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DOE/NRC Natural Phenomena Hazards (NPH) Workshop

Implications of Discretization and Quantification Assumptions in Natural Hazard Probabilistic Risk Assessment

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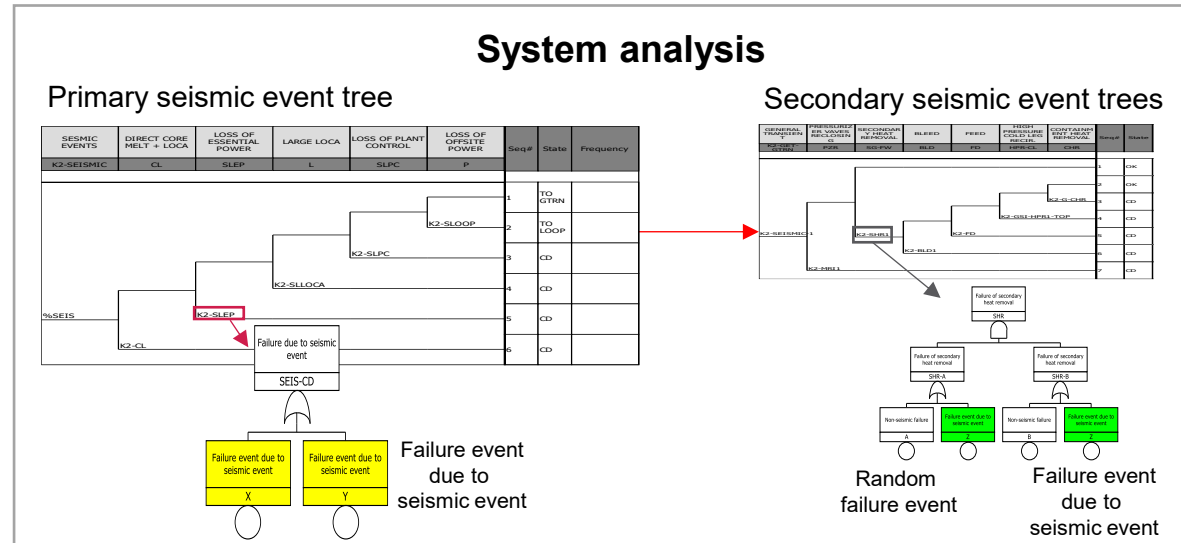
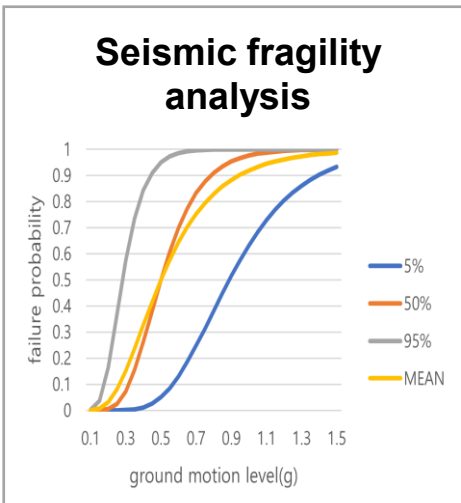
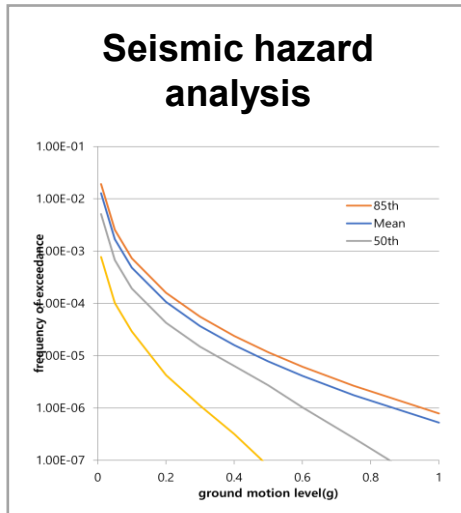
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Introduction

Seismic Probabilistic Risk Assessment (PRA)



Quantification

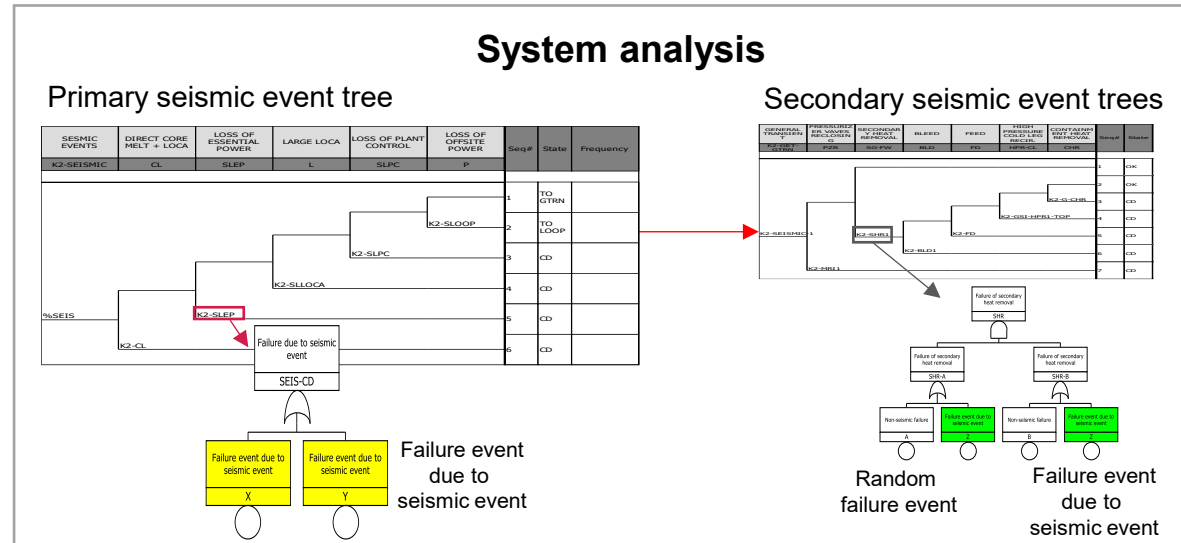
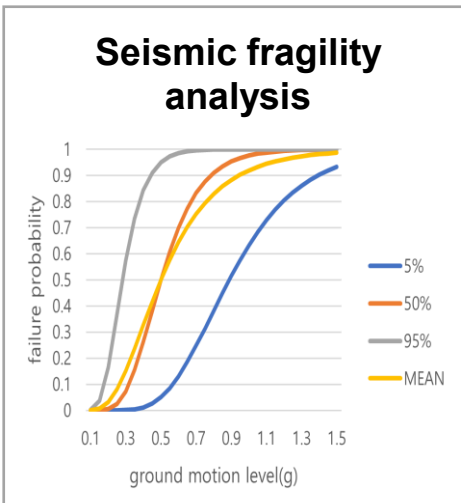
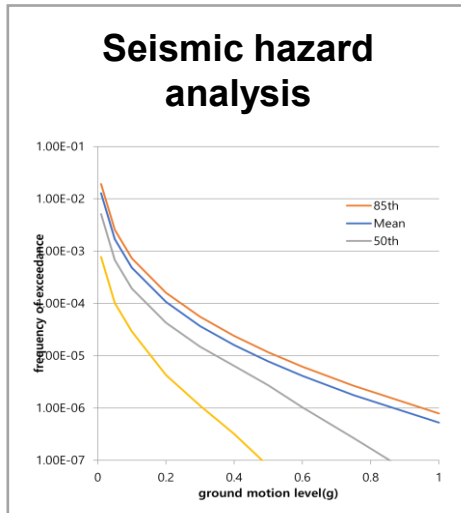
$$P(x) = \int_0^{\infty} -\frac{dH(x)}{dx} F(x) dx$$

