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# A Survey of Existing Momentum Exchange Models in Two-Phase Flows for Use in the Simmer Fast Reactor Safety Analysis Code

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#### ABSTRACT

Models which describe momentum exchange in two-phase, vapor-liquid flows are surveyed. These models are related to the momentum exchange coefficient formulations used in the SIMMER code. The result is a set of model-dependent exchange coefficients for various flow regimes. Criteria for flow regime transitions and experimental needs in momentum exchange modeling are also discussed.



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A SURVEY OF EXISTING MOMENTUM EXCHANGE MODELS IN  
TWO-PHASE FLOWS FOR USE IN THE SIMMER FAST REACTOR  
SAFETY ANALYSIS CODE

CHAPTER I  
INTRODUCTION

The SIMMER (S<sub>n</sub>, Implicit, Multifield, Multicomponent, Eulerian, Recriticality) computer code<sup>1</sup> is being developed at Los Alamos Scientific Laboratory (LASL) for the United States Nuclear Regulatory Commission (NRC). The code is designed to analyze the complex neutronic and thermal hydrodynamic phenomena that might occur in a hypothetical core disruptive accident (HCDA). As part of SIMMER development, a program has been initiated to verify the code. The process of verification includes a critical review of models used in the code, suggestions for improvements and model addition, as well as the design and execution of experiments to confirm models or to provide insights into the phenomena which are characteristic of HCDAs. Sandia Laboratories is a participant in the verification program. The purpose of this report is to examine in detail the SIMMER coefficients for momentum exchange between the liquid, vapor, and structure fields and to relate these to existing models in the two-phase flow literature. The following presentation assumes that the reader has a general familiarity with the implicit multifield (IMF) method used in the KACHINA code<sup>2</sup> and with the structure of the SIMMER code.

Figure I-1 is a flow chart which illustrates a general approach that may be taken to verify SIMMER momentum exchange coefficients. The formulation of the momentum equations in SIMMER is first reviewed and expressed in a form similar to more conventional and simplified treatments. These simplified models depend upon flow regime. The major flow regimes are bubbly, slug (churn-turbulent), annular (including annular mist), and dispersed (drop) flow. Since little information is available on slug flow, it is not considered in this report. However, some modeling work in slug flow is described in References 3 and 4. The existing models for each of the other flow regimes are considered. Each of these models uses one of the following basic flow-related assumptions:

1. Homogeneous flow -- the liquid and vapor are treated as an average fluid with no slip between phases,
2. Separated flow -- the continuity, momentum, and energy equations are written separately for each phase, and

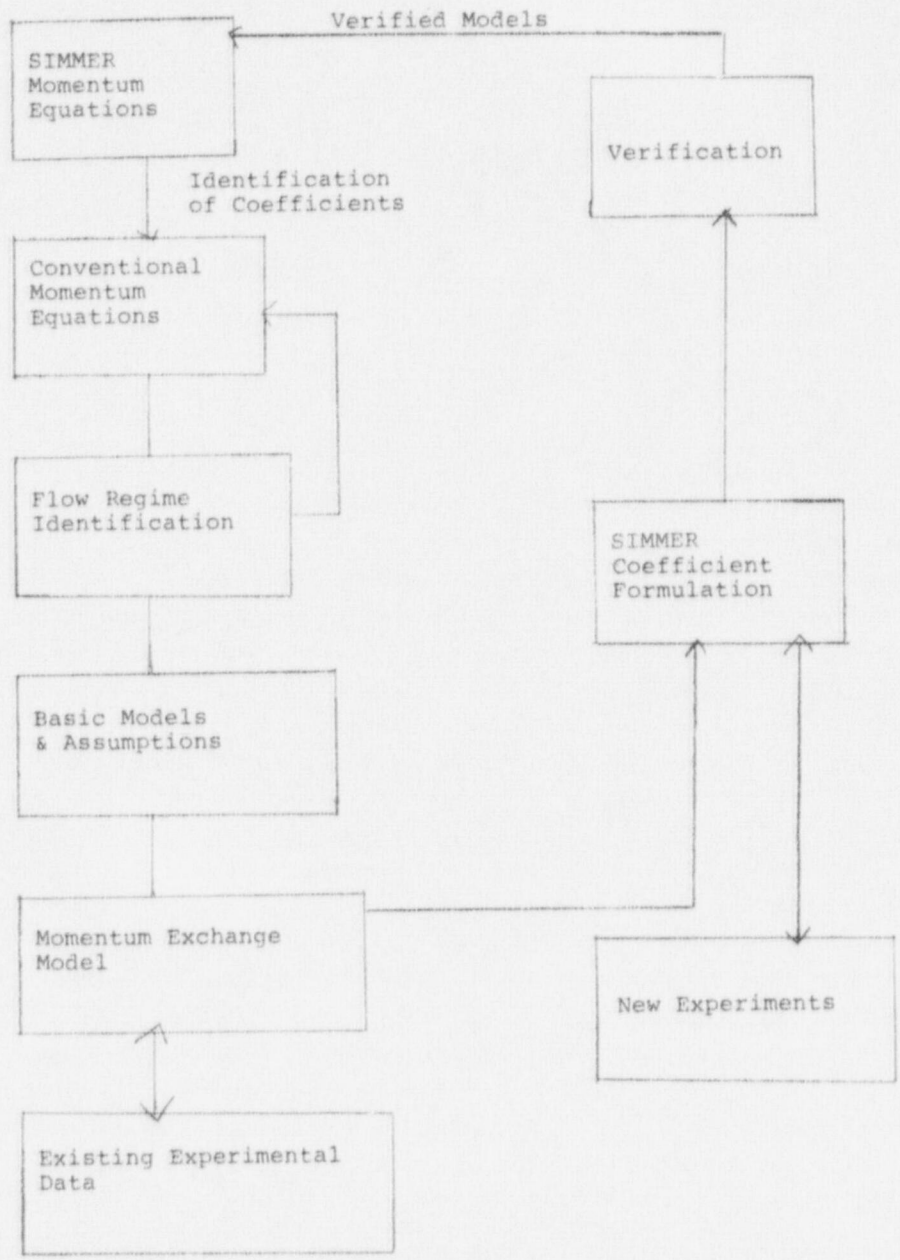


Figure I-1. Flow Chart for Verification of SIMMER Momentum Exchange Coefficients



3. Flow pattern flow model -- similar to the separated flow model with the exception that the constraints related to the flow regime geometry are included.

These assumptions and their ranges of applicability will be examined in detail. For the moment, it is useful to note that while the present SIMMER-I momentum exchange is basically a separated flow model, later versions of SIMMER will undoubtedly become more flow-pattern oriented. For each particular model, certain mechanisms for momentum exchange are postulated. In this report, care is taken to mention these mechanisms and to stress their implications. Finally, the relationship of a particular model to the SIMMER exchange coefficients is examined. A summary of the momentum exchange models is given in Table I-1.

TABLE I-1  
Summary of Momentum Exchange Models

<u>Flow Regime</u>	<u>Specifically Treated in SIMMER-I?</u>	<u>Other Models</u>	<u>Where Examined</u>
Bubbly	Yes	Homogeneous, Drift Flux	Chapter IV
Slug	No	Griffith and Wallis	Ref. 3 and 4
Annular	No	Armand, Martinelli	Chapter III
Annular Mist	No	Wallis, Levy	Chapter III
Drop	Yes	Homogeneous, Drift Flux	Chapter IV

Following a discussion of the specific models for each flow regime, the problem of flow regime transitions is considered. A number of criteria are examined with the objective of defining the transition point between two flow regimes. No specific recommendation for model incorporation into SIMMER is made. Next, experimental needs related to momentum exchange processes are discussed. Here the concern is to identify the major problems rather than to define specific experiments although some of the latter is done.

At the outset it should be noted that the general case treated by SIMMER, i.e., two-dimensional multiphase flow with heat transfer and phase change, is considerably more complicated than those cases normally investigated by the researchers in two-phase flow. In fact, the preponderance of available data actually pertains to two-component (air-water), adiabatic, axisymmetric flow. As might be expected, most of the available models also were developed for this system. When heat addition is examined, it is usually in relation to the water-steam system at high pressure. One of the reasons for this apparently narrow range of effort is that the problems inherent in the investigation and description of multiphase flows are extremely complicated. Hence, it is probably overly optimistic to expect to develop new, detailed models for momentum exchange coefficients. Instead, it is more practical to adapt existing models and

correlations. The advantages of this approach are that the degree of effort is tractable and that the SIMMER momentum exchange calculations will agree with accepted methods for the problems where data are available.

Because of time limitations and the lack of knowledge noted previously, a number of problems are not discussed in this report. These include

1. Two-dimensional effects,
2. The behavior of solids in the fluid fields,
3. Particle (drop, bubble) size distributions, and
4. Inlet and area change effects.

Further effort will be needed in these areas.

