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Detecting Component Failure Potential Using Proportional Hazard Model

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Abstract

The proportional hazard model (Cox, 1972) provides a framework for incorporating measurable parameters (covariates) into the description of failure rates. In this report a methodology is presented for calculating the failure probability based on the parameters measured at a routine surveillance test of the component. These parameters are expected to vary in an identifiable way before a failure indicating an impending failure, and are termed symptomatic emitters. A decision framework for a repair/replacement policy based on such information is presented.

The use of symptomatic emitter information in determining a component failure potential has great utility, particularly in highly reliable systems like those in nuclear power plants. Further work on application of this methodology is recommended. If actual data are not readily available, simulated data can be developed using the simulation approach presented in this report.

EXECUTIVE SUMMARY

The operability of standby safety system components is assured through periodic testing of components by detecting any failures that may have occurred during the standby time period. However, in many cases, a test may not detect a failure but may detect certain symptoms indicating a degraded condition of the components. One useful approach to prevent component failure is to identify the failure potential of the component using the identifiable symptoms that are exhibited before a failure. In this report, a methodology to develop a repair/replacement strategy is presented incorporating information on the parameters (symptoms exhibited by a component) observed during a surveillance test.

The methodology is based on proportional hazards models (developed by D. Cox, 1972) which express the failure rate as a function of the variables that are identified to provide symptoms of component failure. As adapted for component failure analysis, this formulation assumes that a component failure rate at any time will be the basic failure rate, if the variables being measured on the component remain at their normal operating values. However, the fluctuations on those variables will influence the failure rate through a linear function multiplicative of the basic failure rate. This ability to express the failure rate as a function of measurable variables in the component allows one to calculate the higher failure rate assumed to be exhibited by the component before a failure through changes in the parameters observed during a surveillance test. The calculated higher failure rate is used to develop a remedial maintenance/extensive repair or replacement strategy. The constraint in developing such a strategy was to prevent unnecessary replacement or extensive repairs.

The methodology presented in this report has a strong appeal in component failure analysis since it holds promise in preventing failures of risk-important components in nuclear power plants. At the same time, the underlying assumption in the approach that the inherent failure rate of a component will show significant departure before a failure and that such departures can be identified using the measurable parameters collected during the test of the component has not been validated by actual failure data analyses. Recognizing the potential utility of such an approach in analyzing the information obtained in routine component testing, a study on the application of the methodology to nuclear safety system components is recommended.

CONTENTS

1. INTRODUCTION

The standby safety system components play a very important role in case of an abnormal event in a nuclear power plant. The basic role of these components is to prevent a routine abnormal event from transforming into a serious safety significant one. A failure in one or more of these important components may result in such situations. In the operation of nuclear power plants, measures are taken in terms of test and maintenance of these components to assure their operability. Nevertheless, these components are subject to failure. The probabilistic risk assessment of nuclear power plants calculates the frequency of expected accidents in the plant, and such analyses develop the failure rates of those components which contribute to the accident frequency. The failure rates are developed from the failures of the components observed during testing and actual demands. The objective of this report is to present a methodology that can predict when a failure of a component is imminent. The ability can then be used to develop a repair/replace policy to forestall the failure and thus prevent the chance of any serious consequence.

The basic notions of this study are as follows:

a) Components exhibit some identifiable symptoms before they actually fail. That is, there are observables in a component that can be analyzed to determine the failure proneness.

b) The state of a component before a failure can be represented by a higher failure rate and this failure rate can be determined based on the symptoms exhibited by the component.

The methodology presented expresses failure times as a function of certain variables. These variables are, in general, measures of a component's ability to operate and changes in these variables are considered to be symptoms of failure. Modern interest in expressing failure rate as a function of explanatory variables (as termed in regression analysis) was developed by D. Cox (1972), and since then significant studies have been conducted in survival analyses. The Cox formulation as adapted for component failure analysis assumes that the component failure rate at any time will be the basic failure rate, if the variables being measured on the component remain at their normal operating value. However, the fluctuations on these variables will influence the failure rate through a linear function multiplicative of the basic failure rate. This ability to express the failure rate as a function of measurable variables in a component will allow one to calculate the higher failure rate expected to be exhibited by the component before a failure. The use of the Cox model in developing such a model is presented in Chapter 2.

The components for which failure proneness is analyzed are standby safety system components inspected or tested at regular intervals. The explanatory variables or the symptoms of failures are expected to be observed or measured at each of the tests. The requirement imposed by such an approach is to collect the explanatory variables every time the component is tested. Currently, such a data base is not available but can be developed from plant records. Another alternative at this stage of model development is to simulate data to test the applicability of such an approach. Chapter 3 presents a simulation procedure for analyzing component failures with symptomatic emitter information.

For this approach to be practical and implementable, the failure probability of the component calculated, based on the explanatory variables should reliably change when the component is truly changing for a degraded situation signaling an impending failure and should not deviate significantly if the component performance has not changed. Chapter 4 presents a decision framework for implementing a repair/replace policy based on the failure probability calculated at each test. Obviously, the decision rule must be used as an alarm to decide the repair necessary. The repair may be performed such that the explanatory variables are brought back to the normal level. Besides forestalling an impending failure, this may serve another purpose. At any time during plant operation, the plants may maintain the list of important components that are exhibiting higher probability of failures. In case of an accident, the operators can monitor these components more closely to take corrective actions in case the component actually fails.

2. USE OF PROPORTIONAL HAZARD MODELS AND COX-STYLE FITTING IN DETECTING COMPONENT FAILURE POTENTIAL

In this section a brief overview is presented of proportional hazard models and how they can be used to detect the failure potential of a component.

A component in a nuclear power plant is subject to failure at any time. The failure of a standby component to perform its function during an accident can have serious consequences, and accordingly the standby components are tested at regular intervals to detect any failures that may have occurred. However, in many instances, a component emits symptoms of failures that can be detected during the test before being in a failed state. The objective here is to use these symptoms to prevent an outright failure of the component. The proportional hazard model is presented for incorporating the failure symptoms into the description of failure rates.

2.1 Problem Setting

In performing surveillance testing on risk important standby components, the operability is assured through observation of component functional parameters. The observations can be divided into two categories:

a. direct observations are made periodically on operability; e.g., a component is inspected and tested once a month for operability, and

b. associated measurements of system environmental or related parameters are also made at the same intervals or else possibly evidence their changes on instrument readings or otherwise indirectly.

The idea of (a) is to verify that the system is currently capable of performing properly, while concomitant measures (b) may be organized to provide warning that the desirably low failure probability associated with (a) is about to change unless something is done.

Let us consider the monthly surveillance testing of an emergency core cooling pump. The pump is tested for its operability and the direct observation on operability will be, for example, the flow rate developed by the pump and the time required to develop the required flow rate. These measurements fall under (a) and can be considered direct observations. If actual values of these parameters are maintained, they can also be used as indicators of future performance. In addition to these observations, one can also measure other related parameters. Examples of such parameters are the vibration of the pump, the pump motor current, any leakage, or accumulation of any particles, corrosion, etc. These measurements fall under Category (b) and can provide indication of an impending failure when analyzed adequately.

Therefore, at each surveillance testing, the following information is obtained:

a. the actual capability of the item, i.e., whether it can indeed carry out its mission, if required, and

b. the values or states of certain indicators that can influence the operability of the component. In this report, these indicators are termed symptomatic indicators.

The symptomatic indicators provide symptoms of an impending failure and, from a component failure point of view, are those variables that influence the failure rate of a component. In proportional hazard models such parameters are termed covariates or explanatory variables. In this report, the terms symptomatic emitters, covariates, and explanatory variables are synonymous and are used interchangeably.

