

MPR ASSOCIATES, INC.

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CALCULATION TITLE PAGE

CLIENT Consumers Power		PAGE 1 OF 8	
PROJECT BIG ROCK POINT PLANT		TASK NO. 98-114	
CALCULATION TITLE AMPLIFICATION FACTOR FOR SINUSOIDAL FORCING FUNCTION FOR A SECOND-ORDER MECHANICAL SYSTEM		CALCULATION NO. (OPTIONAL) 98-114-HWH-1	
PREPARER(S)/DATE	CHECKER(S)/DATE	REVIEWER(S)/DATE	REV. NO.
HWH McCurdy 2-17-92	Michael Francis 2-17-92	John W. Zimany 2-17-92	0

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PREPARED BY

HW McCurdy

CHECKED BY

TH Francis

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Following is a calculation of the amplification factor resulting from a sinusoidal forcing function for a second-order mechanical system (see Figure 1).

The system equation is,

$$\frac{M}{g_c} \ddot{x} + C \dot{x} + Kx = F(t)$$

where,

M = system mass, lb

g_c = gravitational constant, ft-lb/lb-sec²

$x, \dot{x}, \ddot{x}$ = position (ft), velocity (ft/sec) and acceleration (ft/sec²)

C = system damping, lb-sec/ft

K = system spring constant, lb/ft

F = applied force, lb

Divide the system equation by K ,

$$\frac{M}{g_c K} \ddot{x} + \frac{C}{K} \dot{x} + x = \frac{F(t)}{K}$$

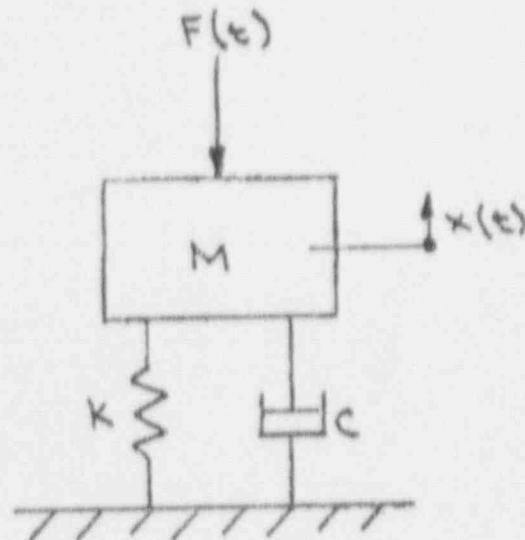
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M J. J. J.

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FIGURE 1

SECOND-ORDER MECHANICAL SYSTEM

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DEFINE THE FOLLOWING TERMS,

$$\omega_n^2 = \frac{K_{gc}}{M}$$

$$\frac{2\zeta}{\omega_n} = \frac{C}{K}$$

WHERE,

ω_n = SYSTEM NATURAL FREQUENCY, RAD/SEC

ζ = SYSTEM DAMPING COEFFICIENT

SUBSTITUTE IN THE SYSTEM EQUATION,

$$\frac{\ddot{x}}{\omega_n^2} + \frac{2\zeta}{\omega_n} \dot{x} + x = \frac{F(t)}{K}$$

TAKE LAPLACE TRANSFORM AND ASSUME INITIAL CONDITIONS ARE ZERO (SEE REFERENCE 1),

$$\left[\frac{s^2}{\omega_n^2} + \frac{2\zeta s}{\omega_n} + 1 \right] x(s) = \frac{F(s)}{K}$$

OR,

$$\frac{K x(s)}{F(s)} = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

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M Francis

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ASSUME THAT $F(t) = F_0 \cos \omega t$,

$$F(s) = F_0 \frac{s}{s^2 + \omega_0^2}$$

THE SYSTEM EQUATION IS,

$$\frac{Kx(s)}{F_0} = \frac{\omega_0^2 s}{(s^2 + 2\zeta \omega_0 s + \omega_0^2)(s^2 + \omega_0^2)}$$

$$\frac{Kx(s)}{F_0} = \frac{\omega_0^2}{2\zeta \omega_0} \cdot \frac{2\zeta \omega_0 s}{(s^2 + 2\zeta \omega_0 s + \omega_0^2)(s^2 + \omega_0^2)}$$

$$\frac{Kx(s)}{F_0} = \frac{\omega_0}{2\zeta} \cdot \left[\frac{1}{s^2 + \omega_0^2} - \frac{1}{s^2 + 2\zeta \omega_0 s + \omega_0^2} \right]$$

$$\frac{Kx(s)}{F_0} = \frac{1}{2\zeta} \left[\frac{\omega_0}{s^2 + \omega_0^2} - \frac{\omega_0}{(s + \zeta \omega_0)^2 + \omega_0^2(1 - \zeta^2)} \right]$$

TAKE THE INVERSE TRANSFORMS,

$$\frac{Kx(t)}{F_0} = \frac{1}{2\zeta} \left[\sin \omega_0 t - \frac{e^{-\zeta \omega_0 t}}{(1 - \zeta^2)^{1/2}} \sin \omega_0 (1 - \zeta^2)^{1/2} t \right]$$

ASSUME THAT $\zeta^2 \ll 1$, THE AMPLIFICATION FACTOR IS

GIVEN BY,

$$\frac{Kx_{MAX}}{F_0} = \frac{F_{MAX}}{F_0} = \frac{1}{2\zeta}$$

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H W McCurdy

CHECKED BY

TH J. HANCO

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FROM REFERENCE 2, A TYPICAL DAMPING VALUE FOR A BOLTED STEEL STRUCTURE (LIKE THE BIG ROCK POINT BAFFLE PLATE / CONCRETE PLATE STRUCTURE) IS 4 PERCENT. THE AMPLIFICATION FACTOR IS,

$$\frac{F_{MAX}}{F_0} = \frac{1}{2.04} = 12.5 \quad (\text{NOTE THAT THE 4 PERCENT COEFFICIENT IS CONSERVATIVELY LOW AND THEREFORE THE AMPLIFICATION FACTOR IS HIGH.})$$

FOR THE BIG ROCK POINT INLET NOZZLE / BAFFLE PLATE ARRANGEMENT, THE APPROPRIATE FORCE IS THE MOMENTUM FORCE EXERTED BY THE FLOW ENTERING THE NOZZLE. THE FORCE ON THE BAFFLE IS,

$$F(t) = f \cdot \frac{VM}{g_c} \cos \omega_n t = F_0 \cos \omega_n t$$

WHERE,

f = FRACTION OF TOTAL MOMENTUM FORCE WHICH CAUSES LOADING OF BAFFLE

V = FLUID VELOCITY EXITING THE NOZZLE, FT/SEC

M = MASS FLOW RATE ENTERING NOZZLE, LB/SEC

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THE AMPLIFIED FORCE IS,

$$F_{MAX} = 12.5 \cdot \frac{4VM}{g_c}$$

AS AN EXAMPLE, IF $F = \frac{1}{12.5}$,

$$F_{MAX} = \frac{VM}{g_c}$$

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98-114-HWH1

PREPARED BY

HWMCCORDY

CHECKED BY

M. Davis

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REFERENCES:

- (1). CONTROL OF NUCLEAR REACTORS AND POWER PLANTS, M.A. SCHULTE, MCGRAW-HILL BOOK COMPANY, SECOND EDITION, 1981, SEE TABLE B-3.
- (2) USNRC REGULATORY GUIDE 1.61, "DAMPING VALUES FOR SEISMIC DESIGN OF NUCLEAR POWER PLANTS", OCTOBER 1973.