CHAPTER II

REVIEW OF SIMMER-I MOMENTUM EQUATION

In order to relate the SIMMER exchange coefficients to more conventional two-phase flow models, it is necessary to briefly review the formulation used in the code. For the Eulerian control volume, the momentum equation of the vapor field is<sup>1</sup>

Vapor Field

$$\begin{aligned} \frac{\partial}{\partial t} (\bar{\rho}_g \vec{V}_g) + \nabla \cdot (\bar{\rho}_g \vec{V}_g \vec{V}_g) = & \quad [3] \quad [4] \quad [5] \quad [6] \\ & -\alpha_g \nabla P + \Gamma_{el} \vec{V}_l - (\Gamma_{cg} + \Gamma_{cs}) \vec{V}_g + \vec{g} \rho_g \\ & + K_{gl} (\vec{V}_l - \vec{V}_g) - K_{gs} \vec{V}_g \end{aligned} \quad (II-1)$$

where

$$\bar{\rho}_g = \alpha_g \rho_g$$

$\alpha_g$  = vapor fraction

$$\Gamma_{el} = \sum_{m=1}^3 \Gamma_{elm}$$

$\Gamma_{eli}$  = mass rate of vaporization for liquid component i

$$\Gamma_{cg} = \sum_{m=1}^3 \Gamma_{cgm}$$

$\Gamma_{cgi}$  = mass rate of vapor component i per unit volume on liquid i

$$\Gamma_{cs} = \sum_{m=1}^3 \sum_{k=1}^3 (\Gamma_{cgm})_{sk}$$

$(\Gamma_{cgm})_{sk}$  = mass rate of condensation of vapor component on structure component k

Then term by term, the various parts of the equation are

- 1 = temporal vapor acceleration,
- 2 = convective acceleration--momentum change across the control volume,
- 3 = pressure gradient,
- 4 = addition of liquid to vapor field at velocity  $\vec{V}_l$ ,



- 5 = loss of vapor to liquid field at velocity  $\vec{V}_g$ ,  
 6 = body force due to gravity,  
 7 = drag force between vapor and liquid, and  
 8 = drag force between vapor and structure

Similarly, for the liquid field,

$$\begin{aligned}
 \frac{\partial}{\partial t} \left( \overset{[1]}{\bar{\rho}_l \vec{V}_l} \right) + \nabla \cdot \left( \overset{[2]}{\bar{\rho}_l \vec{V}_l \vec{V}_l} \right) = & \overset{[3]}{-\alpha_l \nabla p} + \overset{[4]}{\Gamma_{cg} \vec{V}_g} - \overset{[5]}{(\Gamma_{el} + A_s) \vec{V}_l} \\
 & + \overset{[6]}{\bar{\rho}_l \vec{g}} + \overset{[7]}{K_{gl} (\vec{V}_g - \vec{V}_l)} - \overset{[8]}{K_{ls} \vec{V}_l} \quad (II-2)
 \end{aligned}$$

where

$$A_s = \sum_{m=1}^3 \sum_{k=1}^3 (A_{lm})_{sk}$$

$(A_{lm})_{sk}$  = mass rate of adhesion of liquid m to structure component k

and the other terms have the same meaning as they do in Eq. (II-1). Note that term [5] in Eq. II includes both the effects of vaporization and the effects of freezing.

In both equations, terms [7] and [8] contain momentum exchange coefficients. If, for the moment, it is assumed that these coefficients do not change appreciably in the presence of phase changes (or that this effect is somehow included later), and the temporal acceleration terms are neglected, then the following equations are obtained:

Vapor

$$-\alpha \nabla p = \nabla \cdot (\bar{\rho}_g \vec{V}_g \vec{V}_g) - \vec{g} \bar{\rho}_g + K_{gl} (\vec{V}_g - \vec{V}_l) + K_{gs} \vec{V}_g \quad (II-3)$$

Liquid

$$-(1 - \alpha) \nabla p = \nabla \cdot (\bar{\rho}_l \vec{V}_l \vec{V}_l) - \vec{g} \bar{\rho}_l + K_{gl} (\vec{V}_l - \vec{V}_g) + K_{ls} \vec{V}_l \quad (II-4)$$

where

$$\alpha = \alpha_g$$

$$1 - \alpha = \alpha_l$$

A standard technique in two-phase flow calculation is to write

$$\frac{dp}{dz} = \left(\frac{dp}{dz}\right)_F + \left(\frac{dp}{dz}\right)_A + \left(\frac{dp}{dz}\right)_G \quad (\text{II-5})$$

where

$$\left(\frac{dp}{dz}\right)_F = \text{frictional pressure drop}$$

$$\left(\frac{dp}{dz}\right)_A = \text{accelerational pressure drop}$$

$$\left(\frac{dp}{dz}\right)_G = \text{gravitational pressure drop}$$

Combining these definitions with Eqs. (II-3) and (II-4) (for one-dimensional only\*) yields

$$\left. \begin{aligned} -\left(\frac{dp}{dz}\right)_{A,g} &= \frac{1}{\alpha} \frac{d}{dz} \left( \bar{\rho}_g V_{g,z} V_{g,z} \right) \\ -\left(\frac{dp}{dz}\right)_{A,l} &= \frac{1}{1-\alpha} \frac{d}{dz} \left( \bar{\rho}_l V_{l,z} V_{l,z} \right) \end{aligned} \right\} \quad (\text{II-6a})$$

$$\left. \begin{aligned} -\left(\frac{dp}{dz}\right)_{G,g} &= \frac{1}{\alpha} \bar{\rho}_g g = \rho_g g \\ -\left(\frac{dp}{dz}\right)_{G,l} &= \frac{1}{1-\alpha} \bar{\rho}_l g = \rho_l g \end{aligned} \right\} \quad (\text{II-6b})$$

$$\left. \begin{aligned} -\left(\frac{dp}{dz}\right)_{F,g} &= \frac{1}{\alpha} \left[ K_{gl} (V_{g,z} - V_{l,z}) + K_{gs} V_{g,z} \right] \\ -\left(\frac{dp}{dz}\right)_{F,l} &= \frac{1}{1-\alpha} \left[ K_{gl} (V_{l,z} - V_{g,z}) + K_{ls} V_{l,z} \right] \end{aligned} \right\} \quad (\text{II-6c})$$

Note that all of the exchange coefficients are contained in Eq. (II-6c). Moreover, the problem of finding the exchange coefficient set is seen to be equivalent to the common problem in two-phase flow of evaluating  $(dp/dz)_{F,g}$  and  $(dp/dz)_{F,l}$ .

Clearly, the equation sets (II-1, II-2) and (II-3, II-4) are written for separated flow. However, these equations can be rearranged to yield several other useful approximations. Adding Eqs. (II-3) and (II-4) gives

$$-\nabla p = \nabla \cdot \left( \bar{\rho}_g \vec{V}_g \vec{V}_g + \bar{\rho}_l \vec{V}_l \vec{V}_l \right) - \vec{g} (\bar{\rho}_g + \bar{\rho}_l) + K_{gs} \vec{V}_g + K_{ls} \vec{V}_l \quad (\text{II-7})$$

\* For equations describing the z-component of momentum exchange, the convention that  $V_{g,z} = V_g$  and  $V_{l,z} = V_l$  will be used where no ambiguity will arise.

where the liquid-vapor interactions disappear. If it is assumed that  $\vec{V}_g = \vec{V}_l$ , then the following homogeneous momentum equation is obtained:

$$-\nabla p = \nabla \cdot (\rho_m \vec{V}_m \vec{V}_m) - \vec{g} \rho_m + K_{ms} \vec{V}_m \quad (\text{II-8})$$

where  $\rho_m$  is the mixture density.\* Since distinction is not made between liquid and vapor,

$$K_{gs} + K_{ls} = K_{ms} \quad (\text{II-8a})$$

Quite often the homogeneous flow assumption is used, particularly for bubbly and dispersed flow.<sup>5,6</sup> This assumption will be discussed in Chapter IV.

If Eqs. (II-3) and (II-4) are subtracted from one another, then (in one-dimension)

$$\frac{d}{dz} \left[ \alpha \rho_l V_l^2 - (1-\alpha) \rho_g V_g^2 \right] = g \left[ (1-\alpha) \rho_g - \alpha \rho_l \right] + K_{gl} (V_g - V_l) + K_{gs} (1-\alpha) V_g - K_{ls} \alpha V_l \quad (\text{II-9})$$

Eq. (II-9) shows that the rate at which the vapor and liquid exchange kinetic energy ( $\rho V^2$ ) is proportional to the velocity difference ( $V_g - V_l$ ) between the phases. A slightly different form of Eq. (II-9) is the starting point for the steam slip momentum model developed by Levy.<sup>7</sup>

Various models for momentum exchange in different flow regimes will be examined in the next several chapters. The purpose of these examinations will be to identify terms in the models which are similar to Eq. (II-6) for the frictional pressure drop and thus to obtain model-dependent definitions of the exchange coefficients.

---

\* In these equations,  $\alpha_l + \alpha_g = 1$ , and the structure volume fraction ( $\alpha_s$ ) is not explicitly considered. When the structure fraction is neglected, the volume of the control volume is considered as the flow volume only.



CHAPTER III  
ANNULAR FLOW

The annular flow regime is not considered explicitly in SIMMER-I but is scheduled for inclusion in SIMMER-II. A typical annular flow situation is shown in Figure III-1. The walls of the channel are in contact with a liquid film and the gas flows in a central core. There may be appreciable amounts of liquid in the gas core, in which case the flow pattern is said to be annular mist. The orientation of the channel is important. If the gas is flowing vertically upward, the liquid film may be flowing either upward (cocurrent flow) or downward (countercurrent flow) depending upon the relative flow rates. In vertical annular flow, the pressure drop is strongly dependent upon the liquid film flow direction. The boundary between downward and upward film flow is known as flooding and is accompanied by a sharp increase in pressure drop and entrainment.

