NRC Question RAI 14 – Additional Response

Background

The NRC staff's current understanding of the process used to calculate the reconciled uncertainty is that this calculation does not result in the variance of the reconciled measurement, but rather the variance in the mean of the reconciled measurement. Suppose n samples of the reconciled measurement were made, and then the mean of those n samples was generated. The reconciled value represents that mean, and the reconciled variance is the variance in that mean. However, the quantity of interest is not the variance in the reconciled mean, but the variance in the reconciled measurement itself. To get the variance in the reconciled measurement we need to multiply the variance in the reconciled mean (the outcome of the DVR process) by n.

Response

To establish a common, unambiguous context for discussion, it is useful to define and clarify some of the specific data reconciliation terminology that is used in the field, especially around inputs and outputs of the DVR method.

Figure 1 shows two areas relevant in this RAI: data preprocessing and actual application of DVR.

In the preprocessing step measurement data is averaged over a certain time interval. Averaged data then is used in DVR to calculate reconciled values.

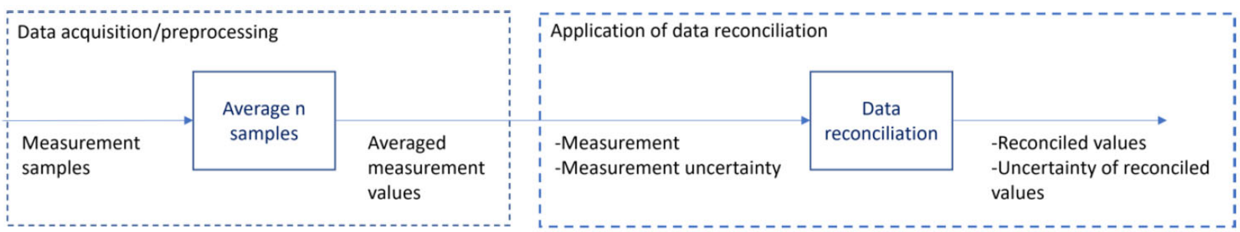

Figure 1: Data preprocessing and application of process data reconciliation

In general, the input of DVR is referred to as **measurements** stated with their respective **measurement uncertainty**. Although this concerns averaged measurements and consequently uncertainty of averaged measurements, this is normally not explicitly mentioned.

The results of DVR are reconciled values together with the uncertainties of those reconciled values. Uncertainties in DVR are normally expressed as a 95% confidence interval.

Note that the output of DVR is called reconciled **'value'**, not reconciled 'measurement'. This helps to avoid confusion between in- and output of DVR. In addition, many reconciled values are unmeasured process values and consequently the term 'reconciled measurement' would be inadequate to describe them.

The uncertainty of the reconciled value is often referred to as 'reconciled uncertainty' although this is strictly spoken not correct, since DVR does not reconcile uncertainties.

Adopting the terminology above, the RAI background statement would read:

The NRC staff's current understanding of the process used to calculate the **uncertainty of a reconciled value** is that this calculation does not result in the variance of the reconciled **value**, but rather the variance in the mean of the reconciled **value**. Suppose n samples of the measurement were made, and then the mean of those n samples was generated. The reconciled value represents that mean, and the variance of the reconciled value is the variance in that mean. However, the quantity of interest is not the variance in the reconciled mean, but the variance in the **reconciled value** itself. To get the variance in the **reconciled value** we need to multiply the variance in the reconciled mean (the outcome of the DVR process) by n.

Response

Interpretation of the given measurement uncertainties

In this RAI there seems to be a misconception of the estimated measurement uncertainty as input to DVR. In the example calculation presented, the measurements are stated as 245 kg/s ± 12.25 and 250 kg/s ± 12.50.

In the RAI example calculation, the uncertainty intervals ± 12.25 and ± 12.50 are divided by n (after conversion to variance) before input to the DVR method, in order to convert to an uncertainty of the mean value. They are apparently interpreted as (random) uncertainty of the single measurement.

