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October 22, 1999

Document Control Desk
U. S. Nuclear Regulatory Commission
11555 Rockville Pike
Rockville, MD 20852

Subject: Project No. 669 – Review of RETRAN-3D
Response to RAI Letter Dated August 25, 1999

By letter dated August 25, 1999, the NRC issued a second request for additional information related to the review of the RETRAN-3D code, which was submitted for review by the RETRAN Maintenance Group on July 8, 1998. Please find attached a response to this request for additional information.

Sincerely,

G. B. Swindlehurst

G. B. Swindlehurst, Chairman
RETRAN Maintenance Group Steering Committee

Attachment

cc: Mr. L. J. Agee (EPRI)
Mr. M. P. Paulsen (CSA)
Mr. W. J. Boatwright (TXU)
Mr. S. J. Peng (Illinova)

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RESPONSE TO
REQUEST FOR ADDITIONAL INFORMATION
EPRI TOPICAL REPORT NP-7450
RETRAN-3D
PROJECT NO. 669

A number of clarifications and minor corrections have been included in Revision 5 of Volume 1 of NP-7450 as a result of the questions asked in this request for additional information (RAI). The pertinent portions of Revision 5 are included with this response as Attachment 1. The individual RAI is included and printed in "italics" followed by the response.

1. The "tube-and-tank or node-flow-path basis" is not fully illustrated by Figure II.3-1 which shows a piece of straight pipe. Difficulties occur when the "tank" or "tube" has a complicated shape and there are changes in flow area and direction from one side to the other (as in the lower plenum of a PWR) or when there are multiple connections to a single node.

Please give examples of more complicated nodes, typical of reactor applications, and show how variables are defined in each case.

Attachment 2 is a description of the node flowpath methodology used to model hydraulic systems with RETRAN-3D. It contains modeling examples for complex geometries typical of reactor applications (pages 5 through 10 of Attachment 2).

2. As used on page II-69 "volumes" and "junctions" are rather misleading terms. Both the mass and momentum cells have "volume" and are connected to adjacent cells by "junctions". When the geometry is anything other than a straight pipe the evaluation of properties in the volumes and at the junctions is not straightforward. Interpolation to give values in the "staggered mesh" is OK for a pipe but not for something like the lower plenum where flow in/out from/to the downcomer is perhaps better characterized by having the velocity in the downcomer, rather than some interpolation between the lower plenum and the downcomer, while the temperature may perhaps be better described by some interpolation when flow is towards the downcomer and by "upwinding" when the flow is from the downcomer.

This is discussed at the bottom of page II-86 where there is also mention of instability. Are there clear rules or is this a matter of user s discretion?

Please describe the rules for evaluating "staggered" properties and indicate if there are ad hoc rules for each component other than a straight pipe.

Attachment 2 is a description of the node flowpath methodology used to model hydraulic systems with RETRAN-3D. Page 4 begins the descriptions of the closure relationships that are used to define the property and mass flow rate

values that arise from the staggered mesh but are not obtained directly from the solution of the balance equations or pressure equation of state.

The application of these models is also described in Attachment 2.

3. Figure II.3-2 shows a "slight generalization" when a "complete generalization" is needed to cover the shapes and conditions of all nodes. The indication of "volumes" and "junctions" is poor - the boundaries should not cut the pipe wall but should enclose the fluid that is actually considered to be in the volume.

Please show typical volumes and junctions for reactor geometries, such as the lower plenum and downcomer, indicating boundaries and giving a description of how properties are evaluated.

Attachment 1 contains a revision to Figure II.3-2 that clearly indicates the momentum cell boundaries (page II-71).

Attachment 2 is a description of the node flowpath methodology used to model hydraulic systems with RETRAN-3D. It contains modeling examples for complex geometries typical of reactor applications, including the lower plenum and downcomer. Also included are descriptions of the closure relationships that are used to define the property and mass flow rate values that arise from the staggered mesh but are not obtained directly from the solution of the balance equations or pressure equation of state.

4. Equation II.3-2a is an energy equation requiring a recipe for D_{ms} at every node, which may not be available. It would seem simpler, if mechanical energy is to be included at all, to use the overall energy balance equation in the form that includes kinetic energy in both the stored energy and flow energy terms. In this case the dissipation and "work done accelerating the fluid" terms are discarded.

Please explain how D_{ms} is evaluated. If it is neglected, then explain if the term involving (p_i-p_{i-1}) is retained and why.

Equation II.2-3a contains a source term D_{ms} that accounts for heating due to viscous dissipation. Section II.2.4.1 includes a discussion of the various components of viscous dissipation. The dissipation due to wall friction is generally small for the problems for which RETRAN-3D is used, and is therefore neglected. However, the dissipation of viscous forces due to moving surfaces such as centrifugal pumps and turbines can be significant. This effect is not neglected and is discussed in Sections VI.1.1 and VI.6.0, respectively.

The $(p_i - p_{i-1})$ term is retained in the development for completeness, but is currently neglected as noted in Section VIII.2.2.3

5. Equation II.3-2a is said to come from Equation II.2-107. In the latter the "mechanical work done on the fluid" term has all variables with the subscript "j", indicating evaluation at boundaries, while in the former equation the pressure is evaluated at boundaries (subscript i) but the flow is evaluated at the volume. (subscript k).

Please explain why the subscripts have been changed in a way that changes the physics.

The general form of the mechanical work term is shown in Eq. II.2-107. Equation II.3-2a is specific to the geometry shown in Figure II.3-2. It is obtained from Eq. II.2-107 by assuming that the pressure varies linearly across the volume, giving $(p_{i-1} - p_k) = (p_k - p_i) = \frac{1}{2}(p_{i-1} - p_i)$. Substituting these identities for the pressure gradient terms lead to Eq. II.3-2a. To avoid confusion in the future, Revision 5 replaces Eq. II.3-2a with a form that follows directly from Eq. II.2-107 (see page II-54 and II-71 to II-72 of Attachment 1).

6. In Equation II.2-107, cited above, the term Δp is not defined nor is the origin of this equation given. A similar term appears in Equation II.2-97, again without defining Δp, and is said to come from Equation II.2-49a. Demonstrate how the volume integral that appears there, representing mechanical work that is done on the fluid to accelerate it rather than to compress it, can be decomposed to two separate factors.

Please explain how the final term in Equation II.2-97 follows from Equation II.2-49. Define Δp and show how the final term in Equation II.2-97 differs, if it does, from the similar term in the enthalpy flux that is the first summation on the right hand side.

The development of Eqs. II.2-49 has been revised (pages II-30 through II-33 of Attachment 1) to clarify the origin of the components comprising the reversible work term. The revisions made to Sections II.2.4.1 and II.2.7.3 also define the Δp terms. Equations II.2-97 and II.2-107 are obtained directly from Eq. II.2-49b.

The final terms in Eqs. II.2-97 are related to the enthalpy as described in the revised derivation of Eq. II.2-49b.

7. It is not clear how to interpret Equation II.3.2c for a node such as the lower plenum where v, A and W are not obviously defined.

Please explain how to interpret Equation II.3-2c for a node of complex geometry, such as the lower plenum, upper head, downcomer, etc.

Equation II.3-2a applies to the geometry of Figure II.3-2, specifically for situations where there is one junction flowing into a volume and one junction flowing out. Equation II.3-2a has been revised (Question 5 above) to use junction flow values. Attachment 2 describes the models for $\overline{W}$.

Equation II.3-2c is an equation for the volume average flow. The calculation of this parameter in complex regions such as a plenum or downcomer is described in Attachment 2.

8. Equation II.3-4 is only valid if the velocity is normal to the surface at boundaries. There is a statement at the bottom of the page "in RETRAN the velocity at a junction surface is assumed to be perpendicular to the surface" but this seems to be contradicted in the analysis of the tee later in this section where we find terms like W_x^2 and W_y^2 multiplying the same area.

Please explain if v is always perpendicular to the surface in RETRAN. If so, explain the apparent breaking of the rule in analyzing the tee junction.

Equation II.3-4 is a special case of Eq. II.2-106 where the velocity is normal to the surface at the momentum cell boundaries. Equations II.3-4 through II.3-7 have been revised to include the more general situation where the velocity is not normal to the surface. The revised text and equations are included in Attachment 1 (see pages II-73 and II-74).

The tee example is addressed in the response to question 14.

9. A picture is needed to explain the difference between A' and A and the various angles used in Equation II.3-6. If the momentum cell flow surfaces are parallel to the same direction (ϕ, the direction of the momentum in the cell)" then they are not the same as A for a bend. If a clear sketch is given it will show that Equation II.3-7 does not follow from Equation II.3-6. Equation II.3-7 has A and not A' in it but the difference between them is not clear. If A is interpreted as the pipe cross-section area for a bend, then Equation II.3-7 is wrong, as there are two factors of cosine of an angle in the momentum flux terms when there should be one. Moreover, the angles have somehow disappeared from the pressure terms. If all the surfaces are perpendicular to ϕ, then the condition of flow being perpendicular to the boundaries is violated for a bend. There is no interpretation of the geometry and definitions which will validate Equation II.3-7 for anything other than a straight pipe.

Please give a rigorous derivation of Equation II.3-7 with clear indication by figures of how it applies to a bend (like Figure II.3-8) or other geometry where the direction of outflow is not the same as the direction of inflow and neither corresponds to the mean direction of flow in the node.

Figure II.3-2 has been revised to more clearly illustrate the momentum cell with the associated junction and boundary orientations. The angles that define the orientations are normal to the associated areas. The derivation for Eq. II.3-7 has also been revised. It starts with Eq. II.2-106 and uses the revised Figure II.3-2 to illustrate the application of the equation to the momentum cell. The revisions are included in Attachment 1 (see pages II-71 through II-74).

The momentum flux terms in Eq. II.3-7 were originally written as $\overline{W}^2_{k,\psi}$ rather than $\overline{W}_k\overline{W}_{k,\psi}$ as revised in Attachment 1 (see page II-74). For straight pipes and orthogonally oriented components, the two products are equal. This is demonstrated by combining Eqs. II.3-32 and II.3-33 from Attachment 1 (see page II-85) to obtain

$$\overline{W}_k\overline{W}_{k,\psi} = \overline{W}^2_{k,\psi} \left| \frac{\cos(\Theta_k - \Phi_k)}{\cos(\Theta_k - \Psi_j)} \right| .$$

The absolute value of the cosine ratio will be unity as long as ϕ_k and ψ_j lie in a straight line or are orthogonal to each other.

A trouble report has been filed to notify users that an error in the momentum flux component of the momentum equation pressure drop can occur if angles other than 0, 90, 180, or 270 degrees are used with the vector momentum option. To assess the potential impact of this error on previous analyses, those presented in the RETRAN-3D Assessment Manual [4] and the RETRAN-3D sample problems were reviewed. It was determined that all used straight pipes and orthogonally oriented components (angles of 0, 90, 180 and 270 degrees). For these angles, the momentum flux terms are evaluated correctly. This is the approach normally applied by users when modeling reactor systems for plant transient analyses. The plant models used to generate results included in Reference 4 were provided by U.S. and foreign utility owners, indicating that commonly used modeling practices produce correct results.

In summary, an error in the momentum equation when using the vector momentum option has been identified for situations in which angles other than 0, 90, 180, or 270 degrees are used. These angles are rarely modeled since they do not routinely exist in the reactor systems for which RETRAN-3D is used. This error is also applicable to RETRAN-02. A trouble report has been issued to RETRAN users to communicate the situation, and a correction to the codes will be made available for any users with an urgent need to address the error. The error will be corrected in future code releases. The error will also be discussed in the upcoming RETRAN User Group meeting on October 19, 1999.

Appendix 4 contains a development for a scalar macroscopic momentum balance for multi-dimensional flow that was written by Dr. T. A. Porsching. The equations given in the development are consistent with those that are given in Attachment 1.

10. Equations such as II.3-7 and II.3-10 cannot be used without recipes for the forces from the boundaries on the fluid. For the lower plenum, for example, the momentum balance must include the reaction from the lower surface, which turns the fluid from the downcomer into the core. It is not just form losses that are involved.

The example of flow around a bend on pp.II-89 and II-90 illustrates this point. There is no attempt there to evaluate the force on the bend due to normal stresses. Therefore the vector momentum balance that is claimed as a basis for analysis is actually not being used at all, but something else.

Please explain what is actually used as a "momentum balance" for nodes, other than pieces of straight pipe, in RETRAN. If it does not follow from the rationale given in the documentation, then supply appropriate new documentation.

The pressure forces acting normal to the wall are included in the surface integrals for Eq. II.2-34 and are subsequently included in the $F_{i,G}^{wa}$ term described with equation II.2-70. In Eqs. II.3-7 and II.3-10, the wall pressure forces are included in the F_{loc} terms by definition. They are geometry dependent and are included in the form or energy loss coefficients found in standard references such as Idlechik [1] or those obtained from measurement.

Bird, Stewart, and Lightfoot [2] present a development where the viscous and pressure forces are related to the energy loss coefficient. This loss coefficient is the same form as the term used in RETRAN-3D. The authors break the losses into two components, one that accounts for pressure drops due to viscous wall shear in piping sections, and another that accounts for pressure drops due to geometric effects for components such as valves, meters, and elbows. This is the same approach used in RETRAN-3D, where the wall shear effects are included in F_w and wall pressure and geometry related dissipation effects are included in the form loss term F_{loc}.

A number of other computer programs used for system analyses of LOCA and plant transient licensing applications, such as LOFTRAN, versions of RELAP4, RELAP5, and versions of RETRAN, all represent the wall pressure and geometry related dissipation effects by use of form losses. All of these codes have been used for a number of years to successfully model the behavior of experimental facilities and operating plants.

The development of the RETRAN-3D flow equation has been revised to clarify its use. Attachment 1 (see pages II-75 through II-80) includes the revised text. Equation II.3-26 is used to compute the flow rates for all junctions.

Attachment 2 (see page 5) contains illustrated examples of momentum cells for junctions connecting to the lower plenum. These include the downcomer to lower plenum junction and the lower plenum to core junction.

11. "The equations presently in RETRAN-3D can be obtained for the geometry shown in Figure II.3-3 which is a representation of a flow channel containing an abrupt area change". The special case of an area change in a straight pipe is not a basis for deducing a general equation for use in more complicated geometries.

Please explain if these derivations are intended to be "general." If so, please derive them for a general node and not only for the straight area change example.

With RETRAN-3D's one-dimensional momentum equation, all area changes are represented as an idealized abrupt area change. When developing an input model for a hydraulic system, junctions are typically located where changes in geometry and area occur. All area changes will look like abrupt area changes to RETRAN-3D. This may be a good approximation for some situations, for others the form loss term may require adjustment to more accurately model the pressure distribution. The selection of the proper input is the responsibility of the user developing the model.

RETRAN-3D solves a one-dimensional momentum equation for the junction flows. Since reactor systems contain components where the flow may be three-dimensional, some approximations will be necessary when developing the input for a plant model. Modeling guidelines are provided in Reference 3 for various components encountered in reactor systems. Sensitivity studies are often performed to gain understanding about the effects of the modeling approximations in RETRAN-3D input models.

Geometry dependent form loss coefficients can be supplied by the user through input. They can be obtained from handbooks or selected to match component or system pressure drop data.

12. Equation II.3-7 is an approximation for the straight area change because all flows are in the same direction and one does not have to deal with the vector nature of the momentum balance. p_i+ and p_i- are mean pressures that may be assumed for this particular geometry, as long as the mixing length needed to establish p_i+ is short compared with the length of the node. There is no way to generalize this to a more complicated variation of area with position or a geometry in which all velocities are not in the same direction. This is an ad hoc treatment of an area change in a straight pipe, not a general derivation for any node. Explain how all junctions can fit this pattern, as suggested in Figure II.3-5. Equation II.3-28 seeks to change a vector momentum flux to a scalar one in the direction ψ. If all momentum terms are resolved in the direction θ_i then the force from the structure in that direction has to be included and it is not just a frictional force, as in Equation II.3-30. Moreover, the pressures will not lose the areas by which they are multiplied. Equation II.3-30 appears to be a strange compromise between momentum and energy conservation but it is not rigorously derived.

Please explain how Equation II.3-30 is justified, using a proper general control volume, or even examples such as a bend or lower plenum. It is not a momentum conservation equation because it contains only scalar terms and includes energy dissipation, which is not a force.

As discussed in the response to question 11, the abrupt area change form of the momentum equation is used for all junctions where area changes occur. The development for the RETRAN-3D momentum equation, Eq. II.3-26, has been revised in Attachment 1 (see page II-75) to clarify the original development, to identify the assumptions and use of the equation in RETRAN-3D. The revised development starts with a uniform area channel with a bend and continues through the abrupt area change and situations where multiple inlet and outlet junctions connect to a volume.

The wall pressure forces are treated separately from the viscous shear or wall friction terms. The response to question 10 provides more details concerning the treatment of these wall pressure forces and their relation to the energy dissipation term.

Equation II.3-26 in Attachment 1 (see page II-79) is obtained by summing two momentum equations that are first divided by the areas associated with the pressure terms. The mechanical energy equation is then used to relate the pressure drop across the area change (static and velocity heads) to a recoverable change and the irrecoverable dissipation loss. This approach is similar to that presented in Reference 3. Attachment 1 (see page II-75) contains revisions that have been made to the theory manual to clarify the assumptions and applications for the momentum equation.

13. Figure II.3-6 does not represent a general node because no areas for flow are included. It would be nice to see the momentum conservation equation derived for a general node with several flows in and out. This is not done. The explanations in the text leave too much up to the reader to try to envisage how this momentum balance might look.

Please draw a more realistic general node and derive the momentum balance for it in a way that is compatible with the actual treatments in RETRAN.

Figure II.3-6 does not represent a momentum cell. It represents a control volume with multiple junctions connected, and is used to illustrate the calculation of the volume centered flows that are used as the momentum cell boundary flows. The figure and text have been revised in Attachment 1 (see page II-81) to clarify these points.

14. The treatment of the tee on page II-87 is very strange. The whole "staggered mesh" treatment has been abandoned, apparently, as the momentum and mass control volumes do not appear to have different subscripts. To evaluate the momentum balance for the shaded volume in figure (a) we need the flows across boundaries which are only indicated for W_{15} (though later addressed in Equations II.3-35b and c. Pressures are presumably evaluated at the boundaries of the shaded volume, but they have the same subscripts as those for mass flow evaluated at the centers of the

momentum volumes. The mass flows with overbars are stated to be "volume average values" whereas they are actually area averages across boundaries and cannot be computed directly from the volume averages for momentum nodes. It is presumably assumed that no x-momentum (not defined but presumably in the direction of the straight pipe) goes out through the piece of this volume that leads to the branch pipe. If the momentum balance is complete around the boundaries of the volume there is no need for the final two terms in Equation II.3-35a that introduce energy dissipation. The only new term justifiable in a momentum balance is a force or momentum flux.

Equation II.3-35c does not account for flow that goes out the side branch.

Equation II.3-36a is more peculiar. Is A_{12} the area of the whole main pipe or one half of it? Is flow entering there with y-momentum (there is no reason why this should be $1/2W_{15}$ as in Equation II.3-36b)? How about the y-momentum that comes in the bottom of the volume or leaves in the direction 13? Provide justification for the energy dissipation terms. The attempt seems to be to use a "general" form of equation that doesn't really fit the situation.

Is there any empirical test or check of these formulations? How are the empirical "dissipation" coefficients evaluated?

