OCT 24 1978

INTERIM REPORT

Accession No. __________
Contractor's Report No. __________

Contract Program or Project Title: Material Control at Licensed Processing Facilities

Subject of this Document: Progress reported for June 1978

Type of Document: Informal letter progress report

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Date of Document: August 21, 1978

Responsible NRC Individual and NRC Office or Division:

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Prepared for
U.S. Nuclear Regulatory Commission
Washington, D.C. 20555
NRC Research Order No. 60-98-097
NRC FIN No. A0115

7810310367

INTERIM REPORT

LAWRENCE LIVERMORE LABORATORY

August 21, 1978
MC 78-738

RECEIVED
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Mr. William M. Murphey
Technical Support Branch
Division of Safeguards, Fuel Cycle
and Environmental Research
Office of Nuclear Regulatory Research
U.S. Nuclear Regulatory Commission
Washington, D.C. 20555

Dear Bill:

This letter constitutes the Monthly Letter Report for June, 1978 for our LLL Safeguards Project in Material Control at Licensed Processing Facilities.

The activities are organized relative to the major areas of development as listed in our Program Milestone Chart of June, 1978.

Assessment Methodology Development

(LLL Contributors: H. Lambert, A. Parziale, I. Sacks, R. Sanborn, and S. Weissenberger.)

An internal LLL paper was written entitled, "Modelling Adversary Tampering of a Safeguard System with a Petri Net". The paper discusses automatic synthesis of a Petri Net from input network/matrix adjacency information, state transitions and dynamics of Petri Nets, qualitatively strong safeguard system designs, and the Petri Net as a base model for a quantitative analytical analysis and Monte Carlo simulation.

Two approaches have been proposed for the analysis of large fault trees employing the FTAP2 code on LLL's 7600 system, one using the FTN compiler for an enlarged FTAP2 code when a larger array in LCM becomes available in three to six months, and the other using the CHAT compiler for a modified version of FTAP2 in the near term.

Adversary and Societal Consequence Models

(LLL Contributors: S. Weissenberger, G. Corynen, and I. Sacks.)

Discussions were held between S. Weissenberger and Craig Kirkwood of Woodward-Clyde on a new adversary "archetype" list aggregated from the over one hundred categories generated in earlier LLL/Woodward-Clyde work. The new "archetypes" were mapped onto the earlier I. Sacks adversary categories.

A review of the Aggregated Systems Model was presented by S. Weissenberger to F. Arsenault, J. Durst, L. Evans, et al. at LLL on June 9.

Further modifications were made by Bruce Judd of Applied Decision Analysis and S. Weissenberger to the Executive Report on PBR settings, focusing on the optimal setting of probabilities of detection and interruption of certain adversaries in terms of system cost versus cost of diversion consequences.

Facility and Components Characterization

(LLL Contributors: J. Candy, D. Dunn, J. Huebel, G. Morris, and R. Rozsa.)

Further checks were performed on the final version of the Estimator code, but no bugs were found.

Progress was made in identifying SNM sources (material types) and appropriate removal nodes, in material and target attractiveness ranking, and in generating a computer code to give graphs of the attractiveness.

Discussions were held with Ralph Keeney of Woodward-Clyde concerning general parameterizations for material attractiveness based on a new material/target attractiveness function elicited for use in the Facility 'X' exercise. Elicitations of probabilities of detection were performed by Bruce Judd with S. Weissenberger and the Facility 'X' team.

Testing of Assessment Methodology

(LLL Contributions: F. Gilman, J. Candy, D. Dunn, J. Huebel, H. Lambert, J. Lim, A. Parziale, and R. Sanborn.)

D. Dunn visited the NRC in Silver Spring, Maryland, for general discussions with R. Shepard, J. Durst, et al. concerning Facility 'X' and monitors.

Discussions were held with D. Richardson and S. Scala of SRI concerning monitor characterization for Facility 'X', and a statement of work was drawn up.

A description of the Facility 'X' accounting system was compiled, and the structure and operation of the Wood River Junction accounting system was established to characterize error analysis and detection performance for incorporation into the digraphs.

Progress was made in developing a generic unit model digraph of the accounting system, and in defining the outputs of Target ID for Facility 'X' to assure their compatibility for the Digraph Fault Tree task.

Systems Integration, Documentation, and Training Program

No effort was expended in this area during the month of June.