However, as mentioned above, the inputs to DVR are mean measurements and uncertainties of mean measurements, and the uncertainty intervals provided (±12.25 and ±12.50) already represent the uncertainties of the mean values, including any effects of sampling (see "Meaning of the measurement uncertainty as input to DVR", below). Consequently, no conversion (division by n) is required before input to DVR and no reciprocal conversion of the result (multiplication by n) is required.

Figure 2 illustrates this issue.

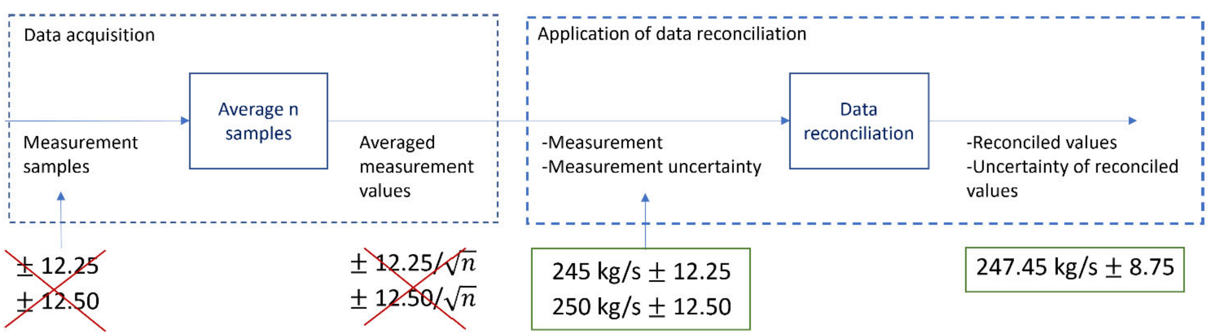

Figure 2: Illustration of the incorrect interpretation of the example uncertainties – these are uncertainties of the mean measurements and not (random) uncertainties of the single measurement.

Meaning of the measurement uncertainty as input to DVR

It is helpful to understand that the uncertainty of the mean measurement (the resulting uncertainty after sampling and averaging) represents the uncertainty of the single measurement due to systematic error.

This can be explained as follows:

Measurements contain both random and systematic errors. When calculating a mean value of n samples, the random error in the measurement vanishes with large enough sample size (the random variance is divided by n), but the systematic error of the measurement remains in the mean value (see Attachment A in the EPRI response).

Since DVR uses mean measurement values, the estimated uncertainties in DVR must include the uncertainty of the single measurement due to systematic error.

This is illustrated in Figure 3. On the left, the measurement uncertainty is given for both the systematic and random component. The random error is normally much smaller than the systematic error. Here a value of 1.0 kg/s is taken as an example for the uncertainty due to random error. The uncertainty due to systematic error is calculated to match the example. As can be seen, due its smaller size and due to averaging, the random component almost completely vanishes, and the systematic error dominates the error of the mean value. It becomes clear that for sufficiently large n, the uncertainty of the mean is the uncertainty of the measurement due to systematic error and that this is independent from n.

