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REACTOR THEORY
AND NUCLEAR
PHYSICS

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TABLE OF CONTENTS

PAGE

~~1. MATHEMATICS.~~

2. ATOMIC STRUCTURE

3. PHYSICAL CHEMISTRY

PAGE

10. REACTIVITY CONSIDERATIONS IN WATER MODERATED REACTORS

11. HEAT TRANSFER EFFECTS IN WATER MODERATED REACTORS

12. OPERATING CHARACTERISTICS OF LIGHT WATER POWER REACTORS

13. CHEMICAL, RADIOCHEMICAL, AND WASTE DISPOSAL CONSIDERATIONS IN WATER REACTOR OPERATION

14. NUCLEAR INSTRUMENTATION

CHAPTER 2

ATOMIC STRUCTURE

STRUCTURE OF MATTER

All matter is made up of certain basic substances called elements. There are 90 naturally occurring elements, of which hydrogen, oxygen, copper, iron, gold, and uranium are examples. The majority of the remainder of the substances found in nature are formed by chemically combining these elements in various ways to form compounds. Examples of common compounds are water, salt, sulfuric acid, oil, etc.

In the early 19th century, it was theorized that the elements were made up of tiny particles called atoms. All of the atoms of a given element have essentially the same chemical and physical properties[1], and for most applications they can be considered to be identical to each other. An atom of an element cannot be decomposed by any ordinary type of chemical change, nor can it be synthesized by chemical union. From the point of view of the chemist, therefore, the atom is the fundamental unit of matter.

If we take two or more individual atoms and allow them to react chemically, we will have formed a compound. In particular, we will have formed the smallest unit which possesses all of the chemical and physical properties of the compound in question. This unit is called a molecule. In contrast to their effect upon atoms, ordinary chemical means can change molecules of one compound into molecules of another compound, or can even decompose molecules back into their component atoms. Typical examples are the burning of natural gas (primarily methane) to form the new compounds carbon dioxide and water, and the electrolysis of water to produce the elements hydrogen and oxygen.

STRUCTURE OF ATOMS

In the early days of the atomic theory, atoms were considered to be small hard spheres similar to microscopic billiard balls. However, it was quickly recognized that this description was incorrect and that atoms themselves were actually complex arrangements of even smaller "subatomic" particles. Further investigations have shown that the subatomic particles may be made up of still smaller particles called among other things, "strange" particles. The ultimate structure of the atom has not yet been determined, and much present day physics research is directed toward this goal. In fact, in 1964 physicist Dr. Murray Gel-Mann postulated on purely mathematical grounds that there were three fundamental particles which he called "quarks." One quark has a charge of (+2/3e) and the other two have a charge of (-1/3e). Their

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[1] Chemical properties describe how an atom reacts chemically with other types of atoms to form compounds, whereas physical properties refer to a substance's boiling point, color, hardness, solubility, etc.

[2] The symbol e stands for the magnitude of the unit electrical charge which is found on other subatomic particles (see next paragraph). Its magnitude is 1.6 x 10^{-19} coulombs.

masses are supposed to be rather large by subatomic particle standards. This is a truly revolutionary theory because heretofore it was not thought possible to have fractional electrical charges. Experimental physicists have been searching in vain for quarks ever since the theory was proposed, and many believe that they do not exist. However, in September 1969, an Australian physicist reported finding evidence of five quarks of the 2/3e variety after studying 66,000 photographs of cloud chamber tracks. The tracks were made by the products of high energy cosmic ray interactions with the earth's atmosphere. Other physicists are now trying to confirm his work.

In spite of the uncertainties which surround the subject, it is still possible to describe an atomic structure which explains most of the observed experimental facts with which we will be concerned. The atom can thus be considered as being made up of three primary subatomic particles: protons, neutrons, and electrons. Protons are particles which possess one unit of positive electrical charge. Electrons are particles which possess one unit of negative electrical charge. Neutrons, as their name implies are particles which possess no electrical charge and are therefore electrically neutral. Protons and neutrons are relatively heavy particles and have approximately the same mass. The electron, conversely, is a relatively light particle and has only about 1/1846 as much mass as the proton or neutron. Every kind of atom has about the same general structure. At the center is a nucleus in which almost all of the weight of the atom is concentrated. On the outskirts of an atom are the electrons which circle constantly around the nucleus much as the planets circle around the sun. The nucleus is made up of protons and neutrons tightly bound together. The nucleus has a diameter of about one ten-thousandth's of the atom's diameter (for purposes of illustration, if the nucleus were blown up to the size of a basketball, the nearest electron would be about one mile away). The actual diameter of a typical atom is about 1×10^{-8} centimeters. The sketches in Figure 2-1 represent two of the simplest atoms found in nature - hydrogen and helium.

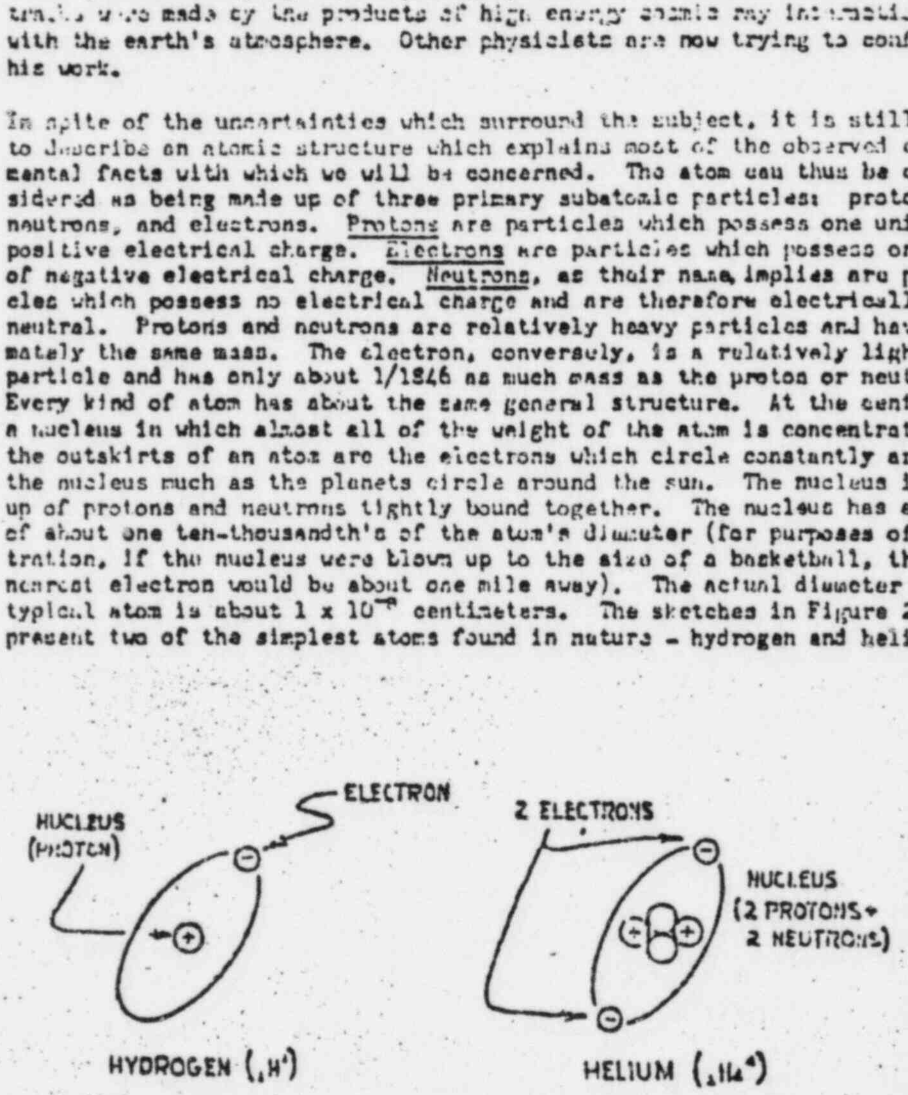

Figure 2-1: Schematic Diagrams of Hydrogen and Helium Atoms

PROPERTIES OF ATOMS

It is an obvious fact that the vast majority of the objects encountered in everyday life are electrically neutral. The implication of this is that the atoms which make up these objects must themselves be electrically neutral. Since electrons and protons both contain unit electrical charges, differing only in sign, atoms must have equal numbers of protons and electrons so that the charges cancel and the atom is electrically neutral. The number of neutrons that an atom possesses has no effect upon the overall charge of the atom since the neutron has no charge.

Since there are 90 naturally occurring elements, there must be at least ninety different types of atoms. Actually, there are many more than this, but we will consider this problem later. These atoms differ only in the number of protons, neutrons, and electrons that they contain. It has been verified experimentally that the chemical and physical properties of an atom (or in other words, the determination of which element a particular atom represents) are determined primarily by the number of electrons that the atom possesses. Since neutral atoms contain equal numbers of protons and electrons, it is also permissible to say that an element can be identified by the number of protons which exist in the nucleus of its atom. Thus an atom of hydrogen can contain one, and only one, proton in the nucleus or electron in the outer orbit. Conversely, since all hydrogen atoms have one electron, they all have the same chemical and physical properties. If we add an additional proton to the nucleus and electron to the outer orbit, we will no longer have hydrogen but will have the new element helium. The largest and most complex naturally occurring element is uranium, which has ninety-two protons in the nucleus.

ISOTOPES

To this point, we have ignored the effects of neutrons upon the properties of atoms. Suppose we take the simplest hydrogen atom containing one proton in the nucleus and one electron orbiting around this nucleus. If we were sufficiently clever, it might be possible to add a neutron to the nucleus of this simple atom. Our previous discussion tells us that this operation will not markedly disturb the chemical and physical properties of the atom since it still contains one proton and one electron. Therefore, this new atom will still be hydrogen. Atoms which have the same number of protons and electrons, but different number of neutrons, are called <u>isotopes</u>. Thus, the atom with one proton and the atom with a proton and a neutron are both isotopes of hydrogen.

It is nearly impossible to distinguish chemically between different isotopes of an element. All isotopes of a given element have essentially the same color, taste, melting point, solubility, etc. There are only two areas in which isotopes of an element differ significantly. The first is mass. Every additional neutron which is added to an atom increases the mass of the atom. The difference in mass between isotopes of the same element leads to slight differences in their chemical and physical properties. However, the differences are small and are usually ignored. In general, our previous statement that the chemical and physical properties of an isotope are determined solely by the number of electrons or protons it possesses is valid. The second major area in which isotopes of the same element differ significantly is in nuclear properties. Much of this course will be involved with the study of the behavior of the atomic nucleus. At this point, it will be sufficient to say that the number of neutrons that a nucleus contains must be within certain limits for the atom to be stable. If the number of neutrons is either too high or too low, the atom will be unstable and will disintegrate. Nevertheless, for most elements, there is a range of neutrons which will result in stable atoms. Therefore, most naturally occurring elements exist in two or more isotopic forms. As a matter of fact, there are about 300 naturally occurring isotopes of the 90 elements.

IDENTIFICATION OF ATOMS

In writing about elements and isotopes it is convenient to adopt a standard set of symbols to avoid confusion. The following simple conventions should be learned.

1. Each element is identified by a symbol which is commonly an abbreviation of the name of the element. Examples are: H - hydrogen, O - oxygen, U - uranium, He - helium, etc.

2. The number of protons in the nucleus is known as the <u>atomic number</u> and is given the symbol Z. The atomic number also represents the number of electrons in the neutral atom.

3. The total number of protons and neutrons in the nucleus is known as the <u>mass number</u> and is given the symbol A.

If X is the symbol for the element, the method of identifying the isotope is to write $_ZX^A$. Thus the two isotopes of hydrogen we have discussed would be designated $_1H^1$ and $_1H^2$.

It should be evident that the number of neutrons in the nucleus is given by A-Z. Thus $_{92}U^{238}$ is an isotope of uranium which contains 92 protons and 146 neutrons (238 - 92 = 146) in its nucleus. Similarly, $_{92}U^{235}$ is another isotope of uranium which has 92 protons and only 143 neutrons in its nucleus.

Since all isotopes of any given element have the same number of protons in the nucleus, the atomic number and the chemical symbol are really two ways of saying the same thing. Once it is known that we are dealing with an isotope of uranium, for example, it follows immediately that the atom has 92 protons. Because of this, you will frequently see the symbol for an isotope written with the atomic number omitted. Thus $_{92}U^{238}$ becomes U^{238} or simply U-238. However, the mass number A must never be omitted if the particular isotope of the element is to be identified.

In general, there are no special names given to different isotopes of the same element. An exception to this is hydrogen. The isotope $_1H^1$ (which amounts of 99.98% of all naturally occurring hydrogen) is commonly known simply as hydrogen. The isotope $_1H^2$ (which accounts for the remaining .02%

of naturally occurring hydrogen) is often called deuterium or heavy hydrogen and is often given the symbol $_1D^2$. The isotope $_1H^3$, often called tritium and symbolized as $_1T^3$, is not naturally occurring but can be produced artificially.

Following the same convention that was used for the elements, the symbols for the proton and the neutron are $_1p^1$ and $_0n^1$ respectively. When dealing with the electron, however, it is convenient to adopt a slightly different convention regarding symbols. We note that the atomic number, Z, not only represents the total number of protons in the nucleus, but also represents the total positive charge on the nucleus since each proton has a charge of +1. Therefore, the subscript in the symbols written previously could just as well have been interpreted as charge rather than as number of protons. Adopting this latter interpretation, the symbol for the electron is written as $_{-1}e^0$ where the subscript, -1, indicates the electron charge, and the superscript, 0, represents the fact that an electron contains no protons or neutrons (or equivalently that it contains practically no mass in comparison to a proton or neutron).

ATOMIC WEIGHTS

One can easily appreciate the difficulty involved in determining the actual weight of a single atom. Therefore a system was developed by which the relative weights of atoms could be expressed without knowledge of the actual weights. It is easy, for example, to determine by chemical means that an oxygen atom weighs about 16 times as much as a hydrogen atom, even though the actual weight of an oxygen atom is unknown. The basis of this new system of weights was taken to be the most common isotope of oxygen, $_8O^{16}$. Since this atom consists of 16 major particles of approximately equal mass (8 protons and 8 neutrons, the 8 electrons having practically no mass at all) it was arbitrarily decided to define the atomic weight of the $_8O^{16}$ atom as 16.000 atomic mass units (abbreviated amu). The atomic weight of the $_{92}U^{238}$ atom is 238.125 amu. This means that the $_{92}U^{238}$ atom is $\frac{238.125}{16.000} = 14.88$ times as heavy as the $_8O^{16}$ atom. Based on this system, the atomic weights of the three basic subatomic particles have been found to be:

	Atomic Wgt. (amu)
Proton	1.00758
Electron	0.00055
Neutron	1.00897

Since the atomic weights of protons and neutrons are almost equal to 1 amu, the mass number A is very nearly equal to the atomic weight of the atom. In fact, if the atomic weight is rounded off to the nearest whole number, it will be equal to the mass number. For example, the atomic weight of $_{92}U^{238}$ is 238.125 amu and the mass number is equal to 238. However, it is important that these two quantities are not confused with each other. The atomic weight is equal to the actual weight of the atom expressed in atomic mass units, whereas the mass number, A, is simply the total number of protons and neutrons in the nucleus.

Later scientific investigations have shown that 1 amu is equal to 1.66×10^{-24} grams. Thus, the actual weight of a single $_8O^{16}$ atom is $(16.000)(1.66 \times 10^{-24}) = 2.66 \times 10^{-23}$ grams. The use of powers of ten notation tends to hide the fact that this is a very small number. If it is written out in decimal form, the weight is 0.0000000000000000000000266 grams. This latter example shows the obvious advantage of using powers of ten notation. It is also obvious that using an atomic weight of 16.000 is much simpler from a calculational standpoint than using the actual atomic weight of the atom in grams. Tables giving the symbols, atomic number, mass number, and atomic weights of the isotopes can be found in most chemistry, physics, or scientific reference books.

PROBLEMS

✓1. Describe protons, neutrons, and electrons with respect to mass, charge, and location within the atom.

✓2. What are isotopes?

✓3. How many neutrons, protons, and electrons do each of the following isotopes contain: ${}_{19}K^{39}$ (potassium), ${}_{28}Ni^{58}$ (nickel), ${}_{12}Mg^{24}$ (magnesium), ${}_{16}S^{32}$ (sulfur)?

✓4. The ${}_{7}N^{14}$ isotope of nitrogen has an atomic weight of 14.00752 amu. How many grams does one atom of this isotope weigh?

5. Define: atomic number, mass number, and atomic weight.

6. Suppose we have an isotope of silicon, ${}_{14}Si^{28}$. If we removed a neutron from its nucleus but made no other change, would the chemical properties of the isotope change significantly? What new isotope would remain?

7. Using a chemistry text as a reference, discuss the differences between the physical properties of normal water (H_2O) and heavy water (D_2O).

CHAPTER 3

PHYSICAL CHEMISTRY

ELECTRONIC STRUCTURE OF ATOMS

Before proceeding on with a study of the nucleus, it is of interest to look briefly at the orbital electrons and some of the important phenomena associated with them. In Chapter 2 an atom was pictured as consisting of a small, positively charged nucleus surrounded by a number of rotating electrons. In a neutral atom, the number of protons in the nucleus is equal to the number of electrons in the outer orbits.

The orientation of the electrons in their orbits is what determines the chemical properties of an atom. An accurate description of this orientation is very difficult, but a simplified description will suffice. For our purposes, it can be assumed that the electrons are arranged in a series of circular orbits. Each particular orbit has a definite maximum number of electrons which it may contain. The first electrons added to an atom go into the innermost orbit nearest the nucleus. When it is filled, succeeding electrons will go into the next largest orbit. This process of filling succeedingly larger orbits continues until all of the electrons in the atom are accounted for. The innermost orbit is called the K orbit (or shell) and may contain a maximum of 2 electrons. The second orbit, or L shell, may contain 8 electrons. The third, or M shell, may contain eighteen electrons, and the fourth, or N shell, may contain 32 electrons.

Following this pattern, the hydrogen atom will have one electron in the K shell and the helium atom will have two electrons in the K shell. The next largest atom, lithium, will have 3 electrons - two in the K shell and one in the L shell. The L shell will continue to be filled until the atom neon, with 10 electrons, is reached. Neon will have a completely filled K shell with two electrons, and a completely filled L shell with 8 electrons. The next element, sodium, will have its eleventh electron in the M shell. Among larger elements, this orderly filling of successively larger shells tends to break down. For example, Table 3-I shows that xenon has 8 electrons in the O shell when its M shell is still unfilled. However, the complexities of electronic structure are beyond the scope of this manual.

CHEMICAL PROPERTIES OF ATOMS

It has been mentioned that the chemical properties of an element are determined by the electron configuration of its atoms. Actually, the most important factor is the number of electrons in the outermost shell. Thus all elements which have atoms with three electrons in the outermost shell will have similar chemical properties (but they will not be exactly the same because many other factors, such as overall size of the atom, enter into it). A case of particular importance is when an atom has eight electrons in its outer shell. Under these circumstances it has been found that the atom is exceedingly stable and has no tendency to undergo chemical reactions. These atoms are said to be chemically inert. Since all of the elements which possess this property are gases they are often called inert gases or noble gases. Helium is an exception to the rule that inert gases contain eight electrons in the outer orbit. Helium only contains two outer electrons, but since they represent a completely filled K shell, they give helium the properties of an inert gas. The inert gases, and the number of electrons in their electron orbits, are tabularized below.

TABLE 3 - I: ELECTRONIC CONFIGURATION OF THE INERT GASES

GAS	ELECTRON SHELL K	L	M	N	O	P
Helium	2					
Neon	2	8				
Argon	2	8	8			
Krypton	2	8	18	8		
Xenon	2	8	18	18	8	
Radon	2	8	18	32	18	8

IONIC CHEMICAL REACTIONS

When atoms undergo chemical reactions, they do so in an attempt to gain the stable electronic structure of an inert gas. Consider for example, the atom of sodium. Sodium has 11 protons in the nucleus and 11 electrons in the outer orbits. A schematic drawing of a sodium atom is shown in Figure 3-1.

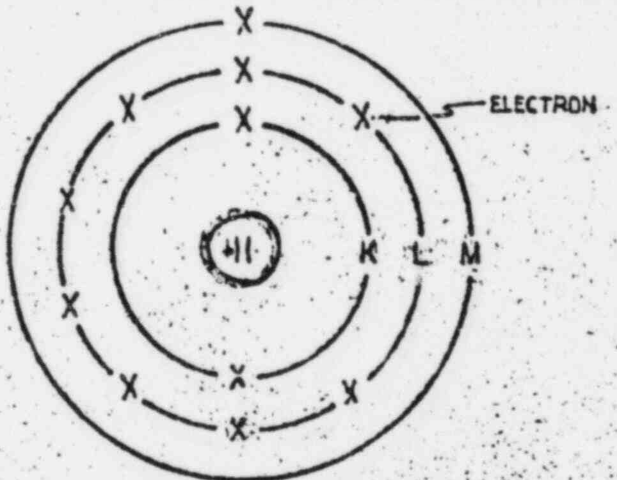

Figure 3-1: Sodium Atom

Notice that if the sodium atom were to get rid of the single electron in the M shell, it would be left with the same stable electron structure as the neon atom. If such a transformation were to occur, the resulting atom would no longer be electrically neutral. It would have a sodium nucleus with a charge of +11 and a neon electron structure with a charge of -10. Thus the net charge on the atom would then be +1. Such a system is no longer properly called a sodium atom, but is called a sodium ion. The process of removing one or more electrons from an atom (or, as we shall discuss shortly, the process of adding additional electrons) is called ionization.

Now let us consider an atom of chlorine, containing 17 protons and electrons. A sketch of this atom is shown in Figure 3-2. It can be seen that if the chlorine atom could obtain an additional electron it would then have the inert gas electron structure of argon. The result of such a transfer would be the formation of a chloride ion with a net charge of -1. Just as the group of noble gas elements with eight electrons in the outer shell have similar properties, so the group of elements with seven electrons in the outer shell have many similar properties. One of these is, of course, their tendency to form ions with a charge of -1. These elements, consisting of chlorine, flourine, bromine, and iodine, are given the special name halogens.

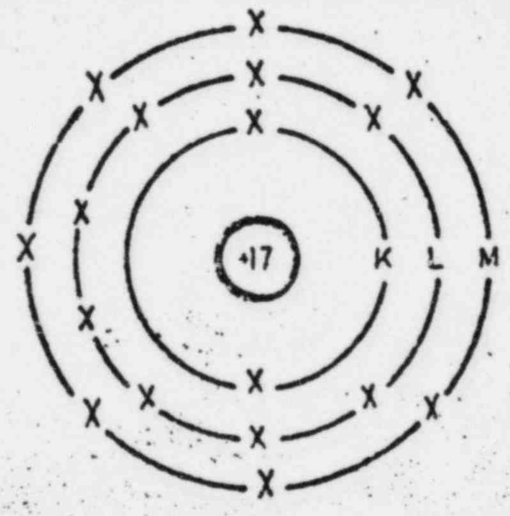

Figure 3-2: Chlorine Atom

The implication of the above discussion is that if an atom of chlorine were placed in the vicinity of a sodium atom the aforementioned electron transfer would take place as each attempted to gain an inert gas electron structure. Such is indeed the case, although in general, reactions of this type will occur only under special conditions. The result of this chemical reaction is that a sodium atom has transferred an electron to a chlorine atom to form a positively charged sodium ion and a negatively charged chloride ion. Since the two ions have opposite charges they are attracted to each other (like electrical charges repel, unlike charges attract) and bind together to form a molecule of sodium chloride. Sodium chloride is nothing more than ordinary table salt.

An ionic reaction such as the above can be written down using the chemical symbols for sodium (Na) and chlorine (Cl).

$$Na + Cl \longrightarrow Na^{+1}Cl^{-1} \qquad (3\text{-}1)$$

Usually the superscripts indicating the charges on the ions are omitted and the reaction is written simply as:

$$Na + Cl \longrightarrow NaCl \qquad (3\text{-}2)$$

COVALENT REACTIONS

When the atoms of two elements have little difference in their tendencies to lose or gain electrons, evidence indicates that they may attain stable electron structures by uniting to share one, two, or three pairs of electrons. For example, one cannot imagine a difference in the electron affinities of two chlorine atoms, and so one would never predict that one chlorine atom would lose electrons to another. Instead, two chlorine atoms gain stable structures by sharing a pair of electrons as shown in Figure 3-3.

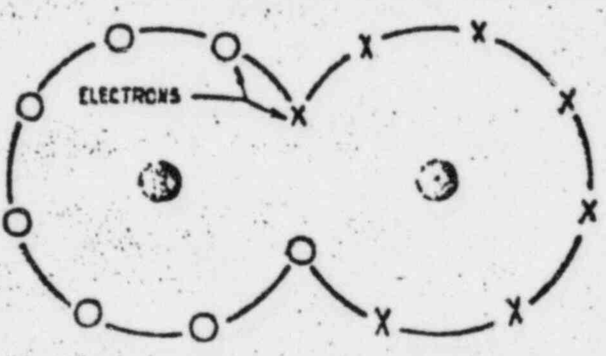

Figure 3-3: Covalent Bond Between Two Chlorine Atoms
(Inner Electron Shells are Omitted)

In the diagram, the electrons in the inner shells have been omitted for clarity since they don't enter into the reaction. The shared pair of electrons tends to travel around both chlorine nuclei and fools them both into believing that they have eight electrons in their outer shell. An example of a molecule with two shared electron pairs is the oxygen molecule shown in Figure 3-4. Each oxygen atom contains six electrons in its outer shell. A chemical bond formed by sharing electrons in this manner is called a covalent bond.

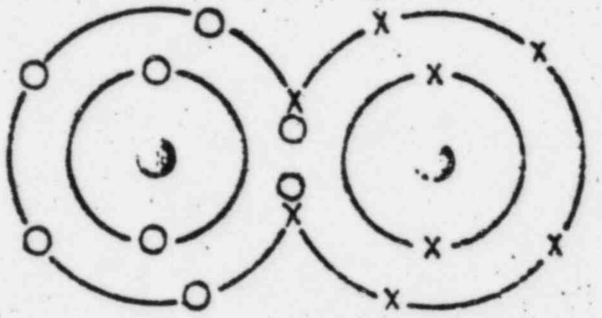

Figure 3-4: Double Covalent Bond Between Two Oxygen Atoms

All elemental gases (except the noble gases) exhibit covalent bonding and exist in nature as diatomic molecules. In other words, molecules of elemental gases contain two atoms of the gas. Symbolically this can be written as H_2, O_2, Cl_2, and N_2 for hydrogen, oxygen, chlorine, and nitrogen respectively. The subscript 2 indicates that the molecule contains two atoms.

Many other molecules are formed by covalent bonds. For example, a schematic of a water molecule is shown in Figure 3-5. As can be seen, two hydrogen atoms each share an electron with an oxygen atom. In this manner, each hydrogen atom is fooled into thinking it has two electrons, while the oxygen atom is satisfied because it thinks it has eight electrons in the outer shell.

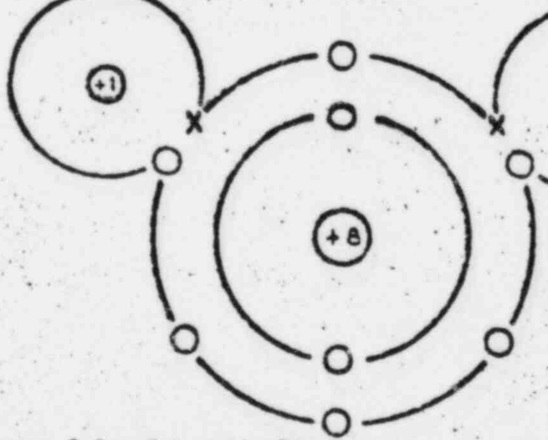

Figure 3-5: Schematic Diagram of a Water Molecule (H_2O)

Since the water molecule contains two hydrogen atoms and an oxygen atom its chemical symbol is H_2O. One possible reaction which could be proposed to produce water would be the reaction of two hydrogen atoms and one oxygen atom as given below:

$$2H + O \longrightarrow H_2O \qquad (3-3)$$

However it is known that hydrogen and oxygen occur in nature as diatomic molecules. Therefore the actual reaction would be that two molecules of hydrogen plus an oxygen molecule combine to form two molecules of water[1].

$$2H_2 + O_2 \longrightarrow 2H_2O \qquad (3-4)$$

This reaction can be made to occur under specialized conditions.

In general, when covalent bonding occurs between two different elements, the shared electrons are not shared equally between the partners. That is, invariably one element will exert a stronger attractive force on the shared electrons than the other, and so the electrons will spend more of their time in the vicinity of the stronger attracting element. This is basically the same behavior which produces ionic bonds, but the disparity in attraction is insufficient to allow complete ionization. Bonds with unequally shared electrons are called <u>polar covalent bonds</u>.

In the case of the water molecule, it has been observed that the shared electrons tend to prefer the oxygen atom to the hydrogen atoms. Thus the molecule tends to be negatively charged on the side occupied by the oxygen atom and positively charged on the side occupied by the hydrogen atoms. This non-uniform distribution of charge on the water molecule gives it many interesting properties, not the least of which is its ability to dissolve a great many materials. The solvent action of water on a molecule of NaCl is illustrated in Figure 3-6. The water molecules tend to congregate so

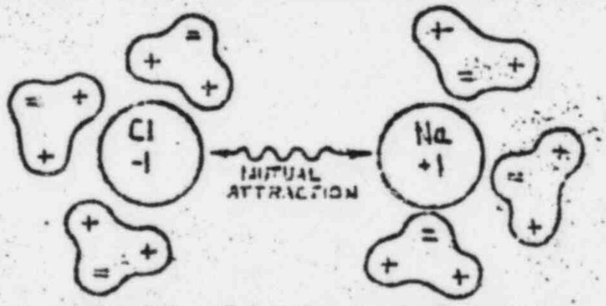

Figure 3-6: Action of a Polar H_2O Molecule in Dissolving an Ionic Salt

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[1] Similarly, equation (3-2) should be more properly written as $2Na + Cl_2 \longrightarrow 2NaCl$ since chlorine also exists as a diatomic molecule.

that their positive sides are adjacent to the negative chloride ion and the negative sides are adjacent to the positive sodium ion. The water molecules thus apply a force to the NaCl molecule which overcomes the attractive force between the Na^+ and Cl^- ions and causes it to be ionized and go into solution.

Many elements exhibit both ionic and covalent behavior depending upon the particular reaction in which they are involved. Furthermore, there are many covalent compounds, such as SO_3 (sulfur trioxide, which is a gas) which have a strong tendency to gain or lose electrons to form "complex ions." For example, sulfur trioxide has a strong tendency to accept two electrons and become the sulfite ion SO_3^{-2} with two minus charges. Figure 3-7 illustrates this behavior. Both sulfur and oxygen contain six electrons in their outer shells. In the SO_3 molecule, we have one double covalent

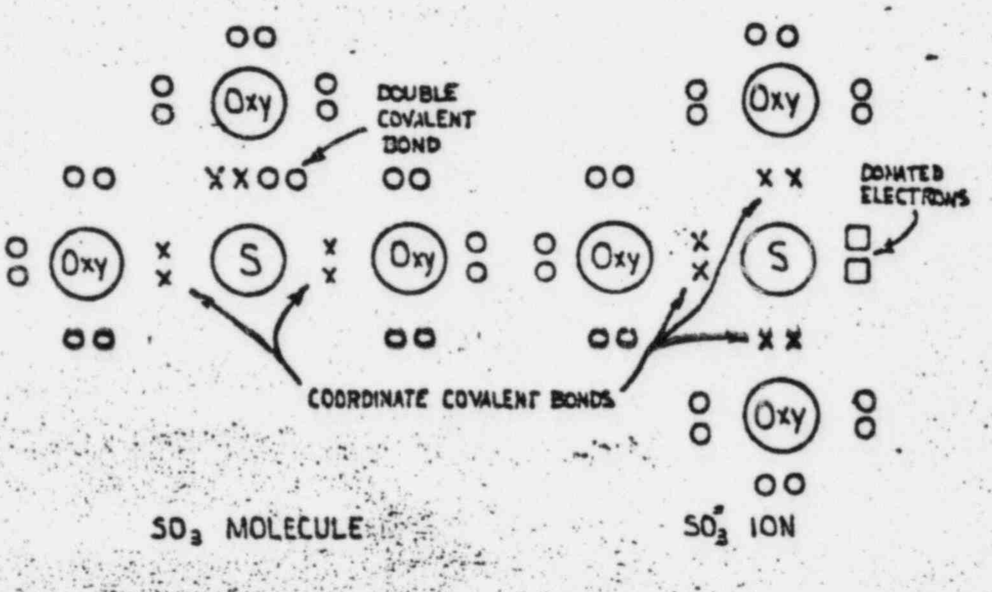

Figure 3-7: Electron Structure of SO_3 Molecule and SO_3^{-2} Ion

bond similar to that shown in Figure 3-4. Both the sulfur and oxygen atoms contribute a pair of electrons to this bond. In addition, we have two other bonds called coordinate covalent bonds. In each of these, two electrons are shared between the sulfur and oxygen atoms. However, in this type of bond, the shared electrons are provided by only one of the atoms – in this case the sulfur atom. In the SO_3^{-2} ion, we have three coordinate bonds joining the sulfur and oxygen atoms. Two additional electrons are donated to the sulfur atom to complete its outer shell from any nearby atoms which are easily ionized (two sodium atoms, for example). Thus, the SO_3 acquires a charge of -2 and acts as an ion. The chemical reaction can be written as:

$$2\,Na + SO_3 \longrightarrow Na_2SO_3 \qquad (3\text{-}6)$$

where the Na_2SO_3 molecule is made up of two Na^{+1} ions and one SO_3^{-2} ion.

The details of the subject of chemical bonding are complex and better left to chemistry texts. The important thing to see from this discussion is that chemical reactions involve alterations in the outer-electron shells, with no changes in the nuclei of the reacting substances.

X-RAYS

We have spent considerable time discussing transformations among the outer orbital electrons, i.e. chemical reactions. The outer electrons are the easiest to rearrange for two reasons; 1) the electrostatic attractive force between the nucleus and an electron decreases rapidly as the two bodies are separated, 2) the negative charges on the interior electrons have a tendency to shield the outer electrons from the full positive charge of the nucleus. The fact that the outer electrons are thus bound to the atom rather loosely accounts for the relative ease with which the vast number of chemical reactions can be made to occur. Conversely, the electrons in the inner orbits are more tightly bound to the atom and displacing them is generally more difficult. Nevertheless, transformations among the interior electrons can be accomplished under the proper conditions.

Before considering these transformations it is necessary to expand briefly upon the subject of electron orbits. The electron orbits about which we have been speaking represent more than simply paths for the electrons to travel in. Each orbit, or shell, also represents an energy level that the electrons occupy. That is to say, the electrons in an atom may have various discrete amounts of energy. The energy which a particular electron possesses determines which shell it will travel in. The farther a shell is from the nucleus, the greater is the energy of the electrons which occupy it. If an atom is left to its own devices, it will eventually orient itself into its "ground state," or condition of lowest energy. That is, the electrons will always move to fill the innermost shells because it is under these circumstances that they possess the lowest energy. This explains our earlier observations that when the electrons are added to an atom they fill succeedingly larger shells until all are accounted for. The exceptions to this rule result from the fact that complicating factors result in some overlapping of energies among the larger shells. For example, the first few electrons placed in the O shell have less energy than the last few electrons placed in the N shell. This is why xenon has electrons in the O shell while the N shell is still partially unfilled.

If an atom is bombarded by high energy electrons (as in the X-ray tube) or high energy light (as in a photoelectric cell) it is possible to knock an electron out of the inner electron shells. Since the atom is now left with a vacancy in one of its low energy inner shells, a higher energy electron from one of the outer shells will eventually take this opportunity to reduce its energy and will drop into this vacancy. In the process, the electron will emit the excess energy it possessed, and this emitted energy is called an X-ray. Since each electron shell has a discrete energy associated with it, the electron transitions between these shells will result in X-rays of certain discrete energies being emitted. The energies of these X-rays are a characteristic of the substance, and are often used as a means of identifying the substance. Although there are methods of producing characteristic X-rays other than the methods discussed above, the important thing to remember about them is that they result from reorientations of the interior orbital electrons of an atom. We will discuss X-rays and other types of atomic emissions further in Chapter 4.

PROBLEMS

1. Define: noble gas, halogen, ionization, characteristic X-ray.

2. Magnesium (Mg) is the element for which Z = 12. Draw a schematic diagram of its electron structure. Write the equation you would expect for the reaction of magnesium and chlorine. What is the charge on the magnesium ion?

3. Give two ways in which the oxygen atom (Z=8) might be ionized to give an inert gas electron structure. What would be the ionic charge in each of these cases? Which of these would you expect to be the most likely alternative? Why?

4. Nitrogen (Z=7) exists as a diatomic molecule with a triple covalent bond. Draw a diagram of the electron structure of the nitrogen molecule.

5. Consider the sodium atom as shown in Figure 3-1. Suppose the energies associated with the K, L, and M shells were 1.5, 3.7, and 4.2 energy units respectively (the energy units used here are ficticious, and are used only for purposes of this problem). Suppose further that an electron from the K shell was removed from the atom. Following this, one of the L shell electrons dropped into the K shell, and one of the M shell electrons dropped into the newly created vacancy in the L shell. What are the energies of the X-rays which would be emitted during these two transitions?

6. The nitrate ion, NO_3^{-1}, has a double bond and two coordinate bonds. Draw a diagram of the electron structure of this complex ion, showing the electron which was donated to it.

7. Draw a diagram of the electron structures for ammonia (NH_3) and the ammonium ion (NH_4^{+1}).

CHAPTER 4

RADIOACTIVE PARTICLE EMISSION

NATURAL RADIOACTIVITY

In 1896 Henri Becquerel observed that a piece of photographic film which had been wrapped in black paper to protect it from exposure to light had nevertheless become heavily blackened after standing near a sample of a uranium salt. In this experiment it appeared that the uranium emitted an invisible radiation which possessed, in common with visible light, the property of blackening photographic film. It was also apparent that this radiation must have a high degree of penetrating power since it had succeeded in getting through the paper cover to the film. The term "radioactive" was coined by Madame Curie to describe the behavior which Becquerel had observed.

If some radium is placed in an apparatus such as that shown in Figure 4-1, it will be found that the photographic plate is exposed in three fairly distinct areas. It was concluded from experiments of this type that there must be at least three types of radiation given off by naturally occurring radioactive materials. Furthermore, it was possible to conclude that one of the radiations must have a positive charge because it was attracted toward the negative pole of the battery (remember that unlike electrical charges attract and like charges repel), another must be negatively charged because it was attracted toward the positive pole, and the third must be electrically neutral because its path was not deflected toward either pole. The three most important types of radiation thus discovered were called alpha, beta, and gamma respectively (named for the first three letters in the Greek alphabet).

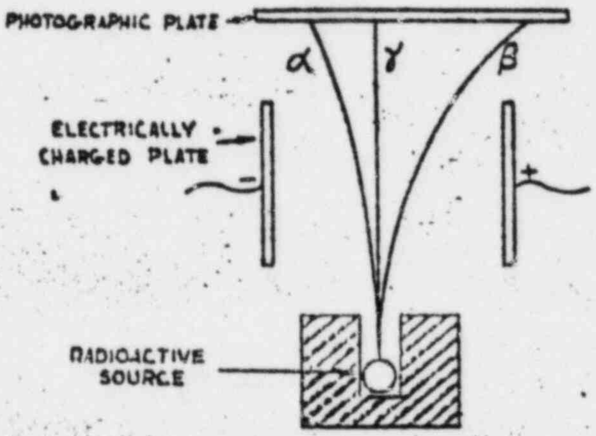

Figure 4-1: The Paths of the Three Types of Radiation in an Electric Field

Further investigations have shown that about 40 of the approximately 300 naturally occurring isotopes undergo spontaneous radioactive disintegration. Nearly all of the naturally occurring radioactive isotopes are isotopes of elements heavier than lead, such as uranium, radium, and thorium. All of these isotopes emit one or more of the three types of radiation previously described. Furthermore, it has been shown that the radiations are emitted from the nucleus of the radioactive atom. In fact, it was the study of these radiations which led to the theory that atoms contain a nucleus. The reason why such radioactive emissions occur is a complex subject, the rudiments of which will be covered in Chapter 5. This chapter will be concerned primarily with the properties of the radiations once they have been emitted.

ALPHA PARTICLES (α, $_2He^4$)

The positively charged radiation was found to be a stream of minute particles which were called alpha particles and given the symbol α. Research has shown that an alpha particle is structurally the same as the nucleus of a helium atom. That is, an alpha particle is made up of two protons and two neutrons. As such, it has an electrical charge of +2 due to the two protons. The alpha particle is emitted from the nucleus of the radioactive atom with a very high velocity, typically about 1.6×10^9 cm per second or ~ 1/20 of the speed of light. However, in spite of its high velocity, the penetrating power of the alpha particle is not great. The range of a typical alpha particle in air is about 2 inches, whereas in a more dense material, such as aluminum, the range is only about 0.001 inches. The subject of alpha particle ranges will be discussed in greater detail later in this chapter.

Since the alpha particle is a helium nucleus, an alternative symbol for it which is commonly used is $_2He^4$. Because this is also the symbol for a helium atom, it is necessary to infer from context whether the symbol represents helium or an alpha particle.

BETA PARTICLES (β, β^-, $_{-1}e^0$)

The negative radiation was also found to be made up of particles, and these were called beta (β) particles. A beta particle has since been found to be identical to an electron, having practically no mass and a charge of -1. The beta particle originates within the nucleus of the radioactive atom at the instant of its disintegration and is emitted at an extremely high velocity, usually [illegible] of the speed of light. Once the beta particle has been slowed down, it is impossible to distinguish it from the other electrons which are always present in the atoms of any material. As we shall see, the range of a beta particle is not as clearly defined as that of an alpha particle. Nevertheless, as a rule of thumb it can be assumed that a beta particle is about 500 times more penetrating in a given material than is an alpha particle of equivalent energy.

Since a beta particle is identical to an electron, except for the fact that it originates within the nucleus, it is often given the symbol $_{-1}e^0$. Here again, it must be inferred from context whether a beta particle or an electron is being referred to. A third symbol commonly used for the beta particle is β^-, where the minus sign indicates the particle is negatively charged. This latter symbolism is used to avoid confusion of the negatively charged beta particle with a positively charged beta particle called the positron (β^+). We will discuss this particle further in this chapter and in Chapter 5. Since the negatively charged beta particle is the most common of the two types, the symbol β without a sign is always assumed to be a negatively charged beta particle.

GAMMA RAYS (γ, ${}_0\gamma^0$)

When a nucleus emits an alpha particle or a beta particle, the event is nearly always accompanied by the emission of a third type of radiation called the gamma ray. Gamma rays are electromagnetic radiation similar to X-rays, visible light, ultraviolet rays, radio waves, etc. However, the typical gamma ray has much more energy and penetrating power than any of the other classes of electromagnetic radiation. The next most energetic class of electromagnetic radiation is the X-ray. There is no exact energy above which X-rays are called gamma rays. Thus high energy X-rays and low energy gamma rays are identical. Usually, X and gamma rays are differentiated by their origin. Electromagnetic radiations arising from transformations within the nucleus are called gamma rays, and those arising from transformations within the outer electron structure of the atom are called X-rays.

The gamma ray has no mass and no charged (hence the fact that it is not deflected by an electric field). The symbol for the gamma ray is γ or ${}_0\gamma^0$ to indicate its lack of charge of mass. The penetrating power of the gamma ray is very great. As in the case of beta particles, the concept of a definite range does not apply. However, as a rule of thumb, they can be considered to be about 50,000 times more penetrating than an alpha particle of equal energy.

RADIOACTIVE CHANGE

By radioactive decay, an unstable nucleus attempts to obtain stability. The naturally occurring radioactive elements accomplish this by emitting either an alpha or beta particle. These transformations are generally accompanied by the emission of one or more gamma rays. In general, a radioactive isotope always emits the same type of particle in its decay. Thus, ${}_{92}U^{238}$ always decays by the emission of an alpha particle and ${}_{90}Th^{234}$ always decays by the emission of a beta particle. Occasionally an isotope will emit an alpha particle on some of its disintegrations and a beta particle on others. An example of this is ${}_{84}Po^{218}$ which decays by alpha emission 99.94% of the time and by beta emission 0.06% of the time. However, no isotope emits both an alpha and a beta simultaneously in any single disintegration.

Radioactive decay is a type of nuclear reaction since it involves a transformation of the nucleus of an atom. In reactions of this type, the atom which decays by particle emission is usually called the parent and the new atom which is formed as a result of the decay is called the daughter. As we shall see, the daughter product of an alpha or beta decay is always an isotope of a different element than the parent, and in general possesses completely different chemical and physical properties.

A basic rule governing radioactive decay nuclear reactions is that the total number of neutrons and protons in the parent nucleus must equal the sum of the total number of neutrons and protons in the daughter nucleus and any particles which have been emitted. An alternative way of expressing this rule is to say that the charge on the parent nucleus must equal the sum of the charges on the daughter nucleus and any emitted particles. In the following sections we will apply these rules to alpha and beta decay.

ALPHA DECAY

Figure 4-2 shows a schematic drawing of the disintegration of a $_{92}U^{238}$ nucleus by alpha particle emission.

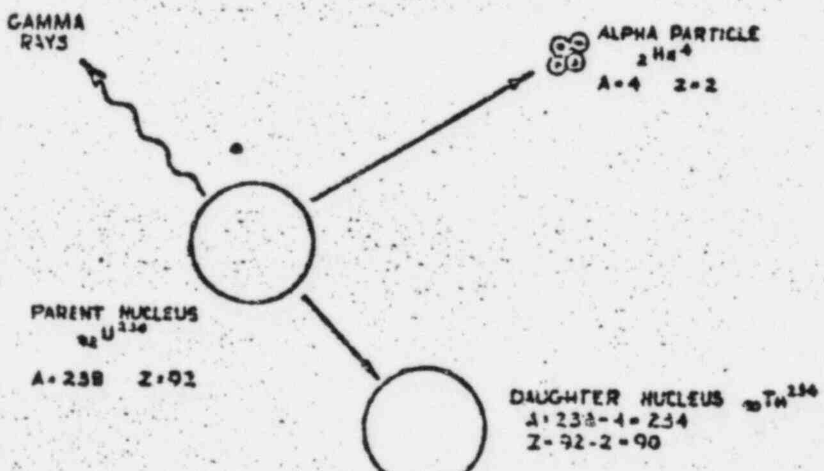

Figure 4-2: Schematic Diagram of $_{92}U^{238}$ Decay by Alpha Emission

Since there are 238 protons and neutrons in the parent nucleus, there must be a total of 238 in the resultant alpha particle, gamma ray, and daughter nucleus. The gamma ray contains no protons or neutrons, so we can neglect it. The alpha particle contains a total of 4 protons and neutrons, and so the daughter nucleus must contain 234 to make the reaction balance. Since the alpha particle represents the removal of 2 protons from the parent nucleus, the daughter nucleus must contain 90 protons. Alternatively, since the uranium nucleus contains a charge of +92, the products of the reaction must also have a charge of +92. The gamma ray possesses zero charge, the alpha particle has a charge of +2, and so the daughter nucleus must have a charge of +90.

The decay reaction is most easily visualized by use of an equation:

$$ {}_{92}U^{238} \longrightarrow {}_{90}Th^{234} + {}_{2}He^{4} + {}_{0}\gamma^{0} \qquad (4\text{-}1) $$

where the parent nucleus is written on the left of the arrow and the products of the reaction are written on the right. Since the superscripts in the equation represent the total number of protons and neutrons involved, they must balance in accordance with the previously stated rule. Thus: 238 = 234 + 4 + 0. Similarly, the subscripts also must balance since they represent the charges on the nuclei involved in the reaction. Thus: 92 = 90 + 2 + 0.

The use of these rules enables us to preduct the products of any decay reaction. For example, consider the reaction:

$$ {}_{90}Th^{230} \longrightarrow ? + {}_{2}He^{4} + {}_{0}\gamma^{0} \qquad (4\text{-}2) $$

where we are concerned with determining the daughter nucleus. We can immediately say that the mass number of the daughter nucleus is 226 since the superscripts must balance and 230 = 226 + 4 + 0. Similarly, the atomic number must be 88 since 90 = 88 + 2 + 0. To identify the element which has an atomic number of 88, it is necessary to consult some reference which gives a table of the elements. Upon doing this we could discover that the element in question is radium. Therefore, the daughter product of the reaction is the isotope ${}_{88}Ra^{226}$ and the reaction can be written as:

$$ {}_{90}Th^{230} \longrightarrow {}_{88}Ra^{226} + {}_{2}He^{4} + \gamma \qquad (4\text{-}3) $$

BETA DECAY

In writing reactions for beta decay, the same rules apply. Thus, in the reaction:

$$ {}_{83}Bi^{214} \longrightarrow {}_{84}Po^{214} + {}_{-1}e^{0} + {}_{0}\gamma^{0} \qquad (4\text{-}4) $$

we see that the superscripts balance (214 = 214 + 0 + 0) and the subscripts balance (83 = 84 - 1 + 0).

We know that the nucleus is made up of protons and neutrons. Since there are no electrons in the nucleus, what is the source of the beta particles? If we examine the previous reaction, we see that the parent bismuth atom contained 83 protons and 131 (214 - 83 = 131) neutrons. The daughter polonium atom contained 84 protons and 130 neutrons. This suggests to us that one of the bismuth neutrons was converted into a proton as a result of the reaction. At the instant of radioactive decay, the following reaction must take place within the nucleus:

$$_0n^1 \longrightarrow {}_1p^1 + {}_{-1}e^0 \qquad (4\text{-}5)$$

The beta particle thus created is emitted from the nucleus in the decay process. Beta decay can therefore be thought of as the transformation of a neutron into a proton within the nucleus of the parent atom.

It is interesting to note that the men who proposed the theory that a neutron is transformed into a proton and beta particle were able to prove strictly on theoretical grounds that such a transformation could not possibly occur unless another particle was emitted in the process. Such a particle, they said, must have no charge and essentially no mass (or at least a mass that was very much smaller than the mass of the beta particle). This particle was called the <u>antineutrino</u> and given the symbol $_0\nu^0$. Although such a particle would seem practically impossible to detect, a great many experiments were launched in an effort to observe one. It has only been in recent years that success in these experiments has been obtained and the existence of the antineutrino proven. Therefore, to be technically correct, equations (4-4) and (4-5) should both have the antineutrino included in them. That is,

$$_{83}Bi^{214} \longrightarrow {}_{84}Po^{214} + {}_{-1}e^0 + {}_0\nu^0 + {}_0\gamma^0 \qquad (4\text{-}6)$$

Usually, however, the antineutrino is simply left out of the equations since it has no practical importance.

RATE OF RADIOACTIVE DECAY

Each radioactive isotope has been found to disintegrate at a precise rate which is a characteristic of the particular isotope. The decay process is a statistical one. That is, it is impossible to say when any individual atom of an isotope will disintegrate. However, it is possible to say that in any given period of time, a certain percentage of the total atoms present will decay. An analogy can be drawn to the case of the life insurance company. No one can predict when any individual person is going to die. However, the life insurance company does say that among the people with a specific age, there will be a certain percentage of deaths within the following year.

Radioactive decay is an exponential process. This means that the rate of disintegration of an isotope is proportional to the number of atoms of the isotope present at that time. If we have two samples of a radioactive material, and one sample is twice as large as the other, then the larger sample will be decaying at a rate twice as fast as the smaller sample. These ideas can be expressed mathematically in the following manner:

$$\text{Rate of decay} = \lambda N \qquad (4\text{-}7)$$

where: λ = decay constant (in units of sec^{-1}, min^{-1}, hr^{-1}, etc)
N = number of atoms present

Note that the units of decay rate are atoms/sec, atoms/hr, etc. These units are equivalent to disintegrations/sec, disintegrations/hr, etc. because each disintegration involves one atom.

If we have a sample of a particular radioactive isotope, then it will be disintegrating at a rate given by equation (4-7). Of course, each disintegration reduces the number of atoms of the isotope in question that are present in the sample. That is, the factor N in equation (4-7) is not constant, but is continually decreasing as time passes. Using relatively advanced mathematics it is possible to derive a formula which expresses the manner in which N varies with time. Thus:

$$N = N_0 \, e^{-\lambda \Delta t} \qquad (4\text{-}8)$$

where: N_0 = number of atoms present at some initial reference time, t_0

λ = decay constant

N = number of atoms present at any later time t.

$\Delta t = t - t_0$ = elapsed time from initial reference time to any later time of interest t.

Note that this is simply one form of the general exponential equations discussed in Chapter 1. Hence our remark that radioactive decay is an exponential process.

A plot of the number of atoms, N, of a typical isotope versus time is given in Figure 4-3. Figure 4-3 is nothing more than a plot of equation (4-8) with $\lambda = 0.139\ hr^{-1}$ and N_0 = 100 atoms for illustrative purposes. This curve is similar in appearance to Figure 1-7, and has the property of asymptotically approaching zero as time increases. There is one important feature of this curve which is common to all radioactive decay curves. Notice that after 5 hours, the number of atoms originally present was reduced by a factor of 2 (from 100 to 50). Notice further that in the next 5 hour time interval, the number of atoms present fell from 50 to 25, or again by a factor of 2. Further investigation reveals that the curve shows a similar behavior in the third, fourth, and fifth 5 hour time intervals. In general, in any 5 hour time interval, 50% of the atoms present at the beginning of the interval will decay. Such a time interval is called the half-life of the isotope. Our hypothetical isotope, therefore, has a 5 hour half-life. In general, we can define <u>half-life</u> as the length of time required for one-half of the atoms in a sample of a radioactive isotope to

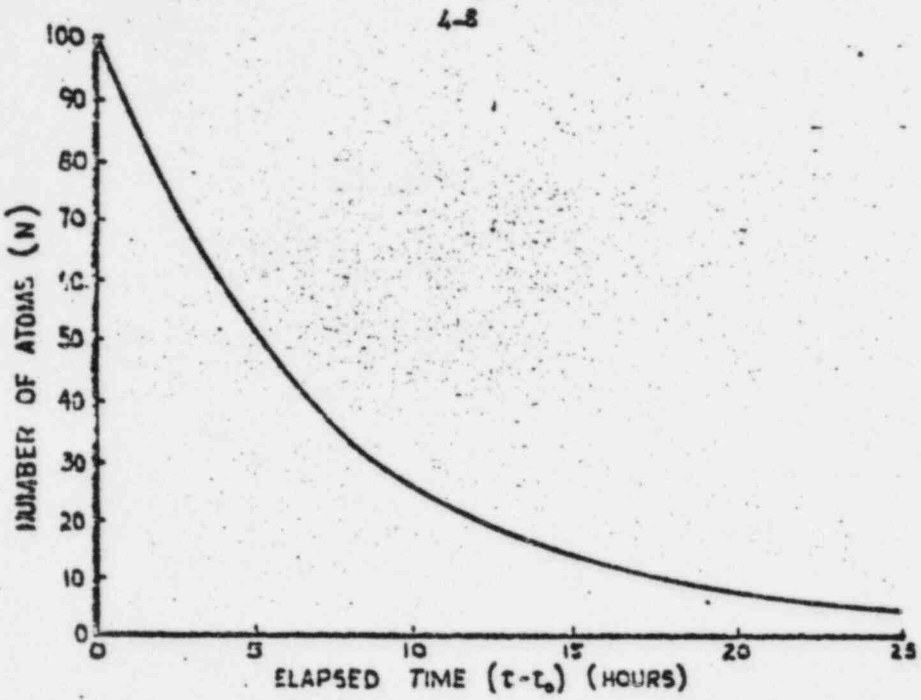

Figure 4-3: Radioactive Decay Curve for a Sample with a 5 Hour Half-Life

decay. This behavior is also consistent with our general remarks in Chapter 1 about exponential processes. That is, in equal increments of time there are equal percentage changes in the number of atoms.

The most commonly used symbol for half-life is $T_{1/2}$. Half-life can be related to the decay constant by the formula:

$$\lambda = \frac{0.693}{T_{1/2}} \qquad (4\text{-}9)$$

Using equation (4-9), we can rewrite equation (4-8) in terms of half-life as follows:

$$N = N_0 \, e^{-0.693 \, \Delta t/T_{1/2}} \qquad (4\text{-}10)$$

Each radioactive isotope has a characteristic half-life. Thus the isotope U-238 decays by alpha emission with a half-life of about 4.5 billion years. Every sample of U-238, no matter what its origin, exhibits this half-life. Half-lives of the various radioactive isotopes range from less than one millionth of a second to well over a billion years. Although there are several isotopes which have nearly the same half-lives, the half-lives of most isotopes are unique and are not possessed by any other isotope. This fact is extremely useful when it is desired to identify a particular isotope. If measurements of the amount of radiation being emitted by the substance are made over a period of time, the half-life of the substance can be determined. It is then necessary to consult a reference book which lists the half-lives of the various isotopes to determine which one is contained in the substance in question.

The unit which expresses the rate at which a substance decays is the curie. One <u>curie</u> of any radioactive isotope is that quantity of the isotope which is decaying at the rate of 3.7×10^{10} disintegrations per second. If the half-life of a material is very short, the amount of the material which represents one curie is vanishingly small. For a substance which decays very slowly, one curie may represent several tons of the substance. Table 4-I gives the amount of material which represents one curie for three typical isotopes.

TABLE 4-I: AMOUNT OF MATERIAL REPRESENTING ONE CURIE OF THREE TYPICAL ISOTOPES

ISOTOPE	HALF-LIFE	AMOUNT REPRESENTING ONE CURIE
${}_{92}U^{238}$	4.5×10^{9} years	3.3 tons
${}_{88}Ra^{226}$	1620 years	1.02 grams
${}_{53}I^{131}$	8.05 days	8.15×10^{-6} grams

We are now in a position to consider the solutions to a number of problems concerned with radioactive decay.

Example 1: At 2:00 P.M., a sample of Te-133 ($T_{1/2} = 2$ min.) contains 10^{10} atoms. Each disintegration represents the emission of one β^- particle. How many β^- particles are being emitted per second?

> From equation (4-7), Rate = λN where $\lambda = 0.693/T_{1/2}$. Since we are interested in disintegrations per <u>second</u>, we must convert $T_{1/2}$ into seconds in order to obtain the proper units. Thus:
>
> $$\lambda = \frac{0.693}{2}\left[\frac{1}{\text{min}}\right] = \frac{0.693}{(2)(60)}\left[\frac{1}{\text{sec}}\right] = 5.78 \times 10^{-3}\left[\frac{1}{\text{sec}}\right]$$
>
> $$R = N\lambda = (5.78 \times 10^{-3})(10^{10}) = 5.78 \times 10^{7}\ \frac{\text{disintegrations}}{\text{second}}$$
>
> Since each disintegration yields one β^- particle, the β^- emission rate is $5.78 \times 10^{7}\ \beta^-/\text{sec}$.

Example 2: How many atoms will remain of the previous sample at 2:08 P.M.?

> There are two methods of working this problem. The first is simply to use the fact that the time interval from 2:00 P.M. to 2:08 P.M. represents the passage of 4 half-lives. Thus:

TIME	ATOMS
2:00	10^{10}
2:02	5×10^{9}
2:04	2.5×10^{9}
2:06	1.25×10^{9}
2:08	$.625 \times 10^{9}$ or 6.25×10^{8}

The second method is to use the general formula given in equation (4-2).

$$N = N_o e^{-\lambda \Delta t}$$

$$N_o = 10^{10}$$

$\lambda = 5.78 \times 10^{-3}$ sec^{-1} (from Example 1)

Δt = 8 minutes = 480 seconds (note that we must work in seconds since we have expressed λ in sec^{-1})

$$N = (10^{10})\ e^{-(5.78 \times 10^{-3})(480)} = 10^{10}\ e^{-2.78}$$

From a table of logarithms or a slide rule it can be determined that $e^{-2.78} = .0625$. Therefore:

$$N = (10^{10})(.0625) = 6.25 \times 10^{8}$$

In this example, method 2 was the more difficult. However, if the time interval in question had not been an even number of half-lives, method 1 would not have worked and method 2 would have been necessary in order to obtain the solution.

Example 3: How many curies of Te-133 were present at 2:00 P.M.?

Since the sample was undergoing 5.78×10^{7} disintegrations/sec (see Example 1), and since each curie represents a disintegration rate of 3.7×10^{10} disintegrations/sec, then the number of curies present at 2:00 P.M. is:

$$\frac{5.78 \times 10^{7}}{3.7 \times 10^{10}} = 1.56 \times 10^{-3} \text{ curies} = 1.56 \text{ millicuries}$$

Example 4: How many curies of Te-133 were present at 2:08 P.M.?

At 2:08 P.M. there are 6.25×10^{8} atoms. Therefore the number of curies present is:

$$\frac{(6.25 \times 10^{8})(5.78 \times 10^{-3})}{(3.7 \times 10^{10})} = 9.8 \times 10^{-5} \text{ curies} = .098 \text{ millicuries}$$

Note that this is just 1/16 the number of curies present at 2:00 P.M. Therefore, just as the number of atoms decreased by a factor of 16 in four half-lives, so the number of curies also decreased by a factor of 16.

Example 5: How many curies are represented by 10^{-3} grams of cobalt-60 ($T_{1/2} = 5.27$ years, atomic weight = 59.95 amu)

Although this problem is not difficult, it is lengthy and so an outline of the solution will be helpful so that we always keep in mind what we are doing.

Step 1: Calculate number of atoms, N.

Step 2: Calculate decay constant, λ.

Step 3: Calculate decay rate, Nλ.

Step 4: Convert disintegrations/sec. into curies.

Step 1: The atomic weight of the Co-60 atom is 59.95 amu. Since 1 amu = 1.66×10^{-24} grams, each atom weighs $(59.95)(1.66 \times 10^{-24}) = 99.5 \times 10^{-24}$ grams. The number of atoms in 10^{-3} grams will then be:

$$\frac{10^{-3} \text{ grams}}{99.5 \times 10^{-24} \text{ grams/atom}} = .01 \times 10^{21} = 1 \times 10^{19} \text{ atoms.}$$

Step 2: The decay constant is given by equation (4-7). Since we will be interested in disintegration/sec., we will first convert half-life into seconds.

$$5.27 \text{ years} = (5.27 \text{ years})\frac{(365 \text{ days})}{(\text{year})}\frac{(24 \text{ hours})}{(\text{day})}\frac{(60 \text{ minutes})}{(\text{hour})}\frac{(60 \text{ seconds})}{(\text{minute})}$$

$$= (52.7)(365)(24)(60)(60) \text{ seconds} = 1.66 \times 10^{8} \text{ seconds}$$

therefore:

$$\lambda = \frac{0.693}{1.66 \times 10^{8}} = 4.18 \times 10^{-9} \text{ sec}^{-1}$$

Step 3: The decay rate is given by equation (4-1) as:

$$R = N\lambda = (1 \times 10^{19})(4.18 \times 10^{-9})\frac{\text{atoms}}{\text{sec}} = 4.18 \times 10^{10} \text{ atoms/sec}$$

Step 4: The number of curies is given by:

$$\text{Number of curies} = \frac{4.18 \times 10^{10}}{3.7 \times 10^{10}} = 1.13 \text{ curies}$$

INTERACTION OF CHARGED PARTICLES WITH MATTER

The interaction of radiations with matter is a subject of considerable importance because it forms the basis for such diverse fields as shielding design, radiation detection instrumentation, and health physics. In this section, we will consider the interaction of charged particles with matter. In the following section we will consider the interaction of gamma rays with matter.

In passing through a substance, alpha particles lose energy chiefly by interaction with the orbital electrons of the atoms making up the substance. The process is pictured schematically in Figure 4-4. As the alpha particle passes in the vicinity of an electron of a target atom, the two particles exert an attractive force upon each other due to their opposite electrical charges. This force tends to slow down the alpha particle, and also tends to pull the electron free of its atom. When an electron is pulled completely free of its target atom, the target atom is said to be ionized (this terminology is consistent with the definition of ionization given in Chapter 3). The combination of the free electron and the resultant positive ion is termed an ion pair.

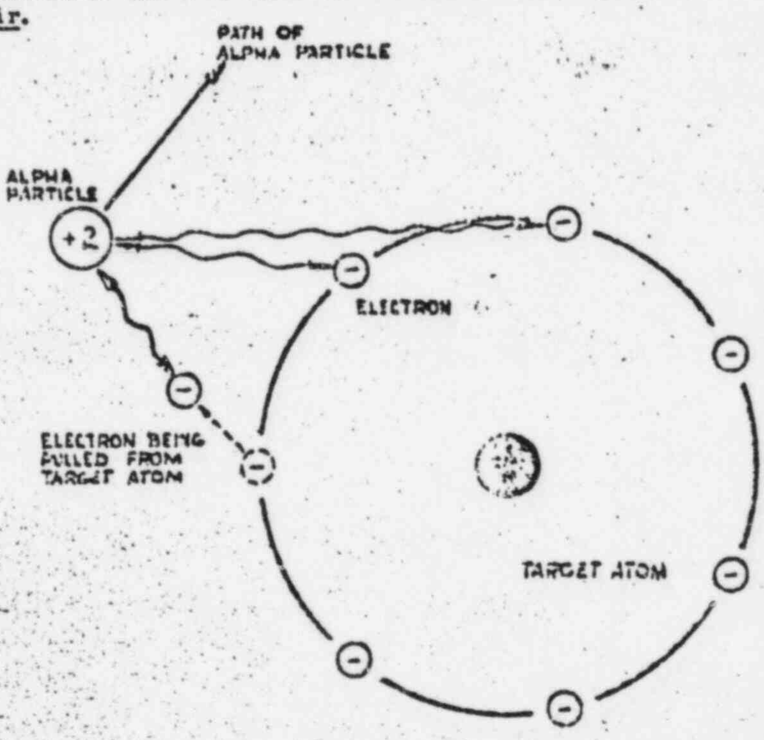

Figure 4-4: Passage of Alpha Particle Through a Target

The amount of energy required to pull an electron free of its atom and thus form an ion pair is about 35 ev[1] in most gases. Thus, a typical alpha particle with an energy of 5 mev would result in the formation of about $5 \times 10^6/35 = 1.4 \times 10^5$ ion pairs along the length of its track. The length of the track will be governed by the rate, expressed in ion pairs formed per centimeter of travel, at which ionization takes place. The higher the rate, the more rapidly will the alpha particle be slowed, and the shorter will be the track.

The rate at which a charged particle such as an alpha loses its energy through ionization is a very complex subject. However, some qualitative remarks about the factors involved are in order. First, the rate at which a charged particle causes ionization is proportional to the electron density (electrons per unit volume) of the target. This simply means that if a charged particle encounters a large number of electrons it will have a greater opportunity to cause ionization. Therefore, a charged particle will be stopped faster in a dense material with high atomic number, like lead, than in a less substantial material like a gas. The second important quantity is the charge on the particle. As one would expect, the greater the charge on the particle, the greater the force exerted on the electrons, and hence the greater the rate of ionization. Other things being equal, one would thus expect an alpha particle to be more highly ionizing than a proton (protons are not emitted in natural radioactive decay, but can be produced in certain artificial nuclear reactions which will be discussed in later chapters). The final quantity of importance is the particle's velocity[2]. The higher the velocity of a particle, the smaller will be its rate of ionization. The reason for this is that at a high velocity, the particle spends less time in the vicinity of a given electron and its attractive force is applied to the electron for a shorter period of time, thereby reducing the chances of causing an ionization.

We noted earlier in this chapter that the alpha particle possessed a very short range in comparison to the beta particle. The previous discussion supports this observation. The relatively high charge of +2 is one factor which increases the ionizing ability of the alpha particle. The other factor is that the large mass of the alpha particle gives it a relatively low velocity in comparison to other nuclear radiations. It should be noted that

---

[1] ev = electron volt. The electron volt is a unit of energy which is defined in Chapter 5. Mev = million electron volts.

[2] It is more common to speak of a particle's energy than its velocity. For most work, a particle's energy and velocity are related by the equation $E = (1/2)mv^2$ where E is the energy, v is the velocity, and m is the mass of the particle. Solving this equation for velocity gives $v = \sqrt{2E/m}$. Thus a particle's velocity is related not to its energy, but to the ratio of E/m. The greater the energy of a particle, the greater will be its velocity. However, if two different particles have the same energy, the smaller one will have the greatest velocity. This is why a beta particle has a much higher velocity than an alpha particle of equal energy.

because the alpha particle is so much larger than the electrons with which it is interacting, these interactions cause very little deflection of the alpha particle from its path. Therefore, the path of an alpha particle is nearly straight and its length is fairly easy to determine. The range of alpha particles in several common materials is given in Table 4-II.

TABLE 4-II: RANGE OF ALPHA PARTICLES IN VARIOUS SUBSTANCES

ENERGY (Mev)	RANGE (inches)			
	Air	Tissue[1]	Ordinary Concrete	Lead
1	.2	.0003	.0002	.00005
2	0.4	.0006	.0003	.00010
5	1.4	.0017	.0010	.00025

As determined above, an alpha particle with 5 million electron volts of energy will cause the formation of about 1.4 x 10^5 ion pairs along its 3.5 cm path in air. The average rate of ionization in air is then about 140,000/3.5 = 40,000 ion pairs/cm. As the alpha particle nears the ends of its path, and its velocity is very slow, it will pick up two electrons and form a helium atom. Thus deprived of its charge, its ability to cause further ionization is reduced to zero.

The interaction of beta particles with matter is similar in many respects to the interaction of alpha particles previously discussed. However, there are important differences arising from the very small mass of beta particle and from its resulting high velocity. The beta particles interact with orbital electrons as did alpha particles except that since the beta particle and the electron are both negatively charged the force acting between them is one of repulsion rather than attraction. Therefore, the beta particle can be thought of as causing ionization by pushing an electron from its orbit rather than by pulling it.

The combination of high velocity and single charge makes the rate at which the beta particle causes ionization much less than for an alpha particle. A 5 mev beta particle will still produce about 1.4 x 10^5 ion pairs in air, since the amount of energy necessary to produce an ion pair is independent of the nature of the ionizing particle, but they will be spread more thinly over a considerably longer track length. Typically, the rate at which a beta particle causes ionization in air is about 80 ions pairs/cm.

In contrast to alpha particles, the beta particles emitted by a particular substance do not show any definite range. There are two major reasons for this. First, because of its small size, the beta particle may have its path altered appreciably by the repulsive forces exerted upon it. Therefore, the typical beta particle travels a very crooked path, as shown in Figure 4-5.

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[1] To a first approximation, the range in tissue is also approximately equal to the range in water.

As can be seen in the figure, the two beta particles traveled about the same total distance, but one penetrated the shield and the other didn't.

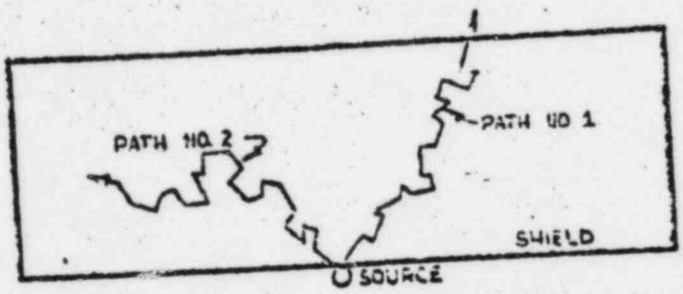

Figure 4-5: Typical Paths of Two Beta Particles from the Same Source

To the shielding designer at least, the concept of range is cloudy at best when applied to beta particles. The second complicating factor is that any given beta source emits beta particles with a continuous spectrum of energies up to a particular maximum. In contrast, an alpha emitter produces alpha particles of only a few well defined energies. This phenomena will be discussed in greater detail in Chapter 6, but its effect upon the range of beta particles from a given source is readily apparent. Nevertheless, it is possible to say that a beta particle of some particular energy will on the average travel a certain total distance in some particlar material. We can define this distance as the range of the beta particle, keeping in mind that in general, the total distance traveled will usually not be in a straight line. The ranges of beta particles in typical substances are given in Table 4-III.

TABLE 4-III: RANGE OF BETA PARTICLES IN VARIOUS SUBSTANCES

	RANGE (inches)			
ENERGY (Mev)	Air	Tissue	Ordinary Concrete	Lead
1	115	.132	.060	.014
2	260	.335	.183	.034
5	665	.830	.598	.089

In addition to ionization, beta particles lose energy by one other important mechanism. As it traverses a target, the beta particle is acted upon not only by the orbital electrons it encounters, but also by the charges on the target nuclei. For reasons beyond the scope of this manual, the beta particles emit a continuous stream of X-rays when acted upon by these nuclear forces. This mechanism for energy loss becomes significant when the beta particle is traversing a target made up of heavy nuclei, with large nuclear charges. The X-rays thus emitted are called bremsstrahlung (German for "braking radiation"). Alpha particles may also lose energy by this mechanism, but the process is insignificant when compared to energy loss by ionization.

INTERACTION OF GAMMA RAYS WITH MATTER

Gamma rays (and X-rays) do not carry an electric charge and therefore do not exert electrical forces upon the orbital electrons in a target material. Nevertheless, in their passage through matter these rays eject electrons and the latter, being electrically charged, are able to produce ionization. Gamma rays (and X-rays) thus cause ionization indirectly. However, before considering the mechanisms by which gamma rays interact with matter, it is necessary to look briefly at the makeup of the gamma ray. The popular conception of electromagnetic radiation is some sort of continuous wave-like substance. This view was held by physicists as well prior to the turn of the century. However, in 1900, Max Planck made the revolutionary pronouncement that electromagnetic radiations were actually streams of discrete "particles" called photons. The photon is a sort of energy packet which possesses many wave-like properties as well as a number of properties normally associated with particles. It is the latter properties of the photon which will be of primary interest to us, although it is not strictly correct to think of a photon as a particle since it does not possess any mass in the usual sense of the word.

There are three ways in which gamma photons interact with matter. In the first of these, the photoelectric effect, the gamma photon is absorbed by one of the orbital electrons in a target atom. Thus supplied with a considerable quantity of excess energy, the orbital electron is ejected from the atom with considerable velocity. It is this ejected electron which causes the ionization associated with the gamma ray. Most commonly, the photoelectric effect involves the ejection of one of the K shell electrons. Following the ejection of this electron, there is a rearrangement of other orbital electrons to refill the hole in the K shell. This rearrangement is accompanied by the emission of characteristic X-rays for the atom in question. However, these are usually of a much lower energy than the original gamma ray, and so for all practical purposes the photoelectric effect can be considered as resulting in the complete disappearance of the gamma ray with the resultant emission of a high speed electron. Figure 4-6 shows a schematic diagram of the photoelectric effect. Photoelectric absorption is most likely with gamma rays of low energy, and has a relatively low probability in most materials for energies above about 1 mev.

For more energetic gamma rays, a second type of interaction mechanism called Compton scattering predominates. In this case the photon interacts with one of the outer orbital electrons in the target. The interaction is a type of billiard ball collision between the photon and the electron as shown in Figure 4-7.

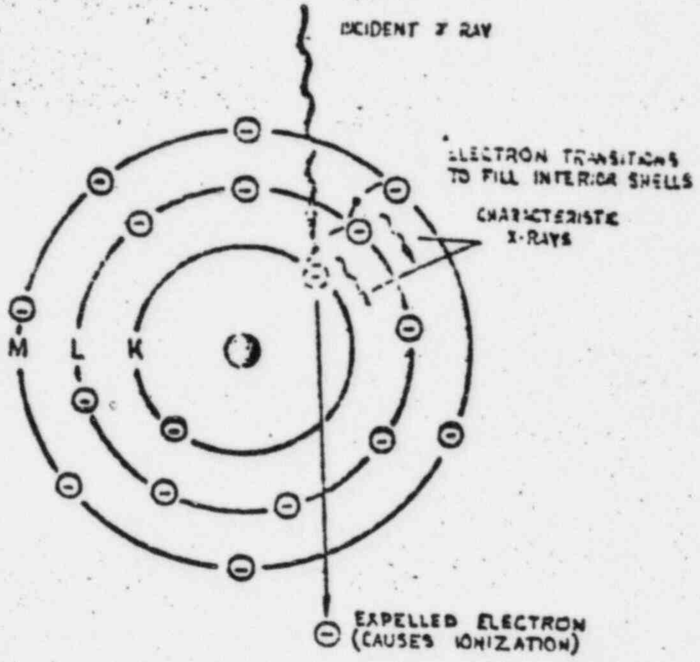

Figure 4-6: Diagram of Photoelectric Effect

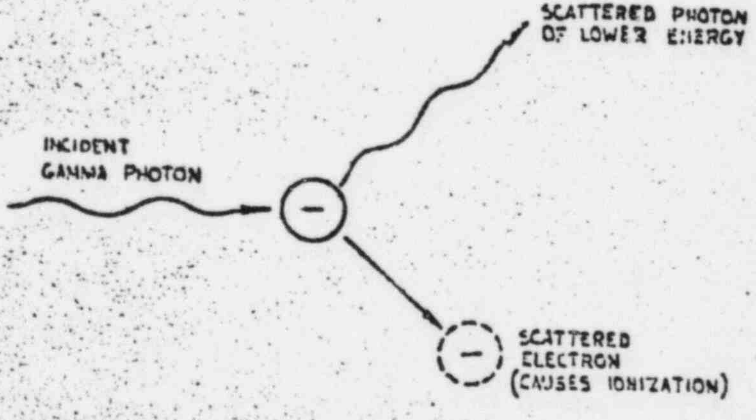

Figure 4-7: Diagram of Compton Scattering

The result is that the incident photon goes off at some angle to the initial line of travel, and the electron is ejected in another direction. The gamma loses some of its energy as a result of the collison, but is not completely eliminated as in the case of the photoelectric effect. Note that in Compton scattering, the gamma photon behaves very much like a particle.

At higher energies, a third and rather startling phenomenon known as pair production takes place. If a high energy gamma ray enters the vicinity of a nucleus, it can give rise to the creation of a pair of electrons as shown in Figure 4-8. One of these is the ordinary type and the other is a positive electron or positron. The minimum gamma ray energy necessary for pair production is 1.02 mev[1].

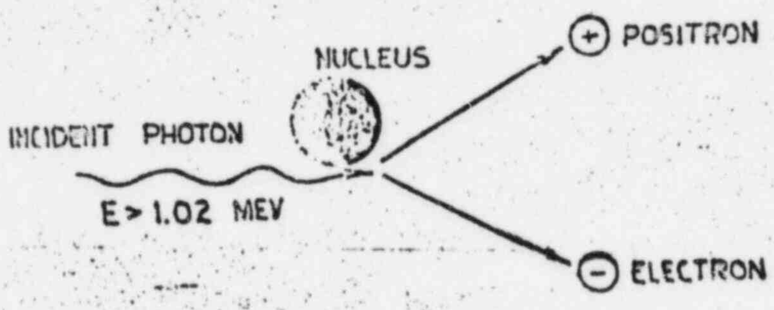

Figure 4-8: Diagram of Pair Production

As gamma rays penetrate a shield, their intensity decreases exponentially with distance. One characteristic of an exponential decrease is that it

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[1] The reason for this involves the fact that mass and energy are really two forms of the same thing, and one cannot be created without the disappearance of an equivalent amount of the other. It turns out that 1 amu is equivalent to 931 mev of energy. Since a positron and an electron each have .00055 amu of mass, the minimum energy necessary to product these two particles is (2)(.00055)(931) = 1.02 mev. The equivalence of mass and energy will be discussed further in Chapter 5. It is of interest to note that the reverse process to pair production also occurs. That is, whenever a positron encounters an electron, the two particles annihilate each other and two 0.51 mev gamma rays are created in their place. Since all matter contains an abundance of electrons, the lifetime of a positron is always very short.

asymptotically approaches zero (see Figure 4-3). In theory therefore, the concept of a definite range as applied to gamma rays is meaningless, since no amount of shielding will ever eliminate 100% of them. In practice, of course, there is a finite thickness of any shielding material which will reduce the gamma intensity to a value so low as to be negligible. Nevertheless, rather than define a gamma ray range in various materials, it is more common to define a tenth-value thickness of the material for gamma rays. Analagous to half-life, the tenth-value thickness of a material is that thickness of shielding which will reduce any incident gamma radiation to one-tenth its original intensity. The tenth-value thickness of a material depends upon the energy of the incident gamma radiation. The higher the gamma energy, the greater its penetrability, and hence the greater the tenth-value thickness. Table 4-IV gives the tenth-value thickness for several common materials. As can be seen, the gamma ray possesses considerably more penetrating power than either the alpha or beta particle. Usually, therefore, a shielding designer only has to be concerned with the removal of the gamma rays from a source, for by so doing he will automatically have removed any alpha or beta particles which were present. The three most common shielding materials for gamma rays are water, concrete, and lead. Use of the values 2 feet of water, 1 foot of concrete, and 2 inches of lead as rule of thumb tenth-value thickness will provide conservative shielding for essentially all applications.

TABLE 4-IV: GAMMA RAY TENTH-VALUE THICKNESS FOR VARIOUS MATERIALS

	TENTH-VALUE THICKNESS (inches)				
ENERGY (Mev)	Air (in miles)	Water	Ordinary Concrete	Iron	Lead
1	.18	13.0	6.1	1.92	1.15
2	.25	18.5	8.7	2.70	1.75
5	.41	30.0	13.2	3.68	1.82

FUNDAMENTALS OF GAMMA RAY SHIELDING

The subject of radiation dose rates is discussed in detail in the radiation protection training manuals. For this discussion it is only necessary to state that the gamma dose rate at a particular location is dependent upon the gamma ray energy (the higher the energy, the higher the dose rate) and the gamma ray flux. The gamma ray flux, f_g, is defined as the number of gamma rays passing through a unit area per second. The units of gamma flux are therefore (gammas)/(area)(second).

Ordinarily, the shielding designer has no control over the gamma ray energy. This depends upon the particular radioactive isotopes which are present, which in turn are governed by the particular processes which are occurring. For example, fission results in the creation of certain specific radioactive isotopes as a byproduct. In any case, the problem of reducing the gamma dose rate thus becomes one of reducing the gamma flux at a particular location.

In order to make any intelligent decision regarding shielding, the designer must know what his starting point is. That is, he must know the gamma flux at some reference location. In addition, he must know something about the physical size of the gamma ray source. With this knowledge, he can then calculate the gamma flux at any other location of interest with the various shields he has chosen to employ. In order to do this, he makes use of the general formula:

$$\phi = \phi_R GS \qquad (4\text{-}11)$$

where: ϕ = gamma flux at location of interest [gammas/(area)(sec)]
ϕ_R = reference gamma flux [gammas/(area)(sec)]
G = geometric reduction factor (no units)
S = shielding reduction factor (no units)

To understand the significance of this formula, we will consider several common examples.

1. Unshielded Point Source

Consider a point source suspended in space, as shown in Figure 4-8. and surrounded by no shielding material other than air. Such a source will emit gamma rays uniformly in all directions, and these rays will travel in reasonably straight paths radially outward from the source. In general the shielding effect of air is so small that it can be neglected, and so the source can be assumed to be essentially unshielded. In terms of equation 4-11, this means that there is no reduction in gamma flux due to shielding, and so the factor $S = 1.0$, and $\phi = \phi_R G$. Now then, to determine the factor G, we must compare the flux at some arbitrary reference distance from the source (assumed to be at radius d_R) with the flux at the point of interest(assumed to be at radius d). To do this, we look at how many gamma rays are passing through equal sized areas at the two locations, for this is what determines the gamma flux. In this case, the reference flux was chosen at a distance somewhat closer to the source than was the point of interest. As can be seen from the figure, more gamma rays pass through the area at the reference distance than through the same area at the point of interest. This is due to the spreading of the gamma rays as they move outward from the source. Thus, even in the absence of any shielding, the dose rate from a point source can be significantly reduced by moving father from the source. To be more specific, it can be shown from geometric considerations that the geometric factor is given by the equation: $G = d^2/d_R^2$ so that for an unshielded point source, equation 4-11 becomes:

$$\phi = \phi_R \left[\frac{d_R^2}{d^2}\right] \qquad (4\text{-}12)$$

To illustrate the use of equation 4-12, assume that at a distance of 1 foot from a point gamma source we measure a gamma flux of 4×10^3 (gammas)/(ft^2)(sec). At a point two feet from the source, the gamma flux would be:

$$\phi = 4 \times 10^3 \,(1^2/2^2) = 4 \times 10^3 \times 1/4 = 1 \times 10^3 \text{ gammas/(ft}^2\text{)(sec)}$$

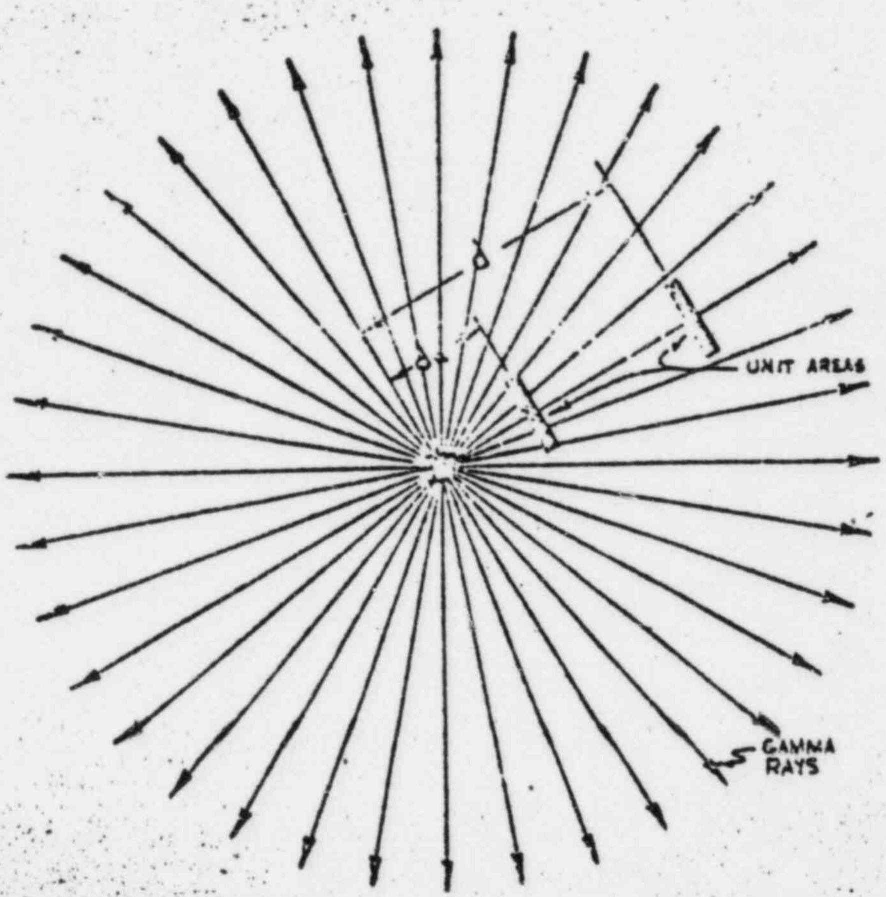


Figure 4-8: Gamma Radiation From an Unshielded Point Source

Since dose rate is directly related to the gamma flux, it would likewise decrease by a factor of four as we doubled our distance from the source. The behavior described by equation 4-12, where doubling the distance reduces the radiation by four, is known as the inverse square law. It is important to note that it theoretically applies only to point sources, where we get maximum benefit from the spreading of the gamma rays. However, in practice it is a useful approximation for any physically small radioactive source.

2. Beam Source

Figure 4-9 shows a beam of gamma rays passing through a shield. Most

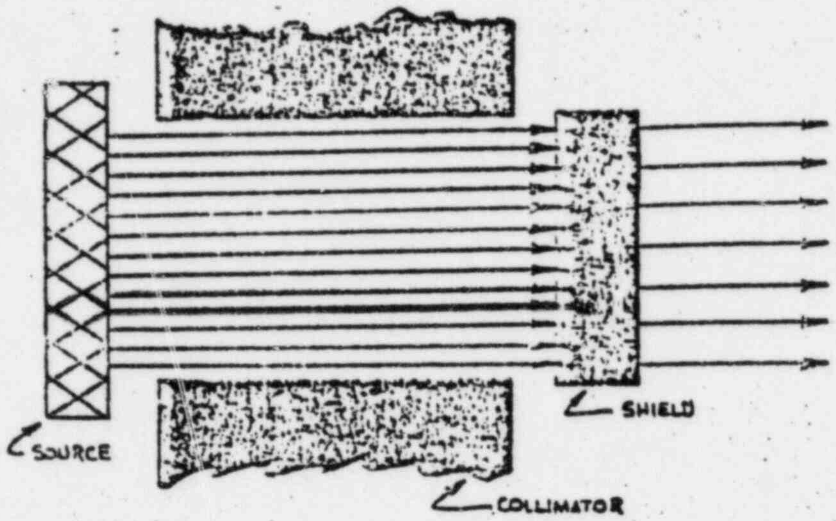


Figure 4-9: Collimated Gamma Ray Beam

sources do not emit a perfectly collimated beam, but rather, emit gamma rays in all directions. A beam can be produced, however, by allowing the gamma rays to fall upon a collimator. A collimator is usually a thick shield with a small hole through it. If the source is placed upon one side of the shield, a collimated beam will emerge out of the hole on the other side. This results from the fact that only gamma rays which were emitted in the same direction as the axis of the hole will be able to penetrate the shield. In practice, reasonably collimated beams are found where pipes penetrate shielding, at doorways, etc. The main difference between a collimated beam and a point source is that there is very little spreading of the gamma rays in a beam. Thus, as long as we remain within the confines of the beam, and in the absence of any shielding, the dose rate will not vary as we move closer to, or farther from, the source. Thus, we have no purely geometrical reduction of the dose rate as we did in the case of a point source, and therefore the G term in equation 4-11 is equal to 1.0. The only way in which we can reduce the intensity of a beam is to insert a shield in the beam.

As discussed in the previous section, gamma rays undergo an exponential reduction in intensity when passing through a shield. That is:

$$\phi = \phi_B e^{-\mu D} \qquad (4\text{-}13)$$

where: ϕ = gamma flux leaving shield
ϕ_R = reference gamma flux entering shield
μ = linear absorption coefficient (cm^{-1})
D = shield thickness (cm)

The linear absorption coefficients for several materials are shown on Figure 4-10. In general, the more dense the material, the greater is the coefficient, indicating that dense materials are more effective shields than light materials. As gamma ray energy increases, the coefficient is reduced. This indicates that high energy gamma rays are more penetrating than low energy gamma rays.

To illustrate the use of equation 4-13, consider the following examples.

Example 1: A beam of 0.8 mev gamma rays is incident upon a 1-inch thick lead shield. What fraction of the gamma rays will be removed from the beam in traversing the shield?

$$\phi = \phi_R e^{-\mu D}$$

$$\mu = 1.0 \text{ cm}^{-1} \text{ (Figure 4-13)}$$
$$D = 1 \text{ inch} = 2.54 \text{ centimeters}$$

$$\phi/\phi_R = e^{-(1)(2.54)} = e^{-2.54}$$

$$\ln \phi/\phi_R = -2.54$$

$$\phi/\phi_R = 0.079$$

Since the beam leaving the shield is only 7.9% as strong as the beam entering the shield, 92.1% of the beam was removed.

Example 2: How thick must a normal concrete shield be in order to reduce the intensity of a 0.5 mev gamma ray beam to 1/100 of its unshielded value?

$$\phi/\phi_R = e^{-\mu D}$$

$$\phi/\phi_R = 1/100 = .01 \text{ (from conditions of the problem)}$$

$$\mu = 0.205 \text{ cm}^{-1}$$

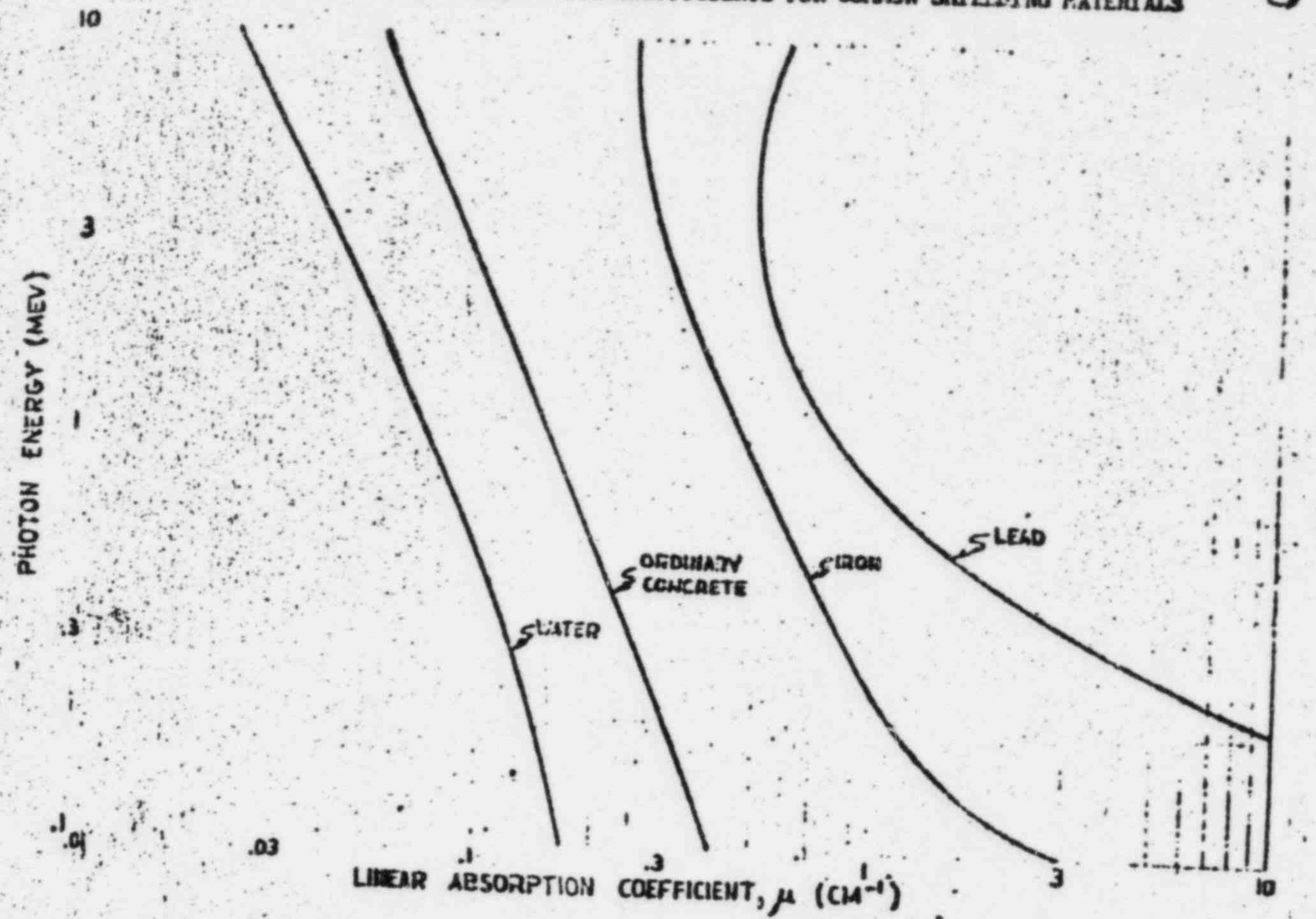
$$.01 = e^{-(0.205)D}$$

$$\ln .01 = -(.205)D$$

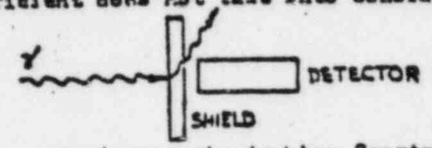
$$-4.6 = -.205\,D$$

$$D = -4.6/-.205 = 22.5 \text{ cm} = 8.85 \text{ inches}$$

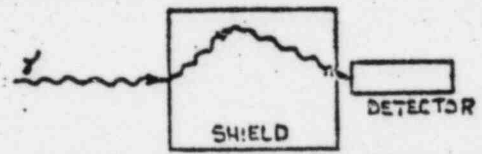
FIGURE 4-10: LINEAR ABSORPTION COEFFICIENTS FOR COMMON SHIELDING MATERIALS



It should be noted that the use of equation 4-13 will underestimate the amount of shielding required for a source, or equivalently, will overestimate the effectiveness of a given shield. This results from an effect called buildup. In general, attenuation coefficients are measured by placing thin sheets of absorber between a source and a detector and noting the decrease in detector count rate. The geometry for such an experiment is shown in Figure 4-10a. In general, with the detector very close to the source, any gamma ray which undergoes a scattering reaction will be deflected out of the sight of the detector. The detector will thus consider this gamma ray as being removed from the beam. However, if the shield is thick, some of the gamma rays which originally scattered out of the beam may undergo additional scattering reactions and return to the beam. The measured value of the attenuation coefficient does not take into consideration such multiple events, and so will overestimate the removal of gamma rays by the shield. To account for buildup, equation 4-13 is modified as follows:



a) Thin absorber experiment showing how Compton scattered gamma rays generally are not detected



b) Thick absorber experiement showing how gamma ray which has undergone multiple Compton scatterings may be detected

Figure 4-10: Illustration of Gamma Ray Buildup

$$\phi = \phi_R D e^{-\mu D} \qquad (4\text{-}14)$$

where: B = buildup factor (no units)

The magnitude of the buildup factor is 1.0 for thin shields and increases as the shield gets thicker. Figure 4-11 shows the buildup factors for a point source in water (the values for water can also be used for concrete with only a small error) and lead.

3. Shielded Point Source

We have considered an unshielded point source, to show how geometric considerations can effect the dose rate, and also gamma ray attenuation in shields. To complete the discussion, we will consider a case where both effects come into play. For a shielded point source, equation 4-11

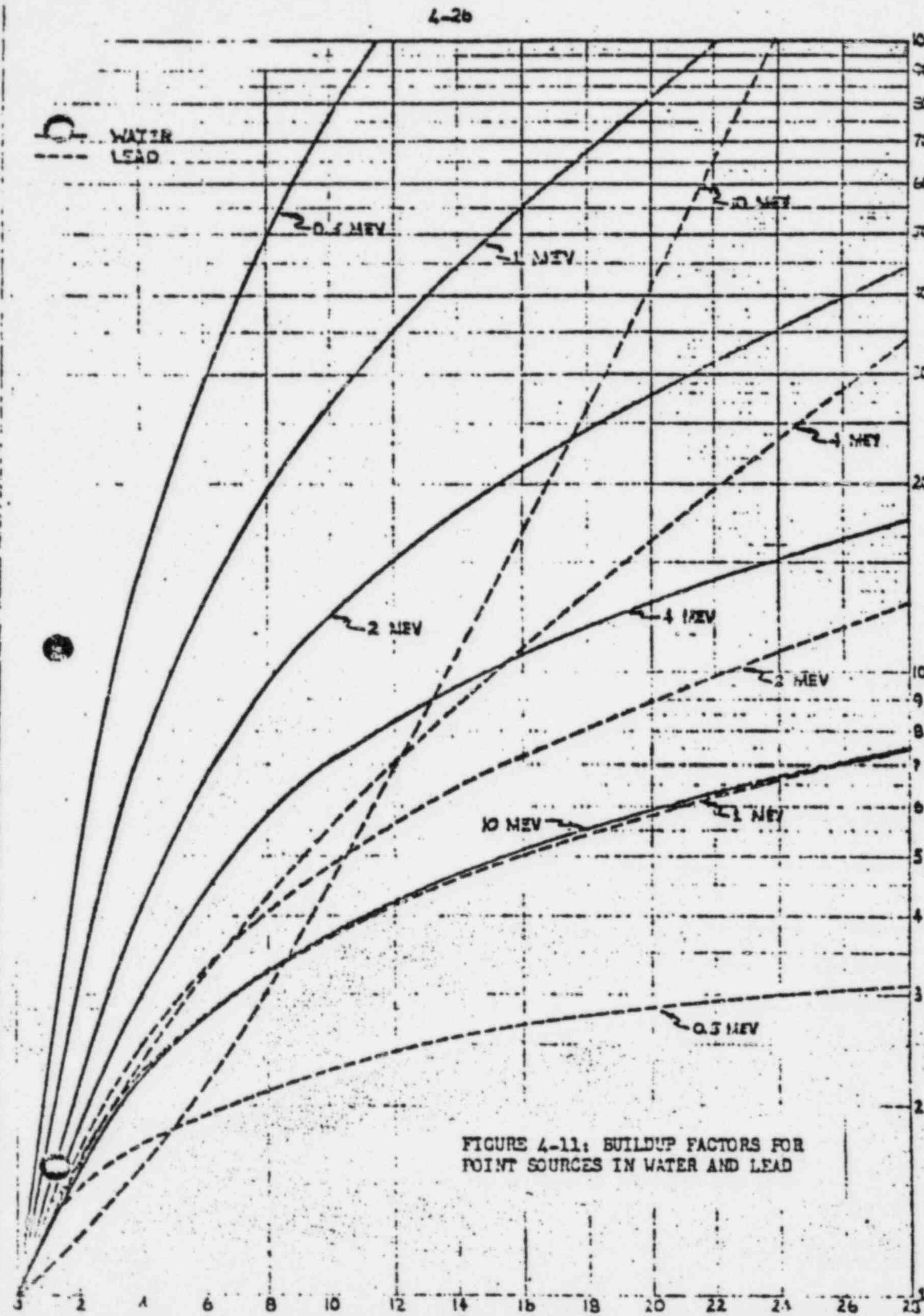


FIGURE 4-11: BUILDUP FACTORS FOR POINT SOURCES IN WATER AND LEAD

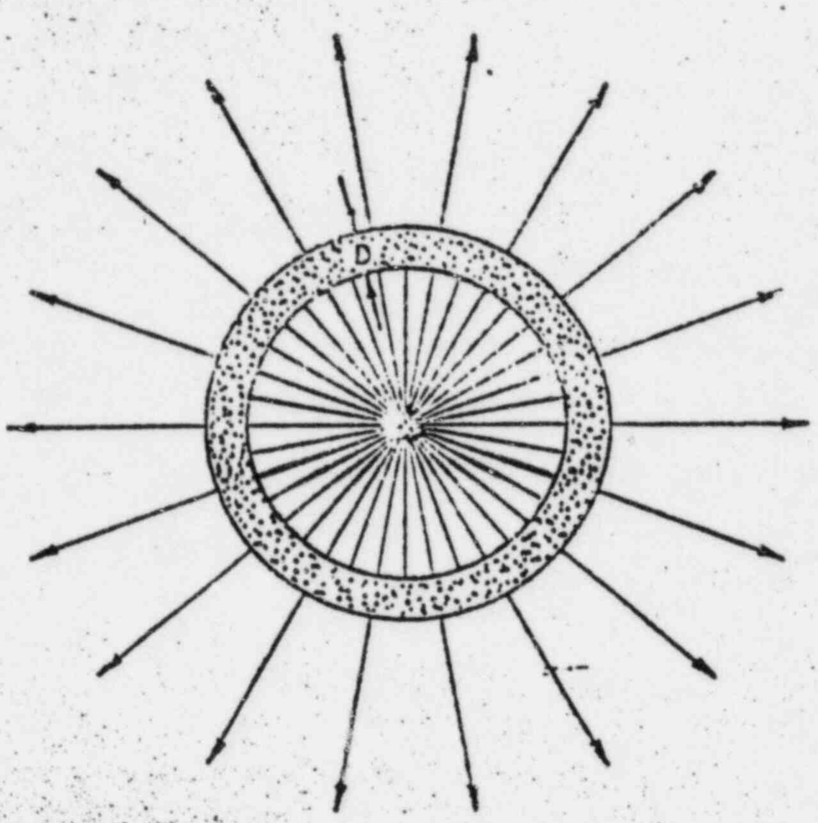


Figure 4-11: Shielded Point Source

becomes:

$$\phi = \phi_R \frac{d_R^2}{d^2} B e^{-\mu D} \qquad (4\text{-}15)$$

Example: The gamma dose rate 6 inches from an unshielded small source (E_γ = 1 mev) is 3 R/hr. We surround the source with a piece of lead sheet 0.25 inches thick. What would the dose rate be 2 feet from this source?

$$\mu_{Pb} = 0.82\ \text{cm}^{-1}\ \text{(Figure 4-10)}$$

$$D = 0.25\ \text{in.} = (0.25)(2.54) = 0.635\ \text{cm}$$

$$e^{-\mu D} = e^{-(0.82)(0.635)} = e^{-0.521} = 0.594$$

$$DR = (DR_{Ref})(d_{Ref}^2/d^2)\, e^{-\mu D}$$

$$= (3)(0.5/2)^2(0.594) = 0.11\ \text{R/hr}$$

This solution ignores dose buildup. For a thin shield the buildup effect is not large and in fact, Figure 4-11 shows it to be 1.15. So a more accurate value of the dose rate would be (0.11)(1.15)= 0.13 R/hr.

PROBLEMS

1. Describe alpha particles, beta particles and gamma rays as to mass, charge, and penetrating power.

2. Fill in the blanks as appropriate.

   a) $_{82}Pb^{214} \longrightarrow {}_{Z}Bi^{A} + \beta^{-} + \gamma$ Z = ____ A = ____

   b) $_{90}Th^{230} \longrightarrow {}_{88}Ra^{226} + \_\_\_\_ + \gamma$

   c) $_{Z}Rn^{A} \longrightarrow {}_{84}Po^{214} + \alpha + \gamma$ Z = ____ A = ____

   d) $_{91}Pa^{234} \longrightarrow {}_{92}U^{234} + \_\_\_\_ + \gamma$

   e) $_{Z}Tl^{A} \longrightarrow {}_{82}Pb^{206} + \beta^{-} + \gamma$ Z = ____ A = ____

3. What is an antineutrino?

4. Define half-life, half-thickness, curie, bremsstrahlung, and ion pairs.

5. Suppose we have 10^{10} atoms of an isotope with a 4 minute half-life. What is the rate of radioactive disintegration (in disintegrations/sec.) of this isotope? How many curies does this represent?

6. How many curies of the previous isotope will remain 16 minutes later?

7. An average 150 lb. man contains 245 grams of potassium, of which 0.0119% is the naturally occurring β^{-} emitter potassium -40 (atomic weight = 40 amu, $T_{1/2} = 1.3 \times 10^{9}$ years, $\lambda = 1.7 \times 10^{-17}$ sec^{-1}). How many curies of potassium -40 does the man contain? Recall that 1 amu = 1.66×10^{-24} grams.

8. How many half-lives does it take for a quantity of a radioactive isotope to be reduced to less than 1% of its original value?

9. Describe the processes by which each of the three types of radiation produce ionization.

10. Why does a beta particle travel farther in matter than an alpha particle?

11. Why don't the beta particles from a given emitter show a fairly definite range in matter? Conversely, why do alpha particles from a given emitter show a fairly definite range?

12. It is desired to reduce the gamma ray intensity from a particular source by submerging the source in water. To what approximate depth must it be submerged to reduce the gamma intensity by a factor of 10^{6}? What approximate thickness of lead would be required to give equivalent shielding?

13. A small, unshielded radioactive source emits 1.2 mev gamma rays and produces a dose rate of 10^4 R/hr at 6 inches. It is encased in a lead shield which has walls 8 inches thick. What is the dose rate 2 feet from the source (shield is between you and the source)?

CHAPTER 5

MASS, ENERGY, AND NUCLEAR STABILITY

THE EQUIVALENCE OF MASS AND ENERGY

In 1905, Albert Einstein proposed his famous theory of relativity. One of the most significant portions of this theory was Einstein's statement that mass and energy were really two different forms of the same thing, and that it should be possible (at least in theory) to convert mass into energy or to convert energy into mass. In support of his contention, he proposed his famous equation relating mass and energy:

$$E = MC^2 \qquad (5\text{-}1)$$

where:

M = mass of a body

E = equivalent energy of the body

C = speed of light

The implications of this equation were of immediate interest to physicists throughout the world. They recognized that because the factor C^2 is an exceedingly large number, even very small amounts of mass must be equivalent to tremendous amounts of energy. For example, let us calculate the energy which could be obtained from the complete conversion of one pound of mass. The speed of light is 186,000 miles/sec or 9.82×10^8 ft/sec. Therefore:

$$E = (1\ \text{lb})(9.82 \times 10^8\ \text{ft/sec})^2 = 96.3 \times 10^{16}\ \frac{\text{lb ft}^2}{\text{sec}^2}$$

The energy unit lb ft^2/sec^2 is not very common, but if we convert our answer to kilowatt-hours we get $E = 11.3 \times 10^9$ KWH. For the year 1963, P.G. and E. generated 27.9×10^9 KWH. Therefore, the complete conversion to energy of one pound of mass would have supplied P.G. and E. with 41% of its total 1963 electrical energy generation requirements. However, such examples are of purely academic interest because it is generally impossible to carry out such a conversion with masses this large. As we shall see, when such conversions are made the amount of mass involved is usually infinitesimally small.

It should be noted here that any process which results in the liberation of energy does so at the expense of a small amount of mass. For example, the complete combustion of a pound of coal yields about 13,000 BTU of heat. Therefore the products of combustion must weight less than one pound by an amount equal to the mass equivalent of 13,000 BTU. However, it requires the conversion of only 1/3 of a billionth of a pound of mass to product 13,000 BTU, and this loss of mass is so small that it goes undetected.

THE ELECTRON VOLT UNIT OF ENERGY

Before further consideration of the implications of equation 5-1, it is convenient to define a new unit of energy known as the <u>electron volt</u>. One electron volt (abbreviated ev) is equal to the energy acquired by any particle carrying a unit electrical charge when it passes without resistance through a potential difference of one volt. There are two other units which are commonly used:

1 mev (million electron volts) = 1,000,000 ev

1 kev (kilo electron volts) = 1,000 ev (5-2)

The unit of mass with which we will be working is the atomic mass unit (see Chapter 2). We can convert 1 amu to energy using equation (5-1) providing we are careful to use the proper units and conversion factors in the problem. The calculation is somewhat tedious and will not be carried out here, but the results are that:

1 amu = 931 mev (5-3)

NUCLEAR FORCES

It has been common knowledge among scientists for many years that unlike electrical charges attract and that like charges repel. When the present theory of the nucleus was proposed early in the twentieth century, one immediate question arose in many people's minds. Why don't the repulsive forces among the protons in the nucleus cause the nucleus to be very unstable and fly apart? The conclusion drawn was that there must be a new type of force which acts within the nucleus to hold it together. Furthermore, since this attractive force is only observed in nuclei, it must operate only over the very short distances which exist between particles in the nucleus. Thus if two nuclear particles are separated by any significant distance, the attractive force between them must rapidly decrease to zero. This latter property is in contrast to electrical forces, which act upon charged bodies even though they may be separated by relatively large distances.

No universally acceptable theory has yet been advanced as to the ultimate nature of the nuclear force. Although the details of modern theories are highly complex, they are based upon the idea that protons and neutrons are not truly fundamental particles, but in fact are rather complex systems in themselves (the fact that a neutron can be transformed into a proton in beta decay gives some evidence of this). Apparently, one of their constituents is a small particle called the pi meson. Pi mesons have a mass of ~ 270 times that of an electron and can have a charge of +1, -1, or 0. They decay (with intermediate steps) to electrons and neutrons. When two nucleons (neutrons or protons) are in the immediate vicinity of each other, it appears that they can transfer pi mesons back and forth. This interaction of their mesons is thought to be responsible for the force of attraction between them.

As further evidence for the complexity of the proton and neutron, one recent group of experiments appears to show that both the neutron and proton are composed of a common core of radius ~ 0.2 f[1] with a charge of about +0.35e.

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[1] The letter f stands for a unit of length called the fermi, which is equal to 10^{-13} cm.

Surrounding this core to a radius of ~ 0.8 f is a cloud of mesons. In the proton this cloud has a charge of +0.5e and in the neutron it is -0.5e. Finally, both particles possess a third meson cloud, extending to a radius of ~ 1.4 f, which has a charge of +0.15e. The overall proton charge is thus +1e, and the neutron has no net charge. The fractional charges apparently are the result of something like the quark.

The idea of forces arising from the exchange of "particles" is not new. The electrical and magnetic forces have been shown to result from the exchange of photons between the charged bodies (hence the term electromagnetic radiation for X and γ rays). Similarly, the force of gravity is thought to result from the exchange of an as yet undiscovered "particle" called the graviton.

NUCLEAR STABILITY

Although we have not been able to present a very clear picture of the factors involved in the short range nuclear forces, it is still possible to draw some conclusions regarding nuclear stability. Suppose we were to make a nucleus entirely of protons. It is well known that the electrical force existing between charged particles increases rapidly as the particles are brought closer together. In our hypothetical proton nucleus, the individual protons would have to be very close to each other in order for the short range nuclear attractive forces to be effective. However, under these conditions where the distances between protons are small, the repulsive electrical forces get very large and it is easy to see that such a configuration would tend to be unstable. The obvious solution to this problem is to add some neutrons to our nucleus. This operation would leave the attractive forces essentially unchanged because the attractive force between a neutron and a proton is just as strong as the attractive force between two protons) but would tend to decrease the repulsive forces because, on the average, the protons would then be spaced farther apart. Thus it is that the ${}_2He^2$ nucleus is so unstable that it doesn't exist, whereas the ${}_2He^3$ nucleus is stable and occurs naturally (although it is very rare) and the ${}_2He^4$ nucleus is exceedingly stable (and accounts for ~ 100% of all natural helium). However, there is a limit to the number of neutrons we could add and still maintain a stable nucleus. A nucleus with too many neutrons can become just as unstable as a nucleus with too many protons. Again using helium as an example, we find that the ${}_2He^5$ and ${}_2He^6$ nuclei are not found in nature. They can be produced artificially but are both found to decay by β^- emission. The ${}_2He^7$ nucle is so unstable that it doesn't exist. The reason why an excessive amount of neutrons causes instability involves concepts which are beyond the scope of this manual.

The conclusion to be drawn from the previous discussion is that nuclear stability is in large part determined by the relative number of neutrons and protons in the nucleus. However, for a nucleus with a given number of protons there is usually a range of neutrons which will result in stable nuclei. This is equivalent to saying that most elements exhibit more than one stable isotope.