The objective of using the proportional hazard model incorporating the symptomatic emitter information is primarily to develop a repair/replacement policy so that an actual failure may be avoided. In achieving that overall goal, the additional information necessary to make the judgment can be summarized as follows:

a. Suitable measurements of symptomatic emitters, which may be combinations of direct measurements and associated measurements, should be identified. These symptomatic emitters can then be systematically combined to provide a numerical assessment of the failure proneness of the system during a subsequent period. The intent here is to find a set of explanatory variables or symptomatic emitters that will, in some combination, effectively and reliably represent the index of failure proneness when the component is truly changing for the worse but will not change much when all is as it should be.

b. Increase in the failure probability of the component should be measured incorporating the symptomatic emitter information to develop the repair/replace policy of the component. Expectedly, there will be fluctuations in the symptomatic emitter information and these will induce some change in any formula-calculated probability of failure or survival from period to period even though nothing has really changed. Presumably, a considerable increase in the failure probability signals intervention. A decision framework for component repair/replace policy using changes in failure probability is presented in Chapter 4.

2.2 The Proportional Hazards (Cox) Model for Symptoms of Failure

In an important paper, David Cox (1972) systematically studied a regression model for incorporating explanatory variables into the description of failure rates. His particular representation for the failure rate (in continuous time) was

$$\lambda(t;\underline{z}) = \lambda_o(t)\exp(\underline{z}\ \underline{\beta}) \quad , \qquad (2.1)$$

$\lambda(t;\underline{z})dt$ is the probability of failure of an item that has lived for time t and exhibits the characteristics described by z_k, the kth component of the explanatory variable profile or vector $\underline{z} = (z_1, z_2, \ldots, z_k, \ldots z_p)$. $\lambda_o(t)$ is an arbitrary non-negative function of t representing the unspecified baseline hazard function. The unknown parameters $\underline{\beta}$ describe the influence of each such variable upon the instantaneous hazard, $\lambda(t;\underline{z})$; furthermore, Cox states that a given, known function of t can be incorporated into the explanatory variables, e.g., $z_2 = tz$. Other indicators which remain fixed throughout the analysis, possibly identifying manufacturer or operating company or location in the case of equipment (nuclear) reliability, can be used.

Conditional Likelihood Estimates of Regression Coefficients

A unique contribution of the Cox (1972) paper was to estimate the regression parameters , $\underline{\beta}$, from failure time data taken in continuous or discrete sampling time without assuming a specific form for $\lambda_o(t)$. Here is the basic idea. Suppose there are J items of age t susceptible to failure, and a particular one fails; mark his/her distinguishing explanatory z-vector by a prime ($\underline{z}'$). Now the probability that one of the J fails in (t, t+dt) is assuming independence of their failures and the model (2.1),

$$\sum_{\ell \varepsilon \underline{J}} \lambda_o(t)\exp(\underline{z}(\ell)\underline{\beta})dt \quad . \tag{2.2}$$

Here we have used $\underline{J}$ to denote the set, the J items susceptible at t. The probability that the item with description $\underline{z}'$ actually failed is

$$\lambda_o(t)\exp(\underline{z}'\,\underline{\beta})dt \quad . \tag{2.3}$$

Hence the conditional probability that the item with $\underline{z}'$ actually failed is the ratio of (2.3) to (2.2) or

$$\frac{\exp(\underline{z}'\underline{\beta})dt}{\sum_{\ell \varepsilon \underline{J}(t)} \exp(\underline{z}(\ell)\underline{\beta}}\quad , \tag{2.4}$$

since the $\lambda_o(t)$-term factors out. This term is now viewed as a contribution to a likelihood for $\underline{\beta}$. The methodology discussed by Cox addresses the general situation in which any known number of items are at risk at the time, $t_{(i)}$, of the ith failure to occur. In his notation, and ours, at time $(t_{(i)}$, $\underline{z}' = \underline{z}(i)$, the description of the item actually failing at $t_{(i)}$. Consequently, the conditional likelihood becomes

$$L(\underline{\beta}) = \prod_{i=1}^{k} \frac{\exp(\underline{z}(i)\underline{\beta})}{\sum_{\ell \varepsilon \underline{J}(t(i))} \exp(\underline{z}(\ell)\underline{\beta}} \quad . \tag{2.5}$$

One can next take logarithms and proceed to get reasonable estimates, at least asymptotically, i.e., after many failures have occurred.

2.3 Application of the Cox Model: Discrete Time Adaptation Using the Logistic Representation

Suppose a system is examined at intervals T time units apart; let T = 1 for convenience for the moment. Think of the explanatory profile $\underline{z}(t)$ observed at t as influencing the probability of failure in the interval t: namely the times from (t, t+1). In reliability applications $z_1,\ldots,z_r$ may be fixed factors identifying the manufacturer, location of the system, etc., while the remaining factors vary with time: $z_{r+1}(t)$, $z_{r+2}(t),\ldots z_p(t)$ are quantities that change from period to period; for instance current levels of vibration, oil leakage, and temperature are candidates to explain failure propensity for pumps.

Because of the discrete time sampling, it is convenient, even necessary, to consider the logistic model for $p(t,\underline{z}(t))$, the probability of failure during interval t, given the explanatory variable values relevant to that interval, i.e., $\underline{z}(t)$. That is, represent model odds for a failure at t as follows:

$$\left(\frac{p(t,\underline{z})}{1-p(t,\underline{z})}\right) = \exp(\underline{z}\ \underline{\beta}) \cdot \frac{p_o(t)}{1-p_o(t)} \quad , \tag{2.6}$$

or, calling the log-odds the logit, as is usual,

$$\text{logit } p(t,z) = \underline{z}\ \underline{\beta} + \gamma_o(t) \quad . \tag{2.7}$$

Note that if $p(t,\underline{z}) = \lambda(t,\underline{z})dt$ and $p_o(t)=\lambda_o dt$, then as $dt\to 0$ the discrete time model and the continuous time model approach each other. The above implies that the probability of failure is:

$$p(t,\underline{z}) = \frac{e^{\underline{z}\ \underline{\beta}+\gamma_o(t)}}{1+e^{\underline{z}\ \underline{\beta}+\gamma_o(t)}} \tag{2.8}$$

The primary objective is to estimate the coefficients $\underline{\beta}$ at this stage.

Conditional Likelihood Estimates; Single Component

The Cox method for estimating $\underline{\beta}$ appears to be best adapted to situations for which many nearly identical units journey through time experiencing about the same initial and environmental conditions. Such may not often be a natural situation in reliability applications.

Under some circumstances it may be necessary to estimate $\underline{\beta}$ using Cox conditional likelihood arguments when a single system is observed. To do so, think of the successive times to failure for the systems in question as representing independent and identically distributed sample histories or realizations of a group of identical systems, all started simultaneously; this is, of course, immediately questionable if any kind of reliability growth occurs between successive failures. Nevertheless, proceed as follows by analogy to the earlier arguments. Suppose first that ties are unlikely, and can be ignored. Then if a (the ith) failure occurs in time (actually, age) interval (t_i, t_i+1) the conditional probability that it is in the unit observed that has description $\underline{z}'$ is proportional to

$$\frac{\exp(\underline{z}'\underline{\beta})}{\sum_{\ell \epsilon \underline{T}(t)} \exp(\underline{z}(\ell)\underline{\beta})} , \tag{2.9}$$

which again eliminates the time-dependent factor $\gamma_o(t)$ in (2.8). If ties are present, i.e., if the system has failed two or more times in the interval (t_i, t_i-1), a form similar to (2.9) can be derived by thinking of the multinomial sampling features of the situation; details omitted for now.