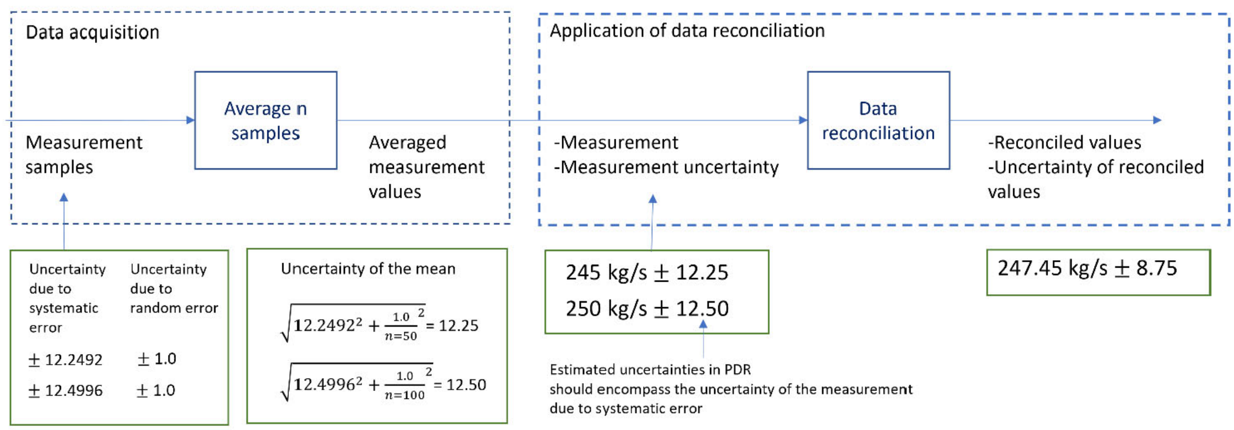

Figure 3: Illustration of the role of the systematic error of the measurement

Correction of the inputs to the example calculation

To further clarify to the response above, a number of corrections to Table 1 from RAI 14 are proposed. Table 1 from RAI 14 is reproduced below.

Table A: Reproduction of Table 1 from RAI-14

Flow Meter	$\bar{x}$ mean flow rate (kg/sec)	$\bar{x} \pm 1.96\, s$ 95% confidence* Interval (kg/sec)	s standard deviation (kg/sec)	s^2 variance (kg^2/sec^2)	s_x^2 variance in the mean (kg^2/sec^2)	n (number of samples to obtain the mean)
Flow Meter A	245.00	245 ±12.25	12.25/1.96=6.25	39.06	39.06/50=0.7812	50
Flow Meter B	250.00	250 ±12.50	12.50/1.96=6.38	40.67	40.67/100=0.4067	100

*Difference between tolerance interval and confidence interval is discussed later in the response

Column 3 presents the confidence interval of the mean flow rate $\bar{x}$:

$$\bar{x} \pm 1.96\, s$$

The values from the example being 245 ± 12.25, 250 ± 12.50.

Since s represents a confidence interval around $\bar{x}$, and it would be appropriate to use $s_{\bar{x}}$ instead of s:

$$\bar{x} \pm 1.96\, s_{\bar{x}}$$

In Table 1 from RAI 14, s is treated as the (random) standard error of single flow rate measurements. s2 is subsequently divided by the number of samples n to derive a variance of the mean, but it already is the variance of the mean. The adjustment for sample size is therefore not required.
Table B below shows a corrected version of Table 1 from RAI 14.

Table B: Corrected version of Table 1. Corrections are bolded.

Flow Meter	$\bar{x}$ mean flow rate (kg/sec)	$\bar{x} \pm 1.96\, s_{\bar{x}}$ 95% confidence* Interval **of the mean flow rate** (kg/sec)	$s_{\bar{x}}$ standard deviation **of the mean flow rate** (kg/sec)	${s_{\bar{x}}}^2$ variance **of the mean flow rate** (kg^2/sec^2)		
Flow Meter A	245.00	245 ±12.25	12.25/1.96=6.25	39.06		
Flow Meter B	250.00	250 ±12.50	12.50/1.96=6.38	40.67		

*Difference between tolerance interval and confidence interval is discussed later in the response

Direct Response to first Question

"For such a situation in which the mean of all measurements is not calculated using the same number of samples, how is the reconciled variance calculated?":

As explained in the response above, no adjustment for the number of samples is required. The uncertainties in the example represent the uncertainties of the mean and are input to DVR without correction.

Direct Response to second Question

"Conversely, what is the correct reconciled variance that should be used from Table 2, and why?":

Using the variances from Table B (39.06, 40.67) as input to the DVR calculation without division by n yields correct and unambiguous results. The resulting reconciled variance is 19.93. See also Attachment A of the prior RAI-14 response for the detailed calculation.

