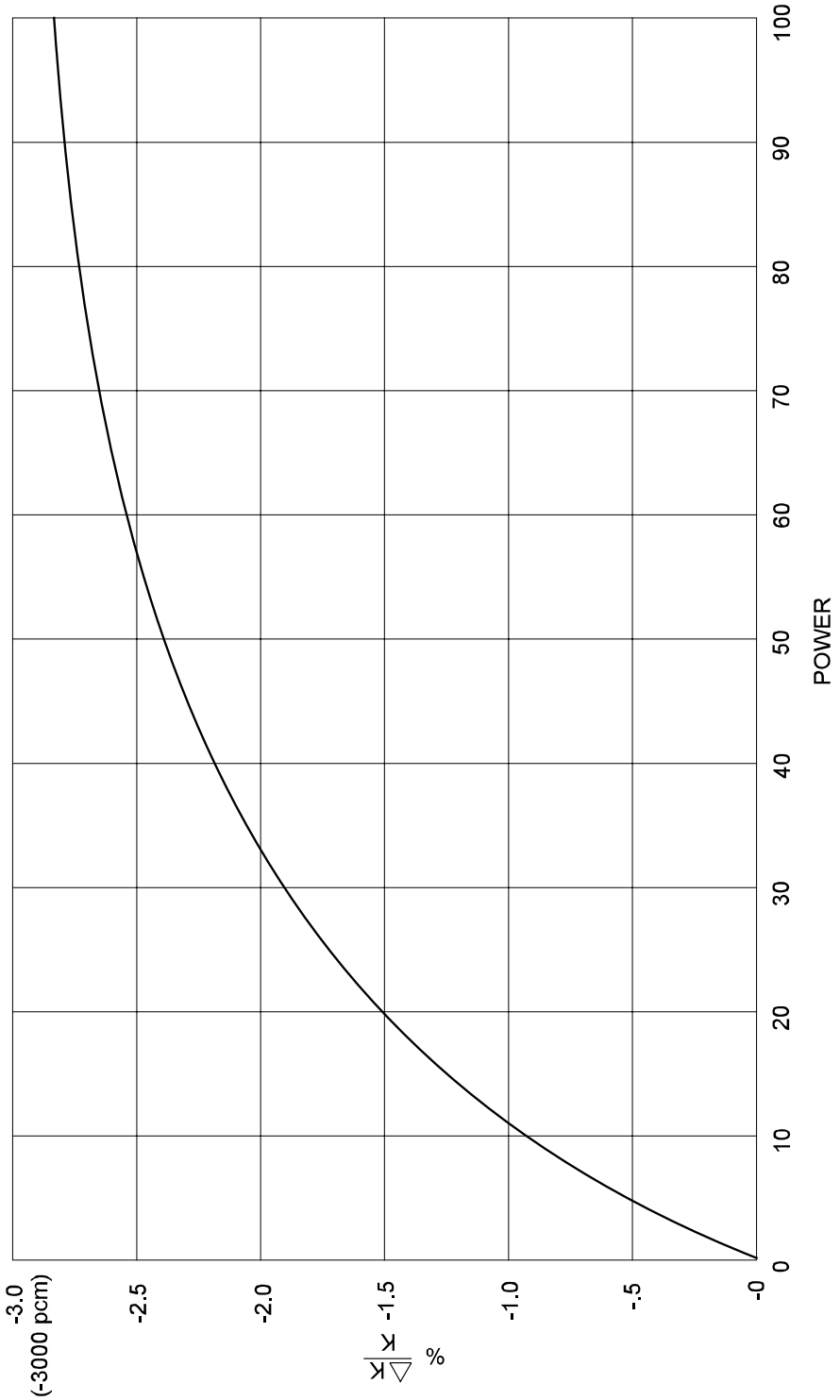


Figure 2.1-12 Equilibrium Xenon Worth vs Percent of Full Power

