NUREG/CR-5044
LA-11179-MS

Estimation Techniques for Common Cause Failure Events

Los Alamos

Los Alamos National Laboratory is operated by the University of California for the United States Department of Energy under contract W-7405-ENG-36.

Edited by Wilma Bunker, Group A-4

An Affirmative Action/Equal Opportunity Employer

NUREG/CR-5044
LA-11179-MS

RG

Estimation Techniques for Common Cause Failure Events

Elizabeth J. Kelly
Geralyn M. Hemphill

Manuscript submitted: October 1987
Date published: March 1988

Prepared for
Division of Risk Analysis and Operations
Office of Nuclear Regulatory Research
US Nuclear Regulatory Commission
Washington, DC 20555

NRC FIN No. A7225

Los Alamos Los Alamos National Laboratory
Los Alamos, New Mexico 87545

ESTIMATION TECHNIQUES FOR COMMON CAUSE FAILURE EVENTS

by

Elizabeth J. Kelly and Geralyn M. Hemphill

ABSTRACT

Common cause failure probability estimation techniques, including β-factor, basic parameter, binomial failure rate, multiple Greek, and C-factor estimators, are evaluated and compared using simulation data that captures the real world problem of sparse data from different plants. The effects on the estimators' performances from underlying factors such as common cause shock rates, lethal shock rates, probability of failing given a shock, independent failure rates, and system operational time are discussed. Worst case results are reported, and it is seen that for extremely small common cause failure probabilities the binomial failure rate estimators are best. However, these estimators can underestimate the true probabilities when the failures deviate from the binomial failure rate model. The β-factor technique is shown to be conservative, and in some cases to overestimate the true probability by several orders of magnitude. When there are observed failures for each failure event, the basic parameter technique is best and is easily calculated. This estimator is investigated in detail and is used to develop an estimator for the probability of K or more units failing due to a common cause. Uncertainty limits for this probability are also developed.

1.0 INTRODUCTION

The design principle of redundancy has been applied in nuclear power plants to assure a high degree of reliability for safety critical systems. The essential assumption is that multiple units will fail independently, thus greatly reducing the likelihood of the loss of a safety critical function. However, probability risk assessments (PRAs) and operating experience have shown that the assumption that redundant systems fail only by a series of independent failures is not valid. Rather, multiple failures due to a common initiating event such as a design flaw, an environmental condition, an operator error, or faulty maintenance dominate system unavailability and plant risk. Such multiple failures are called common cause failures and are the subject of this paper. The problem investigated is how to best quantify the common cause failure probabilities. Currently, several techniques are used by the nuclear power industry to estimate common cause failure probabilities, and there is a great deal of controversy and confusion about which method to use. Since data is extremely sparse, it is very difficult to evaluate these estimation techniques using real data. In this study, we use a Monte Carlo simulation to generate common cause failure data and then use this data to evaluate the β-factor, multiple Greek, basic parameter, binomial failure rate, and C-factor estimators. This simulation captures the real world problem of sparse data from dissimilar plants and allows the failure data to be generated by various failure models.

Appendix A contains the recommended procedures for common cause event analyses. Also in this Appendix, the details of the recommended procedures are developed and uncertainty intervals described.

2.0 THE MODEL

In the context of this discussion, a system is any collection of M redundant units. A unit can be thought of as a component, such as a valve or pump, or as a collection of components, such as a train. A basic event, E_k, is a shock to the system such that k (k = 1, ..., M) specific units fail as a result of this shock. We make the simplifying assumption that the rate of occurrence of such events, λ_k, is the same for any group of k units. For example, if a system contains three redundant pumps, the rate of simultaneous failures of pumps one and two is the same as the rate for pumps two and three. Let N_k represent the number of events in time T, the total system operational time, for a population of systems. If the events occur independently of one another from system to system and λ_k is constant over time from one system to the next, then the distribution of N_k is Poisson with parameter $\lambda_k T$. (We restrict ourselves to time-related failures in the discussion that follows; however, the analysis is identical for demand-related failures with T replaced by total system demands and λ_k viewed as the probability of k specific units failing on demand. In this case one assumes that there are N_D independent system demands and consequently $M{\bullet}N_D$ unit demands. If these assumptions are violated as in staggered sampling plans (Parry, G. W., 1986), the estimators derived in this paper are not valid, and one must examine the parameter definitions to determine the appropriate estimators.) When a single unit fails, it results from either an underlying event (a potential common cause failure), or from causes restricted to that unit alone (an independent failure). The independent failure rate is denoted λ, and the number of independent failures in time T is assumed to be Poisson with parameter λT. We are interested in calculating the probabilities that in time t there are events, E_k, $k = 1, ..., M$. These are the probabilities needed for the fault tree analysis and are given by

$$P_k = 1 - e^{-\lambda_k t}, \quad k = 1, ..., M .$$

In the Introduction we noted that common cause events range from failures resulting from design flaws and environmental factors to maintenance errors. The grouping together of these diverse common cause events is necessary because of the paucity of data. As common cause data bases improve, factor analysis studies should be employed to determine more appropriate common cause groupings.

The basic parameter (BP) and multiple Greek (MLG) methods are based on this Poisson model. These techniques require estimating λ_k, $k = 1, ..., M$. The binomial failure rate model (Atwood, 1980) restricts this general model, assuming that the basic events or shocks occur with constant rate, μ, and that, given a shock, the units fail independently, each with probability p. These assumptions reduce the number of parameters to estimate to λ, μ, and p. The λ_k are given by

$$\lambda_1 = \lambda + \mu\, p\, (1-p)^{M-1}$$

and

$$\lambda_k = \mu\, p^k (1-p)^{M-k}, \quad k = 2, ..., M . \qquad (2.0.1)$$

Both maximum likelihood binomial failure rate (BFR) and Bayes (BFRBM - BFR Bayes Mode or Mean) solutions can be found for the parameters of this model. (In this study, only the Bayes mode is evaluated). Atwood (1980) found that this model underestimated the probability of all M units failing and introduced the notion of lethal shocks. A lethal shock causes all M units to fail simultaneously; that is, given a lethal shock, $p = 1$. The rate of lethal shocks, ω, is assumed to be constant, giving $\lambda_M = \mu p^M + \omega$. This model has four parameters to estimate, and we denote these as the BFRLS (maximum likelihood – BFR Lethal Shocks) and BFRBMLS (BFR Bayes Mode Lethal Shocks) estimators. When potential common cause failures cannot be identified, we must restrict the cases to $M > 2$. However, if the potential common cause failures can be identified, this restriction is not necessary and the estimators are called BFRSF (maximum likelihood – BFR Single Failures) and BFRBMSF (BFR Bayes Mode Single Failures). If both potential common cause failures and lethal shocks can be identified, we denote the estimators as BFRSFLS (maximum likelihood – BFR Single Failures and Lethal Shocks) and BFRBMSFLS (BFR Bayes Mode Single Failures and Lethal Shocks).

3.0 ESTIMATORS

This analysis compares several of the common cause estimation techniques that have been suggested for use in PRA analyses. These techniques include basic parameter (BP), multiple Greek (MLG), various binomial failure rate estimators, the β-factor, and the C-factor estimators. It is easy to show that, in the case of data from systems with the same number of units (M), the MLG and BP techniques are equivalent. In Appendix B, we show that, for $M = 3$, the MLG, BP, and BFR estimators are equal.

3.1 Basic Parameter

The BP estimation technique uses the maximum likelihood estimators (MLEs) for the Poisson model described in 2.0. In this case the number of events E_k, where k specific units fail simultaneously in time T, is Poisson with parameter $\lambda_k T$. The MLEs for λ_k are

$$\hat{\lambda}_k = \frac{n_k}{\binom{M}{k} T}, \quad k = 2, \ldots, M . \qquad (3.1.1)$$

The n_k are the observed number of events with k units failing simultaneously and T is the total system operational time. This technique allows zero estimates if there are no observed failures. To avoid the problem of zero estimates, Bayes estimators can be used.

One approach is to assume that the λ_k's have gamma prior distributions with parameters *a* and *b*. The resulting estimators are

$$\hat{\lambda}_k = \frac{n_k + a}{\binom{M}{k} T + b} \quad , \; k = 2, \ldots, M \; . \qquad (3.1.2)$$

For the noninformative prior, $a = 1/2$ and $b = 0$. In the simulation stud;·, this estimator is denoted BP Bayes NI. Other priors can be used. Welker and Lipow (974) suggest an iterative technique for deriving priors beginning with a simple prior joini ; 0 and $2T$ and deriving the posterior. The posterior is then used as a prior and the pro:ess continues. We investigated one of these estimators that is equivalent to setting $a = 0.175$ and $b = 0$. This estimator is denoted BP Bayes 0.175.

3.2 Multiple Greek

The BP estimators require knowledge of the total system operational time, T. The MLG estimators were derived to avoid the need to know T or, in the case of demand-related data, the total number of system demands. Fleming (Picard, Lowe, & Garrick, 1985a) uses a four-unit system to illustrate the use of the MLG estimators. The parameters of interest are

λ_T = the failure to operate rate for each unit resulting from all independent and common cause events,

β = the conditional probability that two or more units will fail resulting from a common cause given that there is a failure,

γ = the conditional probability that three or more units will fail resulting from a common cause given that two or more units fail resulting from a common cause, and

δ = the conditional probability that all four units fail resulting from a common cause given that three or more units fail resulting from a common cause.

In terms of the λ_k's defined in section 2.0, the MLG parameters can be written

$$\lambda_T = \lambda_1 + 3\lambda_2 + 3\lambda_3 + \lambda_4 \ ,$$

$$\beta = \frac{3\lambda_2 + 3\lambda_3 + \lambda_4}{\lambda_1 + 3\lambda_2 + 3\lambda_3 + \lambda_4} \ ,$$

$$\gamma = \frac{3\lambda_3 + \lambda_4}{3\lambda_2 + 3\lambda_3 + \lambda_4} \ ,$$

and

$$\delta = \frac{\lambda_4}{3\lambda_3 + \lambda_4} \ .$$

In these equations, Fleming uses rates to represent probabilities. This simplification is justified since the events are rare (λ_k's small). The λ_k's can be written in terms of the MLG parameters,

$$\lambda_1 = (1 - \beta)\,\lambda_T \ ,$$

$$\lambda_2 = 1/3\,(1 - \gamma)\,\lambda_T\,\beta \ ,$$

$$\lambda_3 = 1/3\,(1 - \delta)\,\lambda_T\,\beta\,\gamma \ ,$$

and

$$\lambda_4 = \lambda_T\,\beta\,\gamma\,\delta \ .$$

One can see where the name multiple Greek comes from; clearly large *M* necessitates a change of notation. In general,

$$\lambda_j = \frac{1}{\binom{M-1}{j-1}}\,[1 - \mathrm{MLG}(j+1)]\prod_{i=1}^{j}\mathrm{MLG}(i) \ , \qquad (3.2.1)$$

where MLG (j) is the jth MLG parameter [MLG (1) = λ_T, MLG (2) = β, MLG (3) = γ, and MLG (4) = δ]. Using the generalized notation, the estimators for the MLG parameters are

$$\widehat{MLG}(k) = \frac{\sum_{j=k}^{M} j\, n_j}{\sum_{j=k-1}^{M} j\, n_j} \quad \text{for } k = 2, \ldots, M$$

and

$$\widehat{MLG}(1) = \frac{\sum_{j=1}^{M} j\, n_j}{MT} \quad , \tag{3.2.2}$$

where the n_j are the number of common cause failures where j units fail simultaneously.