Additional Meetings

J. Lim attended the INMM 1978 Annual Meeting held in Cincinatti, Ohio, June 27 - June 29, and presented a paper coauthored by F. Gilman, H. Lambert, and herself entitled, "The Results of a Directed Graph and Fault Tree Assessment of an MC&A System".

A. Maimoni and J. Lim made a presentation of "Overview of the LLL Assessment Procedure for MC&A Systems" to F. Arsenault, J. Durst, L. Evans, et al. at LLL on June 9.

A Workshop on Decision Analysis and Social Risk was held at Asilomar, California, June 28 - June 30, and attended by NRC Material Control & Accountability Project decision analysis subcontractors and LLL decision analysis personnel. The purpose of the convocation was to allow personnel from both NRC projects to present and debate basic methodological issues concerning the use of decision analysis in the assessment of social risk. A report containing copies of the working papers, and describing the results of the workshop is forthcoming.

This month's Technical Highlights features a memo by S. Weissenberger on the subject of "Optimal Safeguard System Resource Allocation".

Sincerely,

John H. O'Brien

JOHN H. O'BRIEN
Assistant Program Manager
Material Control Project

ctn

Copy to:
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OPTIMAL SAFEGUARD SYSTEM RESOURCE ALLOCATION

Background and Abstract

A major task of the MC&A program is to develop tools to assist the NRC in setting safeguard regulation criteria. The approach used by LLL is guided by decision analysis: a set of quantitative procedures for making decisions under uncertainty. A basic product of this work is called the aggregated systems model (ASM), which represents an integration of probabilistic assessments of:

- adversary characteristics
- physical security and MC&A performance
- social impact of consequences

together with information on safeguard system cost and the value of impacts.

In addition, the model contains linkages with regulatory decisions which enable the prediction of the value/impact of these decisions and hence permit the recommendation of the best decision from among a given set. Relevant data has been brought together from various sources in forming this model, which is described in several of the references* in the following report.

Although the ASM is relatively aggregated (e.g., compared to the LLL detailed assessment model), it is nevertheless overly complex for illustrating certain aspects of the methodology. Hence, with this illustrative objective in mind, the ASM is further simplified in the following report, and certain methodological questions and results studied in a relatively clean analytical context.

*The most complete source of information on the ASM and its application is contained in B. Judd, "Methodology and Preliminary Models for Analyzing Nuclear Safeguards Decisions," August, 1978.

Interdepartmental letterhead

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August 21, 1978
MC 78-742

To: Distribution

From: S. Weissenberger

Subject: Optimal Safeguard System Resource Allocation

1. Introduction and Summary

The purpose of this note is to discuss the basic problem of how best to allocate safeguard system (S/S) resources to counter the threats posed by different categories of adversaries. The first formal approach to this problem, using a similar model, was taken in [1]. The model used here is a simplified version of that in [2], [3], and [illegible] and is constructed solely to illustrate certain methodological [illegible] of this problem, as well as to gain some intuitions and insigh[illegible] om this exercise.

One of the main objectives is to compare two approaches to this problem: one may be roughly termed "overall cost minimization", and is the approach taken in [4]; the other will be denoted "risk equalization" and is the course recommended in [1]. It is argued here that the former approach [4] is to be preferred.

Section 2 outlines the basic problem; in Section 3 solutions are given for the case where adversary probabilities are constant; this assumption is removed in Section 4 to demonstrate that the preferred cost - minimization methodology remains conceptually valid even in this case where adversaries are permitted to respond in a certain sense to S/S decisions.

A fundamental concluding insight is that "overall cost minimization" leads to a condition where expected diversion costs cannot be further reduced while keeping a constant S/S cost budget; in contrast, "risk equalization" produces an allocation such that a re-allocation at constant S/S budget can in general decrease total expected diversion costs. A numerical example (Appendix A) demonstrates the diversion cost penalty which is imposed by risk equalization.

It should be noted here that it is already assumed in this model that resources are allocated with technical efficiency, in the sense that, for example, increases in probability of interruption cannot be obtained without increases in corresponding costs. Further, it should be pointed out that most of the analysis proceeds with S/S cost as a decision variable, rather than probability of interruption, as would be more natural for a discussion of regulation criteria setting, as in [4]. However, because of the one-to-one relation between the two variables, the two descriptions are entirely equivalent.