In the lighter elements, stability is achieved when the number of neutrons and protons in the nucleus are approximately equal, or in other words, when the ratio of neutrons to protons is about 1.0. As the number of protons increases, however, it becomes increasingly difficult to overcome the electrical repulsive forces. Therefore, it takes, on a relative basis, more and more neutrons to do the job. For example, in a stable heavy isotope of lead such as ${}_{82}Pb^{208}$ there are 82 protons and 126 neutrons. Thus for every proton it takes 126/82 = 1.54 neutrons to achieve stability. If we locate the stable isotopes on a plot of neutrons versus protons, we see that the neutron to proton ratio increases gradually from about 1.0 for the lighter elements to ~ 1.6 for the heavier elements. Such a plot is shown in Figure 5-1. The stable isotopes fall within the shaded area. An isotope with an equal number of protons and neutrons would fall on the dotted N = Z line. Notice that the shaded area is located somewhat above this line for all but the lightest elements, indicating that the neutron to proton ratio must be greater than 1.0 as we have previously discussed. Notice also that the shaded area is somewhat fattened for the intermediate sized nuclei, indicating that these elements must exhibit many stable isotopes. For example, tin (Z = 50) occurs naturally with ten stable isotopes (${}_{50}Sn^{112}$, ${}_{50}Sn^{114}$, ${}_{50}Sn^{115}$, ${}_{50}Sn^{116}$, ${}_{50}Sn^{117}$, ${}_{50}Sn^{118}$, ${}_{50}Sn^{119}$, ${}_{50}Sn^{120}$, ${}_{50}Sn^{122}$, and ${}_{50}Sn^{124}$).

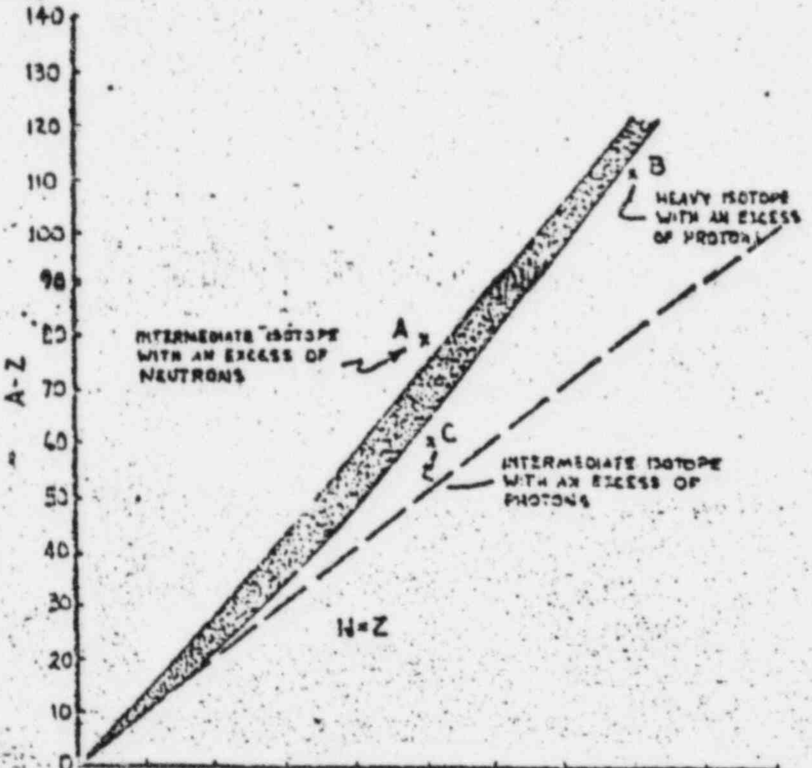

Figure 5-1: Chart of Stable Nuclides

Any isotope which falls outside of the shaded area is unstable and will emit radiations in an attempt to reach a stable condition within the shaded area. Consider, for example, an isotope which is unstable due to an excess of neutrons in the nucleus. Such an isotope might be located at point A on Figure 5-1. Its mode of decay will be such that it reduces its neutron to proton ratio. It could do this directly by emitting a neutron, but this process has not been found to occur (there is an exception to this in the case of several fission products, but this will be discussed further in Chapter 8). The process by which this isotope will decay is by β^- emission. We have already seen that this is equivalent to converting a neutron into a proton, thereby decreasing the neutron to proton ratio as was required. This can be seen by considering the following example:

$$_{53}I^{131}\ \text{(iodine)} \longrightarrow {}_{54}Xe^{131}\ \text{(xenon)} + \beta^- + \gamma$$

$$(N/P)_I = 78/53 = 1.47 \qquad (N/P)_{Xe} = 77/54 = 1.43$$

The isotope $_{54}Xe^{131}$ is stable.

Now consider an isotope of small or intermediate size which has too many protons. Such an isotope can be pictured as being at point C on Figure 5-1. It will decay in such a manner as to increase its neutron to proton ratio. Again, the isotope could take the direct approach and decay by proton emission, but this process never occurs. Instead this isotope will probably decay by positron (β^+) emission and thereby convert one of its protons into a neutron. Again using iodine as an example we see that:

$$_{53}I^{124} \longrightarrow {}_{52}Te^{124}\ \text{(tellurium)} + \beta^+ + \nu$$

The neutron to proton ratio for I-124 is 71/53 = 1.34, and for Te-124 it is 72/52 = 1.38. Te-124 is stable. Positrons are not one of the three types of naturally occurring radiations discussed in Chapter 4. All positron emitters are artificially produced radioactive isotopes. The methods by which they are produced will be discussed in Chapter 6.

There is a second method by which a proton heavy nucleus can reach stability. This process is known as orbital electron capture. In this process, the nucleus attracts one of its own orbital electrons (usually from the nearest, or K, shell so this process is often called K capture) which in turn neutralizes one of the excess protons and converts it to a neutron. The net effect is seen to be similar to positron emission. This procedure is favored when, because of other reasons, not enough energy is available for positron emission. As an example of K capture,

$$_{29}Cu^{64}\text{ (copper)} + {}_{-1}e^{0}\text{ (from K shell)} \longrightarrow {}_{28}Ni^{64}\text{ (stable nickel)} + \nu$$

Such a transition would be followed by the emission of the characteristic X-rays of the nickel atom as its electrons rearranged themselves.

There is still a third method by which a proton heavy isotope can reach stability. This method involves the emission of an alpha particle and is the preferred method for heavier nuclei (such as that represented by point B on Figure 5-2). It seems reasonable that a heavy nucleus would have a tendency to be unstable because of its complexity and because of the large repulsive forces which must be overcome. In fact, all isotopes above bismuth (Z = 83) are radioactive although several of them (such as U-238) can be considered stable because of their extremely long half-lives. Thus we might expect that a heavy isotope would decay in such a manner as to lower its mass as well as increasing its neutron to proton ratio. It is obvious that alpha emission results in a reduction of the mass although it is not quite so obvious that alpha emission increases the neutron to proton ratio. This latter fact can be verified by considering the following example.

$$_{92}U^{238} \longrightarrow {}_{90}Th^{234} + {}_{2}He^{4} + \gamma$$

Here we see that $(N/P)_U = 146/92 = 1.59$ and $(N/P)_{Th} = 144/90 = 1.60$.

Although most heavy elements decay by alpha emission, there are some which decay by positron emission. For those heavy elements which have an excess of neutrons, β^- emission is the mode of decay.

ENERGY AND MASS CONSIDERATIONS IN NUCLEAR STRUCTURE

If we could gather atomic particles individually and build them into an atom, we would find that the assembled atom weighed less than the total weight of the original individual neutrons, protons, and electrons. For example, consider an atom of the aluminum isotope $_{13}Al^{27}$. Experimentally, the atomic weight of this atom has been found to be 26.99722 amu. Now let us calculate the weight of 13 protons, 14 neutrons, and 13 electrons.

Particles	Mass per particle (amu)	Total amu
Proton, 13	1.00758	13.09854
Neutrons, 14	1.00897	14.12558
Electrons, 13	.00055	.00715
Total weight of individual particles		27.23127
Actual weight of $_{13}Al^{27}$ atom		26.99722
Difference		.23405 amu

This difference between the summation of individual particle weights and the actual weight of the atom is called the mass defect of the atom. It represents the amount of mass that must have been lost at the instant the atom was created. This mass was converted to energy in accordance with Einstein's equation and probably appeared as gamma rays and kinetic energy of the newly created atom.

Now let us take an assembled Al-27 atom and try to break it back up into its forty component parts. In order to do this, we must resupply the atom with the mass which was lost during its creation. That is to say, we cannot break the Al-27 atom completely up unless we return the 0.23405 amu of mass that it lost during its creation. It follows from this that the mass defect of the atom is related to the strength of the forces acting to hold it together. The greater the mass defect, the more difficult it is to break the atom up, and hence the stronger the forces must be which are holding the atom together[1].

We have seen that mass is destroyed whenever an atom is assembled from its component parts. The question arises as to which of the three major atomic particles are involved in this loss of mass. We can eliminate the orbital electrons on the following grounds. If an orbital electron lost some mass during the creation of the atom, it is reasonable to assume that we would hav to resupply this mass (or its equivalent in terms of energy as determined by Einstein's equation) in order to remove the electron from the atom. We know from our study of chemical reactions that it is relatively easy to remove orbital electrons from an atom, at least in comparison to the difficulty involved in removing a proton or neutron. Our conclusion must therefore be that the amount of mass lost by the orbital electrons is infinitesimally small. Thus, we can ignore it and consider the entire mass defect to be lost by the protons and neutrons in the nucleus.

Since the mass defect is a measure of the strength of the short range nuclear force, much study has been devoted to it to resolve some of the problems concerned with the nature of this force. Although we will not pursue the subject further, it should be mentioned that through use of the mass defect it is possible to predict whether a particular isotope will be stable or radioactive, the type of radiation it will emit if it is radioactive, and the energy of the emitted radiation. The existance of the mass defect also suggests that transformations within the nucleus represent a way in which significant quantities of mass can be converted into energy in accordance with equation (5-1).

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[1] The mass defect can thus be thought of as a measure of the total binding forces holding the atom together. For this reason the energy equivalent of the mass defect is often called the binding energy. The binding energy of Al-27 is (931)(.23405) = 217.9 mev.

PROBLEMS

1. There are four stable isotopes of iron (Z = 26) with mass numbers 54, 56, 57 and 58. What type of radioactive decay would you expect the iron isotope with A = 53 to undergo. What about the one with A = 59? Why?

2. What type of decay does a heavy isotope with a relatively low neutron to proton ratio usually undergo?

3. What is K capture? If both ${}_{54}Xe^{123}$ and ${}_{54}Xe^{137}$ are known to be radioactive, which of the two would you expect to under K capture? What type of decay would the other be likely to exhibit?

4. Define mass defect. What particles contribute to the mass defect?

5. How much mass is converted into energy during the creation of a ${}_{3}Li^{7}$ (lithium) atom? The atomic weight of Li-7 is 7.01322 amu. How many mev of energy does this amount of mass represent?

CHAPTER 6

NUCLEAR REACTIONS

INTRODUCTION

The discovery of natural radioactivity, and its subsequent identification as a transformation within the nucleus, led immediately to attempts to produce artificial nuclear transformations. Further impetus was provided for the study of nuclear transformations by the realization that they represented the most likely way in which significant quantities of mass could be converted to energy in accordance with Einstein's equation. The difficulties involved in producing artificial nuclear reactions can be appreciated by recalling several facts.

1. The nucleus presents an extremely small target to any projectile we choose to hit it with. Its diameter is only about 1/10,000 as large as the atomic diameter.
2. In most nuclei, the particles are held together by extremely strong forces. It is therefore much more difficult to remove a proton or neutron from a nucleus than it is to remove an electron from an atom.
3. It is difficult to shoot any positively charged particle into a nucleus, or to make two nuclei collide with each other because of the electrical repulsive forces which exist.

Most nuclear reactions occur when a particle is shot into the nucleus and disrupts it to a point where it disintegrates in one way or another (an exception to this is natural radioactive decay in which the nucleus disintegrates because it is naturally unstable). The reluctance of nuclear particles to interact can be overcome in several ways.

1. Thermonuclear reactions: These occur where a temperature of several million degrees is achieved and nuclei have enough kinetic energy to overcome their mutual electrostatic repulsion. This type of reaction occurs on the sun and stars and represents their energy source. Although research is being carried out on thermonuclear reactions, the experimental difficulties involved make this method impractical for most applications at the present time.
2. Acceleration of charged particles: Protons, alpha particles, electrons, etc. can be accelerated in atom smashers like the cyclotron, betatron, synchrotron, and linear accelerator until they have enough kinetic energy to cause a nuclear reaction when they hit a target nucleus. Through use of atom smashers, many nuclear reactions have been studied, nearly a dozen new elements have been synthesized, and a great wealth of information concerning the structure of the nucleus has been obtained.
3. Nuclear reactors: When it is desired to shoot a neutron into a nucleus, a nuclear reactor provides a convenient source of neutrons. We will discuss this method in great detail throughout the remainder of the manual.

4. Radioactive sources: Much of the early work with nuclear reactions was performed by placing a substance near a naturally occurring radioactive source such as radium and allowing it to be bombarded with alpha particles from the source.

TRANSMUTATION PROCESS

A nuclear reaction is believed to occur in two distinct steps:

1. A nuclear particle, such as a proton, neutron, or alpha particle, is hurled at and strikes an atomic nucleus known as the target nucleus. The incident particle and the target nucleus are assumed to combine with each other momentarily to form a "compound" nucleus. Formation of the compound nucleus can be given by the equation:

   Target nucleus + incident particle ⟶ compound nucleus

2. The incident particle transfers its kinetic energy and mass defect[1] (or binding energy) to the compound nucleus. This excess energy causes the compound nucleus to become unstable, whereupon it breaks up into products of the reaction. The lifetimes of compound nuclei are very short - typically on the order of 10^{-12} to 10^{-14} seconds. A given compound nucleus may disintegrate in several different manners. The way in which it does disintegrate depends strongly upon the kinetic energy of the incident particle. The higher the kinetic energy of the incident particle, the more unstable the compound nucleus becomes, and hence the greater are the number of variations in its mode of decay. In any case, the products of decay usually include a product nucleus and one or more ejected particles. This second step in the reaction can thus be given by the equation:

   Compound nucleus ⟶ product nucleus + ejected particles

ATOMIC PROJECTILES

There are several common projectiles which are used to produce nuclear reactions. These are listed in Table 6-I.

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[1] Transfer of the mass defect is difficult to visualize. Remember that a particle weighs less when combined in a nucleus than it does when it is separated. Therefore, once the incident particle is captured, it possesses some mass that it no longer needs. This mass is converted to energy and will be emitted later, but at the instant of formation the compound nucleus contains this extra mass (or its equivalent in energy).

TABLE 6-I: COMMON NUCLEAR PROJECTILES

Particle	Symbol
Proton	p or $_1H^1$
Deuteron	d or $_1H^2$ or $_1D^2$
Alpha particle	α or $_2He^4$
Neutron	n or $_0n^1$
Gamma ray	γ or $_0\gamma^0$

The deuteron is the nucleus of a deuterium (heavy hydrogen) atom and consists of a proton and a neutron. Chapter 7 will be concerned with neutron induced reactions. Reactions induced by the other projectiles will be considered in the following sections of this chapter.

PROTON BOMBARDMENT

A typical proton induced reaction is the bombardment of fluorine.

$$_9F^{19} + {}_1H^1 \longrightarrow \left[{}_{10}Ne^{20}\right] \longrightarrow {}_8O^{16} + {}_2He^4 \qquad (6\text{-}1)$$

In this reaction the compound nucleus is $_{10}Ne^{20}$. Under normal circumstances, the Ne-20 isotope is highly stable and accounts for ~ 91% of all naturally occurring neon. However, this artificially produced neon nucleus contains the kinetic energy and mass defect of the proton, and this is sufficient to cause it to disintegrate into the products of the reaction. The brackets around the neon nucleus are included to indicate that it is highly unstable and exists for only a moment. The lifetime of the compound nucleus is so short that in practice its existance is seldom noticed. Therefore it is customary to write the reaction with the compound nucleus omitted, and to simply remember that in reality it does exist as an intermediate step. In other words, the previous reaction would normally be written as:

$$_9F^{19} + {}_1H^1 \longrightarrow {}_8O^{16} + {}_2He^4 \qquad (6\text{-}2)$$

Notice that this reaction is balanced in the same way as were the radioactive decay reactions previously discussed. That is, the mass numbers (superscripts) and charges (subscripts) balance on both sides of the equation.

There is a shorthand notation which is frequently used when writing reactions of this type. In this shorthand system, the previous equation would be written as:

$$F^{19}\ (p,\ \alpha)\ O^{16} \qquad (6\text{-}3)$$

Inside the brackets, the symbol for the incident particle is placed on the left and is separated from the symbol for the ejected particle by a comma. The symbols outside the brackets indicate the target and product nuclei in the reaction. In speaking of this process we would say that "fluorine-19 undergoes a proton-alpha reaction to form oxygen-16."

To illustrate the variety of products which can be obtained from the same reactants, consider the bombardment of copper with protons. The following reactions have been observed:

$$ {}_{29}Cu^{63} + {}_{1}H^{1} \longrightarrow {}_{30}Zn^{63} + {}_{0}n^{1} \qquad (6\text{-}4)$$

$$ {}_{29}Cu^{63} + {}_{1}H^{1} \longrightarrow {}_{30}Zn^{62} + 2{}_{0}n^{1} \qquad (6\text{-}5)$$

$$ {}_{29}Cu^{63} + {}_{1}H^{1} \longrightarrow {}_{29}Cu^{62} + {}_{1}H^{1} + {}_{0}n^{1} \qquad (6\text{-}6)$$

$$ {}_{29}Cu^{63} + {}_{1}H^{1} \longrightarrow {}_{29}Cu^{61} + {}_{1}H^{1} + 2{}_{0}n^{1} \qquad (6\text{-}7)$$

As might be expected, the simpler reactions are dominant with low proton kinetic energies and the more complex reactions only become important at higher energies. More specifically, the (p, n) reaction is most common for protons of ~ 10 mev, the (p, 2n) and (p, pn) reactions dominate at ~ 25 mev, and the (p, p2n) reaction is most important at ~ 40 mev. Nevertheless, above ~ 15 mev there is a definite probability of any of the four reactions occurring, so the process must be considered to be somewhat statistical in nature. All of the product nuclei in these reactions are radioactive isotopes which do not exist in nature. Several hundred artificial radioisotopes have been produced through reactions of this type. It is of further interest to note that all of the product nuclei are positron emitters (zinc-62 also undergoes K capture). In Chapter 4 it was mentioned that there are no naturally occurring positron emitters.

One final example of proton bombardment is of interest. Consider the reaction:

$$ {}_{6}C^{12} + {}_{1}H^{1} \longrightarrow \left[{}_{7}N^{13} \right] \longrightarrow {}_{7}N^{13} + \gamma \qquad (6\text{-}8)$$

In this case, the compound nucleus, ${}_{7}N^{13}$, does not possess enough excess energy to allow it to disintegrate by emission of a heavy particle, so it simply gives up its excess energy as a gamma ray to produce a more stable form of ${}_{7}N^{13}$. However, even under the most favorable conditions, ${}_{7}N^{13}$ is an unstable isotope and it decays by β^{+} emission to ${}_{6}C^{13}$ with a 10-minute half-life.

ALPHA-PARTICLE BOMBARDMENT

An example of a reaction involving an alpha particle is the following:

$$ {}_{4}Be^{9} + {}_{2}He^{4} \longrightarrow {}_{6}C^{12} + {}_{0}n^{1} \qquad (6\text{-}9)$$

In the shorthand notation this would be:

$$ Be^{9}\ (\alpha,\ n)\ C^{12} \qquad (6\text{-}10)$$

This reaction has historical significance because it led to the discovery of the neutron by Chadwick in 1932. In the original experiments, the neutrons were allowed to fall on blocks of paraffin. In so doing, they collided with the protons in the paraffin (paraffin has a lot of hydrogen in it, and is therefore a good source of loosely bound protons) and knocked them free. Measurements of the energy received by the protons indicated that they must have been struck by a particle with approximately the same mass as the proton. The fact that no ionization was noticed along the track of the new particle indicated that it had zero charge.

DEUTERON BOMBARDMENT

A typical example of deuteron bombardment is given below:

$$ {}_6C^{12} + {}_1H^2 \longrightarrow {}_7N^{13} + {}_0n^1 \qquad (6\text{-}11) $$

In shorthand this becomes:

$$ C^{12}\ (d,n)\ N^{13} \qquad (6\text{-}12) $$

Notice that this is another method of producing the isotope ${}_7N^{13}$. There are quite often a number of alternative methods of producing any given isotope[1].

GAMMA RAY BOMBARDMENT

We have already noted that gamma ray photons possess many properties in common with particles. One of these is the ability of induce nuclear reactions. For example:

$$ {}_{12}Mg^{25} + {}_0\gamma^0 \longrightarrow {}_{11}Na^{24} + {}_1H^1 \qquad (6\text{-}13) $$

which in shorthand becomes:

$$ Mg^{25}\ (\gamma,p)\ Na^{24} \qquad (6\text{-}14) $$

Gamma induced reactions are often called photonuclear reactions.

ENERGY AND MASS CONSIDERATIONS IN NUCLEAR REACTIONS

In any nuclear reaction (or for that matter, in any earthly process) a fundamental rule applies. This rule states that the sum total of mass and energy must remain constant throughout the process. For example, if we have a process by which we create mass, we do so at the expense of an equivalent amount of energy. Applying this rule to nuclear reactions, we see that the total mass and energy on one side of a nuclear equation must equal the total mass and energy on the other side. This rule is very useful in determining the feasibility of nuclear reactions.

Consider the reaction:

$$ {}_7N^{14} + {}_2He^4 \longrightarrow {}_8O^{17} + {}_1H^1 \qquad (6\text{-}15) $$

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[1] Another reaction in which N-13 is produced is: $O^{16}(p,\alpha)N^{13}$. This reaction occurs extensively within the core of water-cooled reactors. As a result, N-13 is a major contributor to the radioactivity found in the condenser off-gas of power plants which use boiling water reactors. This subject will be discussed in greater detail in a later chapter.

From a table of isotopes we find that the atomic weights[1] of the isotopes involved are:

$$_7N^{14} = 14.00752 \text{ amu}$$

$$_2He^4 = 4.00387 \text{ amu}$$

$$_8O^{17} = 17.00453 \text{ amu}$$

$$_1H^1 = 1.00814 \text{ amu}$$

We see in this reaction that the mass of the products is 18.01267 and that of the reactants is 18.01139 amu. Therefore 18.01267 - 18.01139 = 0.00128 amu has been created in this reaction. This represents (0.00128)(931) = 1.19 mev of energy which must be supplied. Consequently, the reaction is not possible unless the alpha particle has a kinetic energy of at least 1.19 mev. In actual practice it takes somewhat more than this, primarily due to the fact that the alpha particle must have enough energy to overcome the electrostatic repulsion forces of the nucleus. For the case in question, the alpha particle requires about 4.4 mev to overcome the electrostatic repulsion forces of the nucleus. Since only 1.19 mev is needed in the reaction, the difference between these values will be given off as gamma rays and kinetic energy of the products. Therefore, gamma rays usually accompany any of the nuclear reactions we have discussed. Usually we leave them out when writing the equation unless they play an important part such as in a photonuclear reaction.

We can also apply the conservation of mass-energy principle to radioactive decay reactions. Consider, for example, the decay of $_{90}Th^{228}$ by alpha emission.

$$_{90}Th^{228} \longrightarrow {}_{88}Ra^{224} + {}_2He^4 \qquad (6\text{-}16)$$

The masses involved are as follows:

$$_{90}Th^{228} = 228.0998 \text{ amu} \qquad {}_{88}Ra^{224} = 224.0900 \text{ amu}$$

$$_2He^4 = 4.0039 \text{ amu}$$

$$\text{Total} = 228.0939 \text{ amu}$$

It is seen that the products of the reaction weigh less than the $_{90}Th^{228}$ nucleus by 0.0059 amu. Therefore there must be a total of (0.0059)(931) = 5.516 mev given off during the reaction. Most of this energy appears as kinetic energy of the alpha particle and some of it appears as gamma rays (a small portion is also given off as kinetic energy of the $_{88}Ra^{224}$ nucleus). The fact that energy is given off in the reaction is not surprising since this has to be true in order for the reaction to proceed spontaneously.

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[1] The nuclear weights, rather than atomic weights, should be used in these calculations since only the nuclei take part in the reaction. However, atomic weights may be used since there must be equal number of electrons on both sides of the equation and their masses will cancel out.

The next question is how this available energy is distributed between the alpha particle and the gamma rays. In order to explain this we must digress for a moment. When we spoke of orbital electrons, we found that their orbits could be more accurately described as energy levels. Thus, when an electron was moved from the K shell to the L shell, this process represented an increase in the energy of the electron. If such a process took place, the atom was said to be excited. Although the atom can exist in an excited state for a moment, eventually it will try to return again to its lowest energy state (termed the ground state). When this occurs, the excited electron will drop back into the K shell and the excess energy it possessed will be emitted as a characteristic X-ray. It now appears that the particles in the nucleus also exist in certain distinct energy shells. Thus it is possible to excite a nucleus by giving it energy. Eventually, however, the nucleus will also return to its ground state and any excess energy it possessed will be emitted as gamma rays.

Now let us return to our example. We saw that there was 5.516 mev given off during the reaction. Of this, 0.095 mev is given to the $_{88}Ra^{224}$ nucleus, leaving 5.421 mev to be divided up between the alpha particle and the gamma rays. We might expect that the proportion of the total energy taken by each of these particles would be quite arbitrary and that $_{90}Th^{228}$ would thus emit alpha particles with a continuous spectrum of energies ranging from 0 to 5.421 mev. Experimentally, however, 72% of the alpha particles have an energy of 5.421 mev (in which case there is no γ ray accompanying the reaction) and 28% of the alpha particles have an energy of 5.333 mev (in which case a 0.084 mev gamma ray accompanies the reaction). The explanation for this is that in the case of the 5.421 mev alpha particle, the decay reaction proceeds directly to the ground state of $_{88}Ra^{224}$. In the case of the 5.338 mev alpha particle, however, the $_{90}Th^{228}$ nucleus has decayed to an excited state of $_{88}Ra^{224}$. The 0.084 mev gamma ray must be given off by the excited $_{88}Ra^{224}$ nucleus as it goes to its ground state. Thus $_{88}Ra^{224}$ must have an excited state that represents 0.084 mev of energy above its ground state. Careful scrutiny of the decay reaction indicates that $_{88}Ra^{224}$ actually has several excited states. If we consider the ground state of $_{88}Ra^{224}$ as having 0.000 mev excess energy, we can show its excited states on a plot such as Figure 6-1. This figure shows that the decay of the $_{90}Th^{228}$ is actually quite complex, although 99% of the decays take place by one of the two modes previously discussed. This explains why alpha particles and gamma rays of a few discrete energies are obtained from the decay of an alpha emitter. This fact is very useful in the identification of an unknown alpha emitter. To identify the isotope, it is only necessary to place the unknown sample in a detector which will measure the energy of the radiations emitted and then consult reference books which contain information such as that presented in Figure 6-1 (this is usually not as easy as it sounds because there are a great many isotopes to choose from and the results are often clouded by experimental error).

Similar considerations hold for reactions involving β emitters. An energy level diagram for the decay of $_{11}Na^{24}$ (sodium) to $_{12}Mg^{24}$ by β^- emission is given in Figure 6-2. Based on our previous arguments we would expect to find

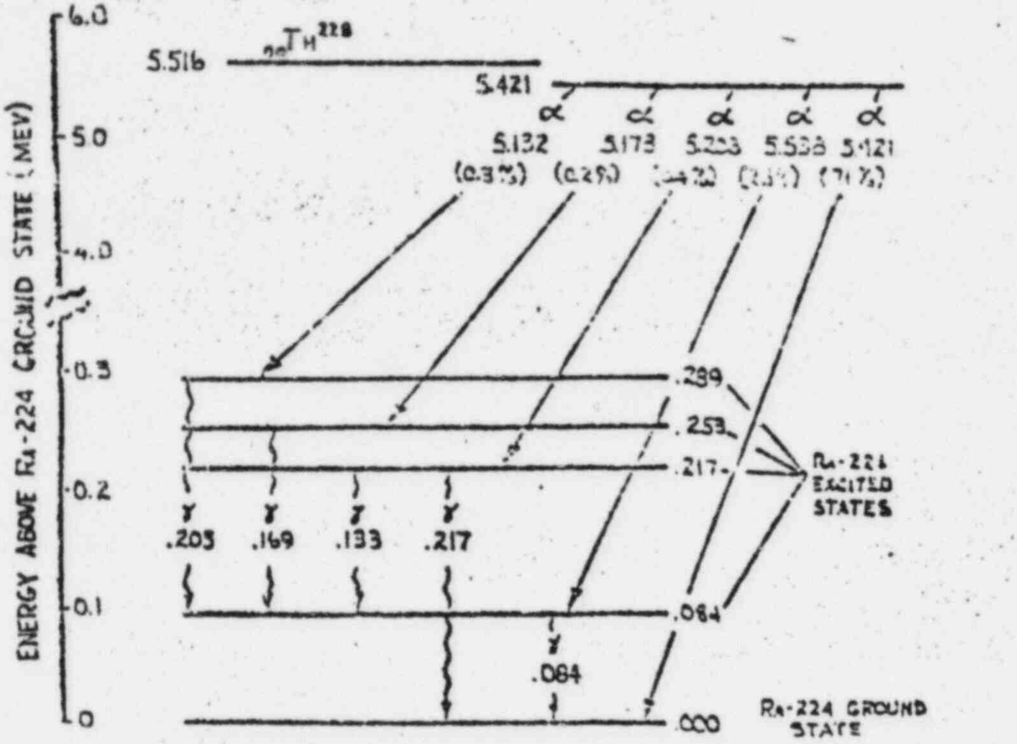

Figure 6-1: Energy-level Diagram for $_{88}Ra^{224}$ as Obtained from the Observed α and γ Radiation of $_{90}Th^{224}$

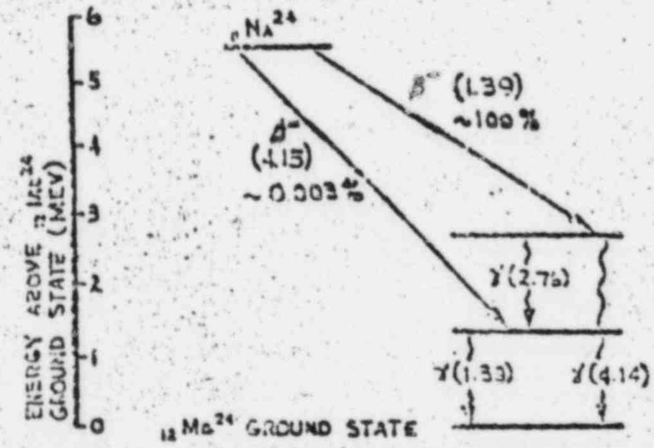

Figure 6-2: Decay Scheme of $_{11}Na^{24}$

$_{11}Na^{24}$ emitting β^- particles with an energy of 1.39 mev (neglecting the rare 4.15 mev β^- particle). However, we actually find a continuous spectrum of β^- energies ranging from 0 to a maximum of 1.39 mev. What is the reason for this? The answer is that the antineutrino carries away some of the β^- particle's energy. Thus in this example the sum of the kinetic energies of the β^- particle and the antineutrino must equal 1.39 mev. However, in any single decay, there is no way of knowing how this energy will be split up between the two particles.

We can see that observation of the β^- energies from a given isotope is of little use in identification of the isotope because the β^- particle may have any energy up to a certain maximum. However, isotope identification may be made on the basis of the gamma ray energies since these have discrete values. In most cases, alpha emitter identification is also based on the gamma energies rather than the alpha energies. This is primarily because the exceedingly short range of alpha particles in most substances complicates the necessary experimental techniques. Identification by use of γ's also has economic advantages because the same apparatus can be used for both α and β emitters. The process of identifying radioactive isotopes by observing their γ ray energies is called *gamma ray spectrometry*. A gamma ray spectrometer measures the intensity of the gamma radiations from a source at different energy levels. The data thus obtained is usually plotted as shown in Figure 6-3, which is drawn for Na-24.

We see from the figure that peaks in the gamma ray intensity occur at the three energies corresponding to the gamma rays emitted in the decay process.

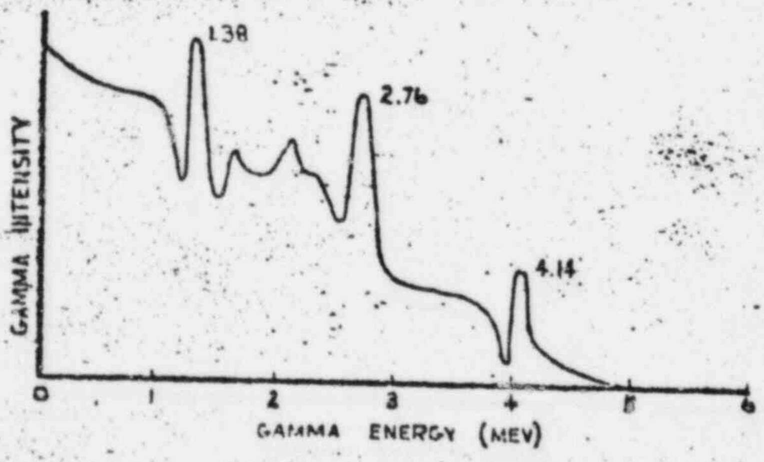

Figure 6-3: Gamma Ray Spectrum for $_{11}Na^{24}$

1. Fill in the blanks in the following reactions:

   a) ${}_{15}P^{31}$ + ____ $\longrightarrow$ ${}_{15}P^{32}$ + ${}_{1}H^{1}$

   b) ${}_{28}Ni^{58}$ + ____ $\longrightarrow$ ${}_{29}Cu^{58}$ + ${}_{0}n^{1}$

   c) ${}_{11}Na^{23}$ + ____ $\longrightarrow$ ${}_{13}Al^{26}$ + ${}_{0}n^{1}$

   d) ${}_{15}P^{30}$ + γ $\longrightarrow$ ____ + ${}_{0}n^{1}$

   e) ${}_{18}A^{40}$ + ${}_{2}He^{4}$ $\longrightarrow$ ${}_{20}Ca^{43}$ + ____

2. Plutonium-239 undergoes α decay by the following reaction:

$${}_{94}Pu^{239} \longrightarrow {}_{92}U^{235} + \alpha + \gamma$$

The masses involved are: Pu-239 = 239.1265 amu, U-235 = 235.1170 amu, α = 4.0039 amu. The U-235 nucleus is known to receive 0.067 mev of kinetic energy in the reaction. Studies have shown that alpha particles of three discrete energies are emitted in the reaction. Furthermore, in 11% of the disintegrations a gamma ray of 0.051 mev is emitted, in 17% of the disintegrations a gamma ray of 0.013 mev is emitted, and in 72% of the disintegrations there is no gamma ray. What are the three alpha particle energies? Draw an energy level diagram similar to Figure 6-1 which shows the excited states of U-235.

3. Why aren't β emitters ordinarily identified by measuring the energies of the β particles they emit?

CHAPTER 7

NEUTRON BEHAVIOR

INTRODUCTION

The neutron is particularly well suited to cause nuclear reactions because it is not affected by the charge on the nucleus. Even neutrons of very low kinetic energy can penetrate the nucleus, and as a result, neutron induced reactions are considerably more common than the other types discussed in Chapter 6. In fact, it is the ability of the neutron to cause the fission reaction in U-235 which has led to the development of nuclear power as it exists today. Therefore, because of the great practical importance of the neutron and its reactions, we will consider these topics in some detail in this chapter.

NEUTRON SOURCES

Before studying neutron reactions, let us briefly consider some of the methods used to produce neutrons.

1. Neutrons can be produced by bombarding certain light elements, like beryllium and lithium, with alpha particles from an alpha emitter like radium, polonium, or plutonium. For Example:

$$ {}_4Be^9 + {}_2He^4 \longrightarrow {}_6C^{12} + {}_0n^1 \qquad (7\text{-}1)$$

2. Neutrons can be produced in several photonuclear reactions. For example:

$$ {}_4Be^9 + {}_0\gamma^0 \longrightarrow {}_4Be^8 + {}_0n^1 \qquad (7\text{-}2)$$

The isotope ${}_4Be^8$ disintegrates immediately into two alpha particles, so the overall reaction can be written as:

$$ {}_4Be^9 + {}_0\gamma^0 \longrightarrow 2\ {}_2He^4 + {}_0n^1 \qquad (7\text{-}3)$$

This reaction is actually used in the neutron sources for many nuclear reactors. Because of the importance of this reaction, it is worth our while to determine the minimum gamma ray energy which will allow the reaction to occur. The masses involved are:

$${}_4Be^9 = 9.01504 \text{ amu}$$

$$\begin{aligned} {}_2He^4 &= 4.00387 \text{ amu} \\ {}_2He^4 &= 4.00387 \text{ amu} \\ {}_0n^1 &= 1.00897 \text{ amu} \\ \hline \text{Total} &= 9.01671 \text{ amu} \end{aligned}$$

As can be seen, the products of this reaction weigh more than the original ${}_4Be^9$ atom by 0.00167 amu. Since this figure represents mass which was created in the reaction, the gamma ray must have sufficient energy to supply this mass. Therefore, the minimum gamma ray energy is (0.00167)(931) = 1.56 mev. A 1.56 mev gamma ray would allow the reaction to occur, but would not have any energy left over to give to the ${}_2He^4$ and ${}_0n^1$ products as kinetic energy. It can be shown that the products of the reaction must have a small amount of kinetic energy to satisfy certain theoretical requirements. Therefore the minimum gamma ray energy is slightly higher than the value we calculated. The actual figure is about 1.6 mev.

3. Neutron beams can be produced by bombarding certain target materials with high energy charged particles in various accelerators such as the cyclotron.

4. Neutrons are produced in great numbers in all nuclear reactors. We will, of course, devote much of the remainder of this manual to the study of nuclear reactors.

NEUTRON REACTIONS

There is really nothing about neutron reactions that is different from the things we learned about nuclear reactions in Chapter 6. The reactions are written and balanced in the same way, there is a compound nucleus formed as an intermediate step, and the conservation of mass-energy holds. The only reason we are discussing neutron reactions separately is because of their particular importance in the nuclear power field. There are several important types of reactions brought about by neutrons. These include:

1. Radiative capture (n, γ) in which the excited compound nucleus emits its excess energy as gamma radiation. For example:

$$ {}_{49}In^{115} + {}_0n^1 \longrightarrow \left[{}_{49}In^{116} \right] \longrightarrow {}_{49}In^{116} + {}_0\gamma^0 \quad (7\text{-}4) $$

In shorthand, this would be: In^{115} (n, γ) In^{116}. We have written the reaction as though only one gamma ray is emitted. In general, however, the excited nucleus drops to intermediate excited states on its way to the ground state, and will emit a series of gamma rays which add up to the total available excitation energy. In many cases the product nucleus is neutron heavy, and so decays by β^- emission (usually accompanied by γ's). For the example cited, indium-116 decays by β^- emission with a 54 minute half-life.

Radiative capture reactions are very common and most of the processes whereby radioactive isotopes are produced for experimental research or industrial uses involve this type of reaction. A stable target element is inserted into a nuclear reactor and is bombarded with neutrons for a period of time, thus producing the desired radioisotopes. A nuclear reactor especially designed for radioisotope production is located in Oak Ridge, Tennessee. Radiative capture reactions also play an important part in nuclear reactor operation because they are involved in breeding and fission product poisoning among other things. And finally, in designing shielding it is necessary to compute the rate of generation of capture gammas since they often are the chief source of biological hazard during reactor operation.

2. Charged particle emission [(n,p), (n,α), etc.] in which the compound nucleus decays by emission of a charged particle such as a proton or alpha particle. These reactions are rare with low energy neutrons because expulsion of a charged particle usually requires a considerable amount of energy. As neutron energy is increased, however, they become more 'probable.' One such reaction which does occur with low energy neutrons is:

$$ {}_5B^{10} + {}_0n^1 \longrightarrow {}_3Li^7 + {}_2He^4 \qquad (7\text{-}5)$$

This is an extremely important one because, as we shall see in later chapters, it provides the basis for the detection of slow neutrons and also is an important reaction for the control of nuclear reactors.

3. Fission in which the nucleus of a fissionable material such as U-235 absorbs a neutron and splits into two large fragments. The products of this reaction, in addition to the two fission fragments, are two or three new neutrons plus a considerable quantity of energy. Because of its particular importance, the fission reaction will be discussed separately in Chapter 8.

The three types of reactions discussed above are often grouped together under the general heading of absorption (or capture) reactions. The reasons for this is readily apparent since in each type the incident neutron is absorbed by the target nucleus and other particles leave the resultant compound nucleus.

There is a second important type of neutron reaction known as scattering. In this reaction, the neutron can be thought of as colliding with the target nucleus and rebounding with less energy (speed) than it had before the collision. The reduction in neutron energy appears as added motion of the target nucleus. The amount of energy lost by the neutron depends upon two things: the mass of the target nucleus and the angle at which the neutron strikes the nucleus. These facts can be easily visualized by considering the neutron to be a billiard ball. It is well known that if one billiard ball suffers a head-on collision with another billiard ball of equal mass, the incident ball will come to a dead stop and the second ball will roll away with the same velocity as the incident ball originally had. In other words, the incident ball will transfer its entire kinetic energy to the ball it strikes if the two balls are the same size. Now consider a billiard ball suffering a head-on collision with a ball of much greater mass. An example of this would be dropping a billiard ball onto the sidewalk (i.e. the second ball is the earth). In this case, the billiard ball will rebound with essentially the same velocity it had when it struck the sidewalk, indicating that it suffered no appreciable loss of energy in the collision. We can generalize this idea and say that a neutron is capable of losing a large portion of its energy if it scatters off a particle of nearly equal mass (a proton, for example) but can only lose a small portion of its energy if it scatters off a much larger body (an uranium nucleus, for example). The effect of the angle of collision can also be illustrated by considering the collision between two identical billiard balls. If the collision is a glancing blow, the incident ball will continue to move, but with a reduced velocity. In general, it can be stated that a neutron will lose more of its energy in a head-on collision than in a glancing blow collision. It is easy to see that scattering can produce a wide variety of neutron energies, depending upon the mass number of the scattering nuclei, and the initial energies and scattering angles of the incident neutrons.

The "billiard ball" type of scattering discussed above is more properly called elastic scattering. There is also a second type of scattering known as inelastic scattering. Inelastic scattering is really an (n,n) reaction, in which a neutron is absorbed by a target nucleus to form a compound nucleus which then disintegrates by emitting a neutron of lower energy. The bulk of the energy lost by the incident neutron appears as excitation energy of the target nucleus. This energy is subsequently emitted by the target nucleus as gamma rays. Elastic scattering is the most important type for light scattering nuclei and for neutrons of low energy. With heavy elements and high neutron energies, inelastic scattering becomes important. However, both types of scattering produce the same end result, which is the reduction of the neutron energy. Therefore, in future discussions we will not differentiate between the two mechanisms.

CROSS SECTION

The designer of a nuclear facility must know not only what reactions are possible when neutrons strike a particular material, but also to what extent these reactions will occur. To provide this latter information, the concept of cross section was devised. The cross section for a given nucleus can be defined as being a measure of the probability that the nucleus will undergo a nuclear reaction with a neutron. The greater the cross section of a nucleus, the greater the probability it will react with a neutron. The unit of cross section is the barn (abbreviated b). The term barn may seem strange. It arose when scientists said that a nucleus with a high probability of undergoing a reaction was as easy to hit with a neutron as the broad side of a barn.

One might expect that the probability that a neutron will undergo a reaction with a nucleus would be dependent upon the size, or cross sectional area, of the nucleus. That is, the larger the target area that a nucleus presents, the greater the chances that a neutron will hit it and cause a reaction. Hence, the adoption of the term cross section to describe this probability. However, it has been found that the actual cross sectional area of a nucleus is of secondary importance in the determination of its cross section for neutron reactions. Of far greater importance are quantities such as the energies of the excited states that a nucleus may occupy. Thus it is that certain nuclei have such an affinity for neutrons that they seem to attract them much as a magnet attracts iron. These nuclei have cross sections millions of times greater than their cross sectional areas would suggest. Conversely, other nuclei seem almost to repel neutrons, and have cross sections much smaller than their actual areas would suggest. However, since the probability of reaction does bear a small relationship to actual physical size, the unit of the cross section should be related to the approximate cross sectional area of a nucleus. Since cross sectional area is related to the square of the nuclear radius ($A = \pi r^2$ for a circle), and since the nuclear radius is on the order of 10^{-12} cm, the barn was defined as being 10^{-24} cm^2.

Suppose we have two atoms, A and B. Suppose further that we know that atom A has a cross section of 5 barns and atom B has a cross section of twenty barns. If we shoot a neutron at each of these atoms, what are the chances that a reaction will occur with atom B as opposed to a reaction occurring with atom A? The answer is that the chances of a reaction occurring with atom B are four times as great as the chances of a reaction occurring with atom A. We conclude this because we know that the cross section for atom B is four times as large as the cross section for atom A.

The concept of cross section has far greater usefulness than the simple problem above would suggest. We will show later in this chapter how the cross section can be used to determine the precise reaction rate when many neutrons strike a target containing a large number of target nuclei.

Although the probability that a neutron will react with a nucleus is of interest, it is often even more important to know what type of reaction it will undergo. Therefore, the total cross section is usually broken down into its two major component parts. That is, we say that the total cross section is the sum of the absorption cross section and the scattering cross section. We can break things down still further and say that the absorption cross section is the sum of the radiative capture, charged particle emission, and fission cross sections. Now we not only know the probability that a neutron will undergo a reaction with a nucleus, but also the probability of occurrence of each particular type of reaction.

So far we have been speaking strictly of the cross section for an individual nucleus. This cross section is more correctly called the <u>microscopic cross section</u> and is given the symbol σ. In the real world, however, we are never concerned with the reaction between a neutron and a single nucleus, but are concerned with the reactions between a number of neutrons and a target containing billions of nuclei. The cross section for a cubic centimeter of such a target is termed the <u>macroscopic cross section</u>, Σ, and is the product of the microscopic cross section and the total number of nuclei in one cubic centimeter of the target. This is:

$$\Sigma = N\sigma \qquad (7\text{-}6)$$

where: N = total number of target nuclei/cm^3 of target
σ = microscopic cross section (barns/nucleus)
Σ = macroscopic cross section (barns/cm^3 or cm^{-1})

By subdividing the total cross section into its component parts, we obtain the relationship:

$$\Sigma_t = \Sigma_s + \Sigma_{n,\gamma} + \Sigma_{cpe} + \Sigma_f \qquad (7\text{-}7)$$

where: Σ_t = total macroscopic cross section

Σ_s = macroscopic scattering cross section

$\Sigma_{n,\gamma}$ = macroscopic radiative capture cross section

Σ_{cpe} = macroscopic charged particle emission cross section

Σ_f = macroscopic fission cross section

The total macroscopic absorption cross section, Σ_a, would be given by:

$$\Sigma_a = \Sigma_{n,\gamma} + \Sigma_{cpe} + \Sigma_f \qquad (7\text{-}8)$$

Equations (7-7) and (7-8) can also be written in terms of microscopic cross sections if desired. For example, in terms of the microscopic cross section, equation (7-7) becomes:

$$\sigma_t = \sigma_s + \sigma_{n,\gamma} + \sigma_{cpe} + \sigma_f \qquad (7\text{-}9)$$

where all subscripts are consistent with those previously used.

The calculation of cross sections is exceedingly difficult and the values used are based primarily upon experimental data. Almost every book on the subject of nuclear energy contains tables of cross section data for various isotopes. The tabulated values are generally the microscopic cross sections since these are independent of the quantity of material and depend only upon the isotope in question.

DEPENDENCE OF CROSS SECTIONS UPON NEUTRON ENERGY

The magnitude of the microscopic cross section of an atom depends not only upon the particular isotope in question, but also upon the energy of the neutrons involved. We will first consider the energy dependence of the absorption cross section, and then the behavior of the scattering cross section.

For many elements, especially those of mass number exceeding 100, an examination of the variation of the absorption cross sections with energy reveals the existence of three regions. There is first a low-energy region below about 0.1 ev where the absorption cross section decreases steadily with increasing neutron energy (or velocity). In this region, the cross section is inversely proportional to the neutron velocity, v. The region is therefore called the 1/v region and the cross sections are said to obey the 1/v law. Physically, this behavior may be explained by the idea that the chance that a slow neutron will be captured depends, among other things, upon the length of time that the neutron spends in the vicinity of the nucleus. Thus, the higher the velocity of the neutron, the more quickly it passes out of the range of influence of the nucleus and the smaller its chances of being absorbed.

Following the 1/v region for slow neutrons, the elements under consideration exhibit a resonance region, usually for neutrons of roughly 0.1 to 1000 ev energy. This region is characterized by the occurrence of peaks where the absorption cross section rises fairly sharply to high values for certain neutron energies and then falls again. Most isotopes exhibit several resonance peaks, although the peaks are generally more numerous, and more important, for the larger elements.

Resonance absorption may be illustrated by reference to Figure 7-1. We noted in Chapter 6 that a nucleus could exist for short periods of time in any of several excited energy states before decaying back to the ground state. The process of adding a neutron with zero kinetic energy to a target nucleus is shown in Figure 7-1 by the arrow labeled "Neutron Capture with Zero Kinetic Energy." For a moment, the compound nucleus contains the entire mass of the neutron (or its energy equivalent of 931 mev) and the energy of the system can be thought of as being raised to the point marked E_0. Of course, once the neutron is combined, it will lose some of its mass, as discussed in Chapter 5, and this excess mass will be emitted as energy by the compound nucleus as it decays to its ground state. This process is shown

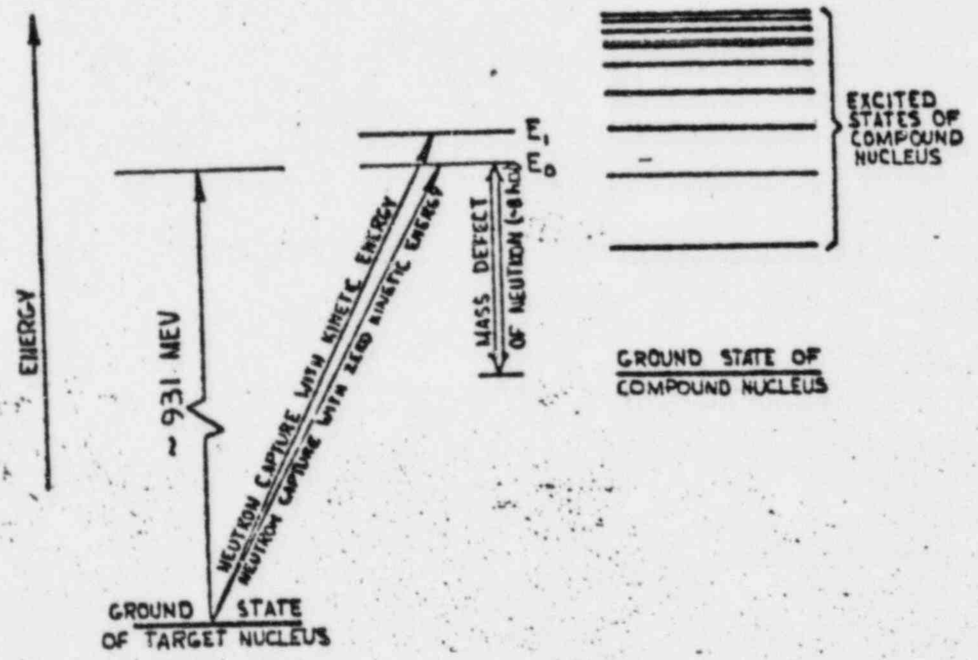

Figure 7-1: Compound Nucleus Formation and Nuclear Energy Levels

by the arrow labeled "Mass Defect of the Neutron." Now suppose that the neutron had, in addition to its mass, enough kinetic energy to just bring the energy of the system up to E_1 instead of E_0. Notice that E_1 is equal to the energy of one of the compound nucleus' excited states whereas E_0 was not. When the kinetic energy of the neutron is just at a value which will allow a capture reaction to go directly to an excited state of the compound nucleus, the probability of the reaction occurring is greatly enhanced. Thus, a peak in the cross section occurs at this neutron energy and we have a resonance. Notice that there will be other resonances for those neutron energies that create a compound nucleus in some of its other excited states.

Beyond the resonance region, the absorption cross sections decrease steadily with increasing neutron energy and are usually insignificant compared to the scattering cross sections at energies above a few mev. With the exception of the resonance region, we noted a general decrease in the absorption cross section as energy increased. Once again, this can be attributed to the fact that faster neutrons spend less time in the vicinity of a nucleus and therefore have less opportunity to react with it. As an illustration of our discussion on absorption cross sections, Figure 7-2 shows a plot of σ_a versus energy for indium.

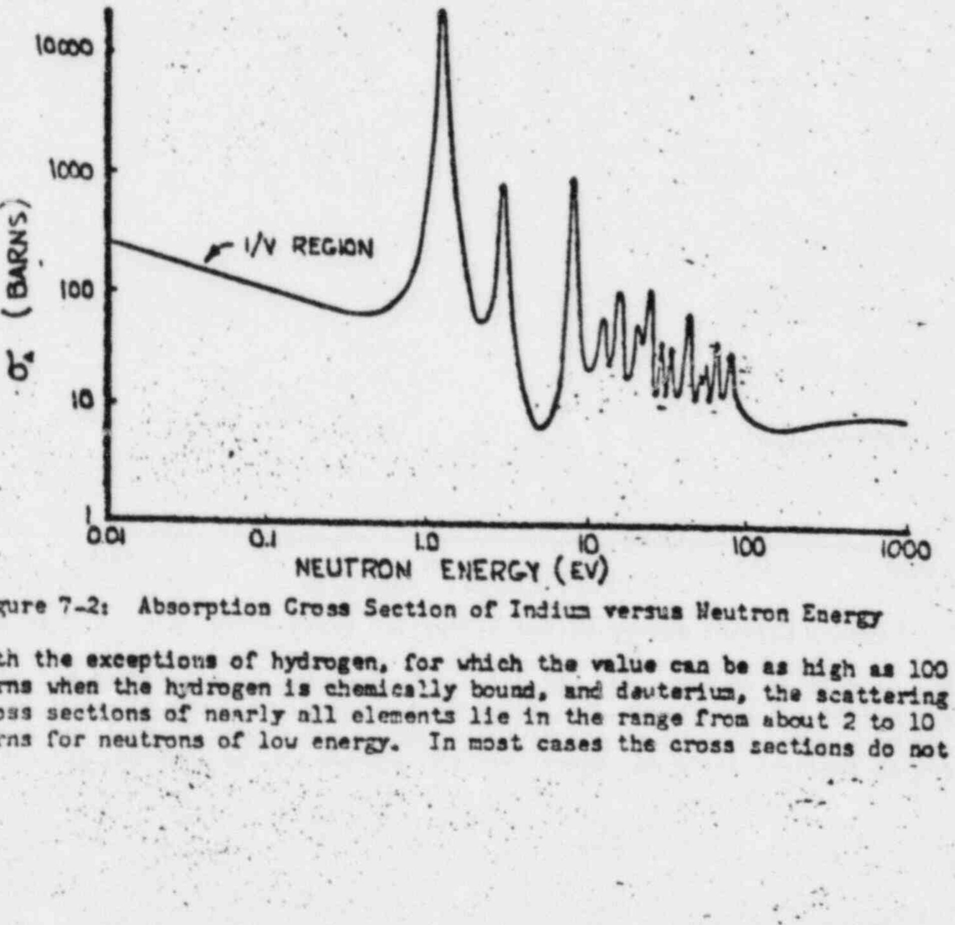

Figure 7-2: Absorption Cross Section of Indium versus Neutron Energy

With the exceptions of hydrogen, for which the value can be as high as 100 barns when the hydrogen is chemically bound, and deuterium, the scattering cross sections of nearly all elements lie in the range from about 2 to 10 barns for neutrons of low energy. In most cases the cross sections do not vary markedly with neutron energy, although there is a general tendency for the values to decrease for neutrons of high energy.

CLASSIFICATION OF NEUTRON ENERGIES

In nuclear work, there are several terms which are commonly used to classify neutron energies. Neutrons with energies below ~ 1 ev are referred to as slow neutrons. This category essentially covers the 1/v region of absorption cross sections. Neutrons with energies in the range of ~ 1 ev to ~ 10,000 ev, which essentially covers the resonance region, are called intermediate neutrons. Finally, neutrons with energies above ~ 10,000 ev (0.01 mev) are called fast neutrons. There is no upper limit on the fast region, but from a practical standpoint, neutrons in the reactor rarely exceed 10 mev.

Thermal neutrons are slow neutrons which are in temperature equilibrium with their surroundings. This definition requires some explanation. In our discussion of scattering we tacitly assumed that the target nuclei were at rest. Actually, the atoms in any target will have some vibrational kinetic energy, and this energy will be determined primarily by the temperature of the target. The higher the temperature, the higher will be the kinetic energy of the target nuclei. Now consider a collision between a neutron and a nucleus where the kinetic energy of the neutron is less than the kinetic energy of the nucleus. In this case, the nucleus will transfer some of its energy to the neutron, with the result that the neutron will gain energy in the collision. On the average the neutrons can never slow down to where they have energies less than the energies of the target nuclei, for if they do, they will begin to gain energy in scattering collisions and will continue to do so until equilibrium is reached. Therefore, at the completion of the slowing down process, the neutrons will also be in temperature equilibrium with the surroundings. Hence the term thermal neutrons. Neutrons with energies just above the thermal region, i.e., in the upper range of the slow region, are often called epithermal neutrons.

Tables of cross section data commonly list the thermal neutron cross sections. It has been standard practice to base these thermal cross sections on neutrons having an energy of 0.025 ev, which corresponds to a neutron velocity of 2200 meters/sec and an ambient temperature of ~ 72°F. Actually, water cooled power reactors operate with coolant temperatures of ~ 550°F. As we shall see in Chapter 9, the coolant is also used to slow neutrons down. Thus the true thermal energy in a power reactor will correspond to a temperature of ~ 550°F, which is ~ .047 ev.

NEUTRON FLUX

The neutron flux, ϕ, is defined as the total distance traveled by all of the neutrons in one cubic centimeter of a material in one second. In one second, an individual neutron will travel a distance equal in magnitude to its velocity, v. That is, if a neutron is traveling 220000 cm/sec, it will travel 220000 cm in each second. The total track length of all the neutrons in a cubic centimeter of material will simply be the product of the neutron velocity, and the total number of neutrons in this volume. That is,

$$\phi = n\,v \qquad (7\text{-}10)$$

where: ϕ = neutron flux (neutrons/cm^2-sec)

n = neutron density (neutrons/cm^3)

v = neutron velocity (cm/sec)

From a practical standpoint, neutron flux is usually taken simply to be a measure of neutron population density. That is, a region of high neutron flux is ordinarily taken to mean a region of high neutron density. This interpretation is strictly correct, however, only if we specify a neutron velocity. For example, we might be discussing the thermal neutron flux (i.e. $v \approx 220000$ cm/sec). With all neutrons having essentially constant velocity, the v term in equation 7-10 is a constant, and ϕ is directly proportional to n.

NEUTRON REACTION RATE

The true usefulness of neutron flux and cross sections comes in the calculation of the reaction rate between a group of neutrons and a mass of target material. The rigorous mathematical derivation of the reaction rate expression is beyond the scope of this manual, but intuition will lead us to the correct result with little difficulty.

It seems reasonable that the reaction rate between neutrons and target atoms should be dependent upon their collision rate. The collision rate in a target is dependent upon the total travel distance (per second) of neutrons in the target. To see this, assume that target atoms are spaced, on the average, 2 centimeters apart[1]. If a neutron travels 100 centimeters in a second (i.e. v = 100 cm/sec), it could be expected to suffer 50 collisons per second. Similarly, if it traveled 200 cm/sec, it should undergo 100 collisions/sec. Thus, the collision rate should be directly proportional to the neutron flux.

The example above also indicated that collision rate is also dependent upon target atom spacing. A good measure of target atom spacing is the target atom density, N (target atoms/cm^3). The higher the density, the more closely are the atoms spaced, and the higher the collision rate for any given neutron flux. Thus collision rate should be related to the product ϕN.

The collision rate, however, does not necessarily equal the reaction rate. To obtain this, we multiply the collision rate by the probability that a collision will produce a reaction, which is measured by σ. Thus, the reaction rate for neutrons in a cubic centimeter of target material is given

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[1] The distances here are for illustrative purposes only. Except for gaseous media at extremely low pressures, the target atom spacing would be much smaller than this.

by:

$$RR\ (\text{reactions/cm}^3\text{-sec}) = \phi N \sigma = \phi \Sigma \qquad (7\text{-}11)$$

Consistent units require that σ be expressed in cm^2 rather than barns in this equation.

If the target contains more than one cubic centimeter of material, which is the usual case, then the total reaction rate is equal to the reaction rate per cubic centimeter times the target volume, V, in cubic centimeters.

$$TRR\ (\text{reactions/sec}) = \phi \Sigma V \qquad (7\text{-}12)$$

If we are interested in only one type of reaction, say fission, then we would use the appropriate fission cross sections in equations 7-11 and 7-12.

Example: The thermal neutron flux in a block of graphite is 5×10^{12} neuts/cm^2-sec. The atomic density of the graphite is 8×10^{22} atoms/cc. The microscopic absorption cross section for carbon is 4×10^{-3} barns. What is the neutron absorption rate per cubic centimeter of target material?

$$RR = \phi N \sigma_a$$

$$= (5 \times 10^{12})(8 \times 10^{22})(4 \times 10^{-27}) = 1.6 \times 10^{9} \text{ absorptions/cc-sec}$$

The only thing to be careful about in this problem is that you convert barns to cm^2 ($1\ b = 10^{-24}\ cm^2$).

PROBLEMS

1. The iron (chemical symbol = Fe) isotope with Z=26, A=54 is known to undergo a radiative capture reaction. Write the equation for this reaction.

2. The aluminum isotope, $_{13}Al^{27}$, undergoes a charged particle emission reaction to the sodium isotope, $_{11}Na^{24}$. Write the equation for this reaction.

3. What is the daughter product of the (n,p) reaction with $_{12}Mg^{24}$? Compare this result with question 2.

4. Define microscopic cross section, absorption reaction, scattering reaction, resonance, thermal neutron.

5. Bursts of 10^{8} neutrons are directed against two equally sized targets, A and B. In target A there are $3 \times 10^{\text{[illegible]}}$ atoms, with $\sigma_a = 6.1$ barns. In target B there are $4.5 \times 10^{\text{[illegible]}}$ atoms, with $\sigma_a = 4.1$ barns. Which target will absorb the greatest number of neutrons? Which type of atom has the best chance of absorbing a neutron?

6. What is meant by the statement that B-10 is a 1/v absorber?

7. Under what conditions is it likely that a target nucleus will exhibit a resonance in its σ_a?

8. In general, how do scattering cross sections vary with neutron energy?

9. What are the conditions which will result in the neutron losing a large amount of energy in a single scattering collision?

10. Discuss the physical significance of neutron flux in terms of neutron reaction rates and neutron population.

11. A large nuclear reactor produces heat by the fission of U-235 ($\sigma_{f,\,thermal} = 580$ b). There are 1.6×10^{20} atoms/cc of U-235 in the core, and the average thermal neutron flux in the core is 3×10^{13} n/cm^2-sec. The core can be considered to be a cylinder 12 feet long and 10 feet in diameter. It is known that it requires 3×10^{10} fissions/sec to produce 1 watt of power. At what power level (in megawatts) is the reactor operating. Also note that 1 inch = 2.54 centimeters.

CHAPTER 8

NUCLEAR FISSION

INTRODUCTION

Shortly after the discovery of the neutron, Enrico Fermi in Italy bombarded uranium with neutrons in an effort to produce elements with atomic numbers greater than 92, the so-called transuranium elements. His theory was that the neutron bombardment would produce (n,γ) reactions in the uranium, and the neutron heavy uranium nuclei thus produced would then decay by β^- emission to the higher elements. These attempts eventually succeeded, but interpretation of early experiments was difficult because of a new and unexpected result. Fermi found that neutron-irradiated uranium was radio- active and that the half-lives of the radioactive nuclides present did not correspond with any known half-life of the heavy elements. For this reason, Fermi believed that transuranium elements had been produced. However, in 1939, the German scientists Hahn and Strassmann chemically separated the radionuclides from the uranium and proved that they were actually isotopes of barium ($_{56}Ba^{139}$) and lanthanum ($_{57}La^{140}$) instead of the transuranium elements that were anticipated. The results could only be explained by assuming that the uranium nucleus split into two parts during the reaction with the neutron. This process of splitting a heavy nucleus into two parts was termed fission.

The announcement of uranium fission led to intense research by scientists throughout the world. It was soon shown that fission is accompanied by the release of large amounts of energy, and this fact led to the development of the atomic bomb. Since that time, the fission process has been applied to the commercial generation of electric power.

FISSION CROSS SECTIONS

The discovery of uranium fission by neutrons led to a number of investigations to determine whether other nuclei were fissionable and if particles other than neutrons could be used. It was discovered that any of the nuclear projectiles we have discussed can cause fission, and that almost any heavy nucleus will undergo fission if the energy of the projectile is large enough. Thus tantalum (atomic number 73) has been fissioned by the use of 400 mev alpha particles.

From an industrial standpoint, fission by neutrons is the only important type. Furthermore, although fast neutron fission technology has advanced steadily in recent years, processes using thermal neutron fission still predominate in industry. Therefore, much work has been done with three nuclides - U-235, U-233, and Pu-239 - which are known to undergo thermal fission. U-235 is the only one of the three which occurs in nature. Natural uranium contains 0.712% U-235, 99.282% U-238, and 0.006% U-234.

As explained in Chapter 7, fission is one type of absorption reaction. A plot of the microscopic fission cross section for U-235 versus neutron energy, as shown in Figure 8-1, has the general characteristics of absorption cross

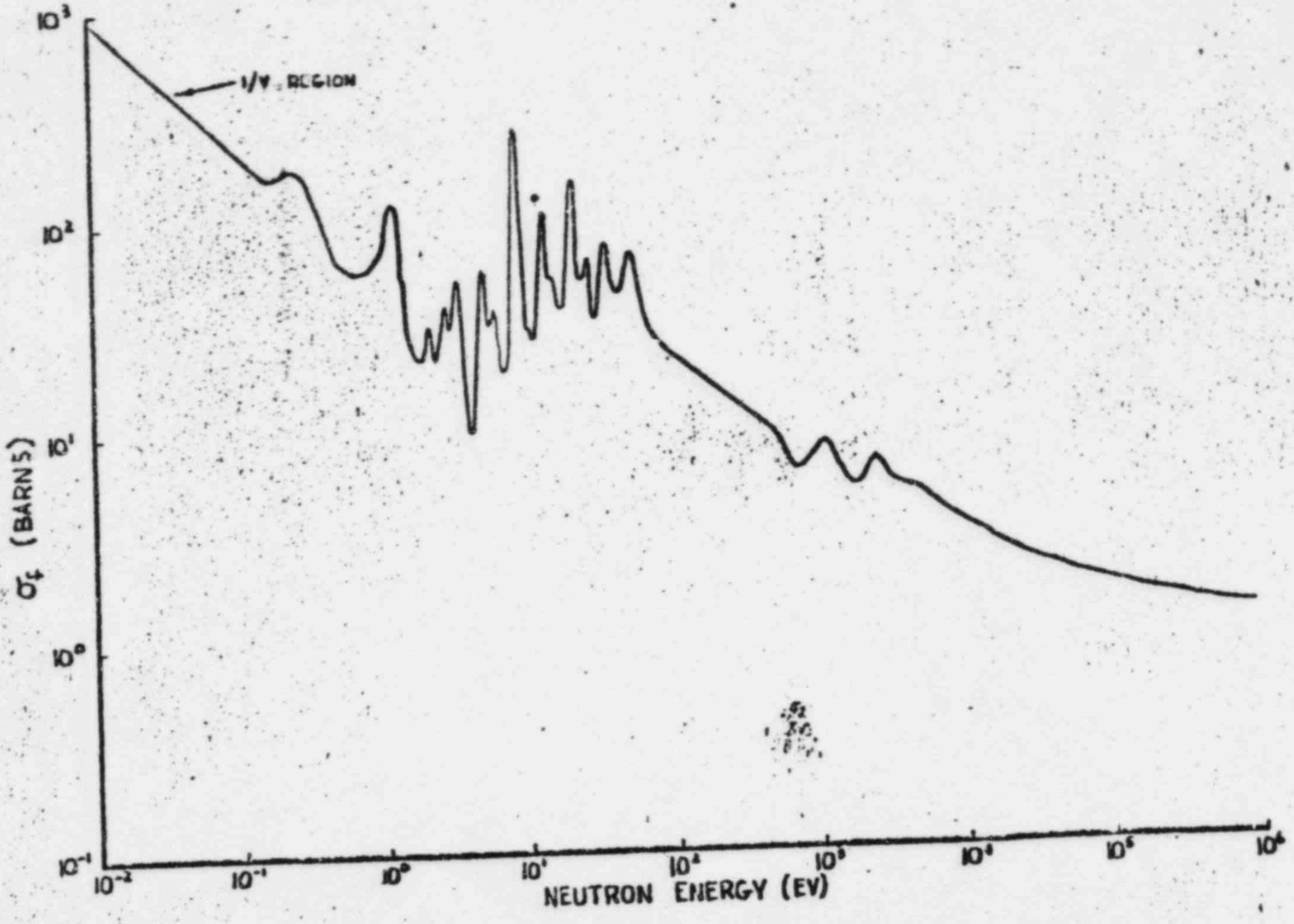

Figure 8-1: Fission Cross Section for U-235 Versus Neutron Energy

sections discussed in Chapter 7. That is, in the low energy regions the cross section is very nearly described by the 1/v law, there are resonances in the intermediate energy region, and the cross section is small at higher energies. The thermal neutron (0.025 ev) cross sections for U-235 are given in Table 8-I. We see from this data that when a thermal neutron strikes a U-235 nucleus, it will cause a fission 580/696 = 83.3% of the time, will undergo an (n,γ) reaction to U-236 107/696 = 15.4% of the time, and will scatter 9/696 = 1.3% of the time.

Figure 8-2 shows the fission cross section for Pu-239. Its distinguishing feature is a large resonance at 0.29 ev which causes it to deviate significantly from 1/v behavior in the slow neutron region.

TABLE 8-I: THERMAL NEUTRON CROSS SECTIONS OF THE URANIUM ISOTOPES AND Pu-239

Cross Section[1]	U-235	U-238	U-233	Pu-239
σ_f	580	0	533	750
$\sigma_{(n,\gamma)}$	107	2.75	52	315
σ_s	9.0	8.3	-	9.6
σ_{total}	696	11.05	-	1074.6

Table 8-I shows that the thermal neutron fission cross section for U-238 is 0 barns. In other words, U-238 cannot be fissioned by thermal neutrons. The fission reaction involving U-238 is a so-called threshold reaction. This means that there is a threshold neutron energy below which the reaction cannot occur and above which the reaction can occur. In Chapter 6 we considered the reaction Be^9 (γ,n) $2\ He^4$ and found from mass-energy balance considerations that the minimum gamma ray energy was ~ 1.6 mev. This is an example of a threshold reaction. The threshold for fission of U-238 is 1.0 ± 0.1 mev. A plot of the fission cross section versus neutron energy for U-238 is given in Figure 8-3.

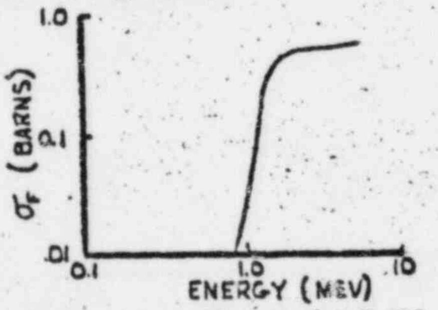

Figure 8-3: Fission Cross Section of U-238 versus Neutron Energy

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[1] Cross sections for charged particle emission are negligible in all cases.

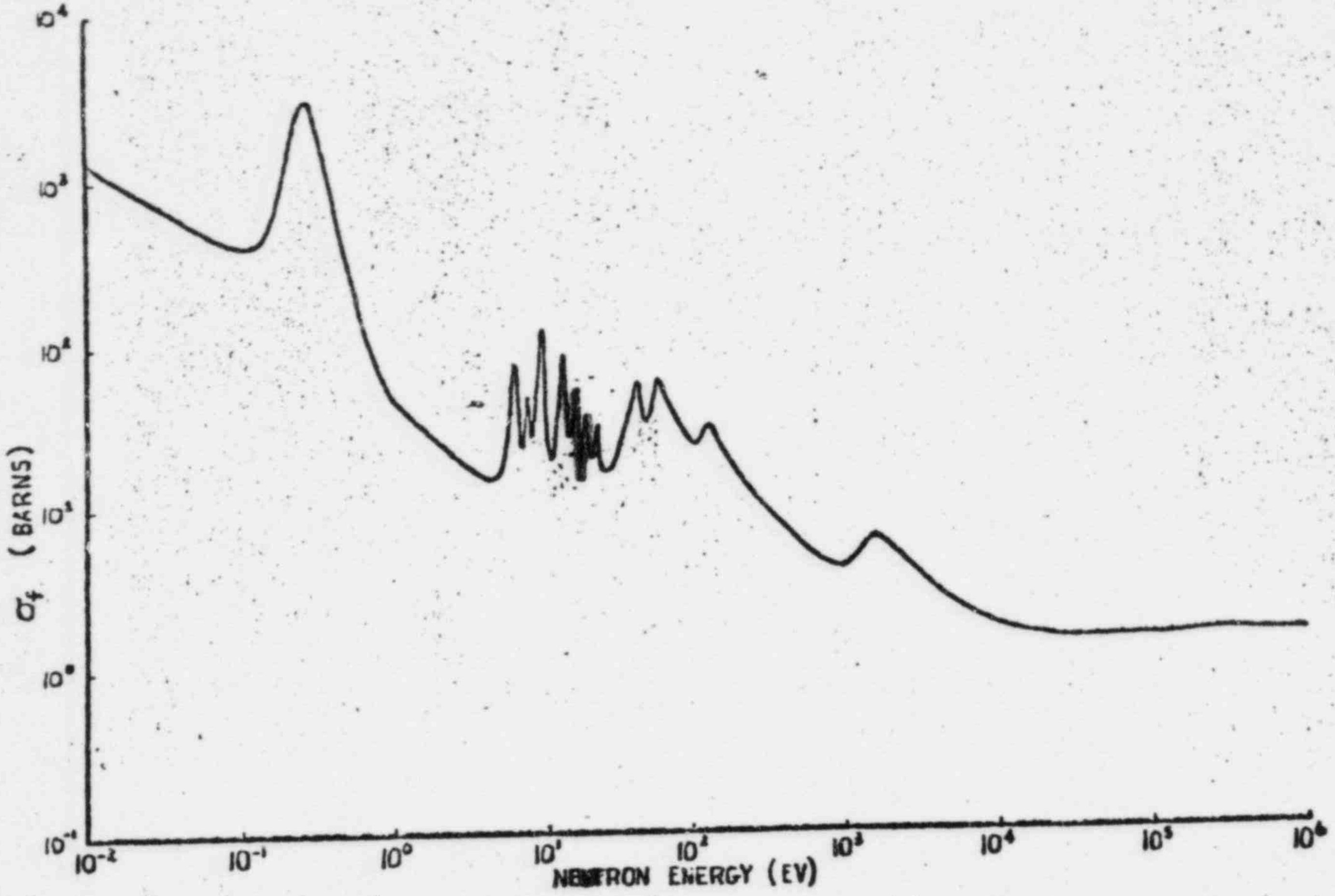

Figure 8-2: Fission Cross Section for Pu-239 Versus Neutron Energy

In addition to the three nuclides listed in Table 8-I, there are many others which have high cross sections for thermal fission. Several examples are ${}_{90}Th^{227}$, ${}_{91}Pa^{232}$, ${}_{92}U^{231}$, ${}_{93}Np^{238}$, and ${}_{94}Pu^{241}$. The only one of any foreseeable importance is Pu-241, which may be of importance in plutonium-fueled power reactors which operate with high usage of fuel. All those nuclides fissionable by thermal neutrons are, of course, also fissionable by fast neutrons. In addition, there are several nuclides which, like U-238, have fission thresholds of about 1 mev. Among these are ${}_{90}Th^{232}$, ${}_{91}Pa^{231}$, and ${}_{93}Np^{237}$.

FERTILE MATERIALS

Table 8-I indicates that ${}_{94}Pu^{239}$ and ${}_{92}U^{233}$ are both potentially useful fissionable materials. While neither of these isotopes is naturally occurring, both can be produced in useful quantities by neutron bombardment of ${}_{92}U^{238}$ and ${}_{90}Th^{232}$ respectively. U-238 and Th-232 are called fertile materials; that is, they are non-fissionable materials (to thermal neutrons) which can be converted into fissionable materials by neutron bombardment. In the case of U-238, the reactions involved are:

$$ {}_{92}U^{238} + {}_{0}n^{1} \longrightarrow {}_{92}U^{239} + \gamma \tag{8-1} $$

$$ {}_{92}U^{239} \xrightarrow{T_{1/2} = 23.5 \text{ min}} {}_{93}Np^{239} + \beta^- + \gamma \tag{8-2} $$

$$ {}_{93}Np^{239} \xrightarrow{T_{1/2} = 2.3 \text{ days}} {}_{94}Pu^{239} + \beta^- + \gamma \tag{8-3} $$

Notice that these are the reactions Fermi was trying to produce when fission was discovered.

The absorption cross section of U-238 is shown on Figure 8-4. Notice that it is characterised by a number of large resonance peaks. This characteristic has a significant effect upon the operation of nuclear reactors as will be discussed in Chapter 10.

In the case of Th-232, the reasons are:

$$ {}_{90}Th^{232} + {}_{0}n^{1} \longrightarrow {}_{90}Th^{233} + \gamma \tag{8-4} $$

$$ {}_{90}Th^{233} \xrightarrow{T_{1/2} = 23.3 \text{ min}} {}_{91}Pa^{233} + \beta^- + \gamma \tag{8-5} $$

$$ {}_{91}Pa^{233} \xrightarrow{T_{1/2} = 27.4 \text{ days}} {}_{92}U^{233} + \beta^- + \gamma \tag{8-6} $$

Use of fertile materials in nuclear reactors will be discussed more fully in Chapter 9. They can, for example, be placed in a reactor running on U-235 and converted into fissionable isotopes at a significant rate. Such a reactor has the potential of producing a significant portion of its own fuel. This process is called breeding and reactors designed for this use are called breeder reactors.

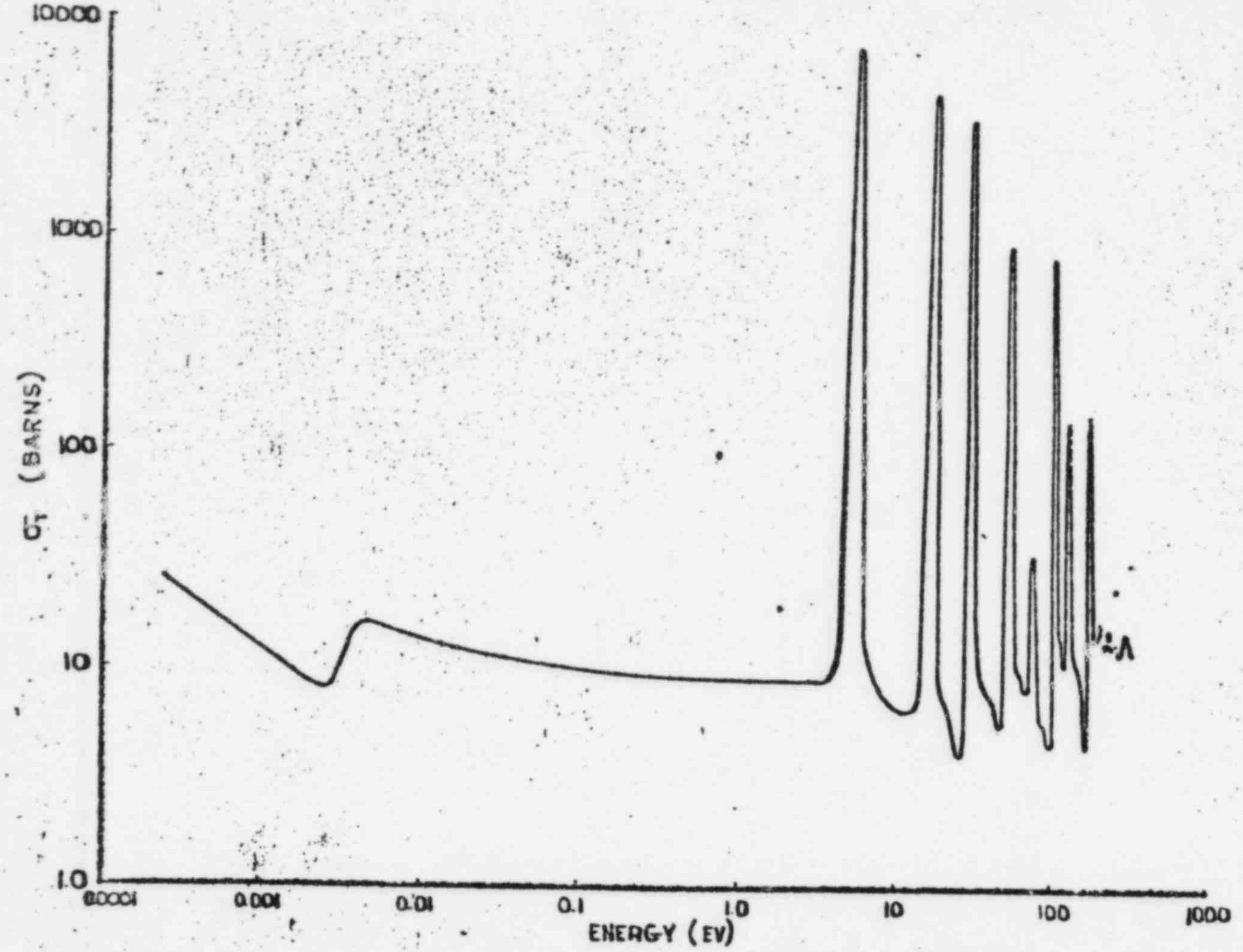

Figure 8-4: Total Cross Section of U-238 vs. Neutron Energy

THE FISSION REACTION AND ENERGY RELEASE

The fission reaction is written and balanced in the same way as are all nuclear reactions we have discussed so far. For example, consider the most common fission reaction - that of U-235 and a thermal neutron:

$$ {}_{92}U^{235} + {}_{0}n^{1} \longrightarrow {}_{z}X^{a} + {}_{z}Y^{a} + 2\,{}_{0}n^{1} \qquad (8\text{-}7) $$

The two isotopes, ${}_{z}X^{a}$ and ${}_{z}Y^{a}$, are called the fission products of the reaction[1]. The U-235 nucleus can split in a great many different ways, and so it is impossible to say in any given fission reaction just exactly what the fission products will be. Equation (8-7) shows that two neutrons were emitted as a result of the fission. Again, however, it is impossible to say in any particular fission reaction just how many are actually emitted. In some cases no neutrons are emitted; in others there may be one, two, three or even more. On the average, there are 2.46 neutrons emitted for each thermal fission of U-235. The average numbers of neutrons given off in Pu-239 and U-233 thermal fissions are 2.88 and 2.54 respectively.

Although we have seen that it is impossible to predict the exact products of any particular fission reaction, it is of interest to take a typical example and calculate the energy release which accompanies it. Such a reaction would be:

$$ {}_{92}U^{235} + {}_{0}n^{1} \longrightarrow {}_{36}Kr^{95} + {}_{56}Ba^{139} + 2\,{}_{0}n^{1} $$

The masses involved are: U-235 = 235.124 amu, n = 1.009 amu, Kr-95 ~ 94.945 amu, Ba-139 ~ 138.955 amu. Calculating the total masses before and after the reaction we find that:

Masses Before Fission		Masses After Fission	
U-235	235.124	Kr-95	94.945
n	1.009	Ba-139	138.955
Total	236.133	2 n	2.018
		Total	235.918

Mass converted into energy = 236.133 - 235.918 = 0.215 amu.
Energy released per fission = (0.215)(931) = 198 mev.

Although this calculation was made for one particular mode of fission, it may be regarded as typical. While there are slight variations from one mode to another, a rule of thumb estimate of about 200 mev of energy released per

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[1] In the overwhelming majority of cases, there are two fission products produced as indicated in equation 8-7. However, in about 1 out of every 12500 fissions, a third fission product, tritium, is also produced. This reaction, called ternary fission, is illustrated by the equation:

$$ {}_{92}U^{235} + {}_{0}n^{1} \longrightarrow {}_{z}R^{a} + {}_{z}S^{a} + {}_{1}H^{3} + 2\,{}_{0}n^{1} $$

The major significance of this reaction involves the unique waste disposal and health physics implications of tritium, which is a β^- emitter. The subject of tritium will be discussed further in Chapter 13.

U-235 fission is satisfactory for most work. The same value may also be used for the fission of U-233 and Pu-239.

The major portion - over 80 percent - of the energy of fission appears as kinetic energy of the fission fragments, and this immediately manifests itself as heat as these heavy, highly charged particles slow down. Part of the remaining 20 percent or so is liberated in the form of instantaneous gamma rays and as kinetic energy of the fission neutrons. The rest is released gradually as energy carried by β^- particles and γ rays emitted by the radioactive fission products as they decay over a period of time. The approximate distribution of the fission energy is summarized in Table 8-II.

TABLE 8-II: DISTRIBUTION OF FISSION ENERGY

Kinetic energy of fission fragments	168 ± 5
Instantaneous gamma-ray energy	5 ± 1
Kinetic energy of fission neutrons	5 ± 0.5
Beta particles from fission products	7 ± 1
Gamma rays from fission products	6 ± 1
Antineutrinos	10
Total fission energy	201 ± 6

The 10 mev energy of the neutrinos, which accompany the beta radioactivity, is not available for producing power since these particles do not interact appreciably with matter. Therefore the primary fission energy is only about 191 mev per fission. However, there is a secondary source of energy which arises when fissions occur in a nuclear reactor. Some of the excess fission neutrons are captured by the nuclei of various materials such as U-238, structural materials and moderator (see Chapter 9), present in the reactor, to undergo radiative capture (n,γ) reactions. Thus neutrons are absorbed and energy is released in the form of gamma rays. Since neutrons capture and emission of gamma radiation occur within a very short time, this energy may be regarded as being released instantaneously, i.e. at the moment of fission. The energy of the capture gamma radiation will depend upon the materials present in the reactor, but may be taken to be 3 to 7 mev/fission. Since the products of the (n,γ) reactions are frequently radioactive, another 1 or 2 mev, on the average, is released over a period of time as beta and gamma ray energy while these products decay. This secondary energy discussed above is thus approximately equal to the energy lost to the neutrinos, and so the total fission energy available in a reactor can still be thought of as being roughly 200 mev per fission. It is of interest to break this energy down into the parts which appear immediately and those which are delayed. This is done in Table 8-III.

TABLE 8-III: LIBERATION OF HEAT DUE TO FISSION

Instantaneous	Mev
Energy of fission fragments	168
Energy of fission neutrons	5
Instantaneous gamma rays	5
Gamma rays from (n,γ) reactions	7
	185
Delayed	
Beta particles from fission products	7
Gamma rays from fission products	6
Radiations from (n,γ) reaction products	2
	15

There are several interesting conclusions to be drawn from the data of Table 8-III. In the first place, when the reactor is started up only 185 mev of heat will be released per fission. In the course of time, as the fission products and capture products accumulate and decay, the heat will increase gradually to 200 mev, at which value it will remain constant provided the system is in equilibrium. When the reactor is shut down and the fission process is brought to a virtual stop, the delayed heat will continue to be released for some time while the various radioactive nuclei decay. Anytime a large reactor is shut down, it is necessary to cool the fuel in order to remove this delayed heat (or decay heat as it called). Figure 8-5 shows the decay heat generation in a reactor versus time after shutdown for the case where the reactor operated for an infinite number of full power days preceding the shutdown. This latter condition is equivalent to saying that a truly equilibrium fission product inventory existed in the reactor prior to shutdown. Table 3-IV gives more precise values for the decay heat at various times during the first few hours after shutdown.

TABLE 8-IV: RATE OF PRODUCTION OF DECAY HEAT FOLLOWING SHUTDOWN FROM FULL POWER

Time After Shutdown	% of Full Power
1 sec	6.0
1 min	4.5
30 min	2.0
1 hour	1.6
8 hours	1.0
24 hours	0.7
48 hours	0.6

Another point to note regarding the fission energy is that because neutrons and gamma rays travel considerable distances in matter, a large portion of the energy they possess is liberated as heat at some distance from the place where fission occurs. The fact that a considerable quantity of heat is liberated in the moderator and structural materials of the reactor has important bearing on their design from a heat removal standpoint.

Using a value of 200 mev released per fission, and applying the appropriate conversion factors, it can be shown that it takes 3.1×10^{10} fissions per second to produce 1 watt of power.

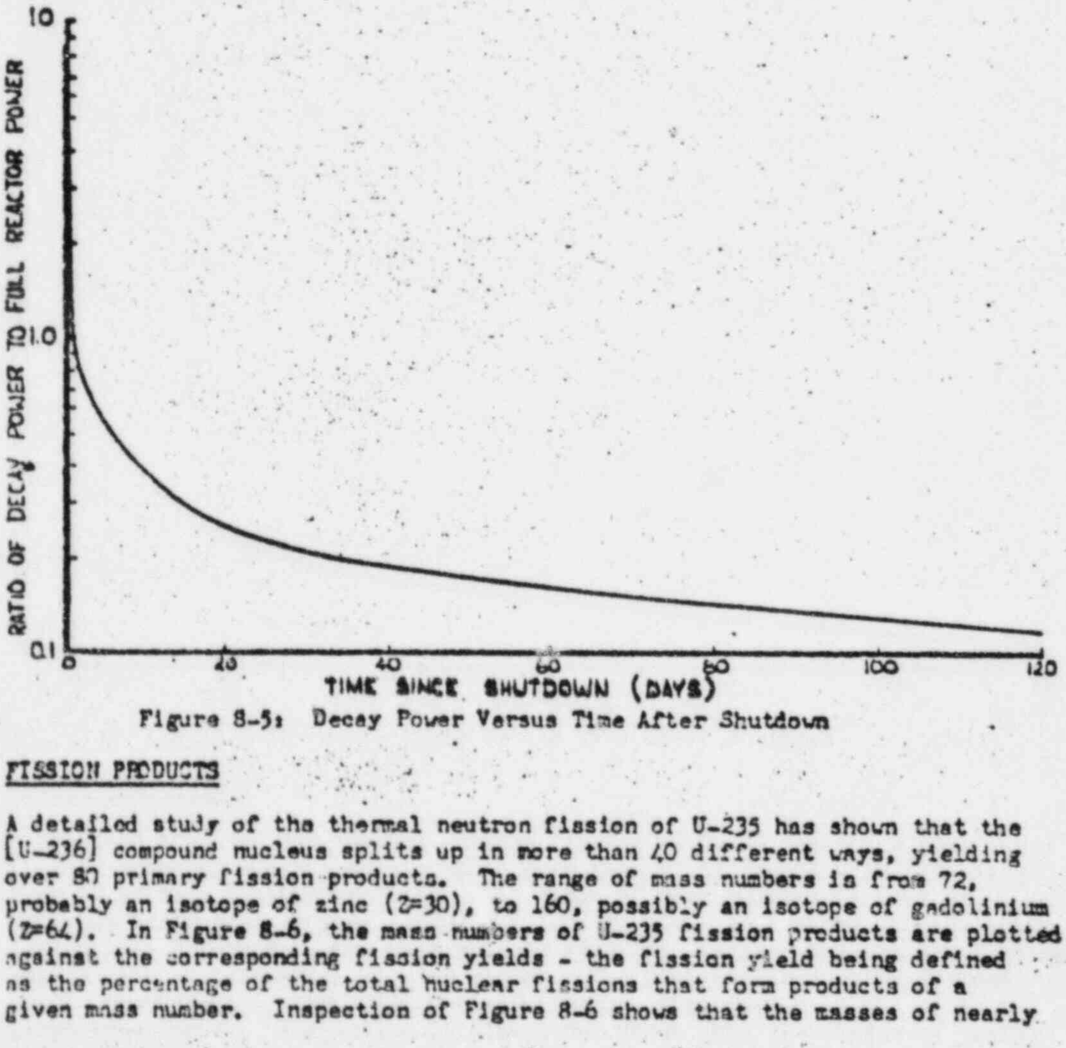

Figure 8-5: Decay Power Versus Time After Shutdown

FISSION PRODUCTS

A detailed study of the thermal neutron fission of U-235 has shown that the [U-236] compound nucleus splits up in more than 40 different ways, yielding over 80 primary fission products. The range of mass numbers is from 72, probably an isotope of zinc (Z=30), to 160, possibly an isotope of gadolinium (Z=64). In Figure 8-6, the mass numbers of U-235 fission products are plotted against the corresponding fission yields - the fission yield being defined as the percentage of the total nuclear fissions that form products of a given mass number. Inspection of Figure 8-6 shows that the masses of nearly all the fission products fall into two broad groups, a "light" group, with mass numbers from 80 to 110, and a "heavy" group, with mass numbers from 125 to 155. The most probable type of fission, comprising nearly 6.4% of the total, gives products with mass numbers 95 and 139. Therefore, it is apparent that in the great majority of cases, the fission of U-235 is unsymmetrical. Pu-239 and U-233 have similar fission product yield curves which are also shown on the figure.

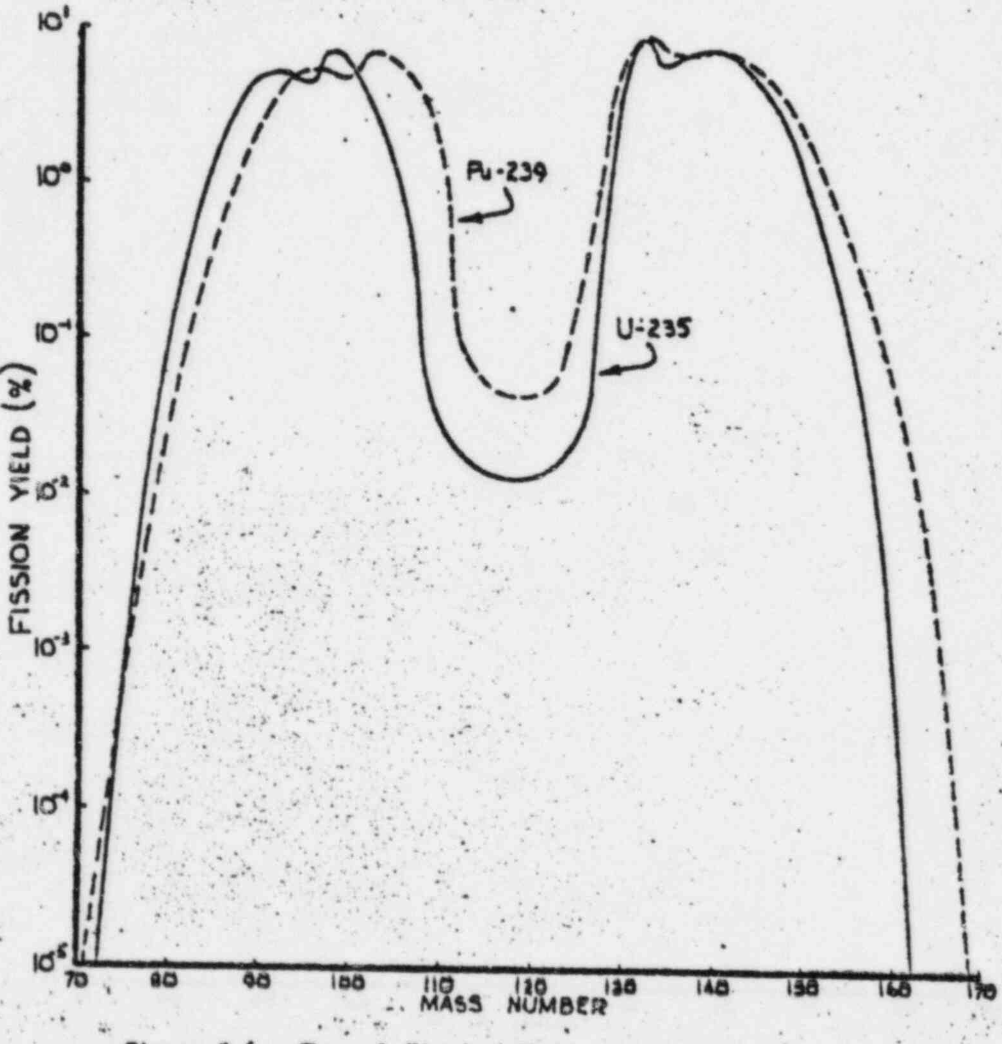

Figure 8-6: Thermal Fission Yield versus Mass Number

At the instant of fission, the fission product nuclei are ejected with such a high velocity that they have a tendency to leave some of their orbital electrons behind. Therefore, the fission products act as highly charged particles passing through matter. The lighter fragments carry an average positive charge of about 20 units, whereas the heavy fragments carry some 22 positive charges. In Chapter 4 we noted that the ionizing ability of a charged particle increases with the charge and mass of the particle. Therefore, the fission fragments produce a great deal of ionization and their range is relatively short for particles of such high energy. The ranges in air of the light and heavy groups of fission fragments have been found to be about 2.5 cm and 1.9 cm, respectively. This is slightly less than the range of an alpha particle from a typical radioactive source. In Chapter 4 we calculated that an alpha particle produced about 40,000 ion pairs/cm in air. It is of interest to calculate the average rate of ionization for a fission fragment. On the average, the heavy fragment possesses about 61 mev of energy. In Chapter 4 it was noted that the amount of energy required to form an ion pair in air was ~ 35 mev. Therefore the total number of ion pairs formed by the heavy fragment is:

$$\frac{61 \times 10^6}{35} = 1.74 \times 10^6 \text{ ion pairs}$$

This represents an <u>average</u> ionization rate along its 1.9 cm path of:

$$\frac{1.74 \times 10^6}{1.9} = 920{,}000 \text{ ion pairs/cm}$$

RADIOACTIVITY OF FISSION PRODUCTS

Nearly all of the fission products are radioactive and emit negative beta particles. The reason for this is readily apparent from a consideration of the neutron to proton ratios involved. Recall that heavy isotopes require about 1.6 neutrons for every proton in order to overcome the large electrostatic repulsive forces. The fission products, since they are formed from uranium, will also have a neutron to proton ratio of ~ 1.6. Intermediate sized isotopes, however, require a neutron to proton ratio of ~ 1.3 for stability. It is evident, then, that the fission products will possess an excess of neutrons. As we discussed in Chapter 5, neutron heavy isotopes decay by β^- emission. The fission products are so neutron heavy that on the average it requires about three β^- emissions to reach stability. This gives rise to decay chains, a typical one being the decay of the xenon-140 isotope to a stable isotope of cerium.

$${}_{54}Xe^{140} \xrightarrow[16\ sec]{\beta^-} {}_{55}Cs^{140} \xrightarrow[66\ sec]{\beta^-} {}_{56}Ba^{140} \xrightarrow[12.8\ days]{\beta^-} {}_{57}La^{140}$$

$$\xrightarrow[40\ hr]{\beta^-} {}_{58}Ce^{140} \qquad (8\text{-}8)$$

Since there are some 80 different radioisotopes produced in fission and each produces through its decay chain an average of two additional radioisotopes, there are over 200 radioactive species present among the fission products after a short time.

The fission products play an important role in the operation of any reactor. First, their intense radioactivity makes a shutdown reactor completely inaccessible for refueling or maintenance unless extensive shielding is provided. Secondly, several fission products, notably xenon-135 and samarium-149 have large absorption cross sections for thermal neutrons. We shall see in Chapter 10 that these isotopes have a large effect on the operation of the reactor because of the competition they represent to U-235 for the absorption of neutrons.

FISSION NEUTRONS

Let us return our consideration for a moment to the fission reaction represented by equation (8-7). As in any nuclear reaction there is a compound nucleus formed as an intermediate step. For the fission reaction with U-235, the compound nucleus would be an excited state of U-236. The implication of equation (3-7) is that the compound nucleus disintegrates into the products of the reaction; namely, two fission products and two neutrons. Actually, this is not strictly correct. What does happen is that the compound nucleus only breaks up into two fission fragments. The fragments thus produced are so neutron heavy that they decay by neutron emission rather than by beta emission. Therefore, it is the decay of the fission fragments which produce the neutrons observed in the fission process. The products of the reaction that are given in equation (8-7) are really the resultant products after the original fission fragments have undergone decay by neutron emission. In most cases, the fission fragments eject their neutrons with about 10^{-14} seconds after the fission. For all intents and purposes this is instantaneous and so we never observe the original fission fragments but only the products after neutron emission. It is for this reason that we ignore the original fission fragments and write the fission equation using the products of the neutron decay.

The neutrons which are emitted by the fission fragments within about 10^{-14} seconds are called prompt neutrons. For U-235 fission, prompt neutrons account for about 99.36% of all fission neutrons. The remaining 0.64% of the fission neutrons are called delayed neutrons. Delayed neutrons do not appear instantly, but at a gradually decreasing rate over a period of minutes after the fission. Research has shown that the decayed neutrons accompanying fission of U-233, U-235, and Pu-239 fall into six major groups, each of which is characterized by a definite half-life. Table 8-V shows the characteristics of each of these groups.

The fact that the delayed neutrons have definite decay half-lives suggest that their origin is associated with the disintegration of certain radioisotopes. The 55.6 second half-life has been definitely associated with the decay of bromine-87 and the 22.7 second half-life with iodine-137. The scheme shown in Figure 8-7 has been proposed to account for the 55.6 second delayed neutron group, and a similar mechanism is probably applicable in the

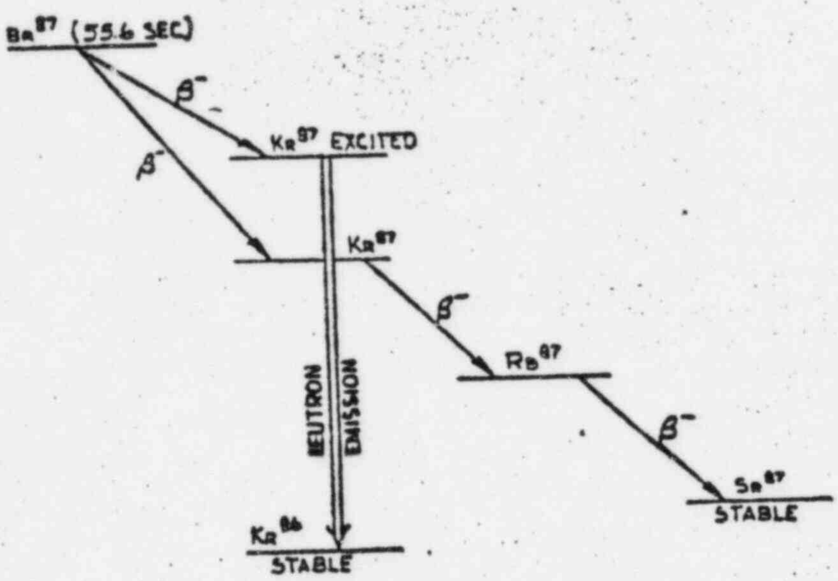

Figure 8-7: Mechanism of Delayed Neutron Emission

TABLE 8-V: CHARACTERISTICS OF DELAYED FISSION NEUTRONS IN THERMAL FISSION

			Fraction of Total Fission Neutrons (%)		
Group	Half-life (sec)	Energy (mev)	U-233	U-235	Pu-239
1	55.6	.25	.022	.02	.007
2	22.7	.46	.078	.14	.063
3	6.2	.41	.066	.13	.044
4	2.3	.45	.072	.25	.069
5	0.6	.42	.013	.07	.018
6	0.2	-	.009	.03	.009
Total		-	0.26	0.64	0.21

other cases. One of the products of fission is the β^- emitter, bromine-87, decaying with a 55.6 second half-life to produce krypton-87. The latter is evidently formed in a highly excited state, with sufficient energy to permit it to immediately eject a neutron and leave a stable krypton-86 nucleus. Any excess energy available appears as kinetic energy of the neutron. The observed rate of emission of the delayed neutron is determined by the rate of formation of the neutron emitter, krypton-87, and this is dependent on the rate of decay of the bromine-87. This means that the neutron emission also exhibits a 55.6 second half-life.

There is one more important fact concerning delayed neutrons. Table 8-V shows that although U-235, U-233, and Pu-239 produce delayed neutrons in the same six groups, the amount of delayed neutrons varies considerably between the three types of fuel[1]. We will see later that this is of great importance in the control of a nuclear reactor. In general, the more delayed neutrons there are, the more easily a reactor can be controlled.

It is of interest to consider the energy of the neutrons born in the fission reaction. No definite energy can be assigned to the prompt neutrons because this energy will be dependent upon the mode of fissioning (i.e. the particular set of fission products which are formed, etc.). Instead, the prompt neutrons show a continuous energy spectrum which has a peak at ~ 1 mev and an average value of ~ 2 mev. The prompt neutron energy spectrum for U-235 is shown in Figure 8-8. The energies of most of the prompt neutrons are considerably higher than the energies of the various delayed neutron groups which are given in Table 8-V.

SPONTANEOUS FISSION

All of the isotopes with atomic number 90 and above are sufficiently unstable that they have a definite probability of undergoing spontaneous fission - that is, fission without neutron bombardment. For uranium and plutonium isotopes, this probability is very small - spontaneous fission half-lives being on the order of 10^{15} - 10^{17} years. Table 8-VI gives some spontaneous fission data for the important uranium and plutonium isotopes.

TABLE 8-VI: SPONTANEOUS FISSION DATA FOR U AND Pu ISOTOPES

Isotope	Spontaneous Fission Half-life (years)	Spontaneous Fission Rate per Ton of Material (fiss/sec-ton)
U-235	1.8×10^{17}	280
U-238	8.0×10^{15}	6300
Pu-239	5.5×10^{15}	9100

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[1] In addition to the values given in the table, the delayed neutron fraction for the fast fission of U-238 is 1.68%.

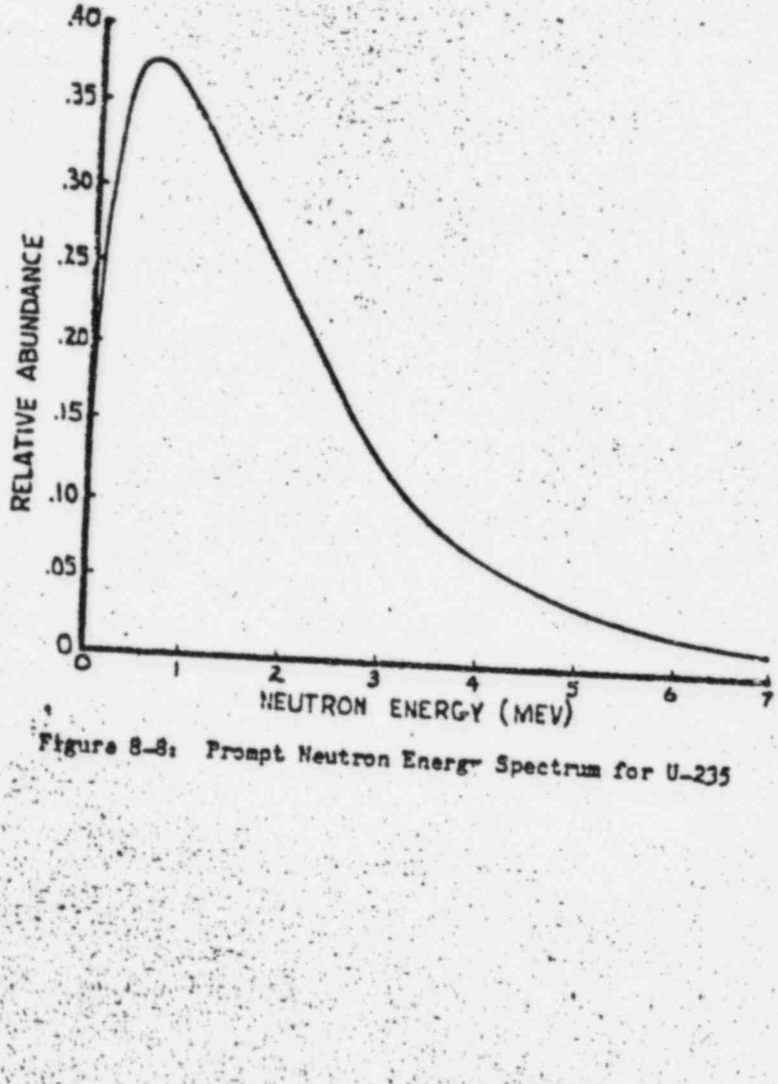

Figure 8-8: Prompt Neutron Energy Spectrum for U-235

PROBLEMS

1. Define: nuclear fission, fertile material, prompt neutron, delayed neutron, thermal neutron.

2. Write the equations for the conversion of U-238 into Pu-239.

3. What is the source of the decay heat which continues to be generated after the shutdown of a nuclear reactor? Immediately after shutdown, about what fraction of the total reactor power does decay heat represent?

4. In many reactor facilities, the concrete biological shielding which surrounds the reactor pressure vessel (a picture of a typical reactor and pressure vessel arrangement is given in Chapter 9) is provided with cooling coils. Why is this necessary?

5. What is the average number of neutrons produced per thermal neutron fission in U-235? In U-233? In Pu-239?

6. How many fissions per second are taking place in a reactor which is producing 200 megawatts of power?

7. Describe the mechanism by which delayed neutrons are produced.

8. How do prompt and delayed neutrons compare with regard to energy?

9. Why are fission products β^- emitters? Write the decay chain for the fission product ${}_{36}Kr^{90}$ (use a chart of the nuclides).

10. Could U-238 be used as the fuel for a nuclear reactor in which the majority of the neutrons had energies near thermal? Why?

11. How much energy is released in the U-235 fission reaction in which ${}_{38}Sr^{94}$ (atm. wgt. = 93.935 amu) and ${}_{54}Xe^{140}$ (atm. wgt. = 139.949 amu) are the fission products?

12. A reactor core contains 60 tons of uranium enriched to 3%. What is the spontaneous fission rate in this core? How many watts of power does this represent?

CHAPTER 9

INTRODUCTION TO NUCLEAR REACTORS AND NUCLEAR POWER PLANT CYCLES

INTRODUCTION

Thus far we have talked almost exclusively about pure nuclear physics. From this point on, we will concern ourselves with one of the practical applications of nuclear physics. In this chapter we will introduce the nuclear reactor and discuss its application to the generation of electric power. The material in this chapter is intended to present a broad picture of the nuclear power plant. The remainder of the manual will discuss various aspects of nuclear power plants in greater detail.

THE CHAIN REACTION

In Chapter 8, we discussed nuclear fission as a more or less isolated event - what happens when a single nucleus splits. The fission reaction is useful only when the reaction takes place in a mass of fissionable material containing billions of fissionable nuclei. Under these conditions it is possible that at least one of the 2.5 neutrons emitted during the fission reaction will be captured by another fissionable nucleus and cause a second fission. If each fission causes at least one more fission, the process will be self-sustaining and will constitute what is called a chain reaction. Since there are, on the average, 2.5 neutrons emitted per fission of uranium 235[1], it is possible that two neutrons will be captured by fissionable isotopes and cause fissions. In this instance, our chain reaction will not only continue, but will increase in magnitude. Figure 9-1 shows schematic diagrams of two chain reactions.

At first glance it might seem that it would be very easy to create a chain reaction. After all, it only requires that one of the 2.5 neutrons produced be absorbed in fissionable material and cause another fission. Therefore, **we have, on the average, 1.5 excess neutrons to play with after each fission.** Nevertheless, in practice it is not particularly easy and it requires special materials and a considerable amount of engineering skill. The problems can be overcome, however, and chain reacting systems can be made. The device in which a controlled nuclear fission chain reaction takes place is called a <u>nuclear reactor</u>[2]. In the following sections of this chapter we will

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[1] Uranium-233 and plutonium-239, the other two common materials fissionable by thermal neutrons, emit 2.54 and 2.88 neutrons per fission respectively.

[2] An atomic bomb is a specially constructed device in which a highly uncontrolled nuclear chain reaction can be made to occur for a sufficiently long time to yield a significant energy release. Although both nuclear reactors and nuclear weapons have in common the use of fissionable material, their construction and behavior are entirely different. The gas flame on a kitchen stove burner and a raging uncontrolled forest fire are both combustion processes but they are also different in construction and behavior.

consider some of the problems of nuclear reactors and their solutions.

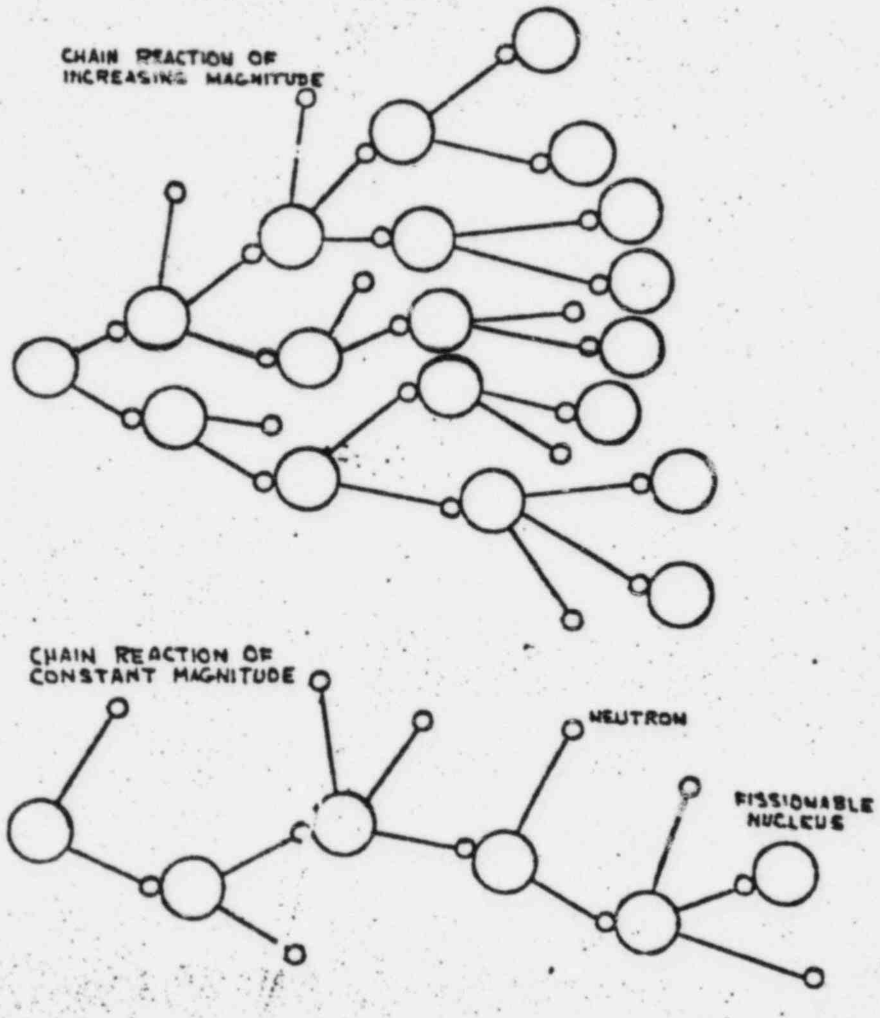

Figure 9-1: Schematic Diagrams of Chain Reactions

TYPE OF FUEL

Natural uranium (99.3% U-238, 0.7% U-235) by itself cannot be used for the construction of a nuclear chain reactor. It is impossible to establish a chain reaction with fast neutrons in pure natural uranium because, at the average fission neutron energy of ~ 2 mev, the fission cross section of U-238 is only about 0.5 barns as compared with the scattering cross section of about 6 barns. Thus, in an assembly of natural uranium, most fission neutrons would simply be reduced in energy by scattering collisions to an energy below the 1 mev fission threshold of U-238. But a chain reaction based on thermal neutrons is also impossible because U-238 has very high resonance peaks in the intermediate energy range for the absorption of neutrons by radiative capture, as was illustrated on Figure 8-4. Essentially all neutrons would be lost by resonance capture in U-238 before reaching thermal energies where U-235 has a high fission cross section.