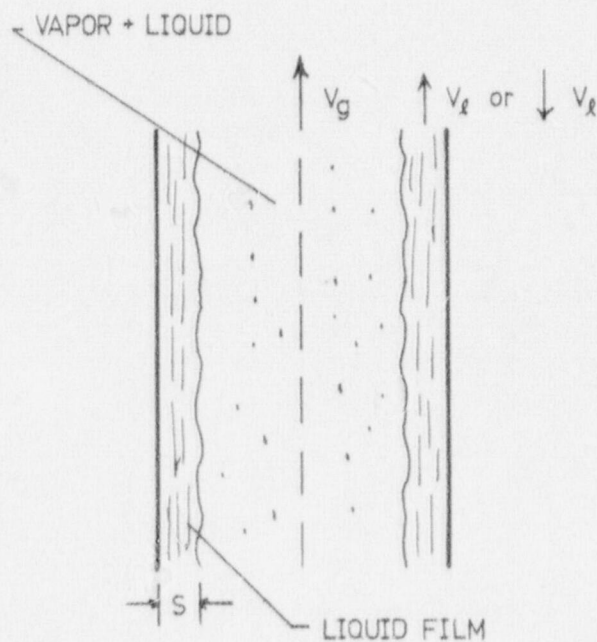


Figure III-1. Vertical Annular Flow

On the basis of the known flow geometry, some characteristics of the momentum exchange coefficient set are immediately apparent. The vapor-structure coefficient,  $K_{gs}$ , is zero since the vapor is not in contact with the structure.

If there is a mobile solid component, then  $K_{gs(m)}$  is nonzero; however, some form of this exchange coefficient will be needed for all the flow patterns. The wall shear stress depends only upon  $K_{ls}$ . Finally, the vapor-liquid drag is composed of two parts: one is related to the drag at the film-core interface; the second describes the interaction between liquid drops and gas in the core. Clearly, the second interaction is the same as  $K_{gl}$  in dispersed (drop) flow.

Several models have been proposed to define the pressure drop in annular flow. The purpose of a model is normally actually twofold:

1. To predict the pressure drop, and
2. To predict the void fraction  $\alpha$ .

In most cases  $\alpha$  is not known separately. The models considered here are those developed by Armand, Martinelli et al., Wallis, and Levy. Although various other models exist, the preceding models illustrate the general approaches to the problem.

#### Armand Model for Annular Flow

One of the earliest models developed for annular flow was that of Armand.<sup>8,9</sup> The model was originally intended for horizontal, simple annular flow, as shown in Figure III-2. For this model, the following assumptions are made:

1.  $\partial p / \partial r = 0$ , and
2.  $dp/dz)_A + dp/dz)_G \ll dp/dz)_F$ ; therefore,
3. The pressure drops in both phases are equal.

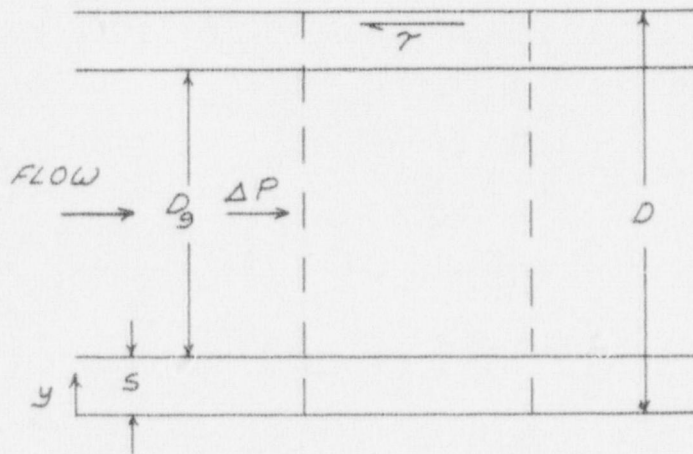


Figure III-2. Horizontal Annular Flow Pattern Used by Armand

Based on these assumptions, the momentum equation for the total flow is just

$$\tau_w = \frac{r}{2} \left( \frac{dp}{dz} \right)_{TP} \quad (III-1)$$

where  $\tau_w$  is the wall shear stress and  $(dp/dz)_{TP}$  is the two-phase pressure drop gradient. Armand further assumes that the film velocity profile is similar to that in turbulent single-phase flow so

$$v_y^{\ell} \propto \left( \frac{\tau_w}{\rho \ell} \right)^{4/7} \left( \frac{y}{\ell} \right)^{1/7} \quad (III-2)$$

After  $v_y^{\ell}$  is found in terms of the liquid mass flow rate and some algebra is applied, the following expression for the pressure drop is obtained:

$$\left( \frac{dp}{dz} \right)_{TP} = \left( \frac{dp}{dz} \right)_{\ell} \cdot \frac{H'}{\left(1 - \sqrt{\alpha}\right)^2 \left(1 + \frac{8\sqrt{\alpha}}{7}\right)^{7/4}} \quad (III-3)$$

which for  $\alpha > 0.7$ , can be approximated as

$$\left( \frac{dp}{dz} \right)_{TP} = \left( \frac{dp}{dz} \right)_{\ell} \cdot \frac{H}{(1 - \alpha)^2} \quad (III-4)$$

where  $H'$  and  $H$  are constants and the term  $(dp/dz)_{\ell}$  is the friction drop that would exist if the liquid film were to flow alone and turbulently in the channel. This is expressed by

$$\left( \frac{dp}{dz} \right)_{\ell} = \frac{f}{2D_g} \rho \left( \frac{V_{\ell}}{1 - \alpha} \right)^2 \quad (III-5)$$

The friction factor is that normally used in single-phase flow and is a function of Reynolds number and pipe roughness.

Pressure drop equations which take the form of Eq. (III-4) are often written as

$$\left( \frac{dp}{dz} \right)_{TP} = \left( \frac{dp}{dz} \right)_{\ell} \phi_{\ell}^2 \quad (III-6)$$

where  $\phi_{\ell}^2$  is the two-phase friction multiplier based upon the liquid. In Armand's model,

$$\phi_{\ell}^2 = \frac{H}{(1 - \alpha)^2} \quad (III-7)$$



The fact that

$$\phi^2 \propto \frac{1}{(1 - \alpha)^2}$$

has been noted independently by Levy.<sup>10</sup>

As pointed out earlier, the void fraction is not normally known, so Eq. (III-4) alone is not adequate to find the pressure drop. Armand assumes that the interfacial shear stress depends upon the relative velocity between the gas and liquid so

$$\left(\frac{dp}{dz}\right)_{TP} = \left(\frac{dp}{dz}\right)_{F,g} = 2f_g \frac{\rho_g (V_g - V_{lm})^2}{D \sqrt{\alpha}} \quad (\text{III-8})$$

where  $f_g$  is the gas friction factor and  $V_{lm}$  is the liquid velocity at the interface. For the assumed film velocity profile

$$V_{lm} \approx \frac{8}{7} V_l$$

so

$$\left(\frac{dp}{dz}\right)_{F,g} = 2f_g \frac{\rho_g \left(V_g - \frac{8}{7} V_l\right)^2}{D \sqrt{\alpha}} \quad (\text{III-9})$$

Taken together Eqs. (III-9) and (III-4) can be solved for both  $\alpha$  and the pressure drop, provided that  $H$  and a relationship for  $f_g$  can be found.

Eq. (III-9) can be solved for  $\alpha$ , using

$$\alpha = 1 - \frac{4 + \frac{8}{7} S}{5 + S \left( \frac{\psi}{1 - \psi} + \frac{8}{7} \right)} \quad \psi > 0.9 \quad (\text{III-10})$$

where

$$\psi = \frac{\frac{m_g}{\rho_g}}{\frac{m_g}{\rho_g} + \frac{m_l}{\rho_l}}$$

$$S = 4 \operatorname{Re}^{1/8} \sqrt{\frac{\rho_g}{\rho_l}} a'$$

$$a' = \frac{\sqrt{f_g}}{\text{const.}} \quad (\text{III-10a})$$

The empirical constant  $a'$  is found to be

$$a' = 0.69 + (1 - \psi) (a + 21.9 \sqrt{Fr_{\ell}}) \quad (\text{III-10b})$$

where  $Fr_{\ell}$  is the liquid Froude number

$$Fr_{\ell} = \frac{V^2}{dg}$$

Accordingly, the procedure is to find  $\alpha$  from Eq. (III-10) and then solve Eq. (III-5) for  $(dp/dz)_{TP}$ .

Although the analysis above applies only to pure annular flow, it is further assumed that an equation of the form

$$\left(\frac{dp}{dz}\right)_{TP} = \left(\frac{dp}{dz}\right)_{\ell} \frac{H}{(1 - \alpha)^i} \quad (\text{III-11})$$

applies for all flow regimes. For  $\psi < 0.9$ , the void fraction relationship is taken as

$$\alpha = 0.834$$

$$0 < \psi < 0.9$$

Values of  $H$  and  $i$  for various flow regimes are given in Table III-1.

TABLE III-1  
Values of  $H$  and  $i$  in the Armand Model<sup>10</sup>

Flow Regime	$\alpha$ Range	$H$	$i$
Bubbly, Slug	$0 < \alpha < 0.65$	1	1.42
Annular	$0.65 < \alpha < 0.9$	0.478	2.2
Annular Mist	$0.9 < \alpha < 0.99$	1.73	1.64

#### Application to SIMMER

The Armand correlations were initially developed for horizontal annular flow and were later extended to other flow regimes in horizontal two-phase flow. Direct application of these correlations to vertical annular flow is probably not warranted. However, the simplicity of the results is attractive, and correlations of a similar form might be used. Therefore, it is worthwhile to see how the Armand analysis can be related to SIMMER exchange coefficients.

