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Development of an Engineering Definition of the Extent of J Singularity Controlled Crack Growth

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PRIOR REPORTS

Prior reports in this series are listed below:

1. J. A. Joyce, "Application of the Key Curve Method to Determining J–R Curves for A533B Steel," NUREG/CR–1290, U.S. Naval Academy, Annapolis, MD (January 1980)

2. J. P. Gudas, M. G. Vassilaros, J. A. Joyce, D. A. Davis, and D. R. Anderson, "A Summary of Recent Investigations of Compact Specimen Geometry Effects on the J_I–R Curve of High Strength Steels," NUREG/CR–1813, DTNSRDC, Annapolis, MD (November 1980).

3. J. A. Joyce, "Instability Testing of Compact and Pipe Specimens Utilizing a Test System Made Compliant by Computer Control," NUREG/CR–2257, U.S. Naval Academy, Annapolis, MD (March 1982).

4. J. A. Joyce, "Static and Dynamic J–R Curve Testing of A533B Steel Using the Key Curve Analysis Technique," NUREG/CR–2274, U.S. Naval Academy, Annapolis, MD (July 1981).

5. M. G. Vassilaros, J. P. Gudas and J. A. Joyce, "Experimental Investigation of Tearing Instability Phenomena for Structural Materials," NUREG/CR–2570, Rev. 1, DTNSRDC, Annapolis, MD (August 1982).

6. J. A. Joyce, "Development and Verification of Conditions for Ductile Tearing Instability and Arrest," NUREG/CR–4528, U.S. Naval Academy, Annapolis, MD (February 1986).

7. J. A. Joyce and E. M. Hackett, "Application of the Key Curve and Multispecimen Techniques to Dynamic J–R Curve Testing of Alloy Steel," NUREG/CR–4579, U.S. Naval Academy, Annapolis, MD (April 1986).

8. J. A. Joyce and C. S. Schneider, "Application of Alternating Current Potential Difference to Crack Length Measurement During Rapid Loading," NUREG/CR–4699, U. S. Naval Academy, Annapolis, MD (August 1986).

9. J. A. Joyce and E. M. Hackett, "Development of an Analytic Key Curve Approach to Drop Tower J–R Curve Measurement," NUREG/CR–4782, DTNSRDC, Annapolis, MD (December 1986).

10. J. A. Joyce and E. M. Hackett, "Transition Range Drop Tower J–R Curve Testing of A106 Steel," NUREG/CR–4818, DTRC, Annapolis, MD (February 1987).

11. J. A. Joyce, "Ductile to Brittle Toughness Transition Characterization of A533B Steel," NUREG/CR–5142, DTRC, Annapolis, MD (June 1988).

12. J. A. Joyce, D. A. Davis, E. M. Hackett, and R. A. Hays, "Application of the J–Integral to Cases of Large Crack Extension," NUREG/CR–5143, DTRC, Annapolis, MD (February 1989).

ABSTRACT

An experimental definition is proposed for the extent of singularity controlled behavior in a J-resistance fracture test. The singularity zone is defined in terms of a constant ratio of plastic crack opening displacement and normalized crack extension. Justification for this definition is given in terms of experimental results on compact specimens of three steel alloys of varying material toughnesses and in terms of a simple analytical model.

The experimental limit can be evaluated from the data normally obtained during an unloading compliance single specimen J integral resistance curve experiment. Generally the experimental singularity limit extends the region of test validity well beyond that which is presently allowed by the ASTM J-R test standard, E1152.

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ACKNOWLEDGEMENTS

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Introduction

The objective of this report is to present recent work which attempts to define the limits of the J singularity controlled crack extension in a bend type fracture mechanics test specimen. Both analytical(1) and computational(2)(3) techniques have been applied to this task in the past and have led to size criteria presently utilized in the J_{IC} and J–R curve test standards, i.e. ASTM E813 and ASTM E1152, respectively. When applied to experimental data these limitations have not corresponded to observed experimental phenomena which could be identified as due to a loss of singularity for the particular test. This would seem to lead to the conclusion that either the singularity was not present before the aforementioned criteria was reached, or that it still existed after the criteria were exceeded.

1.0 BACKGROUND

1.1 Observations from Experiment

Experimental work was described in a previous report(4) in which standard unloading compliance J–R curve tests were conducted to large crack extensions. The results used the J equations of ASTM E1152 and the J_M (J modified) formulation of Ernst(5). The observations of this previous work can be stated briefly as follows:

1. For scaled compact specimens of materials with a range of toughnesses, the deformation J–R curve was found to be remarkably size independent to crack extensions as large as 60% of the initial uncracked ligament.

2. Deformation J–R curves continued to rise even to these large crack extensions.

3. No limit to the J singularity was apparent from the J–R curves for any of the materials tested.

4. The J_M–R(4) curves, on the other hand, demonstrated strong size dependence with small specimens developing a sigmoidal shape rising distinctly above the corresponding J_M–R curves of larger specimens. These differences amongst specimens occurred after about 30% of crack extension and were most distinct in the low toughness alloys.

5. The J_M–R curves were relatively size independent only for the highest toughness alloy which exceeded all standard J size requirements before measurable crack extension was found.

Additional experimental work performed recently has verified the above observations. A major objective of this work is to look more closely at this data set, to clarify this rather confusing situation, and where possible to generate meaningful limitations to the useful extent of the J–R curve. In a later section, a simple model for J is developed giving J in terms of two simpler relationships which can be readily fit to experimental data. This model is then used in the following sections to answer some basic "what if" questions – which lead, in due course, to some desired answers.

1.2 Deformation J Equations

The original J integral formulation by Rice[6] was that (see Fig. 1):

$$J = \oint_{\Gamma} \left[W dy - T_i \cdot \frac{\partial \bar{u}_i}{\partial x} ds \right] \tag{1}$$

where

$W = \int \sigma_{ij} \, d\epsilon_{ij}$ is the strain energy density

Γ = the path of the integral

ds = increment of distance along the contour Γ

T_i = tractions on the contour Γ

$\bar{u}_i$ = a displacement component in the direction of T_i .

This equation is useful for analysis and for computational methods but it is not a good starting point to develop J estimates for laboratory specimens. An equivalent form for J was presented by Rice[7] as:

$$J = -\int_0^{\delta} \frac{\partial P}{\partial a} \, d\delta = \int_0^{P} \frac{\partial \delta}{\partial a} \, dP \tag{2}$$

where P is the load applied to the specimen and δ is the resulting load point displacement.

Equation (2) was used successfully by Begley and Landes[8][9] to do experimental J integral work, but it is far from convenient. A further simplification was obtained by Rice et. al.[7] who combined Eq. (2) with the observation that for bending geometries

$$\theta = F\left[\frac{M}{b^2}\right] \tag{3}$$

where b is the remaining uncracked ligament and M is the applied moment, and obtained for J that:

$$J = \frac{2}{b} \int_0^{\theta} M d\theta = \frac{2}{b} \int_0^{\delta} P d\delta \, . \tag{4}$$

Equation (4) is only exact for the case of deeply cracked bend bars. It does relate the J integral directly, however, to an easily measured quantity, the area under the specimen load displacement record.

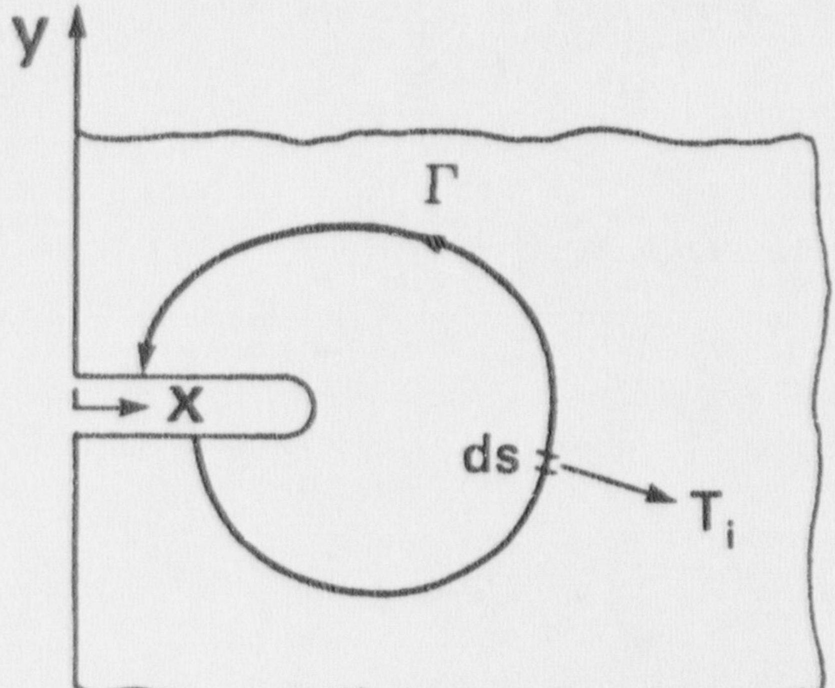

Figure 1 - Contour Path Integral Definition of J

For the standard compact specimen, Merkle and Corten[10] presented a modification of Eq. (4) to account for the additional tensile loading component which can be expressed in the form[11]:

$$J = \frac{\beta A}{B b} \tag{5}$$

where:

$\beta = 2(1 + \alpha)/(1 + \alpha^2)$

B = specimen thickness

A = area under specimen load displacement record

$\alpha = \left[\left[\frac{2a}{b}\right]^2 + 2\left[\frac{2a}{b}\right] + 2\right]^{1/2} - \left[\frac{2a}{b} + 1\right]$

a = crack length

b = uncracked ligament

While the above equations are adequate for experimental work for initiation J, i.e. J_{IC} evaluation as per ASTM E813, they are not suitable for the case of crack growth as is the case in J resistance (J–R) curve measurement. For the case where crack growth is present, Ernst et. al.[12] proposed an incremental evaluation of J as:

$$J_{i+1} = \left[J_i + \left[\frac{\eta}{b}\right]_i A_{i,i+1}\right]\left[1 - \left[\frac{\gamma}{b}\right]_i \left[a_{i+1} - a_i\right]\right] \tag{6}$$

where γ and η are geometry dependent parameters and $A_{i,i+1}$ is the area under the specimen load displacement record between point i and point i+1. Equation (6) is based on deformation plasticity assumptions and calculates J at a crack length $a_f = a_o + \Delta a$ as if the specimen had the final crack length from the start of loading. The applicability of deformation plasticity to elastic plastic fracture testing was discussed in the previous report[4] in terms of a series of blunt notched specimen tests. The principle conclusions of that report were that deformation plasticity appeared to work well even when large crack extensions were present, but not so well when large specimen deformations were present.

Equation (6) was utilized in ASTM E1152 with only one modification, the separation of J into elastic and plastic components. In this case then

$$J = J_{EL} + J_{PL} \tag{7}$$

$$J_{EL(i)} = \frac{(K_i)^2(1-\nu^2)}{E} \tag{8}$$

where

$$K_i = \left[P_i/(BB_N W)^{1/2}\right] \cdot f(a_i/W) \tag{9}$$

with:

$$f(a_i/W) = \frac{\left[(2 + a_i/W)(0.886 + 4.64\, a_i/W - 13.32(a_i/W)^2 + 14.72(a_i/W)^3 - 5.6(a_i/W)^4)\right]}{(1 - a_i/W)^{3/2}}$$

and:

$$J_{pl(i)} = \left[J_{pl(i-1)} + \left[\frac{\eta_i}{b_i}\right] \frac{A_{pl(i)} - A_{pl(i-1)}}{B_N}\right] \cdot \left[1 - \gamma_i \frac{(a_i - a_{i-1})}{b_i}\right]$$

where:

$$\eta_i = 2.0 + 0.522\, b_i/W, \text{ and}$$
$$\gamma_i = 1.0 + 0.76\, b_i/W. \tag{10}$$

The integration of Eq. (2) is obtained in Eqs. (7–9) by trapezoidal approximations over which the crack length is assumed constant and small steps are required for accurate J evaluation using this form. This becomes more crucial as the remaining ligament, b, becomes small.