RAI-14: Additional Response

After additional review of RAI-14 some errors were noticed in the example problem. Below are Tables and Equations from RAI-14 pointing out where the suspected error is and its implications on the calculation. Additional discussion is included regarding the interpretation of input and output uncertainty values applied with the DVR method. There is a key mistake made in Table 1 and then it seems the same mistake is being implied at the end of the RAI-14 example.

Below is RAI-14 Table 1. The highlighted term appears to be a misinterpretation of the input uncertainty as a tolerance interval and it should be a confidence interval. This causes errors in adjacent columns. On its own the highlighted equation is correct for assessing a tolerance interval, but that is not how the input uncertainty value is treated.

RAI-14 Table 1: Example Values

Flow Meter	$\bar{x}$ mean flow rate (kg/sec)	$\bar{x} \pm 1.96 \cdot s$ 95% tolerance Interval (kg/sec)	s standard deviation (kg/sec)	s^2 variance (kg^2/sec^2)	$s_{\bar{x}}^2$ variance in the mean (i.e., the standard error squared) (kg^2/sec^2)	n (number of samples to obtain the mean)
Flow Meter A	245.00	245 ± 12.25	$\frac{12.25}{1.96} = 6.25$	39.06	$\frac{39.06}{50} = 0.7812$	50
Flow Meter B	250.00	250 ± 12.50	$\frac{12.50}{1.96} = 6.38$	40.67	$\frac{40.67}{100} = 0.4067$	100

Confidence Interval vs Tolerance Interval

This comes down to the difference between confidence interval and tolerance interval. The confidence interval is intended to express how well a sample mean ($\bar{x}$) is an estimate of a true population mean value (μ) by assigning confidence bounds ($\varepsilon_{95\%}$). So we end up with a statement that the population mean μ has a 95% probability of falling into the confidence interval $\bar{x} \pm \varepsilon_{95\%}$. In the case of DVR we are taking measurements of process parameters (flow, pressure, etc.) and sample mean values of these measurements. We are using the mean value (typically a 1-hour average) as our best estimate for the "true" value of that process parameter (the population mean) for a given time period. We need to assign uncertainty to these inputs to express how good the estimate is and this takes the form of a 95% confidence interval. This is the intended purpose of the confidence interval, to express how well a sample mean estimates an unknown population mean. A confidence interval can be expressed as:

$$\bar{x} \pm t_{(1-\gamma,n-1)} \frac{s}{\sqrt{n}}$$

Where: $\bar{x}$ = sample mean
s = sample standard deviation

$t_{(1-y,n-1)}$ = Student's t-test value for selected confidence level γ at (n -1) degrees of freedom (at 95% confidence approximately 2, approaches 1.96 as n→∞)

One important consideration with instrumentation and measurement is that there can be multiple unknown error sources that affect how well the measured sample mean approximates the population mean (or the "true" process value). These error sources (systematic, random, potential biases) are all estimated as 95% confidence intervals and combined to form a single confidence interval and a more complete estimate about how close the observed sample mean is to the true value.

With a tolerance interval there is no concern for how a sample mean compares to the population mean. The tolerance interval is focused just on the spread of the sampled data and makes a statement about spread of the population based on the sample data. The tolerance interval is typically centered around the sample mean value (making it very similar to a confidence interval) and requires specifying a confidence level and a fraction of the population. A tolerance interval could be stated as: "we have γ% confidence that (L,U) contains π% of the population." A tolerance interval can be expressed as:

$$\bar{x} \pm ks \text{ (100γ,100π)} \quad \text{or} \quad \bar{x} \pm ks_{(\gamma\%/\ \pi\%)}$$

Where: $\bar{x}$ = sample mean

s = sample standard deviation

k = *k-factor* from a reference table lookup, based on selected γ confidence level, selected π fraction of population, and sample size n [1]

L = $\bar{x} - ks$

U = $\bar{x} + ks$

If a tolerance interval is applied to a measured process parameter it will just examine the noise, or spread, of the instrument based on a sample. The statement made by the tolerance interval can tell us what process values we are most likely to measure in the future and the spread we expect to measure. This can be useful some situations, but it only considers the repeatability of the measurement (precision) and does not give us any space to account for the measurement accuracy. The tolerance interval cannot account for any errors in measurement and effectively assumes the data is without any error or bias.