- **Sampling approach**
 - Monte Carlo simulation, Latin Hypercube
 - ex) PRASSE
- **Discrete approach**
 - Discretization of the ground motion level

- **Seismic hazard analysis** develops hazard curves that indicate the exceedance frequencies of the seismic event depending on the ground motion intensity.
- **Seismic fragility analysis** develops fragility curves that quantify the conditional probability of SSC failure depending on the ground motion intensity.
- **System analysis** models plant response following a seismic initiating event using event trees and fault trees that incorporates seismically induced SSC failures.
- **Quantification** integrates the hazard, fragility, and system models over the range of ground motion intensity to estimate the seismic core damage frequency.

Introduction

Seismic Probabilistic Risk Assessment (PRA)



Quantification

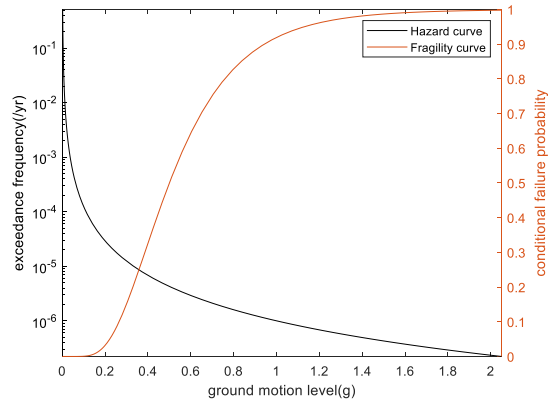
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- **Sampling approach**
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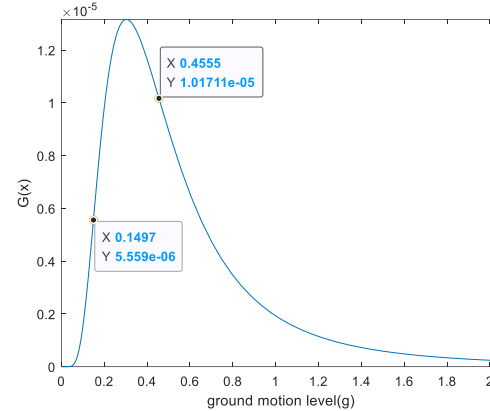
- The discrete approach divides the continuous range of ground-motion intensity into a finite number of intervals.
- Each interval is assigned a representative ground-motion level and modeled as a separate seismic initiating event.
- This formulation is compatible with conventional PRA methods and software.
- The resulting minimal cut sets identify the combinations of failures that lead to core damage.

Problem Statement

K_I : a constant
 K_H : a slope parameter
 A_m : median ground motion level capacity
 β_c : composite variability



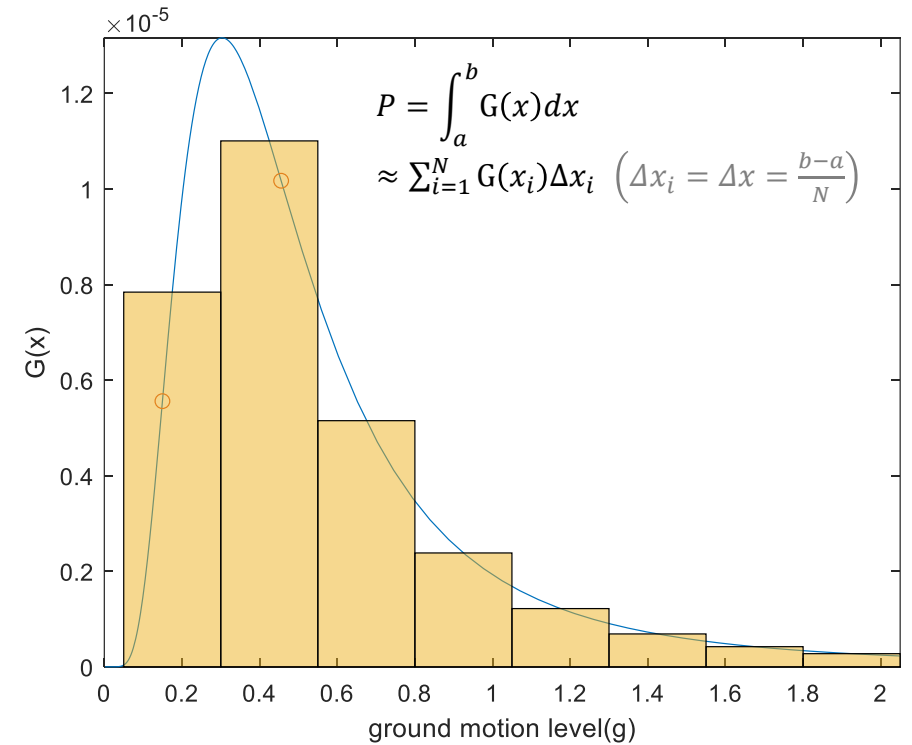
- $H(x) = K_I x^{-K_H}$
- $F(x) = \Phi\left(\frac{\ln(x) - \ln(A_m)}{\beta_c}\right)$



$$P = \int_a^b -\frac{dH(x)}{dx} F(x) dx$$



Discrete
probability
distribution
method



- Seismic risk is evaluated through the continuous convolution of the hazard and fragility over the ground-motion intensity.
- The discrete approach approximates this continuous integral using a finite number of intervals.
- This approximation requires analysts to specify:
 - the number of intervals,
 - the interval boundaries, and
 - the representative ground-motion level used for each interval-specific PRA evaluation.
- Existing guidance provides limited technical criteria for these selections, which often rely on expert judgment.
 - Different practices : Diablo Canyon(16), Sequoyah(9), Surry(8), US SPAR model(3→5)
- Different selections can lead to underestimation or overestimation, particularly when only a small number of intervals is used.

