
Optimization of Public and Occupational Radiation Protection at Nuclear Power Plants

A Calculation Method

Prepared by W. H. Horton

Science Applications, Inc.

Prepared for
U.S. Nuclear Regulatory
Commission

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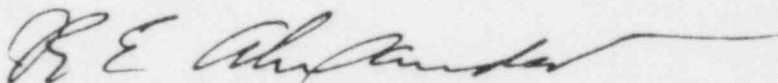
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Until recently decision makers on the Nuclear Regulatory Commission staff have had to evaluate proposals for new maintenance and inspection requirements at nuclear power plants without the benefit of quantitative comparisons between the risk potential averted by the new requirement and the occupational risk created at the same time. While it was fully recognized that the generation of quantitative information of high precision would not be possible, it was also recognized that improved analytical techniques for quantitative comparisons could contribute substantially to the decision making process. Therefore funding was requested for a research project to develop an appropriate technique, to document it, and to provide comprehensive supporting material which would enable users to understand its strengths and weakness and to evaluate the rationale on which it is based. The project was awarded to SAI, Inc., and it has, I believe, been very ably carried out by the SAI staff.



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ABSTRACT

The methodology presented in this report formulates an approach for the optimization of benefits resulting from NRC decision making processes. Recent increases in occupational exposures in nuclear power plants resulting from NRC regulatory practices have led to the questioning by NRC of the overall benefit of specific regulations. The optimization methodology in this report provides a tool for the determination of the cost-benefit of proposed NRC regulations. Detailed methods are presented for the modeling of plant safety systems undergoing inspection, testing, and/or repair. This methodology utilizes dynamic Markov modeling techniques with extensive additional model development associated with operator errors involved in the inspection, test, and repair activities of the plant. Closed form solutions to the Markov models are provided. The report appendix presents the Markov model solution process in detail sufficient for model verification. Other methods necessary for the optimization process are discussed in lesser detail. An application of the methodology dealing with steam generator inspection frequency and steam generator tube rupture events is presented. The example determines the steam generator inspection intervals which minimize expected costs and total expected occupational and public dose.

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1. INTRODUCTION

This report presents a summary of the tools that can be utilized to optimize practices associated with public and occupational radiological risk. A major portion of the report presents an expansion of a particular method used to model nuclear power plant inservice inspection, testing, and repair activities. This methodology is developed in depth because it is not a standard approach and thus is not well known and has not been utilized in past efforts associated with risk analysis. A specific example related to steam generator tube inspection practices is presented to demonstrate the techniques involved.

1.1 BACKGROUND

Following the Three Mile Island II accident, the Nuclear Regulatory Commission implemented many procedures and retro-fit design changes to prevent similar incidents in other reactors. In addition, normal NRC regulatory practices required utilities to perform numerous plant activities that involved radiation exposure to their personnel. The resulting efforts of the NRC to reduce public risk seemed to be effective, however, the occupational exposures that occurred during the years following the TMI II accident were higher than previous years [1-3]. It became a concern that the public risk reduction activities were leading to significant increases in occupational radiation exposure and risk.

About this time, the NRC began to search for some methods that would allow them to make decisions that dealt with trade-offs between public risk and occupational risk. An approach was required to make sound decisions with the aim of an overall reduction in risk. The tremendous costs associated with the TMI accident put another factor into the decision making process; costs of various actions and/or inactions.

An initial attempt at the methodology for making "optimum" decisions was developed in support of the new value-impact analysis approach that was being required by NRC in its decision process [4]. This format provided the decision maker with information dealing with costs, risk reduction, occupational impacts, as well as other factors. Although this step was a major advance, a means of combining this information into a single measure or rating was still not available. This single measure was needed so that radiation exposure information could be evaluated in the same process as cost information.

The ICRP's recent publication of ICRP-26 [5] and ICRP-37 [6] began to formulate a method for the incorporation of cost factors and doses. These documents presented the information as the recommended approach of a commission both knowledgeable in the area and well respected. Thus, this stamp of approval increased the likelihood of the optimization concepts being accepted by the nuclear industry. The efforts of the ICRP were directed at occupational doses

with little mention of public risks. The ICRP methods do not directly address NRC requirements which have more emphasis on concerns dealing with public risks.

Many organizations and individuals have directed efforts at filling this gap associated with public and occupational risks. All of these efforts have formulated a basis for the methods and concepts presented in this report. The following discussions will not redevelop methods that already exist and are well documented. Instead, the established methods will be put into the proper perspective for use in optimization and the methodology that appears to be missing will be developed and documented for the user of this report.

1.2 USES OF THE OPTIMIZATION PRINCIPLES IN THIS REPORT

The optimization project started with a literature search for incorporation of occupational dose considerations in risk analyses. The results of the literature search are reported in the first [7] of the three volumes of this study. The basic findings were that little effort has been made in the past to consider the expected occupational dose associated with accidents or normal releases from nuclear power plants. It was pointed out that the effort associated with incorporating occupational exposures into risk assessments would be major due to the complexity of modeling operator actions and location during accidents and cleanup activities. Occupational dose has recently been included in the NRC value-impact analysis efforts as a line item in a listing of important variables and has not been directly incorporated into a cost/benefit structure as proposed by this study.

Based on the preceding results, it is recommended that risk analysis procedures should only incorporate occupational doses for cleanup operations and not for accident responses. Since this optimization study is being conducted to support NRC in regulatory decision making, the methodology must be geared to comparisons between current practices and proposed practices. In these comparisons, the accident analyses will be weighted by their respective probabilities which will reduce the impact of occupational dose exclusion. In addition, the dose to operators in accident response should be small compared to public doses and cleanup doses due to the relatively small number of personnel involved and the short time duration of the accident. Therefore, for the purposes of comparison with regard to NRC decision making, the impact of excluding occupational dose associated with accident response from the overall risk computation is not significant and should not result in significant errors in rule making by NRC.

The second volume [8] of this optimization study provided discussion dealing with a dollar cost equivalent for collective radiation dose and with the comparative worth of public and occupational radiation dose. The results of the study recommended that public and occupational dose be treated equally; that is, a man-rem of public dose is equivalent to a man-rem of occupational dose. Collective dose was recommended as a measure of radiation detriment. High consequence events of low probability (such as core melts) should be treated as equivalent to low consequence events of high probability (such as routine releases or waste handling accidents) that have equal expected risk. No discounting of future detriment should be included in the methodology. Early deaths attributed to non-stochastic effects should be assigned a surrogate

value of $2.5E+4$ man-rem for the purposes of cost/benefit calculations and the cost effectiveness of dose reduction should be based on a value of \$100/man-rem. These recommendations are based on the review of a wide range of literature. The formulations presented in the second volume are supported by discussions of the review findings. The equation for computation of the net collective dose equivalent provides for computations under assumptions other than those presented above to allow sensitivity analyses for assumption impact.

The methodology presented in this volume of the report was developed to utilize the results of the first two volumes. The intent behind the development of these methods was to provide a tool for use in NRC decision making processes involving multiple options, as was stated earlier. These methods are not intended to be used to give precise estimates of risk or cost but are to be used in the comparisons of these decision making options. This usage allows simplification of the models, to some extent, and removes much of the impact of data uncertainty from consideration. Sensitivity studies should still be done in those cases where the data uncertainty is high to assure the analyst that the conclusions drawn are insensitive to the data variations. If this is not the case, the high uncertainty input data should become a key assumption in the analysis. Sensitivity results and justification of the final data base selection should be presented along with the chosen option.

In the selection of options for study, the null option should always be included. The null option is the result of doing nothing or making no changes to the current process. Thus, if a retro-fit study was to be conducted, the options would not only include the retro-fit design but also the option of not changing the current design. The null option can be used as the base option for comparison of all other options.

Often, the options selected may require multiple applications of the methodology. If a problem exists which has many different potential solutions and the solutions themselves have variations, the methodology must be used in a multi-step process to arrive at the final decision. For example, if a particular valve is found to be creating a risk-related problem due to its low reliability, options for solution can include; do nothing (null option), test the valve frequently, or replace the valve with a higher reliability valve. The testing option has its own variations due to the need for a selection of a test frequency. This problem would be solved in stages. First, the do-nothing option would be analyzed for cost (both for operations and exposures). The replace option would be easily analyzed utilizing the base case option by changing the valve reliability and including the costs associated with the installation of the new valve as well as occupational exposure costs from old valve removal and new valve installation. The testing option could be analyzed next by finding the test interval for valve testing which minimizes costs and then using the resulting minimum cost test option in comparisons with the other two options. Methods to perform the test optimization are presented in this report along with guidance on the procedures for other option cost analysis.

1.3 REPORT ORGANIZATION

The remainder of this report presents a discussion of a methodology to perform cost/benefit analysis dealing with nuclear power plant risks to both the public and plant personnel. The methods presented rely heavily on existing

methodologies except in the area of dynamic modeling of plant activities.

Section 2 of the report presents a discussion of the approach to be used as a basis for the formulation of a cost/benefit methodology. Example applications to the nuclear power industry are given along with discussions of the approaches to be taken in solving example related problems.

Section 3 discusses the results in the second volume of this report and their application to the methods used in this volume. Simple occupational dose models are provided. This section primarily deals with occupational dose modeling but does not address the problems associated with data base support for most occupational models other than to mention that the problem exists.

Public dose modeling is discussed in Section 4. Existing methods are presented along with the formulation of a methodology for the modeling of system availability as a function of inspection, test, and repair plant activities. This methodology uses Markov modeling techniques. Due to the limited application of Markov techniques in previous studies, the Markov models developed in this report are presented in great detail to permit direct application of the overall approach without extensive research efforts on the part of future users of these methods.

Section 5 briefly covers hardware and labor cost models, and a summary of the report is presented in Section 6. These sections are followed by a detailed example of an application of the methods to the problem of steam generator tube rupture and inspection frequency. The example is presented in Section 7 followed by a reference section and an appendix which presents the details of the development of the Markov models in Section 4.

2. FORMULATION OF APPROACH

The following section discusses the basis of the approach adopted by this methodology as it applies to nuclear power plants. A general cost-benefit formula is presented and discussed followed by specific situations in nuclear power plant operations that could utilize the methodology.

2.1 GENERAL COST-BENEFIT FORMULA

The ICRP presented a systematic approach for dose limitation in its publication of ICRP-26 [5] in 1977. This system was expanded upon and its definition was improved in the recent publication of ICRP-37 [6] in 1983. The cost-benefit approach taken in this study on nuclear power plant dose optimization or minimization uses the ICRP system as a basic framework.

The ICRP system was developed to ensure two objectives which relate to the work presented in this report. First, an activity associated with radiation exposure should only be conducted if the activity produces a benefit that exceeds the costs associated with the activity. An activity which costs more in actual production costs and/or in dose detriment costs than the reduction in costs (of a similar nature) resulting from the activity does not produce a net benefit. Second, an activity associated with radiation exposure should be conducted such that the resulting dose is kept as low as is reasonably achievable. This implies that the activity can have an optimum. There is some point where the protection costs and resulting dose are marginally equivalent; i.e., a protection dollar spent before this point buys more than a dollars worth of dose benefit and the same dollar spent after the point buys less than a dollars worth of dose benefit. Figure 2-1 indicates this relationship. The upper curve represents the sum of the two underlying curves. The lower curve which starts high and decreases represents dose cost, and the lower curve which starts low and increases represents protection costs. The point at which the upper curve is at a minimum is where the marginal cost of the dose curve equals the marginal cost of the protection curve.

The NRC could utilize both of the above objectives in making licensing/regulatory decisions. For example, the first objective may be applicable to decisions dealing with retro-fit situations. Would a retro-fit produce a sufficient reduction in expected public dose and accident costs to warrant the costs associated with the retro-fit and the occupational doses encountered in the retro-fit process? This is primarily a yes or no decision situation and does not cover a wide range of options that would lend themselves to an optimization approach. However, a situation which requires a decision on frequency of inspection would cover a wide range (almost continuous) of options and would lend itself to determining an optimum. This would be similar to the second objective discussed above. Therefore, the decision maker could determine a frequency of inspection which results in the lowest overall cost associated with both actual dollar cost items, such as labor, and dose detriment

costs for public and occupational exposures.

The above approach assumes that the net benefit of an activity involving radiation exposure is equivalent to the difference between the gross benefit of the activity and the sum of three other components: 1) the basic production costs, 2) the cost of achieving a selected level of protection over the basic production cost, and 3) the cost of the detriment due to radiation exposure associated with the activity. Inherent in this methodology is the assumption that the detriment due to radiation exposure can be represented as a cost and thus be another input into the overall cost of an activity. The ICRP presented the relationship for net benefit in the following form:

$$B = V - (P + X + Y)$$

where

B = net benefit
V = gross benefit
P = production costs
X = incremental protection costs
Y = radiation detriment costs

The above formulation assumes that the detriment due to radiation is no different than any other cost.

For the purpose of nuclear power plant decision making, the ICRP formula is overly broad and does not reflect the trade-offs associated with the NRC licensing/regulatory decision process. However, the concept represented by the formula is valid and will be used as a basis for the proposed formulation. The ICRP system attempts to find the optimum by maximizing the net benefit. In nuclear power plant applications, the net benefit is usually not of interest. The benefit to society of the power being produced by the plant is not of concern to the NRC in its decision making. Thus, most activities of nuclear power plants do not result in positive net benefits. The activities are attempts to reduce the overall costs but do not get incorporated into the power plants gross/net benefit. The reason that this occurs stems from the neglecting of the true benefits of the power production capability of the nuclear power plant. It is basically assumed that the nuclear power plant does produce a positive net benefit if all factors are incorporated in the benefit computation. For the purposes of this analysis technique, the gross benefit of nuclear power is not considered. The aim of this application is to reduce the detrimental costs associated with nuclear power and not to increase its gross benefit. Thus, the V term in the above equation is not of interest for most decision making situations. If the decision making process dealt with shutting a plant down or allowing a plant to operate, it might be of value to reincorporate the gross benefit portion of the equation to determine if the decision is optimizing the result for the public or if the decision is made outside the true benefits of the power production.

The equation is no longer a benefit equation but deals only with detriment. The objective is to minimize this detriment through the selection of proper decision alternatives or by finding optimum situations associated with various activities. The base production costs are not of interest in this process since

they tend to be constant. Only the costs associated with each alternative and the dose detriment associated with each alternative remain as variables of interest. The sum of the costs of these variables is a measure of sorts for a particular alternative. This sum will be denoted as an alternative's Figure of Merit. The equation presented earlier can be rewritten and will now look like the following for each alternative:

$$\begin{aligned} \text{Alternative Figure of Merit} &= \text{Alternative Dollar Cost} \\ &+ \text{Alternative Dose Detriment Cost} \\ \text{or} \\ \text{FM(A)} &= \text{DOLL(A)} + \text{DOSE(A)} \end{aligned}$$

For cases where alternatives are not based on variables that can be treated as continuous but deal with specific discrete options, the Figure of Merit of each alternative can be computed and comparisons made. For alternatives with a continuous variable or one which can be modeled as such (inspection intervals, shielding thickness, etc.), the equation can be differentiated with respect to the variable, set equal to zero, and solved for the value of the variable which is optimum. Many situations may have a combination of these two variable types and may thus require initial optimization of some options for a later comparison with other options which are separate from the continuous variable optimized situations.

2.2 TRADE-OFF SITUATIONS IN NUCLEAR POWER PLANT DECISION MAKING

The remaining discussions in this section deal with the variety of trade-off situations that occur in nuclear power plant decision making. Four general categories of trade-offs are presented. These categories are:

- 1) occupational dose -- protection cost
- 2) occupational dose -- public dose
- 3) occupational and public dose -- protection cost
- 4) public dose -- protection cost

Example situations for each category will be discussed in the following subsections.

2.2.1 Occupational Dose -- Protection Cost

Plant activities that occur in radiation environments all fall into this trade-off category even though they may be activities associated with safety and public protection. Most exposures to personnel that are associated with a routine activity can be reduced with an increase in expenditures for shielding, ventilation, training, special tooling, special radiation protection gear, or remote devices. It is only a question of how great an expenditure is reasonable for the protection given. The following discussion describes how problems associated with this trade-off situation may be solved using the methodology

presented in this study.

Increased expenditures in shielding, ventilation, and training can be evaluated using optimization principles in most cases. Special tooling, special radiation gear, and remote devices are more discrete options and thus can be evaluated by option comparisons rather than solving for some minimum or optimum value. In all the above cases, the data base to support the evaluations relies heavily on plant experience dealing with times to perform plant activities and on accurate measurements of the radiation environments in the area of the plant activity being addressed.

In the case of shielding, the parameter which can be used as the basis for optimization is the thickness of the shield. Other shield options can be evaluated which deal with shield materials and the number and placement of shields. Shield thickness can be directly related to cost based on the extra material involved in construction. If the radiation environment is well defined, the effect of the shielding on occupational exposure over specified time intervals can be evaluated using standard shielding equations. The exposure to the plant personnel can be equated to a cost and thus the minimum cost solution can be found as a function of the shield thickness.

Ventilation system sizing and air flow can be important, in some plant areas, for reducing the amount of occupational radiation exposure due to inhalation. Thus, the air flow through a room can be used as a parameter for optimization. Air flow increases raise ventilation system costs but reduce inhaled radioactive particulates and thus reduce costs associated with occupational exposure. The option of no ventilation system should also be examined as a comparative option with the optimum air flow result.

The amount of time spent in training to perform a particular plant activity can impact the resulting length of time that the plant personnel are exposed to radiation. Increased training time raises the costs of training due to the manpower costs for the trainer and for the personnel involved. Also, the training may require mechanical aids which would increase costs but can be included in an optimization process. The minimization process for the costs associated with the materials used as training aids would have to deal with the aids as comparative options within the optimized training time situations. The increase in training time will decrease plant activity time and exposure. The formula describing the relationship between the time in training and the time in performing the plant activity would be difficult to derive and may require testing to formulate the curve. The curve should look like a decreasing exponential curve indicating that as training time increases the amount of reduction in activity time decreases. This is a case of diminishing returns but it is of value to determine the optimum within the limitations of the data base utilized.

The discrete options associated with tooling, protection gear, and remote devices must be evaluated on a case by case basis. Options can then be compared to find the minimum cost option. As stated earlier, cases may exist where discrete options and options dealing with continuous variables exist for the same problem. In these cases, optimize the continuous variables first and then compare the remaining options with the optimized results to find the overall optimum.

2.2.2 Occupational Dose -- Public Dose

The trade-off between occupational dose and public dose usually includes costs of equipment and/or manpower but the options are dictated by the increasing of occupational dose in order to reduce public risk. Retention of gaseous radwaste, inservice inspection, maintenance, testing, and retro-fits all can be associated with this category of trade-offs.

Often, situations which fall in this category are decided by costs other than those associated with exposure detriment. The example in Section 7 which deals with steam generator tube inspection begins by looking for the optimum inspection interval for steam generators to reduce risk associated with rupture events. The optimum solution to the problem is nearly independent of the dose aspects of inspection, repair, and accident consequences and response but is dominated by the costs associated with inspection outages and accident outages.

In the case of gaseous radwaste retention, short-half-life isotopes of elements in the form of radioactive gases which are generated by the processing of the primary coolant can be retained prior to release to the environment. The longer the retention, the greater the decay of the radioactive gas to non-radioactive gas. Retention would require the construction of large tanks and compression systems and would result in increases of occupational exposure during plant operation. In this case, the variable which can be optimized is time of retention. Note that material costs are a part of the problem and must be included in the analysis. The study should not be restricted to dose computations alone.

The reliability or availability of the components or systems of the plant can be increased by inservice inspection, maintenance, and testing. These operator intervention activities can discover certain failures which are precursors to major failures or which are major failures in standby systems. This ability to detect and resolve failures before they become critical to plant safety and operation reduces the risk to the public of major releases of radioactivity. However, the operator actions often result in occupational exposure and thus increase the risk to operators of the plant. The trade-off between occupational exposure and public exposure can be optimized by determining the frequency of inspection, maintenance, or test which minimizes the total occupational and public dose. This particular problem requires a methodology for assessing the availability of systems as a function of the frequency of operator intervention. Section 4 of this report presents the details of such a methodology for inspection and testing. Maintenance activity modeling methodology is not covered in this report but could be developed without significant effort from the models presented.

Certain retro-fit design actions fall into this category of trade-off. A system change to reduce system failure probability and thus reduce public risk may require the installation of new equipment in systems which are contaminated. The installation activity would result in an occupational dose to the workers involved. Depending on the retro-fit, this dose to personnel may be significant and require numerous workers each receiving the allotted dose limit. The problem can be analyzed by comparing the two alternatives or options involved using their costs of materials and labor as well as dose detriment.

2.2.3 Occupational and Public Dose -- Protection Cost

Trade-offs that reduce both occupational and public doses with increases in other costs are covered by this category. The reduction in occupational dose is a net reduction. Often, the implementation of the option may result in an initial occupational dose but the option will eventually reduce operational doses such that the net occupational dose commitment is negative. Situations that fall into this category are not normally found in plant studies. The plants are designed in a manner that reduces the likelihood of the identification of an option of this sort. General examples include new designs for future plants, some retro-fit designs, add-on facilities or equipment, and some plant operations.

Future plants may change designs in a manner that reduces both the occupational dose and public dose expected from plant operation. These new designs could cost more than current designs which leads to the trade-off situation. This particular example is not generally in the scope of NRC rule making as long as the plant meets the current guidelines and regulations. It would provide an interesting alternative study for future plants or for comparisons to plant designs of foreign countries.

Assume a particular component that has applications in nuclear power plants has a major design breakthrough which would greatly reduce the need for inspection, maintenance, testing and repair. The installation of the new component may result in an initial occupational dose with a long term reduction in occupational dose and public risk. This type of retro-fit situation is an example of this category of trade-offs.

The use of robotics for certain plant activities may permit more frequent inspections, automatic and frequent testing, and some remote maintenance activity with a corresponding decrease in the current occupational dose. These add-ons would be expensive and may require their own maintenance and repair but could result in a net decrease in dose.

Plant operations that fall into this category can be illustrated by an example dealing with steam generator tube problems. The addition of a very high quality secondary water chemistry program would reduce the degradation of steam generator tubing. This reduction would decrease the need for inspection of steam generators which has high occupational dose consequences. It would also decrease the amount of tube repair required during a steam generator inspection and thus decreases occupational dose. More reliable steam generators would reduce the risk to the public of rupture sequences leading to major releases of radioactivity. Thus, a secondary water chemistry program could fall into this type of trade-off due to its reduction of both occupational and public doses at the expense of equipment, materials, and training associated with the program's implementation.

2.2.4 Public Dose -- Protection Cost

The reduction of public dose with no impact on occupational dose is another situation with a limited number of actual examples. Examples may include spent fuel transportation cask design and improvements in containment designs. These cases would be evaluated using the techniques discussed in subsections 2.2.1 through 2.2.3, as appropriate. The extreme example of this category is the shutting down of an operating plant due to some licensing issue. Costs would

include replacement power costs, nuclear power plant capital expenditures, etc. Dose reduction would have to be evaluated against the current expected public and occupational doses associated with the faulted plant. It would be interesting to evaluate some of the current plants with major licensing issues such as Diablo Canyon and its earthquake fault and the several plants with extreme quality control problems. The difficulty in the latter cases is determining the current design weaknesses in an accurate manner. Poor quality control does not necessarily mean poor quality; it simply increases the likelihood of poor quality.

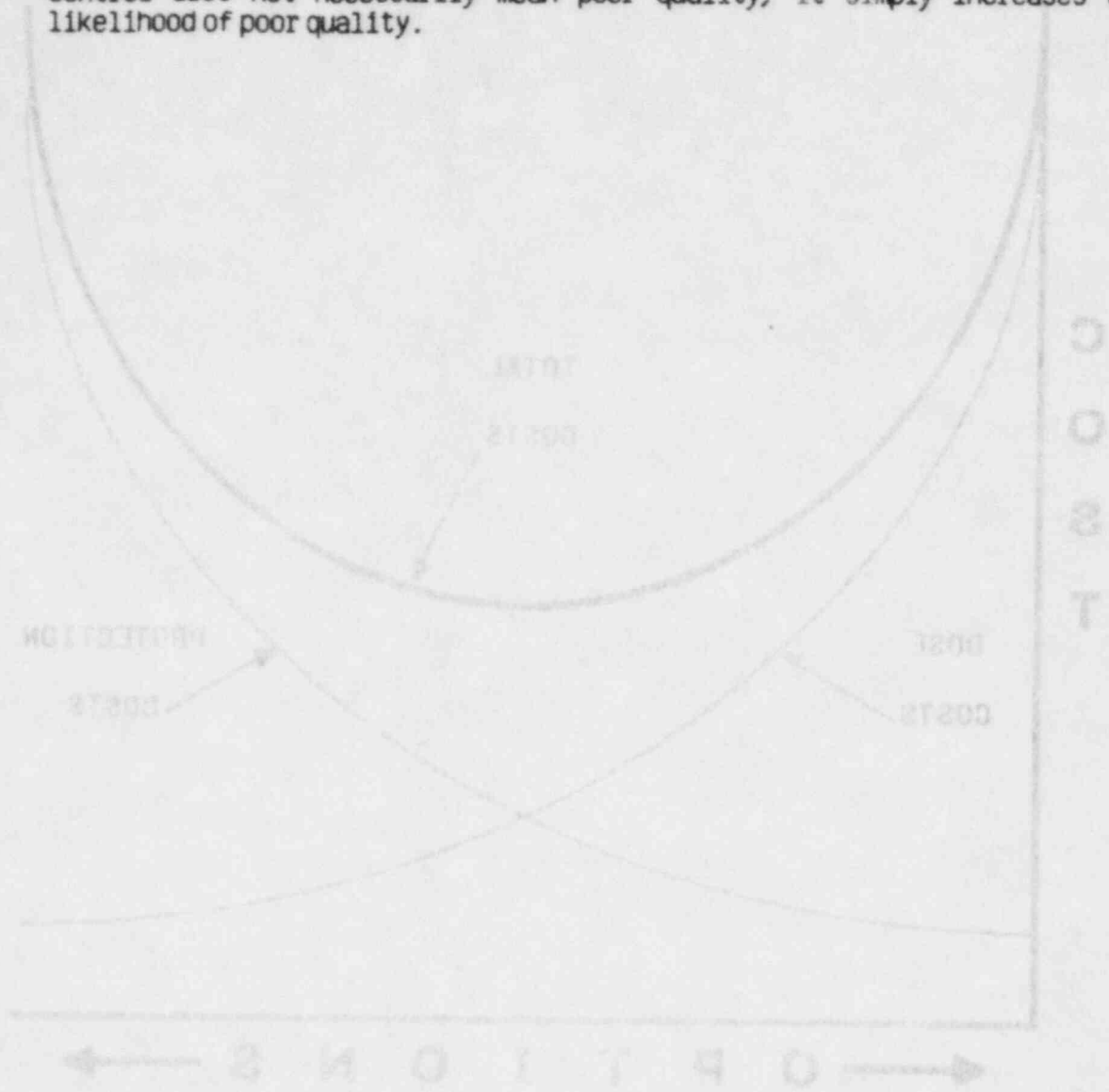


FIGURE 2-1. TOTAL COSTS FOR WHICH A DESIGN IS A FUNCTION OF DOSE AND PROTECTION COSTS

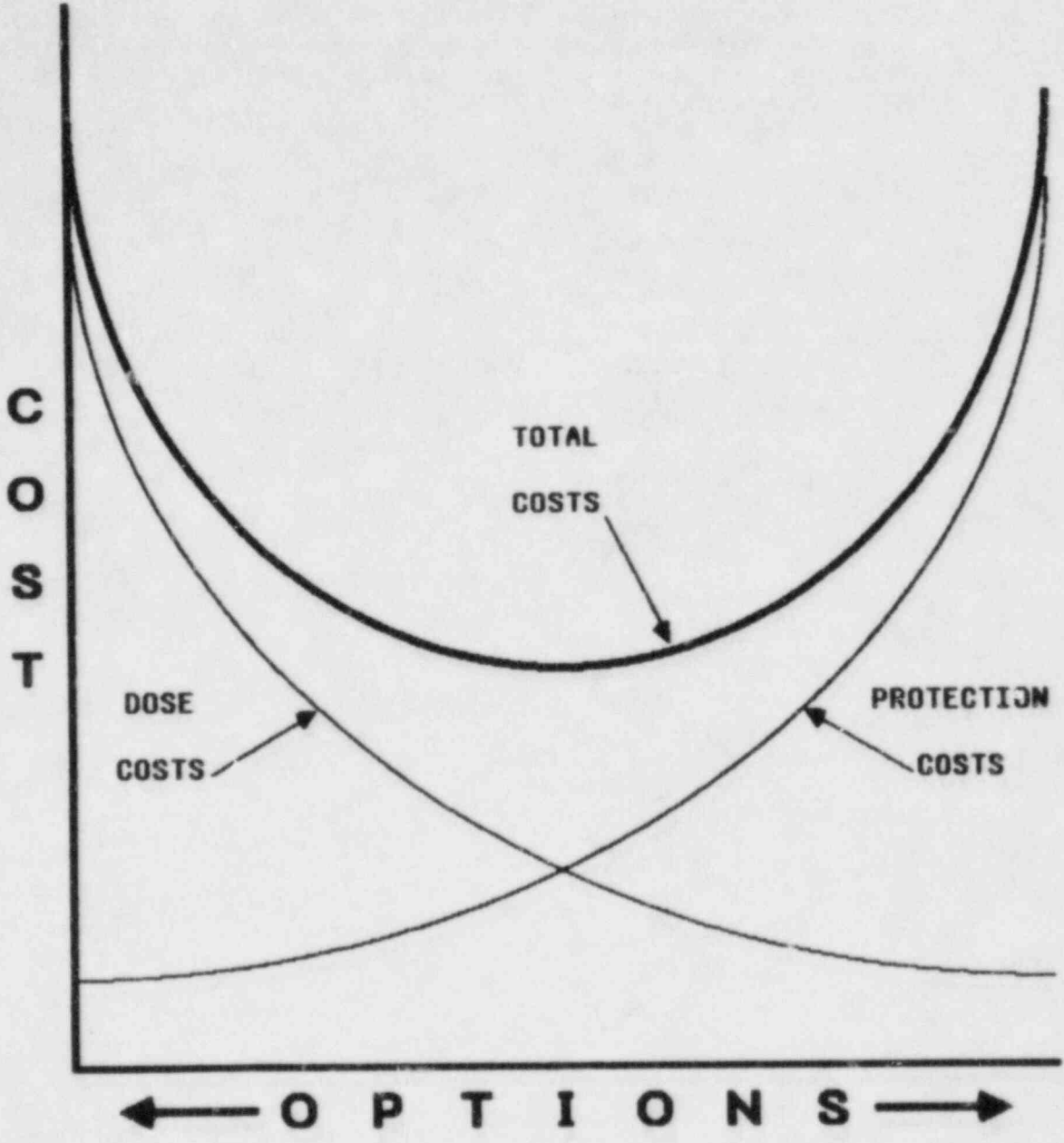


FIGURE 2-1. TOTAL COSTS FOR VARIOUS OPTIONS AS A FUNCTION OF DOSE AND PROTECTION COSTS

3. RADIATION DOSE MODELING

This section of the report provides a discussion of some approaches for the modeling of radiation exposures to the public and plant personnel. No new methodology is presented for this topic and the methods discussed can be found in the literature. The section begins with a recap of some of the information presented in Volume 2 of the optimization report [8] dealing with the modeling of radiation exposure in cost-benefit applications. An extrapolation of the Volume 2 methods is presented to provide a more general tool for modeling doses and dollar costs. This is followed by a description of some models dealing with shielding and ventilation systems. These models are used in evaluation of occupational exposures.