Because the theme as a whole here is the influence of adversary threats on optimal S/S allocation, a final section (5) directly addresses the issue of the sensitivity of allocations with respect to adversary probabilities. Two of the simple and intuitive results are that, i) (Section 5b), the unconditional probability of an adversary act does not influence allocations under a safeguard budget constraint, and ii) (Section 5c), there are conditions under which this unconditional probability of an adversary act does not influence the proportion of allocations, with or without budget constraints.

Although the approach here is somewhat formal, the mathematics is simple and it should be stressed once more that the point of this exercise is not explicit quantitative results, but rather the methodological approach and the derivative insights.

2. Basic Problem

Consider the simplified model of a safeguard system represented by the probability tree of Figure 1. It is assumed that one malevolent act

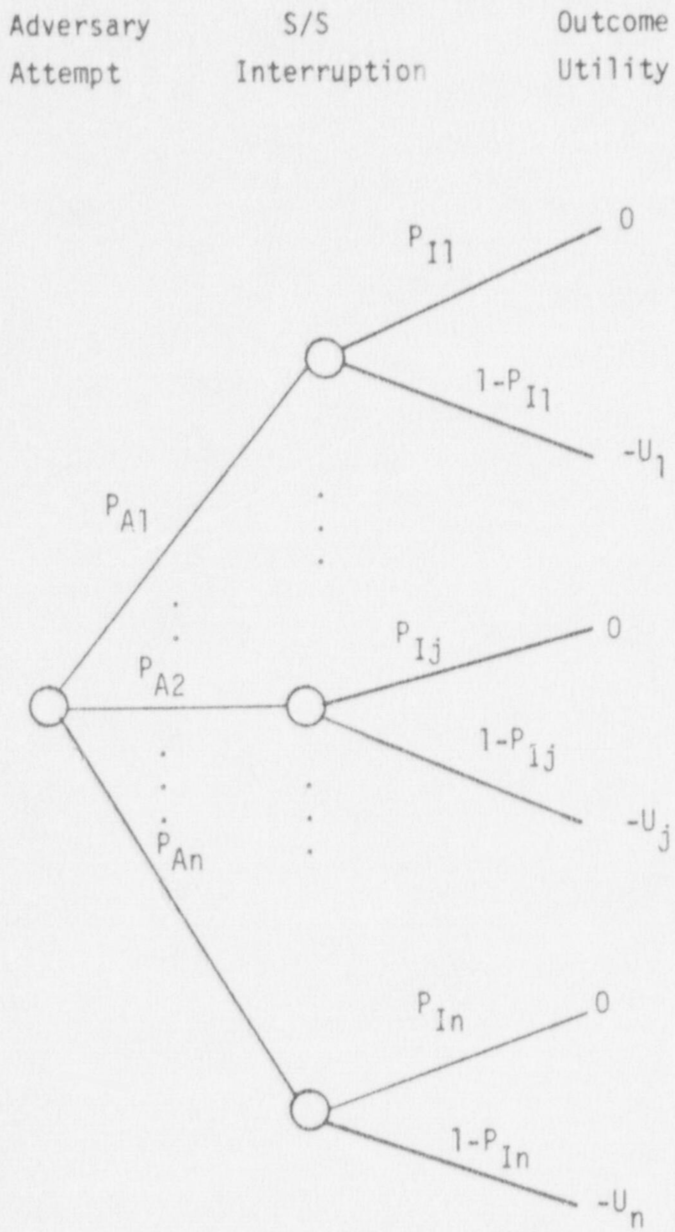

Figure 1 - Probability Tree for Simplified Safeguard System Model

(say an attempted diversion of SNM) may occur per year, with probability P_{Aj} by the $j^{\underline{th}}$ category of adversary (A_j), for j = 1,2,...,n.* (These categories will be characterized by objectives, resources, information, authority level, etc, as described in [2]). In Section 3 these probabilities will be constant; in Section 4 they will be permitted to be functions of S/S parameters. For an act by the $j^{\underline{th}}$ adversary, there is a probability of interruption of this act of P_{Ij}, where the combined effectiveness of both MC&A and physical protection systems are reflected in this single number, for each adversary. If interrupted, the expected utility of all consequences of this event are taken as zero; if not interrupted, the expected utility is given by ($-U_j$). (The assumption of zero disutility for all interrupted acts does not of course hold in practice except approximately; it does, however, simplify the subsequent exemplary analysis, and nothing essential would be added by its avoidance).