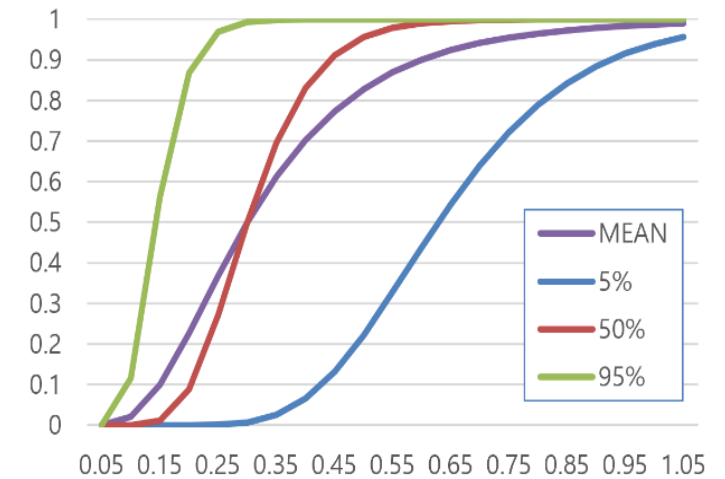
Problem Statement

- **High Conditional Failure Probabilities**

- SSC failure probabilities increase with ground-motion intensity.
- REA can then substantially overestimate the top-event probability.

- **Potential Overestimation from Neglecting Success Branches**

- The approximation ($P(S) = 1 - P(F) \approx 1$) is reasonable only when ($P(F) \ll 1$).
- As failure probabilities increase, deleting success terms (e.g. delete-term approximation) can lead to risk overestimation.
- **BDD Calculation**
 - Full BDD quantification may require substantial memory and computational resources.
 - Hybrid implementations use BDD for a prespecified subset of dominant minimal cut sets and MCUB for the remainder.
- **Negate-down options in FTREX** retains selected success branches in the primary seismic event tree and avoids reliance on the delete-term approximation.

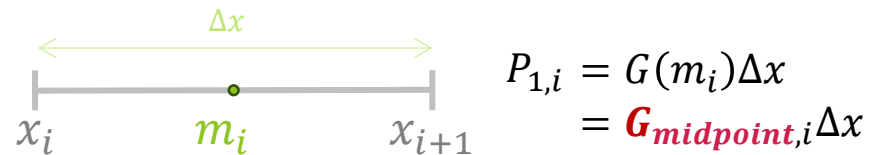


Seismic failure probability
when $A_m = 0.3$, $\beta_r = 0.3$, $\beta_u = 0.45$

Effect of the Number of Ground-Motion Subintervals

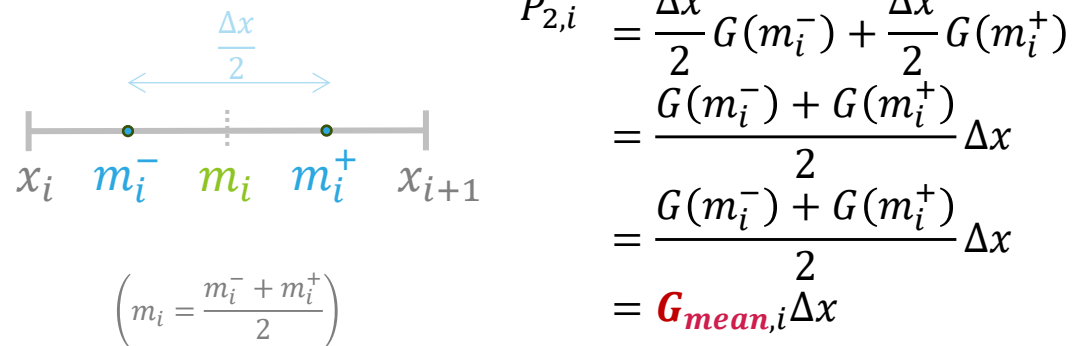
$P_1: 2^n$ intervals

One midpoint evaluation in the original interval



$P_2: 2^{n+1}$ intervals

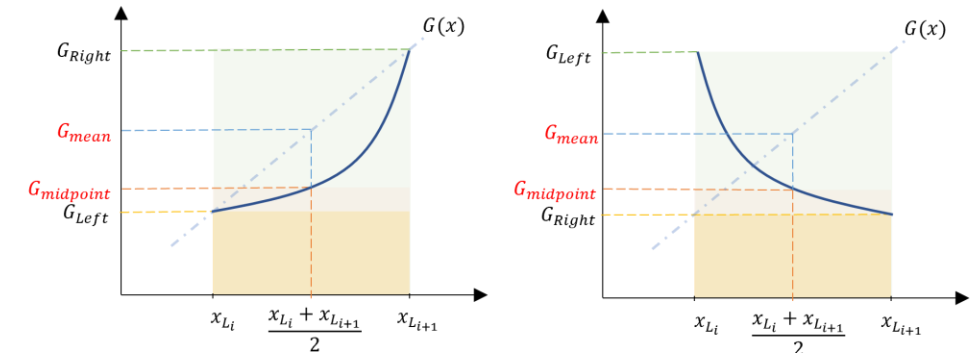
The same interval split into two equal halves



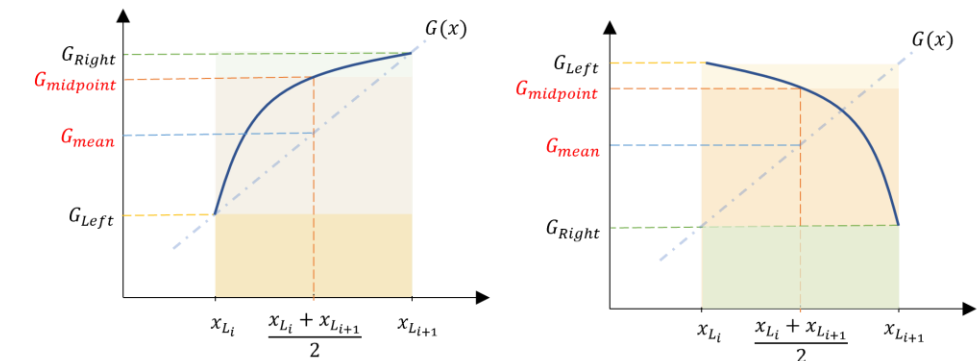
Assumptions

- Subintervals are equally spaced
- Midpoints of subintervals are selected

Convex curve



Concave curve



Convex region ($G''(x) > 0$) $\Rightarrow G_{\text{mean},i} > G_{\text{midpoint},i} \Rightarrow \mathbf{P_1} < \mathbf{P_2}$

Concave region ($G''(x) < 0$) $\Rightarrow G_{\text{mean},i} < G_{\text{midpoint},i} \Rightarrow \mathbf{P_1} > \mathbf{P_2}$