3.1 GENERAL COST AND RADIATION EXPOSURE MODEL

An in-depth discussion of the treatment of radiation exposure modeling in cost-benefit analysis is provided in the second volume of this optimization report. The basic formulation for the determination of the net collective dose equivalent is as follows:

$$CD_n = (p \cdot CD_p^a) + q(p \cdot CD_o^a)$$

where

CD_n is the net collective dose equivalent

CD_p is the public collective dose

CD_o is the occupational collective dose

p is the probability associated with the option

q is an equivalence factor for relating occupational dose to public dose

a is the risk aversion factor

The risk aversion factor is used to emphasize high consequence events even though their probability may be low. If the value of the factor is greater than one, the high consequence events are given more weight than low consequence events because the factor is raising the consequences to a power. The factor can be removed from the analysis by simply assigning it the value of one. More complete descriptions of the terms can be found in the second volume.

The Volume 2 report provides recommendations for the values of some of the factors in the above formula. The only change from these recommendations is to assign a value of one to the risk aversion factor. The recommendation of the earlier report was to use a value of 1.2 for the risk aversion factor. This

factor is oriented to perceptions of risk rather than actual risk. The approach taken in this volume is to initially perform an objective analysis which may be followed by subjective analysis dealing with perceived risks. If the recommended values of the above factors are placed in the formula, the net collective dose formula simplifies considerably to the following:

$$CD_n = p \cdot (CD_p + CD_o)$$

This formulation assumes that the risk aversion factor is one and the equivalence factor for occupational and public dose is one. Sensitivity studies can be conducted on the values of these parameters for specific applications.

This initial formulation is the basis of the overall formula presented below. The dose equation is expanded to include other costs such that the net cost of an option can be determined. Costs arise from initial implementation, operation, and accident response and consequences. A formula for the expected net cost of an option is:

$$C_N = C_I + C_{ON} + C_{RN}$$

where

C_N is the expected net cost of an option

C_I is the cost of initial implementation

C_{ON} is the net cost of operation

C_{RN} is the expected net cost associated with accidents

The net cost of operation is found by summing the dollar costs of the operation and the cost equivalence of the occupational and public exposures due to normal operation. This is shown in the following formula:

$$C_{ON} = C_O + C_g(CD_{po} + q \cdot CD_{oo})^a$$

where

C_O is the dollar cost of operation

CD_{po} is the public collective dose during operation

CD_{oo} is the occupational collective dose during operation

C_g is the cost equivalence of dose in \$/man-rem

The expected net cost associated with accidents is found by summing the expected costs of each accident:

$$CRN = CRN1 + CRN2 + \dots + CRNn$$

where each expected accident cost is found by adding accident dollar costs with dose equivalent costs and weighting the result by the accident probability as shown below:

$$CRN1 = p_1 [C_g (CD_{p1} + q \cdot CD_{o1})^a + CR_1]$$

where

CD_{p1} is the public collective dose for the i th accident

CD_{o1} is the occupational collective dose for the i th accident

CR_1 is the dollar cost of the i th accident

p_1 is the probability of the i th accident

The above formulation can be used for most evaluations associated with NRC decision making. The recommendation of this report is to assign values of one for the risk aversion factor (a) and for the occupational and public dose equivalence factor (q). The cost equivalence factor is recommended to be set at \$100/man-rem. All other variables are found during the evaluation process for specific applications. It is also recommended that the above factors be tested for sensitivity and that the results indicate the findings of the sensitivity studies.

3.2 OCCUPATIONAL EXPOSURE MODELS

The following subsections present several models related to the analysis of occupational exposures. In general, the modeling of occupational dose is dependent on the data base associated with the particular plant activity. This data base is all that is needed in many cases. However, there are some cases which require additional model development and lend themselves to optimization studies. The cases discussed below deal with shielding problems and ventilation system problems and are presented in limited detail.

3.2.1 Shield Models

Shielding is used to reduce personnel exposure to radiation sources near work areas. The cost of shielding can be computed easily and the reduction in radiation can also be determined readily. A source of uncertainty in shielding models results from not knowing the actual location of the recipient of the radiation as well as not knowing the duration of the exposure. The shield model that is presented below assumes an average dose rate for the work area and predicts duration and frequency of exposure using an average occupancy factor. If better data is available, the exposure portion of the model could be improved significantly.

The occupational collective dose as a function of shield wall thickness can be expressed in the following manner:

$$CD_{00} = H \cdot f \cdot N \cdot T \cdot e^{-ut}$$

where

H is the existing annual dose rate with current shielding

f is the occupancy factor for the area per person

N is the number of personnel in the exposed group

T is the life of the facility in years

u is the shield's effective absorption coefficient

t is the shield wall thickness

This standard formula has been presented by the ICRP [6] as well as in a NUREG/CR dealing with ALARA programs [9]. Shield costs are primarily functions of the amount of material used in the construction of the shield. The costs also vary with the type of material used in the shield. Additional costs can arise from support facilities and/or equipment needed for shield wall construction. The corresponding formula for the cost of the shield is simply:

$$C_I = C_S \cdot h \cdot l \cdot t + C_{OS}$$

where

C_S is the cost of installed shielding per volume

h is the height of the shield

l is the length of the shield

t is the shield wall thickness

C_{OS} is other support costs associated with shield wall installation

Using the formulas presented in the preceding discussions, the net cost of the shielding option is:

$$C_N = C_I + C_{CN}$$

and

$$C_{CN} = C_g \cdot CD_{00}$$

yielding

$$C_N = C_S \cdot h \cdot l \cdot t + C_{OS} + C_g \cdot H \cdot f \cdot N \cdot T \cdot e^{-ut}$$

Thus the cost of the detriment, C_{DN} , and the cost of implementation, C_I , are both functions of the shield wall thickness, t . This lends itself to an optimization process by differentiation of C_N with respect to the shield wall thickness, t . This differentiated sum is set to zero and solved for t , the optimum shield thickness which will minimize the net cost.

3.2.2 Ventilation System Models

Ventilation may not be as great a concern in a nuclear power plant as it is in other fuel cycle facilities. However, cases do arise where workers must function in areas that may contain some airborne radioactive particles. The impact of the airborne particles can be reduced by increasing the air flow through the area. The ventilation system model shown below assumes that the cost of the operation of the ventilation system over the life of the facility will greatly exceed the cost of system installation. This model is also presented in an ICRP publication [6].

The collective dose associated with people in a confined space is proportional to concentration of airborne radioactive particles in the area. This concentration can be shown to vary inversely with the area ventilation flow rate. The occupational collective dose in this case can be calculated as:

$$CD_{OO} = f \cdot N \cdot T \cdot F_D \cdot A / Q$$

where

f is the occupancy factor for the area per person

N is the number of personnel in the exposed group

T is the life of the facility in years

F_D is a dosimetric factor converting activity concentration to dose

A is the input rate of airborne radioactive particles

Q is the ventilation flow rate

Under the assumption that the operating costs of the system dominate the overall system costs, the ventilation flow rate can be directly related to the system costs. Thus, the operating costs can be represented by the following formula:

$$C_O = C_e \cdot E_c \cdot F_O \cdot T \cdot Q$$

where

C_e is the cost of electricity

E_c is the energy expended to circulate a volume of air

F_O is the fraction of time the ventilation system operates

T is the life of the facility in years

Again, using the above formulas for net cost, the following equations can be obtained:

$$C_N = C_{ON}$$

where

$$C_{ON} = C_0 + C_g \cdot CD_{00}$$

yielding

$$C_N = C_e \cdot E_c \cdot F_0 \cdot T \cdot Q + C_g \cdot f \cdot i \cdot T \cdot F_d \cdot A / Q$$

Thus, the detriment, CD_{00} , and the operating costs, C_0 , are both functions of the ventilation flow rate, Q . An optimum solution can be found by differentiating the net cost equation, C_N , with respect to Q , setting the result equal to zero and solving for the optimum ventilation flow rate value of Q which minimizes the net cost.

4. PUBLIC DOSE MODELING

This section of the report provides a discussion of approaches to the modeling of public risk. The discussion refers to probabilistic risk assessments, consequence modeling, and other methods associated with the performance of public risk analysis. New methodology is presented for the modeling of dynamic plant activities (activities which have the potential for operator alteration of the system failure status at a given point in time). These activities generally include inservice inspection, maintenance, repair and testing. Each of these activities has the potential to change the failure status of the components of the system involved or to initiate another activity which would change component status. This methodology can be used as an input for existing PRAs to provide estimates of system failure probability.

4.1 EXISTING METHODOLOGY FOR PUBLIC DOSE MODELING

Current approaches to modeling public risk resulting from normal plant activities and from accident situations are discussed in the following subsections. Details of the methods are not presented due to their widespread usage in the industry.

4.1.1 Normal Releases

Nuclear power plants do have some release of radioactivity resulting from the day-to-day operation of the plants. These releases generally originate from the radwaste handling portion of the plant. Primary coolant undergoes a type of cleaning operation which removes unwanted chemicals and radioactive particles that are picked up from the fuel elements in the reactor core. Much of the radwaste that is recovered in this process is in the form of gaseous elements of some radioactive isotopes. The gaseous portion of the radwaste is held for some length of time to reduce activity levels and then is released out the plant stack. The final release still contains radioactivity but is at levels that are acceptable for public risk.

The methodology used in the analysis of the risk associated with releases of this type is generally termed consequence modeling. Gaseous plume dispersion models have been developed and utilized for this problem for many years. The dispersion models generally assume a Gaussian distribution of the plume at any point with the rate of dispersion dependent on the distance from the release point and the wind and weather conditions at the time of the release. The models account for temperature inversions, stack height, population demography, wind direction and turbulence. From the application of these models, an analyst can determine the whole body dose and the inhalation dose for the surrounding population. Normal release analysis generally assumes that the weather conditions are average for the site and do not attempt to develop worst case conditions since the releases occur regularly and will tend toward the average over the long term.

Other normal releases include some liquid effluent primarily from the radwaste handling section of the plant or from the secondary steam side of a PWR and the turbine system of a BWR.

4.1.2 Accidental Releases

Accident analysis methodology is extensive and has been applied in major programs such as WASH-1400, RSSMAP, IREP, and RMIEP. The techniques generally used include event tree methodology, fault tree methodology, external event analysis, human factors analysis, consequence modeling, Failure Modes and Effects Analysis (FMEA), and other methods for specific problem solution.

The consequence modeling discussed under the normal release methodology section is also applied to the accident modeling situations. The weather conditions are not averaged, however. The analysis utilizes worst case weather conditions for the evaluation in order to determine an upper bound for the accident risk.

System failure probability is generally determined using fault tree analysis methods. The IREP program has recently provided detailed procedures for the development of system fault trees for applications in risk analysis. Most PRAs of nuclear power plants utilize this form of system modeling. The shortcoming of this approach is the limited capability of fault tree models to analyze operator interaction with the system. The methods presented in the following section provide a tool for performing this type of analysis.

The plant failure model is generally developed using event tree methods. System failure probabilities, initiating event probabilities, and the likelihood of certain operator actions are inputs into the event tree. These methods develop a listing of accident scenarios showing all possible outcomes of a specific initiating event. Each outcome can be evaluated for its resulting public dose by using the appropriate consequence models.

The methodology developed in the following section provides a system failure probability which can be used as an input into the event tree and thus can impact the consequence model by changing the likelihood of the scenario and the resulting risk. The methods are used to model operator diagnostic actions and their impact on the system failure probability. These methods are appropriate for risk analysis but have a tendency to be difficult to develop and apply. This difficulty has resulted in the usage of the less complex fault tree methods instead of dynamic modeling techniques such as Markov modeling. The following sections describe Markov modeling techniques in detail and provide Markov models for application.

4.2 A DYNAMIC MODEL FOR APPLICATION IN PUBLIC DOSE MODELING

The material in this subsection describes the steps involved in the application of the Markov models developed to analyze inspection, testing, and repair activities at nuclear power plants. A detailed discussion of the development of the models is presented in the appendix of this report. An example application is provided in Section 7 to aid in the understanding of the approach.

4.2.1 Inspection and Testing of Direct Failures

Operator activities associated with inspection and testing provide an opportunity for the discovery and repair of failures which occur in a system. This interaction with the system creates inaccuracies in static models of the system failure probability. Static failure models provide estimates of system reliability which is the likelihood of the system being capable of performing its function at the end of some time interval. In general, this implies that there is no interaction with the system during the time interval. Systems in nuclear power plants, particularly standby systems, can be modeled more accurately using a dynamic model which provides estimates of system availability. Availability is the likelihood that the system will be available to perform its function at any point in time. Thus, when the system goes down for test and repair, it becomes unavailable and the dynamic model would be able to account for the outage.

Reliability models can be used to model dynamic situations but become very cumbersome. This is particularly true of systems composed of redundant elements which have alternating test or inspection intervals. A reliability estimate could be computed for the inspection interval and then the analyst could keep track of the redundant train reliabilities after each interval. Updates would be performed at each interval and probability would be assigned to the multitude of possible outcomes of the system status. The accounting system associated with this process quickly becomes excessive. Dynamic models, although more complex and difficult to apply than a normal reliability model, greatly simplify the modeling of the the above process.

The following section addresses a Markov model which can be used to evaluate the availability of a system susceptible to direct component failures. Direct failures are failures which, if observed by some diagnostic activity, are true failures of the component rather than precursors of failure. A crack in a pipe is an example of a precursor to a failure of the pipe. A rupture of the pipe is an example of the true failure of the pipe. The precursor failure modeling can be done using dynamic models and a Markov model for that purpose is developed in the subsection following this Markov model

The model presented is a closed form solution to the defined Markov model. A closed form solution does not require computer usage to solve the set of simultaneous differential equations normally associated with Markov analysis models. The results of the closed form solution are sets of exponential equations which can be solved by hand. This aspect of the developed methodology is particularly of value because it makes application of the methods straightforward once the method is understood.

4.2.1.1 Model description

The model was developed to be applicable to many situations found in nuclear power plant systems. The system that is modeled is assumed to be composed of two redundant legs or sections of similar components which have another section of components or leg in common. The failure of the system would require failure of both redundant legs or failure of the common leg. Figure 4-1 presents a simplified diagram of the system. As shown in the figure, each leg of the system has an associated set of failure rates, one for detectable failures and one for undetectable failures. The failure rates associated with these two modes of failure are dependent on the diagnostic activity being modeled. A detectable

failure is one which the operator can discover in the process of performing the diagnostic activity. An undetectable failure cannot be discovered by the operator in the process of the activity. Thus, the failure rates for detectable and undetectable failures for inspection activities would probably be different from those associated with testing activities. In fact, the testing detectable failure rate for a leg should be greater than or equal to the inspection detectable failure rate for the same leg. This is because testing should uncover more failures than inspection on most equipment and should never uncover fewer failures.

A mechanism for the determination of detectable and undetectable failure rates for a piece of equipment is to utilize an FMEA for the component, if one exists. Each failure mode of the component would be listed in the FMEA. These failure modes can be examined to determine if the particular diagnostic activity being modeled would detect the failure associated with the failure mode. Once each failure mode is evaluated, the failure rates associated with the failure modes that are detectable can be summed to yield the detectable failure rate for the component for that diagnostic activity. The undetectable failure rate would be found by taking the component overall failure rate and subtracting the detectable portion. If no FMEA exists for the component, the overall failure rate of the component could be partitioned using engineering judgement applied to past failure records of the component. In cases where there is uncertainty associated with this process, it is recommended that the split be treated as a parameter of the study and that model result sensitivity to this parameter should be determined by performing numerous model computations for multiple values of the parameter and statistically evaluating the results for significant variation. The purpose of the analysis is to provide additional information for a decision making process. This should be kept in mind when developing the data base.

Returning to Figure 4-1, the failure rates for the legs are assigned variable names of "A", "B", "C", and "D". "A" is the undetectable failure rate of the common leg of the system. "B" is the corresponding detectable failure rate for the common leg. If "A" and "B" are both assigned the value of zero, the common leg has no failure rate at all and the system simplifies down to a simple parallel system composed of two redundant legs as shown in the figure. Likewise, "C" is the undetectable failure rate of one of the redundant legs (both redundant legs have the same failure rates) and "D" is the detectable failure rate for a redundant leg. If both "C" and "D" are equal to zero, the system becomes a simple series system. This capability of the model allows the evaluation of many configurations of system piping. It is not necessary to have the common section of piping preceding the redundant legs of the system, for example. The common piping can be fed by the redundant legs or can exist on both sides of the redundant legs. If the common leg is actually on both the suction and discharge sides of the system, the diagnostic activities associated with the two sections of actual piping must be equivalent and simultaneous. If this is not the case, the model as stated in this report is not sufficient and the actual evaluation would have to apply multiple Markov models of the type in this report to determine the system availability.

In order to model this system, two Markov models were developed. The first Markov model, which is called the Primary Model, is used to evaluate the entire system but treats the two redundant legs as being indistinguishable. The second model, the Supporting Model, evaluates only the redundant legs of the system but treats them as distinguishable legs. If the two models were combined, the

resulting Markov model would be too large to find a closed form solution without extensive and lengthy evaluation. The splitting of the model results in a slightly more cumbersome product, but reduces the time for the determination of a closed form solution by at least a factor of five. The appendix gives some indication of the effort involved in finding the current model solutions.

The reason the redundant legs must be treated as distinguishable legs in the system arises from the normal sequence of performing diagnostic activities. Usually, one leg of a redundant system is inspected or tested at a time. The next inspection or test is performed on the other redundant leg. To keep track of which leg has been inspected or tested and which is due to be inspected or tested, the model must be able to identify each leg as a separate entity. The discussion in subsection 4.2.1.2, which deals with model application, provides more details associated with the need for redundant leg identification.

Markov models are represented by definitions of system states and transitions between the defined states. Table 4-1 presents the state definition for the Primary Model. As can be seen, the model is composed of 15 states. Each state is unique and all states together define the entire range of possibilities for the system. In other words, at any time, the system can be shown to be in one and only one of the defined states. The differences between states deal with various combinations of detectable and undetectable failures in the three legs of the system. State 15 is defined as having either an undetectable failure in the common leg or both redundant legs having an undetectable failure. The table does not define state 15 in this detail. State 1 is the all okay system state with no failures of any type. This is usually used as the starting state of the Markov analysis. The system changes state by the occurrence of failures in the legs of the system. States with detectable failures have the potential of repair and thus of moving to a state with fewer failures. Of the 15 states, states 1, 2, 4, and 5 do not represent system failure. Note that the redundant legs are referred to as trains and that there is no distinction between redundant legs.

The state definition for the Supporting Model is shown in Table 4-2. State 13 is defined as both redundant legs having an undetectable failure. In this model, each redundant leg of the system is defined separately. This can be seen by comparing states 2 and 3 which have the same type of failure but in different legs. Both states 2 and 3 in the Supporting Model are represented by state 2 in the Primary Model in Figure 4-1.

It is interesting to note that in both the models shown in the tables, the elimination of failure rates by assigning a failure rate variable to zero will greatly reduce the complexity of the model in many cases. If "A" is zero, there is no impact on the Supporting Model and limited impact on the Primary Model. However, if "B" is zero, the Primary Model will not contain states 8 - 14. If "C" is zero, the Primary Model loses states 4, 5, 6, 7, 11, 12, 13, and 14. The Supporting Model would lose states 5, 6, 7, 8, 9, 10, 11, 12 and 13. If "D" is zero, the Primary Model loses states 2, 3, 5, 6, 7, 9, 10, 12, 13, and 14 and the Supporting Model loses states 2, 3, 4, 7, 8, 9, 10, 11, and 12. The models don't actually lose the states in these cases, it just becomes impossible to be in the identified states under the stated assumption. The modeling of a simple series system by setting "C" and "D" to zero will eliminate the need for the Supporting Model.

The assumptions associated with the transitions between states for the above models are presented in the appendix along with the details of the model

solution process. The remaining discussion presents the results of the solution of the models. The results are presented in the form of exponential equations for the probability of being in a particular state at a specified time. The equations used in their present form would provide an estimate of the availability of the system at a specified time. Often, it is more meaningful to find the average availability over a specified time interval. This can be done by integrating the formulas for state probabilities and dividing by the time interval. These integrated formulas are not presented in this report.

Table 4-3 presents a tabulated version of the probability equations for the states of the Primary Model. The table is divided into three basic groupings of columns. The "P" column is used to identify the state probability being defined. Most state probabilities require more than one row of the table to express the exponential equation. A blank row separates one state probability definition from another. The "K1" through "K15" columns show the coefficients of the initial condition matrix associated with the state probability. The "A" through "D" columns show the coefficients of the failure rates that appear in the exponent of the exponential expression. As an example, the probability of the system being in state 5 at time T can be written:

$$\begin{aligned}
 P_5 = & 2K_1 \cdot e^{-(A+B+2C+2D)T} \\
 & - (2K_1 + K_2) \cdot e^{-(A+B+2C+D)T} \\
 & - (2K_1 + K_4) \cdot e^{-(A+B+C+2D)T} \\
 & + (2K_1 + K_2 + K_4 + K_5) \cdot e^{-(A+B+C+D)T}
 \end{aligned}$$

where

P_5 is the probability of the system being in state 5 of the Primary Model

K_x is the probability of the system starting in state x of the Primary Model

A-D are the failure rates of the system legs as defined earlier

T is the time of interest when a diagnostic activity is to occur

The " K_x " values are the entries in the initial conditions matrix. The system model has the potential to start in states which represent failures. This is particularly true when the failures have occurred in the leg of the system which is not being inspected or tested. The first application of the model would find that the value of K_1 would be one and the other K values would be zero. This corresponds to the assumption that the system starts with everything in a ready condition, not failed. As the model application repeats for intervals between diagnostic activities that occur further in the future, these K values will not be a one and zeros but will have various probabilities allocated among the states. The K values must add up to one.

Similar equations can be written for the other tabulated state probabilities. An additional example is presented below for state 14 of the Primary Model.

$$\begin{aligned}
 P_{14} = & 2K_1 \cdot e^{-(A+B+2C+2D)T} \\
 & - (4K_1 + 2K_2) \cdot e^{-(A+B+2C+D)T} \\
 & + (2K_1 + 2K_2 + 2K_3) \cdot e^{-(A+B+2C)T} \\
 & - (2K_1 + K_4) \cdot e^{-(A+B+C+2D)T} \\
 & + (4K_1 + 2K_2 + 2K_4 + K_5 + K_6) \cdot e^{-(A+B+C+D)T} \\
 & - (2K_1 + 2K_2 + 2K_3 + K_4 + K_5 + K_6 + K_7) \cdot e^{-(A+B+C)T} \\
 & - (2K_1 + 2K_8) \cdot e^{-(A+2C+2D)T} \\
 & + (4K_1 + 2K_2 + 4K_8 + 2K_9) \cdot e^{-(A+2C+D)T} \\
 & - (2K_1 + 2K_2 + 2K_3 + 2K_8 + 2K_9 + 2K_{10}) \cdot e^{-(A+2C)T} \\
 & + (2K_1 + K_4 + 2K_8 + K_{11}) \cdot e^{-(A+C+2D)T} \\
 & - (4K_1 + 2K_2 + 2K_4 + K_5 + K_6 + 4K_8 + 2K_9 + 2K_{11} + K_{12} + K_{13}) \\
 & \quad \cdot e^{-(A+C+D)T} \\
 & + (2K_1 + 2K_2 + 2K_3 + K_4 + K_5 + K_6 + K_7 + 2K_8 + 2K_9 + 2K_{10} + K_{11} \\
 & \quad + K_{12} + K_{13} + K_{14}) \cdot e^{-(A+C)T}
 \end{aligned}$$

The state probability for Primary Model state 15 is found by summing the values of the first 14 states and then subtracting the sum from one. As can be seen from the two examples, a listing of the equations written as above would be cumbersome. Therefore, the probability expressions are tabulated as found in Table 4-3.

An equivalent probability expression development for the Supporting Model is found in Table 4-4. The Supporting Model defines its state probabilities as "Sx" values instead of "Px" values which are used in the Primary Model. Also, the state starting probabilities are labeled as "Lx" in the Supporting Model versus the "Kx" values in the Primary Model. Note that the failure rates associated with the Supporting Model are only the "C" and "D" failure rates for the redundant legs. As in the Primary Model, the state probability for the final state, state 13, is found by summing the state probabilities for states 1

through 12 and subtracting the sum from one. An example state probability equation for the Supporting Model is as follows:

$$\begin{aligned}
 S_{12} = & - L_1 \cdot e^{-(2C + 2D)T} \\
 & + (2L_1 + L_2 + L_3) \cdot e^{-(2C + D)T} \\
 & - (L_1 + L_2 + L_3 + L_4) \cdot e^{-(2C)T} \\
 & + (L_1 + L_6) \cdot e^{-(C + 2D)T} \\
 & - (2L_1 + L_2 + L_3 + 2L_6 + L_8 + L_9) \cdot e^{-(C + D)T} \\
 & + (L_1 + L_2 + L_3 + L_4 + L_6 + L_8 + L_9 + L_{12}) \cdot e^{-CT}
 \end{aligned}$$

Similar equations can be developed for the other state probabilities of the Supporting Model.

These state probability equations for the Primary and Supporting Models could easily be computerized to allow rapid application of the methodology. An additional set of equations associated with average availability computations could also be developed by the integration of the above formulas over the time interval of evaluation and division by the time interval value.

4.2.1.2 Model application

The application of the dynamic system model described above begins with an initialization of the Primary Model (PM). It is appropriate to start the analysis assuming that the system is in perfect condition. An examination of Table 4-1 indicates that PM state 1 represents the state in which the system has no failures. Thus, the probability assigned to the starting probability of PM state 1 is one. The remaining state starting probabilities are assigned zero values. Therefore, the following listing is developed:

K1 = 1.0	K2 = 0.0	K3 = 0.0	K4 = 0.0
K5 = 0.0	K6 = 0.0	K7 = 0.0	K8 = 0.0
K9 = 0.0	K10 = 0.0	K11 = 0.0	K12 = 0.0
K13 = 0.0	K14 = 0.0	K15 = 0.0	

Note that the sum of the "K"s is always equal to one. This is also true of the sum of the "P"s, "L"s, and "S"s.

The PM is now exercised to find solutions for the state probabilities at a specified time interval. The time interval is set as the first time that an operator diagnostic activity is to be performed on the system. Inputs to this step of the analysis are the "K" values shown above and the failure rates for the system legs, "A", "B", "C", and "D". This input information is applied in the tabulated equations of Table 4-3 to calculate the PM state probabilities. The value of P15 is found by summing the values of the P1 through P14 results and subtracting from one. The solution process now has a set of "K" values and a set

of "P" values. The probability of being in a particular state at the end of the time interval of interest is now known.

Before determination of the results of the diagnostic activity on the model states, the state probabilities for the PM must be partitioned to reflect the likelihood of being in a leg of the system which is associated with the diagnostic activity. The partitioning process is only necessary if the values of either "C" or "D" are not zero. If both values are zero, the analyst should proceed to the application of Tables 4-9 and 4-10 or Tables 4-17 and 4-18, directly. These tables are discussed later in the text. The partitioning for the case when either "C" or "D" are not zero is accomplished by application of the Supporting Model (SM). Initialization of the SM is performed following the assignments found in Table 4-5. The SM is to be exercised twice which explains the usage of the A and B following the "Lx" identifiers. The A and B following the "Kx"s in the table can be ignored for this first iteration. On this iteration, the values of L1A and L1B are one and the remainder of the "L"s are zero.

The SM is now ready to be evaluated two times. Using the "LxA" values first, a set of "SxA" values is determined from application of Table 4-4 formulas. Then using the "LxB" values, a set of "SxB" values is also determined. This distinction is necessary in those cases where the common leg of the system has its own diagnostic interval which is different from the redundant leg diagnostic intervals. Otherwise, the A and B solution sets should be equivalent.

It is now possible to partition the PM state probabilities to reflect the distinction between the two redundant legs. The rationale behind the partitioning process can best be illustrated with an example. Assume that the system is composed only of two redundant legs. Also, assume that the diagnostic activity being considered is the alternate inspection of the two legs every month. That is, at the end of the first month, the first leg will be inspected; at the end of the second month the other leg is inspected; at the end of the third month, the first leg is reinspected; etc.. The example will only look at what happens with state 2 of the PM. This state represents a single detectable failure in one of the redundant legs of the system. For ease of discussion and understanding, the two redundant legs will be called the east and west leg of the system.

The first month elapses and the inspection of the east leg is to take place. The likelihood of being in state 2 is computed to be 0.4. But state 2 encompasses failures in either the east leg or the west leg. So, the likelihood for having a failure in the east leg should be one-half of the state 2 likelihood since the two legs have equivalent failure rates. Assuming that the inspection process, as well as the repair process, is perfect, 0.2 of the state 2 probability can be removed and placed in the state 1 probability bin. This leaves 0.2 probability in state 2 at the start of the next interval. This remaining probability, which may include other probabilities that are added to it from transitions from other states following the inspection, is called the starting probability of the state or the state initial condition.

The next interval is addressed by computing the state probabilities as before but the starting values are different. Now the state 2 probability is found to be 0.35. We know that the state 2 probability was 0.2 at the start of the interval and that all of that probability was associated with a failure in the west leg. We also know that one-half of the probability entering from state 1 will be

associated with the west leg and the other half will correspond to east leg failure. What is not known is how much of the original 0.2 probability was associated with transitions to other states as well as how much of the new probability from state 1 is associated with transitions out of state 2. Using only the PM model does not permit the analyst to evaluate this situation. There is no way to determine how much probability is associated with west leg failure in state 2. Thus there is no way to assess the impact of the inspection on the system. The need for the SM becomes apparent at this point.