One logical way to get around this difficulty is to separate the isotopes of uranium so that substantially pure U-235 is available. This not only eliminates capture of neutrons by U-238 but also increases the fission cross section at low neutron energies so that a chain reaction is possible. The process of separating the uranium isotopes is exceedingly difficult and expensive (remember that ordinary chemical separations cannot be used because U-238 and U-235 have the same chemical properties), but is done at three special plants in the U.S.A., one of which is at Oak Ridge, Tennessee. The separation of isotopes will be more fully discussed in a later chapter. Uranium in which the percentage of U-235 has been artificially increased above the natural level is called <u>enriched uranium</u>. Enriched uranium is available in any isotopic concentration from that of normal uranium up to essenticlly pure U-235.

Generally, it is desireable from a fuel cost standpoint to operate the reactor with as low enrichment as possible. The ability of uranium to sustain a chain reaction can be improved by adding other materials to the reactor which help to prevent neutrons from being wasted in non-fission reactions. These techniques will be discussed in detail later in this chapter and in later chapters. As a result, some reactors do employ natural uranium fuel, and even those reactors which employ enriched uranium usually have an enrichment of < ~ 4% U-235[1].

CRITICAL MASS

Neutrons will be lost from any nuclear chain reactor merely by escape or leakage from its outer surface. The larger the physical size of the reactor, the smaller is the fraction of neutrons which leak from its boundaries. If we start with a very small amount of fuel, so many neutrons are lost through leakage that no chain reaction is possible. As additional fuel is added, the importance of leakage is reduced until finally there is just enough fissionable material present to initiate a chain reaction. At this point the system is said to go <u>critical</u> and the amount of fuel is termed a <u>critical mass</u>. When the reactor is just critical, the production of neutrons through fission is exactly balanced by the loss of neutrons through leakage, absorption in fuel, and absorption in all other materials present in the reactor.

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[1] We have already mentioned that U-235 is not the only fissionable material which could be used as a reactor fuel. Two other possibilities are U-233 and Pu-239. However, since neither of these are naturally occurring isotopes, U-235 was the logical choice for fuel in the early reactors and continues to be by far the most widely used fuel. Use of the other isotopes, notably Pu-239, is expected to become more important in future years.

REACTOR CONTROL

For practical considerations, a reactor which is to operate at any appreciable power level must contain more fuel than simply a minimum critical mass. This excess fuel is necessary for overcoming temperature effects as the power is raised, for overcoming the poisoning effects of fission products that gradually build up in the reactor fuel, and to make available additional fuel to compensate for the depletion of fissionable material as the reactor operates. Let us consider what would happen if we were to bring together a supercritical (i.e. larger than the minimum critical mass) assembly of fissionable material. A chain reaction would immediately start because of the presence of a stray neutron from cosmic radiation or some other source. The power would rise at a rate determined by the degree of supercriticality until temperature effects within the reactor halted the increase. For highly supercritical reactors, these temperature effects could include core damage (physical distortion of a reactor core will frequently result in a less favorable arrangement of components which will make the assembly subcritical).

In any case, it is evident that some reactor control mechanism must be provided in any practical reactor facility to give the operator the means to shut the plant down, vary the steady state power level, and insure that no reactor damage will occur due to excessive power generation.

There are several ways in which a reactor can be controlled, but by far the simplest and most commonly employed method is the use of an adjustable neutron poison. A poison material is a material with a high neutron capture cross section. When such a material is inserted into a reactor, it enters into competition with the fuel for the available neutrons. Each neutron which is absorbed by the poison is just one less neutron which is available for sustaining the chain reaction, and if the poison material absorbs a sufficient number of neutrons, the chain reaction will be terminated. There are a number of neutron poisons which can be used as control materials. One of the most commonly employed is boron (σ_a = 4010 barns for thermal neutrons). Adjustable control rods containing boron are inserted the proper distance into the reactor to maintain the reactor critical at the desired power level. As the fuel burns up and fission product poisons accumulate, it is necessary to pull out control rods gradually to keep reactor power constant. If it is desired to shut the reactor down, the control rods are inserted their maximum distance into the reactor and the chain reaction dies out.

INTRODUCTION TO POWER REACTORS

There are a number of ways in which power reactors are classified. These classifications will be given at this point to provide a basis for further discussions.

1. Physical arrangement of moderator and fuel

   The concept of a moderator will be discussed below and will help to clarify the following remarks. However, reactors in which the fuel and moderator are placed together in a uniform mixture are called homogeneous reactors. For example, a homogeneous reactor can be made by dissolving a uranium salt in a water moderator. Conversely, a heterogeneous reactor, which is by far the most common type for industrial uses, is one in which the fuel and moderator are separate bodies. An example would be lumps of uranium fuel embedded at various locations in a large block of graphite.

2. Type of coolant used

   Reactors are frequently spoken of as being water cooled, sodium cooled, gas cooled, etc.

3. Neutron energy spectrum

   A third way in which reactors are classified is by neutron energy spectrum. Reactors in which most of the fissions are caused by high energy neutrons are known as fast reactors. No moderator is used with this type of reactor. Reactors in which most of the fissions are caused by moderated neutrons are called thermal reactors.

4. Use of fuel

   Reactors are sometimes classified in terms of the way in which they utilize fissile material[1]. A breeder reactor produces more fissile material than it consumes by providing neutrons for conversion of fertile material to fissile material. Breeding has been demonstrated in fast reactors and may be possible in thermal reactors which are specially designed to avoid non-productive neutron losses. A converter reactor produces some new fissile material but less than the amount consumed. (A typical light water moderated converter will extract only about 2% of the available energy in its fuel whereas a breeder can potentially utilize 100% of the energy). Finally, a burner reactor produces no new fissile material. By definition, it has little or no fertile material available for conversion.

   The bulk of present day commercial power reactors are converters. Breeder reactors are still in the experimental stage, and several prototypes are being constructed throughout the world. It is expected that they will come into general use in the mid 1980's, barring any unforeseen technical difficulties. Their general use will make the known uranium reserves a virtually limitless source of energy.

A schematic diagram of a hypothetical heterogeneous power reactor is shown in Figure 9-2. This sketch is not intended to illustrate any particular type of reactor, but to simply show all of the components that a reactor may (but no always) employ.

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[1] Fissile materials are those isotopes which can practically be used as reactor fuel. Fissile materials are U-233, produced from the naturally occurring fertile material Th-232; naturally occurring U-235; and Pu-239, produced from the naturally occurring fertile material U-238.

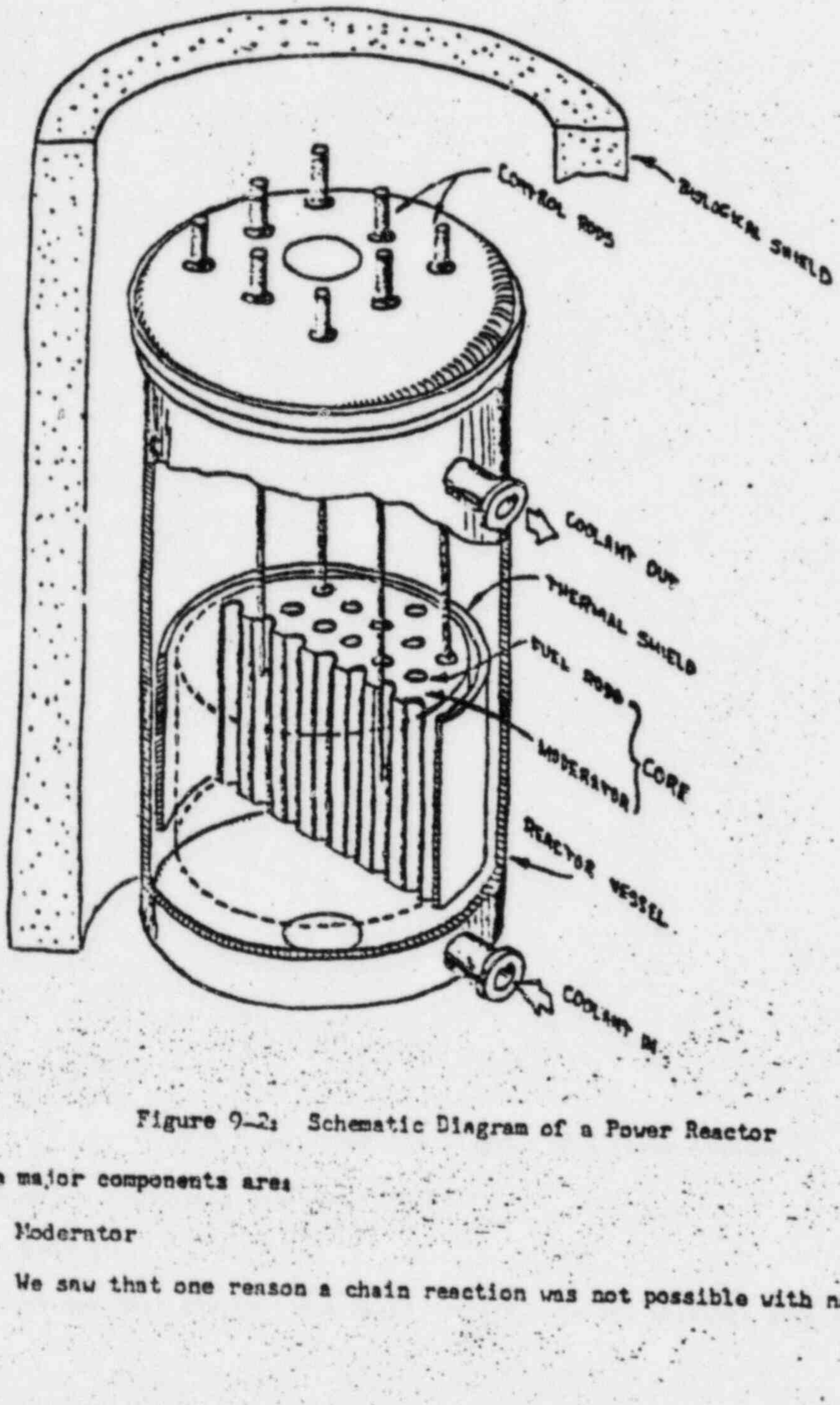

Figure 9-2: Schematic Diagram of a Power Reactor

The major components are:

1. Moderator

We saw that one reason a chain reaction was not possible with natural uranium was that so few neutrons could be slowed down past the U-238 resonances to take advantage of the large U-235 thermal fission cross section. However, a chain reaction is possible if natural uranium is mixed with certain materials which will rapidly slow down the high energy fission neutrons. In this way, the neutrons spend relatively little time with energies in the resonance region, with the result that resonance capture in U-238 is minimized and the high thermal fission cross section of U-235 can be utilized. A material which rapidly slows down fast neutrons is called a moderator, and the process of slowing neutrons is called moderation or thermalizing.

The mechanism by which the moderator slows neutrons down is scattering. A good moderator must therefore have a high scattering cross section. In order to be effective, a moderator must also allow the neutron to lose a large part of its energy in a single collision. This means that the moderator must be a light material (recall our discussion of scattering in Chapter 7). The reasons for this latter restriction are twofold. The more energy that can be lost by a neutron in a single collision, the fewer, on the average, collisions will be necessary to thermalize the neutron. Since even the best moderators have a small absorption cross section, every unnecessary collision that is required simply increases the chances that the neutron will be absorbed by the moderator and lost. Secondly, a neutron which can lose most of its energy in a single collision spends less time with energies in the U-238 resonance region. For example, in a single collision with a hydrogen nucleus a neutron may drop from ~ 1 mev, which is above the resonance region, to thermal (0.025 ev), which is below the resonance region, without having spent any time at all actually in the resonance region. If, however, the moderator functions to slow the neutron by a great number of small energy losses, there is a good chance that after some collision the neutron will be right at the energy of one of the resonances, in which case it will almost always be absorbed. A third requirement of a good moderator is, as we have already implied, that it have a low absorption cross section. The smaller the absorption cross section, the smaller is the chance that the neutron will be lost by absorption in the moderator during the slowing down process. Finally, increased density improves the moderating ability of a material since the higher the density the more likely it is for a neutron to encounter a moderator atom. For example, neutrons will be moderated more efficiently in liquid water than in steam.

It should also be noted that in addition to the general criteria discussed above, the suitability of a particular material for use as a moderator, or for that matter for use in any capacity in the reactor, is also influenced by its ability to withstand a high temperature environment, ability to withstand the intense radiation field which is present in the core, and its chemical and corrosion characteristics.

Some common moderators are water, heavy water (D_2O), graphite (carbon), and, to a lesser extent, beryllium. Some properties of these various material are given in Table 9-I.

TABLE 9-I: PROPERTIES OF TYPICAL MODERATORS

Material	Atomic Weight	σ_s (barns)	σ_a (barns)
Water	18	~ 48	6.6×10^{-1}
Heavy Water (D_2O)	20	~ 10.5	9.2×10^{-4}
Graphite	12	4.8	3.3×10^{-3}
Beryllium	9	7	1×10^{-2}

2. Reflector

The leakage from a reactor can be reduced significantly by surrounding it with a reflector. A reflector, as its name implies, is a material which reflects neutrons back into the reactor after they have leaked out. Use of a reflector reduces the critical mass of a reactor core and results in lower fuel costs, and savings resulting from smaller reactor components.

The reflector works by scattering the neutrons, and so a material with a high scattering cross section is preferred. The reflector must also have a low absorption cross section since any neutrons which are absorbed by it are lost just as effectively as if they had leaked out. Since these are similar requirements to those for a moderator, it is common practice in many actual reactors to have the same material serve both purposes. Thus, in the figure, the moderator at the extreme edge of the core[1] would serve as a reflector. Coolant which is located around the outer core surfaces also acts as a reflector.

3. Coolant

With many industrial nuclear reactors, the heat produced in the fission reaction is the important commercial product. For example, nuclear reactors are used as heat sources for the commercial generation of electric power, for power generation in space, and have been proposed for sea water desalinization plants. In order for a reactor to be used as a heat source, provisions must be included in its design for extracting the heat from the fuel, and this is the basic function of the coolant. In addition, most nuclear reactors, even those used for purposes other than the generation of heat (isotope production, for example) operate at high enough power levels that some form of cooling must be provided simply to prevent structural damage to the core due to excessive temperature.

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[1] The core is the name given to the region of the reactor system in which the fuel is located.

The basic requirements for a coolant are that it have suitable heat transfer properties, have a low absorption cross section, and meet the general requirements regarding behavior in the reactor environment which were given in the discussion of moderators.

Favorable heat transfer requirements invariably require that the coolant be a fluid because heat is transferred more efficiently in flowing systems and because the heat energy must be transported from the reactor to other apparatus, such as heat exchangers and turbines, where it is used. In addition, it is necessary to have a fluid which can absorb a large amount of heat while operating at a temperature and pressure within the limits of the common structural materials.

Coolants which are used in reactors include ordinary water, heavy water, gases such as helium and carbon dioxide, liquid metals (principally sodium), organic materials, and molten salts.

4. Thermal shield

To protect the reactor vessel from possible damage from the heat liberated upon absorption of radiation, a thermal shield is normally included between the core and the vessel walls. It is usually made of a substantial thickness of a dense material of fairly high melting point (usually steel). Since the thermal shield is a heavy metal, it is effective for absorbing γ rays and for scattering fast neutrons. These two types of radiation carry most of the energy leaking from the reactor core. The absorption of the primary gamma rays and the secondary radiations accompanying the capture of the slowed down neutrons produces a considerable amount of heat in the thermal shield. In power reactors this is removed by the coolant and so contributes to the available energy.

5. Fuel rods

The fuel may be in any of several chemical forms, the most common being uranium dioxide (UO_2). The fuel rods are physically oriented so that coolant can circulate in and around them to remove the heat. The core lattice structure also has provisions for guiding the control rods into the appropriate regions.

Since the coolant ordinarily comes into direct contact with the fuel rods, the latter are generally encapsulated into metallic containers called <u>cladding</u>. The cladding protects the fuel from deterioration due to chemical attack or erosion by flowing coolant, provides structural rigidity for the fuel rods, and provides a barrier which prevents radioactive fission products from reaching the coolant and being transported to all parts of the primary system.

6. Control rods.

We have already discussed the functions of the control rods. They are moved by some type of actuating mechanism which is located external to the reactor vessel (not shown on the figure). Both electrically operated and hydraulically operated control rods are in use. The drives may enter through the top of the vessel, as shown, or the bottom.

Provisions are included for positioning the rods at a number of intermediate positions between full in and full out so that fine control of the reactor is achieved. In addition, they are provided with a rapid insertion feature which can be used to quickly shut the reactor down if conditions warrant. Such a rapid shutdown is called a reactor trip, or scram, and ordinarily is accomplished in ~ 1 - 3 seconds. For safeguards reasons, a number of features are provided which will automatically trip the reactor at the first sign of an abnormal condition. Such a condition, for example, might be high reactor power, which would be monitored by instruments sensitive to neutron flux.

7. Pressure vessel

The entire core is generally contained within some sort of pressure vessel. Ordinarily it is made of steel, but other materials such as concrete have been used. The rating of the vessel varies considerably with the type of coolant used. For water cooled reactors, the vessels commonly have to withstand pressures on the order of 2500 psig.

8. Biological shield

As reactor operation proceeds, a substantial inventory of fission products accumulates in the fuel. The fission products are intensely radioactive, emitting β^- particles and γ rays. In addition, nuclear reactions occurring in the coolant and structural material produce a number of radioisotopes, and there is generally a substantial number of neutrons which escape from the reactor vessel. Thus it is necessary from the standpoint of protection of individuals working in the vicinity of the reactor to provide some biological shielding. Ordinarily this is made of concrete although other materials may be used. (Where neutrons are a problem, a small amount of polyethelyne may be used to moderate the neutrons so that other shielding materials may absorb them more effectively.)

POWER REACTOR SYSTEMS

In recent years, the nuclear reactor has found increasing application in the commercial generation of electric power. In simplest terms, the reactor replaces the boiler in a conventional power plant cycle. There are, however, a large number of reactor types, and the design of the overall plant will vary considerably depending upon which type of reactor is used. In the following sections of this chapter we will look at the basic power plant cycles for a few of the more common reactor types and will discuss some of the advantages and disadvantages of each. It should be recognized that the discussions are very brief and that many features in these plants which are concerned with reactor control, safeguards, and handling and disposal of radioactive materials, and which are fundamental to the overall operation are not discussed. Some of these are considered in greater detail in later chapters of this manual, notably chapters 12, 13, 14 and 15.

WATER REACTORS

1. Pressurized water (PWR)

In the pressurized water reactor concept shown schematically in Figure 9-3, ordinary water at temperatures up to about 600°F and pressures up to about 2600 psig is pumped through one or more[1] primary coolant loops. Each loop contains a coolant pump and a shell and tube heat exchanger called the steam generator. The circulating water serves as the reactor coolant, moderator, and reflector. The heat which is absorbed by the coolant as it passes through the core is removed from the coolant as it passes through the steam generator. This heat is transferred to water on the secondary (shell) side of the steam generator which boils and is sent to the turbine. The secondary side of the plant is virtually identical to conventional steam plant systems and contains the usual condenser, feedwater heaters, condensate and feed pumps, etc.

Although steam is generated in the secondary cycle, little or no bulk boiling of the primary coolant is allowed. This is prevented by maintaining the primary coolant several degrees below the boiling point. Another way of saying this is that the coolant is maintained at a pressure somewhat higher than the saturation pressure corresponding to the maximum coolant temperature. For example, 600°F coolant would begin to boil at about 1530 psig, whereas the actual coolant pressure would be kept at ~ 2300 to 2600 psig.

The coolant pressure is controlled by the pressurizer which is a vessel connected to one (and only one) coolant loop through a relatively small piece of pipe. The pressurizer is partly filled with water and is heated with electric emersion heaters until it is at saturation conditions at the primary system operating pressure (i.e., ~ 2300-2600 psig, ~ 650-670°F). The pressurizer thus acts like a piston, exerting pressure on the primary coolant through the surge line. The pressure applied to the coolant at this point is transmitted throughout the remainder of the primary system since the latter is filled solid with water. The fact that the pressurizer is only partially filled with water, the remainder being saturated steam, enables it to function as a pressure controller. When pressure starts to fall, pressurizer liquid flashes in order to attempt to maintain the steam space pressure. Conversely, if pressure starts to rise, steam condenses in order to minimize the steam space pressure increase.

Present day PWR's use a combination of control rods and chemical shim for reactor control. The latter consists of boric acid dissolved in the

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[1] Current PWR's are designed to produce between 250 and 350 MWe per coolant loop, depending on the manufacturer. Thus a 1000 MWe plant would utilize 3 or 4 primary coolant loops.

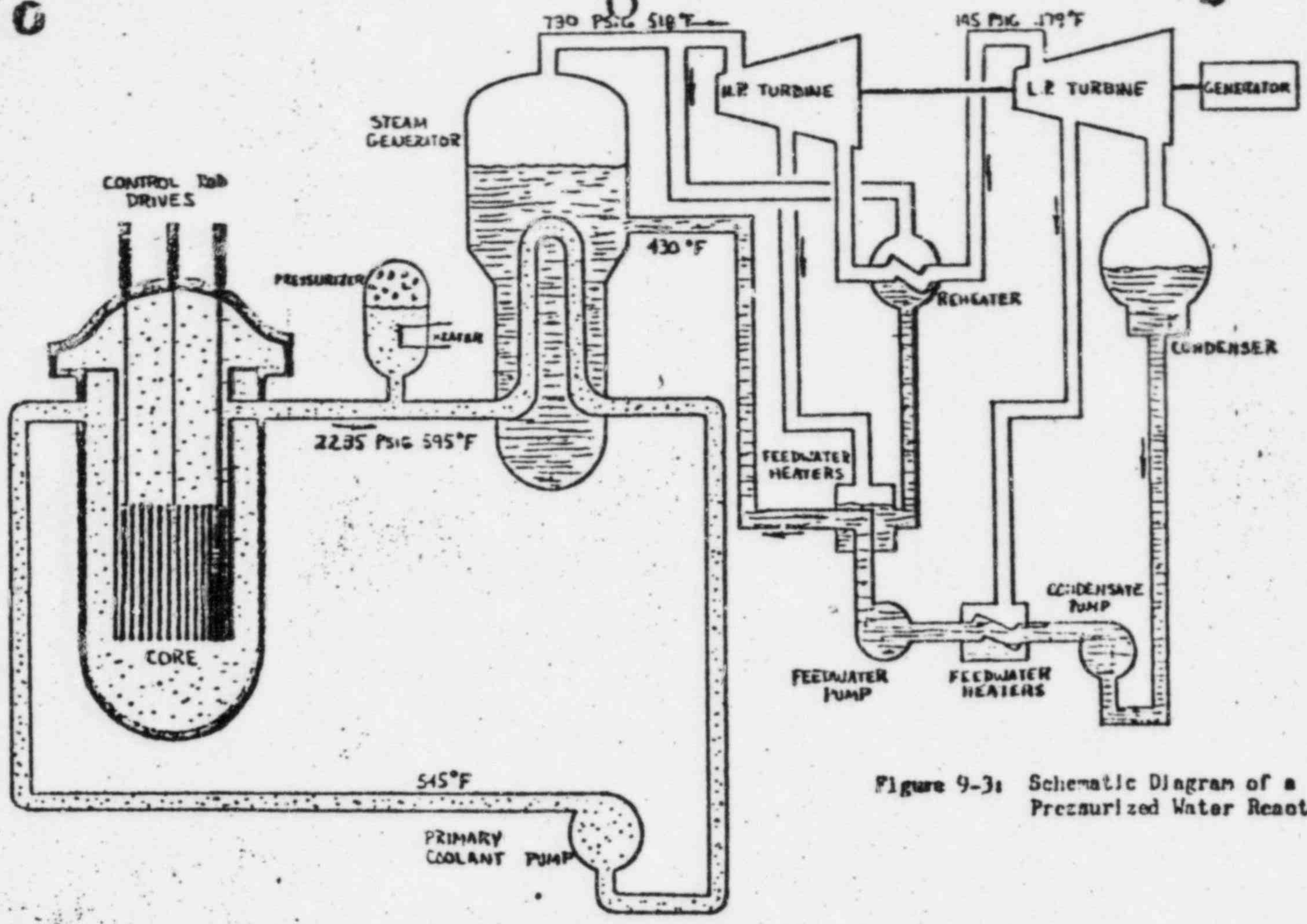

Figure 9-3: Schematic Diagram of a Pressurized Water Reactor

coolant. The control rods are essentially fully withdrawn at full power and are used mainly for load swings and to provide rapid shutdown capability. The boric acid is used to compensate for the extra fuel which is included when the core is new, and the boric acid is gradually removed from the coolant (by dilution and demineralization) as power operation progresses and the extra fuel is depleted. The use of a uniformly distributed poison produces a more uniform power distribution than was possible in earlier PWR's which only used control rods (the latter tend to strongly reduce the power in their general vicinity and have a lesser effect upon other regions of the core).

Load changes on a PWR are normally accomplished in the following manner. The turbine control valves are manually opened or closed as desired, thereby changing secondary flow. This causes a change in the amount of heat extracted from the primary coolant in the steam generator and results in a change in primary coolant temperature. An automatic control system senses the change in coolant temperature and moves the control rods to compensate for it, thus adjusting reactor power to match the desired electrical output.

Since the pressurized water reactor concept seemed economically sound and the materials of construction were available, these reactors received early emphasis. They are used almost exclusively for nuclear ships. In addition, the Shippingport plant which went into operation in 1957, was the first U.S. nuclear plant to be built as part of a public utility. Through 1970, roughly half of the light water reactors which have been, or are presently being built in the world, are PWR's. The total capacity of these plants is in excess of 30,000 MWe.

Some advantages of the PWR include:

A. The principle materials used (water, steel, etc.) are commonly employed in many industries and their properties are well known. This also minimized (but did not eliminate) the additional development work for reactor peripheral equipment such as pumps, heat exchangers, etc. Much of this equipment are virtually off-the-shelf items.

B. Separation of the secondary and primary systems means that no radioactivity enters the turbine plant (in the absence of steam generator tube leaks). This results in a savings on shielding and makes this equipment more accessible during operation.

C. A PWR operates at a somewhat higher core power density (MW/ft^3) than does the other major light water reactor (the BWR - see next section) and requires a correspondingly smaller fuel inventory. This higher power density is the result of heat transfer considerations. The cost advantage of the smaller fuel inventory is partially offset by the fact that a PWR requires a slightly higher enrichment.

Disadvantages of the PWR include:

A. In order to obtain coolant temperatures which are sufficient to provide overall plant cycle efficiencies which are acceptable, extremely high pressure equipment must be used. The technology of building the large pressure vessels (both for PWR's and BWR's) has been one of the limiting factors on maximum plant size.

B. The use of steam generators and a considerable quantity of equipment associated with boric acid control, adds to the overall cost and complexity of the plant compared with direct cycle plants.

C. In order to prevent bulk core boiling, maximum coolant temperatures are limited to ~ 600°F. In order to transfer the heat to the secondary fluid, the temperature of the latter must be still lower, typically ~ 515°F. This low temperature (and pressure) steam results in a considerably poorer cycle efficiency than is possible with modern fossile fired plants or some other reactor types. In some cases this disadvantage has been overcome by using fossil fueled boilers to superheat the secondary steam, but in general, this approach has not been economically, or ecologically attractive.

D. The high power density and large stored energy in PWR's makes them rather sensitive to coolant loss or pumping failure.

2. Boiling water (BWR)

Most boiling water reactors are direct cycle plants similar to that shown in Figure 9-4. Water is converted directly to steam in the core, eliminating the need for a steam generator. Since boiling is allowed, these systems operate at considerably lower pressures, typically about 1000 psig, than PWR systems.

As the coolant passes through the core, only a portion of it boils. The remainder is mixed with the incoming feedwater and is recirculated back to the core. Virtually all BWR's employ forced circulation, that is, the coolant is pumped back to the vessel. In some of the early small plants, notably Humboldt Bay, natural circulation was used. This depends upon the difference in density between the coolant in the core, which contains a substantial amount of steam voids, and the solid water which is being recirculated.

The large new BWR's use jet pumps in addition to centrifugal pumps for coolant recirculation. The figure shows a plant of this type. The principle reason for adopting this scheme is simply to reduce the number and size of the centrifugal recirculation pumps. This results in savings in reduced pumping power costs and in maintenance.

The steam leaving the reactor vessel must be dried so that its quality is satisfactory for turbine operation and so that the amount of radioactivity transported to the turbine plant equipment is minimized (many radioactive isotopes, notably the iodines and cesiums, are soluble in water and prefer to stay in the liquid). The older BWR's used screen mesh dryers, but the new plants use standpipes with internal swirling vanes which separate the steam-water mixture by centrifugal force.

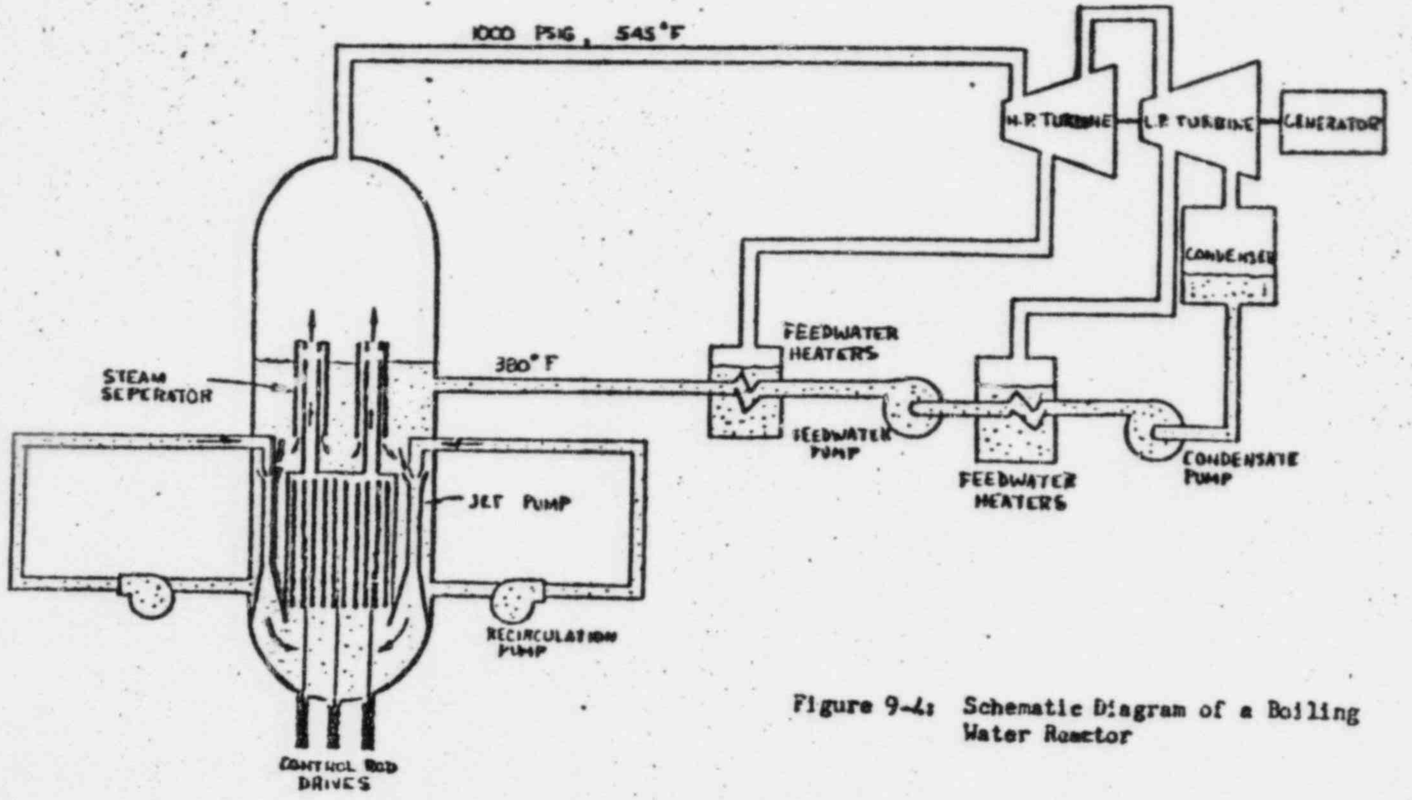

Figure 9-4: Schematic Diagram of a Boiling Water Reactor

BWR shutdown capability comes entirely from control rods. These rods ordinarily enter the bottom of the core because the top of the core is already partially poisoned due to the presence of steam voids. The reasons why voids act as a poison will be considered in detail in Chapter 10, but in simplest terms it results from the fact that steam is a less efficient moderator than water, and as a result neutrons take longer to slow down in voided regions and more of them get captured in U-238 resonances. Chemical shim is not used because of reactor stability considerations resulting from poison displacement by steam voids.

Load changes are accomplished by two general techniques in a BWR. The first is by control rod movement. The turbine control valves are operated by a reactor pressure sensor. When control rods are withdrawn, reactor power rises, producing more steam, and causing reactor pressure to start rising. The reactor pressure sensor will see this increase and will open the turbine control valves sufficiently to accept the extra steam which is being produced. The second control technique involves varying the coolant recirculation flow rate. As the recirculation flow rate is increased, the steam which is produced in the core represents a smaller and smaller fraction of the total coolant in the core. In effect, at high liquid flows, the core is more efficiently moderated and reactor power tends to rise. This would produce a response by the turbine in the manner described above. The reverse is true for reductions in recirculation flow rate.

The BWR and PWR plants are exceedingly close together from an economic standpoint, and since neither concept enjoys a clear-cut advantage, utility orders have been split essentially 50-50 between the two types of plants. In 1970, the capacity of BWR's either in service or on order totaled more than 30,000 MWe.

The major advantages of the BWR include:

A. Advantages resulting from use of conventional materials as discussed for PWR.

B. Simple cycle with minimum of auxiliary equipment. In particular, elimination of chemical shim results in a substantial reduction in plant equipment and potential maintenance problems. The size of the primary coolant pumps is another area in which the BWR has an advantage over the PWR.

Disadvantages of the BWR concept include:

A. Radioactive materials reach turbine during operation, requiring additional shielding and limiting access during operation. However, the bulk of the activity is short-lived and does not interfere with maintenance during outages.

B. The BWR cannot be operated with excessive core steam vapor volume fractions. As a result the power density is limited to values lower than with PWR's.

C. Although turbine steam temperatures and pressures are higher than those of a comparable PWR, they are still lower than those of modern fossile plants and some of the other reactor types. Since the coolant does boil, however, it is technically possible to superheat it within the reactor. Nuclear superheat has been tried in a few cases (Pathfinder, VESR, Bonus) but all of these projects in the U.S.A. have been abandoned. There is simply not enough economic incentive to warrant the additional R. & D. costs, particularly since the breeder reactors which are on the horizen will employ superheated steam.

3. Homogeneous reactors

The homogeneous reactor concept is one in which the fuel is intimately mixed with the water coolant which also acts as the moderator. There are no fuel elements and generally no control rods are in the core during operation. The coolant and fuel circulate through the reactor vessel, coolant circulating pump and steam generator. Heat is removed from the fuel solution in the steam generator. The fission products in the solution make these components highly radioactive. The primary system must be operated at high pressures comparable to those of the pressurized water systems in order to prevent boiling. A side stream of primary system solution is circulated through a low-pressure system where evaporators are utilized to adjust the concentration of the fuel solution for the high-pressure loop. Equipment is also provided in the low pressure system for fission product removal.

No aqueous homogeneous reactors are operating in this country at present, although there were four reactor experiments that operated at one time or another in recent years. The uranyl sulfate system (HRE-2) operated at a maximum temperature of 570°F and a pressure of 1800 psig. Steam pressure was about 520 psig.

Homogeneous reactors have many advantages due to the mobility of the fuel. The core can be designed simply and compactly with a high power density. The need for fabrication of fuel elements is eliminated. One of the most outstanding features of the system is the ease of control. There is a very sensitive negative temperature coefficient caused by changes in fuel density with temperature which provides a large amount of load following control.

There are some formidable problems inherent with aqueous homogeneous systems. One problem is that of corrosion because of the strongly acid and oxidizing solutions investigated to date. Solution stabilities limit the maximum temperatures attainable and require proper flow conditions to prevent stagnant areas and local overheating. In addition, the solutions become highly radioactive during reactor operation, and the hazard of a solution leak is a large problem. Precautions must be taken to prevent accumulation of a critical mass in parts of the system other than the reactor core.

4. Heavy water reactors

The principle advantage of moderation by heavy water is that natural uranium can be used as a reactor fuel whereas the light water reactors typically operate with enrichments in the range of 2 - 4%. Heavy water is a superior moderator to light water due to its extremely low absorption cross section. The principle disadvantage of heavy water reactors is the cost of the heavy water itself (~ $28 per pound). A second major disadvantage is that a substantial amount of tritium is produced due to neutron activation of deuterium, which adds to the radiation protection problems.

In a heavy water moderated reactor, the fuel rods do not have to be packed as closely together as they do in a light water reactor. This has led to the use of "pressure tubes" rather than pressure vessels to contain the fuel and coolant.

Figure 9-5 shows a schematic diagram of the 100 MWe SGHWR (steam generating heavy water reactor) in England. The fuel is located in pressure tubes and sent to a steam drum and then to the turbine. A portion of the steam is sent to a special superheater fuel bundles and the resulting superheated steam is mixed with the main body of the saturated steam enroute to the turbine. The pressure tubes pierce a low pressure vessel containing the D_2O moderator. This vessel is at atmospheric pressure. Control is obtained from a combination of dissolved boric acid in the moderator, liquid poison in separate tubes in the fuel clusters, and varying moderator level outside the fuel. Individual pressure tubes can be cut out of service for fuel replacement while the plant is at power.

GAS COOLED REACTORS

Gases such as helium and carbon dioxide have been extensively used as reactor coolants. The early interest in this concept arose because it is possible to utilize natural uranium as a fuel when graphite is used as a moderator. Thus countries like England, which had an early need for nuclear power but had no large scale uranium enrichment facilities, adopted the gas cooled reactor concept.

Use of a gaseous coolant permits the attainment of high temperatures at substantially lower primary system pressures than are possible with a water cooled system. Thus it becomes feasible to obtain cycle efficiencies which are comparable to those of modern fossile fueled plants. In addition, the gas cooled reactor can be adopted to either thermal converter or fast breeder plants. The former employs a graphite moderator whereas the latter would not. The applicability of a gas to a fast reactor is brought about

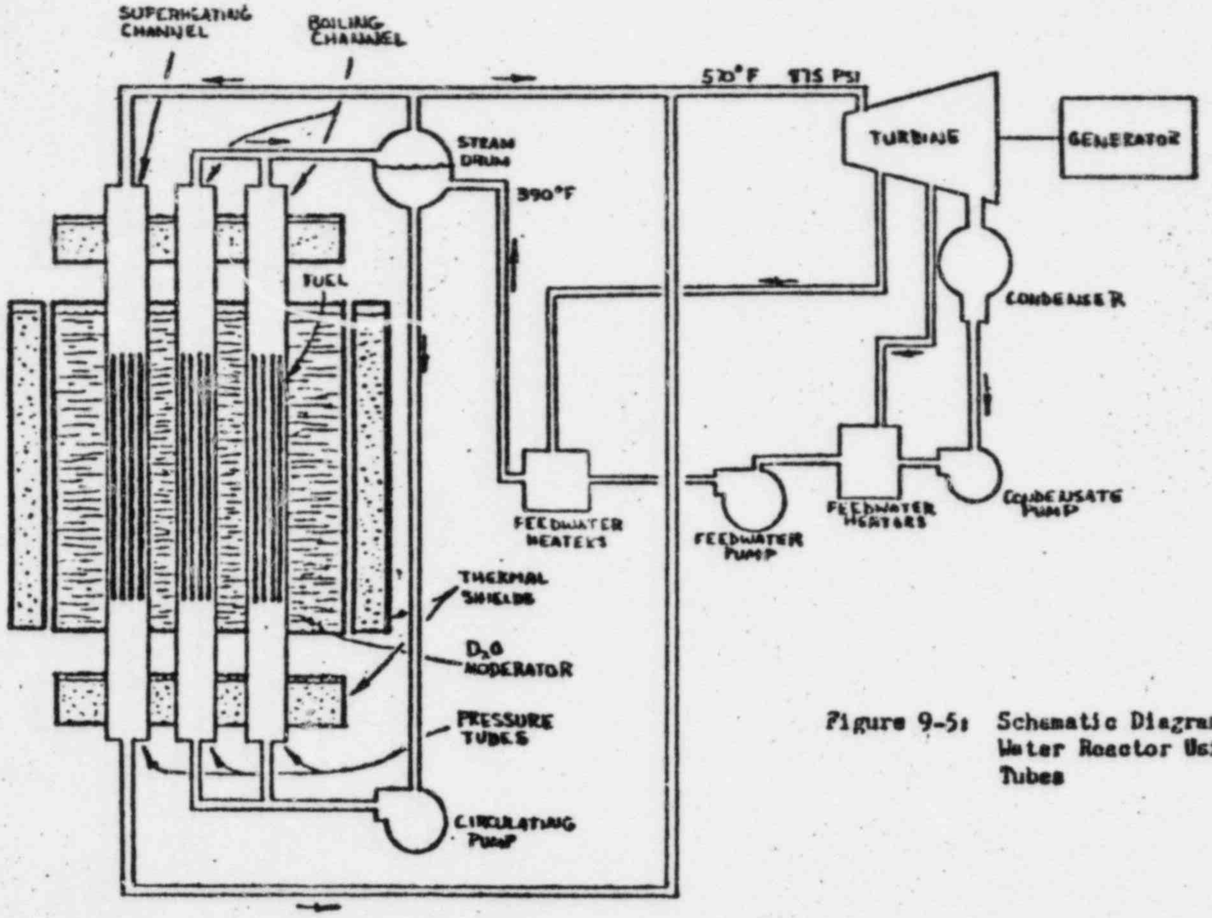

Figure 9-5: Schematic Diagram of a Heavy Water Reactor Using Pressure Tubes

by the relatively low density of the gas, which makes it a poor moderator.

Figure 9-6 shows a schematic of a gas-cooled advanced thermal converter reactor similar to the Fort St. Vrain plant in Colorado. This plant has several unique features including:

1. Use of uranium-thorium fuel cycle. The initial loading will be about 4% U-235 and 96% Th-232. Subsequent fuel loadings will use the U-233 formed from the thorium. At Fort St. Vrain, the fuel is in the form of very small particles (~ .01 inch diameter) of the metal carbides which are coated with layers of pyrocarbons for cladding. These particles are then packed into 0.5 inch diameter graphite cylinders to form fuel rods and the latter placed in drilled hexagonal graphite blocks to form fuel bundles. The graphite blocks also have coolant and control rod passages drilled in them. The core is then made up of a large number of fuel bundles sitting side by side.

2. Use of pre-stressed concrete reactor vessel, which not only houses all the major components of the nuclear steam supply system, but also provides the biological shielding.

For a high temperature gas cooled reactor, it is necessary to use helium as a coolant because carbon dioxide reacts chemically with graphite at temperatures in excess of ~ 650° F.

Control for a HTGR is accomplished entirely with control rods. The load is varied by regulating the turbine control valves. The turbine steam flow is then fed back to the control rod positioners and they are moved to match reactor power to the generator output.

One of the main disadvantages of the concept has been the economic factor imposed by the low volumetric heat capacity of gases and the consequent high coolant pumping power and large primary system equivalent sizes. Another disadvantage results from the fact that it is difficult to design fuel elements with a reasonable life expectancy at the high temperatures of the advanced gas-cooled systems.

LIQUID METAL REACTORS

Liquid metals, principally liquid sodium, have applications for reactor coolants because they allow the attainment of high operating temperatures without the need for high pressures. The coolant system pressure is usually less than 100 psig[1] while temperatures up to 1300° F are obtained. This is made possible because of the excellent thermal properties of the metal

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[1] The 100 psig is due principally to pump head required to overcome system pressure losses. The actual vapor pressure of liquid sodium is only ~ 1 psia at 1200° F.

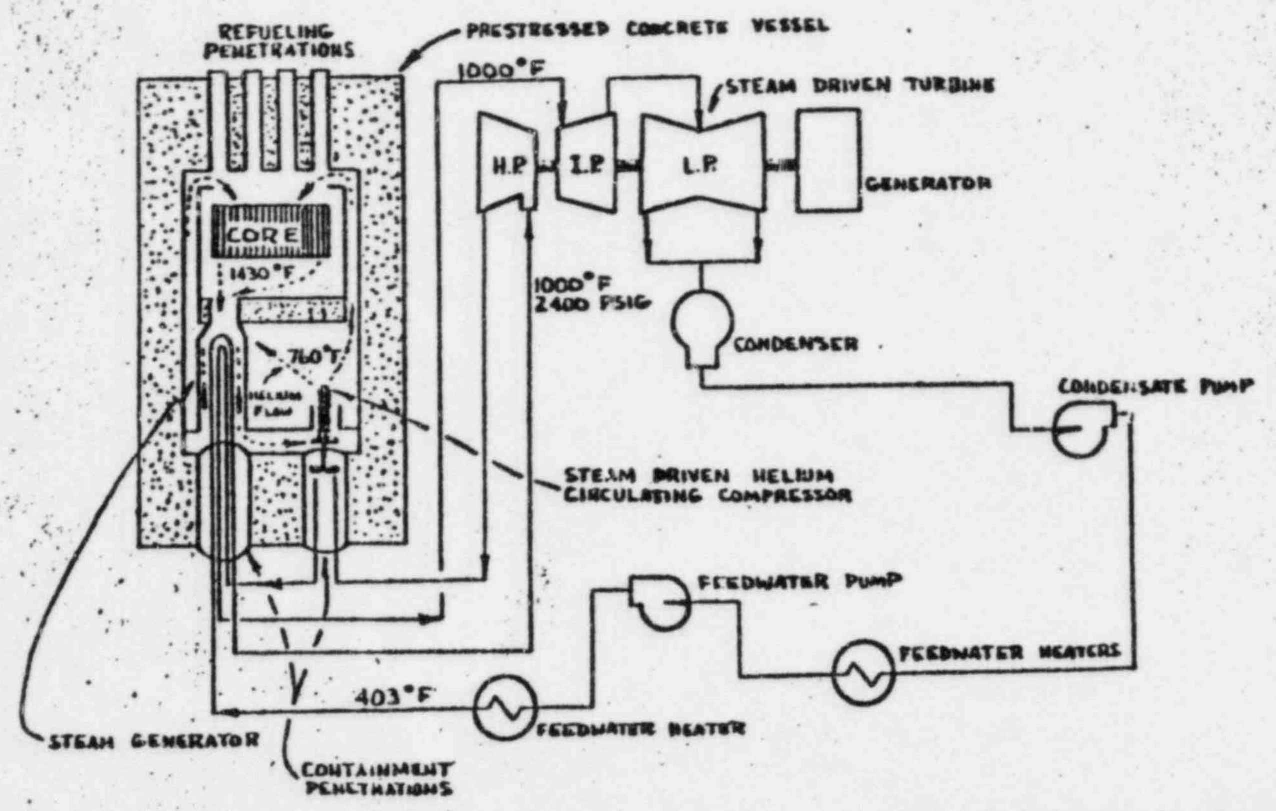

Figure 9-6: Schematic Diagram of a High Temperature Gas-cooled Reactor

coolants (high specific heats and volumetric heat capacity, and low vapor pressure). The high temperatures allow attainment of steam conditions (approaching steam temperatures of 1000° F) which give net plant efficiencies of greater than 35%.

Although liquid metal cooled, graphite moderated, thermal reactors have been built (notably the 75 MWe Hellam plant in Nebraska, which was decommissioned following failure of moderator elements in the core), their widespread use will be as fast breeder reactors. As discussed previously, a fast reactor does not employ a moderator (liquid sodium is too heavy to be an effective moderator) and depends upon fission by fast neutrons to sustain the chain reaction. Breeder reactors are reactors which produce more fissionable material by neutron absorption in fertile material than they consume. It is expected that these reactors will achieve breeding ratios[1] of ~ 1.4, which means that they will double their initial charge of fissile material in about 7 years of operation. Since the world's supply of fertile material (U-238 and Th-232) is quite large, this will result in sufficient fissile material being produced to supply man's needs for centuries to come.

A schematic diagram of a typical LMFBR is shown in Figure 9-7. Notice that it contains an intermediate sodium loop between the primary coolant loop and the steam system. This is necessitated by the fact that:

1. Sodium readily activates as it passes through the core due to the reaction Na^{23} (n,γ) Na^{24} and it is necessary to prevent any escape of primary coolant from the system.

2. Sodium reacts vigorously with water to form sodium hydroxide and hydrogen gas ($2Na + 2H_2O \rightarrow 2NaOH + H_2$). In a closed container such as a steam generator, a leak can result in high pressures from the hydrogen formed and form the rapid heating of the water and steam. (However, there is no explosion if oxygen is excluded from the hydrogen). Thus it is possible that some sodium from the primary system could be released if it came into direct contact with water. The use of an intermediate loop prevents this.

Sodium is an opaque fluid, and as a result, fuel handling and maintenance in the core must be done "blind." Automatic fuel handling machines are used for this purpose. In addition, sodium reacts chemically with air, and so the reactor vessel must be maintained in an inert atmosphere of helium, argon, or nitrogen.

Special attention must be paid to the control systems of LMFBR's since they do not possess as many inherent safety characteristics as do light water reactors. For example, the moderator temperature and void coefficients are positive since any reduction in moderating characteristics of the core.

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[1] Breeding ratio is defined as number of fissile atoms produced per fissile atom consumed.

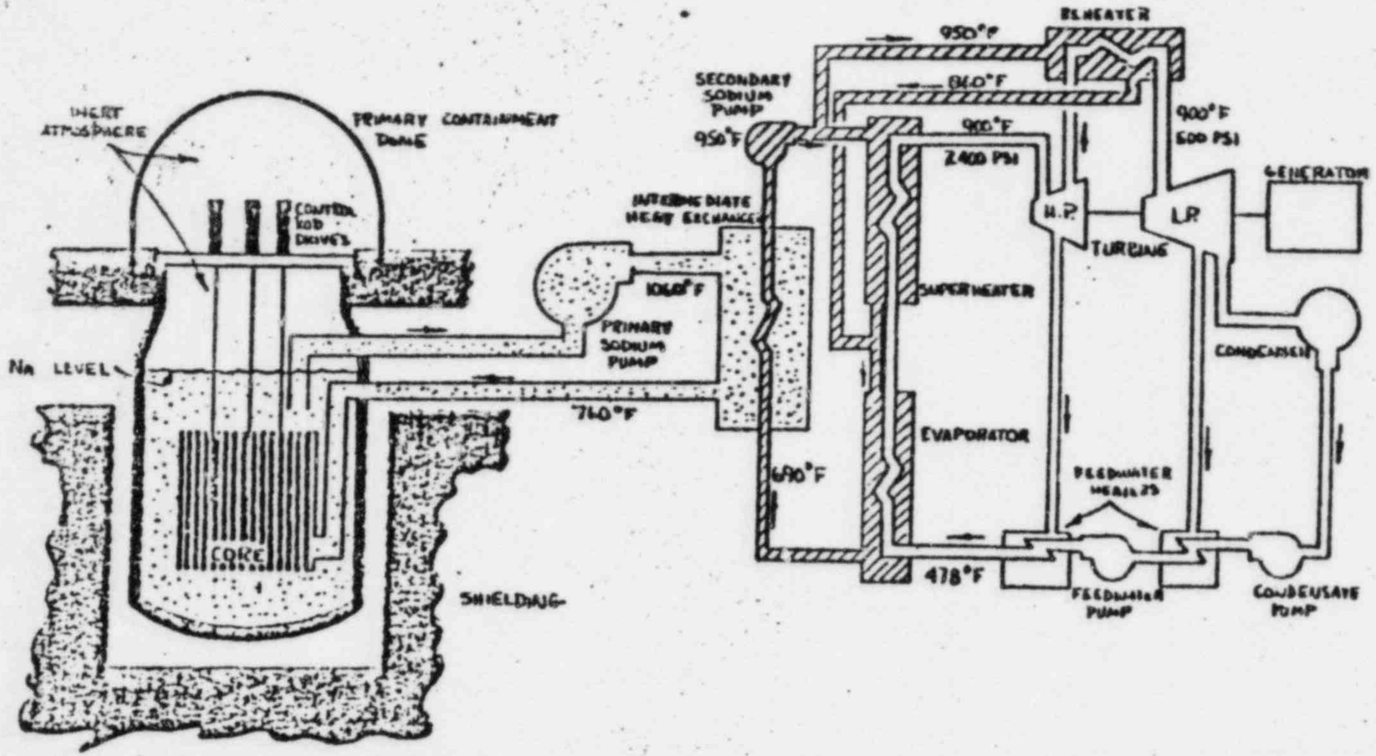

Figure 9-7: Schematic Diagram of a Liquid Sodium Fast Breeder Reactor

tends to increase reactivity in a fast reactor. The doppler coefficient is negative, but less important than in a water reactor because the majority of the neutrons in a fast reactor remain at energies above the resonance peaks of U-238. In addition, the neutron lifetime in a fast reactor is shorter than in a thermal reactor, and Pu-239 (which will be the ultimate breeder reactor fuel) has a smaller delayed neutron fraction than does U-235. The significance of these statements will be made clearer by the discussions of later chapters.

CHAPTER 10

REACTIVITY CONSIDERATIONS IN WATER MODERATED REACTORS

INTRODUCTION

To this point our discussion of nuclear reactors has been limited, with the major emphasis being directed toward a brief description of how the various types of reactors are utilized for the generation of electric power. In this chapter we will look in detail at the processes occurring within one general class of reactors - the water moderated reactors. However, many of the principles which we will discuss are common to all types.

MULTIPLICATION FACTOR

As a first step in the development of a quantitative picture of the nuclear reactor, it is necessary to define a new term. The multiplication factor, k, is defined as the ratio of the number of neutrons in one generation to the number of neutrons in the previous generation. A generation is the period of time between successive fissions. Figure 10-1 illustrates chain

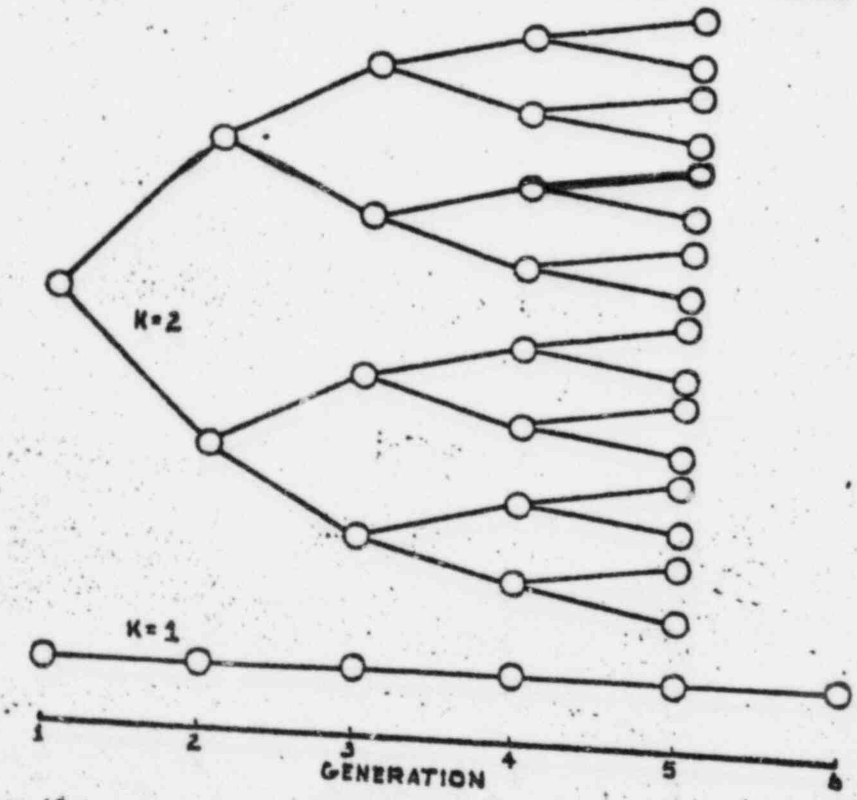

Figure 10-1: Number of Neutrons in Successive Generations for Multiplication Factors of k=2 and k=1

reactions for k=2 and k=1. The value k=2, as will be shown later in this chapter, is unrealistically high and would never be achieved nor desired in a real reactor. It is only given as an example because it is easy to draw. Study of Figure 10-1 indicates that if k is exactly equal to 1, a chain reaction will proceed with constant power level and the reactor will be just critical. On the other hand, if k is greater than 1, the reactor power will continue to increase with time and the reactor is said to be supercritical. Similar reasoning leads us to the conclusion that if k is less than 1, the number of neutrons present in the reactor will be continuously decreasing. In this case, the chain reaction is not self-sustaining and the reactor is said to be subcritical.

In order to obtain a better understanding of the operation of a reactor let us consider the sequence of major events which neutrons undergo from their "birth" through fission to their "death" through absorption in some material or leakage from the reactor. Consider N fast neutrons (≈2 mev) released through thermal[1] fission of U-235. Since the fuel is generally lumped together in the form of rods or plates, there is a definite probability that some of the original N neutrons will strike other uranium atoms before they have had a chance to reach the moderator and slow down. Since the neutrons are still above the threshold energy for fission in U-238 (≈1 mev), and since U-235 also has a small fission cross section for fast neutrons, some fissions will occur and the fast neutron population will be increased. After fast fission has occurred there will be $N\epsilon$ fast neutrons, where ϵ, the fast fission factor, is a number greater than one which measures the increase in the fast neutron population due to fast fissions in U-235 and U-238 (or any other fissionable material which is present in the reactor). Now the $N\epsilon$ fast neutrons are ready to enter the moderator and begin to slow down. But before they do, there is a definite chance that some of them, in particular those born near the edge of the core, will leak out of the reactor and be lost. The probability that a fast neutron will <u>not</u> leak out of the core is called the fast non-leakage probability, $\mathcal{L}_f$. Since $\mathcal{L}_f$ represents the fraction of the total number of fast neutrons which remain in the core, a total of $N\epsilon\mathcal{L}_f$ neutrons actually begin the slowing down process. As the fast neutrons slow down, they encounter the U-238 absorption resonances (see Figure 8-4 in Chapter 8). The probability that a neutron will <u>not</u> be captured in the U-238 resonances is called the resonance escape probability, p. Since p represents the fraction of the total number of neutrons which remain in the core, $N\epsilon\mathcal{L}_f p$ neutrons will reach thermal energy. As the thermal neutrons move around the reactor, a certain percentage of them will leak out. The probability that a thermal neutron will <u>not</u> leak out of the core is called

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[1] It has become common terminology to speak of "thermal" fission, absorption, and leakage when discussing the six factor formula. The student should realize, however, that "thermal" truly refers to a specific neutron energy, as discussed in Chapter 7, whereas the "thermal" quantities discussed above are actually averages over the slow range. The "thermal" terminology is thus not strictly correct, but it is so widespread that we will continue to use it.

the thermal non-leakage probability, $\mathcal{L}_t$. Therefore, after thermal leakage has occurred there will be $N\varepsilon\mathcal{L}_f p\mathcal{L}_t$ remaining in the core. All of the thermal neutrons which do not leak from the core will end up being absorbed in one of the various materials in the core. The only neutrons which will be of any use in furthering the chain reaction will be those which are absorbed in fissionable material. The probability that a thermal neutron will be absorbed in fissionable material as opposed to all other reactor materials is termed the thermal utilization, f. Therefore, the number of thermal neutrons absorbed in fissionable material is $N\varepsilon\mathcal{L}_f p\mathcal{L}_t f$. Of the thermal neutrons absorbed in fissionable material, some will cause fissions and some will be wasted in (n,γ) reactions. The fuel quality, η, is defined as the number of fission neutrons produced per thermal neutron absorbed in fuel. Thus the total number of new fission neutrons will be $N\varepsilon\mathcal{L}_f p\mathcal{L}_t f\eta$. This latter expression represents the number of fast neutrons available at the start of the next generation. The ratio of this figure to the original number of neutrons, N, will be the multiplication factor k. That is:

$$k = \frac{N\varepsilon\mathcal{L}_f p\mathcal{L}_t f\eta}{N} = \varepsilon\mathcal{L}_f p\mathcal{L}_t f\eta$$

The multiplication factor given in the previous formula is usually called the effective multiplication factor, or k_{eff}. Thus:

$$k_{eff} = \eta\varepsilon p f\mathcal{L}_f\mathcal{L}_t \qquad (10\text{-}1)$$

Equation (10-1) is often referred to as the six factor formula. The derivation of the six factor formula is illustrated in Figure 10-2. The figure assumes that there are 1000 neutrons at the start of the hypothetical generation. Typical values for each of the six factors are assumed for illustrative purposes. Although the values chosen are realistic, there can be considerable variation in any of them depending upon the particular reactor, temperature, fuel burnup, and control rod position, etc.

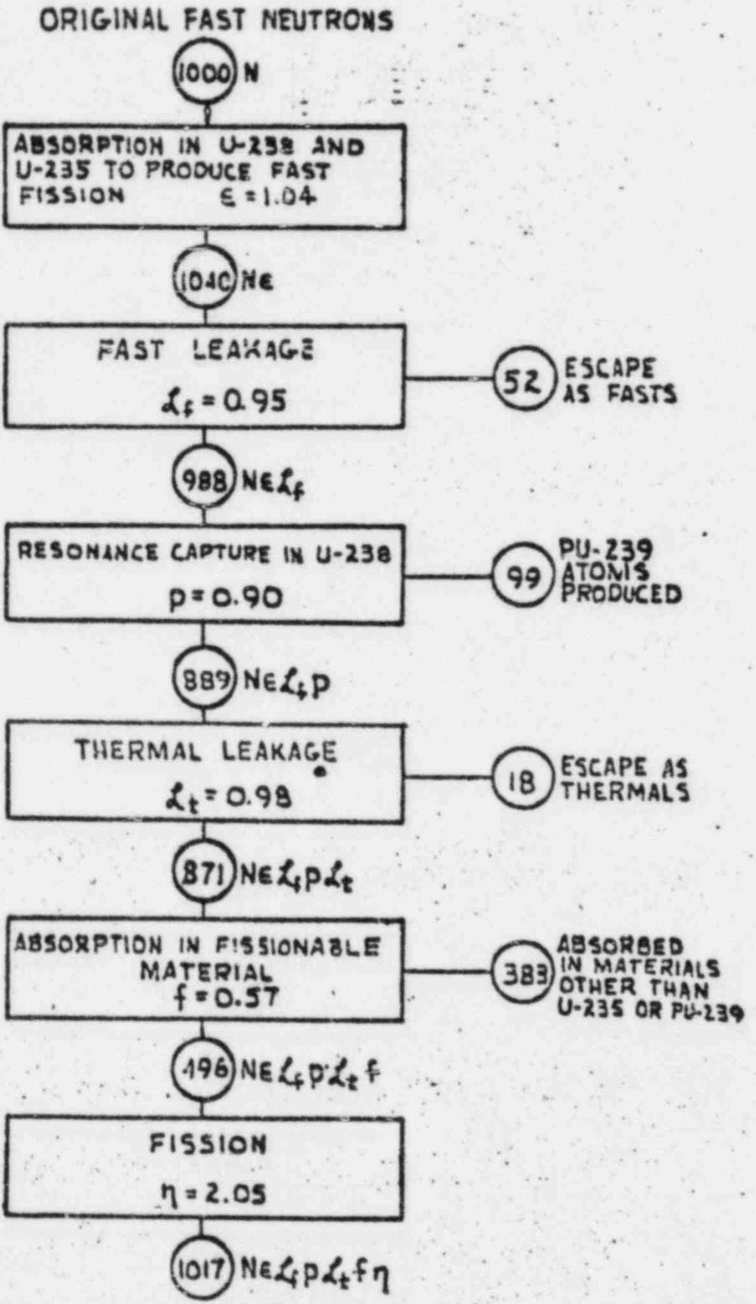
Leakage can never be completely eliminated in a real reactor because a real reactor must of necessity be finite in size and therefore some neutrons will always be able to reach its boundaries. However, to aid in the solution of certain problems, the reactor physicist often finds it convenient to consider a reactor in which there is no leakage. Such a reactor would have to be infinite in size so that a neutron could never reach its boundaries. The multiplication factor for such a reactor is called the infinite multiplication factor, or k_∞. Equation (10-1) will hold for k_∞ except, of course, the two non-leakage probabilities $\mathcal{L}_f$ and $\mathcal{L}_t$, will both be equal to 1. Therefore:

$$k_\infty = \eta\varepsilon p f \qquad (10\text{-}2)$$

Equation (10-2) is often referred to as the four factor formula.

If equation (10-2) is substituted into equation (10-1), we see that:

$$k_{eff} = k_\infty\mathcal{L}_f\mathcal{L}_t \qquad (10\text{-}3)$$



$$K_{eff} = \frac{1017}{1000} = 1.017$$

Figure 10-2: Life History of Neutrons in a Typical Reactor

Since $\mathcal{L}_f$ and $\mathcal{L}_t$ are both numbers less than 1, it follows from equation (10-3) that k_{eff} must always be less than k_∞.

Before considering each of the six factors in equation (10-1) in greater detail, it is well to summarize the discussion by giving a more precise definition of each of the terms introduced to this point. Thus:

$$k_{eff} = \frac{\text{number of neutrons in any generation of a finite sized reactor}}{\text{number of neutrons in the previous generation}} \qquad (10\text{-}4)$$

$$k_\infty = \frac{\text{number of neutrons in any generation of an infinite reactor}}{\text{number of neutrons in the previous generation}} \qquad (10\text{-}5)$$

$$\eta = \frac{\text{number of fast neutrons produced by thermal fission}}{\text{number of thermal neutrons absorbed in fissile material}} \qquad (10\text{-}6)$$

$$f = \frac{\text{number of thermal neutrons absorbed in fissile material}}{\text{number of thermal neutrons absorbed in all reactor materials}} \qquad (10\text{-}7)$$

$$\varepsilon = \frac{\text{number of fast neutrons produced by fissions of all energies}}{\text{number of fast neutrons produced by thermal fission}} \qquad (10\text{-}8)$$

$$p = \frac{\text{number of fast neutrons which slow down to thermal}^{1}}{\text{number of fast neutrons which start to slow down}} \qquad (10\text{-}9)$$

$$\mathcal{L}_f = \frac{\text{number of fast neutrons which remain in the core after leakage}}{\text{total number of fast neutrons in the core before leakage}} \qquad (10\text{-}10)$$

$$\mathcal{L}_t = \frac{\text{number of thermal neutrons which remain in the core after leakage}}{\text{number of thermal neutrons in the core before leakage}} \qquad (10\text{-}11)$$

In general, the evaluation of the six factors is a complex mathematical problem with which we will not concern ourselves. Nevertheless, it is instructive to look at how various reactor parameters affect the six factors, at least in a qualitative sense.

1. η

This factor can be calculated for a particular type of fuel providing the number of neutrons given off per fission, ν, is known as well as the appropriate absorption and fission cross sections. The value for η is then calculated from the following relationship:

$$\frac{\text{neutrons produced}}{\text{absorption}} = \frac{\text{neutrons produced}}{\text{number of fissions}} \times \frac{\text{number of fissions}}{\text{number of absorptions}}$$

$$\eta = \nu \frac{\sigma_f}{(\sigma_f + \sigma_{n,\gamma})} \qquad (10\text{-}12)$$

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[1] The numerator of this term is calculated on the assumption that there is no leakage during the slowing down process. This leakage is accounted for in $\mathcal{L}_f$.

At the standard thermal energy of 0.025 ev (70° F), the appropriate cross sections for U-235 are: σ_f = 580 b, $\sigma_{n,\gamma}$ = 107 b. The value of ν for the fission of U-235 is 2.43, so η = (2.43)(580)/(687) = 2.05. The corresponding values of ν and η for Pu-239 are 2.89 and 2.08, respectively.

As discussed in Chapter 7, the energy of a thermal neutron varies considerably as the moderator temperature is increased from room temperature to the full power value. The average neutron energy in the slow range varies even more because it includes effects such as incomplete thermalization when steam voids displace liquid moderator. Thus, it is instructive to look at the variation of η with neutron energy over the slow neutron energy range (in which the majority of the fissions occur) as shown in Figure 10-3. The variations shown in the figure are principally due to variations in the ratio $\sigma_f/(\sigma_f + \sigma_{n,\gamma})$. The values for ν do increase somewhat as neutron energy increases, but are essentially constant for both U-235 and Pu-239 over the slow neutron range. It is seen that η for U-235 is nearly independent of energy, but that the value for Pu-239 shows a generally decreasing trend with a significant depression near the absorption resonance peak at 0.29 ev (see Figure 8-2).

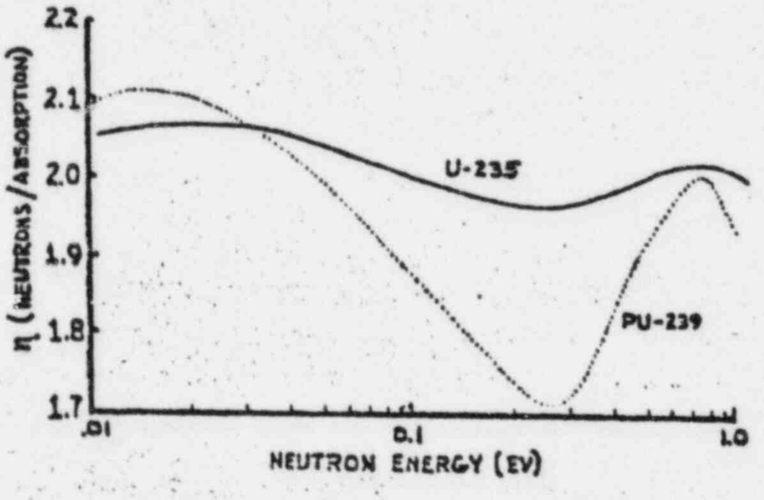

Figure 10-3: Variation of η with Neutron Energy

The average neutron energy in the core is dependent basically upon two quantities - the relative amounts of moderator and fuel, and the temperature of the system. In general, the more moderator which is available, the more effective will be the slowing down process and the lower will be the average neutron energy. Similarly, the lower the temperature, the lower will be the average neutron energy. The significance of these observations will be discussed later in this chapter.

2. f

The thermal utilization can be calculated if the thermal neutron absorption rates in all reactor materials are known. The reaction rate in any material is given by equation (7-12). The absorption rate in U-235 is therefore $\phi^{fuel} \Sigma_a^{235} V^{fuel}$ and the absorption rate in the moderator is $\phi^{mod} \Sigma_a^{mod} V^{mod}$. Finally, neutrons are absorbed in numerous other reactor materials such as U-238, control materials, cladding, structural materials, and fission products. For illustrative purposes, we will lump these all together and assume the absorption rate is given by $\phi^{other} \Sigma_a^{other} V^{other}$. In the previous expressions, the ϕ terms are the average thermal neutron fluxes in the various materials and the V terms are the volumes of the various materials. The thermal utilization is the U-235 absorptions[1] divided by the total absorptions, or:

$$f = \frac{\phi^{fuel} \Sigma_a^{235} V^{fuel}}{\phi^{fuel} \Sigma_a^{235} V^{fuel} + \phi^{mod} \Sigma_a^{mod} V^{mod} + \phi^{other} \Sigma_a^{other} V^{other}} \qquad (10\text{-}13)$$

Sometimes the top and bottom of this expression are divided by ϕ^{fuel} to give:

$$f = \frac{\Sigma_a^{235}}{\Sigma_a^{235} V^{fuel} + (\phi^{mod}/\phi^{fuel}) \Sigma_a^{mod} V^{mod} + (\phi^{other}/\phi^{fuel}) \Sigma_a^{other} V^{other}} \qquad (10\text{-}14)$$

The ratio (ϕ^{mod}/ϕ^{fuel}) is often called the disadvantage factor, and is usually about 1.1 to 1.2 in water moderated UO_2 lattices. What this means is that the average thermal neutron flux in the fuel is lower than that in the moderator. A typical flux distribution through a fuel rod is shown in Figure 10-4. The thermal neutrons which are incident upon the surface of the fuel rod are rapidly absorbed because of its high cross section. As a result, the central regions of the fuel rod are somewhat depleted of thermal neutrons.

Note that macroscopic cross sections and volumes must be used in equations (10-13) and (10-14) because the various materials in the reactor are not present in equal quantities. Using the cross section per atom, σ_a, will not give the correct result because it does not take into consideration that there are more atoms of some materials than of others.

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[1] This discussion assumes U-235 is the only material in the fuel which undergoes thermal fission. If Pu-239 is also present, its cross section must also be included in the fuel terms.

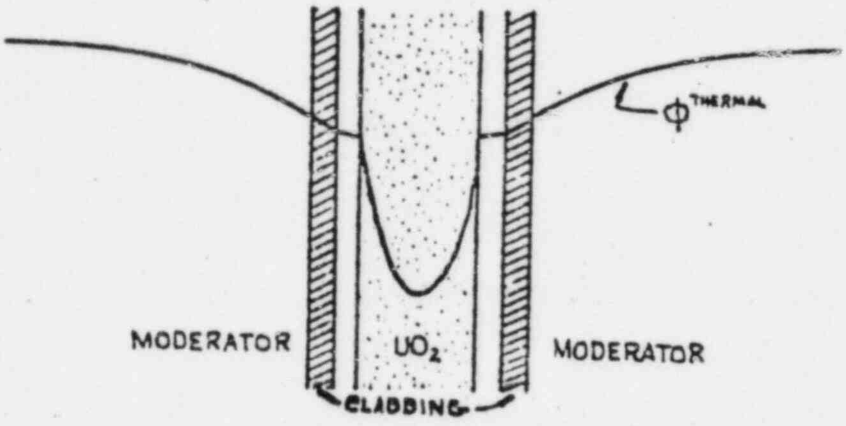


Figure 10-4: Thermal Neutron Flux Distribution in the Vicinity of a Fuel Rod

We will not carry out any calculations using the equations for f. The main reason for presenting them here is that they very conveniently show the effect of a poison (i.e. a material with a high absorption cross section) upon the reactor. The introduction of a poison into the reactor, such as the buildup of certain fission products, markedly increases Σ_a^{other}. Since Σ_a^{other} appears in the denominator of the fraction, any increase in it will result in a decrease in f and a corresponding decrease in k_{eff}. The major effect of a control rod is its influence on the thermal utilization.

It should be evident from the expressions for thermal utilization that it is strongly dependent upon the relative amounts of various reactor materials, the principle ones of which are moderator and fuel. One question the fuel designer initially faces is the determination of the proper volume ratio of moderator to fuel. Most reactors now being constructed have a moderator to fuel volume ratio of about 2:1 to 3:1. Once the fuel is built, this ratio is fixed. However, it is still possible to vary the ratio of moderator to fuel on a weight or molecular basis. The weight or molecular ratio changes significantly as the temperature and steam void content of the moderator changes. As moderator temperature increases, or steam voids are produced, the density of the moderator falls and the weight of the moderator (or number of molecules) in the portion of the core occupied by the moderator is reduced.

Figure 10-5 shows how variations in the ratio of moderator molecules to fuel molecules influence the thermal utilization. As N_{H_2O}/N_{UO_2} approaches 0 (i.e. when there is little or no moderator in the core) the probability that a neutron will be absorbed in fissile material

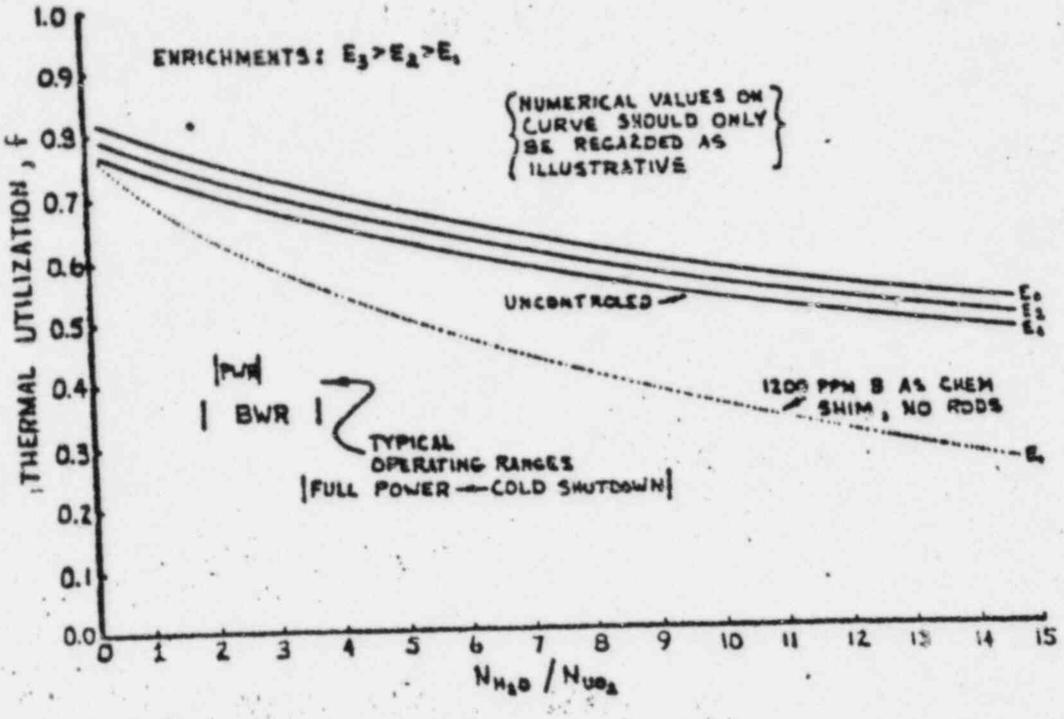


Figure 10-5: Effect of Moderator to Fuel Ratio and Enrichment Upon Thermal Utilization for Typical Water Moderated Reactors Operating With and Without Chemical Shim

is improved. Hence, the thermal utilization increases. Conversely, as N_{H_2O}/N_{UO_2} increases (i.e. the relative amount of fuel in the core is reduced) the thermal utilization must decrease. The figure also shows that an increase in enrichment increases the thermal utilization due to the fact that an increase in enrichment involves the substitution of a fissile absorber, U-235, for a non-fissile absorber, U-238. Figure 10-5 has two sets of curves, one for a core operating without boric acid dissolved in the coolant, and the other for a core operating with about 1200 ppm of boron in the coolant in the form of dissolved boric acid. The former curve would be representative of boiling water reactors, which do not use chemical shim for reactivity control, and the other is representative of a present day pressurized water reactor at beginning of core lifetime. The presence of the boric acid markedly increases Σ_a^{other} in equation (10-13) due to the high thermal neutron absorption cross section of boron, and thus results in a significant reduction in thermal utilization. The point to note about the shape of the two curves is that as the moderator to fuel ratio is reduced, the thermal utilization increases much more rapidly when the core contains chemical shim than in the case where no chemical shim is employed. The reason for this is that the removal of moderator from the core in the case where chemical shim is employed simultaneously removes an equivalent amount of dissolved boron. Thus, the reduction in the amount of moderator in the core also reduces the poison effect of the boron and the thermal utilization rises substantially. The significance of this will be discussed further in later sections of this chapter.

The figure also shows typical values of N_{H2O}/N_{UO2} for PWRs and BWRs over the operating range from cold shutdown to full power. Cold shutdown would be located at the right hand side of the range, and the operating point would move to the left as a system was heated and power raised. For reasons which will be considered later, the BWR generally has a higher moderator to fuel volume ratio than a PWR, and therefore has a higher value of N_{H2O}/N_{UO2} at shutdown. However, the introduction of voids into the moderator of a BWR gives it a wider spread of operating values, and therefore the two reactor types end up at about the same place at full power.

The previous discussions are based on the assumption that U-235 is the only fissile material in the core. If Pu-239 is present, it must also be considered in expressions for f. Since Pu-239 has a higher microscopic thermal absorption cross section than U-235 (1029 b vs. 683 b), the direct substitution of Pu-239 for U-235 would result in an increase in f. In practice, however, Pu-239 is only produced as the reactor is operated, and thus it only appears at the expense of U-235 depletion. For breeder reactors, Pu-239 is built in faster than U-235 is depleted, and f will actually increase throughout much of core life (ultimately the increase will come to an end because as the concentration of Pu-239 increases its depletion rate through fission will also increase until it eventually balances the production rate). This behavior, where f increases as the fuel burns is also possible in convertor reactors for about the first 1000 MWD/Ton if the initial production rate of Pu-239 per U-235 atom destroyed is about 0.8 or so. To accomplish this, the enrichment must be less than ≈2% (as enrichment is increased, there is less U-238 in the core and hence less Pu-239 produced). Otherwise, the factor f decreases continuously throughout core life. However, even in this case, f decreases more slowly throughout core life than it would without the contribution from Pu-239 buildup. These ideas are illustrated in Figure 10-6 below.

3. ε

The fast fission factor varies from about 1.02 to about 1.1 for water moderated reactors. It is most strongly affected by the moderator to fuel ratio as shown in Figure 10-7. As the amount of moderator is reduced, the fast fission factor increases due to the fact that neutrons are less likely to encounter a moderator molecule and be slowed down before they can cause a fast fission.

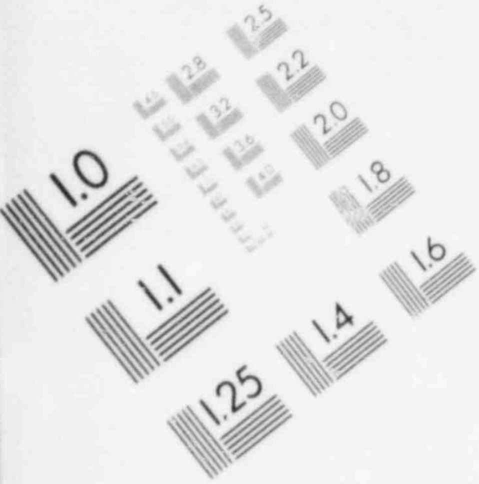
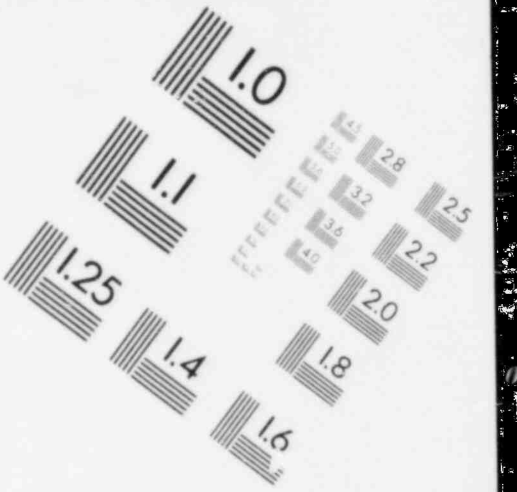
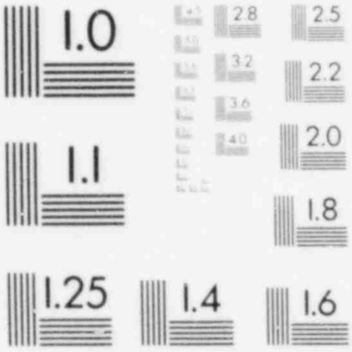
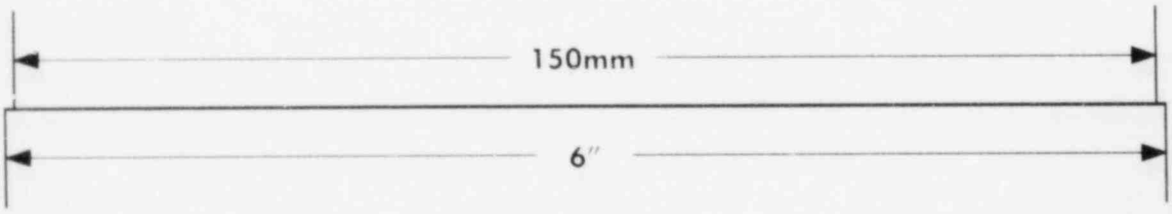
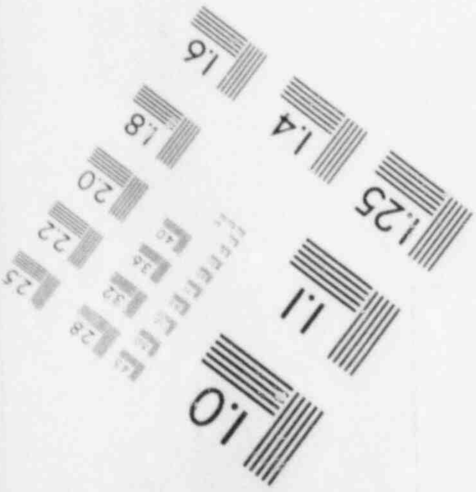


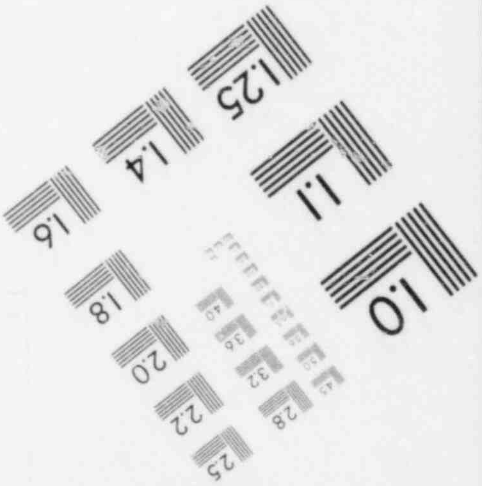
IMAGE EVALUATION TEST TARGET (MT-3)











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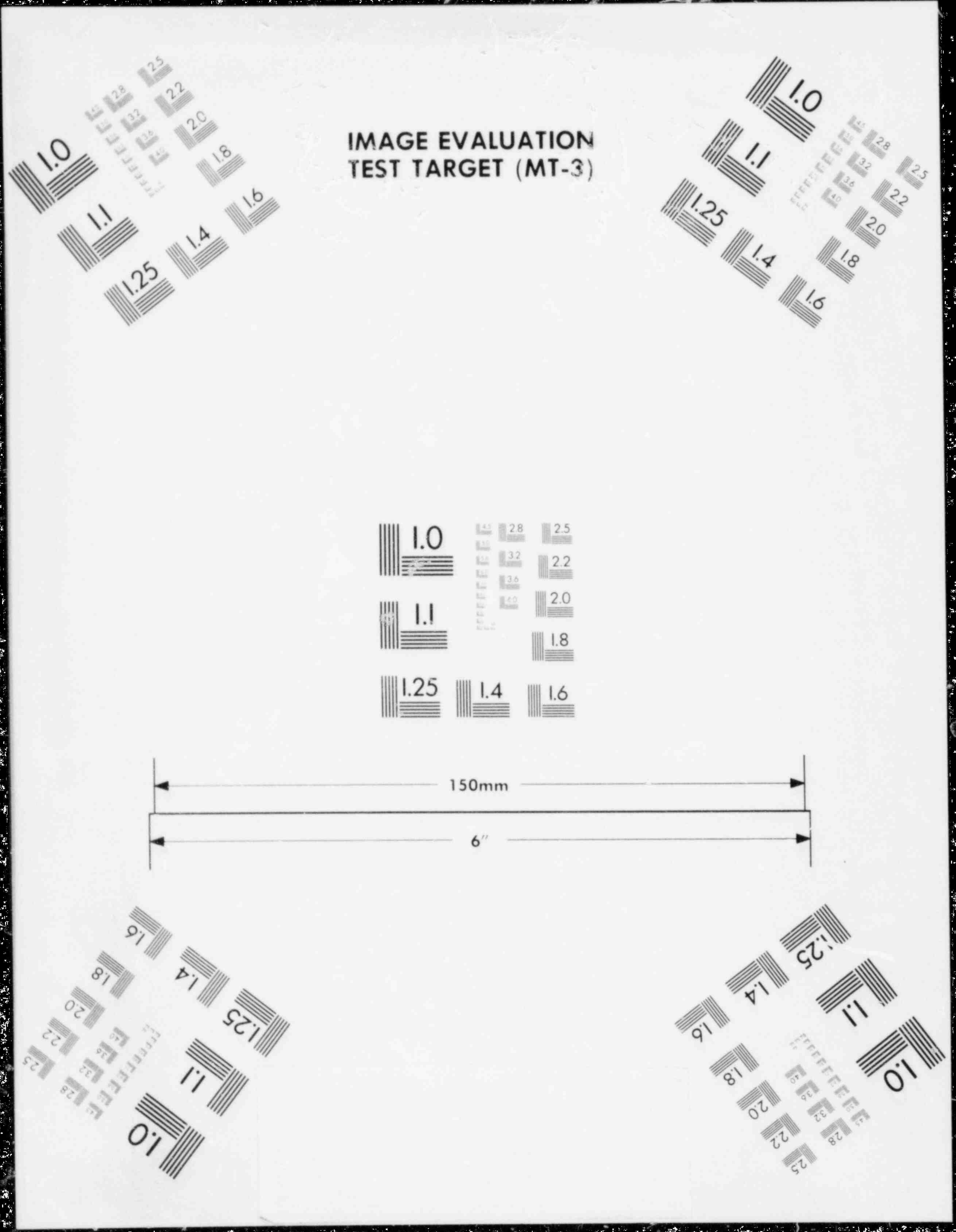

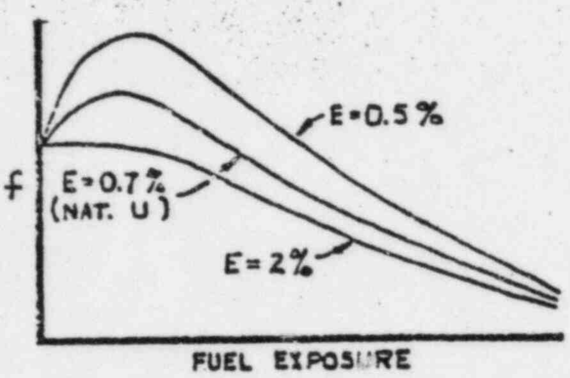

Figure 10-6: Effect of Pu-239 Buildup Upon f as Fuel Depletion Progresses

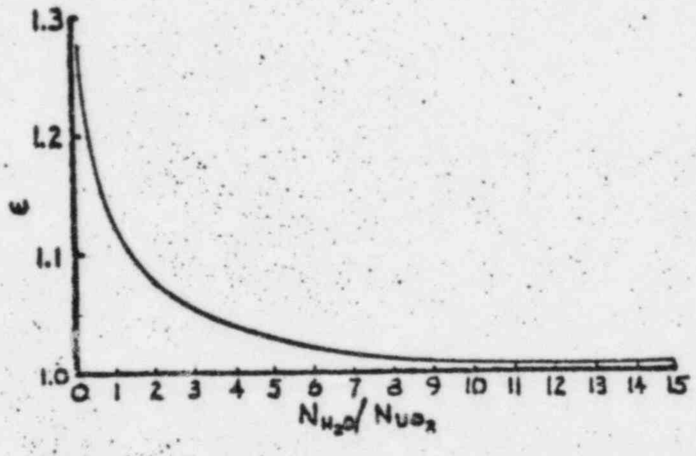

Figure 10-7: Effect of Moderator to Fuel Ratio Upon the Fast Fission Factor

In the range of enrichments (≈1.5% to ≈3.5%) which are used in present day water moderated reactors, the fast fission factor is essentially independent of enrichment. This is because the fast σ_f for U-235 and U-238 do not differ greatly (the value for U-235 is ≈1 b and for U-238 is ≈0.4 b) so small variations in their relative abundance have no great effect. In addition, the buildup of plutonium as core lifetime progresses has no appreciable effect upon ε.

4.

Expressions for resonance escape probability are too complex to present here. Again, however, some general statements can be made about the effects of various parameters on resonance capture. The resonance escape probability is very strongly dependent upon the moderator to fuel ratio as shown in Figure 10-8. As the amount of

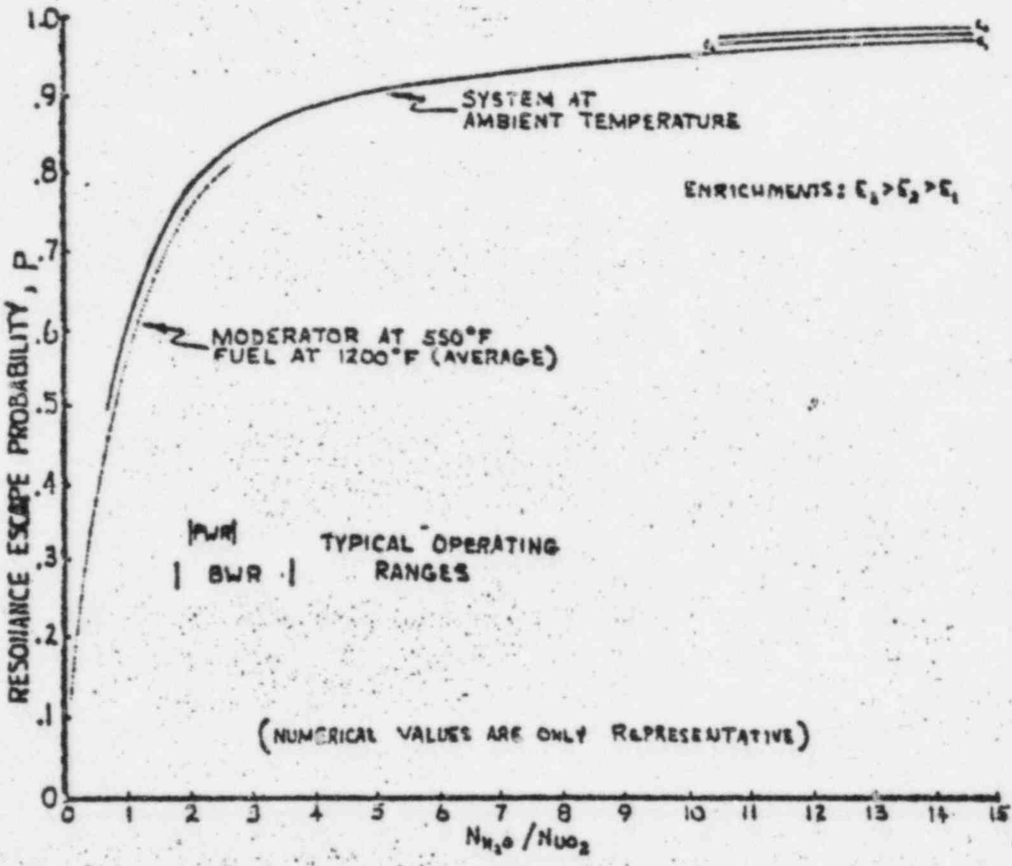

Figure 10-8: Effects of Water to Fuel Ratio, Enrichment, and System Temperature upon Resonance Escape Probability

moderator in the core is reduced, the fast neutrons are not slowed down as efficiently and they spend more time with energies in and about the resonance region. As a result, more of them are captured in the resonances and the escape probability decreases.

The resonance escape probability is also strongly affected by fuel temperature. In general, as fuel temperature increases, resonance absorption increases and the escape probability decreases. This behavior is called the doppler effect and will be discussed in detail later in this chapter.

As discussed previously, there are basically two methods to vary the ratio of moderator molecules to fuel molecules. The first is the selection of the relative volumes of each material by the fuel designer. The basis for his choice will be considered later in more detail, but basically it involves the necessity of providing adequate fuel cooling when the reactor is operating at full power, the desire to get something close to the maximum k_{eff} out of the fuel material while at the same time providing a negative moderator temperature coefficient (discussed below). In any case, one curve on Figure 10-8 shows the effect upon p of varying the relative volumes of moderator and fuel while the system remains at ambient temperature (70° F). A second curve shows the behavior of p when the water to fuel volume ratio is held constant but the moderator and fuel are heated to operating temperatures. Note that this curve drops considerably below the curve for a 70° F system due to the doppler effect.

The resonance escape probability is also dependent upon the composition of the fuel. In general, the higher the enrichment, the higher will be p. This is simply because an increase in enrichment reduces the amount of U-238 in the core and therefore reduces the chances that resonance absorption will take place in this material. As the fuel burns, fission products build up and these tend to reduce p because they include resonance absorbers which were not initially in the fuel. The buildup of plutonium also makes a negative contribution to the resonance escape probability. This is principally the result of the buildup of the non-fissionable isotope, Pu-240, which has a large absorption resonance at 1.025 ev. The cross section of Pu-240 is shown in Figure 10-9.

5. $\mathcal{L}_F$ and $\mathcal{L}_T$

Intuitively, one would expect the non-leakage probabilities to depend upon two quantities: (1) The average distance the neutrons travel in the reactor, and (2) the size of the reactor. In this regard, three new terms are introduced: (1) the thermal diffusion length, L_T, which is 41% of the average crow flight path of a thermal neutron in the core, (2) the fast diffusion length[1], L_F, which is 41% of the average crow flight path that neutrons travel with energies above thermal, and (3) the geometric buckling, Bg^2, which is a

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[1] A more common terminology in reactor theory texts is the square of the fast diffusion length, which is called the fermi age, τ.

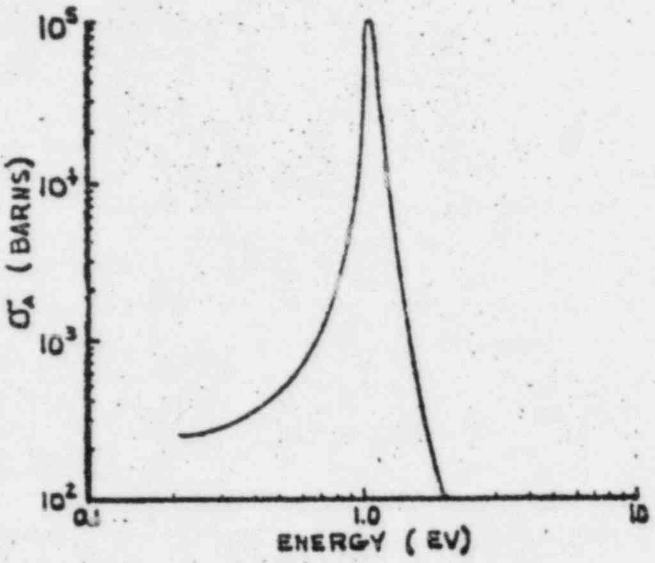

Figure 10-9: Absorption Cross Section of Pu-240

measure of the reactor size. The exact formula for geometric buckling is arranged so that as reactor size increases, geometric buckling decreases.[1] These three terms appear to be rather strange choices, but are arrived at because of the precise mathematical formulation of the non-leakage probabilities.

Figure 10-10 shows typical values of L_F and L_T versus moderator to fuel ratio. It is seen that L_F increases substantially as moderator is removed from the core. This is simply because the neutrons are less likely to encounter moderator atoms and be slowed down, so the slowing process takes longer and the neutrons travel farther. Remember, the primary mechanism for removal of fast neutrons from the core is thermalization rather than absorption, since the absorption cross sections for most materials are small for intermediate and fast neutrons (except at the resonance peaks). Thus L_F should be expected

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[1] The geometric buckling formulae for cylindrical and parallelepiped (boxlike) shaped cores are:

Cylindrical: $B_g^2 = (2.405/r)^2 + (\pi/h)^2$ where r = effective radius, h = effective height

Parallelepiped: $B_g^2 = (\pi/a)^2 + (\pi/b)^2 + (\pi/c)^2$ where a, b, c = effective lengths of the three sides

The effective distances equal the true distances plus a small additional correction to account for the reflector (since a reflected reactor has less leakage than a bare reactor of equal size, the reflected reactor is effectively larger).

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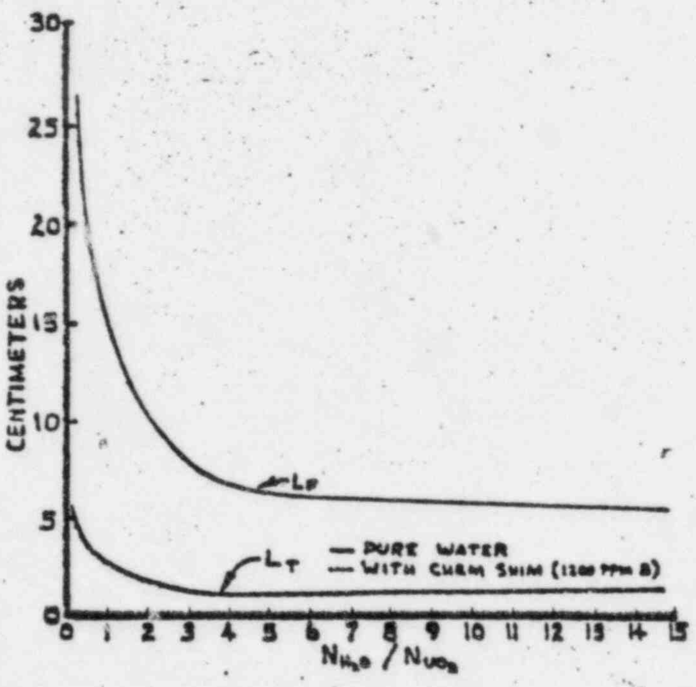

Figure 10-10: Typical Values of Fast and Thermal Diffusion Lengths in a Heterogeneous Water - UO_2 System

to depend rather strongly upon the moderating ability of the core. However, there is another factor involved in this behavior, and that involves the fact that scattering reactions also frequently change the direction of the neutron as well as its energy. Thus, instead of traveling in a straight line, the neutrons travel a crooked path as they slow down. This crooked path tends to reduce their crow flight path length. Thus the removal of water also results in an increase in $\mathcal{L}_F$ because the neutron path is straighter. This is basically saying that the water in the core has its reflecting properties reduced.

Similar arguments hold for the behavior of $\mathcal{L}_T$ with N_{H_2O}/N_{UO_2}. The major difference is that with thermal neutrons, the principle removal mechanism is absorption. As the moderator is removed, it absorbs fewer neutrons and the neutrons travel a straighter path. Both of these effects tend to increase $\mathcal{L}_T$. The isotopic composition of the core can have a significant influence upon $\mathcal{L}_T$. For example, the presence of a strong thermal neutron absorber, such as chemical shim, tends to reduce $\mathcal{L}_T$.

It is seen that typical values of $\mathcal{L}_T$ and $\mathcal{L}_F$ at room temperature are ≈1.2 cm and ≈6.6 cm. The average crow flight paths of thermal and above thermal neutrons are thus ≈2.9 cm (≈1.25 inches) and ≈16 cm (≈6.4 inches) respectively. Under operating conditions, these values would be increased by ≈20-50%.

Assuming core dimensions of ≈12' high and 8' diameter, the variation of fast and thermal non-leakage probabilities are shown in Figure 10-11. As can be seen, thermal leakage is virtually non-existant from large present day cores. As expected, however, both fast and thermal non-leakage probabilities decrease as moderator is removed from the core.

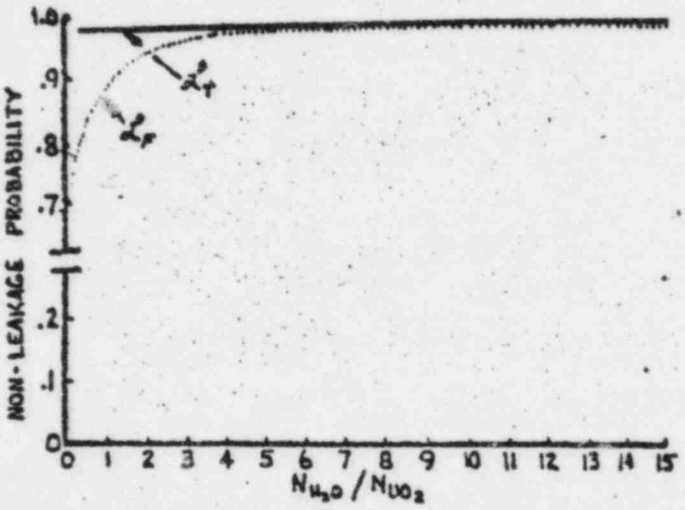

Figure 10-11: Typical Variation of Fast and Thermal Non-leakage Probabilities with N_{H2O}/N_{UO_2} for a 12' High, 8' Diameter Core

The fast non-leakage probability is not greatly affected by changes in fuel composition as the fuel burns since the travel distance is determined principally by the scattering properties of the moderator. However, the non-leakage probability is dependant upon the power distribution in the core. That is, the more power that is generated on the edges of the core, the higher will be the neutron population on the edge of the core, and the greater will be neutron leakage. It will be shown in Chapter 12 that near the end of core life the power distribution in the core is frequently shifted towards the edges.

In theory, the thermal non-leakage probability is affected by both the power distribution and the buildup of strong thermal absorber fission products. In practice, however, thermal leakage is already so low that minor variations in it have no significant effect upon the reactor.

6. k_{eff}

The effective multiplication factor is, of course, the product of the six factors previously discussed. A plot of k_{eff} versus moderator to fuel ratio is given in Figure 10-12. It can be seen that for low

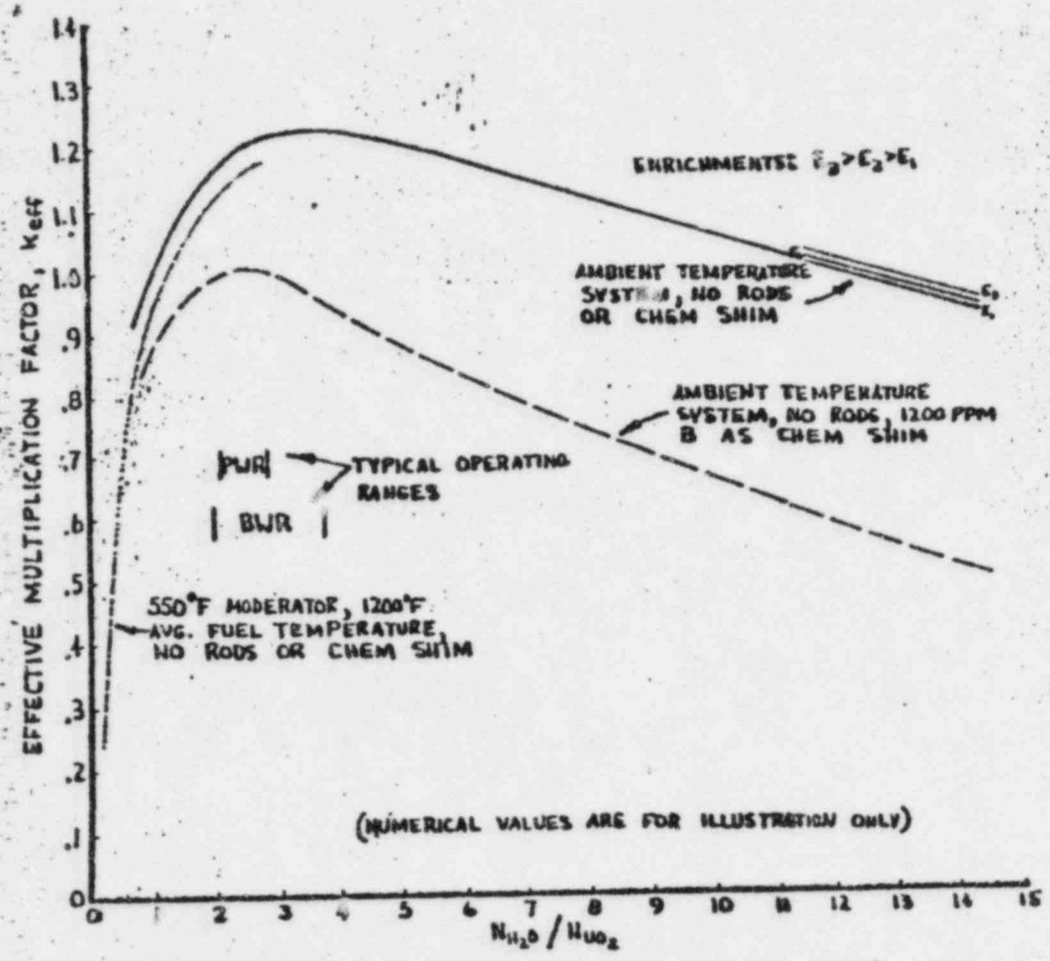

Figure 10-12: k_{eff} Versus Moderator to Fuel Ratio for Reactors Operating With and Without Chemical Shim Control

values of N_{H_2O}/N_{UO_2}, k_{eff} decreases rapidly due to the rapid decrease in p (see Figure 10-8). Similarly, at high values of N_{H_2O}/N_{UO_2}, k_{eff} also decreases due primarily to the decrease in f (see Figure 10-5). There is an intermediate point where k_{eff} is a maximum for any given reactor and set of operating conditions. This is the point of optimum design, for at this point the maximum k_{eff} is achieved for any given quantity of fuel. When N_{H_2O}/N_{UO_2} is less than the optimum value, the core is often said to be undermoderated, i.e. it has too little moderator to achieve the maximum k_{eff}. Conversely, when N_{H_2O}/N_{UO_2} is greater than the optimum value, the core is said to be overmoderated.

There are several curves shown on the figure. The solid curve at the top shows the effect of varying the volumes of moderator and fuel in an uncontrolled reactor at ambient temperature. The peak will usually occur at a water to fuel volume ratio of ≈1.5:1 to 3:1 depending upon the fuel rod diameter and enrichment. The dotted curve shows the effect of increasing the system temperature to typical full power values. At a given value of N_{H_2O}/N_{UO_2}, the dotted curve falls below the solid curve due to the reduction in p caused by the doppler effect. Finally, the dashed curve shows the effect of dissolving a rather substantial amount of boric acid chemical shim in the coolant. This curve falls well below the previous ones because of the reduction in f brought about by the B-10 poison.

It is interesting to note that in a core with chemical shim, the optimum point occurs at a lower value of N_{H_2O}/N_{UO_2} than in a core without chemical shim (assuming the two cores are identical in other respects). This is because the moderator contains a poison, and can be explained by the following reasoning. When the moderator is pure water, adding some of it to the fuel substantially improves p without significantly hurting f (because water has a low absorption cross section). This results in an increase in k, which continues as we add water until we get p pretty close to 1. In the chemical shim case, the addition of water helps p the same as before. But this time the high absorption cross section of poison in the water starts killing us on f (notice from Figure 10-5 that f falls a lot faster with chemical shim than without). Thus, the benefit we got from increasing p is much more quickly offset by the reduction in f, so the optimum point occurs at a lower water to fuel ratio. Since PWRs are designed to operate with chemical shim throughout much of core lifetime, this is one reason why they generally have a "tighter" fuel lattice (i.e. more fuel per unit core volume) than BWRs.

Figure 10-12 is drawn for an enrichment of about 2.6%. If the enrichment is increased, the curves would all be translated upward since both f and p increase with enrichment. Conversely, if enrichment is decreased, the curves would be translated downward.

EPITHERMAL CONTRIBUTIONS TO k_{eff}

The six factor formula discussed above somewhat simplifies the actual processes taking place in the reactor. The major effect which has been ignored in our description is resonance capture in fissile material. When the fuel rods are closely packed, as is the case in present day reactors, the relatively low moderator to fuel ratio results in incomplete thermalization of many neutrons before they are absorbed. As a result, resonance fission will represent a significant fraction of the total power - generally 10 to 20%. Usually this is accounted for by adjusting f with the addition of an epithermal cross section to the numerator of equation (10-14). In addition, we have ignored the relatively small amount of fast and resonance energy absorption in the moderator and structural materials. This is usually accounted for by making appropriate adjustments to the resonance escape probability. Finally, boron control material (either rods or boric acid), which is basically a slow neutron absorbing material, also results in a small amount of epithermal absorption. This is usually accounted for by an appropriate correction to f. Thus, while it is convenient to retain our simplified six factor approach, it should be realized that the factors must be adjusted somewhat in actual practice. Figures 10-13 and 10-14 show neutron balances similar to that in Figure 10-2, but including the effects just discussed.

REACTIVITY

Reactivity, ρ, is defined by the equation:

$$\rho = \frac{k_{eff} - 1}{k_{eff}} \qquad (10\text{-}15)$$

This is simply a new term which is used to describe the state of the reactor with respect to criticality. It is more commonly used than k_{eff}, but it really doesn't provide any additional information. If k_{eff} is known, reactivity can be calculated from equation (10-15), and vice-versa.

When the reactor is exactly critical, $k_{eff} = 1$ and $\rho = 0$. Under these conditions the reactor is said to have zero reactivity. Suppose $k_{eff} = 1.003$. Under these conditions, the reactor will be supercritical and the power will be rising. The reactivity will be:

$$\rho = \frac{1.003 - 1}{1.003} = \frac{0.003}{1.003} = 0.00299 = 0.299\%$$

Under these conditions the reactor is said to possess 0.299% reactivity or 0.299% "excess" reactivity. Note that reactivity has a positive sign when the reactor is supercritical.