Conditional Likelihood Estimates; Several Component Copies

Suppose there are a number of copies of the same component design in use in various locations. In other words, $S_1, S_2, \ldots, S_n$ denotes these component copies, and their failure rates are influenced by variables $\underline{z}_1, \underline{z}_2, \ldots \underline{z}_i, \ldots \underline{z}_n$. The discrete time-sample proportional hazards model would attempt to describe failure probability for each copy by use of model (2.8):

$$p_j(t,\underline{z}) = \frac{\exp[\underline{z}_j\underline{\beta}) + \gamma_o(t)]}{1 + \exp[(\underline{z}_j\underline{\beta}) + \gamma_o(t)]} , \tag{2.10}$$

i.e., the same regression coefficients hold for all copies, meaning that the explanatory variables $\underline{z}_j$ exert the same leverage upon probability of failure for each copy.

In order to explain or represent the extra variability in failure rate or probability not explained by known explanatory variables $\underline{z}$, one can consider using an empirical Bayes version of (2.8). For this, one assumes, for example, that

$$p_j(t,\underline{z}) = \frac{\exp[(\underline{z}_j\underline{\beta}) + \varepsilon_j]}{1 + \exp[(\underline{z}_j\underline{\beta}) + \varepsilon_j]} , \tag{2.11}$$

where ε_j is a realization of a random variable (or sample) from a distribution. The assumption that ε_j comes from a Normal distribution with mean μ and variance σ^2 can be entertained. The problem then is to use the data to 1) estimate μ and σ^2, and 2) to use the estimates to improve the estimate of the individual component. This can possibly be checked if enough data are available -- then the Cox conditional likelihood method can be applied to estimate $\underline{\beta}$. The idea is to use (2.5) as the likelihood, and to maximize it.

The above discussion suggests that the semi-nonparametric feature of the Cox conditional likelihood fitting procedure may not be necessary for current purposes, e.g., to predict what may occur during the next time (sampling) period, one month, for example. Rather, one might better simply assume $\lambda_o(t) = \lambda_o$ a constant, to be incorporated into $\underline{z}\ \underline{\beta}$ in (2.8), and that the logistic model could then be fit into $p_j(t,\underline{z})$ probabilities. Time-dependent information on the covariates can also be treated within this approach. Appendix A provides a discussion on the extended Cox model for time-dependent covariates following the approach of Prentice and Gloeckler (1978).

3. SIMULATION OF FAILURES AND RELATED SYMPTOMATIC EMITTERS

It is recognized that data required to analyze the effectiveness of a Cox-type proportional hazard model are not easily available. At the same time, before one gets into such data collection, which may be expensive, the efficacy of such an approach should be carefully studied to determine the conditions or situations under which such an approach will produce meaningful practical results. Simulated data can be very effective in such circumstances, and in this section the process of simulating failures along with symptomatic emitter is presented.

3.1 The Idea of Symptomatic Emitters and Some Notation

A component or subsystem is inspected at regular intervals, T, and its state recorded. The state is taken to be $\underline{X}_{n+1}$ at the time of the (n+1)th inspection following a failure, and

$$X_{n+1} = \left(I_{n+1}; z_1(n), z_1(n-1), \ldots; z_2(n), z_2(n-1), \ldots; z_i(n), z_i(n-1), \ldots\right)$$

$$I_{n+1} = \begin{cases} 0 \text{ if the item is up, i.e., no failure at } (n+1) \\ 1 \text{ if the item is down, i.e., has failed at } (n+1) \end{cases}$$

$(i=1,2,\ldots,p)$

while $z_i(n)$ is the strength of the ith symptomatic emitter (S.E.) at the time of the nth inspection after the last failure event. The idea of an S.E. is that it is a physical evidence of change in failure propensity of the item; and its numerical value or strength is taken to be 1) an indicator function of the event that such a physical change has occurred, or 2) a measurement of the magnitude of such a change, or a suitable transformation thereof.

Example: A symptomatic emitter of physical change in a pump is considered to be an oil leak, so the sequence of events and corresponding values might be

n:	1	2	3	4	5	6	7
Oil leak/No oil leak :	N	N	N	N	N	Y	Y
z(n):	0	0	0	0	0	1	1

indicating no oil leak (N) at the first 5 inspections, strengths (indicators) thus being 0, and finally a noticeable oil leak at n=6 (Y), with strength now $z_6=1$, and again at n=7, so $z_7=1$, etc.

A more refined description might well include the magnitude of the oil leak (for example, volume in a drip pan), or even color or chemical composition of the oil leaking. In this case z_n is potentially a real number.

Of course, the notion behind recording S.E. strengths is that an unusual or abnormal strength signals an important change in the hazard rate of the item, and hence can be used as information upon which to base removal or repair. It stands to reason that pump oil leaks, like considerable increase in human blood pressure, "should" signify a much increased hazard rate or death rate unless there is intervention. But a quantitative study is necessary 1) to derive appropriate rules for combining the various possible emitter values to provide a sensitive signal of impending failure, and then 2) to derive decision aiding rules for removal or replacement in advance of actual failure.

3.2 The Simplest Emitter Behavior; Step-Function Change of One Emitter's Rate

In order to illustrate the ideas described above consider the following illustrative case.

Situation 1: A single emitter signals in a exponentially distributed time W ($W \sim \exp[\alpha]$) preceding a shift of failure rate from λ_0 to $\lambda_1 (>\lambda_0)$. The strength of the emitter is taken to be z.

For example, the item survives to n with conditional probability

$$P\{I_n = 0 \mid W>n\} = (e^{-\lambda_o T})^n = e^{-n\lambda_o T} \quad . \tag{3.1}$$

Otherwise and unconditionally, it survives to n with probability

$$P\{I_n=0\} = (e^{-\lambda_o T} e^{-\alpha T})^n + \sum_{n'=0}^{n-1} (e^{-\lambda_o T})^{n'} \nu_o (e^{-\lambda_1 T})^{n-n'-1} \quad , \tag{3.2}$$

where

$$P\{\text{Transition from } \lambda_o \text{ to } \lambda_1\} = \nu_o = \int_o^T e^{-\lambda_0 x} e^{-\lambda_1 (T-x)} e^{-\alpha x} \alpha dx$$

$$= \frac{\alpha}{\lambda_0 - \lambda_1 + \alpha} [e^{-\lambda_1 T} - e^{-(\lambda_0+\alpha)T}] \quad . \tag{3.3}$$

Summing out the geometric series gives the formula

$$P\{I_n=0\} = e^{-(\lambda_o+\alpha)Tn} + \nu_o e^{-\lambda T(n-1)} \frac{1-e^{-(\lambda_o-\lambda_1)Tn}}{1-e^{-(\lambda_o-\lambda_1)T}} = P\{T_s>n\} \quad , \tag{3.4}$$

where T_s is the survival time of the component, assuming no usage of the emitter's signals.