Reference to Eq. (II-6c) makes evident that

$$\left(\frac{dp}{dz}\right)_{TP} = \left(\frac{dp}{dz}\right)_{F,g} = \frac{1}{\alpha} \left[ K_{gl} (V_g - V_l) \right] \quad (III-12)$$

where  $K_{gs}$  is taken to be zero. Hence, from Eq. (III-9),

$$K_{gl} = 2f_g \rho_g \frac{(V_g - \frac{8}{7} V_l)^2 \sqrt{\alpha}}{D(V_g - V_l)} \quad (III-13)$$

where  $f_g$  is found from Eqs. (III-10a) and (III-10b). Alternatively, Eq. (III-4) can be used to obtain

$$K_{gl} = \frac{f_l}{2D_g} \frac{H}{(1 + \alpha)^2} \rho \left( \frac{V_l}{1 - \alpha} \right)^2 \frac{\alpha}{(V_g - V_l)} \quad (III-14)$$

The second form is probably easier to use. In either case, the empirical constants  $a'$  or  $H$  need to be found for vertical annular flow. Suitable correlation can probably be developed from existing data and other correlations or could be developed experimentally.

The Armand model does not directly address the exchange coefficient  $K_{ls}$ . It is important to note that Eq. (III-4) states that

$$\tau_w TP \propto \tau_w l$$

but does not define the proportionality. If it is assumed that  $dp/dz)_A$  and  $dp/dz)_G$  can be neglected (Assumption No. 2 above), then

$$\tau_w TP = \frac{\tau_i}{\sqrt{\alpha}} \quad (\tau_w + \tau_i \text{ as } \alpha = 1) \quad (III-15)$$

in order to satisfy Assumption No. 3. Since  $\tau_i$  is known in the Armand model,  $\tau_w$  and hence a value for  $K_{ls}$  can be obtained. It should be emphasized, however, that neglecting the gravitational term for upward annular flow is a poor assumption.

11,6

Annular Flow Model of Martinelli et al.

A series of well-known correlations was proposed by Martinelli and his coworkers in which several basic assumptions are made:

1. The two-phase flow is considered as the sum of two single-phase flows,
2. The static pressure drop is caused by friction alone,



3. The radial pressure gradient is zero, and
4. The cross sections occupied by the single-phase flows remain approximately constant.

Assumption No. 1 above implies an annular flow pattern. In addition, the static pressure drops in both phases are equal.

The two-phase pressure drop is defined as

$$\left(\frac{dp}{dz}\right)_{TP} = 2f_l \frac{V_l^2 \rho_l}{D_l} \quad (\text{III-16a})$$

and

$$\left(\frac{dp}{dz}\right)_{TP} = 2f_g \frac{V_g^2 \rho_g}{D_g} \quad (\text{III-16b})$$

where the diameters  $D_l$  and  $D_g$  are defined as

$$D_l^2 = \frac{4A_l}{\pi\gamma_l}$$

$$D_g^2 = \frac{4A_g}{\pi\gamma_g}$$

The two empirical factors  $\gamma_l$  and  $\gamma_g$  are the ratios between the true cross-sectional flow areas and the areas associated with  $D_l$  and  $D_g$ . The absolute mean velocities are

$$V_l = \frac{\dot{m}_l}{A_l \rho_l} \quad (\text{III-17a})$$

$$V_g = \frac{\dot{m}_g}{A_g \rho_g} \quad (\text{III-17b})$$

and the friction factors are

$$f_l = C_l (\text{Re}_l)^{-r} = \frac{C_l (\pi/4)^r}{\left(\dot{m}_l / \gamma_l \rho_l D_l\right)^r} \quad (\text{III-18a})$$

$$f_g = C_g (\text{Re}_g)^{-s} = \frac{C_g (\pi/4)^s}{\left(\dot{m}_g / \gamma_g \rho_g D_g\right)^s} \quad (\text{III-18b})$$

where  $r$ ,  $s$ ,  $C_l$  and  $C_g$  are functions of the liquid and vapor Reynolds numbers.

Combining these equations yields

$$\left(\frac{D_l}{D_g}\right)^{5-r} = \frac{C_l}{C_g} \left(\frac{\rho_g}{\rho_l}\right) \left(\frac{\mu_l}{\mu_g}\right)^r \left(\frac{\dot{m}_l}{\dot{m}_g}\right)^{2-r} \left(\frac{\gamma_l}{\gamma_g}\right)^{r-2}$$

Finally, combining them with Eq. (III-16a) and noting that  $A_l + A_g = A$ ,

$$\left(\frac{dp}{dz}\right)_{TP} = \left(\frac{dp}{dz}\right)_g \left(\frac{D}{D_g}\right)^{5-s} \gamma_g^{s-2} \quad (\text{III-19a})$$

and

$$\left(\frac{dp}{dz}\right)_{TP} = \left(\frac{dp}{dz}\right)_g \gamma_g^{s-2} \left[ \gamma_g \left(\frac{D_l}{D_g}\right)^2 + \gamma_g \right] \frac{5-s}{2} \quad (\text{III-19b})$$

where  $(dp/dz)_g$  would be the pressure drop of the gas if it were flowing alone in the pipe.

If it is assumed that the flow in both phases is turbulent, then  $r = s = 0.2$  and  $C_l = C_g$ , so that

$$\frac{dp/dz}{dp/dz}_g = \gamma_g^{1.5} \left[ 1 + \left(\frac{\gamma_l}{\gamma_g}\right)^{0.25} \left(\frac{\rho_g}{\rho_l}\right)^{0.416} \left(\frac{\mu_l}{\mu_g}\right)^{0.653} \left(\frac{\dot{m}_l}{\dot{m}_g}\right)^{0.75} \right]^{2.4} \quad (\text{III-20})$$

For annular flow,  $\gamma_g \approx 1$  and  $\gamma_l$  is determined from experimentally measured pressure drops. Martinelli et al., found that

$$\frac{dp/dz}{dp/dz}_g = \phi_{tt,g}^2 = \left[ 1 + \gamma_l^{1/4} (\chi_{tt}) (\chi_{tt})^{3/4} \right]^{2.4} \quad (\text{III-21})$$

where  $\phi_{tt,g}^2$  is the two-phase friction multiplier based upon turbulent flow and the gas. The factor  $\chi_{tt}$  is

$$\chi_{tt} = \left(\frac{\mu_l}{\mu_g}\right)^{0.111} \left(\frac{\rho_g}{\rho_l}\right)^{0.555} \left(\frac{\dot{m}_l}{\dot{m}_g}\right) \quad (\text{III-22})$$

A similar expression for the pressure drop can be written in terms of the liquid-only pressure drop,

$$\phi_l^2 = \frac{dp/dz)_{TP}}{dp/dz)_l} \quad (III-23)$$

Finally, the two single-phase pressure drops are related by

$$X^2 = \frac{dp/dz)_l}{dp/dz)_g} \quad (III-24)$$

The value of X known as the Lockhart-Martinelli parameter varies with the flow regime. For turbulent-turbulent flow,

$$\begin{aligned} X_{tt} &= X_{tt}^{0.9} \\ &= \left(\frac{\dot{m}_l}{\dot{m}_g}\right)^{0.9} \left(\frac{\rho_g}{\rho_l}\right)^{0.5} \left(\frac{\mu_l}{\mu_g}\right)^{0.1} \end{aligned} \quad (III-25)$$

and  $\phi_{tt,l}^2$  or  $\phi_{tt,g}^2$  is found from graphs of  $\phi$  versus X. Such a graph is shown in Figure III-3.

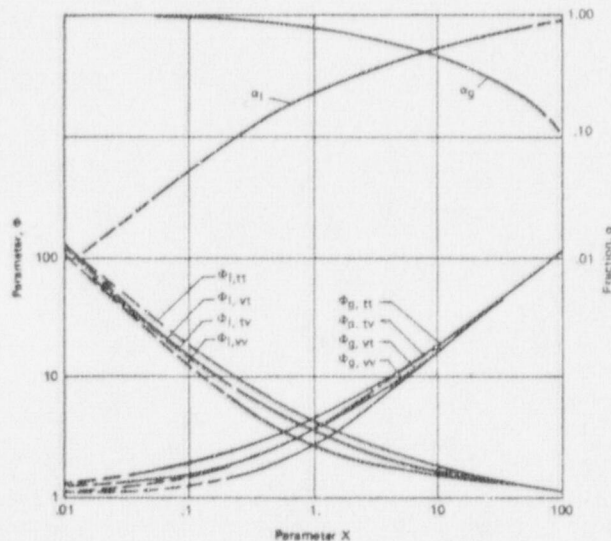


Figure III-3. Lockhart-Martinelli Parameter (See text for nomenclature; Figure reproduced from Reference 6)

As pointed out by Hsu and Graham,<sup>6</sup> the original work of Lockhart and Martinelli was done for adiabatic, horizontal annular flow at low pressure. Provided that the frictional term dominates, the correlation also applies to vertical flow at low pressure. For high-pressure boiling systems, improved correlations have been proposed by Martinelli and Nelson,<sup>12</sup> by Thom,<sup>13</sup> and



by Baroczy.<sup>14</sup> These are attempts to correct for known problems associated with flow pattern transition and the effects of mass velocity and quality. An additional criticism of the Lockhart-Martinelli correlation is that

$$\phi^2 \propto \frac{1}{(1 - \alpha)^3}$$

rather than, as noted earlier,

$$\phi^2 \propto \frac{1}{(1 - \alpha)^2}$$

However, this discrepancy does not appear to influence the actual accuracy of the correlation.