This model is consistent in all important respects with that of [3] and [4], and amounts to an aggregation of all events to just two (adversary attempt and S/S interruption) and of all outcome attributes to a single pair of expected utilities (0, $-U_j$).

The expected disutility arising from this safeguard system is then given by

$$C_D \equiv E\left\{U\right\} = \sum_{j=1}^{n} P_{Aj} (1 - P_{Ij}) U_j \tag{1}$$

where the notation C_D arises from its origin as "diversion costs" for material diversion acts.

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* Although it is not important for most of the following discussion, we can stipulate that there is some probability of <u>no</u> attempt in a year, so that

$$\sum_{j=1}^{n} P_{Aj} < 1$$

Now we assume that S/S decisions can be characterized by resources C_j (measured in annual \$) applied in various ways to counter the $j^{\underline{th}}$ adversary. These resources might be employed to a) improve the interruption probability and/or b) decrease the consequence disutility; hence, we may consider P_{Ij}, U_j as functions of expenditures C_j to counter (interrupt and ameliorate, respectively) A_j, i.e.,

$$\begin{aligned} P_{Ij} &= P_{Ij}\,(C_j) \\ U_{Ij} &= U_{Ij}\,(C_j) \end{aligned} \tag{2}$$

(Clearly, in general, specific resources will be effective in <u>either</u> of these ways but not both; as a result, for improved accuracy we would describe separate resources for each effect, but for simplicity we ignore this complication).*

In general, we can expect that a change in the interruption probability P_{Ij} will change A_j's <u>perception</u> of this probability, which in turn will effect the probability of an act from this adversary class, i.e., an increase in P_{Ij} will produce the <u>deterrent</u> effect of a decrease in P_{Aj}. In Section 4 this effect is modeled and its implications explored.

The basic problem, now, is to determine how best to allocate the S/S resources $\$C_j$ across the n adversaries, j = 1, 2, ..., n. If society acts as an expected utility maximizer, and if social utility is additive in U_j and C_j and linear in C_j**, then the problem is to

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* We also ignore for simplicity the possibilities of interdependence between P_{Ij}. See [4] for a situation where this interdependence can be plausibly ignored. It should be noted that many of the specific qualitative results found here are valid only for this case of independence, although the general method can also be effectively applied to the more complex situation.

** These assumptions are all plausible, and their relaxation does not add any significant features to the problem. In (3) it has also been assumed that U_j have been scaled in \$ units.

$$\underset{\underline{c}}{\text{Minimize}}\ \overline{U}(\underline{c}), \tag{3}$$

where

$$\overline{U}(\underline{c}) = \sum_{j=1}^{n} P_{Aj}\,[1 - P_{Ij}(C_j)]U_j(C_j) + \sum_{j=1}^{n} C_j \tag{4}$$

is the expected total cost (disutility) resulting from adversary acts and from operation of the safeguard system. (The notation $\underline{c}$ indicates the vector whose components are C_j, $j = 1,2,\ldots,n$).

3. <u>Problem Solution for Constant P_{Aj}</u>

The solution to (3), assuming that the function C_D (Eqn(1)) is convex in C_j away from the origin (see [4] for example), is obtained simply by differentiating (3) successively with respect to C_j and setting these derivatives equal to zero:

$$\frac{\partial}{\partial C_j}\left\{P_{Aj}[1 - P_{Ij}(C_j)]U_j(C_j)\right\} = -1,\ j = 1,2,\ldots,n \tag{5}$$

Solution of these n equations in n unknowns will yield the optimum allocations C_j^*, $j = 1,2,\ldots,n$.

Note that if we had posed either of the <u>constrained</u> minimization problems,

a) $\underset{\underline{c}}{\text{minimize}}\ C_s(\underline{c})$ (6a)

such that $C_D(c) = \text{constant} = K_D$ (6b)

(minimize safeguard cost subject to diversion cost constraint) or

b) $\underset{\underline{c}}{\text{minimize}}\ C_D(\underline{c})$ (7a)

such that $C_s(\underline{c}) = \sum_{j=1}^{N} C_j = \text{constant} = K_s$ (7b) (minimize diversion cost subject to safeguard budget constraint), it is easy to show that the necessary conditions, analogous to (5), are

$$\frac{\partial}{\partial C_j} \left\{ P_{Aj}[1 - P_{Ij}(C_j)]U_j(C_j) \right\} = - \lambda = \text{constant}, \quad j = 1,2,\ldots,n, \tag{8}$$

constituting effectively (n-1) equations in n unknowns C_j, $j = 1,2, \ldots, n$ together with either one of the constraint equations (6b) or (7b). The unconstrained solution (5) then is simply a special case of the solutions (8) to (6) or (7), with $\lambda = 1$ and the budget constraints removed.