Substituting the MLG estimators of Eq. (3.2.2) into the equation for the λ_k's, Eq. (3.2.1), it is easily seen that the MLG and BP estimators are identical. However, these estimators can differ when there are multiple systems and the number of units varies between systems.

In many applications MLG(1) or λ_T is known; therefore, total system operational time or total number of system demands need not be specified. In all the cases we considered, MLG estimation with known λ_T gave either no improvement or very slight improvement over the general MLG estimators; therefore, it is not discussed further.

The MLG estimators also permit zero estimates when there are no observed failures. To avoid this problem, Fleming (Picard, Lowe & Garrick, 1985a) suggests Bayes estimation. The Bayes estimators are determined by assuming a multinomial prior distribution. He illustrates the technique for a system with three units. The prior distribution is

$$f(\beta,\gamma) = h\, \beta^{A-1} (1-\beta)^{B-1} \gamma^{C-1} (1-\gamma)^{D-1} \quad ,$$

where h is a normalizing factor. The posterior distribution is also multinomial, and the Bayes estimators are

$$\hat{\beta} = \frac{A + \sum_{j=2}^{M} j\, n_j}{A + B + \sum_{j=1}^{M} j\, n_j}$$

and

$$\hat{\gamma} = \frac{C + 3\, n_3}{C + D + \sum_{j=2}^{M} j\, n_j} \quad .$$

As is often the case with Bayes estimation, the problem of what prior to use arises. Fleming suggests using the noninformative prior : $A = B = C = D = 1$.

3.3 Binomial Failure Rate

The BFR model for modeling common cause failures in a system was introduced by Vesely (1977) and further developed and applied by Atwood (1980, 1982, 1983a,b,c). In this paper, we will not go into great detail about the BFR estimators since the mathematics are complex and are described in detail in Atwood's 1980 paper "Estimators for the Binomial Failure Rate Common Cause Model." For the purposes of illustration and comparison, we present the MLEs for the basic case (potential common cause and lethal shocks are not identified), and briefly describe the Bayes estimation techniques.

3.3.1 Maximum Likelihood Estimators for the Binomial Failure Rate Model

Atwood introduces the parameters λ_s – the rate of single failures (which he calls λ_1) – and λ_+ – the rate of common cause occurrences. These rates are defined as

$$\lambda_s = M\lambda + \mu\, r_1 \qquad (3.3.1.1)$$

and

$$\lambda_+ = \mu\,(1 - r_0 - r_1) \ , \qquad (3.3.1.2)$$

where μ is the rate of common cause shocks, λ is the rate of independent failures respectively, r_0 is the probability of no common cause failures given a shock,

$$r_0 = (1-p)^M \quad ,$$

and r_1 is the probability of one and only one failure given a shock,

$$r_1 = M\,p\,(1-p)^{M-1} \quad .$$

If $M > 2$, the MLEs for μ, λ, and p are found by reparameterizing the likelihood function in terms of λ_s, λ_+, and p, solving for these quantities and using Eqs. (3.3.1.1) and (3.3.1.2) to determine μ and λ. The restriction $M > 2$ is not necessary if potential common cause failures can be identified. The MLEs for λ_s and λ_+ are

$$\hat{\lambda}_s = \frac{n_1}{T} \qquad (3.3.1.3)$$

and

$$\hat{\lambda}_+ = \frac{n_+}{T} \quad , \qquad (3.3.1.4)$$

where n_1 is the number of single failures and n_+ is the number of common cause events,

$$n_+ = \sum_{j=2}^{M} n_j \quad .$$

The MLE for p is the unique solution of

$$s = M\,n_+\,p\,\frac{1 - q^{M-1}}{1 - r_0 - r_1} \quad , \qquad (3.3.1.5)$$

where $q = (1\text{-}p)$ and s is the total number of unit failures due to common cause events,

$$s = \sum_{j=2}^{M} j\,n_j \quad .$$

Atwood (1980) shows that for $s = 2n_+$, $\hat{p} = 0$. This result seems contradictory since common cause failures have occurred. The situation arises when the only common cause events observed are those with two units failing. This condition was quite common in the simulation study and in these cases, rather than using $p = 0$, we say the estimators cannot be evaluated. If potential common cause events can be identified, the definitions of s, λ_+, and n_+ are modified appropriately (Atwood, 1980, p.39), and this problem does not arise. When lethal shocks can be identified, the estimators are the same as those described in Eqs. (3.3.1.3), (3.3.1.4), and (3.3.1.5), except that the number of lethal shocks, n_L, is subtracted from n_M and

$$\hat{\omega} = \frac{n_L}{T} .$$

Atwood also develops confidence intervals for these estimators (Atwood, 1980, pp.10-16).

3.3.2 Bayes Binomial Failure Rate Estimators

The Bayesian estimation is much more complicated than the MLE technique. Atwood selects gamma distributions as a suitable class of priors for λ, λ_+, and ω:

$$f(\lambda_+) = \frac{b_+ \lambda_+^{a_+ - 1} e^{-b_+ \lambda_+}}{\Gamma(a_+)} ,$$

$$f(\lambda_s) = \frac{b_s \lambda_s^{a_s - 1} e^{-b_s \lambda_s}}{\Gamma(a_s)} ,$$

and

$$f(\omega) = \frac{b_\omega \omega^{a_\omega - 1} e^{-b_\omega \omega}}{\Gamma(a_\omega)} .$$

The prior for p is a beta distribution defined as

$$f(p) = \frac{\Gamma(c + d)}{\Gamma(c)\,\Gamma(d)} p^{c-1} (1-p)^{d-1} .$$

Using the modes of the posterior distributions to determine the Bayes estimators give

$$\hat{\omega} = \frac{n_L + a_w - 1}{T + b_w} ,$$

$$\hat{\lambda}_s = \frac{n_s + a_s - 1}{T + b_s} ,$$

and

$$\hat{\lambda}_+ = \frac{n_+ + a_+ - 1}{T + b_+} .$$

The estimator for p is the unique solution of

$$s + c - 1 = p \left\{ c + d - 2 + M n_+ \frac{1 - q^{M-1}}{1 - q^M - M p q^{M-1}} \right\}.$$

If the number of observed single failures is too small, it can lead to a negative estimate for λ. In these cases the value of p that maximizes the likelihood function must be found by a numerical procedure such as a simplex search.

The means of the posterior distributions give the following Bayes estimators:

$$\hat{\lambda}_s = \frac{n_s + a_s}{T + b_s} ,$$

$$\hat{\lambda}_+ = \frac{n_+ + a_+}{T + b_+} ,$$

and

$$\hat{\omega} = \frac{n_L + a_\omega}{T + b_\omega} .$$

The mean of the posterior distribution of p can only be found by numerical integration. The formulas can be quite difficult to evaluate. In this study, we do not report these estimators; however, there is some indication that the Bayes mean estimators will be slightly more conservative than either the MLEs or Bayes mode estimators (Atwood, 1980, pp. 50-54). The results of our simulation indicate that the differences between the Bayes mode estimators using a noninformative prior and the MLEs are not important in practical applications.

3.4 β-Factor

The β-factor was introduced by Fleming (1975) and is widely used in common cause estimation in PRA analyses (American Nuclear Society and the Institute of Electronic and Electrical Engineers, 1983). This estimator is valid for systems with two units and adequate for systems with three units, but overestimates the P_k's as the number of units increases. The method is popular because it is conservative and allows subjective estimates of the β-factor when data is not available. The β-factor is defined as the ratio of the rate of unit failures resulting from common cause events, λ_c, to the total unit failure rate, λ_T;

$$\beta = \frac{\lambda_c}{\lambda_T}.$$

Note that λ_c is not the rate of common cause events. The parameter, λ_T, is the sum of the common cause failure rate and the independent failure rate, $\lambda_T = \lambda_c + \lambda_I$. The β-factor can be viewed as the percentage of total failures resulting from common cause events or the conditional probability that a failure is owing to a common cause event, given that a failure has occurred. The estimate for the β-factor is

$$\hat{\beta} = \frac{N_c}{N_T},$$

where N_T is the total number of failures and

$$N_C = \sum_{j=2}^{N} j\, n_j \quad \text{[s in the BFR equations – (3.3.1.5)].}$$

The β-factor estimator does not distinguish between multiple failures, providing only one estimate for the failure rates;

$$\hat{\lambda}_k = \hat{\beta}\, \lambda_T = \frac{N_c}{M\,T},\ k = 2, \ldots, M\ .$$

If the total failure rate is known, then the common cause failure rate is

$$\lambda_k = \beta\, \lambda_T,\quad k = 2, \ldots, M \qquad (3.4.1)$$

and

$$\hat{\lambda}_k = \frac{N_c}{N_T}\lambda_T,\ k = 2, \ldots, M\ .$$

We evaluated the β-factor estimator with known λ_T in the simulation study; however, it gave only slightly improved results and is not discussed further. The appeal of the β-factor technique is understood by noting that when no common cause data are available, but the unit failure rate is known, only one parameter (β) has to be estimated by "expert opinion." Given the cost and effort involved in eliciting experts' estimates, it is much more practical to ask for one estimate from each expert than several (Meyer et. al., 1982).

3.5 C-Factor

There is more confusion over this estimator than any of the other estimators. We believe that we have uncovered the source of the confusion. This technique was introduced in the Ringhals 2 Probabilistic Safety Study (Gyllenbaga et.al, 1983). In this study the C-factor is defined as "the ratio of the number of common cause events to the number of independent failure events." The authors claim that "it is more correct since it predict(sic) directly the item of interest, the probability (or rate) of multiple failure events." Using the Ringhals notation, these statements indicate that

$$C = \frac{\lambda_E T}{\lambda_I T} = \frac{\lambda_E}{\lambda_I}$$

and

$$\lambda_E = C\,\lambda_I\ ,$$

where λ_E is the rate of multiple failure occurrences (λ_+) and λ_I is the rate of independent failures (λ). This definition leads to the estimators

$$\hat{C} = \frac{\frac{N_E}{T}}{\frac{N_I}{MT}} = \frac{M N_E}{N_I} \qquad \text{and} \quad \hat{\lambda}_k = \hat{\lambda}_E = \frac{N_E}{T} = \sum_{j=2}^{M}\frac{n_j}{T},\ \ k = 2, \ldots, M\ , \qquad (3.5.1)$$

where N_E is the total number of common cause events ($N_E = n_+$), N_I is the number of independent failures, and M is the total number of units in the system. However, the authors define $\hat{C}$ as

$$\hat{C} = \frac{N_E}{N_I} ,$$

and this leads to

$$\hat{\lambda}_k = \hat{\lambda}_E = \frac{N_E}{MT} = \sum_{j=2}^{M} \frac{n_j}{MT} , \quad k = 2, \ldots, M ,$$

which does not agree with the authors definition. The expression in Eq. (3.5.1) does indeed lead to an estimate of the probability of multiple failure events; however, since $\sum_{j=2}^{M} \frac{n_j}{T} \geq \sum_{j=2}^{M} \frac{jn_j}{MT}$ (the β-factor estimate), this estimator does not accomplish the authors' goal of reducing the large positive bias that the β-factor method produces.