The tolerance interval is describing how the process affects the measurement and the sample data is treated as an outcome of the process. The confidence interval is focused on using the measurement and sample data to make an inference about the process: an estimate of the process parameter. The DVR process then seeks to make an improvement on this estimate using redundant information and process equations.

[1] Note that with a 95% confidence level and 95% of population, the k-factor will approach a value of 1.96 as n approaches infinity. This would match the formula shown in RAI-14 Table 1 for tolerance interval

Corrected Table 1

Below is a corrected version of RAI-14 Table 1 with what we consider the correct interpretation. Additional text and columns are added to explain all the values.

Corrected RAI-14 Table 1: Example Values

Flow Meter	$\bar{x}$ mean flow rate (kg/sec)	$\bar{x} \pm 1.96 \cdot \frac{s}{\sqrt{N}} = 1.96 \cdot \bar{s}$ 95% Confidence Interval (kg/sec)	$\bar{s}$ standard deviation in the mean (kg/sec)	s Combined standard deviation of sample (kg/sec)	s^2 Combined Variance of sample (kg^2/sec^2)	$s_{\bar{x}}{}^2$ Variance in the mean (i.e., the standard error squared) (kg^2/sec^2)	n (number of samples to obtain the mean)
Flow Meter A	245.00	245 ± 12.25	$\frac{12.25}{1.96} = 6.25$	$6.25 * \sqrt{50}$ =44.19	44.19^2= 1953.125	6.25^2=39.06	50
Flow Meter B	250.00	250 ± 12.50	$\frac{12.50}{1.96} = 6.38$	$6.38 * \sqrt{100}$ =63.78	63.78^2= 4067.89	6.38^2=40.7	100

The uncertainty values in Table 1 (±12.25, ±12.50) represent the total 95% confidence interval for a sample mean value of each of the flow meters. They are intended to be an estimation of unknown true flow values that fall within the interval. These uncertainty values are the Square Root Sum of Squares (SRSS) of the systematic (or instrument) and random uncertainty elements and can only apply to the mean measurement value:

$$12.25 = \pm\sqrt{\varepsilon_{A,sys}^2 + \varepsilon_{A,rand}^2} \qquad 12.50 = \pm\sqrt{\varepsilon_{B,sys}^2 + \varepsilon_{B,rand}^2}$$

Where:

$\varepsilon_{A,sys}$, $\varepsilon_{B,sys}$ = Systematic, or instrument channel uncertainties for flow meters

$\varepsilon_{A,rand}$, $\varepsilon_{B,rand}$ = Random uncertainty of flow meter measurements ($= 1.96 \cdot \frac{s_{sample}}{\sqrt{N}}$)

If the uncertainty of a single flow reading was desired, instead of a mean value, the random uncertainty portion would need to be considered differently. The benefit of using a mean measurement value as an estimate is that the random uncertainty is reduced with the incorporation of more measurement samples (with higher N, $1.96 \cdot \frac{s_{sample}}{\sqrt{N}}$ becomes smaller). For an individual flow reading, random uncertainty would be ε_{rand}=*1.96*s_{sample}*. This requires treating s_{sample} as an estimate of the measurement population's true standard deviation (with higher N, s_{sample} won't decrease). This individual measurement random uncertainty is higher than uncertainty for the mean value by a factor of $\sqrt{N}$. The DVR process generally uses mean values that are one-hour averages as inputs (T.R. Section 3.2.3, 6.2.10). The DVR results are corrected mean values and individual measurement values are never considered.