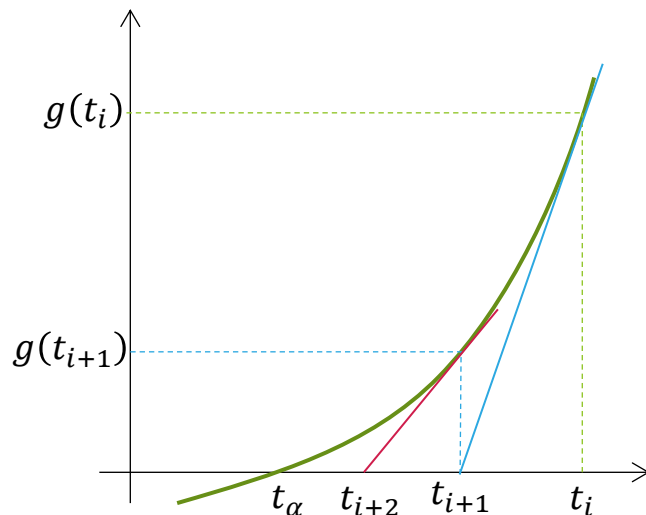
Effect of the Number of Ground-Motion Subintervals

$$G''(x) \rightarrow g(t) = C_1(1 + \operatorname{erf}(t)) - (C_2 + \sqrt{2}t) \exp(-t^2)$$

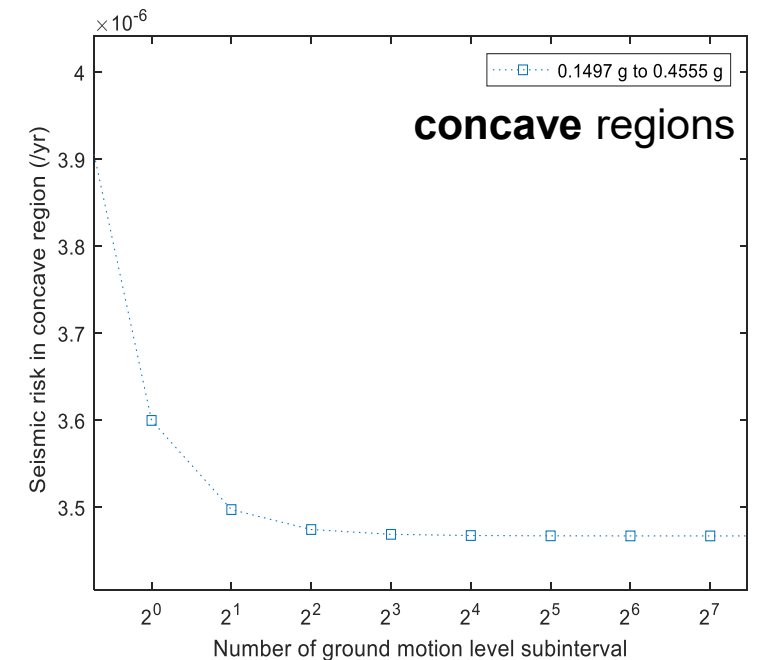
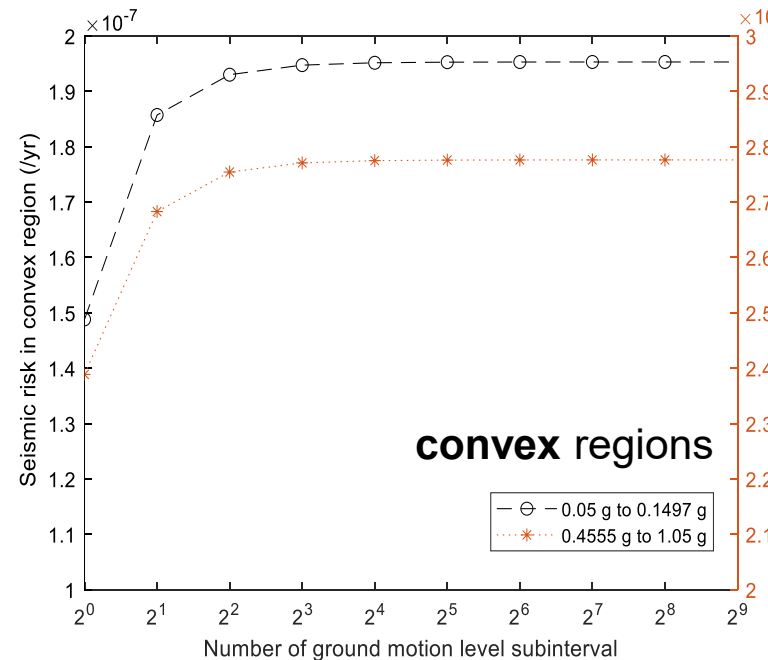
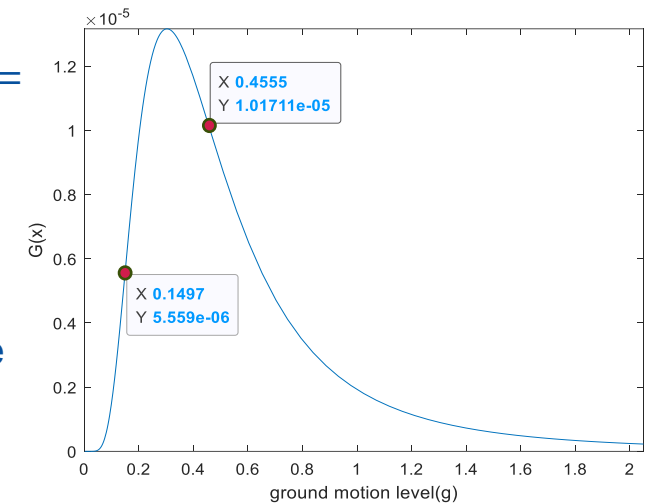
- $t = \frac{\ln(x) - \ln(A_m)}{\sqrt{2} \beta_c}$
- $\operatorname{erf}(t) = \int_0^t e^{-x^2} dx$
- $C_1 = \beta_c^2 \sqrt{2\pi} (K_H + 2)(K_H + 1) \frac{1}{2}$, $C_2 = 2\beta_c K_H + 3\beta_c$

Newton Raphson method

$$t_{i+1} = t_i - \frac{g(t_i)}{g'(t_i)}$$



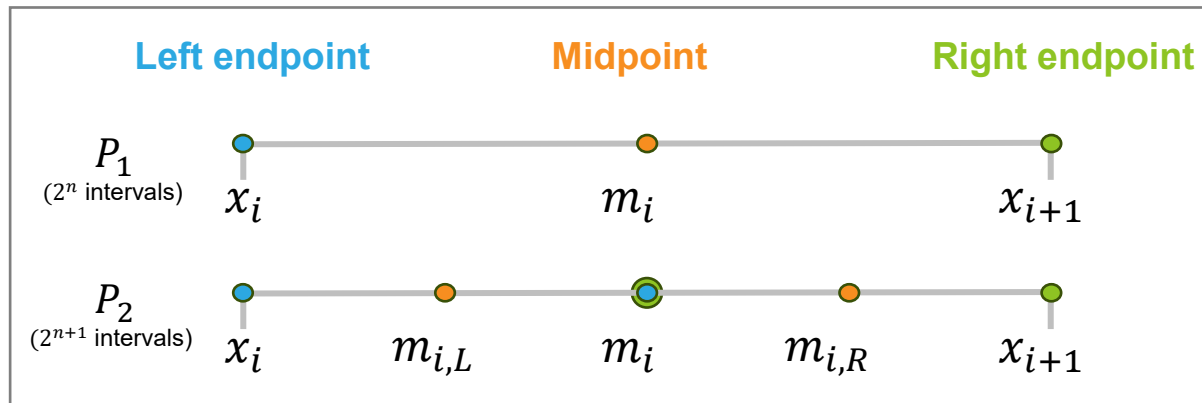
- An example with a hazard curve ($K_1 = 10^{-6}, K_H = \frac{1}{\log(3)}$), and a fragility curve ($A_m = 0.5, \beta_r = \beta_u = 0.35$).
- With Newton Raphson method, **two boundaries** between overestimation and underestimation are obtained as **0.15g** and **0.46g**.