To add to the confusion, some practitioners have mistakenly defined the C-factor as

$$C = \frac{\lambda_c}{\lambda_I} ,$$

where λ_c is the rate of failures resulting from common cause events (see Sec. 3.4). Using this definition and solving for C in terms of β, one finds

$$C = \frac{\beta}{1-\beta} .$$

This definition of C leads to the same estimators for the common cause failure rates, λ_k, as the β-factor method. Thus, there is only one useful definition for the C-factor – Eq. (3.5.1). This definition leads to estimators of the common cause failure probabilities that are greater than the β-factor estimators; therefore, we do not discuss the C-factor method further.

4.0 ANALYSIS

Since actual plant data is extremely sparse, a simulation was developed to generate common cause failure data that could then be used to determine the behavior of the various estimators. Three different models for failure generation were used. The first (Simulation I) generates data using the BFR model for M = 3, 5, and 9 for cases where the data is very sparse (reflecting the real world situation) and for moderate and large samples. The second (Simulation II) uses the general model to generate the samples and mimics the situation where the failures deviate from the binomial model by having higher incidences of two, three, and M failures. The third (Simulation III) allows for variation between plants, assuming that the data comes from three different plants and that the λ_k's vary between plants.

4.1 SIMULATION I (BFR MODEL)

4.1.1 Data Generation

To generate data for the BFR model, three values of p were used – p = 0.1, 0.5, and 0.9. The rates of shocks to the system, μ, were 10^{-3}, 2×10^{-4}, and 10^{-4}. Two lethal shock rates, $\omega = 0$ (no lethal shocks) and $\omega = 10^{-5}$, were considered. The independent failure rates were 10^{-3} and 10^{-4}. The total operational times, T, were 10^4, 10^5, and 10^6 hours. For each M and each combination of parameters, failure data was produced using 1000 iterations of the Monte Carlo simulation. Not all combinations of parameters produced sufficient samples for analysis. For example, for $M = 3$, $p = 0.1$, $T = 10^4$, $\mu = 10^{-4}$, and $\omega = 0$, there were no common cause failures in the 1000 iterations. If a combination of parameters produced less than 200 samples, it was eliminated from the analysis.

Tables 1 and 2 give example summaries of the data generated. Table 1 illustrates a simulation with sparse data sets. The parameters used to generate this data were $M = 3$, $p = 0.1$, $\mu = 10^{-4}$, $\lambda = 10^{-4}$, $T = 10^5$, and $\omega = 10^{-5}$. Out of 1000 iterations, 762 had no common cause failures. Of the remaining 238 cases, the number of potential common cause failures (k = 1) varied from 0 to 8, with 28 cases of no potential failures. The average number of potential common cause failures was 2.4. The number of common cause failures where two units failed simultaneously (k = 2) varied between 0 and 3. There were 9 cases of no failures, and the average number of failures was 1. There were only 12 data sets that had three units (k = 3) failing as a result of common causes other than lethal shocks. In each case, there was only one such failure. The number of

Table 1. COMMON CAUSE FAILURE GENERATION A SPARCE DATA SET

Total number of iterations = 1000
Number of cases with no common cause failures = 762
Number of cases (out of 238) where $s = 2n_+ = 83$

238 Cases with Common Cause (CC) Failures				
k	Average # of failures	Minimum # of CC failures of k units	Maximum # of CC failures of k units	# of times no CC failures of k units
1	2.4	0	8	28
2	1.1	0	3	9
3	0.05	0	1	226
Independent failures	30.0	17	46	0
Lethal shock failures	1.0	0	5	85

Example of sparse data set generated by M = 3, p = 0.1, μ = 10E-4, T = 10E5, λ = 10E-4, and ω = 10E-5.

Table 2. COMMON CAUSE FAILURE GENERATION A LARGE DATA SET

Total number of iterations = 1000
Number of cases with no common cause failures = 54
Number of cases (out of 946) where $s = 2n_+ = 0$

946 Cases with Common Cause (CC) Failures				
k	Average # of failures	Minimum # of CC failures of k units	Maximum # of CC failures of k units	# of times no CC failures of k units
1	24.4	11	41	0
2	3.0	0	8	3
3	0.1	0	2	847
Independent failures	300	244	362	0
Lethal shock failures	10	1	22	0

Example of large data set generated by M = 3, p = 0.1, μ = 10E-4, T = 10E6, λ = 10E-4, and ω = 10E-5.

independent failures ranged from 17 to 46, with an average of 30. The number of lethal shocks varied from 0 to 5, with 85 cases of no lethal shocks. For this data set, the BFR and BFRBM estimators could not be calculated since there were 83 cases where $s = 2n_+$ (leaving only 155 cases for estimating these estimators). Because most of the cases with three units failing were the result of lethal shocks, the BFRLS and BFRBMLS estimators, which analyze lethal shocks separately, had only 12 cases where they could be calculated.

Table 2 illustrates a simulation that generated data sets with a large number of common cause failures. The parameters are identical to those in Table 1, except $T = 10^6$. Out of the 1000 iterations, 946 had common cause failures. The number of potential common cause failures ranged from 11 to 41, and the number of common cause failures with two units failing ranged from 0 to 8, with only 3 data sets having no two-unit failures. Common cause failures of three units ranged from 0 to 2, with 847 cases of zero failures. The number of lethal shock failures ranged from 1 to 22; therefore, when lethal shocks were not identified, there were no cases where $s = 2n_+$. However, when lethal shocks were analyzed separately (BFRLS and BFRMLS), there were only 99 data sets where s was not equal to $2n_+$.

For each case, each combination of parameters, and each estimation technique, there are $k = 1, \ldots, M$ probabilities (P_k) to estimate. These are the M probabilities of the event that k specific units fail. To evaluate the estimation methods, the biases and variances of the parameter estimators were determined and compared. The biases were approximated by

$$\text{BIAS}(\hat{P}_k, \varepsilon) = \frac{1}{R}\sum_{i=1}^{R}(\hat{P}_k(i, \varepsilon) - P_k) ,$$

where $\hat{P}_k(i, \varepsilon)$ is the i^{th} replicate of the estimator for P_k using estimation technique ε, and R is the total number of replications. The variances were approximated by the difference of the mean square error and the square of the bias,

$$\text{VAR}(\hat{P}_k, \varepsilon) = \frac{1}{R}\sum_{i=1}^{R}(\hat{P}_k(i, \varepsilon) - P_k)^2 - \text{BIAS}^2(\hat{P}_k, \varepsilon) .$$

4.1.2 Analysis of Variance

For each M, ω, and k, an analysis of variance (ANOVA) was performed to determine what factors (estimation technique, p, T, μ, λ) were important in determining differences between the BIASs and VARs. Because interactions of these factors influence sample size, the interaction terms were also considered. Tables 3 and 4 are representative of the resulting ANOVA tables. Although there was some variation in which interaction terms were significant, all the ANOVAs showed that all factors except λ (independent failure rate) were important in determining differences in BIAS and VAR. Because interaction terms and all factors other than λ were significant, each case was examined to determine where there were important differences between estimation techniques. To identify the important differences, the ratios of BIAS to probability of failure (BIASR) and the square root of VAR to the average of the probability estimators (VARR – an estimate of relative variance) were determined. BIASR and VARR are defined as

$$\mathrm{BIASR}(\hat{P}_k, \varepsilon) = \frac{\mathrm{BIAS}(\hat{P}_k, \varepsilon)}{P_k}$$

and

$$\mathrm{VARR}(\hat{P}_k, \varepsilon) = \frac{\sqrt{\mathrm{VAR}(\hat{P}_k, \varepsilon)}}{\bar{P}_k(\varepsilon)} ,$$

where

$$\bar{P}_k = \frac{1}{R}\sum_{i=1}^{R} \hat{P}_k(i, \varepsilon)$$

The cases where the value of BIASR was greater than 4 or less than -0.6, or VARR was greater than 2 were identified. The estimators were considered to remain within reasonable bounds if they did not exceed these limits. The estimators that were in reasonable bounds were within an order of magnitude of the true failure probability when they overestimated, or within a factor of 5 when they underestimated, in 95% or more of the samples.

Table 3. ANALYSIS OF VARIANCE FOR BIAS

$M = 3$, $k = 2$, and $\omega = 10^{-5}$

Dependent Variable: BIAS

SOURCE	DF	SUM OF SQUARES	MEAN SQUARE	F-VALUE	PR>F	R^2
Model	86	313	3.64	27	0.0	.88
Error	329	44	0.13			
Corrected Total	415	357				

SOURCE	DF	TYPE 1 SS	F-VALUE	PR>F
λ	1	0.005	0.04	0.8474
μ	2	8.953	33.28	0.0001
p	2	116.185	431.90	0.0000
T	2	10.578	39.32	0.0001
p*T	4	29.306	54.46	0.0001
Estimator	8	25.996	24.16	0.0001
Estimator*p	16	50.548	23.49	0.0000
Estimator*T	16	16.228	7.54	0.0001
Estimator*p*T	27	13.100	3.61	0.0001
T* μ	4	2.719	5.05	0.0006
p* μ	4	39.738	73.86	0.0000

Table 4. ANALYSIS OF VARIANCE FOR VAR

$M = 3$, $k = 2$, and $\omega = 10^{-5}$

Dependent Variable: VAR

SOURCE	DF	SUM OF SQUARES	MEAN SQUARE	F-VALUE	PR>F	R^2
Model	95	68	0.729	56	0.0	.94
Error	370	4	0.012			
Corrected Total	465	73				

SOURCE	DF	TYPE 1 SS	F-VALUE	PR>F
λ	1	0.013	1.01	0.3149
μ	2	5.712	264.94	0.0000
p	2	3.564	140.70	0.0000
T	2	37.504	1480.40	0.0000
p*T	4	5.795	114.36	0.0000
Estimator	9	4.219	37.01	0.0000
Estimator*p	18	1.101	4.83	0.0001
Estimator*T	18	3.899	17.10	0.0001
Estimator*p*T	31	1.119	2.85	0.0001
T* μ	4	3.569	70.44	0.0000
p* μ	4	0.900	17.76	0.0001

4.1.3 RESULTS

After sifting through a large amount of data, we made the following observations. For $M = 3$, all of the estimators (when they could be calculated) gave reasonable results (as described above). There was one exception to this: the case $p = 0.1$, $T = 10^5$, and $\omega = 0$. In this case the bias ratios for the β-factor technique ranged from 18 to 79 (for various T, k, and μ), and for the BP Bayes and MLG Bayes with noninformative priors, they ranged from 13 to 25. In these cases there were very few observations of three-unit common cause failures. The BFR, BFRLS, BFRBM, and BFRBMLS estimators could not be calculated because $s = 2n_+$ in the majority of cases. The MLG and BP techniques had the smallest BIASRs and performed better than the BFRBMSF and BFRBMSFLS estimators, which performed better than the BFRSF and BFRSFLS estimators. However, these differences were not important. The VARRs were all within the prescribed bound.