Based on the data in Table 1, the actual values of the systematic and random uncertainties are not known. The raw sample data used to calculate the mean values would be needed to calculate the standard deviation (s_{sample}) and then apply the standard error formula. Alternatively, the systematic uncertainty could be specified.

In the corrected Table 1 there are now “Combined” standard deviation and variance of the sample. This is because we are taking a “combined” total uncertainty and then treating it as a single standard error to solve for standard deviation and variance values that do not correspond to the actual values that would come from the measurement data itself. This may work mathematically because the SRSS total uncertainty value follows a normal distribution, but the meaning and usefulness of these values is not clear. These values offer no information about the measured data, they just describe the normal distribution of the combined total uncertainty.

The RAI-14 version of Table 1 applies the 1.96σ definition of a 95% tolerance bound and for σ is applying the sample standard deviation. The corrected Table 1 shows the formula relating the confidence interval, population standard deviation (s), standard deviation in the mean ($\bar{s}$), and the number of measurements (N). The definition of probabilistic confidence intervals requires the application of $\bar{s}$ and not s. This is in accordance with numerous industry codes including PTC 19.1, VDI-2048, and GUM.

An Updated RAI-14 Calculation

Below are updated equations and results from RAI-14 based on the changes made to Table 1 above. The original equation numbers were kept and some additional comments are also included. The text in red indicates updated calculations and values based on the corrected Table 1. Note that below the equations are evaluated as written in RAI-14 with the updated values from Table 1. This still produces the wrong answer. To produce the correct answer Equations 1, 4, and 5 should use values for $\bar{s}^2$ and not s^2.

$$f_{DVR}(\bar{x}_A, \bar{x}_B, s_A{}^2, s_B{}^2) = \begin{bmatrix} \bar{y}_A \\ \bar{y}_B \end{bmatrix} = \begin{bmatrix} \left(1 - \frac{s_A{}^2}{s_A{}^2 + s_B{}^2}\right) \cdot \bar{x}_\mathrm{A} + \frac{s_A{}^2}{s_A{}^2 + s_B{}^2} \cdot \bar{x}_\mathrm{B} \\ \frac{s_B{}^2}{s_A{}^2 + s_B{}^2} \cdot \bar{x}_\mathrm{A} + \left(1 - \frac{s_B{}^2}{s_A{}^2 + s_B{}^2}\right) \cdot \bar{x}_\mathrm{B} \end{bmatrix} \quad \text{Eq. 1}$$

From the original RAI-14 Table 1, Equation 1 uses the variance terms (95% tol. Interval/1.96)^2. This was the correct numerical value to use, but it seems to be considered here a sample variance when it should be considered the variance in the mean. We want (95% confidence interval/1.96)^2 yielding $\bar{s}^2$, not s^2. The correct reconciled value would be calculated with the original RAI-14 Table 1 and Equation 1 as the number was correct (it was just labeled wrong). However, when Table 1 is corrected, as discussed above, the result of Equation 1 would no longer be the correct reconciled output because the values of s_A and s_B are different.

$$s_{\bar{y}}{}^2 = \left(\frac{\partial f_{DVR}}{\partial \bar{x}_A}\right)^2 s_{\bar{x}_1}{}^2 + \left(\frac{\partial f}{\partial \bar{x}_\mathrm{B}}\right)^2 s_{\bar{x}_\mathrm{B}}{}^2 \quad \text{Eq. 3}$$

$$\frac{\partial f_{DVR}}{\partial \bar{x}_A} = \left(1 - \frac{s_A{}^2}{s_A{}^2 + s_B{}^2}\right) = \frac{s_B{}^2}{s_A{}^2 + s_B{}^2} = \frac{4067.89}{1953.13 + 4067.89} = 0.6756 \quad \text{Eq. 4}$$

$$\frac{\partial f_{DVR}}{\partial \bar{x}_B} = \frac{s_A{}^2}{s_A{}^2 + s_B{}^2} = \left(1 - \frac{s_B{}^2}{s_A{}^2 + s_B{}^2}\right) = \frac{1953.13}{1953.13 + 4067.89} = 0.3244 \quad \text{Eq. 5}$$