Representative Ground-Motion Levels in Practical Discretization

- Seismic PRA typically represents hazard curves as tabulated data rather than closed-form functions.

$$P \approx P_N = \sum_{i=1}^N [H(x_i) - H(x_{i+1})] F(x_{i,r})$$



With a small number of subintervals:

- Left endpoint:** $P_1 < P_2$
The discrete approximation underestimates seismic risk and produces an **optimistic estimate**.
- Right endpoint:** $P_1 > P_2$
The discrete approximation overestimates seismic risk and produces a **conservative estimate**.
- Midpoint:** $P_1 \leq P_2$
The approximation may underestimate or overestimate seismic risk, **depending on the relative changes in the hazard and fragility curves**.

Midpoint case

$$P_1 - P_2 = \sum_{i=1}^{2^n} \left(\underbrace{(H(x_i) - H(m_i)) (F(m_i) - F(m_{i,L}))}_{\text{positive}} + \underbrace{(H(m_i) - H(x_{i+1})) (F(m_i) - F(m_{i,R}))}_{\text{negative}} \right)$$

$$\frac{H(x_i) - H(m_i)}{H(m_i) - H(x_{i+1})} \leq \frac{F(m_{i,R}) - F(m_i)}{F(m_i) - F(m_{i,L})}$$

$R_{H,i}$ $R_{F,i}$

- $R_{H,i} < R_{F,i} \Rightarrow P_1 < P_2$ (*Optimistic estimation*)
- $R_{H,i} > R_{F,i} \Rightarrow P_1 > P_2$ (*Conservative estimation*)

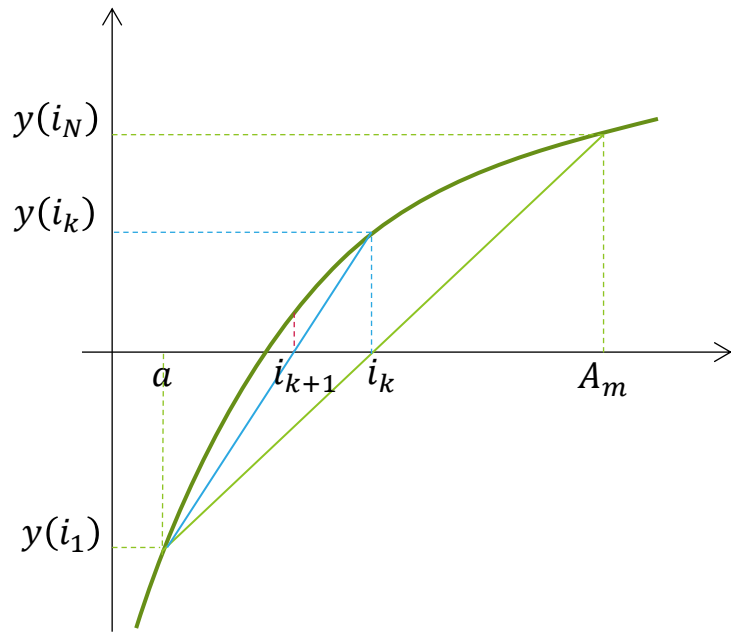
Assumption

- Hazard decreases and Fragility increases with ground-motion intensity.
 - $H(x_i) > H(m_i) > H(x_{i+1})$
 - $F(m_{i,L}) < F(m_i) < F(m_{i,R})$
- Each interval in P_1 is divided into two equal subintervals in P_2 .
- Both discretizations apply the same representative-level rule.

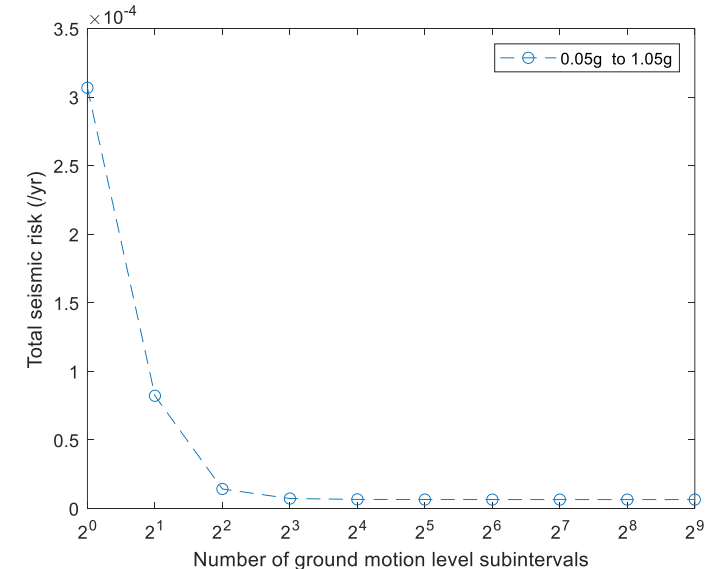
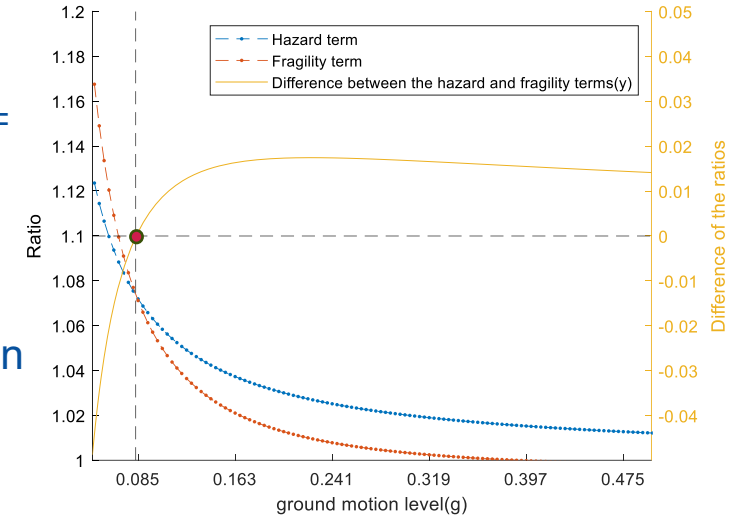
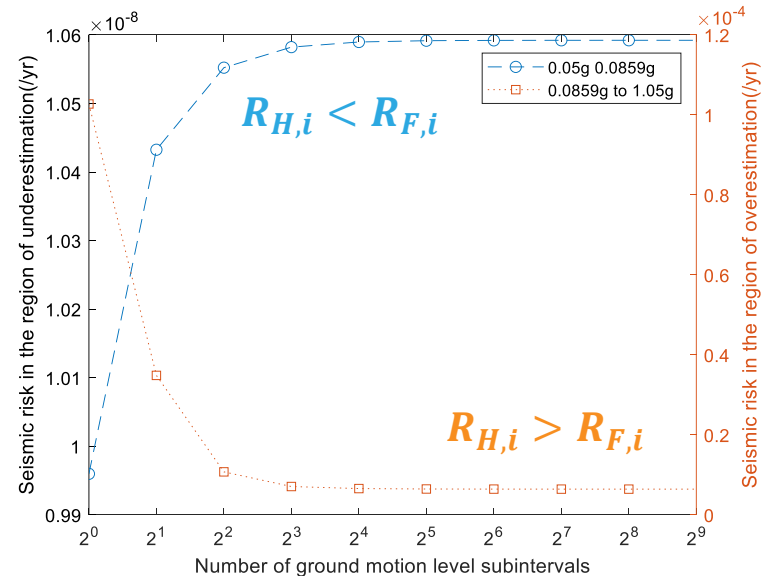
Representative Ground-Motion Levels in Practical Discretization