The tee example has been revised in Attachment 1 (see page II-86) to clarify the use of the staggered mesh, nomenclature, and the use of the vector momentum feature. The results for the original example problem are correct and agree with the momentum equation and surface flow definitions, even though Figure II.3-7 may have been misleading. The results for the revised example differ slightly from those for the original problem, but the differences are due only to revisions made to the figure to clarify the example, not as the result of errors in the original example.

In the question above, the statement that "*The mass flows with overbars are stated to be "volume average values" whereas they are actually area averages across boundaries and cannot be computed directly from the volume averages for momentum nodes*", implies that something is being attempted that in actuality is not, nor is there any implication to that effect in the documentation. The flow terms with overbars are defined throughout the document to be cell centered, surface, or average volume flows. The document specifically states that a model, other than a balance equation, is required to define the volume centered flows. The averaging form of the equation is then used in the example to define the cell-centered flows. The statement that leads to the confusion has been removed from Attachment 1 (see page II-86) and the discussion regarding the application of the equations for defining the volume centered flows has been expanded and clarified.

Contrary to the statement in the question above, the energy dissipation terms must be included in Eq. II.3-26 (and the results for the tee example) since they are related to the wall pressure forces. Refer to the response to question 10 for the justification of the dissipation terms.

The dissipation or form loss coefficients are obtained either from steady-state measurements of the pressure drop across a component such as an elbow, orifice, or tee, or using methods described in References 1 and 2. For locations where pressure drops are known, the RETRAN-3D steady-state initialization feature can be used to calculate loss coefficients.

15. On pages II-89 and II-90 again the "staggered mesh" is not clearly indicated. The "junction 2" for which Equation II.3-37b is written should presumably end on the right hand side as a boundary that cuts the bend in half at 45 degrees.

Equation II.3-37b contains subscripts x in the momentum terms and is presumably supposed to be the x-component of a momentum balance. One would expect $\underline{W}_3$ to be $(W_2+W_3)/2$, like Equation II.3-38a, and $\underline{W}_{3,x}$ to be $\underline{W}_3/\sqrt{2}$, as the x-component of a vector flux. Then the $\underline{W}_{3,x}^2$ term in Equation II.3-37b will be like $W^2/2$ rather than $W^2/4$. In fact the real momentum contribution from this boundary is like $W^2/\sqrt{2}$ because it is a scalar mass flux times the x-component of the vector velocity (see the discussion of Equation II.3-7), so there are two errors here.

If the momentum balance is properly written for the actual control volume, the pressure p_3 acts at an angle and picks up a factor of $1/\sqrt{2}$ which is missing in the text.

In such a situation there is clearly a force from the bend on the flow which should appear in the momentum balance. It is not at all clear that the "energy dissipation" terms give this force. In fact, flow around a bend can be almost frictionless while the x-reaction on the bend is large and not related to friction at all but results from the normal stresses on the wall.

In steady state one would expect p_3 to be halfway between p_2 and p_4, but the equations are not symmetrical.

Presumably these "steady state" equations" are used to illustrate the terms that would also appear in an unsteady flow case with compressibility, when the W's are not equal? In view of the discussion on p.II-73 it would seem that this balance should be written in the direction of the average velocity in the cell, which is at 22.5 degrees to the x-axis. This still faces the problem of the resultant of normal forces on the bend, which are not the same in unsteady flow as in steady flow.

Please clarify these points of error.

Figure II.3-8 has been revised in Attachment 1 (see page II-90) to more clearly illustrate the momentum cells used with the elbow example. The discussion for

the example has also been revised to account for the revised figure and momentum equation. The momentum flux term for volume 3 in the original example was 1/4 W_2^2 (given that $W_2 = W_3$ for steady-state conditions), whereas the revised momentum equation gives $1/(2\sqrt{2})$ W_2^2. The ratio of the revised and original momentum flux terms gives $2/\sqrt{2}$, which agrees with the cosine ratio given in the response to question 9.

Contrary to the statement in the question above, the pressure term for the boundary at the middle of the elbow should not have a cosine of 45 degrees when using the RETRAN-3D momentum equation. If this were the case, a model combining volumes 2 and 3 into a single volume to represent the 90-degree bend and downstream volume as shown in Figure II.3-8 (see Attachment 1 page II-92) would have a cosine of 90 degrees (= zero) multiplying the exit pressure term. This would make the flow equation a function of the upstream pressure only, which only makes physical sense for choked flow. If only the flow surface portion of the pressure surface integral is considered (see page II-73 of Attachment 1) then a cosine term would appear, but a compensating term should appear from the remainder of the surface integral. The use of a simple pressure gradient is consistent with formulations for the energy loss coefficients which include the surface pressure forces.

As described in the response to question 10 above, the wall normal pressure forces are included in the dissipation or form loss term which is dependent on the junction velocity and density. Given these dependencies, the form losses will vary during a transient.

16. As an assessment of the 3-D kinetics capabilities of the RETRAN-3D code please provide a comparison of data versus calculated power for SPERT Hot Standby Case 81 and the Full Power Test 86.

The RETRAN-3D predictions for the SPERT 81 and 86 tests are given in Attachment 3.

References

1. Idelchik, I. E., Handbook of Hydraulic Resistance, 3rd Edition, CRC Press, Boca Raton, 1994.

2. Bird, R. B., Stewart, W. E., and Lightfoot, E. N., Transport Phenomena, John Wiley and Sons, New York, 1965.

3. Harrison, J. F., Farman, R. F., Peterson, C. E., and Jensen, P. J., "RETRAN-02 – A Program for Transient Thermal-Hydraulic Analysis of Complex Fluid Flow Systems - Volume 5: Modeling Guidelines", EPRI NP1850-CCM, November, 1987.

4. Paulsen, M. P., McFadden, J. M., Gose, G. C., Peterson, C. E., Jensen, P. J., Shatford, J. G., McClure, and J. A., Westacott, J. L., "RETRAN-3D – A Program for Transient Thermal-Hydraulic Analysis of Complex Fluid Flow Systems - Volume 4: Assessment Manual", EPRI NP-7450-CCM, Revision 5, July, 1999.

Attachment 1
Volume 1 Revision 5

The results are

$$\frac{d}{dt}\int_{V_a} \rho_a\, u_a\, dV = -\int_{\Sigma A_{aj}} \left[\rho_a\, u_a \left(v_k^a - {}_s v_k^a\right) n_k^a + q_k^a\, n_k^a\right] dS$$

$$+ Q_T^{ab} + Q_{wa} + Q_{\epsilon a} + Q_{ma} + D_{ma} + \dot{W}_a + \int_{V_a} \dot{Q}^a\, dV \qquad \text{(II.2-47a)}$$

where

$$Q_T^{ab} = -\int_{S_{ab}} \left[\rho_a\, u_a \left(v_k^a - {}_s v_k^a\right) n_k^a + q_k^a\, n_k^a\right] dS \quad ,$$

$$= \left(\dot{m}_{ab}\right)_{S_{ab}} u_a - \int_{S_{ab}} q_k^a\, n_k^a\, dS \qquad \text{(II.2-47b)}$$

$$Q_{wa} = -\int_{S_{wa}} q_k^a\, n_k^a\, dS \quad , \qquad \text{(II.2-47c)}$$

$$Q_{\epsilon a} = -\int_{S_{\epsilon a}} q_k^a\, n_k^a\, dS \quad , \qquad \text{(II.2-47d)}$$

and

$$Q_{ma} = -\int_{S_{ma}} q_k^a\, n_k^a\, dS \quad , \qquad \text{(II.2-47e)}$$

where $\left(\dot{m}_{ab}\right)_{S_{ab}}$ is the mass transfer rate defined previously for the continuity equation.

The second term on the right-hand side of Eq. II.2-46 represents the integral of the reversible work over the finite control volume. If the pressure inside the volume integral is taken to be the volume average value and assumed to be uniform over the control volume, it can be moved outside the integral. The resulting integral can then be evaluated as shown in References II.2-9 and II.2-10, to give

$$\dot{W}_a = -\bar{p}_a \left[\sum_j \left(v_a A_a\right)_j \cos\theta_j^a + V \frac{d\alpha_a}{dt}\right] \qquad \text{(II.2-47f)}$$

where

$\bar{p}_a$ = volume average pressure.

The first term on the right-hand side of Eq. II.2-47f represents the reversible work associated with the normal inlet and exit flow paths, while the second term gives the reversible work associated with moving the phasic boundaries due to compression or expansion within a volume.

The third term on the right-hand side of Eq. II.2-46 represents the integral of the viscous dissipation over the control volume. It's contribution to the internal energy equation is usually neglected at the flow surfaces into and out of the control. This assumption is also used in RETRAN-3D. In addition, viscous dissipation due to wall-to-phase friction and interfacial friction is neglected in RETRAN. Dissipation due to friction will almost always be a small term in the types of analyses to which RETRAN is applied. The dissipation of viscous forces due to a moving surface in the control volume can be a significant contribution to the energy equation. In particular, the centrifugal pumps in the primary-side loops of the reactor coolant system are sometimes used to heat the coolant in the systems. This heating is due to viscous dissipation. Thus, the contribution of dissipation due to moving surfaces is retained in the equation. For the present development, the viscous dissipation due to a moving surface is left as a volume integral

$$D_{ma} = \int_{V_a} \sigma_{ik} \frac{\partial}{\partial x_i} v_k^a \, dV \quad . \tag{II.2-47g}$$

The interface jump condition of Eq. II.2-43b shows that the interface energy transfer of Eq. II.2-47b satisfies

$$Q_T^{ab} = - Q_T^{ba} \quad . \tag{II.2-48}$$

It is also interesting to note that the first term on the right-hand side of Eq. II.2-47a contains all the terms of the interface jump condition. This result indicates that the flow areas A_j take on the role of "interfaces" between the finite control volume and its surroundings. The surface integrals of Eqs. II.2-47c and II.2-47d represent energy exchange, in the form of heat transfer, between phase or fluid "a" and the solid surfaces bounding the fluid.

The terms on the right-hand side of Eq. II.2-47a are evaluated by the same procedure employed for the continuity and momentum equations. The major assumptions usually employed in evaluating the surface integrals in Eq. II.2-47a are:

- the contributions of $q_k^a \, n_k^a$ and $\sigma_{ik}^a \, v_k^a \, n_i^a$ are neglected at the surfaces A_j,
- the dissipation is neglected in RETRAN except for the contribution from centrifugal pumps,
- the areas A_j are taken perpendicular to the flow channel walls and the velocity is assumed to be parallel to the flow channel walls, and
- the density ρ_a is assumed to be uniform over A_{aj} and the average of the product $(u_a v_k^a)$ is taken to be the product of average values u_a and v_k^a.

Substituting this information into Eq. II.2-47a gives

$$\frac{d}{dt}\int_{V_a}\rho_a\,u_a\,dV = -\sum_j\left[\left(\rho_a u_a v^a + \bar{p}_a v^a\right)_j A_{aj}\cos\theta_j^a\right] - \bar{p}_a V\frac{d\alpha_a}{dt} + Q_T^{ab} + Q_{wa} + Q_{\epsilon a} + Q_{ma} + D_{ma} + \int_{V_a}\dot{Q}^a\,dV \quad . \tag{II.2-49a}$$

Factoring $\rho_a v_a$ out of the first term on the right-hand side of Eq. II.2-49a, leaves

$$u_a + \frac{\bar{p}_a}{\rho_a} \quad .$$

This term would be the junction (or flow path) enthalpy if the pressure was the junction value p_a rather than the volume average value. By adding and subtracting p_a/ρ_a, and applying the junction enthalpy definition, the internal energy equation can be rewritten as

$$\frac{d}{dt}\int_{V_a}\rho_a u_a dV = -\sum_j\left[\left(\rho_a v^a A_a\right)_j\left(h_a - \frac{p_a - \bar{p}_a}{\rho_a}\right)_j\right] - \bar{p}_a V\frac{d\alpha_a}{dt} + Q_T^{ab} + Q_{wa} + Q_{\epsilon a} + Q_{wa} + D_{ma} + \int_{V_a}\dot{Q}_a dV \tag{II.2-49b}$$

where

$$A_{aj} = (\epsilon\alpha_a A)_j \quad . \tag{II.2-49c}$$

As discussed previously with the continuity equation, the $\cos\theta_j^a$ is always -1 for inflow and +1 for outflow. It therefore has been omitted in the above and the appropriate sign for inflow and outflow is implied in the summation.

The junction pressure that appears in Eq. II.2-49b, should be consistent with the pressure used with the junction enthalpy model. For example, if the junction enthalpy is donored, the junction pressure should also be that of the donor volume. Thus, the Δp term will be zero for junctions flowing out of a volume. For the junctions flowing into a volume, the junction pressure should be that of the upstream volume. The pressure difference for this term will be small for most applications.

The other junction enthalpy models, e.g., enthalpy transport, bubble rise, etc., also neglect the effects of pressure drop when computing the junction enthalpy. Consequently, the argument

given above for using the donor volume pressure also applies to the other junction enthalpy models.

Eqs. II.2-49b is analogous to the macroscopic mass and momentum balances given previously and require an extensive amount of information to be useful.

2.4.2 Mechanical Energy Equation (Generalized Bernoulli Equation)

Another useful form of the energy balance can be obtained from the local form of the kinetic energy equation. The macroscopic form of the kinetic energy equation is developed in the following discussion.

The local form of the kinetic energy equation is obtained by forming the scalar product of $\tilde{v}$ with the local momentum equation to get

$$\frac{\partial}{\partial t}\left(\frac{1}{2}\rho_a v_k^a v_k^a\right) + \frac{\partial}{\partial x_i}\left(\frac{1}{2}\rho_a v_k^a v_k^a v_i^a\right) = -\frac{\partial}{\partial x_k} p_a v_k^a + p_a \frac{\partial}{\partial x_k} v_k^a + \frac{\partial}{\partial x_k} \sigma_{ki}^a v_i^a - \sigma_{ik}^a \frac{\partial}{\partial x_k} v_i^a + \rho_a v_k^a g_k^a \quad . \tag{II.2-50}$$

Substituting the property per unit volume $\psi = \frac{1}{2}\rho_a v_k v_k$ into the Reynolds transport theorem of Eq. II.2-2, operating on Eq. II.2-50 with $\int_{V_a} dV$, and using the Gauss theorem of Eq. II.2-4 gives

$$\frac{d}{dt}\int_{V_a}\frac{1}{2}\rho_a v_i^a v_i^a dV = -\int_{S_a}\left[\frac{1}{2}\rho_a v_i^a v_i^a \left(v_k^a - {}_s v_k^a\right) n_k^a + p_a v_k^a n_k^a - \sigma_{ik}^a v_i^a n_k^a\right] dS + \int_{V_a}\left[p_a \frac{\partial v_k^a}{\partial x_k} - \sigma_{ik}\frac{\partial v_i^a}{\partial x_k}\right] dV + \int_{V_a}\rho_a v_k^a g_k^a dV \quad , \tag{II.2-51}$$

Substituting Eq. II.2-55 into Eq. II.2-51 gives

$$\frac{d}{dt}\int_{V_a}\frac{1}{2}\rho_a\, v_i^a\, v_i^a\, dV + \frac{d}{dt}\int_{V_a}\rho_a\,\hat{Z}_a\, dV$$

$$= -\int_{S_a}\left[\frac{1}{2}\rho_a\, v_i^a\, v_i^a\left(v_k^a - {}_s v_k^a\right) n_k^a\right.$$

$$\left. + \rho_a\,\hat{Z}_a\left(v_k^a - {}_s v_k^a\right) n_k^a + p_a\, v_k^a\, n_k^a - \sigma_{ik}^a\, v_i^a\, n_k^a\right] dS$$

$$+ \int_{V_a}\left[p_a\frac{\partial v_k^a}{\partial x_k} - \sigma_{ik}\frac{\partial v_i^a}{\partial x_k}\right] dV \quad . \qquad \text{(II.2-56)}$$

The last term on the right-hand side of Eq. II.2-56 can be evaluated as follows. The second argument of the integral represents dissipation by viscous stresses (see Section II.2.4.1),

$$D_a = \int_{V_a}\sigma_{ik}^a\frac{\partial v_i^a}{\partial x_k}\, dV \quad . \qquad \text{(II.2-57)}$$

The first argument accounts for compressibility of the fluid. Evaluation of this term must consider the thermodynamic path followed by the fluid, isothermal or isentropic, for example. In addition, because compressibility of the fluid is included in this term, evaluation of the integral will include effects due to the time rate of change of the porosity, ϵ, and the volume fraction, α_a. For the present time the integral of the first argument is left unchanged.

Evaluation of the surface integrals in the first term on the right-hand side of Eq. II.2-56 is carried out as follows. The contributions at the flow areas, A_{aj}, are

$$\int_{\Sigma A_{aj}}\left[\frac{1}{2}\rho_a\, v_i^a\, v_i^a\left(v_k^a - {}_s v_k^a\right) n_k^a + \rho_a\,\hat{Z}_a\left(v_k^a - {}_s v_k^a\right) n_k^a + p_a\, v_k^a\, n_k^a\right] dS \quad , \qquad \text{(II.2-58)}$$

where the contributions of $\sigma_{ki}^a\, v_i^a\, n_k^a$ have been neglected. At the stationary surfaces S_{wa} and S_{ea}, all the surface integrals are zero by use of the no-slip assumption. On the moving surface S_{ma}, the surface integral accounts for the rate of work done by the fluid on the surroundings by means of pressure and viscous forces; that is,

$$\dot{W}_a = \int_{S_{ma}}\left[p_a\, v_k^a\, n_k^a - \sigma_{ik}\, v_i^a\, n_k^a\right] dS \quad . \qquad \text{(II.2-59)}$$

On the interface S_{ab}, the surface integral gives the energy exchange between the fluid phases. This contribution is denoted by

$$Ke_T^{ab} = \int_{S_{ab}} \left[\frac{1}{2} \rho_a v_i^a v_i^a \left(v_k^a - {}_s v_k^a\right) n_k^a + \rho_a \hat{Z}_a \left(v_k^a - {}_s v_k^a\right) n_k^a \right.$$
$$\left. + p_a v_k^a n_k^a - \sigma_{ik} v_i^a n_k^a \right] dS \quad . \qquad \text{(II.2-60)}$$

Substituting Eqs. II.2-57 through II.2-60 into Eq. II.2-56 gives

$$\frac{d}{dt} \int_{V_a} \frac{1}{2} \rho_a v_i^a v_i^a \, dV + \frac{d}{dt} \int_{V_a} \rho_a \hat{Z}_a \, dV$$
$$= - \int_{\Sigma A_{aj}} \left[\frac{1}{2} \rho_a v_i^a v_i^a \left(v_k^a - {}_s v_k^a\right) n_k^a + \rho_a \hat{Z}_a \left(v_k^a - {}_s v_k^a\right) n_k^a + p_a v_k^a n_k^a \right] dS$$
$$+ \int_{V_a} p_a \frac{\partial v_k^a}{\partial x_k} \, dV - \dot{W}_a - D_a - Ke_T^{ab} \quad . \qquad \text{(II.2-61)}$$

Eq. II.2-61 is the macroscopic form of the kinetic energy equation. The steady-state form of Eq. II.2-61 is called the generalized Bernoulli equation and the usual Bernoulli equation is the incompressible form of Eq. II.2-61. Thus, Eq. II.2-61 represents the Bernoulli equation for transient, compressible flow of two-phase fluids in porous media.