The derivative terms for Equation 3 are recalculated above in Equations 4 and 5, applying the “combined variance of the sample” term from the corrected Table 1. The above equations are repeating what was demonstrated in RAI-14. The same issue with Equation 1 applies to Equations 4 and 5, the variance values should be $\bar{s}^2$, not s^2. When these values are applied the results are 0.5103 (Eq 4) and 0.4897 (Eq 5) which, coincidentally, are the values originally calculated in RAI-14.

$$s_{\bar{x}_A}{}^2 = \frac{s_A{}^2}{n_A} = \frac{1953.13}{50} = 39.06 \quad \text{Eq. 6}$$

$$s_{\bar{x}_B}{}^2 = \frac{s_B{}^2}{n_B} = \frac{4067.89}{100} = 40.7 \quad \text{Eq. 7}$$

Equations 6 and 7 are reevaluated with the corrected Table 1 values.

Finally, with Equation 8 an updated variance in the reconciled mean value ($s_{\bar{y}}{}^2$) is determined based on the corrected Table 1 values:

$$s_{\bar{y}}{}^2 = \left(\frac{\partial f_{DVR}}{\partial \bar{x}_A}\right)^2 s_{\bar{x}_A}{}^2 + \left(\frac{\partial f}{\partial \bar{x}_B}\right)^2 s_{\bar{x}_B}{}^2 = (0.6756)^2 \cdot 39.06 + (0.3244)^2 \cdot 40.7 \quad \text{Eq. 8}$$

$s_{\bar{y}}{}^2 = 22.11$ (Variance in the reconciled mean value)

$s_{\bar{y}} = 4.702$ (Std Dev in the reconciled mean value)

$1.96 * s_{\bar{y}} = 9.21$ (95% confidence interval)

The result of the reevaluated Equation 8, $s_{\bar{y}}{}^2 = 22.11$, whereas the initial value calculated in the RAI-14 example was 0.30. The updated result is actually much closer to the value calculated by the DVR method presented in the first response to RAI-14 ($s_{\bar{y}}{}^2 = 19.93$).

After solving for $s_{\bar{y}}{}^2$ RAI-14 moves on to Equation 9 and discussing a need to convert this “variance in the mean of the reconciled value” to “variance in the reconciled measurement itself.”

$$s_y{}^2 = n_y \cdot s_{\bar{y}}{}^2 \quad \text{Eq. 9}$$

This requires coming up with some value for n_y, an equivalent number of measurements for the reconciled value. This is not a task that has a clear method or approach and the DVR process does not take this step. Table 2 of RAI-14 presents possible values of n_y and the resulting values for s_y^2. The original Table 2 and a recalculated Table 2 are shown below.

Table 1: Possible values of $\boldsymbol{n_y}$

$\boldsymbol{n_y}$	$\boldsymbol{s_y}^2$
$= n_A = 50$	15.045
$= \frac{n_A+n_B}{2} = 75$	22.5675
$= n_B = 100$	30.09
$= n_A = n_B$	19.93

Updated Table 2:

$\boldsymbol{n_y}$	$\boldsymbol{s_y}^2$
$= n_A = 50$	1105.5
$= \frac{n_A+n_B}{2} = 75$	1658.2
$= n_B = 100$	2210.9
$= n_A = n_B$	1319.5