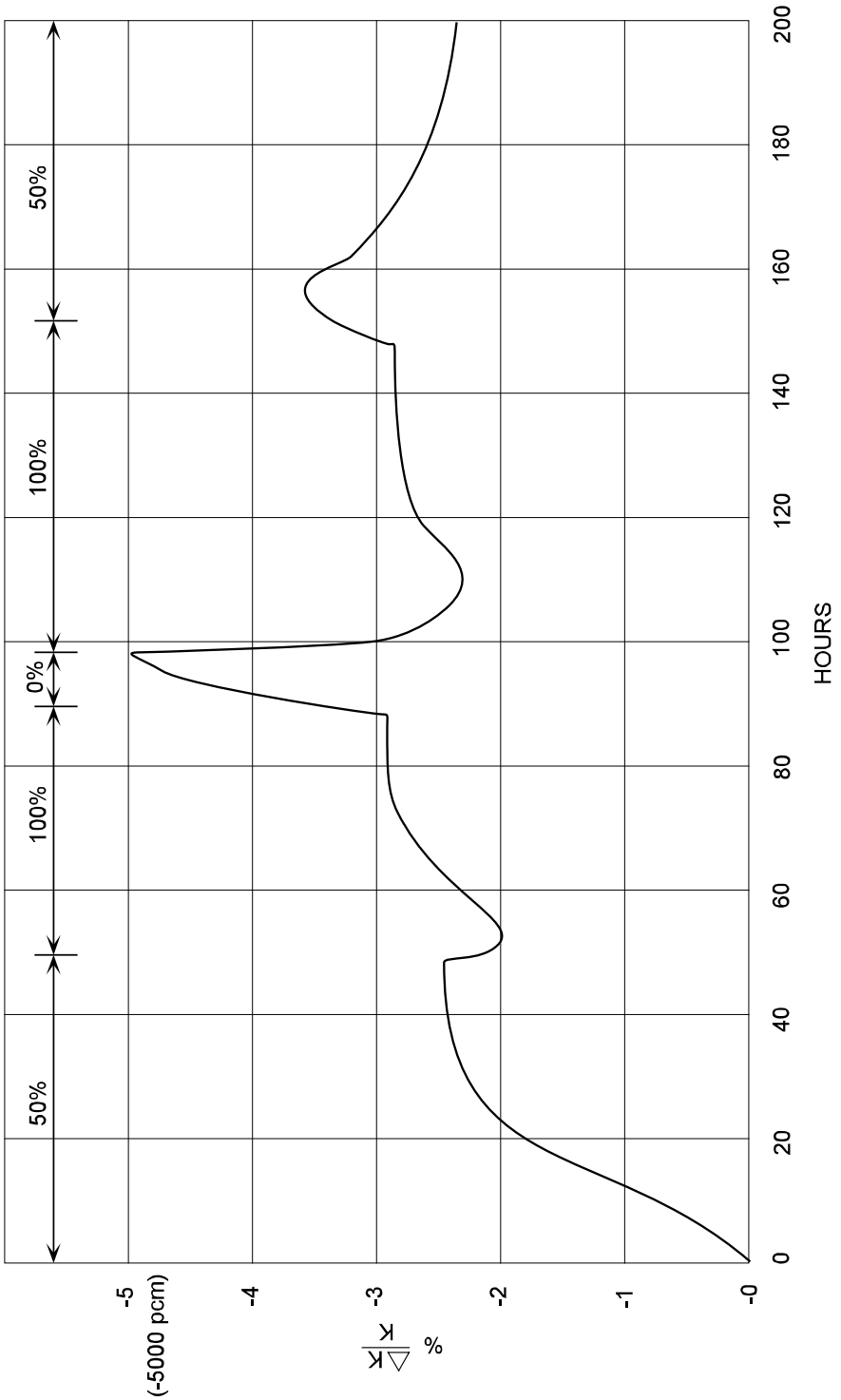


Figure 2.1-13 Xenon Transients

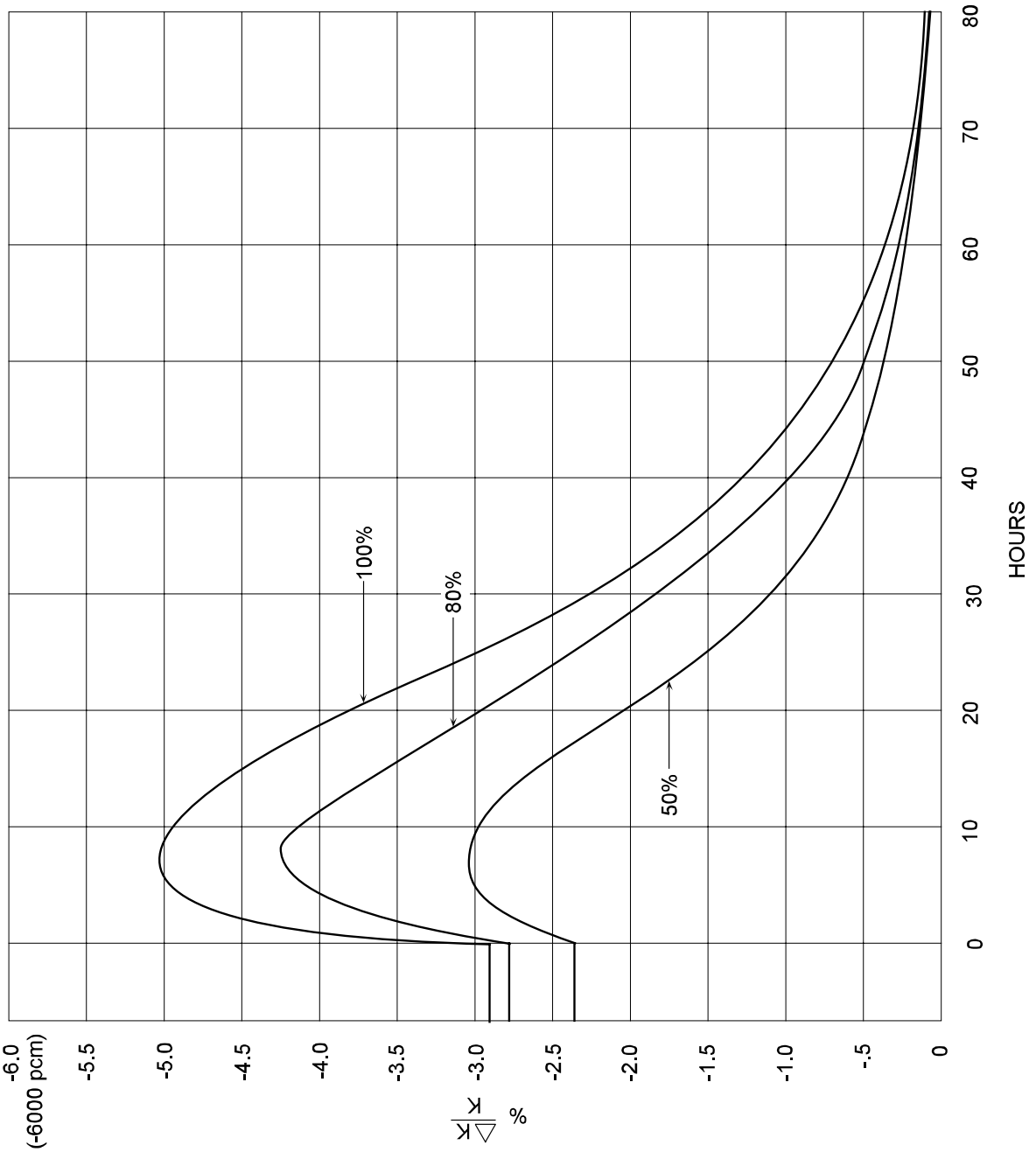