1.3 Modified J Equations

More recent work by Rice et. al.[13] has investigated the evaluation of J for growing cracks. Their results seem to suggest that when appreciable crack extension is present, a J resistance curve can depend on the type and size of the specimen used for its experimental determination. Ernst[5] has proposed a modified J quantity which is corrected to first order to eliminate this proposed dependence on specimen size and type. The details of this analysis are left to reference (5) but the resulting equation is:

$$J_M = J - \int_{a_o}^{a} \left[\frac{\partial J_{pl}}{\partial a}\right]_{\delta_{pl}} da\ . \tag{11}$$

where δ_{pl} is the plastic component of the applied load line displacement. Ernst simplifies this for experimental geometries by substituting the approximate relationship that

$$\left[\frac{\partial J_{pl}}{\partial a}\right]_{\delta_{pl}} = -\, m\, \frac{J_{pl}}{b} \tag{12}$$

where m is a function of crack length and specimen geometry to give:

$$J_M = J + \int_{a_o}^{a} \frac{m}{b} J_{pl} \, da \qquad (13)$$

In the sections that follow, both J and J_M resistance curves are evaluated for large crack extensions for compact specimens of a range of sizes and toughnesses. These evaluations are carried out well beyond accepted J and crack extension limits and the results need careful critical appraisal.

2.0 EXPERIMENTAL PROCEDURE

2.1 On the Presence of a Singularity

The presence of a singularity in an elastic plastic fracture toughness specimen is difficult to experimentally verify. Computational techniques[14] do verify the path independence of J and the equivalence of the various equations for its evaluation. The analysis of Paris and Hutchinson[15] argues that if strain components remain proportional, the J singularity can, in fact, exist even with some crack growth, but does not identify to what extent growth consistent with a J singularity might exist for a given specimen configuration.

Since the nonlinear elastic J integral singularity reduces to the elastic stress intensity factor for cases of fully elastic material behavior, and the singularity has been well verified, it certainly seems likely that a J singularity might exist for low toughness materials, for limited amounts of crack growth. Such a singularity should act to produce the most intense conditions for crack growth, and as the singularity weakens, the amount of crack growth per increment of specimen deformation should be reduced as well.

2.2 Material and Specimen Characterization

The three materials analyzed in this report are a 3% Nickel alloy steel, an A710 high strength low alloy steel, and an A533B pressure vessel steel. The tensile mechanical properties of these steels are shown in Table 1 and the chemistries are shown in Table 2. All specimens were 1/2T, 1T, or 2T compact specimens. The A710 and 3% Nickel alloys were tested in the T–L orientation while the A533B steel was tested in the L–T orientation. All tests were performed using an unloading compliance technique according to ASTM E1152 except that the loading was continued until large crack extensions were present.

2.3 Normalized Load Displacement Records

Data plots which have been looked at to try to gain insight into the existence of a singularity using the data of reference (4) are shown in Figs. 2–4. On these three figures a normalized load, PW/Bb^2, is plotted versus δ_{pl}/W and it can be seen that this format causes the load displacement relationships of various sizes of specimens to plot on a single curve. The three curves demonstrate that this result is insensitive to the material toughness and yield strength. This formulation is a useful observation and will be utilized further below, but it does not yield directly any insight into the presence or loss of a singularity for the materials.

Table 1 Tensile Properties for Alloy Steels 70°F

	Code	.2% Yield Strength (psi)	Tensile Strength (psi)	% Elongation	% R.A.
3% Nickel	FYB	89000.	106000.	32.	80.
A710	GFF	74000.	87000.	23.	63.
A533B	H13	64150.	90000.	26.	60.

Table 2 Chemical Composition of Alloys (Wt%)

	C	Mn	P	S	Cu	Si	Ni	Cr	Mo	V	Ti	Cb
3% Nickel	0.153	0.33	0.012	0.013	0.033	0.18	2.55	1.66	0.37	0.003	<.001	—
A710	0.04	0.59	0.005	0.004	1.17	0.25	0.90	0.70	0.19	0.003	0.06	0.03
A533B-H13	0.19	1.28	0.012	0.013	—	0.21	0.64	—	0.55	—	—	—

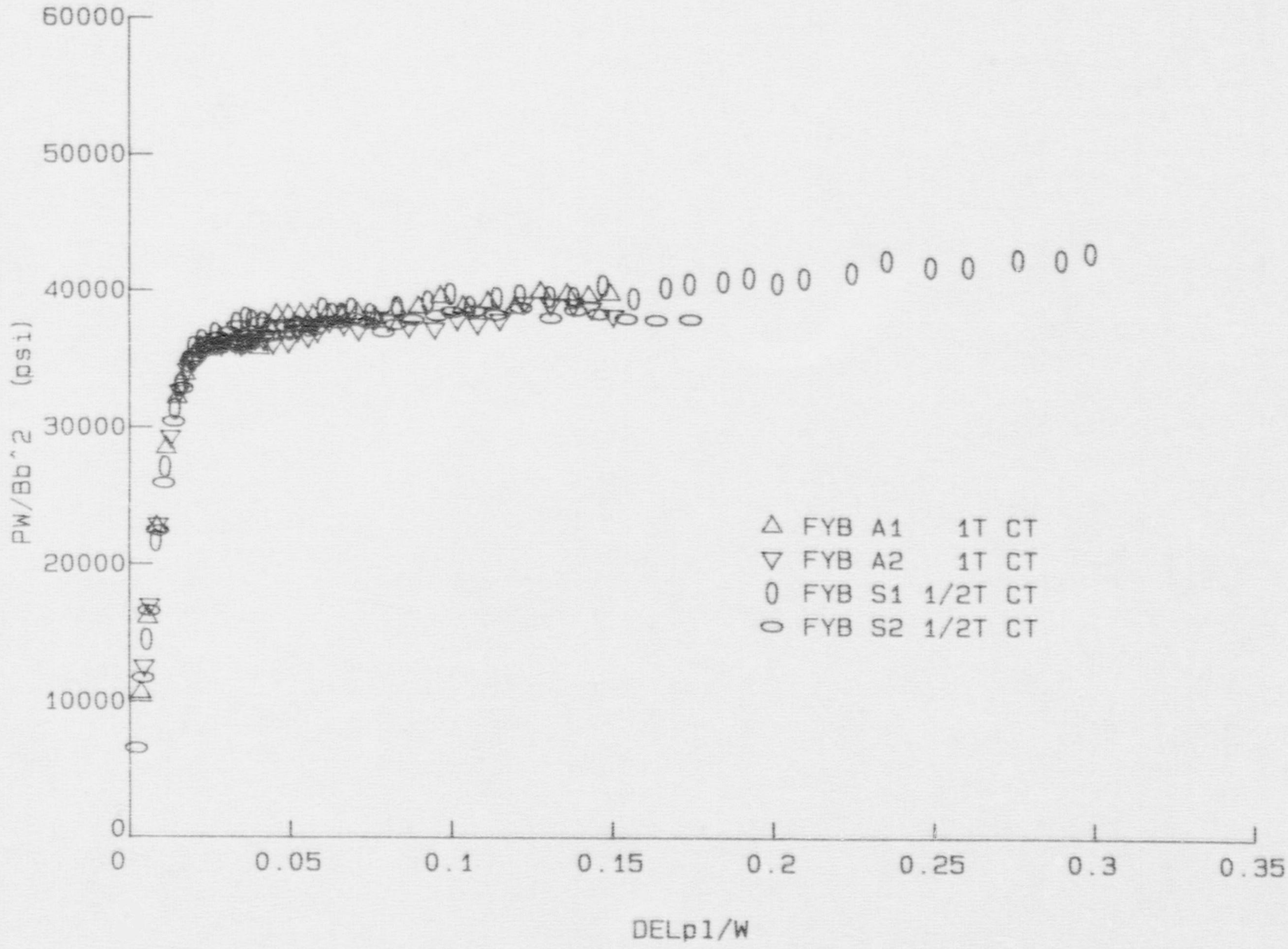

Figure 2. Key Curve Normalized Plot for the 3% Nickel Alloy (FYB).

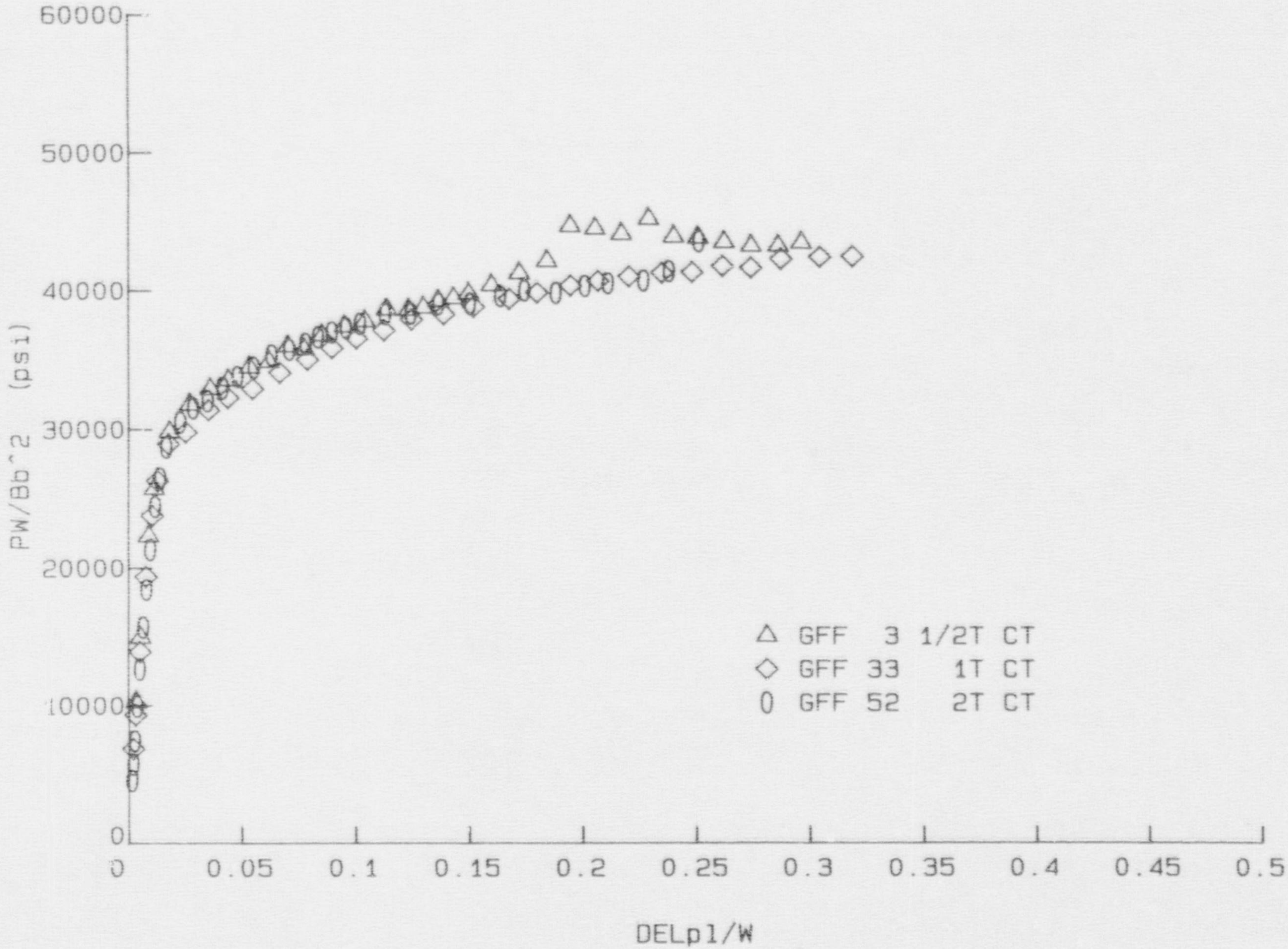

Figure 3. Key Curve Normalized Plot for the A710 (GFF) Alloy.