Now consider a subcritical reactor with $k_{eff} = 0.997$. Power would be decreasing under these conditions. The reactivity would be:

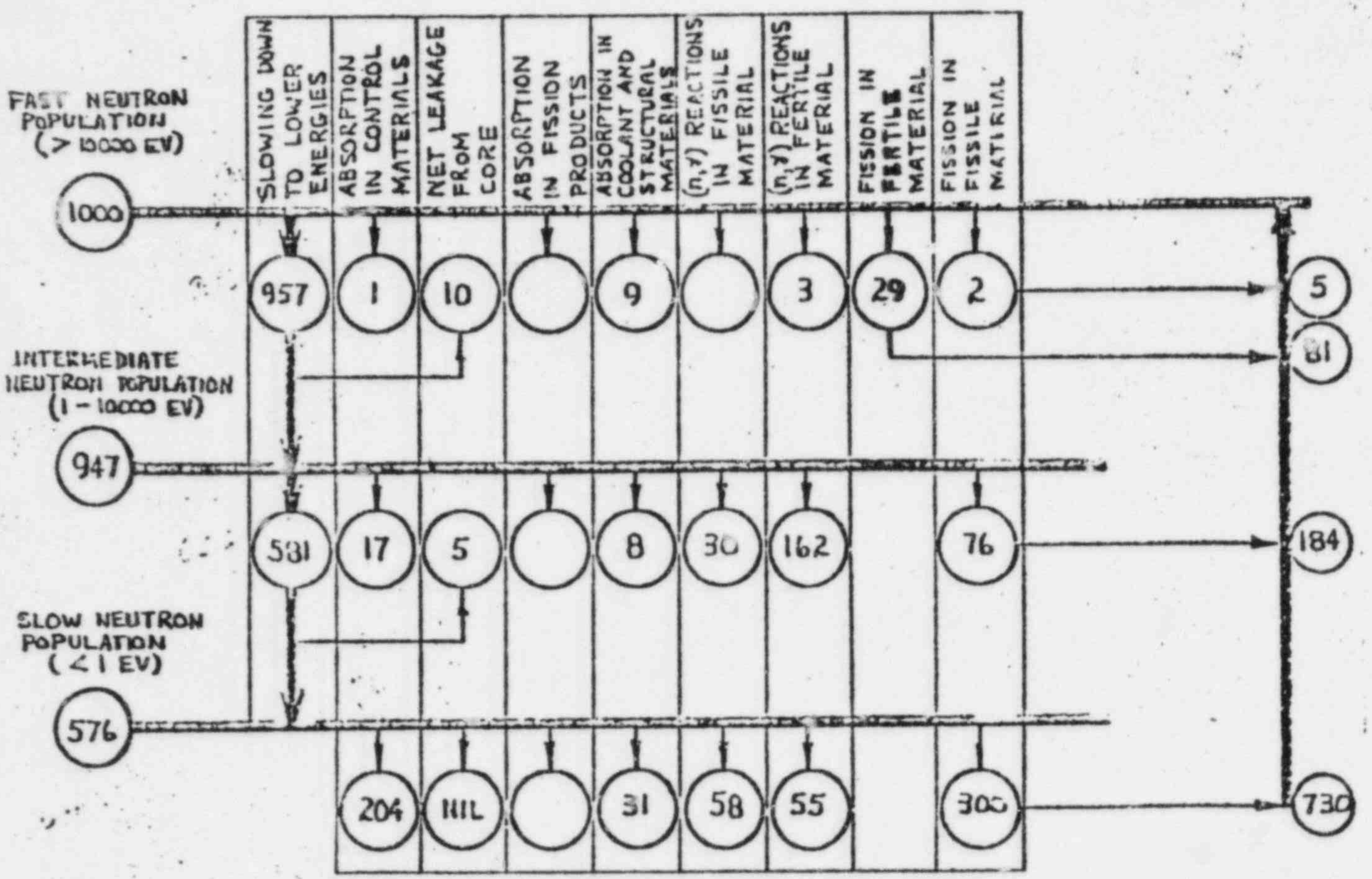

Figure 10-13: Detailed Neutron Balance in a Typical PWR with Fresh Fuel and B_4C Control Rods

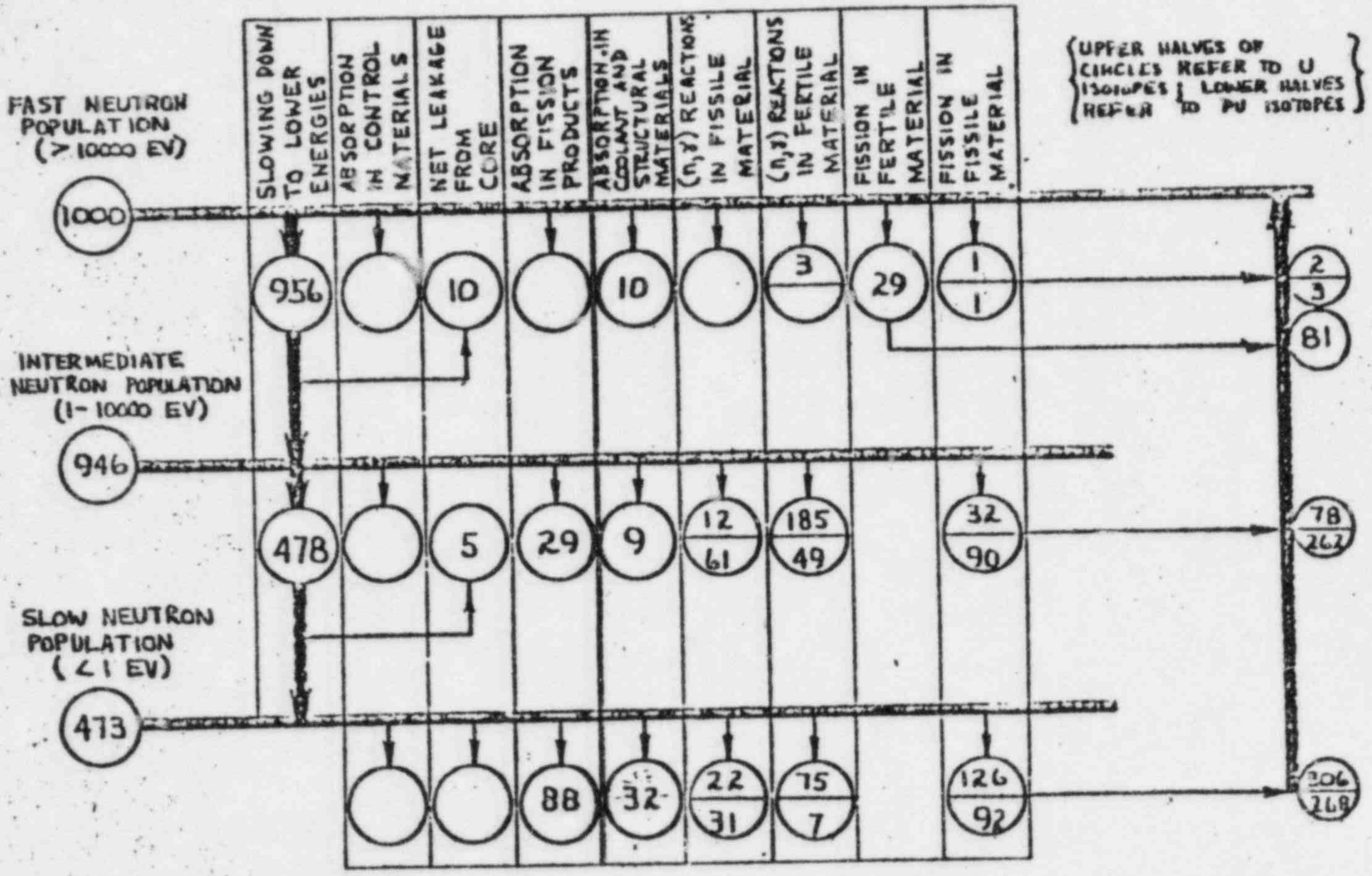

Figure 10-14: Detailed Neutron Balance for a Typical

PWR at End of Core Lifetime

$$\rho = \frac{0.997 - 1}{0.997} = \frac{-0.003}{0.997} = -0.00301 = -0.301\%$$

Notice that when the reactor is subcritical, reactivity is negative. In this instance the reactor is said to possess 0.301% negative reactivity.

The rules concerning k_{eff} and reactivity are summarized in Table 10-1 below.

TABLE 10-1: RELATIONSHIP OF k_{eff}, ρ, AND REACTOR CONDITION

REACTOR CONDITION	k_{eff}	ρ
Subcritical	<1.00	negative (<0)
Critical	1.00	0
Supercritical	>1.00	positive (>0)

Reactivity is often given the symbol $\Delta k/k$ instead of ρ. In this context k without a subscript means k_{eff}. The symbol Δ (greek letter delta) is commonly used in mathematics to mean "the change or difference in." Thus, in this case Δk means "the difference in k_{eff} with respect to critical," or $k_{eff} - 1$. Since k_{eff} is usually very close to 1.00 numerically, and since division by 1.00 does not change the value of a number, reactivity is often approximated simply by Δk rather than $\Delta k/k$. In both of the two previous examples, it is seen that this approximation would have had a negligible effect on the results. However, care must be taken to avoid this approximation if k_{eff} differs significantly from 1.00. In summary:

$$\rho = \frac{k_{eff} - 1}{k_{eff}} = \frac{\Delta k}{k}$$

which for values of K_{eff} close to 1.00 becomes:

$$\rho = k_{eff} - 1 = \Delta k$$

CONTROL RODS

In order to operate a nuclear reactor, the operator must have the capability of varying reactivity at will within certain practical limits. Control rods have traditionally been the most important device for this purpose although in present day reactors their use is being supplemented by such devices as dissolved boric acid and variable speed coolant pumps (both of which will be discussed in detail in later portions of this manual). However, control rods still remain an important method for regulating reactor power, power distribution, and most important, providing the capability for rapid reactor shutdown. In this section we will discuss the principles which govern their effects, while in later chapters we will consider their use in the overall picture of reactor operation.

Both BWRs and PWRs now use control rods employing boron carbide as the poison material[1]. Their performance is based upon the ability of the boron-10 isotope to absorb thermal neutrons in accordance with the equation: ${}_5B^{10} + {}_0n^1 \rightarrow {}_3Li^7 + {}_2He^4$. The microscopic absorption cross section for this reaction is 3810 barns.[2] The principle effect of control rod insertion is the reduction in the thermal utilization factor as discussed previously. However, the fact that control rods are localized poisons, and in general are not uniformly distributed throughout the core (in contrast, for example, to chemical shim) gives them certain unique characteristics which will be discussed below.

As one might expect, the potential strength of a control rod depends primarily upon the percentage of the total core thermal neutron population which has a reasonable chance of interacting with the control rod. In other words, if a control rod is capable of absorbing 50% of the neutrons in the core, it will be considerably stronger than a rod which can only absorb 10% of the neutrons in the core[3]. Our task is therefore to identify the factors which will affect the "absorbability" of a control rod.

Since a control rod is a localized poison, only a limited number of neutrons are available to it; namely, those which are born close enough to the control rod to have a reasonable chance of striking its surface. We have already mentioned that the average crow flight path length of a thermal neutron is related to the diffusion length, L_T, and is on the order of 1 - 2 inches. Thus, one would expect that the bulk of the neutrons absorbed by a control rod were born within 1 - 2 inches

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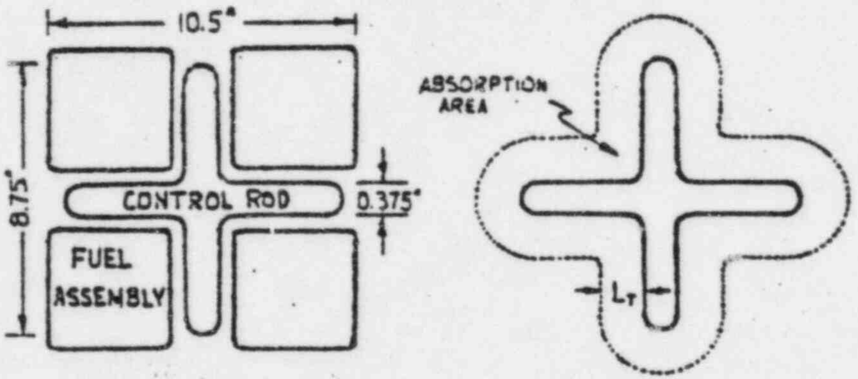
[1] BWRs have traditionally used B_4C control rods, but until recently PWRs employed a silver-indium-cadmium alloy. However, the prohibitive cost of silver has resulted in the adoption of B_4C for new PWRs. The major difference between B_4C rods and Ag-In-Cd rods is that the former are almost exclusively slow neutron absorbers whereas the latter absorb an appreciable number of epithermal neutrons due to Ag and In resonances. The general remarks in this section, however, are applicable to both types.

[2] Naturally occurring boron is 18.8% B-10 and 81.2% B-11. The microscopic thermal absorption cross section of natural boron is 755 barns.

[3] Note that control rod strength is not so much determined by the absolute number of neutrons which a control rod absorbs, but rather by the percentage of the total core population which it absorbs. That is, a rod which absorbs 200 neutrons from a core containing 1,000,000 neutrons will be considerably weaker than a control rod which absorbs 100 neutrons when the total core population is only 500. Thus the student should realize that control rod strength does not necessarily increase when the reactor power is increased.

of its surface. Figure 10-15(a) shows a schematic diagram of a typical BWR lattice cell, in which a cruciform (cross shaped) control rod is surrounded by four fuel assemblies. In an actual reactor, this lattice cell arrangement would be repeated over and over so that the entire core might contain as many as 500 fuel bundles and 125 rods. The dimensions



(a) TYPICAL BWR LATTICE CELL (b) ABSORPTION AREA OF ROD

Figure 10-15: Region of influence of a BWR Control Rod

shown are for the Humboldt Bay core. The region of influence of the control rod (often called the absorption area) would thus be an envelope surrounding the rod as shown in Figure 10-15(b). Theory shows that more precise results are obtained by taking the thickness of this envelope to be L_T rather than the total crow flight path (L_T is 41% of the crow flight path). Thus the percentage of the total cell which would be under the influence of an inserted control rod is given by:

$$\frac{\text{Volume of Absorption Area Envelope Surrounding Rod}}{\text{Total Volume of Cell}} = \frac{L_T\, S_{cr}}{V_{cell}}$$

where: S_{cr} = surface area of control rod = (perimeter of rod) x (length of rod)

V_{cell} = volume of lattice cell = (area of cell) x (length of cell)

L_T = thermal diffusion length

The above expression is applicable to a single controlled cell. Since a real core contains many cells, some of which may be controlled while others remain uncontrolled, it is more appropriate to generalize and say that the percentage of the core under the influence of any control rod(s) of interest is given by:

$$\frac{\text{Volume of Absorption Area Envelope Surrounding All Rods of Interest}}{\text{Total Volume of Core}} = \frac{L_T S_{cr}}{V_{core}}$$

where now: S_{cr} = total surface area of all control rods of interest

V_{core} = total core volume

Now that we have determined how much of the core volume is influenced by the control rods, it is necessary to determine what percentage of the core neutron inventory is likely to be found within this region of influence. That is, insertion of a control rod into a region where there are many neutrons could have a significant effect upon the reactor, whereas insertion of control rod of equal physical size into an area where there are very few neutrons would not be very effective. In Chapter 12 we will consider the distribution of the neutrons in a power reactor core in detail, but a few qualitative remarks are in order at this time. In general, the neutron density near the edges of a reactor is somewhat lower than in the center. The reason for this is that on the edges of the core many neutrons are lost to leakage whereas neutrons born in the interior regions are not likely to leak out. Thus in the absence of other effects, one might expect a control rod inserted into the center of the core to absorb a greater percentage of the core neutron population than a similar sized rod inserted near the edge, and in general, this is true. At any given point in the core, the concentration of thermal neutrons is measured by the thermal neutron flux, ϕ_T (see Chapter 7). Thus the number of neutrons which are potentially influenced by the insertion of a rod is proportional to the product of the volume of the core influenced by the rod and the neutron concentration within this volume, or $L_T S_{cr} \phi_{T,cr}$. In this expression $\phi_{T,cr}$ is the thermal neutron flux at the location of the rod (prior to its insertion). By similar reasoning, the total number of thermal neutrons in the core would be proportional to $V_{core} \phi_{T,avg}$, where $\phi_{T\ avg}$ is the average thermal neutron flux throughout the core. Thus the percentage of the total core neutron population which a control rod or group of rods is capable of absorbing is given by:

$$\frac{L_T S_{cr} \phi_{T,cr}}{V_{core} \phi_{T,avg}} = \frac{L_T S_{cr}}{V_{core}} \times \frac{\phi_{T,cr}}{\phi_{T,avg}}$$

There is one more effect which must be considered before we can determine the reactivity worth of a control rod or rods, and this is the importance of the neutrons which are absorbed. Consider, for example, the absorption of a neutron near the edge of the core. If the neutron had not been absorbed, there is a good chance it would have leaked out of the core and been lost to the chain reaction anyway. Therefore, the importance of this neutron is relatively low. On the other hand, a neutron in the center of the core has a high probability of eventually being absorbed in fuel and is therefore a valuable neutron. Thus two rods which absorb equal fractions of the total neutron population may have somewhat different reactivity worths depending upon the importance of the particular neutrons that capture. The problem then is to somehow identify which are important neutrons and which are not. To do this, we will assume that regions of the core which have a high concentration of neutrons must be regions where a lot of fissions are taking place and must therefore be important regions. Conversely, regions with a low concentration of neutrons must be regions where many non-productive processes are occurring (leakage or absorption in poison materials) and must therefore be relatively unimportant. Thus it can be reasoned that the importance of any particular region must be related to the ratio of the thermal neutron flux in the region of interest to the average thermal flux throughout the core. Therefore, to complete our determination of control rod worth, we must multiply the number of neutrons absorbed by their relative importance factor, $\phi_{T,cr}/\phi_{T,avg}$. In other words, the reactivity worth of a rod or group of rods is described by the following expression:

$$\text{Rod Worth } (\Delta k/k) = L_T \times S_{cr}/V_{core} \times (\phi_{T,cr}/\phi_{T,avg})^{a} \qquad (10\text{-}16)$$

The implications of this equation will be discussed in some detail later in this chapter and also in Chapter 12. However, it is instructive at this time to calculate the control rod system strength for the Humboldt Bay core with the lattice structure shown in Figure 10-15(a). There are 32 of these cells in the core[1]. The rods are very thin and so the perimeter of a rod is approximately equal to 4 x 8.75" = 35". Initially we will assume system temperature to be ≈ 70° F, so L_T = 0.5" (≈ 1.2 cm, see Figure 10-10). The cross sectional area of a cell is $10.5^2 = 121$ in^2. The lengths of both the fuel and the control rods are about 79". Thus:

$$\frac{L_T \times S_{cr}}{V_{core}} = \frac{(0.5)(35)(79)(32)}{(121)(79)(32)} = 0.145$$

Since we are considering a case where rods are inserted into every cell, some will be inserted into high flux regions and some into low flux regions. These will offset each other and for the entire control system, $(\phi_{T,cr}/\phi_{T,avg}) = 1$. Thus the reactivity worth of the system is approximately 14.5% Δk/k. That is, if k_{eff} = 1.200 with all rods withdrawn [i.e., ρ = (1.200 - 1)/1.200 = 0.165], the reactivity would be 0.165 - 0.145 = 0.020 with all rods inserted. This is equivalent to k_{eff} = 1.020. In this case, the reactor would still be supercritical

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[1] Actually, in the Humboldt core there are several bundles around the periphery which do not have control rods associated with them. This will make a small increase in core volume without making a corresponding increase in absorption volume. By neglecting these bundles, our calculated rod system strength will be somewhat high.

with all rods inserted, and some supplementary control scheme is obviously required or the plant could not be shut down. This supplementary control could be chemical shim (PWR only), burnable poison rods, or poison curtains as discussed in Chapter 9.

It may be confusing to the student how expression (10-16) relates to equation (10-14) for the thermal utilization factor. Usually the thermal utilization factor is calculated from (10-16) without considering the control rods. Then this value is multiplied by a correction factor which accounts for the effect of the rods.

Before temporarily leaving the subject of control rods, it is of interest to determine the system worth in the above example for the case where the system temperature is = 550° F. The only difference between the 70° F example and the 550° F example is that L_T has increased to about 0.7" (1.85cm). Thus the control system strength will have increased to about (14.5)(0.7)/(0.5) = 20% Δk/k.

NEUTRON SOURCES

The question is often asked as to where the first neutron necessary to start a chain reaction is obtained. The answer is that there are always plenty of neutrons available from natural sources. Among the most important of these natural sources are:

1. **Cosmic neutrons - The earth's atmosphere is being continually bombarded by cosmic particles from nuclear reactions in the sun. High energy neutrons are among the particles reaching the earth's surface. At sea level the cosmic neutron flux is about 50 neutrons/cm^2-hr and at an elevation of one mile about 500 neutrons/cm^2-hr.**

2. Spontaneous fission - The isotopes of uranium are unstable enough to have a small chance of fissioning spontaneously without capture of a neutron. The process is not very frequent, but does proceed at a definite rate in any mass of uranium. The rates were given in Chapter 8 as 280 and 6300 fissions/sec-ton for U-235 and U-238 respectively. In each case several neutrons are produced for each spontaneous fission. In a large power reactor, where there are several tons of fuel, spontaneous fission is a fairly significant neutron source. For example, the Humboldt Bay reactor contains about 17 tons of U-238 in the core. The spontaneous fission rate would therefore be:

$$(17 \text{ tons})\frac{(6300 \text{ fissions})}{(\text{sec})(\text{ton})} = 1 \times 10^5 \frac{\text{fissions}}{\text{sec}}$$

On the average, 2.19 neutrons are emitted for each spontaneous fission of U-238, so the spontaneous fission source strength is = 2.19 × 10^5 neutrons7sec.

3. Photonuclear reactions - After a water moderated reactor has been operated for a time and therefore contains an inventory of fission

products, high energy gamma radiation reacts with heavy hydrogen nuclei to produce neutrons in the following reaction:

$$\gamma + {}_1D^2 \longrightarrow {}_1H^1 + {}_0n^1 \qquad (10\text{-}17)$$

This is a threshold reaction, requiring a minimum gamma ray energy of 2.21 mev. The extent of this reaction in a light water moderated reactor is limited because there is only one atom of heavy hydrogen (${}_1D^2$) per 6,500 atoms of ordinary hydrogen (${}_1H^1$).

Therefore, in any nuclear reactor there is always a supply of neutrons available to initiate a chain reaction. In spite of this, nearly all reactors built to date have self-contained neutron sources installed in their cores. The reason for this is primarily a matter of safety. In order to control the reactor properly, the operator must have an indication of reactor power on his nuclear instrumentation at all times. This is particularly important on a reactor startup because the operator is withdrawing control rods to attain criticality and he must have some indication of reactor behavior in order to know when to stop. If the operator were to withdraw control rods too far, the reactor could be made highly supercritical and reactor power could begin to increase at an undesirably high rate. The instrumentation that the operator employs to monitor reactor behavior uses detectors which are sensitive to the number of thermal neutrons at the location of the detector. The detectors are generally located outside of the reactor pressure vessel, and therefore, the neutrons must traverse a considerable amount of water, steel, and other materials to reach them. As a result, there is a considerable attenuation of the neutrons as they traverse the distance from the core to the detectors. The neutrons present in the core as a result of the various natural sources are more than sufficient to initiate a chain reaction, but are generally not plentiful enough to provide a significant response on the "out-of-core" nuclear instrumentation. The instrumentation readings are so small, that electronic noise, etc., can completely mask any changes in the readings that result from control rod withdrawals. With a source in the core, the neutron population can be raised to a point where the instruments present a true picture of reactor conditions even though the reactor may be completely shut down with all control rods inserted.

1. Operating Source

There are a large number of nuclear reactions which will produce neutrons. However, both PWRs and BWRs employ a source which is based upon the photonuclear reaction:

$$\gamma + {}_4Be^9 \longrightarrow {}_4Be^8 + {}_0n^1 \qquad (10\text{-}18)^1$$

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[1] It is of interest to note that Be-8 is very unstable and immediately decays to two alpha particles.

In Chapter 6 it was shown that this is a threshold reaction which requires a minimum gamma ray energy of 1.6 mev. Therefore, in order to make the source work there must be a supply of high energy gamma rays available. The material used for this purpose is antimony, which occurs in nature as two stable isotopes, Sb-121 with a 57.25% abundance and Sb-123 with a 42.75% abundance. These stable isotopes, when exposed to a neutron flux in a reactor, undergo (n,γ) reactions. Of interest is the reaction with Sb-123.

$$_{51}Sb^{123} + {}_{0}n^{1} \longrightarrow {}_{51}Sb^{124} + \gamma \qquad (10-19)$$

The isotope Sb-124 undergoes β^- decay with the emission of high energy gamma rays (≈ 1.69 mev) of sufficient energy to cause the neutron producing reaction with Be-9. Its half life is 60 days.

$$_{51}Sb^{124} \xrightarrow{T_{1/2} = 60\ d} {}_{52}Te^{124} + {}_{-1}e^{0} + \gamma\ (1.69\text{mev}) \qquad (10-20)$$

Construction details of the source will vary depending upon the particular facility. At Humboldt Bay, the source is in the form of a cylindrical rod of antimony placed in the center of an annulus of beryllium. Both the antimony and beryllium are encapsulated in stainless steel to improve corrosion resistance. The entire apparatus is contained in a dummy fuel assembly and is located in a spare fuel assembly location within the core. In pressurized water reactors, the sources are usually Sb-Be mixture pellets stacked in stainless steel tubes. The tubes are the same diameter as fuel rods and are located in empty fuel rod locations of selected fuel bundles.

Before an Sb-Be source will work, the antimony must undergo irradiation in an operating reactor in order to produce the necessary Sb-124. As the source sits in the neutron flux, its neutron emission rate will gradually increase until it reaches an equilibrium value at which time the production rate of Sb-124 (governed by the power level of the reactor in which the source is located) matches the Sb-124 removal rate by decay. At this point the source is fully "charged". If it is desired to have the Sb-Be source available for the initial startup of a new plant, the source must be "charged" at some other facility.

Whenever the source is removed from a reactor (or, alternatively, if the reactor in which it is located is shut down), the neutron production rate of the source begins to decay with a 60 day half-life because Sb-124 is no longer being produced. If the source is left out of a reactor (or the reactor in which it is located remains shut down) for several half-lives, the source will have to be reactivated in another reactor. Once the reactor in which it is located returns to power, however, the source will recharge itself back to equilibrium. The choice of gamma emitters for a source is fairly limited, then, by the dual requirements of high gamma ray energy and long enough half-life to permit realistic reactor outage times without the need for recharging the source.

An Sb-Be source has a neutron production rate of ≈ 10^7 neutrons/sec/curie of Sb-124. The exact figure depends somewhat upon the geometric configuration. For BWR applications the source might contain 2000 - 4000 curies of Sb-124 and produce ≈ 10^{10} - 10^{11} neutrons/sec. If the natural source strength is taken to be on the order of 10^5 neutrons/sec, it can be seen that installation of an Sb-Be source of this strength would increase the neutron population by 10^5 to 10^6.

The gamma field surrounding a charged Sb-Be source is very high. At a distance of 1 foot from the source, the dose rate is roughly 10 R/hr/curie of Sb-124. Thus from a 2000 curie source, the dose rate at a distance of 1 foot is on the order of 10^4 R/hr.

2. Startup Source

For the initial fuel loading of the reactor, an artificial neutron source is particularly necessary because at this time even the major natural sources (spontaneous fission and photonuclear reactions) are low because there is very little fuel in the core and it has not yet built up a fission product inventory. In addition, the reactor is new and untried during this operation so it is particularly important that its performance be monitored on the instrumentation. During this period, a special startup source is usually employed instead of the operating source. The major reason for using a special source during this period is to employ a type with a low gamma ray background. The high gamma ray background of the Sb-Be source has two major drawbacks: 1) the high radiation level complicates the entire operation by necessitating special shielding and handling procedures, and 2) the neutron detectors are also sensitive to gamma rays and this needlessly raises the background and makes it more difficult to accurately measure the neutron levels.

The common startup sources employ the (α, n) reaction with beryllium:

$$_4\mathrm{Be}^9 + {}_2\mathrm{He}^4 \longrightarrow {}_6\mathrm{C}^{12} + {}_0n^1 \qquad (10\text{-}21)$$

The alpha particles can come from any of several heavy isotopes which undergo alpha decay, have a relatively long half-life (so the source will not die out too quickly), and do not emit any high energy gamma rays in their decay. The two most commonly used isotopes for startup sources are $_{92}\mathrm{Pu}^{239}$ and $_{84}\mathrm{Po}^{210}$. The decay reactions involved are:

$$_{92}\mathrm{Pu}^{239} \xrightarrow{T_{1/2}\,=\,24300\text{ years}} {}_{92}\mathrm{U}^{235} + {}_2\mathrm{He}^4 \qquad (10\text{-}22)$$

$$_{84}\mathrm{Po}^{210} \xrightarrow{T_{1/2}\,=\,138\text{ days}} {}_{82}\mathrm{Pb}^{206} \quad {}_2\mathrm{He}^4 \qquad (10\text{-}23)$$

A Pu-Be source produces 1.5 x 10^6 neutrons/sec/curie of Pu and has a gamma ray dose rate at a distance of 1 meter of ≈ 1 mr/hr per 10^6 neutrons/sec. A Po-Be source produces 2.6 x 10^6 neutrons/sec/curie of Po and has a gamma ray dose rate at a distance of 1 meter of

0.04 mr/hr per 10^6 neutrons/sec.

Thus, to produce 10^8 neutrons/sec would require ≈ 38 curies of Po-210 and the gamma dose rate at a distance of 1 meter would be ≈ 4 mr/hr. The comparable figures for a Pu-Be source are 67 curies and 100 mr/hr.

The short range of alpha particles necessitates mixing the alpha emitter and the beryllium together, usually in powder form. The mixture is then encapsulated in stainless steel (if Pu is used, the source is first encapsulated in tantalum because it resists chemical attack by Pu). PWRs generally use a Po-Be source in a dummy fuel rod just as for the operating source. This source is simply left in the core as the plant starts up and eventually decays away. On the other hand, a Pu-Be source lasts indefinitely because of the long half-life of Pu-239. When these sources are used during fuel loading, they are invariably removed from the core just prior to startup and are replaced with a charged up operating source. If left in the core, a Pu-Be source would undergo fissioning when the reactor was started up and would be ruined (and very likely melted unless care was taken to provide cooling for it). One disadvantage of a Pu-Be source is the cost of the Pu (which is roughly 20 times as expensive as gold).

For refueling outages after the reactor has been operated for a considerable period of time, the Sb-Be source is used because the high fission product gamma ray background eliminates the main advantage for using the startup source. Secondly, there is generally little or no movement of the source during refueling as there is during initial loading, and strict radiological procedures must be adhered to while handling the highly radioactive (due to the fission products) spent fuel, so the other major objections to the Sb-Be source are minimized. The problem of detector response to gamma rays still exists, and in fact is magnified, during refueling. Frequently it is necessary to combat the problem by temporarily changing to another type of detector which has a higher sensitivity to neutrons relative to its gamma sensitivity than do the normal detectors.

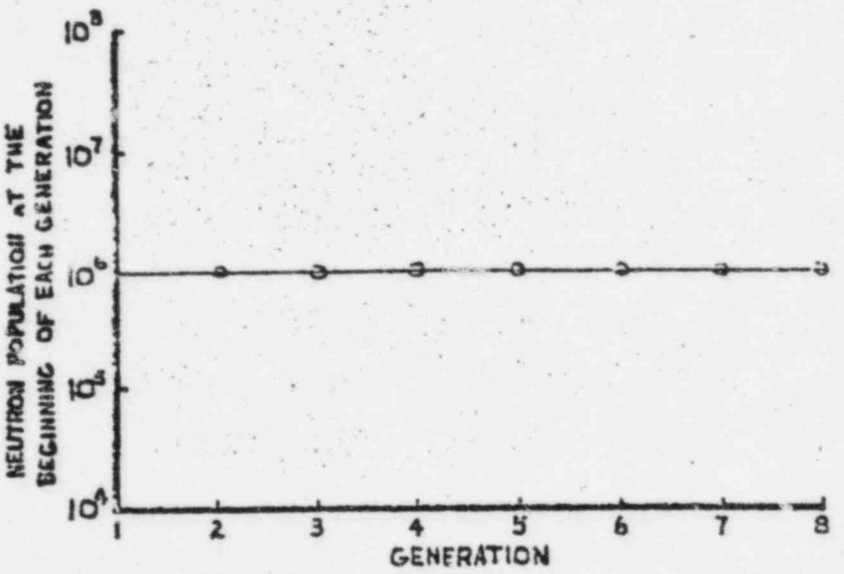
It is of interest to note that for the very large BWR reactors now in the planning and construction stages, all nuclear instrumentation will be located inside the core rather than outside the vessel. Such a move has been necessitated by the fact that it is possible to pull clusters of control rods near the center of very large cores and make small critical volumes which are so far removed from any out-of-core detectors that they simply cannot be monitored. With the detectors located within the core, many small operating sources distributed throughout the core will be used.

NEUTRON MULTIPLICATION IN A SUBCRITICAL REACTOR

Consider a tank of water in which there is no fuel. Suppose that one million neutrons are somehow introduced in to this pool of water. These neutrons will travel about until they are either absorbed by the water or they leak out of the system. In a very short period of time there will be no neutrons left in the pool. Since there were a million neutrons at the beginning of this generation, and zero at the end, k_{eff} would be $0/10^6 = 0$. This result is certainly not unexpected since there is no fuel in the "reactor" (the student should convince himself that the six factor formula also gives the result that $k_{eff} = 0$ for this system because the thermal utilization, f, equals zero). Now suppose that just enough fuel is added to the reactor to make $k_{eff} = 0.5$. If one million neutrons are added to this reactor they will also die by leakage and absorption, but some of the absorptions will produce new neutrons as a result of fissions in the fuel. At the end of the first generation in this reactor there will be (1,000,000)(0.5) = 500,000 neutrons. Similarly, at the end of the second generation there will be (500,000)(0.5) = 250,000 neutrons remaining. Under these circumstances the neutron population is gradually dying out, but several generations will pass before they are completely gone. Again this result is not unexpected because we know that a chain reaction cannot be sustained if k_{eff} is less than one.

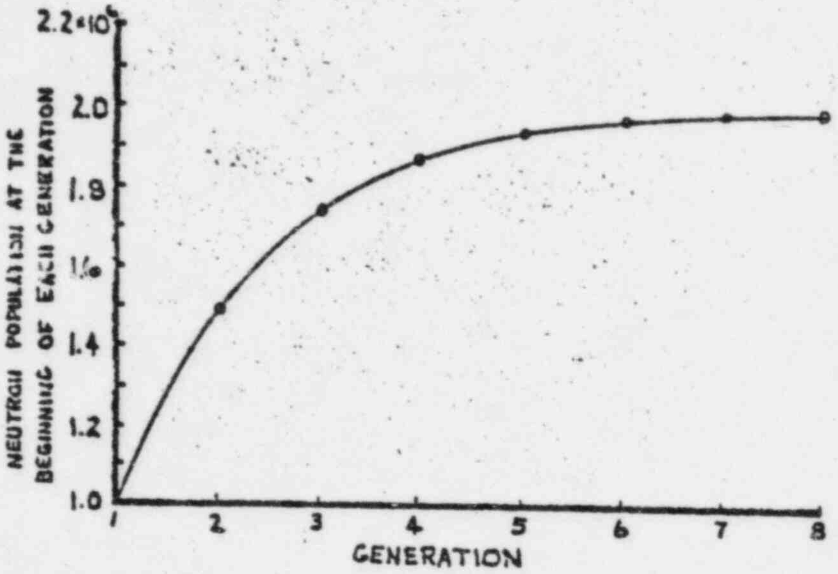
Now consider a slightly different problem. Suppose a continuous source is placed in the reactor, and this source emits neutrons in bursts of 1,000,000. Suppose further that these bursts are timed to coincide with the beginning of each generation in the reactor. Now let us consider the effects of this new source upon the "reactor" with no fuel. At the start of the first generation the source emits a burst of 10^6 neutrons into the reactor. These neutrons will be absorbed and leak out as before, until at the end of the first generation there will be none left. The neutron population does not drop to zero, however, because at the beginning of the second generation the source emits another burst of 10^6 neutrons. The process will continue so that at the start of each generation, there will always be 10^6 neutrons in the core. The student should not be confused into believing that $k_{eff} = 1$ for this reactor simply because there are 10^6 neutrons at the beginning of each generation. k_{eff} is intended to determine what happens to the neutrons from any single burst, i.e. whether they are <u>self-sustaining</u> or not. In this case, the neutron population is artificially sustained, and would immediately die out if the source were removed. The process just described is pictured in Figure 10-16. Notice that the graph of total neutron population versus generation shows that neutron population remains constant at precisely the source strength.

Now consider the same situation except that enough fuel has been added to the reactor to make $k_{eff} = 0.5$. At the start of the first generation there are 10^6 neutrons introduced into the reactor by the source. At the end of the first generation these will have been reduced to 0.5×10^6 because $k_{eff} = 0.5$. In the meantime, however, the source will emit another burst of 10^6 neutrons so that at the beginning of the second generation there will be $(1.0 + 0.5) \times 10^6$ neutrons in the reactor. The process will continue on in this way as shown in Figure 10-17. Notice that an equilibrium value for neutron population is gradually being approached. In this case, if the calculations were carried out for a sufficient number of generations, we would discover that neutron population



SOURCE BURST	GENERATION 1	2	3	4	5	6	7	8
1	10^6	0	0	0	0	0	0	0
2		10^6	0	0	0	0	0	0
3			10^6	0	0	0	0	0
4				10^6	0	0	0	0
5					10^6	0	0	0
6						10^6	0	0
7							10^6	0
8								10^6
TOTAL	10^6	10^6	10^6	10^6	10^6	10^6	10^6	10^6

Figure 10-16: Effect of a Continuous Neutron Source of Strength 10^6 neutrons/generation Upon Total Core Neutron Population when $k_{eff} = 0$



SOURCE BURST	GENERATION 1	2	3	4	5	6	7	8
1	10^6	$.5\times10^6$	$.25\times10^6$	$.125\times10^6$	$.063\times10^6$	$.032\times10^6$	$.016\times10^6$	$.008\times10^6$
2		10^6	$.5\times10^6$	$.25\times10^6$	$.125\times10^6$	$.063\times10^6$	$.032\times10^6$	$.016\times10^6$
3			10^6	$.5\times10^6$	$.25\times10^6$	$.125\times10^6$	$.063\times10^6$	$.032\times10^6$
4				10^6	$.5\times10^6$	$.25\times10^6$	$.125\times10^6$	$.063\times10^6$
5					10^6	$.5\times10^6$	$.25\times10^6$	$.125\times10^6$
6						10^6	$.5\times10^6$	$.25\times10^6$
7							10^6	$.5\times10^6$
8								10^6
TOTAL	10^6	1.5×10^6	1.75×10^6	1.875×10^6	1.938×10^6	1.970×10^6	1.986×10^6	1.994×10^6

Figure 10-17: Effect of a Continuous Neutron Source of Strength 10^6 neutrons/generation upon Total Core Neutrons Population when k_{eff} = 0.5

would equilibrate at 2 times the source strength. In theory it would take an infinite number of generations for equilibrium to be reached, but Figure 10-17 shows that for all practical purposes it has been essentially reached by the eighth generation.

The equilibrium neutron population that is eventually reached depends upon the value of k_{eff} in the reactor. We have seen that when $k_{eff} = 0$, the equilibrium value is exactly equal to the source strength. Similarly, when $k_{eff} = 0.5$, the equilibrium value is just twice the source strength. In general, the equilibrium value will be higher the closer k_{eff} is to 1.00. It can be shown mathematically that the equilibrium neutron population is given by the equation:

$$P = \frac{S}{1 - k_{eff}} \qquad (10\text{-}24)$$

where: P = equilibrium neutron population
S = source strength (neutrons/generation)

The student should convince himself that equation (10-24) gives the correct results for the two cases just discussed.

The quantity $1/(1 - k_{eff})$ is often called the subcritical multiplication factor, M. Equation (10-24) can be rewritten in terms of M as shown below:

$$P = \frac{S}{1 - k_{eff}} = S \times \frac{1}{1 - k_{eff}} = S \times M \qquad (10\text{-}25)$$

Thus, the subcritical multiplication factor is the amount by which the source neutron population is multiplied as a result of the presence of fuel in a subcritical reactor. Inspection of equation (10-24) shows that things seem to go awry when $k_{eff} = 1$. Under these circumstances the denominator of equation (10-24) equals 0 and the equilibrium neutron population approaches infinity. This result simply means that the neutrons from any particular source burst will never die out because $k_{eff} = 1.0$. Since the source continues to emit neutrons, and since none of the neutrons are dying out, it follows that the total neutron population will continue to increase as long as the source is left in the reactor. If the source is left in the reactor for a infinite number of generations the neutron population will build up to infinity as the equation predicts. This brings up an interesting point. If the source is left in the reactor, as it is in practice, the only way in which the reactor power can be held constant is to insert control rods far enough to hold k_{eff} slightly less than 1.0. In practice, however, the value of k_{eff} that is held is so close to 1.0 that the difference can be ignored. For example, the neutron production rate at full power in a typical reactor is on the order of 10^{19} neutrons/generation. A typical source strength might be 10^{8} neutrons/generation. Substituting into equation (10-24) yields:

$$10^{19} = \frac{10^6}{1 - k_{eff}}$$

$$1 - k_{eff} = \frac{10^6}{10^{19}} = 10^{-13}$$

$$k_{eff} = 1 - 10^{-13} = 1 - 0.0000000000001$$

$$k_{eff} = 0.9999999999999$$

Therefore, the assumption that $k_{eff} = 1.0$ when reactor power is constant in a reactor with a source is seen to be justified. The fact that k_{eff} is actually slightly less than 1.0 simply reflects the fact that the reactor does not have to be completely self-sustaining, because the source is making up the difference.

Equation (10-24) fails when $k_{eff} > 1.0$. Under these conditions it predicts a negative neutron population, which is physically impossible.

In the previous discussion of the subcritical multiplication phenomenon, we have made some simplifying assumptions. It would be fortuitous, for example, if a source emitted neutrons in bursts which exactly coincided with the start of each new generation. It would be equally fortuitous if all neutrons had exactly the same lifetime in the reactor. In practice the generations overlap each other, and the source emits a continuous stream of neutrons. All this means is that the calculations must be based upon averages, but this in no way affects the end result.

REACTOR PERIOD AND POWER LEVEL

In the previous section we discussed the behavior of a subcritical reactor. In this section we will discuss the behavior of a supercritical reactor. In Figure 10-1, the behavior of the neutron population in a reactor in which $k_{eff} = 2$ was diagrammed. If there was 1 neutron in the reactor at the start, then there would be 2 at the end of the first generation, 4 at the end of the second, 8 after the third, etc. The student should recognize this as another example of an exponential process. The variation of power level with time for a supercritical (or slightly subcritical) reactor can therefore be expressed with an exponential equation as follows:

$$P = P_0 e^{\Delta t/\tau} \qquad (10\text{-}26)$$

where: P_0 = initial power level

P = power level at any later time

Δt = time interval between P and P_0

τ = reactor period

Equation (10-26) does not have to be written in terms of power level. It could just as correctly be written in terms of neutron flux, since power level is proportional to flux. Thus:

$$\phi = \phi_0 \, e^{\Delta t/\tau} \qquad (10\text{-}27)$$

The concept of reactor period is an extremely important one in the operation of a reactor. The period, τ, is the length of time necessary for reactor power to increase or decrease by a factor of 2.718. The factor 2.718 may seem a strange number to choose for the definition. Recall, however, that 2.718 is simply the numerical value of e, the base of the natural logarithm system. When the sign of the reactor period is positive, reactor power is increasing. When the sign of the period is negative, reactor power is decreasing. For example, if a reactor initially at a power level of 1 watt is placed on a period of +100 seconds, the power level will have increased to 2.718 watts at the end of a 100 second time interval. Conversely, if the reactor was placed on a period of -100 seconds, and the initial power level was 1 watt, the power level after 100 seconds would have fallen to 1/2.718 = 0.368 watts.

It is easy to verify that the verbal definition of period given above is not inconsistant with equation (10-26) which contains the period. Suppose we again work the previous example where the reactor was on a +100 second period, but use equation (10-26). Then:

$$P = P_0 \, e^{\Delta t/\tau}$$

where: P_0 = 1 watt

Δt = 100 seconds

τ = +100 seconds

Substitution of these values into the equation yields:

$$P = (1)[e^{100/100}] = e^1 = 2.718 \text{ watts}$$

which agrees with the previous result. Similarly, equation (10-26) can be used to calculate the result for the second example in which the reactor was on a -100 second period. In this case:

P_0 = 1 watt

Δt = 100 seconds

τ = -100 seconds

and $P = (1)[e^{-100/100}] = (1)(e^{-1}) = 1/2.718 = 0.368$ watts

Equation (10-26) contains four variable quantities, P, P_0, Δt, and τ. If any three of these quantities are known, the fourth can always be determined. Unfortuantely, equation (10-26) cannot be readily solved without reference to a table of natural logarithms or a sliderule which contains logarithm scales. To overcome this difficulty several types of graphs, rules of thumb, and approximations have been developed to aid in the solution of problems involving reactor period and power level.

1. Plot of P/P_0 versus $\Delta t/\tau$

By dividing both sides of equation (10-26) by P_0, the following expression can be obtained:

$$P/P_0 = e^{\Delta t/\tau} \qquad (10\text{-}28)$$

As discussed in Chapter I, a plot of P/P_0 versus $\Delta t/\tau$ on semi-log paper results in a straight line. Such a plot is given in Figure 10-18. Whenever a plot such as this is available, it can be used to work any problem concerning periods. Several examples will illustrate the method.

Example 1: Reactor power increases from 75 MWT to 137 MWT in 70 seconds. What is the period?

$P = 137$ MWT

$P_0 = 75$ MWT

$\Delta t = 70$ seconds

$P/P_0 = 137/75 = 1.83$

When $P/P_0 = 1.83$, Figure 10-18 gives the result that $\Delta t/\tau = 0.604$

Thus:

$70/\tau = 0.604$

$\therefore \tau = 70/0.604 = 116$ seconds

Example 2: Reactor power decreases from 148 MWT to 90 MWT in 82 seconds. What is the period?

$P = 90$ MWT

$P_0 = 148$ MWT

$\Delta t = 82$ seconds

$P/P_0 = 90/148 = 0.608$

From Figure 10-18:

$\Delta t/\tau = -0.497 = 82/\tau$

$\tau = 82/(-0.497) = -165$ seconds

Example 3: The reactor is placed on a 70 second period for 49 seconds. If the initial power was 68 MWT, what is the power after 49 seconds?

0163

1310

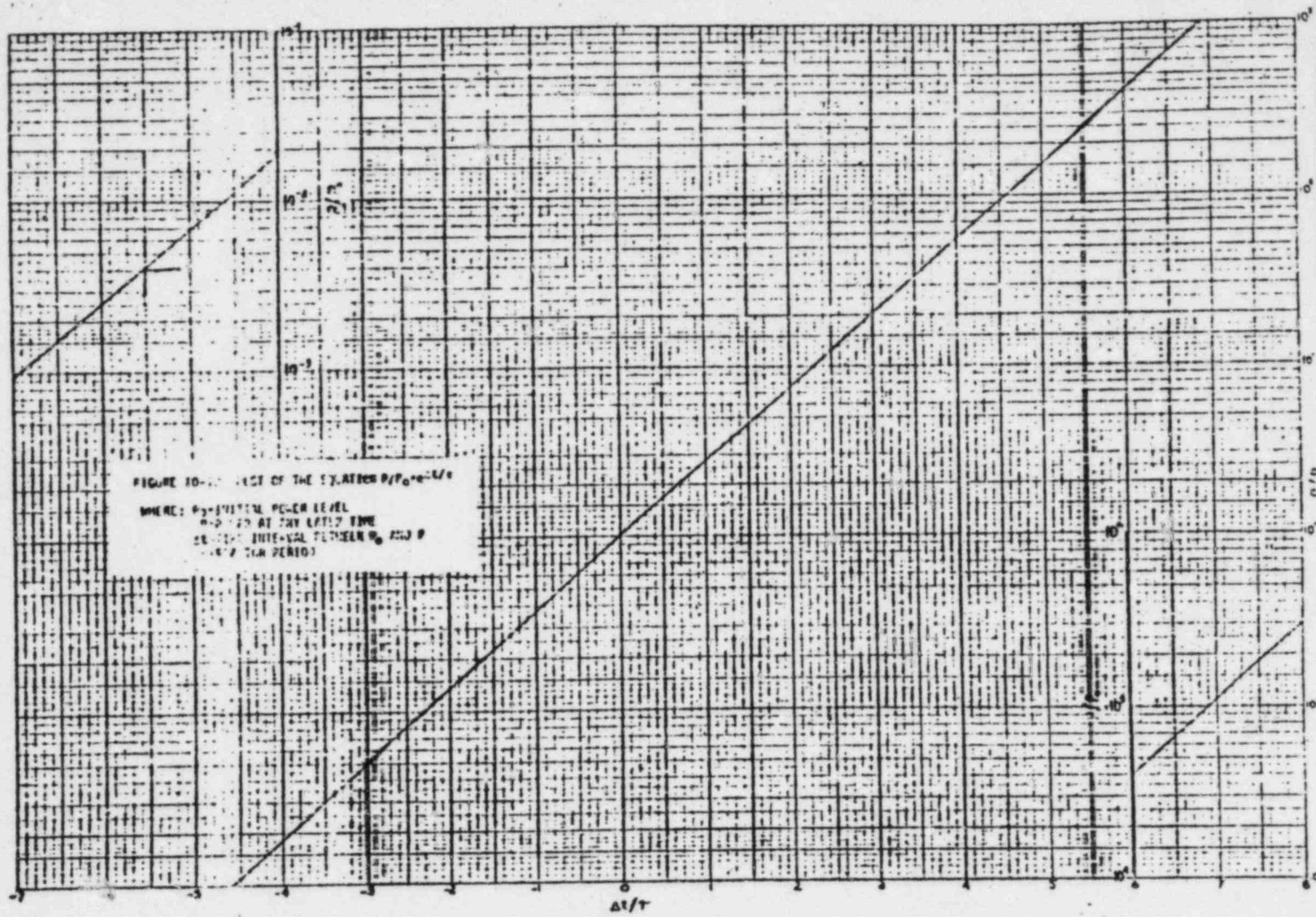


FIGURE 10-[illegible] PLOT OF THE EQUATION $P/P_0 = e^{\Delta t/\tau}$

WHERE: P_0 = INITIAL POWER LEVEL
P = POWER AT ANY LATER TIME
Δt = TIME INTERVAL BETWEEN P_0 AND P
τ = REACTOR PERIOD

$P_0 = 68$ MWT

$\Delta t = 49$ seconds

$\tau = 70$ seconds

$\Delta t/\tau = 49/70 = 0.70$

From Figure 10-18:

$P/P_0 = 2.01 = P/68$

$P = (68)(2.01) = 136.8$ MWT

2. Time interval of interest is an integral number of periods.

If a reactor is placed on a period, then after a time interval equal to the magnitude of the period the power level will have increased or decreased by a factor of 2.718. It is also easy to figure out what the increase or decrease will be when the time interval is exactly two periods, three periods, etc. For a time interval of two periods:

$\Delta t/\tau = 2$

Therefore:

$P/P_0 = e^{\Delta t/\tau} = e^2 = (2.718)(2.718) = 7.39$

If the period is negative:

$\Delta t/\tau = -2$

$P/P_0 = e^{-2} = 1/e^2 = 1/7.39$

For a time interval of three periods:

$\Delta t/\tau = 3$

$P/P_0 = e^3 = (2.718)(2.718)(2.718) = 20$

Similarly, if the period is negative:

$P/P_0 = e^{-3} = 1/20$

It is useful to memorize the figures 7.39 and 20 corresponding to the power changes for two and three periods respectively. Similar results can be derived for four, five or more periods but these values are generally not worth committing to memory.

Example 1: A reactor is at 28 MWT. What will the power level be

3 minutes after the reactor is placed on a 60 second period?

Since 3 minutes represents 3 periods, power will have increased by a factor of 20. That is:

$P/P_0 = 20$

Since $P_0 = 28$ MWT: $P = (28)(20) = 560$ MWT

Example 2: If a reactor initially at 200 MWT is placed on a -200 second period for 400 seconds, what will the final power be?

Since 400 seconds represents two periods:

$P/P_0 = 1/7.39$

Since $P_0 = 200$ MWT: $P = 200/7.39 = 27.1$ MWT

3. Doubling time or halving time.

If the time interval required for the reactor power to either increase or decrease by a factor of 2 is known, the period can be obtained by multiplying this time interval by 1.44. If reactor power is decreasing, the period calculated by this method must be given a negative sign.

Example 1: Reactor power doubles in 63 seconds. What is the period?

$\tau = (1.44)(68) = 98$ seconds

Example 2: Reactor power goes from 42 watts to 21 watts in 62 seconds. What is the period?

$\tau = -(1.44)(62) = -89.3$ seconds

4. Power changes of less than 20%

If the time interval is known for a reactor power change of less than 20%, the period can be calculated from the formula:

$$\tau = \frac{\Delta t}{(P/P_0) - 1} \qquad (10\text{-}29)$$

Example 1: Power falls from 90 MWT to 85 MWT in 20 seconds. What is the period?

$$\tau = \frac{20}{85/90 - 1} = \frac{20}{0.945 - 1} = \frac{20}{-0.055} = -364 \text{ seconds}$$

Notice that if $P/P_0 = 1.1$ or 0.9 (i.e., if there is a 10% power change either direction) then the period is just 10 times the time interval.

This can be verified from equation (10-31).

$$\tau = \frac{\Delta t}{1.1 - 1} = \frac{\Delta t}{0.1} = 10\Delta t$$

$$\tau = \frac{\Delta t}{0.9 - 1} = \frac{\Delta t}{-0.1} = -10\Delta t$$

Example 2: Power goes from 35 watts to 38.5 watts (10% increase) in 9 seconds. What is the period?

$\tau = (10)(9) = 90$ seconds

Other approximate formulas concerning periods and power levels can be derived, but those given above are sufficient for all practical purposes.

STARTUP RATE

On BWRs the meters used to show how fast the reactor power is changing, indicate the period. However, on Westinghouse PWRs, these meters indicate in a new unit called the startup rate. The startup rate, SUR, is defined as the rate of change of reactor power expressed in decades per minute (DPM). A decade is a factor of 10. Thus, if the SUR is 1.0 DPM, the power will increase by a factor of 10 in each minute. Similarly, if the SUR is -1 DPM, the power level will drop to 1/10 its former level in each minute.[1]

When SUR is used instead of period, the variation of power level with time is given by the equation:

$$P = P_0 \times 10^{SUR \times \Delta t} \qquad (10\text{-}30)$$

where: SUR = startup rate (decades per minute)
Δt = time interval between P_0 and P (minutes)
P_0 = initial power level
P = power level at any later time

This equation is analogous to equation (10-26). If the SUR is known, it is possible to calculate the period and vice versa. The relationship between these two quantities is:

$$SUR = \frac{26}{\tau} \qquad (10\text{-}31)$$

where: τ = period in seconds
SUR = startup rate in DPM

Thus, a SUR of 0.5 DPM corresponds to a period of 52 seconds. A 100 second period corresponds to a SUR of 0.26 DPM.

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[1] Later in this chapter we will show that the SUR cannot exist at such a value for any extended period of time.

Equation (10-30) can be solved using a sliderule or a table of common logarithms. Figure 10-19 is a plot of P/P_0 versus SUR x Δt which can be used in an analogous manner to Figure 10-18. However, to eliminate the need for remembering too many formulas, it is suggested that when SUR problems are encountered, the SUR first be converted to period using equation (10-31) and the problems then be solved using the methods discussed in the previous section of this chapter.

On license examinations, the student is occasionally asked to make plots of reactor power versus time on semi-logarithmic paper for various startup rates. These plots will be straight lines on this type of graph paper (see Chapter 1) since the power rise is exponential, and the problem is then to determine the slope of the curve. The easiest way to do this is to determine how long it will take for the power level to increase by one decade. This is accomplished by inverting the SUR. Thus, if the SUR is 0.4 DPM, it will take 2.5 minutes per decade of power increase. Figure 10-20 shows the power response versus time for several SURs. The student

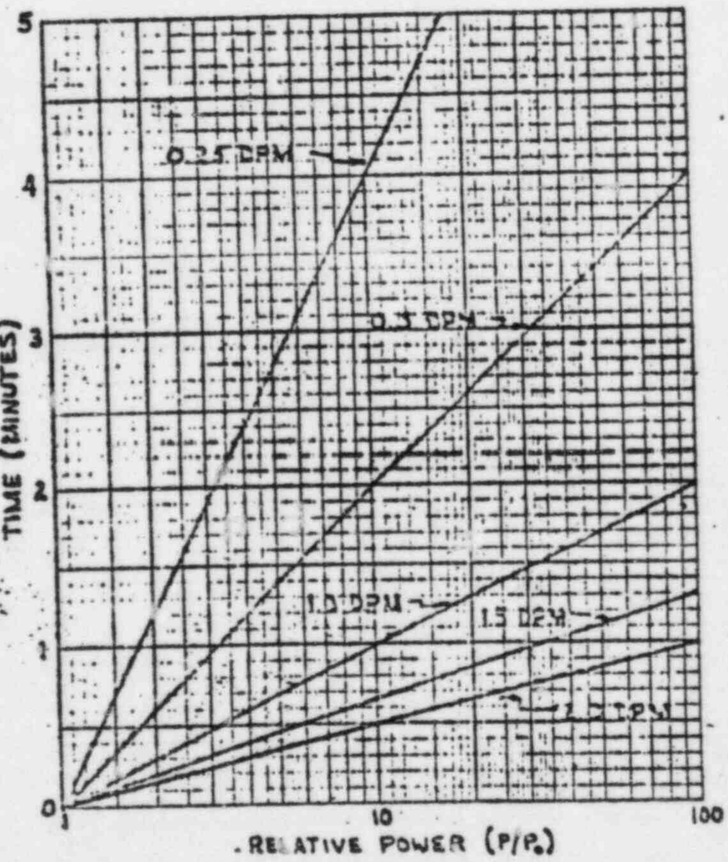

Figure 10-20: Power Versus Time for Various Startup Rates

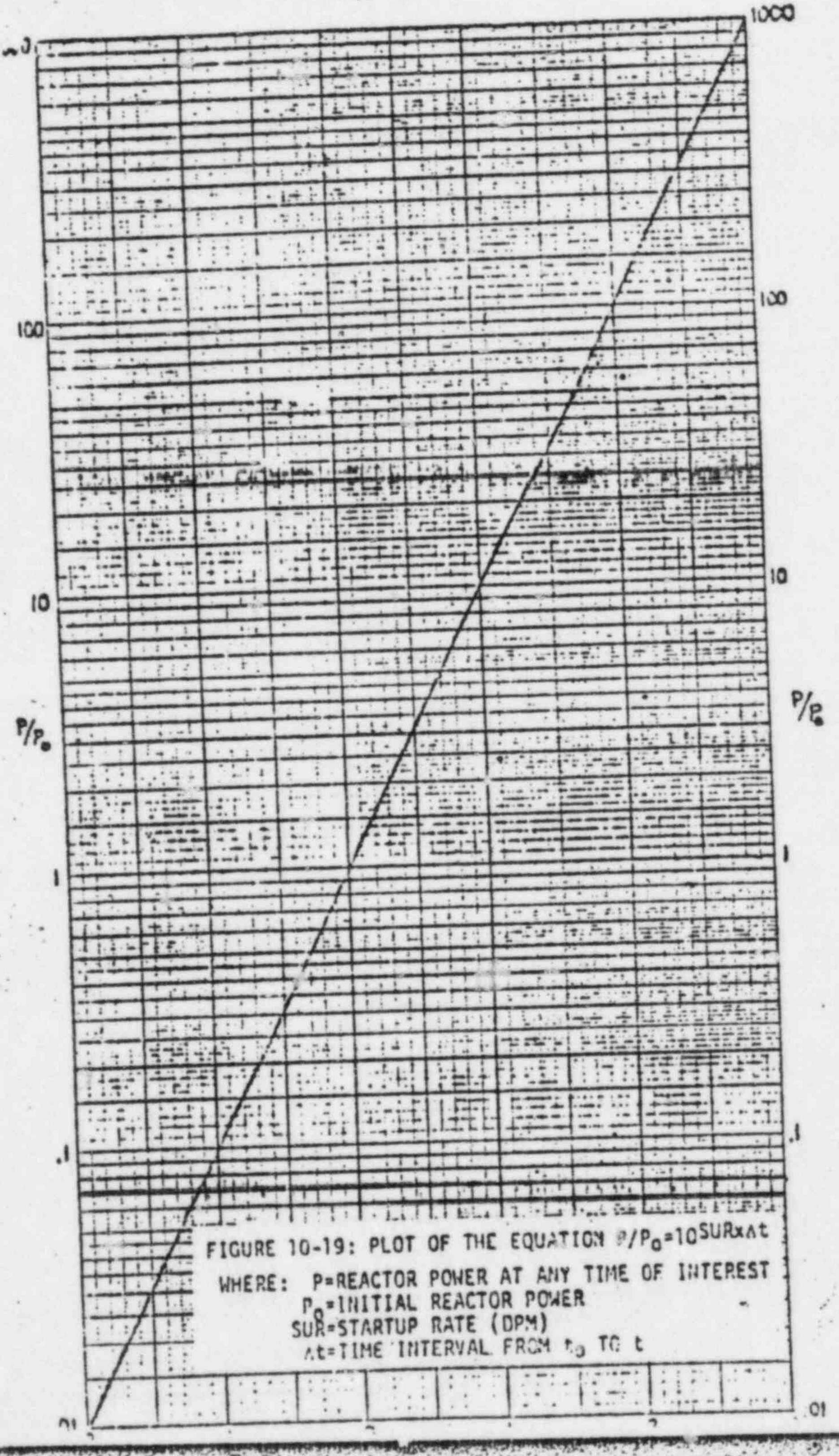

FIGURE 10-19: PLOT OF THE EQUATION $P/P_0=10^{SUR \times \Delta t}$

WHERE: P=REACTOR POWER AT ANY TIME OF INTEREST
P_0=INITIAL REACTOR POWER
SUR=STARTUP RATE (DPM)
Δt=TIME INTERVAL FROM t_0 TO t

should verify that they are drawn with the correct slope by checking the length of time required for power to change by a decade.

Another question which is prequently asked is if it takes a certain number of minutes for power to increase by a decade, by how much will power have increased in one-half of this time interval? The answer is not 5 times its original value, but $\sqrt{10} \approx 3.16$ times. This follows from the fact that in an exponential process, equal percentage changes take place in equal time intervals.

PROMPT NEUTRON GENERATION TIME

In previous sections we have often mentioned neutron generations. Before continuing on, it is worthwhile to discuss the approximate length of time which is involved in each generation. We will define the generation time as the average length of time between successive neutron "births". The total generation time is composed of three parts: 1) the average length of time required to slow the newly born fast neutron down to thermal, 2) the thermal lifetime prior to death by absorption or leakage, and 3) the average length of time between the absorption of a thermal neutron and the birth of a new fast neutron. For low enrichment water moderated reactors, typical values for the three terms are: 1) slowing down time: $\approx 7 \times 10^{-6}$ seconds, 2) thermal diffusion time: $\approx 3.5 \times 10^{-5}$ seconds, 3) time from absorption to birth: $\approx 10^{-12}$ seconds[1]. Thus, a typical prompt neutron generation time for reactors of the type we are discussing would be $\approx 4.2 \times 10^{-5}$ seconds. The precise value will vary somewhat depending upon the size of the reactor, enrichment, moderator to fuel ratio, and fuel burnup. However, the variations are not large.

EFFECT OF DELAYED NEUTRONS UPON REACTOR BEHAVIOR

Consider a reactor in which all neutrons are assumed to be prompt. Let us assume that this reactor is just critical at a power level such that 1,000,000 neutrons are produced in each generation. Now suppose that by control rod withdrawal or some other mechanism, k_{eff} is quickly increased to 1.001. From the definition of k_{eff}, the neutron population at the end of the first generation is $(1{,}000{,}000)(1.001) = 1{,}001{,}000$. Similarly, at the end of the second generation the neutron population is $(1{,}000{,}000)(1.001)^2 = 1{,}002{,}001$. Now suppose it is desired to level off the power by inserting a control rod far enough to return k_{eff} to 1.000. Since control rods cannot be moved instantaneously, this operation will require a small amount of time, say 3 seconds. A typical prompt neutron generation time is $\approx 4 \times 10^{-5}$ seconds, so that in 3 seconds the reactor will have undergone $3/(4 \times 10^{-5}) = 75{,}000$ generations. By the time the

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[1] The last value is for prompt neutrons only. The values for delayed neutrons vary, depending upon the half-life of the precursor, but in all cases are much longer. The significance of this will be discussed in the next section.

reactor has been returned to critical, the neutron population will have risen to $(1,000,000)(1.001)^{75000} = 1.78 \times 10^{38}$. It should be obvious from this example that a reactor operating on prompt neutrons alone is impossible to control effectively.

Now consider the same problem in a reactor using pure U-235 as fuel. In this case, the delayed neutron fraction is 0.64%. With the reactor in its initial critical condition, there will be 1,000,000 neutrons born at the beginning of each generation, of which 993,600 will be prompt and 6400 will be delayed. Of course, the delayed neutrons will not appear at once, but at a gradually decreasing rate over a period of several minutes. By the time all of the delayed neutrons from any particular generation have appeared, the prompt neutrons from the same generation will have undergone a great many additional generations. In other words, the generations overlap by a considerable amount. Nevertheless, with the reactor just critical an equilibrium will eventually be reached so that at the start of each "composite" generation there will be 1,000,000 neutrons, of which 6400 are the result of fissions which occurred in the past. When k_{eff} is increased to 1.001, the first generation will, as before, result in the eventual production of 1,001,000 new neutrons. Of these, 99.36% or 994,594 will appear immediately. The remaining 6406 from this generation will appear later. However, at this time the reactor will still receive 6400 neutrons from the earlier generations when the reactor was just critical. Therefore, at the end of the first "composite" generation there will be 994,594 + 6400 = 1,000,994 neutrons. Notice that this is 6 fewer than would have appeared had all neutrons been prompt. These 1,000,994 neutrons will undergo fission and will eventually become (1,000,994)(1.001) = 1,001,995 neutrons. Again, however, only 99.36% or 995,582, will appear immediately. Another 6400 will also appear as the result of earlier fissions, making a total neutron population of 1,001,982 at the end of the second "composite" generation. This is 19 fewer than appeared at the end of the second generation when all neutrons were assumed to be prompt. Admittedly, the differences between the two cases are very small for the two generations that have been calculated. However, recall that the power buildup in a supercritical reactor is an exponential process, and that exponential processes start slow but quickly build up. If this calculation were carried out for several thousand generations, the differences between the two cases would be found to be truly significant. In fact, it can be shown that the asymptotic period is ≈ 0.1 seconds for the prompt reactor, and about 60 seconds for the reactor with delayed neutrons. In the 3 second time interval required to move a rod, the power will only increase by about 5% if the reactor is on a 60 second period. Under these circumstances it is seen that the reactor is rather easy to control.

In summary, it can be stated that when a reactor is made supercritical, the effect of this excess reactivity is reflected in an immediate increase in prompt neutron population. It is also reflected in an immediate increase in the inventory of delayed neutron precursor fission products. However, the delayed neutron population does not show an immediate increase, but rather, continues for a measurable length of time at a level determined by conditions which existed prior to the time the reactor was made supercritical. Therefore, the rate of increase of the total neutron population is reduced when delayed neutrons are present. Another way of looking at this behavior is that the presence of delayed neutrons markedly increases the average generation time due to the long time delay involved in the birth of delayed neutrons. In effect, the presence of delayed neutrons increases the <u>core average</u> generation time from about 4 x 10^{-5} seconds, which would be the value in a prompt reactor, to about 0.08 - 0.10 seconds. Were it not for this fortunate circumstance, the nuclear reactor would be too difficult to control to make it a practical power producing tool.

Figure 10-21 shows a plot of reactor power versus time for the previous

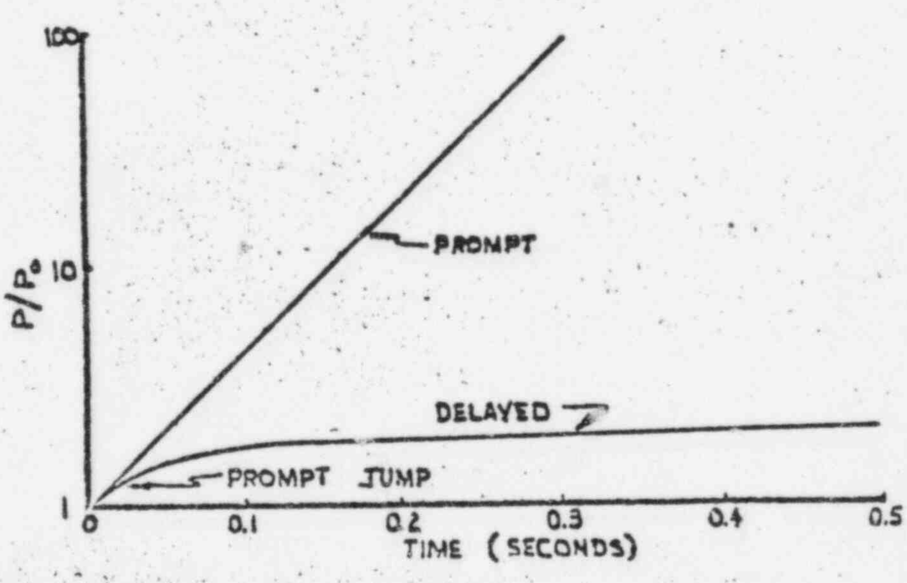

Figure 10-21: Response of Reactors Operating with and without Delayed Neutrons to a Step Change in Reactivity of 0.1%

example. Curves are drawn for both of the cases discussed. Notice that for the first hundredth of a second or so the two curves coincide, but that as time goes on the delayed neutron curve levels off on a much slower period. The fact that the two curves nearly coincide for the first few generations was illustrated in the numerical example, where the difference between the neutron populations in the first two generations was very small. This momentary sharp rise in power following a step change in reactivity is called the prompt jump. Once the delayed neutrons have gained control, the reactor levels out on what is called the stable or asymptotic period. Since the stable period is usually all that we are ever concerned with in practice, the adjective stable is generally ignored and the terms stable period and period are taken to be synonymous. The size of the prompt jump depends upon the magnitude of the step change in reactivity. The larger the reactivity addition, the larger the prompt jump. Table 10-II gives approximate figures for the ratio of power before and after the prompt jump for various reactivity insertions. Occasionally it is possible to see the prompt jump on the power level instrumentation recorder traces, but frequently it is not observable because the control rods move too slow to provide a clean step change in reactivity, the normal reactivity changes are too small, and the recorders are too slow to provide this kind of resolution.

TABLE 10-II: MAGNITUDE OF PROMPT JUMP FOR VARIOUS STEP REACTIVITY INSERTIONS FROM CRITICAL (FOR β = .0075)

ρ	$P_{\text{after jump}}/P_{\text{before jump}}$	Stable period (seconds)
.0002	1.03	445
.0005	1.07	160
.001	1.15	65
.002	1.36	23
.003	1.66	10
.004	2.13	5
.005	2.99	2

PROMPT CRITICAL

In this section we will look at what constitutes the upper limit on k_{eff} in a real reactor. To do this, let us once again consider a critical reactor operating with pure U-235 fuel (i.e., the delayed neutron fraction is 0.64%) and having an equilibrium neutron population of 1,000,000. If k_{eff} is suddenly increased to 1.0064, the first subsequent generation will result in the eventual product of 1,006,400 neutrons, of which 99.36% or 1,000,000 will be prompt. Notice that the reactor is producing enough prompt neutrons to sustain itself without benefit of any delayed neutrons. Under these circumstances the reactor is said to be prompt critical. When the reactor is less than prompt critical it can be seen from the discussion in the previous section that the reactor requires the delayed neutrons to sustain itself. The reactor must, in effect, wait for the delayed neutrons to appear and so the delay time before their birth becomes the limiting factor in determining the rate of power rise. When the reactor is above prompt critical, however, the reactor no longer needs to wait for the delayed neutrons, and so their delay time is no longer the limiting factor. Consider, for example, the behavior of the reactor when $k_{eff} = 1.0074$. The first generation would eventually increase the neutron population to 1,007,400 (assuming an initial population of 1,000,000) of which 1,001,000 would be prompt. If the most pessimistic possible assumption is made, i.e., that the delayed neutrons have an infinite delay time and never appear, the reactor will still behave like a prompt reactor with $k_{eff} = 1.001$. We saw in the previous section that under these circumstances the reactor is uncontrollable by ordinary methods. Therefore, k_{eff} must always be kept below that value which will make the reactor prompt critical. In practice, prompt critical will not even be approached during normal reactor operation. Instead, k_{eff} will be limited to values which will produce a period no shorter than about 20 to 30 seconds.

In water moderated reactors, the fuel is primarily a mixture of three isotopes, U-235, U-238, and Pu-239. The delayed neutron fraction for each of these isotopes is given in Table 10-III. The effective delayed neutron fraction for the entire core will be a weighted average of the delayed neutron fractions for each of the isotopes in the core, with the weighting factor being the relative number of fissions occurring in each isotope. For example, new Humboldt Bay Type I fuel contained 2.6% U-235 and 97.4% U-238. The average delayed neutron fraction for this core was 0.75% which is slightly higher than the value for pure U-235 due to the influence of the U-238. As the fuel burns, there is a gradual buildup of Pu-239. By the time 10,000 MWD/ton of exposure was obtained, and the fuel was about ready to be replaced, its composition was 1.56% U-235, 98% U-238, and 0.32% Pu-239 (the remaining 0.12% is primarily the non-fissionable isotope U-236 formed from (n,γ) reactions in U-235). The average delayed neutron fraction in this core has fallen to 0.60% due to the influence of the Pu-239. Since prompt critical occurs when the excess reactivity in the core equals the delayed neutron fraction, it will occur at a k_{eff} value of 1.0075 for the new Humboldt Bay core, and 1.0060 for the burned core.

TABLE 10-III: DELAYED NEUTRON FRACTIONS FOR REACTOR FUELS

Isotope	(%)
U-235	.64
Pu-239	.21
U-238 (fast fission)	1.68

THE RELATIONSHIP BETWEEN REACTIVITY AND PERIOD

It is of interest to know what period will result for a particular value of k_{eff} in a reactor. This will depend upon the average neutron lifetime (generation time), upon the average half-life of the delayed neutron emitters, and upon how close any particular value of k_{eff} puts the reactor to prompt critical. This latter variable is, of course, measured by the average delayed neutron fraction in the reactor. It is possible to derive a formula which relates k_{eff}, or more commonly reactivity, to period. The derivation is beyond the scope of this manual but the formula is given below:

$$\rho = \frac{L}{k_{eff}\tau} + \frac{\beta}{1 + \lambda\tau} \qquad (10\text{-}32)$$

where: ρ = reactivity ($\Delta k/k$)
L = average prompt neutron lifetime (sec)
β = average delayed neutron fraction[1]
λ = average decay constant for delayed neutron emitters = $0.693/T_{1/2}$ where $T_{1/2}$ is the average half-life of the delayed neutron emitters
τ = reactor period (sec)

The average decay constant depends only upon the relative amounts of each of the six delayed neutron emitters produced per fission. This is reasonably uniform for all fuels, and so an average value for λ of ≈ 0.1 sec can be used with little error for all reactors and for all burnups. The average prompt neutron lifetime will vary somewhat depending upon the composition of the core and its geometrical configuration. For the new Humboldt Bay core, the average lifetime was about 3.8×10^{-5} seconds. This value is only important when the reactor is prompt critical. When the reactor is dependent upon delayed neutrons, their delay time before birth is so much longer than the lifetime after birth, that it is the delay time that is limiting.

Notice that the right hand side of equation (10-32) contains two terms. The first term, $L/k_{eff}\tau$, represents the effect of the prompt neutrons. That is, if all neutrons were prompt, equation (10-32) would only contain this first term on the right hand side. Similarly, the second term on the right hand side accounts for the effect of delayed neutrons. Whenever

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[1] To be strictly correct, a quantity called β_{eff} should be used in these equations instead of the true delayed neutron fraction, β. The quantity β_{eff} takes into account the fact that delayed neutrons are generally born at lower energies (≈ 0.4 mev) than are prompt neutrons (≈ 2 mev). This has two major effects. First, the delayed neutrons cannot cause fast fission in U-233, so in this respect they are ≈ 5% less valuable since ε ≈ 1.05. On the other hand, they do not experience as much fast leakage, and so in this respect they are ≈ 1-2% more valuable since L_f ≈ 0.98. Overall, they end up being ≈ 3% less valuable, and so β_{eff} ≈ 0.97β. Towards end of life, power distribution tends to shift to the edges of the core and fast leakage increases. The fact that delayed neutrons avoid this leakage makes them relatively more valuable at end of life than at beginning of life. Thus at end of life β_{eff} ≈ 0.99β although in extreme cases it can even exceed β. In general, there is so little difference between β and β_{eff} that the subject will not be considered further.

the reactor is significantly less than prompt critical, the prompt neutron term can be ignored and the period can be calculated from the equation:

$$\rho = \frac{\beta}{1 + \lambda\tau} \qquad (10\text{-}33)$$

To illustrate, let us calculate the reactivity which would result in a 60 second period for the new Humboldt Bay core ($\beta = 0.0075$, $L = 3.8 \times 10^{-5}$ seconds, and $\lambda = 0.1$ seconds). From equation (10-32):

$$\rho = \frac{3.8 \times 10^{-5}}{(k_{eff})(60)} + \frac{0.0075}{1 + (0.1)(60)}$$

$$\rho = \frac{0.634 \times 10^{-6}}{k_{eff}} + \frac{0.0075}{1 + 6} = \frac{0.634 \times 10^{-6}}{k_{eff}} + 0.00107$$

Since the period is 60 seconds, the reactor is known to be considerably less than prompt critical. This means that $k_{eff} < 1.0075$ and it is a fair approximation to say that $k_{eff} = 1$. Thus:

$$\rho = 0.634 \times 10^{-6} + 0.00107 = 0.00107$$

Thus it is seen that when the reactor is considerably less than prompt critical, the second term (i.e., the delayed neutron term) is dominant. This is the assumption which led to equation (10-33)

To illustrate the effect of plutonium buildup, let us calculate the period which would result from $\rho = 0.00107$ when $\beta = 0.0060$ instead of 0.0075. Since equation (10-33) is applicable:

$$0.00107 = \frac{0.0060}{1 + 0.1\tau}$$

$$0.00107 + 0.000107\tau = 0.0060$$

$$\tau = \frac{0.0060 - 0.00107}{0.000107} = \frac{0.005}{0.0001}$$

$$\tau = 50 \text{ seconds}$$

which compares with the value of 60 seconds from the previous example.

Equation (10-32) can be used to calculate the period which would be obtained when the reactor is above prompt critical. Consider, for example, the new Humboldt Bay reactor with $k_{eff} = 1.01$. Equation (10-32) gives:

$$\frac{0.01}{1.01} = \frac{3.8 \times 10^{-5}}{1.01\tau} + \frac{0.0075}{1 + 0.1\tau}$$

$$0.0099 = \frac{3.76 \times 10^{-5}}{\tau} + \frac{0.0075}{1 + 0.1\tau}$$

This equation can be solved for τ by relatively advanced algebraic techniques, or by trial and error. We will not complete the calculation here, but for illustrative purposes an approximate result can be obtained by assuming that the delayed neutrons never appear and that the reactor is operating on prompt neutrons alone with $k_{eff} = 1.0100 - 0.0075 = 1.0025$. The delayed neutron term in equation (10-32) can then be ignored and the period can be calculated from the equation:

$$\rho = \frac{L}{k_{eff}\tau}$$

$$\frac{0.0025}{1.0025} = \frac{3.8 \times 10^{-5}}{1.0025\ \tau}$$

$$0.0025 = \frac{3.8 \times 10^{-5}}{\tau}$$

$$\tau = \frac{3.8 \times 10^{-5}}{0.0025} = 0.015 \text{ seconds}$$

Problems of this type are of academic interest only, since the reactor is never operated in the prompt critical condition. For all hand calculations, equation (10-33) will suffice. Although it is instructive for the operator to be able to make rough calculations such as those outlined above, in actual practice they are seldom required. Instead, the operator usually has a "rho-tau" curve at his disposal from which he can obtain the desired results immediately. A "rho-tau" curve is simply a plot of reactivity (rho or ρ) versus period (tau or τ). In other words, it is a plot of equation (10-32), although usually a more accurate form of the equation which considers six separate delayed neutron terms rather than one average term. Figures 10-22 and 10-23 are rho-tau curves for Humboldt Bay for positive and negative reactivities respectively. Notice that there are individual positive reactivity curves for new and old fuel. These simply reflect the buildup of plutonium as we have discussed.

So far we have not considered negative reactivities. Equation (10-32) (or equation (10-33) since prompt critical is not a factor) does not give accurate enough results to be of any practical use when negative reactivities are being considered. The reason for this is that equation (10-32) contains only one delayed neutron term using average delayed neutron properties rather than six terms to account for each of the six groups individually. This does not introduce much error when positive reactivities are being considered, but results in the equation being practically useless for negative reactivities due to the following phenomenon. Suppose the reactor is operating at some steady power level when suddenly all control rods are quickly inserted, thereby making the reactor considerably subcritical. As was the case for positive reactivities, the prompt neutrons will undergo several thousand generations

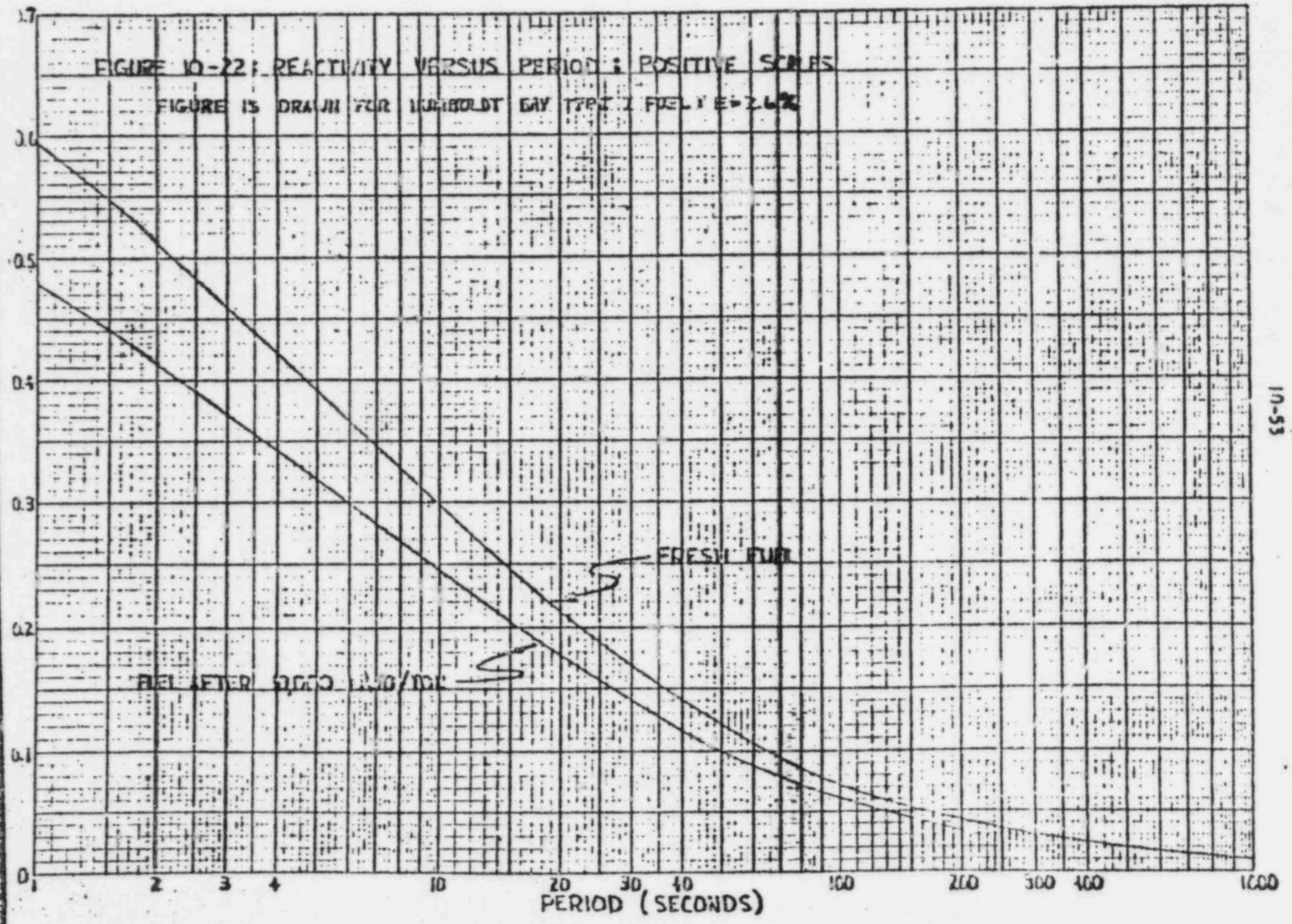

FIGURE 10-22: REACTIVITY VERSUS PERIOD : POSITIVE SCALES

FIGURE IS DRAWN FOR HUMBOLDT BAY TYPE I FUEL : E=2.6%

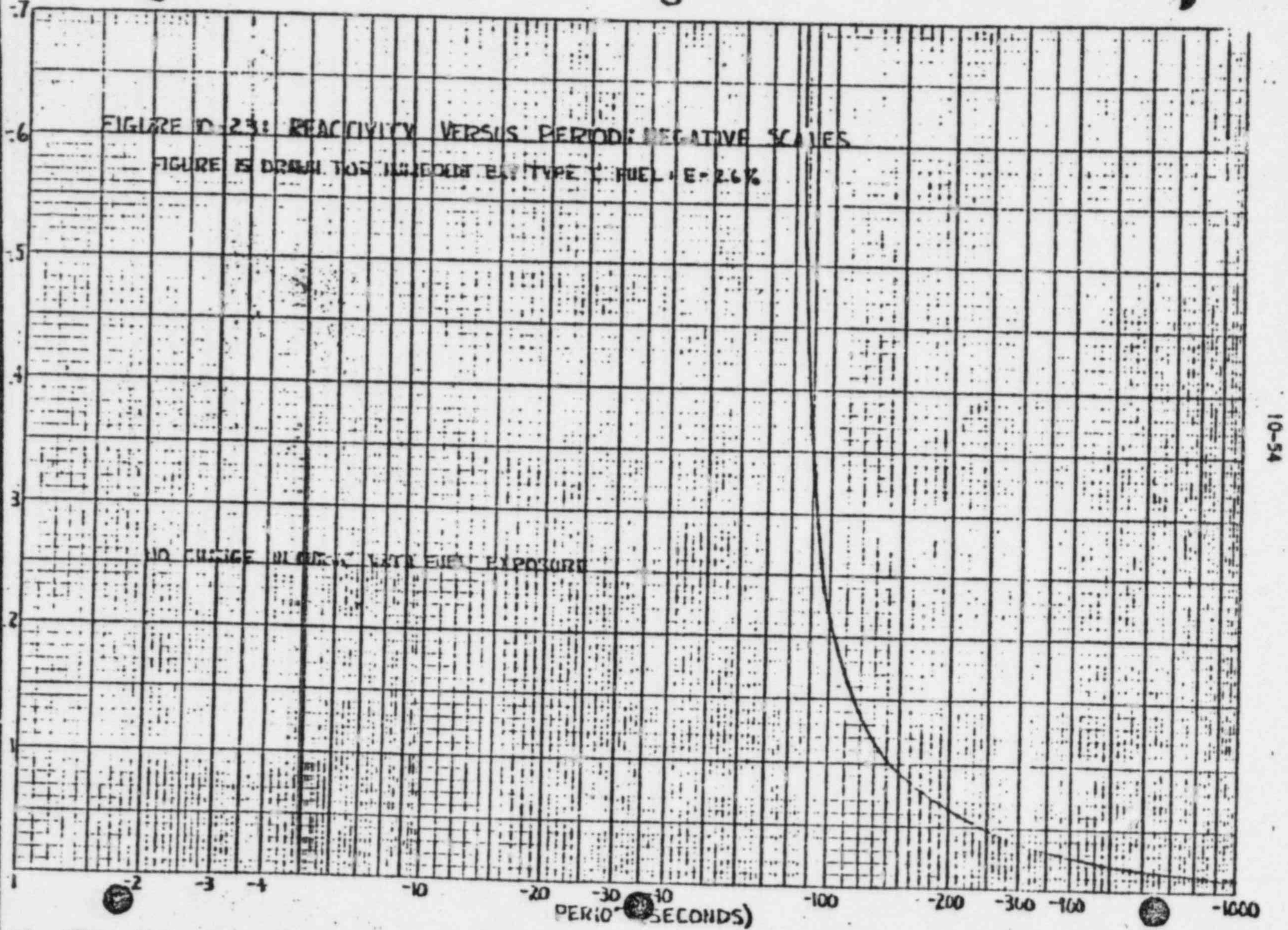

FIGURE 10.23: REACTIVITY VERSUS PERIOD; NEGATIVE SCALES

FIGURE IS DRAWN FOR BURNOUT B4 TYPE I FUEL, E=2.6%

NO CHANGE IN CURVE WITH FUEL EXPOSURE

within a fraction of a second. Since the reactor is far subcritical, the prompt neutron population will thus fall almost immediately to the source multiplication level. This rapid power reduction is called the prompt drop, and is somewhat analogous to the prompt jump. However, there still remains a near equilibrium inventory of delayed neutron precursor fission products in the core. These will continue to emit neutrons at rates determined by their respective half-lives. In the first few minutes after shutdown, the majority of the short half-life delayed neutrons will be born, and then immediately reduced in number to the source level. Finally, the last remaining source of neutrons will be the 55.6 second half-life delayed neutron emitters. Because the neutrons cannot be eliminated until they are born, the rate of reactor power decrease is also limited to a 55.6 second half-life. Since the period is 1.44 times the doubling or halving time, the fastest equilibrium negative period the reactor can be on is -(1.44)(55.6) = -80.2 seconds. The reactor will remain on this period until the neutron population levels off at a value determined by the subcritical source multiplication. Figure 10-24 shows typical recorder traces following scrams (i.e., immediate insertion of all control rods) from full power for a typical BWR and PWR. The curves show the same general behavior and are essentially similar, the only difference being a slightly larger prompt drop in the case of the BWR due to the fact that the strong BWR rods take the core somewhat farther subcritical immediately after the scram. That is, the magnitude of the prompt drop increases as the magnitude of the step negative reactivity insertion increases.

Of course, there is no reason why the reactor must be shut down by complete insertion of all control rods. If the control rods were only partially inserted, the reactor could be leveled off on a longer negative period, say -500 seconds. Further rod insertion might cause the period to shorten to -100 seconds. Still further insertion will cause the reactor to equilibrate at -80.2 seconds as before. Any further rod insertion will have no effect upon the resulting equilibrium period. This does not mean, however, that insertion of just enough control rods to result in a -80.2 second period will shut the reactor down as fast as immediate insertion of all control rods, because with a large reactivity insertion the reactor gets the benefit of a large prompt drop.

REACTIVITY COEFFICIENTS

In our discussion of the six factor formula, we went on at some length about how each of the six factors were affected by variations in such quantities as moderator to fuel ratio and core composition. In this section we will discuss the practical significance of our previous observations.

Suppose a reactor is placed on a 60 second period and allowed to follow its own course without interference. One might expect that reactor power would continue to rise on a 60 second period until the core melted down. However, if the reactor is properly designed, the power would continue to rise until the core began to heat up, at which time the

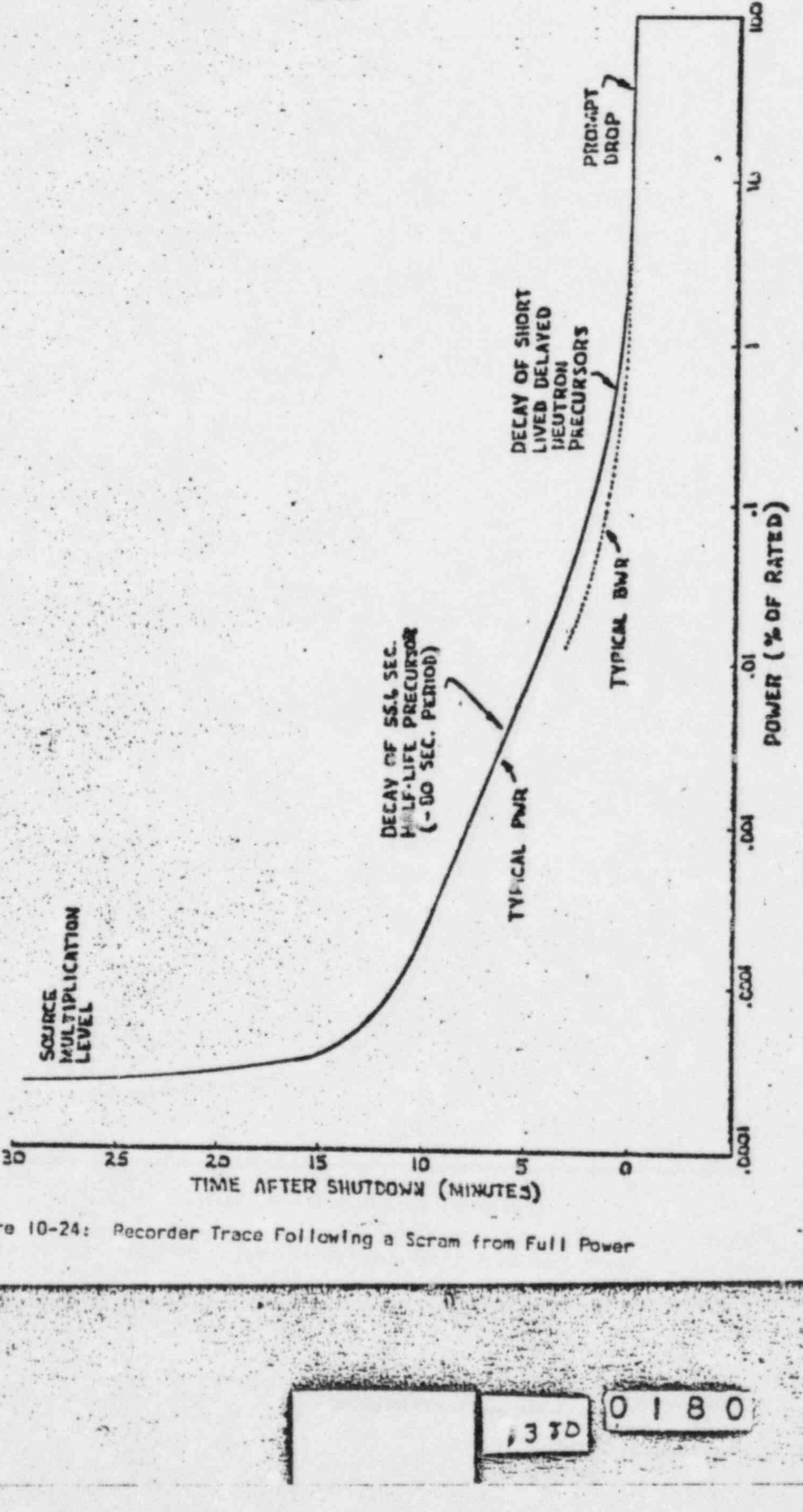

Figure 10-24: Recorder Trace Following a Scram from Full Power

period would begin to lengthen and the power rise would be terminated. Clearly, the effects of the power rise have resulted in a reduction in k_{eff} for the core, and have acted as a kind of internal brake on the reactor. The reactivity effects accompanying a change in power level are mainly the result of changes in three quantities; moderator temperature, fuel temperature, and steam void fraction. The magnitude of the reactivity change which attends a unit change in any of these three quantities is called the reactivity coefficient for the particular quantity. We will consider each of these three reactivity coefficients separately.

1. Moderator temperature coefficient

As the temperature of the water moderator is increased, its density decreases (i.e., it expands) as shown in Figure 10-25. Since the volume in the core which is occupied by moderator remains essentially constant[1], a reduction in moderator density means that there is a

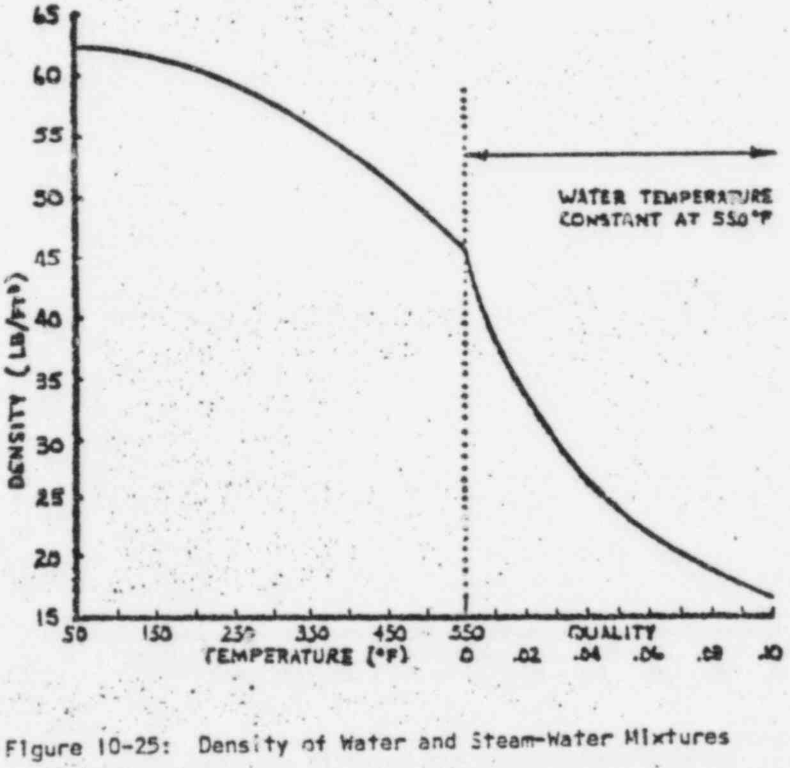

Figure 10-25: Density of Water and Steam-Water Mixtures

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[1] As the core temperature increases, the solid portions of the core

reduction in the number of moderator molecules in the core. The number of fuel molecules in the core remains constant as temperature increases, so the net result of a temperature increase is that the moderator to fuel ratio (on a weight or molecule basis) decreases.

We have already discussed the effects of a change in N_{H_2O}/N_{UO_2} upon the factors which make up k_{eff}, but it is worthwhile summarizing these effects as they apply to both BWRs and PWRs. This is done in Table 10-IV for conditions at beginning of core life and at end of core life. In the table, a negative effect is taken to mean one which tends to reduce k_{eff} when moderator temperature increases. A positive effect is one which tends to increase k_{eff} when moderator temperature increases. There are nine effects listed in the table[1], some of which are positive and some of which are negative. If the negative effects are larger than the positive effects, the overall moderator temperature coefficient is negative. This means that the net effect of an increase in moderator temperature is a reduction in k_{eff}. Conversely, if the positive effects are larger than the negative effects, the reactor will have a positive coefficient and an increase in moderator temperature will tend to increase k_{eff}.

To illustrate, suppose we have $k_{eff} = 1.000$ ($\rho = 0\ \Delta k/k$) and moderator temperature = 500° F. Suppose further that we raise moderator temperature to 550° F without making any other change in the reactor[2]. At this new temperature, k_{eff} would probably no longer be 1.000. If the moderator temperature coefficient is negative, k_{eff} would be less than 1.000, say 0.995 ($\rho = -.005\ \Delta k/k$). The temperature coefficient is defined as the change in reactivity per degree change in temperature, or:

(fuel rods, support structures, etc.) also expand. Therefore, the physical dimensions of a hot reactor are slightly larger than when cold, and the moderator actually occupies a somewhat larger volume. However, the amount that the solid components expand is very much less than the amount that the water expands. Therefore, in our discussion we will assume that the physical dimensions of the core remain essentially constant and that the moderator occupies the same volume under all conditions.

[1] In addition, there are a number of other relatively small effects which are not listed.

[2] In a large plant with forced circulation, this could be accomplished by simply running the reactor coolant pumps and letting the friction heat they generate increase the temperature.

TABLE 10-IV: REACTIVITY EFFECTS OF A DECREASE IN MODERATOR DENSITY

Factor	Effect	Importance to BWR		Importance to PWR	
		BOL	EOL	BOL	EOL
η	Neutrons are not thermalized as efficiently because there are fewer moderator molecules available and the molecules have a higher vibrational energy. Thus average energy of slow neutrons is increased.	Negligible negative effect (η for U-235 nearly independent of neutron energy)	Small negative effect (η for Pu-239 decreases faster than η for U-235 as neutron energy increases)	Same as BWR	Same as BWR
ϵ	Neutrons not slowed down as quickly because there are fewer moderator molecules available. Therefore, neutrons spend more time at high energies and more fast fissions occur.	← Medium positive effect? →			
p	Decreased moderator efficiency means that neutrons spend more time with resonance energies and resonance capture increases.	← Strong negative effect in undermoderated core →			
f	1. Removal of water from core reduces capture in water	← Medium positive effect →			
	2. Average slow neutron energy is increased. More neutrons are captured in Pu-239 resonance at 0.29 ev.	—	Small positive effect	—	Same as BWR
	3. Slow neutrons travel farther and more are able to reach the surface of inserted control rods. Worth of inserted rods increases.	Strong negative effect. Many control rods are still inserted when reactor is operating at full power at BOL.	Small negative effect. Nearly all rods are withdrawn when reactor is at full power at EOL.	Medium negative effect. Throughout core lifetime nearly all control rods are withdrawn when reactor is at full power.	
	4. As moderator expands dissolved boric acid is removed from the core so there is less neutron absorption in chemical shim.	—	—	Strong positive effect at BOL, concentration of chem shim in moderator is high.	Negligible negative effect. Nearly all chem shim has been removed from moderator at EOL.
L_f	Less moderator means neutrons travel farther during slowing down process. Leakage increases.	← Small to medium negative effect →			
L_t	Reflector ability of water is reduced so slow neutrons travel farther. Leakage increases.	← Small negative effect →			

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$$\text{Moderator Temperature Coeff.} = \frac{\rho_{final} - \rho_{initial}}{T_{final} - T_{initial}} \qquad (10\text{-}34)$$

where the temperatures in the denominator refer to the moderator and the subscripts initial and final refer to before and after the temperature change. Thus, for the previous example, the average moderator temperature coefficient over the range from 500° F to 550° F is given by:

$$\text{M.T.C.} = \frac{-.005 - 0}{550 - 500} = \frac{-.005}{50} = -1.0 \times 10^{-4} \ \rho/°\text{F}$$

As we shall see below, the magnitude of the temperature coefficient varies considerably from one reactor to another; and for different operating conditions in the same reactor. However, for light water reactors, the value -1×10^{-4} $\rho/°F$ is a representative figure.

In general, a negative moderator temperature coefficient is a desirable condition. If the reactor power begins to increase, either intentionally or unintentionally, one effect of this increase will usually be an increase in moderator temperature. With a negative moderator temperature coefficient, the increase in temperature would cause a reduction in k_{eff}, thereby acting to stop the rise in power. Thus, a negative temperature coefficient serves as an inherent reactor control mechanism, independent of any mechanical control systems, and can be an important safety feature. Both PWRs and BWRs are designed so that the moderator temperature coefficient is negative for normal "at power" operating conditions.

Before continuing on with our discussion of the coefficient in different reactors, it is well to clarify one point regarding the coefficient which frequently confuses students. Suppose the moderator temperature is held at a constant temperature, say 70° F. The question is, if we maintain a constant 70° F, do we still have a moderator temperature coefficient? The answer is yes. The coefficient is nothing more than a prediction of what will happen to reactivity if temperature is changed. If temperature is held constant, the coefficient still exists, but its effects do not get an opportunity to come in to play.

To help relate the discussion of the moderator temperature coefficient to the earlier discussions in this chapter of the six factor formula, it is instructive to reconsider Figure 10-12, which illustrated the relationship between k_{eff} and N_{H_2O}/N_{UO_2}. To aid in the following discussion, the essential features of the figure have been reproduced in Figure 10-26 in a somewhat exaggerated form. We have already indicated that an increase in moderator temperature causes the operating point to shift to the left on such a figure. Thus, any reactor operates over a range of N_{H_2O}/N_{UO_2} values rather than at a single point on the curve. Three possible operating ranges are indicated on Figure 10-26. In addition, the figure illustrates how the temperature coefficient would behave for reactors operating over each of these ranges.

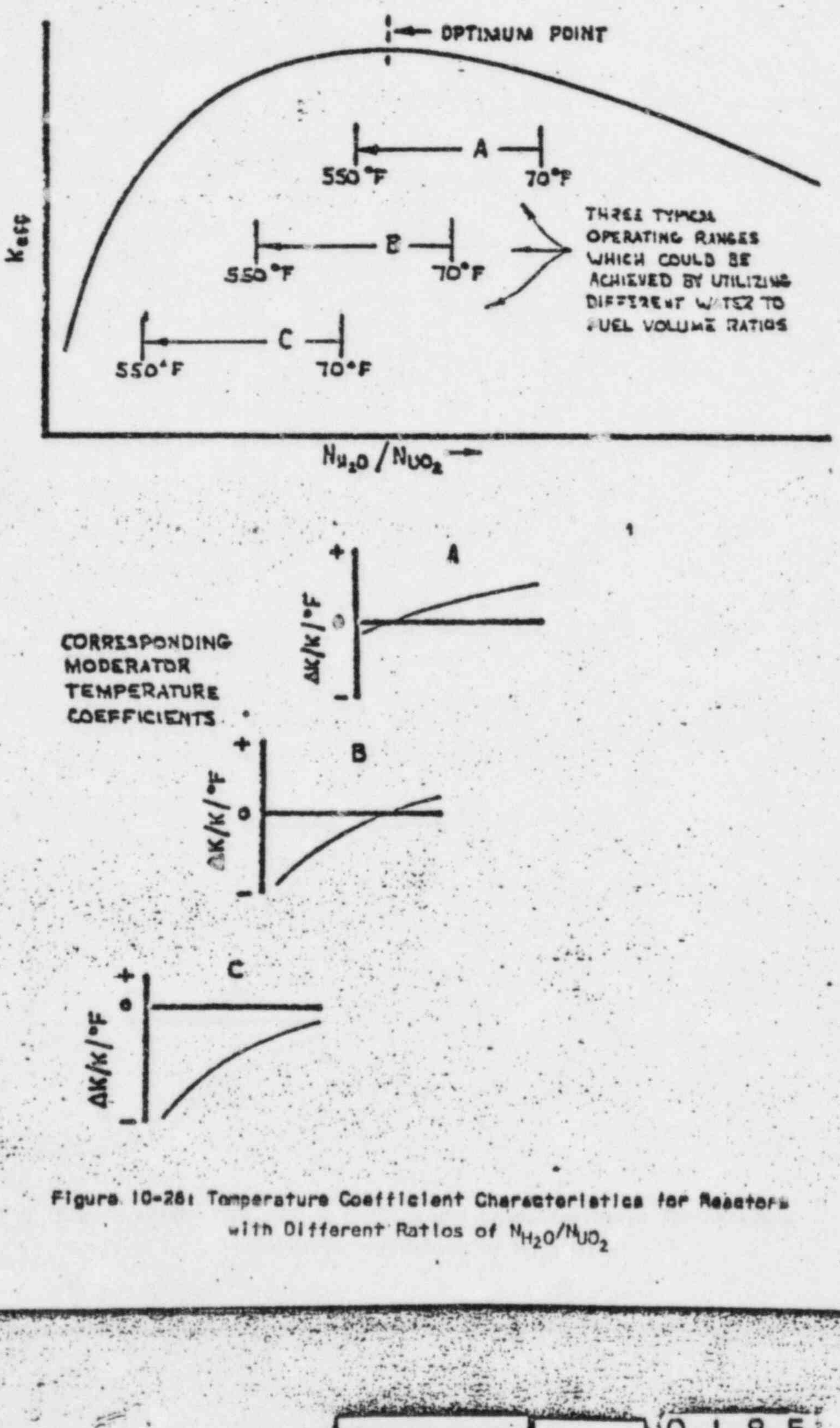

Figure 10-28: Temperature Coefficient Characteristics for Reactors with Different Ratios of N_{H_2O}/N_{UO_2}

Let us first consider the reactor operating over range A. At ambient temperature it is operating in an overmoderated condition (to the right of the optimum point) and it will have a positive coefficient. In fact, it will have a positive coefficient at all operating points in the overmoderated region. However, the magnitude of the coefficient will decrease as the reactor approaches the optimum point. The magnitude of the coefficient depends upon the slope, or steepness of the k_{eff} curve. When the k_{eff} curve is rising steeply, a one degree change in temperature will make a relatively large increase in k_{eff} and the coefficient is large. As the k_{eff} curve flattens out, a one degree change in temperature will make a smaller increase in k_{eff} and the coefficient is smaller. At the optimum point, the coefficient is zero. That is, the slope of k_{eff} curve is horizontal at this point, so a very small change in temperature will not result in a change in k_{eff}. Finally, as temperature is increased beyond the value which corresponds to the optimum point, the coefficient becomes negative. The farther the operating point moves to the left, the more strongly negative the coefficient becomes because the k_{eff} curve begins to fall off at a faster and faster pace.

Construction of the temperature coefficient curves for reactors operating over the B and C ranges follows similar reasoning to that given above. Inspection of the curves indicates the following important observation - no matter what the temperature coefficient is at room temperature, it tends to move in the negative direction as temperature increases.[1]

When the core is new, the peak k_{eff} in an uncontrolled reactor is typically ≈ 1.2 - 1.3. Thus, in a new core, some sort of control would have to be provided. Whether the control takes the form of control rods, as in a BWR, or dissolved boron, as in a PWR, has a significant effect upon the temperature coefficient characteristics of the core. Therefore, the coefficients in the two types of reactors will be considered separately.

A. BWR

In the BWR, control is accomplished by the use of localized poisons - either control rods or fixed burnable poison.[2] In early reactors the burnable poison was in the form of "poison curtains" which

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[1] This effect is strengthened by the fact that a one degree change in temperature causes a larger reduction in density at high temperatures than at low temperatures (see Figure 10-25). Thus, at high temperatures. a one degree temperature rise causes the operating point to shift farther to the left on Figure 10-26 than does a one degree rise at low temperatures.

[2] The reason for splitting the control requirements between control rods and fixed poison is to allow the control rods to be weaker. This lessens the potential consequences of transients which involve the accidental withdrawal of a control rod. For further discussion see Chapter 15.

were thin sheets of borated steel which were sandwiched between fuel bundles. In later designs the poison (usually an element called gadolinium, which is a slow neutron absorber) was mixed with the fuel in selected fuel rods. Earlier in this chapter it was pointed out that the worth of a localized poison is dependent upon how many neutrons can reach its surface. Since the neutrons travel farther as the moderator density is reduced, an increase in moderator temperature results in an increase in the strength of any inserted localized poisons. Thus, early in core life, this effect adds a strong negative contribution to the moderator temperature coefficient (see Table 10-IV). On the other hand, as core lifetime progresses the poisons are removed from the core and the importance of this effect is diminished. Therefore, in a BWR the moderator temperature coefficient is ordinarily more negative at beginning of core life than at the end of core life.

Figure 10-27 shows the temperature coefficient behavior of a typical

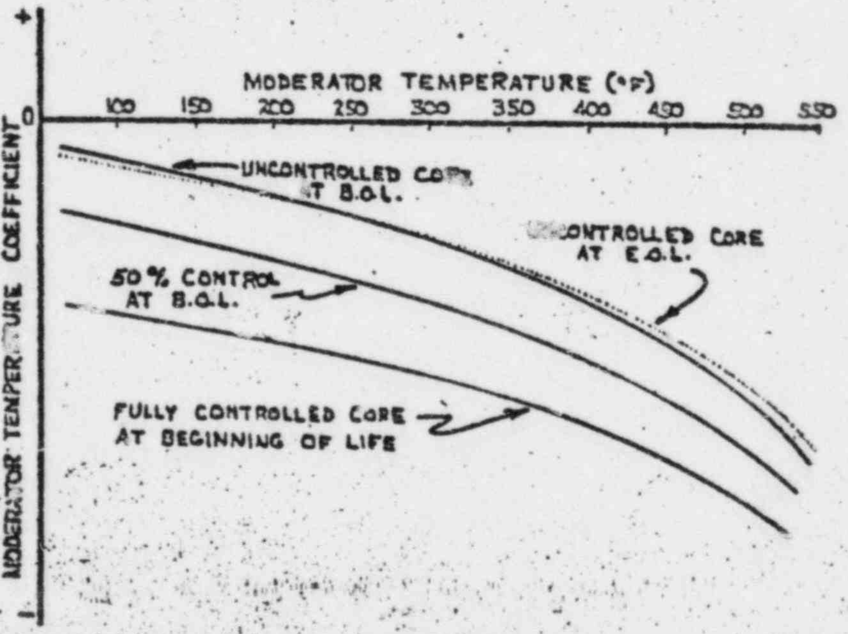

Figure 10-27: Temperature Coefficient Behavior in a Typical BWR for Various Operating Conditions

BWR for various operating conditions. Notice that this particular core has been designed with a volume ratio which gives it a negative coefficient at room temperature even in the uncontrolled case. That is, this reactor would be like that illustrated by range C on Figure 10-26. Figure 10-27 illustrates our previous remarks about the behavior of the coefficient. That is, it is considerably more negative when the control rods are inserted, and it becomes more negative as the temperature increases. The figure also shows the coefficients at beginning of life and end of life for the uncontrolled cases. The difference between these curves is the result of changes in the composition of the fuel. For example, in Table 10-IV it was pointed out that the buildup of Pu-239 made both positive (effect upon f) and negative (effect upon η) contributions to the coefficient. The net effect of the changing core composition is small, hence the two uncontrolled curves nearly coincide. The effect of changing composition as core lifetime progresses would be completely overshadowed by the effect of control rod withdrawal.