Using the same starting conditions associated with the redundant legs, the SM computations yield the fractional split between the east and west legs. The results of the SM computations are not accurate but the relative probability split is thought to be accurate in this model. In the case of the example, the SM starting probabilities would have reflected the 0.2 probability associated with the west leg at the start of the second interval. SM states 2 and 3 would represent the new fractional split of PM state 2 for the east and west legs.

The fractional split of probability is performed with a ratio computation. Table 4-6 presents the formulas for the calculation. Note that the PM starting probabilities are now divided into an A and a B category. The A category always reflects the leg being inspected or tested. Thus, the A and B values must be switched after each computation to provide input for the next interval when the alternate leg is inspected or tested. The formulas shown in the table indicate that the SM must be exercised twice as stated earlier. The "P" values in the formula column are the results of the PM calculations for the appropriate interval. The partitioned probabilities are the values to use in the diagnostic activity evaluation process which is discussed next.

The methodology presented in this report can be used to model inspection and testing activities as well as the associated repair activities. Operator errors related to these activities are also modeled. The activity models presented are the following:

- Inspection of a redundant leg,
- Inspection of the common leg,
- Inspection of the common leg and one redundant leg,
- Inspection of the common leg and both redundant legs,
- Testing of a redundant leg,
- Testing of the common leg,
- Testing of the common leg and one redundant leg,
- Testing of the common leg and both redundant legs.

These eight models can be used in combination or by themselves to model the diagnostic activity. For example, if the system contains all three legs of the full model, the inspection process may look like: 1) inspect a redundant leg, 2) inspect the common leg and the other redundant leg, 3) inspect the first redundant leg, The inspection process may also proceed in the manner: 1) inspect the common leg and a redundant leg, 2) inspect the common leg and the other redundant leg, 3) inspect the common and the first redundant leg, The

application of the above models is up to the analyst but should attempt to provide reasonable coverage of all parts of the system.

The operator errors that can be modeled within the above activity models are the following:

- Operator makes no error,
- Operator fails to detect a detectable error in a redundant leg,
- Operator fails to detect a detectable error in the common leg,
- Operator creates a detectable error in a redundant leg during repair,
- Operator creates a detectable error in the common leg during repair,
- Operator creates a detectable error in a redundant leg during testing,
- Operator creates a detectable error in the common leg during testing,
- Operator creates an undetectable error in a redundant leg during repair,
- Operator creates an undetectable error in the common leg during repair,
- Operator creates an undetectable error in a redundant leg during testing,
- Operator creates an undetectable error in the common leg during testing.

With the inclusion of the no operator error event, it is possible to model the maximum benefit that could result from the diagnostic activity. The operator error modeling allows the analyst to check the sensitivity of the results to various types of operator error. Data bases to support the degree of modeling depth presented here may not be in existence. However, the impact of the errors can still be evaluated or can be ignored by the methodology. The analyst has a choice in the level of application of the models by assigning zero or non-zero values to the inputs.

These models are presented in pairs of tables. The first table of each model shows the potential transitions between states within the PM that can take place following a diagnostic activity. The second of the tables presents the formulas used to calculate the transition probabilities. These tables are presented for

the eight diagnostic activities presented above. The corresponding table numbers for the activity models are:

Tables 4-7 and 4-8	Inspection - One Redundant
Tables 4-9 and 4-10	Inspection - Common
Tables 4-11 and 4-12	Inspection - Common and One Redundant
Tables 4-13 and 4-14	Inspection - Common and Both Redundant
Tables 4-15 and 4-16	Testing - One Redundant
Tables 4-17 and 4-18	Testing - Common
Tables 4-19 and 4-20	Testing - Common and One Redundant
Tables 4-21 and 4-22	Testing - Common and Both Redundant

Only Tables 4-11 and 4-12 will be discussed in any detail. The remaining tables all follow the same format.

Table 4-11 indicates the possible transitions that can occur following inspection of the common leg and one redundant leg. The transitions occur between the partitioned states of the PM. Transitions are indicated by an "X" in a row-column position in the table signifying the (from state) - (to state) transition. For example, the row corresponding to state 10 has seven "X"s shown. This indicates that if the system is in a state 10 condition at the beginning of the inspection of the two legs, there is a potential for the system to be in any of seven states following the inspection. The transitions all depend on the operator errors that are made during the inspection.

The transitions shown in Table 4-11 are defined in detail and computational formulas are tabulated in Table 4-12. This second table is a coded listing of the formulas used to calculate the further partitioning of state probabilities due to the inspection process. The resulting state probabilities are the new starting probabilities for the next inspection interval.

The table is constructed in the form of an equation. The new state probability is represented by the state number in the leftmost column. The next column shows an equal sign and plus signs for the probabilities that will be summed to compute the new state probability. The next column indicates what old state probability is weighted by operator error probabilities reflecting a transition from the old state to the new state. The remainder of the table displays the potential operator errors and indicates the appropriate error for the transition being computed. The error portion of the table uses three symbols to indicate the form of the probability expression. An "X" indicates that the error associated with that column has been committed. An "0" indicates that the error has not been committed and that the computational formula must include the success of the error condition. The "W" is similar to the "0" but is used in the case where the detectable and undetectable failure errors are not being committed. The "W" is always found in a pair that is associated with detectable and undetectable failures. The rationale for this stems from an assumption of the model that an individual operator will not commit more than a single error per leg diagnostic activity other than common mode errors. Thus, the probability of the operator not committing detectable or undetectable failure errors is not simply the

product of the complement of the probabilities of the errors being committed; rather, it is the complement of the sum of the error probabilities. Call the detectable failure error probability OD and the undetectable failure error probability OU. Normally, there are four results possible as outcomes. These outcomes and their probabilities are:

Operator makes a detectable failure error
probability is $OD*(1-OU)$

Operator makes an undetectable failure error
probability is $OU*(1-OD)$

Operator makes both failure errors
probability is $OD*OU$

Operator makes no error
probability is 1-above probabilities

$$\begin{aligned}
 &= 1-OD*(1-OU)-OU*(1-OD)-OD*OU \\
 &= 1-OD+OD*OU-OU+OD*OU-OD*OU \\
 &= 1-OD-OU+OD*OU \\
 &= (1-OD)*(1-OU)
 \end{aligned}$$

However, the assumption that the operator can only commit one error of this type per leg diagnostic activity eliminates one of the four outcomes, that outcome associated with two errors being made by the operator. Thus, the probability of making no errors is:

$$\begin{aligned}
 &\text{Operator makes no error} \\
 &\text{probability is } 1-OD*(1-OU)-OU*(1-OD) \\
 &= 1-OD'-OU'
 \end{aligned}$$

This formulation and simplification aids in the application of the model. The "X" values in the transition probability tables correspond to the OD' and OU' values and the "W" values as a pair correspond to the $1-OD'-OU'$ values from above.

The entire process of computation is best illustrated with an example. Looking at the 2B computation in Table 4-12 indicates that there are six transitions that can occur into state 2B. This is also apparent from the 2B column shown in Table 4-11. Both tables indicate that transitions can occur from states 2B, 3, 7A, 9B, 10, and 14A. Each of these transitions will be discussed in detail and the development of the probability expression for the transitions will be described. The new probability for state 2B is initialized at zero for the purposes of the example and its discussion. The column headings displayed in Table 4-12 which start with the letter "O" indicate operator errors. The next letter in each heading indicates if the error occurs in a redundant leg, "R", or in the common leg, "C". The next letter indicates when the failure occurs. The letter "I" stands for inspection, "R" stands for repair, and "T" stands for testing. The final letter in the column headings indicates either the form of the error - "O" stands for omission - or the type of resulting failure - "D"

stands for detectable failure by commission and "U" stands for undetectable failure by commission. The column heading labeled by "MULT" is used to indicate those cases where more than one error must occur for the transition to take place.

The transition from state 2B to state 2B is marked in Table 4-12 under the column titled "NA". The "NA" stands for not applicable and indicates that the transition is not dependent on the actions of the operator but happens automatically. This is true because the operator is inspecting the other redundant leg and the common leg. The failure in the redundant leg indicated by state 2B is not discoverable during this inspection even though it is a detectable failure. Thus, the new probability for state 2B is incremented by the old probability for state 2B times one.

The transition from state 3 to state 2B is indicated by an "X" in the "NE" column, an "O" in the "ORIO" column, and "W"s in the "ORRD" and "ORRU" columns. The "NE" column stands for no operator error and flags that there will be "O"s and/or "W"s in the row. State 3 is a detectable failure in both of the redundant legs of the system. The transition would require that the operator notice the detectable failure in the inspected redundant leg and correct it without error. The remaining detectable failure would be in the uninspected redundant leg which corresponds to state 2B. The "ORIO" column stands for operator omission error in the inspection of the redundant leg. The usage of an "O" in that column indicates that the error is not committed but that the complement probability must be considered. The "W" columns deal with operator commission errors in the repair of the redundant leg. The usage of the "W" indicates that the errors were not committed but that the complement of the paired probability must be considered. This leads to the following probability formula which is multiplied by the old state 3 probability and then added to the new state 2B summation:

$$(1-ORIO)*(1-ORRD-ORRU)$$

The transition from state 7A to state 2B has a similar construction to the transition associated with state 3 and state 2B. The only difference between state 3 and state 7A is the undetectable failure that is found in the inspected redundant leg. This failure would be corrected when the leg was repaired following the discovery of the detectable failure. The remaining discussion would be equivalent to the state 3 transition discussion and is not repeated.

The transitions from states 9B, 10, and 14A are the same as those discussed above except they require the operator discovery and correction of the detectable failure in the common leg. The probability development would be the same with the additional terms for the common leg correction. Thus, each term would be multiplied by the following expression:

$$(1-OCIO)*(1-OCRD-OCRU)$$

After performing the above probability computations, the new probability for state 2B, K2B, is found as:

$$\begin{aligned} K2B = & P2B + P3*(1-ORIO)*(1-ORRD-ORRU) \\ & + P7A*(1-ORIO)*(1-ORRD-ORRU) \\ & + P9B*(1-OCIO)*(1-OCRD-OCRU) \\ & + P10*(1-ORIO)*(1-ORRD-ORRU)*(1-OCIO)*(1-OCRD-OCRU) \\ & + P14A*(1-ORIO)*(1-ORRD-ORRU)*(1-OCIO)*(1-OCRD-OCRU) \end{aligned}$$

The remaining "Kx" values are found in a similar manner.

The next step in the process is extremely important. The "Kx" probability values associated with the A and B terms must be switched to reflect the change in the redundant leg which is to be inspected. The model always assumes the A leg is the inspected leg of the redundant pair. Thus the old B leg must become the new A leg. The "Kx" values following the switch of the A and B terms are the new starting probabilities for the PM. The process begins again with the reinitialization of the SM as discussed before.

This process can be iterated until the probability values for the PM do not change significantly. This point is called the steady state condition of the system model and would be a good long term estimate of the system availability. The availability is found by summing the steady state probabilities, "Px"s, for states 1, 2A, 2B, 4A, 4B, 5A, and 5B. This same analysis can be conducted using the average availability of the system. Note that the computation of the average availability must be done separately from the computation process described above. Integrated formulations are not used in the partitioning process.

4.2.2 Inspection of Sequential Failures

Many types of components in nuclear power plants have degraded states of operation or degraded conditions prior to the actual failure of the component. These degraded component conditions are often detectable by inspection activities. Thus, an opportunity exists for the operator to replace components that are showing degradation before they actually fail and cause plant outages or safety concerns.

The following Markov model can be used to evaluate the situation described above. It can be applied to components which have a sequential failure mechanism; that is, components that first degrade in a detectable manner and then fail, but that cannot fail without first going through the degraded condition. Piping failures can be modeled under these assumptions. Pipes normally leak or crack prior to rupture. Both leaking and cracking are detectable degradations. Since they do not affect the functioning of the pipe, they are not considered failures but could be called precursors to failure. Equipment which moves in some manner, such as pumps, fans, and compressors, can show signs of wear or vibration prior to failure. This type of equipment can be modeled using the sequential failure Markov model for a portion of its failure modes. Steam generator tubing degrades by thinning which is detectable. This

example is developed in Section 7 and uses the following Markov model.

The model is presented in closed form and thus does not require computer computation. The states of the model are as follows:

State 1	Component(s) okay
State 2	Component(s) degraded in a detectable manner
State 3	Component(s) failed

This simple model can be used to evaluate single components or groups of components that are inspected together.

Two failure rates are required for the model. The first is the rate at which the component becomes degraded in a detectable manner. The second is the rate at which the component fails given that it is degraded in a detectable manner. The first failure rate will be denoted, "D", and the second as, "F". The closed form equations for this model are shown below:

$$P_1 = K_1 \cdot e^{-DT}$$

$$P_2 = (D \cdot K_1 + (D-F) \cdot K_2) / (D-F) \cdot e^{-FT} - D \cdot K_1 / (D-F) \cdot e^{-DT}$$

$$P_3 = 1 - (D \cdot K_1 + (D-F) \cdot K_2) / (D-F) \cdot e^{-FT} + F \cdot K_1 / (D-F) \cdot e^{-DT}$$

where

$P_1 - P_3$ are the state probabilities at time, T ,

$K_1 - K_2$ are the initial probabilities of states 1 and 2,

D is the rate of component degradation to detectability,

F is the rate of component failure from a degraded state.

Application of the model is straightforward. Difficulties arise in the estimation of the failure rates needed as input into the model. Often it is easy to obtain information for the rate of degradation. It is much more difficult to estimate the rate of failure given degradation. If failure data is also available, the rate of failure given degradation can be back computed based on the observed numbers of failures and the known rate of degradation. This process is used in the steam generator tube rupture/inspection interval example presented in Section 7.

FIGURE 4-1. PRIMARY MARKOV MODEL SYSTEM DIAGRAM

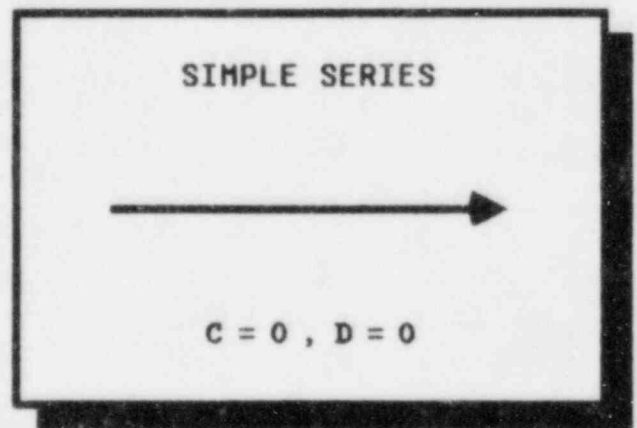
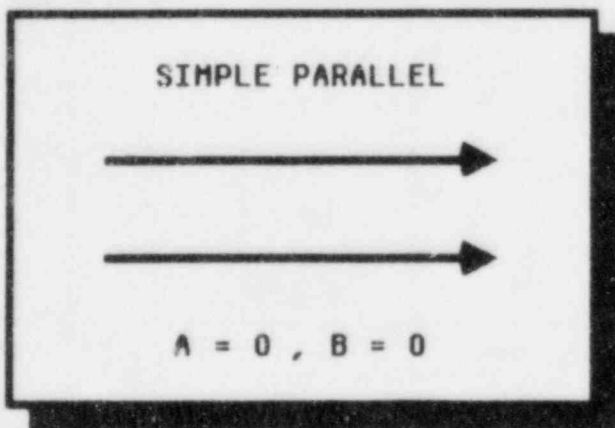
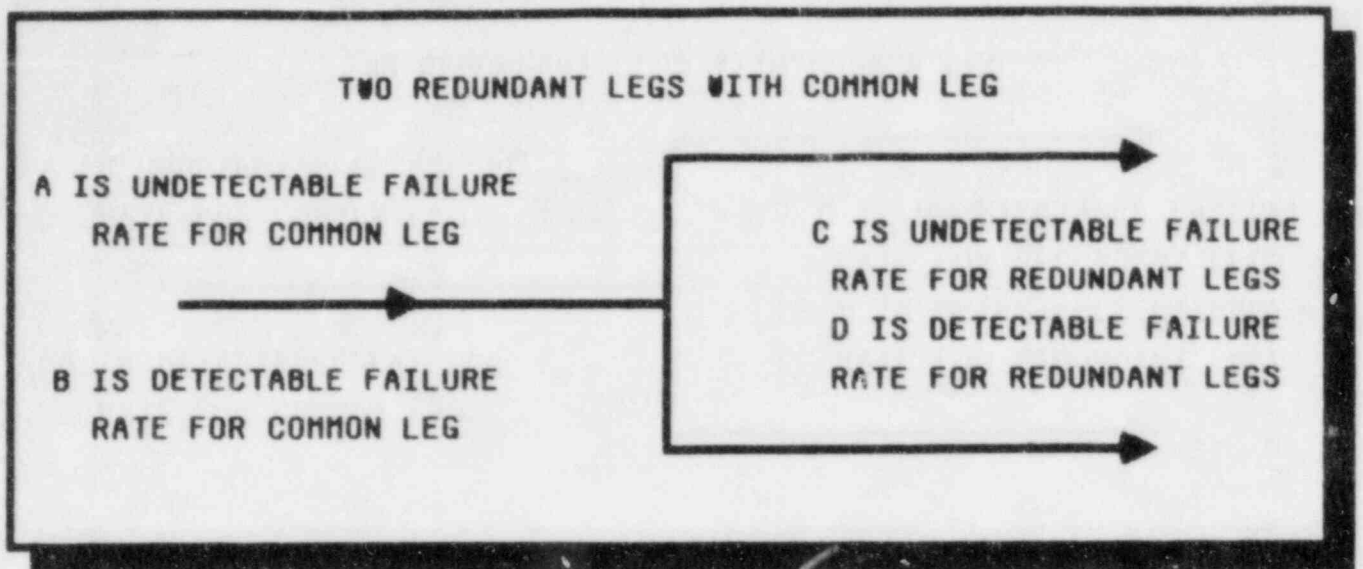


TABLE 4-1. PRIMARY MODEL STATES

PRIMARY MARKOV MODEL STATE DEFINITION						
STATE	SYSTEM CONDITIONS					
	COMMON		ONE TRAIN		OTHER TRAIN	
	DETECTABLE	UNDETECTABLE	DETECTABLE	UNDETECTABLE	DETECTABLE	UNDETECTABLE
1	NO	NO	NO	NO	NO	NO
2	NO	NO	YES	NO	NO	NO
3	NO	NO	YES	NO	YES	NO
4	NO	NO	NO	YES	NO	NO
5	NO	NO	YES	YES	NO	NO
6	NO	NO	YES	NO	NO	YES
7	NO	NO	YES	YES	YES	NO
8	YES	NO	NO	NO	NO	NO
9	YES	NO	YES	NO	NO	NO
10	YES	NO	YES	NO	YES	NO
11	YES	NO	NO	YES	NO	NO
12	YES	NO	YES	YES	NO	NO
13	YES	NO	YES	NO	NO	YES
14	YES	NO	YES	YES	YES	NO
15	--	YES	--	YES	--	YES

TABLE 4-2. SUPPORTING MODEL STATES

SUPPORTING MARKOV MODEL STATE DEFINITION				
STATE	SYSTEM CONDITIONS			
	TRAIN A		TRAIN B	
	DETECTABLE	UNDETECTABLE	DETECTABLE	UNDETECTABLE
1	NO	NO	NO	NO
2	YES	NO	NO	NO
3	NO	NO	YES	NO
4	YES	NO	YES	NO
5	NO	YES	NO	NO
6	NO	NO	NO	YES
7	YES	YES	NO	NO
8	NO	NO	YES	YES
9	YES	NO	NO	YES
10	NO	YES	YES	NO
11	YES	YES	YES	NO
12	YES	NO	YES	YES
13	--	YES	--	YES

TABLE 4-3. PRIMARY MODEL EQUATION DEFINITION

PRIMARY MODEL PROBABILITY EQUATIONS																			
P	K1	K2	K3	K4	K5	K6	K7	K8	K9	K10	K11	K12	K13	K14	K15	A	B	C	D
P1	1															1	1	2	2
P2	-2															1	1	2	2
	2	1														1	1	2	1
P3	1															1	1	2	2
	-2	-1														1	1	2	1
	1	1	1													1	1	2	0
P4	-2															1	1	2	2
	2			1												1	1	1	2
P5	2															1	1	2	2
	-2	-1														1	1	2	1
	-2			-1												1	1	1	2
	2	1		1	1											1	1	1	1
P6	2															1	1	2	2
	-2	-1														1	1	2	1
	-2			-1												1	1	1	2
	2	1		1		1										1	1	1	1

TABLE 4-3. PRIMARY MODEL EQUATION DEFINITION (CONTD)

PRIMARY MODEL PROBABILITY EQUATIONS (CONTINUED)																			
P	K1	K2	K3	K4	K5	K6	K7	K8	K9	K10	K11	K12	K13	K14	K15	A	B	C	D
P7	-2															1	1	2	2
	4	2														1	1	2	1
	2			1												1	1	1	2
	-4	-2		-2	-1	-1										1	1	1	1
	-2	-2	-2													1	1	2	0
	2	2	2	1	1	1	1									1	1	1	0
P8	-1															1	1	2	2
	1							1								1	0	2	2
P9	2															1	1	2	2
	-2	-1														1	1	2	1
	-2							-2								1	0	2	2
	2	1						2	1							1	0	2	1
P10	-1															1	1	2	2
	2	1														1	1	2	1
	-1	-1	-1													1	1	2	0
	1							1								1	0	2	2
	-2	-1						-2	-1							1	0	2	1
	1	1	1					1	1	1						1	0	2	0
P11	2															1	1	2	2
	-2			-1												1	1	1	2
	-2							-2								1	0	2	2
	2			1				2			1					1	0	1	2

TABLE 4-3. PRIMARY MODEL EQUATION DEFINITION (CONT'D)

PRIMARY MODEL PROBABILITY EQUATIONS (CONTINUED)																			
P	K1	K2	K3	K4	K5	K6	K7	K8	K9	K10	K11	K12	K13	K14	K15	A	B	C	D
P12	-2															1	1	2	2
	2	1														1	1	2	1
	2			1												1	1	1	2
	-2	-1		-1	-1											1	1	1	1
	2							2								1	0	2	2
	-2	-1						-2	-1							1	0	2	1
	-2			-1				-2			-1					1	0	1	2
	2	1		1	1			2	1		1	1				1	0	1	1
P13	-2															1	1	2	2
	2	1														1	1	2	1
	2			1												1	1	1	2
	-2	-1		-1		-1										1	1	1	1
	2							2								1	0	2	2
	-2	-1						-2	-1							1	0	2	1
	-2			-1				-2			-1					1	0	1	2
	2	1		1		1		2	1		1		1			1	0	1	1

TABLE 4-4. SUPPORTING MODEL EQUATION DEFINITION

SUPPORTING MODEL PROB EQUATIONS														
P	L1	L2	L3	L4	L5	L6	L7	L8	L9	L10	L11	L12	C	D
S1	1												2	2
S2	-1												2	2
	1	1											2	1
S3	-1												2	2
	1		1										2	1
S4	1												2	2
	-2	-1	-1										2	1
	1	1	1	1									2	0
S5	-1												2	2
	1				1								1	2
S6	-1												2	2
	1					1							1	2
S7	1												2	2
	-1	-1											2	1
	-1				-1								1	2
	1	1			1		1						1	1

TABLE 4-4. SUPPORTING MODEL EQUATION DEFINITION (CONT'D)

SUPPORTING MODEL PROB EQUATIONS (CONTINUED)															
P	L1	L2	L3	L4	L5	L6	L7	L8	L9	L10	L11	L12	C	D	
S8	1												2	2	
	-1		-1										2	1	
	-1					-1							1	2	
	1		1			1		1					1	1	
S9	1												2	2	
	-1	-1											2	1	
	-1					-1							1	2	
	1	1				1			1				1	1	
S10	1												2	2	
	-1		-1										2	1	
	-1				-1								1	2	
	1		1		1					1			1	1	
S11	-1												2	2	
	2	1	1										2	1	
	-1	-1	-1	-1									2	0	
	1				1								1	2	
	-2	-1	-1		-2		-1			-1			1	1	
	1	1	1	1	1		1			1	1		1	0	

TABLE 4-5. INITIALIZATION OF SUPPORTING MARKOV MODEL

<u>SUPPORTING STATE INITIAL PROBABILITY</u>	<u>PRIMARY STATE INITIAL PROBABILITY</u>
L1A	K1
L2A	K2A
L3A	K2B
L4A	K3
L5A	K4A
L6A	K4B
L7A	K5A
L8A	K5P
L9A	K6A
L10A	K6B
L11A	K7A
L12A	K7B
L13A	DIF1*
L1B	K8
L2B	K9A
L3B	K9B
L4B	K10
L5B	K11A
L6B	K11B
L7B	K12A
L8B	K12B
L9B	K13A
L10B	K13B
L11B	K14A
L12B	K14B
L13B	DIF2*

* DIF1 = 1.0 - K1 - K2A - K2B - ... - K7A - K7B

DIF2 = 1.0 - K8 - K9A - K9B - ... - K14A - K14B

TABLE 4-6. INITIAL PARTITIONING OF
PRIMARY STATE PROBABILITIES

<u>PARTITIONED PROBABILITY</u>	<u>PROBABILITY FORMULA</u>
P1	P1
P2A	$P2 = \{S2A / (S2A + S3A)\}$
P2B	$P2 = \{S3A / (S2A + S3A)\}$
P3	P3
P4A	$P4 = \{S5A / (S5A + S6A)\}$
P4B	$P4 = \{S6A / (S5A + S6A)\}$
P5A	$P5 = \{S7A / (S7A + S8A)\}$
P5B	$P5 = \{S8A / (S7A + S8A)\}$
P6A	$P6 = \{S9A / (S9A + S10A)\}$
P6B	$P6 = \{S10A / (S9A + S10A)\}$
P7A	$P7 = \{S11A / (S11A + S12A)\}$
P7B	$P7 = \{S12A / (S11A + S12A)\}$
P8	P8
P9A	$P9 = \{S2B / (S2B + S3B)\}$
P9B	$P9 = \{S3B / (S2B + S3B)\}$
P10	P10
P11A	$P11 = \{S5B / (S5B + S6B)\}$
P11B	$P11 = \{S6B / (S5B + S6B)\}$
P12A	$P12 = \{S7B / (S7B + S8B)\}$
P12B	$P12 = \{S8B / (S7B + S8B)\}$
P13A	$P13 = \{S9B / (S9B + S10B)\}$
P13B	$P13 = \{S10B / (S9B + S10B)\}$
P14A	$P14 = \{S11B / (S11B + S12B)\}$
P14B	$P14 = \{S12B / (S11B + S12B)\}$
P15	P15

TABLE 4-7. POSSIBLE TRANSITIONS FOLLOWING INSPECTION
OF ONE REDUNDANT TRAIN

INSPECTION OF REDUNDANT TRAIN A																										
TO \ FROM	1	2 A	2 B	3	4 A	4 B	5 A	5 B	6 A	6 B	7 A	7 B	8	9 A	9 B	10	11 A	11 B	12 A	12 B	13 A	13 B	14 A	14 B	15	
1	1																									
2A	X	X			X																					
2B			1																							
3			X	X						X																
4A					1																					
4B						1																				
5A	X	X			X		X																			
5B								1																		
6A						X			X																	X
6B										1																
7A			X	X						X	X															
7B								X				X														X
8													1													
9A													X	X		X										
9B															1											
10														X	X							X				
11A																1										
11B																	1									
12A													X	X		X		X								
12B																			1							
13A																	X			X						X
13B																					1					
14A														X	X					X	X					
14B																			X					X	X	
15																										1

TABLE 4-8. TRANSITION PROBABILITIES FOR INSPECTION OF ONE REDUNDANT TRAIN

INSPECTION OF TRAIN A															
NEW INITIAL PRIM PROB	- / +	OLD ENDING PRIM PROB	N A	N E	O R I O	O R R D	O R R U	O R R U	O C I O	O C R D	O C C T D	O C C T U	O C C T U	M U L T	
1	-	1	X												
	+	2A		X	O	W		W							
	+	5A		X	O	W		W							
2A	-	2A			X										
	+	2A			O	X									
	+	5A			O	X									
2B	-	2B	X												
	+	3		X	O	W		W							
	+	7A		X	O	W		W							
3	-	3			X										
	+	3			O	X									
	+	7A			O	X									
4A	-	2A			O			X							
	+	4A	X												
	+	5A			O			X							
4B	-	4B	X												
	+	6A		X	O	W		W							
5A	-	5A			X										

TABLE 4-8. TRANSITION PROBABILITIES FOR INSPECTION OF ONE REDUNDANT TRAIN (CONTD)

INSPECTION OF TRAIN A (CONTINUED)																
NEW INITIAL PRIM PROB	- / +	OLD ENDING PRIM PROB	N A	N E	O R I O	O R R D	O R T D	O R R U	O R T U	O C I O	O C R D	O C C T D	O C C T U	O C C T U	M U L T	
5B	-	5B	X													
	+	7B		X	O	W		W								
6A	-	6A			X											
	+	6A			O	X										
6B	-	3			O			X								
	+	6B	X													
	+	7A			O			X								
7A	-	7A			X											
7B	-	7B			X											
	+	7B			O	X										
8	-	8	X													
	+	9A		X	O	W		W								
	+	12A		X	O	W		W								
9A	-	9A			X											
	+	9A			O	X										
	+	12A			O	X										
9B	-	9B	X													
	+	10		X	O	W		W								
	+	14A		X	O	W		W								

TABLE 4-8. TRANSITION PROBABILITIES FOR INSPECTION OF ONE REDUNDANT TRAIN (CONT'D)

INSPECTION OF TRAIN A (CONTINUED)															
NEW INITIAL PRIM PROB	- / +	OLD ENDING PRIM PROB	N A	N E	O R I O	O R R D	O R T D	O R T U	O R T U	O C I O	O C R D	O C T D	O C T U	O C T U	M U L T
10	-	10			X										
	+	10			O	X									
	+	14A			O	X									
11A	-	9A			O			X							
	+	11A	X												
	+	12A			O			X							
11B	-	11B	X												
	+	13A		X	O	W		W							
12A	-	12A			X										
12B	-	12B	X												
	+	14B		X	O	W		W							
13A	-	13A			X										
	+	13A			O	X									
13B	-	10			O			X							
	+	13B	X												
	+	14A			O			X							
14A	-	14A			X										