For $M = 5$ and 9 (regardless of technique), the estimators with the largest BIASRs and VARRs were those that tried to estimate events with very small probabilities. Thus, for $p = 0.1$, BIASRs and VARRs increased as the number of units failing (k) increased, and for $p = 0.9$, they increased as k decreased. BIASRs and VARRs remained within reasonable bounds for $p = 0.5$, except for the β-factor technique, which had BIASRs that ranged from 14 to 26 for all values of k. These relationships remained true across μ, although the smaller values of μ gave larger BAISRs and VARRs. BIASRs and VARRs were larger for the lethal shock cases ($\omega = 10^{-5}$) than for no lethal shocks ($\omega = 0$), except for $k = M$.

Tables 5 and 6 report the worst case results for $M = 5$. Table 5 summarizes the ranges of BIASRs across T and Table 6 summarizes the ranges of VARRs. In all other cases, all estimators, except for the β-factor, remained within reasonable bounds. The β-factor technique seriously overestimated the failure probabilities in many other cases. In Table 5, for $T = 10^4$ and 10^5, most of the failures for $k > 2$ resulted from lethal shocks; therefore, the BFRLS and BFRBMLS techniques could not be evaluated (too many replications with $s = 2n_+$). The BFR and BFRBM techniques had the same problem when there were no lethal shocks. For $p = 0.9$ and $k = 2$, the β-factor method yielded extremely large BIASRs, ranging from 1100 to 1800. In this case, the noninformative Bayes estimation for MLG and BP techniques produced large BIASRs ranging from 6 to 46 and 8 to 89. The BP Bayes 0.175 estimator had somewhat smaller BIASRs. All the other estimators had BIASRs less than 3.

When failures were observed, BIASRs were consistently smallest for the MLG and

Table 5. BIAS RATIOS (BIASR) – M = 5

ESTIMATION TECHNIQUE	NUMBER OF UNITS IN CC FAILURE (k) AND P_k			
	p = 0.9	p = 0.1		
	$\omega = 10^{-5}$	$\omega = 10^{-5}$		$\omega = 0$
	(2) 1.9E-6	(3) 1.9E-6	(4) 2.2E-7	(5) 2.4E-8
β - Factor	1100 – 1800	160 – 190	1500 – 1700	3500 – 6500
BFR	< 3	5 – 9	147 – 207	7 *
BFRSF	< 3	10 – 11	63 – 96	0.7 – 26
BFRLS	< 3	0.6 *	2.2 *	7 *
BFRSFLS	< 3	< 2	< 5	0.7 – 26
BFRBM	< 3	5 – 7	143 – 171	4 *
BFRBMSF	< 3	10 – 11	62 – 86	0.5 – 15
BFRBMLS	< 3	0.1*	1.1 *	2 *
BFRBMSFLS	< 3	< 1	< 3	0.5 – 15
MLG	< 2	< 1	< 1	-1.0 – 1.0
MLG BAYES NI	6 – 46	< 4	< 5	69 – 420
BP	< 2	< 1	< 1	-1.0 – 1.0
BP BAYES NI	8 – 89	< 6	10 – 110	500 – 5000
BP BAYES 0.175	3 – 18	< 6	3 – 20	175 – 1750

Note: the worst case ranges for the BIASRs, p = 0.1 and $\mu = 10^{-4}$ for k = 3, 4, and 5 and p = 0.9 for k = 2. The independent failure rate was 10^{-4}.

* The only case with enough data for estimation is $T = 10^6$.

Table 6. RELATIVE VARIANCE (VARR) – M = 5

ESTIMATION TECHNIQUE	NUMBER OF UNITS IN CC FAILURE (k) AND P_k			
	p = 0.9	p = 0.1		
	$\omega = 10^{-5}$	$\omega = 10^{-5}$		$\omega = 0$
	(2) 1.9E-6	(3) 1.9E-6	(4) 2.2E-7	(5) 2.4E-8
β - Factor	< 2	< 2	< 2	< 2
BFR	< 6	< 2	< 6	14 *
BFRSF	< 6	< 2	< 6	12 – 13
BFRLS	< 6	< 2 *	6 *	14 *
BFRSFLS	< 6	< 2	< 6	12 – 13
BFRBM	< 8	< 2	< 8	14 *
BFRBMSF	< 8	< 2	< 8	12 – 13
BFRBMLS	< 8	< 2 *	6 *	14 *
BFRBMSFLS	< 6	< 2	8 – 10	12 – 13
MLG	9 – 13	< 2	12 – 21	9 – 45
MLG BAYES NI	< 2	< 2	< 5	< 5
BP	9 – 13	< 2	8 – 10	9 – 45
BP BAYES NI	< 2	< 2	< 5	< 5
BP BAYES [illegible]5	< 2	< 2	< 5	< 5

Note: the worst ca e ranges for the VARRs, p = 0.1 and $\mu = 10^{-4}$ for k = 3, 4, .nd 5 and p = 0.9 for k = 2. The independent failure rate was 10^{-4}.

* The only case with enough data for estimation is $T = 10^6$.

BP estimators. However, failures were not observed in many of the samples from the case $\omega = 0$. This condition led to zero estimates and BIASRs of -1. When Bayes estimation was used to counter this problem, the Bayes estimators seriously overestimated the failure probabilities.

In most cases, the estimators tended to be conservative and overestimate the failure probabilities. The exception to this rule was $\omega = 10^{-5}$, $k = 5$, and was not reported in the tables. In this case, the BFRSF and BFRBMSF techniques underestimated the failure probabilities by at least an order of magnitude. The other BFR techniques also had negative biases, but they remained within a factor of 5 of the failure probability. (The BFRLS and BFRBMLS could only be calculated for $T = 10^6$). The BFR Bayes estimators also had negative biases for $p = 0.9$ and $k = 3$ and 4; however, they remained within a factor of 5 of the true values.

For $M = 9$, all of the estimation techniques, apart from the β-factor estimators, were within reasonable bounds, save for those cases reported in Tables 7 and 8. The results are similar to $M = 5$, except for the much larger BIASRs. Again, most of the estimators overestimated those events with very small probabilities – the events with little or no data. The β-factor method displayed extreme biases with ratios as large as 10^8. Once more, when failures were observed, the best estimators were BFRSFLS, BFRBMSFLS, MLG, and BP. However, in many cases the failure probabilities were so small that there were no observed failures, and the BP and MLG estimators were zero. In such cases, the BFR estimators were much better than the other techniques at estimating these small failure probabilities.

4.2 SIMULATION II

The data generated in Simulation I followed the BFR model. The purpose of Simulation II was to investigate the behavior of the estimators when the data deviated from this model. A condition observed in practice is one where the failures appear to follow a BFR model, but there are increased numbers of two, three, and M failures. The failure data of Simulation II was generated using the worst-case parameters of Simulation I, $p = 0.1$, $\mu = 10^{-4}$ and $\lambda = 10^{-4}$ for $M = 5$, and increasing the failure rates for $k = 2$, 3, and 5. Table 9 summarizes the P_k's used to generate the data. Although $M = 5$ does not have as severe biases as seen for $M = 9$, the trends are the same, and it was felt that the important differences between the estimators and their relative performances could be adequately studied with this value of M.

Table 7. BIAS RATIOS (BIASR) – M = 9

ESTIMATION TECHNIQUE	NUMBER OF UNITS IN CC FAILURE (k) AND P_k							
	p = 0.9			p = 0.1 *				
	(2) 1.9E-10	(3) 1.75E-9	(4) 1.6E-8	(5) 1.6E-8	(6) 1.75E-9	(7) 1.9E-10	(8) 2.2E-11	(9) ω = 0 2.4E-12
β-Factor	E7 **	E6 **	E5 **	E4 **	E5 **	E7 **	E8 **	E8 **
BFR	<0 – 206	<0 – 46	<0 – 12	79 – 84	740–910	E3–E4	E3–E4	5 – 5000
BFRSF	<0 – 201	<0 – 45	<0 – 12	44 – 61	160–380	E3–E4	E3–E4	1 – 64
BFRLS	>0 – 214	>0 – 48	<1 – 13	0.3 – 7	1 – 24	2 – 104	3 – 557	5 – 5000
BFRSFLS	>0 – 209	>0 – 48	<1 – 13	0.2 – 2	<0 – 4	0 – 9	1 – 23	1 – 64
BFRBM	<0 – 85	<0 – 19	<0 – 5	67 – 84	728–784	E3–E4	E3–E4	4 – 2000
BFRBMSF	<0 – 83	<0 – 19	<0 – 5	43 – 56	158–354	E2–E3	E3–E4	1 – 38
BFRBMLS	<0 – 88	>0 – 20	·1 – 5	0.1 – 3	1 – 12	1 – 54	2 – 290	4 – 2000
BFRBMSFLS	>0 – 85	>0 – 20	·1 – 5	0.1 – 1	>0 – 3	>0 – 6	1 – 14	1 – 38
MLG	·1	·1 – 1	·1 – <0	< 1	·1 – 1	·1	·1	·1
MLG BAYES NI	E3–E5	55–4000	3 – 217	2 – 13	26 – 130	E2–E3	E5	E4–E5
BP	·1	·1 – 1	·1 – <0	< 1	·1 – 1	·1	·1	·1
BP BAYES NI	E3–E5	80–8000	6 – 604	5 – 60	81 – 815	E3–E4	E4–E5	E6–E7
BP BAYES 0.175	E3–E4	29–2000	2 – 221	2 – 21	29 – 285	E2–E3	E4–E5	E6–E7

Note: the worst case ranges for the VARRs (p = 0.1 and $\mu = 10^{-4}$ for k = 5, 6, 7, 8, and 9 and p = 0.9 for k = 2, 3, and 4).

The notation <0 and >0 indicates very small numbers either slightly less than or slightly greater than zero.

* The only cases with enough data for estimation are $T = 10^5$ and $T = 10^6$.