We have stressed in this section the fact that inserted control rods tend to make the coefficient more negative because their reactivity worth tends to increase as temperature increases. It is interesting to illustrate this effect upon a curve of k_{eff} versus N_{H_2O}/N_{UO_2} as shown in Figure 10-28. On the figure, the upper

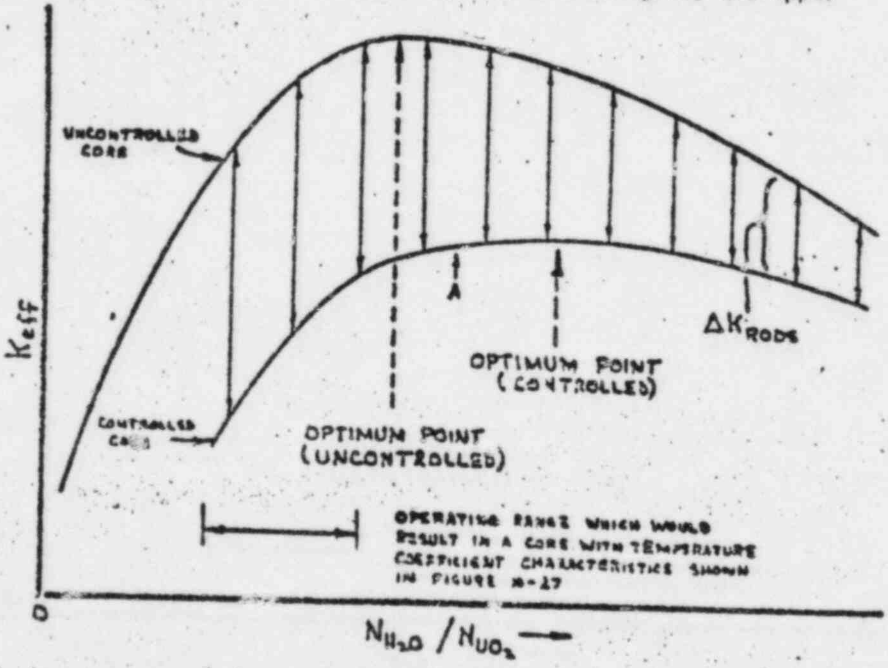

Figure 10-28: Effect of Inserted Control Rods upon The Curve of k_{eff} Versus N_{H_2O}/N_{UO_2}

curve illustrates the behavior of the uncontrolled reactor as N_{H2O}/N_{UO_2} is varied while the lower curve shows the behavior of the controlled reactor. At any given value of N_{H_2O}/N_{UO_2}, the difference between the two curves represents the Δk worth of the rods. The double ended arrows are thus a measure of control system strength. These arrows get longer as N_{H_2O}/N_{UO_2} decreases and the k_{eff} curve for the controlled reactor falls farther and further below that of the uncontrolled reactor, which shifts the peak in the former curve decidedly to the right. As can be seen, a reactor operating at point A would have a positive coefficient when uncontrolled, but a negative coefficient when controlled. This, of course, agrees with the previous remarks regarding the effect of inserted rods.

B. PWR

At the beginning of core lifetime, a PWR has a considerable concentration of boric acid in the coolant. Any mechanism which removes moderator from the core is going to result in the simultaneous removal of an equivalent amount of poison. The removal of poison will tend to increase k_{eff} and will represent a positive contribution to the temperature coefficient. The greater the concentration of boric acid, the more important is this effect and the more the coefficient tends to be positive as shown in Figure 10-29. Thus,

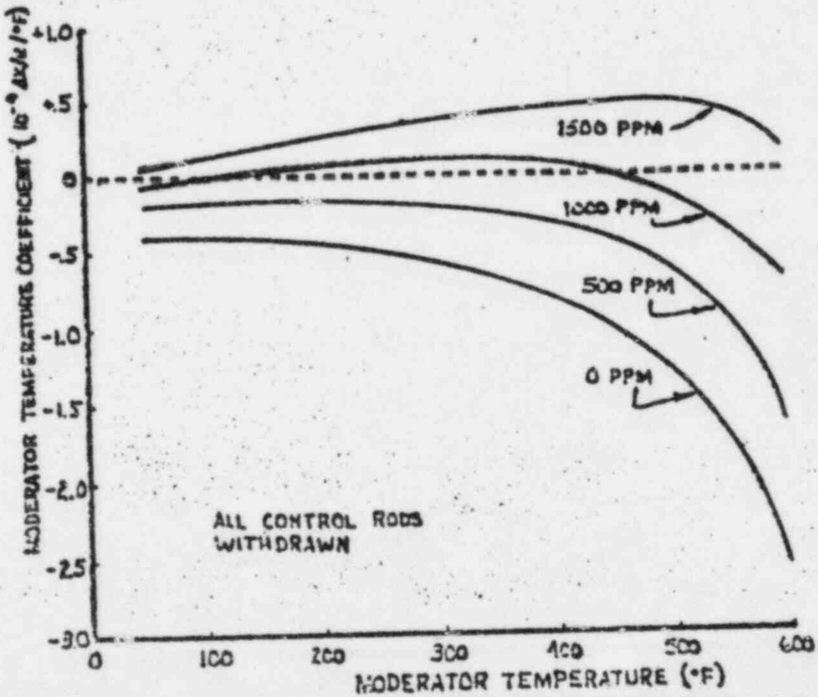

Figure 10-29: Behavior of Moderator Temperature Coefficient in A Typical PWR Employing Chemical Shim

the influence upon the coefficient of a dissolved poison is exactly the reverse of a localized poison. In a PWR the coefficient gets more negative as core lifetime progresses and chemical shim is removed from the core.

The desirability of having a negative temperature coefficient at operating conditions places an upper limit upon the maximum concentration of chemical shim which a PWR can employ. Since this maximum concentration is generally insufficient to provide all of the reactivity control which is required by the core, current PWRs also employ fixed burnable poison rods in the fuel. These localized poisons provide a negative contribution to the coefficient and their use in place of boric acid prevents the coefficient from being positive under operating conditions at beginning of core life.[1]

Pressurized water reactors are always forced circulation reactors. In general, they are heated up to operating temperature and pressure using pump heat rather than nuclear heat. During this process, the reactor is shut down with nearly all control rods inserted.[2] This gives a little break on the coefficient during the low temperature conditions when it is most positive. That is, with rods inserted the coefficient may always be negative where it might not have been at the same boron concentration with all rods withdrawn.

Figure 10-12 shows the effect of dissolved boron on a k_{eff} versus N_{H_2O}/N_{UO_2} curve. The reader should consider this in an analogous manner to Figure 10-28 and verify that as N_{H_2O}/N_{UO_2} is reduced the uncontrolled and controlled curves tend to move closer together because the Δk worth of the shim is reduced. This shifts the optimum point in the controlled curve to the left of that for the uncontrolled curve, which means the coefficient tends to be more positive as discussed above.

2. Moderator Void Coefficient

In this section we will consider the effect of coolant steam voids upon the reactivity of a boiling water reactor core. Most of the principles discussed apply equally well to a PWR, and we will discuss

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[1] Most PWRs have a requirement in their AEC operating license which states that the coefficient must be negative whenever the temperature is above 450° F.

[2] Generally a small number of rods are withdrawn to give the operator the capability of quickly adding some negative reactivity to the core should the need arise. The "need" could arise, for example, by an accidental deboration caused by improper valving in the boration concentration control system.

them briefly at the end of this section, but the importance of voids in a PWR is reduced because there is very little boiling in the core.

Since a BWR is essentially bottled up during the course of heating to rated temperature and pressure, the amount of steam bubbles, or voids, in the core is small. After rated conditions have been reached and reactor power is raised to produce steam, the amount of voids in the core increases until at full load we have a condition called rated voids. At rated voids, the volume of the voids in the core may represent 30% or more of the total moderator volume. At the top of the core, where the amount of steam is at its maximum value, the void volume fraction may be in excess of 70%. In a properly designed reactor, the effect of increasing voids is to add negative reactivity; i.e., increasing voids should tend to shut the reactor down. Such a reactor is said to have a negative void coefficient. The void coefficient is defined in a manner analogous to the moderator temperature coefficient, that is, it is a measure of the reactivity change which accompanies a one percent change in overall core void volume fraction, or:

$$\text{Void Coeff.}(\Delta k/k/\%\ \text{voids}) = \frac{\rho_{final} - \rho_{initial}}{V_{final} - V_{initial}} \qquad (10\text{-}35)$$

where: V = moderator void volume (%)

For example, if an increase in the core average void fraction from 20 to 22% produces a decrease in reactivity of -.002 Δk/k, then the void coefficient will be:

$$\text{V.C.} = \frac{-.002}{22 - 20} = \frac{-.002}{2} = -1.0 \times 10^{-3}\ \Delta k/k/\%\ \text{voids}$$

which is a typical value for a BWR.

The mechanism by which an increase in core voids reduces reactivity is essentially the same as that described for the moderator temperature coefficient. Voids may be thought of merely as being another way of reducing the density of the moderator, and by doing this they result in all of the effects summarized in Table 10-IV. However, introduction of voids into the moderator results in a much greater decrease in effective density than does heating (see Figure 10-25). The result is that the void coefficient is much stronger than the moderator temperature coefficient. Mention was made previously that small positive moderator temperature coefficients were permitted at room temperature. Ordinarily, it is desirable to have a negative void coefficient at any operating conditions which could result in substantial void formation in the core because of the fact that the coefficient is strong and the void content of the core can be changed rather quickly. This usually means the void coefficient must be negative whenever the moderator is above the boiling point at atmospheric pressure (212° F). In general, if the void coefficient is negative, the moderator temperature coefficient is also negative since they are the result of the same effects. Thus, a BWR license frequently requires that the moderator temperature coefficient become negative before the moderator reaches 212° F.

Figure 10-30 shows the void coefficient in a typical BWR versus void volume in the core. The void coefficient becomes more negative as void fraction is increased. As in the case of the moderator temperature coefficient, this is a result of the core becoming significantly undermoderated so that the resonance escape probability begins to decrease very rapidly. In addition, the void coefficient in a BWR tends to become more positive as burnup occurs. Again as in the case of the moderator temperature coefficient, the primary reason for this is the fact that there are fewer control rods in the burned core. This reduces the magnitude of one of the largest negative effects that makes up the coefficient. Plutonium buildup also tends to add a small positive contribution to the void coefficient as burnup occurs due to its effect upon f slightly overshadowing its effect upon η.

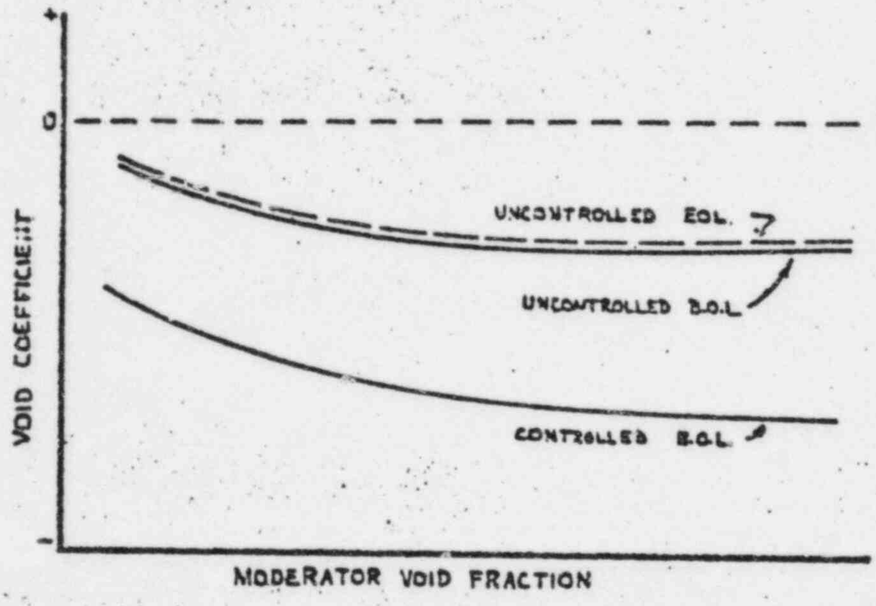

Figure 10-30: Behavior of the Void Coefficient in a Typical BWR

An important consideration with voids is the fact that they are not uniformly present throughout the core. The water entering the bottom of the core is cooled below its boiling point (this is called subcooled water) and must travel a portion of the way up the core before sufficient heat is added to elevate it to the boiling point. All heat added beyond this point goes into boiling the water and increasing the void content, and as a result, the voids tend to act as a poison in the top of the core. It is therefore necessary to properly program movement of control rods so that they act to poison those portions of the core where no poison in the form of voids exists. In this way, the power distribution can be made as uniform as possible. The control of power distribution is discussed further in Chapter 12.

In addition to changes in reactor power level, there are many things which can affect the moderator void volume while a BWR is in operation. Some of these are:

A. An increase in reactor pressure, which causes the reactor power to increase because the voids are compressed and occupy less volume in the core.

B. A decrease in the temperature of the water entering the core (i.e., an increase in subcooling), which causes the reactor power to increase since fewer voids will be produced because more of the reactor's power must go to heating subcooled water. Changes in subcooling can occur by changing feedwater temperature, by cutting feedwater heaters in or out, by transient changes in feedwater flow, by changes in the core recirculation flow as in a variable flow forced circulation reactor, or by changes in subcooling due to changes in the secondary cycle of a dual cycle plant like Dresden.

The operator of a boiling water reactor soon appreciates that in addition to his real control rods, he has "control rods" in the form of his feedwater valve, speed controls on his coolant recirculation pumps, and in the devices which control reactor pressure. Under some conditions, these can act as very potent reactivity control devices.

Although the moderator temperature in a PWR is essentially the same as in a BWR, the moderator is subcooled in a PWR because the reactor is operated at a pressure substantially higher than the saturation pressure. For example, at 550° F, the saturation pressure is ≈ 1030 psig (thus a BWR would operate at this pressure) whereas the operating pressure of a PWR is ≈ 2235 psig. Therefore, a high degree of boiling is impossible in a PWR. Nevertheless, there exists a phenomenon called surface boiling which does occur in PWR cores. The surface of the fuel cladding is somewhat hotter than the bulk coolant temperature and may exceed the boiling point of the coolant at the operating pressure. Therefore, small steam bubbles may be formed on the surface of the cladding. As they are swept away from the cladding by the action of the coolant flow and their own bouyancy, they are rapidly quenched by the main body of the coolant. At full power, surface boiling effects may result in a void volume fraction of ≈ .5% in the coolant. Assuming that the moderator temperature coefficient is negative (as it must be in presently licensed PWRs), the void coefficient will also be negative. However, at beginning of life it will be smaller than the corresponding coefficient in a BWR because of the positive effect of the chem shim and because relatively fewer control rods will be inserted.

3. Fuel Temperature Coefficient

We have already mentioned that reactivity is influenced by changes in fuel temperature. This effect is measured by the fuel temperature coefficient, which is often called the doppler coefficient since it results from a phenomenon called the doppler effect. The doppler effect is a broadening and flattening of the U-238 absorption cross section resonance peaks as U-238 temperature increases.[1] The precise treatment of the physical principles involved is complex, but the following is a simplified explanation of the phenomenon. For purposes of illustration, suppose U-238 exhibits an absorption resonance for neutrons with a velocity of 10 miles per hour (the velocities used here bear no relationship to reality, but are chosen simply for computational ease). Although previous discussions have ignored this fact, the neutron must be traveling at 10 miles per hour relative to the U-238 atom for absorption in this particular resonance. If the U-238 atom is motionless, this can only occur if the neutron is traveling at exactly 10 miles per hour. All of our previous discussions have tacitly assumed this to be the case. In reality, however, all nuclei in a substance are in a state of constant vibrational motion, and the extent of this motion is dictated largely by the temparature of the substance. The higher the temperature, the more vibrational motion exists. Returning to our example, suppose the U-238 nuclei are vibrating back and forth at 5 miles per hour. The neutron will be going 10 miles per hour relative to the U-238 atom if conditions are as shown in either of the two pictures in Figure 10-31.

Figure 10-31: Conditions Which Will Result in Neutron Absorption in A Fictitious U-238 Absorption Resonance at 10 MPH

Thus it is seen that there is a possibility of neutron absorption for neutrons with velocities anywhere between 5 and 15 miles per hour (the intermediate energies are obtained when the neutron and nucleus are traveling at various angles to each other). This is basically why the resonances are broadened. At the same time, the resonance will be flattened somewhat although the reasons are not as easy to explain. The net result of an increase in temperature upon a typical U-238

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[1] The discussion will be principally concerned with U-238 since it is the most important resonance absorber in the core. However, the doppler effect occurs with all resonance absorbers.

absorption cross section resonance is shown in Figure 10-32. As a

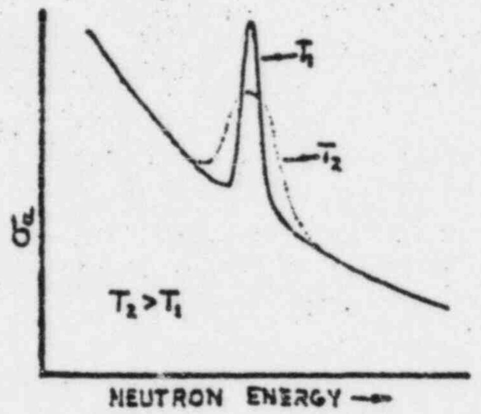

Figure 10-32: Effect of Temperature upon U-238 Absorption Cross Section Resonances

result of this broadening and flattening of the resonances, the neutron sees a significant absorption cross section over a wider range of energies. This results in more resonance capture, a lower resonance escape probability, and a reduction in k_{eff}.

In a low enrichment BWR or PWR, where most of the fuel is U-238, the magnitude of this coefficient is typically on the order of -1×10^{-5} Δk/k/°F. The temperature referred to in this expression is the fuel temperature. At first glance this may not appear to be a very important coefficient since its magnitude in most cases is substantially smaller than the moderator temperature coefficient. However, in the UO_2 rod type fuel employed in both BWRs and PWRs, the fuel temperature increases to very high values as reactor power is increased from 0 to 100%. The maximum fuel temperatures may, in fact, be as high as 4000° F at full power. Thus, although the magnitude of the coefficient is small, the reactivity effects are not necessarily small due to the larger temperature changes which are encountered. However, the fact which makes the doppler coefficient particularly important is that fuel temperature immediately increases following an increase in reactor power, and fuel temperature changes always precede changes in either moderator temperature or void content. Since UO_2 is a relatively poor conductor of heat, and since a cylindrical rod has a small heat transfer surface per unit volume, the time constant of the fuel (the time required for 63% of the heat generated at any instant to be transferred to the moderator) is relatively long--generally five or more seconds. In the event of a large reactivity addition to the reactor, the moderator temperature and void coefficients cannot come into play for several seconds and would have little effect on the termination of the power excursion. The fuel temperature coefficient, on the other hand, starts acting immediately and represents the primary shutdown mechanism for a fast power rise transient. For this reason it is called a prompt coefficient whereas the moderator coefficients are considered to be delayed coefficients. It is one of the most important safety features of present day BWRs and PWRs.

The magnitude of the doppler coefficient becomes smaller (less negative) as fuel temperature increases as shown in Figure 10-33. The

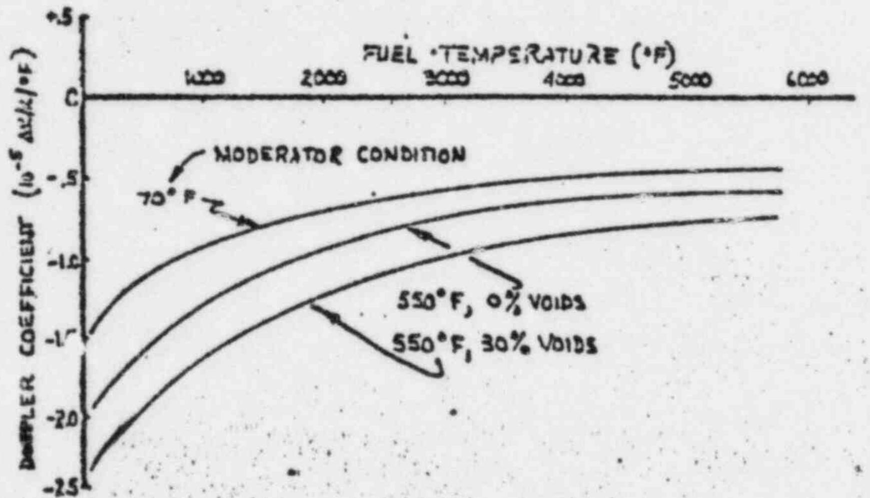

Figure 10-33: Doppler Coefficient vs. Fuel Temperature for Typical BWR or PWR

reasons for this are beyond the scope of this manual. The figure also shows that the magnitude of the coefficient is influenced by moderator condition. As moderator temperature increases, or voids are formed, the average energy of the slow neutrons increases. This means that the absorption cross section of the surface of the fuel rods decreases, which allows more slow neutrons to penetrate into the centers of the rods. That is, the flux is no longer as strongly depressed in the center of the rods and the disadvantage factor is lower. In effect, the U-238 nuclei in the center of the fuel rods get a greater opportunity to contribute to the doppler coefficient, since more neutrons reach these nuclei, and so the coefficient gets more strongly negative.

As burnup occurs the coefficient becomes more negative. This is because of the buildup of resonances in the fuel which were not originally in the new fuel. With more available resonances, the importance of doppler broadening is enhanced. Some of these resonances are fission products (although most of these build up to near equilibrium concentrations fairly early in core life), but one of the most important at high burnups is the isotope Pu-240, which has a resonance of ≈ 10^5 barns at 1.03 ev.

XENON-135

In the preceding sections of this chapter, the reactivity effects of control rods and of the various coefficients were discussed. All of these represent relatively rapid changes in reactivity. For instance, a control rod is moved and its total effect is seen almost instantly. Conversely, the reactivity effect of Xe-135 and the reactivity effects which will be considered in the next sections, samarium-149 and burnup, are things which occur relatively slowly and their influence is felt over periods of time ranging from hours to days and years.

In Chapter 8, fission products were discussed in some detail. Xe-135 is but one of the ≈ 200 possible fission products. Its importance is a result of the fact that its microscopic absorption cross section, σ_a, is ≈ 3,500,000 barns. It is of note that most isotopes, excepting the small group considered along with Xe-135 as nuclear poisons, have values of σ_a in the range of 0.01 to 10 barns. The fact that σ_a for Xe-135 is high would be of no particular consequence if there were little or no atoms of this isotope formed. However, Xe-135 is one of the most common of all fission products. It is formed to a small extent (0.3%) as a direct fission product. The percentage indicates the number of Xe-135 nuclei formed per 100 fissions. The major source of Xe-135, however, is through the decay (by successive β^- emissions) of the fission product chain with mass number 135. The initial fission product in this chain is $_{52}Te^{135}$, which is formed in 5.9% of all fissions. The combination of formation through decay through the chain plus direct yield makes the total effective fission yield of Xe-135 equal to 6.2%. That is, an average of 6.2 atoms of Xe-135 result from each 100 fissions. The production of Xe-135 can thus be summarized as shown in Figure 10-34.

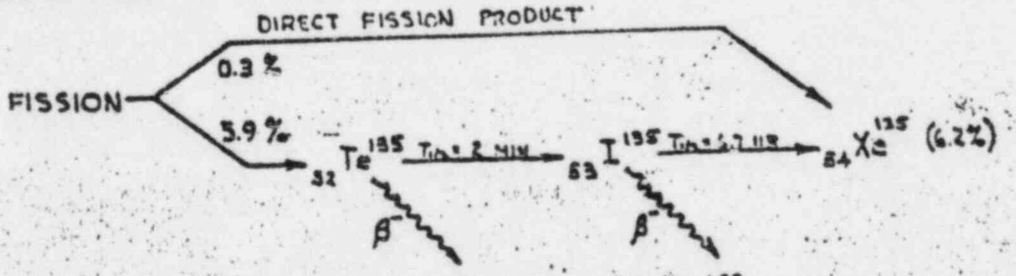

Figure 10-34: Production Mechanisms for Xe-135

Xe-135 and its poisoning effect on the reactor are removed by two mechanisms: 1) its decay by β^- emission with a 9.2 hour half-life to Cs-135 (cesium), and 2) its capture of a neutron with the resulting formation of stable Xe-136. The removal of Xe-135 can thus be summarized as follows:

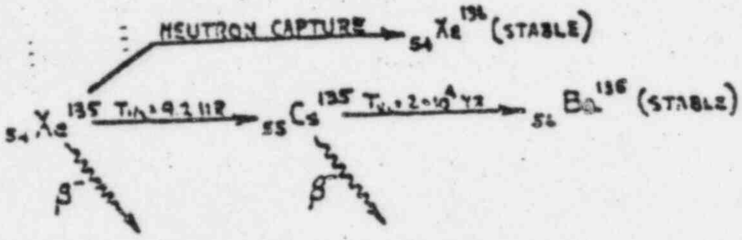

Figure 10-35: Removal Mechanisms for Xe-135

When a reactor of the type we are considering is operated at high power, very roughly half of the Xe-135 formed is removed by each of these mechanisms.

The net amount of xenon in the core is dependant upon the relative rates of its formation and disappearance. In other words:

$$\left[\begin{array}{l}\text{Rate of change}\\ \text{of xenon level}\end{array}\right] = \left[\begin{array}{l}\text{Rate of production from direct}\\ \text{fission and from decay of iodine}\end{array}\right] - \left[\begin{array}{l}\text{Rate of removal by decay}\\ \text{and by neutron capture}\end{array}\right]$$

When the rate of production exceeds the rate of removal, the xenon inventory in the core will be increasing, and vice versa.

Equations (4-7) and (7-12), which express the rates of radioactive decay and the neutron induced reaction rate respectively, can be used to mathematically express the previous word equation. The ultimate production rate of Xe-135 is 6.2% of the fission rate, or $.062\phi\Sigma_f V_{core}$. It must be remembered, however, that most of the xenon is produced via the decay of I-135, so that there is a time delay before the full amount of xenon is produced from any group of fissions. The removal by burnup is given by $\phi\sigma_{a,Xe}N_{Xe}V_{core}$, and the removal by decay is given by $\lambda_{Xe}N_{Xe}V_{core}$. Therefore, the balance equation becomes:

$$\left[\begin{array}{l}\text{Rate of change}\\ \text{of xenon level}\end{array}\right] = .062\phi\Sigma_f V_{core} - \phi\sigma_{a,Xe}N_{Xe}V_{core} - \lambda_{Xe}N_{Xe}V_{core} \qquad (10\text{-}36)$$

where:
ϕ = average core thermal neutron flux
Σ_f = macroscopic fission cross section of the fuel
V_{core} = core volume
$\sigma_{a,Xe}$ = absorption cross section of Xe-135
λ_{Xe} = decay constant of Xe-135
N_{Xe} = concentration of Xe-135 atoms in core

We are now in a position to understand the behavior of xenon in a number of more common operating transients.

1. Startup from the Xe free condition

   A reactor which has never run at power before, or a reactor which has been shut down for several days will be in the Xe free condition.

Figure 10-36 shows how the concentration of xenon in the core (or

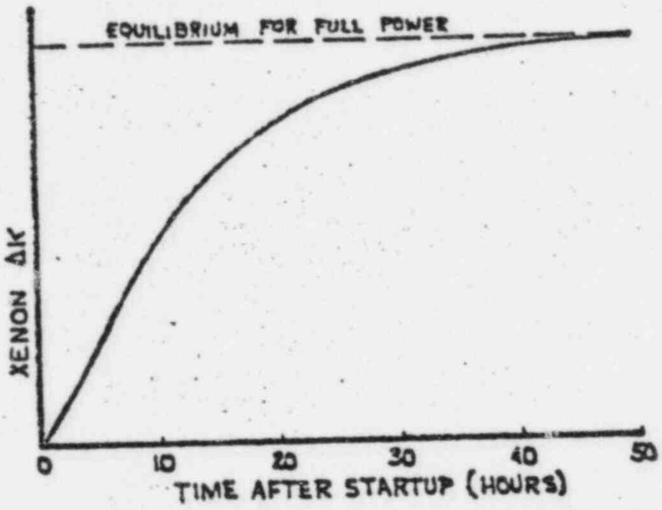

Figure 10-36: Xenon Transient after Startup from Zero Power to Full Power of a Reactor Which Was Initially Xenon Free

alternatively, its reactivity worth) builds up after such a reactor is quickly brought to full power. The figure shows that as soon as the reactor reaches power, the xenon starts building up in the core and gradually approaches an equilibrium value. This equilibrium value is reached when the rate of production of xenon just equals the rate at which it is removed. Although the time required to reach equilibrium varies slightly from one reactor to another depending upon the power density, a value of 50 hours is close enough for all practical purposes. To aid in sketching the buildup curve, it can be further assumed that approximately 90% of the equilibrium value is reached in the first 24 hours after startup.

The buildup of xenon to its equilibrium value from the xenon free condition at startup can be explained by referring to the diagrams of xenon's life history and equation (10-36). As soon as the reactor reaches power, some Xe begins to be formed due to direct fission. In addition, a large amount of I-135 begins to be formed and xenon begins appearing at a gradual rate from its decay. At the outset, when there is little xenon in the core (i.e., N_{Xe} is small in equation (10-36)), the removal mechanisms are too small to balance the production of xenon and so the xenon concentration increases. However, as N_{Xe} gets larger, removal by both decay and burnup increases, tending to cancel the production and causing the curve to start leveling out. When the removal completely catches up to the production, equilibrium is reached.

Now let us consider the situation which exists at equilibrium. In this case the rate of change of xenon is zero, and equation (10-36) becomes:

$$0 = .062\phi\Sigma_f V_{core} - \phi\sigma_{a,Xe}N_{Xe}V_{core} - \lambda_{Xe}N_{Xe}V_{core}$$

solving for N_{Xa} at equilibrium gives:

$$N_{Xe,equil.} = \frac{.062\phi\Sigma_f V_{core}}{\phi\sigma_{a,Xe}V_{core} + \lambda_{Xe}V_{core}} \qquad (10\text{-}37)$$

As can be seen, the equilibrium xenon concentration depends upon the neutron flux, or alternatively the reactor power. The relationship is shown in Figure 10-37. What this figure (or equation (10-37)) tells us is that, in general, the equilibrium xenon level will increase as reactor power increases. This behavior can be seen from the following reasoning. The equilibrium production rate of xenon always increases as the neutron flux increases, thus requiring a higher removal rate to achieve equilibrium. Now an increase in flux automatically increases removal by burnup. But burnup is not the only removal mechanism, and in order to equilibrate at higher fluxes, the removal by decay must also show an appropriate increase. Since decay does not depend upon flux, but only upon N_{Xe}, the latter must increase in order to increase the decay removal mechanism. In general, therefore, the equilibrium xenon concentration will be higher as flux is increased.

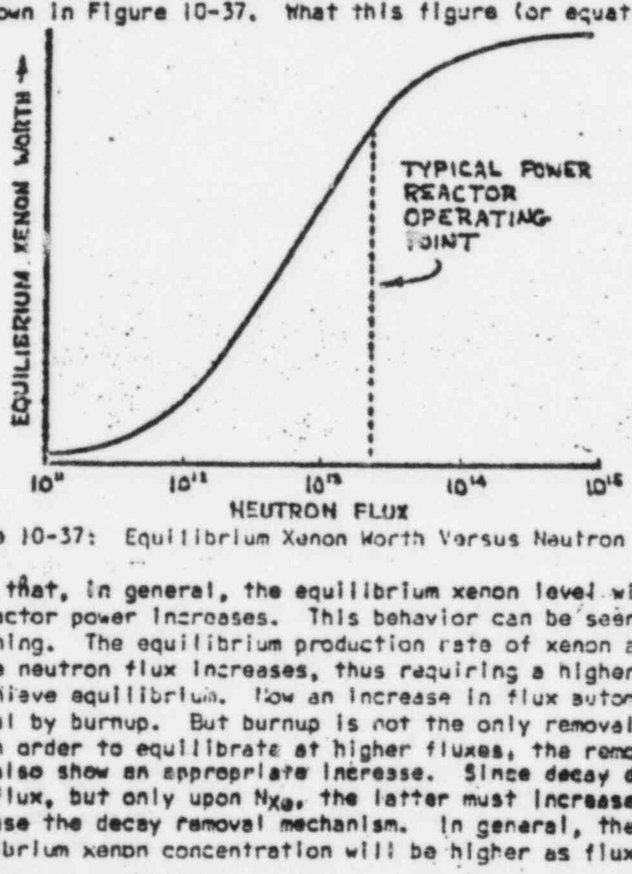

Figure 10-37: Equilibrium Xenon Worth Versus Neutron Flux

The full power reactivity worth of xenon will vary from reactor to reactor depending upon the neutron flux and other factors, but in light water reactors it is typically in the range of 2 to 4% Δk/k.

2. Reactor shutdown from equilibrium xenon

If a reactor is shut down after having run long enough to reach equilibrium xenon, the behavior shown in Figure 10-38 occurs.

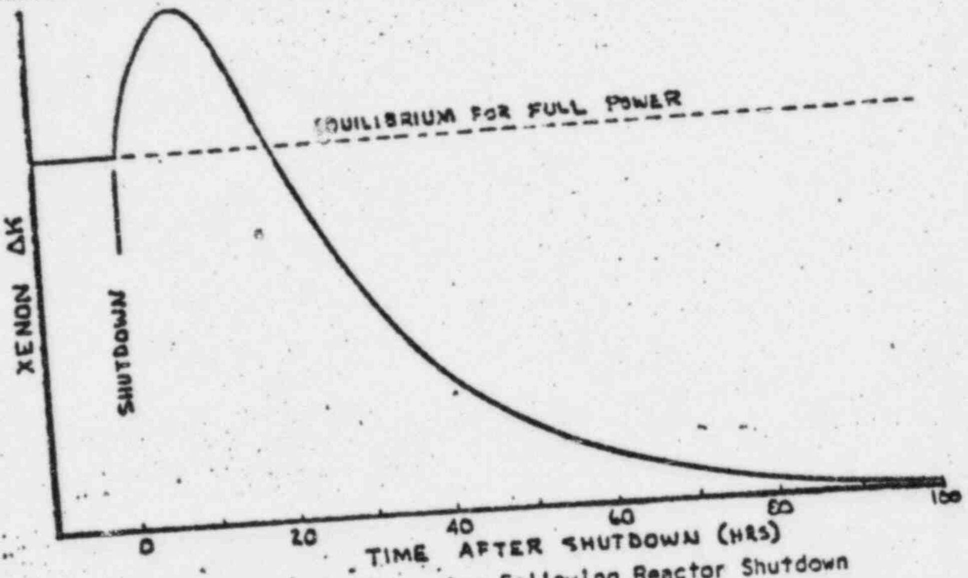

Figure 10-38: Xenon Behavior Following Reactor Shutdown from Full Power, Equilibrium Xenon Condition

Approximately 8 hours after shutdown the amount of xenon in the core, or equivalently its reactivity worth, increases to a peak value and then decays away to the xenon free condition with a half-life that eventually approaches the 9.2 hour half-life of Xe-135. The core can be considered to be xenon free approximately 100 hours after shutdown.

This behavior can be explained as follows:

A. At the time the reactor is shut down, equilibrium amounts of I-135 and Xe-135 are present. At the shutdown, the production of I-135 and Xe-135 by direct fission stops. However, because of the I-135 already in the core, the production of Xe-135 by decay of I-135 continues at essentially the same rate as before the shutdown. Furthermore, after the shutdown, the Xe-135 is no longer removed by neutron capture to any significant extent. The only removal mechanism for Xe-135 is by radioactive decay. Therefore, since the production of Xe-135 continues at almost the equilibrium rate, and since the rate at which Xe-135 is removed is significantly diminished, the concentration of Xe-135 in the core initially builds up following the shutdown.

B. As time passes, the rate at which Xe-135 is being produced diminishes because the I-135 concentration diminishes. On the other hand, the decay rate increases as the concentration of xenon in the core increases. Eventually a point is reached where the Xe-135 production and removal rates are equal, and the amount of Xe-135 in the core levels off. This is the point of peak xenon.

C. Still later in time, the rate at which xenon is removed by decay exceeds the rate at which it is produced from I-135. Therefore, the xenon inventory in the core begins to decrease. Finally, as the I-135 in the core is nearly depleted, the rate at which the Xe-135 inventory decreases is governed by, and approaches, its 9.2 hour half-life.

If a reactor is able to start up when xenon is at its peak, it is said to have sufficient reactivity in control rods to "override" peak xenon. A typical power reactor usually can override peak xenon during early core life. Towards the end of core life, however, it may not be able to do this. After a scram from power, it would be necessary to get started up and reach power within, say two hours, or else the reactor might not be able to achieve criticality for a day or so.

3. Reactor startup with xenon in the core

Let us next consider the common case of a startup which follows a few hours after a shutdown from the full power equilibrium xenon condition. This behavior is shown in Figure 10-39. The first portion of the curve

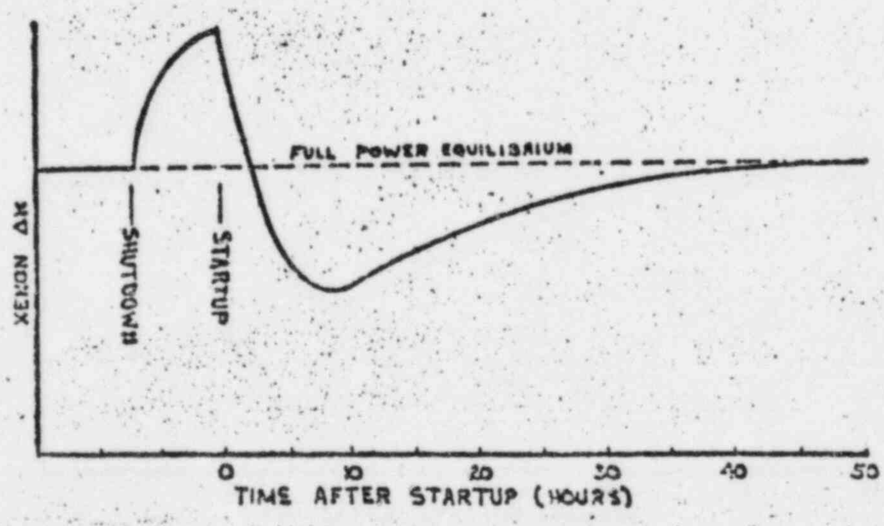

Figure 10-39: Xenon Behavior for Reactor Startup with Xenon in the Core

during which the reactor is shut down, is the same as Figure 10-38. Once the reactor returns to power, however, the xenon inventory begins to drop. This is due to the fact that once the reactor reaches power, xenon again begins to be removed because of neutron capture (as well as by decay). However, its rate of production has fallen off from its equilibrium value because decay during the shutdown time has depleted the I-135 inventory. Temporarily, therefore, the rate of xenon removal exceeds the rate of production. The I-135 inventory starts to recover immediately upon the return to power. Its half-life, however, introduces a lag of several hours before the xenon begins to recover. Eventually, the xenon also returns to the equilibrium value for the power level in question. The initial drop in xenon inventory following a return to power is often called xenon burnout.

4. Xenon behavior on load changes

With the information presented so far, the effect of xenon during load increases and reduction, as shown in Figures 10-40 and 10-41, should be understandable to the reader.

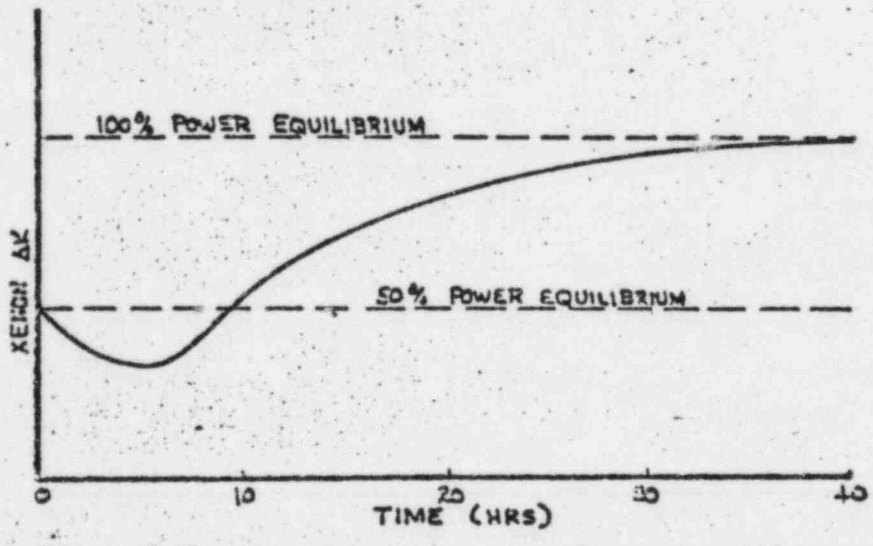

Figure 10-40: Xenon Behavior on a Load Change from 50% Power to 100% Power

SAMARIUM-149

The second fission produce which must be considered because of its large absorption cross section (σ_a = 53,000 barns) is Sm-149. It is a stable rather than a radioactive isotope and is not a direct or primary fission product, but is produced in 1.4% of all fissions by decay from a primary fission product as shown in Figure 10-42.

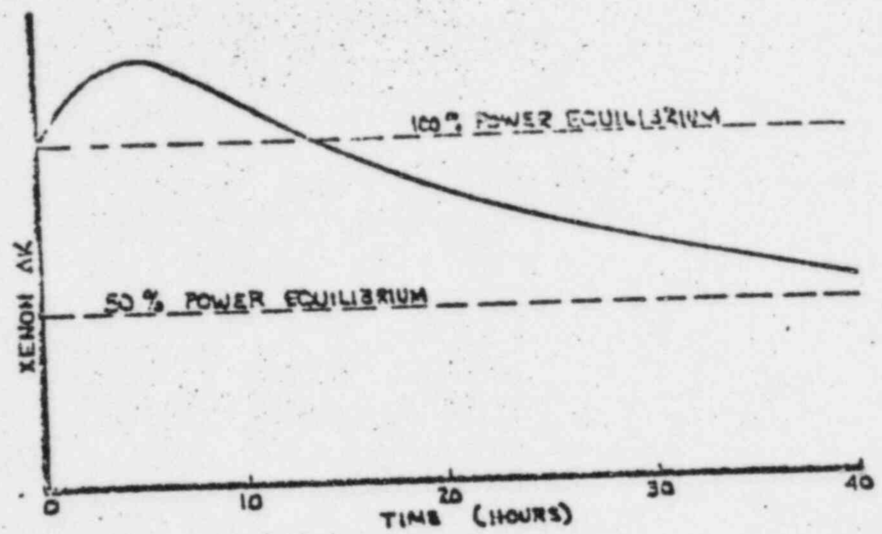

Figure 10-41: Xenon Behavior on a Load Change from 100% Power to 50% Power

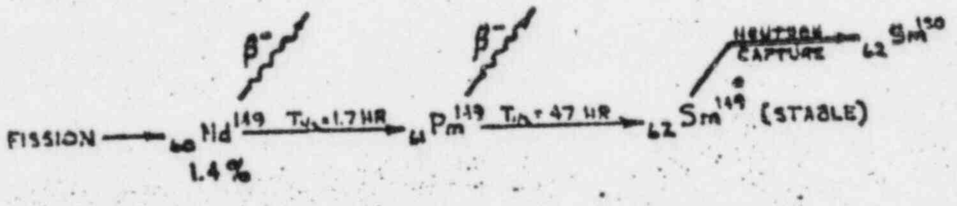

Figure 10-42: Life History of Samarium in a Reactor

Nd is the symbol for neodinium and Pm is the symbol for promethium. The behavior of Sm-149 and its precursor Pm-149 are similar in many respects to Xe-135 and its precursor I-135. Samarium's effect on reactivity is much smaller than xenon's because it has a smaller microscopic cross section and not as much of it is formed. Changes in samarium inventory occur much more gradually because of the longer half-life of its precursor.

1. Startup from the clean condition

A new reactor upon initial startup will build up an equilibrium inventory of Sm-149. The equilibrium Sm-149 inventory, unlike that of Xa-135, is independent of flux or power. The reason is because the isotope is stable. If power doubles, the equilibrium production rate doubles, but the equilibrium removal rate also doubles without requiring any change in samarium concentration. This can be verified by writing a balance equation similar to that for xenon.

$$\begin{bmatrix}\text{Rate of change}\\ \text{of Sm-149}\end{bmatrix} = .014\phi\Sigma_f - \phi\sigma_{a,Sm}N_{Sm} \qquad (10\text{-}38)$$

At equilibrium this becomes:

$$0 = .014\phi\Sigma_f - \phi\sigma_{a,Sm}N_{Sm}$$

or:

$$N_{Sm\ equil} = \frac{.014\phi\Sigma_f}{\phi\sigma_{a,Sm}} = \frac{.014\Sigma_f}{\sigma_{a,Sm}} \qquad (10\text{-}39)$$

Notice that the flux cancels out of equation (10-39), which demonstrates the statement that the equilibrium samarium concentration is independent of power. The equilibrium reactivity worth of Sm-149 in most light water reactors is about 0.7% Δk/k. This number will vary slightly from one reactor to another because the samarium concentration and its reactivity effect depend upon the composition of the core[1] but the variations are small. For all practical purposes however, in any particular reactor the equilibrium reactivity worth is independent of power.

The shape of the Sm-149 buildup curve will look very much like that for Xe-135 (Figure 10-36). The time scale will be lengthened, however, due to the long half-life of Pm-149 and the fact that there is no decay of the Sm-149 to help level it out. Typically, it takes 30-40 days to reach equilibrium.

4. Other effects of Sm-149

Samarium exhibits the other effects which were discussed under Xe-135 but the magnitude of these effects are small and slow acting and are generally masked by other reactivity effects during operation.

A. Sm-149 builds up beyond its equilibrium value after a shutdown from operation where it had attained equilibrium. This is due to the decay of Pm-149 to Sm-149. Since Sm-149 is a stable isotope, it does not decay away after buildup to its maximum value. Figure 10-43 shows this behavior.

B. On startup, Sm-149 burns out until the buildup of Pm-149 and its decay "catches up." Sm-149 then comes back to its equilibrium value.

C. Samarium behaves like xenon on load increases and reductions, except, of course, that it always returns to the same equilibrium value.

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[1] Equation (10-39) shows that it depends upon the fission cross section of the core, for example.

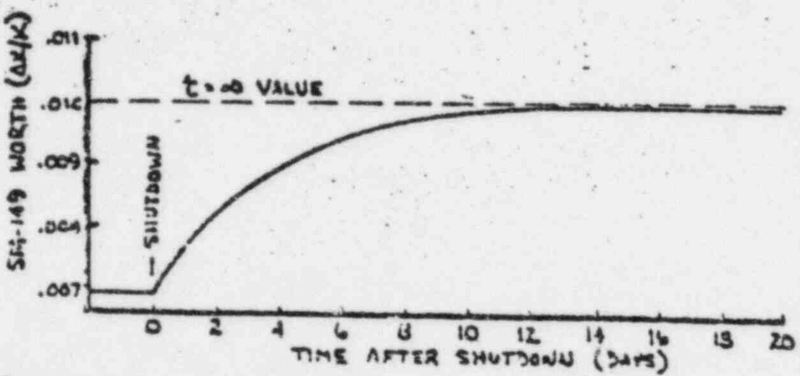

Figure 10-43: Typical Samarium Transient Following a Shutdown from Full Power

FUEL BURNUP

Since each fission results in the destruction of a fissionable nucleus, there is a gradual depletion of the fuel as the reactor produces power. The term used to describe the depletion of the fuel as a result of power operation is fuel burnup. The negative reactivity effect of fuel burnup occurs at a very slow rate. The amount of control rod which must be withdrawn or chemical shim which must be removed each day to compensate for the effects of burnup is small. Nevertheless, over a period of months the reactivity effect of burnup is large, and ultimately it reaches the point where refueling is necessary.

Two of the terms which are used in discussing fuel burnup are:

1. Fuel exposure – expressed in megawatt days per ton of uranium (MWD/T). This term is used to describe the amount of energy release (megawatt days) per unit weight of reactor fuel. It may be applied to an individual fuel element, to the average value for a particular region of the core, or to the whole core. At the present time, most fuel is designed to achieve a burnup on the order of 20,000 to 30,000 MWD/T. However, considerable effort is being made to increase this figure since increased burnup means less reactor down time and more utilization of the fuel, with correspondingly lower power costs.

2. Conversion ratio – usually expressed in percent. This term, as applied to reactors of the type we are discussing, means the number of Pu-239 nuclei which are produced per 100 U-235 nuclei consumed. If the conversion ratio is 50%, 50 fissionable Pu-239 nuclei would be made per 100 U-235 fissions. Conversion ratios for light water reactors are typically in the range of 50% to 70%. Certain breeder reactors are expected to achieve ratios as high as 150%.

Burnup has a number of effects upon the reactor other than reactivity loss. Most of these were discussed earlier in this chapter, but it is worthwhile summarizing some of the more important ones here:

1. The buildup of plutonium reduces the average delayed neutron fraction for the core, and therefore a given reactivity addition results in a slightly shorter period.
2. Burnup effects the reactivity coefficients in the core. The fuel temperature coefficient becomes more negative. The void and temperature coefficients become more negative in a PWR and more positive in a BWR due to the effect of poison removal.
3. The power distribution throughout the core changes with burnup. This subject will be discussed in Chapter 12, however, power distribution changes will affect the core's margin with respect to heat transfer limits, and can also affect the core's stability.

PROBLEMS

1. a) Define k_{eff}.
   b) What is the difference between k_∞ and k_{eff}?
   c) If there are 5,000,000 neutrons in the core, and k_{eff} = 0.95, how many neutrons will there be after two generations have gone by?

2. a) Define each of the six terms which make up k_{eff}.
   b) Explain how each term changes following a reduction in moderator density.
   c) Explain how fuel burnup affects each factor.

3. Calculate k_{eff} and each of the six factors for a reactor which undergoes the following generation:

   1000 fast neutrons
   30 bonus fast neutrons from fast fission
   50 neutrons lost by leakage during moderation
   980 reach resonance energy
   110 neutrons undergo resonance absorption in U-238
   870 neutrons reach slow energies
   60 slow neutrons lost by leakage
   332 neutrons absorbed by U-238, moderator, poisons, etc.
   75 slow neutrons undergo non-fission capture in U-235
   403 slow neutrons absorbed by U-235 to cause fission
   1005 fast neutrons produced from slow neutron fission in U-235

4. a) Define reactivity.
   b) What is the reactivity when k_{eff} = 1.002? k_{eff} = 0.985?
   c) What is k_{eff} when the reactivity is +0.25%Δk/k?

5. At full power in a particular reactor, the insertion of a control rod near the center of the core results in a reduction of 10 MWE in plant output. The insertion of an identical rod near the edge of the core results in a reduction of 6 MWE. Explain.

6. a) Discuss the influence of moderator conditions upon control rod worth.
   b) It is not uncommon for a control rod to have a higher worth at room temperature than at operating temperature. Discuss how this could be possible.

7. A core designer concludes that movement of a control rod causes too large a reactivity change. He decides to replace the rod with two "weaker" rods. When these two rods (A and B) are moved simultaneously, they produce the exact reactivity change the original rod produced. Now the A rod is withdrawn from the reactor and the B rod is left fully inserted. Would you expect the reactivity change to be 1/2, more than 1/2, or less than 1/2 of moving both rods together? Explain.

8. Two control rods are the same length and contain equal amounts of B-10. One control rod is in the form of a cylinder, while the other has a thin bladed cruciform shape. Assuming that the reactors into which they are inserted are identical, which would you expect to have the highest worth? Explain.

9. a) Write the equations involved in the operation of an antimony-beryllium neutron source.
   b) What is the purpose of the source in a power reactor?
   c) What would be the approximate spontaneous fission source strength in a reactor core containing 50 tons of low enrichment uranium?
   d) What is the major source of neutrons in a core 2 minutes after shutdown from full power? 2 days?

10. a) Define and explain subcritical multiplication.
    b) If a source emits 5×10^5 neutrons/generation, and $k_{eff} = 0.98$, how many neutrons are born throughout the core in each generation?
    c) How many times larger will the neutron population be when $k_{eff} = 0.95$ in a particular reactor than when $k_{eff} = 0.90$? Assume constant cource strength.

11. a) Define period.
    b) Define startup rate.
    c) What is the period when the SUR is 0.4 DPM?

12. Power goes from 60 MWT to 82 MWT in 30 seconds. Use Figure 10-18 to determine τ.

13. The following examples are to be worked without using any tables or graphs or other reference material.

    a) Power goes from 20 MWT to 54.3 MWT in 88 seconds. What is τ?
    b) Power goes from 480 MWT to 240 MWT in 200 seconds. What is τ?
    c) If it takes 5.7 seconds for power to increase from 5% of full power to 9.5% of full power, what is τ?
    d) How long does it take for power to increase from 20 KWT to 148 KWT on a 200 second period?
    e) Power is placed on a 60 second period for a total of 3 minutes. At the end of this time period the power was 600 KWT. What was the initial reactor power?
    f) If a reactor operating at 5×10^{-3}% full power is placed on a 50 second period for 100 seconds, what is the final power?

14. On a piece of semilog graph paper, sketch power traces for SURs of 0.2 DPM, 0.4 DPM and 2.0 DPM. Assume 1 watt at start.

15. A reactor's power is equivalent to a 100 watt light bulb and is increasing at the rate of one DPM. How long will it take to reach the power of a 270 hp automobile (assume to be 200 KW)? A 4000 hp locomotive (assume to be 3000 KW)? No reference material is needed. Approximations are necessary. (Hint for second part - consider the $\sqrt{10}$).

13 TO

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16. a) What is the approximate length of a prompt neutron generation?
    b) The majority of a prompt neutron's lifetime is spent in what manner?

17. a) What is the source of delayed neutrons?
    b) What is β in your reactor at BOL and at EOL? Why does it change?
    c) Discuss the effect of delayed neutrons upon a supercritical reactor.

18. a) Calculate the approximate period which would result from ρ = 0.05% $\Delta k/k$.
    b) What would be the period in the previous problem if all neutrons were prompt.
    c) Define prompt critical. What will be the reactivity in the core at prompt critical when β = 0.73%.

19. a) What is the prompt jump?
    b) Why is it that a reactor never has an equilibrium negative period shorter than -80 seconds?

20. a) Describe the important mechanisms which contribute to the moderator temperature coefficient of a BWR.
    b) Discuss the effect of chemical shim upon the moderator temperature coefficient.

21. a) On a curve of k_{eff} vs. N_{H_2O}/N_{UO_2}, show an uncontrolled core, the same core with full rod control, and the same core with full chem shim control. Explain the differences in the curves.
    b) Sketch how the temperature coefficient would behave in the three cases if the core was designed with a water to fuel volume ratio which placed its 70° F operating point at the optimum ratio for the uncontrolled case.

22. If the moderator void coefficient of a BWR is -1.5×10^{-3} $\Delta k/k/\%$ voids at the full power condition of 23% voids, and a rapid pressure increase occurs which reduces the void volume to 20% voids, how much reactivity will be inserted into the core?

23. a) Assume that we are using the frictional heat of a coolant pump to increase moderator temperature. A temperature increase from 460° F to 510° F produces a reactivity decrease of .0075 $\Delta k/k$. What is the average temperature coefficient over this temperature range?
    b) How is the coefficient at 460° F likely to compare with the coefficient at 510° F.

24. a) Describe the effect which produces the fuel temperature coefficient.
    b) Why is this coefficient of particular importance to reactor safety?
    c) What is the approximate magnitude of the fuel temperature coefficient?
    d) Do moderator conditions have any influence upon the fuel temperature coefficient? Explain.

13TD 0210

25. a) How are Xe-135 and Sm-149 produced and removed from the core?
    b) Do the equilibrium amounts of Xe and Sm depend upon flux?

26. Sketch the xenon behavior for the following operating sequence?

    a) Startup to full power from clean condition and operate for 2 days.
    b) Reduce load to 50% for 1 day.
    c) Increase load to full power for 2 days.
    d) Scram and stay shutdown for 8 hours.
    e) Startup to full power and operate for 2 days.

27. What is the conversion ratio?

CHAPTER II

HEAT TRANSFER EFFECTS IN WATER MODERATED REACTORS

INTRODUCTION

The purpose of this chapter is to present some of the significant aspects of reactor operation that are concerned with core heat transfer limitations. During the design of a reactor, consideration is first given to the desired thermal megawatt output, and the basic fuel element and lattice design. Once these are established, the heat transfer and fluid flow engineers in conjunction with the physicists, must consider the question of how large the reactor must be to deliver the required power. It is desirable to make the reactor core as small as possible and to "work" it as hard as possible in order to reduce the size of the vessel and other components, and to minimize the fuel cost. At some point in this process, the designers come up against limits beyond which the reactor would either not operate satisfactorily or the overall cost would increase. The operator of a reactor must be aware of certain of these limits, because it is generally possible to exceed them through improper operation.

EFFECT OF HEAT ADDITION TO A MATERIAL WITH NO CHANGE IN PHASE

When heat energy is added to a material it acts to increase the atomic and molecular vibrational motion of the material. The most familiar observable effect of this increase in atomic and molecular energy is an increase in the temperature of the material. However, considering the wide variations from one material to another in such properties as thermodynamic state (i.e., solid, liquid, or gas), crystal structure, and complexity and size molecule, it is not surprising that different materials are affected to different degrees by the addition of a given quantity of heat energy. That is, the increase in temperature which results from the addition of a Btu of heat to a material will differ from one material to the next. To describe the effect of the addition or subtraction of heat energy upon the temperature of a body, a quantity called the heat capacity of the body is employed. The heat capacity, C_p, is defined as the number of Btu's required to raise one pound of a substance by one °F at constant pressure. If we express the above verbal definition in mathematical form we obtain:

$$C_p = \frac{Q}{m\Delta T} \tag{II-1}$$

where: C_p = heat capacity of a material (Btu/lb-°F)
Q = total heat added or subtracted from the body (Btu)
m = mass of body (lb)
$\Delta T = (T_{final} - T_{initial})$ = temperature change of body (°F)

Equation (II-1) can be rearranged to give:

$$Q = m\, C_p\, \Delta T \tag{II-2}$$

This is the fundamental equation which describes the effect of heat addition to a body under the condition that the body undergoes no change in phase.

(boiling, melting, or freezing). It applies equally well to solids, liquids or gases.

In most power plant applications, heat is added to flowing fluids rather than stagnant bodies of material. For these cases, it is convenient to consider heat and mass flow rates and re-write equation (11-2) in the form:

$$\dot{Q} = \dot{m}\, C_p\, \Delta T \qquad (11\text{-}3)$$

where: $\dot{Q}$ = heat addition (or subtraction) rate (Btu/hr)
$\dot{m}$ = mass flow rate (lb/hr)

Example: In a certain PWR the coolant enters the bottom of the core at 545° F and leaves the top at 595° F. The coolant flow rate is 68,300,000 lb/hr. In the range of 547 to 570° F, the heat capacity of water is ≈ 1.3 Btu/lb-°F. At what power is the reactor operating?

$$\dot{Q} = \dot{m}\, C_p\, \Delta T = (68{,}300{,}000)(1.3)(595-545)$$
$$\dot{Q} = 4.44 \times 10^9 \text{ Btu/hr}$$

To convert this answer to more familiar units, we make use of the fact that 1 MW = 3.413 × 10^6 Btu/hr. Thus:

$$\dot{Q} = \frac{4.44 \times 10^9}{3.413 \times 10^6} = 1300 \text{ MW (Thermal)}$$

There is one major difficulty which is encountered in the use of equations (11-2) and (11-3) and that is the heat capacity of a material varies somewhat depending upon the temperature (and to a much lesser extent the pressure) of the material. For example, near room temperature, the heat capacity of water is ≈ 1.0 Btu/lb-°F, whereas at 550° F it is ≈ 1.3 Btu/lb-°F. Thus it requires about 30% more heat to raise a pound of water from 550° to 551° F than it does to raise it from 70° to 71° F. Therefore, if it is necessary to calculate the heat required to raise the temperature of a material over a wide range of temperatures, it is necessary to account for this variation in heat capacity by employing suitable average values.

The heat capacities of several common materials used in nuclear power plants are given in Table 11-1 below.

TABLE 11-1: HEAT CAPACITIES OF VARIOUS MATERIALS

Material	C_p (Btu/lb-°F)
Water (70° F)	1.0
Water (550° F)	1.3
Steam	0.48
UO_2	0.08
Steel	0.13

EFFECT OF HEAT ADDITION TO A MATERIAL WITH A CHANGE IN PHASE

If a liquid is maintained at a constant pressure and sufficient heat is added to it, the temperature will rise until a definite value, called the saturation temperature or boiling point, is reached. Beyond this point, further addition of heat will cause the liquid to vaporize. Vaporization occurs because the molecules have so much vibrational energy that they can overcome the mutual forces of attraction[1] which hold the main body of the liquid together. Thus the molecules on the surface start breaking away from the main body of fluid.

The vaporization process occurs at constant temperature provided the system is maintained at constant pressure. The amount of heat necessary to vaporize a pound of liquid is called the heat of vaporization, λ_{vap}. Thus, in a constant pressure vaporization process, the following relationship holds:

$$Q = m_{vap}\,\lambda_{vap} \tag{II-4}$$

where: Q = heat added to body (Btu)
m_{vap} = mass of liquid which is vaporized (lb)
λ_{vap} = heat of vaporization (Btu/lb)

In a system containing a mixture of both liquid and vapor, the fraction of the total mass which is in the vapor phase is called the quality of the fluid. Thus, in a liquid-vapor mixture:

$$x = \frac{m_{vap}}{m} \tag{II-5}$$

or:

$$m_{vap} = xm \tag{II-6}$$

where: x = quality
m = total mass of fluid

Thus, $x = 0$ means pure liquid, $x = 1$ means pure vapor, and $x = 0.5$ means the mixture is 50% liquid and 50% vapor on a mass basis.

Substitution of equation (II-6) into equation (II-4) gives:

$$Q = xm\lambda_{vap} \tag{II-7}$$

---

[1] In the case of water, these forces are primarily electrical. Because of the polar nature of the H_2O molecule, the oxygen end of one water molecule tends to be attracted toward, and "bond" with the hydrogen end of an adjacent molecule

If the system is flowing, Q and m would be replaced by the rate terms $\dot{Q}$ and $\dot{m}$.

Frequently the situation is encountered where it is desired to boil a liquid which is initially at a temperature below its boiling point. The first heat which is added to the liquid raises its temperature in accordance with equation (II-2) or (II-3). After the boiling point is reached, the remainder of the heat goes toward making steam (or increasing the quality of the fluid) in accordance with equation (II-7). The total heat added would be the sum of these two, or:

$$Q = m\, C_p\, \Delta T + x m \lambda_{vap} \qquad (II-8)$$

Again, Q and m could be expressed as rates, $\dot{Q}$ and $\dot{m}$.

Example: In a BWR operating at 1035 psig (T_{sat} = 551° F), liquid coolant enters the bottom of the core at 535° F and leaves the top of the core as a steam-water mixture at 13.5% quality. Total coolant flow rate is 50×10^6 lb/hr. Assume that in the temperature range of interest, the heat capacity for liquid water is 1.3 Btu/lb-°F and that the heat of vaporization is 650 Btu/lb. At what power level is the reactor operating?

> To solve this problem we must recognize that part of the reactor thermal output goes toward raising the water temperature and part goes toward the production of steam. Therefore, equation (II-8) applies.
>
> $$\dot{Q} = \dot{m}\, C_p\, (T_{sat} - T_{inlet}) + x\dot{m}\lambda_{vap}$$
>
> $$= (50 \times 10^6)(1.3)(551 - 535) + (.135)(50 \times 10^6)(650)$$
>
> $$= 5.44 \times 10^9 \text{ Btu/hr}$$
>
> Using the conversion factor that 3.413×10^6 Btu/hr = 1 MWT, the power level would be:
>
> $$\dot{Q} = \frac{5.44 \times 10^9}{3.41 \times 10^6} = 1595 \text{ MWT}$$
>
> The reactor would be producing a total steam flow of:
>
> $$\dot{m}_{steam} = x\dot{m}_{coolant} = (.135)(50 \times 10^6) = 6,750,000 \text{ lb/hr}$$

As discussed previously, a constant pressure vaporization process occurs at constant temperature as long as both liquid and vapor are present. When the entire mass of liquid has been vaporized and heat is added to the pure vapor, the temperature of the vapor will begin to increase in accordance with equations (II-2) and (II-3). The heat capacity used in the equations must be that of the vapor, which is ordinarily somewhat smaller than that of the liquid.

HEAT TRANSFER FUNDAMENTALS

In the previous section we discussed the effect of the absorption (or removal) of heat from a given body of matter. Implicit in the discussion was the assumption that heat can, in fact, be transferred back and forth between different bodies. In this section we will discuss the basic fundamentals of the transfer process itself—what requirements must be met before heat may be transferred, how it is transferred, and how easily it is transferred.

The flow or transfer of any quantity, such as matter, electricity, or heat, is governed by three basic laws:

1. There must be a driving force which causes the quantity to flow.
2. The rate of flow increases in direct proportion to the magnitude of the driving force. That is, if we double the driving force, we will double the rate of flow.
3. There will always be some resistive effect which tends to impede the flow. This is basically a restatement of the first law since overcoming the resistive effect is what necessitates a driving force to cause a flow (alternatively, one can think of this as a "you can't get something for nothing" rule which is the reason why perpetual motion machines won't work). Flow rate varies inversely with resistance. That is, if we double the resistance (without changing the driving force) we will cut the flow rate in half.

Using the three previous rules, we can write the general flow equation:

$$\text{Flow Rate} = \frac{\text{Driving Force}}{\text{Resistance}} \qquad (11\text{-}9)$$

In the case of fluid flow, the driving force is a pressure drop and the resistance is the frictional effects of piping, orifices, valves, etc. For the flow of electrons (current flow) the driving force is a voltage drop and the resistance is in the form of resistors and other circuit elements. Finally, the flow of heat, or heat flux (which is simply the heat flow per unit area of heat transfer surface), requires a temperature difference as a driving force. Mathematically, this can be expressed as:

$$\frac{\dot{Q}}{A} = \frac{\Delta T}{R} \qquad (11\text{-}10)$$

where: $\dot{Q}$ = rate of heat transfer (Btu/hr)
A = heat transfer area (ft^2)
$\dot{Q}/A$ = heat flux (Btu/hr-ft^2)
ΔT = temperature difference (°F)
R = heat transfer resistance ($[°F][hr][ft^2]/[Btu]$)

To illustrate the significance of equation (II-10) consider the following cases:

1. An insulated 1000° F steam line is a case where a large driving force exists, namely the difference in temperature between 1000° F and room temperature. Heat flux out of the sides of the pipe, however, is quite low because the resistance to heat transfer of the insulating material is high.

2. In a condenser, heat from the condensing steam is transferred through the condenser tube to the cooling water. Despite the relatively low temperature difference, comparatively large amounts of heat are transferred because the resistances involved are low. These resistances include the resistance of the film of steam and condensate on the outside of the tube, the resistance of the tube material, and the resistance of the stagnant film of water on the inside surface of the tube.

To illustrate the application of equation (II-10), consider typical power reactor fuel rods as shown in Figure II-1. The fuel itself is a cylinder (actually a number of cylindrical pellets stacked on top of each other) of UO_2 about 0.5 inches in diameter. Surrounding the UO_2 is a thin annulus of cladding, usually about 0.02 to 0.03 inches thick, and made of stainless steel or zircalloy. As can be seen, the heat that is generated within the UO_2 must be transferred through four resistances in series--the resistance of the UO_2 itself, the resistance of the small gas space between the

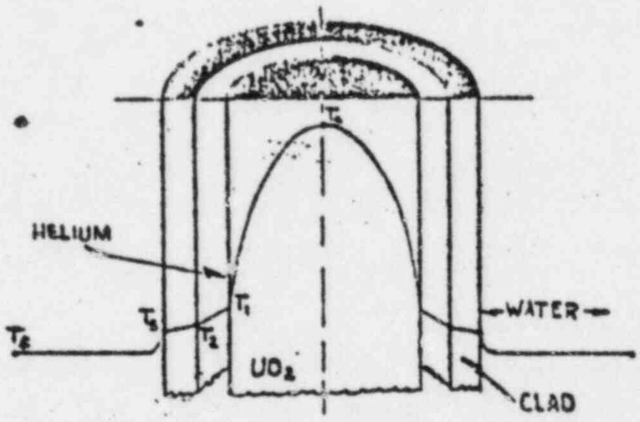

Figure II-1: Schematic Cross Section of a Typical Fuel Rod Showing Fuel Center Temperature (T_C), Fuel Surface Temperature (T_1), Temperature of Interior Cladding Surface (T_2), Cladding Exterior Surface Temperature (T_S), and Bulk Fluid Temperature (T_f)

UO_2 and the cladding[1], the resistance of the cladding, and the resistance of the water. Equation (11-10) can be written for each one of these resistances in turn:

$$T_c - T_1 = (\dot{Q}/A)(R_{UO_2}) \qquad (11\text{-}11)$$

$$T_1 - T_2 = (\dot{Q}/A)(R_{\text{gas space}}) \qquad (11\text{-}12)$$

$$T_2 - T_s = (\dot{Q}/A)(R_{\text{clad}}) \qquad (11\text{-}13)$$

$$T_s - T_f = (\dot{Q}/A)(R_{H_2O}) \qquad (11\text{-}14)$$

The $\dot{Q}/A$ term in each of these expressions is essentially constant. That is, the heat which is transferred through the cladding is the same heat which was transferred through the UO_2 and which will be transferred to the water. The heat transfer area, A, increases as the heat moves radially outward, but for thinly clad rods, the increase in area is small. To obtain the total temperature drop it is necessary only to sum the four individual temperature drops. Therefore, summing the left and right sides of equations (11-11) through (11-14) gives:

$$(T_c - T_1) + (T_1 - T_2) + (T_2 - T_s) + (T_s - T_f) = \\ (\dot{Q}/A)R_{UO_2} + (\dot{Q}/A)(R_{\text{air}}) + (\dot{Q}/A)(R_{\text{clad}}) + (\dot{Q}/A)(R_{H_2O}) \qquad (11\text{-}15)$$

Removing parentheses, factoring, and combining terms in equation (11-15) gives the following expression for the total temperature drop:

$$(T_c - T_f) = (\dot{Q}/A)(R_{UO_2} + R_{\text{air}} + R_{\text{clad}} + R_{H_2O}) \qquad (11\text{-}16)$$

It is seen, therefore, that the total resistance to heat transfer is simply the sum of the four individual series resistances.

We will not attempt to make any exact evaluations of the resistances in equation (11-16), but a few qualitative remarks about the factors involved are in order. One of the most important factors influencing the resistance to heat transfer is the heat transfer mechanism, of which there are three types.

1. Conduction

   Conduction is the transfer of heat by atomic, molecular or electronic action, and is the primary heat transfer mechanism in solids. When

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[1] Fuel rods are evacuated and then filled with helium during manufacture. During operation, fission gases are released into this space.

the temperature of a body increases, the atoms, molecules, and electrons of the body increase their vibrational motion. If a hot, rapidly vibrating molecule is located adjacent to a colder, more stationary molecule, the hotter molecule will ultimately induce vibrations in the colder molecule and will, in effect, have transferred some of its heat. The situation is analogous to having a motionless cube of Jello. If one corner of the cube is wiggled, the wiggles will eventually spread until the whole cube is moving. It is important to recognize that in spite of the fact that the vibrations are transferred throughout the body, there is no permanent relocation of any of the molecules. That is, there is no mixing. Lack of mixing is the distinguishing feature of the conduction process.

The ability of a material to conduct heat is measured by its thermal conductivity. In general, the higher the thermal conductivity, the better the material conducts heat, and the lower is its resistance to heat transfer. As a general rule, the best conductors of heat are metals. They possess large numbers of relatively free electrons which serve as the major heat carriers. The next best conductors are dense, non-porous crystals and other non-metals which have relatively few free electrons and conduct principally by molecular or atomic motion. The thermal conductivities of these materials vary considerably, but are substantially lower (typically about 1/100) than those of most metals. The worst solid conductors are dry, porous non-metals such as asbestos, dry wood, and cork. Since the transmittal of vibrational motion is aided by intimate contact, the pores tend to block the flow of heat. The thermal conductivities of these insulating materials is typically ≈ 1/1000 that of pure metals.

The resistance to heat conduction depends not only upon the thermal conductivity of the material, but also upon its shape. In general, the resistance is lowered by making the body into thin shapes with a large surface area per unit volume. Thick shapes with low surface to volume ratios, such as cylinders and spheres, provide a relatively higher resistance to heat flow because they maximize the path length along which the heat must travel.

For the fuel rod illustrated in Figure 11-1, conduction is the primary heat transfer mechanism in the UO_2 and the cladding. The resistance to heat transfer of the UO_2 is high for two reasons. First its thermal conductivity is relatively low (at elevated temperatures the thermal conductivity of UO_2 is only about 1/4 as great as the thermal conductivity of uranium metal). Second, the cylindrical shape is very poor as far as aiding in heat removal since it is thick and possesses a small surface area relative to its volume. As a result, a large temperature differential is required within a fuel pellet to drive large quantities of heat to the coolant. Were the same quantity of UO_2 pressed into a thin flat plate, the resistance to heat transfer and the resulting temperature differential for a given heat transfer rate would be much smaller. The resistance of the cladding is relatively low because of the high thermal conductivity of the metal and the thin annular shape.

2. Convection

Transfer of heat by mixing of the hot and cold bodies is called convection. This mechanism, of course, is generally limited to heat transfer in fluids (liquids or gases). Where good mixing exists, the resistance to heat transfer is very low. Thus, where it is desirable to promote rapid heat transfer, devices which promote convection are employed to the maximum extent practical. Such devices include high coolant velocities to minimize stagnant areas and promote turbulence, mixing vanes, and nucleate boiling (see below).

Unfortunately, heat transfer in fluids is practically never a pure convective process. This is particularly true in the case of fluids flowing past a metallic surface, as is the case in the reactor core and in most heat exchangers. Fluids have a tendency to adhere to solid surfaces resulting in the formation of a virtually stagnant film on the surface. Although this film is ordinarily very thin, heat transfer through it is principally by conduction rather than convection and its presence can markedly increase the overall heat transfer resistance of the fluid. Thus, in order to evaluate the resistance of a material in which convection plays a part, a number of parameters must be known, including flow rate, flow geometry, fluid temperature[1], thermal conductivity, and steam content.

Under normal conditions, the resistance of the coolant to heat transfer is low in comparison to that of the UO_2. What little resistance there is occurs in the water film at the surface of the cladding. As a result, there is a measurable temperature drop as the heat passes through this film as was illustrated in Figure II-1. Once the heat penetrates the film it is transferred rapidly throughout the remainder of the water and the resistance is so low that there is virtually no temperature variation throughout the bulk of the coolant at any given elevation along the fuel rod (the temperature of the coolant or its void content, does increase steadily as it progresses upward along the rod because of the continual addition of heat to the coolant).

In the gas space surrounding the fuel pellet, heat transfer is by both conduction and convection, with conduction playing an important part because the gas is relatively stagnant. Although the thermal conductivities of gases are very low, and convection currents in this region are small, the resistance to heat transfer is still relatively low due to the thin annular shape of the region.

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[1] The fluid temperature strongly influences its viscosity, which in turn influences its susceptability to formation of films on surfaces.

3. Radiation

Every body of matter which possesses heat energy (as reflected by its temperature), continuously emits this energy in the form of electromagnetic radiation. At the same time, every body is continuously absorbing radiant energy which was emitted by neighboring bodies. The rate at which the radiation is emitted is dependent upon the temperature of the emitter (it increases as the fourth power of the absolute temperature) and the rate at which it is absorbed depends upon the temperature of the surroundings. If there is no large temperature difference between a body and its surroundings, the rates of emission and absorption will be essentially equal and the *net* radiant heat exchange from the body will be small. In most heat exchange equipment, which is specifically designed to promote heat exchange by conduction and convection and is designed to minimize the temperature differentials between various locations in the apparatus, net radiant heat transfer is small and is neglected. However, because radiant heat losses increase rapidly as the temperature of a body rises (because of the fourth power relationship), bodies which operate at very high temperatures, such as reactor fuel rods, may lose a substantial amount of heat by this mechanism.

The student may be confused by the connection between radiant heat transfer and X or γ radiation. X and γ radiation are photons which are emitted by bodies which possess energy above their ground state as a result of nuclear and electron shell processes which have occurred. Radiant heat emission is an exactly analogous process whereby a body rids itself of heat energy which it possesses in order to return itself to the "ground state" temperature of its surroundings. The only difference is in the relative magnitudes of the energies involved in the various processes. Radiant heat transfer involves relatively low energy photons (frequently in the visible light energy range), whereas X and γ radiation involves higher energy photons. In fact, it was the study of radiant heat transfer by Max Planck which led to his formulation of the photon theory (more commonly called the quantum theory) which was subsequently used by Albert Einstein to explain x-rays.

HEAT TRANSFER IN BOILING WATER

Along a considerable portion of the typical BWR fuel rod, boiling is occurring at the coolant-cladding interface. As long as this boiling is of a type known as nucleate boiling, the resistance to heat transfer at the coolant-cladding interface (and the temperature differential between the cladding surface and the bulk of the coolant) is very low. Nucleate boiling is pictured in Figure II-2. In nucleate boiling, a steam bubble

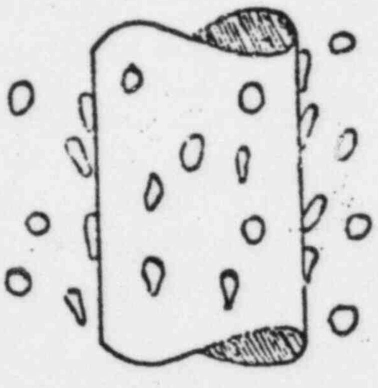

Figure 11-2: Nucleate Boiling

forms on the heat transfer surface and then breaks away into the bulk fluid. At any given time only a small portion of the total heat transfer surface is covered with steam bubbles. Nucleate boiling results in a low resistance to heat transfer because the agitation produced by the bubbles breaks up the thin film of stagnant water which adheres to the surface of the cladding and thereby promotes mixing and convective heat transfer.

If the heat generation within the fuel is increased, the rate of bubble formation upon the cladding surface will also be increased. As a result, the bubbles will occupy a greater percentage of the cladding surface area at any given time. Further increases in the heat flux will increase steam formation to a point where it is being produced faster than it is being swept away by its bouyancy and the coolant circulation. At this point, the steam completely blankets the heat transfer surface and separates it from contact with the bulk coolant. This phenomenon, known as film boiling, is illustrated in Figure 11-3. Since the thermal conductivity

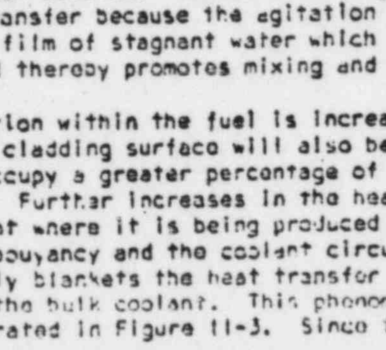

Figure 11-3: Film Boiling

of steam is very low, the steam blanket results in a large increase in the resistance to heat transfer at the coolant-cladding interface. In order to continue transferring the same amount of heat across the interface, the temperature of the cladding must increase sharply[1].

The effect upon cladding surface temperature of the transition from nucleate to film boiling is illustrated in Figure II-4. Referring to the figure,

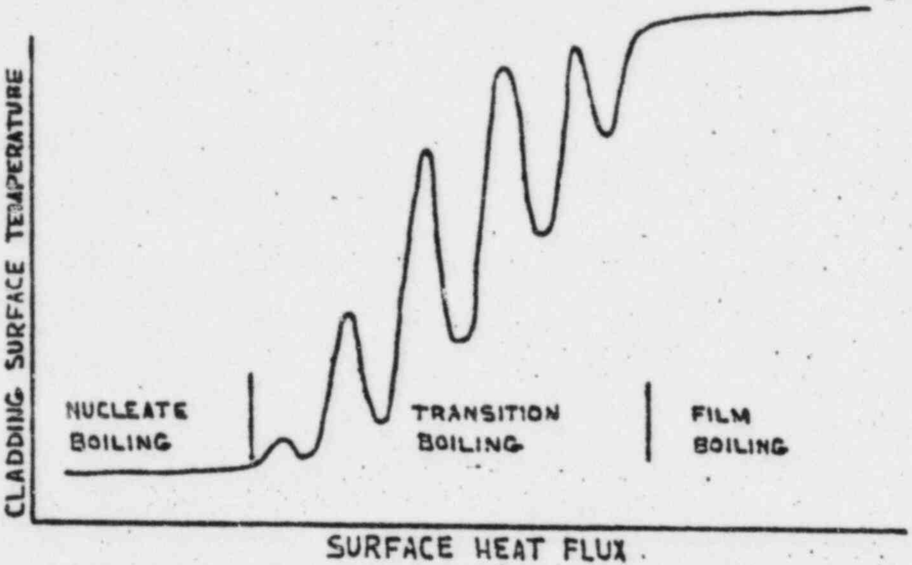

Figure II-4: Effect upon Cladding Temperature of the Transition from Nucleate Boiling to Film Boiling

It is seen that as the heat flux is increased, the cladding surface temperature increases slowly until the critical heat flux is reached. The critical heat flux is considered to be the point at which the transition from nucleate boiling to film boiling begins. The critical heat flux is characterized by localized flow and temperature oscillations brought about by the changing steam bubble conditions. The cladding surface temperature rises rapidly to an equilibrium value for the film boiling condition. If, as is frequently the case, the temperature increase attending the transition from nucleate to film boiling causes the cladding to exceed

---

[1] This latter statement can be verified by considering equation (II-14). If R_{H_2O} increases, and Q/A and T_f remain constant (T_f, the bulk fluid temperature, depends only upon the operating pressure of the reactor), T_s must increase.

its melting point, a failure will occur[1]. Cladding failure resulting from the onset of film boiling is called burnout or departure from nucleate boiling (DNB).

Although a PWR operates with subcooled coolant, there still is a small amount of local or surface boiling which is possible. This type of boiling can occur if the cladding surface temperature at any point exceeds the saturation temperature of the coolant. In this case, a steam bubble might form at this location and then be swept into the main body of the coolant where it would quench. Such a situation is particularly likely on a rapid power increase which results in an increase in cladding surface temperature without appreciably affecting reactor pressure (and corresponding saturation temperature). It is estimated that under transient conditions, local quality in a hot channel may get as high as 5% (as compared to BWR peak channel exit qualities of 20%). Ordinarily, this local boiling effect would have the characteristics of nucleate boiling, but there is a heat flux limit beyond which a PWR would be subject to DNB.

The magnitude of the critical heat flux depends upon many variables, including fuel rod geometry, coolant flow rate, coolant subcooling[2], coolant quality, and reactor pressure. The relationships which relate the critical heat flux to these various parameters are complex, and are based predominately upon a large mass of experimental data. In general, however, the following qualitative remarks apply:

1. The CHF increases with increasing coolant flow rate because the higher velocity is more effective in sweeping away steam bubbles from the cladding.. In effect, this means that steam bubbles can be generated at a faster rate before the cladding will begin to be blanketed.

2. If the coolant is saturated, as in a BWR, an increase in pressure will decrease the CHF. This results from the fact that as pressure increases, the density of the steam bubbles approaches the density of the liquid coolant. This reduces the bouyancy of the steam bubbles and makes them more difficult to sweep away.

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[1] Experiments have shown that if the melting point of the cladding is not exceeded, short term operation above the critical heat flux may not produce burnout. Nevertheless, the cladding begins to weaken at temperatures well below the melting point, and long term operation under these conditions will ultimately result in failure--if only due to inability to withstand contained fission gas pressure (see next section).

[2] The subcooling is the difference between the actual coolant temperature and the saturation temperature. For example, if the coolant temperature is 540° F, and the saturation temperature is 550° F, the coolant is said to be subcooled by 10° F.

3. If the coolant is subcooled, as in a PWR, an increase in the subcooling will increase the CHF. This results from the fact that steam bubbles will quench more readily in highly subcooled water than in water which is near the boiling point. Thus, removal of steam bubbles from the surface of the cladding by quenching action is improved as the subcooling increases. In a PWR, subcooling can be increased by two means:

   A. If the reactor pressure is held constant (i.e., saturation temperature is held constant), a decrease in coolant temperature will result in an increase in subcooling.

   B. If reactor coolant temperature is held constant, the subcooling will increase as the reactor pressure increases (because of the corresponding increase in the saturation temperature). Thus, in contrast to a BWR, an increase in the operating pressure of a PWR may result in an increase in the CHF.

Because of variations from one reactor to another in the parameters listed above, it is impossible to give a value for the CHF unless the precise operating conditions are known. However, Figure II-5 shows a typical graph of CHF versus coolant quality. The typical operating range of a PWR would be 0-5% quality, where the CHF is on the order of 10^6 Btu/hr-ft^2. On the

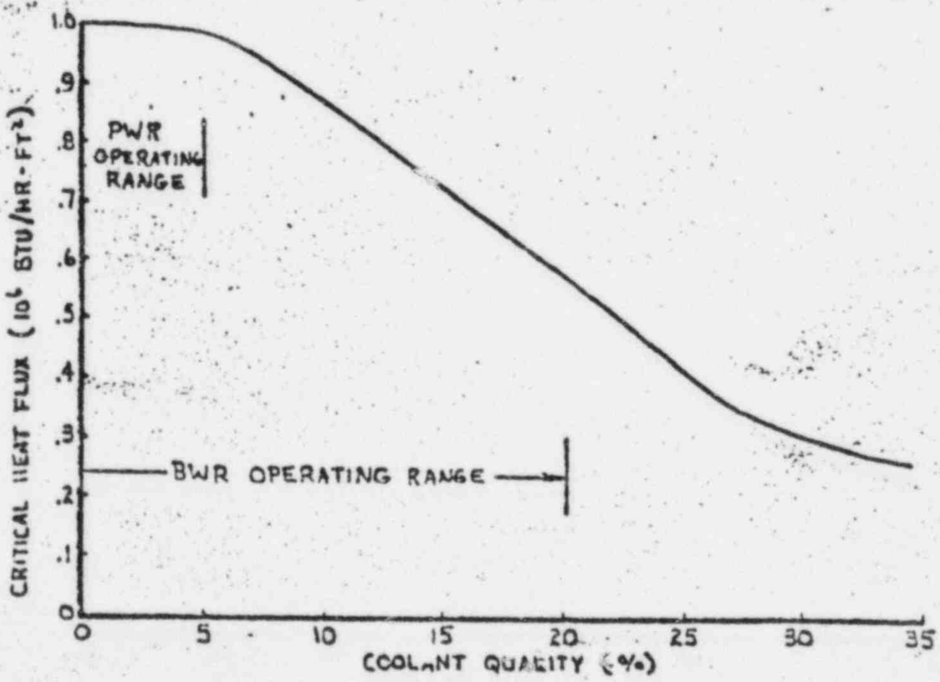

Figure II-5: Variation of Critical Heat Flux with Coolant Quality other hand, the large change in coolant quality from entrance to exit of a BWR produces a significant variation in the CHF along the length of the fuel bundle. At the entrance to the core, it would be on the order of 10^6 Btu/hr-ft^2 as for a PWR, but near the core exit it may only be 600,000-700,000 Btu/hr-ft^2 because of the high quality.

It should now be clear that if cladding failure is to be avoided, the reactor must be operated in such a manner that the critical heat flux is not exceeded at any point in the core. Therefore, all power reactors have this restriction written into their operating license. The limit is usually expressed in terms of a quantity called the critical heat flux ratio (CHFR). The CHFR at any location in the core is defined as follows:

$$\text{CHFR} = \frac{\text{critical heat flux at location of interest}}{\text{actual heat flux at location of interest}} \qquad (11\text{-}17)$$

If the critical heat flux ratio is 1.00 at any location in the core, the heat flux at that location has reached the value at which DNB can be expected. If the critical heat flux ratio is >1.00 at a given core location, the heat flux at this point is less than the critical heat flux and DNB should not occur.

Although many experiments have been performed to measure the critical heat flux for different operating conditions, there is still a small statistical uncertainty associated with the results. Therefore, in order to provide a degree of conservatism, most reactor licenses specify that the CHFR at all locations in the core must never fall below ~ 1.3 to 1.5 during steady state operation, normal operational transients, or anticipated abnormal transient conditions. As the DNB phenomenon becomes better understood, the need for conservatism is reduced, and licenses may be written with minimum CHFR limits of 1.00 at sometime in the future.

The CHFR varies considerably from one location in the core to another. The location of greatest concern, of course, is that at which the ratio is at its minimum (MCHFR). Inspection of equation (11-17) indicates that the CHFR can be reduced in two ways: 1) by reducing the critical heat flux, or 2) by increasing the actual heat flux.

In determining the MCHFR, one location which must always be investigated is the point in the core which is operating at the highest heat flux. In both BWRs and PWRs this frequently occurs below the vertical centerline of the core[1]. In the lower regions of the core, the coolant is relatively

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[1] The reason for this will be discussed in detail in the following chapter. For a BWR this results from the presence of voids in the top half of the core. Voids tend to reduce reactivity (negative void coefficient) and power in this region. In a PWR, the control rods enter from the top of the core. Whenever control rods are partially inserted they selectively poison the upper portion of the core and the power tends to be generated preferentially in the lower portions.

cool and free of voids, which are conditions favorable to a high value of the critical heat flux. Thus, the point of maximum heat flux is not necessarily the point at which the critical heat flux is a minimum. This is particularly true in BWRs where there is a significant decrease in the critical heat flux in highly voided regions. As a result, in locating the MCHFR, one must also investigate those locations nearer to the top of the core which combine a relatively high actual heat flux with a high coolant temperature or void content and correspondingly lower critical heat flux to see if they are limiting. In general, the determination of the location and magnitude of the MCHFR is a complex problem which requires detailed knowledge of the coolant flow in each fuel bundle and the reactor power distribution.

Current PWR and BWR designs have a MCHFR of ≈ 1.8-1.9 at normal full power operating conditions. Since the CHF for a PWR is typically ≈ 10^6 Btu/hr-°F, this means that peak heat fluxes can be somewhat in excess of 500,000 Btu/hr-ft^2. For BWRs, where the CHF may fall to 700,000-800,000 Btu/hr-ft^2 in regions of relatively high flux, the peak heat fluxes must be held to somewhat lower values--typically 400,000-450,000 Btu/hr-ft^2. Thus, the core of a PWR can be somewhat smaller in physical size, with smaller diameter fuel rods and less heat transfer surface area than a BWR of comparable thermal output.

FUEL PELLET CENTER MELTING

Historically, two heat transfer limits have been prescribed for water moderated reactors. The first is the MCHFR limit previously discussed, which is intended to prevent cladding failure due to DNB. The second is a limit on fuel rod center temperature which is intended to avoid fuel pellet melting during normal steady state and expected transient conditions. The melting point of UO_2 fuel pellets is ≈ 5000° F. Since the fuel pellet temperature is not directly measured, equation (II-16) is employed to establish the limit. Thus the limit might be expressed in terms of the heat flux (Q/A), or alternatively a fuel rod power density (which is essentially Q per gram of fuel) which would produce a fuel center temperature near the melting point. The coolant heat transfer resistance used in establishing this limit is that for the expected normal coolant-cladding interface conditions (nucleate boiling or pure liquid at the interface) since the core is designed to simultaneously meet the previously discussed DNB limit.

Both PWRs and BWRs operate with peak fuel center temperatures at full power of ≈ 4000°F. Under transient conditions, the values may approach ≈ 4500° F. Core average fuel temperature values are considerably lower--typically ≈ 1100° F.

The temperature profile across a fuel rod can be calculated from equations (II-11) through (II-14) if the various resistances are known. The approximate temperature profile across a Humboldt Bay (BWR) fuel rod operating under nucleate boiling conditions is shown in Figure II-6 for two operating

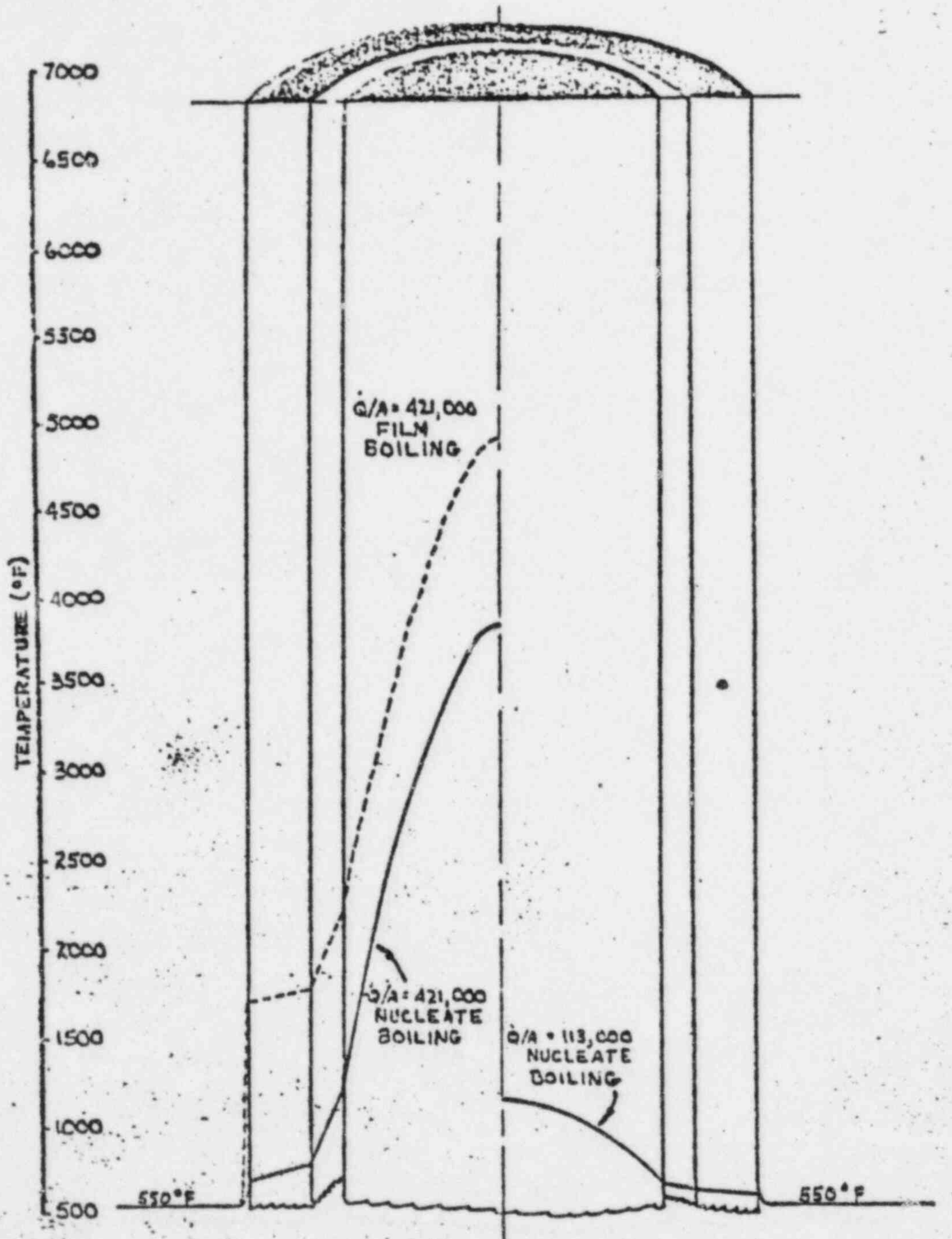

Figure II-6: Temperature Profile Across a BWR Fuel Rod for Nucleate and Film Boiling Condition heat fluxes. This figure shows that when power generation (i.e., the heat flux) within a fuel rod increases, the fuel center temperature rises sharply. The bulk coolant temperature of a BWR, of course, is not affected by power, but is simply equal to the saturation temperature of water at the reactor operating pressure. The temperature drops across the four major resistances are, of course, dependent upon such parameters as dimensions, materials, coolant flow rate, etc. For Humboldt Bay it has been calculated that 495,000 Btu/hr-ft^2 will result in a fuel center temperature just below the UO_2 melting point assuming nucleate boiling and this has been set as the limit. Although a small amount of fuel center melting can occur without adversely affecting the fuel, a high degree of center melting will result in failures. The major reason is that the melted UO_2 will no longer retain its fission product gases and the internal pressure in the rods due to these gases will cause the cladding to rupture. A structurally sound sintered UO_2 pellet will retain over 75% of the total fission product gases produced in its lifetime. If, however, its integrity is weakened by melting, these gases will be released.[1] Although it is reasonable to expect that it will ultimately be possible to operate oxide fuel with some portion of the fuel molten (thereby eliminating the melting point as a design limit) there are other difficulties which must be combated before significant increases in fuel power density can be achieved over present day designs. Among these is excessive fuel pellet thermal expansion, which could result in overstressing the cladding.

Which of the two basic heat transfer limits, MCHFR or center melting, is the most conservative must be determined by calculation. For BWRs, the MCHFR is almost always the limiting factor. That is, for normal power distributions the MCHFR will probably be less than the license limit of ≈ 1.3-1.5 before any point in the core is operating at the maximum heat flux limit. For PWRs, the two limits are about equally conservative and which is limiting is dependent upon the nature of the occurrence which causes them to be approached. On a rapid power increase, where the fuel temperature increases but the fuel time constant delays an increase in coolant temperature, the center melting limit may be the one which is reached first. On slower (and somewhat more likely) power increase, where the coolant temperature rises appreciably, the MCHFR limit may be reached first (since the CHF decreases as the coolant temperature increases). There is an incentive, of course, to design the core in such a manner that the two limits are approximately equally limiting. It would be inefficient to have a core limited to a low power level due to MCHFR considerations when there was substantial margin on fuel center temperature and vice-versa.

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[1] Actually, the ability of the fuel to retain gases is reduced significantly when the fuel temperature exceeds 3000° F (called the fission product redistribution temperature). Even in normal operation, a small amount of fuel may exceed 3000° F. However, the fuel is designed with sufficient plenum volume within the cladding to accept the gases released from this fuel. If the fraction of fuel operating above 3000° F increases much beyond the design value, however, fuel cladding failures will be likely.

It should be recognized that if the critical heat flux is reached both DNB and center melting may occur even though the critical heat flux is less than the maximum heat flux limit. This is so because the coolant-cladding interface resistance increases dramatically when the critical heat flux is reached. This results in a significant increase in fuel center temperature over that which would exist under nucleate boiling conditions. Superimposed on Figure II-6 is a temperature profile which might exist in the case of DNB being reached.

HEAT TRANSFER COEFFICIENT

Equation (II-16), which is the basic heat transfer relationship, is often written in the form:

$$\frac{\dot{Q}}{A} = \frac{1}{(R_1 + R_2 + \ldots R_n)} \Delta T = U \Delta T \qquad \text{(II-18)}$$

where: U = overall heat transfer coefficient (Btu/hr-ft^2-°F)

The overall heat transfer coefficient is thus seen to be the reciprocal of the total resistance to heat transfer. When the heat transfer coefficient is high, the resistances are low, and a favorable heat transfer condition exists. Conversely, when the heat transfer coefficient is low, resistance is high, and conditions for heat transfer are unfavorable. In this terminology then, DNB would result in a significant reduction in the heat transfer coefficient of the fuel.

PROBLEMS

1. Define or explain:

   a) Heat flux
   b) Critical heat flux
   c) Nucleate boiling
   d) Film boiling
   e) Convection
   f) Conduction
   g) Heat Capacity

2. a) A PWR operates at 3200 MWT. The coolant temperature increases from 545° F to 600° F as it passes through the core. What coolant flow rate is required? Assume $C_p = 1.3$ Btu/hr-ft^2. Also 1 MWT = 3.413×10^6 Btu/hr.
   b) What would be the temperature rise at 50% power? Assume constant flow rate.
   c) If the average core heat flux is 205,000 Btu/hr-ft^2, how many square feet of heat transfer surface area are required?

3. a) At an elevation 3 feet up from the bottom of the core, the peak heat flux in a particular BWR fuel bundle is 300,000 Btu/hr-ft^2. The CHFR at this point is 3.2. What is the critical heat flux at this point?
   b) Two feet farther up the bundle there is a point where the heat flux is 250,000 Btu/hr-ft^2. However, the CHFR is 3.0. Discuss why this could occur.

4. Discuss what occurs at DNB and of what significance it is to reactor operation.

5. In a PWR you wish to include a safety feature which will sense reactor power and will trip the reactor before a power level is reached which will result in DNB. Assume that under the conditions that reactor pressure is 2235 psig and coolant enters the core at 550° F and leaves at 600° F, the appropriate trip setpoint has been determined to be 110% of full power.

   a) If the reactor pressure is raised to 2335 psig, but the coolant temperatures are not altered, would it be legitimate to raise the trip setpoint? Why?
   b) If the pressure is allowed to remain at 2235 psig, but the reactor is operated in such a manner that the inlet and outlet temperatures are 530 and 580° F respectively, what could you do to the setpoint and why?

6. What is the effect of pressure on the CHF in a BWR and why?

7. a) The major resistance to heat transfer in a typical fuel rod is provided by what?
   b) Why is there a sharp drop in coolant temperature in the immediate vicinity of the cladding, but a near uniform temperature as you move farther away from the cladding?

8. Why is extensive center melting undesirable?

9. a) The Humboldt Bay core operates at 230 MWT. Reactor coolant flow is ≈ 12×10^6 lb/hr. Coolant entering the core is subcooled by ≈ 10° F. Saturation temperature is ≈ 550° F. What is the approximate core exit quality? Assume C_p = 1.3 Btu/lb-°F, λ_{vap} = 650 Btu/lb.
   b) What is the approximate steam flow rate?

10. What effect does an increase in heat transfer resistance have on fuel pellet center temperature? Assume constant power.

CHAPTER 12

OPERATING CHARACTERISTICS OF LIGHT WATER POWER REACTORS

INTRODUCTION

In the previous two chapters we considered some of the basic theory involved in power reactor design and performance. In this chapter we will discuss the practical application and significance of this theory to plant operation.

INITIAL FUEL LOADING

The first task which must be accomplished in the life of a power reactor is to load the individual fuel bundles into the core. A typical PWR or BWR fuel bundle has a clean, uncontrolled (no control rod, chemical shim, or fission products) $k_\infty = 1.3$ at room temperature. However, the dimensions of a single fuel bundle are relatively small and as a result, excessive leakage makes $k_{eff} < 1$ for a core composed of a single bundle. As additional bundles are added, the core dimensions increase and leakage decreases. With some minimum number of uncontrolled fuel assemblies, leakage would be reduced to the point that $k_{eff} = 1$, and the reactor would go critical. This core is called the minimum critical core, and it seldom consists of more than ≈ 15 fuel assemblies, and usually less. The remainder of the core loading, which generally consists of several hundred fuel assemblies, is used to reduce leakage to negligible proportions in order to provide sufficient excess reactivity to overcome the negative power coefficient, fission product buildup, and to provide for an appropriate amount of fuel depletion as operation progresses. In large present day cores, leakage is small and generally represents ≈ 2-3% Δk/k. Obviously, once additional fuel is loaded beyond the minimum critical core, some form of control must be employed to keep the reactor shut down.

The initial fuel loading programs differ somewhat for BWRs and PWRs. In a BWR the control rods are first installed and inserted into the empty core.[1] Fuel elements are then loaded one at a time around the inserted rods in a prescribed sequence. At periodic intervals, the loading is stopped and some or all of the control rods in the loaded region are carefully withdrawn to check the status of the core with respect to criticality. Usually the core loading is temporarily suspended when the minimum critical size (uncontrolled) is reached and a series of measurements are performed to verify certain physics design parameters such as k_∞, moderator temperature and void coefficients, and control rod strength. As the loading progresses beyond the minimum critical size, additional physics data is obtained at regular intervals so that by the time the core loading is completed its characteristics are reasonably well known.

During the fuel loading, the head must be removed from the reactor vessel in order to provide access to the core. In contrast to a BWR, the control

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[1] To physically support the rods until fuel is loaded around them, dummy fuel assemblies are also loaded into selected core locations.

rods in a PWR enter through the upper vessel head and must be disconnected when the head is removed. To avoid the inconvenience of providing some sort of temporary actuating mechanism during the loading process, physics testing is postponed until after the core is fully loaded. Instead, the water in the core is borated to a relatively high value (called the refueling concentration) to eliminate any possibility of the core achieving criticality during the loading. In addition, control rod clusters are installed in the appropriate fuel bundles (about one bundle in three gets a control rod) prior to loading, and so the core is fully controlled by rods as well as boric acid during the loading. The total control strength provided during loading is sufficient to prevent k_{eff} from exceeding ≈ 0.9 at any time. Bundles are installed one at a time, on essentially a non-stop schedule, until the core is completely loaded. At the completion of loading, the vessel head is installed and the control rods are connected. The procedure for achieving the initial criticality is then to: 1) withdraw part of the control rods, 2) reduce the boron concentration by dilution of the coolant with pure water until a predetermined concentration is reached, and 3) continue control rod withdrawal until $k_{eff} = 1$. At this time a program of testing is begun to measure the important physics parameters.

Throughout the fuel loading process, the core neutron population is continuously monitored to guarantee that the reactor will never inadvertently achieve criticality and (in the case of a BWR) to predict the size of the minimum critical core. Thus, prior to the installation of any fuel, several neutron detection instruments are placed in service, either inside the core itself at vacant fuel bundle locations or directly outside the vessel in their normal holders. Second, a neutron source[1] is loaded into the core. In BWRs the source is in a special holder and is installed in a vacant fuel bundle location. In PWRs, the source is installed integrally with a fuel bundle. The bundle with the source is the first one loaded into the core. Once the source is in the core, the detectors will begin to respond, and the magnitude of their response will provide an indication of the total core neutron population.

In general, we are dealing with a subcritical reactor during the loading process. As a result, the core neutron population is given by equation (10-24), which is repeated below:

$$P = \frac{S}{1 - k_{eff}}.$$

where: P = total-core neutron population
S = source strength (neutrons/generation)

---

[1] The neutron detectors are generally spread around so that they monitor different sides of the core. In order to provide an initial response on all of the widely separated detectors, more than one neutron source may be used.

Of course, it is impossible to count every single neutron in the core. Instead, the neutron detectors see only a small fraction of the total core neutron population (detectors located outside the core would see a certain percentage of those which leaked from the core). Our data then consists of a detector count rate, which in turn is equal to the total core neutron population multiplied by a detector efficiency factor to account for such variables as detector location with respect to the core, detector size, and detector sensitivity. That is:

$$CR = E \times P = \frac{ES}{1 - k_{eff}} \qquad (12\text{-}1)$$

where: CR = detector count rate
E = detector efficiency factor

Equation (12-1) has four variables (CR, E, S, and k_{eff}). If any three of these are known, the fourth can be calculated. For example if CR, E, and S are known, it is possible to calculate k_{eff}[1]. Frequently however, calculations of k_{eff} from this equation are subject to rather large errors resulting from the difficulty associated with determining a precise value of the E factor. However, in spite of the uncertainty associated with the calculation of k_{eff} from this relationship, it can yield valuable information on whether or not the reactor is nearing criticality or it can serve as a means of predicting where it will go critical if the reactor is to be brought critical. This information is obtained by examining the ratio of the detector count rate at any given point in time to the detector count rate at some arbitrary reference time. This ratio is generally called the count rate ratio. From equation (12-1), the count rate ratio is seen to be:

$$CRR = \frac{CR_n}{CR_{ref}} = \frac{E_n S_n/(1 - k_{eff,n})}{E_{ref}\, S_{ref}/(1 - k_{eff,ref})} =$$

$$\frac{E_n S_n (1 - k_{eff,ref})}{E_{ref}\, S_{ref} (1 - k_{eff,n})} \qquad (12\text{-}2)$$

where: CRR = count rate ratio
CR_n = detector count rate for core condition n
CR_{ref} = detector count rate for reference core condition
E_n = detector efficiency for core condition n
E_{ref} = detector efficiency for reference core condition
S_n = source strength for core condition n
S_{ref} = source strength for reference core condition
$k_{eff,n}$ = k_{eff} for core condition n
$k_{eff,ref}$ = k_{eff} for reference core condition

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[1] The student should verify algebraically that $k_{eff} = 1 - [ES/(CR)]$.

The subscript n can stand for the value of any quantity which is being varied in order to change k_{eff}, such as number of fuel elements loaded, control rod position, and chemical shim concentration.

Equation (12-2) can be simplified somewhat by assuming that the loading, or approach to critical, takes place during a period of time which is short in comparison to the half-life of the source (which ranges from 60 days for an Sb-Be source to 24,600 years for a Pu-Be source). Thus, if the source undergoes no significant decay between the time the reference data was obtained and the time the remaining data was obtained, $S_{ref} = S_n$ and the source strength can be cancelled from equation (12-2) to give:

$$CRR = \frac{CR_n}{CR_{ref}} = \frac{E_n(1 - k_{eff,ref})}{E_{ref}(1 - k_{eff,n})} \quad \text{[constant source strength]} \tag{12-3}$$

Under some circumstances, it is also legitimate to assume that the detector efficiency is constant for a particular set of count rate data, so that it too can be cancelled to give:

$$CRR = \frac{CR_n}{CR_{ref}} = \frac{(1 - k_{eff,ref})}{(1 - k_{eff,n})} \quad \begin{bmatrix}\text{constant detector geometry,}\\ \text{constant source strength}\end{bmatrix} \tag{12-4}$$

However, as we shall see below, the assumption of constant detector efficiency is often a poor one due to changes in the detector-core-source geometry during the loading process. Nevertheless, the loading procedure is usually planned in such a manner as to retain constant detector efficiency to the maximum extent possible.

The application of the CRR to fuel loading (or an approach to critical by rod withdrawal or boron dilution, for that matter) utilizes the fact that as the reactor approaches criticality (i.e., when $k_{eff,n}$ approaches 1.0) the CRR gets very large because the $(1 - k_{eff,n})$ term in the denominator approaches zero. In theory, when criticality is achieved CR_n will continue to rise forever[1] and the CRR will approach infinity. This fact is true no matter what the numerical value of CR_{ref}[2]. To predict the point at which criticality occurs, we might make a plot of CRR versus number of fuel elements loaded (or control rod position or chemical shim concentration if we are using these devices to achieve criticality) and extrapolate this curve to a $CRR \rightarrow \infty$. Obviously, extrapolating any curve to infinity is impossible so this method isn't very useful. However, when a number approaches ∞, its reciprocal approaches zero (i.e., $1/\infty \rightarrow 0$) and

---

[1] Because the source is continuously injecting neutrons into the core and they never die out. That is, subcritical multiplication becomes infinite as discussed in Chapter 10.

[2] This is the same as saying ∞/N or $N/0$, where N is any finite number, are always infinitely large.

it is easy to extrapolate a curve to zero. Therefore, it is useful to define the inverse count rate ratio (ICRR) as the reciprocal of the CRR. Thus:

$$\text{ICRR} = \frac{CR_{ref}}{CR_n} = \frac{E_{ref}(1 - k_{eff,n})}{E_n(1 - k_{eff,ref})} \quad \text{[constant source strength]} \tag{12-5}$$

$$\text{ICRR} = \frac{CR_{ref}}{CR_n} = \frac{(1 - k_{eff,n})}{(1 - k_{eff,ref})} \quad \begin{bmatrix}\text{constant source strength,} \\ \text{constant detector efficiency}\end{bmatrix} \tag{12-6}$$

To predict criticality, we can thus extrapolate a plot of ICRR versus number of bundles loaded (or control rod position or boron concentration) to ICRR = 0. The following examples will illustrate the mechanics of the method as well as its limitations.

Example 1: In the loading of a BWR, the startup source and a neutron detector were first installed in the vessel at specified locations and the detector was observed to read 25 counts per second (cps). Four fuel bundles were then loaded into the core and the detector count rate increased to 45 cps. Then four more fuel bundles were loaded and the detector count rate was determined to be 125 cps. All readings were taken with the control rods in the loaded region of the core fully withdrawn. Based upon this data, predict the size of the minimum critical uncontrolled core. Also estimate k_{eff} with 8 bundles in the core.

To solve the first part of the problem, we will make a plot of ICRR versus number of fuel bundles loaded. We can use any one of the data points as the reference condition, but the most common practice is to use the first one , and we will arbitrarily do this.

Elements	Count Rate (CR_n)	ICRR (CR_{ref}/CR_n)
N = 0	$CR_0 = CR_{ref} = 25$	25/25 = 1.00
N = 4	$CR_4 = 45$	25/45 = 0.57
N = 8	$CR_8 = 125$	25/125 = 0.20

The plot of ICRR versus number of fuel elements is shown in Figure 12-1. Note that the starting point of the curve is an ICRR of 1.0 obtained by dividing the reference count rate by itself. This the customary starting point for plots of this type. Examination of Figure 12-1 indicates that criticality can be expected with an uncontrolled core containing ten elements. Notice that had we extrapolated the curve after the initial loading increment of four elements, the predicted critical loading would have been nine rather than ten. Generally, the loading procedure includes the restriction that the number of elements loaded between ICRR determinations cannot exceed one-half of the number of additional elements predicted for criticality (this restriction will not apply when only one more element is required since it is impractical

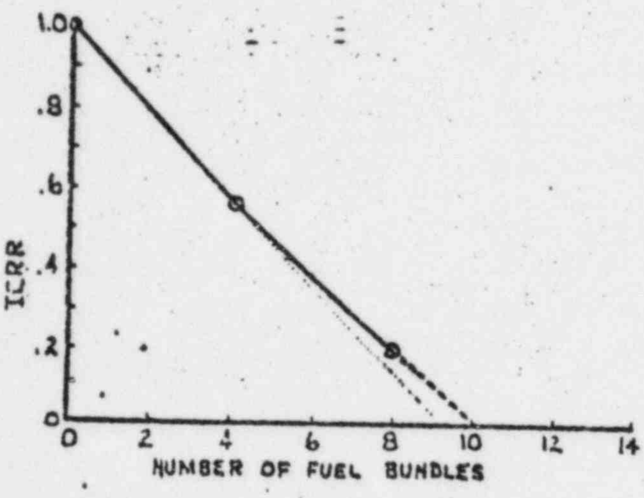

Figure 12-1: ICRR Curve for Example Problem

to load a fraction of a bundle). Thus, a more conservative approach would have been to take data after six bundles were loaded as well as after eight bundles. Since the data for eight bundles extrapolates to ten, data would be taken after each bundle was loaded beyond the eighth. The extrapolations would, of course, become increasingly accurate as criticality was neared.

Notice that in order to predict the size of the minimum critical core, it was not necessary to actually calculate k_{eff}. Inspection of equations (12-5) and (12-6) indicates that in order to determine $k_{eff,n}$ the value of $k_{eff,ref}$ must be known. In many cases $k_{eff,ref}$ is not known and the ICRR cannot be used to determine $k_{eff,n}$, even though it can be used to predict criticality. In this instance, the reference data was taken with no fuel in the core so that we know $k_{eff,ref} = 0$. In order to determine k_{eff} after 8 bundles, we must decide whether equation (12-5) or equation (12-6) is applicable. Since no information regarding the relative magnitude of the detector efficiency factors at the two data points was given in the problem, the only thing we can assume is that the efficiency was constant throughout the loading and that equation (12-6) applies. If this assumption is not warranted, the results will be erroneous. In any case:

$$\frac{25}{125} = \frac{(1 - k_{eff,8})}{(1 - 0)}$$

$$0.20 = 1 - k_{eff,8}$$

$$k_{eff,8} = 1 - 0.20 = 0.8$$

In the previous example the data was purposely adjusted so as to produce a fairly linear ICRR plot for illustrative purposes. The next two examples will consider more realistic cases where variations in detector efficiency come into play.