#### Application to SIMMER

The Martinelli-type correlations offer a simple method of calculating  $K_{gl}$ . Based on reference to Eq. (II-6c),

$$K_{gl} = \frac{\alpha}{V_g - V_l} \left[ \left( \frac{dp}{dz} \right)_g \phi_{ij,g}^2 \right] \quad (\text{III-26a})$$

or

$$K_{gl} = \frac{\alpha}{V_g - V_l} \left[ \left( \frac{dp}{dz} \right)_l \phi_{ij,l}^2 \right] \quad (\text{III-26b})$$

where  $i$  and  $j$  denote the flow regimes of the two phases. The single-phase pressure drop for the liquid phase is

$$\left( \frac{dp}{dz} \right)_l = \frac{2f_l}{D} \rho_l V_l^2 (1 - \alpha)^2 \quad (\text{III-27a})$$

and for the vapor is

$$\left( \frac{dp}{dz} \right)_g = 2 \frac{f_g}{D} \rho_g V_g^2 \alpha^2 \quad (\text{III-27b})$$

The friction factors have the normal single-phase values and are functions of the liquid and vapor Reynolds numbers.

As with the Armand model, no explicit consideration is given to the true wall shear stress  $\tau_w)_{TP}$ . However, since the basic assumptions in the two models are similar, Eq. (III-15) also applies here. Hence, a value for  $K_{lS}$  can be found. An improved approach for finding  $K_{lS}$  will be discussed in the next section.

## Annular Flow Model of Wallis

The annular flow model developed by Wallis<sup>15</sup> includes, in a straightforward manner, the effects of entrainment and the phenomena associated with flooding in vertical annular flow. This theory is described in detail in Reference 3. Only those results relevant to momentum exchange modeling are discussed here.

Review of the previous models makes it clear that definition of the interfacial and wall shear stresses is the primary goal. Ideally, the various physical processes at work are included directly in the formulations.

### Interfacial Shear

In vertical flow, it is necessary to include the gravitational component in the momentum equation. For the gas core, if negligible temporal and convective acceleration are assumed,

$$\frac{dp}{dz} + \rho_g g = -\frac{4\tau_i}{D\sqrt{\alpha}} \quad (\text{III-28})$$

where the interfacial shear stress is defined as

$$\tau_i = f_i \frac{\rho_g}{2} V_g^2 \quad (\text{III-29})$$

As the first approximation, the friction factor at the interface is related to the liquid film thickness  $s$ , by

$$f_i = 0.005 \left[ 1 + 300 \frac{s}{D} \right] \quad (\text{III-30})$$

and to the void fraction by

$$f_i = 0.005 \left[ 1 + 75(1 - \alpha) \right] \quad (\text{III-31})$$

Both of the equations assume the presence of waves on the liquid film and are approximately correct for  $s/D > 0.005$ .

Based on Eq. (III-31), the momentum equation can be written as

$$-\left[ \frac{dp}{dz} + \rho_g g \right] = 0.005 \left[ 1 + 75(1 - \alpha) \right] \frac{2\rho_g V_g^2}{D\sqrt{\alpha}} \quad (\text{III-32})$$

A number of corrections can be made to this equation to allow for various effects.

The effect of the liquid film velocity is included by writing

$$\tau_i = \frac{1}{2} f_i \rho_g (V_g - 2V_l)^2$$

where the interface velocity is taken to be twice the average liquid velocity. The effect of entrainment is accounted for in the following manner. The mass flow in the core is

$$\dot{m}_c = \dot{m}_g + \dot{m}_{le} \quad (\text{III-33})$$

where  $\dot{m}_{le}$  is the entrained liquid flow. The average core density is approximately

$$\rho_c = \frac{\dot{m}_c}{\dot{m}_g} \rho_g \quad (\text{III-34})$$

and the equation for the interfacial shear becomes

$$\tau_i = \frac{f_i}{2} \rho_g \left( \frac{\dot{m}_c}{\dot{m}_g} \right) (V_g - 2V_l) \quad (\text{III-35})$$

where  $f_i$  is defined in Eq. (III-31). Experiments reveal, however, that for high entrainment rates or large gas flows, the assumption of a constant  $f_g (=0.005)$  is incorrect. This error is corrected by using the homogeneous friction factor for the core

$$f_c = \nu/0.79 \text{Re}_c^{-0.25} \quad (\text{III-36})$$

where the core Reynolds number is defined as

$$\text{Re}_c = \frac{4 \dot{m}_c}{\pi D \mu_g}$$

Combining Eqs. (III-36), (III-35), (III-31), and (III-28) yields

$$\tau \left[ \frac{dp}{dz} + \rho_g g \right] = \frac{2}{D \sqrt{\alpha}} \rho_g \left( \frac{\dot{m}_c}{\dot{m}_g} \right) (V_g - 2V_l)^2 0.079 \left( \frac{4 \dot{m}_c}{\pi D \mu_g} \right)^{-0.25} \quad (\text{III-37})$$

Correction for the boundary layer in the gas core can also be included if necessary.

#### Wall Shear Stress

In vertical annular flow, the wall shear stress can be positive (upward) or negative (downward) depending upon the film thickness and the interfacial shear stress. This is because of the importance of the gravitational force in the film



equation of motion. Hence, an equation such as Eq. (III-15) is not suitable for the prediction of the wall shear stress.

The wall shear stress can be written as<sup>16</sup>

$$\tau_w = \tau_i \left( \frac{r_i}{r} \right) + \frac{1}{2} \left( \rho_l g + \frac{dp}{dz} \right) \left( \frac{r_i^2 - r^2}{r} \right) \quad (\text{III-38})$$

where  $r - r_i$  is the film thickness,  $s$  ( $2s = D(1 - \alpha)$ ). Since the second term is negative ( $r > r_i$ ), it is clear that  $\tau_w$  can change sign. Experiments show that as the gas flow rate is reduced during vertical cocurrent flow, the pressure drop decreases to a minimum and then increases until film flow reversal takes place. For thin films ( $s \ll r$ ), the minimum pressure drop corresponds to the point at which the shear stress is zero. Since the wall friction factor is

$$f_w = \frac{2\tau_w}{\rho_l V_g^2} \left( \frac{1 - \alpha}{\alpha} \right)^2 \quad (\text{III-39})$$

it is clear that the value of  $f_w$  (and hence  $K_{ls}$ ) can also vary considerably.

To account for this effect, Wallis writes the momentum equation as

$$-\left[ \frac{dp}{dz} + \rho_g g \right] = \frac{2f_w' V_g^2 \rho_f}{D} + B(1 - \alpha)g(\rho_f - \rho_g) \quad (\text{III-40})$$

where  $f_w'$  is given by Hewitt<sup>16</sup> and is very nearly

$$\begin{aligned} f_w' &\approx 16/\text{Ref} && \text{Ref} < 2000 \\ f_w' &\approx 0.079 \text{Ref}^{-0.25} && \text{Ref} > 2000 \end{aligned}$$

The value of  $B$  is

$$B = \frac{0.684}{1 + 0.183 \text{Re}_f} \quad \begin{aligned} &\text{Re}_f < 1000 \\ &1000 > \text{Re}_f > 8000 \\ &\text{Re}_f > 8000 \end{aligned}$$

Note that in horizontal flow  $B = 1$ , and Eq. (III-40) is simply the momentum equation for the entire flow.

#### Application to SIMMER

The momentum equation for the vapor is given by Eq. (III-37) and reference to Eq. (II-6c) shows that

$$\frac{1}{\alpha} K_{gl}(V_g - V_l) = \frac{2}{D\sqrt{\alpha}} \rho_g \left( \frac{\dot{m}_c}{\dot{m}_g} \right) (V_g - 2V_l)^2 0.079 \left( \frac{4\dot{m}_c}{\pi D V_g} \right)^{-0.25}$$

where  $\dot{m}_c$  is defined in Eq. (III-36). If there is no entrainment, then  $\dot{m}_c = \dot{m}_g$ . In this model then,

$$K_{gl} = \frac{2\sqrt{\alpha}}{D} \rho_g \left( \frac{\dot{m}_c}{\dot{m}_g} \right) (V_g - 2V_l)^2 (V_g - V_l)^{-1} \\ \cdot 0.079 \left( \frac{4\dot{m}_c}{\pi D u_g} \right)^{-0.25} \quad (\text{III-41})$$

The situation for the liquid-structure exchange coefficient is somewhat more complicated since Eq. (III-40) is written for the combined flow and Eq. (II-4) is written only for the liquid. Assuming for the sake of simplicity that  $B = 1$ , Eq. (III-40) then becomes

$$-\frac{dp}{dz} = \frac{2f_w'}{D} V_l^2 \rho_l + (1 - \alpha) \rho_f g + \alpha \rho_g g \quad (\text{III-42})$$

and Eq. (II-4) can be written as

$$-\frac{dp}{dz} = g \rho_l + \frac{K_{gl} (V_l - V_g)}{1 - \alpha} + \frac{K_{ls} V_l}{1 - \alpha} \quad (\text{III-43})$$

for the same assumptions used by Wallis. Equating the two equations and solving for  $K_{ls}$  yields

$$K_{ls} = \frac{2(1 - \alpha) f_w' V_g^2 \rho_l}{D V_l} + \frac{(1 - \alpha) \alpha}{V_l} g (\rho_g - \rho_l) \\ - K_{gl} \frac{(V_l - V_g)}{V_l} \quad (\text{III-44})$$

Although this result is somewhat more complicated than that for  $K_{gl}$ , it is still useful. Note that  $K_{ls} = f(K_{gl})$ . This condition arises from the coupling between interfacial and wall shear stress in the film momentum equation.

#### Levy Model for Annular Flow with Entrainment

Levy's model for annular flow with entrainment<sup>10</sup> is an extension of his work on the application of mixing length theory to two-phase flow. Only a brief summary of Levy's work is given here; the details are contained in Reference 10.