** Order of magnitude

Table 8. RELATIVE VARIANCE (VARR) – M = 9

ESTIMATION TECHNIQUE	NUMBER OF UNITS IN CC FAILURE (k) AND P_k							
	p = 0.9			p = 0.1 *				
	(2) 1.9E-10	(3) 1.75E-9	(4) 1.6E-8	(5) 1.6E-8	(6) 1.75E-9	(7) 1.9E-10	(8) 2.2E-11	(9) ω = 0 2.4E-12
β-Factor	< 1	< 1	< 1	< 1	< 1	< 1	< 1	< 1
BFR	0.7 – 5	< 3	< 2	< 1	< 1	< 1	< 2	3 – 7
BFRSF	0.7 – 5	< 3	< 2	< 1	< 2	< 2	< 4	2 – 6
BFRLS	0.7 – 6	< 3	< 2	< 1	< 2	< 3	< 5	3 – 7
BFRSFLS	0.7 – 6	< 3	< 2	< 1	< 2	< 3	< 4	2 – 6
BFRBM	0.7 – 6	< 4	< 3	< 1	< 1	< 1	< 2	3 – 8
BFRBMSF	0.7 – 6	< 4	< 3	< 1	< 2	< 2	< 4	2 – 6
BFRBMLS	0.7 – 6	< 4	< 3	< 1	< 2	< 3	< 5	3 – 8
BFRBMSFLS	0.7 – 6	< 4	< 3	< 1	< 2	< 3	< 4	2 – 6
MLG	**	** – 11	** – 11	3 – 9	** – 9	**	**	**
MLG BAYES NI	**	< 1	< 1	< 1	< 1	**	**	**
BP	**	** – 11	** – 11	3 – 9	** – 9	**	**	**
BP BAYES NI	**	< 1	< 1	< 1	< 1	**	**	**
BP BAYES 0.175	**	< 1	< 1	< 1	< 1	**	**	**

Note: the worst case ranges for the VARRs (p = 0.1 and $\mu = 10^{-4}$ for k = 5, 6, 7, 8, and 9 and p = 0.9 for k = 2, 3, and 4).

* The only cases with enough data for estimation are $T = 10^5$ and $T = 10^6$.

** All estimators identical (each replication has zero failures).

Table 9. THE PROBABILITIES USED TO GENERATE THE FAILURE DATA FOR SIMULATION II

	k				
	1	2	3	4	5
P_k	2.6E-3	2.4E-3	2.6E-5	2.2E-7	2.4E-4

Note: P_1 includes the probability of independent failures. These probabilities are for 24 hours of operation.

Table 10 reports the results for Simulation II. The BIASRs and VARRs are given for the three values of T. When only one number is reported, the values did not vary significantly across T. Once more, for $T = 10^4$, the BFR, BFRLS, BFRBM, and BFRBMLS techniques could not be evaluated because there were too many cases where $s = 2n_+$.

The maximum likelihood BFR techniques had negative BIASRs (-0.6) for $k = 2$. However, the VARRs are small, and the data showed that the estimators were generally within a factor of 5 of the true value. Although all of the BFR techniques underestimated the probability of P_2, there were no other serious biases or large VARRs.

For $k = 3$, the β-factor estimator overestimated P_3 by over two orders of magnitude for all T, and the BFR techniques overestimated this probability by more than an order of magnitude. The MLG and BP estimators were essentially unbiased, and the MLG Bayes and BP Bayes techniques had BIASRs less than 3. The VARRs were small except for the MLG and BP techniques.

The most serious biases were for $k = 4$. In this case, the β-factor estimator had a BIASR of 47,000, while most of the BFR estimators had BIASRs of 1000 or greater and did not improve with increased T. The MLG and BP estimators had many cases where there were zero estimates (no observed failures), and the MLG and BP Bayes techniques overestimated by one to two orders of magnitude.

For $k = 5$, the BFR and BFRBM techniques seriously underestimated P_5. The remaining estimators were always within reasonable bounds. These simulation results indicated that when the data deviated from the BFR model, the MLG and BP estimators

Table 10. SIMULATION II

ESTIMATION TECHNIQUE	NUMBER OF UNITS IN CC FAILURE (k) AND P_k							
	(2) P_k = 2.6E-3		(3) P_k =2.6E-5		(4) P_k = 2.2E-7		(5) P_k = 2.4E-4	
	BIASR	VARR	BIASR	VARR	BIASR	VARR	BIASR	VARR
β-Factor	< 3	< 1	400	< 1	47000	< 1	40	< 1
BFR *	-0.6	< 1	11	< 1	550	< 1	-0.8	< 1
BFRSF	-0.6	< 1	20	< 1	1400	< 1	-0.3	< 1
BFRLS *	-0.6	< 1	11	< 1	500	< 1	< 1	< 1
BFRSFLS	-0.6	< 1	19	< 1	1300	< 1	< 1	< 1
BFRBM *	-0.4	< 1	39	< 1	3800	< 1	-0.8	< 1
BFRBMSF	-0.4	< 1	18	< 1	1100–1300	< 2	-0.3	< 1
BFRBMLS *	-0.4	< 1	39	< 1	3700	< 1	< 2	< 1
BFRBMSFLS	-0.4	< 1	18	< 1	1200–1300	< 1	< 1	< 1
MLG	>0	< 1	>0	1 – 9	-1 – >0	4 – 15	< 0.1	< 4
MLG BAYES NI	< 0.5	< 1	< 2	< 1	5 – 149	< 1	< 0.5	< 1
BP	>0	< 1	>0	1 – 9	-1 – >0	4 – 15	< 0.1	< 4
BP BAYES NI	< 1	< 1	< 3	< 1	7 – 700	< 1	< 3	< 1
BP BAYES 0.175	< 0.5	< 1	< 1	< 1	3 – 300	< 1	< 1	< 1

Note: the notation <0 and >0 indicates very small numbers either slightly less than zero or slightly greater than zero.

* Cannot be evaluated for $T = 10^4$.

were preferable. If there were no observed failures, the BP Bayes and MLG Bayes estimators should be used.

4.3 SIMULATION III

Simulation III dealt with the problem of data from different systems. (In practice this usually means different plants). In the first case, the failure data was generated for three systems using the worst case parameters $p = 0.1$ and $T = 10^4$, increasing the failure rate for $k = 2$, and letting lethal shock rates (ω) and common cause shock rates (μ) vary for each system. The simulation parameters are described in Table 11. The data from all three systems were combined to estimate the VARRs and BIASRs for each estimator for each of the three plants. In this simulation, a plant with no common cause failures would still be included in the analysis if there were common cause failures from at least one of the three plants. Including plants with no common cause failures gives lower, more accurate estimates, but is not done in practice. Although data was combined from three plants, there were many cases where $s = 2n_+$ and the BFR, BFRLS, BFRBM, and BFRBMLS estimators could not be calculated.

Table 12 reports the worst-case results for this simulation. For this analysis T is fixed (10^4), and the worst-case results are for the system with the smallest common cause shock rate ($\mu = 10^{-4}$). Again the serious biases were for the small P_k 's ($k = 3$ and 4). The β-factor method had the largest BIASRs ranging from 534 to 4800. The BFR techniques had BIASRs ranging from one to two orders of magnitude. The MGL and BP estimators had very small biases,and the MLG and BP Bayes techniques had BIASRs ranging from 4 to 37. None of the techniques had large VARRs except for MLG and BP for P_4 where the VARR was 11. The data showed that the BFRSFLS, BFRBMSFLS, MLG and BP estimators overestimated the true probabilities and always remained within two orders of magnitude of the true value. The MLG and BP estimators (both MLE and Bayes) had a higher percentage of estimates within an order of magnitude of the true probability than did the BFRSFLS and BFRBMSFLS estimators.

The second case also used a modified BFR model to generate the failure data for the three plants; however, this time the three different systems had three different values for p (0.1, 0.5, and 0.9) and different lethal shock rates. The simulation parameters are reported in Table 11. The combined data was then used to evaluate BIASRs and VARRs for each estimator and each system. System 1 ($p = 0.1$ – the system with the least data) had estimators with large BIASRs as seen in Table 13. In this case, although the β-factor technique was the worst, all techniques overestimated the true probabilities. Excluding the β-factor, all estimators for system 2 ($p = 0.5$) had BIASRs less than 3 and VARRs

Table 11. FAILURE RATES USED TO GENERATE FAILURE DATA FOR SIMULATION III

k	Simulation III Case 1			Simulation III Case 2		
	System			System		
	1	2	3	1	2	3
1	6.5E-6	1.3E-5	6.5E-5	6.5E-6	3.1E-6	9.0E-9
2	1.0E-6	1.0E-5	5.0E-5	1.0E-5	1.0E-5	1.0E-6
3	8.1E-8	1.6E-7	8.1E-7	8.1E-8	3.1E-6	7.3E-7
4	9.0E-9	1.8E-8	9.0E-8	9.0E-9	3.1E-6	6.6E-6
5	1.0E-9	2.0E-9	1.0E-8	1.0E-9	3.1E-6	5.9E-5
Independent	1.0E-3	2.0E-4	1.0E-4	1.0E-3	2.0E-4	1.0E-4
Lethal shock	1.0E-7	1.0E-6	1.0E-5	1.0E-7	1.0E-6	1.0E-5

Table 12. SIMULATION III - CASE 1

ESTIMATION TECHNIQUE	NUMBER OF UNITS IN CC FAILURE (k) AND P_k			
	(3) P_k = 3.9E-6		(4) P_k = 4.3E-7	
	BIASR	VARR	BIASR	VARR
β-Factor	534	< 1	4800	< 1
BFR	30	< 1	122	< 1
BFRSF	24	< 1	87	< 1
BFRLS	*	*	*	*
BFRSFLS	22	< 1	71	< 1
BFRBM	33	< 1	121	< 1
BFRBMSF	22	< 1	76	< 1
BFRBMLS	*	*	*	*
BFRBMSFLS	20	< 1	62	< 1
MLG	< 2	3	< 2	11
MLG BAYES NI	4	< 1	28	< 1
BP	< 2	3	< 2	11
BP BAYES NI	10	< 1	37	< 1

* These estimators could not be calculated because $s = 2n_+$.

Table 13. SIMULATION III - CASE 2, SYSTEM 1 (p =0.1)

ESTIMATION TECHNIQUE	NUMBER OF UNITS IN CC FAILURE (k) AND P_k					
	(3) P_k = 1.9E-6		(4) P_k = 2E-7		(5) P_k = 2.4E-6	
	BIASR	VARR	BIASR	VARR	BIASR	VARR
β-Factor	900	< 1	8000	< 1	700	< 1
BFR	52	< 1	657	< 1	140	< 2
BFRSF	49	< 1	610	< 1	106	< 2
BFRLS	51	< 1	615	< 1	158	< 2
BFRSFLS	47	< 1	565	< 1	127	< 2
BFRBM	44	< 1	571	< 1	126	< 2
BFRBMSF	42	< 1	540	< 1	96	< 2
BFRBMLS	43	< 1	530	< 1	129	< 2
BFRBMSFLS	41	< 1	499	< 1	102	< 2
MLG	15	< 2	383	< 2	239	< 1
MLG BAYES NI	22	< 1	393	< 1	219	< 1
BP	15	< 2	383	< 2	239	< 1
BP BAYES NI	28	< 1	500	< 1	300	< 1

less than 2. For system 3 (p = 0.9), all BFR techniques had negative BIASRs (-0.8) for P_5. The only other serious misestimates for this system were for the β-factor, which had large BIASRs for k = 2,3, and 4.