Figure 2.1-14 Xenon Transients Following a Reactor Trip

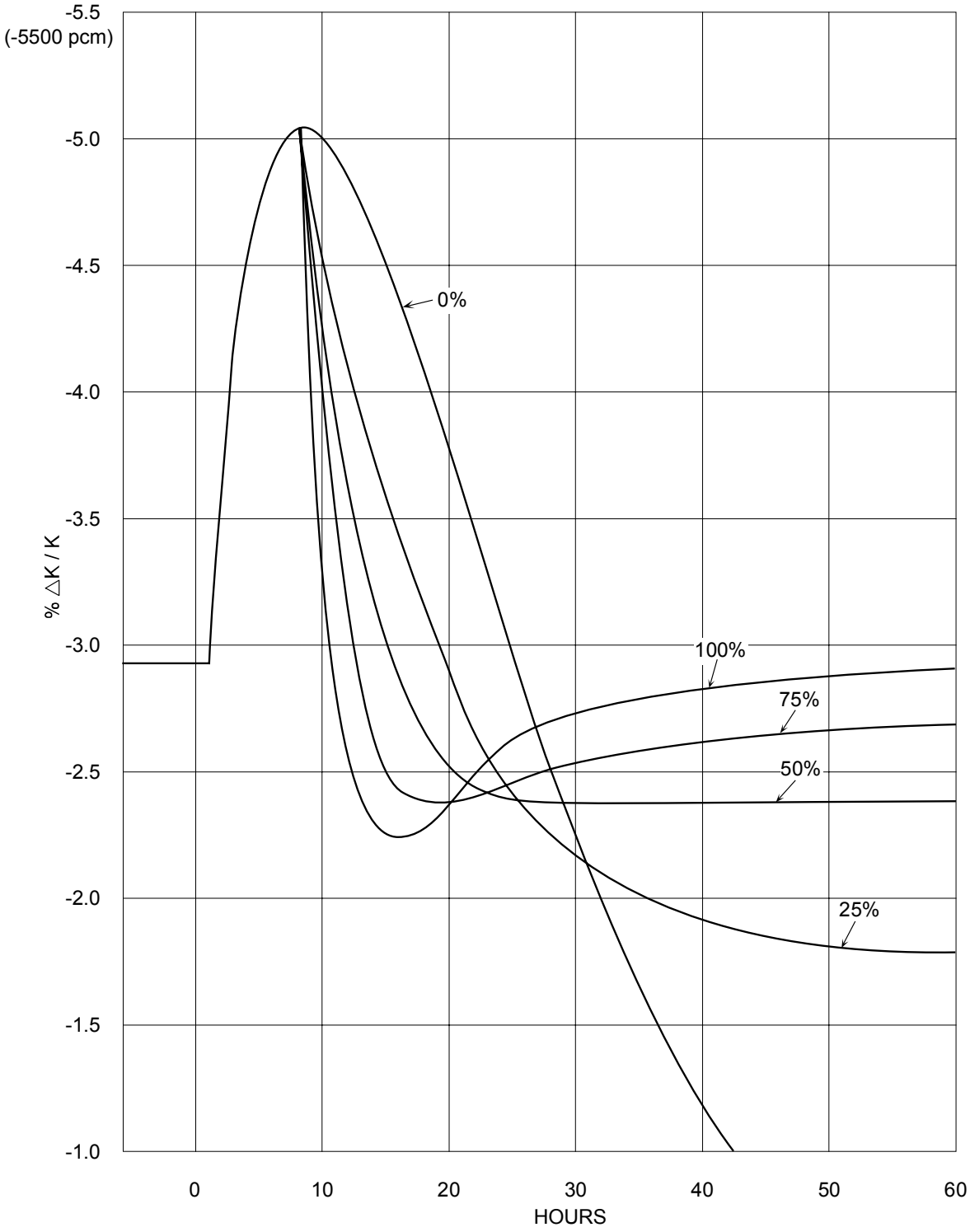


Figure 2.1-15 Xenon