The surface integrals remaining on the right-hand side of Eq. II.2-61 are evaluated by the same procedures employed for the continuity, momentum, and energy equations. The major assumptions usually employed in evaluating the surface integrals are

- the areas A_j are taken perpendicular to the flow channel walls and the velocity is assumed to be parallel to the flow channel walls, and
- the density is assumed to be constant over A_{aj} and the average of products, $v_i^a v_i^a v_k^a$ for example, are taken to be the product of the average values of the quantities.

With these assumptions, Eq. II.2-61 can be written

$$\frac{d}{dt}\int_{V_a}\frac{1}{2}\rho_a v_i^a v_i^a dV + \frac{d}{dt}\int_{V_a}\rho_a \hat{Z}_a dV$$

$$= -\sum_j \left\{\left[\frac{1}{2}\rho_a (v^a)^3 + \rho_a \hat{Z}_a v^a + p_a v^a\right]_j A_{aj}\cos\theta_j^a\right\}$$

$$- \dot{W}_a - D_a - Ke_T^{ab} + \int_{V_a} p_a \frac{\partial v_k^a}{\partial x_k} dV \quad , \qquad \text{(II.2-62a)}$$

where

$$A_{aj} = A\epsilon\alpha_{aj} \quad , \qquad \text{(II.2-62b)}$$

and θ_j^a is the angle between $\tilde{v}^a$ and $\tilde{n}^a$ Eqs. II.2-62 are analogous to the macroscopic mass, momentum, and energy equations given previously and require an extensive amount of information in order to be useful.

2.5 Fluid Equation of State

The equations of state of the fluid are obtained from the fundamental relation of the material. The equations of state are usually expressed in terms of two independent variables in the form

$$\rho_a = \rho_a(p_a, u_a) \qquad \text{(II.2-63a)}$$

for the density and

$$T_a = T_a(p_a, u_a) \qquad \text{(II.2-63b)}$$

for the temperature, with similar expressions for phase or fluid "b". If the fluid is at the saturation state, the state properties are functions of the pressure only, and the equations of state can be written

$$\rho_{as} = \rho_{as}(p_a) \quad , \qquad \text{(II.2-63c)}$$

and

$$T_{as} = T_{as}(p_a) \quad . \qquad \text{(II.2-63d)}$$

$$Q_T^{ab} = - \int_{S_{ab}} \left[\rho_a u_a \left(v_k^a - {}_s v_k^a \right) n_k^a + q_k^a n_k^a \right] dS \quad . \tag{II.2-77a}$$

The macroscopic mechanical energy balance, Eq. II.2-61, contains the interphase energy exchange of Eq. II.2-60a

$$Ke_T^{ab} = \int_{S_{ab}} \left[\frac{1}{2} \rho_a v_i^a \left(v_k^a - {}_s v_k^a \right) n_k^a + \rho_a \hat{Z}_a \left(v_k^a - {}_s v_k^a \right) \right.$$
$$\left. + p_a v_k^a n_k^a - \sigma_{ik}^a v_k^a n_k^a \right] dS \quad . \tag{II.2-77b}$$

Inspection of Eqs. II.2-77 shows that the interface energy exchanges consist of a contribution due to mass exchange plus other thermal-hydraulic effects. As in the case of the mass and momentum balances, specific expressions for the interface energy exchange are not required for the present section. Eqs. II.2-77 are simply rewritten as

$$Q_T^{ab} = \hat{E}_{(\dot{m})}^{ab} + q^{ab} + \dot{W}_a \quad , \tag{II.2-78a}$$

and

$$Ke_T^{ab} = \hat{Ke}_T^{ab} + \dot{W}_a \quad , \tag{II.2-78b}$$

respectively. Eqs. II.2-78 are substituted into Eqs. II.2-49a and II.2-61, respectively. The resulting equations are not repeated here.

2.7 Summary of the Macroscopic Balance Equations

The macroscopic balance equations for general models of two-phase flow have been derived from the local, instantaneous Navier-Stokes equations. As in the case of single-phase or homogeneous two-phase flow, the macroscopic balance equations require a large amount of additional information to be useful. Averaging procedures always eliminate information and in the case of macroscopic balances, the information loss is severe. Practical applications of the macrobalances require experience and engineering judgment. In the following discussion we summarize the results obtained previously and specialize the equations to the models used in RETRAN. In particular, the mixture equations are obtained from the component equations and the definitions of various quantities of interest are introduced.

The two-phase flow model in RETRAN-3D is designed for thermal-hydraulic analyses of single-phase and two-phase boiling water with or without noncondensables. In the following discussions, the sub- and superscripts for phase "a" or "b" will be changed to "ℓ" for the liquid

The details of incorporating the vector momentum balance of Eq. II.2-96 into RETRAN are given in Section II.3.

2.7.3 Energy Conservation Equations

RETRAN utilizes two energy equations:

- the internal energy equation and
- the kinetic energy (generalized Bernoulli) equation.

The specialization of the energy conservation equations proceeds in the same manner as the mass and momentum equations and, thus, not as many details will be given for the energy equations. For the following developments, assume $\epsilon = 1$, $S_{\epsilon g} = S_{\epsilon \ell} = 0$, $S_{mg} = S_{m\ell} = 0$ and $p_g = p_\ell = p$. As with the preceding momentum equation development, this discussion of energy balances can be regarded as complete with respect to the composition of the gaseous mixture. That is, the subscript "g", which refers to the gaseous mixture, is generic in nature and is appropriate regardless of gas phase composition (i.e., noncondensables and/or water vapor).

The internal energy equation, Eq. II.2-49b, for the gas phase, is

$$\frac{d}{dt}\int_{V_g} \rho_g\, u_g\, dV = -\sum_j \left[\alpha_g\, \rho_g\, v^g\, h_g\, A\right]_j + Q_T^{g\ell} + Q_{wg} - p\,\frac{d}{dt}\,\alpha_g\, V + \sum_j \left[\alpha_g\, v^g\, A\, (\Delta p)\right]_j \quad , \tag{II.2-97a}$$

and for the liquid phase

$$\frac{d}{dt}\int_{V_\ell} \rho_\ell\, u_\ell\, dV = -\sum_j \left[\alpha_\ell\, \rho_\ell\, v^\ell\, h_\ell\, A\right]_j + Q_T^{\ell g} + Q_{w\ell} - p\,\frac{d}{dt}\,\alpha_\ell\, V + \sum_j \left[\alpha_\ell\, v^\ell\, A\, (\Delta p)\right]_j \quad , \tag{II.2-97b}$$

where

$$\Delta p = p_j - \bar{p} \quad . \tag{II.2-97c}$$

In Eqs. II.2-97, the volumetric generation and pump work terms have been dropped. The RETRAN models for these contributions are discussed in Sections V and VI, respectively. Evaluation of the last terms in Eqs. II.2-97 is given later in this report as the specific equations in RETRAN are listed.

and Eq. II.2-104a, as

$$\alpha_g = \frac{1}{1 + \left[\left(1 - X_g^m\right)/X_g^m\right]\left[\rho_g/\rho_\ell\right]} \quad . \tag{II.2-105b}$$

The mixture momentum balance of Eq. II.2-96, obtained with $v^g = v^\ell = v$, is slightly generalized to include the pump term as

$$\frac{d}{dt}\int_V \rho\,\tilde{v}\,dV = -\sum_j \left[\rho\,\tilde{v}\,v\,A + p\,A\,\tilde{n}\right]_j - \tilde{F}_f^{wm} - \tilde{F}_{loc}^{wm} - \Delta\tilde{p}_p + \int_V \rho\tilde{g}\,dV \quad . \tag{II.2-106}$$

Equation II.2-106 is a vector equation and the details of the RETRAN model are given in Section II.3.

The equation for conservation of internal energy is

$$\frac{d}{dt}U = -\sum_j (W\,h)_j + Q_{wm} + \sum_j \left[\frac{W}{\rho}(\Delta p)\right]_j + \dot{D}_{ms} + Q \quad , \tag{II.2-107}$$

where

$$\Delta p = p_j - \tilde{p} \tag{II.2-107a}$$

The over bar on the pressure designates the volume average pressure which typically will have a volume subscript. When a volume subscript is used with the pressure, the over bar will be omitted.

The contributions due to wall heat transfer and volumetric heat generation have been included. Note that the viscous dissipation term D_{ms} includes the effects of mechanical shear introduced by components such as centrifugal pumps and turbines as discussed in Sections VI.1.1 and VI.6.0, respectively. Viscous dissipation associated with wall friction is ignored as discussed in Section II.2.4.1.

The equation of state, in terms of the macroscopic balance properties is

$$p = p(M,U) \quad . \tag{II.2-108}$$

When noncondensables are present, the equation of state becomes

$$p = p(M, U, M_{nc}) \tag{II.2-108a}$$

and the gas properties correspond to those for the gas mixture. In general neither the gas nor liquid will be saturated for situations where noncondensables are not present.

Additional information is necessary in order to obtain a form of the momentum balance of Eq. II.2-106 that can be used for engineering analyses. Further development of the momentum balance is given in Section II.3. The mass conservation equation, Eq. II.2-103a, is complete as written. The energy conservation equation, Eq. II.2-107, requires information about Q_{wm}, $\dot{Q}$ and energy dissipation due to pumps, D_{ms}. Models and equations for these terms are given in other sections of this report.

unknown quantities. Application of the macroscopic balance equations to analyses of hypothetical transients in complex thermal-hydraulic loops has resulted in generalization of the usual engineering application procedures associated with these equations.

In order to conduct the generalized thermal-hydraulic analyses, many of the results obtained from the steady-state applications are used in the transient form of the equations. In particular, the steady-state equations are generally employed with experimental data in order to determine the wall friction, heat transfer coefficients, local losses, and pump characteristics, for example. The correlations and models which result from this process are then used in the general equations for transient analyses. As previously mentioned, in some cases the local equations can be solved for the detailed distributions of the fluid state and flow and the integrals obtained in the macroscopic balances, evaluated exactly. In most cases of practical interest, analytical solution of the local equations is not possible at the present time and numerical solutions would be required.

The macroscopic balance equations require that a large amount of information be supplied in order to produce a closed equation system. The derivation of the balance equations has introduced the following quantities:

- geometric details of the flow channel represented by the volume V, the flow areas A_j, and the wetted areas S_w and S_m;
- momentum exchange and forces due to wall friction, $\tilde{F}_w$;
- wall normal pressure forces and dissipation due to local flow perturbations, $\tilde{F}_{loc}$;
- momentum exchanges due to moving surfaces, $\Delta\tilde{p}_p$;
- energy exchanges between the fluid and the flow channel bounding surfaces, Q_w;
- work done on or by the fluid due to moving surfaces, $\dot{W}_p$; and
- volumetric generation of energy in the fluid, $\dot{Q}$.

All of these quantities must be specified before the general equation system can be solved. In addition to these specific engineering quantities, implementation of the equation of state of the fluid, in a manner consistent with the variety of processes encountered in transient analyses, can present problems of a practical nature. Models and correlations for all of the quantities required for closure of the equation system are given in the following sections of this volume.

Many transients of interest involve two-phase flow of the system fluid. The assumption that the two-phase mixture can be treated as a single homogeneous fluid has been explicitly introduced into the derivation of the equations. In RETRAN-3D, the two-phase nature of the fluid is introduced into the equation system through the models and correlations employed for the constitutive equations required for system closure. That is, the friction, heat transfer, pump characteristics, and the other quantities discussed in the preceding paragraph contain an accounting of the two-phase nature of the coolant. In addition, several models are employed

which account for some of the more important clearly nonhomogeneous situations encountered in system analysis. These models are of a specialized nature and are described in this volume.

RETRAN-3D has four basic forms of the general equations, each with or without noncondensables. They are

- the homogeneous equilibrium model,
- the dynamic slip model,
- the algebraic slip model, and
- the five-equation model.

The simplest form is given by the homogeneous equilibrium model. In this model, each phase in a two-phase mixture is assumed to move at the same velocity ($v^g = v^\ell = v$) and to have the same temperature ($T_g = T_\ell$). Thus, countercurrent flows, subcooled boiling and similar physical phenomena associated with transient two-phase flow cannot be completely described.

In order to relax the equal-velocity assumption, two optional model forms for the velocity difference are provided in RETRAN-3D. The first is based upon a differential equation for velocity difference obtained from algebraic manipulation of the phasic momentum-balance equations. This model is called a "dynamic slip" model and is further described in Section II.3.2. The second velocity difference option is an algebraic equation for velocity difference based on a drift flux approach and is called an "algebraic slip" model. This method is presented in Section II.3.3. Regardless of the velocity difference model selected, the mixture momentum and internal energy equations contain the same accounting of terms due to unequal-phase velocities.

The fourth model removes in part the limitation imposed on the homogeneous equilibrium model, the dynamic slip, and algebraic slip options by the assumption of equal-phase temperatures. The five-equation option of RETRAN-3D assumes the vapor in a two-phase mixture is saturated (in the absence of noncondensables) and allows the liquid phase to be subcooled, saturated, or superheated. This option computes a vapor-phase mass using a separate balance equation and allows the modeling of some nonequilibrium phenomena. Either the dynamic slip equation or the algebraic slip equation can be used to model unequal-velocity conditions in the five-equation option. This option is described in Section II.3.4.

The above equation forms are generally valid whether or not noncondensables are present. When the homogeneous equilibrium model is used, all fluid components (liquid, vapor, and noncondensable) exist at single temperature. When either of the slip models is used, the vapor and noncondensable form a homogeneous gaseous mixture that moves at a single velocity. When the five-equation model is used, the noncondensable and vapor exist at the single temperature that is mass weighted between the endpoints T_1 (only noncondensable) and $T_{sat}(p)$ (only vapor). The noncondensable tracking model adds another continuity equation to the

overall equation set. It differs from the earlier RETRAN-3D accumulator in that the noncondensable is not confined to the volume in which it originates. It differs from the generalized transport model in that the noncondensable does affect the overall hydraulic solution.

3.1 Homogeneous Equilibrium Model

The application procedure for the homogeneous equilibrium model is given in the following discussion. In this model the fluid is characterized by a single velocity and temperature. In this section, as following sections, the discussion is typically generalized to include noncondensables.

3.1.1 Geometry of the Application Procedure

The generalized macroscopic balance equations are applied in RETRAN-3D on a tube-and-tank or node-flow-path basis that are illustrated by Figure II.3-1. A portion of a straight, constant-area, vertical flow channel is shown in the figure. The regions bounded by the flow-channel walls and the dashed lines, with centers labeled "k" and "k+1", correspond to the tanks (or nodes or volumes). The regions bounded by the flow-channel walls and the solid lines correspond to the tubes (or flow paths or junctions). The volumes are sometimes referred to as "mass and energy cells" and the junctions as "momentum cells". The conservation equations for mass and energy and the equation of state are applied to the volumes, and the momentum balance is applied to the junctions.

In addition to being an application procedure for the macroscopic equations, this "staggered mesh" representation is related to the numerical solution procedure. As is the case for almost all solution procedures, the macroscopic balances sometimes require information from spatial locations where it is not directly calculated. Use of the RETRAN-3D application procedure provides information that would not otherwise be available. A hydraulic network is represented in RETRAN-3D by a number of control volumes connected by junctions or flow paths. All the geometric characteristics of the network must be supplied to the macroscopic equations along with the interactions between the walls (heat transfer) and pumps or turbines and the fluid.

A slight generalization of the flow channel of Figure II.3-1 is given in Figure II.3-2 which introduces the vector momentum model. The mass and energy conservation equations are scalar equations and are unaffected by the vector nature of the momentum balance. They only require that the sign of the mass flow rate be correct. For the case of flow into volume "k" at junction "i-1" and flow out of volume "k" at junction "i", the mass conservation of Eq. II.2-103a gives