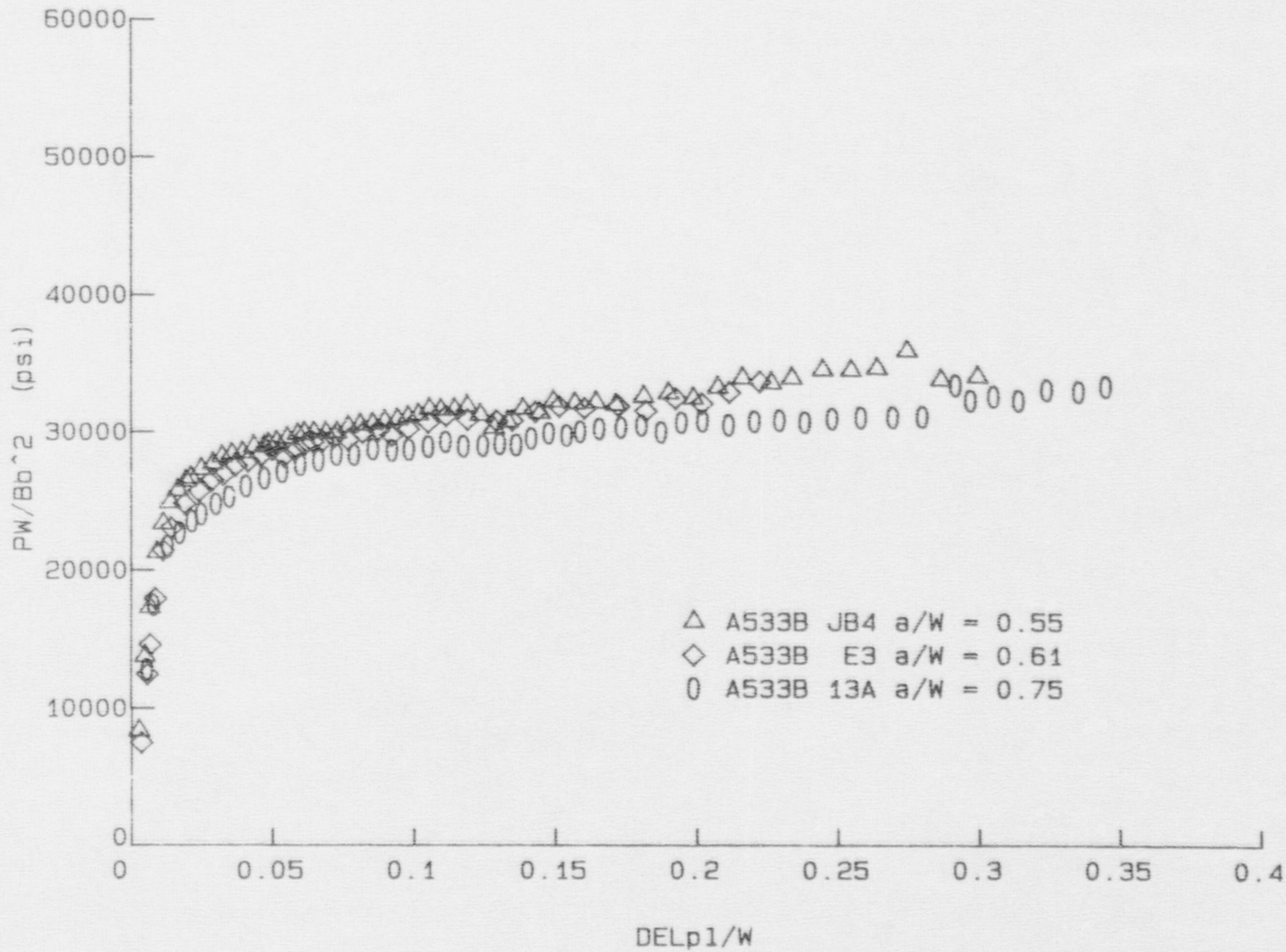

Figure 4. Key Curve Normalized Plot for the A533B -H13 Alloy.

The singularity expected near an elastic–plastic notch or crack tip would be predominantly a strain singularity. The experimentally measured load is thus likely to be very weakly dependent on whether or not a singularity is present at a crack tip in a standard test geometry. On the other hand, if the specimen is at its limit load, this is no guarantee that a strain singularity is not still present and controlling the local conditions for crack growth.

2.4 J Resistance Curves

Figures 5 to 10 show typical deformation J and modified J resistance curves presented previously[4]. Here the deformation J–R curves seem to maintain the desired uniformly rising shape and are consistent for various specimen sizes. The modified J resistance curves, on the other hand, show a tendency to rise after approximately 30% growth of the crack has occurred, and the resulting resistance curves are then strongly size dependent. Careful crack length measurements, and blunt notch specimen tests, were done as part of the work in reference (4) to verify the accuracy of the J resistance curves developed, and this work has shown that accuracies of both J and Δa should be within 10% even after the large crack extensions had occurred.

For the lower toughness materials, like the 3% Nickel steel of Fig. 5, a J singularity is expected for the initial region of crack growth, but as shown, the J–R curve continues to consistently rise to large J values and large amounts of crack extension. The labeled boxes on the figures show the valid ASTM E1152 regions presently thought to define bounds to the region of J controlled growth. Nothing seems to occur on the J deformation resistance curve which could be taken to imply a loss of singularity even well beyond this "valid" box. Does the singularity not exist inside the box? Does the singularity continue to exist outside the box? How can one define meaningful engineering limits to the region of J? These questions need a more complete answer before J is used in critical fracture analyses, and a partial answer now seems to be forming – as shown in the next section.

2.5 Definition of a Singularity Region

Figure 11 shows a typical plot of the normalized crack opening displacement versus normalized crack extension for various size specimens of the 3% Ni material.

Figure 11 shows a region of slow crack extension as a function of specimen bend angle, referred to here as initial crack blunting, followed by a region of more intense crack growth, which is then followed by a third region of ever slower crack extension. For this alloy, each size specimen generates a separate curve, as shown in Fig. 12, with smaller specimens giving more elevated results. It is shown in the next section that this observation is consistent with the presence of a singularity for these specimens. The third region also seems consistent with a gradual loss of singularity conditions and the point where this region starts is labeled on Fig. 11 as the "limit of singularity" region for these specimens.

The initial blunting line shown on Fig. 11 is taken to have the functional form:

$$\frac{\delta_{pl}}{W} = 2\,\frac{\Delta a}{W}\,. \qquad (14)$$

The slope of two was chosen here because of observed agreement with data obtained for the high toughness A710 alloy (Fig. 13). This alloy does not appear to demonstrate a

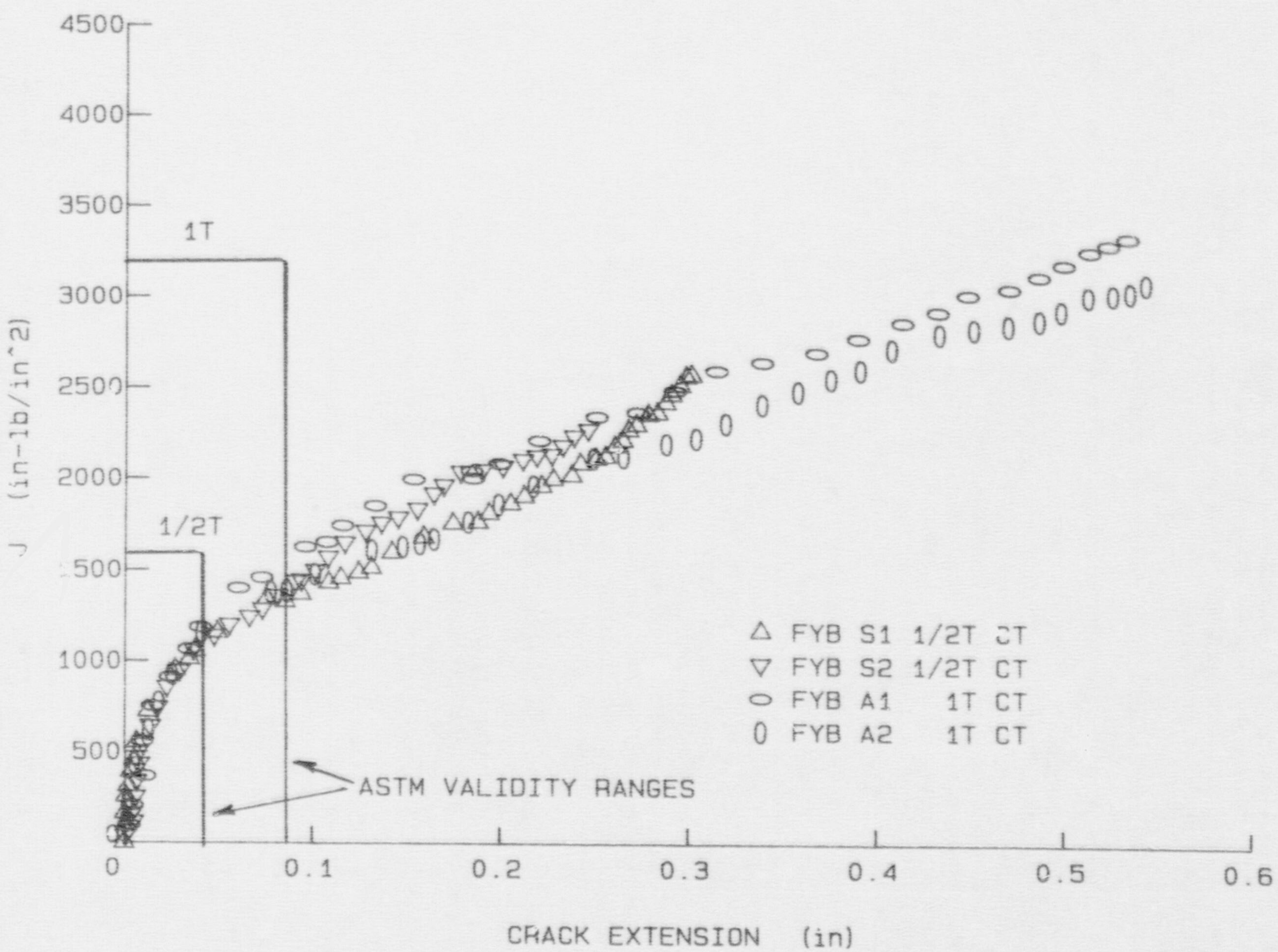

Figure 5. Deformation J Resistance Curves to Large Crack Extensions for the 3% Nickel Alloy.