Transients Following a Reactor Trip and Return to Power

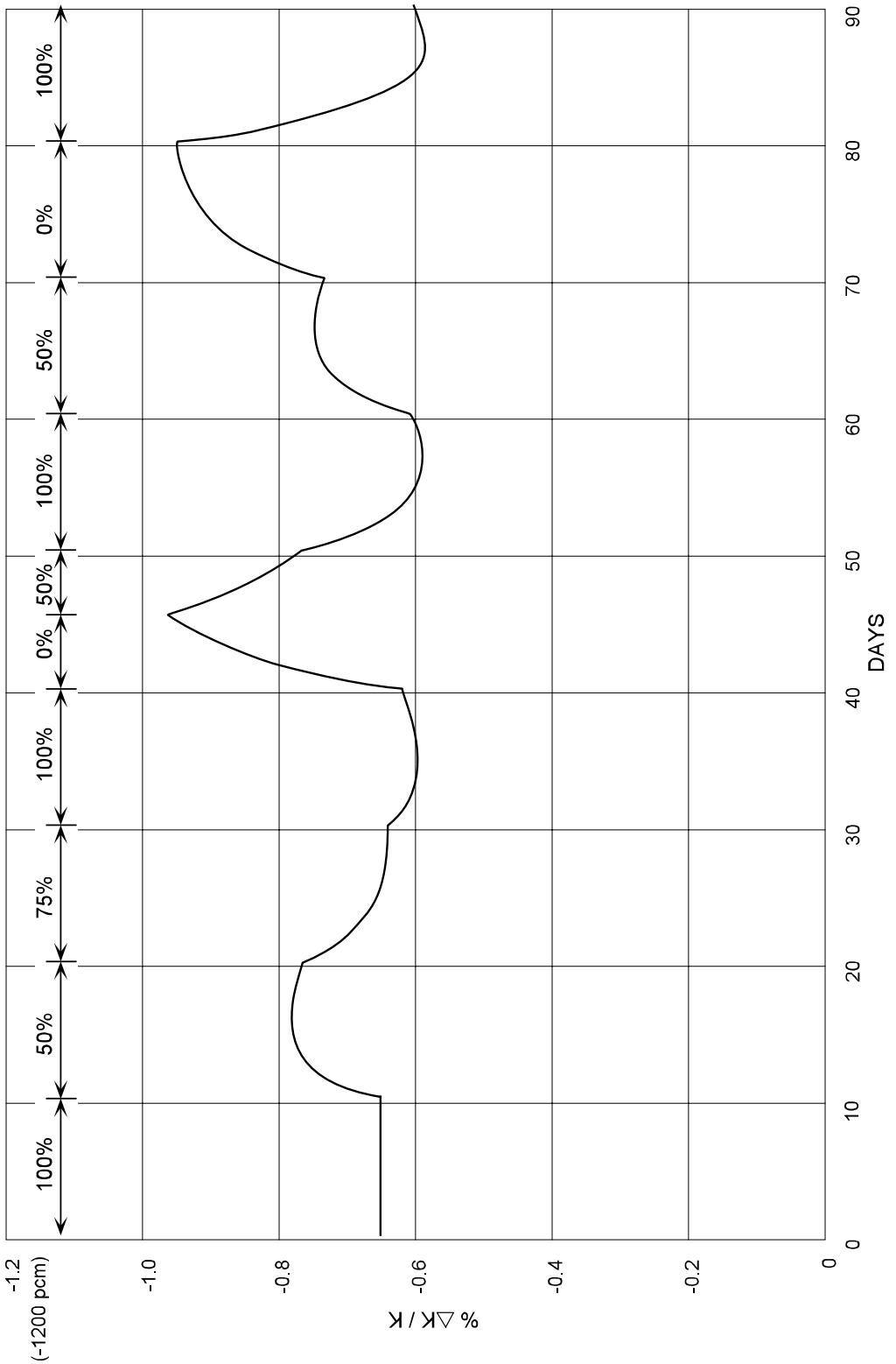


Figure 2.1-16 Samarium Transients