$$\frac{dM_k}{dt} = W_{i-1} - W_i \quad , \qquad \text{(II.3-1a)}$$

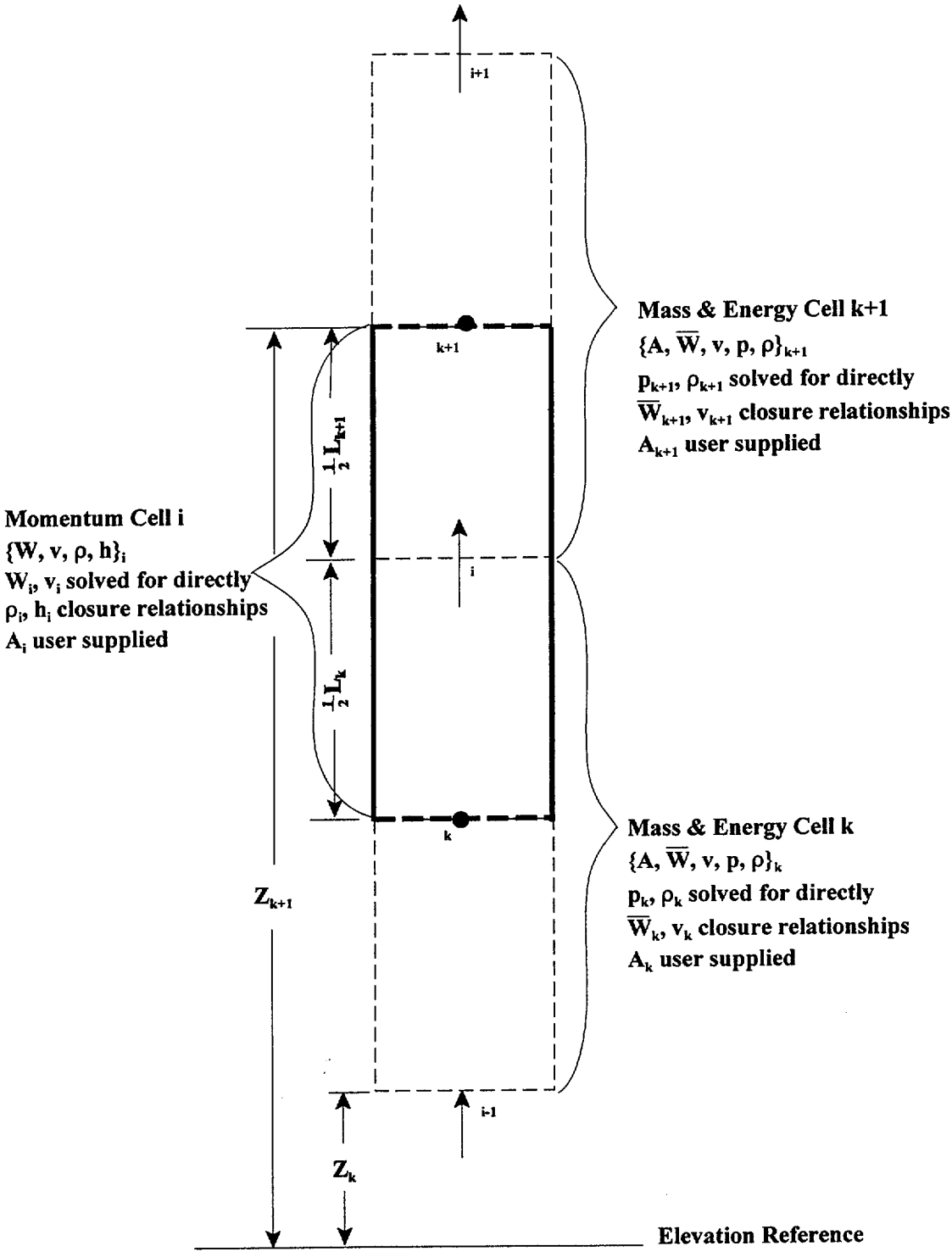


Figure II.3-1. Balance Equation and Staggered Mesh Description

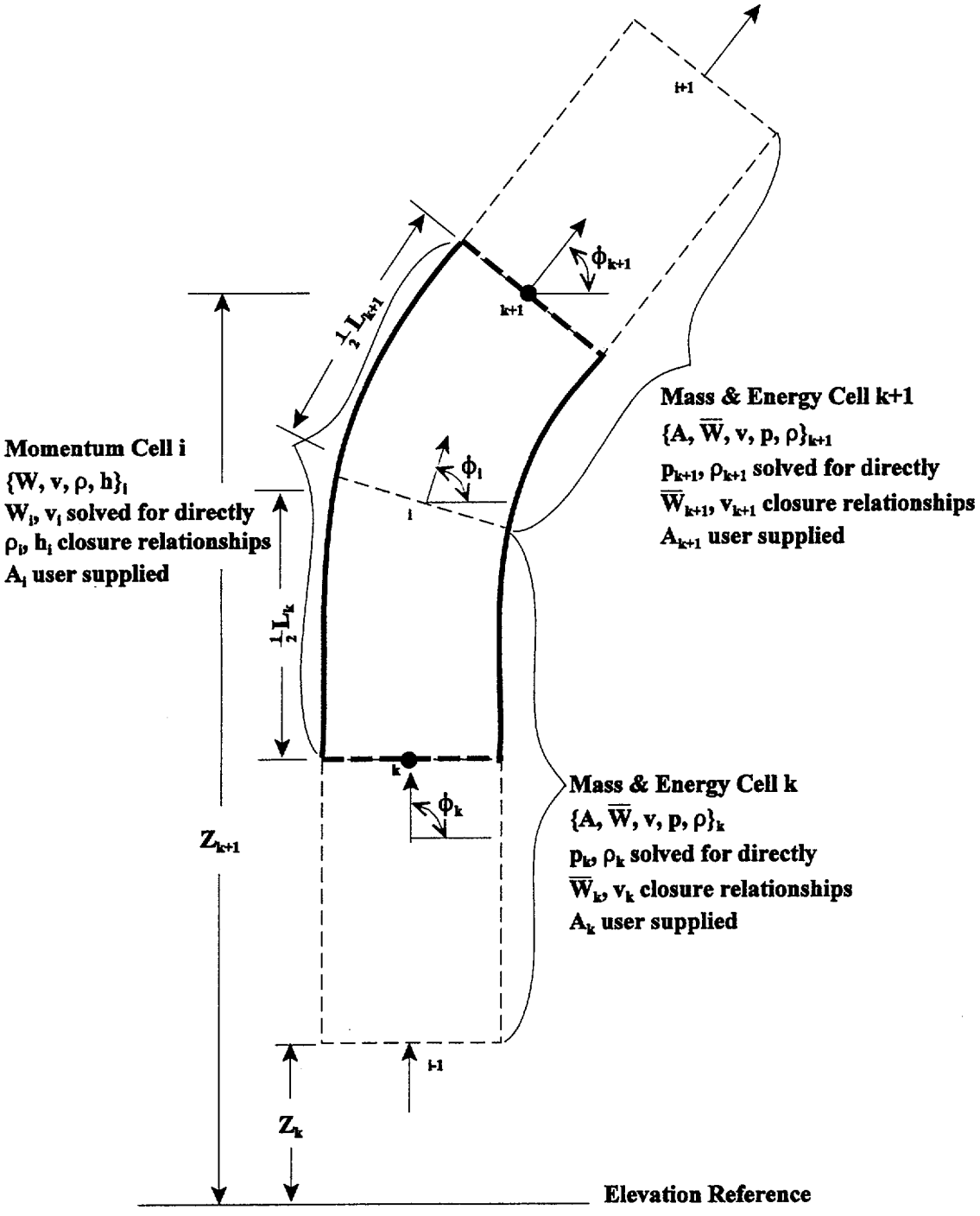


Figure II.3-2. Momentum Cell Description

where

$$M_k = \rho_k V_k \quad , \tag{II.3-1b}$$

$$W_{i-1} = (\rho v A)_{i-1} \quad , \tag{II.3-1c}$$

and

$$W_i = (\rho v A)_i \quad . \tag{II.3-1d}$$

The energy conservation of Eq. II.2-107 gives

$$\frac{dU_k}{dt} = (W_{i-1}\ h_{i-1}) - (W_i\ h_i) + Q_{wm} - \frac{W_{i-1}}{\rho_{i-1}}(p_{i-1} - p_k) + \frac{W_i}{\rho_i}(p_i - p_k) + D_{ms} + \dot{Q} \quad . \tag{II.3-2a}$$

The equation of state (omitting noncondensables), Eq. II.2-108, gives the pressure in the volume as

$$p_k = p_k(M_k, U_k) \tag{II.3-3a}$$

where

$$U_k = M_k\ u_k \quad , \tag{II.3-3b}$$

and the temperature, T_k, as

$$T_k = T_k(M_k, U_k) \quad . \tag{II.3-3c}$$

When noncondensables are included an additional mass conservation equation is required, and the noncondensable mass is added as an independent parameter input to the equation of state.

Eqs. II.3-1, II.3-2, and II.3-3 show that the thermodynamic state of the fluid is determined in the volumes. These equations also illustrate the problem discussed regarding the availability of information at spatial locations at which the thermodynamic state is not calculated. The energy conservation of Eq. II.3-2a, for example, requires the fluid enthalpy, h in Eq. II.3-2d, at the

junctions. The fluid state is determined in the volumes so that a method of obtaining state information in the junctions from the volume information is necessary. Similar requirements are associated with the momentum balance.

3.1.2 Mixture Momentum Balance

The flow equations used in RETRAN-3D are derived using the one-dimensional momentum equation and mechanical energy equation. First, the flow equation is developed for constant area channels that may include bends. The second step extends the development to variable area channels, and finally the development is extended to situations where multiple junctions connect to a given control volume. The result is the RETRAN-3D flow equation. Each of the development steps is summarized in the following sections. Examples of the vector momentum balance for several geometric arrangements are given following the development of the flow equations. Additional information is given in References II.3-1, II.3-2, and II.3-3.

3.1.2.1 Constant Area Channels. The general macroscopic momentum balance equation, Eq. II.2-106, is written in the RETRAN form as:

$$\frac{d}{dt}\int_v \rho\tilde{v}\, dV = -\sum_j (\rho v A)_j\, \tilde{v} - \tilde{F}_w - \tilde{F}_{loc} - \Delta\tilde{p}_p - \sum_j (pA)_j\tilde{n} + \rho\, V\, \tilde{g} \quad . \qquad \text{(II.3-4)}$$

where v_j is normal to A_j and the summations are over the flow surfaces into and out of the momentum cell. During the development of the local momentum equation, the pressure surface integrals (see Eq. II.2-34) were split into two pieces, one for the solid surface and another for the flow surface. They correspond to the third and fifth term on the right-hand side of the above equation. Recombining the two surface integrals into a single integral gives

$$-\tilde{F}_{loc} - \sum_j (pA)_j\, \tilde{n}_j = -\int_{S_{tot}} p\, \tilde{n}\, dS$$

which when substituted into Eq. II.3-4 gives

$$\frac{d}{dt}\int_v p\tilde{v}dV = -\sum_j (\rho v A)_j\, \tilde{v} - \tilde{F}_w - \Delta\tilde{p}_\rho + \rho V\tilde{g}\int_{S_{tot}} p\, \tilde{n}\, dS \qquad \text{(II.3-5)}$$

The component of momentum in the direction of the junction ϕ_i is obtained by taking the dot product of Eq. II.3-5 with a unit normal vector oriented in that direction, $\tilde{n}_\phi$. This gives the momentum equation

$$\frac{d}{dt}\int_{v}(\rho_i\tilde{v}_i\cdot\tilde{n}_\phi)\,dV = (\rho vA)_k\,\tilde{v}_k\cdot\tilde{n}_\phi - (\rho vA)_{k+1}\,\tilde{V}_{k+1}\cdot\tilde{n}_\phi$$

$$- F_w - \Delta p_\rho - M_i g_z - \int_{S_{wet}} p\tilde{n}\cdot\tilde{n}_\phi\,dS \qquad \text{(II.3-6)}$$

The pressure surface integral is then applied to the region upstream of the junction, k, and the region downstream k+1 assuming a uniform pressure along the surface within each region. This gives the RETRAN-3D flow equation

$$\frac{d}{dt}(\rho_i\,v_i\,V_i) = \rho_k\,v_k\,A_k\,v_{k,\psi} - \rho_{k+1}\,v_{k+1}\,A_{k+1}\,v_{k+1,\psi} - F_w - F_{loc} + (p_k - p_{k+1})\,A_k - M_i\,g_z\ , \qquad \text{(II.3-7)}$$

where

$$v_{k,\psi} = v_k\cos(\phi_k - \phi_i)$$

is the component of volume centered velocity lying in direction ϕ_i. The effect of the local form losses due to the bend is F_{loc}, which includes the effects of wall pressure forces not included in the pressure difference and other geometric changes associated with the flow turning through bends.

If the bend and other geometric changes are absent, Eq. II.3-7 reduces to

$$\frac{d}{dt}(\rho_i\,v_i\,V_i) = \rho_k\,v_k^2\,A_k - \rho_{k+1}\,v_{k+1}^2\,A_{k+1} - F_w + (p_k - p_{k+1})\,A_k - M_i\,g_z\ , \qquad \text{(II.3-8)}$$

which is the standard RETRAN straight-pipe momentum equation.

The areas A_k are constant in space and time and the volume V_i is

$$V_i = A_k\left[\frac{L_k}{2} + \frac{L_{k+1}}{2}\right]\ . \qquad \text{(II.3-9)}$$

The volume centered mass flow rate is

$$\overline{W}_k = (\rho Av)_k$$

and the component of the volume centered flow in the direction of the junction is

$$\overline{W}_{k,\psi} = (\rho A)_k v_{k,\psi}\ .$$

Using these definitions, Eq. II.3-7 can be written

$$\frac{1}{2}\left[\frac{L_k + L_{k+1}}{A_k}\right]\frac{dW_i}{dt} = \frac{\overline{W}_k\,\overline{W}_{k,\psi}}{\rho_k\,A_k^2} - \frac{\overline{W}_{k+1}\,\overline{W}_{k+1,\psi}}{\rho_{k+1}\,A_k^2} - \frac{1}{A_k}F_w - \frac{1}{A_k}F_{loc} + p_k - p_{k+1} - \frac{M_i}{A_k}g_z \quad . \tag{II.3-10}$$

Eq. II.3-10 is valid for the case of flow in a channel of constant cross-sectional area. Specific models are required for the mass flow component $\overline{W}_{k,\psi}$ in direction ψ, the wall friction $F_{w,k}$, the local flow perturbation $F_{loc,k}$ and the mass M_i in volume V_i. An example application of Eq. II.3-10 is given in Section II.3.1.2.4.

Eq. II.3-10 is an equation for the mass flow W_i, the quantity that appears in the advection terms for the continuity and energy equations, Eqs. II.3-1a and II.3-2a, respectively. The mass flows $\overline{W}_k$ and $\overline{W}_{k,\psi}$ on the right-hand side of Eq. II.3-10 are not available from a differential equation and must be obtained as functions of the mass flows W_i. A model is required for $\overline{W}_k$ and $\overline{W}_{k,\psi}$ to close the set of equations. It is given in Section II.3.1.2.3.

3.1.2.2 Variable-Area Channels. The momentum equation developed in the previous section is for constant area channels. Many situations encountered in a hydraulic system or reactor vessel model will have area changes between adjacent control volumes. An example is shown in Figure II.3-3 which is a representation of a flow channel containing an abrupt area change. The RETRAN-3D momentum equation for an abrupt area change is obtained by applying Eq. II.3-7 to the portions of the flow channel labeled $\frac{1}{2}L_k$ and $\frac{1}{2}L_{k+1}$. The two regions are separated by a small region where the area change occurs. The velocity through this region is assumed to be normal to the junction area.

The upstream half volume k, extends from the volume center to a location just upstream of the junction. The downstream half volume extends from a location just downstream of the junction to the center of the downstream volume k+1. The junction region is assumed to have no volume and the upstream side is designated by "i^-" and the downstream side is "i^+". For the upstream half volume, the momentum equation is

$$\frac{d}{dt}\frac{1}{2}(\rho v)_k V_k = \rho_k A_k v_k v_{k,\psi} - (\rho A v^2)_{i^-} - F_{w,k} - F_{loc,k} + (p_k - p_{i^-})A_k - M_{1/2\,V_k}\,g_{z,k} \quad . \tag{II.3-11}$$

where the wall friction $F_{w,k}$ and the mass $M_{1/2V_k}$ are understood to refer to the distance $\frac{1}{2}(L_k)$ and volume $1/2(V_k)$, respectively. Applying Eq. II.3-7 to the downstream half volume, the momentum equation is

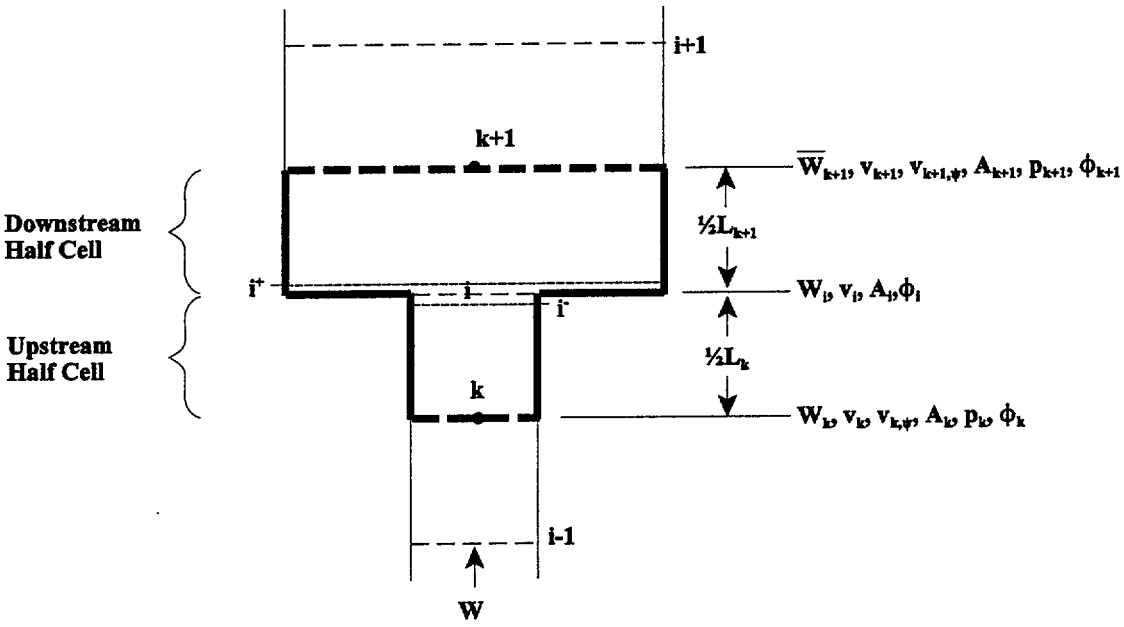


Figure II.3-3. Geometry for RETRAN Momentum Equation with an Abrupt Area Change

$$\frac{d}{dt}\frac{1}{2}(\rho v)_{k+1} V_{k+1} = (\rho A v^2)_{i^+} - \rho_{k+1} A_{k+1} v_{k+1} v_{k+1,\psi} - F_{w,k+1} - F_{loc,k+1} + (p_{i^+} - p_{k+1}) A_{k+1} - M_{1/2\, v_{k+1}} g_{z,k+1} \quad . \tag{II.3-12}$$

In Eqs. II.3-11 and II.3-12 the superscripts "i^-" and "i^+" refer to positions on either side of the transition portion of the flow channel as shown in Figure II.3-3. The volumes $\frac{1}{2}(V_{k+1})$ and $\frac{1}{2}(V_{k+1})$ are

$$\frac{1}{2} V_k = \frac{1}{2} L_k A_k \quad , \tag{II.3-13a}$$

and

$$\frac{1}{2} V_{k+1} = \frac{1}{2} L_{k+1} A_{k+1} \tag{II.3-13b}$$

respectively.

The RETRAN-3D model for the flow-area-transition region of the flow channel is based upon the macroscopic mechanical energy balance of Eq. II.2-102a. The latter equation is used because

the available information relating to the effects of local flow perturbations such as area changes are usually given in terms of irreversible energy loss coefficients.

Equation II.2-102a incorporates the assumption that the flow in the transition region can be considered to be steady-state flow and that the wall friction forces can be neglected. In addition, the model which is described here incorporates the assumption that the fluid is incompressible for purposes of applying the mechanical energy equation.

Applying the assumption of steady flow to the integral mass balance of Eq. II.3-1a, for the area transition regions gives

$$0 = W_{i-} - W_{i+} \quad , \tag{II.3-14}$$

or

$$(\rho v A)_{i-} = (\rho v A)_{i+} = W_i \quad . \tag{II.3-15}$$

The incompressible-flow assumption gives

$$\rho_{i-} = \rho_{i+} \quad . \tag{II.3-16}$$

The macroscopic mechanical energy balance for steady incompressible flow, Eq. II.2-102a, applied to the area-transition region, gives

$$0 = \frac{1}{2} \rho \left[(v_{i-})^2 - (v_{i+})^2 \right] + p_{i-} - p_{i+} - \frac{1}{2} \rho (v_{i+})^2 e_i \quad , \tag{II.3-17}$$

where e_i is the irreversible loss coefficient. These coefficients are available from tables or experimental data for common geometric changes. The irreversible loss coefficients are measures of the amount of turbulence induced into the flow field by the geometric-flow perturbations. Thus, the value of e_i for an area expansion is not the same as the value for an area contraction.