The local shear stress in a fluid can be written as

$$\tau = \rho_l u_{\rho u}^l \left( \frac{du}{dy} \right)^2 + u_{\rho u}^l \rho_l \frac{du}{dy} \left| \frac{dp}{dy} \right| \quad (\text{III-45})$$

where  $u$  is the local velocity,  $y$  is the length dimension normal to  $u$ , and  $\lambda_u$  and  $\lambda_{\rho u}$  are the velocity and momentum mixing lengths. The second term is normally neglected in compressible single-phase flow since the density gradient near the wall is small. However, in annular flow it is important to include the mass-transfer contribution to the shear at the interface where  $dp/dy$  is nonnegligible.

Using this approach, the shear stress at the interface can be related to the relative film thickness by

$$\left( \frac{\tau_i}{U_c (\rho_l - \rho_c) (U_c - U_l)} \right)^{1/2} R = F' \left( \frac{2s}{D - 2s} \right) \quad (\text{III-46a})$$

for  $-\frac{dp}{dz} \geq g\rho_l$  or by

$$\left( \frac{\tau_i}{U_c (\rho_l - \rho_c) (U_c - U_f)} \right)^{1/2} R \left( \frac{g\rho_l}{-dp/dz} \right)^{-1/3} = F' \left( \frac{2s}{D - 2s} \right) \quad (\text{III-46b})$$

for  $-\frac{dp}{dz} < g\rho_l$

where  $R \approx \left( \frac{\rho_l}{\rho_g} \right)^{1/3}$

and  $U_c$  and  $\rho_c$  are the velocity and density of the core which contains entrained liquid. The function  $F'$  is found from experimental data.

#### Application to SIMMER

The original purpose of Eq. (III-46) was to find the film thickness from the pressure drop. However, in advanced versions of SIMMER, assuming that the amount of entrained fluid is known, the film thickness is available. Hence, Eq. (III-46) can be solved for  $\tau_i$  by the following:

$$\tau_i = F' \left( \frac{2s}{D - 2s} \right)^2 \left( \frac{U_c (\rho_l - \rho_c) (U_c - U_l)}{(\rho_l/\rho_g)^{2/3}} \right) \quad (\text{III-47})$$

If the simplification is made that

$$2s \approx D(1 - \sqrt{\alpha}), \quad U_c \approx V_g, \quad \rho_c \approx \rho_g \left( \frac{\dot{m}_c}{\dot{m}_g} \right)$$

$$\begin{aligned} \frac{dp}{dz} F &= \frac{4\tau_i}{D\sqrt{\alpha}} \\ &= \frac{4}{D\sqrt{\alpha}} F' \left( \frac{1 - \sqrt{\alpha}}{\sqrt{\alpha}} \right)^2 \frac{V_g (\rho_l - \rho_g \frac{\dot{m}_c}{\dot{m}_g}) (V_g - V_l)}{(\rho_l/\rho_g)^{2/3}} \end{aligned} \quad (\text{III-48})$$



and based on Eq. (II-6c)

$$K_{g\ell} = \frac{4\sqrt{\alpha}}{D} F' \left( \frac{1 - \sqrt{\alpha}}{\sqrt{\alpha}} \right)^2 \frac{V_g \left[ \rho_\ell - \rho_g \left( \frac{\dot{m}_c}{\dot{m}_g} \right) \right]}{(\rho_\ell / \rho_g)^{2/3}} \quad (\text{III-49})$$

To find  $K_{\ell s}$ , the same procedure as that noted in the Wallis model is followed. As a first approximation, the value of  $K_{g\ell}$  can be substituted from Eq. (III-49) into Eq. (III-44) to find  $K_{\ell s}$ .

It should be noted that the function  $F'$  is found from experimental data. Levy's values for  $F'$ , which were mainly calculated from experiments with air-water and water-steam systems, do not agree with those found in sodium systems.<sup>17,18</sup>

#### Summary

An overview of the models for momentum exchange in annular flow is given in Table III-2. Also included is the SIMMER-I formulation for dispersed flow which is discussed in the next chapter. The SIMMER model does not require that in the steady state the pressure drops in both phases be equal. This is incorrect. The models of Wallis and Levy appear to be the most sophisticated; however, the Martinelli model often gives surprisingly good results. When Table III-2 is used, care should be taken to recall the assumptions and limitations associated with each model.

TABLE III-2  
Momentum Exchange Models

Annular Flow (see text for nomenclature)

#### 1. SIMMER-I

$$K_{g\ell} = \frac{3 \bar{\rho}_g \alpha_\ell}{2 (\alpha_g)^2 (r_p)^2} \left[ \frac{3\mu_g}{\rho_g} + \frac{r_p C_D}{4} \left| \vec{V}_g - \vec{V}_\ell \right| \right] \quad \text{Eq. (IV-1)}$$

$$K_{gs} = \frac{f_g \alpha_g \rho_g \left| \vec{V}_g \right| \alpha_s}{2D} \quad \text{Eq. (IV-6a)}$$

$$K_{\ell s} = \frac{f_\ell \alpha_\ell \rho_\ell \left| \vec{V}_\ell \right| \alpha_s}{2D} \quad \text{Eq. (IV-6b)}$$

Comments:

1. Intended for dispersed flow,
2. Does not preserve equal static pressure drops in steady state,
3.  $K_{gs}$  nonzero,

TABLE III-2 (Continued)

4. Incorrect treatment of interfacial shear, and
5. No flow regime transition difficulties.

2. Armand

$$K_{gl} = \frac{2 f_g \rho_g (V_g - \frac{8}{7} V_l)^2 \sqrt{\alpha}}{D (V_g - V_l)}$$

Eq. (III-13 or

Eq. (III-14)

$$= \frac{f_l}{2D_g} \frac{H}{(1-\alpha)^2} \rho \left( \frac{V_l}{1-\alpha} \right)^2 \frac{\alpha}{(V_g - V_l)}$$

$$K_{gs} = 0$$

$$K_{ls} - \text{See discussion of Eq. (III-15)}$$

Comments:

1. Requires H or  $f_g$  from experiment,
2. Intended for horizontal flow, and
3. No treatment of entrainment, wave phenomena.

3. Martinelli

$$K_{gl} = \frac{\alpha}{V_g - V_l} \frac{2f_l}{D} \rho_l V_l^2 (1-\alpha)^2 \phi^2$$

Eq. (III-26a)

$$K_{gs} = 0$$

$$K_{ls} - \text{See discussion of Eq. (III-15)}$$

Comments:

1. Separated flow model originally for horizontal flow,
2.  $\phi^2$  is partially empirical, and
3. For additional assumptions, see page 20.

4. Wallis

$$K_{gl} = \frac{2\sqrt{\alpha}}{D} \rho_g \left( \frac{\dot{m}_c}{\dot{m}_g} \right) (V_g - 2V_l)^2 (V_g - V_l)^{-1} 0.079 \left( \frac{4m}{\pi D u_g} \right)^{-0.25}$$

Eq. (III-41)

$$K_{gs} = 0$$

$$K_{ls} = \frac{2(1-\alpha)f_w V_g^2 \rho_l}{D V_l} + \frac{(1-\alpha)(\alpha)}{V_l} g (\rho_g - \rho_l)$$

$$- K_{gl} \frac{(V_l - V_g)}{V_l}$$

Eq. (III-44)

TABLE III-2 (Continued)

Comments:

1. Includes effects of entrainment,
2. Includes vertical flow, and
3. Interfacial shear assumes disturbance waves.

5. Levy

$$K_{gl} = \frac{4\sqrt{\alpha}}{D} F' \left( \frac{1 - \sqrt{\alpha}}{\sqrt{\alpha}} \right)^2 \frac{v_g \left| \rho_l - \rho_g \left( \frac{\dot{m}_c}{\dot{m}_g} \right) \right|}{(\rho_l / \rho_g)^{2/3}} \quad \text{Eq. (III-49)}$$

$$K_{gs} = 0$$

$K_{ls}$  = same as in Wallis model; Eq. (III-49) is used for  $K_{gl}$

Comments:

1. Suitable for vertical flow,
2. Deals directly with entrainment, and
3. Requires experimental data for  $F'$ .



CHAPTER IV  
BUBBLY AND DISPERSED FLOW

SIMMER-I contains formulations for momentum exchange coefficients in bubbly and dispersed flow. These formulations are also used in the range of void fractions characteristic of slug, churn-turbulent, and annular flow where, because of the different interfacial conditions, their application is unwarranted. In this chapter, the general concern is with momentum transfer between vapor and structure and between liquid and structure. However, some comments concerning vapor-liquid interactions are also made.

In SIMMER-I, the formulation for the usual exchange coefficient between liquid and vapor, with vapor as the continuous phase,<sup>1</sup> is

$$K_{g\ell} = \frac{3\bar{\rho}_g \alpha_\ell}{2\alpha_g^2 r_p^2} \left[ 3 \frac{u_g}{\rho_g} + \frac{r_p C_D}{4} \left| \vec{V}_g - \vec{V}_\ell \right| \right] \quad (\text{IV-1})$$

where  $r_p$  is the liquid drop radius. Although not mentioned in Reference 1, the first term in Eq. (IV-1) is a simplified form of the Hadamard-Rybczynski drag model for deformable particles.<sup>19</sup> The second term describes the form drag. This definition of  $K_{g\ell}$  requires that the Weber number for the liquid droplets

$$W_\ell = \frac{2r_p \rho_g}{\sigma_\ell} \left| V_g - V_\ell \right|^2$$

be sufficiently small so that little distortion from sphericity occurs.<sup>20</sup> By the time appreciable distortion does begin, the form drag predominates so that little error results. Unfortunately, this does not apply for bubbles in a continuous liquid phase. In this case, there is a sharp transition from Hadamard-Rybczynski drag to form drag at small bubble distortion.<sup>20</sup> This is because of flow separation at the bubble. Accordingly, the use of the analog of Eq. (IV-1) for bubbly flow is probably not justified.