Now introduce the notion of the emitter's signal. Suppose that when the rate λ_0 is in force, $z_1(n)=z_0$, while when λ_0 switches to λ_1 ($\lambda_1>\lambda_0$) then $z_1(n)=z_0+\Delta$, $\Delta>0$. For the moment assume that as soon as a $\lambda_0 \to \lambda$, shift occurs, it is detected. This is relatively simplistic, but it can be handled most easily. For example, z switches from 0 to 1 as soon as a noticeable oil leak becomes evident. This corresponds to the Cox-type model of survival through an inspection cycle of

$$e^{-[\lambda_o T+(\lambda_1-\lambda_o)Tz]} \tag{3.5}$$

Remember that the system state is only visible or detectable <u>at</u> inspection times, while the switch occurs <u>between</u> two such times. Presumably action is taken the moment the oil leak is noticed, so we are interested in the <u>probability of detection before failure</u>. In this case notice that

P{detection at end of period of rate switch, before failure} equals

$$\nu_o = \frac{\alpha}{\lambda_o-\lambda_1+\alpha}\left[e^{-\lambda_1 T} - e^{-(\lambda_o+\alpha)T}\right] . \tag{3.6}$$

3.3 <u>Simulation of the Previous Model</u>

<u>Situation 1</u>: In order to simulate the emitter signals, failures, and replacement or repair actions, follow this procedure:

<u>Simulation Procedure for Situation 1</u>

1. Specify λ_0 (basic rate), λ_1 (degraded rate), T (inspection interval), α (rate of emitter occurrence; α^{-1} is expected (average or mean) time until λ_0 changes to λ_1).

2. Obtain random number with exponential distribution and rate $(\lambda_0+\alpha)$:

$$V = \frac{1}{\lambda_o+\alpha} \ln(U_1) \quad . \tag{3.7}$$

   U_1~Uniform (0,1), i.e., U_1 is an ordinary random number. V is the time until <u>either</u> a failure occurs or the emitter signals.

3. Obtain the <u>identification</u> of V: Let U_2 be an independent (of U_1) random number, and identify time V or time to

   <u>Failure</u> if $0 \leq U_2 < \frac{\lambda_o}{\lambda_o+\alpha}$; then $V = V_F$ (3.8a)

   <u>Emitter Signal</u> if $\frac{\lambda_o}{\lambda_o+\alpha} \leq U_2 < 1$; then $V = V_E$ (3.8b)

4. If a <u>failure</u> is identified under (3), then the component is <u>down</u> from time

   V_F until NT , (3.9)

where

$N = \{n:(n-1)T \leq V < nT\}$

V_F denotes a time V identified or ending with a failure.

5. If an <u>Emitter Signal</u> is identified, obtain an exponentially distributed number

$$H = \frac{1}{\lambda} \ln (U_3) \quad , \tag{3.10}$$

where U_3~Uniform (0,1) and independent of U_1, U_2. This is the time to failure, measured from the moment of emitter signal beginning.

6. Now, if

$$V_E + H < NT \tag{3.11}$$

then the component is down (failed) from time V_E+H until NT. Here V_E stands for the time V when the latter is identified as an emitter (see Step (3)). Note that at time NT, start again from (2), following a repair.

7. If

$$V_E + H > NT \tag{3.12}$$

there is no failure. the emitter is assumed to be noted, and a repaired unit is put into effect before a failure can occur.

In this case, start again with (2).

<u>Comments</u>:

a) The above setup can be completely analyzed by Markov chain methods, so that one can, for instance, find the long-run expected time that the system is down, or the time until some independent Poissonian initiating event occurs during a down period. This can also be done by simulation.

b) For this simple setup there is just one explanatory variable, z_1, and, <u>if no replacements are made until a failure actually occurs</u> (Policy: replacement at failure in force) then if n represents the number of inspections after the installation of a new component (equivalent to complete repair) we have

$$z_1(n) = 0 \text{ for } nT < V \tag{3.13}$$

$$z_1(n) = 1 \text{ for } V_E \leq nT < V_E + H \tag{3.14}$$

Thus we could see the following state combinations

1) $\underline{X}_{n+1} = \left(I_{n+1} = 0,\ z_1(n) = 0\right)$ if <u>either</u>

$\{(n+1)T < V\}$ <u>or</u> $\{nT < V_E \leq (n+1)T$ <u>and</u> $V_E + H > (n+1)T\}$

2) $\underline{X}_{n+1} = \left(I_{n+1} = 1,\ z_1(n) = 0\right)$ if

$\{(nT < V \leq (n+1)T\}$ <u>and</u> $\{V_E + H \leq (n+1)T\}$.

3) $\underline{X}_{n+1} = \left(I_{n+1} = 0,\ z_1(n) = 1\right)$ if

$\{(n-1)T < V_E < nT\}$ <u>and</u> $\{V_E + H > (n+1)T\}$.

4) $\underline{X}_{n+1} = (I_{n+1} = 1,\ z_1(n) = 1)$ if

$\{(n-1)T < V_E < nT\}$ <u>and</u> $\{V_E + H \leq (n+1)T\}$.

One could presumably setup a discrimination procedure to show that knowledge of z, e.g., that $z_1(n)=1$, is a strong predictor that $I_{n+1}=1$. Of course, $I_n=1$ may already have occurred; if the failure rate λ_1 associated with time H is too high then failure may occur too soon after the emitter signals to be useful. The only countermeasure is to attempt to reduce T or improve the sensitivity of the emitter.

<u>Situation 2</u>: In order to generalize the above model in a realistic way, one can proceed to introduce other S.E. factors. Let α_i be the rate of occurrence of S.E. $i(i=1,2,\ldots,P)$. There are $p \geq 1$ emitters, all acting independently; the ith emitter signals a rate increase of δ_i at an exponentially distributed time $W_i(W_i \sim \exp[\alpha_i])$. Thus, if no emitters have occurred the rate is λ_o; if emitter 3 only has occurred then the rate is $\lambda_o+\delta_3$; if emitters 3 and 5 have occurred, the rate is $\lambda_o+\delta_3+\delta_5$, <u>etc</u>.

Let

$$z_i(t) = \begin{cases} 1 \text{ if the ith S.E. has occurred} \\ 0 \text{ otherwise} \end{cases}$$

Then the Cox proportional hazard model for survival to t is

$$P\{T_s > t\} = e^{-[\lambda_o + \sum_{i=1}^{P} \delta_i z_i(t)]t}$$

where T_s is the time to failure of the component.

In order to conveniently simulate this situation assume that $\delta_i=\delta$, so that each emitter's appearance signals the same increase in the hazard. Also, let $\alpha_i=\alpha$, so the rate of emitter occurrence is the same. Such assumptions can be relaxed. The present simulation procedure depends on its relative simplicity upon the above assumptions.

Simulation Procedures for Situation 2

1. Specify λ_o, δ, p (the number of emitters), T, α.

2. Obtain a random number with exponential distribution having rate $\lambda_o+p\alpha$:

$$V(1) = \frac{1}{\lambda_o+p\alpha} \ln(U_1)$$

U_1 is an ordinary random number.

3. Obtain the identification of V:

Failure if $0 \leq U_{11} < \frac{\lambda_o}{\lambda_o+p\alpha}$; then $V(1) = V_F(1)$

Emitter Signal if $\frac{\lambda_o}{\lambda_o+p\alpha} \leq U_{11} \leq 1$; then $V(1) = V_E(1)$

U_{11} is an independent (of U_1) random number.

4. If a failure is identified under (3), then the component is down from

$V_F(1)$ until N_1 , where: $N_1 = \{n:(n-1)T \leq V(1) < nT\}$

(Note: The component is down for time $\{N_1T-V_F(1)\}$ out of a total exposure time of N_1T, presuming, as seems reasonable, that the process begins again after repair.)

5. If an Emitter Signal is identified, so $z_1(n)=1$ for $n \geq N_1$, $z_2(n) = 0$, etc., obtain an exponentially distributed random number

$$H(1) = \frac{1}{\lambda_o+\delta} \ln (U_{12}) = \frac{1}{\lambda_1} \ln (U_{12}) \quad ,$$

where $\lambda_1=\lambda_0+\delta$, is the new failure rate, and U_2 is a random number, (independent of U_1, U_{11}).

6. If

$$V_E(1) + H(1) \leq N_1T \quad ,$$

then the component is down (failed or unavailable) from time

$V_E(1) + H(1)$ until N_1T .