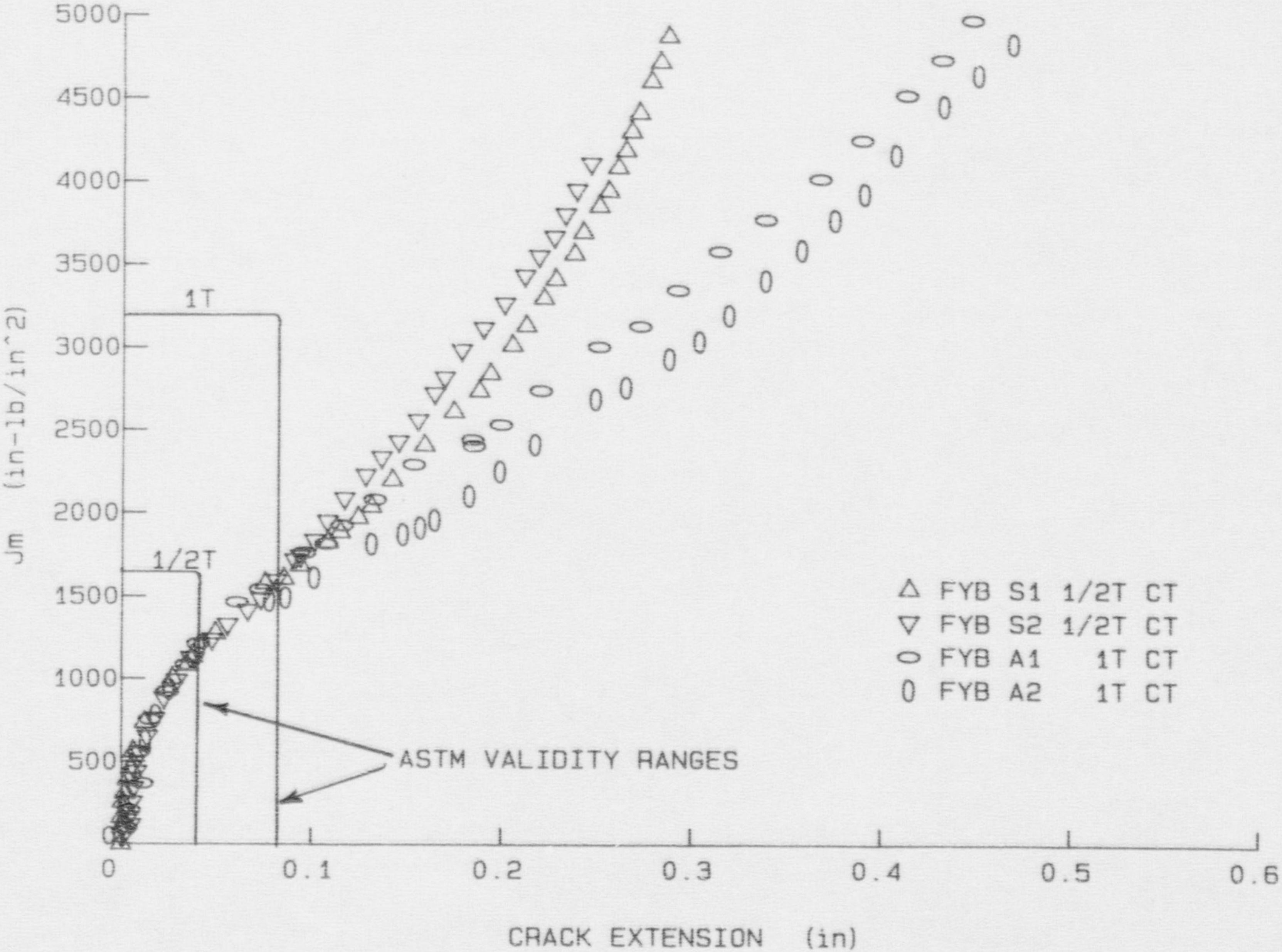

Figure 6. Modified J Resistance Curves to Large Crack Extensions for the 3% Nickel Alloy.

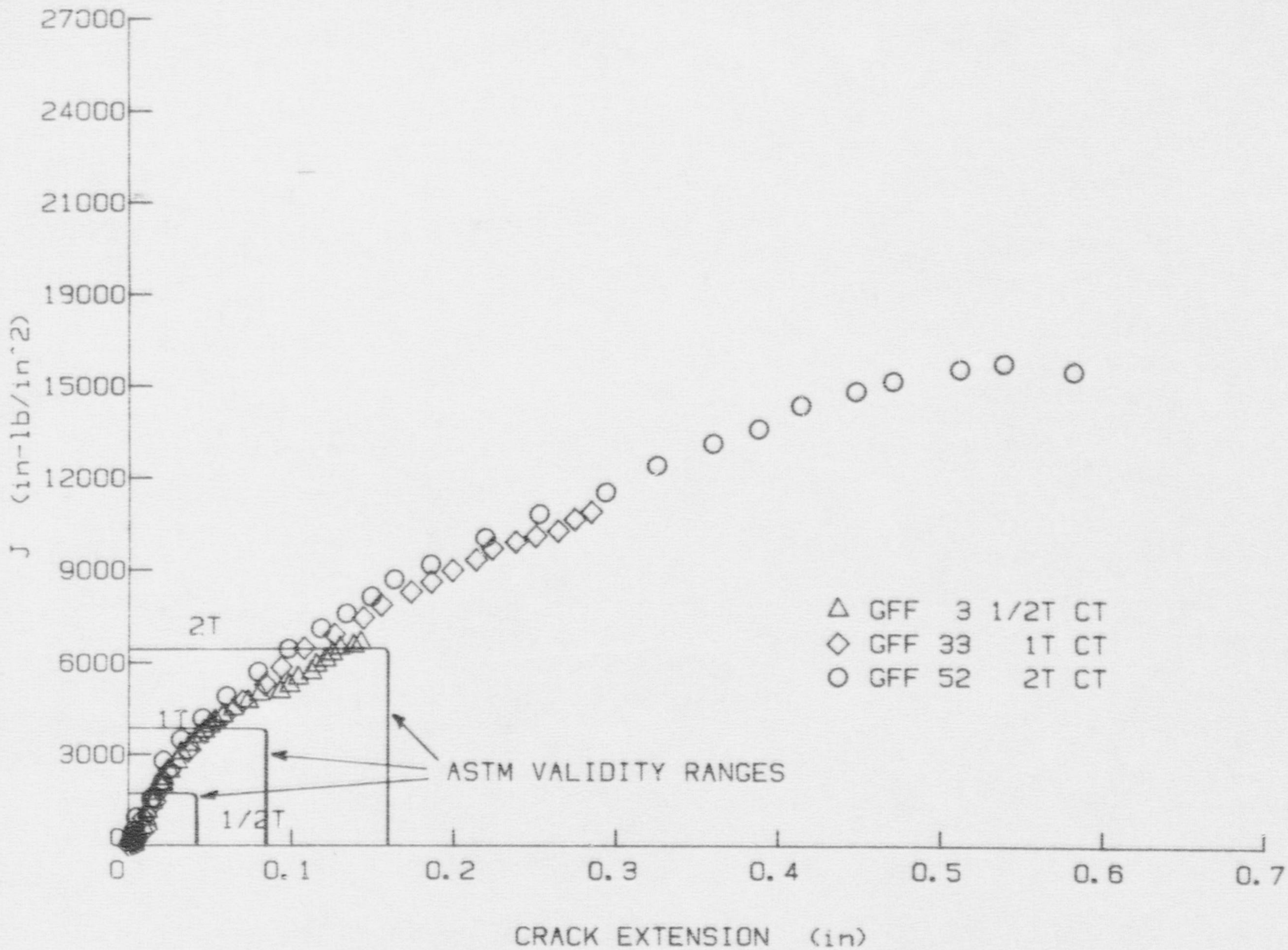

Figure 7. Deformation J Resistance Curves to Large Crack Extensions for the A710 Alloy.

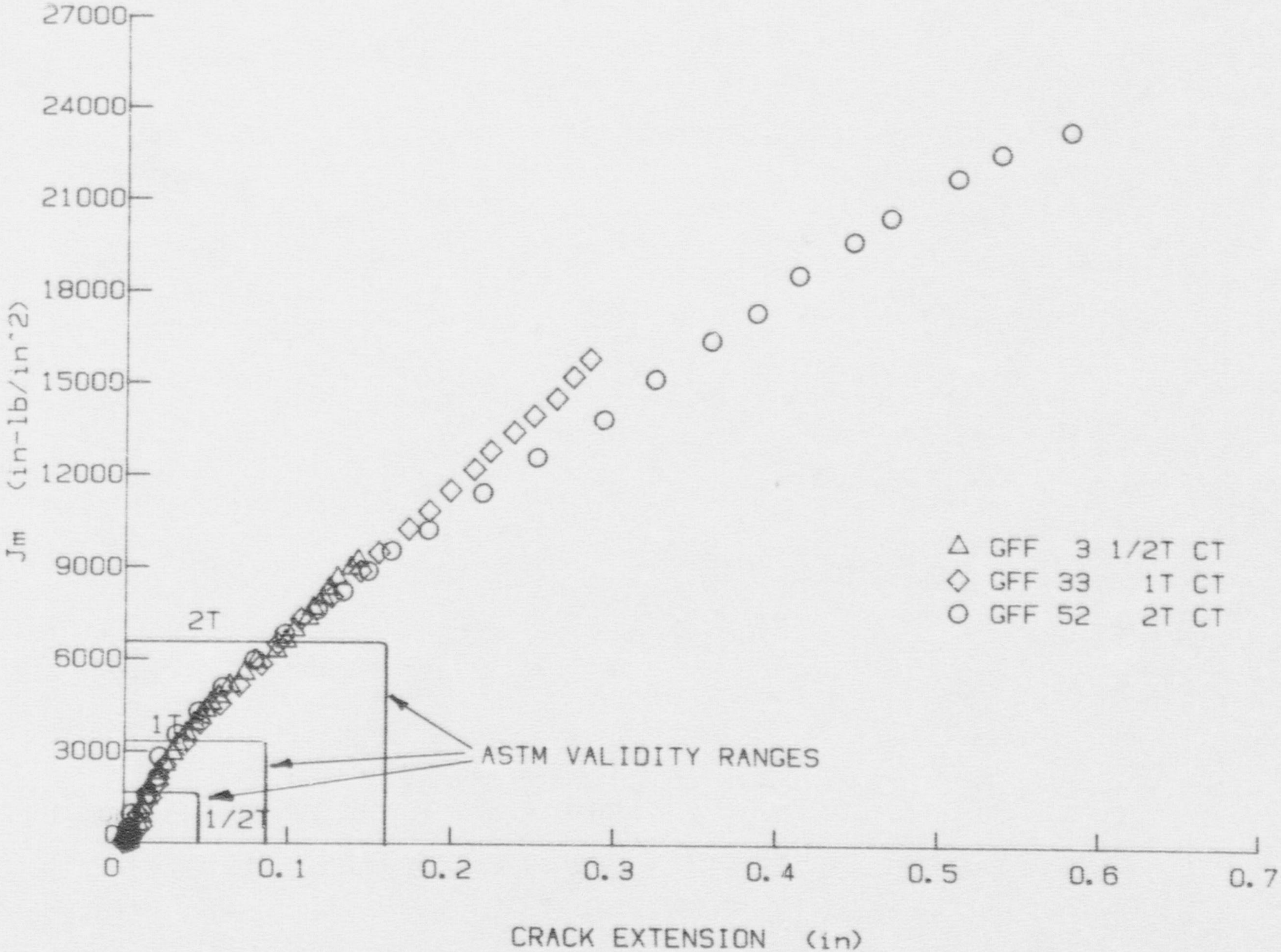

Figure 8. Modified J Resistance Curves to Large Crack Extensions for the A710 Alloy.