Although Eq. (IV-1) is a reasonable formulation for dispersed flow in the steady state, it does not include all the effects present when droplet acceleration takes place. Since strong accelerations in the flow fields can be expected in an HCDA calculation, their omission represents a code deficiency. When a particle is accelerated, it generates a two-dimensional flow about it. This flow field possesses kinetic energy and work is done on the field by the particle.

For a particle with acceleration,  $a$ , in a stationary liquid, the force required to produce the acceleration is<sup>3</sup>

$$F = \frac{4}{3} \pi r_p^3 \left( \rho_p + \frac{\rho_l}{2} \right) a \quad (\text{IV-2})$$

This is known as the apparent mass effect since the mass to be accelerated is larger than the particle mass. In addition to the potential field produced by the accelerating particle, a viscous flow field also results. The additional force which results, known as the Basset force<sup>21</sup>, is difficult to calculate since it depends upon the earlier particle history. Wallis points out that for laminar flow, and with constant acceleration, the ratio of the Basset force to the steady state drag is

$$\frac{F_B}{F_D} = \frac{d}{\sqrt{\frac{\pi \rho_l t}{\mu_l}}} \quad (\text{IV-3})$$

where  $d$  is the particle diameter and  $t$  is the time measured from the beginning of the acceleration.<sup>3</sup> Hence, for small time periods the Basset force can be significant. When many particles (bubbles, drops, solid fragments) are present, the magnitudes of the apparent mass and Basset forces are different. It appears prudent then to examine accelerating systems and to determine proper models for  $K_{gl}$  under such conditions.

At low velocities, the drag predictions of Eq. (IV-1) or of other models can be compared to those obtained using the drift flux model.<sup>3</sup> By definition, the drift flux is

$$j_{gf} = \alpha(1 - \alpha)(V_g - V_l) \quad (\text{IV-4})$$

The drift flux physically represents the rate at which the vapor is moving relative to the superficial velocity of the combined flow. As pointed out by Hsu and Graham,<sup>6</sup> equations for the drift flux imply the use of the momentum equations for the two fields since the relative velocity ( $V_g - V_l$ ) depends upon momentum exchange. Hence, the relative velocity predicted by SIMMER can be compared with the drift flux model as a check on the code vapor-liquid momentum exchange model. As an example, the drift velocity for small bubbles in vertical flow is<sup>4</sup>

$$V_g - V_l = 1.53(1 - \alpha)^2 \left[ \frac{\sigma_g(\rho_l - \rho_g)}{\rho_l^2} \right]^{1/4} \quad (\text{IV-5})$$

Similar equations exist for large bubble, churn-turbulent and slug flow. The drift flux model is also a useful tool in correlating experimental data on two-phase flows, and its use in analyzing verification experiments is worth consideration.

For momentum exchange between the fluid fields and the structure, the exchange coefficients in SIMMER-I are defined as<sup>22</sup>

$$K_{gs} = f_g \frac{\alpha_g \rho_g |\vec{v}_g| \alpha_s}{2D} \quad (\text{IV-6a})$$

$$K_{ls} = f_l \frac{\alpha_l \rho_l |\vec{v}_l| \alpha_s}{2D} \quad (\text{IV-6b})$$

where  $\alpha_s$  represents the amount of structure in a computational cell. The friction factors are found from the liquid and vapor Reynolds numbers using

$$f_{g,l} = \frac{16}{\text{Re}_{g,l}} \quad (\text{Laminar Flow}) \quad (\text{IV-7a})$$

$$f_{g,l} = (0.08 \text{Re}_{g,l})^{-0.25} \quad (\text{Turbulent Flow}) \quad (\text{IV-7b})$$

The associated pressure drops due to friction are

$$\frac{dp}{dz} F_{g \rightarrow s} = \frac{1}{\alpha_g} K_{gs} V_g = f_g \frac{\rho_g V_g^2}{2D} \alpha_s \quad (\text{IV-8a})$$

$$\frac{dp}{dz} F_{l \rightarrow s} = \frac{1}{\alpha_l} K_{gl} V_l = \frac{f_l \rho_l V_l^2}{2D} \alpha_s \quad (\text{IV-8b})$$

Actually, as written, Eqs. (IV-8a) and (IV-8b) are incorrect. It can be readily shown that the frictional term in the SIMMER momentum equations should not depend on  $\alpha_s$ . For a typical subassembly,  $\alpha_s \approx 0.6$ ; therefore, the effect of drag is underestimated.<sup>23</sup> Further, for the friction factor as defined in Eq. (IV-7) (Fanning friction factor), the correct relationship for the pressure drop is

$$\frac{dp}{dz} F_{g \rightarrow s} = \frac{2f_g \rho_g V_g^2}{D} \quad (\text{IV-9a})$$

$$\frac{dp}{dz} F_{l \rightarrow s} = 2 \frac{f_l \rho_l V_l^2}{D} \quad (\text{IV-9b})$$

In the following discussion, Eq. (IV-9) and the associated definitions for  $K_{gs}$  and  $K_{gl}$  will be used.

The total pressure drop due to drag on the structure is the sum of  $K_{gs} V_g$  and  $K_{ls} V_l$  in Eq. (II-7). That is, it is assumed that the fluid-structure drag arises as if two totally independent flows acted on the structure. It is worthwhile to examine this assumption in the light of conventional treatments of bubbly and mist flow. The standard technique is to consider such flows homogeneous; no slip is allowed between the vapor and liquid. Corrections for nonzero slip can be obtained using the drift flux model and radial effects can be accounted for using the variable density model.<sup>24</sup>



The starting point for the homogeneous model is the mixture momentum equation (II-8). The frictional pressure drop depends only upon  $K_{ms} V_m$  since no vapor-liquid interaction is assumed to occur. The friction factor is defined in terms of a mixture viscosity,  $\bar{\mu}$ . One common definition is

$$\bar{\mu} = x\mu_g + (1 - x)\mu_l \quad (IV-10)$$

where  $x$  is the quality

$$x = \frac{\dot{m}_g}{\dot{m}_g + \dot{m}_l} \quad (IV-11)$$

The friction factor for the two-phase flow is

$$f_{TP} = 0.08 \text{Re}_m^{-1/4} \quad (IV-12)$$

where

$$\text{Re}_m = \frac{\rho_m V_m D}{\bar{\mu}}$$

Based on this definition, it is easy to show that

$$\left(\frac{dp}{dz}\right)_{F,m} = \left(\frac{dp}{dz}\right)_{F,l_0} \phi_{l_0}^2 \quad (IV-13)$$

where

$$\phi_{l_0}^2 = \left[ 1 + x \left( \frac{\rho_l}{\rho_g} - 1 \right) \right] \left[ 1 + x \left( \frac{\mu_l}{\mu_g} - 1 \right) \right]^{-1/4} \quad (IV-14)$$

and

$$\left(\frac{dp}{dz}\right)_{F,l_0} = 2 \frac{f_{l_0} \rho_m V_m}{D} \left( \frac{\rho_m}{\rho_l} \right) \quad (IV-15)$$

The friction factor is based upon the total flow and the liquid viscosity

$$f_{l_0} = 0.08 \left( \frac{\rho_m V_m D}{\mu_l} \right)^{-1/4} \quad (IV-16)$$

Based on the homogeneous model, it is straightforward to write

$$K_{gs} = \alpha \phi_{l0}^2 \frac{2f_{l0}}{D} \rho_m V_m \left( \frac{\rho_m}{\rho_l} \right) \quad (IV-17)$$

$$K_{gl} = (1 - \alpha) \phi_{l0}^2 \left( \frac{2f_{l0}}{D} \right) \rho_m V_m \left( \frac{\rho_m}{\rho_l} \right) \quad (IV-18)$$

There is of course no term for liquid-vapor interaction since the velocities are equal. If there is some slip but  $V_g/V_l - 1$  is small,\* then as a first approximation,

$$K_{gs} = \alpha \phi_{l0}^2 \left( \frac{2f_{l0}}{D} \right) \rho_m V_m \left( \frac{V_m}{V_g} \frac{\rho_m}{\rho_l} \right) \quad (IV-19)$$

$$K_{ls} = (1 - \alpha) \phi_{l0}^2 \left( \frac{2f_{l0}}{D} \right) \rho V \left( \frac{V_m}{V_l} \frac{\rho_m}{\rho_l} \right) \quad (IV-20)$$

The homogeneous model is often used successfully for bubbly and mist flows. It is therefore interesting to see how the SIMMER formulation compares with this model for  $V_l = V_g$ . The total frictional pressure drops are

#### SIMMER

$$\begin{aligned} \frac{dp}{dz} F, s &= 2f_g \frac{\alpha \rho_g}{D} V_m^2 + 2f_l \frac{(1 - \alpha)}{D} \rho_l V_m^2 \\ &= \frac{2V_m^2}{D} \left[ \alpha f_g \rho_g + (1 - \alpha) f_l \rho_l \right] \end{aligned} \quad (IV-21)$$

#### Homogeneous Model

$$\frac{dp}{dz} F, s = 2 \frac{f_{l0}}{D} \left( \phi_{l0}^2 \right) \rho_m V_m^2 \left( \frac{\rho_m}{\rho_l} \right) = 2 \frac{V_m^2}{D} \left[ \rho_m f_{l0} \phi_{l0}^2 \frac{\rho_m}{\rho_l} \right] \quad (IV-22)$$

The terms of interest in both equations are those in square brackets. After some algebraic manipulation and the use of the relationship between quality and void fraction, it can be shown that

$$\left[ \alpha \rho_g f_g + (1 - \alpha) \rho_l f_l \right] = \alpha \rho_g Re_g^{*-1/4} + (1 - \alpha) \rho_l Re_l^{*-1/4}$$

\* There is some evidence that  $V_g/V_l$  seldom exceeds 2 due to apparent mass effects (3).

and

$$\left[ \rho_m f_{l0} \phi_{l0}^2 \left( \frac{\rho_m}{\rho_l} \right) \right] = (\text{Re}_g^* + \text{Re}_l^*)^{-1/4} [\alpha \rho_g + (1 - \alpha) \rho_l]$$

where the reduced Reynolds numbers are

$$\text{Re}_g^* = \alpha \text{Pe}_g$$

$$\text{Re}_l^* = (1 - \alpha) \text{Re}_l$$

Clearly, the two quantities in brackets are not equal. The SIMMER formulation does not reduce to the well-known homogeneous model when the vapor and liquid velocities are identical.