Example 2: Consider the BWR loading shown in Figure 12-2. The applicable data is presented below. All count rates were obtained with control rods in the loaded region fully withdrawn.

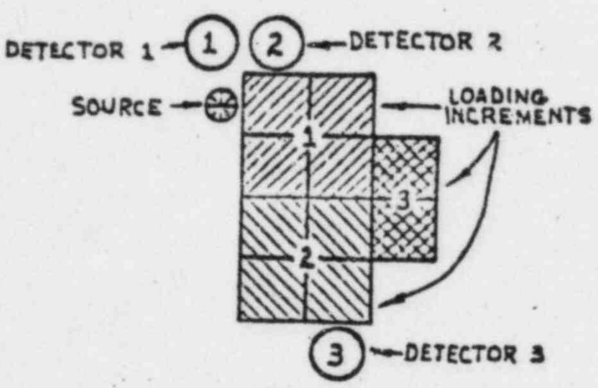

Figure 12-2: Locations of Source, Detectors and Fuel for Sample Loading Scheme

Loading Increment	Number of Elements in Increment	Total Number of Elements in Core	Detector Count Rates 1	2	3
0	0	0	600	150	7
1	4	4	672	500	70
2	4	8	792	620	175
3	2	10	1000	1070	230

The problem is to predict the size of the minimum critical core from this data.

As in the previous example, we will choose the reference data to be that obtained before any fuel was loaded. With this basis, the CRR and ICRR values for the various detectors are found to be:

	Detector 1		Detector 2		Detector 3	
Increment	CRR	ICRR	CRR	ICRR	CRR	ICRR
0	1.00	1.000	1.00	1.000	1.00	1.000
1	1.12	.893	3.33	.300	10.00	.100
2	1.32	.758	4.13	.242	25.00	.040
3	1.67	.600	7.14	.140	32.90	.034

The ICRR values can then be plotted as shown in Figure 12-3. As can be seen, the three curves are far from being straight lines which point directly to the number of elements needed for criticality. Let us first consider detector No. 1. The initial count rate of this detector is high because of its proximity to the source. Recall that the average crow-flight path length of neutrons in a cold reactor is about 6 inches. Since typical BWR fuel elements are ≈ 4-5 inches square, it can be seen that there will be a considerable attenuation of neutrons trying to reach detector 1 from all fuel elements other than the one in the upper left hand corner. When the core is far subcritical, as is the case for the first few loading increments, the neutron production rate in the fuel is low because of the low value of multiplication. Since there are few neutrons in the core, and since there is a considerable attenuation in going from the core to detector 1, it should be apparent that detector 1 will not show any significant increase in count rate for the first few loading

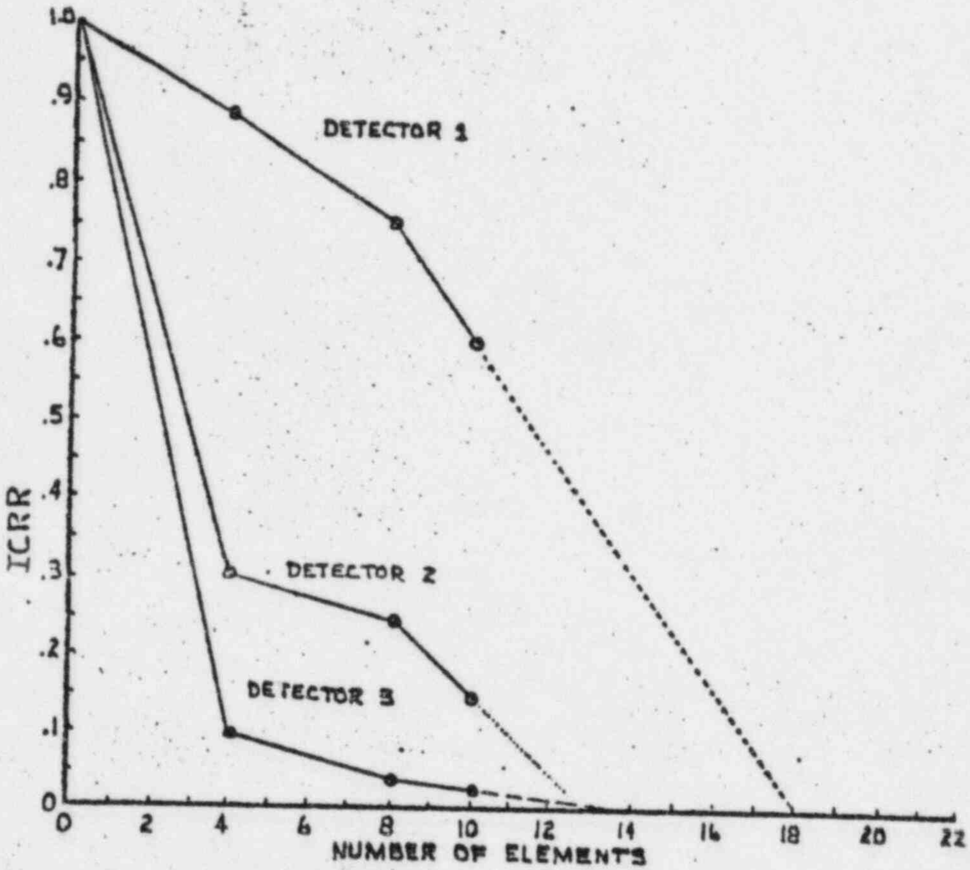

Figure 12-3: ICRR Curve for Sample Problem

increments. However, as criticality is approached and subcritical multiplication is high, the neutron population in the core is high enough that detector 1 begins to see significant changes in count rate in spite of the large attenuation. Under these conditions its ICRR curve will begin to extrapolate to the correct value.

Now let us consider detector 2. Its initial count rate is lower than detector 1's because it is located farther from the source. However it shows a fairly substantial increase in count rate when the first increment is loaded. The main reason is simply that the detector sees a much larger portion of the loaded core than did detector 1. It therefore responds more correctly to the neutron multiplication. Actually, detector 2 will overestimate the multiplication of the fuel. This is because the upper left hand corner of the loaded volume is directly in the line of sight between the source and the detector. Not only is this corner of the fuel increasing the count rate due to subcritical multiplication, but it is also displacing some of the water that the neutrons must travel through before they reach the detector. Since water moderates the neutrons, and thereby exposes them to the relatively high thermal absorption cross sections of the various reactor materials, its presence tends to attenuate the neutrons. The displacement of water therefore increases the travel length of the neutrons and would result in an increase in detector count rate even if there was no multiplication. Therefore, whenever fuel is loaded between the source and the detector, count rate will be increased by a greater factor than would be predicted simply by considering neutron multiplication. Notice that detector 2 did not respond as greatly to the second increment of fuel. This is simply because the change was made farther away from this particular detector. By the end of Increment 2, detector 2 should be reading about right since it overestimated the first step and underestimated the second. From this point on, it should reflect changes in the core with reasonable accuracy and extrapolating its curve to predict a critical loading of ≈ 13 elements is probably not too much in error.

Detector 3 was initially placed too far from the source. As a result its initial count rate was quite low and probably unreliable. In addition, the previously discussed effect of moderator displacement causes the detector to grossly overestimate the effect of the first two increments. After the third increment of fuel is added, however, this detector will begin to give fairly reliable results since it sees the core very well and the source-core-detector geometry is no longer being altered appreciably.

Although the previous example was purely fictitious, it shows that a considerable amount of interpretation must be factored into the use of an ICRR curve because of the large changes in detector efficiency brought about by shifting core-source-detector geometry throughout the loading. Linearity of the curves can be improved somewhat by averaging the various detector responses or by plotting ICRR versus geometric buckling rather than number of fuel elements. The locations of the detectors can also be changed throughout the course of the loading to keep them in favorable geometric positions. Of course, any time a detector is moved its count rate will change (because its efficiency changes) and this must be accounted for in subsequent ICRR calculations by applying a correction factor.

The two previous examples were typical of BWR loadings where it is intended to actually achieve criticality at some point in the process. As discussed previously, the core remains subcritical throughout the course of a typical PWR loading because the core is so heavily poisoned that k_∞ of the poisoned fuel is <1.0. Thus, no matter how much fuel we load, the reactor remains subcritical. Typically, in the fully loaded core, $k_{eff} \approx 0.9$. The general shape of an ICRR plot for this type of loading is shown in Figure 12-4 below. When the first bundle is loaded, k_{eff} will be relatively low (typically ≈ 0.4) because leakage is high in this single bundle core. Addition of a second bundle will double the size of the core and cause a substantial decrease in leakage. This will be reflected as a significantly higher k_{eff} and a significant decrease in the ICRR. As the loading continues, however, each bundle makes a progressively smaller percentage change in core size (addition of a single bundle to a ten bundle core increases core size by about 10%, but in a twenty bundle core it represents only about a 5% change), with a correspondingly smaller change in leakage, k_{eff} and ICRR. As a result, the ICRR curve tends to flatten out as bundles continue to be loaded, and it approaches a limiting value determined by the k_{eff} of the fully loaded core. Of course, the precise shape of the curve is also affected by source-detector-core geometry just as was the case of the BWR example considered previously.

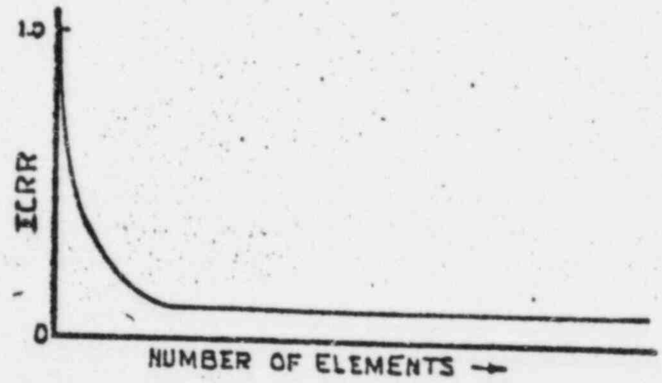

Figure 12-4: Typical ICRR Curve for the Loading of a Subcritical Reactor

Example: Figure 12-5 shows the fuel loading sequence of a small PWR which employs the early style (for PWRs) cruciform rods. Control rods were

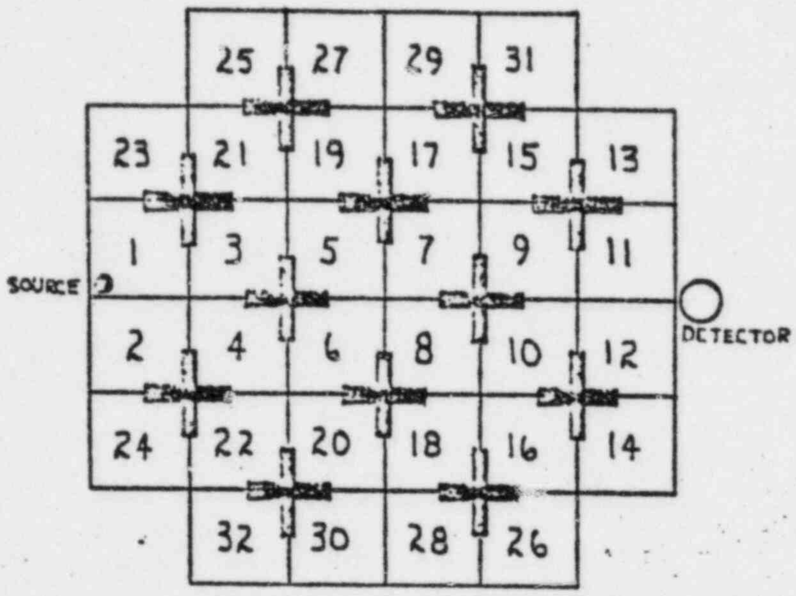

Figure 12-5: Core Loading Sequence for Small PWR

inserted in the core throughout the loading and the coolant was highly borated. The source was contained in the first fuel bundle which was loaded. The following count rate data was obtained.

Bundle	CR	Bundle	CR	Bundle	CR	Bundle	CR
1	100	9	667	17	2190	25	2120
2	154	10	870	18	2210	26	2100
3	189	11	1670	19	2270	27	2080
4	173	12	2000	20	2150	28	2120
5	370	13	2380	21	2180	29	2110
6	358	14	2450	22	2160	30	2050
7	360	15	2220	23	2130	31	2310
8	417	16	2200	24	2100	32	2400

The problem is to make two ICRR plots--one using the count rate data for the first bundle as a reference, and the second using the data for the twelfth bundle as a reference.

The two plots involved are shown in Figure 12-6. The arithmetic necessary to construct the plots will be left to the reader. Looking first at the detector response during loading of the first twelve bundles, we see a curve which is similar to that of detector 3 in the previous example. The first few fuel bundles make a substantial

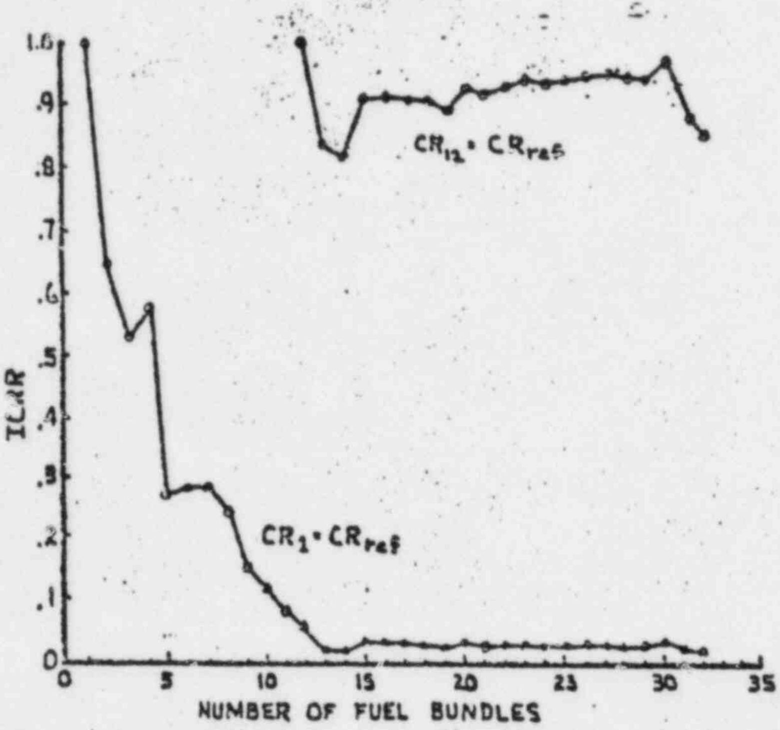

Figure 12-6: ICRR Plots for Example Problem

percentage increase in core size and k_{eff}, and should result in a significant increase in detector count rate. But in addition, the bundles are located directly in the line of sight from the source to the detector and thus the neutron attenuation between the core and the detector also decreases. In effect, the detector is moved closer to the core and becomes more efficient. This produces an increase in count rate over and above that which can be attributed to the increase in core neutron population alone. We also see the effects of the inserted control rods on detector efficiency. For example, two control rods shield the detector from the first four bundles. However, the fifth bundle is only shielded from the detector by one control rod. Thus, loading the fifth bundle produces a substantial increase in count rate. This same effect is also observed when the ninth bundle is loaded.

After twelve bundles have been loaded, the source-core-detector geometry is reasonably constant. In addition, the core is now large enough so that leakage is fairly small already and the loading of additional fuel does not make a very large increase in k_{eff}. Thus the ICRR curve tends to level out. However, the curve using the first bundle as a reference is now so close to zero that it is a little difficult to really spot any trend in the data. Since the loading of the twelfth bundle represents a reasonable stabilization of the geometry, the data for this bundle is a good choice for a new reference. This second curve once again starts at an ICRR of 1.0. This shifts the curve upward and gets it out of the mud, thereby making trends somewhat easier to see. Of course, if criticality is approached, this second curve must also approach zero, just as will the original curve. Adopting new reference count rates from time to time during loading is a common practice in order to make the data easier to interpret.

APPROACH TO CRITICAL IN THE LOADED CORE

After the core is fully loaded, it is brought to critical by adjusting its various control mechanisms. In a BWR, this is accomplished by withdrawal of individual control rods in a prescribed sequence until criticality is reached. In a PWR, both control rods and chemical shim concentration must be adjusted. The general procedure is to first withdraw a group of rods called the shutdown bank,[1] and then follow with a boron dilution[2]. Assuming that the reactor had been borated to a refueling concentration prior to the startup, k_{eff} would typically be ≈ 0.9 with all rods inserted. The shutdown bank is generally worth $\approx 2\%\ \Delta k/k$, so $k_{eff} \approx 0.92$ when it is withdrawn. The dilution would be planned to add $\approx 5\%\ \Delta k/k$, or to bring k_{eff} up to ≈ 0.97. Criticality would then be achieved by the withdrawal of additional control rods.

Startup of a fully loaded reactor by boron dilution and/or rod withdrawal is governed by the same rules as startup by loading fuel.. The only real difference is that k_{eff} is changed in the former by varying thermal utilization, whereas in the latter it is changed by reducing leakage. As chemical shim is removed or control rods are withdrawn, the count rate on the out-of-core nuclear instrumentation will increase due to the increase in subcritical multiplication. Equations (12-3) and (12-5) apply to this process, and if we can assume constant detector geometry, equations (12-4) and (12-6) also apply. The latter assumption is a very good one during boron dilutions since the reactivity change is being made uniformly throughout the core. On the other hand, the instrument response to rod

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[1] The PWR control rod system is designed so that groups of rods are withdrawn simultaneously instead of individually as they are in a BWR. Group sizes range from 4 to 8 rods.

[2] The shutdown bank must always be withdrawn prior to initiating the dilution. This is to provide the capability of making a rapid negative reactivity insertion in case the dilution is inadvertently carried too far and criticality is approached.

withdrawal is strongly influenced by the source-rod-detector geometry and so the use of equations (12-4) and (12-6) may give erroneous results.

Example 1: In a PWR with all rods inserted and sufficient chemical shim to make $k_{eff} = 0.9$, the out-of-core neutron detectors read 50 cps. The shutdown bank, consisting of 8 control rods worth a total of 2.1% Δk, are withdrawn as the first step in an approach to critical. Estimate the readings on the detectors when these rods are fully withdrawn.

The problem does not state whether or not the rods are located in such a manner that detector geometry is strongly influunced by their movement. In the absence of such information we can only assume that detector efficiency is constant. If this is true, we can get the required answer from either equation (12-4) or equation (12-6). The obvious reference conditions are $CR_{ref} = 50$ and $k_{eff,ref} = 0.9$. To determine the count rate after the rods are withdrawn, it is necessary to know k_{eff} after the rods are withdrawn. Since the worth of the shutdown bank was given, we can determine that after the bank is withdrawn, $k_{eff,sd} = 0.900 + 0.021 = 0.921$. Thus:

$$\frac{50}{CR} = \frac{(1 - 0.921)}{(1 - 0.900)} = \frac{0.079}{0.100} = 0.79$$

$$CR = \frac{50}{0.79} = 63 \text{ cps}$$

Example 2: A dilution was then begun in the above reactor and was continued until the count rate was 200 cps. What was k_{eff} at the completion of the dilution? What was the reactivity worth of the boron which was removed from the core?

We must again use equation (12-4) or (12-6) to solve this problem. For the reference conditions we can use either CR = 50, $k_{eff} = 0.9$ or CR = 63, $k_{eff} = 0.921$. We will arbitrarily use the original values (however the student should also work the problem with the new reference conditions in order to verify that it gives the same result) so that:

$$\frac{50}{200} = \frac{(1 - k_{eff,dilute})}{(1 - 0.900)}$$

$$0.25 = \frac{(1 - k_{eff,dilute})}{0.100}$$

$$(1 - k_{eff,dilute}) = 0.025$$

$$k_{eff,dilute} = 1 - 0.025 = 0.975$$

To answer the second part of the question, we must determine the reactivity before and after the dilution. Thus:

$$\rho_{before} = \frac{1 - 0.921}{0.921} = -0.0859$$

$$\rho_{after} = \frac{1 - 0.975}{0.975} = -0.0257$$

Thus the reactivity worth of the boron removed from the core was:

$$\Delta\rho = \rho_{after} - \rho_{before} = -0.0257 - (-0.0859) = 0.0602$$

$$= 6.02\% \ \Delta k/k$$

Example 3: After completion of the above dilution, control rods were pulled in increments of .003Δk until the reactor was supercritical. After each increment of control rod, the rod motion was halted for a sufficient time to let the count rate reach its steady state value. Determine the steady state value of count rate after each increment of rod withdrawal.

The solution of this problem is completely analogous to the solution of the first example. The reference figures can now be the k_{eff} and CR values before rod withdrawal, after the shutdown bank is withdrawn, or after the dilution. The calculations will be left to the student, but the results are:

Increment	k_{eff}, after Increment	Count Rate after Increment
0	0.975	200
1	0.978	227
2	0.981	263
3	0.984	313
4	0.987	385
5	0.990	500
6	0.993	714
7	0.996	1250
8	0.999	5000
9	1.002	Continuous Increase

In the previous three examples, we have calculated the ideal response of an out-of-core detector during a typical approach to criticality of a PWR. The results are idealized because they assume constant detector efficiency throughout the entire process, and in general this will not be the case. Later in this section we will consider some of the factors which affect the data, but for the moment we will ignore them. Figure 12-7 is a sketch of the data from the previous examples as it might appear to the operator on a strip chart recorder. The most obvious feature of the curve is that the neutron flux responds quickly to each increment of rod withdrawal, whereas there is a gradual increase throughout the dilution process. This is to be expected since the reactivity increase brought about by rod withdrawal approximates a step change[1] whereas the reactivity

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[1] Actually, it takes a minimum of ≈ 3.5 minutes to completely withdraw

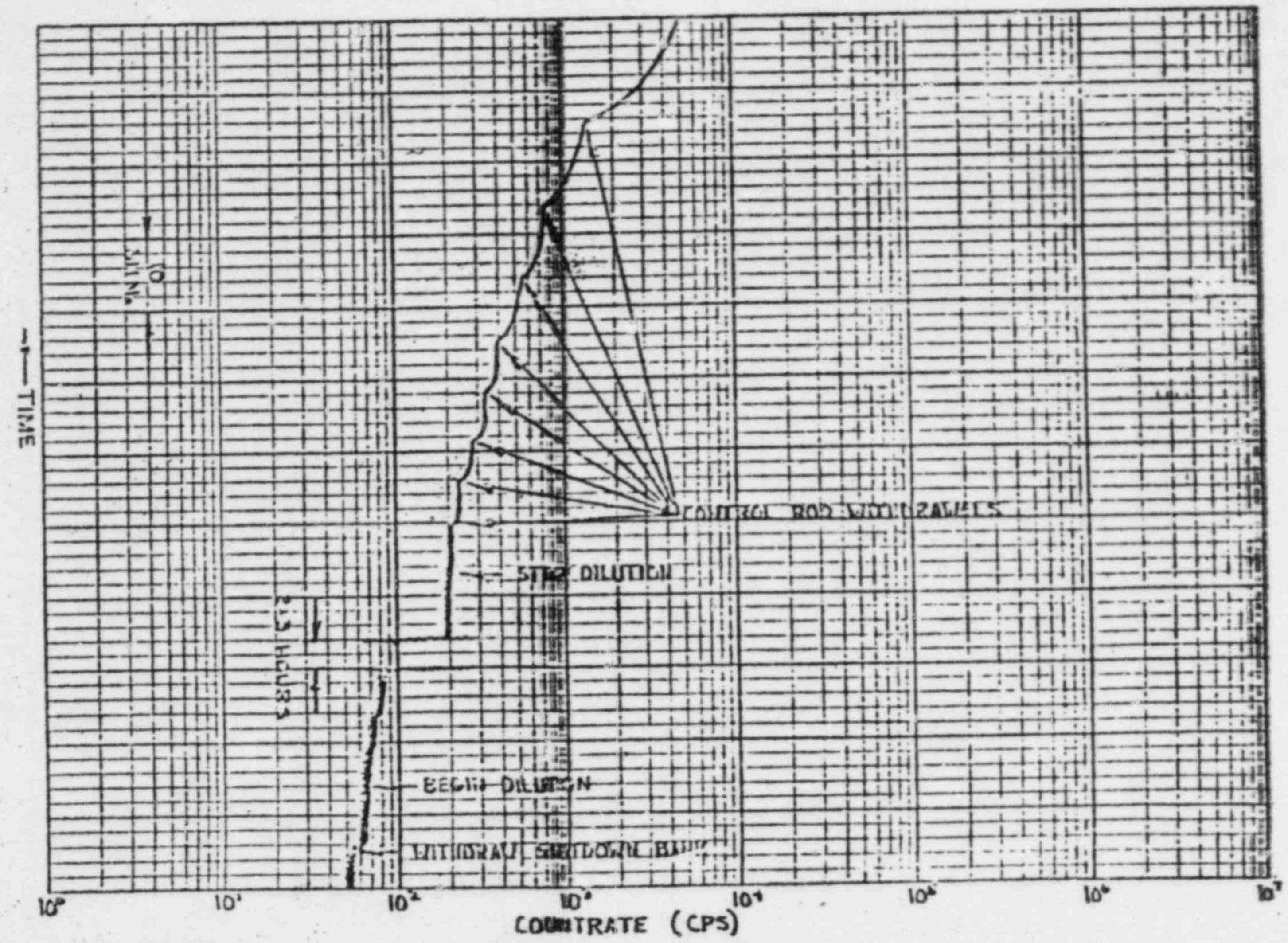

Figure 12-7: Idealized Out-of-Core Detector Response During Approach to Critical on a PWR

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increase during dilution takes place very slowly.

Now consider the response of the instruments to the equally sized reactivity additions which take place after the dilution. Notice that the increase in count rate gets larger and larger as the reactor gets closer to critical. This is simply a reflection of the fact that the CRR approaches ∞ as criticality is neared. In addition, it takes longer and longer for the count rate to reach the equilibrium value. This is particularly noticeable in the last two or three increments where it takes several minutes to level off even though the control rod movement may have taken less than a minute.

In theory, the lengthening of the equilibration time would occur in a reactor even if all neutrons were prompt. This point is illustrated in the discussion of subcritical multiplication in Chapter 10 (pgs 10-32 ff.). In this discussion, where delayed neutrons had not yet been considered, it was seen that for the case where $k_{eff} = 0.5$ (Figure 10-17) it took the neutron population about eight generations to reach a near equilibrium total (although in theory it is never reached). By constructing a similar table for a higher value of k_{eff}, say 0.9, the student can verify that considerably more than eight generations will be required. The equilibrium neutron population in this case would be 10×10^6 for $k_{eff} = 0.9$[1], but the population after eight generations would only be $\approx 5.7 \times 10^6$, or a little more than half way. However, if all neutrons were prompt, it would only take a short period of time to equilibrate even though many thousands of generations were required because the prompt neutron generation time is so short. If all neutrons were prompt, therefore, it is doubtful whether we would notice this effect even though it is occurring.

What really stretches out the observed equilibration time in an actual reactor is the presence of delayed neutrons. To illustrate this we will consider a hypothetical reactor in which we can arbitrarily turn the delayed neutrons on and off. Whenever the delayed neutrons are turned off, k_{eff} is reduced to 99.3% of its true value because we are eliminating 0.7% of the neutrons which are produced in each generation. Assuming a source strength of 10^6 neutrons per generation, we will calculate the equilibrium neutron population for true (delayed neutrons turned on) k_{eff} values of 0.95 and 0.999 and for these same cases without delayed neutrons (in which case the k_{eff} values are 0.943 and 0.992 respectively). The calculations are made using equation (10-25).

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a bank of rods in a large PWR. The addition of 0.3% Δk/k would generally only require the withdrawal of part of a bank and it may take anywhere from ≈ 10 seconds to well over a minute depending upon how many rods are in the bank and the axial location of the bank.

[1] Because $P = S/(1 - k_{eff}) = 10^6/(1 - 0.9) = 10 \times 10^6$

	$k_{eff} = 0.95$	$k_{eff} = 0.999$
Equil. pop. when all neuts are considered	20,000,000	1,000,000,000
Equil. pop. assuming no delayed neuts appear	17,500,000	125,000,000

Now let us consider what we would see when we quickly bring k_{eff} to 0.95. Within a very short period of time the prompt neutrons would build up to a near equilibrium population of 17,500,000 neutrons. Then there would be a slower increase from 17,500,000 to 20,000,000 as the delayed neutrons gradually build into the core. In effect, we see a prompt jump that takes us 87.5% of the way to equilibrium, which when coupled with the rapid buildup of the short half-life delayed groups means that we reach a near equilibrium population in a very short time. On the other hand, if we had quickly brought k_{eff} to 0.999, our prompt jump would only take us 12.5% of the way to equilibrium. During the period that the delayed neutrons were building in, the count rate would increase 800%. Thus, a considerable time passes before the count rate will stop rising.

This characteristic of taking longer and longer for the count rate to equilibrate when reactivity additions are made to a subcritical core is one of the surest signs available to the operator that criticality is being neared.

There is no reason why an ICRR plot could not be plotted during the startup of a fully loaded reactor. During a boron dilution process, in fact, the ICRR data is very good because of the constant detection geometry. During rod withdrawals, however, the data is frequently not very good because of geometry problems. One of the major difficulties is involved in picking a quantity against which ICRR can be plotted. The logical choice is against number of control rods withdrawn. However, since the reactivity strength of different control rods varies considerably, the k_{eff} of the core depends strongly upon which control rods are withdrawn as well as simply how many control rods are withdrawn. This throws an additional complicating factor into the plot.

The difficulties associated with an ICRR plot can best be seen by considering the data from a typical BWR startup. The Humboldt Bay core is shown in Figure 12-8. Although there are several out-of-core neutron detectors which

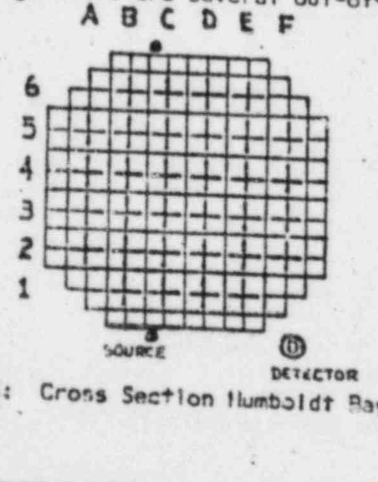

Figure 12-8: Cross Section Humboldt Bay BWR Core

are employed in this reactor, we will look at the response of just one, which is located adjacent to the southeast corner of the core as shown in the figure. The k_{eff} of the fully controlled core is not precisely known, although it is probably ≈ 0.9. The sequence of rod withdrawal, detector response, and ICRR are tabulated below.

Rod Position	Total Number of Rods Withdrawn	Detector cps	ICRR
All rods in	0	40	1.000
C-3 fully withdrawn	1.00	40	1.000
D-4 fully withdrawn	2.00	40	1.000
C-1 fully withdrawn	3.00	50	0.800
D-6 fully withdrawn	4.00	50	0.800
F-3 fully withdrawn	5.00	60	0.667
A-4 fully withdrawn	6.00	60	0.667
F-5 withdrawn 58%	6.58	60	0.667
A-2 withdrawn 58%	7.16	60	0.667
E-1 withdrawn 58%	7.74	75	0.534
B-6 fully withdrawn	8.74	85	0.470
E-1 fully withdrawn	9.16	90	0.444
A-2 fully withdrawn	9.58	90	0.444
F-5 fully withdrawn	10.00	90	0.444
C-5 withdrawn 19%	10.19	140	0.286
D-2 withdrawn 19%	10.38	150	0.267
B-3 withdrawn 19%	10.57	200	0.200
E-4 withdrawn 19%	10.76	300	0.134
D-2, B-3, E-4 withdrawn 23%	10.88	Slightly supercritical	

Figure 12-9 shows an ICRR plot for this data. As can be seen, extrapolation of the curve to estimate a critical rod pattern is virtually impossible until we are within about one rod of reaching criticality. The first two rods were pulled from the center of the core and since the core is still considerably subcritical at this point (probably ≈ 2% Δk/k), and since the increase in total core neutron population brought about by the withdrawal of these rods is concentrated near the center of the core, the detector does not "see" these rods. On the other hand, rod C-1 does produce an increase in count rate because it is located on the south edge of the core near the detector and because there is a neutron source also located in the vicinity. The source is actually located at the axial midplane of the core, and the majority of the increase in count rate caused by rod C-1 occurs when the upper tip of the blade moves past the midplane, thereby "uncovering" the source. A similar situation occurs when rod E-1 is withdrawn. Notice that the count rate increases more when E-1 is first withdrawn 58% than it does when E-1 is withdrawn the remaining 42% even though the core is somewhat closer to critical during the second increment. Of course, as criticality is neared, the neutron population in the core gets quite high and the population increases as increments of rod are withdrawn become large enough that the detectors respond to all rod movement. By this time, however, a competent operator would be well aware that the reactor was nearly critical simply by observing the nuclear

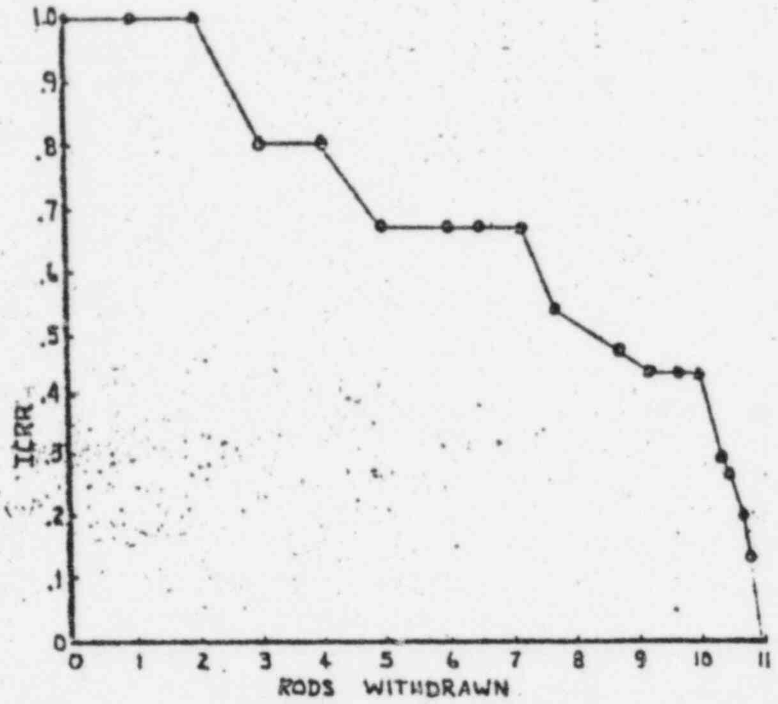

Figure 12-9: Out-of-Core Detector Response for Typical Approach to Critical at Humboldt Bay by Rod Withdrawal

instrumentation. In the opinion of the author, it is a waste of time to make plots like this during approaches to critical with standard rod patterns.

Figure 12-10 shows one final example of an actual approach to critical. This rocorder trace was obtained at a 500 MWE PWR during a rapid return to critical following a reactor trip from high power. In this case, the chemical shim concentration was already adjusted to the proper operating concentration so dilution was not necessary. The rods are being manually withdrawn in banks. As is typical of a PWR, there is an overlapping of banks at the ends (except for the shutdown bank). That is, bank B begins to withdraw simultaneous with bank A when bank A is ≈ 75% withdrawn. There are three interesting things to note in the figure. First, there is a substantial increase in count rate as bank A uncovers the source. Secondly, notice the prompt jump when the reactor was put on the 0.5 DPM SUR. And third, notice how quickly the doppler coefficient stopped the power rise.

CONTROL ROD WORTH CHARACTERISTICS OF BWRS

In the BWR the reactivity worth of control rods is relatively high and there exists a potential for undesirably large reactivity insertions caused by misoperation of the system which could result in large power

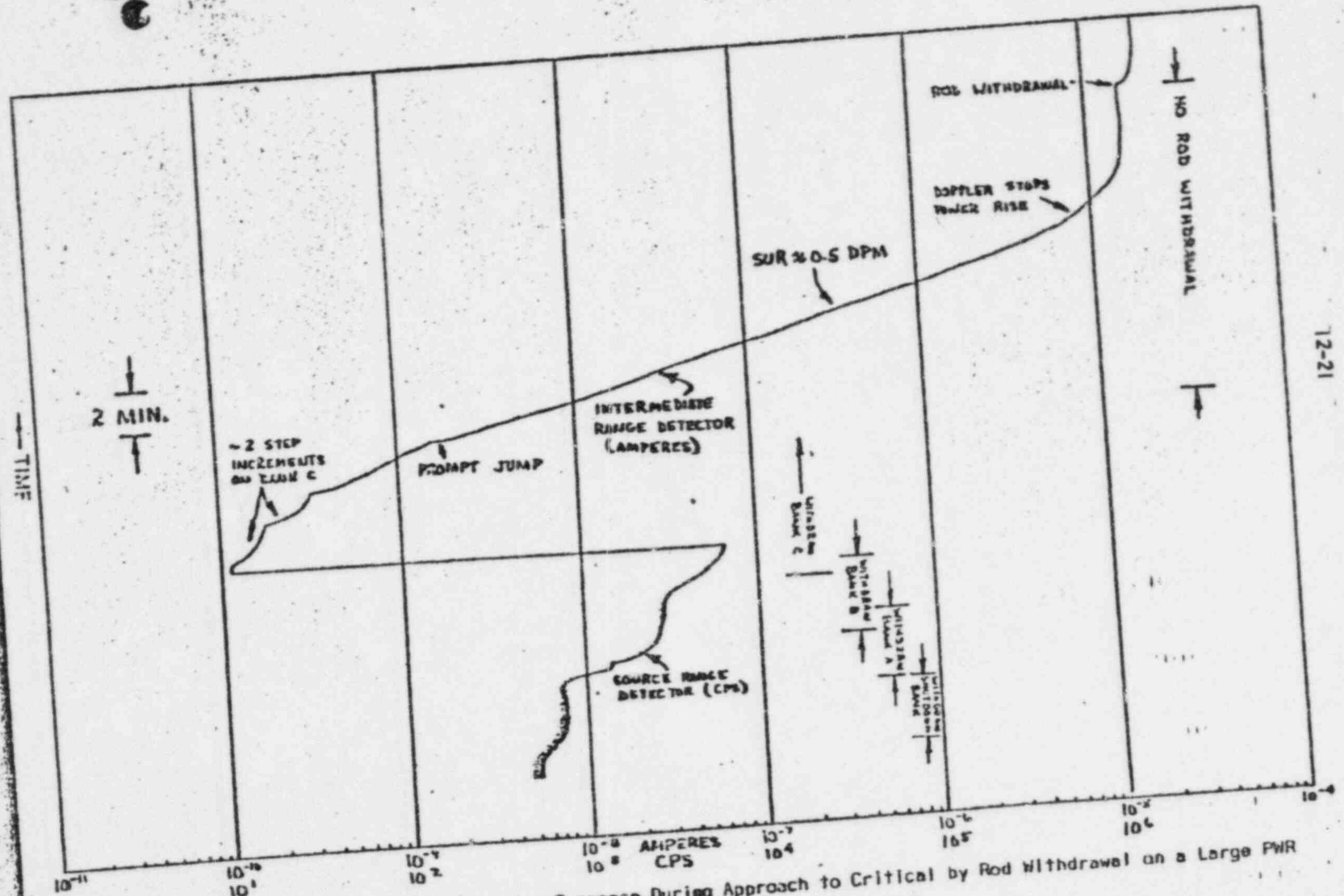

Figure 12-10: Actual Out-of-Core Detector Response During Approach to Critical by Rod Withdrawal on a Large PWR

transients. The potential is particularly great during an approach to critical when the core is at cold, zero power conditions and the reactivity coefficients have not yet come into play. Of course, measures are taken to avoid this type of incident which include equipment design features as well as operating procedures, but in the final analysis the best safeguard is an alert operator who thoroughly understands the significance of the equipment he is using.

In Chapter 10, we developed a relationship which showed some of the factors which influence the strength of control rods. In this chapter we will apply this relationship to some practical situations to better illustrate its full significance. In addition we will discuss some of the criteria which are used to develop acceptable control rod withdrawal sequences. Although the discussion will be directed principally toward BWRs, the physical principles involved apply equally well to PWRs.

In Chapter 10, we stated that control rod worth was dependent upon the expression:

$$\text{Rod worth } (\Delta k/k) = L_t \times \frac{S_{cr}}{V_{core}} \times \left[\frac{\phi_{t,cr}}{\phi_{t,core}}\right]^2$$

We used this expression to show that the total control system strength of a BWR such as Humboldt Bay might be = 15% Δk/k (and in newer designs ranges up to = 20% Δk/k). The number of control rods typically ranges from = 30 in small cores like Humboldt Bay to = 130 in large cores. Thus the strength of an "average" rod might run from = 0.2-0.5% Δk/k[1]. It is found, however, that the worth of any particular rod is extremely sensitive to the geometric arrangement of other control rods in the core, and that in certain configurations a particular rod may exceed the average rod worth by a factor of 10 to 20. It is important that the operator be able to recognize patterns which can produce these high rod worths.

To illustrate this behavior, consider the first part of the startup of a typical BWR. Figure 12-11 shows a schematic of the core cross section. In a normal startup sequence, the rods are categorized into groups (or banks in PWR terminology). The groups are withdrawn in sequence, and within each group, rods are withdrawn individually. Two such rod groups are illustrated on the figure--the five rods enclosed in solid boxes being group A and the four rods in the dashed boxes being group B. Assume the core is freshly loaded and all rods are inserted. Under these conditions, the core is ordinarily subcritical by = 4-5% Δk/k (i.e., k_{eff} = 0.94-0.95). Now assume that the center rod is withdrawn. With this rod completely out of the core, the core will still shutdown by = 1% Δk/k (i.e., k_{eff} = 0.99).[2]

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[1] Determined by dividing total control strength by the number of control rods. That is: 15% Δk/k/32 rods = 0.5% Δk/k/rod.

[2] All reactors are designed in this manner to insure that the core can still be shutdown if a rod gets stuck in the fully withdrawn position.

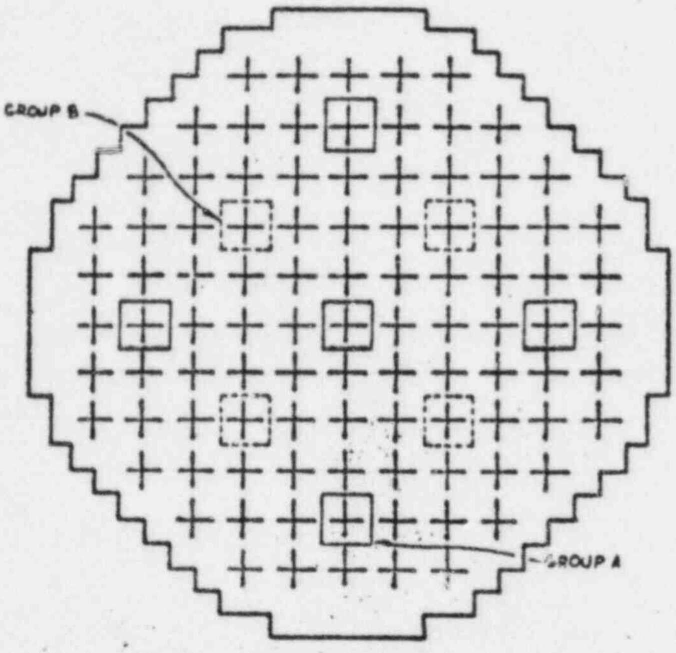

Figure 12-11: Beginning of Rod Withdrawal Sequence In Typical BWR

The withdrawal of this rod has brought the reactor from $k_{eff} = 0.94-0.95$ to $k_{eff} = 0.99$, and thus it is worth ≈ 3-4% $\Delta k/k$, which is 10 to 20 times the "average" rod worth. The large worth of this rod is of little concern, however, since the reactor remains subcritical. Nevertheless, it is of interest to try and explain the worth of this rod on the basis of our previous expression. The key involves the size of the core when the rod is withdrawn. What we really have in this case is a four bundle reactor surrounded by a reflector composed of fully controlled fuel. The effective volume of the core is thus very small, and equals the true volume of four fuel bundles plus an appropriate volume of reflector. The control rod extends into virtually every region of this effective core, thereby making the ratio S_{cr}/V_{core} very high and the rod very strong.

As the second rod in group A is removed, experience shows that the reactor will still remain subcritical. Therefore, the worth of the second rod must be less than 1% $\Delta k/k$. The reason is that the second rod is for removed from the first rod. The effective core size now comprises the eight bundles surrounding the two withdrawn rods, plus twelve controlled bundles between the two rods, plus an appropriate amount of reflector. Since the two rods are the same physical size, but the first was withdrawn from a core of 4+ bundles, whereas the second was withdrawn from a core of 20+ bundles, the second is worth less than 20% (4/20) as much as the first rod. When the third rod is pulled, the effective core size increases in such a manner as to include the twelve bundles surrounding the three withdrawn rods, plus the bundles in controlled cells within the triangular region bounded by the three rods, plus an appropriate amount of reflector. Thus the worth of this third rod is very low. By the time the last rod in group A is pulled, the effective core size has increased to a point where it includes essentially the entire core and the worth of this last rod is very low.

This behavior illustrates one important consideration in the development of a rod withdrawal sequence. The first few rods which are withdrawn should be scattered throughout the core so that the effective core size approaches the true core size early in the sequence. In this way, the worth of the rod which takes the reactor critical is minimized because it is only controlling a small fraction of the total effective core volume.

In an actual reactor it takes from 10 to 20 withdrawn rods to go critical in a normal scattered pattern. For purposes of illustration, however, assume that the reactor is just subcritical with the five group A rods out. Now assume the withdrawal of one rod in group B. Since this is the rod which takes the reactor critical, its worth has some practical importance. Because the location of this rod is within the region of the effective core which was formed during the withdrawal of group A, the ratio of S_{cr}/V_{core} is about the same as that for the last group A rod. However, although the worth of the first group B rod is small (generally <1% $\Delta k/k$), it is worth somewhat more than the last group A rod. The reason for this is that when the B rod is withdrawn, the core flux distribution will shift rather strongly to its vicinity. Thus $(\phi_{T,cr}/\phi_{T,core})^2$ will be high for this rod. The second group B rod which is withdrawn would usually be diagonally across from the first rod. Normally, we would not be able to immediately withdraw this second rod because the reactor period would be too short.[1] However, presuming we could withdraw it, its worth would be somewhat less than the first rod in the group because it tends to equalize the flux distribution across the core. In other words, the flux would not be peaked as strongly around the second rod as it was around the first rod, but would be more evenly divided between the two. Thus, $(\phi_{T,cr}/\phi_{T,core})^2$ is lower for the second rod than for the first.

Figure 12-12 is a qualitative graph of the rod worth behavior as we pull a typical scattered rod pattern such as that discussed above. Within the first groups we reduce the worth of each succeeding rod because we increase the effective core size. After the effective core size has been stabilized, we continue to see a similar shaped curve because we alternatively peak the flux around the first rod in a group and then spread it throughout the core as succeeding rods in the group are withdrawn.

Before leaving the topic of rod worths during approaches to critical, it is of interest to illustrate examples of undesirable rod patterns. In

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[1] It would be gradually withdrawn to overcome the effects of reactor power escalation and heating.

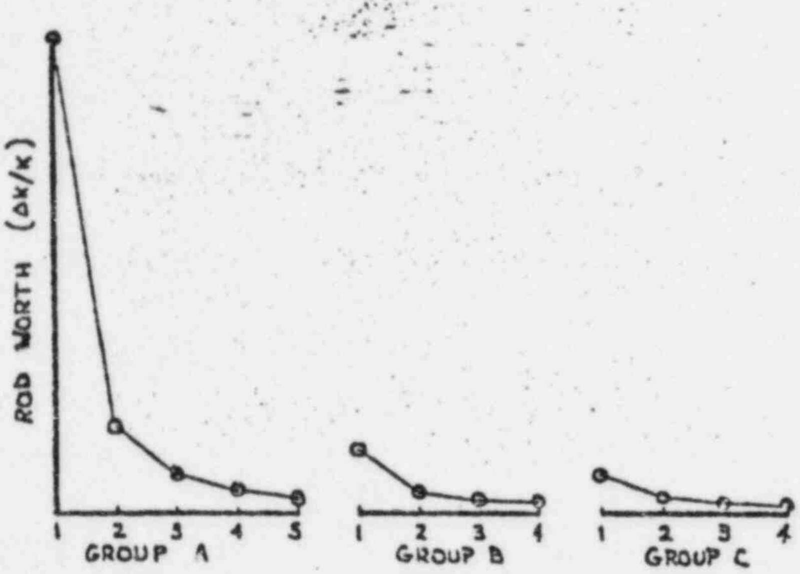

Figure 12-12: Typical Rod Worths During Scattered Withdrawal Pattern

general, these are patterns which confine the effective critical volume to a small region of the core, thereby resulting in high rod worths. For example, consider the two rod clump pattern shown in Figure 12-13. We have already seen that the first rod is worth ≈ 4% Δk/k, although this high worth is of little importance since the reactor cannot go critical with only one rod withdrawn. In many cases, however, the reactor can go critical in the cold condition with a two rod clump. Since the second rod essentially doubles the size of the core, it should be worth ≈ 50% as much as the first rod, or ≈2% Δk/k. This is a very strong rod to go critical on, and the core could end up highly supercritical if it were fully withdrawn.

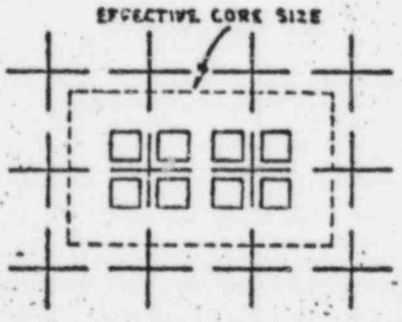

Figure 12-13: Two Rod Clump Control Rod Pattern

There is one other condition which is extremely important from the standpoint of control rod worths. This is the hot, standby condition where a BWR is shut down with all rods inserted, but is at rated temperature and pressure. Such a condition exists immediately following a scram from full power. It is occasionally necessary to start the reactor up from this condition following spurious scrams from faulty instruments, etc. The effects of the negative temperature coefficient mean that it will be possible to withdraw several more rods before reaching critical than was possible in the cold reactor. Usually, it will be possible to withdraw something equivalent to the "rod cross" shown in Figure 12-14 and still have the reactor be slightly subcritical. In this case, the rod left in the center of cross will be extremely strong and its complete withdrawal may place the reactor in a highly supercritical condition resulting in severe damage to the core and the reactor vessel[1]. The reason that this rod has a high worth can be seen by considering the following factors. When the rod is inserted, the "effective" core consists of not one, but four little four-bundle reactors which are essentially unconnected because neutrons from one cannot travel through the inserted blade to intermingle with neutrons from another (there is a small amount of coupling on the diagonals, but this is weak). Normally, when a control rod is withdrawn, it adds the four fuel bundles which surround it to the effective size of the core. But in this case, if we look at one of the small four bundle reactors, the withdrawal of the control rod adds not four bundles to it, but sixteen. In other words, the withdrawal of one control rod transformed

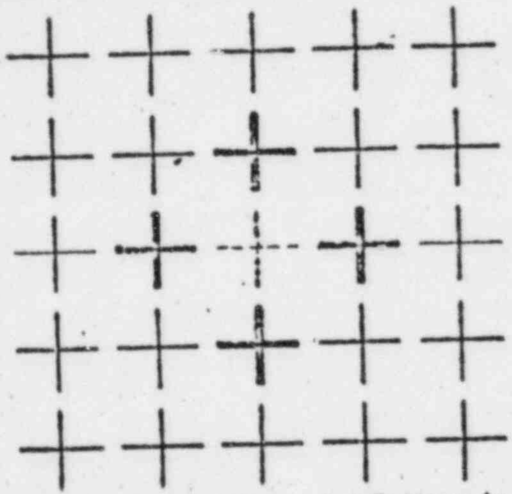

Figure 12-14: Five Rod Cross Rod Pattern in a BWR

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[1] The vessel can be severely damaged by shock waves generated in the water following a severe power excursion.

a four-element reactor into a twenty-element reactor and for all intents and purposes, the size of this rod is much bigger than its actual physical size would suggest. Also, it should be noted that when the rod is withdrawn, the flux will be peaked rather strongly in the center of this twenty-element reactor, and $(\phi_{T,cr}/\phi_{T,core})^2$ will be high. Finally, the fact that the reactor is hot increases L and further enhances rod worth. A rod such as this may be worth 4-5% Δk/k as compared to typical rod worths of 0.5-1.0% Δk/k in the normal full power rod pattern and 2-3% Δk/k in the cold two rod clump. Finally, the reactor is very close to critical when this rod is withdrawn. Thus nearly the entire worth of the rod goes towards making the core supercritical. In the two rod clump, the core was still at least 1% Δk/k shutdown prior to withdrawing the second rod. Thus, half of its worth is required just to get critical and k_{eff} is much lower when it is fully withdrawn.

POWER ESCALATION

When the reactor achieves criticality, its power level is very low--typically less than a watt.[1] As a result, the fuel temperature is essentially equal to the bulk coolant temperature.[2] After the reactor is brought critical, it is usually placed on a period of ≈ 40-60 seconds (0.65 and 0.43 DPM respectively) by appropriate rod withdrawal beyond the just critical point. With no further rod motion, the power will rise several decades--typically to ≈ 2-5% power depending upon the size of the core, diameter of fuel rods, magnitude of the fuel temperature coefficient, and how far the reactor was supercritical--and then level off. This behavior results from the fact that when power gets high enough, the fuel temperature begins to rise significantly, thereby causing a reduction in k_{eff} due to the doppler effect. When k_{eff} reaches 1.0, the power rise will be terminated. It is important to stress that it is the fuel temperature increase which initially acts to stop the power increase. Changes in moderator temperature, if they occur at all, will require that the reactor remain at an elevated power for a measurable length of time. The termination of the initial power rise by the fuel temperature coefficient was illustrated on Figure 12-10. It should be noted that once the power has leveled off, it will remain at this level indefinitely as long as there is no change in rod position or moderator conditions. If power starts to fall, the fuel temperature will drop and cause k_{eff} to exceed 1.0 long enough to return the power back to the equilibrium point, and vice versa. It should also be noted that once the coefficients come into play, it is necessary to withdraw rods (or deborate) in order to make further power increases.

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[1] We are talking now of heat produced from fission. There may be some decay heat if the fuel contains a significant inventory of fission products.

[2] This can be verified by considering equations (11-16) or (11-18). Since the reactor is producing no heat, $\dot{Q}/A = 0$, and $\Delta T = 0$. That is, no temperature driving force is required since no heat is flowing.

In a BWR plant, the reactor is generally brought critical at or near room temperature, and nuclear heating is used to achieve rated coolant temperature and pressure.[1] Therefore, after criticality is achieved the power is raised to a level called "heating power", which is typically ≈ 10% of full power. The instrument trace for this process would look like Figure 12-10, and several increments of rod would have to be withdrawn to overcome to fuel temperature increase that occurs in reaching heating power. Once heating power is achieved, the moderator temperature begins to increase at a slow, steady rate. The vessel is bottled up during the heating, so there is no boiling in the coolant. Instead, the increase in moderator temperature is accompanied by an appropriate increase in system pressure. Throughout the heating process, reactor power is held virtually constant. In spite of this, however, it will be necessary to withdraw control rods from time to time. This results from the fact that the continuously rising moderator temperature acts to try and shut the reactor down. In other words, rods must be withdrawn in order to overcome the moderator temperature coefficient (assuming it is negative).[2] A typical recorder trace during a BWR heatup is shown in Figure 12-15. The thing to note is that once heating power is reached, the power does not level off after each increment of rod withdrawal as it did during the approach to heating power because the moderator temperature is continuously rising since we are adding heat to the coolant but are not yet removing any significant amount. In other words heat addition and removal are not balanced.

Figure 12-16 shows a BWR heatup for the undesirable case of a positive temperature coefficient. Note that rods must be periodically inserted to keep the reactor under control.

Once the BWR is heated to rated conditions, a flow of steam is started to the turbine and pressure is held essentially constant. If additional rods are withdrawn, the power will start to rise and then level off due to the formation of moderator voids and the increase in fuel temperature. The power will stay leveled off after each rod withdrawal increment and will not fall off again as it did during the heatup. This is because the system is no longer bottled up and so an increase in coolant heat addition (reactor power) results in a corresponding increase in heat removal (steam flow to turbine) and a balance is achieved which prevents a continual increase in moderator void content. Recall that this balance did not occur in the heatup phase and that moderator temperature continued to increase even when the power was leveled off. Figure 12-17 shows the trace which would accompany a power increase by rod withdrawal in a BWR.

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[1] Assuming a normal "cold" startup. Occasionally the reactor will be brought critical at an elevated temperature and pressure during the recovery from a plant trip. This is a so-called "hot" startup.

[2] Actually, during the heating process the fuel temperature is rising

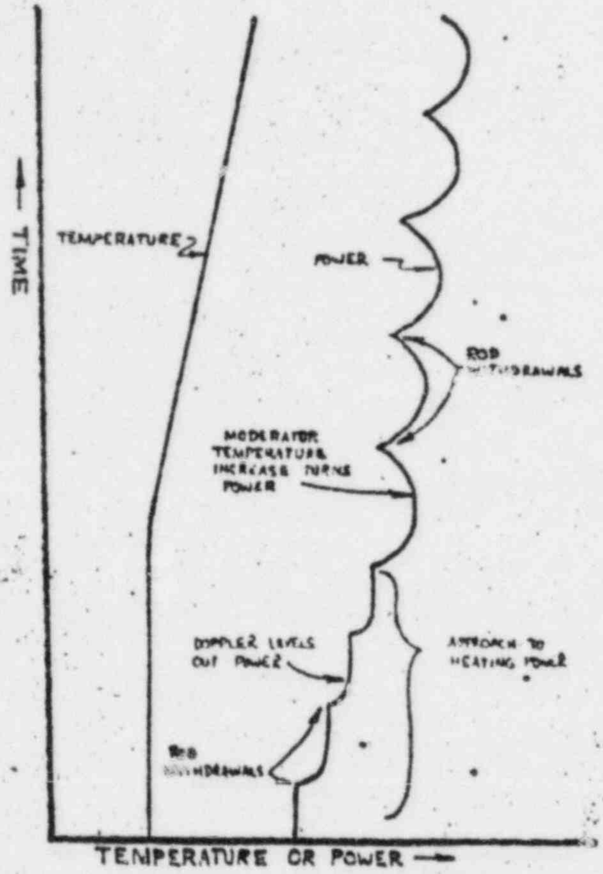

Figure 12-15: Reactor Power and Moderator Temperature Traces Obtained During BWR Heatup with Negative Moderator Temperature Coefficient

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right along with the moderator temperature even though we are maintaining essentially constant power. This can be verified by inspection of equation (11-10), which says that if we are transferring a fixed quantity of heat to increasingly hotter water, the fuel temperature must also rise to maintain the necessary driving force. Thus during heatup, we lose some reactivity due to the doppler effect as well as the moderator temperature coefficient.

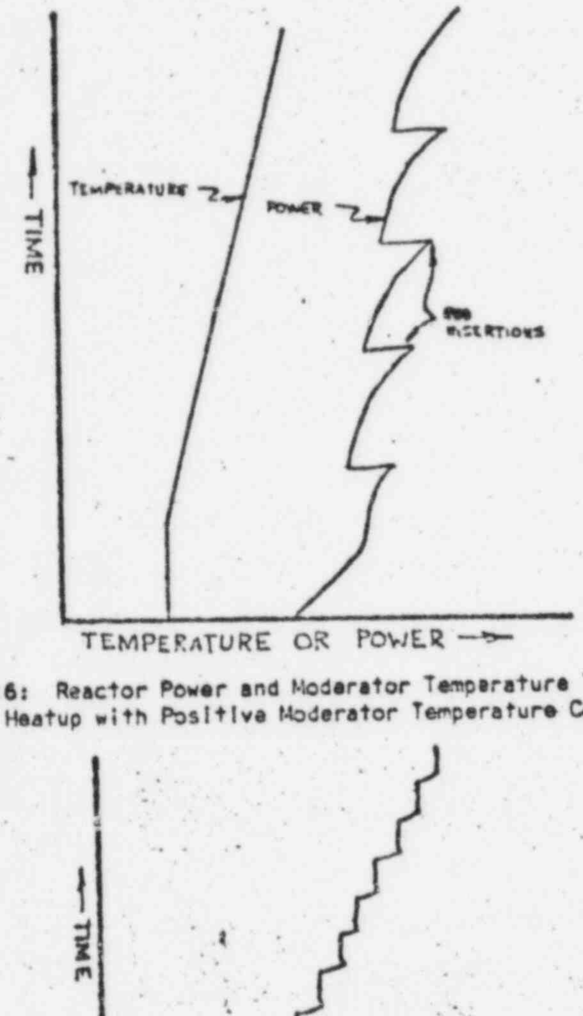

Figure 12-16: Reactor Power and Moderator Temperature Traces Obtained During BWR Heatup with Positive Moderator Temperature Coefficient

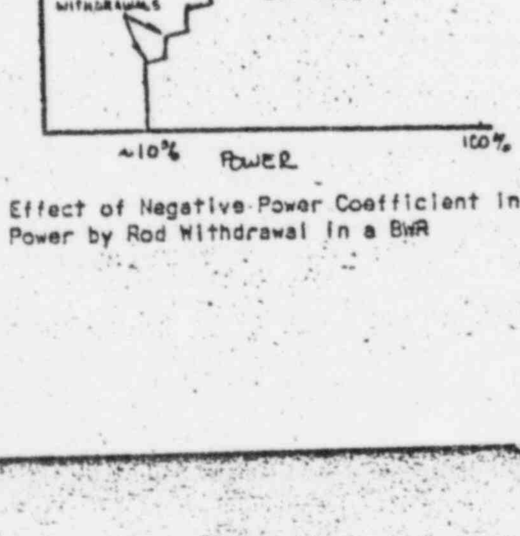

Figure 12-17: Effect of Negative Power Coefficient in Going from Heating Power to Rated Power by Rod Withdrawal in a BWR

A PWR is ordinarily heated to rated conditions with coolant pump friction heat before the reactor is brought critical. Therefore, the heating phase is not necessary. Instead, we can go directly to full power providing the secondary system has been warmed up. The power escalation will require rod withdrawal to overcome the doppler effect and a rather small amount of moderator temperature increase.[1] The power trace during the escalation will look similar to Figure 12-17, except that the power rise from each increment of rod withdrawal (or turbine load increase, see next section) will be terminated by doppler and moderator temperature rather than doppler and voids as would be the case in a BWR.

OPERATING CHARACTERISTICS OF A PWR AT HIGH POWER

In Chapter 9 we looked briefly at the type of control systems employed on PWRs and made the following general observations:

1. Load changes are made by changing the position of the turbine control valves.
2. The automatic control rod system then acts to reposition the rods to maintain the proper coolant temperature.
3. The steady state coolant temperature increases as load increases, and the steady state secondary steam temperature and pressure decrease as load increases.

We will now derive the relationship which relates load, coolant temperature, and secondary temperature (and pressure) and show why they are programmed to change as they do. Consider the simplified diagram of a PWR cycle shown in Figure 12-18. As the coolant passes through the core, it is heated

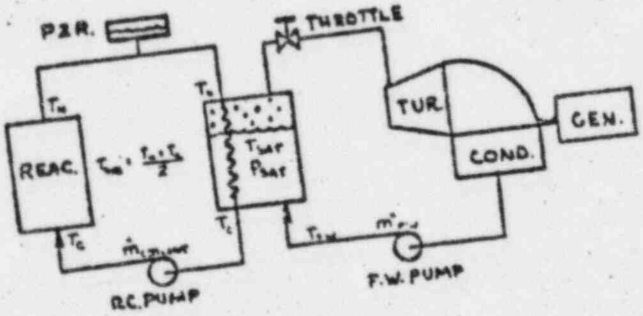

Figure 12-18: Simplified PWR Cycle

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[1] As discussed in the next section, moderator temperature only increases by about 20-30° F in going from zero power to full power.

from the cold leg temperature, T_c, to the hot leg temperature, T_h. The rate at which heat must be added to the coolant in order to produce this temperature increase is given by equation (11-3). That is:

$$\dot{Q} = \dot{m}\, C_p\, (T_h - T_c) \qquad (12\text{-}7)$$

where:
$\dot{Q}$ = heat addition rate from reactor to coolant (Btu/hr)
$\dot{m}$ = coolant flow rate (lb/hr)
C_p = average coolant heat capacity in the temperature range from T_h to T_c (Btu/lb-°F)
T_h = core exit (hot leg) temperature (°F)
T_c = core inlet (cold leg) temperature (°F)

The heat addition rate, $\dot{Q}$, is of course, the reactor power. The coolant flow rate is a fixed quantity and the heat capacity of water is also essentially constant over the normal range of coolant temperatures. Thus the product of $\dot{m}C_p$ is virtually constant and we can make the statement that the temperature rise across the core is directly proportional to reactor power.

Example: At full power in a PWR, T_c = 550° F and T_h = 600° F. At 50% power, we observe that T_c is still 550° F. What is T_h at 50% power?

> Since $(T_h - T_c)$ = 50° F at full power, we know that $(T_h - T_c)$ = 25° F at 50% power. Thus, at 50% power:
>
> $$T_h = T_c + 25 = 550 + 25 = 575° \text{ F}$$

While heat is being added to the primary coolant by the reactor, it is being simultaneously removed in the steam generator. At any point in the steam generator, heat is being transferred from the primary coolant to the secondary fluid at a rate determined by equation (11-18). That is:

$$\dot{Q}_x = UA_x(T_{pri,x} - T_{sec,x}) \qquad (12\text{-}8)$$

where:
$\dot{Q}_x$ = heat transfer rate at some arbitrary location x in the steam generator (But/hr)
U = Heat transfer coefficient for steam generator (Btu/hr-ft^2 - °F)
A_x = heat transfer surface area at location x (ft^2)
$T_{pri,x}$ = primary coolant temperature at location x
$T_{sec,x}$ = secondary coolant temperature at location x

In this expression, T_{sec} is essentially uniform throughout the steam generator[1] and is equal to the saturation temperature of the secondary

---

[1] We are speaking now of the secondary temperature throughout the steam generator at a particular plant load. If load changes, the secondary temperature will change (discussed later in this section) but as long as saturation conditions exist, all of the fluid will be at the same temperature, whatever it might be.

steam[1]. On the other hand, the temperature of the primary fluid is continually changing as the primary fluid passes through the steam generator. As a result, the heat transfer rate through the steam generator tubes varies as the fluid moves through the steam generator. At the point where the primary fluid first enters the steam generator, the heat transfer rate is high because T_{pri} is high. At the primary fluid exit, the heat transfer rate is lower because T_{pri} is lower. However, an average heat transfer rate over the entire steam generator can be determined by using an average primary coolant temperature in equation (11-18). The total heat transfer rate throughout the steam generator will then be:

$$\dot{Q} = UA_{tot}(T_{avg} - T_{sat}) \tag{12-9}$$

where: $\dot{Q}$ = total heat transfer rate in the steam generator (Btu/hr)
A_{tot} = total heat transfer surface area (ft^2)
T_{avg} = average primary coolant temperature in the steam generator (°F) = $(T_h + T_c)/2$
T_{sat} = saturation temperature of the secondary system

At steady state (but not necessarily during transients) the $\dot{Q}$ in equation (12-9) is equal to the $\dot{Q}$ in equation (12-7). In effect, we have two independent relationships which relate steady state reactor power to the various temperatures in the primary and secondary systems. Using these relationships we can investigate the behavior of PWRs with various control systems.

1. Control System Designed to Maintain Constant T_c

The automatic reactor control system of most large present day PWRs is designed to maintain nearly constant T_c at all steady state loads. How this task is accomplished will be considered later in this section. In any case, if we specify this requirement, the steady state values of the remaining variables are determined from equations (12-7) and (12-9). Figure 12-19 shows the steady state characteristics of a PWR being controlled in this manner. The numerical values on the figure are typical values, but vary somewhat from plant to plant.

Let us first consider conditions at zero power. Equation (12-7) indicates that there will be no primary coolant temperature rise across the core so that $T_c = T_h = T_{avg}$. Intuition tells us that this

---

[1] Actually, the feedwater enters the steam generator at a lower temperature than saturation temperature. However, the amount of heat necessary to raise the temperature up to the saturation point is small in comparison to the heat which is added to the fluid as it boils. Therefore, during most of the time that the secondary fluid is in the steam generator it is at the boiling point. As a result, it is justified to assume that the secondary fluid temperature is constant.

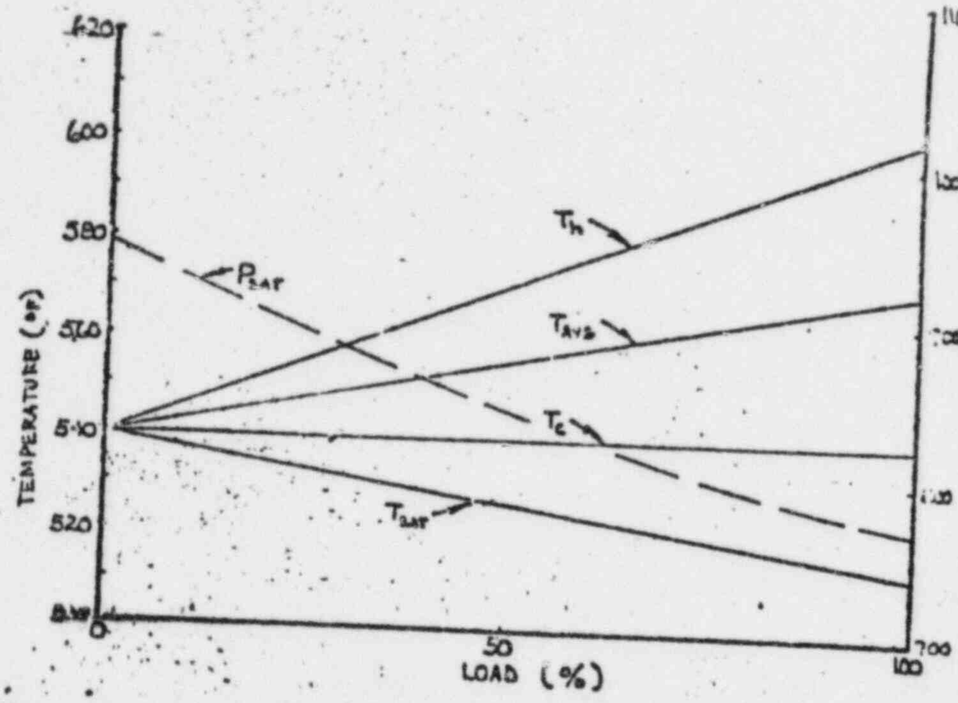

Figure 12-19: Steady State Characteristics of a PWR Operating with a Constant T_c Control Program

must be the case since zero power indicates that the reactor is not adding heat to the primary coolant. In addition, in order to maintain steady state conditions we cannot be transferring any heat to the secondary system, so $T_{sat} = T_{avg}$ (from equation (12-9)). That is, at zero power there is no steam flow to the turbine and the secondary system is effectively a bottled up volume of fluid physically located adjacent to a volume of high temperature primary coolant. As a result, the temperature of the secondary fluid will increase until it is in equilibrium with that of the primary system. Once temperatures are equalized, there will be no heat transferred between the primary and secondary systems.

Now let us assume that the plant is brought to 50% power. For the time being we will ignore the mechanism for achieving this power increase and will concern ourselves only with how system conditions will equilibrate at this power level. Since T_c is assumed to remain constant, equation (12-7) indicates that T_h must rise substantially. Since T_{avg} is always half-way between T_c and T_h, it also rises. Once steady state is reached, all of the heat added to the primary coolant by the reactor must be transferred to the secondary fluid in accordance with equation (12-9). Thus, T_{avg} must be higher than

T_{sat}. We have already seen that T_{avg} will rise from its no-load value which provides some of the required driving force. However, if known values of $\dot{Q}$ (reactor power), U (obtained from design information on the steam generator), A_{tot} (physical size of steam generator), and T_{avg} (from equation (12-7)), are substituted into equation (12-9), it can be shown that T_{sat} also has to fall somewhat from its no-load value in order to provide the necessary temperature differential. As a result, PWRs operating with a constant T_c control philosophy show a reduction in secondary temperature as load is increased. Since the secondary fluid is saturated, the secondary temperature uniquely determines the secondary steam pressure. That is, the turbine throttle pressure will also fall as load is increased. Secondary steam pressure is also shown on Figure 12-19.

The fact that secondary temperature has to have a steady state value lower than the primary system temperature means that we can no longer leave the secondary system bottled up. Otherwise, the temperature of the secondary system would rise until it equalled the primary system temperature as in the no-load case. Thus, to maintain the proper temperature relationships for 50% power we would have to open the turbine throttle valves and the feedwater valve and establish a secondary system flow. The flow would be sufficient to allow as much energy to be extracted from the secondary fluid in the turbine and condenser as was being added to the secondary system in the steam generator. In other words, as in every other steam plant, steam flow from the steam generator of a PWR would increase as the load increases.

Similar reasoning indicates that when load is increased from 50% power to 100% power the following will occur:

A. T_h and T_{avg} will rise. $(T_h - T_c)$ will be twice as large at 100% power as it was at 50% power.

B. T_{sat} will drop causing P_{sat} to drop.

C. Turbine steam flow will increase to approximately twice the 50% power value.

All of the above analyses were based on the assumption that T_c remained constant. The next question is how can we be sure that this will be the case? The answer is that we can provide the reactor with an automatic rod system which regulates the reactor power in response to electrical load demands in such a manner that T_c does in fact remain constant. Such a system is illustrated schematically in Figure 12-20.

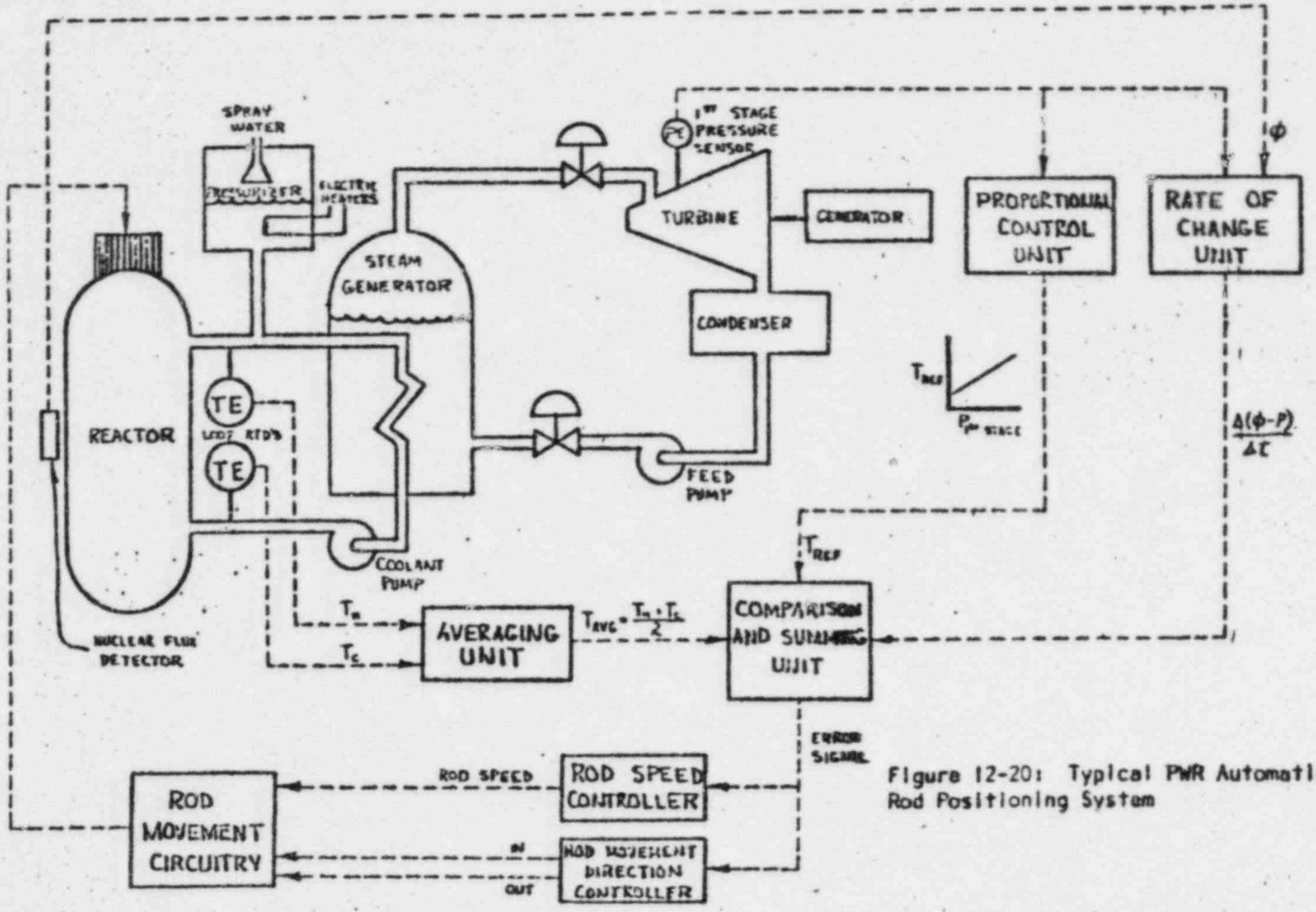

Figure 12-20: Typical PWR Automatic Rod Positioning System

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If we maintain constant T_c, we can determine from equation (12-7) that T_{avg} will vary with load as shown in Figure 12-19. Therefore, if we control the reactor so that T_{avg} follows Figure 12-19, we will be assured that T_c is constant. The control systems are designed to maintain T_{avg} at the proper value rather than working directly with T_c[1]. As a result, one essential piece of data which must be fed into the control system is the actual measured T_{avg}. To obtain this, temperature sensors are located in both the hot and cold legs of the coolant loop. The measured values of T_h and T_c are sent to an averaging unit which computes the true T_{avg} in the core. This value is then fed to a comparison and summing unit.

The control system now knows that T_{avg} actually is, but before it can determine whether or not the rods need to be moved it must also know what T_{avg} is supposed to be. In other words, we must generate some reference value of T_{avg} against which the control system can compare the actual measured T_{avg}. This reference T_{avg} which we will now call T_{ref}, is simply going to be the curve of T_{avg} versus load that was determined for Figure 12-19 since this is what we are trying to get the control system to match. Therefore, if load is known, T_{ref} can be calculated just as was T_{avg} on the Figure.

As a measure of load, the control system uses turbine first stage pressure. This value is fed into a proportional unit which converts first stage pressure into T_{ref} in accordance with the appropriate linear (straight line) relationship. The output from this unit, which is the T_{ref} for the existing turbine load, is fed to the comparison and summing unit.

The comparison and summing unit compares the desired T_{ref} against the actual T_{avg}, and if they are not equal it generates an output signal (called an error signal) which is sent to the control rod movement circuitry. The error signal can be either positive or negative, depending on whether T_{avg} is higher or lower than T_{ref}. The sign of the signal indicates whether the rods should move in or out. The magnitude of the signal indicates how fast the rods will move. That is, if T_{avg} and T_{ref} differ by a small amount the rods will move slowly to make the correction, whereas if they differ by large amounts (large being = 5° F or more) the rods will begin moving at high speed to adjust reactor power. Then as T_{avg} and T_{ref} begin to come together, the rods will slow down and finally stop moving altogether when the error signal is eliminated.[2]

---

[1] Because T_{avg} is influenced by what is occurring at both core inlet and exit rather than just core inlet. Therefore, T_{avg} is a more reliable indicator of overall core performance than is T_c, and it can be used in other control systems besides the automatic control rod system.

[2] Actually, in order to keep the rods from continually "hunting" there

There is one other feature of the control system which needs to be discussed. The only way T_{avg} can change is if we disturb the balance between heat added to the coolant by the reactor and heat extracted from the coolant by the secondary system. Thus, a change in T_{avg} must be preceded by a change in reactor power or a change in turbine load. There is, however, a time delay between the change in power or load and the resulting change in T_{avg}. To smooth out the operation, it is advantageous to immediately get the rods moving following a rapid change in power or load in anticipation of an eventual change in T_{avg}. In this way, we get a lot of our rod movement out of the way before T_{avg} gets a chance to get very far away from T_{ref}. Therefore, the control system looks at reactor power as measured by an out-of-core neutron detector, and at turbine load. These signals are sent to a unit which looks at the rate of change[1] of their difference. If these two quantities are in balance this unit has no output signal. However, if the two quantities become unbalanced, caused by either one being changed without a corresponding change in the other, than this unit will send a signal to the summing unit which will get the rods started moving in the proper direction. The biggest use of this anticipation feature is to start the rods moving inward at full speed following a large turbine load rejection. To illustrate, assume that the unit is at steady state full power. In this condition, turbine load and reactor power are steady. Thus any small difference between them[2] is not changing and this unit sends no signal to the summing unit. Now if the turbine trips, turbine load will drop almost immediately to zero, but reactor power will stay at about the full load value. Thus a rapidly increasing unbalance between the turbine and the reactor is noted and this feature sends a signal to the summing unit to start the rods moving inward at full speed.

To complete this discussion of PWR operating characteristics, let us see how the entire plant would respond to a rapid increase in load from 90% power to 100% power. This increase in load would be initiated by the operator opening the control valves so that turbine steam flow increased. The effect of this increase in load upon several plant parameters is shown in Figure 12-21. The numerical values have in most cases been omitted because they vary widely from facility to facility.

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is a deadband associated with the error signal. Ordinarily rods will not begin to move until the error signal is at least ± 1.5°, and they will stop moving when it is reduced back within this band.

[1] In calculus, the determination of the rate of change of a quantity is called "taking the derivative". Therefore, this unit is often called a derivative unit.

[2] Reactor power and turbine load are never balanced exactly since some of the reactor power is wasted in heat losses of one sort or another.

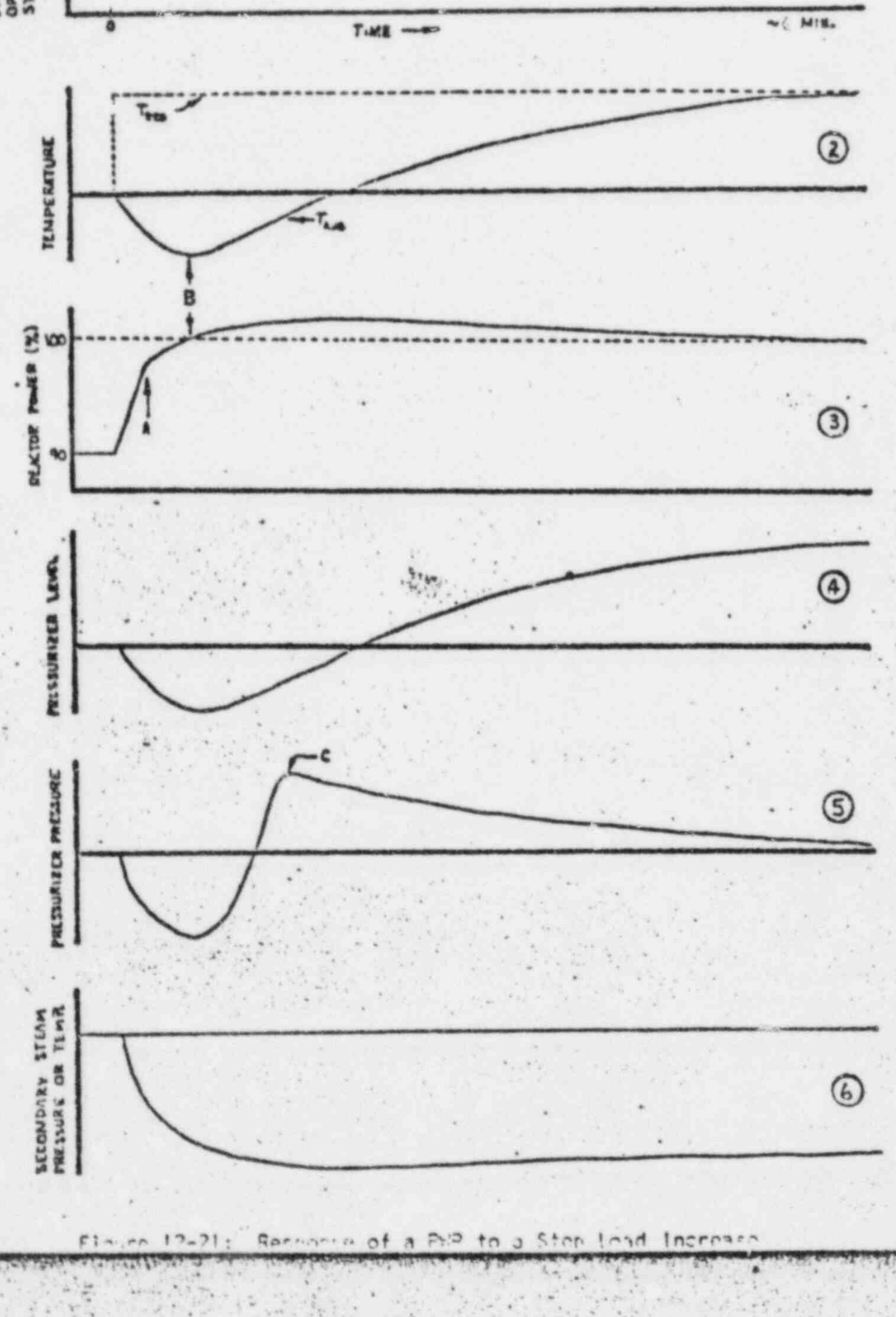

Figure 12-21: Response of a PWR to a Step Load Increase

The initiating event is the step increase in steam flow shown on curve 1. This immediately causes secondary steam pressure and temperature to start falling (curve 6). The reduction in secondary temperature causes more heat to be extracted from the primary coolant in the steam generator (equation (12-9)) and the immediate effect is to reduce T_{avg} (curve 2). That is, we are initially removing ≈ 100% power from the primary coolant in the steam generator and are only supplying 90% power from the reactor. Since heat removal exceeds heat addition, the coolant temperature starts dropping.

The initial response of the reactor to this transient is a rapid power increase. This results from the following:

A. The reduction in T_{avg} increases reactivity because of the negative moderator temperature coefficient.

B. The anticipation feature in the control system sees a rapid increase in turbine load without a corresponding increase in reactor power. This rapidly increasing imbalance sends a signal to the rods calling for rapid withdrawal.

C. The step load increase results in a step increase in T_{ref} (dotted line curve 2). Thus T_{avg} is now too low and this results in an error signal calling for rod withdrawal.

After a few seconds, the anticipation feature signal dies out and T_{avg} is beginning to be turned, so the rate of power increase slows down (point A on curve 3). However, the (T_{avg} - T_{ref}) error signal calls for additional rod withdrawal, so power continues to rise, but at a slower rate.

When reactor power reaches 100% (point B curve 3), T_{avg} will be levelled out because we are now putting just as much heat into the coolant as we are extracting. Although T_{avg} is levelled out, it is still too low so reactor power continues to rise above 100%. This means we are now adding more heat to the coolant than we are taking out in the secondary system and T_{avg} will begin to increase.

As T_{avg} increases toward T_{ref}, the error signal will gradually disappear and rod withdrawal will gradually come to a halt. The negative temperature coefficient will act to limit the power rise, and its effects coupled with automatic trimming of the rods as necessary, will bring the reactor power back to 100% when T_{avg} equals T_{ref}. At this time the system will once more be in equilibrium.

Typically the above transient will take 5-8 minutes to dissipate itself. The initial coolant temperature drop will probably be 3-4° F depending on the system, and the final coolant temperature will be about 3° F higher at full power than it was at 90% power[1]. The control rods will be farther out at the end of the transient to compensate for the negative reactivity affects of increased moderator and fuel temperature.

Before leaving the topic, it is instructive to consider the effects of the above transient on the pressurizer. The density of water is directly related to its temperature (see Figure 10-25). Thus, when T_{avg} drops, the volume of the primary coolant gets smaller. This causes the level in the pressurizer to fall. On the other hand, when T_{avg} starts rising, the primary coolant swells, and the pressurizer level increases. Since the primary system is filled with solid water, this expansion and shrinkage would cause violent pressure fluctuations if the pressurizer was not available to accomodate the coolant volume changes.

Although the action of the pressurizer limits the magnitude of pressure surges to acceptable levels, pressure does fluctuate during the transient. When pressurizer level initially falls, the volume of the pressurizer steam space increases and the steam pressure drops. Since the liquid in the pressurizer is saturated, it begins to flash in an attempt to maintain its equilibrium vapor pressure. However, the flashing process causes the pressurizer liquid to cool because it must supply the necessary heat of vaporization to the steam. Thus, the equilibrium vapor pressure (which is at all times equal to the reactor pressure) of the pressurizer liquid will drop during a reduction in level. The pressure drop will be retarded by electric heaters in the pressurizer liquid which automatically come on as pressure falls. Since they supply some or all of the necessary heat of vaporization, they prevent the liquid from cooling (and the pressure from falling) as far as it would have in their absence.

Conversely, as level starts to rise, the steam in the top of the pressurizer will be compressed and will begin to condense. The condensation process supplies the liquid with the heat of vaporization possessed by the condensing steam, thereby causing the liquid temperature and the corresponding vapor pressure to increase. To help stop the pressure increase, cold[2] water is sprayed into the pressurizer steam space when the pressure increases above a particular value. This cools the pressurizer liquid and rather abruptly turns the pressure transient (point C on curve 5). The spray system then

---

[1] If $(T_h - T_c)$ is 60° F at full power, T_{ref} will change by 30° F in going from zero load to full load. Thus, a 10% change in load will produce a 3° F change in T_{ref}.

[2] Relatively speaking. Actually the spray comes from the cold leg of a coolant loop and is therefore 540-550° F compared to the pressurizer fluid temperature of = 651° F.

continues to function until the pressure returns to the pre-transient value.[1]

Finally, curve 6 shows that the secondary steam pressure falls as a result of the power increase. We have already seen that this is a consequence of equation (12-9). Actually, since we are transferring essentially full power to the secondary system while T_{avg} is still below its ultimate equilibrium value, the secondary temperature must initially fall slightly below its equilibrium value to provide the necessary ΔT to satisfy equation (12-9). Then as T_{avg} rises, the secondary temperature can also rise by a corresponding amount.

2. Constant T_{avg} Program

In the previous section we discussed the typical PWR control philosophy, but gave no indication as to why it was chosen. In this section and the next, we will discuss two alternative control schemes and will show why they are ordinarily rejected in favor of the previous scheme.

One possible control scheme is to maintain constant T_{avg} under all steady state loads. Figure 12-22 shows the variation in several variables when the reactor employs this type of control scheme. The advantages of this program are:

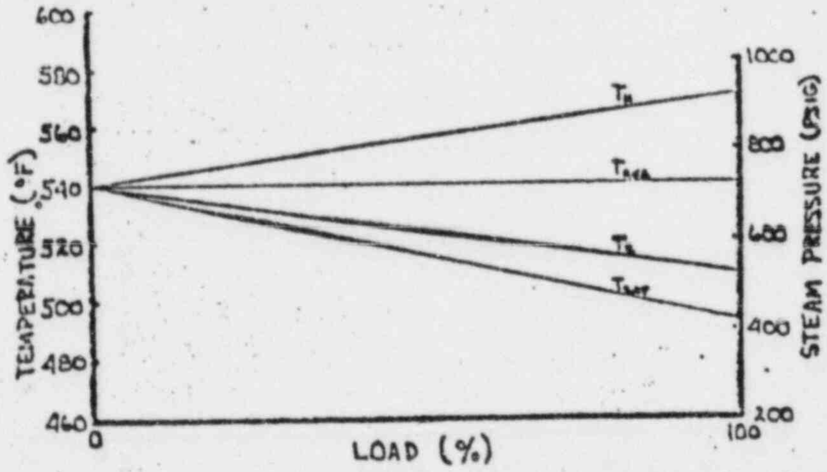

Figure 12-22: Steady State Characteristics of a PWR Using a Constant T_{avg} Program

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[1] PWRs are normally operated at a constant pressure for all loads. Typically it is ≈ 2235 psig.

A. Volume of coolant is constant at steady load so that required size of the pressurizer is minimized.

B. Control rod movement to increase load is minimized. The rods only need to overcome the doppler effect. There is no steady state moderator temperature coefficient effect to be overcome.

C. More of the core's excess reactivity can be used for fuel burnup, since loss of it goes for overcoming the moderator temperature coefficient.

The disadvantages are:

A. At full power, steam pressure is very low since T_{sat} must fall substantially in order to allow the heat to be transferred in the steam generator. This low pressure means low thermodynamic cycle efficiency.

B. The physical size of the turbine must be very large to handle the low pressure steam with its corresponding high specific volume (ft^3/lb).

The poor secondary system performance with this type of control scheme has caused this method to be abandoned for large power reactors.

3. Constant Secondary Temperature (and Pressure) Program

The characteristics of this system are shown in Figure 12-23. The

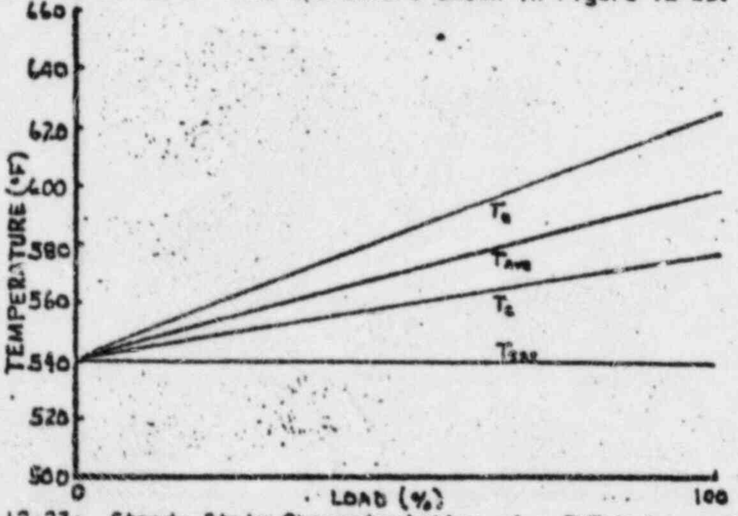

Figure 12-23: Steady State Characteristics of a PWR Using a Constant T_{sat} Program

principal advantage of this approach is that it optimizes the secondary system because it provides a high secondary temperature and pressure at full load. Its disadvantages include:

A. T_{avg} must rise substantially in order to provide the required temperature differential in the steam generator at full load. This means that the pressurizer must be very large to accommodate the large coolant swell.

B. Considerable rod movement is required to overcome the negative temperature coefficient when changing load.

C. Considerable excess reactivity is tied up in overcoming the temperature coefficient.

D. The core exit temperature (T_h) is very high at full power. This puts the reactor closer to the DNB limit.

Now we are in a position to understand why something approximating the constant T_c program is ordinarily the one chosen for an actual plant. It is basically a compromise between the constant T_{avg} and constant T_{sat} programs, necessitated because the former results in undesirably poor secondary system performance and the latter is too severe on the primary system.

OPERATING CHARACTERISTICS OF A BWR AT HIGH POWER

A simplified schematic diagram of a direct cycle BWR system is shown in Figure 12-24. In the early plants using the direct cycle

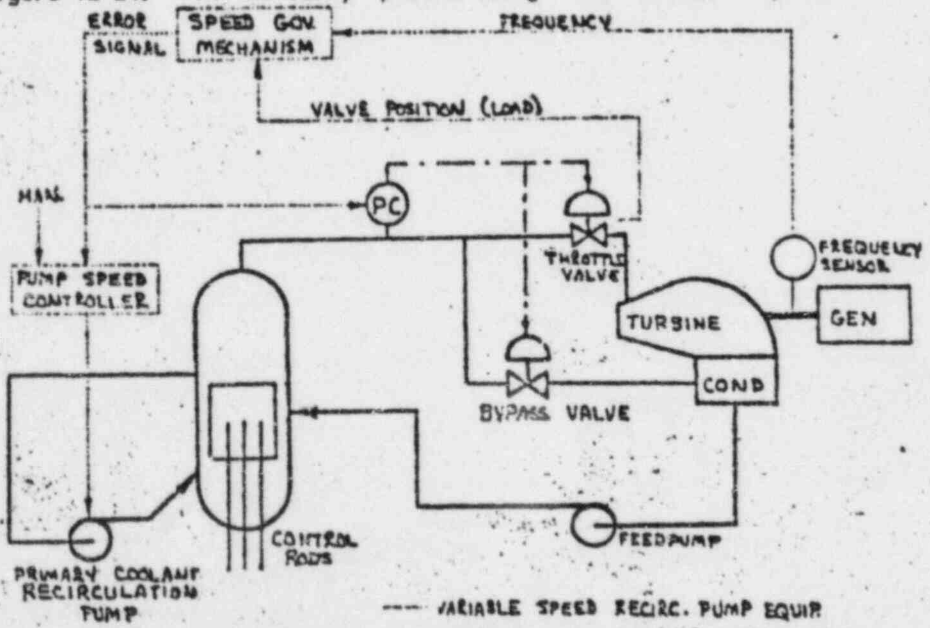

Figure 12-24: Simplified Schematic Diagram of a Direct Cycle BWR

approach[1], the normal method for making load changes is manual insertion or withdrawal of control rods. The turbine is designed to automatically follow the reactor (which is essentially backwards from the PWR control setup) through the action of a pressure controller on the steam line.

To illustrate the response of a BWR to rod movement at high power, we will consider the case where a group of partially inserted control rods are quickly withdrawn from the lower half of the core[2] to give a small "step" power increase. The response of several important plant variables is shown in Figure 12-25. The immediate effect of the rod withdrawal is a reactor power increase which continues until essentially the entire reactivity addition is cancelled by the doppler effect (Curve 1). Then the power settles back somewhat as the additional heat being generated in the fuel makes its way to the coolant and increases the coolant void fraction. The increase in void content causes the reactor to momentarily go subcritical until power falls to a point where the void and fuel temperature effects share in cancelling the rod withdrawal instead of doppler doing the whole job. Since the fuel time constant is generally in the range of 5 - 15 seconds (see Chapter 11), the process of building in the equilibrium voids takes place over a time interval which may approach a half minute or so.

When the reactor power increases, the steam production rate in the reactor momentarily exceeds the rate at which steam is being accepted by the turbine. Since this excess steam has nowhere to go, it accumulates in the steam space in the top of the vessel and is compressed, causing a rise in reactor pressure (Curve 5). A pressure controller located on the main steam line senses this increase and automatically begins to open the turbine throttle valves (Curve 6). When the turbine catches up with the reactor, that is, when the throttle valves are open far enough to accept all of the steam which is being produced, the pressure stops rising and the controller stops

---

[1] As opposed to the "dual cycle concept used at Dresden 1, but abandoned in favor of direct cycle with variable recirculation flow control for recent units. Basically, the dual cycle concept involves placing a heat exchanger in the primary coolant recirculation loop. Part of the feedwater on its return trip to the vessel is routed through this heat exchanger. Some of this rerouted feedwater boils and is sent to a low pressure stage of the turbine. The remainder of the rerouted feedwater is returned to the main feedwater stream. By varying the amount of feedwater sent through this heat exchanger, the feedwater temperature and temperature of the recirculated primary coolant are altered, which produces power changes due to the void coefficient. Although the principle is basically the same as variable recirculation flow, which will be discussed later in this section, the latter has the advantages of providing a greater range of load follow capability and of producing smaller changes in power distribution, which is beneficial in terms of reactor stability.

[2] Recall that the rods enter a BWR from the bottom.

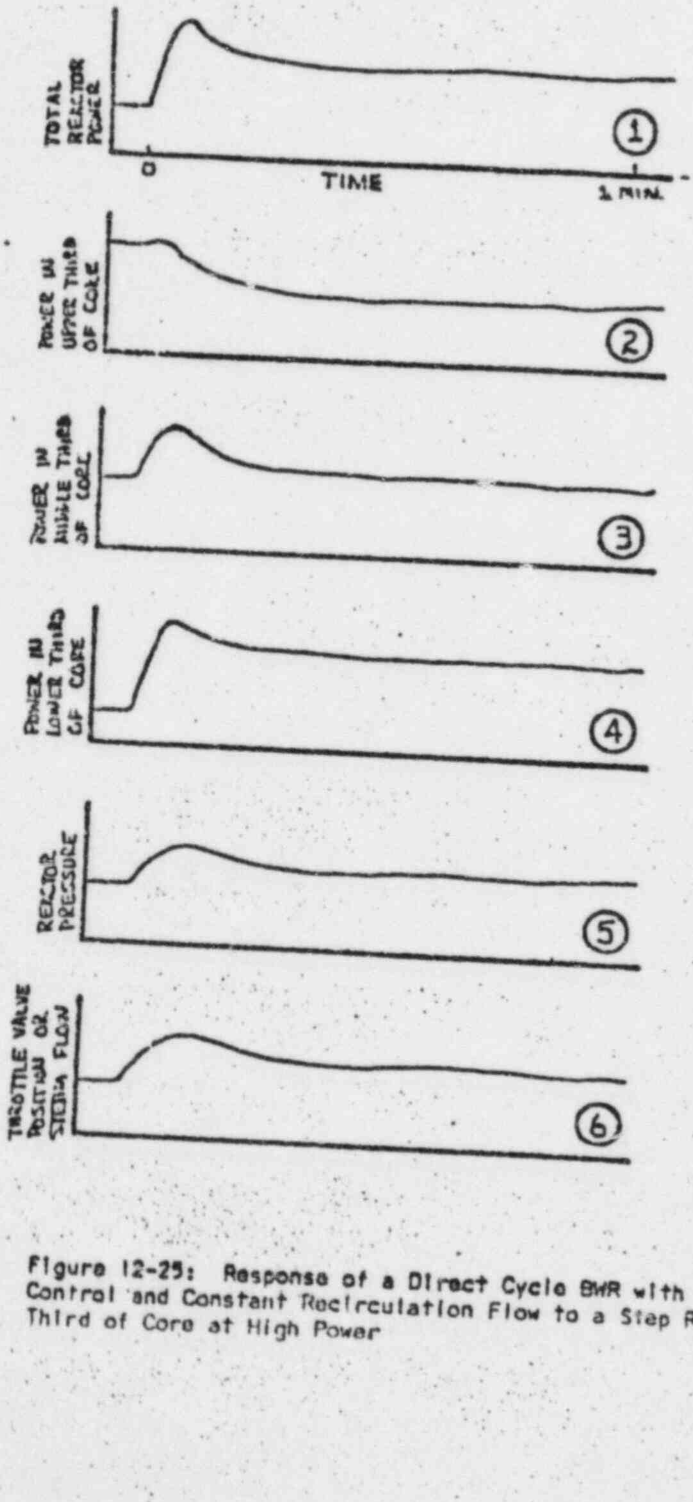

Figure 12-25: Response of a Direct Cycle BWR with Proportional Pressure Control and Constant Recirculation Flow to a Step Rod Withdrawal from Lower Third of Core at High Power

opening the valves.[1] During the period when reactor power falls as voids build in, the pressure also falls somewhat and the throttle valves back off a bit as well.

Curves 2, 3, and 4 on Figure 12-25 show the power traces for different elevations in the core. The thing to notice is that the power increase took place in the lower regions of the core from which the rods were withdrawn. In the top of the core there was no reactivity increase due to rod withdrawal, but rather a reactivity decrease due to higher void content, and so the power actually dropped somewhat. The core power distribution will be discussed in detail later in this chapter.

We see from above that rod withdrawal is accompanied by a pressure increase due to the nature of the control scheme. Although the operator would ordinarily readjust the reactor pressure back to the normal operating value at the completion of the load change, the effects of the transient increase must still be considered. Basically, an increase in reactor pressure causes a reactivity increase due to the compression of core voids. That is, since the void coefficient is negative, any effect which reduces core void volume will increase reactivity. Therefore, when rod withdrawal initiates a power increase, the resulting pressure increase causes the power to peak somewhat higher than it would have had pressure remained constant. In fact, if the operator attempted to make a large load increase, it is not at all unlikely that the combined reactivity effects of the rod withdrawal and pressure increase would cause a reactor trip on high power (see Chapter 15). In any case, it is desirable to minimize transient reactor pressure fluctuations since they produce undesirable reactivity swings. To aid in this effort, the plants are provided with turbine bypass valves which dump steam directly to the condenser. The bypass valves are set to automatically begin opening when reactor pressure exceeds the normal setpoint by a small amount, typically = 10 - 15 psi. Thus in the previous example, had the rate of reactor power increase outstripped the turbine controller's ability to keep pace, the bypass valves would have opened on high pressure and accepted some of the excess steam until the turbine caught up.

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[1] This description assumes that the pressure regulator is a proportional controller, as is usually the case, and contains no reset action. The amount that the pressure must increase in order to cause the throttle valves to come open is an adjustable feature called the "proportional band" of the controller. Ordinarily, in going from no load to full load with no change in setpoint, the pressure may increase by 100 psi or more. In order to avoid such swings in the reactor operating pressure, the operator would manually readjust the controller setpoint after a change in load so that he carried essentially constant pressure at all steady state power levels.

Actually, the true value of the turbine bypass system is not to help out on load pickups, since the operators ordinarily have the good judgment to limit their load pickup rates to acceptable values. The true value comes on turbine load rejections (caused by generator protective relay action in most cases) where the rapid closure of the throttle valves by governor action would cause a large pressure increase and subsequent reactor trip (on high power due to void compression with a backup trip on high pressure). In a situation like this, the bypass valves would fly open[1] to accept the steam being rejected by the turbine. Assuming the valves are designed to handle a large load rejection, the action of the valves would limit the pressure increase, thereby averting the high power trip, and allowing the operator time to manually insert sufficient control rods to reduce the reactor power down to a low, standby value.

The previous description of the response of a BWR to rod withdrawal, assuming essentially constant recirculation flow, is basically valid for the new BWR plants as well as the earlier ones. The newer plants, however, are equipped with variable speed recirculation pumps which improve the plants' load follow characteristics by enabling load to be varied over a significant range without necessitating the rather time consuming movement of control rods. The newest plants can vary load over a range of about 35% of full power without rod movement, and can accomplish this swing in about 70 seconds. The additional control equipment ordinarily used for load following by this method is included on Figure 12-24.

To illustrate the basic principle of this method, consider Figure 12-26.

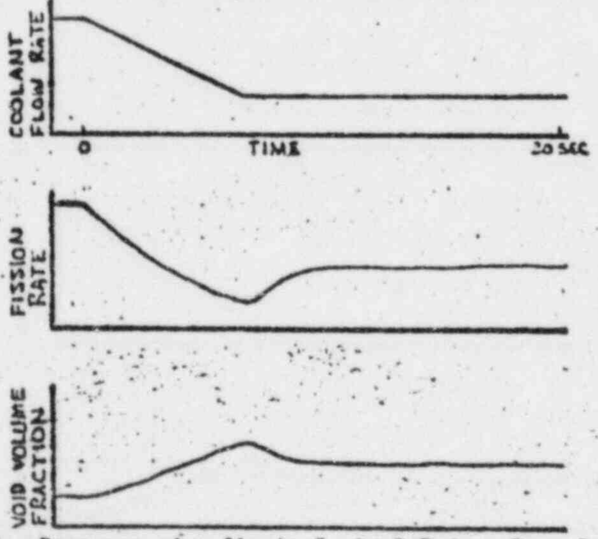

Figure 12-26: Response of a Single Cycle BWR to a Ramp Reduction in Primary Coolant Flow

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[1] Generally on a signal from the generator OCB. The valves would open

which shows the response of a BWR to a negative coolant flow ramp. As flow is reduced, the core average void fraction rises. This is simply because the reactor is adding its heat to a smaller amount of water, thereby causing a greater percentage of it to boil. As the void fraction increases, the reactor goes subcritical and power falls until the gain in reactivity resulting from the reduction in fuel temperature cancels the loss in reactivity resulting from the increase in void fraction. Recall that since we have not moved the rods, the excess reactivity which is available to voids and doppler is roughly the same before and after the flow change. Notice that there is a small undershoot on power and overshoot on voids. This is because of the time lag between a reduction in fission rate and a corresponding reduction in heat transfer rate to the coolant. Even though the fission rate falls rapidly, it takes a few seconds for all of the heat stored up in the fuel rods to be dissipated to the coolant.

In the actual systems which are in use, a change in coolant flow can be initiated either manually or automatically. The automatic system senses generator speed (frequency) and responds to the small frequency changes which occur when system load demand is not matched by system generation. For example, a reduction in system frequency indicates the necessity of an increase in generator output. Assuming that the plant was at less than full load and therefore capable of picking up load, the reduction in frequency would result in an error signal being sent to the recirculation pumps calling for more pump speed (increased flow). This would be just the reverse of the previous example and would cause a reactor power increase as desired. To speed the overall plant response, the error signal is also sent to the steam pressure regulator and momentarily lowers its setpoint. This results in a rapid opening of the turbine throttle valves as they attempt to reduce reactor pressure, and immediately flashes some additional steam from the large reservoir of water in the vessel. This setpoint reduction is only a temporary measure, however, to rapidly pick up the steam flow while waiting for the increased recirculation flow to take effect. Long term reduction of pressure would tend to increase the void volume and work against the power increase which is trying to be achieved by increased flow.

One thing which must be guarded against in a variable flow system is excessive power increase by rod withdrawal while operating at reduced flow, since this could lead to DNB. Control rod withdrawal interlocks are provided to guard against this condition.

POWER DISTRIBUTIONS IN UNIFORM CORES

A given reactor has a certain core volume and number of square feet of heat transfer surface. If the reactor could be operated in the ideal manner, it would be operated so that all portions of the core were working

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as soon as the OCB opens rather than waiting for a pressure increase. This gives them a little head start.

equally hard and producing power at the maximum rate consistent with the heat transfer limits discussed in the previous chapter. Under these conditions, the fuel would be burned uniformly, the core size necessary to produce a given amount of power would be minimized, and the lowest fuel cost per MWT would be realized. Unfortunately, however, there are a number of unavoidable phenomena which make it impossible to achieve a perfectly uniform power distribution. In this section and those which follow, some of the more important of these phenomena will be discussed.

To begin the discussion, we will consider a very simple uncontrolled, homogeneous reactor surrounded by a vacuum (a so-called "bare" reactor). In this reactor we would discover that the power density (power generated per unit volume) drops substantially as we move outward from the center to the edge in any direction. The reason for this is that a neutron born near the edge of the core has a far greater probability of leaking out of the reactor than does a neutron born near the center. Since neutrons which leak from the core are no longer available for producing a fission, the fission rate (or power production) is depressed on the edges of the core. In this simple reactor, we would find that at any particular elevation, the power distribution would look like one hump on a cosine curve as we moved horizontally from one side of the core to the other[1]. Similarly, the power distribution from top to bottom at any particular radial location is also shaped like a cosine curve. This idealized power distribution is called a cosine distribution and is illustrated on Figure 12-27. The figure actually shows the thermal neutron flux distribution, which is directly proportional to the power distribution within the core.[2] The only difference is that there are thermal neutrons outside the core which do not result in fissions. Thus the power distribution drops abruptly to zero at the edge of the core, whereas the thermal flux distribution gradually falls to zero outside the core (dashed extrapolation on the figure).

The first complication to this cosine distribution occurs if the core is surrounded by a reflector. The reflector operates in two ways. First, it scatters some of the thermal neutrons which leak from core back into the fuel. Secondly, it moderates fast neutrons which leak from the core. This moderating ability produces a big pileup of thermal neutrons just outside the core boundary (called a reflector peak), and many of these thermal neutrons reenter the core. Both of these effects tend to increase

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[1] Since the shape of most fully loaded power reactors approximates that of a right circular cylinder, the horizontal direction from one side of the core to the other is generally called the radial direction. The vertical direction from top to bottom is called the axial direction.

[2] Assuming uniform fuel composition throughout the core. That is, $P = \phi \Sigma_f V$, where Σ_f and V are constants.

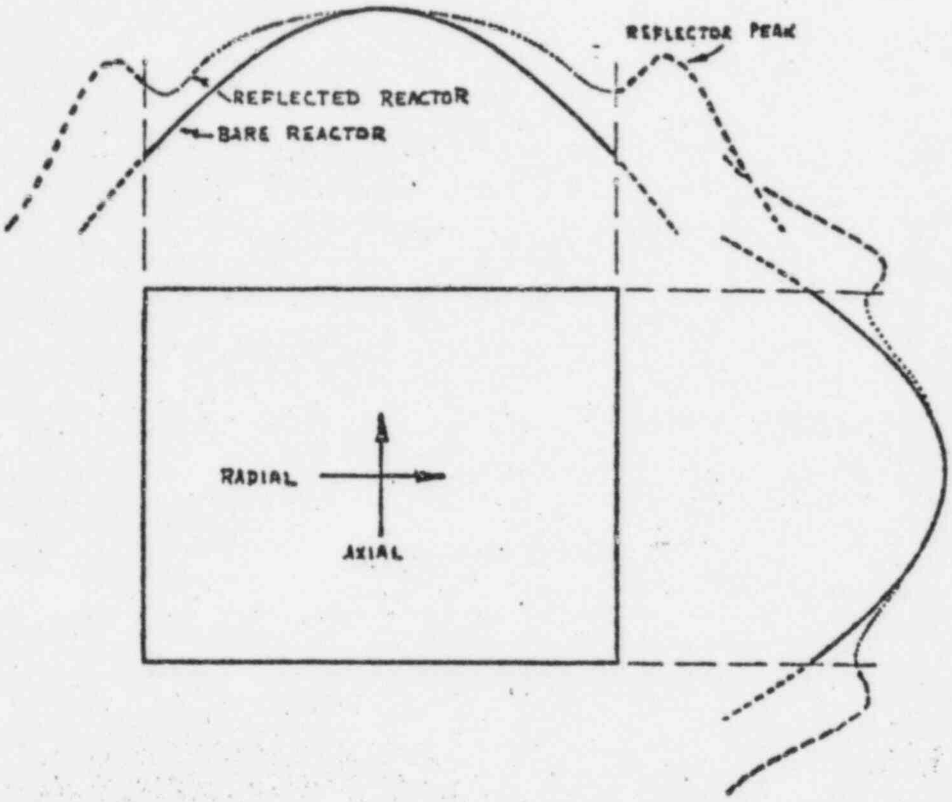

Figure 12-27: Thermal Neutron Flux Distributions in a Homogeneous Reactor

the power generation on the edges of the core relative to what it would have been in a bare reactor and thus tend to flatten the power distribution.

