AA50-2
PDR

UNITED STATES
NUCLEAR REGULATORY COMMISSION
WASHINGTON, D. C. 20555

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NOTE TO: Bob Dube

FROM: Mary Miller

SUBJECT: CALCULATION OF THE STANDARD DEVIATION OF THE $\bar{X}$

The point was made at our contractors meeting last April that the calculation of a standard deviation of a material test statistic must include an estimate of the uncertainty of the $\bar{X}$ for a material control test of the form $AT = 5 + \bar{X} - k\sigma$ where:

AT = Alarm Threshold

$\bar{X}$ = mean value of test statistic

k = 1.28 for 90% pod; 2.33 for 99% pod

σ = standard deviation of test statistic

The calculation of a value to be associated with the uncertainty of the $\bar{X}$ is not a trival matter. Intuitively it might be thought that a reasonable approach would be to calculate the standard deviation of the series of $\bar{X}$'s associated with a series of inventory periods. However, this is a vary arbitrary division of the data set of X values into subsets and really has no statistical validity.

A better approach is to invoke the Central Limit Theorem which states that "if random samples containing a fixed number n of measurements are repeatedly drawn from a population with finite mean μ and standard deviation σ, then if n is large, the sample means will have a distribution that is, approximately normal with mean $\mu_{\bar{X}} = \mu$ and standard deviation $\sigma_{\bar{X}} = \sigma/\sqrt{n}$ ".[1]

This says that the standard deviation of $\bar{X}$ is equal to the standard deviation of the population of X's (a quantity already calculated, i.e., the standard deviation of the test statistic) divided by the square root of n (i.e., the number of observations that went into the calculation of $\bar{X}$ and σ.) Thus the more observations on which $\bar{X}$ and σ are based the smaller is the uncertainty in $\bar{X}$. The only problem with this line of reasoning is that the Central Limit Theorem assumes that the population values are all independent, which of course is not true in our application. In order to properly account for the covariant effects it would be necessary to perform a much more complex calculation such as is done in the AMASS program. The equation for estimating the variance of the $\bar{X}$ has been shown by Fred Tingey to be[2]

$$\sigma_{\bar{X}} = \sum_{j=1}^{n} \frac{\sigma_{e_j}^2}{n^2} + \sum_{j=1}^{n} \frac{\sigma_{Y_j}^2}{n^2} + \sum_{j \neq k} \frac{\sigma_{e_j e_k}}{n^2} + \frac{2(n-1)\,\sigma_{Y_i,Y_{i+1}}}{n^2}$$

where $\sigma_{e_j}^2$ = measurement system variances

σ_{Y_j} = nonmeasurement system variances

σ_{e_j,e_k} = measurement system covariances

$\sigma_{Y_i \cdot Y_{i+1}}$ = nonmeasurement system covariances

It should be noted that this equation reduces to the Central Limit Theorem approximation ($\sigma_{\bar{X}} = \sigma/\sqrt{n}$) if the covariances are ignored.

John Jaech has also addressed this problem[3] and has reached essentially the same answer. His assessment of the situation and one with which Fred Tingey agrees is that the use of the approximation $\sigma/\sqrt{n}$ is probably sufficiently close to the correct answer especially if n is large. Using this approximation if n=25 the net estimated standard deviation would be $\sqrt{\sigma^2 + \sigma^2/25} = 1.0198\sigma$ i.e., the effective σ is increased by about 2%. I have recommended that SAI use this approach in the analysis they are performing for us and believe that it should be recommended in the guidance documents which are under development. Of course, if a licensee has the capability to properly account for the covariances this would also be acceptable.

Mary

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1. Mendenhall and Ott. Understanding Statistics Duxbury Press, 1980, p. 117.
2. NUSAC, Inc. "Statistical Guidance for Material Control Detection Tests" NUSAC Report No. 712, April 1982, pp. 23-24.
3. Jaech, J. L., Statistical Methods in Nuclear Material Control, USAEC, 1973, pp. 44-45.