Eqs. II.3-11 through II.3-17 can be rearranged as follows. In Eq. II.3-11, $A_{i-} = A_k$ and it can be written

$$\frac{1}{A_k} \frac{d}{dt} \frac{1}{2} (\rho v)_k V_k = \rho_k v_k v_{k,\psi} - \rho_{i-} (v_{i-})^2 - \frac{1}{A_k} F_{w,k} - \frac{1}{A_k} F_{loc,k} + p_k - p_{i-} - \frac{1}{A_k} M_{1/2\, v_k} g_{z,k} \quad . \tag{II.3-18}$$

In the same manner, $A_{i^+} = A_{k+1}$ and Eq. II.3-12 is rewritten as

$$\frac{1}{A_{k+1}} \frac{d}{dt} \frac{1}{2} (\rho v)_{k+1} V_{k+1} = \rho_{i^+} (v_{i^+})^2 - \rho_{k+1} v_{k+1} v_{k+1,\psi} - \frac{1}{A_{k+1}} F_{w,k+1} - \frac{1}{A_{k+1}} F_{loc,k+1} + p_{i^+} - p_{k+1} - \frac{1}{A_{k+1}} M_{1/2\, v_{k+1}} g_{z,k+1} \quad . \tag{II.3-19}$$

Adding Eqs. II.3-18 and II.3-19 gives

$$\frac{1}{A_k} \frac{d}{dt} \frac{1}{2} (\rho v)_k V_k + \frac{1}{A_{k+1}} \frac{d}{dt} \frac{1}{2} (\rho v)_{k+1} V_{k+1} = \rho_k v_k v_{k,\psi} - \rho_{k+1} v_{k+1} v_{k+1,\psi} + p_k - p_{k+1} - \frac{1}{A_k} F_{w,k} - \frac{1}{A_{k+1}} F_{w,k+1} - \frac{1}{A_k} F_{loc,k} - \frac{1}{A_{k+1}} F_{loc,k+1} - \frac{1}{A_k} M_{1/2\, v_k} g_{z,k} - \frac{1}{A_{k+1}} M_{1/2\, v_{k+1}} g_{z,k+1} + \rho_{i^+} (v_{i^+})^2 - \rho_{i^-} (v_{i^-})^2 + p_{i^+} - p_{i^-} \quad , \tag{II.3-20}$$

where the wall friction $F_{w,k}$ and $F_{w,k+1}$, refer to $1/2(L_k)$ and $1/2(L_{k+1})$, respectively. Eq. II.3-17 is substituted into Eq. II.3-20 using Eqs. II.3-14 through II.3-16 to obtain

$$\frac{1}{A_k} \frac{d}{dt} \frac{1}{2} (\rho v)_k V_k + \frac{1}{A_{k+1}} \frac{1}{2} (\rho v)_{k+1} V_{k+1} = \rho_k v_k v_{k,\psi} - \rho_{k+1} v_{k+1} v_{k+1,\psi} + p_k - p_{k+1} - \frac{1}{A_k} F_{w,k} - \frac{1}{A_{k+1}} F_{w,k+1} - \frac{1}{A_k} M_{1/2\, v_k} g_{z,k_{k+1}} - \frac{1}{A_{k+1}} M_{1/2\, v_{k+1}} g_{z,k+1} + \frac{1}{2} \frac{W_i^2}{\rho_i} \left[\frac{1}{A_{k+1}^2} - \frac{1}{A_k^2} \right] - E_{v_i} - E_{v_k} - E_{v_{k+1}} \quad , \tag{II.3-21}$$

where

$$E_v = \frac{1}{2} \rho_i (v_{i^+})^2 e_i \quad , \tag{II.3-22a}$$

$$E_{v_k} = \frac{1}{A_k} F_{loc,k} \quad , \text{ and} \tag{II.3-22b}$$

$$E_{v_{k+1}} = \frac{1}{A_{k+1}} F_{loc,k+1} \quad . \tag{II.3-22c}$$

If E_{v_k} and $E_{v_{k+1}}$ are assumed to have the same form as E_{v_i}, the following is obtained

$$E_{v_i} + E_{v_k} + E_{v_{k+1}} = \frac{1}{2} \rho_i \left(v_i\right)^2 \left(e_k + e_i + e_{k+1}\right) . \quad \text{(II.3-22d)}$$

Combining the three form or energy loss coefficient for the upstream volume, area change, and downstream half volume into an equivalent coefficient gives

$$e_i^* = \left(e_k + e_i + e_{k+1}\right) . \quad \text{(II.3-22e)}$$

The total form loss for a momentum cell is then

$$E_{v_i} + E_{v_k} + E_{v_{k+1}} = \frac{1}{2} W_i \left| W_i \right| \frac{e_i^*}{\rho_i A_i^2} . \quad \text{(II.3-23)}$$

where the coefficient e_i^* is usually referenced to conditions downstream of the flow perturbation. The irreversible loss term of Eq. II.3-23 is used in RETRAN-3D based on the conditions at location "i".

Substituting Eqs. II.3-13 into the time derivatives on the left-hand side of Eq. II.3-21 gives

$$\frac{1}{A_k} \frac{d}{dt} (\rho v A)_k \frac{L_k}{2} + \frac{1}{A_{k+1}} \frac{d}{dt} (\rho v A)_{k+1} \frac{L_{k+1}}{2} = \left[\frac{1}{2} \frac{L_k}{A_k} + \frac{1}{2} \frac{L_{k+1}}{A_{k+1}}\right] \frac{dW_i}{dt} , \quad \text{(II.3-24)}$$

where the momentum within each half-volume is approximated as the product of an average mass flow times the path length. These half-volume average flows are further approximated as being equal to the flow W_i. These approximations for the temporal derivative of the momentum terms implicitly introduce assumptions associated with mass flow and density distributions.

The volume average flows are defined as

$$\overline{W}_k = (\rho v A)_k \quad \text{(II.3-25a)}$$

and

$$\overline{W}_{k,\psi} = (\rho A)_k \, v_{k,\psi} \quad \text{(II.3-25b)}$$

Substituting Eqs. II.3-23, II.3-24, and II.3-25 into Eq. II.3-21 gives

$$\left[\frac{1}{2}\frac{L_k}{A_k} + \frac{1}{2}\frac{L_{k+1}}{A_{k+1}}\right]\frac{dW_i}{dt} = \frac{\overline{W}_k\,\overline{W}_{k,\psi}}{\rho_k A_k^2} - \frac{\overline{W}_{k+1}\,\overline{W}_{k+1,\psi}}{\rho_{k+1} A_{k+1}^2} + p_k - p_{k+1} - \frac{1}{A_k}F_{w,k} - \frac{1}{A_{k+1}}F_{w,k+1}$$
$$- \frac{1}{A_k}M_{1/2\,v_k}\,g_{z,k} - \frac{1}{A_{k+1}}M_{1/2\,v_{k+1}}\,g_{z,k+1}$$
$$+ \frac{1}{2}\frac{W_i^2}{\rho_i}\left[\frac{1}{A_{k+1}^2} - \frac{1}{A_k^2}\right] - \frac{1}{2}W_i|W_i|\frac{e_i^*}{\rho_i A_i^2} \quad . \qquad \text{(II.3-26)}$$

Equation II.3-26 is for an abrupt area change. In RETRAN-3D, the available geometric information describes control volumes in a one-dimensional sense. When volumes with different flow areas are connected in a model, they appear as abrupt area changes in the momentum equation and there is a momentum flux correction (next to last term in Eq. II.3-26) associated with this area change. Other pressure losses are associated with geometric effects and are included through the energy loss or form loss term (last term of Eq. II.3-26).

Equation II.3-26 is used as the general RETRAN-3D flow equation and several approximations have been used to obtaining this equation. For flow in a constant area channel, Eq. II.3-26 reduces to the more nearly exact form given by Eq. II.3-10. Eq. II.3-26 also reduces to the cases of incompressible, steady flow in channels with various flow perturbations.[II.3-2]

3.1.2.3 Volume with Multiple Junctions. Some components that are modeled with one-dimensional assumptions in RETRAN-3D actually contain regions of flow which are truly multidimensional. Several examples may be found in a typical nuclear reactor system: the downcomer, plenum, and core regions in the reactor pressure vessel; the plenum and secondary side of a steam generator, and piping networks with branches in the water makeup and treatment systems. The shapes of these regions are many and varied; they cannot always be classified as simple geometrical figures. Thus, the shape of the control volume must be considered in a more general sense than axially aligned tubes.

In developing the generalized momentum balance equation with vector information, the groundwork laid by deriving Eq. II.3-26 is used as fully as possible. The vector-momentum equation is still a combination of the macroscopic form of the momentum and kinetic energy equations, but it requires additional information to describe the direction of vector components. By taking the vector component of momentum in the same direction as the junction, the terms for inertia, area change, form loss, and friction remain the same, and only the momentum flux terms are affected.

Consider two amorphous volumes, labeled k and ℓ, shown in Figure II.3-4. In general, each of the volumes has several flow paths (junctions) associated with it. In this example, there are six total junctions for the two volumes, and the two volumes are connected only by junction i. Junction i leads from volume k to volume ℓ. The angle for the direction of the effective normal

vector for a junction surface is defined with respect to the upstream ("from") volume. Thus, junction angles for junctions 1, 2, and i are defined as normal vectors for volume k, as shown by the open arrows in Figure II.3-4, junction angles for junctions 4 and 5 are defined as normal vectors for volume ℓ, and junction 3 is for another volume.

The flow vector is assumed to be normal to the junction surface, and the direction of positive flow is defined in the direction of the normal vector so ϕ_i is the direction of positive flow for W_i. The flow vectors show the true direction of flow through each junction in Figure II.3-4. W_1, W_5, and W_i are positive, and W_2, W_3, and W_4 are negative. The volume central flows for volumes k and k+1 are not necessarily normal to the volume or junction directions, as shown in Figure II.3-5. They are vector quantities, determined by the junction flows connected to the volume. A model equation is used to combine the junction flows to determine the magnitude and direction of the cell centered flow, $\overline{W}_{k,\theta}$ and θ_k, respectively. It will be presented subsequent to a discussion of the application of the scalar momentum equation.

Volumes k and k+1 are control volumes for the continuity equation, but not for the momentum equation. The momentum cell is centered over the junction and overlaps half of each volume connected by the junction. As shown in Figure II.3-5, a momentum cell is constructed by placing planes across the center of volumes k and k+1. The normal to this plane is parallel to the flow channel boundary.

Volumes k and k+1 are described as idealized volumes of length $\frac{1}{2}L_k$ and $\frac{1}{2}L_{k+1}$ and uniform cross-sectional areas A_k and A_{k+1}. The momentum cell for junction i is shown in Figure II.3-5. The axes of the cylindrical volumes are in directions ϕ_k and ϕ_{k+1}, neither of which is required to be the same as ϕ_i. The full momentum equation for the component of flow through junction i is developed using the vector nature as discussed in developing Eq. II.3-10 and in the above paragraphs and considering the aspects of an area change as discussed in developing Eq. II.3-26. The resulting equation is

$$\frac{1}{2}\left[\frac{L_k}{A_k}+\frac{L_{k+1}}{A_{k+1}}\right]\frac{dW_i}{dt} = \frac{\overline{W}_k\overline{W}_{k,\psi}}{\rho_k A_k^2} - \frac{\overline{W}_{k+1}\overline{W}_{k+1,\psi}}{\rho_{k+1} A_{k+1}^2} + p_k - p_{k+1} - \frac{1}{A_k}F_{w,k} - \frac{1}{A_{k+1}}F_{w,k+1} - (\Delta p)_p$$

$$- \frac{1}{A_k}M_{1/2\,v_k}\,g_{z,k} - \frac{1}{A_{k+1}}M_{1/2\,v_{k+1}}\,g_{z,k+1}$$

$$+ \frac{W_i^2}{2\rho_i}\left[\frac{1}{A_{k+1}^2} - \frac{1}{A_k^2}\right] - \frac{1}{2}W_i\,|W_i|\,\frac{e_i^*}{\rho_i A_i^2}\quad , \qquad \text{(II.3-27)}$$

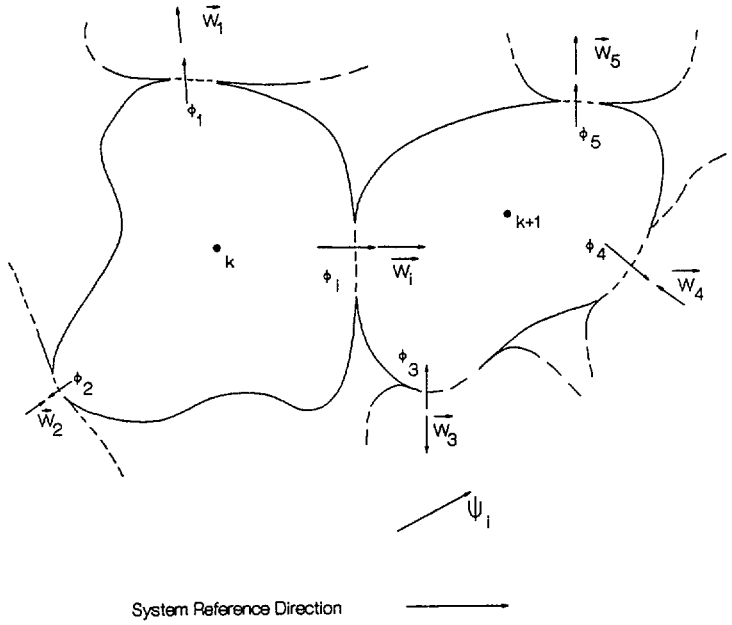


Figure II.3-4. Generalized Volumes and Junctions

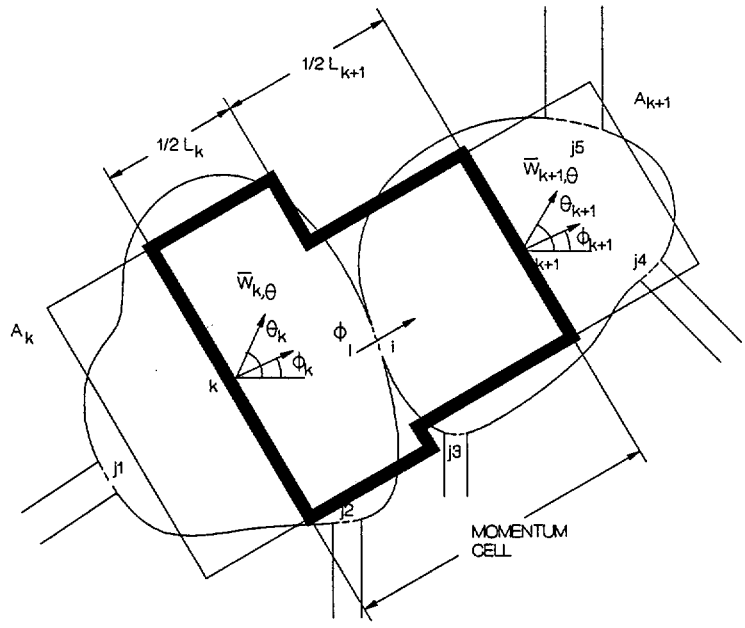


Figure II.3-5. Generalized Control Volumes with Several Flow Paths for Each Volume

where

W_i = mass flow at center of momentum cell,

$\overline{W}_k$ = mass flow at center of mass and energy cell in the direction of the volume,

$\overline{W}_{k,\psi}$ = mass flow at center of mass and energy cell in direction ψ,

$F_{w,k}$ = friction force in $1/2(V_k)$ over length $1/2(L_k)$,

$(\Delta p)_p$ = pressure drop associated with flow past a moving surface such as a pump,

$M_{1/2\ V_k}$ = mass of fluid in volume ½ (V_k),

e_i^* = irreversible loss coefficient due to flow perturbation, and

p_k = pressure in volume V_k.

Flow-field models are required for F_w, and e_i^*. If the momentum cell contains moving surfaces such as those associated with pumps, compressors, and turbines, the model must be incorporated into the momentum equation. The RETRAN momentum equation includes models for pumps and turbines as given in Section VI. The flow-field models for the wall friction and local-loss are given in Section III. The models for $\overline{W}_k$ and $\overline{W}_{k,\psi}$ are described in the following.

To evaluate the terms for momentum flux, values must be obtained for the surface flows $\overline{W}_{k,\theta}$ and $\overline{W}_{k+1,\theta}$ as shown in Figure II.3-5. These vector mass flow rates are used to obtain the cell centered flow normal to the boundary flow area and the cell centered flow in the direction of the junction, $\overline{W}_k$ and $\overline{W}_{k,\psi}$ for the upstream boundary and $\overline{W}_{k+1}$ and $\overline{W}_{k+1,\psi}$ for the downstream boundary. Consider the generalized volume k as shown in Figure II.3-6. The volume is divided by the plane at the volume center and normal to the direction ϕ_k. For purposes of discussion, the half volumes are called upstream and downstream with ϕ_k pointing to the downstream region. All eight possible relationships between a junction flow, a junction angle, and the volume regions are shown. A junction may be associated with either the upstream or downstream region, a junction direction may be inward or outward with respect to the volume, and the flows may be positive or negative with respect to the junction direction ϕ_i. Each junction i connected to volume k has a unit normal vector N_{ik} directed away from volume k. This normal vector is not shown in Figure II.3-6.

The first step in obtaining the flow rates required to evaluate the momentum flux, is to evaluate the volume centered flows $\overline{W}_{k,\theta}$ and $\overline{W}_{k+1,\theta}$. The normal flows $\overline{W}_k$ and $\overline{W}_{k+1}$ and the components in the direction of the junction $\overline{W}_{k,\psi}$ and $\overline{W}_{k+1,\psi}$ are then obtained directly from the cell centered values.

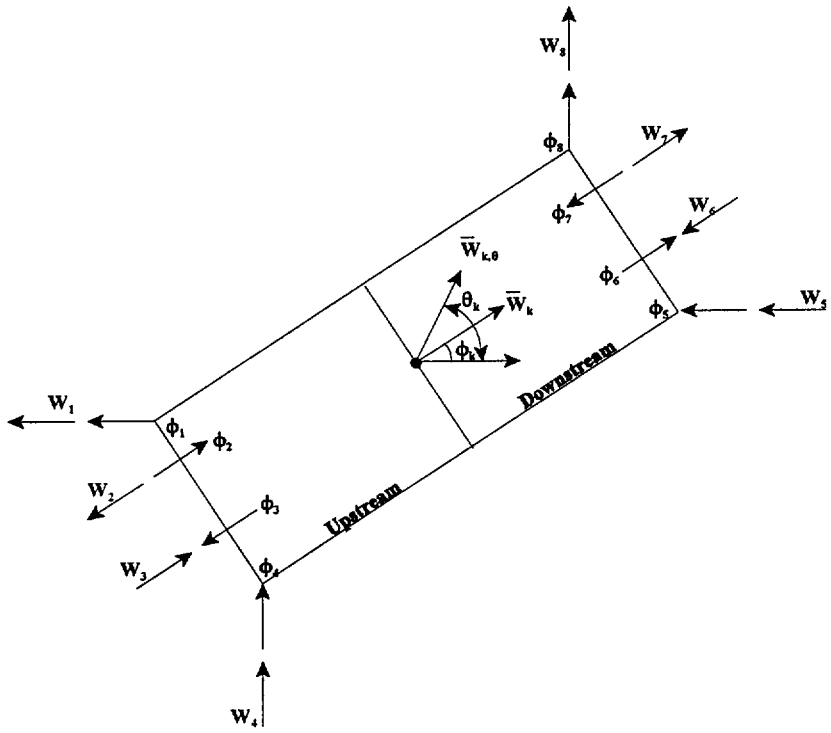


Figure II.3-6. Volume with Multiple Junction Connections

For the example control volume with multiple junction connections shown in Figure II.3-6, the junction flows are oriented in the direction indicated and are normal to the junction area. The volume is divided into an upstream and downstream half based on the volume angle, which is equal to ϕ_k. The normal vectors N_{ik} which are directed outward form the surface of volume k, are determined by the from-volume/to-volume specification and angle assigned with each junction as follows,

N_{ik} = ϕ_i for the "from" side of a junction where positive flow is out of the "from" volume k

= $(\phi_i + \pi)$ for the "to" side of a junction where positive flow is into the "to" volume k

Both the averaging and donor forms of the volume centered flow equation models are obtained using a general form. The equations used to obtain the components of the flow in the x and y directions are

$$\overline{W}_{k,x} = \zeta_{kuj}\,\overline{W}_{kuxj} + \zeta_{kdj}\overline{W}_{kdxj} \qquad \text{(II.3-28a)}$$

$$\overline{W}_{k,y} = \zeta_{kuj}\,\overline{W}_{kuyj} + \zeta_{kdj}\overline{W}_{kdyj} \qquad \text{(II.3-28b)}$$

where the ζ are coefficients dependent on the model (averaging or donor) selected and the upstream and downstream components of the volume flow are given by the following equations. The summations are over all junctions connecting to the volume that also satisfy the requirement for Φ which is defined as $\Phi = N_{ik} - \phi_j$,

$$\left.\begin{aligned} \overline{W}_{kuxj} &= -\sum_{i\epsilon k} W_i \left| \cos(\phi_i) \right| \cos(N_{ik} - \phi_i) \\ \overline{W}_{kuyj} &= -\sum_{i\epsilon k} W_i \left| \sin(\phi_i) \right| \cos(N_{ik} - \phi_i) \end{aligned}\right\} \quad \text{for } \frac{\pi}{2} < \Phi \le \frac{3\pi}{2} \qquad \text{(II.3-29a)}$$

and

$$\left.\begin{aligned} \overline{W}_{kdxj} &= \sum_{i\epsilon k} W_i \left| \cos(\phi_i) \right| \cos(N_{ik} - \phi_i) \\ \overline{W}_{kdyj} &= \sum_{i\epsilon k} W_i \left| \sin(\phi_i) \right| \cos(N_{ik} - \phi_i) \end{aligned}\right\} \quad \text{for } -\frac{\pi}{2} < \Phi \le \frac{\pi}{2} \; . \qquad \text{(II.3-29b)}$$

The volume centered velocity and direction are then given by

$$\overline{W}_{k,\Theta} = \left[\overline{W}_{k,x}^2 + \overline{W}_{k,y}^2\right]^{1/2} \qquad \text{(II.3-30)}$$

and

$$\Theta = \begin{cases} \tan^{-1}\left(\dfrac{\overline{W}_x}{\overline{W}_y}\right) & \text{for } \overline{W}_x \ge 0 \\ \pi + \tan^{-1}\left(\dfrac{\overline{W}_x}{\overline{W}_y}\right) & \text{for } \overline{W}_x < 0 \end{cases} \qquad \text{(II.3-31)}$$

The component of the volume centered mass flow rate that is normal to the volume flow area is

$$\overline{W}_k = \overline{W}_{k,\Theta} \left| \cos(\Theta_k - \Phi_k) \right| \qquad \text{(II.3-32)}$$

and the component that is in direction ψ is

$$\overline{W}_{k,\psi} = \overline{W}_{k,\Theta} \left| \cos(\Theta_k - \psi_k) \right| \; . \qquad \text{(II.3-33)}$$

The coefficients used to obtain the averaging and donor models are:

Arithmetic Average

$$\zeta_{kuj} = 0.5 \qquad \text{(II.3-34a)}$$

$$\zeta_{kdj} = 0.5 \qquad \text{(II.3-34b)}$$

Donor Cell

$$\left.\begin{aligned}\zeta_{kuj} &= 1.0\\ \zeta_{kdj} &= 0.0\end{aligned}\right\} \quad \text{for } W_j \geq 0.0 \qquad \text{(II.3-34c)}$$

$$\left.\begin{aligned}\zeta_{kuj} &= 0.0\\ \zeta_{kdj} &= 1.0\end{aligned}\right\} \quad \text{for } W_j < 0.0 \qquad \text{(II.3-34d)}$$

The volume flow terms required for the solution of the momentum equation are obtained using the above equations. They are evaluated for both boundaries of the momentum cell.