CHAPTER V  
FLOW REGIME TRANSITION CRITERIA

With the availability of multiflow regime capability in SIMMER-II, it becomes necessary to establish criteria for switching the calculation from one flow pattern to another. A number of methods have been proposed for defining the transition points:

1. Relative flow rates,
2. Mass velocity and quality,
3. Volumetric flow fraction and Froude number,
4. Dimensional analysis, and
5. Interfacial wave stability.

In addition, separate criteria for each flow transition have been suggested. It is not clear which of these approaches is best; however, it appears that the use of a separate criterion for each transition is the most straightforward. On the other hand, a consistent method is easier to implement within a computer code. Several methods will be reviewed briefly here. For a discussion of special criteria for individual transitions see References 5 and 6.

Establishing a consistent method for treating flow regime transitions is made difficult by the fact that many factors, not easily included in a model, are at work. For example, the transition from bubbly to slug flow depends upon the rate of bubble coalescence. Entrance effects are therefore important. Similarly, the manner in which liquid is introduced into a vapor flow can strongly affect entrainment in the development of annular flow. This should be kept in mind while the methods which follow are considered.

Dimensional Analysis<sup>25</sup>

According to the dimensional analysis approach, the flow transitions can be examined in terms of gravitational, pressure, viscous, and surface forces, that is, in terms of the Froude, Euler, Reynolds, and Weber numbers. The classification in terms of these dimensionless numbers is shown in Figure V-1. It should be noted that the Reynolds number is contained implicitly because of the use of  $f_{TP}$  in the method. The general validity of the actual transition parameters shown has not been established. However, the simplicity of the system and its suitability for experimental verification (or modification) is certainly attractive.

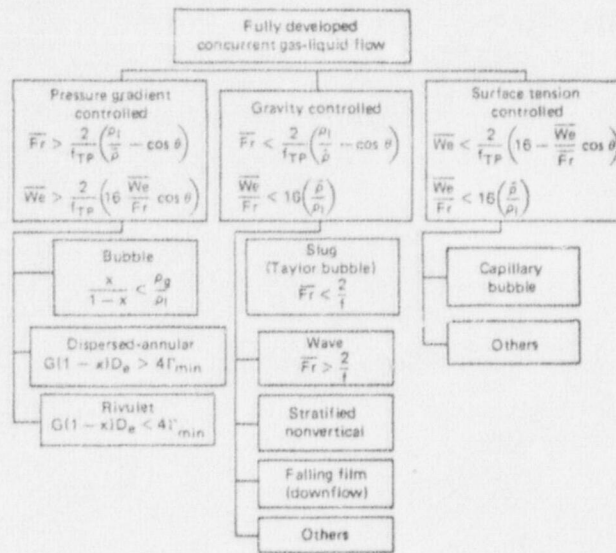


Figure V-1. Flow Transition Criteria Based on Dimensional Analysis (Figure from Reference 25).

### Volumetric Flow Fraction and Froude Number<sup>26</sup>

The volumetric flow fraction and Froude number approach is to correlate the vapor volumetric flow fraction

$$\beta = \frac{j_g}{j_g + j_f} \tag{V-1}$$

with the Froude number

$$Fr = \frac{[(j_l + j_g)/A]^2}{gD} \tag{V-2}$$

The transition points are

1. Bubble-Plug --  $\beta = 0.05 Fr^{0.2}$
2. Plug-Dispersed Plug --  $\beta = 0.12 Fr^{0.15}$
3. Dispersed Plug-Emulsion --  $\beta = 0.5 Fr^{0.1}$
4. Emulsion-Film Emulsion --  $\beta = 0.65 Fr^{0.05}$
5. Film Emulsion-Drop --  $\beta = 0.85 Fr^{0.02}$

The second through fourth flow patterns encompass churn-turbulent and slug flow. This general approach has been used by Griffith and Wallis to define the slug flow region.<sup>4</sup>

The interaction between phases of different velocities leads to waves. An example of this is the well-known Kelvin-Helmholtz instability. According to Kutateladze,<sup>28</sup> the wave stability depends on the ratio of dynamic to surface forces and can be expressed as

$$K = \frac{u^* \sqrt{\rho_c}}{[g \sigma_H (\rho_H - \rho_L)]^{1/4}} \quad (V-3)$$

where  $u^*$  is the critical velocity of the lighter phase,  $\rho_c$  is the density of the continuous phase, and the subscripts H and L denote the lighter and heavier fluids. The critical values of K are given in Table V-1. This model has been used by Fauske<sup>27</sup> to describe boiling flow regimes in an HCDA.

TABLE V-1  
Flow Transition Criteria Based on Wave Stability<sup>27</sup>

<u>Flow Transitions</u>	<u>K-Value</u>	<u>Continuous Phase</u>
Breakdown of Bubbly Flow	0.3	$\rho_c = \rho_H$
Breakdown of Churn-Turbulent Flow	0.14	$\rho_c = \rho_L$
Flooding of Liquid Film (onset of annular flow)	3	$\rho_c = \rho_L$
Liquid Film Entrainment (onset of annular mist flow)	3.7	$\rho_c = \rho_L$

It would certainly be useful to compare the models outlined above with one another and with existing data on flow pattern transition. The need for new experiments in this area is not clear although it certainly cannot be ruled out at this time. With respect to the inclusion of transition models in SIMMER, it should be noted that the momentum models do not agree at the transition points. Hence some smoothing is needed to avoid numerical difficulties when the flow regime changes.



CHAPTER VI  
EXPERIMENTAL NEEDS

A series of generic needs related to momentum exchange can be readily identified, i.e., major uncertainties exist which are independent of a particular model or flow regime. These uncertainties are directly related to the simplifying assumptions made in the models discussed above. In particular, the following questions need to be considered:

1. What is the effect of rapid acceleration on momentum exchange?  
The normal assumption is that steady state models and correlation can be applied to transients with rapid acceleration. Because of apparent mass and Basset forces, this may be a serious error. In addition, the nature of flow transitions may be seriously affected. Very little experimental data are available for such conditions.
2. How strongly do heat transfer processes affect momentum exchange?  
The particular manner in which mass or density changes occur is often neglected. However, in boiling systems for example, the effects of bubble formation, increased momentum transfer near the wall, and nonequilibrium vapor quality are all significant. How these effects can be included in SIMMER is unclear, and experiments appear necessary.
3. What is the effect of a two-dimensional flow field?  
Many models assume that the flow is essentially one-dimensional. The application of these models to a two-dimensional flow field may be unwarranted. Evidence of pressure drops at bends and area changes in two-phase flow suggest that such effects can be significant. One would expect such effects to be important during sodium voiding, at partial blockages, and when sub-assembly walls fail.
4. How important are inlet effects?  
Because of the coupling between heat transfer and fluid mechanics in diabatic two-phase flow, there are no true steady-state velocity or temperature profiles. Entrance effects can exist for very large length-to-diameter (L/D) ratios. This can also strongly influence the range at which flow regime transitions occur. These effects are often neglected in computer codes although their importance may be great.

Although not discussed in this report, the problems of particle-particle and particle-fluid drag which will arise in SIMMER-II will probably also require experimental efforts.

In addition to these general concerns, specific model-related problems exist which require verification. Whenever possible, the verification should be accomplished through a comparison of a SIMMER calculation, using the model in question, with available experimental data. An example of this is the drift velocity of bubbles in bubbly flow which was mentioned in Chapter IV. New experiments are required where the data are insufficient or if new information is needed in the course of model development. Until the requisite code-experiment checks are performed, it is not possible to define new detailed experiments. However, in addition to the generic needs indicated above, several areas are worth noting. The behavior of liquid-vapor shear in bubbly flow should be very carefully verified. Similarly, it is important to have adequate models for flooding, flow reversal, and rates of entrainment in annular flow. Other concerns may exist in slug and drop flow but they have not yet been identified.

## CHAPTER VII

### SUMMARY

The purpose of this report has been to review existing models used for momentum exchange coefficients in SIMMER. A variety of models was considered for the bubbly, annular, and drop flow regimes. For each of the models, the forms of  $K_{gs}$ ,  $K_{gl}$  and  $K_{ls}$  were developed and, where possible, compared to the formulations contained in SIMMER-I. Care was taken to state the assumptions associated with each model and to relate them to the types of problems addressed by SIMMER. In addition, the question of phase transition criteria was briefly examined, and several methods for establishing the transition points were noted. Finally, the question of experimental needs was addressed. Those needs are related to the validity of the model assumptions under HCDA conditions. The point is made that in some cases the models can be verified by comparing SIMMER calculations to existing experimental data, while in others new experiments will often be required. The appropriate approach can only be determined by carrying out the necessary calculations. It is felt that by systematically testing existing models where possible and by performing experiments and model development only where necessary the verification process can be most efficiently carried out.



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