TABLE 4-9. POSSIBLE TRANSITIONS FOLLOWING INSPECTION
OF COMMON TRAIN

INSPECTION OF COMMON TRAIN																										
TO \ FROM	1	2 A	2 B	3	4 A	4 B	5 A	5 B	6 A	6 B	7 A	7 B	8	9 A	9 B	10	11 A	11 B	12 A	12 B	13 A	13 B	14 A	14 B	15	
1	1																									
2A		1																								
2B			1																							
3				1																						
4A					1																					
4B						1																				
5A							1																			
5B								1																		
6A									1																	
6B										1																
7A											1															
7B												1														
8	X												X												X	
9A		X												X												X
9B			X												X											X
10				X												X										X
11A					X												X									X
11B						X												X								X
12A							X												X							X
12B								X												X						X
13A									X												X					X
13B										X												X				X
14A											X												X			X
14B												X												X	X	
15																										1

TABLE 4-10. TRANSITION PROBABILITIES FOR INSPECTION OF COMMON TRAIN

INSPECTION OF COMMON															
NEW INITIAL PRIM PROB	- / +	OLD ENDING PRIM PROB	N A	N E	O R I O	O R R D	O R T D	O R R U	O R T U	O C I O	O C R D	O C T D	O C R U	O C T U	M U L T
1	-	1	X												
	+	8		X						O	W		W		
2A	-	2A	X												
	+	9A		X						O	W		W		
2B	-	2B	X												
	+	9B		X						O	W		W		
3	-	3	X												
	+	10		X						O	W		W		
4A	-	4A	X												
	+	11A		X						O	W		W		
4B	-	4B	X												
	+	11B		X						O	W		W		
5A	-	5A	X												
	+	12A		X						O	W		W		
5B	-	5B	X												
	+	12B		X						O	W		W		
6A	-	6A	X												
	+	13A		X						O	W		W		

TABLE 4-10. TRANSITION PROBABILITIES FOR INSPECTION OF COMMON TRAIN (CONTD)

INSPECTION OF COMMON (CONTINUED)															
NEW INITIAL PRIM PROB	- / +	OLD ENDING PRIM PROB	N A	N E	O I O	O R O	O R O	O R O	O R O	O C O	O C O	O C O	O C O	O C O	M U L T
6B	-	6B	X												
	+	13B		X						O	W		W		
7A	-	7A	X												
	+	14A		X						O	W		W		
7B	-	7B	X												
	+	14B		X						O	W		W		
8	-	8								X					
	+	8								O	X				
9A	-	9A								X					
	+	9A								O	X				
9B	-	9B								X					
	+	9B								O	X				
10	-	10								X					
	+	10								O	X				
11A	-	11A								X					
	+	11A								O	X				
11B	-	11B								X					
	+	11B								O	X				

TABLE 4-11. POSSIBLE TRANSITIONS FOLLOWING INSPECTION
OF COMMON AND ONE REDUNDANT TRAIN

INSPECTION OF COMMON TRAIN AND REDUNDANT TRAIN A																									
TO \ FROM	1	2 A	2 B	3	4 A	4 B	5 A	5 B	6 A	6 B	7 A	7 B	8	9 A	9 B	10	11 A	11 B	12 A	12 B	13 A	13 B	14 A	14 B	15
1	1																								
2A	X	X			X																				
2B			1																						
3			X	X						X															
4A					1																				
4B						1																			
5A	X	X			X		X																		
5B								1																	
6A						X			X																X
6B										1															
7A			X	X					X	X															
7B							X				X														X
8	X											X													X
9A	X	X			X							X	X			X									X
9B			X										X												X
10			X	X					X					X	X						X				X
11A					X											X									X
11B						X											X								X
12A	X	X			X		X					X	X			X		X							X
12B							X											X							X
13A					X			X									X			X					X
13B									X										X						X
14A			X	X					X	X				X	X						X	X			X
14B							X				X							X						X	X
15																									1

TABLE 4-12. TRANSITION PROBABILITIES FOR INSPECTION OF COMMON AND ONE REDUNDANT TRAIN (CONT'D)

INSPECTION OF COMMON AND TRAIN A (CONTINUED)																
NEW INITIAL PRIM PROB	- / +	OLD ENDING PRIM PROB	N A	N E	O R I O	O R R D	O R T D	O R R U	O R T U	O C I O	O C R D	O C T D	O C R U	O C T U	M U L T	
3	-	3			X											
	+	3			O	X										
	+	7A			O	X										
	+	10			X					O	W		W			
	+	10			O	X				O	W		W			
	+	14A			O	X				O	W		W			
4A	-	2A			O			X								
	+	4A	X													
	+	5A			O			X								
	+	9A			O			X		O	W		W			
	+	11A		X						O	W		W			
	+	12A			O			X		O	W		W			
4B	-	4B	X													
	+	6A		X	O	W		W								
	+	11B		X						O	W		W			
	+	13A		X	O	W		W		O	W		W			
5A	-	5A			X											
	+	12A			X					O	W		W			
5B	-	5B	X													
	+	7B		X	O	W		W								
	+	12B		X						O	W		W			
	+	14B		X	O	W		W		O	W		W			

TABLE 4-12. TRANSITION PROBABILITIES FOR INSPECTION OF COMMON AND ONE REDUNDANT TRAIN (CONT'D)

INSPECTION OF COMMON AND TRAIN A (CONTINUED)																
NEW INITIAL PRIM PROB	- / +	OLD ENDING PRIM PROB	N A	N E	O I O	O R D	O R D	O R U	O R U	O C O	O C D	O C D	O C U	O C U	M U L T	
6A	-	6A			X											
	+	6A			O	X										
	+	13A			X					O	W		W			
	+	13A			O	X				O	W		W			
6B	-	3			O			X								
	+	6B	X													
	+	7A			O			X								
	+	10			O			X		O	W		W			
	+	13B		X						O	W		W			
	+	14A			O			X		O	W		W			
7A	-	7A			X											
	+	14A			X					O	W		W			
7B	-	7B			X											
	+	7B			O	X										
	+	14B			X					O	W		W			
	+	14B			O	X				O	W		W			
8	-	8								X						
	+	8								O	X					
	+	9A			O	W		W		X						
	+	9A			O	W		W		O	X					
	+	12A			O	W		W		X						
	+	12A			O	W		W		O	X					

TABLE 4-12. TRANSITION PROBABILITIES FOR INSPECTION OF COMMON AND ONE REDUNDANT TRAIN (CONTD)

INSPECTION OF COMMON AND TRAIN A (CONTINUED)														
NEW INITIAL PRIM PROB	- / +	OLD ENDING PRIM PROB	N A	N E	O R I O	O R R D	O R R T D	O R R T U U	O C I O	O C C R D	O C C T D	O C C R U	O C C T U	M U L T
11A	-	9A			O			X	X					2
	+	9A			O			X	O	X				2
	+	11A							X					
	+	11A							O	X				
	+	12A			O			X	X					2
	+	12A			O			X	O	X				2
11B	-	11B							X					
	+	11B							O	X				
	+	13A			O	W		W	X					
	+	13A			O	W		W	O	X				
12A	-	12A			X				X					2
	+	12A			X				O	X				2
12B	-	12B							X					
	+	12B							O	X				
	+	14B			O	W		W	X					
	+	14B			O	W		W	O	X				
13A	-	13A			X				X					2
	+	13A			X				O	X				2
	+	13A			O	X			X					2
	+	13A			O	X			O	X				2

TABLE 4-13. POSSIBLE TRANSITIONS FOLLOWING INSPECTION
OF COMMON AND BOTH REDUNDANT TRAINS

INSPECTION OF COMMON TRAIN AND BOTH REDUNDANT TRAINS																									
FROM \ TO	1						2						3						4						
	1	2A	2B	3	4A	4B	5A	5B	6A	6B	7A	7B	8	9A	9B	10	11A	11B	12A	12B	13A	13B	14A	14B	15
1	1																								
2A	X	X			X																				
2B	X		X			X																			
3	X			X																					X
4A					1																				
4B						1																			
5A	X	X			X		X																		
5B	X		X			X		X																	
6A						X			X																X
6B					X					X															X
7A	X			X							X														X
7B	X			X								X													X
8	X												X												X
9A	X	X			X								X	X		X									X
9B	X		X			X							X		X		X								X
10	X			X								X				X									X
11A					X											X									X
11B						X											X								X
12A	X	X			X		X						X	X		X		X							X
12B	X		X			X		X				X		X			X		X						X
13A						X			X								X				X				X
13B					X					X						X						X			X
14A	X			X							X		X			X							X		X
14B	X			X							X	X				X							X	X	X
15																									1

TABLE 4-14. TRANSITION PROBABILITIES FOR INSPECTION OF COMMON AND BOTH REDUNDANT TRAINS

INSPECTION OF COMMON AND BOTH TRAINS														
NEW INITIAL PRIM PROB	- / +	OLD ENDING PRIM PROB	N A	N E	O R I O	O R R T D	O R R T U	O R R T U	O C I O	O C R D	O C C T D	O C C T U	O C C T U	M U L T
1	-	1	X											
	+	2A		X	O	W		W						
	+	2B		X	O	W		W						
	+	3		X	O	W		W						
	+	5A		X	O	W		W						
	+	5B		X	O	W		W						
	+	7A		X	O	W		W						
	+	7B		X	O	W		W						
	+	8		X					O	W		W		
	+	9A		X	O	W		W	O	W		W		
	+	9B		X	O	W		W	O	W		W		
	+	10		X	O	W		W	O	W		W		
	+	12A		X	O	W		W	O	W		W		
	+	12B		X	O	W		W	O	W		W		
	+	14A		X	O	W		W	O	W		W		
	+	14B		X	O	W		W	O	W		W		
2A	-	2A			X									
	+	2A			O	X								
	+	5A			O	X								
	+	9A			X				O	W		W		
	+	9A			O	X			O	W		W		
	+	12A			O	X			O	W		W		

TABLE 4-14. TRANSITION PROBABILITIES FOR INSPECTION OF COMMON AND BOTH REDUNDANT TRAINS (CONT'D)

INSPECTION OF COMMON AND BOTH TRAINS (CONTINUED)															
NEW INITIAL PRIM PROB	- / +	OLD ENDING PRIM PROB	N A	N E	O R I O	O R R D	O R T D	O R T U	O R T U	O C I O	O C R D	O C T D	O C R U	O C T U	M U L T
2B	-	2B			X										
	+	2B			O	X									
	+	5B			O	X									
	+	9B			X					O	W		W		
	+	9B			O	X				O	W		W		
	+	12B			O	X				O	W		W		
3	-	3			X										
	+	3			O	X									
	+	7A			O	X									
	+	7B			O	X									
	+	10			X					O	W		W		
	+	10			O	X				O	W		W		
	+	14A			O	X				O	W		W		
	+	14B			O	X				O	W		W		
4A	-	2A			O			X							
	+	4A	X												
	+	5A			O			X							
	+	6B		X	O	W		W							
	+	9A			O			X		O	W		W		
	+	11A		X						O	W		W		
	+	12A			O			X		O	W		W		
	+	13B		X	O	W		W		O	W		W		

TABLE 4-14. TRANSITION PROBABILITIES FOR INSPECTION OF COMMON AND BOTH REDUNDANT TRAINS (CONT'D)

INSPECTION OF COMMON AND BOTH TRAINS (CONTINUED)														
NEW INITIAL PRIM PROB	- / +	OLD ENDING PRIM PROB	N A	N E	O R I O	O R R D	O R R U	O R R U	O C I O	O C C D	O C C D	O C C U	O C C U	M U L T
4B	-	2B			O			X						
	+	4B	X											
	+	5B			O			X						
	+	6A		X	O	W		W						
	+	9B			O			X	O	W		W		
	+	11B		X					O	W		W		
	+	12B			O			X	O	W		W		
	+	13A		X	O	W		W	O	W		W		
5A	-	5A			X									
	+	12A			X				O	W		W		
5B	-	5B			X									
	+	12B			X				O	W		W		
6A	-	6A			X									
	+	6A			O	X								
	+	13A			X				O	W		W		
	+	13A			O	X			O	W		W		
6B	-	6B			X									
	+	6B			O	X								
	+	13B			X				O	W		W		
	+	13B			O	X			O	W		W		

TABLE 4-14. TRANSITION PROBABILITIES FOR INSPECTION OF COMMON AND BOTH REDUNDANT TRAINS (CONTD)

INSPECTION OF COMMON AND BOTH TRAINS (CONTINUED)																
NEW INITIAL PRIM PROB	- / +	OLD ENDING PRIM PROB	N A	N E	O R I O	O R R D	O R T D	O R R U	O R T U	O C I O	O C R D	O C T D	O C R U	O C T U	M U L T	
7A	-	7A			X											
	+	14A			X					O	W		W			
7B	-	7B			X											
	+	14B			X					O	W		W			
8	-	8								X						
		8								O	X					
		9A			O	W		W		X						
		9A			O	W		W		O	X					
		9B			O	W		W		X						
		9B			O	W		W		O	X					
		10			O	W		W		X						
		10			O	W		W		O	X					
		12A			O	W		W		X						
		12A			O	W		W		O	X					
		12B			O	W		W		X						
		12B			O	W		W		O	X					
		14A			O	W		W		X						
		14A			O	W		W		O	X					
		14B			O	W		W		X						
		14B			O	W		W		O	X					

TABLE 4-14. TRANSITION PROBABILITIES FOR INSPECTION OF COMMON AND BOTH REDUNDANT TRAINS (CONTD)

INSPECTION OF COMMON AND BOTH TRAINS (CONTINUED)															
NEW INITIAL PRIM PROB	- / +	OLD ENDING PRIM PROB	N A	N E	O R I O	O R R D	O U R T D	O R R U	O R R U	O C I O	O C R D	O C T D	O C R U	O C T U	M U L T
9A	-	9A			X					X					2
	+	9A			X					O	X				2
	+	9A			O	X				X					2
	+	9A			O	X				O	X				2
	+	12A			O	X				X					2
	+	12A			O	X				O	X				2
9B	-	9B			X					X					2
	+	9B			X					O	X				2
	+	9B			O	X				X					2
	+	9B			O	X				O	X				2
	+	12B			O	X				X					2
	+	12B			O	X				O	X				2
10	-	10			X					X					2
	+	10			X					O	X				2
	+	10			O	X				X					2
	+	10			O	X				O	X				2
	+	14A			O	X				X					2
	+	14A			O	X				O	X				2
	+	14B			O	X				X					2
	+	14B			O	X				O	X				2

TABLE 4-14. TRANSITION PROBABILITIES FOR INSPECTION OF COMMON AND BOTH REDUNDANT TRAINS (CONTD)

INSPECTION OF COMMON AND BOTH TRAINS (CONTINUED)															
NEW INITIAL PRIM PROB	- / +	OLD ENDING PRIM PROB	N A	N E	O I O	O R O	O R O	O R O	O R O	O C O	O C O	O C O	O C O	O C O	M U L T
11A	-	9A			O				X	X					2
	+	9A			O				X	O	X				2
	+	11A								X					
	+	11A								O	X				
	+	12A			O				X	X					2
	+	12A			O				X	O	X				2
	+	13B			O	W			W	X					
	+	13B			O	W			W	O	X				
11B	-	9B			O				X	X					2
	+	9B			O				X	O	X				2
	+	11B								X					
	+	11B								O	X				
	+	12B			O				X	X					2
	+	12B			O				X	O	X				2
	+	13A			O	W			W	X					
	+	13A			O	W			W	O	X				
12A	-	12A			X					X					2
	+	12A			X					O	X				2
12B	-	12B			X					X					2
	+	12B			X					O	X				2

TABLE 4-14. TRANSITION PROBABILITIES FOR INSPECTION OF COMMON AND BOTH REDUNDANT TRAINS (CONTD)

INSPECTION OF COMMON AND BOTH TRAINS (CONTINUED)															
NEW INITIAL PRIM PROB	- / +	OLD ENDING PRIM PROB	N A	N E	O R I O	O R R D	O R T D	O R R U	O R T U	O C I O	O C R D	O C T D	O C R U	O C T U	M U L T
15	=	3			O			X							
	+	6A			O			X							
	+	6B			O			X							
	+	7A			O			X							
	+	7B			O			X							
	+	8								O			X		
	+	9A								O			X		
	+	9B								O			X		
	+	10			O			X		-					
	+	10			-					O			X		
	+	11A								O			X		
	+	11B								O			X		
	+	12A								O			X		
	+	12B								O			X		
	+	13A			O			X		-					
	+	13A			-					O			X		
	+	13B			O			X		-					
	+	13B			-					O			X		
	+	14A			O			X		-					
	+	14A			-					O			X		
	+	14B			O			X		-					
	+	14B			-					O			X		
	+	15	X												

TABLE 4-15. POSSIBLE TRANSITIONS FOLLOWING TESTING OF ONE REDUNDANT TRAIN

TESTING OF REDUNDANT TRAIN A																										
FROM \ TO	1					2					3					4					5					
	1	2A	2B	3	4	4A	4B	5A	5B	6A	6B	7A	7B	8	9A	9B	10	11A	11B	12A	12B	13A	13B	14A	14B	15
1	X	X			X																					
2A	X	X			X																					
2B			X	X						X																
3			X	X						X																
4A					X		X																			
4B						X			X																	X
5A	X	X			X																					
5B								X				X														X
6A						X			X																	X
6B										X	X															
7A			X	X						X																
7B								X				X														X
8													X	X			X									
9A													X	X			X									
9B															X	X							X			
10															X	X							X			
11A																	X		X							
11B																		X			X					X
12A													X	X			X									
12B																				X				X	X	
13A																		X			X					X
13B																						X	X			
14A															X	X						X				
14B																				X				X	X	
15																										1

TABLE 4-16. TRANSITION PROBABILITIES FOR TESTING OF ONE REDUNDANT TRAIN

TESTING OF TRAIN A															
NEW INITIAL PRIM PROB	- / +	OLD ENDING PRIM PROB	N A	N E	O I O	O R D	O R T	O R U	O R T	O R U	O I O	O C D	O C T	O C U	M U L T
1	-	1		X				W		W					
	+	2A		X			W			W					
	+	5A		X			W			W					
2A	-	1						X							
	+	2A					X								
	+	5A					X								
2B	-	2B		X			W			W					
	+	3		X			W			W					
	+	7A		X			W			W					
3	-	2B						X							
	+	3						X							
	+	7A						X							
4A	-	1								X					
	+	2A							X						
	+	4A		X			W			W					
	+	4A								X					
	+	5A							X						
4B	-	4B		X			W			W					
	+	6A		X			W			W					
5A	-	4A						X							

TABLE 4-16. TRANSITION PROBABILITIES FOR TESTING
OF ONE REDUNDANT TRAIN (CONTD)

TESTING OF TRAIN A (CONTINUED)															
NEW INITIAL PRIM PROB	- / +	OLD ENDING PRIM PROB	N A	N E	O R I O	O R R D	O R T D	O R R U	O R T U	O C I O	O C R D	O C C T D	O C R T U	O C C T U	M U L T
5B	-	5B		X				W		W					
	+	7B		X			W		W						
6A	-	4B						X							
	+	6A					X								
6B	-	2B											X		
	+	3							X						
	+	6B		X			W		W						
	+	6B											X		
	+	7A							X						
7A	-	6B						X							
7B	-	5B						X							
	+	7B					X								
8	-	8		X			W		W						
	+	9A		X			W		W						
	+	12A		X			W		W						
9A	-	8						X							
	+	9A					X								
	+	12A					X								

TABLE 4-16. TRANSITION PROBABILITIES FOR TESTING OF ONE REDUNDANT TRAIN (CONTD)

TESTING OF TRAIN A (CONTINUED)															
NEW INITIAL PRIM PROB	- / +	OLD ENDING PRIM PROB	N A	N E	O R I O	O R D	O R T D	O R T U	O R T U	U C I O	O C R D	O C T D	O C R U	O C T U	M U L T
9B	-	9B		X			W		W						
	+	10		X			W		W						
	+	14A		X			W		W						
10	-	9B						X							
	+	10					X								
	+	14A					X								
11A	-	8							X						
	+	9A							X						
	+	11A		X			W		W						
	+	11A							X						
	+	12A							X						
11B	-	11B		X			W		W						
	+	13A		X			W		W						
12A	-	11A						X							
12B	-	12B		X			W		W						
	+	14B		X			W		W						
13A	-	11B						X							
	+	13A					X								

TABLE 4-17. POSSIBLE TRANSITIONS FOLLOWING TESTING
OF COMMON TRAIN

TESTING OF COMMON TRAIN																									
TO \ FROM	1	2 A	2 B	3	4 A	4 B	5 A	5 B	6 A	6 B	7 A	7 B	8	9 A	9 B	10	11 A	11 B	12 A	12 B	13 A	13 B	14 A	14 B	15
1	X												X												X
2A		X												X											X
2B			X												X										X
3				X												X									X
4A					X												X								X
4B						X												X							X
5A							X												X						X
5B								X												X					X
6A									X												X				X
6B										X												X			X
7A											X												X		X
7B												X												X	X
8	X												X												X
9A		X												X											X
9B			X												X										X
10				X												X									X
11A					X												X								X
11B						X												X							X
12A							X												X						X
12B								X												X					X
13A									X												X				X
13B										X												X			X
14A											X												X		X
14B												X												X	X
15													X												X

TABLE 4-18. TRANSITION PROBABILITIES FOR TESTING OF COMMON TRAIN

TESTING OF COMMON															
NEW INITIAL PRIM PROB	- / +	OLD ENDING PRIM PROB	N A	N E	O R I O	O R D	O R T D	O R T U	O R T U	O C I O	O C R D	O C T D	O C T U	O C T U	M U L T
1	-	1		X								W		W	
	+	8		X							W		W		
2A	-	2A		X								W		W	
	+	9A		X							W		W		
2B	-	2B		X								W		W	
	+	9B		X							W		W		
3	-	3		X								W		W	
	+	10		X							W		W		
4A	-	4A		X								W		W	
	+	11A		X							W		W		
4B	-	4B		X								W		W	
	+	11B		X							W		W		
5A	-	5A		X								W		W	
	+	12A		X							W		W		
5B	-	5B		X								W		W	
	+	12B		X							W		W		
6A	-	6A		X								W		W	
	+	13A		X							W		W		

TABLE 4-18. TRANSITION PROBABILITIES FOR TESTING OF COMMON TRAIN (CONTD)

TESTING OF COMMON (CONTINUED)														
NEW INITIAL PRIM PROB	- / +	OLD ENDING PRIM PROB	N A	N E	O R I O	O R R D	O R R U	O R R U	O C I O	O C R D	O C R D	O C R U	O C R U	M U L T
6B	-	6B		X									W	W
	+	13B		X							W		W	
7A	-	7A		X									W	W
	+	14A		X							W		W	
7B	-	7B		X									W	W
	+	14B		X							W		W	
8	-	1											X	
	+	8											X	
9A	-	2A											X	
	+	9A											X	
9B	-	2B											X	
	+	9B											X	
10	-	3											X	
	+	10											X	
11A	-	4A											X	
	+	11A											X	
11B	-	4B											X	
	+	11B											X	

TABLE 4-18. TRANSITION PROBABILITIES FOR TESTING OF COMMON TRAIN (CONT'D)

TESTING OF COMMON (CONTINUED)															
NEW INITIAL PRIM PROB	- / +	OLD ENDING PRIM PROB	N A	N E	O R I O	O R D	O R D	O R T U	O R T U	O C I O	O C R D	O C T D	O C R U	O C T U	M U L T
15	-	1												X	
	+	2A												X	
	+	2B												X	
	+	3												X	
	+	4A												X	
	+	4B												X	
	+	5A												X	
	+	5B												X	
	+	6A												X	
	+	6B												X	
	+	7A												X	
	+	7B												X	
	+	8											X		
	+	9A											X		
	+	9B											X		
	+	10											X		
	+	11A											X		
	+	11B											X		
	+	12A											X		
	+	12B											X		
	+	13A											X		
	+	13B											X		
	+	14A											X		
	+	14B											X		
	+	15	X												

TABLE 4-19. POSSIBLE TRANSITIONS FOLLOWING TESTING
OF COMMON AND ONE REDUNDANT TRAIN

TESTING OF COMMON TRAIN AND REDUNDANT TRAIN A																									
FROM \ TO	1						2						3						4						
	1	2A	2B	3	4A	4B	5A	5B	6A	6B	7A	7B	8	9A	9B	10	11A	11B	12A	12B	13A	13B	14A	14B	15
1	X	X			X								X	X		X									X
2A	X	X			X								X	X		X									X
2B			X	X					X					X	X							X			X
3			X	X					X					X	X							X			X
4A					X		X									X		X							X
4B						X			X								X			X					X
5A	X	X			X								X	X		X									X
5B								X				X							X					X	X
6A						X			X									X			X				X
6B									X	X												X	X		X
7A			X	X					X					X	X							X			X
7B								X				X							X					X	X
8	X	X			X								X	X		X									X
9A	X	X			X								X	X		X									X
9B			X	X					X					X	X							X			X
10			X	X					X					X	X							X			X
11A					X		X									X		X							X
11B						X			X								X			X					X
12A	X	X			X								X	X		X									X
12B								X				X							X					X	X
13A						X			X									X			X				X
13B									X	X												X	X		X
14A			X	X					X					X	X							X			X
14B								X				X							X					X	X
15																									1

TABLE 4-20. TRANSITION PROBABILITIES FOR TESTING OF COMMON AND ONE REDUNDANT TRAIN

TESTING OF COMMON AND TRAIN A															
NEW INITIAL PRIM PROB	- / +	OLD ENDING PRIM PROB	N A	N E	O R I O	O R R D	O R T D	O R T U	O R T U	O C I O	O C R D	O C T D	O C R U	O C T U	M U L T
1	=	1		X				W		W				W	W
	+	2A		X			W		W					W	W
	+	5A		X			W		W					W	W
	+	8		X				W		W			W		W
	+	9A		X			W		W				W		W
	+	12A		X			W		W				W		W
2A	=	1						X					W		W
	+	2A					X						W		W
	+	5A					X						W		W
	+	8						X					W		W
	+	9A					X						W		W
	+	12A					X						W		W
2B	=	2B		X				W		W			W		W
	+	3		X			W		W				W		W
	+	7A		X			W		W				W		W
	+	9B		X				W		W			W		W
	+	10		X			W		W				W		W
	+	14A		X			W		W				W		W

TABLE 4-20. TRANSITION PROBABILITIES FOR TESTING OF COMMON AND ONE REDUNDANT TRAIN (CONTD)

TESTING OF COMMON AND TRAIN A (CONTINUED)															
NEW INITIAL PRIM PROB	- / +	OLD ENDING PRIM PROB	N A	N E	O I O	O R D	O R D	O R T U	O R T U	O C I O	O C R D	O C T D	O C R U	O C T U	M U L T
3	-	2B						X					W	W	
	+	3					X						W	W	
	+	7A					X						W	W	
	+	9B						X				W		W	
	+	10					X					W		W	
	+	14A					X					W		W	
4A	-	1							X				W	W	
	+	2A							X				W	W	
	+	4A		X			W		W				W	W	
	+	4A							X				W	W	
	+	5A							X				W	W	
	+	8							X			W		W	
	+	9A							X			W		W	
	+	11A		X			W		W			W		W	
	+	11A							X			W		W	
	+	12A							X			W		W	
4B	-	4B		X			W		W				W	W	
	+	6A		X			W		W				W	W	
	+	11B		X			W		W			W		W	
	+	13A		X			W		W			W		W	
5A	-	4A						X				W		W	
	+	11A						X				W		W	

TABLE 4-20. TRANSITION PROBABILITIES FOR TESTING
OF COMMON AND ONE REDUNDANT TRAIN (CONTD)

TESTING OF COMMON AND TRAIN A (CONTINUED)															
NEW INITIAL PRIM PROB	- / +	OLD ENDING PRIM PROB	N A	N E	O R I O	O R D	O R T D	O R T U	O R T U	O C I O	O C R D	O C C D	O C C T U	O C C T U	M U L T
5B	=	5B		X			W		W				W		W
	+	7B		X		W		W					W		W
	+	12B		X			W		W			W		W	
	+	14B		X		W		W				W		W	
6A	=	4B						X					W		W
	+	6A				X							W		W
	+	11B					X					W		W	
	+	13A				X						W		W	
6B	=	2B							X				W		W
	+	3						X					W		W
	+	6B		X			W		W				W		W
	+	6B							X				W		W
	+	7A						X					W		W
	+	9B							X				W		W
	+	10						X					W		W
	+	13B		X			W		W				W		W
	+	13B							X				W		W
	+	14A						X					W		W
7A	=	6B					X						W		W
	+	13B					X						W		W

TABLE 4-20. TRANSITION PROBABILITIES FOR TESTING OF COMMON AND ONE REDUNDANT TRAIN (CONTD)

TESTING OF COMMON AND TRAIN A (CONTINUED)																
NEW INITIAL PRIM PROB	- / +	OLD ENDING PRIM PROB	N A	N E	O I O	O R D	O R D	O R U	O R U	O C O	O C D	O C D	O C U	O C U	M U L T	
7B	-	5B						X					W		W	
	+	7B				X							W		W	
	+	12B					X				W			W		
	+	14B				X					W			W		
8	-	1					W		W				X			
	+	2A				W			W				X			
	+	5A				W			W				X			
	+	8					W		W			X				
	+	9A				W			W			X				
	+	12A				W			W			X				
9A	-	1					X						X		2	
	+	2A				X							X		2	
	+	5A				X							X		2	
	+	8					X					X			2	
	+	9A				X						X			2	
	+	12A				X						X			2	
9B	-	2B					W		W				X			
	+	3				W			W				X			
	+	7A				W			W				X			
	+	9B					W		W			X				
	+	10				W			W			X				
	+	14A				W			W			X				

TABLE 4-20. TRANSITION PROBABILITIES FOR TESTING OF COMMON AND ONE REDUNDANT TRAIN (CONT'D)

TESTING OF COMMON AND TRAIN A (CONTINUED)															
NEW INITIAL PRIM PROB	- / +	OLD ENDING PRIM PROB	N A	N E	O R I O	O R D	O R T D	O R T U	O R T U	O C I O	O C R D	O C T D	O C R U	O C T U	M U L T
10	-	2B					X					X			2
	+	3				X						X			2
	+	7A				X						X			2
	+	9B					X					X			2
	+	10				X						X			2
	+	14A				X						X			2
11A	-	1							X			X			2
	+	2A						X				X			2
	+	4A					W		W			X			
	+	5A						X				X			2
	+	8							X			X			2
	+	9A						X				X			2
	+	11A					W		W			X			
	+	12A						X				X			2
11B	-	4B					W		W			X			
	+	6A				W		W				X			
	+	11B					W		W			X			
	+	13A				W		W				X			
12A	-	4A					X					X			2
	+	11A					X					X			2