The macroscopic balance equations require a large amount of information in order to determine the time rate of change of the state of the fluid in the mass, momentum, and energy cells. Basically, all the information at the boundaries of the cells must be specified. Application of the momentum equation to a control volume bounded by the center of the mass and energy cells allows some of the boundary information to be calculated with a differential equation. The RETRAN-3D macroscopic balance equations form a system of coupled differential-difference equations for the quantities M_k, W_i, U_k, p_k, and T_k (and M_v and M_{nc} for five-equation and noncondensable, respectively, when present). All other information required by the equation system must be expressed in terms of these quantities, or other equations must be added to the system. The RETRAN-3D models for these variables are described in Section III.

3.1.2.4 Examples Using Vector Model for a Tee and Elbow. The following examples are provided to illustrate the application of the RETRAN-3D momentum equation to typical hydraulic components. In particular, the examples illustrate the calculation of the momentum cell boundary flows used to compute the momentum flux terms, using the vector momentum option.

For the examples, the pressures and densities are assumed to be known at the volume centers or momentum cell surfaces. Figure II.3-1 illustrates the locations where the various quantities are defined for mass, energy, and momentum cells. The surface areas are parallel to the boundary

surfaces shown in the illustration and the junction flows are normal to the junction flows. The volume and junction orientations (angles) are relative to the axes shown on the example figures.

Tee Example

Common geometries found in hydraulic networks and reactor systems are those of tees or manifolds. Since a manifold can be thought of as a number of connected tees, the tee is used to illustrate the application of Eq. II.3-27 for something other than a straight pipe. The examples assume steady-state flow, which allows the time derivative to be dropped. The wall friction and hydrostatic head are neglected for the purpose of simplification.

For the horizontal path of the tee (Junction 2) shown in Figure II.3-7(a), the momentum equation reduces to

$$p_1 - p_2 + \frac{\overline{W}_1 \overline{W}_{1,\psi}}{\rho_1 A_1^2} - \frac{\overline{W}_2 \overline{W}_{2,\psi}}{\rho_2 A_2^2} - \frac{W_2^2}{2\rho_2}\left[\frac{1}{A_2^2} - \frac{1}{A_1^2}\right] - \frac{W_2 |W_2|}{2\rho_2 A_{j_2}^2} e_2^* = 0 \ . \qquad \text{(II.3-35a)}$$

Equations II.3-29 are used to define the upstream and downstream flows in the x and y directions for Volume 1 as

$$\overline{W}_{1ux} = W_1 \qquad \overline{W}_{1dx} = W_2$$

$$\overline{W}_{1uy} = 0 \qquad \overline{W}_{1dy} = W_4$$

The volume average flow components in the x and y directions are then obtained using Eq. II.3-28 with the coefficients defined for the averaging method. Also applying the assumptions of steady-state conditions gives

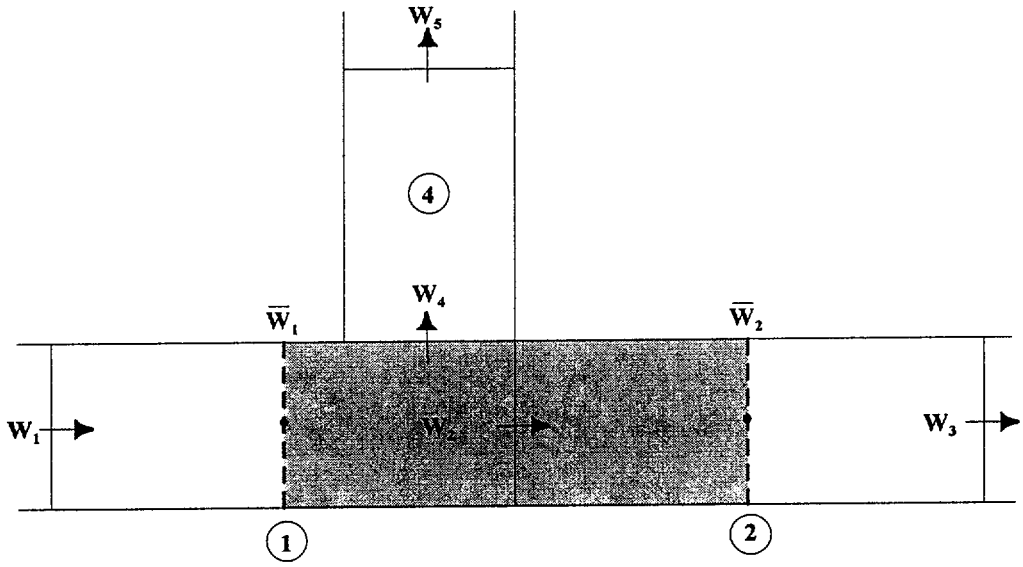
$$\overline{W}_{1,x} = \frac{1}{2}(W_1 + W_2)$$

and

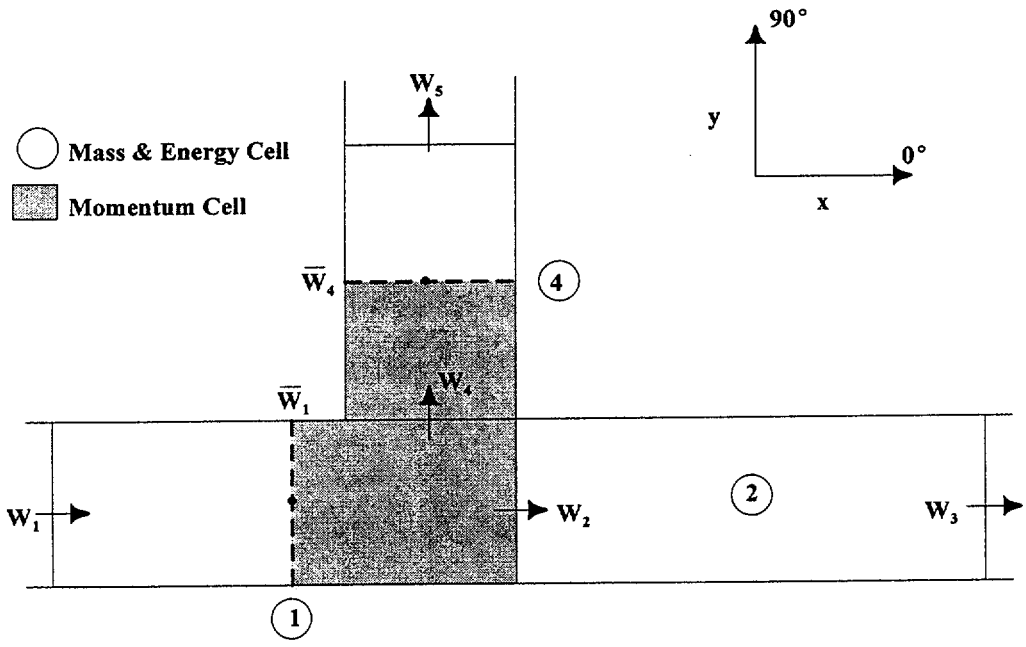
$$\overline{W}_{1,y} = \frac{1}{2} W_4 = \frac{1}{2}(W_1 - W_2) \ .$$

Equation II.3-30 is then used to define the magnitude of the volume centered flow, giving

$$\overline{W}_{1,\theta} = \frac{1}{2}\left[(W_1 + W_2)^2 + (W_1 - W_2)^2\right]^{1/2} \ .$$



(a) Momentum Cell for Junction 2



(b) Momentum Cell for Junction 4

Figure II.3-7. Momentum Cells for an Example Tee

Note that $W_{1,\theta}$ is not required for this example since the junctions are oriented along the x and y axes.

The terms needed to evaluate the momentum flux terms for the upstream boundary (center of Volume 1) for momentum cell or Junction 2 are available from the components of the volume flow already defined. They are

$$\overline{W}_1 = \overline{W}_{1,x} = \frac{1}{2}\left(W_1 + W_2\right)$$

and

$$\overline{W}_{1,\psi} = \overline{W}_{1,x} = \frac{1}{2}\left(W_1 + W_2\right) .$$

The volume centered flows for the downstream boundary of the momentum cell are obtained similarly for Volume 2. They are

$$\overline{W}_{2ux} = W_2 \qquad \overline{W}_{2dx} = W_3$$

$$\overline{W}_{2uy} = 0 \qquad \overline{W}_{2dy} = 0$$

Given the assumption of steady-state conditions

$$\overline{W}_{2,x} = \frac{1}{2}\left(W_2 + W_3\right) = W_2$$

and

$$\overline{W}_{2,y} = 0 \ .$$

The volume average flow is then

$$\overline{W}_{2,\theta} = W_2 \ ,$$

and the angle is zero degrees. The flow components for the downstream momentum flux evaluation are

$$\overline{W}_2 = \overline{W}_{2,x} = W_2$$

and

$$\overline{W}_{2,\psi} = \overline{W}_{2,x} = W_2 \ .$$

Substituting the boundary flow definitions into Eqs. II.3-35a gives

$$p_1 - p_2 + \frac{(W_1 + W_2)^2}{4\rho_1 A_1^2} - \frac{W_2^2}{\rho_2 A_2^2} - \frac{W_2^2}{2\rho_2}\left[\frac{1}{A_2^2} - \frac{1}{A_1^2}\right] - \frac{W_2 |W_2|}{2\rho_2 A_{j_2}^2} e_2^* = 0 \ . \qquad \text{(II.3-35b)}$$

The flow equation for the vertical flowpath (Junction 4) can be developed using the same process. The junction is illustrated in Figure II.3-7(b) and the associated momentum equation is

$$p_1 - p_4 + \frac{\overline{W}_1 \overline{W}_{1,\psi}}{\rho_1 A_1^2} - \frac{\overline{W}_4 \overline{W}_{4,\psi}}{\rho_4 A_4^2} - \frac{W_4^2}{2\rho_4}\left[\frac{1}{A_4^2} - \frac{1}{A_1^2}\right] - \frac{W_4 |W_4|}{2\rho_4 A_{j_4}^2} e_4^* = 0 \ . \qquad \text{(II.3-36a)}$$

The x and y direction flows for Volume 1 were defined above for the horizontal flowpath (Junction 2) and are used again here. For Volume 4, the downstream side of Junction 4

$$\overline{W}_{4ux} = 0 \qquad \overline{W}_{4dx} = 0$$

$$\overline{W}_{4uy} = W_4 \qquad \overline{W}_{4dx} = W_5$$

Given the assumption of steady-state conditions

$$\overline{W}_{4,x} = 0$$

and

$$\overline{W}_{4,y} = \frac{1}{2}(W_4 + W_5) = W_4 \ .$$

The flow components needed for evaluation of the downstream momentum flux are

$$\overline{W}_4 = \overline{W}_{4,y} = W_4 = (W_1 - W_2)$$

and

$$\overline{W}_{4,\psi} = \overline{W}_{4,y} = W_4 = (W_1 - W_2) \ .$$

Substituting these values, and the y components for the upstream side (Volume 1) of the momentum cell, gives

$$p_1 - p_4 + \frac{(W_1 - W_2)^2}{4\rho_1 A_1^2} - \frac{(W_1 - W_2)^2}{\rho_4 A_4^2} - \frac{(W_1 - W_2)^2}{2\rho_4}\left[\frac{1}{A_4^2} - \frac{1}{A_2^2}\right] - \frac{(W_1 - W_2)\,|(W_1 - W_2)|}{2\rho_4 A_{j_4}^2}\, e_4^* = 0 \ . \qquad \text{(II.3-36b)}$$

The local loss coefficient e_4^* must account for the turning of the flow from Volume 1 to Volume 4.

A typical application of Eq. II.3-36b in a system model is for a pressurizer surge line in a PWR. At steady-state, $W_4 = 0$ and $W_1 = W_2$. For these conditions, the momentum flux terms are zero and do not effect the pressure gradient which is primarily due to hydrostatic head (neglected in the example). If the vector feature isn't used for the surge line example (all angles 0 or the same), the surge line pressure will be elevated by $W_1^2/\rho_1 A_1^2$, which could be significant.

Elbow Example

Applying Eq. II.3-30 to the elbow in Figure II.3-8, shows how the vector balance reduces to a simple form with a well-known result. The steady-state mass balance gives

$$W_1 = W_2 = W_3 = W_4 \ . \qquad \text{(II.3-37a)}$$

For steady-state conditions, zero wall friction, and no elevation change, application of Eq. II.3-27 to Junction 2 gives

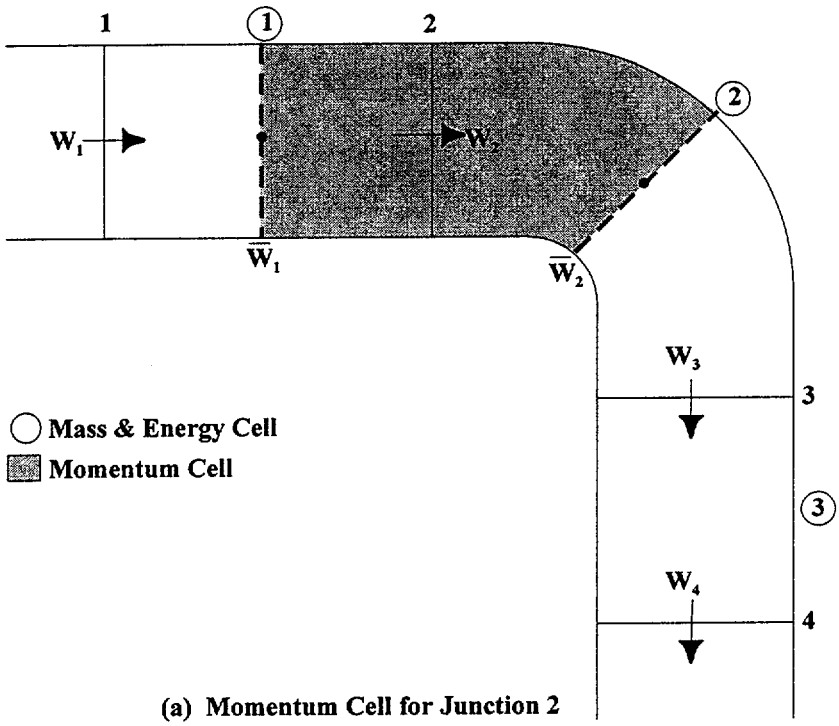
$$p_2 - p_1 = \frac{\overline{W}_1 \overline{W}_{1,\psi}}{\rho_1 A_1^2} - \frac{\overline{W}_2 \overline{W}_{2,\psi}}{\rho_2 A_2^2} - \frac{W_2^2}{2\rho_2}\left[\frac{1}{A_1^2} - \frac{1}{A_2^2}\right] - \frac{W_2|W_2|}{2\rho_2 A_{j_2}^2}\, e_2^* \ . \qquad \text{(II.3-37b)}$$

Since all of the junctions connecting to Volume 1 are horizontally oriented, the volume flow will be in that direction and there will not be a vertical component. The volume centered flow is then obtained from Eqs. II.3-28 and II.3-29

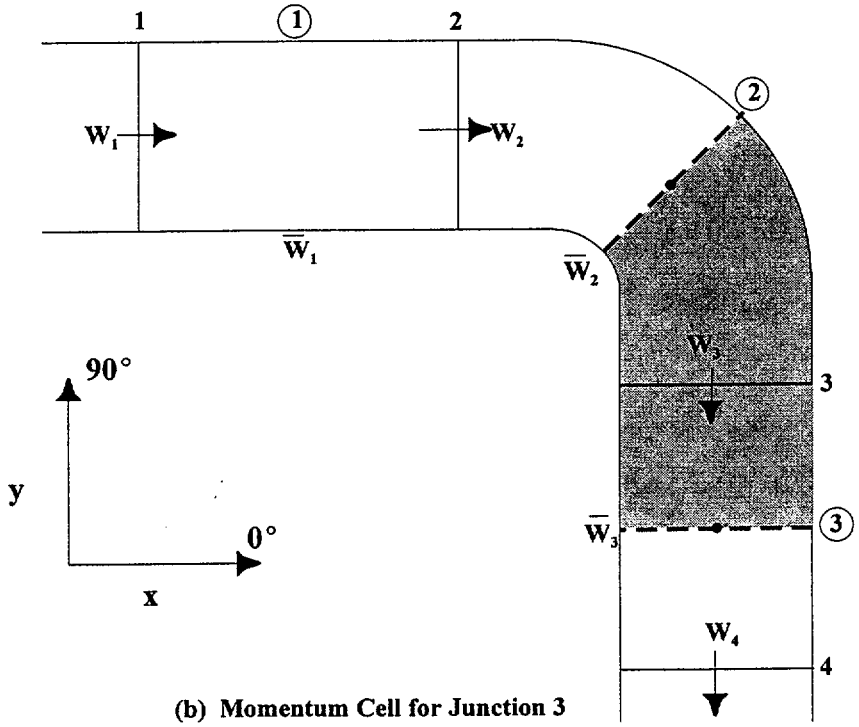
$$\overline{W}_{1ux} = W_1 \ ,$$

$$\overline{W}_{1dx} = W_2 \ ,$$

$$\overline{W}_1 = \overline{W}_{1,x} = \frac{1}{2}(W_1 + W_2) = W_2 \ , \text{ and}$$