The process starts over (after a repair) at stage (2).

7. Otherwise, if

$$V_E(1) + E(1) > N_1T$$

there is no failure, the emitter presence is noted, and the process continues as follows.

8. Obtain an exponential random number

$$V(2) = \frac{1}{\lambda_1+(p-1)\alpha} \ln (U_2) \quad ,$$

where $\lambda_1=\lambda_0+\delta$, and U_2 is an independent random number.

9. Obtain the identification of V(2):

<u>Failure</u> if $0 \leq U_{21} < \frac{\lambda_1}{\lambda_1+(p-1)\alpha}$; then $V(2) = V_F(2)$

<u>Emitter Signal</u> if $\frac{\lambda_1}{\lambda_1+(p-1)\alpha} \leq U_{21} \leq 1$; then $V(2) = V_E(2)$

where U_{21} is an independent random number.

10. If a <u>failure</u> is identified, above (9), then the system is <u>down</u> from

$V_F(2)$ until N_2T ,

where

$$N_2 = \{n:(n-1)T \leq V_E(1) + V(2) < nT\}$$

11. If an <u>Emitter Signal</u> is identified

$$H(2) = \frac{1}{\lambda_2} \ln (U_{22})$$

where $\lambda_2=\lambda_0+2\delta$, and U_{22} is an independent random number.

12. If

$$V_E(1) + V_E(2) + H(2) \leq N_2T \quad ,$$

then the component is down (failed) from time

$$V_E(1) + V_E(2) + H(2) \text{ until } N_2T \quad .$$

The process starts over (after repair) at stage (2).

13. Otherwise, if

$$V_E(1) + V_E(2) + H(2) > N_2T$$

there is no failure, the second emitter's presence is noted ($z_2(n)=1$ for $n \geq N_2$) and the process continues, now with p-2 emitters to signal, and with failure rate $\lambda_2=\lambda_0+2\delta$.

In general, suppose i emitters have signalled, and failure has not occurred. Then the next simulation step proceeds as follows:

14. Obtain an exponential random number

$$V(i+1) = \frac{1}{\lambda_i+(p-i)\alpha} \ln (U_i) \ ,$$

where $\lambda_i=\lambda_0+i\delta$.

15. Obtain the identification of V(i+1):

<u>Failure</u> if $0 \leq U_{i+1,1} < \frac{\lambda_i}{\lambda_i+(p-i)\alpha}$; $V(i+1) = V_F(i+1)$

<u>Emitter Signal</u> if $\frac{\lambda_i}{\lambda_i+ (p-i)\alpha} \leq U_{i+1,1} \leq 1$; $V(i+1) = V_E(i+1)$.

16. If a <u>failure</u> is identified, above (9), then the system is <u>down</u> from

$$V_F(i+1) \text{ until } N_{i+1}T \quad ,$$

where

$$N_{i+1} = \{n:(n-1)T \leq V_E(1) + V_E(2) + \ldots + V(i+1) < nT\} \ .$$

17. If an <u>Emitter Signal</u> is identified, obtain

$$H(i+1) = \frac{1}{\lambda_{i+1}} \ln (U_{i+1,2}) \quad ,$$

where $\lambda_{i+1}=\lambda_i+\delta$.

18. If

$$V_E(1) + V_E(2) + \ldots V_E(i+1) + H(i+1) \leq N_{i+1}T \quad ,$$

then the component is down (failed) from

$$V_E(1) + V_E(2) + \ldots V_E(i+1) + H(1+1) \text{ until } N_{i+1}T \, .$$

and the process starts over from stage (2).

19. Otherwise, if

$$V_E(1) + \ldots + V_E(i+1) + H(i+1) > N_{i+1}T$$

and the process continues by advancing i+1 to i+2 and starting at stage 14. Also, $z_1(n)=z_2(n)= \ldots = z_{i+1}(n)=1$, other emitter values = 0 for $n \geq N_{i+1}$.

Comments:

The above process, involving $p(\geq 1)$ emitters is relatively simple. Various embellishments are possible: for instance, the observed emitter value may be obtained by adding a normally distributed error value to $z_i(n)$; one could let the mean be zero, while the variance of error, σ^2, varies; for large σ^2 it will be impossible to tell the true emitter rate state, and hence the prevailing failure rate.

3.4 Discussion

The setup described permits generation of synthetic data to be analyzed by survival analysis with covariates or explanatory variable techniques, e.g., by use of the Cox analysis of a proportional hazards model. Of course it would not be assumed in such a data analysis that p is known, and the variables might well not be taken to be simple indicators. But such an analysis might well help provide an indication of a much increased failure rate.

It remains to specify a good, or possible optimum, decision rule to follow concerning actions to be taken, given the values of $z_1, z_2, \ldots, z_g$ (where g=p if the true mechanism is as described). That is, if

$$\sum_i z_i < 3$$

no action is necessary, but if

$$\sum_i z_i \geq 3$$

one might repair or replace in anticipation of failure, since failure is quite likely. Such a pre-failure repair is no doubt expensive both in time and money, but it may help to forestall a failure between inspection periods.

4. DECISION FRAMEWORK FOR SYSTEM MAINTENANCE-REPAIR CONTROL WITH "SYMPTOMATIC EMITTER" INFORMATION

In the previous sections a methodology is presented for using certain symptoms, such as oil leaks, vibrations, temperature rises, etc., that precede component failures. These symptoms, when interpreted by appropriate statistical methods, can be used to determine the appropriate action - remedial maintenance or expensive repair or replacement, to forestall an impending failure. The symptomatic emitter information is used by the statistical methodology to obtain a failure rate, representing the degraded state based on the emitter information. The important issue that remains is whether this information can be effectively used to define a maintenance/repair policy to reduce the component unavailability by reducing the outright failure frequency. In this section, a quantitative formulation of the problem of operationally evaluating a maintenance-repair based on existence of symptomatic emitters is presented.

4.1 A Three-State System Model

Consider a system or component that can be in one of three states at any time t:

- $S(t) = 0$: the system is up (operative) at time t, and possesses a failure rate of λ_0, and transfer rate ν_0; transfer refers to a change to state 1 or state 2.
- $S(t) = 1$: the system is up at time t, emits symptoms, and posseses a failure rate $\lambda_1 > \lambda_0$.
- $S(t) = 2$: the system is down (inoperative) at t.

Note that the system can change from State 0 to either State 2 (failure) or State 1 (symptomatic).

The system is observed at regular intervals, T. In practice, the interval T may be one month. Let $S_n = S(nT)$ denote the true state of the system at the nth inspection or observation.

A realistic feature of the inspection process is that it is error-prone: if the system is in State i, there is a probability d_{ij} that it is estimated to be in State $j \neq i$. Call $\{d_{ij}\}$ the diagnosis probabilities. Ideally, of course, $d_{ii}=1$ and $d_{ij}=0$, $i \neq j$, but this sharpness cannot always be achieved: the extent to which sharpness is achieved will depend upon the strength and clarity of the indication that the system is particularly, in State 1, when symptomatic emitter scores are expected to be noticeably different from their values when the system is in State 0. Presumably $d_{22}=1$, since it should be quite evident that the component is actually inoperative. Multivariate statistical analysis helps to furnish rules that imply values of $\{d_{ij}\}$. Details of a completely statistical approach can be developed.

4.1.1 A Decision Rule

If the system is estimated to be in State j at inspection time, then action is to be taken to transfer the state of the system to State 0, associated with the lowest failure rate λ_0. Any such action requires ("costs") time, even if the action is unnecessary. Suppose that during the action time the system is as totally unavailable as if it had failed. It follows that if knowledge of the actual state is error-prone, then such unavailability is sometimes incurred unnecessarily. Of course economic cost will also be associated with such transfers, both directly (cost of replacements or parts) and indirectly (resources used to examine and repair one system are diverted from another).