5.0 CONCLUSIONS

This analysis looked at factors such as common cause shock rate, probability of failure given a shock (p), lethal shock rate, total operational time, independent failure rate, and the interactions of these factors to determine the important influences driving estimator bias and variability. The results indicated that all factors except independent failure rate were important influences. The interaction terms were also significant. Biases were larger in the case of lethal shock effects except for estimators of the probability of all units failing. Biases were also larger for small common cause shock rates and for extreme values of p (p = 0.1 and 0.9).

The study showed that for three-unit systems with data derived from the same or similar sources, the BP, MLG, BFR, and β-factor estimators all gave reasonable estimates for the probabilities of common cause events. For systems with more than three units, the β-factor method had extremely large positive biases when event probabilities were small. Even when event probabilities were on the order of 10^{-4}, biases could be an order of magnitude or greater. However, the β-factor technique was always conservative.

If the underlying failure data could be reasonably described by the BFR model, then the BFR estimators performed extremely well as long as potential common cause failures and lethal shocks could be identified. The BFR techniques that required knowledge of potential common causes and lethal shocks (BFRSFLS and BFRBMSFLS) were particularly good at estimating small probabilities where there was no data. The MLG and BP techniques gave zero estimates in these cases. The BFR estimators that did not identify potential common causes – BFR, BFRLS, BFRBM, and BFRBMLS – could not be evaluated in a large portion of cases. Those estimators that did not identify lethal shocks (BFRSF and BFRBMSF) could be negatively biased if there were lethal shocks, a condition that occurs frequently. Although the Bayes mode estimator with a noninformative prior generally outperformed the MLE, the improvements did not appear to be worth the considerable extra effort required to calculate the Bayes estimators. It is possible, however, that for data from plants with considerable variability in the failure rates, an empirical Bayes estimator would significantly improve the results. Such an estimator would use the data from the plants to determine the parameters of the prior distribution (Atwood, 1982).

If the data does not follow a BFR model then, not too surprisingly, the BFR techniques have problems and can underestimate the true probabilities. There are techniques for checking the appropriateness of the binomial model (Atwood, 1980, 1982, 1983a,b,c). Unfortunately, these techniques require fairly large data sets and are not practical for many real applications where data is sparse.

If data is available, even when it is scant, the MLG and BP estimators have the smallest biases. These estimators performed well when the BFR estimators were seriously biased; however, they allowed zero estimates when there were no failures. When Bayes techniques were used to avoid this problem, they tended to overestimate the failure probabilties. Some further study of appropriate priors for Bayes estimation is needed.

The BP estimator is particularly simple to use and extends nicely to the case where the number of units varies from system to system (Appendix A). Often the probability of interest is the probability that K or more components fail owing to a common cause. The BP technique is easily extended to estimate this probability and is recommended in this case, since the problem of zero estimates does not occur. Appendix A describes this estimator and a technique for evaluating its uncertainty limits.

The MLG estimators are much more complicated and do not extend easily to cases where data comes from systems with varying units. This technique is identical to BP estimation when data comes from one system and is recommended when system operational time or demand data are not available.

We were impressed with how well the BFRSFLS, BFRBMSFLS, MLG and BP techniques performed, even when the failure data were scarce. The results reported in this paper were worst-case results. In all the other cases we considered, these estimators did not overestimate the true probabilities by more than an order of magnitude nor underestimate by more than a factor of 5. The most severe problems were for the case where data came from different systems (plants) with considerable variability between the systems. The MLG and BP estimators were the most robust to this condition. When the study plant had failure rates similar to the plant contributing the least data, none of the estimators performed well. All had biases of at least two orders of magnitude. These biases were conservative since the plant contributing the least data had the smallest failure rates. Although we did not evaluate Bayes estimators with priors determined from the data, we believe that, in the case of highly variable data sources, these estimators should be used. Appendix A introduces such a technique for the BP method, and Atwood (1982) describes the technique for the BFR estimators.

After completing this study, it is our conclusion that if the assumption of constant failure rates are valid (no aging effects), then the binomial failure rate and basic parameter estimation techniques are appropriate and adequate for estimating common cause failure probabilities. (There is a need to look more carefully at uncertainty estimation for these existing estimators, and we propose to do this in a future study.) The overwhelming problem for the nuclear power industry is the lack of reliable common cause data. We believe that the focus of future efforts should be on gathering this data. Our first impulse is to suggest identifying those systems that are most important to common cause analyses and then devising an appropriate sampling scheme to go into the plants and gather the data. However, we are not at all sure that, even if we could execute such a scheme, the plant data would be obtainable. It is our impression that there is no organized effort within the plants to collect the information necessary to make meaningful estimates of common cause failure probabilities. A concerted effort is needed to formalize data-gathering procedures and to institute them in at least a sample of plants so that germane common cause data may be gathered in the future.

APPENDIX A

RECOMMENDED PROCEDURES

A.1 INTRODUCTION

The β-Factor method requires the least information and gives conservative estimators; therefore, it is recommended for preliminary screening in PRAs. However, when accurate estimates are needed and there is sufficient data, the BP estimators are preferred. This technique is very simple to use and extends readily to the case of multiple systems with varying numbers of units. This technique is also easily applied to estimate the probability that K or more units fail resulting from a common cause and to determine uncertainty bounds for this estimator. In the simulation study, the maximum likelihood BP estimators were best in the sense of smallest bias and variability in all cases where there were observed failures. In the cases where no failures were observed for a particular common cause event, Bayes estimators were used. In our study these Bayes BP estimators were conservatively biased, and the large biases were for very small probabilities.

There may be cases where the data is not adequate for the BP estimators. In these cases, if the model is appropriate and the lethal shock and potential common cause information is available, the BFRSFLS technique is recommended. If potential common causes cannot be identified and the data has sufficient failures ($s \neq 2n_+$), then the BFRLS estimators should be used. Bayes estimation is only recommended when the data come from different plants and there is considerable variability between the plants. In this case prior distributions should be derived from the data (Atwood, 1982). The Bayes estimators we studied used noninformative priors and did not differ dramatically from the MLEs. They did not appear to merit the increased work necessary to calculate them.

A.2 MODEL VERIFICATION

In the simulation study the BFR estimators were not reliable when the failure data was generated by a model that deviated from the BFR model by increased numbers of two and M failures. In practice the data is often so sparse that it is impossible to use statistical tests to determine if the model is appropriate. In those cases where data is available, Atwood (1980, 1982, 1983a,b,c) gives techniques for testing the appropriateness of the BFR model. The BP estimators are based on the more general Poisson model; however, they do require a constant failure rate and independence. To verify the Poisson model for the BP estimators, one can use the classical chi-square test (Hahn and Shapiro, 1967, ch.8) or a graphical procedure suggested by Hoaglin (1979). The Bayes BP estimators assume that the failure rates derive from a gamma distribution, and that given the failure rate, the number of failures is Poisson. If the estimators for the gamma parameters (A.2.1.3.1 and A.2.1.3.2) are negative, it is an indication that this model is not appropriate.

A.3 BASIC PARAMETER ESTIMATION FOR MULTIPLE SYSTEMS

In Sec. A.3.1, we present the MLE and the Bayes BP estimators for the case where there is data from different systems (plants) with varying units (M_i) and no plant-specific data. In Sec. A.3.2 an empirical Bayes estimator is introduced for the case where there is some plant-specific data available. Techniques for estimating the confidence and tolerance limits for these BP estimators are also presented.

In many PRA analyses, the probability needed is the probability that K or more units fail in time t due to a common cause event. The BP estimator for this probability is developed in Sec. A.4 as well as a technique for estimating its uncertainty limits.

A.3.1 NO PLANT-SPECIFIC DATA

A.3.1.1 MAXIMUM LIKELIHOOD ESTIMATOR

Often there is no plant-specific common cause data available. In this case, data from several plants can be used to give a generic estimate for the plant of interest. If N represents the number of systems (plants) from which the data is derived, and these systems and the system of interest have the same common cause rates (not too different anyway), then the data can be viewed as N independent observations from a population with constant common cause event rates, λ_k. The MLEs are

$$\hat{\lambda}_k = \frac{\sum_{i=1}^{N} n_{ik}}{\sum_{i=1}^{N} \binom{M_i}{k} T_i} \quad , \quad k = 2, \ldots, M_i \; , \; i = 1, \ldots, N \; .$$

Here, n_{ik} is the number of events with k units failing in system i, M_i is the number of units, and T_i is the total operational time. Note that the n_{ik} are distributed as Poisson variables,

$$n_{ik} \sim P\left(\binom{M_i}{k} T_i \lambda_k \right) .$$

The relationship between the Poisson and the χ^2 allows us to estimate the $\alpha\%$ ɲd $(1\text{-}\alpha)\%$ confidence bounds by

$$\lambda_k^{\alpha} = \frac{\chi^2_{\alpha}(2\, n_k)}{2\, T(k)} \quad ,$$

$$\lambda_k^{1-\alpha} = \frac{\chi^2_{1-\alpha}(2 n_k + 2)}{2\, T(k)} \quad , \qquad \text{(A.3.1.1.1)}$$

where

$$n_k = \sum_{i=1}^{N} n_{ik} \, , \qquad T(k) = \sum_{i=1}^{N} \binom{M_i}{k} T_i \quad .$$

(The definition for n_k is different from that in the main text, where n_k represented either data from one system or data from multiple systems that could be grouped together as one.)

The chi-square variate with $2n_k$ degrees of freedom at the α cumulative probability is $\chi^2_{\alpha}(2n_k)$, and $\chi^2_{1-\alpha}(2n_k + 2)$ is the chi-square variate with $2n_k + 2$ degrees of freedom at the $(1-\alpha)$ cumulative probability.

If $n_k = 0$, the BP estimator for λ_k is also zero. In this case many alternatives have been suggested. Borkowski et al. (1983) suggest the median of the chi-square with one degree of freedom for n_k, giving

$$\hat{\lambda}_k = \frac{0.227}{T(k)} .$$

Another estimator that has been suggested (Welker and Lipow, 1974) is $0.3333/T(k)$. If one assumes the λ_k are random variables with gamma distributions and uses a noninformative prior, then the estimator is $0.5/T(k)$. Other priors result in estimators with smaller numerators; for example, one evaluated in the simulation study is $0.175/T(k)$ (Welker and Lipow, 1974). An estimator suggested by the Empirical Bayes estimators (A.2.2.1) is

$$\hat{\lambda}_k = \frac{\bar{n}_j}{T(k) + \bar{T}(j)} ,$$

where

$$\bar{n}_j = \frac{n_j}{N} , \quad \bar{T}(j) = \frac{T(j)}{N} ,$$

and j is the index of the closest event to k with observed failures. We call this estimator the *nearest neighbor estimate*. In the simulation study, all of the Bayes estimators were conservative. The nearest neighbor estimator was developed after the simulation study was completed and was only evaluated on a small subset of cases; however, it also proved to be conservative.The nearest neighbor estimator and the 0.175 estimator had the smallest biases among the nonzero BP estimators.