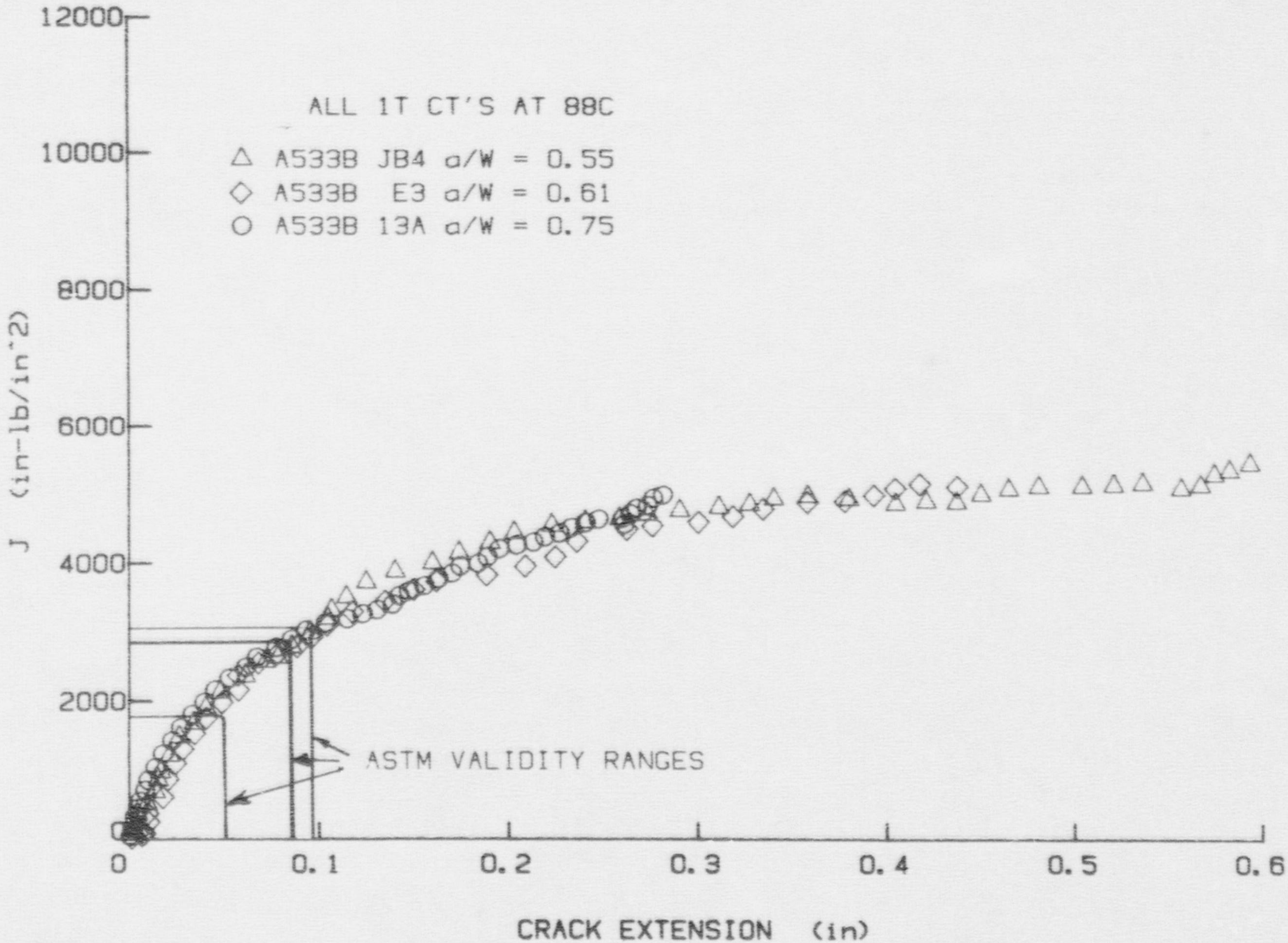

Figure 9. Deformation J Resistance Curves to Large Crack Extensions for the A533B -H13 Alloy.

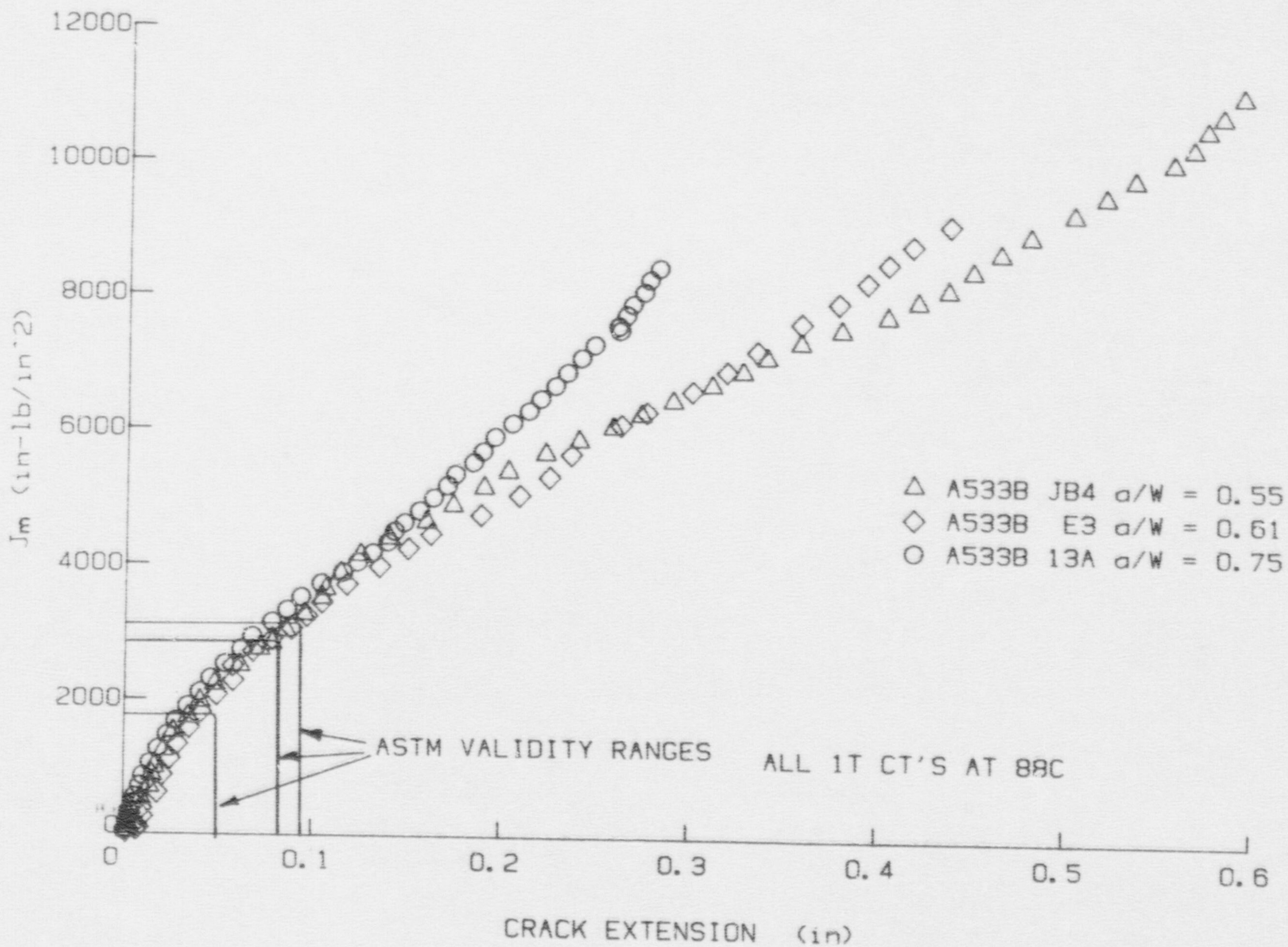

Figure 10. Modified J Resistance Curves to Large Crack Extensions for the A533B -H13 Alloy.

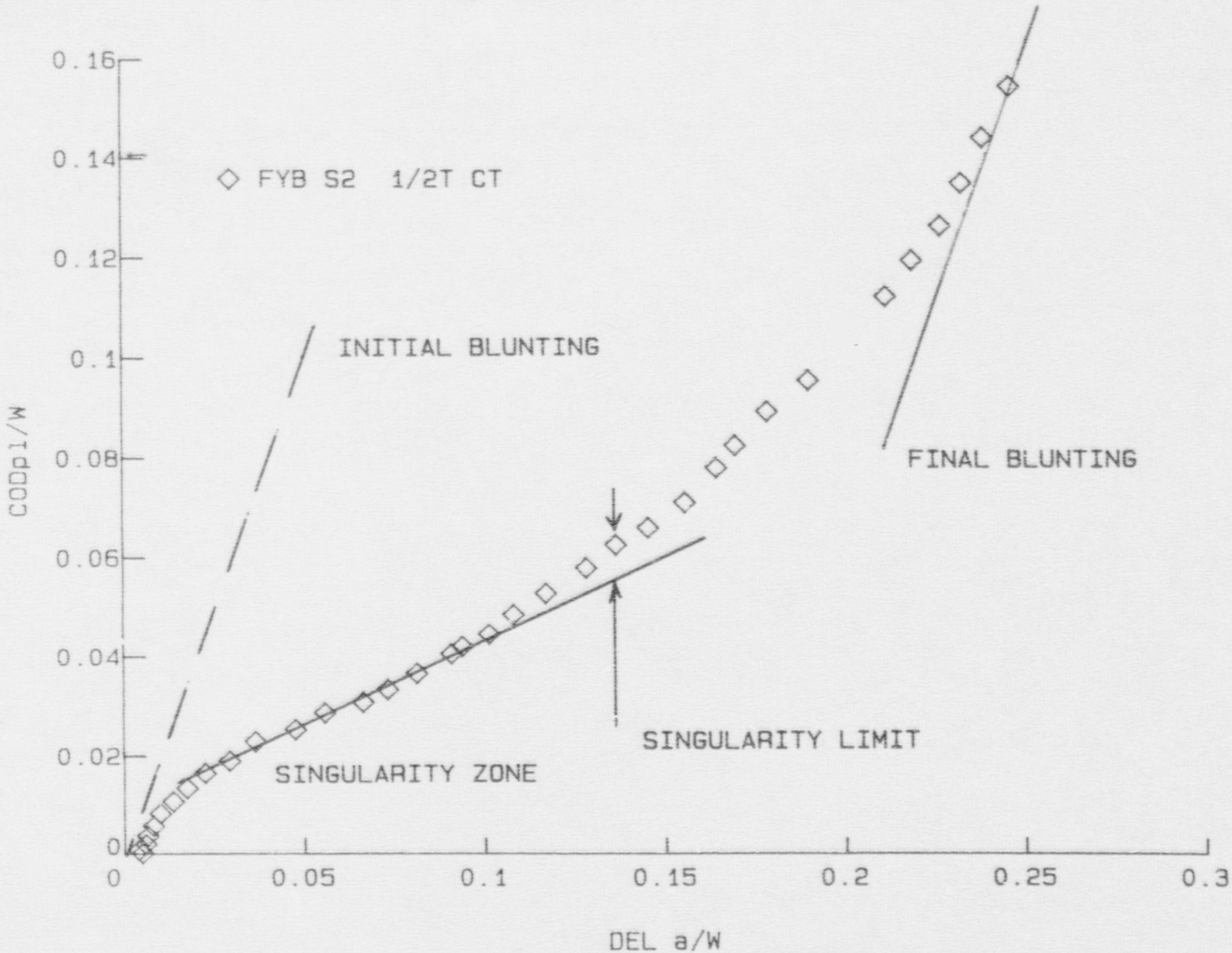

Figure 11. Definition of an Engineering Singularity Limit in Terms of the Loss of Intense Crack Growth.