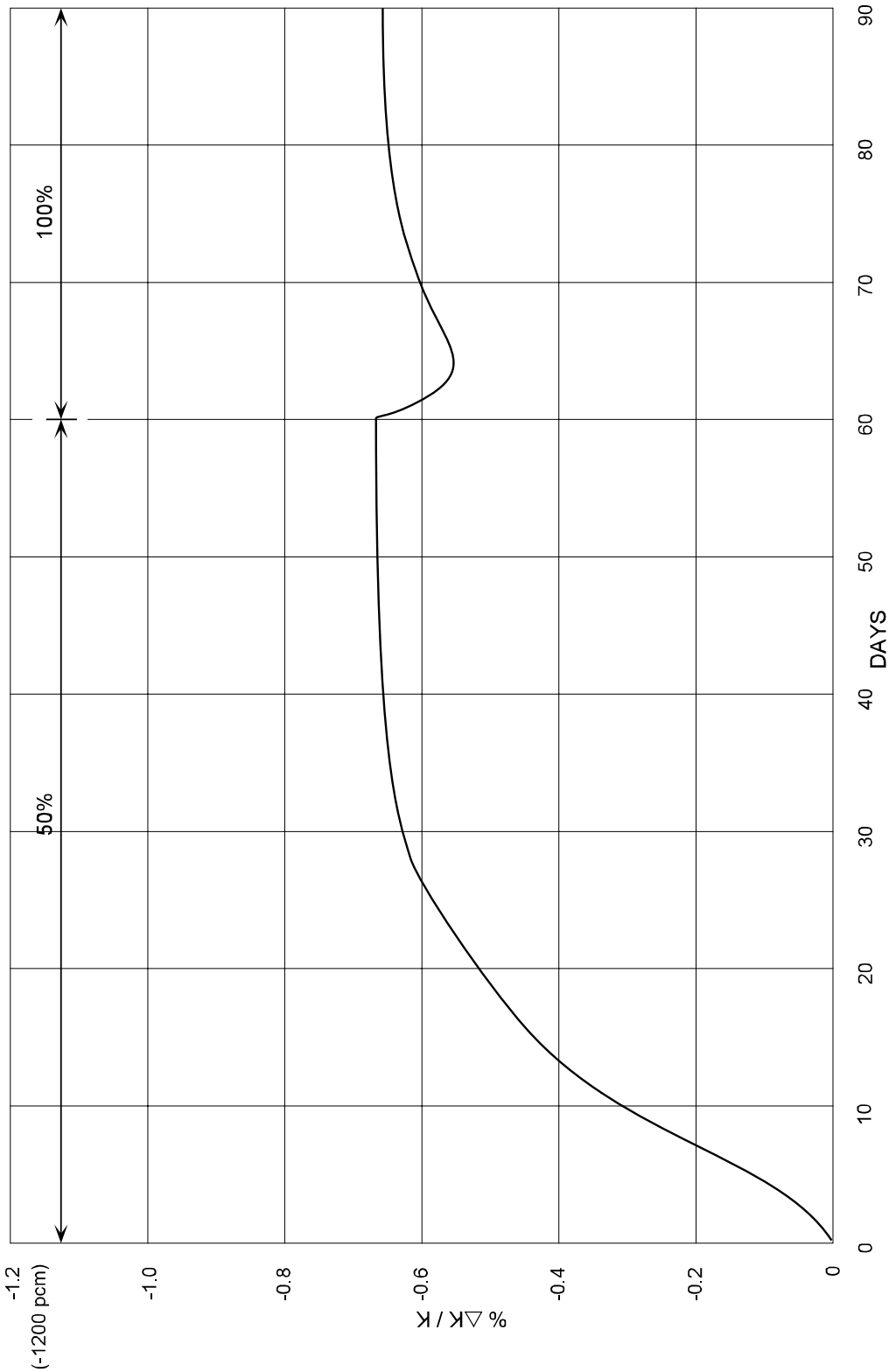


Figure 2.1-17 Samarium Transients Starting with a Clean Core