It is of interest to compare the solution (5) or (8) with that attained by "equalizing the risk" across adversary types as recommended in [1]; the prescription then would be to distribute costs such that

$$P_{Aj}\,[1 - P_{Ij}(C_j)]U_j(C_j) = \text{constant}, \; j = 1,2,\ldots,n \tag{9}$$

presumably together with a specification of a constraint of either form (6b) or (7b), e.g., a safeguard budget constraint

$$C_{s(c)} = K_s \tag{10}$$

Note immediately that the "equal risk" criterion <u>requires</u> the specification of some such (possibly arbitrary) constraint as (10), while the expected utility minimization criterion (3) does <u>not</u> require any such constraint and leads directly to a specification of optimum overall level of S/S expenditure as well as the "relative" expenditure to each adversary class. Thus the only "fair" comparison of the criterion (9) is the constrained cost minimization problem (7) with its solution (8), (10).

The important observation here is a comparison of the form of the general cost minimization result (8) with the "equal risk" result (9) (both have in common the auxiliary equation (10),: clearly the two are different and will in general lead to different allocations of resources. A simple numerical example is worked out in the Appendix to illustrate this difference. (Also see [4] for a more detailed and somewhat more realistic example of the application of the cost-minimization approach.) Note that overall cost minimization leads to the equating of what could be termed "marginal risks" due to each adversary class, while the alternative simply equates the "risks"; that the former is to be preferred in an overall sense can be seen from the following argument: Imagine that "marginal risks" (8) are not equal (as would be the case if risks were equated (9)); then we can always re-allocate a pair C_i, C_j such that total C_s is unchanged, but C_D is decreased (by shifting from the class with the more positive derivative (8) to the other); thus for constant S/S budget we could lower the overall diversion risk. If "what counts" to society is overall diversion cost C_D then society should prefer the cost minimization result.

Note that cost minimization leads to the admission of unequal "risks" arising from individual adversary categories. This inequality is simply a consequence of differing cost/effectiveness relations for different adversaries, as well as differing probabilities of occurrence and utility of consequence; overall cost minimization natually permits appropriately different individual risks to occur in order to achieve the desired overall objective. If there are independent (say political) reasons for compromising this objective, they may be accounted for within the same structure, e.g., a constraint could be added to the cost minimization problem that the risk from some particular adversary is not to exceed some specified amount or proportion of the whole; it would be understood, then, that there might be a sacrifice in overall cost to satisfy this added requirement (and this cost increase could in fact be determined, as it is for the extreme case of risk equalization across all adversaries in Appendix A.)

A particular objection that is sometimes raised against the overall-cost-minimization solution is that it "leaves holes" against particular adversaries (of which these adversaries can presumably in some sense take advantage). The next section considers this problem and shows this concern to be erroneous.

4. Problem Solution for Variable P_{Aj}

Precisely the same kind of cost minimization as Section 3 can be applied in case the adversary probabilities P_{Aj} are functions of other variables. The obvious variables for such dependency here are P_{ij} and U_J:

$$P_{Aj} = f(P_{Ij}, U_j), \ j = 1,2,\ldots,n \tag{11}$$

It is assumed in (11) that the adversaries' perceived probabilities of interruption are identical with the actual probabilities of interruption, and furthermore that their perceived utility is equal to society's disutility. It is also assumed that interruption probabilities and utilities in one adversary category have no effect on the choice behavior of adversaries in another category. This is clearly valid for many adversaries, for either purely definitional reasons, or else because there can be no advantage for the particular adversary to change categories. A review of the categories in [2] and [4] suggests that this is more often the case than not. In any event, the assumption can be relaxed, and the general approach of this note still applied, but the specific results of Section 3 will not hold because of the added coupling between terms in C_D.

From (11) we see that P_{Aj} now becomes a function of C_j:

$$P_{Aj} = P_{Aj}(C_j) = f(P_{Ij}(C_j), U_j(C_j)), \tag{12}$$

and hence that the optimality conditions (5) and (8) remain valid. Hence the entire discussion of Section 3 holds.