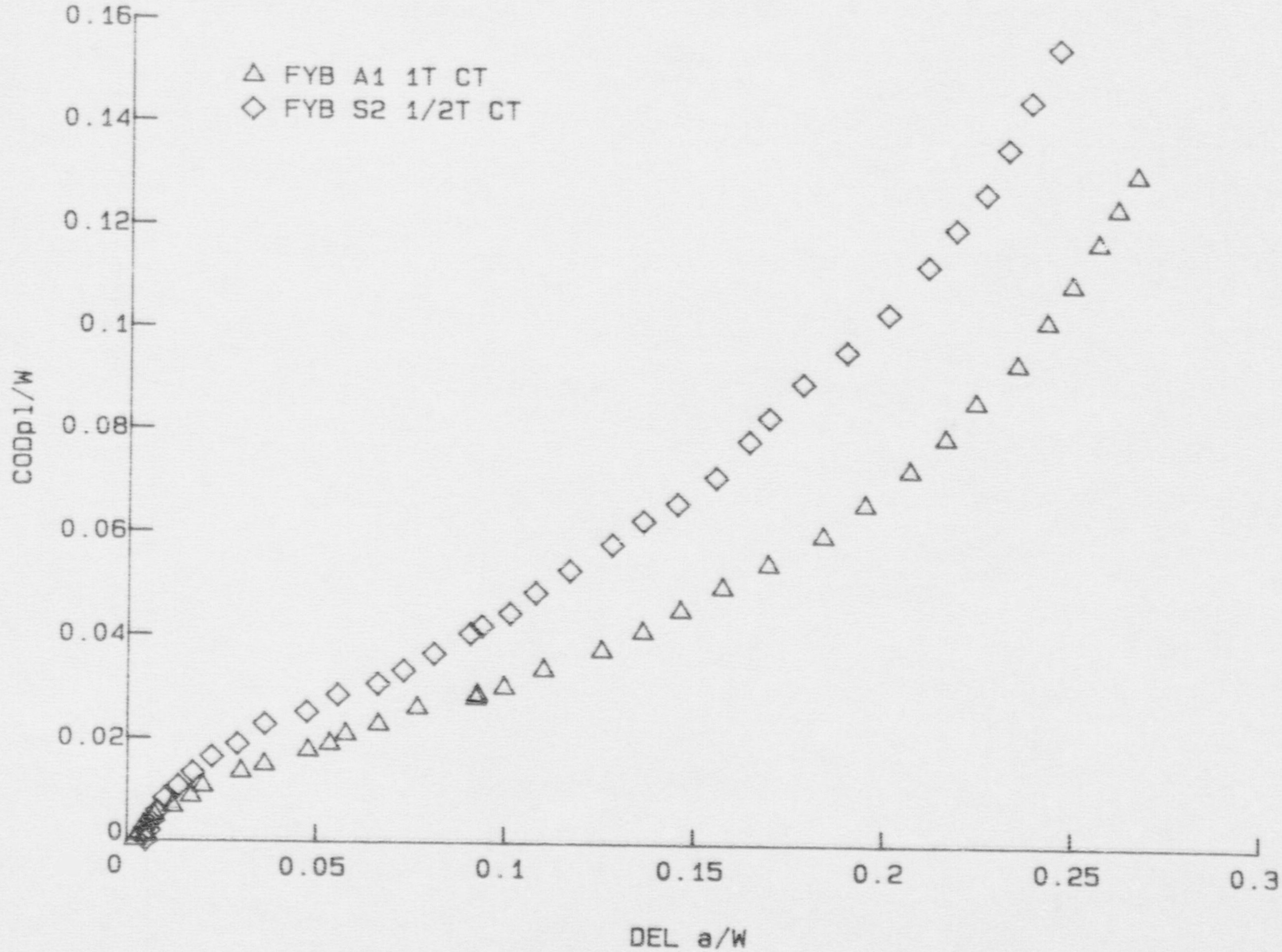

Figure 12. The Effect of Specimen Size on the Plot of δ_{pl}/W Versus Crack Extension.

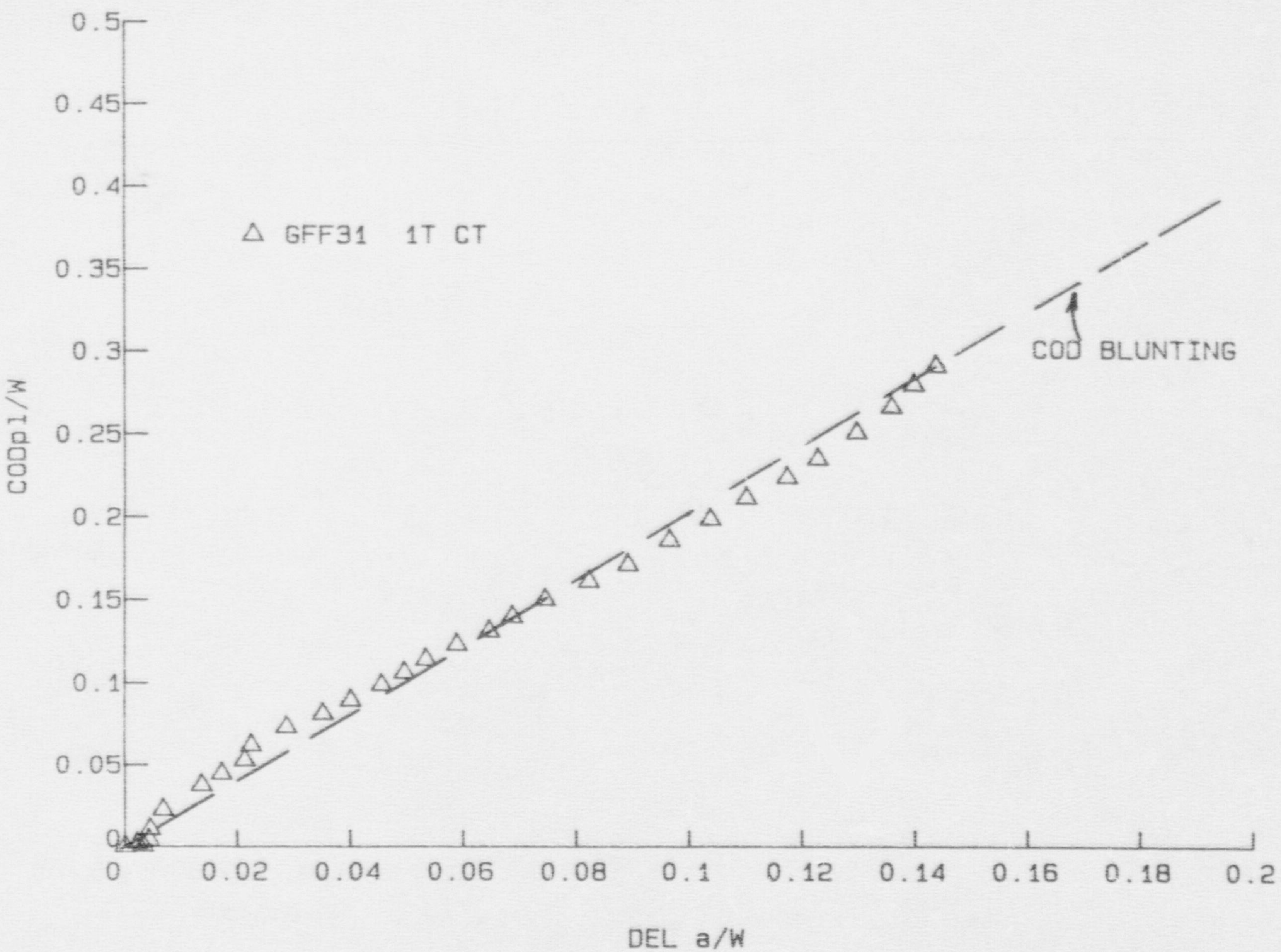

Figure 13. Blunting Only Behavior as Demonstrated by the High Toughness A710 Alloy.

region of singularity controlled, crack growth, at least for this specimen size. Larger values of slope are probably present for other geometries. In any case, the slope of two corresponds to a specimen in which large amounts of plasticity are present and little fracture is occurring.

Figure 14 shows the same type of plot for the high toughness A710 material including results for three scaled compact specimen sizes. The resulting curves for the $\frac{1}{2}$T and 1T scale specimens are now coincident and correspond well throughout their length with the initial blunting behavior as defined by Eq. (14). The 2T compact shows a region of separation from the initial blunting slope but it is not very distinct or very extensive for this alloy.

While the 3% Nickel alloy of Fig. 11 clearly defines a singularity controlled region, the A710 alloy seems to show only notch or COD blunting. The 3% Nickel alloy starts with some blunting, develops a singularity, and then the singularity dissipates as the crack again returns to blunting near the specimen back wall. These changes cannot be seen in terms of load dominated plots like the J–R curve or the key curve normalized load plots, but they are defined clearly by a plot like that of Figs. 11 or 12 and also by the J_M–R curve, which is sensitive to this final loss of singularity. Figures 13 and 14 do not show these distinct regions for the HSLA alloy.

Figure 15 shows the corresponding plot for three 1T specimens with different crack lengths for the A533B –H13 alloy. This figure demonstrates the increased constraint demonstrated by this alloy for crack lengths near the center of the specimen ligament. The two specimens with a/W values of 0.55 and 0.61 show singularity behavior while the specimen with a/W in excess of 0.75 shows only blunting behavior, apparently feeling the proximity of the specimen back wall.

3.1 <u>Development of a Single Analytical Mode</u>

The incremental equations used to evaluate J from experimental data presented in the earlier section as Eqs. (6 to 10) are very useful but they do not give much insight into the effects of the various quantities which contribute to the final J value. An analytical model based on simpler relationships which can be demonstrated experimentally can give some of this insight. For example, if a load displacement relationship of the form

$$\frac{PW}{B\,b^2} = Fl = k(\delta_{pl}/W)^n \tag{15}$$

accurately represents the experimental observations for a series of specimens as described above, J can be evaluated from Eq. (2) as:

$$J = J_{EL} + \int_0^{\delta_{pl}} \frac{2\,b\,k}{W}\left[\frac{\delta_{pl}}{W}\right]^n d\delta_{pl} - \int_{a_o}^{a} \frac{2\,k}{n+1}\left[\frac{\delta_{pl}}{W}\right]^{n+1} da \tag{16}$$

If only small crack extension was present the second integral can be ignored and J could be evaluated in terms of the initial uncracked ligament b_o as:

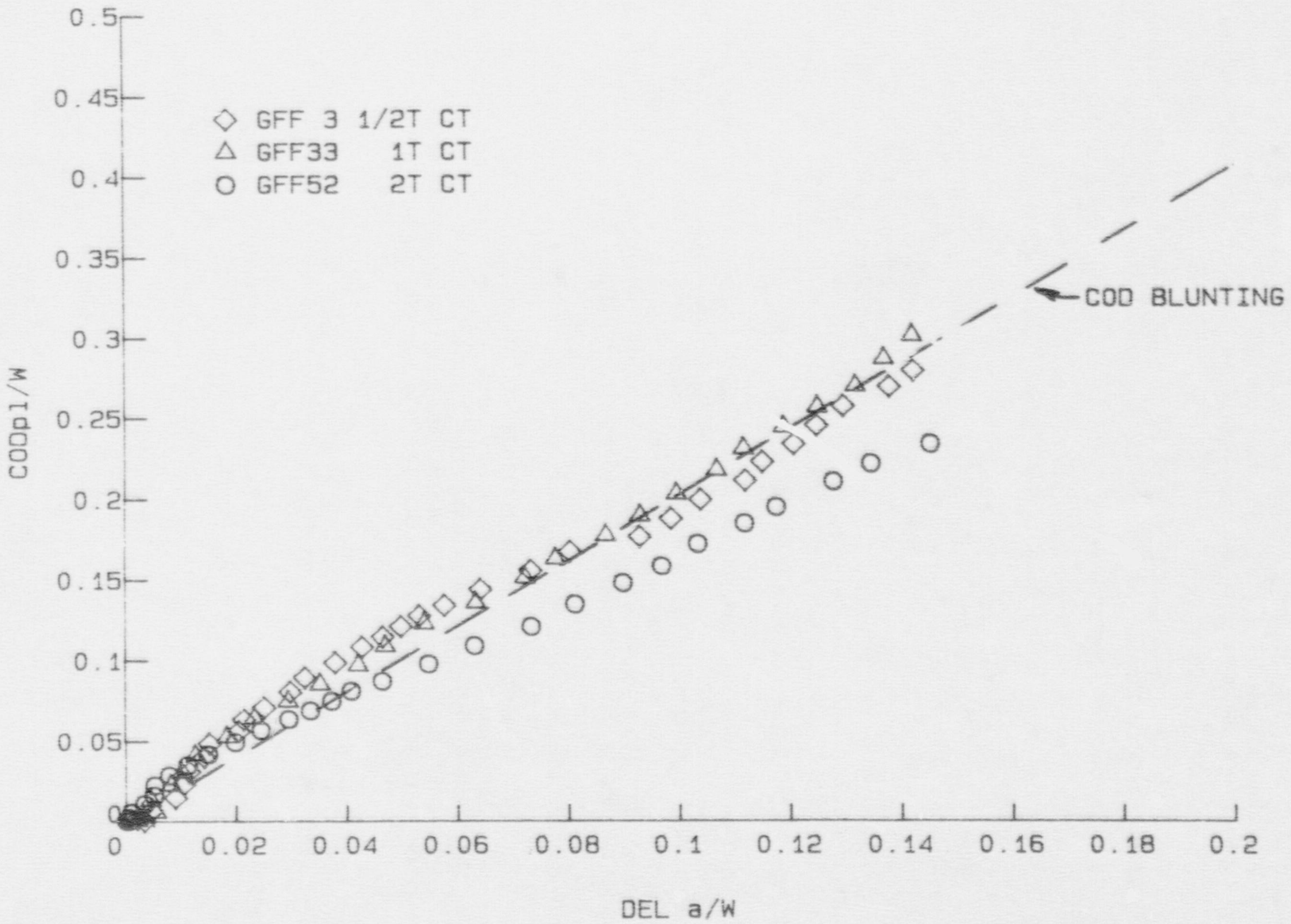

Figure 14. The Effect of Specimen Size on the Plot of δ_{pl}/W Versus Crack Extension for the High Toughness Alloy.