Here are some specific assumptions concerning the operation of such a decision rule. Let j be the observed state at the time of an inspection. Whatever the j-value, the result of the corresponding action, A_j, is to place the system in actual state 0. Let L_j denote the time lost, or unavailability time associated with A_j. Then,

j	A_j	L_j
0	Do nothing	0
1	Maintain	L_1
2	Repair/replace	L_2

It may be reasonable to assume $0<L_1<L_2<T$. Also let L_1 and L_2 be taken to be deterministic, not random. The action taken at the beginning of a period between inspections governs the exposure time of the item to unavailability. If a period begins with a maintenance, then there is a time equal to $T-L_1$ until the next inspection, during some part of which the system is subject to failure and unavailability. Of course the system is totally unavailable during L_1. The same situation holds when a period begins with repair/replacement, but now L_1 is replaced by L_2, and the system is unavailable for at least $T-L_2$, perhaps reasonably at least equal to $T-L_1$.

The use of the terms "maintain" and "repair/replace" are illustrative and not to be taken too literally. Also, there may possibly be three or more actions to choose from in practice.

4.2 Markov Chain Analysis of Decision Rules

The long-run availability of the system depends upon the failure and transfer rate λ_0, ν_0, and λ_1, upon inspection interval T, and upon the diagnostic probabilities $\{d_{ij}\}$; the latter will often implicitly depend upon the relationship of λ_0 and λ_1. If λ_1 is much greater than λ_0, then the d_{ij}-values should become quite sharp. Note, however, that if $\lambda_1 T$ is very high, so that the survival probability $e^{-\lambda_1 T}$ is very small, then knowledge that the system is in the emission state (State 1) can seldom be useful, for the item fails soon after the change is made from "health" (State 0) to "sickness" (State 1), and inspection cannot detect the new state in a time that is useful. The only remedy is to decrease the inspection interval.

In what follows, various decision rules will be evaluated in terms of the Markov chain $\{S_n, n=1,2,...\}$, where $S_n=S(nT+)$, i.e., the state of the unit just following an inspection. The use of the Markov model has been studied in deciding the maintenance policies (Pau, 1975). Of particular use will be the long-run properties of such chains, i.e., their behavior as $n\to\infty$. Let these be denoted by $\{\pi_i(R), i=1,2,3\}$, where R denotes the decision rule that is in effect; then

$$\pi_j(R) = \sum_{i=1}^{3} \pi_i(R)P_{ij}(R) \ , \ j = 1,2,3, \quad , \tag{4.1}$$

or, in matrix notation, $\underline{\pi}(R)$ being a row-vector,

$$\underline{\pi}(R) = \underline{\pi}(R)\underline{P}(R) \quad , \tag{4.2}$$

where the one-step matrix $\underline{P}(R)$ is implied by the decision rule R; examples are presented shortly. Of course

$$\sum_{j=1}^{3} \pi_j(R) = 1 \quad .$$

Having $\underline{\pi}_R$ allows evaluation of the long-run expected point availability, i.e., the expected fraction of time during which the unit is operative and can respond to an instantaneous demand for response. There follow some useful building-block expressions for evaluating the Markov chains and the conditional expectations of availability over an inspection cycle.

4.3 Markov-Chain Probability Components

Markov chain methods can be used to analyze the implications of system control policies that depend upon knowledge of system states. In order to write down the Markov transition probabilities economically, introduce the following auxiliary conditional probabilities,

$$\{a_{ij}(\tau), \ i = 0,1,2; \ j = 0,1,2; \ \tau>0\} \tag{4.3}$$

where

$$a_{00}(\tau) = e^{-(\lambda_0+\nu_0)\tau} \tag{4.4a}$$

$$a_{01}(\tau) = \int_0^{\tau} e^{-(\lambda_0+\nu_0)x} \nu_0 dx e^{-\lambda_1(\tau-x)} = \frac{\nu_0}{\nu_0+\lambda_0-\lambda_1} [e^{-\lambda_1\tau} - e^{-(\nu_0+\lambda_0)\tau}] \tag{4.4b}$$

$$a_{02}(\tau) = \int_0^\tau e^{-(\lambda_0+\nu_0)x}\lambda_0 dx + \int_0^\tau e^{-(\lambda_0+\nu_0)x}\nu_0 dx[1-e^{-\lambda_1(\tau-x)}]$$

$$= 1-e^{-(\lambda_0+\nu_0)\tau} - \frac{\nu_0}{\nu_0+\lambda_0-\lambda_1}[e^{-\lambda_1\tau}-e^{-(\nu_0+\lambda_0)\tau}] \quad (4.4c)$$

and

$$a_{11}(\tau) = e^{-\lambda_1\tau} \quad (4.5a)$$

$$a_{12}(\tau) = 1-e^{-\lambda_1\tau} \quad (4.5b)$$

These can be regarded as describing the transition probabilities in a pure death process $\{S(t;d)\}$ that passes through the same states as described, but never experiences inspection or repair. Explanations are simple: for instance $a_{01}(\tau)$ simply expresses the probability of a transfer from event rate λ_0 to λ_1 without failure, while $a_{02}(\tau)$ refers to the probability of transition to the failed state, either directly or in the intermediate, symptomatic emitter state.

Rule 0: Repair Only Upon Failure (State 2)

Suppose inspection is only capable of detecting failure (State 2), or non-failure (States 0 or 1). In particular, no action is taken if the unit is in State 1. Here is the matrix of one-step Markov transition probabilities for the S_n-process; denote this by $\underline{P}(R) = \{P_{ij}(R)\}$. We have

$$\underline{P}(0) = \begin{array}{c} i/j \\ 0 \\ 1 \\ 2 \end{array} \begin{array}{ccc} \underline{0} & \underline{1} & \underline{2} \\ a_{00}(T) & a_{01}(T) & a_{02}(T) \\ 0 & a_{11}(T) & a_{12}(T) \\ a_{00}(T-L_2) & a_{01}(T-L_2) & a_{02}(T-L_2) \end{array} \quad (4.6)$$

An explanation of the P_{2j} entries is simply that if the system is seen to have failed (State 2) at the nth inspection, then repair occurs for an interval L_2 thereafter, after which the unit is returned to perfect condition (State 0), and the process acts like a pure death process until the time of the next inspection. The P_{1j} entries reflect the fact that the symptomatic emission is occurring (State 1) is unrecognized--unintentionally or intentionally.

Now the use of (4.6) in connection with either (4.1) or (4.2) yields $\pi_j(0)$, which describes the situation prevailing at the start of an inspection interval in the long run. In order to obtain the long-run unavailability we first calculate the expected time down during an interval, conditional upon starting in State j. For convenience, define

E[Fraction of time down during interval | System in State j at the beginning of interval]

$= f_j(T;R)$.