False Position Method

$$\begin{cases} y(i_k)y(i_N) < 0 : i_{k+1} = \frac{i_k y(i_N) - i_N y(i_k)}{y(i_N) - y(i_k)} \\ y(i_k)y(i_N) > 0 : i_{k+1} = \frac{i_1 y(i_k) - i_k y(i_1)}{y(i_k) - y(i_1)} \end{cases}$$



- An example with a hazard curve($K_1 = 10^{-6}, K_H = \frac{1}{\log(3)}$), and a fragility curve($A_m = 0.5, \beta_r = \beta_u = 0.35$)
- With False position method, **a boundary** between overestimation and underestimation is obtained as **0.085g**.

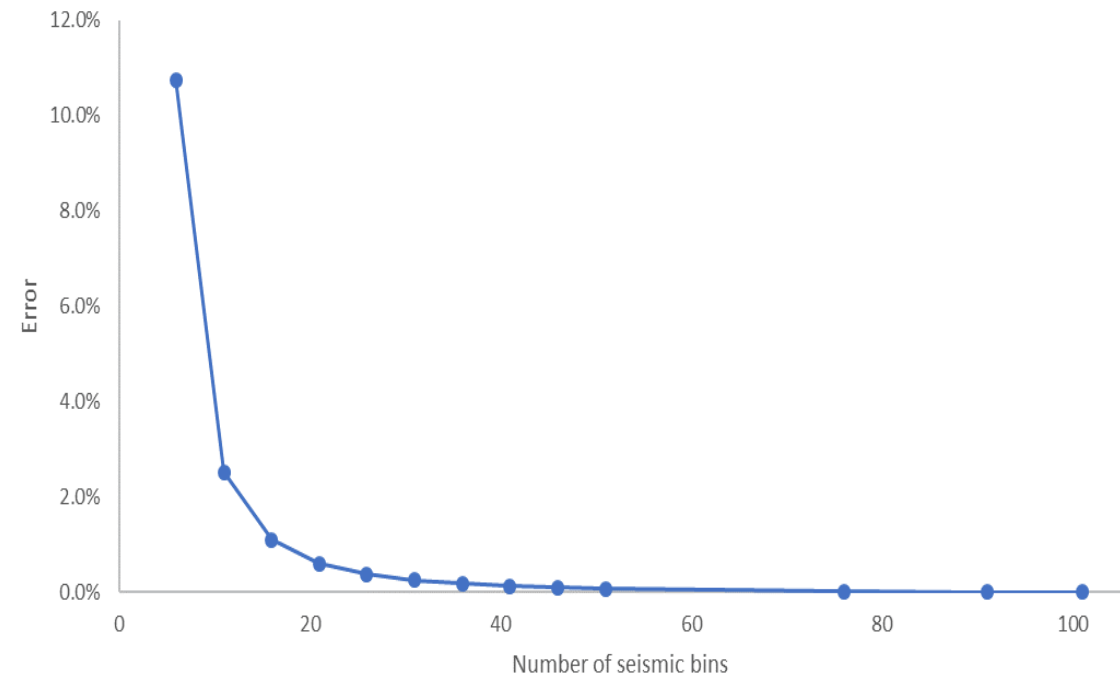


Quantification with a sufficiently large number of subintervals

- The numerical study uses a Seismic PRA Pilot model.
- The result obtained with **101 ground motion subintervals** serves as the reference.

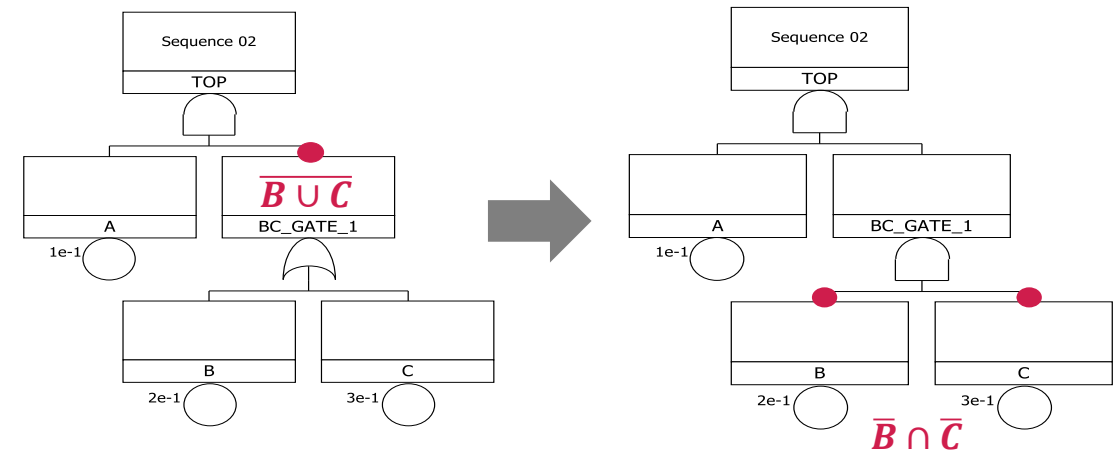
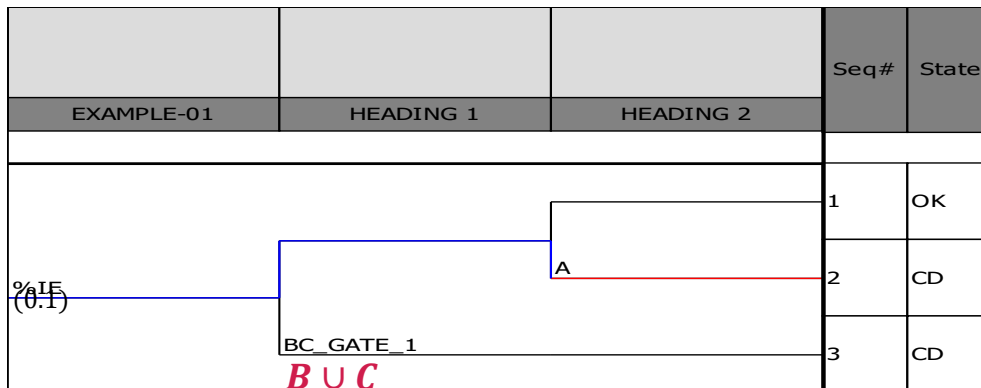
$$\text{Relative difference}(N) = \frac{|P_N - P_{101}|}{P_{101}} \times 100\%$$