(a) Momentum Cell for Junction 2



(b) Momentum Cell for Junction 3

Figure II.3-8. Momentum Cells for an Example Elbow

$$\overline{W}_{1,\psi} = \overline{W}_{1,x} = W_2 \ .$$

The volume centered flows for Volume 2 will have both x and y components. Equation II.3-29 gives

$$\overline{W}_{2ux} = W_2 \qquad W_{2dx} = 0$$

$$\overline{W}_{2uy} = 0 \qquad W_{2dy} = W_3$$

Applying the averaging form of Eqs. II.3-28 gives

$$\overline{W}_{2,x} = \frac{1}{2}W_2$$

and

$$\overline{W}_{2,y} = \frac{1}{2}W_3 = \frac{1}{2}W_2 \ .$$

The magnitude of the volume average flow is given by Eq. II.3-30

$$\overline{W}_{2,\theta} = \frac{1}{\sqrt{2}}W_2$$

and the angle of the flow will be at -45° since the x and y components are equal. If the downstream boundary of the momentum cell is at -45° also

$$\overline{W}_2 = \overline{W}_{2,\theta} = \frac{1}{\sqrt{2}}W_2 \ .$$

Since Junction 2 is horizontal (angle $\phi_j = 0$) the component of the volume flow in the direction of the junction is

$$\overline{W}_{2,\psi} = \overline{W}_{2,x} = \frac{1}{2}W_2 \ .$$

Substituting the volume flow components into the momentum flux terms of Eq. II.3-37b gives

$$p_2 - p_1 = \frac{W_2^2}{\rho_1 A_1^2} - \frac{W_2^2}{2\sqrt{2}\rho_2 A_2^2} - \frac{W_2^2}{2\rho_2}\left[\frac{1}{A_1^2} - \frac{1}{A_2^2}\right] - \frac{W_2 |W_2|}{2\rho_2 A_{j_2}^2} e_2^* \ . \qquad \text{(II.3-37c)}$$

Applying Eq. II.3-27 to Junction 3 gives

$$p_3 - p_2 = \frac{\overline{W}_2\overline{W}_{2,\psi}}{\rho_2 A_2^2} - \frac{\overline{W}_3\overline{W}_{3,\psi}}{\rho_3 A_3^2} - \frac{W_3^2}{2\rho_3} - \frac{W_3^2}{2\rho_2}\left[\frac{1}{A_2^2} - \frac{1}{A_3^2}\right] - \frac{W_3|W_3|}{2\rho_3 A_{j_3}^2} e_3^* . \quad \text{(II.3-38a)}$$

From the development for Junction 2 above,

$$\overline{W}_2 = \overline{W}_{2,\theta} = \frac{1}{\sqrt{2}} W_2 .$$

Since Junction 3 is oriented at 270 degrees (-90), the component of the volume flow in the junction direction is simply the y component, or

$$\overline{W}_{2,\psi} = \overline{W}_{2,y} = \frac{1}{2} W_2 .$$

For the downstream boundary for Junction 3, the flow will be in the direction of the junction since Junction 4 is oriented in the same direction. As a result

$$\overline{W}_3 = \frac{1}{2}\left(W_3 + W_4\right) = W_2$$

and

$$\overline{W}_{3,\psi} = \overline{W}_3 = W_2 .$$

Substituting the values of the volume center flows into Eq. II.3-37d gives

$$p_3 - p_2 = \frac{W_2^2}{2\sqrt{2}\rho_2 A_2^2} - \frac{W_2^2}{\rho_3 A_3^2} - \frac{W_2^2}{2\rho_3}\left[\frac{1}{A_2^2} - \frac{1}{A_3^2}\right] - \frac{W_2|W_2|}{2\rho_3 A_{j_3}^2} e_3^* . \quad \text{(II.3-38b)}$$

Summing Eqs. II.3-37c and II.3-37e gives

$$p_3 - p_1 = \frac{W_2^2}{\rho_1 A_1^2} - \frac{W_2^2}{\rho_3 A_3^2} - \frac{W_2^2}{2\rho_2}\left[\frac{1}{A_1^2} - \frac{1}{A_2^2}\right] - \frac{W_2^2}{2\rho_3}\left[\frac{1}{A_2^2} - \frac{1}{A_3^2}\right]$$

$$- \frac{W_2|W_2|}{2\rho_2 A_{j_2}^2} e_2^* - \frac{W_2|W_2|}{2\rho_3 A_{j_3}^2} e_3^* . \quad \text{(II.3-38c)}$$

For a uniform flow area and a constant density, the above reduces to

$$p_3 - p_1 = \frac{W_2 |W_2|}{2\rho A^2}\left(e_2^* + e_2^*\right) = \frac{-W_2 |W_2|}{2\rho A^2} e^* \qquad \text{(II.3-39)}$$

which is commonly used to determine local-loss coefficients from experimental data.

It should be noted that Eq. II.3-37c shows that there will be a pressure recovery in p_2 due to the momentum flux terms. However, an equal and opposite recovery (loss) occurs between the middle of the elbow and the exit as shown by Eq. II.3-38b.

While the elbow example given above is useful for illustrating the use of the vector momentum model, it is not illustrative of a typical use in RETRAN-3D. RETRAN-3D control volumes usually represent long piping sections containing elbows and flow restrictions or expansions. For such models, the loss coefficients for the related junctions must be equal to the sum of the losses for the various components that are associated with the adjacent control volumes. Reference II.3-17 gives guidelines for modeling LWR systems.

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Attachment 2
The Node-Flowpath Methodology

The Node-Flowpath Methodology

The traditional method used in the numerical simulation of nuclear reactor systems has the piping system geometry represented as a series of nodes or volumes that are connected by a flowpath or junction. This approach has come to be known as the "node-flowpath" or "volume-junction" representation of the reactor system. The components represented with this method include the core and plena regions of the reactor vessel, the hot and cold legs, steam generators, and components such as pumps, pressurizer and accumulator. The representation is basically a one-dimensional model of the reactor system, in that flow enters one end of the node, exits at the other end, and the properties are assumed to be homogeneous within the node.

The early applications of programs of this type were undertaken by reactor vendors to obtain the system response to postulated loss-of-coolant accidents (LOCA). The node-flowpath representation of the reactor system has remained as an integral part of the computer programs that have followed the early LOCA codes. Today, the programs generally applied for system analyses of LOCA and plant transient licensing applications, such as LOFTRAN, versions of RELAP4, RELAP5, and versions of RETRAN, all rely on this representation to model the piping system. Programs used to represent three-dimensional geometries such as TRAC and some vendor programs use different methods to solve the conservation equations for special applications.

Some of the requests for additional information ask questions regarding the node-flowpath approach and how it is applied in complex regions for a RETRAN model of the reactor system. In the following discussion, the homogeneous mixture equations of RETRAN are applied to the node-flowpath approach first for a section of straight pipe and then for regions of complex geometry such as those involving the downcomer and lower plenum. The models used to compute values of certain variables needed for closure of the system equations are summarized. Special considerations related to geometric input for the volumes and junctions in the region of complex geometry are discussed and the form of the momentum flux terms for these two geometries are presented. It is noted that the nodalization model presented for the complex region discussion represents one method of modeling this region, and it is used herein for discussion of the vector momentum model in RETRAN-3D.

Application to Simple Pipe Geometry

The node-flowpath concept uses two basic types of elements, the first is a control volume, also denoted as a node. Nodes are connected to each other via flowpaths or junctions. The geometric representation of a node is defined by the volume, flow area, elevation, and height, and other model parameters such as the wetted perimeter and hydraulic diameter. A flowpath is defined by a flow area, inertia, elevation, and energy loss coefficient. It also uses some of the geometric information of the two nodes it connects, including the flow length for the connected nodes and the flow area and elevation. A junction must reside between the vertical bounds of the adjacent control volumes it connects with.

In the node-flowpath method, the equations for conservation of mass and energy are solved in the nodes or volumes, and the one-dimensional momentum equation is solved in the flowpaths. Flowpaths occupy the region bounded by the centers of the nodes connected. Figure 1 illustrates the regions occupied by nodes and flowpaths for a simple pipe geometry.

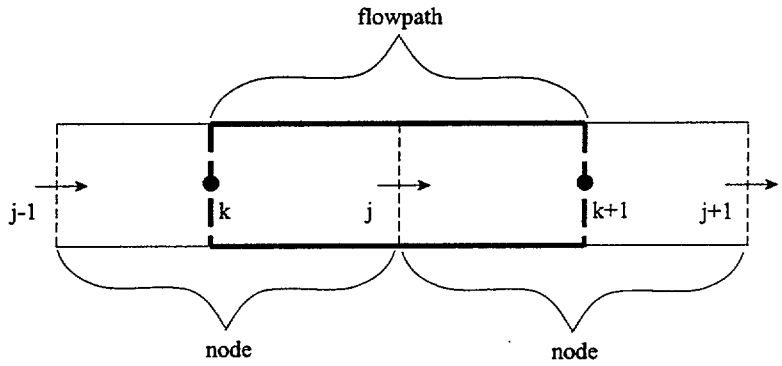

Figure 1. Node-Flowpath Mesh Structure

The mixture mass and energy equations are spatially integrated over a control volume which corresponds to a node in Figure 1. They are solved to obtain the total mass and internal energy for each node. The mixture momentum equation is spatially integrated over a region corresponding to the flowpath and is solved to obtain the average mass flow rate in the flow path. Note that the regions of integration are different for the nodes and the flowpaths. The flow path region extends from the center of the upstream node to the center of the downstream node.

Historically, nodes are referred to as control volumes (or simply volumes) since they correspond to the region over which the mass and energies are integrated. Similarly, the flowpaths are referred to as junctions since they correspond to the connection or junction between volumes. The remainder of the discussion refers to nodes and flowpaths as volumes and junctions, respectively.

Governing Equations

In this discussion, the mixture conservation equations will be considered. While RETRAN includes options to model slip and nonequilibrium conditions, the inclusion of these equations relies on the same node-flowpath representation of the system. RETRAN uses the volume-junction terminology and that is applied in the following. The three governing equations for the RETRAN-3D program are given by Equations 1, 2, and 3. Detailed derivations of these equations for the homogeneous equilibrium mixture model are presented in Reference 1. This discussion presents these equations applied to the straight pipe of Figure 1.

The mixture continuity equation is

$$\frac{d}{dt} M_k = - \sum_j W_j \quad , \tag{1}$$

and the mixture energy equation is

$$\frac{d}{dt} U_k = - \sum_j (W\ h)_j + Q_{wm} + \sum_j (W/\rho)_j\ (\Delta p)_j \quad , \tag{2}$$

where

$$\Delta p = p_j - p_k \ .$$

The junction flow equation is obtained from the one-dimensional mixture momentum equation and is

$$I_j \frac{dW_j}{dt} = \frac{\overline{W}_k \overline{W}_k}{\rho_k A_k^2} - \frac{\overline{W}_{k+1} \overline{W}_{k+1}}{\rho_{k+1} A_{k+1}^2} - \frac{1}{A_k} F_{w,k} - \frac{1}{A_{k+1}} F_{w,k+1} + p_k - p_{k+1} - \frac{M_{\frac{1}{2}v_k}}{A_k} g_z - \frac{M_{\frac{1}{2}v_{k+1}}}{A_{k+1}} g_z \tag{3a}$$

where

$$I_j = \frac{1}{2}\left[\frac{L_k}{A_k} + \frac{L_{k+1}}{A_{k+1}}\right] . \tag{3b}$$

The k (and k+1) subscripts refer to the node average quantities that are assumed to exist at the geometric center of the volume and the j subscript values are assumed to exist at the junction connecting two nodes. The geometric and model parameters associated with the control volumes are,

A = uniform flow area,
L = flow length,
Z = elevation of the bottom,
Z_{vol} = height,
g_z = gravitational constant, and
$F_{w,k}$ = wall friction loss.

The dependent variables (unknowns) associated with each of the three balance equations are

Balance Equation	**Dependent Variable**
Mixture Continuity	Total Mass - M_k
Mixture Energy	Total Energy - U_k
Mixture Momentum	Junction Flow - W_j

Pressures for each control volume are obtained using the water property functions and an iterative search procedure that determines the pressure given the total mass and energy in a volume. The pressure search solutions is discussed in detail is Reference 1 and is given by the function

$$p_k = f(\rho_k, u_k) \tag{4}$$

where

$$\rho_k = M_k/V_k$$

and

$$u_k = U_k/M_k \ .$$

The thermodynamic state of the control volume is also obtained from the pressure search; thus, the following state properties are known

h_k = enthalpy, and
T_k = temperature,

as are the saturation and other state properties.

Closure Relations

For a node-flowpath program, the mass and energy inventories are computed for the volumes and mass flow rates are computed for the junctions. This means that the pressure, enthalpy, and density values are available only in the volumes. To relate these parameters to a junction, some assumptions are required. These assumptions are

1) The pressure for a junction, p_j, is the flow donor node pressure adjusted for hydrostatic head and wall friction losses between the volume center and the junction. This model is used at all junctions. (Section III.2.2 of Reference 1)

2) The enthalpy of a flow path, h_j, is the enthalpy of the upstream (or donor) volume for most applications although there are special models that account for variations in the enthalpy as a result of spatial gradients due to heating or phase separation. The donor

method is the default model and is applied unless the user selects another option through input. (Section III.2.3 of Reference 1)

3) The density of a junction, ρ_j, is the density of the upstream (or donor) volume unless a special model that accounts for variations in the enthalpy as a result of spatial gradients due to heating or phase separation is used. For these special models, the density is computed from the junction pressure and enthalpy using the RETRAN-3D water property functions. (Section III.2.2 of Reference 1)

The solution of the momentum equation requires evaluation of the momentum flux terms (the first two terms on the right-hand side of Equation 3a) at the upstream and downstream surfaces of the momentum cell. The volume centered flow values used in these terms are not available directly from the solution of the balance equations, i.e., they are not one of the dependent variables noted above. Consequently, a separate model is necessary to define these flow rates as a function of the junction mass flow rates; it is given below.

The momentum flux term is given by the product of the volume average flow, $\overline{W}_k$, which is parallel to the volume axis and normal to the volume flow area, and the component of the volume average flow which lies in the direction of the momentum cell. For the straight pipe shown in Figure 1, these two terms are equal and the momentum flux terms become

$$\frac{\overline{W}_k^{\,2}}{\rho_k A_k^2} - \frac{\overline{W}_{k+1}^{\,2}}{\rho_{k+1} A_{k+1}^2} .$$

Assuming steady-state conditions, the volume average flow, $\overline{W}_k = \overline{W}_{k+1} = W_j$, and the momentum flux term reduce to

$$\frac{W_j^2}{\rho_k A_k^2} - \frac{W_j^2}{\rho_{k+1} A_{k+1}^2} .$$

Given steady-state conditions in a constant area pipe, the momentum flux gradient is determined by the density gradient only.

Application to Complex Geometries

The node flowpath geometry shown in Figure 1 is typical of that for piping sections. For RETRAN-3D models of light water reactor (LWR) systems, the vessel is a large three-dimensional component comprised of a number of nearly one-dimensional components (core channels and bypass) that are connected via regions where there are three-dimensional effects, such as the plenums and annuli. Figure 2 is an illustration of such a system, and shows a simplified noding example when the node-flowpath approach is applied to the RETRAN-3D balance equations. The system is represented by connecting a series of one-dimensional volumes. Thus, a cold leg pipe, downcomer, the plenums and other regions in the system would

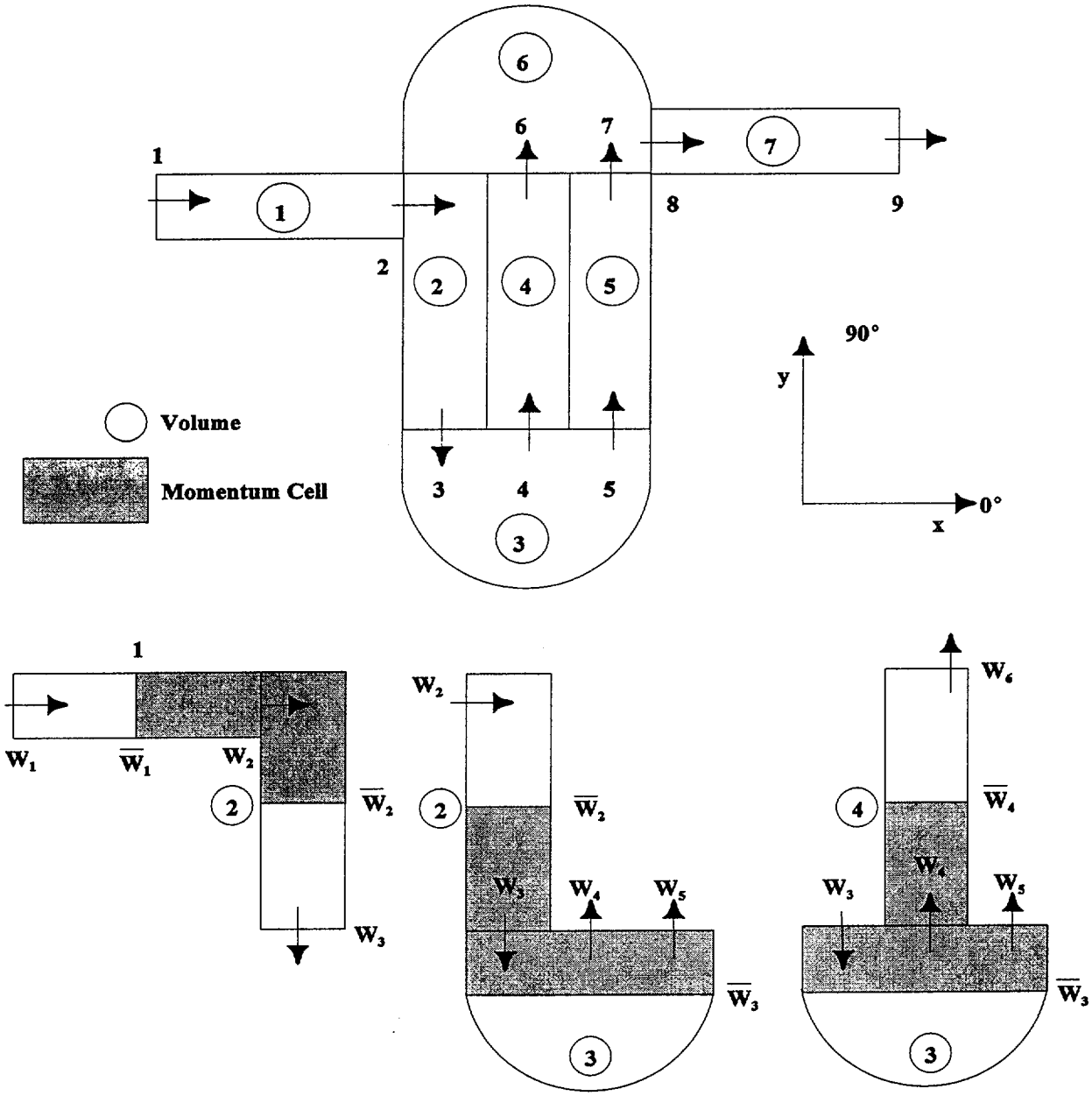

Figure 2. Reactor Vessel Noding

all be represented as one-dimensional volumes, and they would be connected to other volumes by junctions. The volume (node) boundaries are defined by the user through input values for the volume size, equivalent flow area, length and hydraulic diameter. If a particular piping section actually has pipes of varying sizes or elbows, expansion or contraction sections and is represented as a single node, it is the responsibility of the user to provide the appropriate equivalent values for the physical volume occupied by the node as well as the area, length and diameter values. An equivalent loss coefficient may also be required to account for the total pressure drop related to the geometry changes.