TABLE 4-20. TRANSITION PROBABILITIES FOR TESTING OF COMMON AND ONE REDUNDANT TRAIN (CONT'D)

TESTING OF COMMON AND TRAIN A (CONTINUED)																
NEW INITIAL PRIM PROB	- / +	OLD ENDING PRIM PROB	N A	N E	O R I O	O R D	O R T D	O R T U	O R T U	O C I O	O C R D	O C T D	O C R U	O C T U	M U L T	
12B	-	5B					W		W					X		
	+	7B				W			W					X		
	+	12B					W		W			X				
	+	14B				W			W			X				
13A	-	4B					X							X	2	
	+	6A				X								X	2	
	+	11B					X					X			2	
	+	13A				X						X			2	
13B	-	2B							X					X	2	
	+	3						X						X	2	
	+	6B					W		W					X		
	+	7A						X						X	2	
	+	9B							X			X			2	
	+	10						X				X			2	
	+	13B					W		W			X				
	+	14A						X				X			2	
14A	-	6B					X							X	2	
	+	13B					X					X			2	
14B	-	5B					X							X	2	
		7B				X								X	2	
		12B					X					X			2	
		14B				X						X			2	

TABLE 4-20. TRANSITION PROBABILITIES FOR TESTING
OF COMMON AND ONE REDUNDANT TRAIN (CONTD)

TESTING OF COMMON AND TRAIN A (CONTINUED)															
NEW INITIAL PRIM PROB	- / +	OLD ENDING PRIM PROB	N A	N E	O R I O	O R D	O R D	O R U	O R U	O C I O	O C R D	O C T D	O C R U	O C T U	M U L T
15	=	1						-	-					X	
	+	2A						-	-					X	
	+	2B						-	-					X	
	+	3						-	-					X	
	+	4A						-	-					X	
	+	4B							X			-		-	
	+	4B						-	-					X	
	+	5A						-	-					X	
	+	5B							X			-		-	
	+	5B						-	-					X	
	+	6A							X			-		-	
	+	6A						-	-					X	
	+	6B						-	-					X	
	+	7A						-	-					X	
	+	7B							X			-		-	
	+	7B						-	-					X	
	+	8							-					X	
	+	9A						-	-					X	
	+	9B						-	-					X	
	+	10						-	-					X	
	+	11A							-					X	
	+	11B							X			-		-	
	+	11B						-	-					X	
	+	12A						-	-					X	
	+	12B							X			-		-	
	+	12B						-	-					X	

TABLE 4-21. POSSIBLE TRANSITIONS FOLLOWING TESTING
OF COMMON AND BOTH REDUNDANT TRAINS

TESTING OF COMMON TRAIN AND BOTH REDUNDANT TRAINS																									
TO FROM	1	2 A	2 B	3	4 A	4 B	5 A	5 B	6 A	6 B	7 A	7 B	8	9 A	9 B	10	11 A	11 B	12 A	12 B	13 A	13 B	14 A	14 B	15
1	X			X									X			X									X
2A	X	X	X	X	X	X			X	X			X	X	X	X	X	X		X	X				X
2B	X	X	X	X	X	X			X	X			X	X	X	X	X	X		X	X				X
3	X			X									X			X									X
4A					X						X					X						X			X
4B						X						X					X							X	X
5A	X	X	X	X	X	X			X	X			X	X	X	X	X	X		X	X				X
5B	X	X	X	X	X	X			X	X			X	X	X	X	X	X		X	X				X
6A						X		X	X		X						X		X	X			X	X	
6B				X		X			X	X						X		X			X	X			X
7A	X			X									X			X									X
7B	X			X									X			X									X
8	X			X									X			X									X
9A	X	X	X	X	X	X			X	X			X	X	X	X	X	X		X	X				X
9B	X	X	X	X	X	X			X	X			X	X	X	X	X	X		X	X				X
10	X			X									X			X									X
11A					X						X					X						X			X
11B						X						X					X							X	X
12A	X	X	X	X	X	X			X	X			X	X	X	X	X	X		X	X				X
12B	X	X	X	X	X	X			X	X			X	X	X	X	X	X		X	X				X
13A						X		X	X		X						X		X	X			X	X	
13B				X		X			X	X						X		X			X	X			X
14A	X			X									X			X									X
14B	X			X									X			X									X
15																									1

TABLE 4-22. TRANSITION PROBABILITIES FOR TESTING OF COMMON AND BOTH REDUNDANT TRAINS

TESTING OF COMMON AND BOTH TRAINS															
NEW INITIAL PRIM PROB	- / +	OLD ENDING PRIM PROB	N A	N E	O I O	O R D	O R T D	O R T U	O R T U	O C I O	O C R D	O C C T D	O C R T U	O C C T U	M U L T
1	-	1		X			W		W			W		W	
	+	2A		X			W	W	W	W		W		W	
	+	2B		X			W	W	W	W		W		W	
	+	3		X			W		W			W		W	
	+	5A		X			W	W	W	W		W		W	
	+	5B		X			W	W	W	W		W		W	
	+	7A		X			W		W			W		W	
	+	7B		X			W		W			W		W	
	+	8		X				W		W		W		W	
	+	9A		X			W	W	W	W		W		W	
	+	9B		X			W	W	W	W		W		W	
	+	10		X			W		W			W		W	
	+	12A		X			W	W	W	W		W		W	
	+	12B		X			W	W	W	W		W		W	
	+	14A		X			W		W			W		W	
	+	14B		X			W		W			W		W	
2A	-	2A					X	W		W			W		W
	+	2B					W	X		W			W		W
	+	5A					X	W		W			W		W
	+	5B					W	X		W			W		W
	+	9A					X	W		W			W		W
	+	9B					W	X		W			W		W
	+	12A					X	W		W			W		W
	+	12B					W	X		W			W		W

TABLE 4-22. TRANSITION PROBABILITIES FOR TESTING OF COMMON AND BOTH REDUNDANT TRAINS (CONTD)

TESTING OF COMMON AND BOTH TRAINS (CONTINUED)																
NEW INITIAL PRIM PROB	- / +	OLD ENDING PRIM PROB	N A	N E	O R I O	O R R D	O R T D	O R R U	O R T U	O C I O	O C R D	O C T D	O C R U	O C T U	M U L T	
2B	-	2A					W X		W				W		W	
	+	2B					X W			W			W		W	
	+	5A					W X		W				W		W	
	+	5B					X W			W			W		W	
	+	9A					W X		W			W			W	
	+	9B					X W			W			W		W	
	+	12A					W X		W				W		W	
	+	12B					X W			W			W		W	
3	-	1						X					W		W	
	+	2A					X X						W		W	2
	+	2B					X X						W		W	2
	+	3					X						W		W	
	+	5A					X X						W		W	2
	+	5B					X X						W		W	2
	+	7A					X						W		W	
	+	7B					X						W		W	
	+	8						X				W			W	
	+	9A					X X						W		W	2
	+	9B					X X						W		W	2
	+	10					X						W		W	
	+	12A					X X						W		W	2
	+	12B					X X						W		W	2
	+	14A					X						W		W	
	+	14B					X						W		W	

TABLE 4-22. TRANSITION PROBABILITIES FOR TESTING OF COMMON AND BOTH REDUNDANT TRAINS (CONT'D)

TESTING OF COMMON AND BOTH TRAINS (CONTINUED)															
NEW INITIAL PRIM PROB	- / +	OLD ENDING PRIM PROB	N A	N E	O I O	O R D	O R T D	O R T U	O R T U	O C I O	O C R D	O C T D	O C R U	O C T U	M U L T
4B	-	2A				W		W	X			W		W	
	+	2B					W	X	W			W		W	
	+	4B		X			W		W			W		W	
	+	5A				W		W	X			W		W	
	+	5B					W	X	W			W		W	
	+	6A		X		W	W	W	W			W		W	
	+	6A				W		W	X			W		W	
	+	9A				W		W	X		W		W		
	+	9B					W	X	W		W		W		
	+	11B		X			W		W		W		W		
	+	12A				W		W	X		W		W		
	+	12B					W	X	W		W		W		
	+	13A		X		W	W	W	W		W		W		
	+	13A				W		W	X		W		W		
5A	-	6B				W	X	W				W		W	
	+	13B				W	X	W			W		W		
5B	-	6A				W	X	W				W		W	
	+	13A				W	X	W			W		W		

TABLE 4-22. TRANSITION PROBABILITIES FOR TESTING OF COMMON AND BOTH REDUNDANT TRAINS (CONTD)

TESTING OF COMMON AND BOTH TRAINS (CONTINUED)															
NEW INITIAL PRIM PROB	- / +	OLD ENDING PRIM PROB	N A	N E	O R I O	O R D	O R T D	O R R T U	O R T U	O C I O	O C R D	O C T D	O C R U	O C T U	M U L T
6A	=	2A				X			X			W		W	2
	+	2B					X	X				W		W	2
	+	5A				X			X			W		W	2
	+	5B					X	X				W		W	2
	+	6A				X	W		W			W		W	
	+	9A				X			X			W		W	2
	+	9B					X	X				W		W	2
	+	12A				X			X			W		W	2
	+	12B					X	X				W		W	2
	+	13A				X	W		W			W		W	
6B	=	2A					X	X				W		W	2
	+	2B				X			X			W		W	2
	+	5A					X	X				W		W	2
	+	5B				X			X			W		W	2
	+	6B				X	W		W			W		W	
	+	9A					X	X				W		W	2
	+	9B				X			X			W		W	2
	+	12A					X	X				W		W	2
	+	12B				X			X			W		W	2
	+	13B				X	W		W			W		W	
7A	=	4A					X					W		W	
	+	6B				X	X					W		W	2
	+	11A					X					W		W	
	+	13B				X	X					W		W	2

TABLE 4-22. TRANSITION PROBABILITIES FOR TESTING OF COMMON AND BOTH REDUNDANT TRAINS (CONT'D)

TESTING OF COMMON AND BOTH TRAINS (CONTINUED)														
NEW INITIAL PRIM PROB	- / +	OLD ENDING PRIM PROB	N A	N E	O R I O	O R R D	O R R D	O R R U	O C I O	O C R D	O C T D	O C R U	O C T U	M U L T
7B	-	4B					X					W		W
	+	6A				X	X					W		W
	+	11B					X				W		W	
	+	13A				X	X				W		W	2
8	-	1					W		W			X		
		2A				W	W		W	W			X	
		2B				W	W		W	W			X	
		3				W			W				X	
		5A				W	W		W	W			X	
		5B				W	W		W	W			X	
		7A				W			W				X	
		7B				W			W				X	
		8					W		W				X	
		9A				W	W		W	W			X	
		9B				W	W		W	W			X	
		10				W			W				X	
		12A				W	W		W	W			X	
		12B				W	W		W	W			X	
		14A				W			W				X	
		14B				W			W				X	

TABLE 4-22. TRANSITION PROBABILITIES FOR TESTING OF COMMON AND BOTH REDUNDANT TRAINS (CONT'D)

TESTING OF COMMON AND BOTH TRAINS (CONTINUED)															
NEW INITIAL PRIM PROB	- / +	OLD ENDING PRIM PROB	N A	N E	O I O	O R D	O R T D	O R T U	O R T U	O C I O	O C R D	O C T D	O C R U	O C T U	M U L T
11A	-	2A					W	X	W				X		2
	+	2B				W		W	X				X		2
	+	4A					W		W				X		
	+	5A					W	X	W				X		2
	+	5B				W		W	X				X		2
	+	6B				W	W	W	W				X		
	+	9A					W	X	W			X			2
	+	9B				W		W	X			X			2
	+	11A					W		W			X			
	+	12A					W	X	W			X			2
	+	12B				W		W	X			X			2
	+	13B				W	W	W	W			X			
11B	-	2A				W		W	X				X		2
	+	2B					W	X	W				X		2
	+	4B					W		W				X		
	+	5A				W		W	X				X		2
	+	5B					W	X	W				X		2
	+	6A				W	W	W	W				X		
	+	9A				W		W	X			X			2
	+	9B					W	X	W			X			2
	+	11B					W		W			X			
	+	12A				W		W	X			X			2
	+	12B					W	X	W			X			2
	+	13A				W	W	W	W			X			

TABLE 4-22. TRANSITION PROBABILITIES FOR TESTING OF COMMON AND BOTH REDUNDANT TRAINS (CONTD)

TESTING OF COMMON AND BOTH TRAINS (CONTINUED)															
NEW INITIAL PRIM PROB	- / +	OLD ENDING PRIM PROB	N A	N E	O R I O	O R R D	O R T D	O R R U	O R T U	O C I O	O C R D	O C T D	O C R U	O C T U	M U L T
13B	=	2A					X	X				X			3
	+	2B				X			X			X			3
	+	5A					X	X				X			3
	+	5B				X			X			X			3
	+	6B				X	W		W			X			2
	+	9A					X	X				X			3
	+	9B				X			X			X			3
	+	12A					X	X				X			3
	+	12B				X			X			X			3
	+	13B				X	W		W			X			2
14A	=	4A					X					X			2
	+	6B				X	X					X			3
	+	11A					X					X			2
	+	13B				X	X					X			3
14B	=	4B					X					X			2
	+	6A				X	X					X			3
	+	11B					X					X			2
	+	13A				X	X					X			3

TABLE 4-22. TRANSITION PROBABILITIES FOR TESTING OF COMMON AND BOTH REDUNDANT TRAINS (CONTD)

TESTING OF COMMON AND BOTH TRAINS (CONTINUED)																
NEW INITIAL PRIM PROB	- / +	OLD ENDING PRIM PROB	N A	N E	O R I O	O R D	O R T D	O R U	O R U	O C I O	O C D	O C T D	O C U	O C U	M U L T	
15	-	1							X			-		-		
	+	1						-	-					X		
	+	2A							X X			-		-	2	
	+	2A					-	-	-					X		
	+	2B							X X			-		-	2	
	+	2B					-	-	-					X		
	+	3							X			-		-		
	+	3					-		-					X		
	+	4A							X			-		-		
	+	4A						-	-					X		
	+	4B							X			-		-		
	+	4B						-	-					X		
	+	5A							X X			-		-	2	
	+	5A					-	-	-					X		
	+	5B							X X			-		-	2	
	+	5B					-	-	-					X		
	+	6A							X -			-		-		
	+	6A							- -					X		
	+	6B							X -			-		-		
	+	6B							- -					X		
	+	7A							X			-		-		
	+	7A						-	-					X		
	+	7B							X			-		-		
	+	7B						-	-					X		
	+	8							X			-		-		
	+	8						-	-					X		

TABLE 4-22. TRANSITION PROBABILITIES FOR TESTING OF COMMON AND BOTH REDUNDANT TRAINS (CONT'D)

TESTING OF COMMON AND BOTH TRAINS (CONTINUED)															
NEW INITIAL PRIM PROB	- / +	OLD ENDING PRIM PROB	N A	N E	O I O	O R D	O R T	O R T	O R T	O C I O	O C R D	O C T D	O C R U	O C T U	M U L T
15-CTD	=	9A						X	X			-		-	2
	+	9A				-	-	-	-					X	
	+	9B						X	X			-		-	2
	+	9B				-	-	-	-					X	
	+	10						X				-		-	
	+	10				-		-						X	
	+	11A							X			-		-	
	+	11A					-		-					X	
	+	11B							X			-		-	
	+	11B					-		-					X	
	+	12A						X	X			-		-	2
	+	12A				-	-	-	-					X	
	+	12B						X	X			-		-	2
	+	12B				-	-	-	-					X	
	+	13A				-	-	X	-			-		-	
	+	13A				-	-	-	-					X	
	+	13B				-	-	X	-			-		-	
	+	13B				-	-	-	-					X	
	+	14A						X				-		-	
	+	14A				-		-						X	
	+	14B						X				-		-	
	+	14B				-		-						X	
	+	15	X												

5. PROTECTION COSTS

This section gives a brief description of some of the cost considerations that must go into optimization studies. The general types of costs that may be involved in nuclear power plant decision making processes are presented along with a discussion of discounting and how it may relate to nuclear power plant costs.

5.1 EXAMPLE COSTS ENCOUNTERED IN NUCLEAR POWER COST-BENEFIT ANALYSIS

Costs associated with the implementation of NRC guidelines and/or regulations can fall into a number of general categories. There may be materials that must be purchased such as equipment or tooling. Analyses may be necessary prior to implementation which would require payment to consulting services or manpower costs within the plant organization. Any labor associated with the installation, future maintenance, test, inspection and repair of equipment would have to be considered. If plant outages occur as a result of the implementation process, the replacement power costs become important. Any future utility needs of installed equipment, such as electric power or water, should be considered in the analysis. The discussions below attempt to organize the multitude of costs that may be encountered into general categories for ease of consideration.

The first general category in this listing is labor costs. Labor is used through all phases of the implementation process. It can take the form of physical work or mental effort. Initial labor tasks may include the following: engineering analysis, design, evaluation, licensing document preparation, licensing hearings, presentations, and data collection. The actual implementation of the guideline or regulation may include the following forms of labor: data collection, analysis associated with implementation, training, special tool design and fabrication, equipment fabrication and assembly, installation, procedure writing, cleanup, testing, and post installation data collection. Long term labor considerations may include: inspection, maintenance, test, repair, accident response and cleanup, future training, retraining, and future data collection and evaluation. The above listing of labor cost considerations may not be complete but is intended to give users of this methodology a sampling of the types of labor that may be involved in various options under consideration.

The second category of general costs includes costs associated with hardware. At the time of implementation the hardware costs may include the following: special tooling, shielding construction, installed equipment, temporary support equipment, protective gear for workers, and waste handling equipment. Long term hardware costs may include: spare parts, special tooling, refabricated equipment, and replacement equipment.

The third general cost category covers supporting utility costs. These costs include consideration of the following types of support: electric power,

water, oil or other lubricants, fuel oil, pneumatic systems, health physics testing materials, other testing materials, and steam.

The fourth and final general category for costs covers miscellaneous costs such as replacement power, document publication, travel, court fees, lawsuit settlements, accident cleanup, and publicity. As stated earlier, the above lists are not necessarily complete but are intended to give an overview and some guidance on the types of costs that could arise in a cost-benefit analysis.

5.2 DISCOUNTING COSTS

There are two applications of discounting that are presented in this report. The first deals with present worth evaluations and the second covers annualization of present costs.

Discounting to determine the present worth of a future cost requires consideration of a time-related weighting factor. This factor utilization is equivalent to the reciprocal of compounding interest. It determines the amount of money that would have to be invested now in order to obtain a specific future amount. The present worth, PW, of some future cost, FC, in a specified number of years, Y, given a defined discount rate (like an inflation rate or interest rate), R, can be computed using the following formula:

$$PW = \frac{FC}{(1 + R)^Y}$$

This formula can be used to find the sum of costs over a plant's life as well as the sum of benefits or cost reductions over the same time period. By treating the Y value as a variable and summing the above formula for different values of Y up to the plant life value, the resulting summation would give the present worth of all future costs and/or benefits of a particular option. The net value of present worth can be found by treating the costs and benefits separately. Using different discount rates for each may be desired. This would imply that a future cost has a different value in present worth than a future benefit of the same value. Also, the benefit summation may be for a different set of years than the cost summation. In cases where the costs or benefits are constant in value from year to year, the summation can be replaced with an annuity factor which is simply the algebraic solution to the summation of the discount rate formula given above. This annuity factor is defined as follows for N years of plant life:

$$PW = FC * \frac{(1 + R)^N - 1}{R(1 + R)^N}$$

The reciprocal of the annuity factor can be used to calculate the future costs associated with making loan payments on current costs which required borrowed money. Thus, to pay off a loan associated with the purchase of some equipment

and/or associated activity for a specified amount, LA, over the period of a given number of years, Y, the annual cost, AC, is found by:

$$AC = LA * \frac{R(1 + R)^Y}{(1 + R)^Y - 1}$$

The previous forms of discounting are standard techniques. They can be applied to cost-benefit analysis associated with nuclear power plant decision making procedures. It is recommended that usage of discounting be well documented to provide reviewers with sufficient information to perform sensitivity studies associated with discount rates. As with any process that attempts to predict the future, discounting factors add uncertainty to the results of the cost-benefit analysis. The best approach for reducing the impact of this uncertainty on the results is to provide some form of sensitivity study along with the results. In most cases, the results of a cost-benefit analysis dealing with nuclear power will be insensitive to the discount rate unless one option has high initial costs and another option has high operating costs. Situations where options do not differ significantly on their cost distribution with time should be insensitive.

6. SUMMARY OF OPTIMIZATION METHODOLOGY

The methodology presented in this report can be used in the NRC decision making processes to select the optimum of multiple alternatives. This methodology relies heavily on the existence of a supporting data base, particularly in the case of occupational doses related to plant activities. Also, the methodology incorporates many of the past dose and risk analysis methods and presumes that the user has access to these analytical tools. Detailed methods development is only provided in the area of modeling the impact of inspection and testing on system failure probability.

Cost-benefit methods are applicable to problems associated with occupational and public exposures. Dose detriment can be treated as a cost item in decision making processes in order to determine the most cost-effective alternatives for solution to dose related problems. Concerns about proper values for dollar equivalence of dose can be handled by utilizing sensitivity study techniques to determine if the decisions reached by the methods are impacted by the dollar equivalence value chosen. Other factors used in the cost-benefit analysis can be treated in a similar manner.

The key to the usage of this methodology is to realize that the information obtained in the process is nothing more than input into the decision process and is not intended to represent reality beyond what is necessary to make a decision. Thus, cost values obtained for an option are not intended to represent actual costs associated with the option. It may turn out that they do estimate actual expected costs and the objective of the analysis is to come as close to representing reality as is possible. However, the methodology is developed to provide a comparative tool for measuring alternatives and is not developed to provide estimates of actual costs. It is the comparative nature of the methodology that allows simplification. Also, the data base which supports most of the analyses is not sufficient at this time to justify accurate and/or precise estimation of cost or dose.

Another key factor in application of the methods is to always provide for the alternative of no action. It is not safe to assume that every problem's optimum solution necessarily requires a change. Current design and/or plant activity may be the best choice in the long run for the dollars spent. The assumption that a change is necessary to solve every problem rather than determining if the current approach is optimum may have led to the observed increase in occupational exposure at nuclear facilities. The NRC's awareness of this trend and their willingness to examine the problems from a new viewpoint has resulted in the development of this methodology.

The detailed development associated with the Markov models presented in this study provides a new tool for use in both cost-benefit analysis and risk analysis. The models were developed to be applied in both areas and the documentation is detailed enough to instruct users in their proper implementation. Markov analysis has been utilized in risk assessments in past

projects and provides a very valuable tool in determining the impact of operator diagnostic activity on the availability of standby or operating systems. These models could be expanded to cover maintenance activities without difficulty. It is also possible to utilize the models as pieces of larger system models but the application of the methods in this manner requires care.

A detailed example application of the methods is presented in the following section. The sequential failure Markov model is used in an analysis of the optimum inspection interval for steam generator tubing. The example is current and provides a good demonstration of the power of the methods to supply additional information to the NRC decision making process.

7. STEAM GENERATOR TUBE RUPTURE EXAMPLE

This section of the report presents an example application of the optimization methodology that has been discussed in earlier sections. The example chosen deals with the recent safety issue associated with steam generator tube rupture events in PWR plants. The example was selected due to the availability of data, the author's recent involvement in the SGTR value-impact analysis conducted for NRC, and the fact that the problem exercises a portion of the Markov methods discussed in Section 4.

7.1 THE PROBLEM

Pressurized Water Reactors utilize large heat exchangers to transfer heat from the primary coolant system to the feedwater system. This heat transfer converts water to steam on the feedwater side of the heat exchanger which is then used to drive a turbine-generator for power production. Since the heat exchanger is involved in the water-to-steam conversion, this large piece of equipment is called a steam generator. It is composed of thousands of tubes which provide the large surface area for heat transfer. These tubes are the portion of the steam generator which is of concern.

The steam generator tubing experiences tremendous stresses due to high temperatures, water chemistry problems, high pressures, and flow induced vibration. These stresses tend to degrade the tubing over long time periods. Techniques do exist to detect the degraded tubes in a steam generator. A testing technique called eddy-current testing is relatively reliable in determining if the tube examined has experienced extensive wear. Tubes that are found to be defective are repaired by either tube plugging or tube sleeving. Plugging a tube prevents primary water from circulating through the tube and thus eliminates the safety concern associated with tube degradation. Sleeving adds an extra boundary between the primary and secondary sides of the steam generator and thus reduces the likelihood of leakage of the worn tube.

Tube wear, if unchecked or if ignored, will eventually lead to tube rupture. A ruptured steam generator tube creates a small LOCA condition in the primary coolant system as well as providing a direct path for the radioactive primary coolant to go to the atmosphere. This event also places the plant in a situation which relies heavily on the operators' responses rather than relying on the plant automatic safety systems. Without operator response, the LOCA will not be contained and the primary coolant will, in the worst case, decrease to the point of uncovering the core causing a core melt. Thus, the event of an SGTR is of concern, not only because of the potential for a core melt, but also due to the release of radioactive primary coolant to the atmosphere.

The steam generator tubing is inspected to reduce the likelihood of tube rupture. The process is costly to the utility because of the potential power outage as well as the labor and materials costs. The inspection is done inside the steam generator which in older plants can be highly radioactive and lead to

high operator or technician doses during the inspection process and during tube repair. Thus, the inspection process is not done without significant cost and should be done only as often as is necessary to assure safety.

The problem is to find or determine the optimum inspection interval for steam generator tubing. Optimum, in this case, refers to a minimum cost for both dollar expenditures for labor and materials and dose detriments for occupational and public exposures. High inspection frequencies reduce the potential for public dose from an accident, reduce the likelihood of occupational dose from cleanup and repair, reduce costs associated with accident cleanup and repair, increase costs associated with tube inspection and repair, and increase occupational dose from inspection and repair. Low inspection frequencies switch these trends. The methodology for optimization presented in this report is applicable to the solution of this problem.

7.2 THE SOLUTION APPROACH

To begin the solution of this problem, data sources must be identified to determine model depth and model definition. Steam generator repair experience for 26 Westinghouse PWRs exists as part of the working data for NUREG-0886 [10] which deals with steam generator tube experience. This data is displayed in Table 7-1. For the 26 PWRs listed, information dealing with tube experience and plant experience is presented. The first data column gives the number of steam generators at the plant. The second column is an estimate of the number of tubes in each steam generator at the plant. This information was based on a back-calculation from a percentage of tubes repaired. Thus, the numbers are probably not accurate but should be close enough to the actual values to be useful for this study. The next column presents the total number of tubes that have been repaired at each plant during the life of the plant as far as the records went. The final column gives the number of operating months of experience of the plants but does not reduce operation by outage times. Note that the new steam generators in the Surry plants are not included in this data base. This data tends to be dominated by the three reactors with large numbers of repaired tubes.

The consequence data for SGTR events comes from two sources; the recent value-impact analysis dealing with steam generator tube degradation and rupture [11] and the safety issue prioritization guideline document, NUREG/CR-2800 [12]. Tables 7-2 and 7-3 present the data extracted from those documents. The WASH-1400 [13] release categories for PWRs were used as a basis for consequence categorization. The value-impact analysis defined the likelihood of SGTR events resulting in the various release categories with the addition of two partial categories which are fractions of PWR category 9. Categories 3, 4, and 5 deal with core melt sequences with various containment performances. Categories 8, 9, 9(1/2), and 9(1/10) deal with LOCAs with no core melt and reflect degrees of release associated with the event and its response. Each category is assigned a conditional probability given that an SGTR event has occurred. The repair costs for an SGTR are constant and do not vary with the resulting accident severity. Accident cleanup costs do vary, however, with the severity of the accident. Note that core melts have the same costs and that LOCAs are divided into two degrees of cost. The dose to the steam generator repair team is the same for all outcomes. From Table 7-3, the doses to the public and the resulting occupational exposures for cleanup and repair of the reactor (not the steam generator) following the various releases indicate the range of the consequences covered by the release categories. Note that no cleanup or

repair doses are shown for the smaller LOCA accidents.

The only other source of data used in this study is the SGTR experience given in NUREG-0651 [14] and NUREG-0909 [15]. There have been four SGTR events to date. Of the four, two were caused by loose parts in the steam generator which caused degradation due to friction. These two events are not appropriate for this analysis which deals with tube degradation over time due to normal conditions experienced by the tubing. Thus, there have only been two SGTR events of the type evaluated in this example.

Additional data was extracted from the value-impact analysis dealing with costs associated with steam generator inspection and repair. Dose information for the same activities was also obtained from the report. Each steam generator inspection costs $\$2.46E+5$ and results in 12 man-rem of exposure. In addition, each tube that is repaired in the steam generator costs $\$1.6E+3$ and produces one man-rem of exposure.

The approach taken to determine the optimum inspection interval for the prevention of SGTR events proceeds in the following manner:

- 1) determine the tube degradation rate based on the observed experience of the 26 PWRs
- 2) determine the tube rupture rate given tube degradation based on the observed experience of the 26 PWRs
- 3) determine cost and dose expected following an SGTR
- 4) compute the probability of observing various numbers of ruptures for candidate inspection intervals
- 5) compute the expected number of tubes requiring repair for candidate inspection intervals as well as appropriate fractions of inspection intervals
- 6) compute the expected total yearly cost and dose for each candidate inspection interval

Using the above steps, it will be possible to determine the optimum inspection interval for minimizing cost and dose associated with steam generator tube degradation and rupture.

7.3 THE SOLUTION

The tube degradation rate is computed from the data presented in Table 7-1. A total of 10,671 tube repairs have been required in $1.73E+10$ tube hours of experience. The tube hours of experience are determined by multiplying the number of tubes per steam generator times the number of steam generators in the plant times the number of months in operation times the number of hours in a month for each plant, and then summing the results for each plant. The tube degradation rate is simply the ratio of the tubes requiring repair and the number of hours of experience. The ratio yields the result of approximately

6.17E-7 degraded tubes per tube hour of experience.