A.3.1.2 UNCERTAINTY LIMITS FOR THE MAXIMUM LIKELIHOOD ESTIMATOR

The (1-α)% confidence limits (A.2.1.1.1) define intervals such that (1-α)% of the intervals, based on samples of size N, will contain λ_k. These bounds can be used to determine the confidence interval for the probability that there is at least one failure in a specified time, t (often two or 24 hours in PRA studies). Such confidence limits do not give bounds on the failure rates that will be observed in the study plant. Some PRA analyses want these bounds or, equivalently, bounds on the number of failures that will be observed in time t. Since the number of failures is Poisson with parameter λ_k, we could use the (1-α) probability interval for n_k if we knew λ_k. However, we only have an estimate for λ_k and, therefore, can only estimate such intervals with a certain confidence. Hahn and Chandra (1981) give a technique for determining bounds that will contain at least 100P% of the n_k with (1-α)% confidence. These intervals are called tolerance intervals and are the appropriate estimates for the uncertainty limits in this case. A technique for determining conservative, symmetric two-sided intervals is to obtain the (1-α)% confidence limits for the unknown distribution parameter, λ_k, and then obtain a [100(1+P)/2] percent one-sided upper probability bound (N_U) from the Poisson using the previously calculated upper confidence limit and a [100(1+P)/2] percent one-sided lower probability (N_L) bound using the previously calculated lower confidence limit. The uncertainty limits for λ_k are $N_U/T(k)$ and $N_L/T(k)$. The problem with these tolerance limits is that the Poisson distribution is discrete, and exact 100P percent intervals cannot be determined. For extremely small λ_k, the limits are often $N_U = 1$ and $N_L = 0$. These limits may actually represent 99.999...% intervals for small t (such as 2 or 24 hours). In these cases tolerance limits are not useful.

A.3.1.3 BAYES ESTIMATION

An alternate approach is to assume that the λ_k vary between plants, that this variability can be described by a random variable, and that N_k, conditioned on λ_k, has a Poisson distribution. A reasonable assumption (and one that facilitates estimation) is to assume that the λ_k have gamma distributions (priors) with parameters α_k and β_k. It is easy to show that the posterior distribution of λ_k, given the n_{ik}, is gamma with parameters

$$\alpha_k + \sum_{i=1}^{N} n_{ik} \quad \text{and} \quad \sum_{i=1}^{N} \binom{M_i}{k} T_i + \beta_k .$$

The Bayes estimator (the mean of the posterior) is

$$\hat{\lambda}_k = \frac{\alpha_k + \sum_{i=1}^{N} n_{ik}}{\sum_{i=1}^{N} \binom{M_i}{k} T_i + \beta_k} .$$

To estimate α_k and β_k, note that the unconditional distribution of n_k is a negative binomial with parameters α_k and $T(k)/\beta_k$ (Johnson and Kotz, 1969). The mean and variance of n_k are

$$E(n_k) = \frac{\alpha_k T(k)}{\beta_k}$$

and

$$\mathrm{Var}(n_k) = \frac{\alpha_k T(k)}{\beta_k} + \frac{\alpha_k T(k)^2}{\beta_k^2} .$$

Equating the sample mean and variance to the corresponding population values and solving for α_k and β_k gives

$$\hat{\beta}_k = \frac{n_k T(k)}{N S^2 - n_k} \qquad \text{(A.3.1.3.1)}$$

and

$$\hat{\alpha}_k = \frac{n_k^2}{N S^2 - n_k} , \qquad \text{(A.3.1.3.2)}$$

where

$$S^2 = \sum_{i=1}^{N} \frac{(n_{ik} - n_k)^2}{(N-1)} \quad .$$

Note that $\hat{\alpha}_k$ and $\hat{\beta}_k$ are negative if $S^2 < \bar{n}_k$, and if this happens it indicates that the underlying assumptions of the model may not be appropriate for this data. If $\hat{\alpha}_k$ and $\hat{\beta}_k$ are close to α_k and β_k, then one can use a $(1-\alpha)\%$ probability interval from the gamma distribution with parameters $\hat{\alpha}_k$ and $\hat{\beta}_k$ to determine uncertainty limits for λ_k. Aitchison (1975) refers to this as the *estimative interval*. The *estimative interval* is approximate and can be too small (Atwood, 1984). Atwood develops more accurate techniques for determining the appropriate tolerance limits; however, he finds in assessing common cause failures from a real data set (diesel generator data based on 22 common cause events at 58 plants), that "the estimative interval seems to be good enough and that uncertainty due to lack of data is small compared to the inherent variability in λ." The more accurate intervals are quite complex and require considerable effort to calculate; however, if the interval is important, it is probably better to spend the time, effort, and money to get an accurate one.

A.3.2 PLANT-SPECIFIC DATA AVAILABLE

If plant-specific data is available and adequate, then, given the current lack of reliable common cause data, the safest approach is to use the plant-specific data to determine the maximum likelihood BP estimators. However, in many cases the plant data will be insufficient – no observed common cause failures and total system time, or number of demands so small that estimates are unreasonably high. In these cases, one is forced to combine the plant-specific data with other plant data. If the other plant data is reliable, then this information will improve the estimation procedure and reduce uncertainty. When plant-specific data is to be combined with other data, empirical Bayes estimation is recommended. Under the assumptions of A.2.1.3, the empirical Bayes estimators are

$$\hat{\lambda}_{Ik} = \frac{n_{Ik} + \hat{\alpha}_k}{\binom{M_I}{k} T_I + \hat{\beta}_k}, \; k = 2, \ldots, M \; .$$

The subscript I indicates plant-specific data. The values of $\hat{\alpha}_k$ and $\hat{\beta}_k$ and the uncertainty bounds are determined as in A.2.1.3.

A.4 PROBABILITY OF K OR MORE UNITS FAILING AS A RESULT OF COMMON CAUSE EVENTS

PRA analyses are often concerned more with the probability (PK) of at least K out of M units failing in a specified time period (t) due to common cause events than with the probability of exactly K units failing. The BP estimation technique is particularly good in this situation as it can always be evaluated (given that system operational time or demands are known) and does not require the restrictive assumptions of the BFR model. Noting that $\lambda_k t$ is very small (<<1) and making the appropriate approximations gives

$$PK \approx \sum_{k=K}^{M} \binom{M}{k} t \lambda_k$$

and

$$\widehat{PK} = \sum_{k=K}^{M} \binom{M}{k} t \hat{\lambda}_k \ .$$

If adequate plant-specific data is available, then

$$\widehat{PK} = \sum_{k=K}^{M} \frac{n_k t}{T} \ ,$$

where T is the system operational time. Assuming that the n_k are independent, the confidence and tolerance limits are found as in A.3.1.1 and A.3.1.2 with n_k replaced by $\sum_{k=K}^{M} n_k$. If there is no adequate plant-specific data available, then data from other sources must be used to give

$$\widehat{PK} = \sum_{k=K}^{M} \binom{M}{k} t \frac{n_k}{\sum_{i=1}^{N} \binom{M_i}{k} T_i} = \sum_{k=K}^{M} a_k n_k \; . \qquad \text{(A.4.1)}$$

If we assume that (A.4.1) can be reasonably approximated by a normal distribution, then uncertainty limits for PK_{N+1} (the probability that K or more units fail in the N + 1st system) can be approximated by the same technique one uses to find the prediction interval for the next observation of a normally distributed random variable with N independent observations. The underlying assumption is that system N + 1 is similar to the other N systems. Note that

$$\widehat{PK}_{N+1} = \sum_{k=K}^{M} \frac{t}{T_{N+1}} n_{N+1\ k}$$

and

$$\mathrm{Var}\,(\widehat{PK}_{N+1} - \widehat{PK}) = \gamma^2 = \sum_{k=K}^{M} \left[\left(\frac{t}{T_{N+1}} \right)^2 + N a_k^2 \right] \sigma_k^2 = \sum_{k=K}^{M} g_k \sigma_k^2 \; ,$$

where σ_k^2 represents the variance of the n_{ik}, i = 1, ..., N+1. (As defined previously, n_{ik} is the number of k failures in system i, n_k is the total number of k failures, and $n_{N+1,k}$ is the number of k failures in the new system.) Let

$$\hat{\sigma}_k^2 = \frac{\sum_{i=1}^{N} (n_{ik} - n_k)^2}{N-1}$$

and

$$\hat{\gamma}^2 = \sum_{k=K}^{M} g_k \hat{\sigma}_k^2 \; .$$

Using the Satterthwaite (1946) approximation for complex estimates of variance and the normal approximations, the (1-α)% uncertainty limits (probability limits) for $\widehat{PK}_{N+1}$ are

$$\widehat{PK} \pm t_{1-\alpha/2}(n)\,\hat{\gamma} \; ,$$

where $t_{1-\alpha/2}$ is the $(1-\alpha/2)$ percentile from the student's T distribution with n degrees of freedom and

$$n \approx (N-1)\frac{\left(\sum_{k=K}^{M} g_k \hat{\sigma}_k^2\right)^2}{\sum_{k=K}^{M} g_k^2 \hat{\sigma}_k^4} \; .$$

In practice $\hat{\sigma}_k^2$ may be zero for some k. In these cases, an alternative to using zero in the equation for n is to add the coefficients of the zero variance estimates to the coefficient of the smallest nonzero estimate and use this sum as the coefficient of the smallest variance estimate. These intervals are approximate and need further study to determine their coverage probabilities.

APPENDIX B

BFR, BP, AND MLG TECHNIQUES ARE EQUIVALENT, M = 3

For fixed M (the data are from systems that have the same number of units), it is easy to show that the MLG and BP estimators for the λ_k's are identical (refer to Sec. 3.2). For $M = 3$, if single failures occur ($n_1 \neq 0$) and the BFR estimators exist ($s \neq 2\, n_+$), then all three techniques, BP, MLG, and BFR, are equivalent.

To prove this statement, we derive the BFR estimators and show that they are identical to the BP estimators. We assume that common cause events are observed, $n_+ \neq 0$. If $n_3 = 0$ and $n_2 \neq 0$, then $s = 2n_+$, and the BFR estimators cannot be evaluated. Therefore, there are only two cases to consider: $n_2 = 0$, $n_3 \neq 0$ and $n_2 \neq 0$, $n_3 \neq 0$.

Case 1. $n_2 = 0, n_3 \neq 0$

From Eq. (2.0.1) we have

$$\hat{\lambda}_1 = \hat{\lambda} + \hat{\mu}\,\hat{p}\,(1-\hat{p})^2 ,$$

$$\hat{\lambda}_2 = \hat{\mu}\,\hat{p}^2\,(1-\hat{p}),$$

and

$$\hat{\lambda}_3 = \hat{\mu}\,\hat{p}^3.$$

If $n_2 = 0$, then $s = M\, n_+$ and $p = 1$. Substituting the BFR estimators (3.3.1) into the equations for the $\hat{\lambda}_k$ gives

$$\hat{\lambda}_1 = \hat{\lambda} = \frac{n_1}{MT} ,$$

$$\hat{\lambda}_2 = 0 ,$$

and

$$\hat{\lambda}_3 = \hat{\mu} = \frac{\hat{\lambda}_+}{1-\hat{r}_0-\hat{r}_1} = \hat{\lambda}_+ = \frac{n_3}{T} \ .$$

These are precisely the BP estimators.