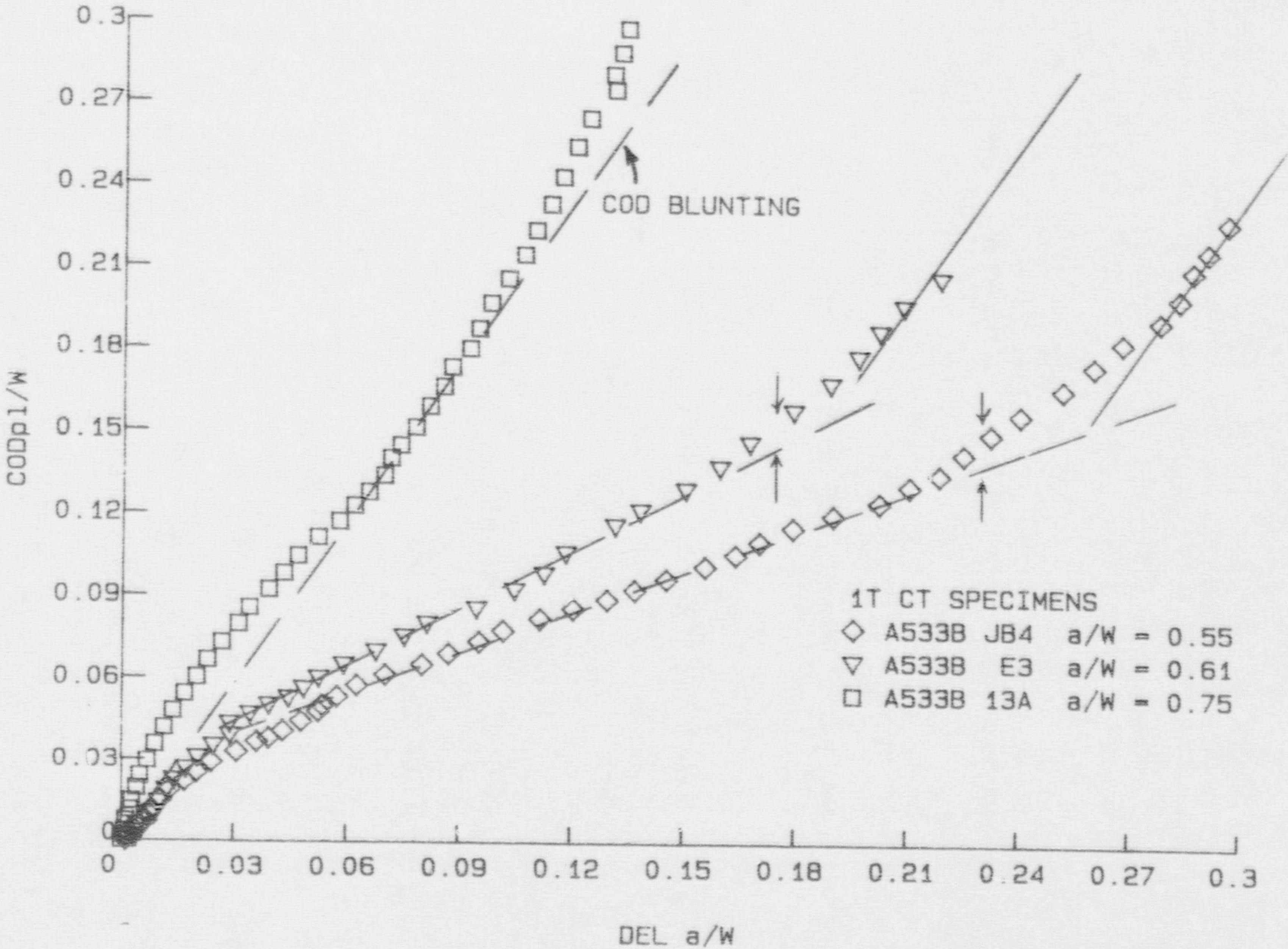

Figure 15. The Specimen Crack Length is Shown to Have a Strong Effect on the Singularity Limit.

$$J = J_{EL} + \frac{2b_o k}{n+1}\left[\frac{\delta_{pl}}{W}\right]^{n+1} \qquad (17)$$

For the case of interest here, however, large crack extensions are involved and the a and b dependent terms of Eq. (16) must be kept. To proceed, then, requires a relationship between $\frac{\delta_{pl}}{W}$ and $\frac{\Delta a}{W}$, i.e.

$$\frac{\Delta a}{W} = F\left[\frac{\delta_{pl}}{W}\right] \qquad (18)$$

Using the trends shown in Figs. 11–15, a linear relationship is taken here of the form

$$\frac{\Delta a}{W} = k_2 \frac{\delta_{pl}}{W} + k_1 \qquad (19)$$

where k_2 and k_1 are fitting coefficients. By noticing that

$$\frac{b}{W} = \frac{b_o}{W} - \frac{\Delta a}{W} = \frac{b_o}{W} - k_2 \frac{\delta_{pl}}{W} - k_1 \qquad (20)$$

and substituting E. (20) in Eq. (16) gives, after some manipulation, that:

$$J = J_{EL} + 2k\left[\frac{b_o}{W} - k_1\right] \int_o^{\delta_{pl}} \left[\frac{\delta_{pl}}{W}\right]^n d\delta_{pl}$$

$$- 2kk_2\left[\frac{n+2}{n+1}\right] \int_o^{\delta_{pl}} \left[\frac{\delta_{pl}}{W}\right]^{n+1} d\delta_{pl} \qquad (21)$$

Carrying out these integrations gives

$$\frac{J}{W} = \frac{J_{EL}}{W} + \frac{2k}{n+1}\left[\frac{\delta_{pl}}{W}\right]^{n+1} \left[\left[\frac{b_o}{W} - k_1\right] - k_2\left[\frac{\delta_{pl}}{W}\right]\right]$$

or

$$\frac{J}{W} = \frac{J_{EL}}{W} + \frac{2k}{n+1}\left[\frac{\delta_{pl}}{W}\right]^{n+1} \left[\frac{b}{W}\right] . \qquad (22)$$

Re-substituting Eq. (19) gives a J-R curve form that

$$J = J_{EL} + \frac{2kW}{n+1}\left[\frac{b_o}{W} - \frac{\Delta a}{W}\right]\left[\frac{\Delta a}{k_2 W} - \frac{k_1}{k_2}\right]^{n+1} \qquad (23)$$

which shows explicitly the crack extension dependence of the model. Because of the W in the $2kW/(n+1)$ coefficient in Eq. (23), a size independent J–R curve will result only if k_2 and/or k_1 are W dependent. If k_1 and k_2 are W independent J/W should plot consistently versus $\Delta a/W$ for specimens of different scales, at least if the b_o/W values are identical.

For small crack extensions Eq. (23) gives a power law shaped J–R curve, but when the crack length changes dramatically, J reaches a maximum and the J–R curve subsequently declines.

Repeating the above analysis for J_M gives:

$$\frac{J_M}{W} = \frac{J_{EL}}{W} + 2k\left[\frac{\delta_{pl}}{W}\right]^{n+1} \left[\left[\frac{b_o}{W} - k_1\right] \frac{1}{n+1} - \frac{k_2}{n+2} \left[\frac{\delta_{pl}}{W}\right]\right]$$

$$= \frac{J_{EL}}{W} + \frac{2k}{n+1} \left[\frac{\delta_{pl}}{W}\right]^{n+1} \left[\frac{b}{W} + \frac{\Delta a}{(n+2)W} - \frac{k_1}{n+2}\right] \tag{24}$$

Equation 24 shows that J_M is elevated over J of Eq. (22) by the presence of the additional terms in the bracket, nonetheless for large Δa, this function will ultimately also reach a maximum J_M value and start to decrease.

3.2 Evaluation of the Model Equations

Using standard fitting procedures the coefficients k, n, k_1 and k_2 can be evaluated for the specimens described in the previous sections. These coefficients are tabulated in Table 3. Points to notice from this Table are the following:

1) The coefficients k and n are consistent between large and small size specimens of all materials.

2) The coefficients k_1 and k_2 are specimen size dependent for lower toughness materials and become less size dependent for higher toughness specimens.

Note that Eq. (23) states that, if all fitting coefficients are the same for two sizes of geometrically scaled specimens, J/W versus $\Delta a/W$ will be identical for the two specimen sizes, while J versus Δa will not. Size dependent k_1 and k_2 coefficients must be present for a singularity based J–R curve, i.e. a size independent J–Δa function to exist.

Figures 16 and 17 show the calculated results from the model Eqns. (23–24) in comparison with the results of the experimental J and J_M equations.

The model curve fits the unloading compliance data in the region where Eq. 19 fits the data of Fig. 11, i.e., in the singularity zone. If Eq. 19 is used beyond the singularity limit the model J resistance curve deviates distinctly from the unloading compliance results, reaches a maximum J value and then falls. Which of these J resistance curves is correct is not presently known and it is felt that only data in the singularity zone should be used at this time.

Similar correspondence between the model and the unloading compliance data is found if J_M is used as shown on Fig. 17. Again, when Eq. 19 is used beyond the singularity limit the model resistance curve deviates distinctly from the unloading compliance result. The unloading compliance data is curving up because of a gradual loss of singularity conditions at the crack tip while the model curves down, probably representing a limit to the specimen capacity.

4.0 DISCUSSION OF RESULTS

It seems logical that singularity controlled crack growth conditions would be the most intense crack growth conditions felt by an elastic–plastic cracked body. This argument is the principal reason the central portion of Fig. 11 is labeled as the singularity controlled zone. Beyond this zone it is felt the singularity conditions are weakening and less intense crack growth is occurring, i.e. the crack growth is returning to a plasticity controlled "blunting" behavior. If this occurs, the amount of driving force per unit crack extension should be elevated, and this corresponds to a tendency for the J–R curves to rise. This effect is demonstrated first in the J_M–R curve by the development of a distinct upward curvature, but it is present also in the deformation J–R curve which artificially continues to rise even when it should reach a maximum and start to fall, if standard incremental equations are used beyond the singularity zone. This could be looked upon as a fortuitous result since it adds to the effective capacity of small specimens – but it is also possibly dangerous if it leads to an application of results without a true understanding of what is occurring.