The tube rupture rate given a degraded tube is back computed using the sequential failure Markov model from Section 4. Several assumptions must be made at this point to allow the modeling of this situation. First the fact that degraded tubes often leak prior to rupture is to be ignored for this analysis due to the complexity that would be encountered in the modeling of the actual situation. This assumption will result in conservative risk estimates for public exposures but in non-conservative estimates in the occupational exposure portion of the model. These errors can be attributed to the overlooking of plant outages and steam generator tube repairs that would be the result of leaking tubes. An average plant is assumed to be representative of the industry for the purposes of this optimization process. The average plant is assumed to have 3.15 steam generators with 3290 tubes per steam generator. The industry is assumed to have had only two SGTR events of the type involved in this exercise. Also, the average plant is assumed to have had an average inspection interval of 2.5 years. This last assumption can be supported to some extent by the observed experience at most plants but it is not calculated in a rigorous fashion.

Using the sequential failure Markov model requires the definition of two failure rates. In this case, the first failure rate is associated with the rate of steam generator tube degradation. The second failure rate is associated with the rate of steam generator tube rupture given that the tube has degraded. It is assumed that two failures have occurred in the recorded experience of the plants in the data base. The rate of tube degradation is estimated and the average inspection interval is assumed to be 2.5 years. From Section 4, the probability of experiencing the second failure in a sequential set of failures is:

$$P = 1. - R_1/(R_1-R_2)*\exp(-R_2*T) + R_2/(R_1-R_2)*\exp(-R_1*T)$$

The R_1 corresponds to the tube degradation rate and the R_2 corresponds to the tube rupture rate. The T is the time between inspections. This formula can be used as the probability of having a single tube rupture during the inspection interval. Multiplying this value times the number of actual tube experience hours would give the expected number of ruptured tubes in the data base history. Since this is assumed to be equal to two, the time between inspection is assumed to be 2.5 years, the tube degradation rate is estimated to be 6.17E-7 failures per tube hour, the average plant has 3290 tubes in its 3.15 steam generators and there have been 199.6/2.5 inspection intervals, the equation can be solved for the rate of tube rupture. The equation is now:

$$2.0 = \{1.0 - 6.17E-7 / (6.17E-7 - R_2) * \exp(-2.5 * 8640.0 * R_2) \\ + R_2 / (6.17E-7 - R_2) * \exp(-2.5 * 8640.0 * 6.17E-7)\} \\ * 3290.0 * 3.15 * 199.6 / 2.5$$

A tabulation of the results for the solution of the above equation for R_2 for various inspection intervals in terms of years is shown below.

<u>INSPECTION</u> <u>INTERVAL (YEARS)</u>	<u>R_2</u>
1.0	4.21E-8
2.0	2.11E-8
2.5	1.69E-8
3.0	1.41E-8
4.0	1.06E-8

The value of 1.69E-8 tube ruptures per degraded tube hour was selected from the above listing representing the inspection interval of 2.5 years on the average.

The expected costs and public and occupational doses that result from an SGTR event can be computed from the data in Tables 7-2 and 7-3. Summing the costs and doses for each PWR release category, weighing each sum by the likelihood of the category, and then summing the weighted values will give the expected doses and costs. The results of this process yield the following results:

Expected public dose	345 man-rem
Expected occupational dose	353.3 man-rem
Total expected dose	698.3 man-rem
Expected accident cost	\$3.77E+5
Expected repair cost	\$3.56E+7
Total expected SGTR cost	\$3.60E+7

The probability distribution for ruptured tubes is the next item requiring definition in the analysis. Using the formula that was used to compute the rupture rate based on an inspection interval of 2.5 years, the probability of a single tube rupturing in a given inspection interval can be computed. The formula is given below with the failure rates that were determined earlier.

$$P = 1.0 - 6.17E-7 / (6.17E-7 - 1.69E-8) * \exp(-N * 8640.0 * 1.69E-8) + 1.69E-8 / (6.17E-7 - 1.69E-8) * \exp(-N * 8640.0 * 6.17E-7)$$

Using this value for P in a binomial probability distribution function, it is possible to compute the likelihood of observing an exact number of tube ruptures in an inspection interval for the average plant with 3290 tubes in each of 3.15

steam generators. The binomial formula used in the computation of these values is:

$$P = M! / \{N! * (M-N)!\} * (1.0-P)^{(M-N)} * P^N$$

where

M is the total number of tubes

N is the number of ruptures

P is the probability of a tube rupture for a single tube

! is the symbol for factorial ... $M! = M * (M-1) * (M-2) * \dots * 2 * 1$

The above procedure was followed and computations were tabulated for inspection intervals from one to ten years in steps of one year. The results are shown in Table 7-4. Note how the likelihood of a single tube rupture increases as the inspection interval increases. This is expected since the longer one goes without checking and making intermediate repairs, the more likely one is to see a major failure. Also, it is interesting to note that the likelihood of observing two failures becomes rather significant for long inspection intervals. The likelihood of going ten years between inspections and not having a tube rupture is only .673. It would appear that the average plant of this example could not endure lengthy inspection intervals without experiencing tube ruptures. Recall that the data base used to develop this example was dominated by three or four plants with high incidence of tube repair. Plants with low tube repair would not have tube failure probabilities of the type shown in Table 7-4.

At each inspection, there is a chance that degraded tubes will be found and thus the expected cost and dose of each inspection is not constant but is a function of variations in the number of tubes requiring repair. Using the intervals created by Table 7-4 by assuming tube ruptures will occur at equally spaced periods within the interval of inspection, the expected number of repaired tubes at the inspection can be computed. For example, for a six year inspection interval, Table 7-4 indicates that there is a possibility of zero, one, two, and three ruptures being observed. For the case of zero ruptures, the inspection will look at tubes degraded for six years. For the case of one rupture, the rupture is assumed to occur at the three year mark and the inspection will look at tubes degraded for the remaining three years. The two rupture case will have an inspection of two year tubes with ruptures occurring at the two and four year marks. Finally, the three rupture case will have ruptures at the 1.5, 3, 4.5 year marks and tubes will have degradation of 1.5 years. This variation in years of degradation results from the assumption of complete steam generator repair following an SGTR event.

Table 7-5 presents the expected number of tubes requiring repair based on the probability of a tube degrading times the number of tubes that could potentially degrade for the intervals resulting from Table 7-4 manipulation as discussed above. Additional values are given for even longer inspection intervals than required for this problem. The formula used for the likelihood

of a single tube degrading is found in Section 4 as State 2 of the sequential failure Markov model. It is repeated below with the appropriate substitutions.

$$P = 6.17E-7 / (6.17E-7 - 1.69E-8) * \{ \exp(-T*8640.0*1.69E-8) - \exp(-T*8640.0*6.17E-7) \}$$

This probability is multiplied by the number of tubes in the average plant after assigning the appropriate value for T (inspection interval fraction).

All the information is now available for the computation of the expected yearly costs and doses due to various inspection intervals. The results of this computation are shown in Table 7-6. The computation process can best be illustrated by an example. To compute the values shown in the table for the row representing the six year inspection interval with one tube rupture:

- 1) the rupture occurs at the three year mark and leaves a three year interval before the inspection
- 2) in a three year interval, 164.3 tubes will require repair from Table 7-5
- 3) the cost of this repair is \$1.6E+3 per tube plus \$7.75E+5 for inspection of the steam generators
- 4) this yields a repair and inspection cost of \$1.04E+6
- 5) the SGTR expected cost is \$3.60E+7
- 6) the total cost would be \$3.70E+7
- 7) the likelihood of a single tube rupture is .1244 from Table 7-4
- 8) the expected costs over six years is \$4.60E+6
- 9) the average yearly cost is approximately \$7.67E+5 (differs from the result in Table 7-6 due to round-off errors)

The remainder of the table is computed in a similar fashion.

7.4 CONCLUSIONS

The results shown in Table 7-6 are plotted in Figures 7-1 and 7-2 for the expected yearly doses and costs, respectively. It turns out that the optimum inspection interval for dose reduction occurs at the five year mark and the optimum interval from a cost standpoint (excluding dose detriment) occurs at the two year interval. If dose is added to cost using either \$100 or \$1000 per man-rem, the minimum cost still occurs at the two year mark. This is due to the dominance of the costs associated with steam generator inspection, repair and rupture repair costs. Particularly in the case of the rupture repair, the costs

associated with replacement power are extremely large. Dose detriment is not a concern in this particular case for the average plant. Plants with lower rates of tube degradation may have different conclusions due to the reduced likelihood of tube rupture.

It is interesting to note that the industry average inspection interval of 2.5 years is almost the optimum inspection interval. Note however, that the results of this analysis are for an average plant which is heavily influenced by plants with poor steam generator performance. Also, the fact that leaking tubes were not included in this analysis results in the introduction of error. The forced outage associated with a leaky tube would drive expected costs up and lower the costs associated with accidents because of the extra chance for repair of degraded tubes prior to actual rupture. The overall results of the analysis, although not exact, do provide insights into the problem of steam generator inspection interval determination and the trade-offs associated with plant operating costs and occupational exposures.

The occupational dose associated with this problem dominates the total dose. The public dose and occupational dose resulting from an SGTR event are nearly equal but the occupational dose from normal inspection and repair is significantly higher than the accidental dose. Thus, the problem is controlled by the costs and doses associated with steam generator inspection and repair for the normal and accident situations. It is in the best interests of the plant owner to optimize to reduce plant operation costs and personnel exposures.

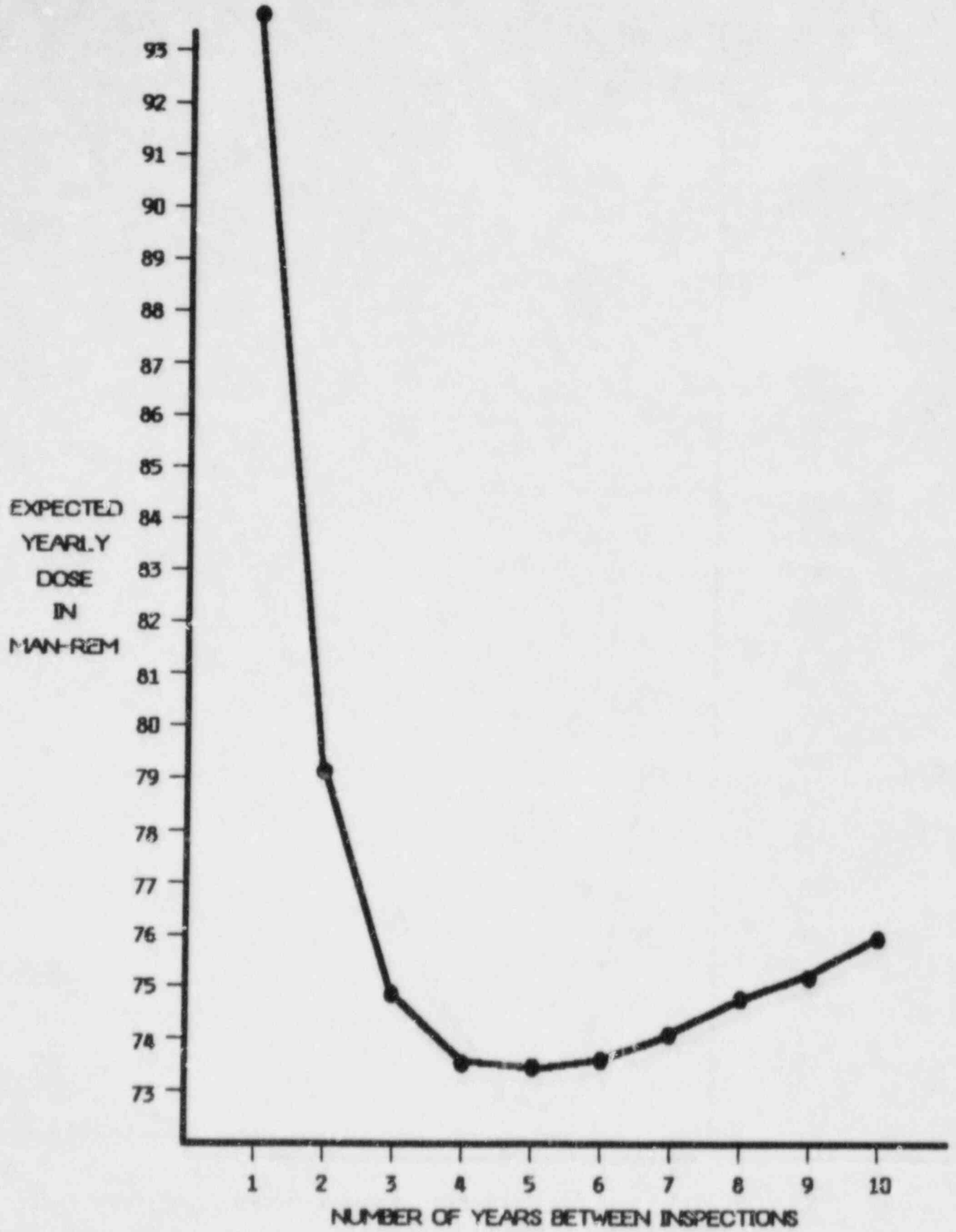


FIGURE 7-1. INSPECTION FREQUENCY VERSUS EXPECTED DOSE

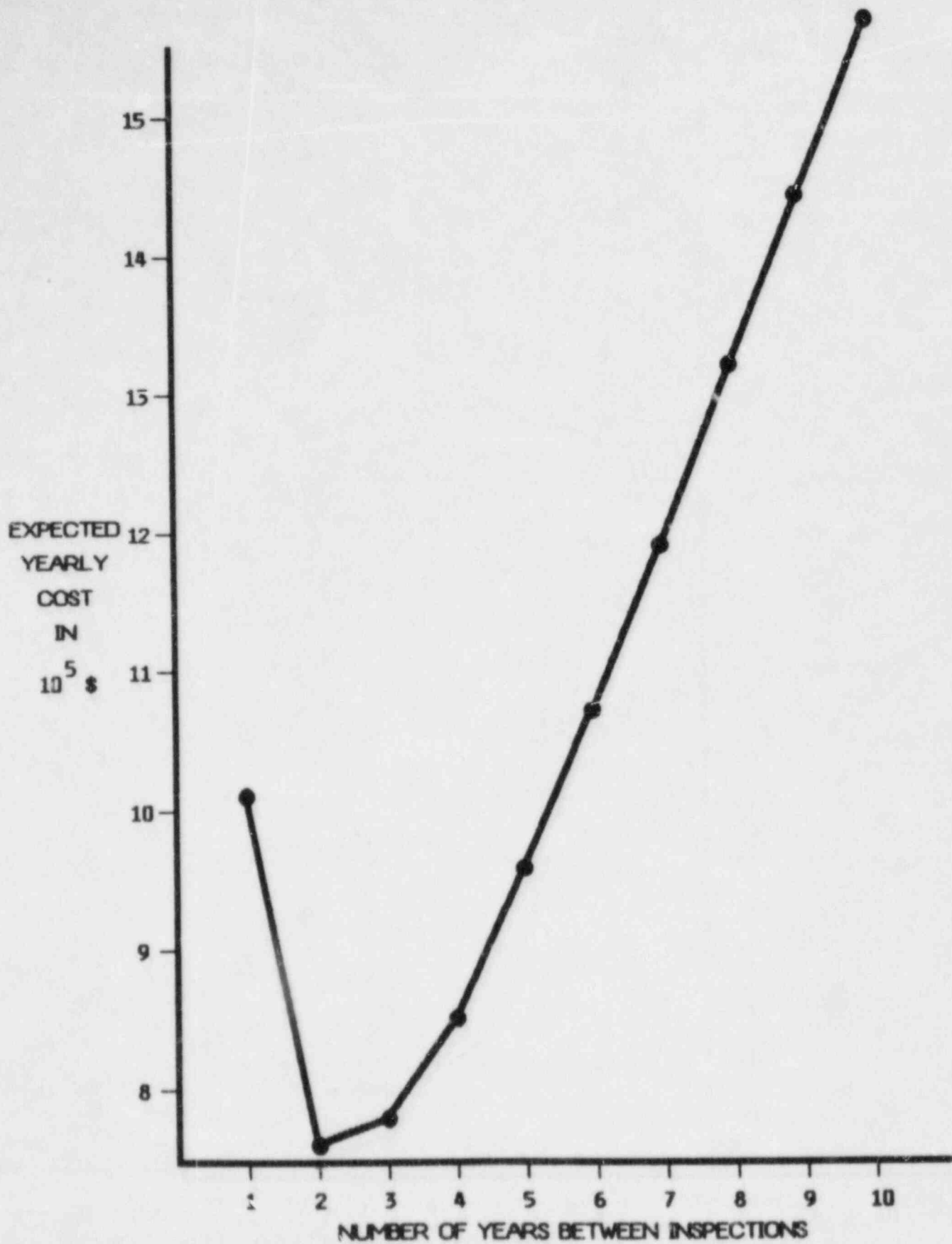


FIGURE 7-2. INSPECTION FREQUENCY VERSUS EXPECTED COST

TABLE 7-1. STEAM GENERATOR EXPERIENCE OF SAMPLED PWR PLANTS

PLANT	NUMBER OF STM GEN	TUBES PER STM GEN	NUMBER OF REPAIRS	OPERATING MONTHS
Yankee Rowe	4	1620	115	210
San Onofre 1	3	3790	2243	154
Haddam Neck	4	4000	71	156
Ginna 1	2	3260	221	140
Robinson	3	3235	1064	134
Point Beach 1	2	3240	767	132
Point Beach 2	2	3240	115	113
Turkey Point 3	3	3180	384	104
Indian Point 2	4	3265	59	95
Surry 1	3	3280	2576	99
Surry 2	3	3280	2154	66
Turkey Point 4	3	3200	400	102
Zion 1	4	3380	25	93
Prairie Island 1	2	3400	13	96
Kewaunee	2	3330	0	77
Zion 2	4	3380	13	94
Prairie Island 2	2	3380	29	83
Cook 1	4	3330	0	81
Trojan	4	3300	42	66
Indian Point 3	4	3260	335	70
Beaver Valley 1	3	3388	0	50
Salem 1	4	3380	30	50
Farley 1	3	3388	8	54
North Anna 1	3	3388	2	22
Cook 2	4	3330	0	46
Farley 2	3	3388	5	8

TABLE 7-2. DATA BASE DEALING WITH SGTR EVENTS FROM
SGTR VALUE-IMPACT ANALYSIS

<u>PWR RELEASE</u> <u>CATEGORY</u>	<u>RELEASE</u> <u>PROBABILITY</u>	<u>RUPTURE</u> <u>REPAIR COST</u>	<u>CLEANUP</u> <u>COST</u>	<u>RUPTURE</u> <u>REPAIR DOSE</u>
3	2.7E-5	\$3.56E+7	\$3.00E+9	350
4	4.5E-5	\$3.56E+7	\$3.00E+9	350
5	2.7E-7	\$3.56E+7	\$3.00E+9	350
8	1.0E-3	\$3.56E+7	\$1.00E+7	350
9	1.5E-2	\$3.56E+7	\$1.00E+7	350
9(1/2)	1.3E-6	\$3.56E+7	\$1.00E+4	350
9(1/10)	4.0E-2	\$3.56E+7	\$1.00E+4	350

TABLE 7-3. DATA BASE DEALING WITH ACCIDENTS
FROM NUREG/CR-2800

PWR RELEASE CATEGORY	ACCIDENT PUBLIC DOSE	REACTOR CLEANUP DOSE	REACTOR REPAIR DOSE
3	5.4E+6	1.2E+4	7.8E+3
4	2.7E+6	1.2E+4	7.8E+3
5	1.0E+6	4.6E+3	3.1E+3
8	7.5E+4	6.7E+2	1.2E+3
9	1.2E+2	--	--
9(1/2)	6.0E+1	--	--
9(1/10)	1.2E+1	--	--

TABLE 7-4. PROBABILITY OF TUBE RUPTURES AT A PLANT
FOR DIFFERENT INSPECTION INTERVALS IN YEARS

INSPECTION INTERVAL (YR)	SINGLE TUBE RUPTURE PROB.	PROBABILITY OF MULTIPLE RUPTURES				
		0	1	2	3	4
1	3.884E-7	.9960	.0040	--	--	--
2	1.551E-6	.9841	.0158	.0001	--	--
3	3.483E-6	.9646	.0348	.0006	--	--
4	6.180E-6	.9380	.0601	.0019	--	--
5	9.639E-6	.9049	.0904	.0045	.0002	--
6	1.385E-5	.8663	.1244	.0089	.0004	--
7	1.882E-5	.8228	.1605	.0157	.0010	--
8	2.454E-5	.7754	.1972	.0251	.0021	.0001
9	3.100E-5	.7252	.2330	.0374	.0040	.0003
10	3.821E-5	.6730	.2665	.0528	.0070	.0007

TABLE 7-5. EXPECTED NUMBER OF TUBE REPAIRS
FOR VARIOUS TIMES BETWEEN INSPECTIONS

<u>TIME</u> <u>INTERVAL(YR)</u>	<u>EXPECTED</u> <u>REPAIRS</u>	<u>TIME</u> <u>INTERVAL(YR)</u>	<u>EXPECTED</u> <u>REPAIRS</u>	<u>TIME</u> <u>INTERVAL(YR)</u>	<u>EXPECTED</u> <u>REPAIRS</u>
.5	27.6	2.5	137.1	5.0	272.4
.67	36.9	2.6	142.6	5.5	299.2
1.0	55.1	2.67	146.4	6.0	326.0
1.25	68.8	2.75	150.7	6.5	352.7
1.33	73.2	2.8	153.5	7.0	379.3
1.5	82.5	3.0	164.3	7.5	405.8
1.6	88.0	3.25	177.9	8.0	432.3
1.67	91.8	3.33	182.2	9.0	485.0
1.75	96.2	3.5	191.5	10.0	537.4
1.8	98.9	3.67	200.7	11.0	589.6
2.0	109.9	3.75	205.0	12.0	641.4
2.2	120.8	4.0	218.5	13.0	693.0
2.25	123.5	4.33	236.3	14.0	744.3
2.33	127.9	4.5	245.5	15.0	795.3
2.4	131.7	4.67	254.6		

TABLE 7-6. EXPECTED COSTS AND DOSES FOR VARIOUS INSPECTION INTERVALS

YEARS BETWEEN STM GEN INSPECT	NUMBER OF RUPTURES IN INT.	EXPECTED COSTS PER YEAR FOR RUPTURE #	EXPECTED TOTAL COSTS PER YEAR	EXPECTED DOSE PER YEAR FOR RUPTURE #	EXPECTED TOTAL DOSE PER YEAR
1	0	\$8.60E+5		92.5	
	1	\$1.47E+5		3.1	
			\$1.01E+6		95.6
2	0	\$4.68E+5		72.7	
	1	\$2.91E+5		6.3	
	2	\$3.64E+3		.1	
			\$7.63E+5		79.1
3	0	\$3.34E+5		65.0	
	1	\$4.28E+5		9.5	
	2	\$1.46E+4		.3	
			\$7.76E+5		74.8
4	0	\$2.64E+5		60.1	
	1	\$5.55E+5		12.7	
	2	\$3.46E+4		.7	
			\$8.54E+5		73.5
5	0	\$2.19E+5		56.1	
	1	\$6.69E+5		15.8	
	2	\$6.56E+4		1.4	
	3	\$4.36E+3		.1	
			\$9.58E+5		73.4
6	0	\$1.87E+5		52.5	
	1	\$7.68E+5		18.7	
	2	\$1.08E+5		2.3	
	3	\$7.26E+3		.1	
			\$1.07E+6		73.6
7	0	\$1.62E+5		49.0	
	1	\$8.50E+5		21.3	
	2	\$1.64E+5		3.5	
	3	\$1.56E+4		.3	
			\$1.19E+6		74.1
8	0	\$1.42E+5		45.6	
	1	\$9.15E+5		23.5	
	2	\$2.29E+5		5.0	
	3	\$2.86E+4		.6	
	4	\$1.81E+3		0	
			\$1.32E+6		74.7

TABLE 7-6. EXPECTED COSTS AND DOSES FOR VARIOUS INSPECTION INTERVALS (CONT'D)

YEARS BETWEEN STM GEN INSPECT	NUMBER OF RUPTURES IN INT.	EXPECTED COSTS PER YEAR FOR RUPTURE #	EXPECTED TOTAL COSTS PER YEAR	EXPECTED DOSE PER YEAR FOR RUPTURE #	EXPECTED TOTAL DOSE PER YEAR
9	0	\$1.25E+5		42.1	
	1	\$9.62E+5		25.4	
	2	\$3.04E+5		6.6	
	3	\$4.84E+4		1.0	
	4	\$4.83E+3		.1	
			\$1.44E+6		75.2
10	0	\$1.10E+5		38.7	
	1	\$9.92E+5		26.9	
	2	\$3.86E+5		8.5	
	3	\$7.63E+4		1.6	
	4	\$1.02E+4		.2	
			\$1.57E+6		75.9

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APPENDIX. DEVELOPMENT OF THE PRIMARY MARKOV MODEL

The purpose of this appendix is to provide the technical details associated with the development of the Primary Model. Similar details are not provided for the Supporting Model and the Sequential Failure Model discussed in Section 4. However, the approach is the same for the development of all the Markov models presented in this report. For this reason, duplication of the development process would not benefit the user of the methods. Confirmation or validation of the models which are not developed in this appendix would require additional documentation or extensive effort on the part of the reviewer. It is not felt that in-depth validation is necessary for this report at this point in time. Validation of the Primary Model material that is to be presented would, by itself, be extremely time consuming and tedious. However, the information is provided for the purpose of validation as well as to demonstrate the complexity and the effort involved in the development of the models in this report.

The appendix assumes that the reader is familiar with the description of the Primary Model already given in Section 4 of the report. Many of the details of the model are not repeated in this discussion unless expansion of a particular aspect of the model is necessary for better understanding of the development process and model assumptions.

The Primary Model is used to evaluate the system shown in Figure A-1. This system is a generalization of many of the systems found in nuclear power plants. The purpose of the Markov model is to aid in the evaluation of the impact of inspection or testing on system availability. The system shown in the figure is developed with this objective in mind. The actual piping of most systems in nuclear power plants does not match the simplistic representation shown in Figure A-1. However, the manner in which they are inspected can be grossly modeled using the provided system which is composed of simple redundant legs with some common elements. Since the methodology is primarily intended for comparative analyses, the inaccuracies created by the simplified model should not impact the decision process significantly.

As presented in the figure, the model is capable of being manipulated to analyze system configurations that are simpler than the general model. This was another objective of the model development; to provide as diverse a tool as possible within the limitations of the development time and costs. Thus, three general system configurations are capable of being modeled using the Primary Model.

Markov models are defined by a set of system conditions or states. This set must be mutually exclusive and collectively exhaustive. This means that the system can never be in two states at the same time and that the system can never not be in a state within the framework of the model. No other states exist within the model framework and each state is distinct. This does not mean that another set of states (i.e., a different model development) does not exist.

The first step in the Markov model development process is to define the system model states. The state definition for the Primary Model is shown in Table A-1. The model is composed of 15 states which represent various combinations of the failures of the system's legs. There is no distinction made by this model between the two redundant legs of the system. A detectable failure in one of the redundant legs is no different to the model than a detectable failure in the other redundant leg. It is simply treated as a detectable failure in a redundant leg. The model complexity increases significantly if a distinction between redundant legs is modeled.

State 15 of the model is an adsorption state in this analysis. Once the system reaches state 15, there is no recovery possible. This is conservative but should not impact the results of the analysis of systems which are well designed and maintained. The likelihood of reaching state 15 should be very small for most applications. Note that the state is a combination of two basic sets of failures. State 15 is either an undetectable failure in the common leg or an undetectable failure in both of the redundant legs. Detectable failure status does not matter in this particular state because of the assumption that state 15 is an adsorption state with no recovery.

The next step in the model development process is to define the transitions that can occur between states and to determine the rates of the transitions. Markov analysis assumes that this transition system behaves as a Markovian process. This process assumption would indicate that the likelihood of leaving any state is independent of the manner in which the state was entered in the first place. A simple example of a Markovian process is the random walk. Given that the individual is at a certain location, the likelihood of the person going left is always the same, no matter how the individual got to the present location. This type of assumption is generally true in nuclear power system analysis. Components whose failure rates change as a function of time would not be modeled accurately under this assumption unless the model was incrementally solved with appropriate alteration of the component failure rate. A case which might not fit this model assumption would be the evaluation of a component which when repaired would not behave the same as a new component. This type of behavioral change of components was not usually modeled in past PRAs and this model also does not accommodate the behavior. In summary, the assumption of a Markovian process does not differ significantly from past analysis assumptions dealing with static models using fault tree analysis techniques.

Another assumption associated with a Markov analysis is that failures occur one at a time. Simultaneous independent failures cannot occur in the model. This does not exclude common mode failures which can be modeled using Markov techniques. This does exclude the transition from one state to another state which has more than one additional failure. Thus, transition from a state with one failure to a state with three or more failures cannot occur unless the transition is due to a common mode failure mechanism. This assumption has little or no impact on the analysis but is important in defining the possible transitions.

Figure A-2 is a graphical representation of the possible transitions associated with the Primary Model. All transitions are shown except for the transitions to state 15. All states in the model can have a transition to state 15. The figure indicates that the model is basically a two tiered system. Transitions among states 1 through 7 are mirrored by transitions among states 8 through 14. Each state in the upper group has an additional transition to its corresponding state

in the lower group. That transition results from the occurrence of a detectable failure in the common leg of the system. The figure indicates, for example, that state 2, which represents a detectable failure in a redundant leg, can progress to state 3, a detectable failure in both redundant legs, to state 5, an undetectable failure in the same redundant leg as the detectable failure, to state 6, an undetectable failure in the redundant leg which does not have the detectable failure, and to state 9, detectable failures in the common leg and one redundant leg. Other state transitions can be determined by examining the figure and Table A-1.

The transitions shown in Figure A-2 are tabulated in Table A-2 along with the rate of transition. The table shows the starting state and the ending state and a number or numbers under columns marked with the four input failure rates. These numbers reflect the coefficients of the column failure rates which yield the rate of transition between the two states. For example, the table shows that state 1 goes to state 2 with a rate of $2D$, or two times the rate of detectable failures occurring in redundant legs. This is because state 1 has no failures and there are two redundant legs, each of which can fail in a detectable manner. Thus the rate of detectable failures in the redundant legs is double the rate of detectable failures in a single leg. Note that the transition of state 2 to state 3 is only a rate D . One detectable failure has already occurred in state 2 and the remaining detectable failure of a redundant leg occurs at the normal rate since there is only one leg left to fail. The transition from state 5 to state 15 occurs with rate $A + C$. State 5 has an undetectable failure in one of the redundant legs and total system failure as represented by state 15 can occur if an undetectable failure occurs in the common leg with rate A or if an undetectable failure occurs in the remaining redundant leg with rate C . Note that the table gives no transitions associated with state 15 to anything else. This is due to the assumption that state 15 is an adsorption state as indicated earlier.