It should be noted that there is an entire economic literature involving such relations as (11), e.g., see [5]*, [6], where these are viewed as (criminal) supply functions. A standard form of a supply function is

$$P_{Aj} = P_{Ajo}\, P_{Ij}^{\varepsilon_{Ij}} \quad U_j^{\varepsilon_{Uj}} \tag{13}$$

where P_{Ajo}, ε_{Ij}, ε_{Uj} are constant and the numbers ε are called supply elasticities. A constant-elasticity supply function of the form (13) has the property that, for example, ε_{Ij} is a constant of proportionality between percentage changes in P_{Ij} and P_{Aj}. Although this is not the only way supply relations can be specified, it is a simple and economically plausible one, and would probably be adequate for most purposes in the current problem.

It should be emphasized that there are econometric methods of assessing the elasticities [5], [6]. Although there is not adequate data for their assessment in the specific (S/S) problem at hand, it is possible to use data from analogous cases to determine numbers which might be considered representative (see, e.g., [5]).

Finally, the important point to be made here is that the cost minimization approach of Section 3, with the supply function (12) (possibly of the form (13), accounts for deterrent/incentive effects of S/S parameters on adversary decisions. Clearly the validity of resulting S/S allocation decisions depend directly on the validity of the model (including the supply function (12)). As always in such cases, it can be argued that an explicit model is of value, if only to provide qualitative insights and to explore sensitivities of assumptions and data. Such sensitivity questions are explored in a preliminary way in the next section.

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* Ref. [5] also contains various qualitative and quantitative analysis of criminal activity and the criminal justice system, carried out in a spirit similar to the one presented here.

5. Sensitivity Analysis

Since assessment of adversary probabilities P_{Aj} (either as constants or as generated by supply functions) will be based in large part on subjective judgments, and hence will always remain to a degree speculative and controversial, it is desirable to examine the sensitivity of the optimal allocations to these probabilities. In this section we study this question qualitatively from several points of view. For simplicity, throughout we consider the case where U_i does not depend on C_i.

A. Free-Budget Sensitivity of C_j* with Respect to P_{Ak}

The $j^{\underline{th}}$ optimality condition (5) can be written (with U_j = constant) as

$$P_{Aj}P'_{Ij}(C^*_j)U_j = 1 \tag{14}$$

Differentiation of (14) with respect to P_{Ak} (with the understanding that C^*_j is a function of independent variables P_{ai}, $i = 1,2,\ldots,n$) gives, after rearrangement,

$$\frac{\partial C_j}{\partial P_{Ak}} = -\begin{cases} \dfrac{P'_{Ij}}{P''_{Ij}} \dfrac{1}{P_{Aj}}, & k = j \\ 0, & k \neq j \end{cases} \tag{15}$$

Thus, <u>optimal allocations to protection against the $j^{\underline{th}}$ adversary are unaffected by revisions in the probabilities of the $k^{\underline{th}}$ ($k \neq j$), when there is a free safeguards budget.</u>* One implication of this result bears on the issue of deterrence: if the $k^{\underline{th}}$ adversary is deterred to a degree unforseen in advance, only the $k^{\underline{th}}$ allocation will be non-optimum and subject to subsequent correction.

* Note that this result cannot be expected to hold in general under a budget constraint, as adjustments in any one C^*_j in that case require adjustments in others.

The righthand side of (15) may be rewritten in the form of a standard sensitivity function as

$$\frac{dC_j^\star/C_j^\star}{dP_{Aj}/P_{Aj}} = -\frac{P'_{Ij}}{P''_{Ij}}\,\frac{1}{C_j^\star}, \tag{16}$$

giving a constant of proportionality between percentage changes in P_{Aj} and percentage changes in $C_j^\star$. The derivatives P'_{Ij}, P''_{Ij} are to be evaluated at $C_j^\star$. Examination of the optimality condition and the computation of U" (and its requirement of being positive definite for a minimum) shows that $P'_{Ij} > 0$ and $P''_{Ij} < 0$ for a minimum. Hence (16) shows the (intuitively reasonable) result that the sensitivity of optimal allocations $C_j^\star$ is increased by increasing P'_{Ij} and decreased by increasing $|P''_{Ij}|$ and $C_j^\star$.

b. Fixed-Budget Sensitivity to "Absolute" and Conditional Adversary Probabilities