The power distribution in a heterogeneous reactor would also tend to assume the modified cosine shape discussed above. In the radial direction, however, the distribution would be "rougher" due to the discontinuities caused by the separation of moderator and fuel. We saw in Chapter 10 (Figure 10-4) for example, that most thermal neutrons are produced in the moderator (by slowing down), but are absorbed before they reach the center of a fuel rod, thereby causing a thermal flux depression in each rod. Conversely, the fast neutrons are thermalized in the water gaps between rods, resulting in thermal neutron flux peaks just outside of each rod. Thus, instead of the smooth modified cosine shape which would exist in the homogeneous core, the heterogeneous core would have a radial distribution as shown in Figure 12-28.

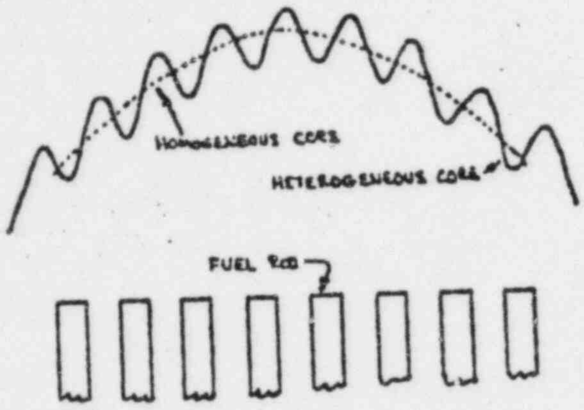

Figure 12-28: Radial Thermal Flux Distribution in a Uniform Heterogeneous Core

The modified cosine flux distribution discussed above is a reasonable "first approximation" to the actual flux shape or power distribution in many power reactors. However, it can be altered substantially by numerous effects, such as void distribution, control rod positions, isotopic composition of the fuel, and coolant flow distribution. In the sections below we will see how these factors affect the power distribution in actual practice.

DISTRIBUTION OF POWER AMONG INDIVIDUAL FUEL BUNDLES

Power reactor cores are ordinarily composed of discrete fuel bundles, each of which contains a great many individual fuel rods. In general, the fuel bundles are handled as units. That is, individual fuel rods are seldom transferred from one bundle to another, but the entire bundle remains intact from the beginning of its useful life to the end. In general, the power produced by individual fuel rods within a bundle varies considerably depending on the position of the rod with respect to the edge of the core, inserted control rods, etc. We will consider the power distribution between rods in detail later in this chapter, but frequently we are only interested in the total power produced by the bundle as a whole. To describe the distribution of power between bundles, a term called the bundle peaking is defined by the equation:

$$(BPF) = \frac{\text{total power generated in the fuel bundle of interest}}{\text{total power generated in an "average" fuel bundle}} \quad (12\text{-}10)$$

For example, if the 172 bundle Humboldt Bay core is producing 240 MWT, an "average" bundle produces 240/172 = 1.4 MWT. If a particular bundle is producing 1.6 MWT, it is operating at a bundle peaking factor of 1.6/1.4 = 1.14. Thus it is producing 14% more power than the "average" bundle.

In the absence of complicating effects, the power distribution across the core will tend to assume the modified cosine distribution for a heterogeneous core shown in Figure 12-28. As a general rule, therefore, the bundles located near the edge of the core will produce less power than bundles located near the center.[1] However, as previously mentioned, it is desirable to try and flatten this distribution as much as possible and equalize the power generation among the various bundles.

In BWRs, where most of the control is taken care of with control rods, a degree of power flattening can be achieved by employing a rod pattern in which the central regions of the core are more heavily controlled than the edges. Such a pattern is shown for the Humboldt Bay BWR in Figure 12-29. Also shown on Figure 12-29 is a plot of the bundle peaking factors across a central row of fuel (the row just above the core centerline). The shape of this plot should be compared to the modified cosine distributions shown previously.[2] The deeply inserted rod is seen to drastically reduce the power within the four surrounding bundles and to make a small reduction in power in the bundles immediately adjacent to these four. In our previous terminology, the region affected by the rod constitutes its absorption area. The difficulty with rods as a power shaping device is apparent from the figure. They are very non-uniform in their effect, and although they have generally depressed the flux in the central region of the core, they have done it by "overkilling" a small number of bundles. One might wonder whether a more uniform effect could be obtained by altering the pattern so that all central rods were partially inserted rather than having a few fully inserted and a few very slightly inserted. The answer is yes on the total bundle power basis that we are considering at the moment, but such a pattern would have a disastrous effect upon the axial flux distribution (to be discussed in next section) inasmuch as it would cause an abnormally high power density in the top of the core where DNB is most likely to occur.

The BWR has one particularly effective inherent power flattening mechanism in the form of voids. An increase in bundle power causes a corresponding increase in the bundle void content, which in turn limits the power increase. Thus the tendency of voids to concentrate in high power regions in preference to low power regions has a definite flux flattening effect.

The absence of core voids and significant numbers of inserted control rods as potential flux flattening devices in PWRs has led to their use of

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[1] Since a bundle's power is therefore loosely related to its radial location with respect to the center of the core, the bundle peaking factor is sometimes called the radial peaking factor.

[2] The reader should recognize that the true distribution when the flux in individual fuel rods is considered would be "smoother" than the step function which results when only total bundle peaking factors are considered.

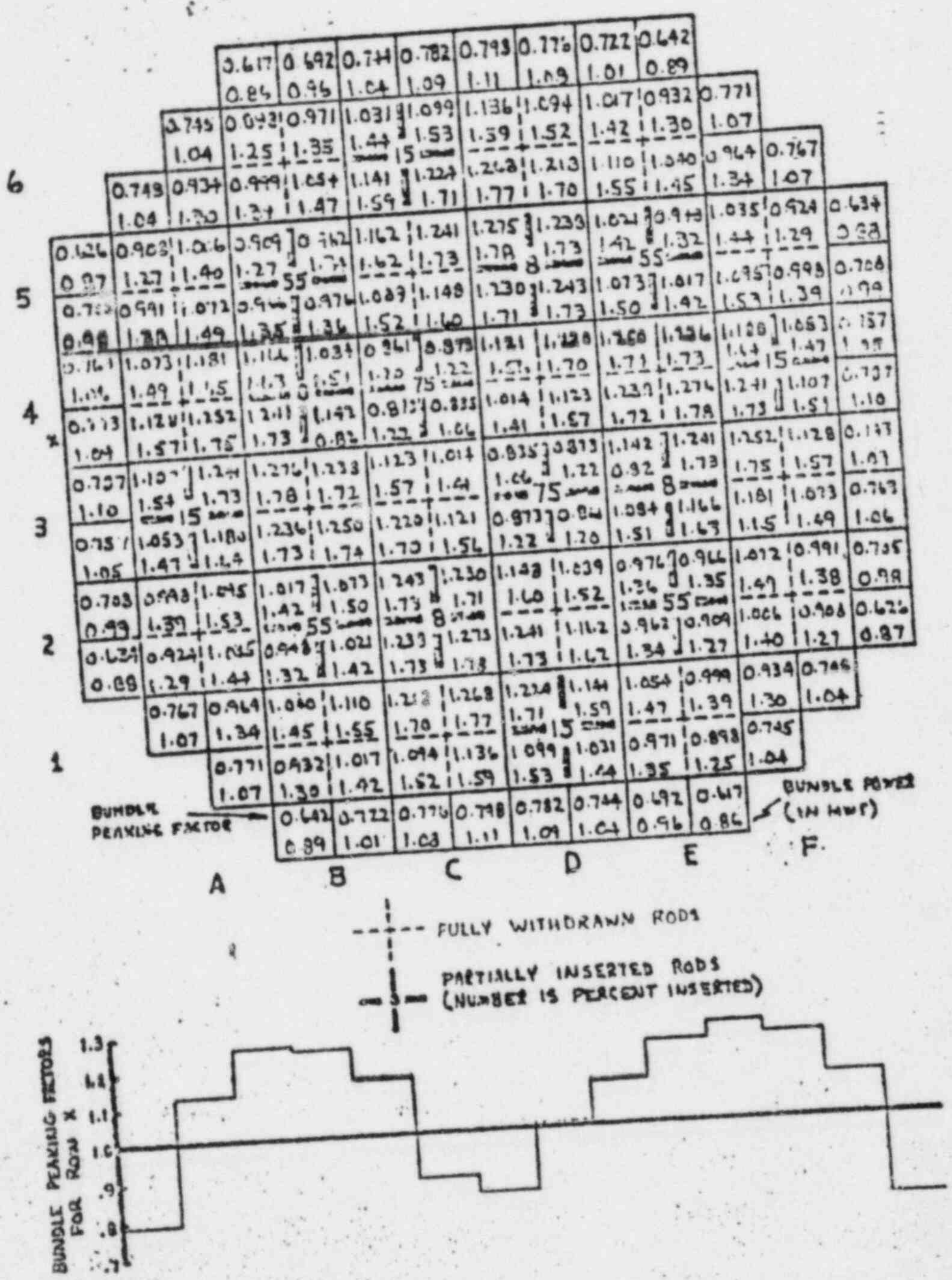

Figure 12-29: Typical Bundle Power Distribution and Control Rod Pattern for Humboldt Bay BWR at 240 MWT

multi-enrichment core loadings to improve the power distribution.[1] In general, three types of fuel, differing only in enrichment, are used. The highest enrichment fuel is loaded on the outside of the core and the lower enrichment fuel is installed in the center. In small and intermediate sized plants (up to ≈ 500 MWE) the two lower enrichment fuels are also separated, with the lowest enrichment fuel going into the center of the core, and the intermediate enrichment fuel into an annulus around the outside of the low enrichment region. Such an arrangement, and the "smoothed out" power distribution which would result, is shown in Figure 12-30. The sharp breaks in the power generation curve actually do exist, although the thermal neutron flux does not show these discontinuities. In fact, at each regional boundary, the fluxes immediately adjacent to the boundary, but on opposite sides of it, are equal. However, even though there is no abrupt change in flux, the power will show an abrupt change because of the abrupt change in enrichment (or Σ_f), since $P = \phi\Sigma_f$. Although it is a valid assumption to say that the power distribution and thermal neutron flux distribution are the same in uniform regions or cores, one must be careful when there is a change in enrichment.

The three zone loading arrangement is not as desirable for the very large cores. This is because the physical size of the core is so much larger than the average crow flight path of a neutron that the regions tend to work independently of each other. That is, in Figure 12-30, what occurs in region 3 has almost no influence upon what occurs in region 1. When the core is operating, it must be borated such that the average k_∞ of the core is ≈ 1 (overall core leakage is very small). Since the boron is uniformly distributed, it makes $k_\infty < 1$ in the center and the center region is in effect a subcritical reactor. The power generation in the center will thus be dependent upon the "source" strength in the center, which in this case is leakage from adjoining regions. When the core is small, the leakage to the center is high and the central region operates at a reasonable power level. However, in the very large cores, this is not the case and power in the central region drops well below that in the outer region as shown in the figure. To overcome this, the two lower enrichment fuels are distributed uniformly in a checkerboard pattern throughout the volume occupied by regions 1 and 2. This produces a two region core with the central region having an "effective" enrichment essentially equal to the average of its two component fuels.

Other means of flattening the bundle power distribution in the core include the use of fixed burnable poison pieces located in the core at appropriate locations, poison material mixed with the fuel in selected fuel bundles,

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[1] The discussion here is referring to a new core with new fuel. However, multi-enrichment cores are "automatic" for both BWRs and PWRs in subsequent cores since they will be composed of a mixture of new and partially depleted fuel. The reloading schemes will be planned to utilize the flux flattening potential of these cores.

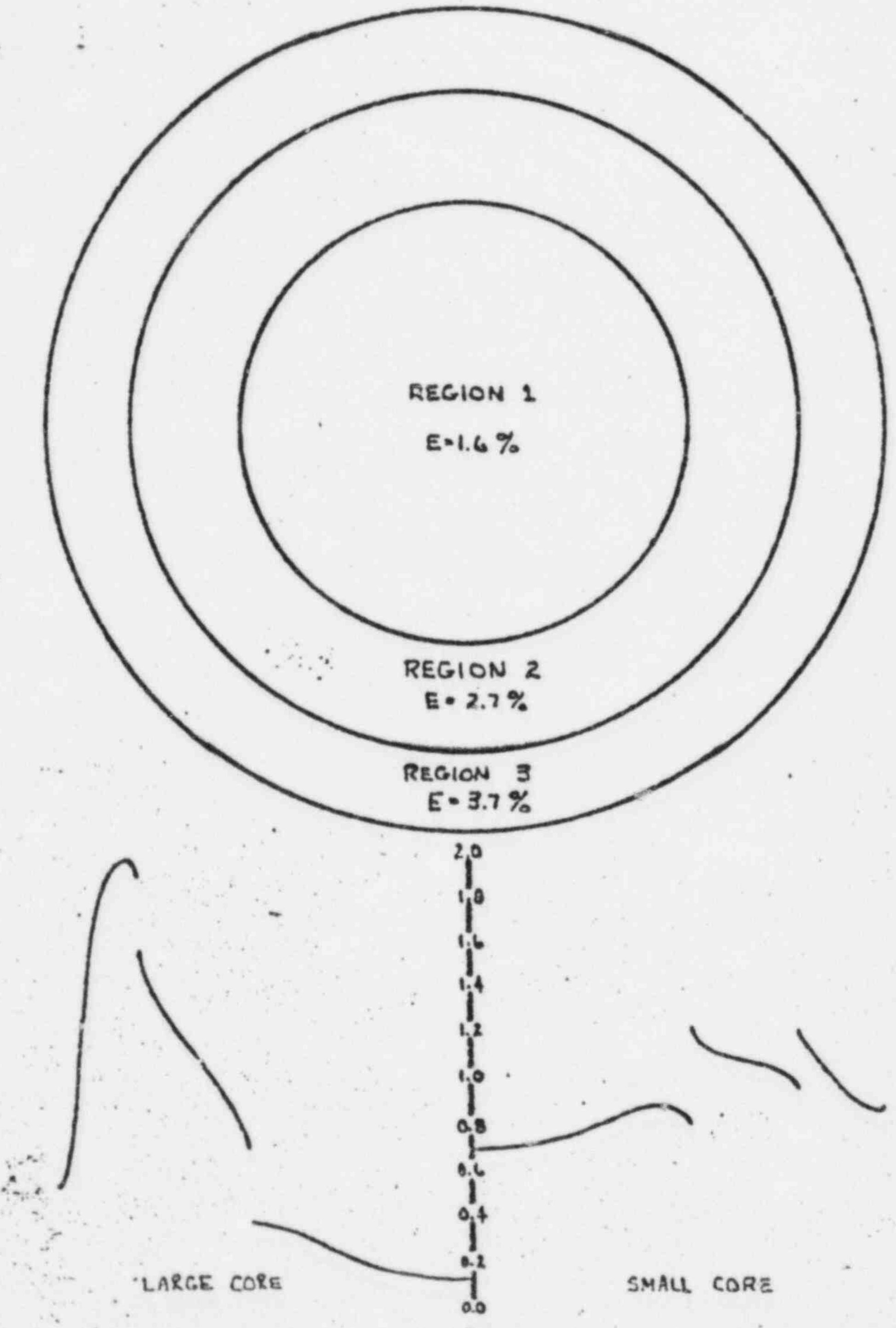

Figure 12-30: Radial Power Distribution In Zone Loaded PWRs with Uniform Poisoning

and non-uniform coolant flow distribution.[1] The actual bundle power distribution will, therefore, vary significantly from one plant to the next, depending on how the core designer has chosen to employ the various options at his disposal. The only general statement which can be made is that bundles in the center tend to work harder than bundles on the edge, unless specific measures are employed to overcome this tendency.

Before moving on, it is of interest to digress for a moment and investigate the effect of power distribution upon rod strength in a BWR. Looking at **Figure 12-20**, it is interesting to speculate on the effect of inserting a rod such as E-4 into a high flux region as opposed to inserting one into a low flux region, like cell A-5. Looking at the terms in equation (10-16) we see that the absorption areas of the two rods are about the same[2] and the effective size of the core is the same for both (it equals the true full size of the core plus a small amount of reflector). The difference between the two rods will be the result of differences in $(\phi_{T,cr}/\phi_{T,avg})^2$ terms. To evaluate this difference, we can assume that the thermal flux distribution is about the same as the power distribution. In this case, $(\phi_{T,cr}/\phi_{T,avg})$ is sort of an average bundle peaking factor for the bundles surrounding each rod. Thus:

$$(\phi_{T,cr}/\phi_{T,avg})\text{ rod E-4} = \frac{1.250 + 1.236 + 1.238 + 1.276}{4} = 1.250$$

$$(\phi_{T,cr}/\phi_{T,avg})\text{ rod A-5} = \frac{0.908 + 1.005 + 0.991 + 1.072}{4} = 0.994$$

$$\frac{(\Delta k/k)\text{ rod E-4}}{(\Delta k/k)\text{ rod A-5}} = \frac{(\phi_{T,cr}/\phi_{T,avg})^2\text{ E-4}}{(\phi_{T,cr}/\phi_{T,avg})^2\text{ A-5}} = \frac{(1.250)^2}{(0.994)^2} = 1.58$$

So rod E-4 would be worth = 60% more than rod A-5 under this set of operating conditions. This is only a rough estimate, but it gives some feel for relative rod worths.

AXIAL POWER DISTRIBUTIONS

In the previous section we concerned ourselves with the total power produced in a bundle without considering what part of the bundle it was generated in. We have already seen that leakage at the top and bottom of the core

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[1] Particularly effective in BWRs because of its effect upon the void distribution. It is accomplished by orificing the inlet nosepieces of the fuel.

[2] L_T would be slightly larger in the center of the core than at the edge because of the higher void content, but the effect is small and is partially offset by the fact that the rods are outside the fuel channels and are surrounded by water rather than a mixture of steam and water.

will depress the power density at the ends of each fuel bundle and will tend to produce a cosine shaped axial distribution. However, this simple distribution will be modified by voids, rods, fuel exposure and other effects as will be discussed below.

To quantitatively describe a vertical, or axial power distribution, the axial peaking factor is employed. The axial peaking factor can best be understood by considering the following example. Suppose a particular Humboldt Bay fuel bundle is generating 1.6 MWT when the reactor is operating at full power (i.e., the bundle peaking factor for this bundle is 1.14). Now let us slice the bundle into ten equal parts as shown in Figure 12-31. Each of these slices is commonly termed a "node" by people who use computers and are engaged in the business of making calculations of this sort. It should be obvious that in this fuel bundle, the "average" node is producing 1/10 of the total bundle power, or 0.16 MWT. The power production in any node in the bundle is related to this so-called "average" node by the axial peaking factor:

$$(\text{APF}) = \frac{\text{power generated in any vertical node of interest in a given bundle}}{\text{average nodal power in the bundle of interest}} \qquad (12\text{-}11)$$

Thus in the example, a node which produces 0.133 MWT has an axial peaking power of 0.83, which means that it is working 17% below the "average" node. Nodal powers and the corresponding axial peaking factors are included in Figure 12-31.

In a BWR, the peak in the axial power density generally occurs below the centerline of the core because of the high void concentration in the top of the core. This tendency is partially offset by the use of bottom entry control rods and a rod pattern which has several rods partially inserted into the lower half of the core. The effect of this control strategy can be seen on Figure 12-32 by comparing the axial flux distribution when

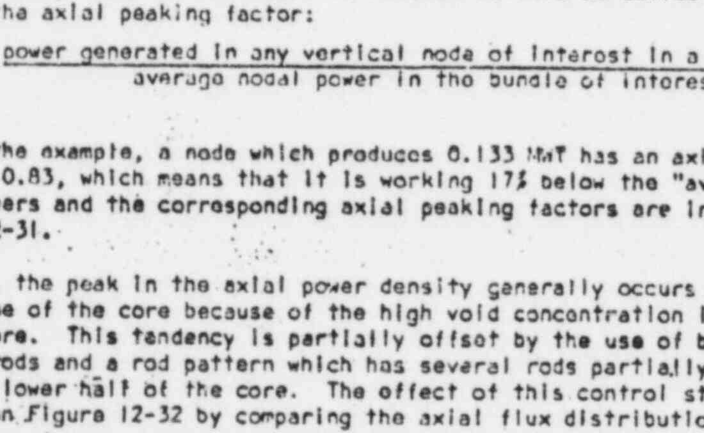


Figure 12-32: Axial Flux Shapes and CHFRs for BWRs Operating with Uniform Axial Control and with Bottom Entry Rods

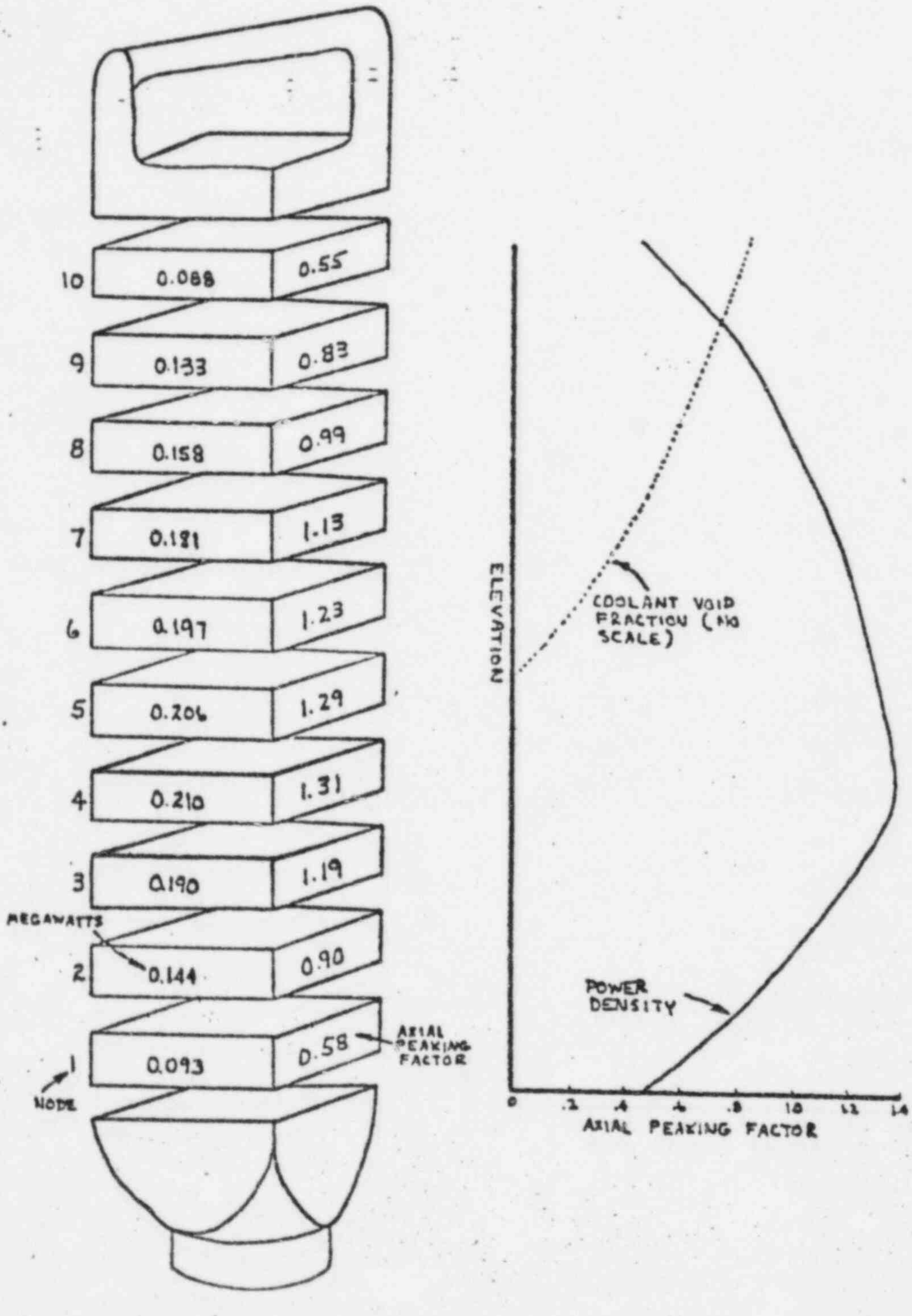


Figure 12-31: Typical Vertical Power Distribution in an Uncontrolled Humboldt Bay Fuel Element for a Total Bundle Power of 1.6MWT and an Average Nodal Power of 0.16 MWT

uniform axial control is used to that when partially inserted bottom entry rods are used. Notice that uniform control results in very high axial peaking in the bottom of the core and results in a low value of the CHFR in this region even though the DNB heat flux is high (since there are few voids). The bottom entry case results in much lower peaking, and a higher minimum CHFR in the bottom of the core. The CHFR is somewhat lower in the upper region of the core, however, because the heat flux is higher.

In a PWR, the effect of higher moderator temperature at the top of the core also tends to push the axial flux peak slightly below the core centerline. However, this effect is not nearly as pronounced as the void effect in a BWR, so uniform axial control is acceptable. In fact, the use of top entry rods means that partial insertion of control rods exaggerates the flux peaking in the lower half of the core. As a result, it becomes advantageous to use uniform axial control to minimize axial peaking, and this is a major reason for employing chemical shim as an alternative to many partially inserted control rods.

LOCAL PEAKING

We looked first at how the total core power is distributed among the individual fuel bundles, then at how the power produced by a given bundle is distributed from one end of the bundle to the other. Finally, we will look at the core in still greater detail and discuss how the power produced in one axial node of a particular bundle is spread among the individual fuel rods.

To illustrate some of the factors which affect the power distribution among various fuel rods, consider Figure 12-33, which shows a typical boiling water reactor lattice arrangement. Note that the bundle is surrounded on all four sides by water. On two of the sides, however, the water gap may be partially filled with a control rod blade depending on whether or not the blade is inserted. The water gaps have a significant effect upon the power distribution among the fuel rods. In general, there is reflector peaking in the water gaps which measurably increases the power generation in the outer row of fuel rods relative to the central rods. This reflector peaking is most strongly felt in the fuel rods which border the wide water gaps which remain after a control rod is withdrawn. Of course, when the control rod is inserted, it overcomes any tendency for reflector peaking (partly because it absorbs thermal neutrons and partly because it displaces the reflector) and these same fuel rods operate at very low power levels relative to the fuel rods in the remainder of the bundle.

Once again, the situation can be described quantitatively through use of a peaking factor--in this case a "local" peaking factor defined by the equation:

$$\text{LPF} = \frac{\text{power in fuel rod of interest in a given vert. node of a particular bundle}}{\text{avg. power in the fuel rods in a given vert. node of a particular bundle}} \qquad (12\text{-}12)$$

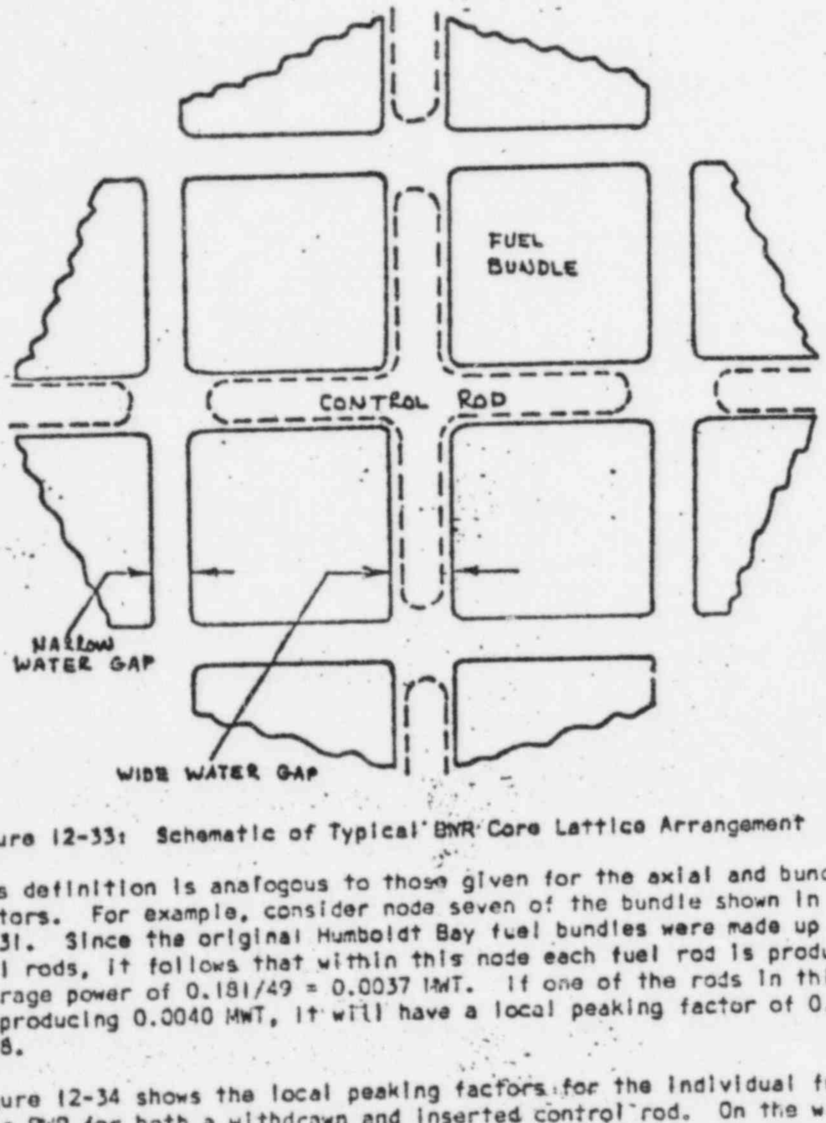
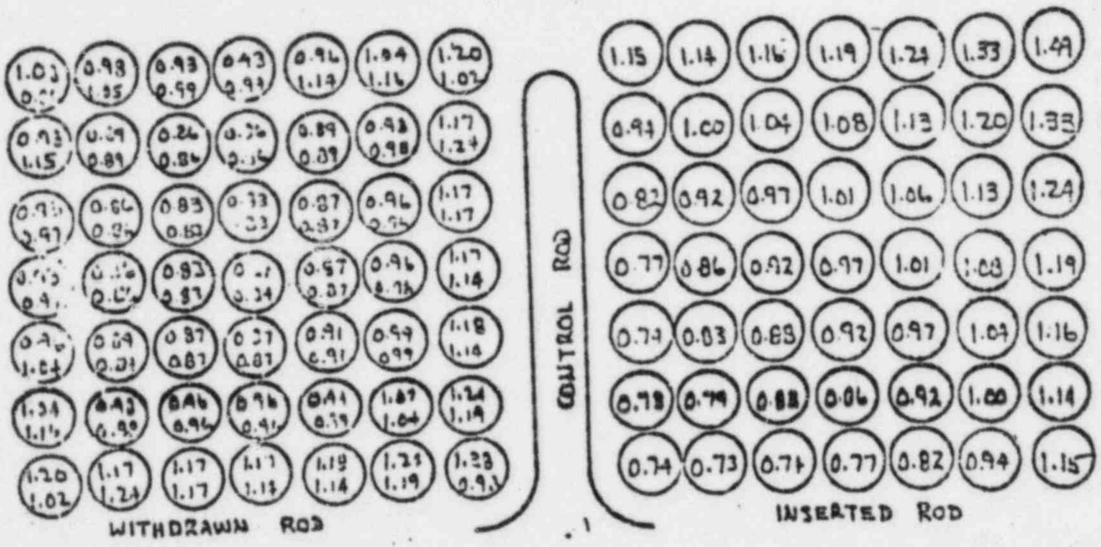


Figure 12-33: Schematic of Typical BWR Core Lattice Arrangement

This definition is analogous to those given for the axial and bundle peaking factors. For example, consider node seven of the bundle shown in Figure 12-31. Since the original Humboldt Bay fuel bundles were made up of 49 fuel rods, it follows that within this node each fuel rod is producing an average power of 0.181/49 = 0.0037 MWT. If one of the rods in this node is producing 0.0040 MWT, it will have a local peaking factor of 0.0040/0.0037 = 1.08.

Figure 12-34 shows the local peaking factors for the individual fuel rods of a BWR for both a withdrawn and inserted control rod. On the withdrawn rod figure, two sets of numbers are given. The upper value shows what would occur if all fuel rods were loaded with fuel of the same enrichment. Notice that the power peaks strongly in the rods located adjacent to the large water gap. To counteract this effect, fuel rods in the vicinity are typically loaded with UO_2 of a slightly lower enrichment. The lower figures show the effect of using 1.9% enriched fuel in the 4 corner rods and 2.4% enrichment throughout the remainder of the bundle. Looking at



1.15	1.14	1.16	1.19	1.24	1.33	1.49
0.94	1.00	1.04	1.08	1.13	1.20	1.33
0.82	0.92	0.97	1.01	1.06	1.13	1.24
0.77	0.86	0.92	0.97	1.01	1.08	1.19
0.74	0.83	0.88	0.92	0.97	1.04	1.16
0.78	0.79	0.83	0.86	0.92	1.00	1.14
0.74	0.73	0.74	0.77	0.82	0.94	1.15

Figure 12-34: Typical Local Peaking Factors for BWR Fuel Showing Effects of Control Rod Water Gaps and Variable Enrichment Loading

the case where the rod is withdrawn, we see that we have reduced the maximum local peaking by ≈ 10% (from 1.38 to 1.24). This is an important consideration because bundles adjacent to withdrawn control rods will be the ones which generate the most power (i.e., have high bundle peaking factors) and would therefore be limiting in terms of CHFR or center melting. Conversely, the high local peaking factors which occur in bundles near inserted rods are of little concern (although they too will be reduced by multiple enrichment fuel) because these bundles are invariably operating at low bundle peaking factors.

The above discussion illustrates the rather large local peaking effects brought about by the discontinuity in the fuel-moderator lattice arrangement due to control rods. In general, any discontinuity in the lattice will cause local flux peaks or depressions. For example, most fuel bundles have one fuel rod which is divided vertically into three or four separate segments for structural support. That is, a fuel rod will consist of a length of pellets, followed by an "end" connector to which is attached some sort of fuel rod support structure, followed by another length of fuel pellets, then another "end" connector, etc. There will usually be a pileup of thermal neutrons in the end connector just as in the reflector unless the connector is made out of special materials. The reason for this is that fissioning acts to remove thermal neutrons from the system and replace them with fast neutrons. As a result, the thermal neutron population is depressed, but the fast neutron population is enhanced by the presence of fuel. The end connector, however, acts very similar to the water moderator. It acts to slow down the plentiful supply of fast neutrons, but does not absorb many of the thermal neutrons which result. Therefore, the thermal neutrons pile up. This effect can be reduced by mixing a small amount of poison material (e.g., a non-fissionable material with high thermal neutron capture cross section such as dysprosium) with the fuel pellets adjacent to the end connectors, or by lowering the enrichment in these pellets.

Present day PWRs, of course, are free of the discontinuities caused by control rods interspaced between fuel bundles. Nevertheless, they have discontinuities within the bundles caused by vacant fuel tubes (into which go control rod clusters, burnable poison inserts, sources, and in-core instrumentation). This will lead to local peaking effects within the bundles just as for BWRs. However, the more uniform PWR lattice results in local peaking factors which are somewhat less than for BWRs.

RELATIONSHIP OF POWER DISTRIBUTION TO THERMAL - HYDRAULIC LIMITS

We have noted in previous sections of this manual the importance of the reactor being operated within the limits on CHFR and fuel center temperature. While calculations of these quantities are not ordinarily made by the operator, he must have a basic understanding of the techniques used to make these determinations. We touched briefly on this subject in the previous chapter, but it is worthwhile to reconsider this topic in somewhat more detail at this time since it directly relates to the subject of power distribution which we have been discussing.

Calculation of the CHFR or fuel center temperature by the standard formulae requires that three pieces of information be known: 1) the physical geometry of the bundle, 2) the power density (or more commonly the heat flux[1]), and 3) the conditions of the coolant at various core locations.

The physical geometry of the system is important for two principal reasons: 1) it influences the resistance to heat transfer and the temperatures of the cladding and fuel [see equations (11-11) through (11-14)], and 2) it influences the pressure drop and coolant flow distribution in the core. However, the physical geometry is fixed by the design and is therefore beyond the control of the operator.

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[1] Heat flux and power density are directly related quantities. The heat generated in the fuel rod becomes heat flux when it flows to the coolant. However, only about 97% of the power generated in a bundle flows through the cladding (thereby being classified as heat flux). The remainder is carried by γ rays and fast neutrons which pass directly through the fuel and cladding without undergoing any interactions which liberate heat until they reach the coolant. Thus 3% of the heat can be considered as being instantly transferred to the coolant without having to flow through the normal paths, and is therefore, not part of the heat flux.

The power density or heat flux must be known because it is the numerator of the CHFR expression (equation (11-17)) and it appears in equation (11-16), from which the fuel center temperature is calculated. Actually, DNB or center melting will occur at some point in the core rather than uniformly throughout the core due to the fact that the heat flux is not uniform throughout the core. Therefore, in these calculations, we are always interested in the heat flux at some particular point in the core, and this determination requires the use of the peaking factors discussed previously. Now, the heat flux at any particular core location is related to the average core heat flux by the equation:

$$(\dot{Q}/A)_y = (BPF)_y \times (APF)_y \times (LPF)_y \times (\dot{Q}/A)_{\text{core avg.}} \qquad (12\text{-}13)$$

where: $(\dot{Q}/A)_y$ = heat flux at point y in the core
$(BPF)_y$ = bundle peaking factor for the bundle in which point y is located
$(APF)_y$ = axial peaking factor in the bundle at the elevation of point y
$(LPF)_y$ = local peaking factor at point y
$(\dot{Q}/A)_{\text{core avg}}$ = core average heat flux

The product (BPF)(APF)(LPF) is sometimes called the total peaking factor, (TPF), so equation (12-13) can be rewritten as:

$$(\dot{Q}/A)_y = (TPF)_y \times (\dot{Q}/A)_{\text{core avg}} \qquad (12\text{-}14)$$

To determine the core average heat flux, it is only necessary to divide the total core power (or more properly 97% of the total core power) by the total heat transfer surface area. For example, Humboldt Bay uses 172 fuel bundles, each of which has about 41 ft^2 of heat transfer surface area. When this reactor is operating at 230 MWT, which is equivalent to 7.84×10^8 Btu/hr[1], the average core heat flux is given by:

$$(.97)(7.84 \times 10^8)/(172)(41) = 108{,}000 \text{ Btu/hr-ft}^2$$

To complete the calculation of the heat flux at a point in the core, we need to know the appropriate peaking factors. These are determined by a combination of computer calculations and experimental techniques such as the determination of the amount of induced radioactivity in test specimens placed in various locations throughout the core (the use of flux wires inserted in the Humboldt Bay reactor core is an example of this), and the use of in-core neutron monitors and in-core thermocouples.

Example: A particular reactor contains 193 fuel assemblies, each having 270 ft^2 of heat transfer surface area. The core operates at 3250 MWT.

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[1] Since 1 MW = 3.41×10^6 Btu/hr.

INCREMENTAL CONTROL ROD WORTH

In earlier sections of this manual, the worth of entire control rods was discussed. Now that we have shown what the neutron flux distribution in the core looks like, we can make some general statements about the incremental rod worth[1]. In both PWRs and BWRs, the control rods can be moved axially in discrete increments (called steps or notches) in order to enable the rods to be positioned in intermediate axial locations between fully inserted and fully withdrawn. The total rod worth will be the sum of the worths associated with each increment of movement.

A first approximation of the relative worth of individual increments of a control rod can be obtained from equation (10-16). The physical size of all control rod increments is uniform and the physical size of the core is also constant. Thus, variations in the worth of different increments of control rod would be expected to be the result of variations in L_T and ϕ_T in their respective vicinities. It seems reasonable to assume, for example, that if a small increment of rod is removed from the extreme top of the core or the extreme bottom of the core, where the flux is low, it will not be worth as much as if it is removed from the center of the core where the flux is high, and in general this is true. As a first approximation, the worth of individual increments along the length of a control rod is proportional to the square of the axial neutron flux distribution. For a perfect cosine flux distribution in the axial direction, and ignoring other complicating effects, the incremental control rod worth (i.e., worth per increment of travel) curve would appear like the bell shaped curve shown on Figure 12-36. Also shown on the figure is the "S" shaped integral

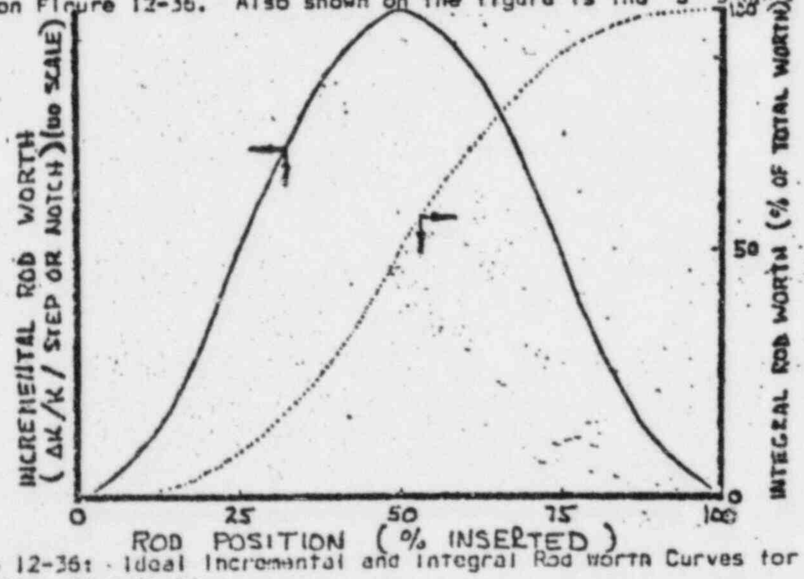

Figure 12-36: Ideal Incremental and Integral Rod Worth Curves for Cosine Axial Flux Distribution

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[1] Incremental worth is also called differential worth in some books.

1. What is the average heat flux throughout the core?
2. What is the average heat flux in a bundle operating at a BPF = 1.40?
3. What is the maximum heat flux in the above bundle if it has a peak APF = 1.65 near the core centerline and the maximum LPF at this elevation is 1.05?

Since 1 MW = 3.41×10^6 Btu/hr, the total reactor power in Btu/hr is:

$$\mathring{Q} = (3250)(3.41 \times 10^6) = 11.1 \times 10^9 \text{ Btu/hr}$$

The total heat transfer surface area is:

$$A = (193)(270) = 52{,}200 \text{ ft}^2$$

Assuming that = 97% of the heat passes through the cladding as heat flux, the core average heat flux is:

$$(\mathring{Q}/A)_{\text{core avg}} = \frac{(11.1 \times 10^9)(0.97)}{5.22 \times 10^4} = 207{,}000 \text{ Btu/hr-ft}^2$$

A bundle operating at a BPF of 1.4 would have an average heat flux of:

$$(\mathring{Q}/A)_{\text{bundle avg}} = (1.4)(207{,}000) = 289{,}000 \text{ Btu/hr-ft}^2$$

The peak heat flux in the bundle would be:

$$(\mathring{Q}/A)_{\text{peak}} = (289{,}000)(1.65)(1.05) = 502{,}000 \text{ Btu/hr-ft}^2$$

The third factor in the determination of CHFR or fuel center temperature is the condition of the coolant. We saw in Chapter 11 that the CHF is dependent upon quality in a boiling system and upon subcooling in a nonboiling system. The condition of the coolant also influences the heat transfer coefficient between coolant and cladding, which in turn affects the fuel center temperature. The most important factor which governs the condition of the coolant is its heat content per unit weight. For practical purposes, the heat content per pound of fluid is known as the fluid enthalpy, H, which has the units of Btu/lb. To completely determine the fluid properties, it is also necessary to know the pressure, since this governs whether or not the heat added to a fluid goes into raising its temperature, producing steam voids, or both. In other words, the pressure tells us whether the fluid is subcooled or saturated.

At every point in the core, there is a maximum allowable coolant enthalpy which can be tolerated without exceeding either of the heat transfer limits. The value of this allowable local enthalpy depends upon the actual heat flux at the point in question. For example, we can only tolerate a low enthalpy if the heat flux is high, but can tolerate a high enthalpy if the heat flux is low. Knowledge of the power distribution from the previously discussed peaking factors, which is equivalent to knowledge of the total heat added to the coolant, is not in itself sufficient to determine the coolant enthalpy. This is because you must also know how much coolant is being heated since enthalpy is defined as Btu per pound. In other words, a high heat flux may be perfectly acceptable if the coolant flow is high, but unacceptable if the coolant flow is low because the first case would result in a low enthalpy, but the second case would result in a high enthalpy.

Therefore, a new peaking factor which takes into account both heat addition and coolant flow is frequently used. This is the enthalpy rise peaking factor, $F_{\Delta H}$, given by the expression:

$$F_{\Delta H} = \frac{\text{enthalpy rise of coolant in flow channel of interest}}{\text{average enthalpy rise of coolant in the core}} \qquad (12\text{-}15)$$

In general, to evaluate the enthalpy rise of a quantity of coolant, the total heat addition to the quantity is divided by the flow rate. Thus, the average enthalpy rise of the coolant in the core (i.e., the denominator of equation (12-15)), would be obtained by dividing the total core power (in Btu/hr) by the total core coolant flow rate (in lb/hr).

Evaluation of the numerator, however, depends upon what one defines as a coolant flow channel. For BWRs, a flow channel always refers to the entire coolant volume contained within the limits of a single fuel bundle. In other words, the calculation is based upon the flow through the entire bundle. This is because the turbulence caused by voids, spacers, and the bundle nose piece produces nearly uniform mixing of the coolant within a bundle at any elevation[1]. For PWRs, however, uniform mixing within a bundle is not assumed. Instead, a coolant flow channel is defined as the region between four fuel rods as shown in Figure 12-35. This is basically

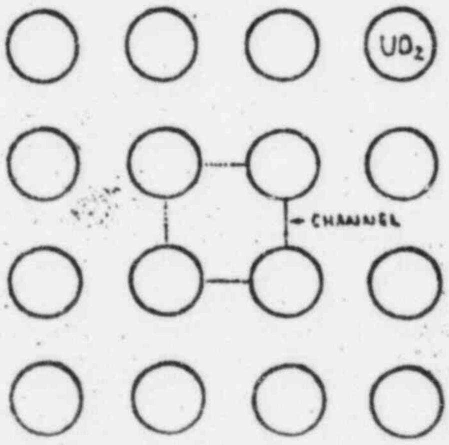

Figure 12-35: PWR Coolant Flow Channel Used in Definition of $F_{\Delta H}$

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[1] It is recognized, however, that around the high powered corner rods the quality or enthalpy will be slightly higher than the bundle average. This small amount of non-uniformity is factored into the CHF correlation.

because the PWR does not have the large void turbulance effect and there is less mixing[1].

Now then, to calculate the enthalpy rise in the peak BWR "channel" (i.e., bundle) we divide the total bundle power by the total bundle flow. The total bundle power is obtained by multiplying the average core heat flux times the bundle peaking factor times the area of the bundle. This would then be divided by the bundle flow rate, which is ordinarily determined by computer calculation. In the case where the fuel geometry is uniform (i.e., there is no orificing), the peak bundle flow rate will usually be slightly lower than the average bundle flow rate. This is because the peak bundle operates with a higher than average void content, and voids have a flow restricting effect. Ordinarily, however, the peak bundle flow rate is only slightly below average, generally 3 to 5%.

Example: In a certain BWR, the peak bundle operates with a BPF = 1.4 and with a coolant flow rate 96% as large as the average bundle flow rate. What is $F_{\Delta H}$ of the peak bundle?

First we will calculate the heat addition in the "average" bundle. Since it operates at a BPF = 1.0,

$$\dot{Q}_{avg} = (\dot{Q}/A)_{core\ avg} \times A_{bundle} \times BPF = (1.0) \times (\dot{Q}/A)_{core\ avg} \times A_{bundle}$$

Since no information was given, we cannot assign numerical values to $(\dot{Q}/A)_{core\ avg}$ or A_{bundle}.

The enthalpy rise in the average bundle, ΔH_{avg}, will be $\dot{Q}_{avg}$ divided by the average bundle flow rate, $\dot{m}_{avg}$, or

$$\Delta H_{avg} = (1.0) \times \frac{(\dot{Q}/A)_{core\ avg} \times A_{bundle}}{\dot{m}_{avg}}$$

The heat addition to the peak bundle is 40% greater than that to the average bundle, or

$$\dot{Q}_{peak} = (\dot{Q}/A)_{core\ avg} \times A_{bundle} \times BPF = (1.4) \times (\dot{Q}/A)_{core\ avg} \times A_{bundle}$$

The peak bundle operates at a flow of 0.96 $\dot{m}_{avg}$, so

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[1] However, some mixing does occur in a PWR, and this is accounted for by multiplying the $F_{\Delta H}$ by a mixing correction factor which is less than one (typically ≈ 0.92). In other words, the actual enthalpy rise in the peak channel is not quite as high as it would be without mixing.

$$\Delta H_{peak} = \frac{(1.4) \times (\dot{Q}/A)_{core\ avg} \times A_{bundle}}{.96\ \dot{m}_{avg}}$$

Then, since $F_{\Delta H} = \Delta H_{peak}/\Delta H_{avg}$,

$$F_{\Delta H} = \frac{(1.4) \times (\dot{Q}/A)_{core\ avg} \times A_{bundle}/.96\ \dot{m}_{avg}}{(1.0) \times (\dot{Q}/A)_{core\ avg} \times A_{bundle}/\dot{m}_{avg}} = \frac{1.4}{.96} = 1.46$$

We see that in a BWR, $F_{\Delta H}$ is nearly equal to the BPF if the flow distribution is reasonably uniform.

In a PWR, we are dealing with subcooled water, and all heat added to the coolant goes into raising the temperature in accordance with equation (11-3). To get enthalpy rise, the heat added is divided by the flow rate, so that:

$$\Delta H_{subcooled} = \frac{\dot{Q}}{\dot{m}} = \frac{\dot{m}C_p\Delta T}{\dot{m}} = C_p\Delta T \qquad (12\text{-}16)$$

Notice that the flow cancels out of the previous expression. That is, any flow imbalance which exists will reflect itself in the channel temperature rise. If two channels receive the same heat, the one with the lowest flow will have the highest temperature rise, and vice versa.

Example 1: The coolant in a PWR enters the core at 545° F and leaves at an average temperature of 595° F. However, the coolant at the outlet of a particular flow channel is 590° F. What is $F_{\Delta H}$ for this channel?

$$F_{\Delta H} = \frac{\Delta H}{\Delta H_{avg}} = \frac{C_p(590-545)}{C_p(595-545)} = \frac{45}{50} = 0.90$$

Example 2: The peak bundle in a particular PWR operates at a BPF = 1.45. Within this bundle, the peak rods operate at a LPF = 1.08. Assume that flow is uniform throughout the core. What is $F_{\Delta H}$?

This example is different from the last one in that no information is given on temperatures so that equation (12-16) is not very useful. However, we can use a procedure analogous to that used in the example on BWRs to calculate the channel enthalpy rises. That is, the heat addition to a channel can be obtained from knowledge of the core average heat flux and the appropriate peaking factors. In this case, we will have to know a BPF and an average LPF for the four rods surrounding the channel (which for this example we will assume to be 1.08). That is, the BPF essentially tells us how much heat is added to the average coolant channel in this bundle, and the LPF tells us what occurs in the hottest channel. Recall that in discussing BWRs, we didn't need to use a LPF in determining coolant properties because we assumed complete mixing within each bundle.

$$\dot{Q}_{peak} = (\dot{Q}/A)_{core\ avg} \times BPF_{peak} \times LPF_{avg\ peak} \times A_{channel}$$
$$= (\dot{Q}/A)_{core\ avg} \times 1.45 \times 1.08 \times A_{channel}$$

The enthalpy rise in the channel is simply the heat addition divided by the flow rate, which for this problem is $\dot{m}_{avg\ chan}$ since all channels receive the same flow.

$$\Delta H_{peak} = \frac{(\dot{Q}/A)_{core\ avg} \times 1.45 \times 1.08 \times A_{channel}}{\dot{m}_{avg\ chan}}$$

The expression for the enthalpy rise in the average channel will be the same as that for the peak channel except the peaking factors are all 1.0 for the average channel.

$$\Delta H_{avg} = \frac{(\dot{Q}/A)_{core\ avg} \times A_{channel}}{\dot{m}_{avg\ chan}}$$

Thus,

$$F_{\Delta H} = \frac{\Delta H_{peak}}{\Delta H_{avg}} = \frac{(\dot{Q}/A)_{core\ avg} \times 1.45 \times 1.08 \times A_{channel}/\dot{m}_{avg\ chan}}{(\dot{Q}/A)_{core\ avg} \times A_{channel}/\dot{m}_{avg\ chan}}$$

$$= 1.45 \times 1.08 = 1.57$$

For a PWR, then, $F_{\Delta H} = (BPF) \times (LPF)$.

For purposes of comparison, Table 12-1 summarizes the design peaking factors for current large (≈ 1000 MWE) BWRs and PWRs. It should be emphasized that these numbers are design figures, and in actual operation it is usually possible to achieve lower peaking. As can be seen, the total peaking factors for the two plants are not greatly different. The BWR has higher local peaking due to the control rod water gaps, but has lower axial and radial peaking due to the flux flattening effects of voids coupled with bottom entry control rods. Although the figures given in the table are representative, they will differ somewhat from plant to plant. In addition, the manufacturers are continually engaged in design work to reduce these peaking factors.

TABLE 12-1: DESIGN MAXIMUM PEAKING FACTORS FOR LARGE PWRs AND BWRs

	BWR	PWR
Local	1.24	1.08
Bundle	1.40	1.46
Axial	1.50	1.72
Enthalpy	--[1]	1.58
Total (LPF x BPF x APF)	2.62	2.72[2]

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[1] Highly dependent upon the precise orificing pattern which is used.

[2] Ordinarily, Westinghouse multiplies this "ideal" figure by an additional 1.04 to account for possible variations brought about by equipment tolerances, etc.

worth curve. This curve is simply the sum of the worth of all inserted notches or steps when the rod is at any particular axial position. The student should be sure that he understands the difference between the bell curve and the "S" curve.

For the ideal case shown in Figure 12-36, the strongest increment occurs when the rod is withdrawn one notch or step at the axial centerline of the core, and the curve is symmetrical about the centerline. Ordinarily, this is not the case, but rather the bell shaped curve is generally skewed toward either the top or bottom of the core depending upon factors such as those listed below:

1. In a BWR, the notches pulled from the top of the core are somewhat stronger than their symmetrical counterparts in the bottom of the core. Thus the peak notch worth occurs above the core centurline (i.e., the bell shaped curve on Figure 12-36 is skewed to the right). The reason for this is that when a rod which is inserted only a small distance into the bottom of the core is withdrawn an additional notch, the power will increase in the bottom of the core. This power increase in the lower core will cause the coolant to start boiling lower down in the core. The voids which are produced in the lower regions of the core will rise throughout the upper regions and tend to poison them, causing the power increase in the bottom of the core to be partially offset by a power reduction in the top of the core so that the net core power increase is small. The major effect of rod withdrawal from the bottom of the core is to shift the power distribution to the bottom. Conversely, when an increment of rod is withdrawn from the top of the core, the extra voids which are produced are already near the core exit and their poisoning effect on other core regions is minimized. Thus, the total core power increase is greater.

2. In a PWR operating at constant power, the strongest steps are usually located below the core centerline (i.e., the bell shaped curve on Figure 12-36 is skewed to the right). The reason for this is that when the rod is partially inserted (or when we are withdrawing steps from below the centerline) the flux is always skewed to the bottom of the core because of the rods in the top and the effect of the warmer water in the top. This enhances the worth of these notches relative to their symmetrical counterparts in the top of the core. One way in which the previous observation can be completely invalidated is if the reactor power distribution is skewed strongly to the top of the core due to insertion of part length rods into the bottom. In this case, the strongest steps may be those withdrawn from the top half of the core.

3. The hotter water or higher void content in the top of the core causes an increase in L_T in this region and tends to increase the notches withdrawn from the top relative to those withdrawn from the bottom.

INCREMENTAL BORON WORTH

Incremental (or differential) boron worth refers to the reactivity effect of each increment of dissolved chemical shim which is added to the core. As a first approximation, it takes about 100 ppm B to reduce reactivity by 1%Δk/k. However, 100 ppm added to a boron free core (i.e., going from 0 ppm to 100 ppm) will have a greater effect than adding 100 ppm to a highly borated core (going from 1000 ppm to 1100 ppm for example). The reason for this is that in the latter case, the newly added boron has to compete for neutrons with the boron already in the coolant, whereas the first boron in the coolant essentially has the field to itself. Thus, the incremental boron worth tends to drop somewhat as concentration increases. This behavior is shown in Figure 12-37.

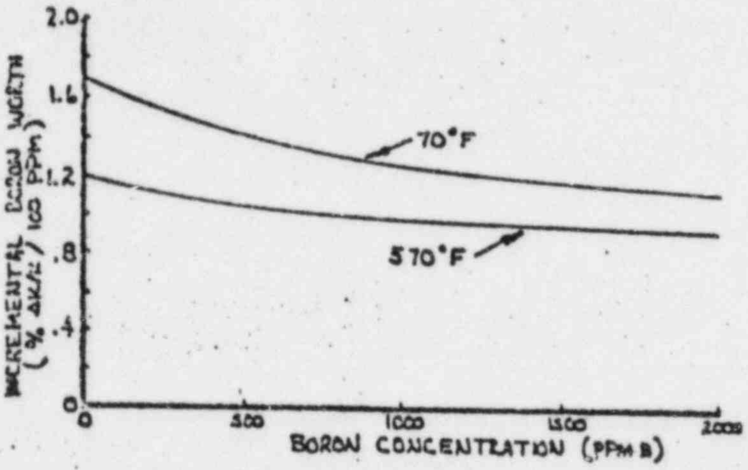

Figure 12-37: Typical Incremental Boron Worth Curves

Also note that the incremental worth drops as moderator temperature increases. This is because of the decrease in moderator density. In effect, 100 ppm B in 70° F represents more boron atoms than 100 ppm in 570° F water. This behavior, of course, is what tends to make the moderator temperature coefficient more positive as discussed in Chapter 10.

REACTOR STABILITY

Any system which operates with feedback mechanisms may become unstable if the feedback serves to amplify disturbances rather than damp them. A power reactor core is an example of a system operating with feedback. For example, reactor power, coolant flow rate (or flow distribution between parallel flow channels), and coolant density are interrelated quantities which feed back upon each other. Thus instability is theoretically possible in the reactor and much design work is done to insure that a given reactor system will, in fact, be stable.

A BWR generally has a lower stability threshold (i.e., is more likely to be unstable) for hydraulic-reactivity coupled instability than a PWR of comparable size because of the larger fluctuations in local coolant density within the core during power operation. As a result, our discussion of this topic will be directed principally toward BWRs, although it should be recognized that PWRs are also susceptible to this type of instability.

Although the reactor may become unstable for many reasons, the sequence of events is usually a variation of the following:

1. A random disturbance causes an increase in reactor power and a subsequent increase in core void fraction.

2. The production of voids causes a power reduction.

3. The power reduction causes a reduction in the core void fraction.

4. The reduction in core voids causes a power increase and the cycle repeats itself.

Oscillations of the type described above occur continuously in the boiling regions of a BWR and are frequently referred to as "boiling noise". In a stable reactor, "boiling noise" oscillations are small (generally <5% of full power, peak to peak) and are not divergent. Divergent means that the amplitude of the oscillations increases with time. In the unstable case, of course, the oscillations do diverge. Under experimental conditions, oscillations of ± 100% around the average power have been observed. The term "chugging" was coined to describe the reactor behavior during early experiments with the "Borax" reactors[1]. In "chugging" void formation was so rapid on the power upswings that much of the unboiled coolant was literally lifted from the core. On the power downswing this large mass of coolant would fall back into the core to restart the cycle.

Power instabilities are frequently accompanied by, or are caused by, hydraulic instabilities (i.e., oscillations in coolant flow). For example, the formation of large amounts of voids in a flowing fluid will markedly increase the frictional losses that the coolant suffers along its flow path. Unless additional pumping power is provided, the increased friction will result in a lower flow. Thus, on the power upswing portion of a large oscillation, the increased void fraction can block off the coolant flow--which compounds the problem by allowing a higher void fraction than would have resulted had flow not dropped. Then on the downswing portion, the voids begin to disappear and suddenly the flow gets switched back on. Oscillations like this do not necessarily affect the core as a whole. That is, the coolant has a choice of many parallel paths as it enters the

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[1] "Borax" refers to boiling reactor experiment. One of the principal early uncertainties with the BWR concept was its ability to operate in a stable manner. A large amount of theoretical work and experimental work including the "Borax" experiments, was therefore done in this area to verify the feasibility of the BWR.

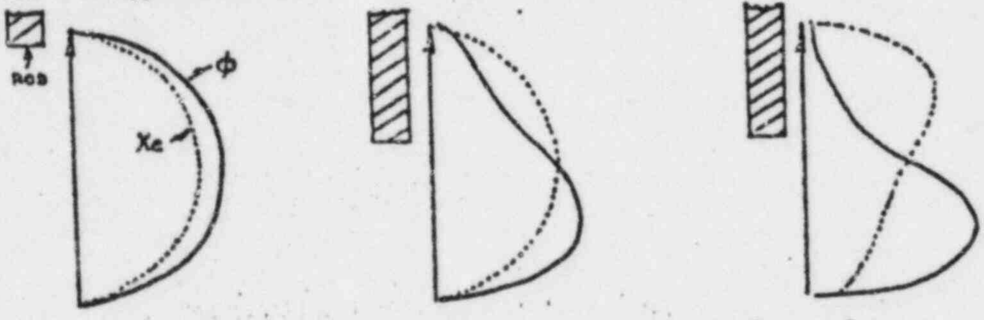
core. It is possible for flow to alternatively be choked off in one portion of the core and then another, so that core flow and core power "slosh" back and forth across the core. In addition to being caused as a result of feedback from power oscillations, hydraulic instability may result when the flow characteristics of the core vary markedly from one region to another due to effects like localized scale buildup, etc. Finally, there is one mechanism by which hydraulic instability may be initiated without warning. This is when something within the vessel breaks or becomes loose. It is easy to envision the effects of a piece of metal rattling around loose in the bottom of the vessel and alternately blocking and unblocking several coolant passages. As a result, the operators must always be alert to any signs of reactor instability. The onset of instability is accompanied by increased fluctuations in feedwater flow, generator output, reactor power, water level, and pressure over and above those attributable to boiling noise and minor fluctuations in systems like the automatic feedwater control. The smoothness of plant response to minor transients brought about by routine testing of such components as the main steam isolation valves also gives a good indication of stability.

The question arises as to what promotes stability in situations such as those described above. The answer is that anything which acts to maintain the status quo promotes stability most of the time. The latter qualification may seem to be nonsense, but it is a fact that a phenomenon which is very effective in maintaining stability under one set of circumstances may cause the reactor to run away under another. Consider, for example, the negative void coefficient. Normally, this coefficient is extremely effective in damping out oscillations because it tends to oppose either a rise or a fall in power level. However, if the magnitude of the coefficient is overly large due to a poor choice of the moderator fuel ratio, reactor power may tend to overcompensate for small changes in void volume and oscillatory conditions may result. Nevertheless, certain factors nearly always aid BWR stability. One of these is high reactor pressure. The higher the pressure the smaller the volume of any given mass of voids, and hence the less the chances of flow choking and overly large reactivity effects. Borax experiments showed that chugging was virtually impossible to initiate when reactor pressure was >600 psig. A second factor is the "time constant" of the fuel for heat transfer. UO_2 rod type fuel tends to retain heat that it generates and releases it rather slowly to the coolant. This tends to damp out rapid fluctuations in core voids. It also brings the doppler coefficient more strongly into play, which acts to maintain the status quo.

The BWR has been proven to be an exceedingly stable device as long as it is operated within its design power and pressure limitations. Nevertheless, any BWR will become unstable if operated at high enough power levels due to excessive void formation and the flow choking and reactivity oscillations this produces. However, tests have shown that this level is generally above the limit which is placed upon the reactor power for purposes of heat transfer capability.

Reactor instability in either a PWR or BWR can be brought about by the automatic control systems. For example, fluctuations in the reactor pressure regulator of a BWR will cause power fluctuations due to alternate compression and decompression of voids. In the design of a complex control system like that employed in a PWR, it is important to recognize the various time lags that are inherent in any physical process. For example, the RTDs used to measure coolant temperature do not respond instantaneously, a rise in reactor power does not produce an immediate increase in T_{avg}, etc. Unless the control system takes these time lags into consideration, the controllers may get out of phase with the plant and cause instability.

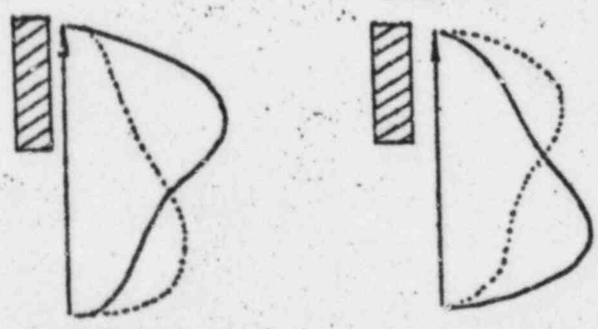
One potential type of reactor instability which has received considerable attention with the advent of the large second and third generation reactors is that brought about by spatial xenon concentration fluctuations. These fluctuations can be initiated by virtually any change in reactor power level and/or power distribution. As an example, consider a PWR operating at full power with all rods essentially withdrawn. The axial flux distribution will approximate a cosine distribution as shown in Figure 12-38(a).



a) Initial Conditions Time = 0

b) Immediately after Rod insertion Time = 0+

c) Time = 6 hours After Rod Insertion



d) Time = 24 Hours After Rod Insertion

e) Time = 48 Hours After Rod Insertion

Figure 12-38: Example of an Undamped Axial Xe Oscillation Following a Change in Axial Power Distribution in a PWR and the xenon distribution will have a similar axial shape. The operator then initiates a boron dilution and allows the rods to drive in to compensate for the chemical shim removal to maintain constant total reactor power. The result of this operation is to skew the power distribution toward the bottom of the core as shown in Figure 12-38(b). Since changes in xenon concentration have a time delay associated with them, the xenon distribution will still retain its initial shape. However, after a few hours, the high flux in the bottom of the core will result in xenon burnout in this region and the reduced flux in the top of the core will result in xenon buildup. Thus the xenon distribution will be skewed to the top of the core, thereby enhancing the flux peak in the bottom of the core as shown in Figure 12-38(c). It is possible that this effect could cause the core to approach DNB and/or center melting limits in the lower region. In any event, the high flux in the bottom of the core relative to the top will eventually result in the xenon distribution shifting to the bottom of the core, which in turn tends to drive the flux back to the top as shown in Figure 12-38(d). Under certain reactor conditions, and in the absence of appropriate operator control action, these axial power oscillations can continue and even diverge.

Although it is desirable to have a reactor which is inherently capable of damping out xenon oscillations without recourse to operator action, this feature is not a necessity because the oscillations are so slow (the period is on the order of a day) that corrective operator action is always possible. Nevertheless, it is instructive to consider some of the factors which will promote stability against xenon oscillations.

Spatial xenon oscillations are concerned with the shift of power from one region of the core to another. Therefore, it is useful to adopt the viewpoint that the total reactor is composed of two (or sometimes more) separate reactors sitting side by side. Those regions in which power is rising must have a local excess reactivity, and those regions where power is falling must have a local negative reactivity. Thus, even though the reactivity for the entire core may be zero (i.e., total core power is constant), it is possible that one region of the core may be very slightly supercritical and another very slightly subcritical.

The rate of change of regional power with respect to time, ΔP/Δt, is directly related to the net reactivity of the region, and can be qualitatively described by the following relationship:

$$\frac{\Delta P}{\Delta t}_{region} = \Delta k_{\infty} + \alpha P + \Delta k_{leakage} - \rho_{Xe} C_{Xe}$$

where: $(\Delta P/\Delta t)_{region}$ = rate of change of regional power

Δk_{∞} = excess reactivity that the region would possess if there was no leakage into or out of the region and if the region was at zero power with no xenon

α = power coefficient of region (Δk/unit power). If the power coefficient is negative, this will be a negative number.

P = regional power

$\Delta k_{leakage}$ = reactivity effect of leakage. Leakage out of the region will have a negative sign, and leakage into the region (from adjacent regions) will be positive.

ρ_{Xe} = reactivity effect of an atom of xenon

C_{Xe} = number of xenon atoms in the region

Thus in determining the net reactivity in the region we start with the reactivity associated with the cold clean k_∞ of the region, which is determined by enrichment, control rod position, chemical shim concentration, etc. To this we must add or subtract the reactivity effect of regional power, which is given by the product of the power coefficient and the power. Ordinarily, this will reduce the regional reactivity since the power coefficient is negative. The next effect is leakage. There will always be some leakage out the sides of the reactor, which is a negative effect. However, leakage can also occur between regions of the core. Leakage always occurs from regions of high neutron population to regions of low population. Thus, if the region of interest is at a lower power than surrounding regions, there could be a net leakage into the region so that leakage constitutes a positive local reactivity effect. Finally, there will be a reactivity loss due to neutron absorption in xenon. The total xenon effect can be considered to be the reactivity effect per xenon atom times the number of xenon atoms available.

If the sum of all of these reactivity effects adds up to zero, the power in the region will be stable. The object is therefore to have the reactor designed so that if any of the reactivity effects changes (i.e., a change in regional power, xenon concentration, etc.) the others will automatically compensate in such a manner as to keep the regional reactivity reasonably well balanced.

Now let us consider how this relationship applies to the lower half of the core in the previous example of a PWR which has undergone a change in power distribution. In the lower half of the core, the local Δk_∞ was initially increased (because chemical shim was removed from the lower half of the core and no control rods were inserted into this region to compensate for it) and power in this region started rising. The faster we can stop this power rise, the smaller will be the upset in the gross core power distribution and the more likely we are to be stable. Stability will thus be enhanced by:

1. More negative power coefficient

   The more negative the power coefficient, the faster the excess reactivity in the lower half of the core will be dissipated. If we prevent a large increase in power in the lower region, we will not suffer as much extra xenon burnout and thus will minimize the shift in the xenon distribution.

2. Increased leakage

As the neutron flux in the lower half of the core increases, the potential for increased neutron leakage out of this region is increased. If we can take advantage of this potential we will terminate the transient more rapidly. Regional leakage will be enhanced by:

A. Small core

Small cores are inherently more stable against xenon oscillations than are large cores.

B. Increased migration length

The farther neutrons travel, the more likely they are to undergo leakage.

C. Non-uniform power distribution

If the power distribution is flat, this means that leakage from one region to another is minimized since the neutron population in adjacent regions is about the same.

3. Lower power density

Xenon is produced by iodine decay and is removed by decay and burnout. The production by iodine decay will increase when the power in the region increases, but there is a rather long time delay before this is felt. Similarly, the removal by decay does not change rapidly when the local power changes. If only these effects had to be considered the changes in xenon concentration would be so small and so slow following a change in regional power that no significant transients could develop. However, at high power densities, removal by burnup is significant. Since this removal mechanism has no time delay associated with it, variations in local xenon burnup rate will cause large changes in local xenon concentration. Thus, as power density increases and removal by burnup becomes the dominant removal mechanism, changes in local power density cause progressively larger changes in local xenon concentration, resulting in larger local reactivity swings, and the reactor tends to be more unstable. Conversely, at lower powers where burnout is less important, the reactor tends to be more stable.

The strong negative power coefficient of BWRs make them somewhat more stable against sustained spatial xenon oscillations than equivalent sized PWRs. Even in the largest BWR cores, xenon oscillations should be highly damped. However, it is not expected that the large twelve foot long PWRs will be inherently stable against axial xenon oscillations early in the life of the first core when the power coefficient is least negative. As a result, these reactors have been provided with part length control rods (rods which have a poison section 25% as long as the active core length) to enable the operator to counteract these oscillations if they occur. If a local flux peak tended to occur, such as in the bottom of the core at the start of the previous example, the part length rods would be moved into this region to act as a local negative reactivity effect in order to help damp out the transient.

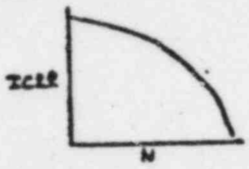
We have only discussed axial xenon oscillations, but radial (side to side) and azimuthal (around the vertical axis of the core) are also possible. Usually, however, the core is more stable against these other types than against axial oscillations. In general, the power distribution is reasonably symmetrical in these directions and there are not large power distribution variations in these directions as sometimes occur in the axial direction.

PROBLEMS

1. In the initial loading of a BWR, the source is loaded prior to any fuel and a countrate of 35 cps is obtained on a detector located in the core. Four fuel bundles are loaded between the source and detector and the countrate increases to 900 cps.

   A. Based upon this data, what is k_{eff} with four bundles?
   B. Discuss the probable accuracy of the previous result.

2. An ICRR curve shaped as shown in the figure would most likely be



   the result of:

   A. Detector too far from source;
   B. Loading between detector and source;
   C. Detector too close to source.

   Explain.

3. A. Explain the general shape of an ICRR curve for a PWR which is borated sufficiently to make $k_\infty = 0.91$.
   B. If the countrate is 50 cps when the source and first bundle are loaded (assume one bundle core has a $k_{eff} = 0.3$), what is the maximum possible countrate for the reactor in part (A) if detector geometry remains constant throughout?

4. A. Predict the critical loading for the following data:

Elements	CR
0	12
4	13
8	14
10	15
12	20
13	30
14	60

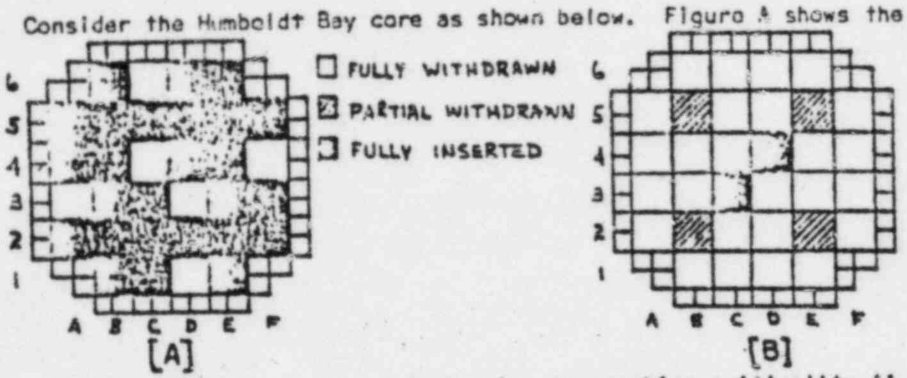
   B. What does the shape of the ICRR indicate about the source-fuel-detector geometry?

5. Assume that a certain BWR has a $k_{eff} = 0.95$ with all rods inserted. Assume further that the worth of the rod notch which takes the reactor just critical is .001 Δk. By what factor will the CRR have increased just prior to pulling the notch which places the reactor critical if detector geometry can be assumed to remain constant throughout the rod withdrawal?

6. As rod withdrawal progresses during an approach to critical, the increases in countrate get progressively larger and take longer to equilibrate. Why does this occur?

7. Sketch the trace you would expect to see at your facility on the most sensitive nuclear instrumentation as the reactor is started up and placed on a 60 second period.

8. Consider the Humboldt Bay core as shown below. Figure A shows the core at ambient temperature just prior to reaching criticality (i.e., criticality can be expected to occur when one more rod is withdrawn). Figure B shows the rod pattern at full power.



   A. If the object is to go critical on a weak rod in order to minimize the chances of a nuclear excursion, which would you pull, C-3, C-1, or D-5? Why?
   B. Would you expect the worth of withdrawing C-3 to be greater in pattern A or pattern B? Why?
   C. In pattern B, which would cause the greatest reduction in power, insertion of A-5 or B-4? Why?

9. Discuss the relative advantages and disadvantages of PWR control programs which result in constant T_{avg}, constant ‾$_c$, and constant $T_{secondary}$.

10. Describe the response of your facility to an opening of the generator OCB at full power.

11. Discuss the response of your plant to a 20% load pickup at the maximum rate by the normal method.

12. A. If a bundle in your core is operating at a BPF = 1.25 at full power, how many MWT is it producing?
    B. What is the heat flux in the above bundle at a point where the APF = 1.4 and the LPF = 1.1?

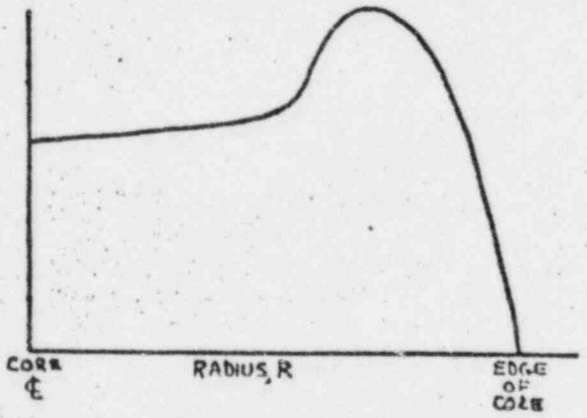
13. Consider a 300 bundle BWR operating a 600 MWT (or 2.05 x 10^9 Btu/hr). Each bundle has a total of 60 ft^2 of heat transfer surface area. At full power, the hardest working bundle in the core is operating at a

1330

bundle peaking factor of 1.35. The corner rod (local) peaking factor is 1.33. At a point 10 inches below the core centerline, the axial peaking factor is 1.43 and the critical heat flux is 750,000 Btu/hr-ft^2. At a point 12 inches above the core centerline, the axial peaking factor is 1.24 and the critical heat flux is 600,000 Btu/hr-ft^2. What is the CHFR at each location at full power? The operating license specifies that the minimum CHFR must be greater than 1.5 at 125% power. Is this limit being met?

14. **Discuss the advantages and disadvantages of control rods and chemical shim in terms of their effectiveness in controlling the radial and axial power distribution, and their effects upon the operating characteristics of the core.**

15. **If the smoothed out radial power distribution in a BWR looked as shown below, what would it indicate about the control rod pattern?**



16. Explain why the worth of a control rod changes if another rod is inserted immediately adjacent to it. This is called rod shadowing.

17. Sketch the axial flux profile you would expect at your facility for normal full power operation at beginning of core life and at end of core life. Explain any differences.

18. Sketch an incremental rod worth curve for your core, and explain its shape and why the peak is located where it is.

19. A 500 MWE PWR is typically zone loaded with three concentric rings of fuel. A 1000 MWE PWR is generally loaded with the three enrichments, but the two lower ones mixed together in checkerboard fashion. Why?

20. A. Define and explain $F_{\Delta H}$.
    B. Discuss what information is required and the general procedure used to determine the MCHFR at your facility.

21. If a PWR operates with T_c = 545° F and T_H = 595° F at full power, what is the core exit temperature of a coolant channel operating with $F_{\Delta H}$ = 1.2?

22. The failure of a core support structure within the reactor vessel causes a significant flow reduction to several fuel assemblies on one side of the core. What are the potential consequences of this event? What, if any, indications do you have to inform you of this occurrence?

23. Discuss the effects of reactor pressure and power level on BWR stability.

24. How do the following influence the threshold for xenon instability?

    A. Power density;
    B. Core size;
    C. Power distribution;
    D. Power coefficient.

CHAPTER 13

CHEMICAL, RADIOCHEMICAL, AND WASTE DISPOSAL CONSIDERATIONS IN WATER REACTOR OPERATION

INTRODUCTION

Over the long run, successful operation of a power plant depends, in large part, upon the success of the chemical control program. The chemical program is basically directed toward four goals:

1. Prevention of equipment failure brought about by chemical attack.
2. Maintaining peak equipment performance levels and minimizing required maintenance.
3. Minimizing the spread of radioactive materials throughout the plant.
4. Control and disposal of process wastes in compliance with established standards.

Most of these goals involve the application of "conventional" chemical principles, particularly those involved with the control of corrosive attack of system materials and the minimizing of scale buildup on heat transfer surfaces. However, these considerations are complicated by the presence of chemical reactions which result from the ionizing effects of reactor radiation energy and the myriad of nuclear reactions which occur in and around the primary system.

In the first part of this chapter we will discuss a number of chemistry-oriented topics to provide a background for later discussions of the particular chemistry programs at reactor plants.

ACIDS, BASES AND pH

Acids and bases are among the most important of all chemical compounds. An acid is defined as a compound which releases the positive hydrogen ion, H^+, to a solution. A base is a compound which releases the negative hydroxyl ion, OH^-, to a solution. An example of a common acid is hydrochloric acid, HCl. This molecule contains a hydrogen ion H^+ bonded to a chloride ion Cl^-. In other words, the hydrogen atom has transferred its electron to the chlorine atom in the manner previously discussed in Chapter 3 under ionic reactions. A common base is sodium hydroxide (or caustic soda), NaOH. This contains the ions Na^+ and OH^-. Acids and bases will <u>neutralize</u> each other if placed together. For example:

$$NaOH + HCl \longrightarrow H_2O + NaCl \qquad (13\text{-}1)$$

In this reaction the H^+ and OH^- ions have combined to form water while the Na^+ and Cl^- ions have combined to form table salt (which would be ionized or dissolved by the water). Thus, in a neutralization reaction, both the acid and the base are destroyed and water is formed.

The acidity or alkalinity of a solution is determined by the relative concentrations of H^+ and OH^- ions in the solution. That is, if the concentration of H^+ ions exceeds the concentration of OH^- ions, the solution is acidic and vice versa. If the concentrations of the two ions are equal, the solution is said to be neutral. The measure of acidity or alkalinity is the pH of a solution. If the pH is 7, the solution is neutral. If the pH is <7, the solution is acidic, and if the pH is >7, the solution is basic. To get a feel for the significance of pH figures, pH 6-8 is essentially neutral, 4-6 is weakly acidic, <4 is strongly acidic, 8-10 is weakly basic, and >10 is strongly basic.

CORROSION OF MATERIALS

The general or uniform corrosion of the most prevalent materials in a closed cycle water cooled reactor, carbon or stainless steel, is illustrated by the corrosion of iron in pure water. At the elevated temperatures found in reactor primary systems iron corrodes to form an oxide called magnetite, Fe_3O_4, according to the following overall equation:

$$3Fe + 4H_2O \longrightarrow Fe_3O_4 + 4H_2 \qquad (13\text{-}2)$$

The detailed mechanism, which has been the subject of much investigation but about which much remains to be learned, is considered to be as follows. The hydrogen ions in water exert a greater attractive force upon the outer electrons in the iron atom than does the iron nucleus, and thus are able to ionize the iron in accordance with the following equation:

$$Fe + \underbrace{2H^+ + 2OH^-}_{2H_2O} \longrightarrow Fe^{++} + 2(OH^-) + H_2 \qquad (13\text{-}3)$$

The Fe^{++} and OH^- ions form the relatively insoluble ferrous hydroxide, $Fe(OH)_2$, which forms a film on the surface of the iron. Ferrous hydroxide decomposes to form magnetite, hydrogen, and water.

$$3Fe(OH)_2 \longrightarrow Fe_3O_4 + 2H_2O + H_2 \qquad (13\text{-}4)$$

The physical picture of the growing oxide film is represented by Figure 13-1. At the metal-oxide interface, ferrous ions and free electrons are

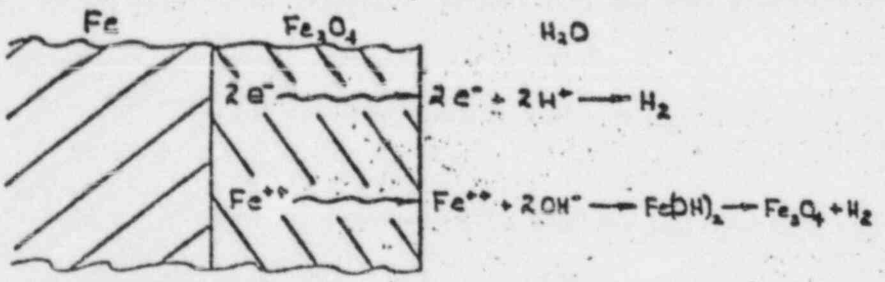


Figure 13-1: Corrosion of Iron at High Temperatures

formed and subsequently diffuse through the oxide film to the oxide-water interface where they are used up in the formation of H_2 and $Fe(OH)_2$. The rate of the reaction appears to be limited by the diffusion time of ferrous ions through the oxide film. As the film builds up the corrosion rate goes down, and the oxide can therefore be considered to be a protective coating for the metal. However, it appears that there are limits to the maximum oxide thickness (usually $<.01$ inch) due to breakdown of the outer surfaces, and so an equilibrium corrosion rate is ultimately reached.

In addition to ferrous ions diffusing through the oxide to the oxide-water interface, hydrogen ions and hydrogen atoms also diffuse in the opposite direction through the oxide to the metal-oxide interface. Hydrogen then enters the metal structure itself and can cause reactions with the carbon in the steel resulting in decarburization and embrittlement of the metal.

One of the important variables which influences the corrosion rate of iron and steel is the pH of the water. The high concentration of H^+ ions in acidic solutions greatly accelerates the ionization of iron in accordance with equation (13-3), and in addition prevents the formation of the $Fe(OH)_2$ protective film on the metal (because the H^+ ions neutralize the OH^- ions and reduce their concentration). Thus, neutral or preferably high pH water is desirable from a corrosion limiting standpoint.

The above discussion is concerned with general corrosion of iron and steel. There are also a number of corrosion mechanisms which occur under various conditions in localized regions. Two of the more important of these are:

I. Chloride Stress Corrosion Cracking

Austenitic stainless steels (300 series) are particularly susceptible to stress-assisted corrosion in high temperature water containing chloride ions and dissolved oxygen. When the protective magnetite film is under tensile stress, the bonds between the iron and oxygen atoms are stretched, often almost to the breaking point. Chlorine atoms which are in the vicinity of stressed magnetite have a tendency to replace oxygen atoms in the magnetite lattice, thus converting it from Fe_3O_4 to $FeCl_3$. The reason this is possible is twofold:

A. Chlorine tends to attract electrons to fill its nearly complete outer electron shell just as does oxygen. Thus chlorine is capable of assuming the electron attraction (from iron) function which was provided by oxygen in the magnetite structure.

B. The chlorine atom is physically larger than the oxygen atom and it tends to fit better in the stretched lattice than does the oxygen atom. That is, when the larger chlorine atom is substituted for oxygen, the bonds can shrink back closer to their normal length in the stressed lattice.

The ferric chloride thus produced in regions of tensile stress is subject to hydrolysis according to the equation:

$$2FeCl_3 + 3H_2O \longrightarrow Fe_2O_3 + 6HCl \qquad (13\text{-}5)$$

which results in a local reduction in pH (due to production of hydrochloric acid). The hydrogen ions then attack the metal.

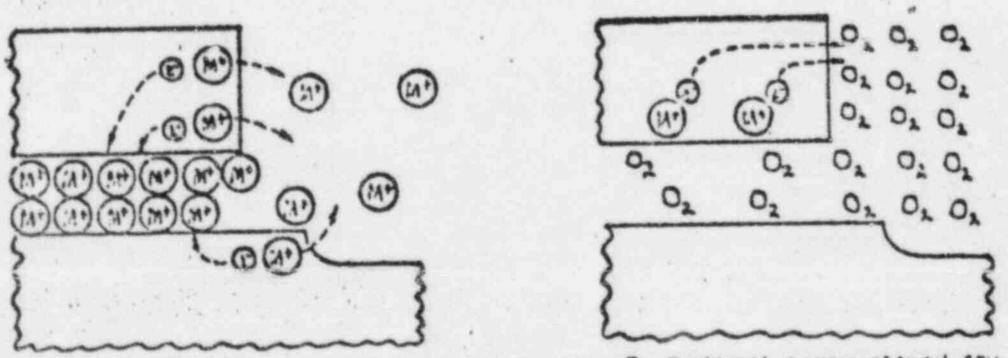
Stress corrosion is greatly enhanced by dissolved oxygen in the water. This appears to result from the fact that oxygen promotes a thicker magnetite film which in turn has a greater capacity to function as a reservoir to concentrate the chloride ions.

2. Crevice Corrosion

Corrosion tends to occur in crevices which are exposed to a corrosive environment. From an engineering standpoint, crevice corrosion is important because it can result in binding or seizure in bearings of rotating equipment, seizure of nuts and bolts, etc.

Corrosion in crevices can occur because of: A) an increase in metal-ion concentration within the crevice (as compared to the concentration outside the crevice), which may then result in concentration cell corrosion; B) a decrease in oxygen concentration within the crevice (as compared to the concentration outside the crevice), which may then result in concentration cell corrosion; or C) increased corrosivity resulting from the accumulation of soluble and insoluble corrosion products formed within the crevice. All three of these mechanisms are the result of little or no fluid circulation through the crevice.

The action of the concentration cells is illustrated in Figure 13-2.



A. METAL-ION CELL

B. OXYGEN CONCENTRATION CELL

Figure 13-2: Concentration Cell Action in Crevice Corrosion

In Figure 13-2(a), the high concentration of metal ions in the crevice (which build up due to general corrosion and are not swept away) tends to attract electrons away from the surface of the metal which bounds the main body of the fluid where there are fewer metal ions. On these latter surfaces, metallic atoms are being ionized and are going into solution. Thus the corrosion takes place along those surfaces which face the main volume of fluid. The principle of the oxygen cell is the same (O_2 attracts electrons to form $O^=$ ions) except that the direction of electron flow is reversed. That is, the high concentration of O_2 molecules in the main body of coolant tends to pull electrons away from the metal surfaces which are adjacent to water which has very little dissolved oxygen. The oxygen content in the crevice would tend to be lower than that in the main body of the coolant because the oxygen would be used up in the formation of metallic oxides and would not be replenished if there was little flow into the crevice. Thus, in the oxygen cell, the corrosion would occur on the metal surfaces which face the liquid in the crevice.

Crevice corrosion is strongly influenced by geometry. The closer the tolerances in a crevice, the greater will be the crevice corrosion. As expected, if the surfaces forming the crevice are in motion relative to each other, crevice corrosion is reduced due to the resultant mixing action. It also follows that the presence of dissolved oxygen greatly increases crevice corrosion.

SOURCES OF RADIOACTIVITY IN REACTOR COOLANT

From a chemical standpoint, the accumulation of activity in the coolant, or the possibility of producing such an accumulation, is one thing which makes a nuclear plant unique from conventional power plants. In dealing with the "ordinary" chemical problems like corrosion, the chemist is severely limited in the chemicals he may add to the coolant to inhibit these processes because of the possibility of them becoming intensely radioactive as they pass through the core. In addition, the buildup of radioactivity in the coolant governs, to a large degree, the radiation levels in which the operators must perform their duties and also dictates the amount of treatment which must be given to process wastes before they can be discharged to the environs.

There are several general sources of activity in the coolant. These include:

1. Activation of Corrosion Products

   The materials used in the construction of the primary system include stainless steel (principally Fe, Cr, Ni, Mn), Inconel (principally Cr and Ni, with lesser amounts of Fe, Mn, and Cu), zircaloy (principally Zr), and carbon steel (Fe; small amount of Mn). In addition, nickel is invariably contaminated with cobalt.

From these constituents, a wide variety of activation products are formed by neutron activation as shown in Table 13-1 below. These activation products are released to the coolant as the structural material corrodes or erodes. Some of this activity remains in solution and is continually circulated throughout the primary system. A large amount of it, however, ends up in various insoluble oxide forms which are commonly known as "crud". Some of the crud is filtered out of the water in the coolant purification systems, and some of it simply stays suspended in the coolant and is continuously recirculated. Much of the crud, however, deposits itself on the metallic surfaces throughout the primary system. The highly radioactive and high temperature fuel heat transfer surface, as well as any relatively stagnant areas such as crevices are particularly susceptible to crud deposition.

TABLE 13-1: SOURCES OF COOLANT CORROSION PRODUCT ACTIVITY

Stable Parent		Activation Product		Activation Product Decay Products
${}_{26}Fe^{58} + {}_{0}n^{1}$	$\longrightarrow \gamma +$	${}_{26}Fe^{59}$	$\xrightarrow{45d}$	${}_{27}Co^{59} + \beta^- + \gamma$
${}_{27}Co^{59} + {}_{0}n^{1}$	$\longrightarrow \gamma +$	${}_{27}Co^{60}$	$\xrightarrow{5.2\ yr}$	${}_{28}Ni^{60} + \beta^- + \gamma$
${}_{25}Mn^{55} + {}_{0}n^{1}$	$\longrightarrow \gamma +$	${}_{25}Mn^{56}$	$\xrightarrow{2.6\ hr}$	${}_{26}Fe^{56} + \beta^- + \gamma$
${}_{24}Cr^{50} + {}_{0}n^{1}$	$\longrightarrow \gamma +$	${}_{24}Cr^{51}$	$\xrightarrow{27d}$	${}_{23}V^{51} + \gamma$ (K capture)
${}_{40}Zr^{94} + {}_{0}n^{1}$	$\longrightarrow \gamma +$	${}_{40}Zr^{95}$	$\xrightarrow{65d}$	${}_{41}Nb^{95} + \beta^- + \gamma$
${}_{29}Cu^{63} + {}_{0}n^{1}$	$\longrightarrow \gamma +$	${}_{29}Cu^{64}$	$\xrightarrow{12.8\ hr}$	${}_{28}Ni^{64} + \beta^+$ (or K capture)
			$\longrightarrow$	${}_{30}Zn^{64} + \beta^-$
${}_{28}Ni^{58} + {}_{0}n^{1}$	$\longrightarrow {}_{1}p^{1} +$	${}_{27}Co^{58}$	$\xrightarrow{71d}$	${}_{28}Ni^{58} + \beta^- + \gamma$
${}_{26}Fe^{54} + {}_{0}n^{1}$	$\longrightarrow {}_{1}p^{1} +$	${}_{25}Mn^{54}$	$\xrightarrow{303d}$	${}_{24}Cr^{54} + \gamma$ (K capture)

In PWRs employing chemical shim, it has been noted that an increase in coolant pH produces a slow (time constant ≈ 10 hr) increase in core reactivity. The reason for this effect is not definitely known, but it is thought to be related to crud deposition in the core. High pH could reduce the crud thickness on the fuel which could A) improve heat transfer coefficient resulting in lower fuel temperatures and doppler reactivity gain, B) reduce parasitic absorption by the crud, and C) reduce surface temperature on fuel and resulting nucleate boiling void formation, causing reactivity gain through the void coefficient. It is likely that all of these mechanisms, and possibly others, contribute to the phenomenon to a greater or lesser degree.

After a plant has operated at full power for several months, it obtains near equilibrium crud levels in the coolant. However, on subsequent startups and sometimes on large load changes, the crud level in the coolant frequently increases by a factor of 10 to 20. These incidents are termed "crud bursts," and are thought to result principally from thermal shocks. Other contributing mechanisms may

1380

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be the result of changes in water chemistry during outages. For example, oxygen is introduced when the primary system is opened for maintenance, and this may influence the chemical structure of the crud. In addition, PWRs are highly borated during outages with a subsequent reduction in pH. One interesting thing about crud bursts is that some plants seem to have more trouble with them than others, even when the plants are very similar, so there must be some very subtle effects involved which have not yet been discovered.

2. Activation of Water

The largest source of coolant activity during actual high power operation results from activation of the water molecule itself. Fortunately, most of the principal contributors to this activity are very short-lived, and so this activity disappears rapidly following a shutdown. Table 13-II lists some of the reactions which occur in the coolant.

TABLE 13-II: ACTIVATION PRODUCTS OF WATER

Parent Nucleus		Activation Product		Activation Product Decay Products
${}_8O^{16} + {}_0n^1$	$\longrightarrow$	${}_1p^1 + {}_7N^{16}$	$\xrightarrow{7.4\ sec}$	${}_8O^{16} + \beta^- + \gamma (= 6\ mev)$
${}_8O^{16} + {}_1p^1$	$\longrightarrow$	${}_2He^4 + {}_7N^{13}$	$\xrightarrow{10\ min}$	${}_6C^{13} + \beta^+$
${}_8O^{18} + {}_1p^1$	$\longrightarrow$	${}_0n^1 + {}_9F^{18}$	$\xrightarrow{110\ min}$	${}_8O^{18} + \beta^+$
${}_8O^{18} + {}_0n^1$	$\longrightarrow$	$\gamma + {}_8O^{19}$	$\xrightarrow{29\ sec}$	${}_9F^{19} + \beta^- + \gamma$
${}_8O^{17} + {}_0n^1$	$\longrightarrow$	${}_1p^1 + {}_7N^{17}$	$\xrightarrow{4.2\ sec}$	${}_8O^{17} + \beta^- + \gamma$
${}_1H^2 + {}_0n^1$	$\longrightarrow$	$\gamma + {}_1T^3$	$\xrightarrow{12.3\ yr}$	${}_2He^3 + \beta^-$

The great abundance of the ${}_8O^{16}$ isotope (99.8% of naturally occurring oxygen) makes the formation of nitrogen by the first two reactions the most prevalent.[1] After their formation, the nitrogen atoms usually have enough energy to break away from the hydrogen atoms in the original water molecules in varying degrees of ionization. They subsequently combine with oxygen and hydrogen in the coolant to form ions or compounds of the following types: NO_2^-, NO_3^-, NH_4^+, NO, NO_2, and N_2.

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[1] Free dissolved oxygen in the coolant from air in-leakage or radiolytic dissociation of water (discussed later in this chapter) are activated by the reactions discussed above, but the oxygen atom in the water molecule is by far the largest source of these activation products.

In PWR systems, all of this activity is confined to the primary coolant. In BWRs, however, about 75% of the N-13 and 5% of the N-16 go with the steam to the turbine. The reader might wonder why there is a difference, since both nitrogen isotopes behave the same from a chemistry standpoint. The difference is due to decay of N-16 during its residence time in the water. The fraction of the nitrogen isotopes which carry over with the steam is dependent upon the specific operating conditions. For example, if there is considerable air in-leakage into the condensate system, there will be a large amount of atmospheric nitrogen dissolved in the water. This will mean that the water is saturated with nitrogen, and newly formed nitrogen will be forced into the steam. The partition factor between water and steam is also effected by pH. If the pH is lowered or raised from pH 7, the radiolytic nitrogen in the steam tends to increase.

From a radiological standpoint during power operation, N-16 is by far the most important of all of the water activation products due to its relatively large formation rate and the extremely hard gamma which accompanies its decay. N-16 is even the major contributor to radiation levels during normal operation around a BWR turbine, in spite of the relatively small percentage of the total N-16 produced which reaches the turbine.

3. Fission Products

There are two general ways in which fission products reach the coolant. The first is by direct recoil, where a fission product nucleus born near the inside surface of the cladding rebounds directly through the cladding. Although the average path length of a fission product nucleus is very short, there still remains a small, but finite, probability that the nucleus will penetrate the cladding.[1] The other way in which fission products travel to the coolant is through cladding defects. The amount of activity which reaches the coolant through defects can vary over many decades depending upon the number and size of the defects and the reactor power level.

There are two general classes of fission products--those which are volatile and tend to separate from the water, and those which are non-volatile and tend to remain in solution or suspension in the water.

Principal among the volatile fission products are isotopes of the noble gases xenon and krypton. Xenon and krypton decay to the non-volatile elements cesium and rubidium respectively. Rubidium decays to strontium and thus is the source of the biologically significant isotope Sr-90.

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[1] So called "tramp" contamination of the outer surfaces of the cladding with minute amounts of uranium during fuel fabrication also contributes to recoil fission product activity.

In the BWR, the majority of the fission product gases are carried with the steam to the turbine and condenser. Those which do not undergo decay during the residence time in the hotwell are then removed from the system, along with other non-condensibles, by the condenser air ejector.

In a PWR, the fission product gases tend to remain trapped in the primary system. There is a tendency for them to come out of solution and accumulate in the gas space of the volume control tank in the chemical and volume control system.[1] However, most of the noble gases are relatively short lived and decay to their non-volatile daughters, which in turn will reenter the coolant. Longer lived noble gases can be vented from the volume control tank whenever required. Venting is a normal procedure following a reactor shutdown (to remove H_2 from the coolant as will be discussed later in this chapter). In addition, the original coolant inventory is gradually expelled from the system and replaced by pure water in order to reduce the chemical shim concentration over the course of an operating cycle. The expelled coolant carries with it dissolved noble gases which are stripped out[2] as a first step in the liquid treatment process.

Principal among the non-volatile fission products are the longer lived iodine isotopes such as I-131 ($T_{1/2}$ = 8 days), I-133 ($T_{1/2}$ = 21 hr), and I-135 ($T_{1/2}$ = 9.2 hr). Other non-volatile fission product isotopes which are commonly found in the coolant include the Cs, Rb, and Sr daughters of the noble gases, plus lesser amounts of Zr-95, Nb-95, Mo-99, Tc-99, Ba-140, La-140, Te-132, and Ce-144.

In both PWRs and BWRs, non-volatile fission products are removed in coolant demineralization systems, although a large percentage of them adhere to vessel and fuel surfaces and also to crud, and are difficult to get rid of. Decay is an important removal mechanism for the short lived isotopes.

During normal operation, the contribution of fission products to the radiation levels around primary system components is small in comparison to N-16. This was true even in the early BWRs which experienced widespread failures of the original stainless steel clad fuel. However

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[1] The system used to maintain the proper mass of coolant in the primary system, and to control the concentration of chemicals (principally chemical shim) in the coolant.

[2] The solubility of a gas decreases as the temperature of the liquid in which it is dissolved increases, and also as the gas pressure above the liquid decreases. Gas strippers are often based upon these principles. The liquid is heated and then sprayed into an evacuated chamber to force the gases out of solution. Using a spray increases the surface area of the liquid and gives the gas a better chance to escape.

many of the fission products are relatively long-lived and markedly increase the radiation levels which exist in the vicinity of primary system components during outages. This, of course, complicates refueling and maintenance work. The longer lived fission products must also be considered in the treatment of process wastes for disposal.

When the chemist measures the fission product activity in the coolant or in the condenser off-gas of a BWR he usually looks not only at the total activity, but also how it is split up amongst the various isotopes. For example, if the fission products are the result of recoil there is, relatively speaking, a large percentage of short lived isotopes because there is little or no delay time between the birth of the fission product and its entrance into the coolant. On the other hand, if there is a pinhole leak in the cladding, there is a greater percentage of long lived isotopes in the coolant because it takes a finite length of time for the fission products to travel from the pellet to the hole and ultimately to the coolant. Finally, if there is a gaping hole in the cladding and a significant quantity of fuel is exposed directly to the coolant, the percentage of short lived isotopes once again increases until it approximates the recoil distribution. The chemist can frequently chart the progress of a fuel defect from this information. It is also of interest to note that a reactor which has operated with large amounts of defective fuel retains a "memory" of the defects for months or years after the defective fuel has been replaced. This is because there will be extensive uranium contamination of the system and fissioning of this contaminant will release fission products directly to the coolant. The fission product distribution from a contaminated system will show a recoil characteristic.

4. Tritium Production

Tritium (${}_1H^3$ or ${}_1T^3$) is an isotope which is produced in nuclear reactors in substantial quantities. It decays by β^- emission in accordance with the following reaction:

$${}_1H^3 \xrightarrow{T_{1/2} = 12.3 \text{ years}} {}_2He^3 + \beta^- + \bar{\nu} \qquad (13\text{-}6)$$

There is no gamma ray accompanying the decay and the β^- energy is very low (18.6 Kev maximum, 5.6 Kev average). By way of comparison, most β^- emitters have maximum β^- energies in the range of 1-2 mev (1000-2000 Kev) in addition to any γ energy. Thus tritium is one of the least hazardous radionuclides and relatively high concentrations can be tolerated in the body and in the environs without adverse effects.

Although experience to date and estimates of future effects of tritium indicate that it will not be a problem, it nevertheless has certain properties which make it worthy of consideration.

A. In the presence of normal water, tritium exchanges rapidly with hydrogen ions to form tritiated water, HTO. Tritiated water (liquid or vapor), rapidly enters the body by skin penetration. Although its biological half-life is relatively short (≈ 12 days), concentration of tritiated water in localized areas may have some biological significance. Actually, there are some scientists who believe that the greatest biological effect of tritium may result from the relatively large mass differences that arise from the substitution of $_1H^3$ for $_1H^1$ in a cell. The mass effects would begin immediately rather than being initiated by decay of the tritium atom.

B. Virtually all of the tritium produced in the reactor is ultimately released to the environs. This is because tritiated water is impossible to separate from normal water by ordinary waste processing techniques (evaporation, demineralization). Actually, more of the plant's tritium inventory would be released by the fuel reprocessor than at the plant site itself, since much of the tritium produced in the core is retained in the fuel.

Tritium is produced in a variety of ways in the core, including:

A. Ternary fission

One out of every 12,500 fissions produces tritium as a fission product. For a 3500 MWT (≈ 1000 MWE) reactor, this amounts to the production of ≈ 13,000 curies per year of full power operation. With stainless steel clad fuel, indications are that about 70% of this activity remains in the fuel and that about 30% diffuses to the coolant. Experience with zircaloy cladding is not as extensive as with stainless steel cladding, but it appears that less than 1% of the tritium is released to the coolant, possibly due to chemical reaction of the tritium with the zircaloy. Thus, for a zircaloy clad core, this source might only contribute ≈ 100 curies per year to the coolant.

B. Activation of boron

The B-10 isotope reacts with fast neutrons to form tritium in accordance with the reaction:

$$_5B^{10} + {}_0n^1 \longrightarrow 2\,{}_2He^4 + {}_1T^3 \qquad (13\text{-}7)$$

The cross section for the reaction is ≈ 0.030 barns in comparison to a cross section of ≈ 3800 barns for the absorption of a thermal neutron by B-10.

This reaction takes place in control rods, burnable poison, and chemical shim. The chemical shim is an important contributor to the tritium activity in a PWR which is not found in a BWR. For a 1000 MWE PWR, it is estimated that ≈ 1000 curies per year will be formed in the coolant as a result of chemical shim.

The boron in burnable poison and control rods is generally encased in stainless steel. Thus, it would be expected that the fraction of the total produced which is released to the coolant from this equipment would be comparable to that released from stainless steel clad fuel rods. For a 1000 MWE PWR, it has been estimated that ≈ 2000 curies per year reach the coolant from control rods and fixed burnable poison.

C. Other reactions

There are a number of other reactions which produce lesser amounts of tritium in the coolant.

1) Activation of deuterium

$$_1D^2 + {}_0n^1 \longrightarrow {}_1T^3 + \gamma \qquad (13\text{-}8)$$

This reaction is very limited (≈ 0.1 curies per year) because of the relatively limited supply of deuterium available in normal water ($_1D^2$ constitutes only 0.15% of naturally occurring hydrogen). In heavy water reactors, however, this is an important source of tritium, amounting to several thousand curies per year formed in the coolant.

2) Activation of lithium

Natural lithium occurs in two isotopic forms, $_3Li^7$ (92.5%) and $_3Li^6$ (7.5%). Both undergo reactions to form tritium.

$$_3Li^7 + {}_0n^1 \longrightarrow {}_1T^3 + {}_2He^4 + {}_0n^1 \qquad (13\text{-}9)$$

$$_3Li^6 + {}_0n^1 \longrightarrow {}_1T^3 + {}_2He^4 \qquad (13\text{-}10)$$

The former has a cross section of about 0.04 barns for fast neutrons and the latter has a cross section of ≈ 7 barns for thermal neutrons.

The largest available source of lithium is the Li-7 isotope formed as a product of the reaction between B-10 and thermal neutrons in control rods and burnable poisons, i.e., B^{10} (n, α) Li^7. The tritium produced from activation of this lithium is included in the previously quoted figures for these pieces of equipment, and represents about half of the total.

In a PWR, LiOH is used for pH control and is used in coolant purification demineralizers and so both Li-6 and Li-7 are introduced into the coolant. Fortunately, this is predominantly Li-7, which has a much lower cross section than does Li-6 (actually, the chemicals containing lithium which are used in PWRs have the Li-7 concentration increased to ≈ 99.9%).

About 20 curies per year of tritium is produced in the coolant of a PWR from these sources.

RADIOLYTIC DECOMPOSITION OF WATER

Water is broken down into hydrogen and oxygen by radiation. The overall equation is given by:

$$2H_2O \xrightleftharpoons{\text{radiation}} 2H_2 + O_2 \qquad (13\text{-}11)$$

In reality, however the reaction takes place in a series of steps which lead to the final products.

Equation (13-11) is drawn with two arrows since it is a reversible reaction. That is, in the presence of radiation hydrogen and oxygen combine to form water.

The dissociation reaction is thought to be attributable primarily to fast neutrons, but gamma radiation is also a contributor, since dissociation continues at a reduced rate following a reactor shutdown. One concern about this reaction is that it results in the production of free oxygen in the coolant, with a subsequent increase in oxygen assisted corrosion. To combat this in a PWR, a small amount of hydrogen is injected into the coolant. This dissolved hydrogen is then available to react with oxygen as soon as it is formed. Thus, an excess of hydrogen promotes the recombination reaction and reduces the equilibrium oxygen level in the coolant.

In a BWR, injected hydrogen would tend to carry over to the turbine with the steam. Although hydrogen injection has been tried on an experimental basis (and it works), it is not ordinarily used because the hydrogen is carried over with the steam and it costs too much to continually make it up.

DEMINERALIZERS

We have shown that virtually all reactor water impurities contribute to the radiation levels around the primary system in addition to contributing to "ordinary" chemical difficulties such as corrosion, scale formation, and crud deposition. Thus it is necessary to treat the coolant to remove as many as possible of the undesirable impurities. Demineralizers are the most commonly used equipment for this service.

A demineralizer works on the principle of ion exchange, wherein the demineralizer removes undesirable ions from the solution and replaces them with more desirable ionic species. There are two general types of demineralizers; 1) cation demineralizers which remove cations (positively charged ions), and 2) anion demineralizers which remove anions (negatively charged ions). Physically, both types consist of millions of microscopic resin beads.