- With **six subintervals**, the relative difference is **10.7%**.
- The relative difference decreases as the number of subintervals increases and **approaches zero relative to the 101-subinterval reference**.



- A success branch introduces a **complemented event (NOT logic)** into the sequence expression; under the **delete-term approximation**, the associated success probability is treated as unity, and the complemented term is removed.
- To **retain the success-branch logic**, the **negate-down option** propagates each NOT gate to the basic-event level using **De Morgan's theorem**:

$$A \bar{\cup} B = \bar{A} \cap \bar{B}, A \bar{\cap} B = \bar{A} \cup \bar{B}$$



When delete-term approximation is applied:

1.00E-02	%IE	A	#EXAMPLE-01-2!		
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When delete-term approximation is avoided (i.e. **FTREX's Negate_down option** is used):

5.60E-03	%IE	A	/B	/C	#EXAMPLE-01-2!
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/BC_GATE

Effects of Retaining Success-Branch Logic on Seismic PRA Quantification

- Competing effects when delete-term approximation is avoided (e.g. Negate-down option is considered) and MCUB-based calculation is used.

1) Decreasing minimal cutset frequency by multiplying $P < 1 \rightarrow \text{CDF} \downarrow$

– (ex) **NOR Gate** : $\overline{(A + B)} = \overline{A} * \overline{B}$

- New minimal cutset: %IE CS· **$\overline{A}$** · **$\overline{B}$**
- Reduction of the frequency of each minimal cutset **due to ‘NOT’ basic events ($\overline{A}$, $\overline{B}$)** whose probabilities are **less than one**

2) Increasing the number of minimal cutsets $\rightarrow \text{CDF} \uparrow$

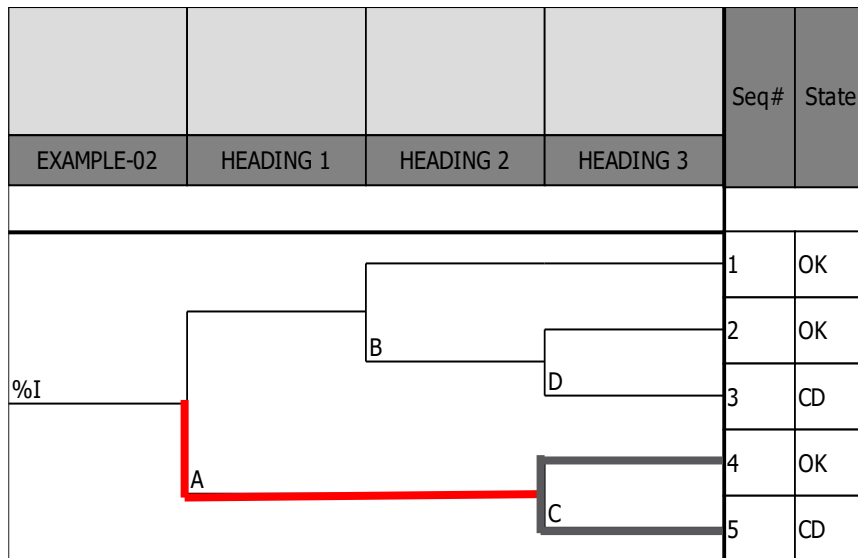
– (ex) **NAND Gate** : $\overline{(A * B)} = \overline{A} + \overline{B}$

- MCUB Result = %IE((1-(1-P(**CS**)·P(**$\overline{A}$**))(1-P(**CS**)·P(**$\overline{B}$**)))
- Doubling of the number of minimal cut sets which **almost doubles MCUB-based calculation results** compared to a single cutset frequency.

→ MCUB-based calculation results decreases as the ground motion level increases.

Effects of Retaining Success-Branch Logic on Seismic PRA Quantification

- When delete-term approximation is applied, BDD-based calculation may not provide exact probabilities/frequencies.



- When delete-term approximation is avoided:
 - $P(AC) + P(/ABD)$ (Exact solution)
- When delete-term approximation is applied after generating minimal cutsets:
 - Minimal cutsets : AC, BD
 - BDD-based calculation :

$$P(AC) + P(/ABD) + \underline{P(A/CBD)}$$

additional term

- BDD-based calculation may produce **slightly conservative** results compared to the exact values when delete-term approximation is applied.
 - Failures paths leads to **core damage** → exact results
 - Additional headings** in failure paths → possibly conservative results

Effects of Retaining Success-Branch Logic on Seismic PRA Quantification

- The effects of avoiding delete-term approximation (e.g. retaining success branch) to BDD-based calculation and MCUB-based can be summarized as follows:

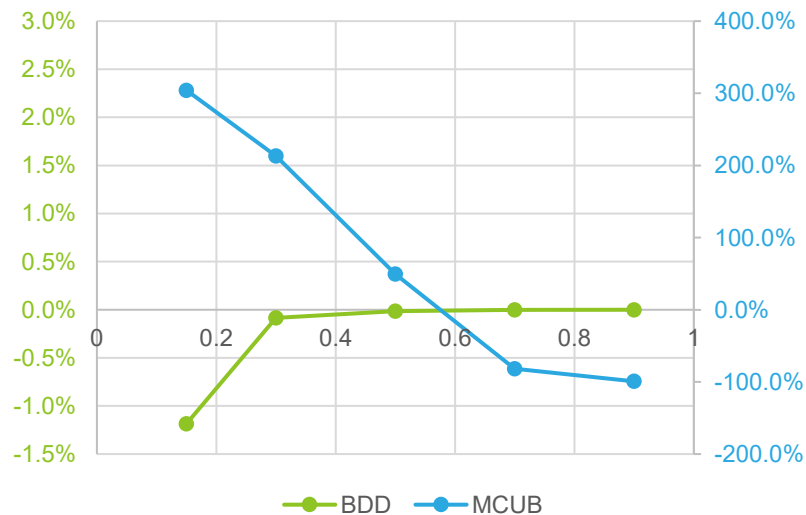
Delete-term Approximation	BDD	MCUB	
		Ignore sequences	Mutually exclusive sequences
Yes	Conservative (Exact under specific condition)	Conservative (exact when all minimal cutsets are independent)	Conservative
No (Application of Negate-down)	Exact	Competing effect	Competing effect (More conservative than “ignore sequences” case)