Write P_{Aj} as

$$P_{Aj} = P_A P_j \tag{17}$$

in such a way that

$$\sum_{j=1}^{n} P_j = 1 \tag{18}$$

Thus P_A is the probability of an adversary act of any type in one year ("absolute" probability) and the P_j are probabilities of various adversary types conditional upon the occurrence of some adversary act. The substitution of (17) into the optimality condition (8) for the constant safeguard budget problem, with auxiliary condition $C_s = K_s$, shows that the resulting equations are independent of P_A. Thus, the optimal allocations in this case are independent of P_A and depend only on the conditional probabilities P_j.

c. Free-Budget Sensitivity to Absolute and Conditional Adversary Probabilities

The optimality conditions for either free or fixed budget give, for all j, k,

$$\frac{P_j P'_{Ij}(C_j^*)U_j}{P_k P'_{Ik}(C_k^*)U_k} = 1 \tag{19}$$

Hence, if $\frac{P_j}{P_k}\frac{U_j}{U_k}$ = constant,

$$\frac{P'_{Ij}(C_j^*)}{P'_{Ik}(C_k^*)} = 1, \tag{20}$$

so that, if

$$\frac{P'_{Ij}(C_j^*)}{P'_{Ik}(C_k^*)} = f\left(\frac{C_j^*}{C_k^*}\right), \tag{21}$$

then

$$\frac{C_j^*}{C_k^*} = \text{constant.} \tag{22}$$

Thus, if (21) holds, then $\frac{C_j^*}{C_k^*}$ = constant for constant P_j, P_k, for any P_A. Note that in this case the absolute value of the C_j^*'s will vary with P_A, but in fixed proportions. A sufficient condition for (21) is that $P_{Ij} = \alpha C_j{}^{\beta}$, a special case of which occurs in the example in the Appendix.

S. Weissenberger

S. WEISSENBERGER
Material Control Project

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[2] I. Sacks, Sequence Categories, LLL Project File, January 9, 1978.

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[4] B. Judd and S. Weissenberger, A Systematic Approach to Safeguards Decision Making, forthcoming.

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[6] J. M. Heineke, "Substitution Among Crimes and the Question of Deterrence", in Economic Models of Criminal Behavior, North Holland Press, 1978.

Appendix: Two-Adversary Example

Consider the case of two adversaries with the data

$$P_{A1} = 1/10 \qquad U_1 = 100 \qquad \alpha_1 = 1$$
$$P_{A2} = 1/4 \qquad U_2 = 8 \qquad \alpha_2 = 1/2 \tag{A-1}$$

where the interruption probabilities are described by

$$P_{Ij} = \alpha_j C_j^{1/2} \; , \; j = 1,2, \tag{A-2}$$

(with suitable restrictions on the ranges of C_1, C_2) and the S/S budget is constrained to be $C_s = 1$. (Let the units of U_i, C_i be 10^5 \$/year.)

For cost relations of the form (A-2), the optimality conditions (8) give

$$\frac{C_i^*}{C_j^*} = \left(\frac{\alpha_i \; P_{Ai} \; U_i}{\alpha_j \; P_{Aj} \; U_j}\right)^2 \quad \text{for all } i,j \tag{A-3}$$

Substituting (A-1) into (A-3) gives

$$\frac{C_2^*}{C_1^*} = \frac{1}{10} \tag{A-4}$$

so that, using the S/S budget cost constraint

$$C_1 + C_2 = C_s = 1, \tag{A-5}$$

we have

$$C_1^* = \frac{10}{11} \tag{A-6}$$

$$\text{and} \quad C_2^* = \frac{1}{11}$$

with corresponding $P_{I1}^* = .95$ and $P_{I2}^* = .15$.

Appendix: Two-Adversary Example (Cont'd.)

Substitution of (A-6) into (1) gives the minimum diversion cost

$$C_D^* = 2.16. \tag{A-7}$$

Alternatively, consider the risk-equating approach which gives, from (9),

$$10(1 - C_1^{1/2}) = 2\ (1 - C_2^{1/2}) \tag{A-8}$$

(A-8) can be solved with (A-5) to get

$$C_1 = .54 \tag{A-9}$$

$$C_2 = .66$$

with corresponding $P_{I1} = .73$ and $P_{I2} = .41$
which finally yield, upon substitution into (1), the diversion cost

$$C_D = 3.84 \tag{A-10}$$

A comparison of the costs (A-10) and (A-7) shows that risk-equating has produced a diversion cost penalty of 1.68, a sizeable increment above the minimum-cost diversion of 2.16.