Clearly, the above fraction depends upon the precise decision rule followed: For R=0 we see that

$$f_0(T;0) = \frac{1}{T}\Big[\int_0^T (T-x)e^{-x(\lambda_0+\nu_0)}\lambda_0 dx + \int_0^T (T-x)\Big(\int_0^x e^{-y(\lambda_0+\nu_0)}$$

$$\cdot\ \nu_0\, dy e^{-\lambda_1(x-y)}\Big)\lambda_1 dx\Big]$$

$$\simeq \frac{\lambda_0 T}{2} - \frac{1}{6}\lambda_0(\lambda_0+\nu_0)T^2\} + \frac{1}{6}\nu_0\lambda_1 T^2 = \frac{\lambda_0 T}{2} + \frac{\nu_0(\lambda_1-\lambda_0)T^2}{6} - \frac{\lambda_0^2 T^2}{6} \quad , \qquad (4.7)$$

and

$$f_1(T;0) = \frac{1}{T}\int_0^\infty (T-x)e^{-x\lambda_1}\lambda_1 dx = \frac{e^{-\lambda_1 T}+\lambda_1 T-1}{\lambda_1 T} \simeq \frac{1}{2}\lambda_1 T \quad , \qquad (4.8)$$

where the approximations are effective if $\lambda_1 T$ becomes small. Finally,

$$f_2(T;0) = \frac{1}{T}\left[L_2 + (T-L_2)f_0(T-L_2;0)\right] \qquad (4.9)$$

for if the system is initially down, then it is subsequently down for repair for time L_2, after which it starts in State 0; $(T-L_2)f_0(T-L_2;0)$ is recognized as the expected downtime for the remainder of the period.

In order to obtain the unconditional expected fraction of downtime, i.e., the desired long-run unavailability, simply evaluate

$$f(T;0) = \sum_{j=0}^{2} \pi_j(0)f_j(T;0) \quad . \qquad (4.10)$$

Explicit formulas can be written down, but they are complicated and uninformative; numerical evaluations are simple and useful, and a small computer program will furnish them. Note that since Rule 0 dictates action only upon failure, the diagnosis probabilities play no role.

Turn next to a consideration of

Rule 1: Maintain When Symptoms are Emitted (State 1); Repair Upon Failure (State 2)

By way of contrast to Rule 0, suppose that it is possible to ascertain system state perfectly, and the rule is to maintain the system as soon as it is found to be in State 1, i.e., to be emitting symptoms. In this case the system may, sometimes at least, be returned to State 0 before failure at the cost of incurring L_1, rather than $L_2>0$, units of downtime.

First, evaluate the transition probabilities, as before.

$$\underline{P}(1) = \begin{array}{c} \underline{i/j} \\ 0 \\ 1 \\ 2 \end{array} \begin{bmatrix} \underline{0} & \underline{1} & \underline{2} \\ a_{00}(T) & a_{01}(T) & a_{02}(T) \\ a_{00}(T-L_1) & a_{01}(T-L_1) & a_{12}(T-L_1) \\ a_{00}(T-L_2) & a_{01}(T-L_2) & a_{02}(T-L_2) \end{bmatrix} \quad (4.11)$$

One now combines the above transition probabilities (4.9) with (4.1) or (4.2) to find the long-run probability distribution $\{\pi_j(1)\}$ to find the long-run unavailability. The conditional unavailabilities are expressed in terms of (4.7), (4.8), and (4.9):

$$f_0(T;1) = f_0(T;0) \quad (4.12a)$$

$$f_1(T;1) = \frac{1}{T}\left[l_1 + (T-L_1)f_0(T-L_1;0)\right] \quad (4.12b)$$

$$f_2(T;1) = \frac{1}{T}\left[L_2 + (T-L_2)f_0(T-L_2;0)\right] \quad (4.12c)$$

Once again the long-run availability is evaluated as:

$$f(T;1) = \sum_{j=0}^{2} \pi_j(1)f_j(T;1) \quad (4.13)$$

It is now possible to compare f(T;1) given by (4.13) to f(T;0) given by (4.10):

- If $f(T;1) < f(T;0)$, there is potential benefit from utilizing symptomatic emitter information, i.e., from information that the component is actually in State (1), a pre-failure state from which failures are likely to occur.

- If $f(T;1) \geq f(T;0)$ there is no use in attempting to utilize symptomatic emitter information, unless conditions can be changed, e.g., unless L_1, the maintenance time, can be decreased enough to make f(T;1) smaller than f(T;0).

Note that the above is an extreme assessment: it pretends that the diagnosis probabilities are all indefinitely sharp, in particular $d_{11} = 1$. A more realistic assessment is given by

Rule 2: Maintain When the System is Estimated to be in the Symptomatic Emission State (State 1); Repair Upon Perceived Failure (State 2).

If statistical methods, e.g., multivariate analyses, are used to establish that the component is in the Symptomatic Emission State (State 1), then it is to be anticipated that errors will occur. The rates of these errors, given the actual states, are summarize by the diagnosis probabilities:

$$d_{ij} = P\{\text{System Estimated in State } j \mid \text{System Actually in State } i\}$$

Of particular importance are d_{0j} and d_{1j}, for it seems probable that if the item is in State 2 (Failed), this will often be evident. Misdiagnosis can lead to loss of time while unnecessary preventive maintenance is carried out (d_{01} "high), or missed opportunity to forestall a failure (d_{10} "high"); in either case unavailability may well be substantially increased.

Evaluation can again be carried out in terms of an appropriate Markov chain. Here are one-step transition probabilities for such a chain.

For ease of writing the one-step transitions are arranged in a table.

$$p_{00}(2) = d_{00}a_{00}(T) + d_{01}a_{00}(T-L_1) + d_{02}a_{00}(T-L_2)$$

$$p_{01}(2) = d_{00}a_{01}(T) + d_{01}a_{01}(T-L_1) + d_{02}a_{01}(T-L_2)$$

$$p_{02}(2) = d_{00}a_{02}(T) + d_{01}a_{02}(T-L_1) + d_{02}a_{02}(T-L_2)$$

$$p_{10}(2) = d_{10}\cdot 0 + d_{11}a_{00}(T-L_1) + d_{12}a_{00}(T-L_2)$$

$$p_{11}(2) = d_{10}a_{11}(T) + d_{11}a_{01}(T-L_1) + d_{12}a_{02}(T-L_2)$$

$$p_{12}(2) = d_{10}a_{12}(T) + d_{11}a_{12}(T-L_1) + d_{12}a_{02}(T-L_2)$$

$$p_{20}(2) = d_{20}\cdot 0 + d_{21}[m_{21}a_{10}(T-L_1)] + d_{22}a_{00}(T-L_2)$$

$$p_{21}(2) = d_{20}\cdot 0 + d_{21}[m_{21}a_{11}(T-L_1)] + d_{22}a_{01}(T-L_2)$$

$$p_{22}(2) = d_{20}\cdot 1 + d_{21}[1-m_{21}) + d_{22}a_{02}(T-L_2)$$

Note that it has become desirable to introduce the new parameter m_{21} which is

m_{21} = probability that if the maintenance action is taken when the item is failed, the system is returned to the pre-failure state (State 1).

This is, of course, a stopgap measure only; a conservative assumption would be that $m_{21}=0$ and certainly a great many other possibilities suggest themselves. The unique feature of the present model is that misperception of the system state induces possibly inappropriate action, which in turn influences the long-run distribution of states, and hence long-run availability.

In order to evaluate the corresponding long-run availability it is only necessary to utilize the conditional expected unavailabilities as given by (4.12); the expected unavailability is then

$$f(T;2) = \sum_{j=0}^{2} \pi_j(2) f_j(T;1) \quad .$$

It is now sensible to compare Rules 0 and 2:

- If $f(T;2) < f(T;0)$, there is evidence of overall benefit from utilizing symptomatic emitter information, even though the latter is imperfect and error-prone, as evidenced by $d_{ii}<1$. That is, there is evidence that a "repair on warning" policy may be beneficial.

- If $f(T;2) \geq f(T;0)$ there appears to be no use in employing <u>the currently available</u> symptomatic emitter information. If either the sharpness of the information were increased (d_{ii} made closer to unity), or the consequences of maintenance made less severe (L_1 shortened), or if maintenance reduced λ_1, then there is justification for following the policy based on symptomatic emitter evidence.