Application to Seismic PRA Pilot model

Ratio of results without DTA to those with DTA

Ground motion level	R_{BDD}	R_{MCUB}	$R_{BDD+MCUB}$
0.15g	0.988	4.042	1.037
0.3g	0.999	3.130	1.004
0.5g	1.000	1.496	1.000
0.7g	1.000	0.184	1.000
0.9g	1.000	0.008	1.000

Relative Change from the DTA Baseline



$$R = \frac{\text{Result without the delete term approximation}}{\text{Result with the delete term approximation}}$$

- BDD is applied to the **first 1,000 minimal cut sets**, and MCUB is applied to the remaining minimal cut sets.

BDD contribution

- Negate-down retains the original success-branch logic, enabling exact BDD quantification for the included cut sets.
- Applying the delete-term approximation introduces additional failure combinations and produces a slightly conservative result.

MCUB contribution: competing effects

- Retaining success terms reduces **individual cut-set frequencies**.
- Negate-down increases the **number of minimal cut sets**.
- At low ground-motion levels, the cut-set-count effect dominates: $R_{MCUB} > 1$.
- At higher ground-motion levels, the cut-set-frequency reduction dominates: $R_{MCUB} < 1$.

Conclusions

- **The discrete approach of seismic PRA** provides a practical framework that can be readily integrated with conventional PRA models, quantification methods, and software.
- When only **a limited number of intervals** is used, the calculated risk may be **underestimated or overestimated** depending on the interval definition and representative-level selection.
- The numerical examples show that increasing the number of intervals reduces discretization differences and improves convergence of the calculated risk.
- At high ground-motion levels, success-branch probabilities may no longer be approximated as unity. Negate-down **retains the success-branch logic** and avoids the delete-term approximation.
- The effects of avoiding the delete-term approximation differ between BDD and MCUB: BDD eliminates the DTA-induced conservative bias, whereas MCUB is affected by the competing effects of cut-set frequency and cut-set count.

References

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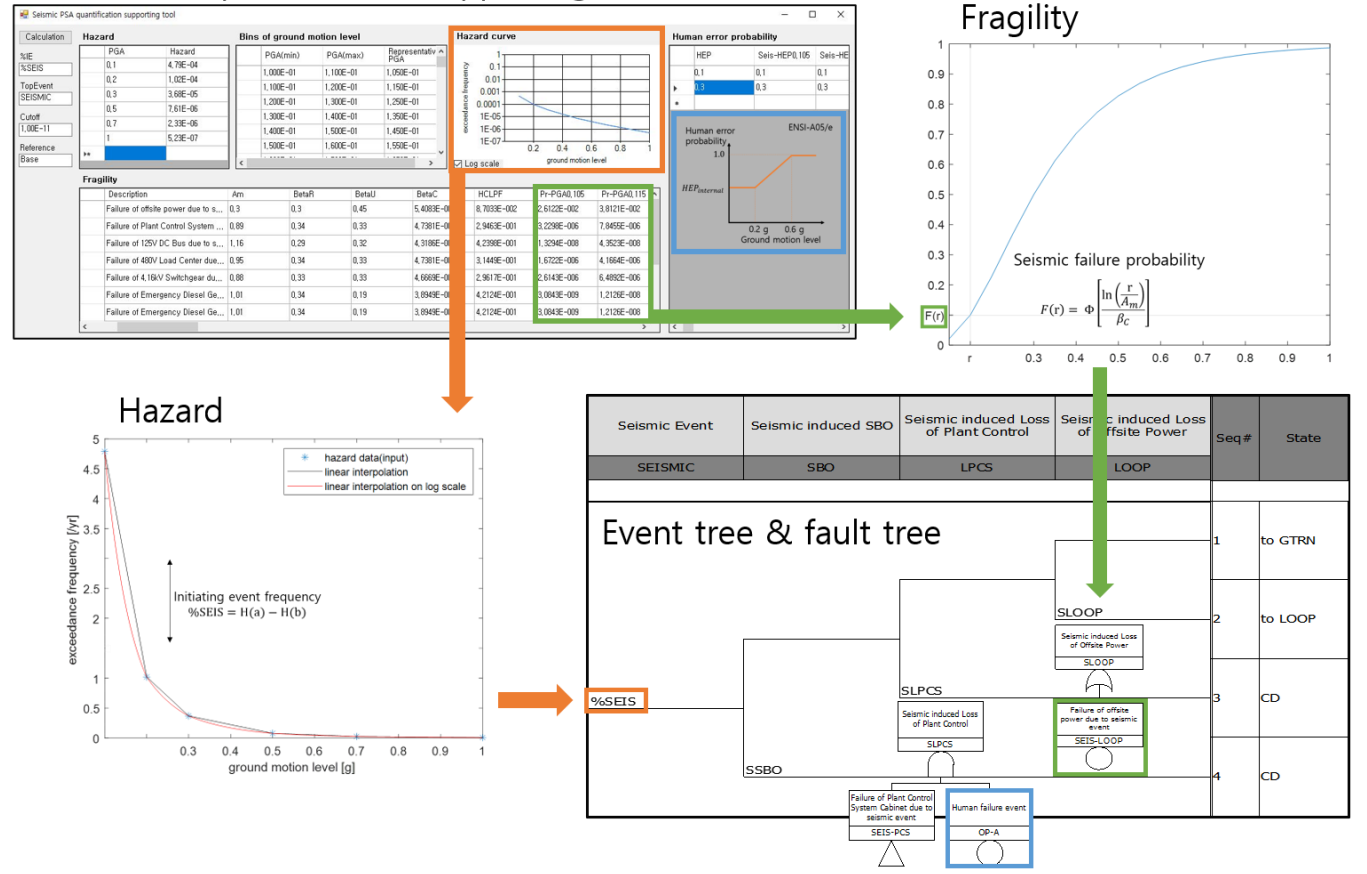
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Quantification with a sufficiently large number of subintervals

Seismic PSA quantification supporting tool



- For each ground motion level subintervals, **the seismic initiating event frequency and seismic failure probabilities of SSCs** need to be calculated
- A considerable amount of time and effort** to analyze the seismic risks as the number of ground motion level subintervals increases.
- Provides the initiating event frequency, the seismic failure probabilities of SSCs, and HEPs under seismic events