The model used in the previous analysis shows, that if one fits the center region of the $\Delta a/W$ versus δ_{pl}/W record with a straight line function, the upward curvature of the J_M and J resistance curves is removed. The model, then, demonstrates what the J resistance curves should be if singularity conditions were in some way maintained – possibly by the use of a larger specimen. Elevation of the experimental curves above the model predictions should be treated with suspicion.

These calculated curves are probably more accurate representations of the J–R curve than that which is obtained from applying standard incremental equations to data which exhibits the apparent reestablishment of crack tip blunting, generating artifically elevated J–R curves.

5.0 CONCLUSIONS

Engineering limits to the applicability of J based resistance curves are not experimentally determinable in terms of load dominated quantities like key curve plots or J resistance curves. Real limits to the applicability of the J integral are much more apparent in a plot of crack extension versus the plastic component of crack mouth opening displacement or specimen bend angle. These plots show a region of initial crack blunting, a region of singularity controlled crack growth, and finally a gradual return to crack blunting. The delineations between these zones are much more apparent on a plot of Δa versus δ_{pl} than on a standard deformation J resistance curve.

Table 3 Model Fitting Coefficients

Specimen	Size	k	n	k_1	k_2
3% Nickel					
FYB A1	1T CT	55870.	0.064	0.005	0.263
FYB A2	1T CT	55070.	0.061	0.002	0.255
FYB S1	1/2T CT	52860.	0.056	0.010	0.328
FYB S2	1/2T CT	53180.	0.059	0.003	0.423
A710					
GFF 3	1/2T CT	54040.	0.107	0.013	1.877
GFF 4	1/2T CT	54630.	0.107	0.016	1.897
GFF 6	1/2T CT	58060.	0.090	-0.02	1.876
GFF 30	1T CT	46800.	0.103	0.017	1.965
GFF 31	1T CT	49670.	0.110	0.007	1.889
GFF 32	1T CT	49516.	0.103	0.012	2.079
GFF 33	1T CT	47420.	0.103	0.006	2.014
GFF 34	1T CT	48317.	0.108	0.005	1.579
GFF 50	2T CT	44450.	0.097	-.003	1.570
GFF 51	2T CT	43680.	0.104	0.001	1.705
GFF 52	2T CT	45160.	0.106	0.008	1.579
A533B-H13					
JB4	1T CT	37800.	0.0798	0.027	0.486
E3	1T CT	38270.	0.0951	0.023	0.697
13A	1T CT	35890.	0.0922	0.039	1.36

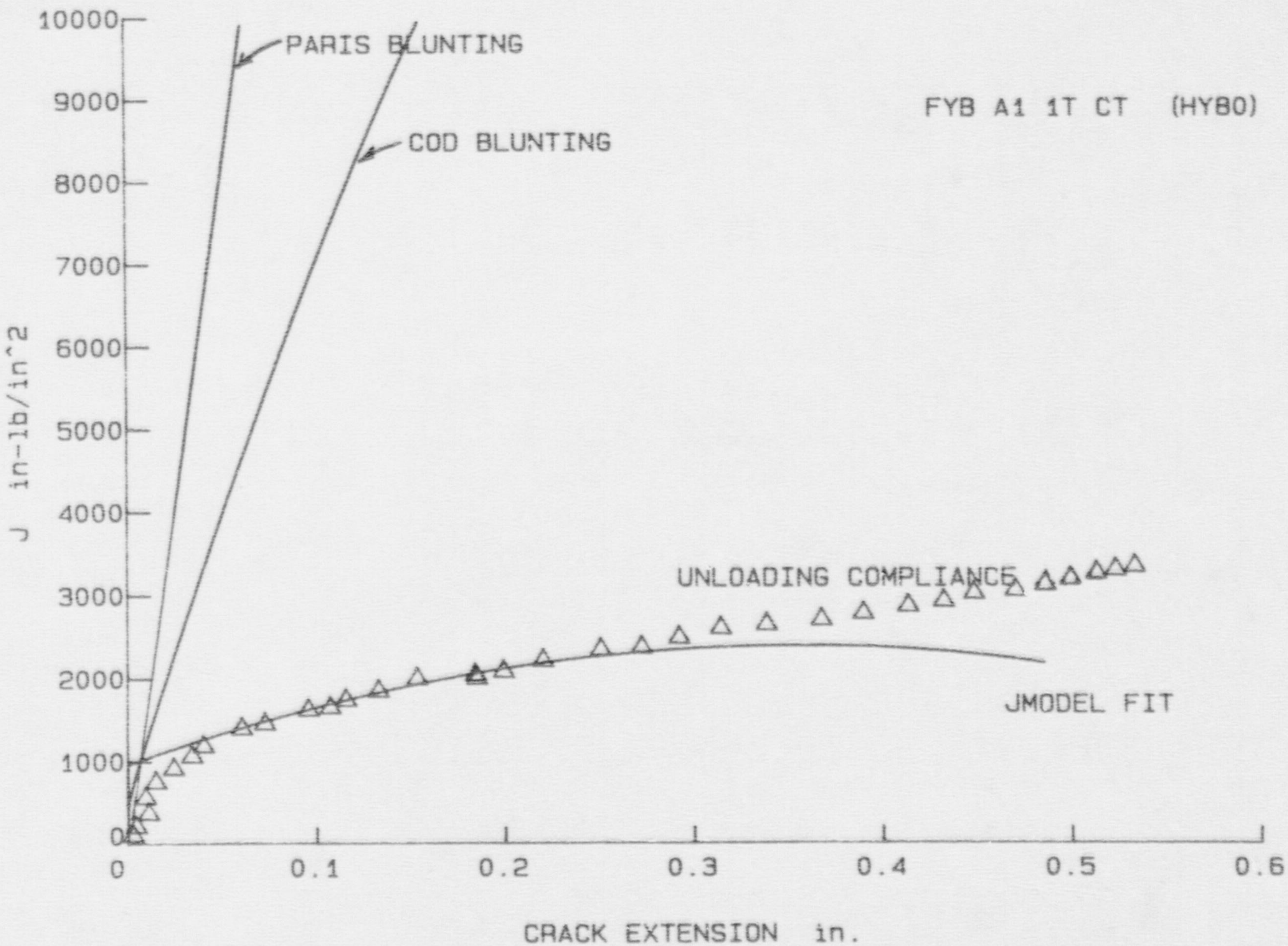

Figure 16. The Simple J Analysis Model Shows that Elevation in J Beyond the Singularity Limit is Due to the Weakening Crack Growth Conditions.

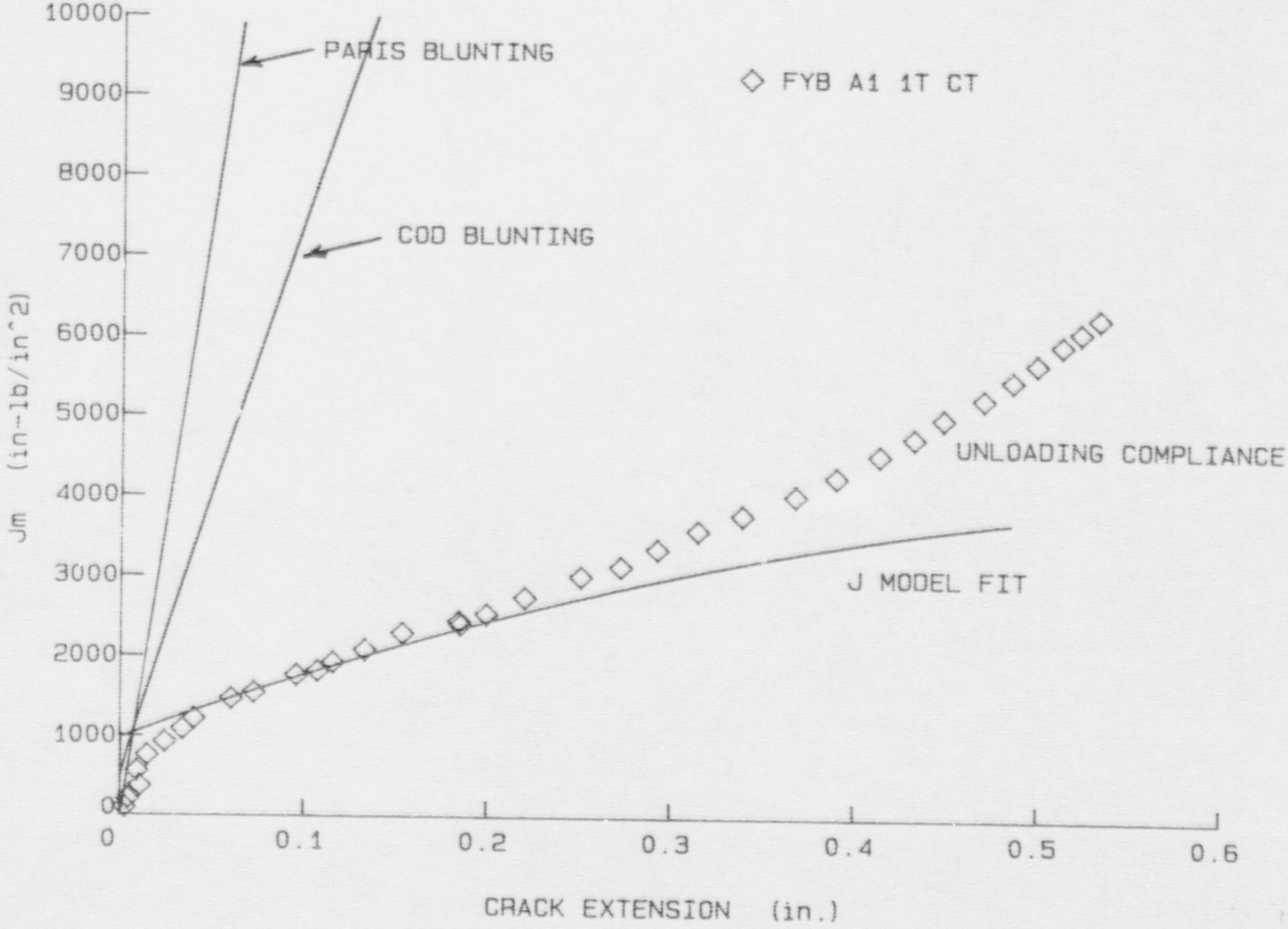

Figure 17. The Model in Terms of Modified J Show More Distinctly the Effect of the Loss of Singularity.

While normalized load displacement plots and J–R curves are generally insensitive to specimen scales, a plot of $\Delta a/W$ versus σ_{pl}/W is strongly scale dependent. While the insensitivity of the former plots is valuable for material characterization and design, the sensitivity of the latter plot should be a valuable aid to the study of the effects of specimen size and constraint, allowing development of methodologies of matching or overmatching constraint to assure that conservative laboratory tests are indeed conducted. It also seems possible that k_1 and k_2 fitting coefficients can be evaluated and related to specimen size so that extrapolations to larger size specimens can be developed in terms of the truly size dependent variables in the fracture process.

The specimen size dependence demonstrated by the modified J of Ernst appears to be a clear indicator of a loss of singularity in these specimens reduced on deformation J resistance curves and can lead to the use of the data outside a region of singularity controlled crack growth. This is now an extrapolation and should be accepted as such and used only with the utmost caution. Other methods of J resistance curve extrapolation based only on data in the singularity controlled region are presently the object of research.

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An experimental definition is proposed for the extent of singularity controlled behavior in a J-resistance fracture test. The singularity zone is defined in terms of a constant ratio of plastic crack opening displacement and normalized crack extension. Justification for this definition is given in terms of experimental results on compact specimens of three steel alloys of varying material toughnesses and in terms of a simple analytical model.

The experimental limit can be evaluated from the data normally obtained during an unloading compliance single specimen J integral resistance curve experiment. Generally the experimental singularity limit extends the region of test validity well beyond that which is presently allowed by the ASTM J-R test standard E1152.

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