5. CONCLUSIONS AND RECOMMENDATIONS

In this report a methodology for detecting failure potential for standby safety system components has been presented. The methodology uses proportional hazard models and provides a framework for incorporating various measurables at a surveillance test. The incorporation of these measurables with information on the operability of the component in calculating the failure rate has a strong appeal. This allows a quantitative framework to develop a decision process, as presented in this report, for component repair/replacement policy in a more structured manner rather than a policy based totally on engineering judgement.

The underlying assumption in the approach presented is that the inherent failure rate of a component will show significant departure before a failure and such departures can be identified using the measurable parameters collected during the test, called the symptomatic emitters. This assumption has not yet been validated by actual data on component failures, but holds great promise in preventing failure of risk important components in nuclear power plants.

The methodology discussed has found wide applications in medical survival analysis over a number of years, typically in those cases where a large amount of data was available. In the nuclear safety area, the number of failures observed on a specific component is few, and the applicability of such a methodology has to be demonstrated for limited failure data.

Recognizing the potential utility of such an approach in analyzing the information obtained in routine component testing, it is recommended that a study be pursued on application both to simulated data and to actual data. In applying the methodology to actual data, a portion of the data should be kept back to check if the analysis can, in reality, predict failures.

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APPENDIX A

INCORPORATION OF TIME DEPENDENT COVARIATES

In the main report, the proportional hazard model of D. Cox (1972)

$$\lambda(t;z) = \lambda_0(t)e^{\underline{\beta}\cdot\underline{z}} \quad ,$$

was analyzed, and the conditional likelihood approach of estimating β was presented. In this appendix, the consideration of time-dependent variables in estimating β is discussed.

The question of time-dependent covariates or symptomatic emitters is important in analyzing the component degradation in nuclear power plants. Typically a component is being tested at a fixed test interval, and the value of the symptomatic emitter will vary from one testing interval to another. The symptomatic emitter information of interest to us is not only the value of the variable at the interval it failed, but also the entire path of the variable.

The analysis of the component degradation and trends can be performed incorporating time-dependent covariates, and the Cox formulation of the proportional hazard model takes the shape

$$\lambda(t;\underline{Z}(t)) = \lambda_0(t)e^{\underline{Z}(t)\underline{\beta}} \quad . \qquad \text{(A-1)}$$

Here, $\underline{Z}(t)$ signifies not only the value of the covariate at time t, but the entire covariate path up to time t. That is, the failure rate depends upon the historical value of the explanatory variable, i.e. values reading from the present value (t) to any previous time point, in this case, the time of first testing. Mathematically speaking, Z(t) is really a functional.

Consider that a component is tested at regular intervals, and t_1, t_2,...,-t_k,... denotes the times of test. At each of the test, one records the explanatory variable vector $\underline{z}$ and also observes any failure of the component. The value of the explanatory variable or the symptomatic emitter is assumed to be constant within a test interval, but can vary from one test interval to another. If a demand occurs within a test interval and the component fails to perform its required function, the failure is assumed to occur at the end of the interval. Therefore, the actual failure time is known to the nearest test interval. The explanatory variable vector for the nth component at the jth interval is

$$\underline{Z}_{nj} = (z_{nj1}, z_{nj2},\ldots, z_{njp})$$

where the component is defined in terms of p variables considered to provide the symptoms of failure.

Let $\underline{Z}$ denote the entire covariate process, i.e., $\underline{Z}_{nk}$ denotes the covariate process up to the kth interval for the nth component.

$$\underline{Z}_{nk} = \{z_{nu} : u \leq k\} .$$

The failure times are grouped into intervals $[0=t_0,t_1)$, $[t_1,t_2),..[t_{k-1},t_k)$, where T_k denote the interval $[t_{k-1},t_k)$, k = 1,2... .

Survivor Function

Let T be the random variable representing the failure time, then the survivor function F(t) is the probability that T is at least as great as t and is given by

$$F(t) = P(T \geq t),\ 0 < t < \infty \quad .$$

For the continuous random variable,

$$F(t) = \exp[-\int_0^t \lambda(u)du] \quad ,$$

where $\lambda(t)$ is the hazard rate at T=t.

In the proportional hazard model, the survivor function of T given $\underline{Z}$ is $F(t;\underline{Z})$ and is given by

$$f(t) = \exp\left[-\int_0^t \lambda_0(u)\exp(\underline{z}\ \underline{\beta})du\right] = [F_0(t)]^{\exp(\underline{z}\ \underline{\beta})} \quad , \qquad \text{(A-2)}$$

where $F_0(t)$ denotes the baseline survivor function for $\underline{z} = 0$.

The hazard in the interval for a component with covariate $\underline{z}$, is:

$$P(T\ t[t_{i-1},\ t_i)/T \geq t_{i-1}) = \frac{F(t_{i-1};\underline{z})-F(t_i;\underline{z})}{F(t_{i-1},\underline{z})}$$

$$= 1 - \exp[e^{-\underline{z}\ \underline{\beta}} \int_{t_{i-1}}^{t_i} \lambda_0(u)du] \quad . \qquad \text{(A-3)}$$

Using

$$\alpha_i = \exp\left[-\int_{t_{i-1}}^{t_i} \lambda_0(u)du\right] \quad , \qquad \text{(A-4)}$$

one obtains,

$$P\left[T \quad (t_{i-1},t_i)/T \geq t_{i-1}\right] = 1 - \alpha_i^{\exp(\underline{z}\ \underline{\beta})} \qquad \text{(A-5)}$$

Note that α_i is the conditional probability of an individual with covariate $\underline{z} = 0$ surviving the interval (t_{j-1}, t_j) given that it has survived up to the interval (t_{j-2}, t_{j-1}).

Maximum Likelihood Estimation of β for Time-Dependent Covariates

The probability of observing a failure time t_i for a component with the covariate path $\underline{Z}_{ni}$ is:

$$P\left\{T \quad (t_{i-1},t_i)/\underline{Z}_{ni},\ T \geq t_i\right\}$$

= [prob. that it fails in the interval T_i] x
[prob. that it survived up to T_{i-1}]

$$= \left[1 - \alpha_i^{\exp(\underline{z}_{ni}\ \underline{\beta})}\right] \prod_{j=1}^{i=1} \alpha_j^{\exp\left(\underline{z}_{nj}\ \underline{\beta}\right)} \qquad \text{(A-6)}$$

where α_i is as defined earlier in Eq.(A-4) and denotes the conditional survival probability in T_i for a component with $\underline{z}_{ni}=0$. Note that the hazard function depends not only on the information about the covariates at the interval it failed, but also on its values at previous intervals.

Kalbfleisch and Prentice (1980) provides further discussion on the use of time-dependent covariates. The reader is also referred to Prentice and Gloeckler (1978) for the maximum likelihood estimation process to develop computationally feasible estimations. At this point, one develops the likelihood function to estimate $\alpha_1, \ldots, \alpha_k$, and $\underline{\beta}$.

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The proportional hazard model (Cox, 1972) provides a framework for incorporating measurable parameters (covariates) into the description of failure rates. In this report a methodology is presented for calculating the failure probability based on the parameters measured at a routine surveillance test of the component. These parameters are expected to vary in an identifiable way before a failure indicating an impending failure, and are termed symptomatic emitters. A decision framework for a repair/replacement policy based on such information is presented.

The use of symptomatic emitter information in determining a component failure potential has great utility, particularly in highly reliable systems like those in nuclear power plants. Further work on application of this methodology is recommended. If actual data are not readily available, simulated data can be developed using the simulation approach presented in this report.

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