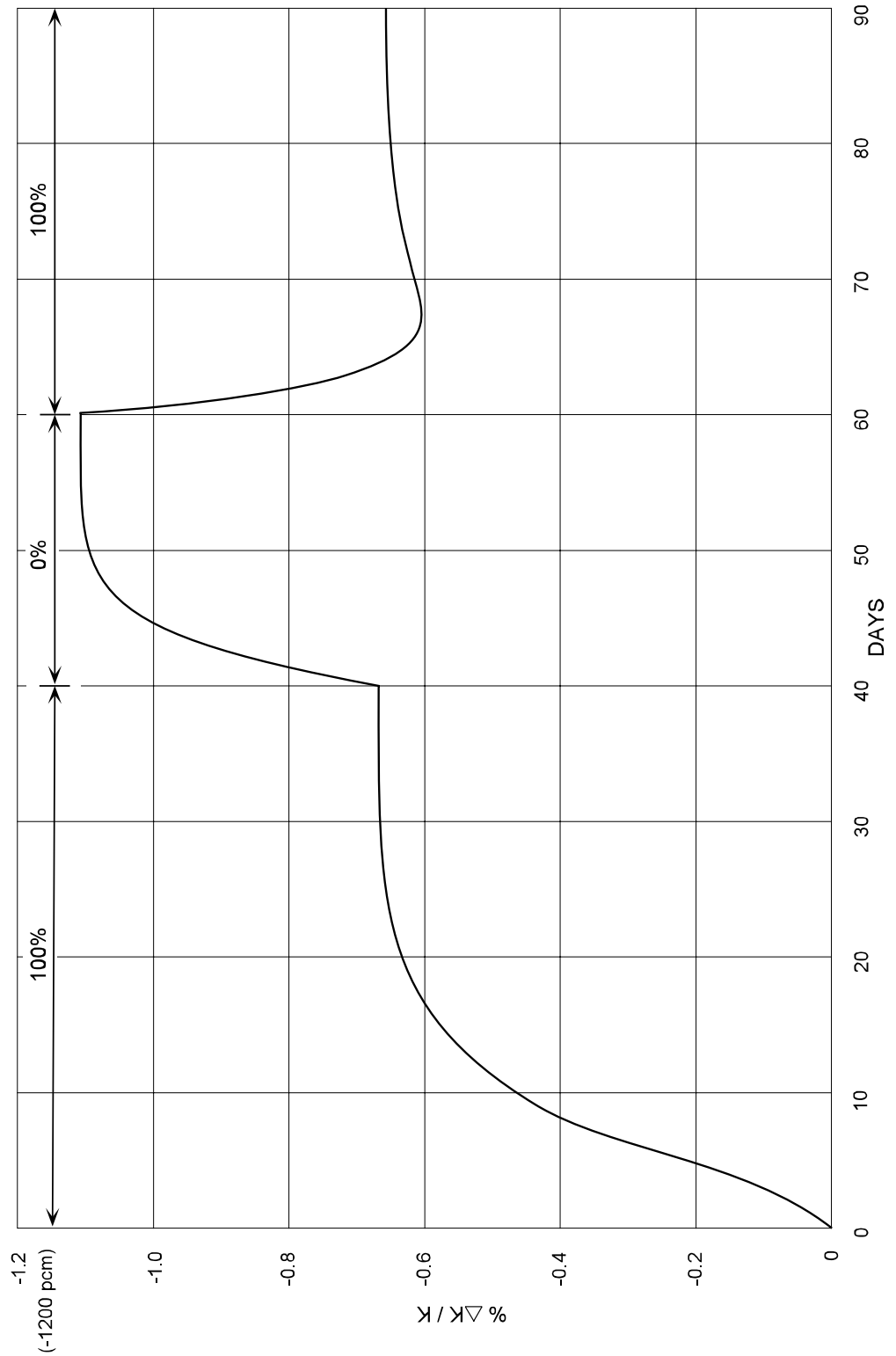


Figure 2.1-18 Samarium Transients Starting with a Clean Core