Once the transitions for the model are defined as functions of the failure rates of the system legs, it is possible to write a set of differential equations which characterize the system model probability structure. This set of differential equations is presented in Table A-3. The table indicates that the rate at which a state changes can be shown as the difference between the rate at which other state transitions to the state occur and the rate at which transitions occur from the state. For example, the rate of change in state 7, $P7'$, is found by summing the rates of transitions to state 7 from state 3, $2C \cdot P3$, state 5, $D \cdot P5$, and state 6, $D \cdot P6$, and subtracting the rate of transition from state 7, $(A+B+C) \cdot P7$. The " Px "s signify the probability of being in state x .

The formulation of Table A-3 equations is a result of the manipulation of the information in Table A-2. Using the example from the preceding paragraph, the rates of transition to state 7 can be found by looking under the TO column of Table A-2 for any entries that are sevens. There are three found; one in the first column set in the row for state 3 with rate $2C$ and two in the second column set for rows corresponding to transitions from states 5 and 6 each with rates of D . The rate of transition from state 7 is found by summing all the rate entries for any rows with a seven in the FROM column of the table.

Often the model development is ended at this point and the information presented in Table A-3 is coded as input into an iterative computer program for solving simultaneous linear differential equations. This is fine if you don't mind having to run a code every time you want an analysis result and don't mind the

expense associated with any iterative computer procedure. Also, the results are not exact. There is some error associated with the process and this must be reduced to acceptable levels by decreasing the time step increment of the code and thus lengthening the analysis runtime and cost.

An alternative to the above iterative process is to find the closed form solution of the differential equations. This can be done by the usage of Laplace transforms. Two transforms are needed for the equations shown in Table A-3. They deal with the transform of a variable and the transform of the differential of the variable. The transform formulas are:

$$L(a \cdot Px) = a \cdot Xx$$

$$L(Px') = Xx \cdot S - Kx$$

where

$L(-)$ is the Laplace transform function

a is a constant

Kx is a constant associated with the transform

Xx is the transform of the Px variable

Applying the above two formulas to the differential equations in Table A-3 yields the results shown in Table A-4. The procedure for evaluation does not require the determination of a solution equation for every variable since the final result of the probabilities must add to one. Thus, one variable can be excluded and the probability of that term can be computed by subtracting the sum of the other variable probabilities from one. The variable chosen for exclusion is $X15$.

Once the $X15$ variable is excluded, the remaining equations can have a substitution of variables as a simplification process. The " $S + A$ " term that would appear in every equation is replaced by " W ". The resulting secondary transform equations are shown in Table A-5. The next step is to solve the equations simultaneously for the " Xx " variables. This process starts by solving for $X1$ which is readily accomplished by dividing both sides of the first equation by $(W + B + 2C + 2D)$. The next step is to substitute the value of $X1$ in the second equation and solve for $X2$. This process is continued until the solution to $X14$ is obtained. The resulting equations are displayed in two sets of tables. The first set shows the numerator terms of the solution and the second table shows the denominator terms of the solution equations. The numerator terms are shown in Table A-6. As shown in the first page of the table, the numerator of the $X1$ solution equation is $K1$. The table shows the coefficient of the Kx values in the numerator and the powers of the W , B , C , and D terms. As an example, the numerator term for the $X2$ solution, shown in the second page of Table A-6, can be written as:

$$K2 \cdot W + K2 \cdot B + 2K2 \cdot C + (2K1 + 2K2) \cdot D$$

The numerator terms for the X3 solution which are shown in the third page of the table can be written as:

$$\begin{aligned}
 &K3*W^2 + [2K3*B + 4K3*C + (K2 + 3K3)*D]*W \\
 &+ K3*B^2 + 4K3*B*C + (K2 + 3K3)*B*D + 4K3*C^2 \\
 &+ (2K2 + 6K3)*C*D + (2K1 + 2K2 + 2K3)*D^2
 \end{aligned}$$

As can be seen, these formulas become larger as each row in Table A-5 is addressed. The numerator terms for the solution for X14 take 24 pages of tables to express. This gives some indication of the reasoning behind stopping at the earlier point and finding computer solutions.

The denominator terms for the equations are made up of combinations of the parenthetical expressions found in Table A-5. Table A-7 displays the term combinations graphically. The parenthetical expressions are shown across the top of the table. The first column stands for $(W + B + 2C + 2D)$. The final column stands for $(W + C)$. Remaining columns follow the same pattern of formulation. If the column is darkened in the row of a particular variable, the expression at the top of the column is found in the denominator of the solution equation for the chosen variable. Thus, X9 has four terms in the denominator and X14 has 12 terms.

The next step is to find the inverse Laplace transform of the identified equations for each variable. This is done by finding the coefficient over each of parenthetical expressions in the denominator of the equation that when recombined would give the numerator term. For example, the equation for X2 has two terms in the denominator; $(W + B + 2C + 2D)$ and $(W + B + 2C + D)$. The numerator terms are $K2*W + K2*B + 2K2*C + (2K1 + 2K2)*D$ as shown before. The solution process finds the values of the "T"s in the following equation:

$$\begin{aligned}
 &T1*(W + B + 2C + D) + T2*(W + B + 2C + 2D) \\
 &= \\
 &K2*W + K2*B + 2K2*C + (2K1 + 2K2)*D
 \end{aligned}$$

Substituting $W = -B - 2C - 2D$ yields:

$$T1*(-D) = 2K1*D$$

Thus $T1 = -2K1$. Substituting $W = -B - 2C - D$ yields:

$$T2*D = (2K1 + K2)*D$$

Thus $T2 = (2K1 + K2)$. The inverse Laplace transform can be performed on the formulas:

$$-2K1/(W + B + 2C + 2D)$$

$$(2K1 + K2)/(W + B + 2C + D)$$

The inverse Laplace transform of interest is:

$$L^{-1}(a/(S + b)) = a \cdot e^{-bT}$$

Therefore, the inverse Laplace of the sum of the above two formulas is as follows:

$$-2K1 \cdot e^{-(A + B + 2C + 2D)T} + (2K1 + K2) \cdot e^{-(A + B + 2C + D)T}$$

This result is the solution to the P2 probability equation where the "Kx"s are the initial probabilities of the x states. The remaining probability expressions are found in the same manner and are displayed in Table A-8. The format for the table has already been described in Section 4.

This concludes the discussion of the development process for the Primary Model. The determination of the final equations was a long involved process and required great patience and painstaking care. It is hoped that the effort produces a product which is utilized by those interested in system availability and are willing to try new techniques for system analysis.

FIGURE A-1. PRIMARY MARKOV MODEL SYSTEM DIAGRAM

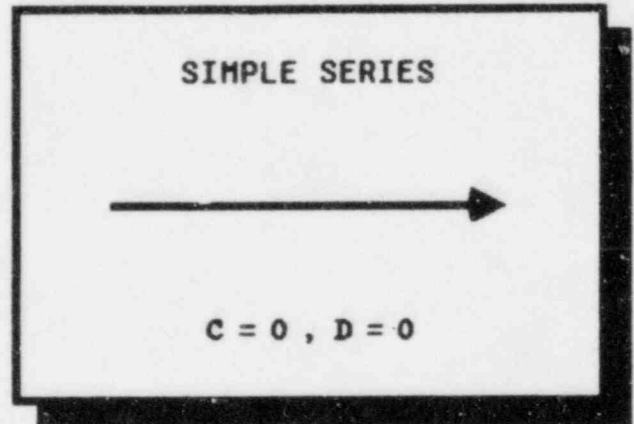
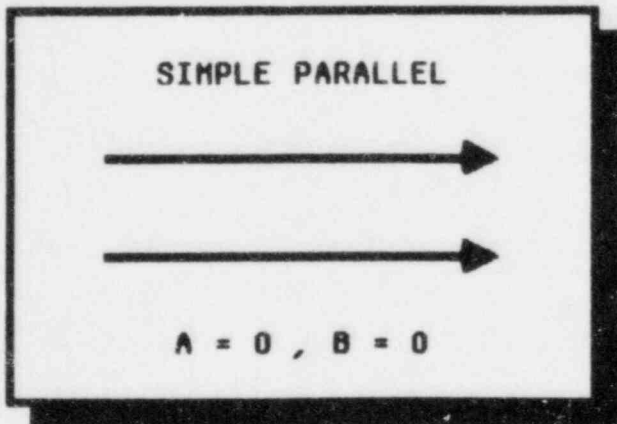
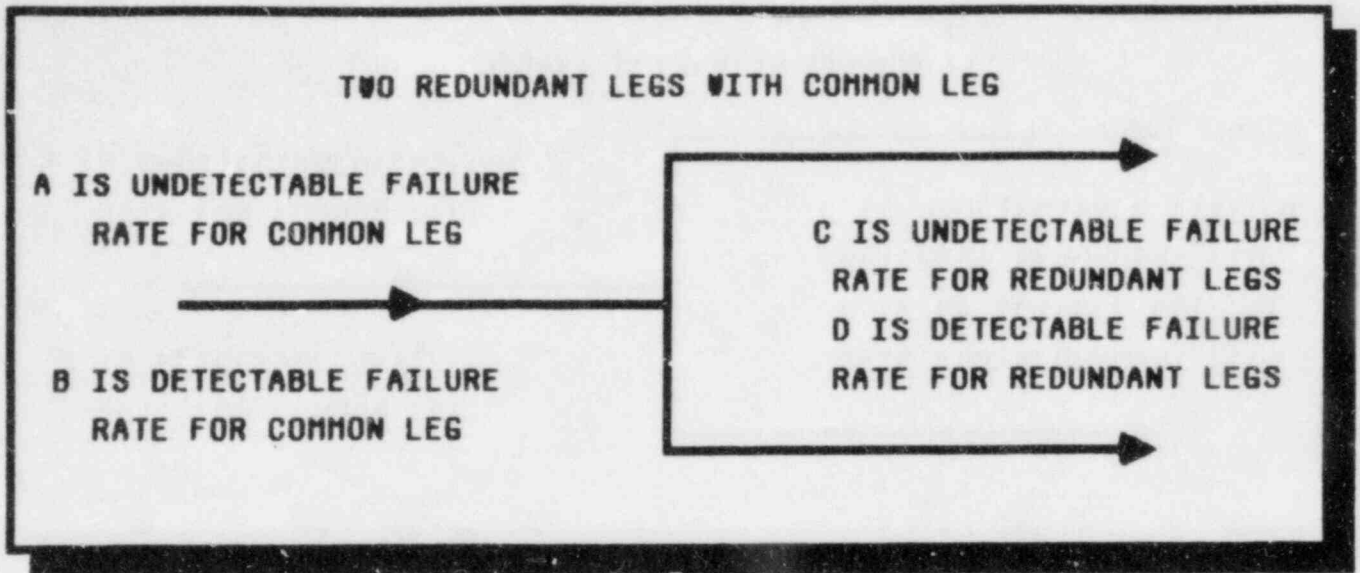


FIGURE A-2. TRANSITION DIAGRAM FOR PRIMARY MODEL
(EXCLUDES STATE 15)

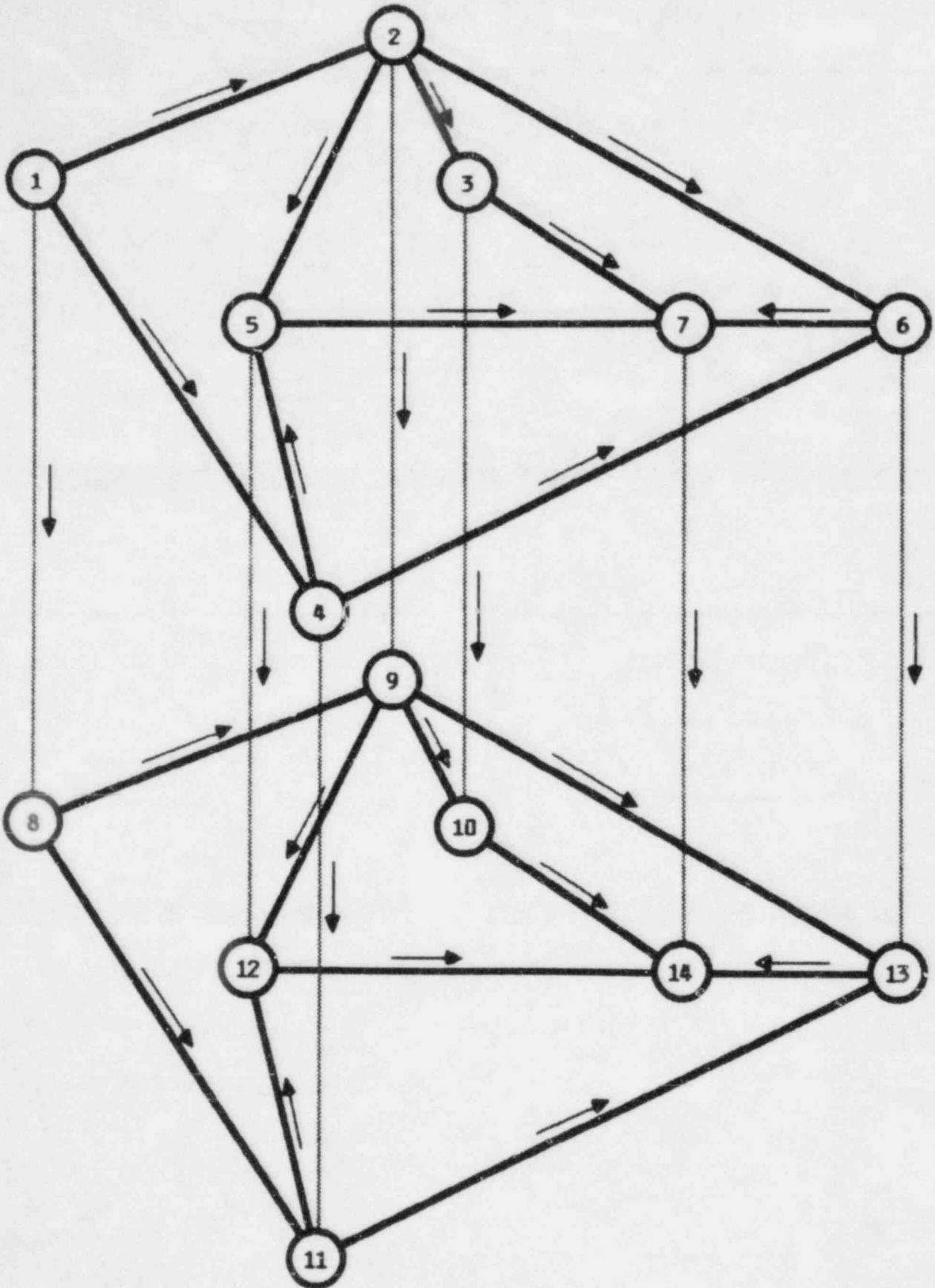


TABLE A-1. PRIMARY MODEL STATES

PRIMARY MARKOV MODEL STATE DEFINITION						
STATE	SYSTEM CONDITIONS					
	COMMON		ONE TRAIN		OTHER TRAIN	
	DETECTABLE	UNDETECTABLE	DETECTABLE	UNDETECTABLE	DETECTABLE	UNDETECTABLE
1	NO	NO	NO	NO	NO	NO
2	NO	NO	YES	NO	NO	NO
3	NO	NO	YES	NO	YES	NO
4	NO	NO	NO	YES	NO	NO
5	NO	NO	YES	YES	NO	NO
6	NO	NO	YES	NO	NO	YES
7	NO	NO	YES	YES	YES	NO
8	YES	NO	NO	NO	NO	NO
9	YES	NO	YES	NO	NO	NO
10	YES	NO	YES	NO	YES	NO
11	YES	NO	NO	YES	NO	NO
12	YES	NO	YES	YES	NO	NO
13	YES	NO	YES	NO	NO	YES
14	YES	NO	YES	YES	YES	NO
15	--	YES	--	YES	--	YES

TABLE A-2. TRANSITION RATES FOR PRIMARY MODEL

FROM	TO	A	B	C	D	FROM	TO	A	B	C	D	FROM	TO	A	B	C	D
1	2				2	4	11		1			9	12				1
1	4			2		4	15	1		1		9	13				1
1	8		1			5	7				1	9	15	1			
1	15	1				5	12		1			10	14				2
2	3				1	5	15	1		1		10	15	1			
2	5			1		6	7				1	11	12				1
2	6			1		6	13		1			11	13				1
2	9		1			6	15	1		1		11	15	1			1
2	15	1				7	14		1			12	14				1
3	7			2		7	15	1		1		12	15	1			1
3	10		1			8	9				2	13	14				1
3	15	1				8	11				2	13	15	1			1
4	5				1	8	15	1				14	15	1			1
4	6				1	9	10				1						

TABLE A-3. DIFFERENTIAL EQUATIONS FOR THE PRIMARY MODEL

$$\begin{aligned}
 P1' &= -(A + B + 2C + 2D)P1 \\
 P2' &= 2D \cdot P1 - (A + B + 2C + D)P2 \\
 P3' &= D \cdot P2 - (A + B + 2C)P3 \\
 P4' &= 2C \cdot P1 - (A + B + C + 2D)P4 \\
 P5' &= C \cdot P2 + D \cdot P4 - (A + B + C + D)P5 \\
 P6' &= C \cdot P2 + D \cdot P4 - (A + B + C + D)P6 \\
 P7' &= 2C \cdot P3 + D \cdot P5 + D \cdot P6 - (A + B + C)P7 \\
 P8' &= B \cdot P1 - (A + 2C + 2D)P8 \\
 P9' &= B \cdot P2 + 2D \cdot P8 - (A + 2C + D)P9 \\
 P10' &= B \cdot P3 + D \cdot P9 - (A + 2C)P10 \\
 P11' &= B \cdot P4 + 2C \cdot P8 - (A + C + 2D)P11 \\
 P12' &= B \cdot P5 + C \cdot P9 + D \cdot P11 - (A + C + D)P12 \\
 P13' &= B \cdot P6 + C \cdot P9 + D \cdot P11 - (A + C + D)P13 \\
 P14' &= B \cdot P7 + 2C \cdot P10 + D \cdot P12 + D \cdot P13 - (A + C)P14 \\
 P15' &= A \cdot P1 + A \cdot P2 + A \cdot P3 + (A + C)P4 + (A + C)P5 \\
 &\quad + (A + C)P6 + (A + C)P7 + A \cdot P8 + A \cdot P9 + A \cdot P10 \\
 &\quad + (A + C)P11 + (A + C)P12 + (A + C)P13 + (A + C)P14
 \end{aligned}$$

TABLE A-4. INITIAL TRANSFORM EQUATIONS FOR PRIMARY MODEL

$$\begin{aligned}
 X1 \cdot S + (A + B + 2C + 2D)X1 &= K1 \\
 X2 \cdot S + (A + B + 2C + D)X2 &= K2 + 2D \cdot X1 \\
 X3 \cdot S + (A + B + 2C)X3 &= K3 + D \cdot X2 \\
 X4 \cdot S + (A + B + C + 2D)X4 &= K4 + 2C \cdot X1 \\
 X5 \cdot S + (A + B + C + D)X5 &= K5 + C \cdot X2 + D \cdot X4 \\
 X6 \cdot S + (A + B + C + D)X6 &= K6 + C \cdot X2 + D \cdot X4 \\
 X7 \cdot S + (A + B + C)X7 &= K7 + 2C \cdot X3 + D \cdot X5 + D \cdot X6 \\
 X8 \cdot S + (A + 2C + 2D)X8 &= K8 + B \cdot X1 \\
 X9 \cdot S + (A + 2C + D)X9 &= K9 + B \cdot X2 + 2D \cdot X8 \\
 X10 \cdot S + (A + 2C)X10 &= K10 + B \cdot X3 + D \cdot X9 \\
 X11 \cdot S + (A + C + 2D)X11 &= K11 + B \cdot X4 + 2C \cdot X8 \\
 X12 \cdot S + (A + C + D)X12 &= K12 + B \cdot X5 + C \cdot X9 + D \cdot X11 \\
 X13 \cdot S + (A + C + D)X13 &= K13 + B \cdot X6 + C \cdot X9 + D \cdot X11 \\
 X14 \cdot S + (A + C)X14 &= K14 + B \cdot X7 + 2C \cdot X10 + D \cdot X12 + D \cdot X13 \\
 X15 \cdot S &= K15 + A \cdot X1 + A \cdot X2 + A \cdot X3 + (A + C)X4 + (A + C)X5 \\
 &+ (A + C)X6 + (A + C)X7 + A \cdot X8 + A \cdot X9 + A \cdot X10 \\
 &+ (A + C)X11 + (A + C)X12 + (A + C)X13 + (A + C)X14
 \end{aligned}$$

TABLE A-5. ALTERNATE TRANSFORM EQUATIONS FOR THE PRIMARY MODEL

$$\begin{aligned}
 (W + B + 2C + 2D)X_1 &= K_1 \\
 (W + B + 2C + D)X_2 &= K_2 + 2D \cdot X_1 \\
 (W + B + 2C)X_3 &= K_3 + D \cdot X_2 \\
 (W + B + C + 2D)X_4 &= K_4 + 2C \cdot X_1 \\
 (W + B + C + D)X_5 &= K_5 + C \cdot X_2 + D \cdot X_4 \\
 (W + B + C + D)X_6 &= K_6 + C \cdot X_2 + D \cdot X_4 \\
 (W + B + C)X_7 &= K_7 + 2C \cdot X_3 + D \cdot X_5 + D \cdot X_6 \\
 (W + 2C + 2D)X_8 &= K_8 + B \cdot X_1 \\
 (W + 2C + D)X_9 &= K_9 + B \cdot X_2 + 2D \cdot X_8 \\
 (W + 2C)X_{10} &= K_{10} + B \cdot X_3 + D \cdot X_9 \\
 (W + C + 2D)X_{11} &= K_{11} + B \cdot X_4 + 2C \cdot X_8 \\
 (W + C + D)X_{12} &= K_{12} + B \cdot X_5 + C \cdot X_9 + D \cdot X_{11} \\
 (W + C + D)X_{13} &= K_{13} + B \cdot X_6 + C \cdot X_9 + D \cdot X_{11} \\
 (W + C)X_{14} &= K_{14} + B \cdot X_7 + 2C \cdot X_{10} + D \cdot X_{12} + D \cdot X_{13}
 \end{aligned}$$

TABLE A-6. NUMERATOR TERMS OF TRANSFORMED EQUATION SOLUTIONS (CONT'D)

NUMERATOR TERMS FOR X5 SOLUTION					PAGE 1 OF 1					
POWERS				COEFFICIENTS						
W	B	C	D	K1	K2	K3	K4	K5	K6	K7
3								1		
*	*	*	*							
2	1							3		
		1			1			5		
			1				1	5		
*	*	*	*							
1	2							3		
	1	1			2			10		
	1		1				2	10		
		2			3			8		
		1	1	4	4		4	17		
			2				3	8		
*	*	*	*							
-	3							1		
	2	1			1			5		
	2		1				1	5		
	1	2			3			8		
	1	1	1	4	4		4	17		
	1		2				3	8		
		3			2			4		
		2	1	6	6		4	14		
		1	2	6	4		6	14		
			3				2	4		
*	*	*	*							

TABLE A-6. NUMERATOR TERMS OF TRANSFORMED EQUATION SOLUTIONS (CONT'D)

NUMERATOR TERMS FOR X6 SOLUTION								PAGE 1 OF 1			
POWERS				COEFFICIENTS							
W	B	C	D	K1	K2	K3	K4	K5	K6	K7	
3									1		
*	*	*	*								
2	1								3		
		1			1				5		
			1				1		5		
*	*	*	*								
1	2								3		
	1	1			2				10		
	1		1				2		10		
		2			3				8		
		1	1	4	4		4		17		
			2				3		8		
*	*	*	*								
-	3								1		
	2	1			1				5		
	2		1				1		5		
	1	2			3				8		
	1	1	1	4	4		4		17		
	1		2				3		8		
		3			2				4		
		2	1	6	6		4		14		
		1	2	6	4		6		14		
			3				2		4		
*	*	*	*								

TABLE A-6. NUMERATOR TERMS OF TRANSFORMED EQUATION SOLUTIONS (CONTD)

NUMERATOR TERMS FOR X7 SOLUTION					PAGE 1 OF 3						
POWERS				COEFFICIENTS							
W	B	C	D	K1	K2	K3	K4	K5	K6	K7	
5										1	
*	*	*	*								
4	1									5	
		1				2				8	
			1					1	1	6	
*	*	*	*								
3	2									10	
	1	1				8				32	
	1		1					4	4	24	
		2				12				25	
		1	1		4	12		7	7	39	
			2				2	5	5	13	
*	*	*	*								
2	3									10	
	2	1				12				48	
	2		1					6	6	36	
	1	2				36				75	
	1	1	1		12	36		21	21	117	
	1		2				6	15	15	39	
		3				26				38	
		2	1			18	54	18	18	93	
		1	2		12	18	26	12	27	27	65
			3					6	8	8	12
*	*	*	*								
1	4									5	
	3	1				8				32	
	3		1					4	4	24	
	2	2				36				4	

TABLE A-6. NUMERATOR TERMS OF TRANSFORMED EQUATION SOLUTIONS (CONT'D)

NUMERATOR TERMS FOR X7 SOLUTION (CONTINUED)								PAGE 2 OF 3		
POWERS				COEFFICIENTS						
W	B	C	D	K1	K2	K3	K4	K5	K6	K7
1	2	1	1		12	36		21	21	117
	2		2				6	15	15	39
	1	3				52				76
	1	2	1		36	108		36	36	186
	1	1	2	24	36	52	24	54	54	130
	1		3				12	16	16	24
		4				24				28
		3	1		26	78		20	20	96
		2	2	36	54	78	24	48	48	106
		1	3	24	24	24	24	30	30	42
			4				4	4	4	4
*	*	*	*							
-	5									1
	4	1				2				8
	4		1					1	1	6
	3	2				12				25
	3	1	1		4	12		7	7	39
	3		2				2	5	5	13
	2	3				26				38
	2	2	1		18	54		18	18	93
	2	1	2	12	18	26	12	27	27	65
	2		3				6	8	8	12
	1	4				24				28
	1	3	1		26	78		20	20	96
	1	2	2	36	54	78	24	48	48	106
	1	1	3	24	24	24	24	30	30	42
	1		4				4	4	4	4
		5				8				3

TABLE A-6. NUMERATOR TERMS OF TRANSFORMED EQUATION SOLUTIONS (CONT'D)

NUMERATOR TERMS FOR X10 SOLUTION								PAGE 1 OF 4			
POWERS				COEFFICIENTS							
W	B	C	D	K1	K2	K3	K4	K5	K6	K7	
5											
*	*	*	*								
4	1					1					
		1									
			1								
*	*	*	*								
3	2					2					
	1	1				8					
	1		1			2	6				
		2									
		1	1								
			2								
*	*	*	*								
2	3					1					
	2	1				12					
	2		1			3	9				
	1	2				24					
	1	1	1			12	36				
	1		2		6	9	13				
		3									
		2	1								
		1	2								
			3								
*	*	*	*								
1	3	1				4					
	3		1			1	3				
	2	2					24				
	2	1	1			12	36				

TABLE A-6. NUMERATOR TERMS OF TRANSFORMED EQUATION SOLUTIONS (CONT'D)

NUMERATOR TERMS FOR X10 SOLUTION								PAGE 2 OF 4		
POWERS				COEFFICIENTS						
W	B	C	D	K1	K2	K3	K4	K5	K6	K7
1	2		2	6	9	13				
	1	3				32				
	1	2	1		24	72				
	1	1	2	24	36	52				
	1		3	12	12	12				
		4								
		3	1							
		2	2							
		1	3							
			4							
*	*	*	*							
-	3	2				4				
	3	1	1		2	6				
	3		2	2	2	2				
	2	3				16				
	2	2	1		12	36				
	2	1	2	12	18	26				
	2		3	6	6	6				
	1	4				16				
	1	3	1		16	48				
	1	2	2	24	36	52				
	1	1	3	24	24	24				
	1		4	4	4	4				
		5								
		4	1							
		3	2							
		2	3							
		1	4							

TABLE A-6. NUMERATOR TERMS OF TRANSFORMED EQUATION SOLUTIONS (CONT'D)

NUMERATOR TERMS FOR X10 SOLUTION								PAGE 3 OF 4			
POWERS				COEFFICIENTS							
W	B	C	D	K8	K9	K10	K11	K12	K13	K14	
5						1					
*	*	*	*								
4	1					3					
		1				10					
			1		1	6					
*	*	*	*								
3	2					3					
	1	1				24					
	1		1		3	15					
		2				40					
		1	1		8	48					
			2	2	5	13					
*	*	*	*								
2	3					1					
	2	1				18					
	2		1		3	12					
	1	2				72					
	1	1	1		18	90					
	1		2	6	12	26					
		3				80					
		2	1		24	144					
		1	2	12	30	78					
			3	6	8	12					
*	*	*	*								
1	3	1				4					
	3		1		1	3					
	2	2				36					
	2	1	1		12	48					

TABLE A-6. NUMERATOR TERMS OF TRANSFORMED EQUATION SOLUTIONS (CONT'D)

NUMERATOR TERMS FOR X10 SOLUTION								PAGE 4 OF 4			
POWERS				COEFFICIENTS							
W	B	C	D	K8	K9	K10	K11	K12	K13	K14	
1	2		2	6	9	15					
	1	3				96					
	1	2	1		36	180					
	1	1	2	24	48	104					
	1		3	12	14	18					
		4				90					
		3	1		32	192					
		2	2	24	60	156					
		1	3	24	32	48					
			4	4	4	4					
*	*	*	*								
-	3	2				4					
	3	1	1		2	6					
	3		2	2	2	2					
	2	3				24					
	2	2	1		12	48					
	2	1	2	12	18	30					
	2		3	6	6	6					
	1	4				48					
	1	3	1		24	120					
	1	2	2	24	48	104					
	1	1	3	24	28	36					
	1		4	4	4	4					
		5				32					
		4	1		16	96					
		3	2	16	40	104					
		2	3	24	32	48					
		1	4	6	8	8					

TABLE A-6. NUMERATOR TERMS OF TRANSFORMED EQUATION SOLUTIONS (CONTD)