When modeling complex geometries with a program that contains a one-dimensional momentum equation, the selection of inputs requires a careful consideration of how those inputs are used in the equations. Reference 2 provides specific guidance for modeling various LWR components. The user may choose to preserve certain geometrical quantities with particular sensitivity to the parameter of interest. Studies of the sensitivity of the model response to input variations are often performed when adapting a one-dimensional equation to a complex geometry. Another consideration is the level of detail that is required to obtain the necessary results. This will dictate the detail that is required for the system nodalization. The nodalization in Figure 2 is used for illustration only.

Input Parameters for Complex Regions

This section presents a short summary of the input considerations for some of the important model parameters and input options for applying RETRAN-3D in regions of complex geometry. It is noted that there may be several ways to represent these regions. The vector form of the momentum equation permits one to account for changes in the momentum flux resulting from a change in the direction of the fluid flow. However, the loss coefficient values for unrecoverable losses for elbows and bends presented in references such as Idelchik [3] include effects due to change in direction. It may be difficult to determine what fraction of the loss coefficient is a loss due to a change in flow direction. Thus, many applications using RETRAN in complex geometric regions use handbook values for loss coefficients or values computed from steady-state pressure drop measurements, and do not account separately for the angle changes when flowing from one volume to the next.

Generally, the volume flow area should be representative of the cross-sectional flow area at the volume center or momentum cell boundary. This value affects the momentum flux terms and can also affect the wall friction.

The volume length and/or the hydraulic diameter may be modified to more correctly represent the wall frictional pressure losses associated with a control volume. In some components, a length other than the physical length may be required to accurately represent the frictional losses.

The geometric inertia is treated as an independent model parameter. As an option, the inertia can be computed directly from the input volume flow areas and lengths for simple one-dimensional regions. This is appropriate for straight pipes, but generally is not a good representation of complex geometries such as plenum and separators. In these components the flow path length

through the component may be significantly different than the physical length (or area) of the volume. The inertia has no effect on the steady-state solution, but it does affect the time constant for the flow during a transient calculation.

Control volume boundaries generally coincide with physical changes in the geometry of the region being modeled, e.g., where the cold leg joins the downcomer. It is common for junctions to have areas that differ from either the upstream or downstream volumes, or both. These local area changes will affect the irreversible wall friction and form losses and the reversible area change momentum flux terms.

Form or energy losses must be included in junctions to account for losses associated with changes in geometry. These geometry dependent loss coefficients are based on measurements and have been collected into various widely used books, such as Idelchik.[3]

Application of Vector Momentum Option

RETRAN-3D provides an option for the code user to approximate the vector nature of the momentum by associating an angle with the momentum cell. This approach has been called "vector momentum". For the system shown in Figure 2, the momentum equation takes the form

$$I_j \frac{dW_j}{dt} = \frac{\overline{W}_k \overline{W}_{k,\psi}}{\rho_k A_k^2} - \frac{\overline{W}_{k+1} \overline{W}_{k+1,\psi}}{\rho_{k+1} A_k^2} - \frac{1}{A_k} F_{w,k} - \frac{1}{A_{k+1}} F_{w,k+1}$$
$$+ p_k - p_{k+1} - \frac{M_{\frac{1}{2}v_k}}{A_k} g_z - \frac{M_{\frac{1}{2}v_{k+1}}}{A_{k+1}} g_z$$
$$+ \frac{W_j^2}{2\rho_j} \left[\frac{1}{A_{k+1}^2} - \frac{1}{A_k^2} \right] - \frac{1}{2} W_j |W_j| \frac{e_j^*}{\rho_j A_j^2} \qquad (5)$$

where $\overline{W}_k$ is the component of the volume average flow that is normal to the flow area and $\overline{W}_{k,\psi}$ is the component that lies in the direction of the junction. Both are obtained from the volume average flow $\overline{W}_{k,\theta}$. Appendix A provides the details of the model equations used to obtain these flows. The form or geometric losses are given by the last term in Eq. 5 where e_j^* is the standard energy loss coefficient.

Figure 2 illustrates the momentum cell and associated control volume relationships for three different junctions. The first is Junction 2 connecting the cold leg piping to the downcomer volume, the second is for Junction 3 connecting the downcomer and the lower plenum, and the third is Junction 4 connecting the lower plenum with the first active core volume. The application of the mixture continuity and energy equations to these geometric configurations is straight forward since they are scalar equations. The momentum flux considerations are discussed below. Tables 1 and 2 list the pertinent information for evaluating the momentum flux contributions, which are also shown (without the ρA^2 terms in the denominators). The evaluations were performed assuming steady-state flow conditions.

Table 1

Junction Related Momentum Flux Terms

Junction	Angle (degrees)	$\overline{W}_{k,\Psi}$	$\overline{W}_{k+1,\Psi}$	$\overline{W}_{k},\overline{W}_{k,\Psi}$	$\overline{W}_{k+1}\overline{W}_{k+1,\Psi}$
1	0	—	—	—	—
2	0	W_2	$\frac{1}{2}W_2$	W_2^2	$\frac{1}{4}W_2^2$
3	270	$\frac{1}{2}W_3$	0	$\frac{1}{4}W_3^2$	0
4	90	0	W_4	0	W_4^2
5	90	—	—	—	—
6	90	—	—	—	—

Table 2

Volume Centered Flow Quantities

Volume	Angle (degrees)	$\overline{W}_{k,x}$	$\overline{W}_{k,y}$	$\overline{W}_{k,\Theta}$	Θ (degrees)	$\overline{W}_{k}$
1	0	W_2	0	W_2	0	W_2
2	270	$\frac{1}{2}W_2$	$\frac{1}{2}W_2$	$1/\sqrt{2}W_2$	315	$\frac{1}{2}W_2$
3	90	0	0	0	0	0
4	90	0	W_4	W_4	90	W_4

Each of the three junctions are discussed below. The momentum flux terms are evaluated using the averaging model for the volume centered flows, where the volume centered flow is the arithmetic average of the inlet and exit flows. The following examples give an intuitive description for the evaluation of the volume average flow and momentum flux terms. The actual equations implemented in RETRAN-3D to perform this task are given in Appendix A.

Cold Leg/Downcomer - Junction 2

Since the cold leg is a pipe and essentially one-dimensional, the inertia for Junction 2 would be one half of the L/A for volume 1 plus the contribution for the associated downcomer half volume. For the downcomer side, the L/A contribution might range from the geometric values to something larger if there is an attempt to account for the flow spreading around the downcomer annulus as well as flowing downward. These are modeling choices rather than code options.

With the assumption of steady-state flow conditions, $W_1 = W_2 = W_3$ and with W_2 substituted for the other values, the value for the average flow for Volume 1 ($\overline{W}_1$) is simply W_2. It is oriented in the x-direction since both the inlet and exit junctions for the volume lie in that direction. For Volume 2, the inlet flow (junction 2) is at 0 degrees, but the exit (Junction 3) is at 270 degrees. Both the x and y components are simply $\frac{1}{2} W_2$. As a result, the averaging algorithm used for the volume flow gives a value of $1/\sqrt{2}\ W_2$ at 315 degrees. The results for the volume center flow components are shown in Table 2. They are combined with the component of the volume flow that lies in the direction of the junction which is shown in Table 1 to give the momentum flux terms which are shown in the last two columns of Table 1. The angular effects for the momentum flux for Junction 2 appear in the downstream term (k+1) which has a multiplier of 1/4 rather than a value of unity that would appear if the angular effects were neglected (as shown for the straight pipe example given above).

Downcomer/Lower Plenum - Junction 3

For Junction 3, the inertia would in general be something other than the geometric value. The upstream contribution from the downcomer could be the geometric value if the flow in the half volume is primarily downward so the length of the flow path is approximately the same as the length of the half volume. On the other side, the flow path length will differ significantly from the half volume length, so the lower plenum contribution to the inertia will typically be larger than the geometric value.

As noted above, $W_2 = W_3$ so W_3 is used for W_2 wherever it is needed. The volume centered flow for Volume 2 was determined in the example for Junction 2. The average flow for Volume 3, the lower plenum, is determined by averaging all of the flows in and out. The orientation of the junctions is either at 90 or 270 degrees, indicating that there will not be a component in the x-direction. Table 2 shows that the volume flow terms are zero since all flows enter and leave the same side of the lower plenum. The result is that the downstream momentum flux term is zero as shown in Table 1.

Lower Plenum/Core Inlet - Junction 4

The inertia for Junction 4 would be the component for the lower plenum half volume which would be the geometric value plus the core node contribution. Since the core node side is primarily one-dimensional, the L/A contribution would be the geometric value.

With the assumption of steady-state flow conditions, $W_4 = W_6$. Consequently, W_4 is substituted for W_6. As indicated above, the volume flow terms are zero for the lower plenum volume as expected for a large tank. The volume flow for the downstream side is W_4 since the core volume has only was one inlet and one exit junction and they are oriented in the same direction. Consequently, the momentum flux term is the equal to the square of the Junction 4 flow rate. This is listed in Table 1.

The examples given above illustrate the use of the node-flowpath approach for some of the more complex geometries that are encountered in reactor vessel models. Reference 2 provides more guidance for specific applications of RETRAN to LWR systems.

References

1. Paulsen, M. P., et. al., "RETRAN-3D – A Program for Transient Thermal-Hydraulic Analysis of Complex Fluid Flow Systems - Volume 1: Theory and Numerics", EPRI NP7450, Revision 5, (to be released).

2. Harrison, J. F., Farman, R. F., Peterson, C. E., and Jensen, P. J., "RETRAN-02 – A Program for Transient Thermal-Hydraulic Analysis of Complex Fluid Flow Systems - Volume 5: Modelling Guidelines", EPRI NP1850-CCM, November, 1987.

3. Idelchik, I. E., Handbook of Hydraulic Resistance, 3rd Edition, CRC Press, Boca Raton, 1994.

Appendix A
Volume Centered Flow Models

The solution of the RETRAN-3D momentum equation requires the mixture velocity (vector quantity) or flow rate at the momentum cell boundaries (or volume centers). These boundary flows are not obtained directly from the solution of the governing equations but are obtained from a simple closure model. Two different options are available in RETRAN-3D. One uses a donor scheme and the other uses an averaging scheme. The choice of the flow model is globally specified by input, but the averaging method is the default. It has been shown that for some atypical problems, notably those where momentum flux is dominant compared to friction and the other terms in the momentum equation, the solution technique in RETRAN-3D can be unstable when the averaging model is used. For these cases, the donor cell provides a stable solution.

The closure equations compute the volume average velocity or flow as a function of the junctions connected to any given volume. The averaging and donor methods available in RETRAN-3D also rely on a knowledge of where a junction connects to a volume, i.e., either in the upstream or downstream half volume. Figure A-1 illustrates a volume with multiple inlet and exit junctions. It also shows the volume flow $\overline{W}_{k,\Theta}$ which is not necessarily parallel to the volume orientation ϕ_k. The component of the volume flow that is in direction ϕ_k is $\overline{\overline{W}}_k$.

For the example control volume with multiple junction connections shown in Figure A-1, the junction flows are oriented in the direction indicated and are normal to the junction area. The volume is divided into an upstream and downstream half based on the volume angle, which is equal to ϕ_k. The normal vectors N_{ik} which are directed outward from the surface of volume k, are determined by the from-volume/to-volume specification and angle assigned with each junction as follows,

$N_{ik} = \phi_j$ for the "from" side of a junction where positive flow is out of the "from" volume k

$= (\phi_j + \pi)$ for the "to" side of a junction where positive flow is into the "to" volume k

Both the averaging and donor forms of the volume centered flow equation models are obtained using a general form. The equations used to obtain the components of the flow in the Ψ, x, and y directions are

$$\overline{W}_{k,\psi} = \zeta_{kuj}\,\overline{W}_{kuj} + \zeta_{kdj}\overline{W}_{kdj} \tag{A-1a}$$

$$\overline{W}_{k,x} = \zeta_{kuj}\,\overline{W}_{kuxj} + \zeta_{kdj}\overline{W}_{kdxj} \tag{A-1b}$$

$$\overline{W}_{k,y} = \zeta_{kuj}\,\overline{W}_{kuyj} + \zeta_{kdj}\overline{W}_{kdyj} \tag{A-1c}$$

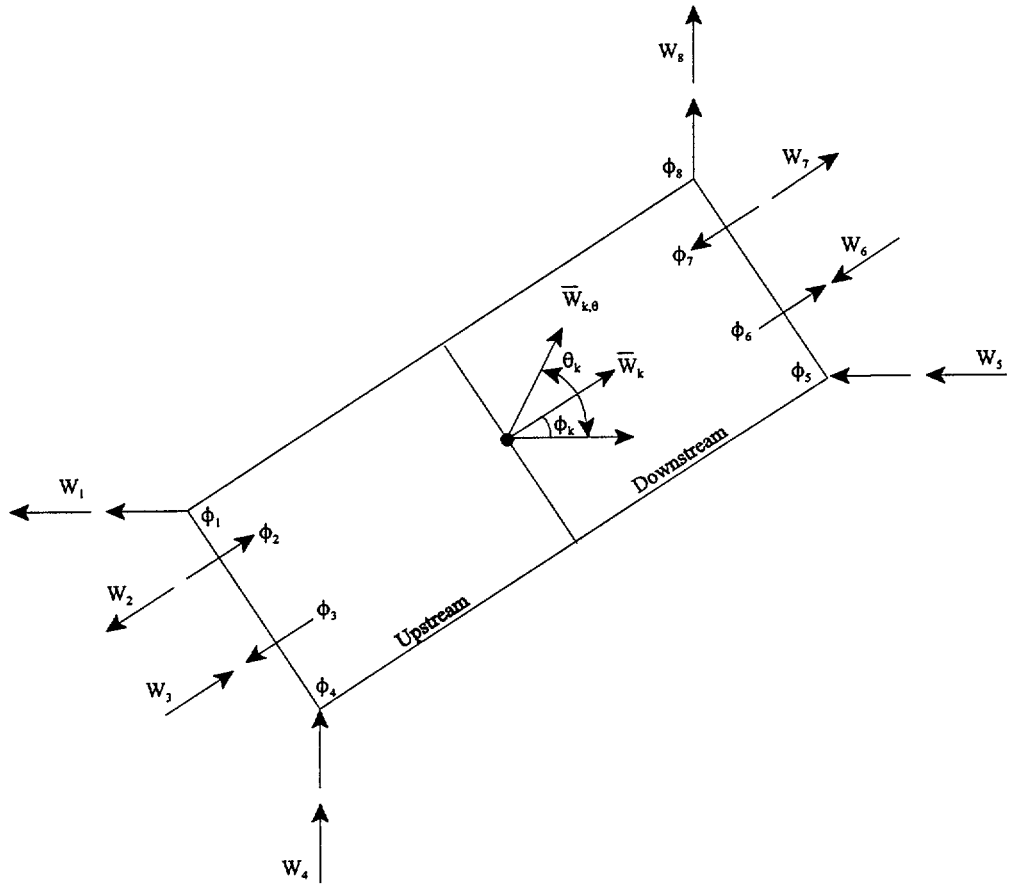

Figure A-1. Volume with Multiple Junction Connections

where the ζ are coefficients dependent on the model (averaging or donor) selected and the upstream and downstream components of the volume flow are given by the following equations. The summations are over all junctions connecting to the volume that also satisfy the requirement for Φ which is defined as $\Phi = N_{ik} - \phi_j$,

$$\left.\begin{aligned} \overline{W}_{kuj} &= -\sum_{i\epsilon k} W_i \left| \cos\left(\phi_i - \phi_j\right) \right| \cos\left(N_{ik} - \phi_i\right) \\ \overline{W}_{kuxj} &= -\sum_{i\epsilon k} W_i \left| \cos\left(\phi_i\right) \right| \cos\left(N_{ik} - \phi_i\right) \\ \overline{W}_{kuyj} &= -\sum_{i\epsilon k} W_i \left| \sin\left(\phi_i\right) \right| \cos\left(N_{ik} - \phi_i\right) \end{aligned}\right\} \quad \text{for } \frac{\pi}{2} < \Phi \leq \frac{3\pi}{2} \qquad \text{(A-2a)}$$

and

$$\left.\begin{aligned}
\overline{W}_{kdj} &= \sum_{i\epsilon k} W_i \left| \cos\left(\phi_i - \phi_j\right) \right| \cos\left(N_{ik} - \phi_i\right) \\
\overline{W}_{kdxj} &= \sum_{i\epsilon k} W_i \left| \cos\left(\phi_i\right) \right| \cos\left(N_{ik} - \phi_i\right) \\
\overline{W}_{kdyj} &= \sum_{i\epsilon k} W_i \left| \sin\left(\phi_i\right) \right| \cos\left(N_{ik} - \phi_i\right)
\end{aligned}\right\} \quad \text{for } -\frac{\pi}{2} < \Phi \leq \frac{\pi}{2} \; . \tag{A-2b}$$

The volume centered velocity and direction are then given by

$$\overline{W}_{k,\Theta} = \left[\overline{W}_{k,x}^2 + \overline{W}_{k,y}^2\right]^{1/2} \tag{A-3}$$

and

$$\Theta = \begin{cases} \tan^{-1}\left(\dfrac{\overline{W}_x}{\overline{W}_y}\right) & \text{for } \overline{W}_x \geq 0 \\ \pi + \tan^{-1}\left(\dfrac{\overline{W}_x}{\overline{W}_y}\right) & \text{for } \overline{W}_x < 0 \end{cases} \tag{A-4}$$

The component of the volume centered mass flow rate that is normal to the volume flow area is

$$\overline{W}_k = \overline{W}_{k,\Theta} \left| \cos(\Theta_k - \Phi_k) \right| \tag{A-5}$$

and the component that is in direction ψ_j is

$$\overline{W}_{k,\psi} = \overline{W}_{k,\Theta} \left| \cos(\Theta_k - \psi_j) \right| \tag{A-6}$$

where ψ_j is the junction angle ϕ_j. Equation A-6 is equivalent to Eq. A-1a.

The coefficients used to obtain the averaging and donor models are:

Arithmetic Average

$$\zeta_{kuj} = 0.5 \tag{A-7a}$$

$$\zeta_{kdj} = 0.5 \tag{A-7b}$$

Donor Cell

$$\left.\begin{aligned}\zeta_{kuj} &= 1.0\\ \zeta_{kdj} &= 0.0\end{aligned}\right\} \quad \text{for } W_j \geq 0.0 \tag{A-8a}$$

$$\left.\begin{aligned}\zeta_{kuj} &= 0.0\\ \zeta_{kdj} &= 1.0\end{aligned}\right\} \quad \text{for } W_j < 0.0 \tag{A-8b}$$

The volume flow terms required for the solution of the momentum equation are obtained using the above equations. They are evaluated for both boundaries of the momentum cell.

Attachment 3
RETRAN-3D Multidimensional Kinetics Calculations for SPERT III E Tests 81 and 86