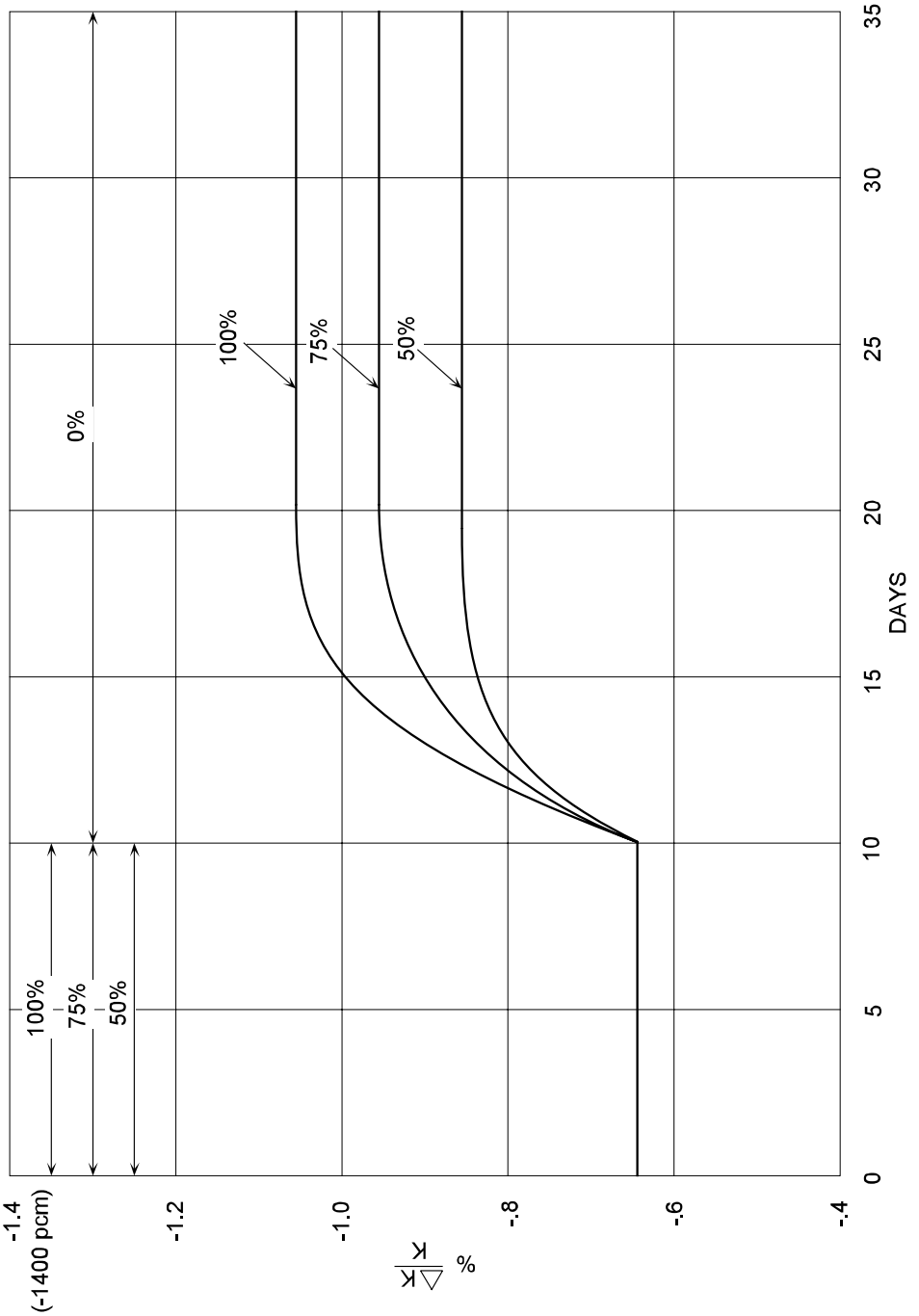
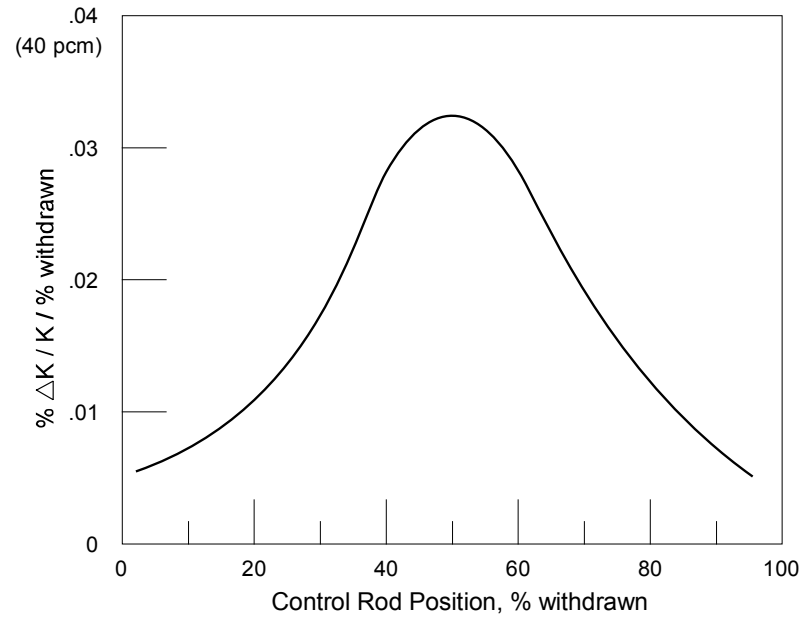
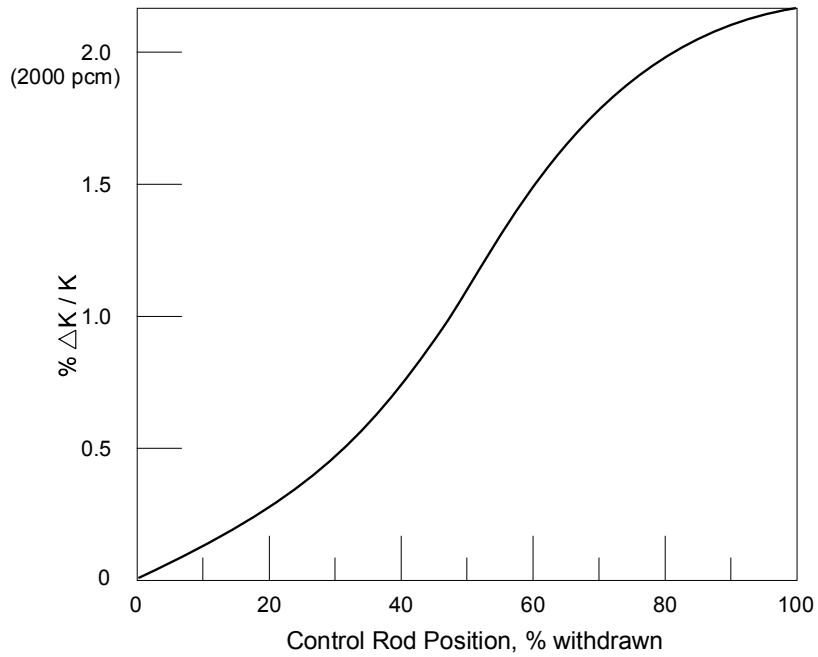