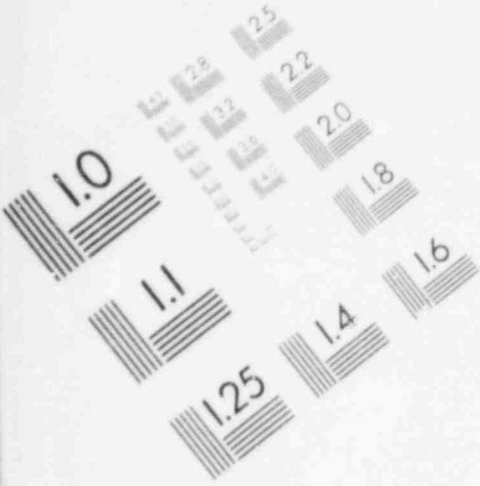
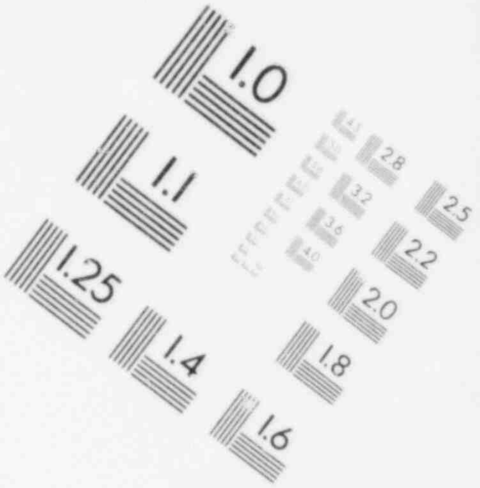
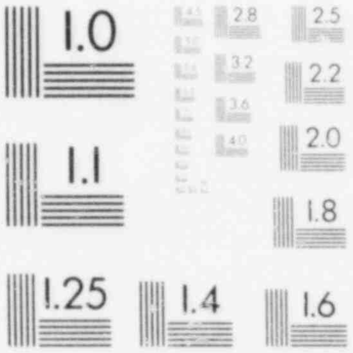
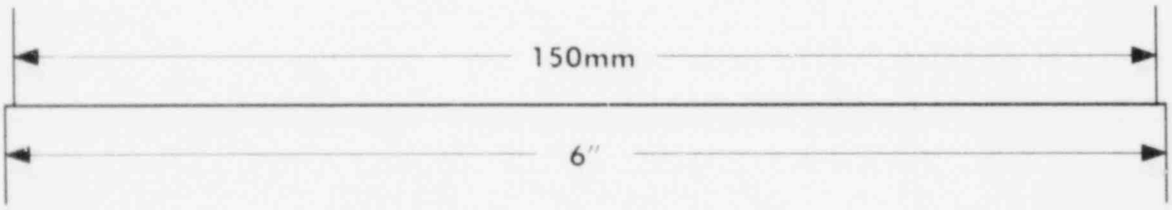
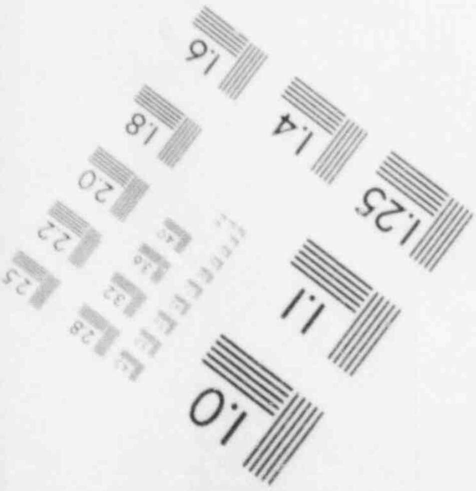


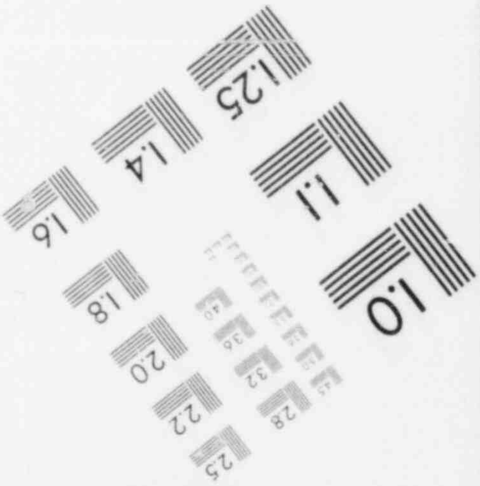
IMAGE EVALUATION TEST TARGET (MT-3)











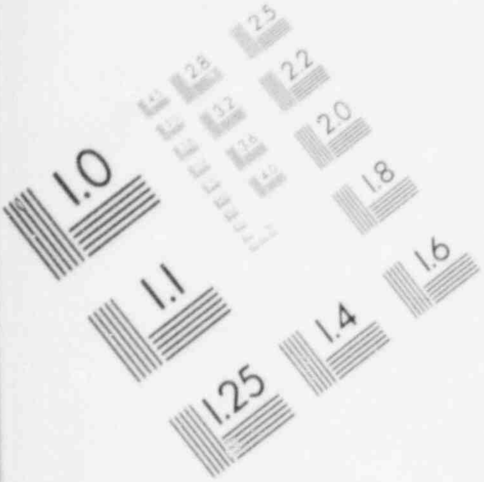


IMAGE EVALUATION
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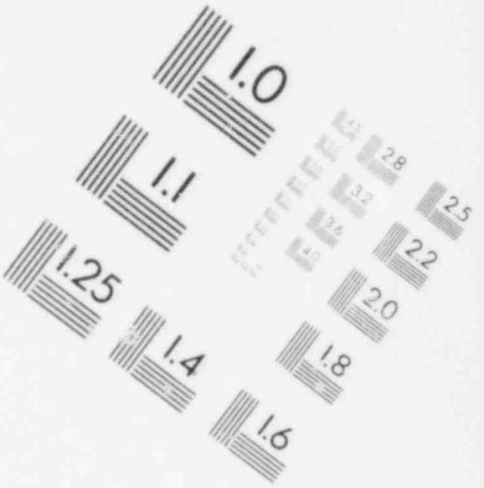

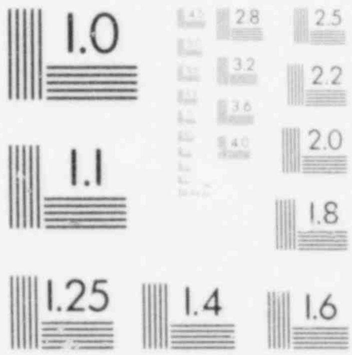

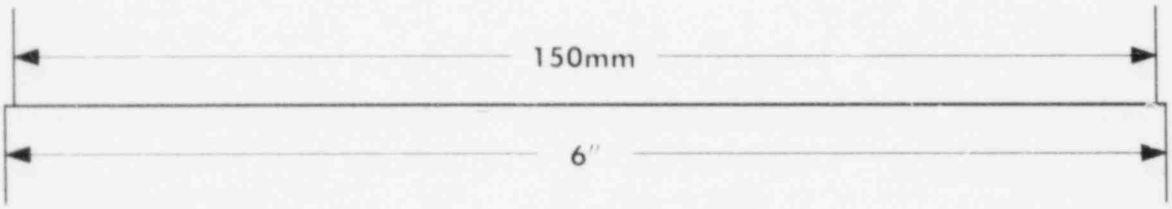

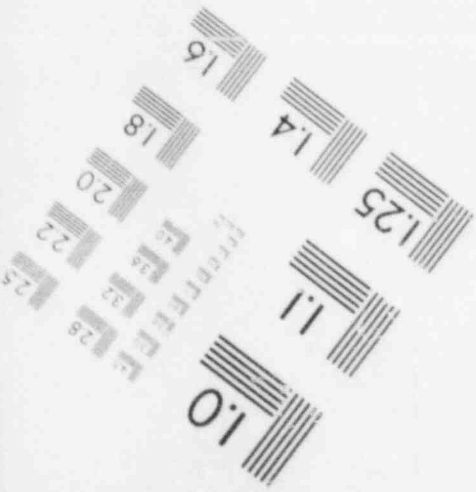

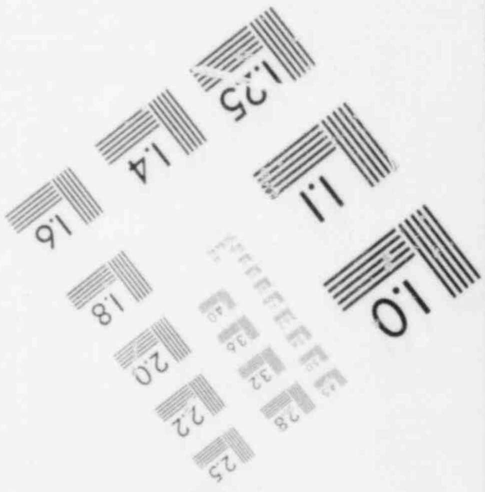

resin.[1] This figure is shown, not because it is important to know the chemical formula for a resin (it isn't), but to show how the resin presents a tangled spider web type of array to nearby ions. This complexity has a large influence upon the ability of the resin to retain ions. Notice that each of the SO_3 functional groups has a single negative charge. The positively charged cations which are held by this resin will be attached to the functional groups.

To simplify further discussions of resin behavior, we will represent the matrix by the letter M and the functional group by the letter F with subscripts a and c to indicate anion and cation resins respectively. Thus the resin shown in the figure would be represented simply as $M_cF_c^-$.

The affinity of a particular resin for different ions varies considerably from one ion to the next. The resins have the ability to reject an ion for which they have a low affinity in favor of one for which they have a higher affinity. Thus if a cation resin has H^+ ions attached to it, and the affinity for hydrogen ions is low, it will exchange its hydrogen ions for nearly any other positive ions, Na^+ for example, which are in the vicinity. The net reaction might be:

$$(M_cF_c)H + Na^+ \longrightarrow (M_cF_c)Na + H^+ \qquad (13\text{-}12)$$

Similarly, an anion resin may be supplied with OH^- ions, for which it has a low affinity, and it will exchange them for other anions for which it has a higher affinity in reactions of the type:

$$(M_aF_a)OH + Cl^- \longrightarrow (M_aF_a)Cl + OH^- \qquad (13\text{-}13)$$

Inspection of the two previous equations indicates that passing the coolant through an appropriate mixture of cation and anion resins would eliminate any NaCl in the solution and replace it with additional water molecules (the H^+ and OH^- ions which are released to the coolant would form water). Thus the water would be purified. In some installations, the anion and cation resins are contained in separate tanks, and in others they are mixed together. The latter are called mixed bed demineralizers.

It is common terminology to identify the particular anion or cation being held by a resin as the "form" of the resin. Thus in the previous equations, resins originally in the hydrogen and hydroxyl forms were converted to the sodium and chloride forms respectively.

A mixed bed demineralizer in which the resins are originally in the hydrogen and hydroxyl forms will have little effect upon the pH of the solution since it will be releasing both hydrogen and hydroxyl ions which will

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[1] On the figure, single lines and double lines joining various atoms represent single and double covalent bonds respectively. Although the bond lines are not shown, the hydrogen atoms are all bound to their respective carbon atoms by single covalent bonds.

Each resin bead is composed of two principal parts; an inert, insoluble matrix, and a chemically active functional group. The latter is basically the device which holds the ion which the resin will use for exchange. The matrix is a complex organic polymer (a long, repetitive chain of individual molecular units joined together). Its principal purpose is to provide an insoluble unit to which ions can be attached and thus be removed from solution. Figure 13-3 shows a typical cation exchange

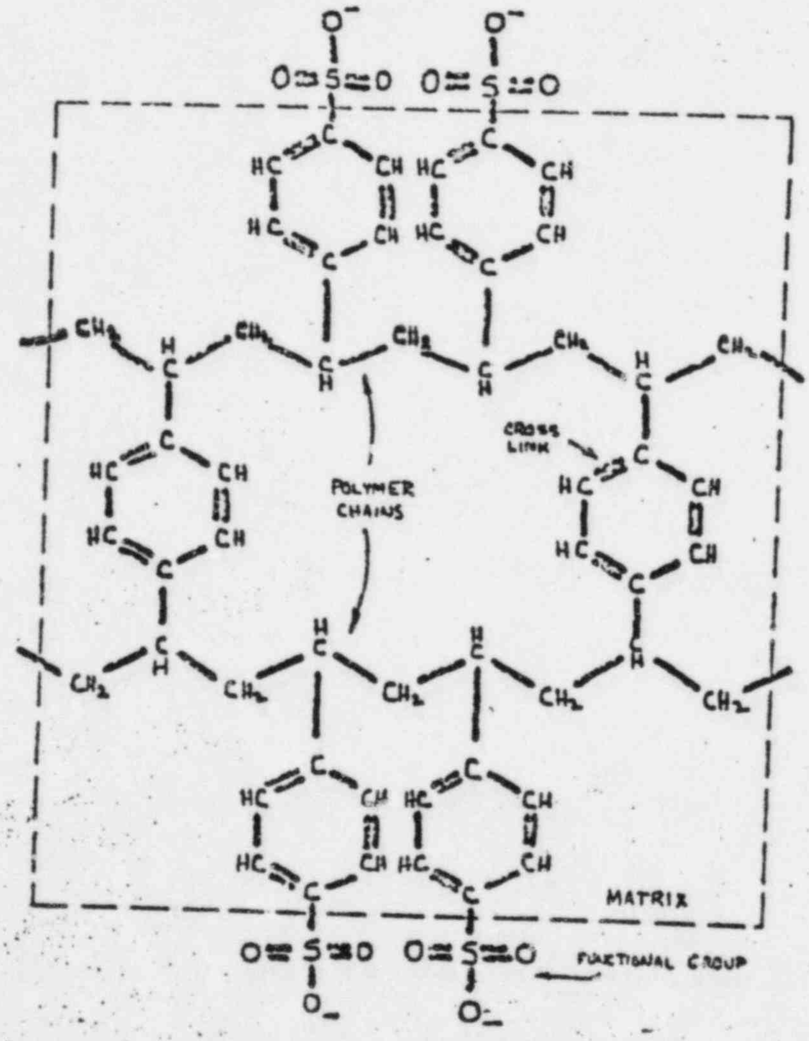

Figure 13-3: Typical Cation Ion Exchange Resin

neutralize each other. In some cases, it is desirable to use the demineralizer to maintain a high or low pH. For example, to maintain a high pH, cation resins in the lithium form might be used in conjunction with anion resins in the hydroxyl form. Action of the demineralizer would result in the release of Li^+ and OH^- ions to the solution which would result in the solution becoming more basic.

The theoretical determination of the relative affinities of demineralizers for different ions is complex and most of the available data is based upon experimentation. Some qualitative observations can be made, however. In general, the larger the charge on an ion, the greater is the resin's affinity for it. This is probably because the ion bonds to more than one functional group and is therefore more tightly held in the resin structure. In addition, larger more complex ions are usually held more tightly than smaller ions of equal charge. This is probably due to their being "entangled" more effectively in the complex polymer structure. It is known, for example, that increased cross linking increases the affinity of a resin for nearly all ions. Cross linking would appear to create many more intermolecular "nooks and cranies" which act to trap ions. However, there are exceptions to these rules and so it is best to consult the literature to determine whether or not a particular resin will remove a given ion from solution. Table 13-III lists a number of common ions in order of their affinity for ion exchange resins. The table shows, for example, that a lithium form cation resin would grab a potassium ion for which it has a relatively high affinity and get rid of the lithium ion. Note that hydrogen or lithium form cation demineralizers will remove cesium and rubidium fission product ions from solution.

TABLE 13-III: RELATIVE AFFINITY OF ION EXCHANGE RESINS FOR COMMON IONS

	Cation Resin	Anion Resin
Low Affinity	Li^+	F^-
	H^+	OH^-
	Na^+	Cl^-
	NH_4^+	Br^-
	K^+	NO_3^-
	Rb^+	I^-
High Affinity	Cs^+	ClO_4^-

When all of the resins in a demineralizer have undergone reactions of the type given above, their usefulness insofar as supplying OH^- and H^+ (or some other ions if they were not originally in the H^+ and OH^- forms) is ended and the bed is said to be depleted. The exhausted resins can be discarded at this time, or they can be regenerated.

To understand regeneration, it is necessary to briefly discuss molecular energies. The ability of a substance to react is related to its energy, with substances ordinarily becoming more reactive as their energy increases.

If we have a group of basically identical molecules in a given region, they will possess a wide distribution of energies about some average value (which is determined by factors such as system temperature and pressure). This is due in large part to the random motion and resulting collisions of the various individual molecules. As collisions occur, molecules are gaining and losing energy in a rather random fashion (the situation is basically similar to that of the "thermal" neutrons in reactor core). Thus, although a given substance may not, on the average, possess the proper energy to have a high probability of undergoing some particular chemical reaction, there will almost always be a few molecules which do have the proper energy and will react. The only problem is to get enough of these molecules to make the reaction proceed at a measurable rate. One way to do this is simply to increase the concentration of molecules in the vicinity. The more molecules there are, the more there will be which have the proper energy for reaction. Thus it is that a reaction which might not otherwise occur to any great extent can be made to occur by increasing the concentration of one or more of the reactants.

Therefore, if we treat a depleted cation bed with a strong acid (i.e., a solution with a high concentration of H^+ ions) reaction (13-12) can be reversed. Similarly, treating a depleted union bed with a strong base will reverse reaction (13-13). Regeneration will therefore return the resins to their original hydrogen and hydroxyl forms in accordance with the following reactions:

$$(M_cF_c)Na + H^+ \longrightarrow (M_cF_c)H + Na^+ \qquad (13\text{-}14)^1$$

$$(M_aF_a)Cl + OH^- \longrightarrow (M_aF_a)OH + Cl^- \qquad (13\text{-}15)$$

Thus the normal preference of the resins for Na^+ and Cl^- is overcome by literally overwhelming them with H^+ and OH^- ions, some of which have the proper energies to make the reverse reaction attractive.

The solutions left over from the regeneration process contain a high concentration of the impurity salts. If these are highly radioactive, they may have to undergo special handling and treatment. Ordinarily, if the regenerant solutions are expected to be highly radioactive, the resins are simply discarded rather than being regenerated.

PRIMARY COOLANT CHEMISTRY PROGRAM

During normal power operation, chemical analyses are run several times a week on the primary coolant. Some of the more important tests and limits are:

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[1] As written, the equation just shows the hydrogen ion since it is the important factor in the reaction. In reality, the acid would have some negative ion associated with it which would also be present in the solution. For example, if sulfuric acid is used, the reaction is $2(M_cF_c)Na + H_2SO_4 \longrightarrow 2(M_cF_c)H + Na_2SO_4$ and the left over solution would contain a high concentration of sodium sulfate.

1. Chloride and Fluoride

Both the chloride and fluoride concentrations must be kept very low--generally <0.1 ppm. The concern with chloride is its detrimental effect upon stainless steel as a result of stress corrosion cracking. The potential for the introduction of chlorides into a BWR system is great if salt water is used for condenser cooling. In addition, many lubricants and solvents contain chlorides.

Fluorides react strongly to dissolve most metals, in particular the zircaloy used for fuel cladding. Fluorine is not as commonly encountered as chlorine, but is found in such materials as teflon and freon and can be released from these materials by thermal or radiolytic decomposition processes. As a result, these materials are not commonly used around the primary system. Fluoride also promotes stress corrosion.

2. Oxygen

It is desirable to maintain the dissolved oxygen levels as low as possible to minimize stress and other types of corrosion. In a PWR, the levels must be <0.1 ppm. During the initial startup, oxygen is scavenged from the primary system with hydrazine in accordance with the equation $N_2H_4 + O_2 \longrightarrow 2H_2O + N_2$. During subsequent power operation, the coolant oxygen concentration is maintained at a low level by maintaining dissolved hydrogen in the coolant (≈ 25-30 cc/kg of coolant).

In a BWR, the dissolved oxygen levels in the coolant may run as high as 0.3 ppm compared to ≈ 0.01 ppm in the feedwater entering the vessel. Short of eliminating condenser air leaks, little can be done to reduce these levels. Sodium sulfite, which is often used as an oxygen scavenger in conventional plants, cannot be used because the stable Na-23 isotope is readily transformed to the highly radioactive Na-24 isotope. In addition, the continual feeding of ionic species into the coolant would quickly deplete the purification demineralizers. Hydrazine is not used because it breaks down in the presence of radiation to nitric acid (HNO_3) unless an excess of hydrogen is present. Hydrogen injection has not been used because of reasons discussed previously.

3. pH

In a BWR, the coolant is kept as pure as possible and the pH is close to neutral. If it varies much from 7.0, it indicates that impurities are present in the water. A low pH (acidic) greatly enhances the corrosion of most materials. In highly oxygenated water, high pH can promote stress corrosion.

In a PWR, the pH will be dependent upon the boric acid concentration in the coolant. At high (≈ 2000 ppm) boric acid concentrations, the pH may be as low as 4.5, whereas without any boric acid it may get as high as 10.5. The reason for it going so high in the absence of boric acid is that LiOH is added to the coolant as a pH control agent. It is of interest to note that the pH values quoted above are those obtained in a lab with a sample temperature of ≈ 70° F. At the actual coolant operating temperature of ≈ 550° F, boric acid ionizes to a much lesser extent than at room temperature. Thus it contributes fewer H^+ ions to the solution and the OH^- ions contributed by LiOH will tend to overwhelm them and produce a neutral or basic solution under these high temperature conditions. Thus, even though the coolant appears to be acidic at room temperature, it will not be so acidic at operating temperatures. We have already seen that high pH is beneficial in terms of reducing corrosion rates.

4. Conductivity

Conductivity is indicative of the amount of ionized substances dissolved in the coolant. In a PWR, the large amount of dissolved substances makes the conductivity relatively high (range <1 to ≈ 40 μmhos/cm) and not particularly meaningful. On the other hand, in the relatively pure BWR coolant, conductivity is a good indication of coolant purity, and it should ordinarily be <1 μmho/cm. In a BWR, high conductivity can indicate a condenser tube leak, depletion of condensate demineralizer resins, etc. The effect of high conductivity depends somewhat upon the particular substance which is causing it. Chloride from a salt leak will enhance stress corrosion as discussed previously. Metallic corrosion product ions, such as Fe^{+++}, Cr^{+++}, Cu^{++}, and Zn^{++} are particularly susceptible to forming oxides such as Fe_3O_4 and plating out as scale on heat transfer surfaces.

5. Boron

The boron concentration in a PWR is carefully monitored as an indication of the reactivity status of the core. It will have a wide range of values, from about 2000 ppm (as B, not boric acid) during refueling shutdowns to a maximum of ≈ 1000 ppm at beginning of cycle power operation, to zero at end of life power operation.

Boron is also monitored at a BWR to indicate leakage of emergency liquid poison solutions into the coolant. A significant amount of boron in the coolant could have undesirable effects upon reactor operation because it tends to make the power coefficient positive. Ordinarily it will be <1 ppm.

6. The primary coolant is occasionally analyzed for iron, copper, nickel, zinc, or other materials which may be present depending upon what alloys are used for construction of system components. These tests help to determine where corrosion is occurring and how much, and whether the corrosion products remain in solution or plate out.

Demineralizers are the principal means for maintaining coolant purity in both the BWR and PWR. In a BWR mixed bed demineralizers in the hydrogen and hydroxl form are installed in the condensate system and in the coolant clean-up system. The latter is a system which takes suction from the reactor vessel and returns the demineralized water back to the vessel on a continuous basis. It prevents the buildup of non-volatile impurities in the vessel itself which otherwise could tend to accumulate there as a result of the distillation effect of the boiling reactor.

In a PWR a small volume of coolant is also continuously recirculated via the chemical and volume control system through mixed demineralizers which are in the Li^+ and the OH^- forms. The borated coolant quickly converts the OH^- anion resin into the borate ($BO_3^{\equiv}$) form and the borate ion then exchanges with impurity ions so that the demineralizer does not continue to remove the chemical shim. The boric acid concentration itself is adjusted by injecting a solution of concentrated boric acid into the coolant when it is desired to increase the concentration or diluting it with pure water when the concentration is to be reduced. However, when the concentration is very low and it is desired to reduce it still further, dilution is impractical because it takes too much water (dilution is an exponential process and the concentration approaches zero asymptotically). In this case, a separate deborating demineralizer containing hydroxyl form anion resins is used. Usually there is also a separate cation demineralizer in the hydrogen form which can be used intermittently to control the buildup of Li^+ if its concentration gets too high.[1]

RADIOACTIVE WASTE DISPOSAL

In the previous sections we discussed the sources of the activity in the primary coolant. With such sources of activity at the plant, the production of radioactive wastes is inevitable. Therefore, all plants have waste treatment facilities to make certain that no radioactive material is released to the environs which could cause a radiation hazard to the public or plant employees. Strict limits are placed upon the discharge of radioactive wastes at each plant by the A.E.C. and various state and local agencies.

1. Gaseous Waste

In a BWR, the major sources of gaseous waste are condenser off-gas and ventilation air from radiation areas. From a volume standpoint, condenser off-gas consists mainly of dissociation products (H_2 and O_2) carried over with the steam plus air from air in-leakage into the condenser. The activity in the off-gas, however, comes from N-13, N-16, and any Kr and Xe isotopes from tramp U or defective fuel. Although there may be considerable activity in this mixture of radioactive gases, the volume involved is nil. After the off-gas is removed from the condenser by the air ejector, it is sent through a large volume of piping and then to the stack. The purpose of the large volume of pipe is to give a traverse time of 20 minutes to an

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[1] Ordinarily Li^+ will not replace H^+ on demineralizers to any great extent, but it will if its concentration increases sufficiently.

hour before the gas is released. This "holdup time" completely eliminates N-16 from the mixture, and a considerable portion of the N-13 and shorter lived noble gases. In the latter group are Kr-90 (3 seconds), Kr-89 (3.2 minutes), Xe-137 (3.8 minutes), Xe-138 (14 minutes) and Xe-135m (15 minutes). At the end of the holdup pipe is placed a high efficiency filter to remove the particulate daughters of these noble gases, in particular Cs-137, Sr-90, and Sr-89 (strontium is the daughter of rubidium, and is also a particulate material). Since Xe-137, Kr-90, and Kr-89 are all short lived, they are essentially transformed completely into particulates before they reach the filter. The strontium isotopes, of course, are particularly hazardous from a biological standpoint because they collect in the bones, and Cs-137 has a long (30 year) half-life which would allow it to build up to unacceptable levels in the environs were it being discharged in significant quantities. After the gas leaves the holdup pipe it is diluted with large quantities of ventilation air and sent up the stack. The stack effectively disperses the dilute mixture to insure that no specific off-site location will receive an undesirably high amount of radiation.

Gaseous waste disposal is less of a problem in PWRs because there is no production of large volumes of dissociation products. From a volume standpoint, the majority of the gas collected in the waste disposal system is nitrogen cover gas which is displaced as various tanks in the CVCS are filled with liquid. The relatively small volume of fission gases which are periodically stripped from the coolant are stored in tanks for long periods of time to allow the shorter lived gases to decay. The bulk of the activity released to the environment consists of the relatively long lived isotopes Kr-85 (10 years) and Xe-133 (5.3 days).

2. Liquid Waste

The primary sources of liquid waste are equipment drains and blowdowns, leaky valves and fittings, and demineralizer regenerant solutions. Several treatment methods are normally used for liquid wastes, including holdup, filtration, demineralization, and evaporation. In the latter, the activity is retained in concentrated form in the slurry. The type of treatment used depends on the condition of the waste. If the water is reasonably pure, it may be economical to demineralize it and return it to the primary system. If the water has high solids content and activity level (regeneration solutions are a good example) concentration may be the best method. Finally, waste which is low in activity but high in solids content (and therefore, not economical to demineralize) may simply be held up to let short lived activity decay and then filtered and pumped overboard to the plant's outfall.

3. Solid Waste

This material will primarily be composed of demineralizer resins, tools, containers, and other equipment that has been used in the radiation area and has become contaminated or activated to such a level that further use would involve a hazard to personnel. Normally there is no treatment for solid waste other than compressing the paper, rags, and other compressible material into compact bales for easier handling and space saving. Special storage buildings are provided for keeping solid waste at the site. Those designed for high activity waste are shielded. Periodically, waste disposal companies are contracted to haul some or all of the solid waste to designated burial grounds in remote desert areas around the country.

PROBLEMS

1. Discuss the principle of the ion-exchange demineralizer. How are these used at your facility?

2. List the limits on reactor water quality for your facility. Discuss the reason for each limit.

3. a) What are the limits for discharge of gaseous and liquid radioactive waste at your facility?
   b) What are the major sources (in terms of volume) of the liquid and gaseous wastes?
   c) What are the major sources of activity in the wastes? What isotopes are normally found in the wastes?

4. Discuss the various types of treatment available at your facility for liquid wastes. Describe under what conditions each is normally used.

5. a) What is the formation mechanism for N-16?
   b) What is its significance during power operation of a BWR? of a PWR?

6. Give examples of each of the following groups:

   a) Radioiodines
   b) Activated corrosion products
   c) Noble gas fission products
   d) Particulates

7. Assume that the concentration of radioiodines in the primary coolant of a BWR is 5 μc/ml due to cladding failures. A packing leak on a primary system valve is discharging 0.5 gallons of water per day. Assuming that all of this water vaporizes on reaching the atmosphere and is then picked up by the reactor building ventilation system and discharged to the stack, what is the radioiodine stack release rate (in μc/sec)? How does this figure compare with the halogen release limit for your facility?

8. Discuss the major sources of tritium in your reactor.

CHAPTER 14

NUCLEAR INSTRUMENTATION

INTRODUCTION

The primary purpose of all nuclear instrumentation systems is to detect and measure the presence of nuclear radiation for indication, record and control purposes in order to protect personnel or equipment. In nuclear power plant applications, alpha particles, beta particles, gamma rays and neutrons are the only important types of radiation from a detection and measurement point of view. The device which actually senses the presence of the nuclear radiation is called a detector. In most detectors of interest, radiation is detected wither directly (as in the case of charged particles) or indirectly (as in the case of gamma rays and neutrons) by the ionization which it causes on passage through a gas, solid or liquid medium.

DETECTORS

Gas Filled Detectors - General

Many of the most common detector types (including the ionization chamber, the proportional counter and the Geiger-Müller tube) employ gas filled chambers and are based upon the behavior in an electric field of the ion pairs formed by ionizing radiation[1] in passing through the chamber. Figure 14-1 is a schematic diagram of such a detector consisting of a cylindrical gas filled chamber containing an axial center wire. The axial center wire is electrically insulated

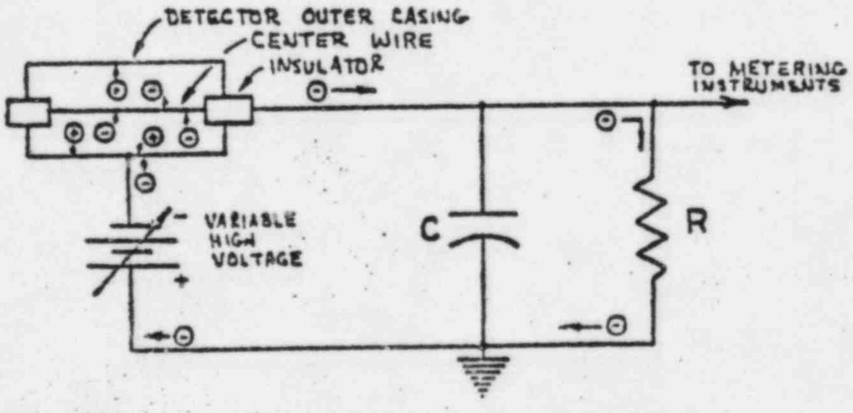

Figure 14-1: Schematic Diagram of a Gas Filled Chamber

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[1] The process of ionization by charged particles and gamma rays is discussed in Chapter 4.

from the cylindrical outer casing and is maintained at a positive potential relative to the casing by an external, variable high voltage power supply.

When a nuclear particle (or photon) passes through the detector, ion pairs are formed in the fill-gas along the path of the particle. The positive and negative ions which are formed within the chamber tend to migrate toward the negatively charged outer casing and positively charged center wire, respectively. The negative ions (electrons) flow through the center wire and the external circuit to the outer casing where they are used to replenish the electrons drawn from the outer casing to neutralize the positive ions. The flow of electrons in the external circuit constitutes an electric current which is measured to indicate the presence and quantity of radiation in the vicinity of the detector.

Fundamental to the operation of a detector of this type is the behavior of the ion pairs formed by the initial ionizing event. One of the important factors in this regard is the voltage gradient[1] between the two electrodes. The voltage gradient determines the rate at which the positive ions and electrons are accelerated toward their respective electrodes[2]. The higher the voltage gradient, the more quickly will the charged particles migrate to the electrodes in any given detector. If the voltage gradient is very low, the positive ions and the electrons do not separate very quickly and there is a good possibility that many of them will recombine before they travel to the electrodes. Thus, the total number of charged particles which reach the electrodes will be less than the number which were formed by the original ionizing event. As the voltage gradient is increased, the charged particles move more quickly toward the electrodes and recombination is reduced. As the voltage gradient is further increased, a point is reached where all of the charged particles produced in the initial ionizing event are collected on the electrodes. That is, there is no recombination. Small increases in voltage gradient beyond this point do not increase the amount of charge collected because

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[1] The voltage gradient is defined as the rate of change of voltage with distance between two points in space. Thus, if two points have a potential difference between them of 10 volts, and they are 5 centimeters apart, the voltage gradient is 2 volts/cm.

[2] The fact that the voltage gradient governs the acceleration of a charged particle can be seen from the following analogy. Consider a ball starting from rest at the top of a hill and rolling down to the bottom. The velocity of the ball at the base of the hill depends solely upon the height of the hill (neglecting friction and wind resistance). However, the rate at which the ball accelerates depends upon how steep the hill is. The steepness of the hill is the same as saying the gradient of the hill in as much as it is a measure of the change in elevation per unit of travel distance. Thus the total energy obtained by a charged particle depends upon the total voltage change that it encounters, but the acceleration it experiences depends upon the voltage gradient.

no more ion pairs are available.

The discussion of the preceding paragraph is pictured in Figure 14-2

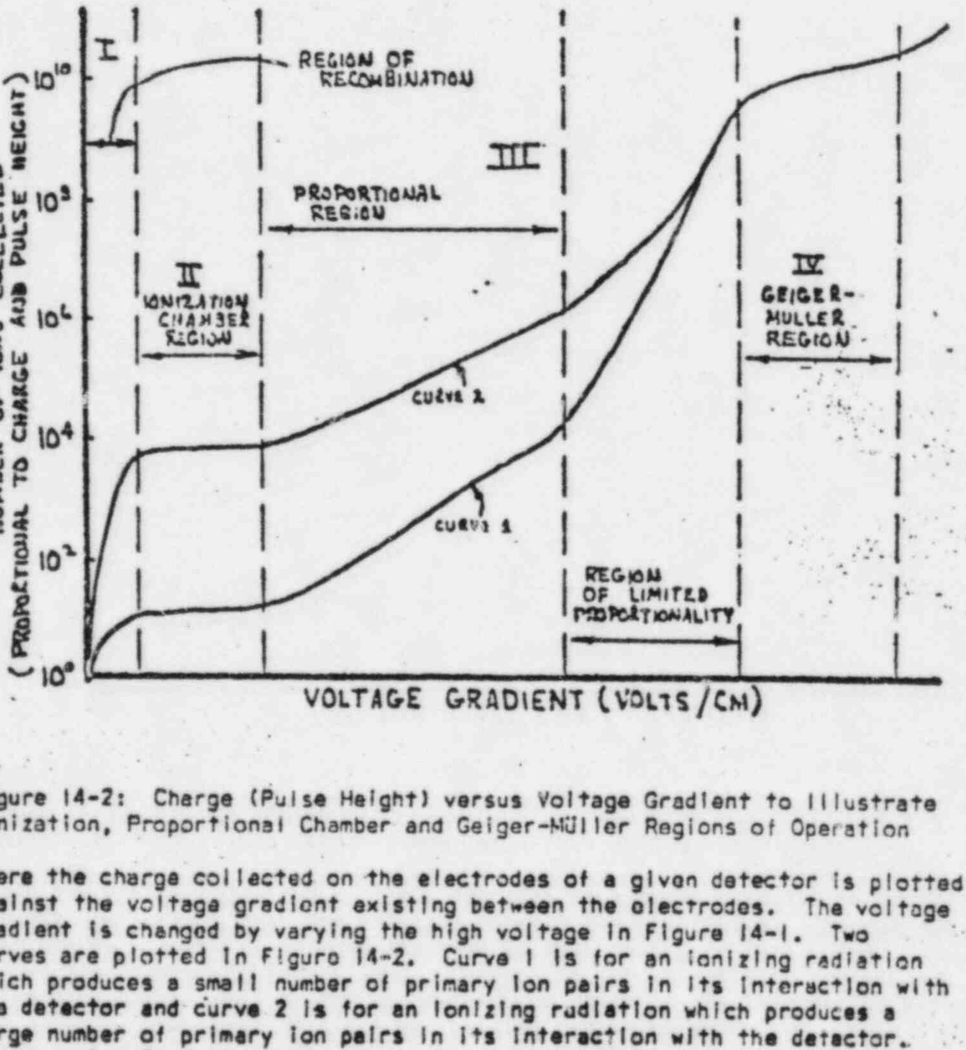

Figure 14-2: Charge (Pulse Height) versus Voltage Gradient to Illustrate Ionization, Proportional Chamber and Geiger-Müller Regions of Operation

where the charge collected on the electrodes of a given detector is plotted against the voltage gradient existing between the electrodes. The voltage gradient is changed by varying the high voltage in Figure 14-1. Two curves are plotted in Figure 14-2. Curve 1 is for an ionizing radiation which produces a small number of primary ion pairs in its interaction with the detector and curve 2 is for an ionizing radiation which produces a large number of primary ion pairs in its interaction with the detector. For example, Curve 1 might represent a γ ray, and Curve 2 might represent an α particle. Region I of the curve is the region of recombination, and the charge collected increases as the gradient increases. Region II is the region where recombination is eliminated, and the amount of charge collected is not dependent upon voltage gradient. This region is commonly referred to as the saturation region or the ionization-chamber region. As discussed before, in this region the amount of charged particles collected is equal to the total number produced by the ionizing radiation in its interaction with the detector.

As the voltage gradient between the center electrode and wall of the chamber is increased beyond that corresponding to the ionization chamber region, the charge collected on the electrodes exceeds that which was produced in the initial event. To understand this phenomenon let us look more closely at the charged particles as they move toward their respective electrodes. Since the entire chamber is filled with gas, there is a good possibility that the charged particles will pass in the vicinity of un-ionized fill-gass atoms during their passage through the detector. As they do so, they exert forces upon the electrons on the fill-gas atoms and attempt to ionize them. If, on the average, the primary charged particles are accelerated to a point where they acquire an energy greater than the ionization potential[1] of the fill-gas before they encounter un-ionized fill-gas atoms, it is likely that these charged particles will, in fact, cause further ionization of the fill-gas. This additional ionization is called secondary ionization. The positive ions and electrons produced by secondary ionization add to the charge collected and may themselves cause additional secondary ionization which also adds to the charge and so on. As a result of this chain or avalanche of ionization, the final charge collected on the electrodes is much larger than that carried by the primary ion pairs formed by the passage of the nuclear particle through the detector. The ratio of the charge ultimately collected to the charge carried by the primary ion pair is called the gas amplification factor. That is:

$$C_{coll} = A \times C_{pri} \qquad (14\text{-}1)$$

where: C_{coll} = total charge collected
C_{pri} = total charge produced in detector by primary ionizing event
A = gas amplification factor

The gas amplification factor is, of course, a number greater than one and may exceed a million[2]. Once gas amplification has begun, then, the total charge collected increases beyond that produced in the primary event as is illustrated by Region III. Over a fairly wide range of voltage

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[1] The ionization potential is the minimum energy required to overcome the attractive force between an electron and its nucleus and completely remove it from the atom. That is, it is the minimum energy necessary to ionize the atom. We saw in Chapter 4 that in most gases the ionization potential is on the order of 35 ev.

[2] Technically speaking, in the ion chamber region, $A = 1$, and in the region of recombination $A<1$.

gradients, the gas amplification factor at any particular value of voltage gradient is independent of the number of ion pairs formed in the initial ionizing event and the two curves on the Figure remain a fixed distance apart. Another way of saying this is that the numerical value of A is the same for both an α particle and a γ ray[1]. This portion of Region III is called the proportional region because within this region the size of the output pulse is proportional to the size of the input pulse.[2] As the voltage gradient continues to be increased, the large, relatively slow moving positive ions begin to pile up between the electrodes, producing a space charge which distorts the electric field in the tube, which in turn begins to affect the gas amplification factor. No longer is the magnitude of A independent of the size of the initial event, and the two curves begin to come together. Since the proportionality between charge collected and the size of the primary event begins to be lost, this portion of Region III is called the region of limited proportionality. In effect, the big primary pulses produce a big pileup of positive ions and the resultant space charge tends to weaken the field and limit further secondary ionization. A smaller initial pulse on the other hand, can tolerate a larger A before it produces as large a pileup of positive ions.

As the voltage gradient is further increased, a point is finally reached where the avalanche of secondary ionization which had previously been confined to a point or small region of the center electrode now spreads over its entire length and the total amount of charge collected is completely independent of the number of ion pairs formed in the initial event. The size of the charge is now limited only by the voltage gradient and the characteristics of the detector and its associated external circuit. The gas amplification factor and therefore the charge collected still increases with the voltage gradient, but the size of the output pulse does not depend on the nature or the energy of the nuclear particle (or photon) causing the initial ionization. This is illustrated by Region IV, which is called the Geiger-Müller region. It should be noted that somewhat of a plateau (flattened area) exists in this region where the charge collected is essentially constant. The beginning of this plateau is called the Geiger threshold.

Increasing the applied voltage beyond the Geiger-Müller region results in a further increase in the charge collected. The potential is so high, however, that once a discharge is initiated, others follow to effect a

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[1] We are speaking now of the numerical value at a given value of voltage gradient. As the voltage gradient increases, A also increases. But in the proportional region, whatever the numerical value of A, it is the same for both particles.

[2] This latter statement is also true in the ion chamber and recombination regions as well, however.

continuous discharge. This region is not utilized in nuclear radiation detection and measurement.

Mode of Operation

As we have already seen, the interaction of a nuclear particle with most detectors results in a flow of electrons through the external electronic circuit of the detector. Referring to Figure 14-1 or 14-3a, it can be seen that for one interaction the flow of electrons through the external circuit will cause a voltage change or pulse across the resistor R. In general, there are two types of nuclear radiation detection systems utilized to detect, measure and record this electric signal. In the first system, which is generally applicable to measurements made in very low radiation fields, an attempt is made to distinguish each individual voltage pulse resulting from an interaction with the detector. This mode is called pulse type operation and the instruments generally read out in interactions per second (or equivalently pulses/sec or "counts"/sec). As the strength of a radiation field increases, the interactions occur so rapidly that it becomes very difficult to distinguish one from another. In a mean level, or current type instrument, no attempt is made to measure individual events, but rather the pulses are allowed to overlap and the average electron flow rate in the external circuit (i.e., the current) is measured.

In principle, the same detector chamber can be made to perform either as a pulse type or current type detector depending upon the characteristics of the electrical circuitry, in particular upon the amount of resistance and capacitance in the circuit. It is beyond the scope of this manual to go into detail about the characteristics of the various electrical circuit elements, but a few general words are in order. In simplest terms, a capacitor can be thought of as a kind of a spring loaded electron storage reservoir. Electrons can flow into and accumulate on the plates of a capacitor. As they do so, however, they "compress the spring of the capacitor" (i.e., build up a voltage across the capacitor). If the charges in the circuit external to the capacitor begin to dissipate, the "spring will be released" and the capacitor will reinject its stored charge back into the circuit. On the other hand, a resistor is simply a device which tends to impede the flow of electrons in a circuit. In order to make electrons move through a resistor, you have to give them a push (i.e., build up a voltage across the resistor).

Now then, let us consider the behavior of the circuit shown in Figure 14-3a. When a burst of ionization occurs within the detector, electrons are driven out of the center wire of the detector by the force (voltage) of the battery. As these electrons reach the junction marked by point A on the figure, the electron flow splits and some flow into the capacitor and some flow into the resistor. Thus, a voltage builds up across the common terminals of these two devices. The electron flow will be proportioned between the two devices depending upon their relative sizes in such a manner that they will have equal voltages across their terminals.

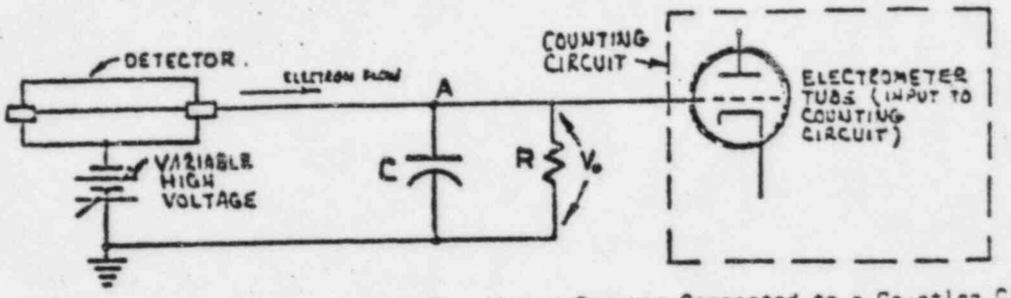
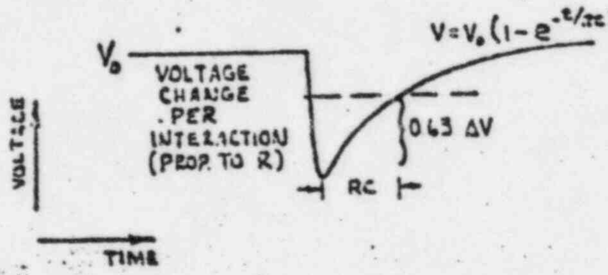


Figure 14-3a: Schematic of a Gas-filled Chamber Connected to a Counting Circuit



When an interaction of a particle within the detector takes place, a pulse is formed. If the original potential at point "A" of Figure 13-3a is V_0, then the potential, V, after a time t, is given by $V = V_0 (1 - e^{-t/RC})$ as shown.

Figure 14-3b: Pulse Formed by Interaction of a Nuclear Particle in Chamber

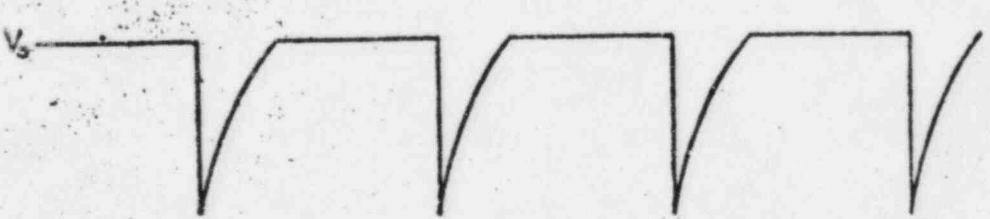


Figure 14-3c: Signal at Point "A" of Figure 14-3a when Circuit Designed for Pulse Type Operation

Figure 14-3d: Signal at Point "A" of Figure 14-3a when Circuit Designed for Current Operation

For example, if the capacitance is large (i.e., its ability to store charges is great), most of the electrons will flow into it rather than trying to force their way through the resistor. Conversely, if the resistor is small, it is so easy (i.e., it takes only a small voltage) for the electrons to pass through it that they will tend to take this path. In any case, the onset of ionization in the detector will result in a rapid buildup of a voltage across the terminals of the resistor and capacitor which is sensed by the counting circuit. This is illustrated in Figure 14-3b by the rapid drop of the potential at point A as seen by the counting circuit.[1]

Now then, once the ionization in the tube ceases, the electron flow out of the center electrode comes to a halt. This is the signal for the capacitor to start unloading its stored charge. Electrons thus flow off the capacitor and through the resistor (they cannot flow back to the detector since the un-ionized fill-gas acts as an open circuit) and the signal persists beyond the time that actual ionization is completed. As the charge on the capacitor dissipates, the voltage signal gradually returns to its steady state value. The rate at which the signal dissipates is governed by a quantity called the RC time constant of the circuit.[2] The larger is the product of R and C, the longer is the time constant and the more slowly will the pulse decay.

If we intend to make a pulse type instrument, we want the potential at point A to recover rapidly after an interaction has occurred so that the next interaction can be recorded as a separate event. As a general rule, therefore, we try to make RC small, usually by minimizing the value of C.[3] With a small value of C, there is very little stored charge and it only takes a short time to dissipate. In principle, we can further reduce RC by reducing R. That is, with a small R, less charge will tend to accumulate on the capacitor in the first place, and that which does accumulate will find it easy to flow back off since it does not have to buck as much resistance. There is a limit to how low we can reduce R, however, because if R gets too small, the electrons will be able to flow through it without having to receive much of a push. That is, we will not develop much of a voltage across R and our counting circuit will not

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[1] The student should not be concerned by the fact that Figure 14-3b shows the voltage at point A dropping while the text talks about voltage buildup. Since our voltage buildup is brought about by an accumulation of negative charges, we are getting a negative voltage buildup, i.e., a voltage drop.

[2] Technically speaking, the time constant measures the time required for 63% of the charge carried by the capacitor to dissipate.

[3] Ordinarily we would not deliberately place a capacitor in the curcuit at all. However, the detector has a small amount of inherent capacitance and so it is impossible to eliminate it completely.

see a very big pulse. The designer of a pulse counting system must, therefore, choose R so that he achieves the best balance between large pulse size and rapid recovery. In any case, a circuit designed with the proper RC time constant for pulse type operation would show a signal similar to Figure 14-3c.

A term which is often used to describe a pulse type system is its resolving time, t. Resolving time is defined as the minimum time which can elapse between the interaction of two successive nuclear particles within a detector if they are to be counted as two successive events. Resolving times of approximately 10^{-6} seconds or less are not uncommon with some systems. If the radiation field the detector is measuring increases to the point that the time interval between the interactions in the detector decreases to less than the resolving time, the detector output can no longer be counted as individual events and the counting system is said to be "saturated" or "jammed".[1] The resolving time may be limited by the characteristics of the detector (long collection time) or by the electronics of the counting system (large RC or inadequate registering device).

It should be apparent by now that current type operation can be obtained by making the resistance R very large so that the RC time constant is longer than the interval between interactions. In this case, the pulses overlap and a continuous current flow passes through R (the detector does not discharge continuously, however). This behavior is shown in Figure 13-3d. No attempt is made to resolve the individual pulses. The signal strength is proportional to the rate of entry of nuclear particles into the detector and the amount of ionization they cause.

Ionization Chambers

An ion chamber detector can be designed in many sizes and shapes and can be used to measure all types of radiation that produce ionization. A typical chamber consists of a cylindrical, conducting, gas filled chamber with a central conducting electrode located on the axis of the chamber and insulated from it as in Figure 14-1. The filling gas in many cases is only air at atmospheric pressure, but nearly any gas may be used for its particular ionization properties. The proper voltage is applied between the wall of the chamber and the center electrode to cause the detector to operate in the ionization chamber region. Since the gas amplification within the chamber is unity, the signal from the ion chamber requires considerable electronic amplification before it can be indicated or recorded. Rather sophisticated and expensive electronic circuitry is sometimes required to do this.

The ionization chamber can be used in either pulse or current type operation. The latter type operation is the most frequent. For either type of operation, the signal current is very small (depending on the application, signal current may vary from approximately 10^{-14} to 10^{-3} ampere). When

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[1] When a counting system becomes saturated, the indication may drift down or even drop to zero.

the current is greater than 10^{-8} ampere, it can be measured by a standard sensitive D'Arsonval type meter, but below about 10^{-8} ampere these instruments are not sufficiently sensitive. As seen in the previous discussion of modes of operation, the detector output can be determined by measuring the voltage drop across a calibrated high resistance. This signal is normally fed to the grid of an electrometer tube (refer to Figure 14-3) which is the first stage of a d-c amplifier. The electrometer tube is a specially designed vacuum tube which draws a very low grid current and consequently has a high input resistance. Maximum grid currents of the order of 10^{-14} ampere can be obtained, giving input resistance of 10^{14} ohms or higher for a 1 volt input. In recent years, with the advent of solid state electronics, the function of the electrometer tube has been taken over by a special type of transistor called a field effect transistor (FET). In any case, one of the major problems encountered in radiation measurements is trying to maintain the extremely high resistance of the detector, calibrated resistors, and associated connecting cables associated with d-c amplifiers.

Ion chambers are also widely used in radiation protection work, as they establish to the highest degree a linear relationship between the ionization formed by the radiation and the energy of the radiation. This is important because the damage to body tissue is, to a first approximation, proportional to the number of ions formed in the tissue.

As mentioned previously, a number of fill gases may be employed in those detectors. Air is often used in chambers designed for the measurement of γ ray dose rate in radiation protection work since the roentgen is defined as a specific amount of ionization in air. In other cases, detectors are filled with pure nitrogen instead of air. This is usually the case if response time is any consideration because the presence of oxygen tends to slow the collection of electrons in the detector. This effect results because oxygen has six electrons in its outer orbital and has a strong tendency to pick up two additional electrons to form the $O^=$ ion. Thus electrons which move rapidly are converted to $O^=$ ions, which move rather slowly because of their size.

Proportional Counters

Detectors operating in the proportional region can be of either pulse or mean level type, as was the case with ionization chambers; however, the pulse type application is by far the most common. Gas filled chambers operating in the proportional region and utilized in pulse type application are called proportional counters. A proportional counter, like the ionization chamber, can be designed in many sizes and shapes, but basically consists of a system of electrodes in a gas filled chamber. The amount of ionization produced per interaction of a nuclear particle is a function of the energy of the particle as well as the chamber characteristics and operating conditions. The gas amplification obtainable in proportional counters varies from unity up to about 10^5.

Once again, virtually any fill-gas can be used in a proportional counter. However, the performance of a proportional counter can be substantially improved by the choice of the proper gas, and therefore the choices are generally somewhat more limited than in the case of an ion chamber. Since proportional counters are usually used in high speed pulse counting systems, oxygen is never included. Ordinarily, the fill-gas contains a large percentage of a monatomic gas like argon. This is because a higher gas amplification is achieved with these gases because the energy available to cause ionization is used more efficiently for this purpose. Polyatomic gaseous molecules are frequently disassociated by charged particles rather than ionized. Usually, however, the fill-gas contains a small amount of a polyatomic gas to help quench the discharge (see next section for a discussion of quenching). A common proportional counter fill-gas is "P-10" gas, composed of 90% argon and 10% methane (CH_4).

Proportional counters offer several advantages over both pulse type ionization chambers and Geiger-Müller tubes (discussed next). Both pulse type ionization chambers and proportional counters maintain proportionality between the output pulse and the number of ion pairs formed in the initial interaction of a nuclear particle in the detector. This porperty makes possible discrimination between the various types of radiation and the determination of the energy of the nuclear particles.[1] However, the proportional counter, because of gas amplification, produces a much larger output voltage pulse than does the ion chamber. As a result, the electronic amplification necessary in a particular application for recording the signal is less for the proportional counter. Therefore the proportional counter often requires simpler, less expensive electronic measuring equipment than that needed by the ion chamber for equivalent service. Actually, pulse type ionization chambers are not sensitive enough for utilization in beta radiation measurements whereas proportional counters are quite adequate. The proportional counter has the advantage over the Geiger-Müller tube (discussed below) in that proportionality is maintained in the proportional counter, whereas in the Geiger-Müller tube, the dependence of the size of the output voltage pulse upon the primary ionization is completely lost. Another advantage of the proportional counter over the Geiger-Müller tube is that for straight counting (where the pulse height does not have to be known accurately) the resolving time of the proportional counter can be made as low as 0.2 to 0.5 μ secs[2] whereas the resolving time of the best Geiger-Müller tube is on the order of 100 to 200 μ secs because of the large discharge which must be quenched after each pulse. Much higher counting rates are therefore possible with proportional counters than with Geiger-Müller tubes. Proportional counters are also somewhat faster responding than pulse type ion chambers because

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[1] Refer to the section on Pulse Height-Analysis on page 14-31 of this section for a discussion of discrimination and particle energy determinations.

[2] Whenever the amplitude of the pulses must be measured accurately, the error in pulse size because of the presence of ionization from the preceding pulse must be avoided. The resolving time of the proportional counter in this case is on the order of 50 to 100 μ secs.

the higher voltage gradient significantly reduces the migration time of the relatively slow positive ions.

Geiger-Müller Tubes

The Geiger-Müller (G-M) tube has been one of the most widely used detectors in the nuclear power industry because of its high sensitivity and reasonable cost. The tube consists of a gas filled envelope containing two electrodes. The voltage applied between the two electrodes is such that the detector operates in Region IV of Figure 14-2. These counters can be used for counting any type of nuclear radiation which will produce ionization within the tube, no matter how small the amount of ionization. The signal from a G-M tube is typically 100 times greater than that from a proportional counter, and may be on the order of 1 volt or more. Consequently, a rather simple, low cost measuring circuit is required in a system utilizing a G-M tube as the detector. It is because of its simplicity, sensitivity and low cost that this type of counter is frequently used. It has, however, certain disadvantages. Because the pulse sizes are independent of the primary ionization, pulse size cannot be used as a measure of particle energy, nor is it possible to discriminate between various types of nuclear particles except by placing varying thicknesses or types of external shielding around the detector. Also, as previously discussed, a G-M counter is approximately 100 times slower than a proportional counter because of the large dead time resulting from the slow moving sheath of positive ions surrounding the center electrode following each ionizing event.

The most common fill-gases for a G-M tube are the noble gases, particularly argon and neon. Unless provision is made to prevent them, multiple discharges will take place for one interaction of a nuclear particle in the tube. Multiple discharges result from the fact that the positive gas (argon or neon) ions are accelerated so rapidly toward the wall in the G-M tube that when they strike the wall they will generally knock out many secondary electrons. These, of course, will cause additional ionizations and allow the tube to discharge continuously. To prevent this from occurring, a small amount of a halogen gas (generally Cl_2 or Br_2) is added to the normal fill-gas to quench the discharge. Although somewhat oversimplified, the basic principle behind halogen quenching is as follows:

1. The positive argon or neon ions have an extremely high ability to attract electrons and thus regain their inert gas structure. Therefore, they will pull electrons from the halogen molecules in the vicinity and neutralize themselves.

2. As a result of step 1, the positive particles which approach the wall of the G-M tube are ionized halogen molecules rather than argon ions. As they near the wall, they attract electrons from it and become neutral halogen molecules. They are highly excited due to the kinetic energy they acquired in traveling to the wall and due to the kinetic energy possessed by the electrons which they attracted. However, halogen molecules are diatomic - being composed of two halogen

atoms held together by covalent bonds. The excited halogen molecules use their energy to break apart into neutral halogen atoms rather than retaining it as kinetic energy. As a result, the cathode does not get bombarded, and secondary emission is averted.

Scintillation Detector

Certain materials, called scintillation phosphors, exhibit the property of emitting flashes of visible or ultraviolet light when struck by charged particles. In a scintillation detector, a scintillation phosphor is coupled to a device called a photomultiplier tube which converts the light flashes into electrical pulses.

There are many types of scintillation phosphors having a variety of properties which make them more or less useful for any particular application. Probably the most widely used scintillator in power plant applications is the sodium iodide crystal with thallium activator, or NaI(Tl). The activator is an impurity which is added in small amounts to an otherwise extremely pure sodium iodide crystal in order to give the latter its light emitting properties. For reasons which will be discussed below, this type of scintillator is used almost exclusively for the detection of γ rays. The general arrangement of a NaI(Tl) scintillation detector is shown in Figure 14-4.

When a gamma ray enters the crystal, it causes ionization within the crystal by the photoelectric effect, Compton scattering, and pair production. The electrons thus liberated within the crystal give up their energy by exciting bound electrons in the crystal structure. When the latter return to the ground state, light flashes are produced.[1] These light flashes pass through the light pipe (which is ordinarily nothing more than a transparent glue which is used to attach the crystal to the photomultiplier tube) and strike the photocathode of the photomultiplier tube. The photocathode is made of a photosensitive material which emits electrons when struck by light.[2] Electrons emitted from the photocathode are attracted to the first dynode of the photomultiplier tube which is at a positive potential. Within the photomultiplier tube is a series of 10 or 12 dynodes arranged so that the electrons emitted from each are focused toward the next. Each dynode has a successively higher potential so that the electrons move from dynode one to dynode two to dynode three; etc. At each dynode secondary emission of electrons take place so that by the time they are collected by the last dynode, called the anode, the number of electrons collected per event is many times the number that left the photocathode. The multiplication factor of present day photomultiplier tubes is around 10^6, so that for each

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[1] The details of the process are complex and beyond the scope of this manual. In addition, details of the process differ from one type of scintillator to another.

[2] The same as in a photoelectric cell. The process by which this occurs is the photoelectric effect.

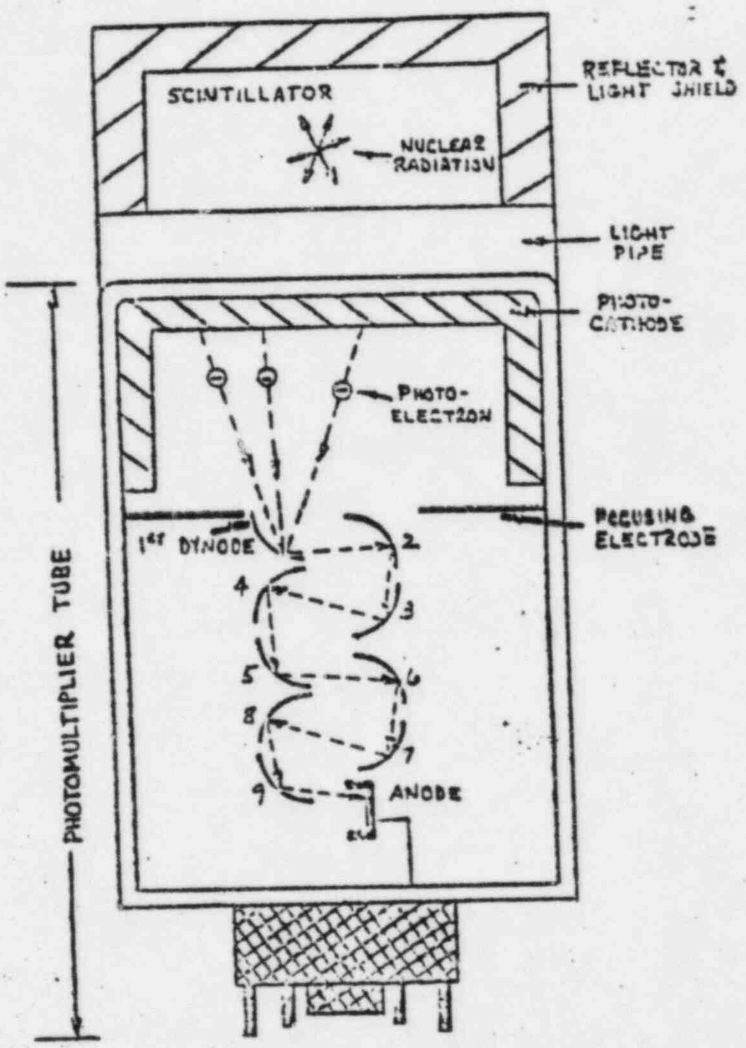

Figure 14-4: Schematic Diagram of a NaI(Tl) Scintillation Detector

electron released from the photocathode, 10^6 electrons would be collected by the anode.

Sodium iodide has several properties which make it useful for the detection of gamma rays. First, it is a relatively dense material (sp. gr. = 3.7) which makes it effective for stopping γ rays. The scintillation process, of course, will not occur if the gamma ray passes through the crystal without undergoing an interaction. In spite of its density, however, a NaI crystal must be relatively large to stop a γ ray. Typically, the crystals have dimensions on the order of 1 to 6 inches in each direction. In order for a light pulse to pass through a crystal of this size to the photocathode, the crystal must be transparent to its own light. Sodium iodide has this property.

Other advantages of NaI(Tl) scintillators, and most other types of scintillators as well, are : 1) very short resolving time (roughly comparable to the proportional counter), 2) very high sensitivity (≈ 100 times greater than a G-M tube as a rule of thumb) due to good light production efficiency coupled with large amplification in P-M tube, and 3) ability to distinguish between incident photons of different energy because the intensity of the light flash is proportional to energy. The main disadvantages are their expense (a 3" x 3" NaI crystal costs ≈ $1,000 compared to ≈ $25 for a typical G-M tube), and the fact that they are relatively fragile.

Theoretically, NaI(Tl) scintillators are sensitive to α and β as well as γ. From a practical standpoint, however, their sensitivity to these radiations is low. One reason for this is that NaI is a hygroscopic material. That is, it readily absorbs moisture from the air which clouds the crystal and makes it less efficient. Therefore, NaI crystals are always hermetically sealed in a metallic container, which tends to shield out all α and most β.

Beta particles are most efficiently detected by organic scintillators such as anthracene and trans-stilbene crystals. Their general physical arrangement is similar to the NaI detector. Actually, these types of scintillators are not commonly employed for power plant work, since the gas filled detectors do an adequate job for most purposes. One common application of organic scintillators, however, is the use of liquid scintillation systems for the detection of tritium. Tritium emits a very weak β which has such low penetrating power and causes so little ionization that it is very difficult to detect. In a liquid scintillator, both the phosphor material and the tritium bearing material (which is ordinarily water) are dissolved in a solvent such as toluene. Thus the tritium comes into intimate contact with the phosphor and can be detected.

Alpha particles are generally detected with zinc sulfide activated with silver, ZnS(Ag). Since an α particle has an extremely short range in a relatively dense material such as ZnS, a thin coating of ZnS applied directly to the glass envelope of the photomultiplier tube will suffice for α counting. Alternative arrangements in which the ZnS is coated on a transparent material such as lucite are also employed. In this event, the lucite would be optically coupled to the P-M tube. The thinness of the ZnS makes its sensitivity to β and γ very low, since both of these pass right through without an appreciable number of interactions. Actually, ZnS is not very transparent to its own radiation, so an amount of it large enough to stop a γ ray would be virtually useless as a detector because no light would reach the P-M tube. For α counting this is no problem, however, because the coating of ZnS is so thin.

Other Detectors

When charged particles or gamma photons pass through photographic emulsions, they produce latent images along their paths. When the film is developed, silver grains appear along the tracks. Calibration procedures have been developed which relate the density of the photographic deposit in the film to the dose of radiation received by it. Photographic film (x-ray film) is used in nuclear power plants for monitoring personnel for the radiation dose due to gamma, beta and neutrons. The photographic film can be worn as a badge or in the form of a finger ring. Photographic film is also used for environmental monitoring. Since specialized equipment is required to make dose determinations from developed film, this service is handled on a contract basis with an outside firm. All personnel film badge results are put on IBM record sheets and maintained as the official record of radiation dose for all employees.

The thermoluminescent dosimeter represents a relatively new advancement in personnel radiation and environmental dosimetry. One type of dosimeter takes the form of a miniature radio tube. The sensitive part of the dosimeter is a cylindrical hollow metal tube covered with an activated fluoride powder (usually calcium fluoride) which is fixed by a vitrified[1] enamel to form a continuous coating. Within the cylinder is a tungsten filament heating element. The entire assembly is enclosed in an evacuated glass bulb which is itself enclosed within a metal or plastic sheath shaped like a fountain pen which is designed to protect the dosimeter against mechanical shocks. The principle of operation is as follows: Irradiation of the activated fluoride powder excites the crystal, creating metastable states or "centers" which consist of electrons or holes trapped at special lattice sites. The number of metastable centers is proportional to the magnitude of the ionizing dose.

To obtain the stored data from the dosimeter, it is inserted into a light-tight cavity and a constant voltage is applied to the heating filament for a predetermined time. When the fluoride powder is heated, the centers are emptied and their charge carriers (electrons or holes) are returned to the ground state accompanied by the emission of light. The intensity of the emitted light is proportional to the magnitude of the ionizing dose. The light output is measured by a photomultiplier tube.

At present the use of thermoluminescent dosimeters has been, for the most part, confined to environmental monitoring. The advantages of these detectors for environmental monitoring are that they are more sensitive than photographic film and for low doses their results are more reproducible.

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[1] The fluoride powder is mixed with the enamel and painted on the metal tube which is then placed in an oven and baked. This gives the enamel a smooth, hard glassy finish.

Another relatively new development is the solid state <u>semiconductor detector</u>. A detailed discussion of doped semiconductor devices is beyond the scope of this manual, but for those who may have some knowledge of the subject, these devices are made of p-doped germanium on the surface of which is drifted an n-type impurity such as lithium. In effect, this results in the formation of a junction diode. For detection purposes it is reverse biased so that it does not conduct. However, ionization of the depletion region by incident radiation will result in momentary current pulses which can be detected. These detectors are ordinarily used to detect γ rays for precise analytical work.

INSTRUMENTATION AND CONTROL SYSTEM APPLICATIONS

Nuclear instrumentation and control applications in a nuclear power plant can generally be divided into four types of systems: neutron monitoring system, reactor protection system, process radiation monitoring system and area and environmental monitoring system. In addition to this is the instrumentation associated with the counting room and personnel radiation protection monitoring.

Neutron Monitoring Systems

Instrument systems must be provided to measure very accurately at all times the reactor power level and the rate of change of reactor power level (reactor period - see Chapter 10 for a discussion of this concept). This information is required by the operator in order for him to intelligently operate the reactor and must be fed into the reactor protection system which operates to shut the reactor down if certain power level or rate of change of power level limits are exceeded. Conventional methods of measuring thermal power output (flow, temperature and pressure measuring instrumentation) do not respond fast enough to protect against the rapid power increase of which a reactor is capable. However, reactor power changes are brought about by changes in the core neutron population, and the latter can be measured accurately and rapidly by a variety of neutron monitoring systems. In general, these systems are composed of a detector, which is usually a gas filled ion chamber or proportional counter, plus an appropriate combination of electronic equipment including high voltage power supplies, amplifiers, and readout devices.

The detectors for the neutron monitoring systems may be located outside of the reactor vessel or they may be located within the reactor core. If it is possible to do so without unduly sacrificing accuracy, the former method is preferred because the detectors tend to be rather fragile and the environment outside the vessel is considerably less hostile than that within the core (in particular, the temperature is much lower and the detector is subjected to less severe bombardment by ionizing radiation which tends to result in gradual deterioration of insulation, etc.). If the detector is located outside of the vessel, it is generally located in a holder located near the external wall of the reactor pressure vessel at the elevation of the core. Located in this manner, the detector responds to those neutrons which reach it after having leaked from the core, passed through the reflector, and penetrated the wall of the vessel. Only a very small percentage of the core neutron population makes this journey. The accuracy of the detector depends upon this percentage remaining nearly constant for all reactor operating conditions. That is, if the core neutron population doubles, the number of neutrons reaching the detector must also double if the detector response is to be of any value. Because the average travel length of neutrons within the core is only a matter of inches, most of the neutrons seen by the out-of-core detectors originated near the edge of the core. The detectors see very few neutrons which originated near the center of the core. This is not a serious deficiency in physically small cores where the entire core tends to work as a single unit and individual regions are not likely to be working independently of each other. Then we can be reasonably sure that a power increase in the center will result in a comparable power increase on the edges as well so that the detectors will respond accurately. Unfortunately, in large present day BWRs the physical size of the core is so large that we can no longer depend upon this behavior. That is, a power change may occur in the center of the core which would go completely unnoticed by an out-of-core detector. Thus these reactors have gone to employing nothing but in-core detectors. Large PWRs, however, continue to use out-of-core detectors since their cores are physically smaller than BWRs of comparable output and since they are less likely to have substantial variations in power distribution (i.e., power increases in one region without a corresponding increase in other regions) because they operate with virtually all control rods withdrawn.

The neutron population in a power reactor will typically change by seven decades or so (i.e., a factor of 10^7) in going from the fully shutdown condition to full power. To design a single instrument which can cover a range this large is virtually impossible. As a result, a series of three neutron monitoring systems is normally employed. These are usually called the source (or startup) range monitors, the intermediate range monitors, and the power range monitors respectively in order of decreasing sensitivity. The exact range covered by each system varies somewhat from plant to plant, but their ranges always overlap so that something is responding at all times between shutdown and full power. Figure 14-5 shows the ranges covered by each system at three representative plants. A more detailed discussion of each type of system is given below.

1. Source range monitors

   As one might expect, the most sensitive monitoring system ordinarily is a pulse type system. The detector for this system is usually a proportional counter since this type of chamber combines the desirable features of high sensitivity, fast response (short resolving time), and ability to distinguish between neutrons and gamma rays. The alert reader, however, must be wondering how a proportional counter can detect neutrons since the latter carry no charge and do not ionize the fill-gas. The answer is that the detector has some boron-10 added to it, which in the presence of thermal neutrons, undergoes the reaction ${}_5B^{10}(n,\alpha)\,{}_3Li^7$. Ionization within the detector is

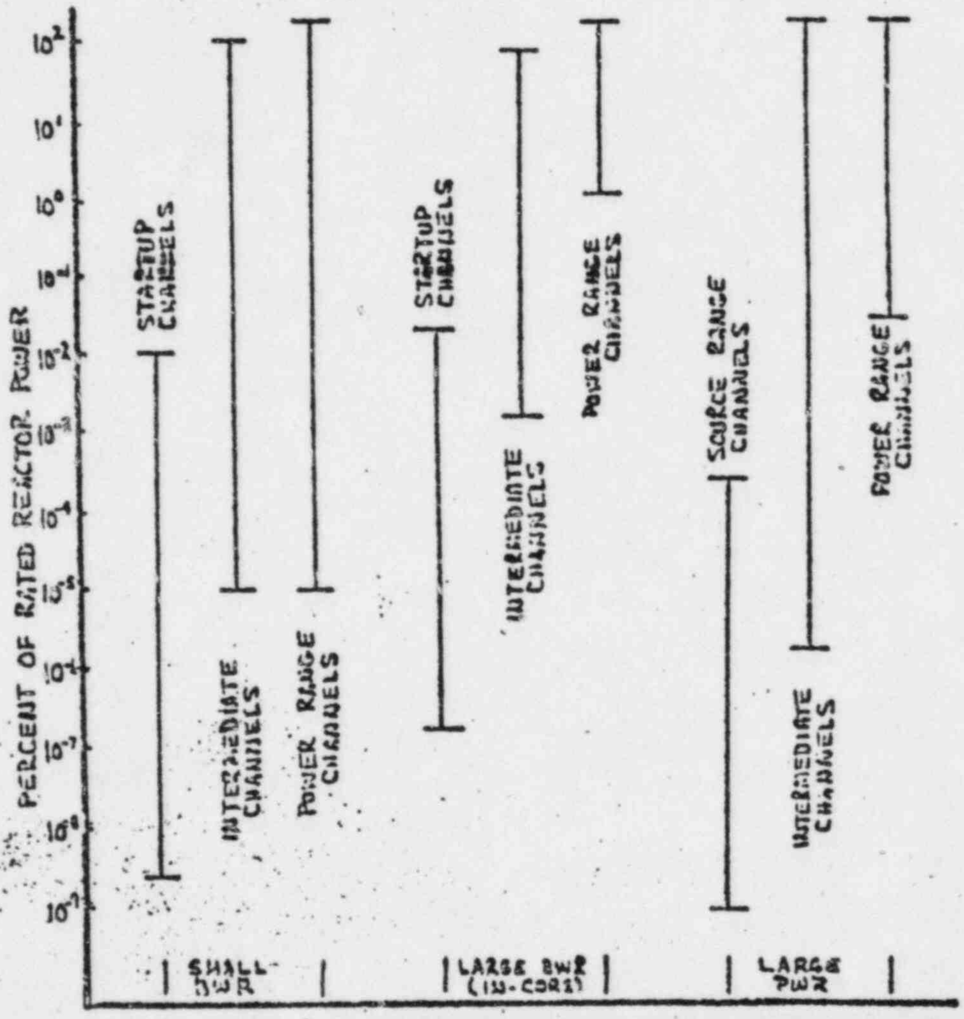

Figure 14-5: Ranges Covered by Neutron Monitoring Systems

produced by the α particle and the Li-7.[1] Since the range of an α particle or a Li-7 ion is very short, the boron must be located within the chamber itself if the particles are to reach the fill-gas. One way of accomplishing this is to fill the detector with a gaseous compound of boron. The most common choice is boron trifluoride (BF_3) and such detectors are known as "BF_3 proportional counters." We saw previously, however, that monatomic fill-gases were preferred for proportional counters. Therefore, it is often desirable to obtain the neutron response by coating the interior of the walls of the detector with a thin film of B-10. Such detectors are known as boron lined proportional counters or "B-10 proportional counters" and they are ordinarily filled predominantly with argon.

The arrangement of electronic equipment in the startup channel varies somewhat from plant to plant. However, a typical arrangement is shown in Figure 14-6. High voltage is applied through a resistor (the same resistor which is shown in Figures 14-1 and 14-3) to the center wire of the detector. The signal from the detector, which is very weak, is sent to a preamp and then to a linear amplifier to increase its size. The signal is then sent to a discriminator. This device looks at the size of the input pulse and blocks those which are smaller than a certain minimum size and passes those which exceed this minimum size. In essence, it is at this point that we separate pulses caused by neutrons and gamma, passing the former on to the counting circuit and eliminating the latter. This is possible, of course, because the neutron pulses are really caused by the heavily ionizing α and Li-7 particles and are much larger than those caused by the less ionizing γ rays. It is important that we do discriminate in the circuit because after a reactor has run for awhile and built up a large accumulation of fission products,

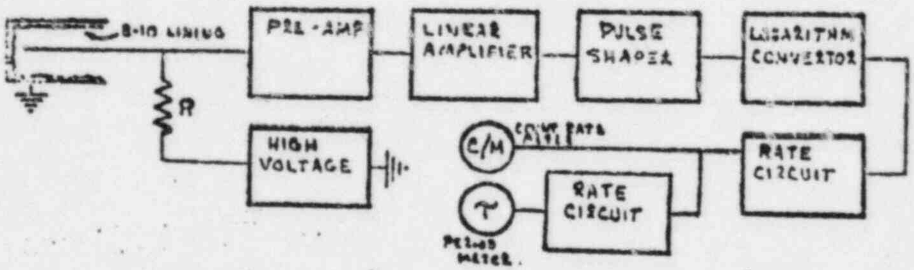

Figure 14-6: Typical Source Range Monitoring System Employing B-10 Proportional Counter

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[1] The Li-7 will ultimately become a neutral atom (as will the α, for that matter), but immediately after the reaction it has a lot of kinetic energy and it "runs away" from its electrons for a moment during which time it ionizes.

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the gamma field around the detector is very high. When the reactor is shut down, the neutron field is very low, so our monitoring system response during the initial phases of a startup would be almost entirely due to γ rays and we would be receiving a false picture of what is really happening to the core neutron population.

After leaving the discriminator, the neutron pulses are sent to a shaper which makes them all exactly uniform and the proper size and duration to be compatible with the remaining electronic equipment. The pulses are then sent to a device which extracts the logarithm of the count rate. This value is displayed on a meter with a logarithmic scale which generally reads out in "counts per second."[1] The final signal is also sent to a device which computes its rate of change, and determines the period (or startup rate). This value is also displayed.

One of the principal problems in the source range is trying to pick out the relatively small number of pulses produced by neutrons from the large number produced by gammas. The discriminator is the major piece of equipment which is used to accomplish this, but its task can be aided by other means. One way is to shield the detector with lead. This stops gamma rays, but has a much smaller effect on neutrons since lead has a relatively small cross section. Another method of improving the response which is generally applicable to out-of-core detectors is to surround the detector with a good moderating material such as polyethelene (which contains a lot of hydrogen). The overwhelming majority of the neutrons which reach the vicinity of the out-of-core detectors are fast neutrons since thermal neutrons simply do not travel far enough. The detector, however, is only sensitive to thermal neutrons since B-10 is principally a thermal neutron absorber. There is considerable thermalization occurring in the various materials in the vicinity of the detector (particularly in the concrete biological shield which surrounds the reactor vessel), but the efficiency of the process can be improved with the polyethelene and so the neutron signal can be increased without a corresponding increase in the γ signal.

When it is desirable to locate the source range detector within the core, a so-called fission chamber is ordinarily used in place of the B-10 proportional counter. The fission chamber is an ion chamber with a coating of U-235 on the inside surface of the detector wall. Thermal neutrons fission this U-235 and the highly charged fission fragments produce ionization in the fill-gas. The reason for using this type of detector is that the size of a pulse produced by a neutron is overwhelmingly larger than that produced by a γ because the fission fragments are so highly ionizing. When a detector is located within the core, the γ field is so high that it is virtually impossible to discriminate them out from pulses produced by α and Li-7. This is because the simultaneous

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[1] A logarithmic scale is used because several decades must be covered and also because it is necessary to take the logarithm of the signal before the period can be determined (for mathematical reasons beyond the scope of this manual).

arrival of many γ's in the tube can produce a pulse which is as large as that produced by the neutron. With the fission chamber, this is less likely to occur because it would require the simultaneous arrival of a much greater number of γ rays.

Finally, most reactors employ at least two independent source range monitoring systems. The detectors are located on opposite sides of the core so that at least one monitor is likely to see a rod withdrawal.

2. Intermediate range monitors

In the intermediate range, the neutron signal is fairly high and so **the most common systems employ ion chambers operating in the current** mode. Since the neutron signal is still relatively low at the bottom end of the intermediate range, however, it is still necessary to provide a means of distinguishing it from the γ signal. A discriminator will not work since these are not pulse instruments. In these instruments, the current produced by γ interactions simply adds to that produced by neutron interactions and it is impossible to tell what fraction of the current came from each source.

Therefore, a slightly different type of detector, called a gamma compensated ion chamber (CIC) is normally used in intermediate range monitoring systems which employ out-of-core detectors. A simplified schematic diagram of a detection system employing a CIC is shown in Figure 14-7. The CIC is really two ion chambers located within a single case. One of the chambers is coated with boron enriched in the B^{10} isotope and is therefore sensitive to neutrons and gamma radiation. The second chamber is uncoated and thus is sensitive only to gamma. By connecting the two chambers so that their output currents buck (the currents electrically oppose each other), then the net electrical output or current from the chamber will be the algebraic sum of the two ionization currents. If the neutron detector is perfectly gamma compensated[1] its output will be proportional to the neutron flux at the detector location. Gamma compensation can be obtained by initially designing the chambers to give a fixed percentage compensation, by designing the chambers so that their volumes can be varied to obtain the desired compensation, or by varying the high voltage supplied to each chamber to obtain the desired compensation.

Normally, CICs are designed so that they are slightly undercompensated (approximately 95% compensated). In an undercompensated chamber the

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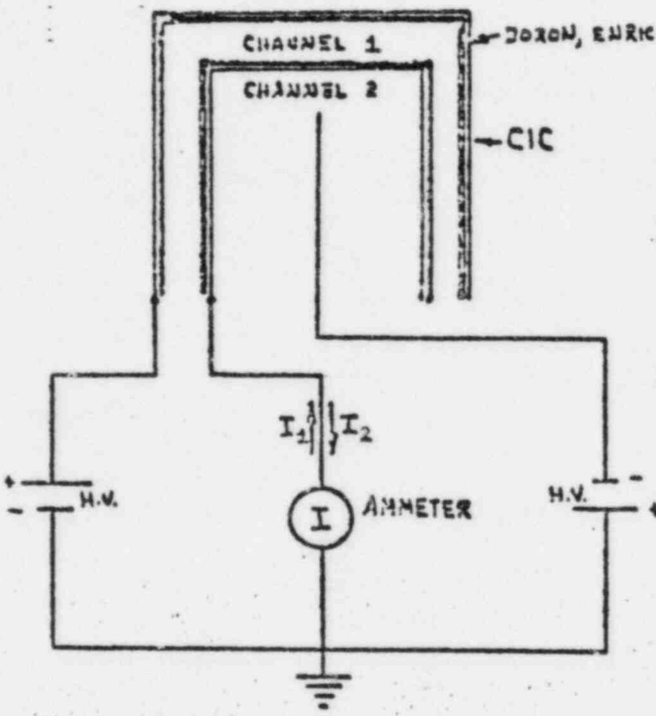
[1] Perfect, or 100%, gamma compensation means that the CIC is designed so that operation in a pure gamma field results in equal output currents from both chambers. With the output currents bucking each other, the net current due to gamma radiation would equal zero. In a gamma and neutron field, the net current would then be proportional to the neutron flux only.



I = Instrument current
I_1 = Current from chamber 1
I_2 = Current from chamber 2

$$I = I_1 - I_2$$

FOR 100% COMPENSATION

$$I_1 = I_N + I_\gamma;\ I_2 = I_\gamma$$

$$I = I_N + I_\gamma - I_\gamma = I_N$$

FOR 95% COMPENSATION

$$I_1 = I_N + I_\gamma;\ I_2 = 0.95\ I_\gamma$$

$$I = I_N + I_\gamma - 0.95\ I_\gamma$$

$$I = I_N + 0.05\ I_\gamma$$

Figure 14-7: Simplified Measurement Circuit Employing a Compensated Ion Chamber

current from the boron lined chamber is greater than the current from the unlined chamber in a pure gamma field. This has the advantage that the indicating and recording instrumentation always reads upscale. It should be pointed out, however, that a grossly undercompensated CIC is hazardous because the instrumentation will incorrectly read upscale at low power levels and consequently will not accurately reflect a change in power level. In an overcompensated CIC the current from the unlined chamber (gamma only) is greater than the current from the boron lined chamber in a pure gamma field. A grossly overcompensated CIC is also hazardous because the instrumentation might be pegged downscale at low power levels. An increase in power level would then be completely masked. With either grossly undercompensated or overcompensated CICs the operator must use the startup channels until he sees a significant neutron reading on his intermediate range instrumentation.

When the reactor is operating at high power level the gamma contribution to the signal from a CIC is very small so that most of the signal current is due to the neutron flux. Under these conditions a loss of high voltage to the gamma only chamber would hardly be noticed. A loss of high voltage to the boron lined chamber would, of course, cause the instrumentation to peg downscale.

To further illustrate the effect of compensation upon the response of a detector, consider Figure 14-8, which shows the response of

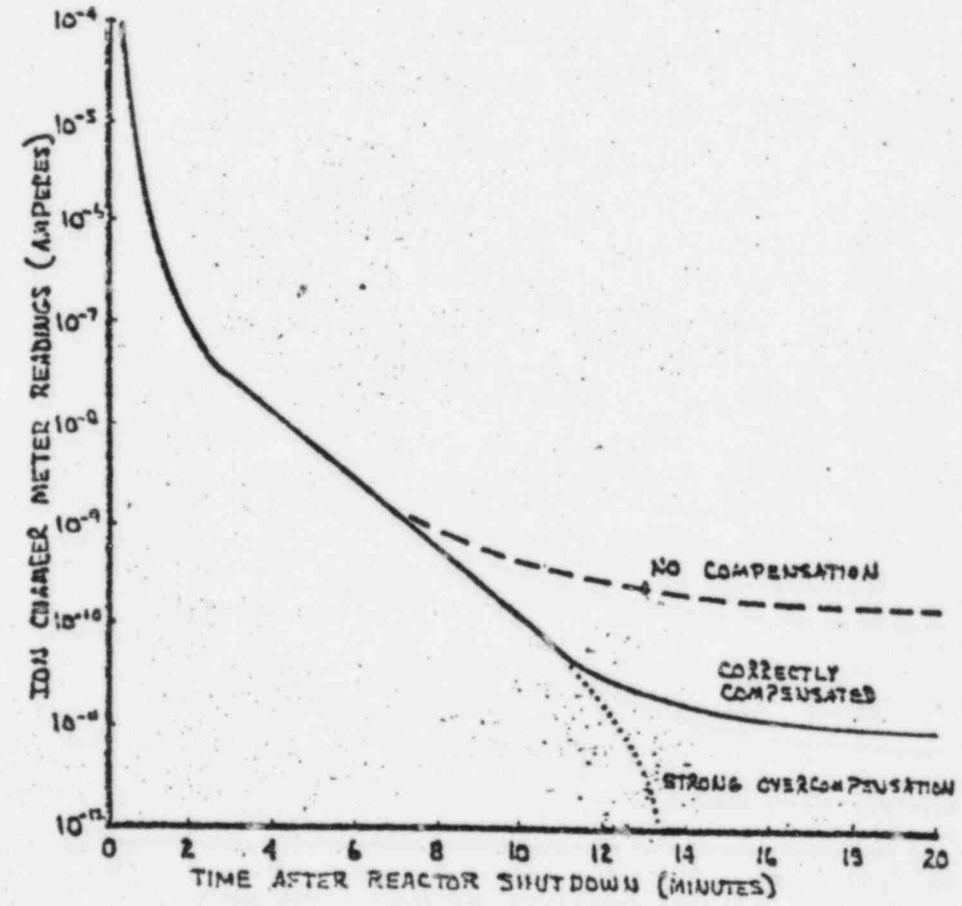


Figure 14-8: Response of a CIC on a Reactor Shutdown

an intermediate channel with a logarithmic display to a rapid shutdown of the reactor. Notice that as the power falls (i.e., the proportion of the signal attributable to neutrons gets smaller), a grossly undercompensated detector reads upscale since it continues to respond to the γ rays. Conversely, the large γ signal from the unlined chamber overwhelms the neutron signal and drives the instrument downscale in the overcompensated case.

As mentioned above, the intermediate range neutron monitoring systems are ordinarily provided with a logarithmic readout since they must indicate over several decades of reactor power. Since a logarithmic readout is desirable, and the logarithm must be taken before the period is determined (again, for mathematical reasons) it is also convenient to go ahead and provide an indication of the period (or startup rate) on these instruments (actually, it is often the other way around - since it is desirable to have period indication, it is convenient to make the instrument logarithmic). A simplified block diagram of a typical system is shown in Figure 14-9.

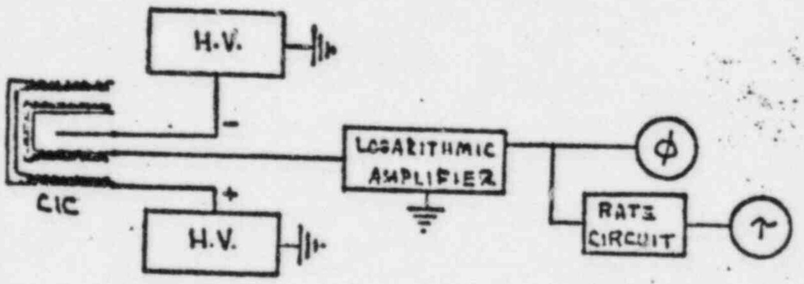


Figure 14-9: Simplified Block Diagram of an Intermediate Range Monitoring System Employing a CIC

Once again, the new large BWRs use in-core fission chambers for the intermediate range monitoring system. This system monitors the neutron flux by a new technique known as the voltage variance method. This method depends upon the fact that neutron interactions with the detector are a statistical process and therefore even at constant power the signal will have a small plus or minus associated with it. The amplitude of this plus and minus variation is related to the size of the signal itself and can be used as a measure of fission rate. Finally, most reactors utilize at least two intermediate range monitoring systems with the detectors appropriately located to look at different regions of the core.

3. Power range monitors

The power range instruments are ordinarily ion chambers connected to linear readouts. Linear readouts are preferred because they are more expanded at the top end of the scale and thus can be read more accurately (a log scale compresses the top of the range and expands the bottom).

There are a number of variations of the power range monitoring channels. The early BWRs like Humboldt use a CIC similar to that employed in the intermediate range channels. This detector is connected to an extremely sensitive ammeter which can measure currents as small as 10^{-12} ampere.[1] The picoammeter is provided with a range switch and a number of scales of different sensitivity. By changing scales, the range of these instruments is extended over many decades (a logarithmic instrument, however, can cover this same range using just one scale). Since these instruments are useful at very low power levels, where the γ signal is significant in comparison to the neutron signal, the use of a CIC is required if the instrument is to read correctly. The instruments are calibrated on the basis of heat balances to read total core power expressed in % of full power. Three of these power range channels are used at Humboldt with the detectors located in holders just outside the vessel and positioned near the axial centerline of the core. The detectors are 120° apart around the circumference of the vessel.

The large PWRs like Diablo Canyon employ uncompensated B-10 lined ion chambers in their power range systems. Compensation is not required because those instruments are not intended to respond until the neutron flux is high enough that it overwhelms the γ signal. These detectors are unusual in that they are very long, stretching the nearly entire axial length of the core. In addition, the detectors are divided in half so that, for all practical purposes, we have two separate ion chambers setting on top of each other. The upper portion of the detector monitors the neutron flux in the upper half of the core and the lower half looks at the bottom of the core. Each half of the long ion chamber feeds its own individual ammeter and readout. In addition, the signals from the two halves are summed and read out. Thus the operator can read not only total core power, but also how it is divided between the upper and lower halves of the core. The reason for providing the latter information is so that the operator can keep an eye out for possible axial xenon oscillations and for distorted power distributions which could adversely affect the CHFR (see Chapters 11 and 12). Four of these monitoring systems are used, with the detectors being located in instrument wells in the biological shielding surrounding the vessel adjacent to each of

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[1] Since the prefix "pico" stands for 10^{-12}, these instruments are often referred to as picoammeters.

the four "corners" of the core.

In addition to the out-of-core power range instruments, both the early BWRs and all PWRs are furnished with miniature in-core fission chambers to measure the power at selected discrete locations within the core in order to determine the power distribution. In the BWRs these small detectors are permanently located at fixed positions within the core while the PWR system uses movable detectors which are inserted into tubes which extend into various core regions.

The large BWRs use miniature in-core fission chambers for both local monitoring and for determining total core power. Some of the detectors are at fixed locations and others can be moved about.

In both PWRs and BWRs a high reading on the power range instruments will produce an automatic reactor trip.

Reactor Protection System

A nuclear power plant reactor protection system encompasses the safety sensors which monitor critical plant parameters and all electrical and mechanical devices, and circuitry involved in generating trip signals associated with the protection functions when a safety sensor responds to an unsafe operating condition. The protection functions include scramming the reactor, blocking control rod withdrawal and actuating engineered safeguards such as containment isolation and emergency core flooding and cooling. One or more of these safety functions may be actuated when the approach to an unsafe operating condition is sensed depending upon the nature and severity of the abnormal condition.

The proposed IEEE Standard for Nuclear Power Protection Systems lists many requirements for these systems. The most significant of these requirements are summarized to show the general philosophy used in the design of these systems:

1. The protection system shall with precision and reliability initiate appropriate action whenever a plant condition monitored by the system reaches a trip level.

2. Any single failure in the system shall not prevent proper protection system action when required.

3. Channels[1] which provide signals for the same plant protection function shall be independent and physically separated.

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[1] A channel in this context is defined as an arrangement of components as required to generate a single trip signal related to a plant condition requiring protective action. A channel loses its identity where single trip signals are combined.

4. Protection system inputs shall be derived from signals which are direct measures of the plant condition monitored (to the extent feasible and practical).

5. Means shall be provided to check the operational availability of each system input sensor during reactor operation.

6. The protection system shall be designed so that once a protection system action is initiated, it will go to completion.

7. Means shall be provided for manual initiation of protection system action.

All reactor protection systems are designed with the coincident trip philosophy[1] to provide a safe and reliable system since a single sensor failure will not defeat the safety operation of the system nor will a single failure cause a spurious trip operation. The sensors are closed under normal operating conditions and open on an abnormal condition. All relays associated with tripping or reactor scram are normally energized. This arrangement provides a "fail safe" type of system since most failures such as broken leads or open or shorted coil would result in returning the relay to its de-energized position and de-energizing the circuit.

One of the most popular type of safety system designs is the dual channel safety system where each safety channel is fed by at least two independent sensors from each measured variable. The operation of one sensor in a safety channel will result in annunciation and a trip signal from that channel (often referred to as half-scram). A simultaneous operation of a sensor in each of the two safety channels results in a reactor scram. The failure to operate of one sensor associated with a measured variable will not impair the ability of the other sensors to operate. Also, a false operation of a sensor cannot cause a reactor scram since it trips only one safety channel. With this arrangement, any sensor can be calibrated, maintained, or tested without causing a scram or without de-activating its trip. Each safety channel receives its supply power from a separate source.

Typical plant parameters that are connected to the reactor protection system are high reactor pressure, low reactor water level, high neutron flux (high power), short reactor period, high containment system pressure and low condenser vacuum. A manual reactor scram is always provided. Other parameters may be tied into the protection system depending on the design of the reactor, the primary system and its containment.

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[1] As an example of coincidence, consider a case where the reactor will trip on high power. The sensor for this is the power range nuclear instrumentation and in a typical PWR there are four of these. In general, however, it would take a simultaneous high reading on at lease two of these instruments to cause a trip. Thus we speak of this as a two out of four coincidence.

Process Radiation Monitoring System

A process radiation monitoring system is provided in nuclear plants to monitor and record the levels of radioactivity in plant process streams and annunciate any significant increase in radioactivity levels in these process lines. With few exceptions, these systems provide no automatic operation, but serve primarily to provide the operator with the information he needs to make prudent decisions in the operation of the plant.

In general, a process monitor consists of a detector connected to a logarithmic remote readout in the control room. The process monitors are, for the most part, monitoring for the presence of fission products and corrosion products which are β-γ emitters. Since the detector is ordinarily located outside of a pipe, or in a well within a pipe, they tend to see more γ than β, but this is not always the case if geometrical considerations are favorable for β detection. In any event, the most commonly used detectors for these instruments are NaI scintillation detectors and G-M tubes because of their sensitivity. If the activity level in the process stream is expected to be high, an ion chamber may be satisfactory.

Typical process streams which are monitored include:

1. Stack or plant vent where gaseous radioactive materials may be released;
2. Liquid waste discharge pipe;
3. Component (or closed) cooling water system;
4. Condenser air ejector;
5. Reactor water;
6. Condensate or steam generator liquid.

In most cases, the purpose of these instruments is simply to inform the operators of the radiation status of the processes, but in a few cases, they perform some automatic safeguards functions. In particular, the monitors on the waste discharge lines generally operate automatic isolation valves to block the release of wastes which exceed predetermined limits.

Area Monitoring System

The primary purpose of the area monitoring system in a nuclear plant is to protect plant personnel by monitoring the gamma radioactivity levels in areas where personnel may be required to work. The system at both Humboldt and Diablo Canyon consists of several channels or stations, each of which consists of a fixed position G-M tube detector and its associated power supply amplifier, and readout. Normally, both local and control room readout on a logarithmic scale are provided for each station. The gamma field at each station is also recorded on a multipoint recorder located in the control room which provides a record of radioactivity as a function of time. The readouts are in mr/hr and cover a range of either 0.01 to 1000 mr/hr or 0.1 mr/hr to 10 R/hr. When the radioactivity level for a station exceeds a preset level, an alarm is displayed at the local station and in the control room.

Sufficient stations are provided so that no area of the plant where personnel may normally work for extended periods of time is left unmonitored. A built-in remotely operated check source is provided at each station to check that it is operating properly. A portable test station provided by the supplier of the system is used to periodically check the calibration of each monitoring station.

Environmental Monitoring System

Several off-site environmental monitoring stations are normally set up approximately two years prior to initial plant operation to determine the background gamma dose rate in the environs. Following initial operation these stations provide the data for determining the plant's contribution to the integrated gamma dose rate to the environs.

Each station at Humboldt contains two 10 mr "stray radiation chambers" (which are nothing more than a sensitive integrating type ion chamber) and a film pack. Considerable work is now being done utilizing thermoluminescent dosimeters for environmental monitoring and it may well be that in a very few years a typical station will consist only of two of these detectors.

The number of stations set up around a plant will depend on the terrain surrounding the plant site. At Humboldt 30 stations were set up initially and after three years of operation the number was increased to 36 stations.

There are many other aspects of an environmental monitoring program, such as air sampling, meteorological data collection, and environmental sampling and sample analysis. Discussion of these topics will be left to texts on radiation protection.

Counting Room Instrumentation

Every nuclear power plant has a counting room which contains equipment for determining the quantity of radioactivity in air, liquid and solid samples collected from the various plant processes, in radiation protection surveys and from the environs. A typical counting room will contain an internal proportional counter, a single channel analyzer, a multichannel analyzer and a liquid scintillation counter.

The internal proportional counters (IPC) are used for alpha and/or beta counting. The detector is a hemispherical gas filled chamber into which the sample to be counted is placed for counting. After inserting a sample and sealing the chamber, the chamber is purged, then filled with P-10 gas (90% argon and 10% methane). A continuous flow of gas through the chamber is maintained while counting to prevent air inleakage. Operating voltages for the detector are in the proportional range so that pulse height is proportional to the primary ionization caused by the radiation being counted. The pulses from the detector are amplified and fed into a discriminator circuit. Only pulses large enough to pass through the discriminator are counted. Pulses due to gamma usually are so small that they are blocked by the discriminator because the specific ionization of gamma (ion pairs/cm) is so low. Therefore, gamma pulses are not normally counted. Some gamma counts are obtained, however, due to the photoelectric effect when gammas are absorbed by the orbital electrons of the atoms in the wall of the detector. The gamma count rate is normally small enough that it can be ignored.

The liquid scintillation system is used for detecting the low energy β from tritium. Most tritium samples are water solutions and measured amounts of the samples are dissolved, along with the scintillation phosphor, in a solvent such as toluene. The vials containing the sample-phosphor solution are then placed in the instrument next to a photomultiplier tube and counted in the usual manner.

Before discussing the single channel and multichannel analyzers, a general discussion of pulse height analysis is desirable. Whenever the output pulse from a counter or ionization chamber is proportional to the energy dissipation in the detector, the measurement of pulse heights may be used to determine the energy of the incident radiation. The simplest pulse height selection can be obtained by utilizing an integral or baseline discriminator which allows only pulses above a certain minimum size to be measured (this is what was done in the startup monitoring system). By calibrating an adjustable integral discriminator, counting rates may be measured as a function of discriminator setting (voltage). Plotting counting rates versus discriminator setting gives a curve like that shown in Figure 14-7a which is called an "integral bias curve." As can be seen from this curve, the higher the discriminator setting, the fewer pulses that are passed. This curve could be differentiated point-by-point to obtain a more useful curve called a "differential bias curve," such as Figure 14-7b, which gives the distribution of pulse heights.

Pulse height distributions such as Figure 14-7b can be obtained directly and much more accurately with a single channel analyzer. In this instrument there are two discriminators (called a differential discriminator) used to pass only pulses of such a height that they fall between the two discriminator settings. The two discriminators can be moved up and down the voltage scale together with a constant voltage interval between them. This voltage interval is commonly referred to as the "window" or "channel." The channel width can be made wider or narrower depending upon the resolution desired. Generally, the narrower the window, the better the resolution. Normally, the pulse height range of a single channel analyzer is 0 to 50 volts or 0 to 100 volts and channel widths

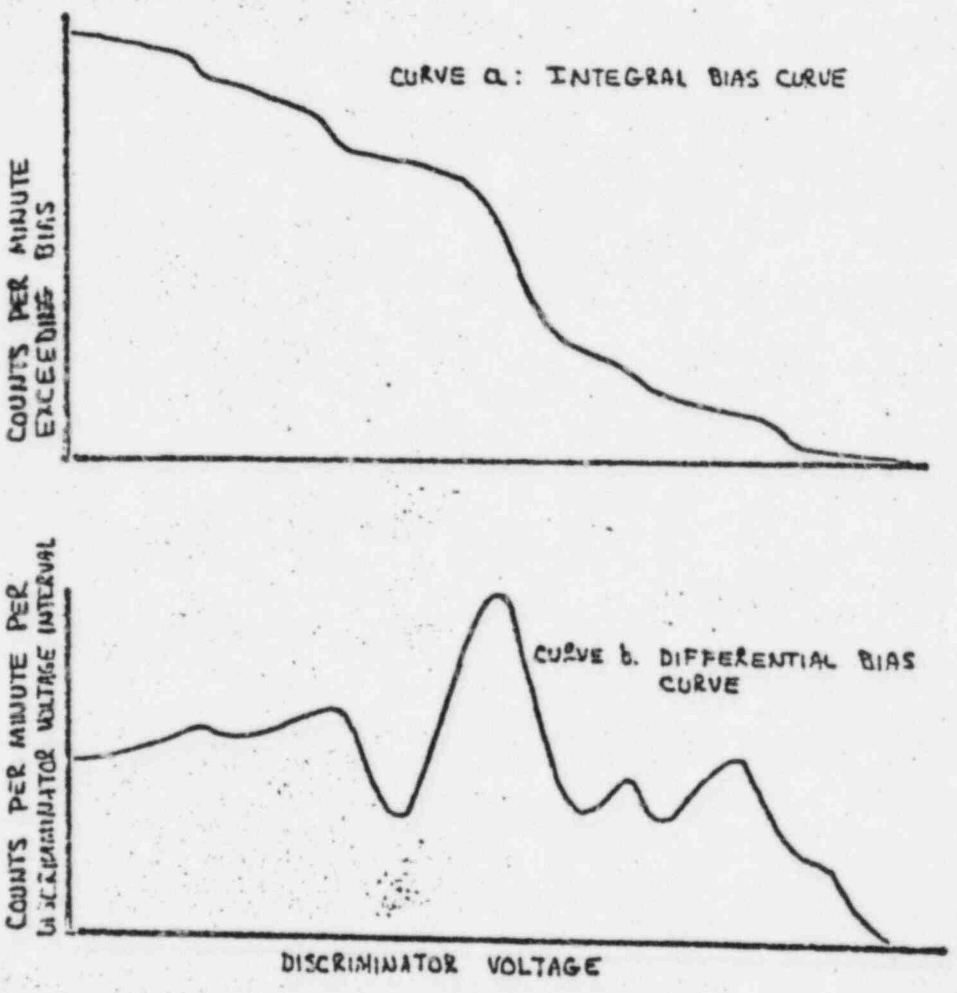

Figure 14-7: Discriminator Curves

of 1 to 5 volts are often used. The voltage scale is normally calibrated in units of particle energy by adjusting the amplifier gain to give a γ ray of known energy a definite sized voltage pulse. By calibrating the system in terms of energy, differential bias curves such as Figure 14-8 are obtained.

Although single channel analyzers are adequate for a large number of measurements, there are many times when a multichannel analyzer is needed.

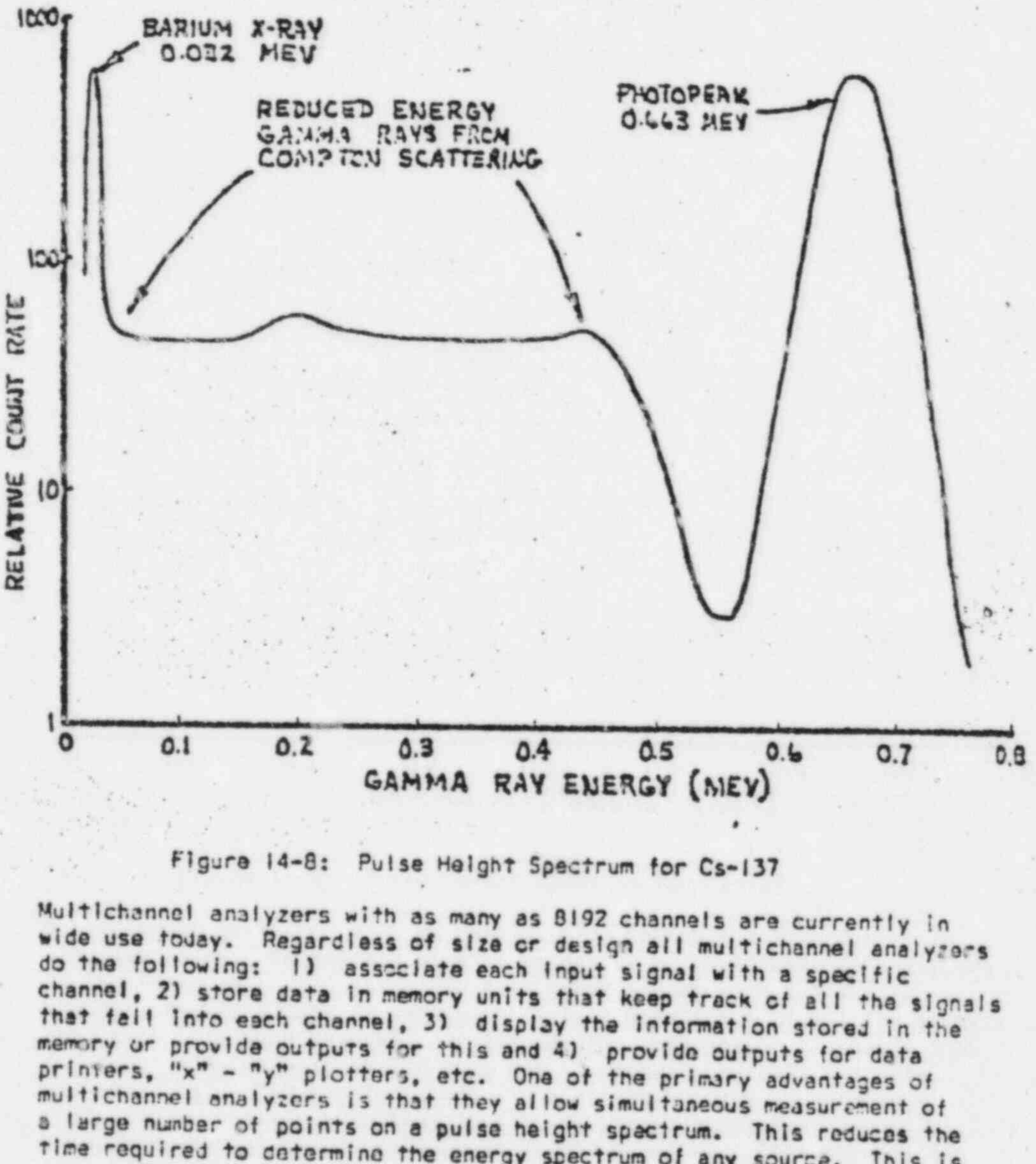

Figure 14-8: Pulse Height Spectrum for Cs-137

Multichannel analyzers with as many as 8192 channels are currently in wide use today. Regardless of size or design all multichannel analyzers do the following: 1) associate each input signal with a specific channel, 2) store data in memory units that keep track of all the signals that fall into each channel, 3) display the information stored in the memory or provide outputs for this and 4) provide outputs for data printers, "x" - "y" plotters, etc. One of the primary advantages of multichannel analyzers is that they allow simultaneous measurement of a large number of points on a pulse height spectrum. This reduces the time required to determine the energy spectrum of any source. This is especially important when the source has a short half-life or when the intensity or character of the radiation being measured varies during the counting interval.

The single channel analyzer is used almost exclusively for gross gamma counting since the multichannel analyzer is available for pulse spectrometry. The detector is a scintillation detector. The detector output is preamplified and fed into a linear amplifier which incorporates a pulse height analyzer. A scaler is used for readout. A separate high voltage power supply provides high voltage for the detector unit.

The multichannel analyzer is used whenever the gamma energy spectrum of a source must be determined. The detector is a scintillation type or a solid state type which feeds a preamplifier. The signal from the preamplifier feeds the multichannel analyzer which incorporates the necessary amplifier and high voltage power supplies. The multichannel output is displayed on an oscilloscope which is part of the analyzer and is also fed to an "x" - "y" plotter where the energy spectrum curve is drawn automatically.

Radiation Protection Monitoring Instruments

I. Radiation Survey Instruments

Many instruments, made by different manufacturers are available for making radiation surveys. Generally, these instruments can be divided into two groups; those used for the detection of radiation which do not provide a reliable quantitative indication of radiation dose rate and those which are used for the measurement of the radiation dose rate. Only the most commonly used survey instruments will be discussed.

A G-M survey meter is generally used for the detection of beta-gamma radiation. The G-M tube is contained in a probe assembly which is attached by an electric cable to a portable instrument box which contains the power supply,[1] electronic amplifying measuring circuitry and indicating meter. The indicating meter usually reads counts per minute although some instruments may have a scale calibrated roughly in mr/hr. The latter is a bad practice, because this dose rate calibration is only good for the particular particle energy for which the scale was calibrated. The probe of this instrument is provided with a metal shield which can be positioned to cover the G-M tube. In this position, the G-M is sensitive to gamma radiation only. With the tube uncovered, the instrument is sensitive to both beta and gamma radiations. Headphones are provided with G-M survey instruments for aural detection of radiation.

For the detection of alpha particles, some form of an alpha survey meter is used. This is sometimes a G-M probe with a very thin window but more often it is a ZnS scintillation detector. In most cases

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[1] Portable G-M and alpha survey meters are normally supplied with battery packs, however, a-c operated instruments are built for use where surveys are to be carried out at a fixed location such as access control.

the alpha detector is just a special probe which attaches to the electronics package for the G-M survey instrument.

The most commonly used instrument for beta-gamma dose rate measurements is called the Cutie Pie (or C.P.). The C.P. is a pistol-shaped instrument with an air filled ionization chamber as the barrel. The ion current produced in the chamber is amplified and read on a microammeter. Since the dose rate is proportional to the ion current, the meter is calibrated in mr/hr. The C.P. is not sensitive to alpha radiation because of the shielding effect of the window in the end of the barrel. A beta shield is attached to the side of the barrel and is placed over the window whenever gamma only measurements are desired. Ordinarily, the C.P. is carried with the window uncovered (window open) to afford maximum protection to the surveyor.

2. Personnel Monitors

Another class of radiation protection monitoring instruments are the personnel monitors which are worn by all personnel who normally are required to work in a radiation area. The personnel monitors most commonly worn are pocket ionization chambers and film badges. Both of these devices measure the total radiation dose received over a period of time.

The pocket dosimeter is an ionization chamber similar to a fountain pen in size and shape. It consists of a plastic or metal case, lined internally with graphite, with a central wire electrode. A special charger-reader is used to measure the potential difference between the center electrode and the inner wall of the case. This charger-reader is also used to charge the electrodes to a known potential difference (voltage). Once charged, the potential difference remains unchanged until ionizing radiation enters the chamber. When this happens, there is a decrease in charge proportional to the amount of radiation received. After a period of exposure, the charge remaining on the electrodes is measured by the charger-reader. The loss in potential is read as the radiation dose absorbed by the chamber. Pocket dosimeters measure gamma and to a slight extent beta radiation. They do not measure alpha particles. By lining the chamber wall with boron-10, pocket dosimeters are produced which are sensitive to slow neutrons.

Pocket dosimeters are also made which are self-reading. They are similar to the device just discussed, but are made so that the charge on the center wire causes a deflection in a quartz fiber. The fiber has the same charge as its support and is moved away from the support by repulsion of like charges. As the charge on the support and quartz fiber reduces because of ionization within the chamber, the quartz fiber moves closer to the support. The movement is indicated on a scale calibrated in mr/hr. The wearer can read on this scale total absorbed dose at any time he wishes. The chamber is recharged using a charger similar to the charger-reader already discussed.

The film badge consists of a plastic case with an identifying name plate or number. A film pack made up of several types of film with varying sensitivities to radiation is inserted into the plastic case. Remember that ionizing radiation produces an effect on photographic film similar to that of light. Various shielding materials are also used over parts of the film to help the film respond more uniformly to gamma radiation of all energies and to absorb low energy particles. Film is made that is sensitive to beta and gamma radiations and also to neutrons.

PROBLEMS

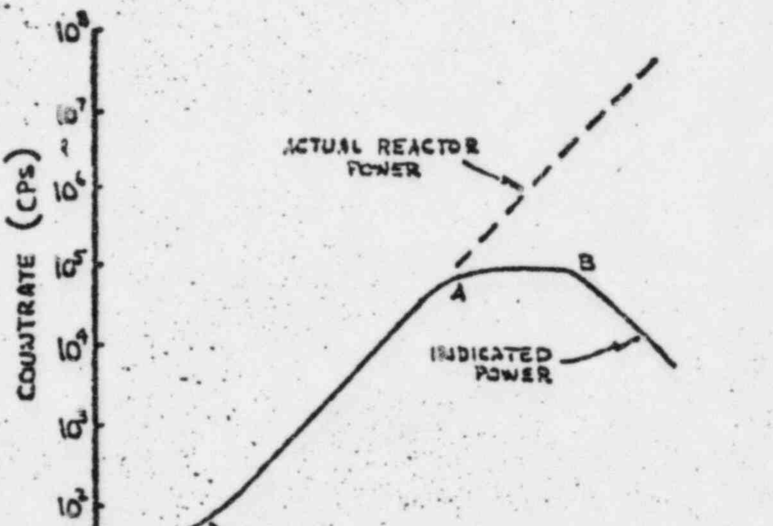
1. Discuss the principle of operation of ion chambers, proportional counters, and G-M tubes. Include relative sensitivities, counting spead, applicability to pulse and mean level usage, and other advantages and disadvantages of each.

2. What is gas amplification? Is it strongly dependent upon the number of primary ion pairs produced in a detector? Is it strongly dependent upon the voltage applied to a detector?

3. a) Discuss the principle of operation of a CIC.
   b) What is meant by the terms overcompensation and undercompensation? Are these conditions desirable or undesirable when a CIC is used in a power range neutron detector? Why?
   c) Occasionally a condition of slight overcompensation can be corrected by adjusting the high voltage to the unlined chamber. Explain what adjustment would be made and why it works.

4. Draw a simple one line block diagram of each type of neutron monitoring channel at your facility. Indicate the type of detector used; alarms, trips, and interlocks associated with the channel; and the approximate range of the channel.

5. The trace obtained on a startup channel employing a B-10 proportional counter and a logarithmic ratemeter with a maximum countrate capability of 10^5 cps is shown below. Discuss the reason for the behavior shown at points A and B.



6. a) Discuss the principle of operation of a scintillation detector.
   b) What are its advantages and disadvantages?
   c) Where are scintillators used at your facility?

7. Consider a B-10 proportional counter connected to a ratemeter which has an integral discriminator. The detector is placed in a field of thermal neutrons and gamma rays. The gamma ray energies are uniformly distributed up to about 1 mev. Assuming the voltage to the detector is set and left at the recommended value, plot a qualitative curve of the detector response which would be obtained in moving the discriminator setting from its lower to its upper limit. Indicate where the discriminator should be placed if the channel is to be used as a startup channel.

8. Discuss some of the things which could affect the calibration of an out-of-core power range channel employing a CIC.

9. Discuss the advantages of fission chambers over boron lined proportional counters for in-core use during refueling operations.

10. Discuss the design of the reactor protection (or safety) system at your facility in terms of the effect of component failure upon its dependability, and features designed to prevent spurious operation.

11. The isotope Zn^{65} decays by positron emission with a gamma ray energy of 1.12 mev. Draw the spectrum you would expect to see if you counted a Zn^{65} sample on a multichannel analyzer. Lable all important features of the plot.

12. Discuss the principle of operation, range, and type of radiation detected for each of the portable survey instruments and personnel dosimeters used at your facility.

13. List all places where radioactive gases and/or liquids are routinely released to the environment or could be released in the event of an accident at your facility. Discuss the monitoring systems employed at these locations.