Case 2. $n_2 \neq 0$, $n_3 \neq 0$

Using the definitions of μ, λ, and r_1 given in 3.3.1, we see that

$$\hat{\lambda} = \frac{\hat{\lambda}_s - \hat{\mu}\,\hat{r}_1}{M} \ ,$$

$$\hat{\mu} = \frac{\hat{\lambda}_+}{1-\hat{r}_0-\hat{r}_1} \ ,$$

and

$$\hat{p}\,(1-\hat{p})^2 = \frac{\hat{r}_1}{M} \ .$$

Substituting these values in the equation for $\hat{\lambda}_1$ gives

$$\hat{\lambda}_1 = \frac{\hat{\lambda}_s - \hat{\mu}\,\hat{r}_1}{M} + \frac{\hat{\mu}\,\hat{r}_1}{M} = \frac{\hat{\lambda}_s}{M} = \frac{n_1}{MT} \ ,$$

which is the BP estimator for λ_1. Substituting for $\hat{\mu}$ in the equations for $\hat{\lambda}_2$ and $\hat{\lambda}_3$ yields

$$\hat{\lambda}_2 = \frac{\hat{\lambda}_+}{1-\hat{r}_0-\hat{r}_1}\,\hat{p}^2(1-\hat{p}) = \frac{n_2+n_3}{T(1-\hat{r}_0-\hat{r}_1)}\,\hat{p}^2(1-\hat{p})$$

and

$$\hat{\lambda}_3 = \frac{\hat{\lambda}_+}{1-\hat{r}_0-\hat{r}_1}\,\hat{p}^3 = \frac{n_2+n_3}{T(1-\hat{r}_0-\hat{r}_1)}\,\hat{p}^3 \ .$$

For $M = 3$, there is a closed form solution for $\hat{p}$:

$$\hat{p} = \frac{3(s - 2n_+)}{2s - 3n_+} = \frac{3n_3}{n_2 + 3n_3} .$$

Substituting this value of $\hat{p}$ into the equations for r_0 and r_1 gives

$$\hat{r}_0 = \frac{n_2^3}{(n_2 + 3n_3)^3} ,$$

and

$$\hat{r}_1 = \frac{9n_3 n_2^2}{(n_2 + 3n_3)^3} .$$

Using these expressions to evaluate $\hat{\lambda}_2$ and $\hat{\lambda}_3$, we find

$$\hat{\lambda}_2 = \frac{n_2}{3T} ,$$

and

$$\hat{\lambda}_3 = \frac{n_3}{T} .$$

These are also the BP estimators.

REFERENCES

American Nuclear Society and the Institute of Electronic and Electrical Engineers (1983), "PRA Procedures Guide; A Guide to the Performance of Probabilistic Risk Assessments for Nuclear Power Plants," NUREG/CR-2300. Sponsored by the U.S. Nuclear Regulatory Commission and the Electric Power Research Institute.

Aitchison, J. (1975), "Goodness of Prediction Fit," *Biometrika*, Vol. 62, pp. 547-554.

Atwood, C. L. (1980), "Estimators for the Binomial Failure Rate Common Cause Model," NUREG/CR-1401, prepared for the U.S. Nuclear Regulatory Commission by EG&G Idaho, Inc.

Atwood, C. L. (1982), "Common Cause Fault Rates for Diesel Generators," NUREG/CR-2099,prepared for the U.S. Nuclear Regulatory Commission by EG&G Idaho, Inc.

Atwood, C. L. (1983a), "Common Cause Fault Rates for Pumps," NUREG/CR-2098, prepared for the U.S. Nuclear Regulatory Commission by EG&G Idaho, Inc.

Atwood, C. L. (1983b), "Common Cause Fault Rates for Valves," NUREG/CR-2770, prepared for the U.S. Nuclear Regulatory Commission by EG&G Idaho, Inc.

Atwood, C. L. (1983c), "Common Cause Fault Rates for Instrumentation and Control Assemblies," NUREG/CR-2771, prepared for the U.S. Nuclear Regulatory Commission by EG&G Idaho, Inc.

Atwood, C. L. (1984), "Approximate Tolerance Intervals, Based on Maximum Likelihood Estimates," *Journal of the American Statistical Association*, Vol. 79, No. 386, pp. 459-465.

Borkowski, R. J., Kahl, W. K., Hebble, T. L., Fragola, J. R., and Johnson, J. W. (1983), "The In-Plant Reliability Data Base for Nuclear Plant Components: Interim Report – The Valve Component," NUREG/CR-3154, prepared for the U.S. Nuclear Regulatory Commission by Oak Ridge National Laboratory.

Fleming, K. N. (1975), "A Reliability Model for Common Mode Failure in Redundant Safety Systems," GA-A13284, *Proceedings of the Sixth Annual Pittsburgh Conference on Modeling and Simulation*, General Atomic Company, San Diego, California.

Gyllenbaga, H., Jolhanson, G., and Lilja, T. (1983), "Treatment of Dependencies in Ringhals 1 & 2 Safety Studies," prepared for the Workshop on Dependent Failure Analysis, Vasteras, Sweden.

Hahn, G. and Chandra, R. (1981), "Tolerance Intervals for Poisson and Binomial Variables," *Journal of Quality Technology*, Vol. 13, No. 2, pp. 100-110.

Hahn, G. and Shapiro, S. (1967), *Statistical Models in Engineering*, John Wiley & Sons, New York, New York.

Hoaglin, D. (1979), "A Poissonness Plot," paper presented at the 1979 Annual Meeting of the American Statistical Association, Washington, D.C.

Johnson, N. L., and Kotz, S. (1969), *Discrete Distributions*, John Wiley & Sons, New York, New York.

Meyer, M. A., Peaslee, A. T., and Booker, J. M. (1982), "Group Consensus Method and Results," LA-9584-MS, prepared by Los Alamos National Laboratory for the US Department of Energy, Assistant Secretary for Defense Programs.

Parry, G. W., "Letter to the Editor: Comments on 'Modeling Uncertainty in Parameter Estimation' in Vol. 26, No.3," *Nuclear Safety*, Vol 27-2, April-June, 1986, pp. 212-214.

Pickard, Lowe and Garrick, Inc. (1985a), "PRA Procedures for Dependent Events Analysis, Volume I and Volume II," PLG-0453, prepared for Electric Power Research Institute.

Pickard, Lowe and Garrick, Inc. (1985b), "Classification and Analysis of Reactor Operating Experience Involving Dependent Events," EPRI NP-3967, prepared for Electric Power Research Institute.

Satterthwaite, F. E. (1946), "An Approximate Distribution of Estimates of Variance Components,"*Biometrics*, Vol. 2, pp. 110-114.

Vesely, W. E. (1977), "Estimating Common Cause Failure Probabilities in Reliability and Risk Analyses: Marshall-Olkin Specializations," *Nuclear Systems Reliability Engineering and Risk Assessment*, Philadelphia: SIAM, pp 314-341.

Welker, E. and Lipow, M.(1974), "Estimating the Exponential Failure Rate from Data with No Failure Events," TRW Systems Group, Redondo Beach, California, 1974 R&M Symposium Proceedings, pp. 420-427.

BIBLIOGRAPHY

Waller, R. A., "A Brief Survey and Comparison of Common Cause Failure Analysis," NUREG/CR-4314, prepared for the US Nuclear Regulatory Commission, Washington, DC.

DISTRIBUTION

	Copies
Nuclear Regulatory Commission, RG, Bethesda, Maryland	198
Technical Information Center, Oak Ridge, Tennessee	2
Los Alamos National Laboratory, Los Alamos, New Mexico	50
	250

NRC FORM 335 (2-84) NRCM 1102, 3201, 3202 — U.S. NUCLEAR REGULATORY COMMISSION

BIBLIOGRAPHIC DATA SHEET

SEE INSTRUCTIONS ON THE REVERSE

1. REPORT NUMBER (Assigned by TIDC, add Vol. No., if any): NUREG/CR-5044, LA-11179-MS

2. TITLE AND SUBTITLE: Estimation Techniques for Common Cause Failure Events

3. LEAVE BLANK

4. DATE REPORT COMPLETED: Month: October; Year: 1987

5. AUTHOR(S): Elizabeth J. Kelly, Geralyn M. Hemphill

6. DATE REPORT ISSUED: Month: March; Year: 1988

7. PERFORMING ORGANIZATION NAME AND MAILING ADDRESS (Include Zip Code): Los Alamos National Laboratory, Los Alamos, NM 87545

8. PROJECT/TASK/WORK UNIT NUMBER

9. FIN OR GRANT NUMBER: A7225

10. SPONSORING ORGANIZATION NAME AND MAILING ADDRESS (Include Zip Code): Division of Risk Analysis and Operations, Office of Nuclear Regulatory Research, U.S. Nuclear Regulatory Commission, Washington, DC 20555

11a. TYPE OF REPORT: Informal

b. PERIOD COVERED (Inclusive dates)

12. SUPPLEMENTARY NOTES

13. ABSTRACT (200 words or less)

Common cause failure probability estimation techniques, including β-factor, basic parameter, binomial failure rate, multiple Greek, and C-factor estimators, are evaluated and compared using simulation data that captures the real world problem of sparse data from different plants. The effects on the estimators' performances from underlying factors such as common cause shock rates, lethal shock rates, probability of failing given a shock, independent failure rates, and system operational time are discussed. Worst case results are reported, and it is seen that for extremely small common cause failure probabilities the binomial failure rate estimators are best. However, these estimators can underestimate the true probabilities when the failures deviate from the binomial failure rate model. The β-factor technique is shown to be conservative, and in some cases to overestimate the true probability by several orders of magnitude. When there are observed failures for each failure event, the basic parameter technique is best and is easily calculated. This estimator is investigated in detail and is used to develop an estimator for the probability of K or more units failing due to a common cause. Uncertainty limits for this probability are also developed.

14. DOCUMENT ANALYSIS — a. KEYWORDS/DESCRIPTORS

b. IDENTIFIERS/OPEN-ENDED TERMS

15. AVAILABILITY STATEMENT: Unlimited

16. SECURITY CLASSIFICATION: (This page) Unclassified; (This report) Unclassified

17. NUMBER OF PAGES

18. PRICE

Available from

Superintendent of Documents
U.S. Government Printing Office
Post Office Box 37082
Washington, D. C. 20013-7982

and

National Technical Information Service
Springfield, VA 22161

120555078877 1 1AN1RG
US NRC-OARM-ADM
DIV OF PUB SVCS
POLICY & PUB MGT BR-PDR NUREG
W-537
WASHINGTON DC 20555

Los Alamos Los Alamos National Laboratory
Los Alamos, New Mexico 87545