Figure 2.1-19 Samarium Shutdown Transients



(a) DIFFERENTIAL CONTROL ROD WORTH



(b) INTEGRATED CONTROL ROD WORTH

Figure 2.1-20 Integral and Differential Rod Worth

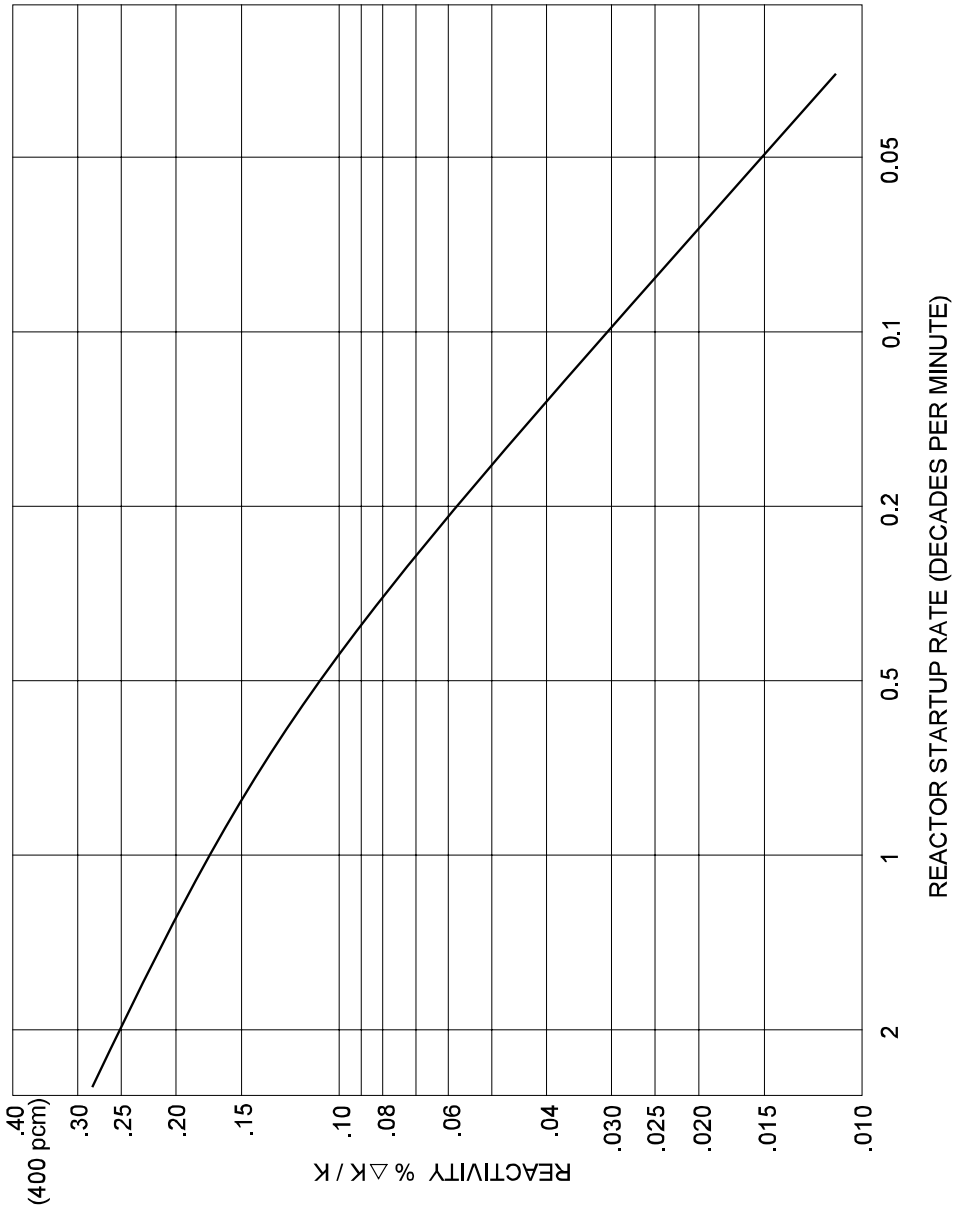


Figure 2.1-21 Reactivity versus Startup Rate