The s_y^2 values in the updated Table 2 are much larger than the initial RAI-14 values and are in line with the “Combined Variance of Sample” values in the corrected Table 1. However, now when the special case where $n_A=n_B$ is applied, the answer no longer matches the DVR result (19.93, as demonstrated in the first RAI-14 response). This goes back to the variance values being implemented in Equations 1, 6, and 7. This is demonstrated below:

$$s_y{}^2 = n_y \cdot s_{\bar{y}}{}^2 \text{ (Eq 9)}$$

$$s_y{}^2 = n_y \cdot \left(\left(\frac{\partial f_{DVR}}{\partial \bar{x}_A}\right)^2 s_{\bar{x}_A}{}^2 + \left(\frac{\partial fDVR}{\partial \bar{x}_B}\right)^2 s_{\bar{x}_B}{}^2\right) \text{ (substitute Eq 3)}$$

$$s_y{}^2 = n_y \cdot \left(\left(\frac{\partial f_{DVR}}{\partial \bar{x}_A}\right)^2 \left(\frac{s_A{}^2}{n_y}\right) + \left(\frac{\partial fDVR}{\partial \bar{x}_B}\right)^2 \left(\frac{s_B{}^2}{n_y}\right)\right) \text{ (substitute Eq 6,7, assuming } n_y=n_A=n_B)$$

$$s_y{}^2 = \left(\frac{\partial f_{DVR}}{\partial \bar{x}_A}\right)^2 s_A{}^2 + \left(\frac{\partial f}{\partial \bar{x}_B}\right)^2 s_B{}^2 \text{ (} n_y \text{ values cancel)}$$

$$s_y{}^2 = (0.6756)^2 \cdot 1953.13 + (0.3244)^2 \cdot 4067.89 = 1319.56$$

The resulting s_y^2=1319.56 is similar in magnitude to the “Combined Variance of Sample” values in the corrected Table 1. This equates to a “Combined Standard Deviation of Sample” s_y=36.32, also similar to the corrected Table 1 values.

Imagine now we wanted to apply this result to our reconciled mean flow value. We need a 95% confidence interval for practical application. We cannot simply take 1.96*s_y, because s_y and the 36.32 value is a standard deviation of some imaginary reconciled sample of data and it would repeat the mistake initially made in Table 1. The 1.96*s_y, would actually be an approximated Tolerance Interval for the reconciled flow telling us an interval that would contain 95% of the reconciled flows with 95% confidence. To make a confidence interval for the reconciled mean we would need to solve for n_y and apply the standard error formula:

$$n_y = \frac{s_y^2}{s_{\bar{y}}^2} = \frac{1319.56}{22.11} = 59.68 \text{ (using Eq 9)}$$

$$95\%\ Conf\ Int = 1.96 \cdot \frac{s_y}{\sqrt{n_y}} = 1.96 \cdot \frac{36.32}{\sqrt{59.68}} = \pm 9.21$$

The resulting 95% Confidence Interval is ±9.21 and this is the same value that is shown above where Equation 8 was reevaluated. It is the initial DVR output. The additional steps of Equation 9 and Table 2 did not produce anything useful. To create an applicable Confidence Interval we had to integrate the n_y value we just solved for back into the standard deviation s_y that we solved for and it ends up where we started at the Equation 8 result.

The inputs to DVR are mean values with confidence intervals that represent estimates of true process values. DVR then makes corrections, when possible, to improve these estimates and provides a corrected mean value and confidence interval that represents an improved estimate of the true process value. The focus and purpose in on estimating the process value and this requires mean values with confidence intervals.

References

1. ASME PTC 19.1-2005. See Appendix A for Confidence and Tolerance Interval discussion
2. NUREG-1475, Revision 1. "Applying Statistics." March, 2011. Chapter 9 addresses Confidence and Tolerance Intervals.
3. "Experimentation, Validation, and Uncertainty Analysis for Engineers," Fourth Edition, Hugh W. Coleman and W. Glenn Steel, 2018. Sections 1-3.1, 2-2.3, 2-2.4, 2-3.3, 2-3.4