NUMERATOR TERMS FOR X12 SOLUTION										PAGE 1 OF 10	
POWERS				COEFFICIENTS							
W	B	C	D	K1	K2	K3	K4	K5	K6	K7	
7											
*	*	*	*								
6	1							1			
		1									
			1								
*	*	*	*								
5	2							3			
	1	1			2			10			
	1		1				2	10			
		2									
		1	1								
			2								
*	*	*	*								
4	3							3			
	2	1			5			25			
	2		1				5	25			
	1	2			15			41			
	1	1	1	12	18		18	84			
	1		2				15	41			
		3									
		2	1								
		1	2								
			3								
*	*	*	*								
3	4							1			
	3	1			4			20			
	3		1				4	20			
	2	2			30			82			

TABLE A-6. NUMERATOR TERMS OF TRANSFORMED EQUATION SOLUTIONS (CONTD)

NUMERATOR TERMS FOR X12 SOLUTION										PAGE 2 OF 10	
POWERS				COEFFICIENTS							
W	B	C	D	K1	K2	K3	K4	K5	K6	K7	
3	2	1	1	24	36		36	168			
	2		2				30	82			
	1	3			44			88			
	1	2	1	72	108		64	278			
	1	1	2	72	64		108	278			
	1		3				44	88			
		4									
		3	1								
		2	2								
		1	3								
			4								
*	*	*	*								
2	4	1			1			5			
	4		1				1	5			
	3	2			18			49			
	3	1	1	16	22		22	101			
	3		2				18	49			
	2	3			66			132			
	2	2	1	108	162		96	417			
	2	1	2	108	96		162	417			
	2		3				66	132			
	1	4			63			104			
	1	3	1	160	238		112	452			
	1	2	2	324	288		288	697			
	1	1	3	160	112		238	452			
	1		4				63	104			
		5									
		4	1								

TABLE A-6. NUMERATOR TERMS OF TRANSFORMED EQUATION SOLUTIONS (CONT'D)

NUMERATOR TERMS FOR X12 SOLUTION								PAGE 3 OF 10			
POWERS				COEFFICIENTS							
W	B	C	D	K1	K2	K3	K4	K5	K6	K7	
2		3	2								
		2	3								
		1	4								
			5								
*	*	*	*								
1	4	2			3			8			
	4	1	1	4	4		4	17			
	4		2				3	8			
	3	3			26			52			
	3	2	1	48	66		40	167			
	3	1	2	48	40		66	167			
	3		3				26	52			
	2	4			63			104			
	2	3	1	160	238		112	452			
	2	2	2	324	288		288	697			
	2	1	3	160	112		238	452			
	2		4				63	104			
	1	5			44			64			
	1	4	1	156	228		96	360			
	1	3	2	480	424		336	764			
	1	2	3	480	336		424	764			
	1	1	4	156	96		228	360			
	1		5				44	64			
		6									
		5	1								
		4	2								
		3	3								
		2	4								

TABLE A-6. NUMERATOR TERMS OF TRANSFORMED EQUATION SOLUTIONS (CONTD)

NUMERATOR TERMS FOR X12 SOLUTION								PAGE 4 OF 10			
POWERS				COEFFICIENTS							
W	B	C	D	K1	K2	K3	K4	K5	K6	K7	
1		1	5								
			6								
*	*	*	*								
-	4	3			2			4			
	4	2	1	6	6		4	14			
	4	1	2	6	4		6	14			
	4		3				2	4			
	3	4			12			20			
	3	3	1	36	48		24	90			
	3	2	2	72	60		60	140			
	3	1	3	36	24		48	90			
	3		4				12	20			
	2	5			22			32			
	2	4	1	78	114		48	180			
	2	3	2	240	212		168	382			
	2	2	3	240	168		212	382			
	2	1	4	78	48		114	180			
	2		5				22	32			
	1	6			12			16			
	1	5	1	56	80		32	112			
	1	4	2	234	204		144	308			
	1	3	3	356	248		248	424			
	1	2	4	234	144		204	308			
	1	1	5	56	32		80	112			
	1		6				12	16			
		7									
		6	1								
		5	2								

TABLE A-6. NUMERATOR TERMS OF TRANSFORMED EQUATION SOLUTIONS (CONT'D)

NUMERATOR TERMS FOR X12 SOLUTION				PAGE 6 OF 10						
POWERS				COEFFICIENTS						
W	B	C	D	K8	K9	K10	K11	K12	K13	K14
7								1		
*	*	*	*							
6	1							4		
		1			1			11		
			1				1	11		
*	*	*	*							
5	2							6		
	1	1			4			38		
	1		1				4	38		
		2			9			51		
		1	1	4	10		10	104		
			2				9	51		
*	*	*	*							
4	3							4		
	2	1			6			48		
	2		1				6	48		
	1	2			30			148		
	1	1	1	16	34		34	302		
	1		2				30	148		
		3			33			129		
		2	1	30	75		41	403		
		1	2	30	41		75	403		
			3				33	129		
*	*	*	*							
3	4							1		
	3	1			4			26		
	3		1				4	26		
	2	2			36			151		

TABLE A-6. NUMERATOR TERMS OF TRANSFORMED EQUATION SOLUTIONS (CONT'D)

NUMERATOR TERMS FOR X12 SOLUTION								PAGE 7 OF 10			
POWERS				COEFFICIENTS							
W	B	C	D	K8	K9	K10	K11	K12	K13	K14	
3	2	1	1	24	42		42	309			
	2		2				36	151			
	1	3			88			302			
	1	2	1	96	204		114	945			
	1	1	2	96	114		204	945			
	1		3				88	302			
		4			63			192			
		3	1	88	220		88	818			
		2	2	180	246		246	1253			
		1	3	88	88		220	818			
			4				63	192			
*	*	*	*								
2	4	1			1			5			
	4		1				1	5			
	3	2			18			62			
	3	1	1	16	22		22	128			
	3		2				18	62			
	2	3			79			233			
	2	2	1	108	189		109	734			
	2	1	2	108	109		189	734			
	2		3				79	233			
	1	4			126			340			
	1	3	1	212	449		188	1453			
	1	2	2	432	513		513	2228			
	1	1	3	212	188		449	1453			
	1		4				126	340			
		5			66			168			
		4	1	126	315		104	916			

TABLE A-6. NUMERATOR TERMS OF TRANSFORMED EQUATION SOLUTIONS (CONTD)

NUMERATOR TERMS FOR X12 SOLUTION								PAGE 8 OF 10			
POWERS				COEFFICIENTS							
W	B	C	D	K8	K9	K10	K11	K12	K13	K14	
2		3	2	396	541		396	1913			
		2	3	396	396		541	1913			
		1	4	126	104		315	916			
			5				66	168			
*	*	*	*								
1	4	2			3			8			
	4	1	1	4	4		4	17			
	4		2				3	8			
	3	3			26			64			
	3	2	1	48	66		40	206			
	3	1	2	48	40		66	206			
	3		3				26	64			
	2	4			75			176			
	2	3	1	160	277		124	761			
	2	2	2	324	327		327	1171			
	2	1	3	160	124		277	761			
	2		4				75	176			
	1	5			88			200			
	1	4	1	204	429		152	1096			
	1	3	2	636	753		564	2295			
	1	2	3	636	564		753	2295			
	1	1	4	204	152		429	1096			
	1		5				88	200			
		6			36			80			
		5	1	88	220		64	536			
		4	2	378	516		312	1432			
		3	3	580	580		580	1952			
		2	4	378	312		516	1432			

TABLE A-6. NUMERATOR TERMS OF TRANSFORMED EQUATION SOLUTIONS (CONT'D)

NUMERATOR TERMS FOR X12 SOLUTIONS								PAGE 9 OF 10			
POWERS				COEFFICIENTS							
W	B	C	D	K8	K9	K10	K11	K12	K13	K14	
1		1	5	88	64		220	536			
			6				36	80			
*	*	*	*								
-	4	3			2			4			
	4	2	1	6	6		4	14			
	4	1	2	6	4		6	14			
	4		3				2	4			
	3	4			12			24			
	3	3	1	36	48		24	108			
	3	2	2	72	60		60	168			
	3	1	3	36	24		48	108			
	3		4				12	24			
	2	5			26			52			
	2	4	1	78	132		52	290			
	2	3	2	240	240		186	612			
	2	2	3	240	186		240	612			
	2	1	4	78	52		132	290			
	2		5				26	52			
	1	6			24			48			
	1	5	1	72	150		48	324			
	1	4	2	306	360		228	870			
	1	3	3	468	414		414	1188			
	1	2	4	306	228		360	870			
	1	1	5	72	48		150	324			
	1		6				24	48			
		7			8			16			
		6	1	24	61		16	128			
		5	2	132	110		96	420			

TABLE A-6. NUMERATOR TERMS OF TRANSFORMED EQUATION SOLUTIONS (CONTD)

NUMERATOR TERMS FOR X13 SOLUTION					PAGE 1 OF 10						
POWERS				COEFFICIENTS							
W	B	C	D	K1	K2	K3	K4	K5	K6	K7	
7											
*	*	*	*								
6	1								1		
		1									
			1								
*	*	*	*								
5	2								3		
	1	1			2				10		
	1		1				2		10		
		2									
		1	1								
			2								
*	*	*	*								
4	3								3		
	2	1			5				25		
	2		1				5		25		
	1	2			15				41		
	1	1	1	12	18		18		84		
	1		2				15		41		
		3									
		2	1								
		1	2								
			3								
*	*	*	*								
3	4								1		
	3	1			4				20		
	3		1				4		20		
	?	2			30				82		

TABLE A-6. NUMERATOR TERMS OF TRANSFORMED EQUATION SOLUTIONS (CONTD)

NUMERATOR TERMS FOR X13 SOLUTION								PAGE 2 OF 10			
POWERS				COEFFICIENTS							
W	B	C	D	K1	K2	K3	K4	K5	K6	K7	
3	2	1	1	24	36		36		168		
	2		2				30		82		
	1	3			44				88		
	1	2	1	72	108		64		278		
	1	1	2	72	64		108		278		
	1		3				44		88		
		4									
		3	1								
		2	2								
		1	3								
			4								
**	**	**	**								
2	4	1			1				5		
	4		1				1		5		
	3	2			18				49		
	3	1	1	16	22		22		101		
	3		2				18		49		
	2	3			66				132		
	2	2	1	108	162		96		417		
	2	1	2	108	96		162		417		
	2		3				66		132		
	1	4			63				104		
	1	3	1	160	238		112		452		
	1	2	2	324	288		288		697		
	1	1	3	160	112		238		452		
	1		4				63		104		
		5									
		4	1								

TABLE A-6. NUMERATOR TERMS OF TRANSFORMED EQUATION SOLUTIONS (CONT'D)

NUMERATOR TERMS FOR X13 SOLUTION								PAGE 3 OF 10			
POWERS				COEFFICIENTS							
W	B	C	D	K1	K2	K3	K4	K5	K6	K7	
2		3	2								
		2	3								
		1	4								
			5								
*	*	*	*								
1	4	2			3				8		
	4	1	1	4	4		4		17		
	4		2				3		8		
	3	3			26				52		
	3	2	1	48	66		40		167		
	3	1	2	48	40		66		167		
	3		3				26		52		
	2	4			63				104		
	2	3	1	160	238		112		452		
	2	2	2	324	288		288		697		
	2	1	3	160	112		238		452		
	2		4				63		104		
	1	5			44				64		
	1	4	1	156	228		96		360		
	1	3	2	480	424		336		764		
	1	2	3	480	336		424		764		
	1	1	4	156	96		228		360		
	1		5				44		64		
		6									
		5	1								
		4	2								
		3	3								
		2	4								

TABLE A-6. NUMERATOR TERMS OF TRANSFORMED EQUATION SOLUTIONS (CONTD)

NUMERATOR TERMS FOR X13 SOLUTION								PAGE 4 OF 10			
POWERS				COEFFICIENTS							
W	B	C	D	K1	K2	K3	K4	K5	K6	K7	
1		1	5								
			6								
*	*	*	*								
-	4	3			2				4		
	4	2	1	6	6		4		14		
	4	1	2	6	4		6		14		
	4		3				2		4		
	3	4			12				20		
	3	3	1	36	48		24		90		
	3	2	2	72	60		60		140		
	3	1	3	36	24		48		90		
	3		4				12		20		
	2	5			22				32		
	2	4	1	78	114		48		180		
	2	3	2	240	212		168		382		
	2	2	3	240	168		212		382		
	2	1	4	78	48		114		180		
	2		5				22		32		
	1	6			12				16		
	1	5	1	56	80		32		112		
	1	4	2	234	204		144		308		
	1	3	3	356	248		248		424		
	1	2	4	234	144		204		308		
	1	1	5	56	32		80		112		
	1		6				12		16		
		7									
		6	1								
		5	2								

TABLE A-6. NUMERATOR TERMS OF TRANSFORMED EQUATION SOLUTIONS (CONTD)

NUMERATOR TERMS FOR X13 SOLUTION								PAGE 6 OF 10			
POWERS				COEFFICIENTS							
W	B	C	D	K8	K9	K10	K11	K12	K13	K14	
7									1		
*	*	*	*								
6	1								4		
		1			1				11		
			1				1		11		
*	*	*	*								
5	2								6		
	1	1			4				38		
	1		1				4		38		
		2			9				51		
		1	1	4	10		10		104		
			2				9		51		
*	*	*	*								
4	3								4		
	2	1			6				48		
	2		1				6		48		
	1	2			30				148		
	1	1	1	16	34		34		302		
	1		2				30		148		
		3			33				129		
		2	1	30	75		41		403		
		1	2	30	41		75		403		
			3				33		129		
*	*	*	*								
3	4								1		
	3	1			4				26		
	3		1				4		26		
	2	2			36				151		

TABLE A-6. NUMERATOR TERMS OF TRANSFORMED EQUATION SOLUTIONS (CONTD)

NUMERATOR TERMS FOR X13 SOLUTION								PAGE 7 OF 10			
POWERS				COEFFICIENTS							
W	B	C	D	K8	K9	K10	K11	K12	K13	K14	
3	2	1	1	24	42		42		309		
	2		2				36		151		
	1	3			88				302		
	1	2	1	96	204		114		945		
	1	1	2	96	114		204		945		
	1		3				88		302		
		4			63				192		
		3	1	88	220		88		818		
		2	2	180	246		246		1253		
		1	3	88	88		220		818		
			4				63		192		
*	*	*	*								
2	4	1			1				5		
	4		1				1		5		
	3	2			18				62		
	3	1	1	16	22		22		128		
	3		2				18		62		
	2	3			79				233		
	2	2	1	108	189		109		734		
	2	1	2	108	109		189		734		
	2		3				79		233		
	1	4			126				340		
	1	3	1	212	449		188		1453		
	1	2	2	432	513		513		2228		
	1	1	3	212	188		449		1453		
	1		4				126		340		
		5			66				168		
		4	1	126	315		104		916		

TABLE A-6. NUMERATOR TERMS OF TRANSFORMED EQUATION SOLUTIONS (CONT'D)

NUMERATOR TERMS FOR X13 SOLUTION								PAGE 8 OF 10			
POWERS				COEFFICIENTS							
W	B	C	D	K8	K9	K10	K11	K12	K13	K14	
2		3	2	396	541		396		1913		
		2	3	396	396		541		1913		
		1	4	126	104		315		916		
			5				66		168		
*	*	*	*								
1	4	2			3				8		
	4	1	1	4	4		4		17		
	4		2				3		8		
	3	3			26				64		
	3	2	1	48	66		40		206		
	3	1	2	48	40		66		206		
	3		3				26		64		
	2	4			75				176		
	2	3	1	160	277		124		761		
	2	2	2	324	327		327		1171		
	2	1	3	160	124		277		761		
	2		4				75		176		
	1	5			88				200		
	1	4	1	204	429		152		1096		
	1	3	2	636	753		564		2295		
	1	2	3	636	564		753		2295		
	1	1	4	204	152		429		1096		
	1		5				88		200		
		6			36				80		
		5	1	88	220		64		536		
		4	2	378	516		312		1432		
		3	3	580	580		580		1952		
		2	4	378	312		516		1432		

TABLE A-6. NUMERATOR TERMS OF TRANSFORMED EQUATION SOLUTIONS (CONT'D)

NUMERATOR TERMS FOR X13 SOLUTION								PAGE 9 OF 10			
POWERS				COEFFICIENTS							
W	B	C	D	K8	K9	K10	K11	K12	K13	K14	
1		1	5	88	64		220		536		
			6				36		80		
*	*	*	*								
-	4	3			2				4		
	4	2	1	6	6		4		14		
	4	1	2	6	4		6		14		
	4		3				2		4		
	3	4			12				24		
	3	3	1	36	48		24		108		
	3	2	2	72	60		60		168		
	3	1	3	36	24		48		108		
	3		4				12		24		
	2	5			26				52		
	2	4	1	78	132		52		290		
	2	3	2	240	240		186		612		
	2	2	3	240	186		240		612		
	2	1	4	78	52		132		290		
	2		5				26		52		
	1	6			24				48		
	1	5	1	72	150		48		324		
	1	4	2	306	360		228		870		
	1	3	3	468	414		414		1188		
	1	2	4	306	228		360		870		
	1	1	5	72	48		150		324		
	1		6				24		48		
		7			8				16		
		6	1	24	60		16		128		
		5	2	132	180		96		420		

TABLE A-6. NUMERATOR TERMS OF TRANSFORMED EQUATION SOLUTIONS (CONT'D)

NUMERATOR TERMS FOR X14 SOLUTION								PAGE 1 OF 24			
POWERS				COEFFICIENTS							
W	B	C	D	K1	K2	K3	K4	K5	K6	K7	
11											
*	*	*	*								
10	1									1	
		1									
			1								
*	*	*	*								
9	2									5	
	1	1				4				16	
	1		1					2	2	12	
		2									
		1	1								
			2								
*	*	*	*								
8	3									10	
	2	1				18				72	
	2		1					9	9	54	
	1	2				54				114	
	1	1	1		12	48		30	30	174	
	1		2				6	21	21	62	
		3									
		2	1								
		1	2								
			3								
*	*	*	*								
7	4									10	
	3	1				32				128	
	3		1					16	16	96	
	2	2				216				456	

TABLE A-6. NUMERATOR TERMS OF TRANSFORMED EQUATION SOLUTIONS (CONTD)

NUMERATOR TERMS FOR X14 SOLUTION (CONTINUED)								PAGE 2 OF 24			
POWERS				COEFFICIENTS							
W	B	C	D	K1	K2	K3	K4	K5	K6	K7	
7	2	1	1		48	192		120	120	696	
	2		2				24	84	84	248	
	1	3				320				476	
	1	2	1		144	576		198	198	1,110	
	1	1	2	48	120	248	84	282	282	806	
	1		3				48	92	92	180	
		4									
		3	1								
		2	2								
		1	3								
			4								
*	*	*	*								
6	5									5	
	4	1				28				112	
	4		1					14	14	84	
	3	2				336				709	
	3	1	1		76	300		187	187	1,083	
	3		2				38	131	131	385	
	2	3				1,120				1,666	
	2	2	1		504	2,016		693	693	3,885	
	2	1	2	168	420	868	294	987	987	2,821	
	2		3				168	322	322	630	
	1	4				1,092				1,289	
	1	3	1		748	2,988		754	754	4,086	
	1	2	2	504	1,260	2,604	510	1,641	1,641	4,539	
	1	1	3	336	496	720	588	1,090	1,090	2,070	
	1		4				154	217	217	321	
		5									

TABLE A-6. NUMERATOR TERMS OF TRANSFORMED EQUATION SOLUTIONS (CONTD)

NUMERATOR TERMS FOR X14 SOLUTION (CONTINUED)					PAGE 3 OF 24						
POWERS				COEFFICIENTS							
W	B	C	D	K1	K2	K3	K4	K5	K6	K7	
6		4	1								
		3	2								
		2	3								
		1	4								
			5								
*	*	*	*								
5	6									1	
	5	1				12				48	
	5		1					6	6	36	
	4	2				252				531	
	4	1	1		60	228		141	141	813	
	4		2				30	99	99	287	
	3	3				1,492				2,218	
	3	2	1		684	2,700		927	927	5,181	
	3	1	2	240	576	1,168	402	1,323	1,323	3,757	
	3		3				228	430	430	834	
	2	4				3,276				3,867	
	2	3	1		2,244	8,964		2,262	2,262	12,258	
	2	2	2	1,512	3,780	7,812	1,530	4,923	4,923	13,617	
	2	1	3	1,008	1,488	2,160	1,764	3,270	3,270	6,210	
	2		4				462	651	651	963	
	1	5				2,364				2,364	
	1	4	1		2,196	8,748		1,824	1,824	9,558	
	1	3	2	2,256	5,616	11,584	1,752	5,400	5,400	14,452	
	1	2	3	3,024	4,464	6,480	3,060	5,484	5,484	10,104	
	1	1	4	936	1,092	1,284	1,620	2,226	2,226	3,210	
	1		5				252	296	296	360	
		6									

TABLE A-6. NUMERATOR TERMS OF TRANSFORMED EQUATION SOLUTIONS (CONTD)

NUMERATOR TERMS FOR X14 SOLUTION (CONTINUED)								PAGE 4 OF 24		
POWERS				COEFFICIENTS						
W	B	C	D	K1	K2	K3	K4	K5	K6	K7
5		5	1							
		4	2							
		3	3							
		2	4							
		1	5							
			6							
*	*	*	*							
4	6	1				2				8
	6		1					1	1	6
	5	2				90				189
	5	1	1		24	84		51	51	291
	5		2				12	36	36	101
	4	3				930				1,380
	4	2	1		450	1,710		585	585	3,240
	4	1	2	180	390	750	270	840	840	2,340
	4		3				150	270	270	510
	3	4				3,630				4,283
	3	3	1		2,536	9,996		2,524	2,524	13,614
	3	2	2	1,800	4,320	8,760	1,758	5,514	5,514	15,119
	3	1	3	1,200	1,716	2,436	2,010	3,654	3,654	6,864
	3		4				518	719	719	1,051
	2	5				5,910				5,910
	2	4	1		5,490	21,870		4,560	4,560	23,895
	2	3	2	5,640	14,040	28,960	4,380	13,500	13,500	36,130
	2	2	3	7,560	11,160	16,200	7,650	13,710	13,710	25,260
	2	1	4	2,340	2,730	3,210	4,050	5,565	5,565	8,025
	2		5				630	740	740	900
	1	6				3,366				2,972

TABLE A-6. NUMERATOR TERMS OF TRANSFORMED EQUATION SOLUTIONS (CONTD)

NUMERATOR TERMS FOR X14 SOLUTION (CONTINUED)								PAGE 5 OF 24			
POWERS				COEFFICIENTS							
W	B	C	D	K1	K2	K3	K4	K5	K6	K7	
4	1	5	1		3,984	15,804		2,904	2,904	14,724	
	1	4	2	5,580	13,770	28,290	3,720	10,980	10,980	28,433	
	1	3	3	11,280	16,596	24,036	8,760	15,174	15,174	27,114	
	1	2	4	7,020	8,190	9,630	7,668	9,949	9,949	13,249	
	1	1	5	1,320	1,376	1,440	1,590	2,044	2,044	3,060	
	1		6				220	232	232	248	
		7									
		6	1								
		5	2								
		4	3								
		3	4								
		2	5								
		1	6								
			7								
*	*	*	*								
3	6	2				12				25	
	6	1	1		4	12		7	7	39	
	6		2				2	5	5	13	
	5	3				264				390	
	5	2	1		144	504		171	171	927	
	5	1	2	72	132	228	90	249	249	663	
	5		3				48	78	78	138	
	4	4				1,800				2,121	
	4	3	1		1,332	5,052		1,278	1,278	6,798	
	4	2	2	1,080	2,340	4,500	966	2,823	2,823	7,543	
	4	1	3	720	952	1,272	1,080	1,858	1,858	3,378	
	4		4				266	353	353	497	
	3	5				5,224				5,224	

TABLE A-6. NUMERATOR TERMS OF TRANSFORMED EQUATION SOLUTIONS (CONT'D)

NUMERATOR TERMS FOR X14 SOLUTION (CONTINUED)								PAGE 6 OF 24			
POWERS				COEFFICIENTS							
W	B	C	D	K1	K2	K3	K4	K5	K6	K7	
3	3	4	1		4,956	19,476		4,072	4,072	21,204	
	3	3	2	5,384	12,836	25,956	4,064	12,128	12,128	32,088	
	3	2	3	7,200	10,296	14,616	7,032	12,308	12,308	22,368	
	3	1	4	2,224	2,536	2,920	3,664	4,948	4,948	7,036	
	3		5				552	640	640	768	
	2	6				6,732				5,944	
	2	5	1		7,968	31,608		5,808	5,808	29,408	
	2	4	2	11,160	27,540	56,580	7,440	21,960	21,960	56,866	
	2	3	3	22,560	33,192	48,072	17,520	30,348	30,348	54,228	
	2	2	4	14,040	16,380	19,260	14,076	18,858	18,858	26,498	
	2	1	5	2,640	2,752	2,880	4,440	5,128	5,128	6,120	
	2		6				440	464	464	496	
	1	7				3,152				2,528	
	1	6	1		4,572	18,036		3,040	3,040	14,928	
	1	5	2	8,232	20,052	40,948	4,992	14,112	14,112	35,368	
	1	4	3	22,320	32,616	47,016	14,880	24,912	24,912	43,164	
	1	3	4	20,944	24,376	28,600	16,144	21,088	21,088	28,864	
	1	2	5	7,920	8,256	8,640	7,752	8,800	8,800	10,308	
	1	1	6	992	992	992	1,568	1,640	1,640	1,736	
	1		7				96	96	96	96	
		8									
		7	1								
		6	2								
		5	3								
		4	4								
		3	5								
		2	6								
		1	7								

TABLE A-6. NUMERATOR TERMS OF TRANSFORMED EQUATION SOLUTIONS (CONTD)

NUMERATOR TERMS FOR X14 SOLUTION (CONTINUED)				PAGE 7 OF 24						
POWERS				COEFFICIENTS						
W	B	C	D	K1	K2	K3	K4	K5	K6	K7
3			8							
*	*	*	*							
2	6	3				26				26
	6	2	1		18	54		18	18	93
	6	1	2	12	18	26	12	27	27	65
	6		3				6	8	8	12
	5	4				378				444
	5	3	1		318	1,110		282	282	1,452
	5	2	2	324	594	1,026	252	639	639	1,608
	5	1	3	216	252	300	270	414	414	696
	5		4				60	72	72	92
	4	5				1,926				1,926
	4	4	1		1,944	7,344		1,548	1,548	7,911
	4	3	2	2,436	5,214	9,974	1,716	4,692	4,692	12,002
	4	2	3	3,240	4,284	5,724	2,898	4,752	4,752	8,292
	4	1	4	996	1,074	1,170	1,446	1,857	1,857	2,529
	4		5				198	220	220	252
	3	6				4,446				3,928
	3	5	1		5,382	21,054		3,888	3,888	19,560
	3	4	2	8,028	18,882	37,962	5,232	14,832	14,832	37,847
	3	3	3	16,152	22,976	32,496	12,192	20,522	20,522	36,042
	3	2	4	10,008	11,412	13,140	9,642	12,657	12,657	17,485
	3	1	5	1,872	1,928	1,992	2,952	3,362	3,362	3,954
	3		6				272	284	284	300
	2	7				4,728				3,792
	2	6	1		6,858	27,054		4,560	4,560	22,392
	2	5	2	12,348	30,078	61,422	7,488	21,168	21,168	53,052
	2	4	3	33,480	48,924	70,524	22,320	37,368	37,368	64,746

TABLE A-6. NUMERATOR TERMS OF TRANSFORMED EQUATION SOLUTIONS (CONT'D)

NUMERATOR TERMS FOR X14 SOLUTION (CONTINUED)								PAGE 8 OF 24			
POWERS				COEFFICIENTS							
W	B	C	D	K1	K2	K3	K4	K5	K6	K7	
2	2	3	4	31,416	36,564	42,900	24,216	31,632	31,632	43,296	
	2	2	5	11,880	12,384	12,960	11,628	13,200	13,206	15,462	
	2	1	6	1,488	1,488	1,488	2,352	2,460	2,460	2,604	
	2		7				144	144	144	144	
	1	8				1,872				1,392	
	1	7	1		3,240	12,696		2,016	2,016	9,600	
	1	6	2	7,236	17,334	35,118	4,128	11,184	11,184	27,144	
	1	5	3	24,696	35,724	51,132	14,976	24,240	24,240	40,728	
	1	4	4	31,068	35,982	42,030	20,592	26,232	26,232	34,972	
	1	3	5	17,712	18,440	19,272	13,392	14,960	14,960	17,184	
	1	2	6	4,464	4,464	4,464	4,152	4,308	4,308	4,516	
	1	1	7	384	384	384	528	528	528	528	
	1		8				16	16	16	16	
		9									
		8	1								
		7	2								
		6	3								
		5	4								
		4	5								
		3	6								
		2	7								
		1	8								
*	*	*	*								
1	6	4				24				28	
	6	3	1		26	78		20	20	96	
	6	2	2	36	54	78	24	48	48	106	
	6	1	3	24	24	24	24	30	30	42	
	6		4				4	4	4	4	

TABLE A-6. NUMERATOR TERMS OF TRANSFORMED EQUATION SOLUTIONS (CONTD)

NUMERATOR TERMS FOR X14 SOLUTION (CONTINUED)								PAGE 9 OF 24			
POWERS				COEFFICIENTS							
W	B	C	D	K1	K2	K3	K4	K5	K6	K7	
1	5	5				264				264	
	5	4	1		306	1,062		228	228	1,116	
	5	3	2	492	882	1,506	312	720	720	1,704	
	5	2	3	648	756	900	504	726	726	1,152	
	5	1	4	192	192	192	228	264	264	324	
	5		5				24	24	24	24	
	4	6				1,080				956	
	4	5	1		1,398	5,250		984	984	4,836	
	4	4	2	2,448	5,112	9,672	1,512	3,852	3,852	9,414	
	4	3	3	4,872	6,380	8,460	3,432	5,348	5,348	8,928	
	4	2	4	2,988	3,222	3,510	2,604	3,228	3,228	4,236	
	4	1	5	552	552	552	732	798	798	894	
	4		6				52	52	52	52	
	3	7				2,072				1,664	
	3	6	1		3,078	11,970		2,032	2,032	9,888	
	3	5	2	5,964	13,746	27,394	3,552	9,552	9,552	23,500	
	3	4	3	16,056	22,596	31,716	10,464	16,920	16,920	28,686	
	3	3	4	14,968	16,996	19,492	11,176	14,248	14,248	19,096	
	3	2	5	5,616	5,784	5,976	5,220	5,834	5,834	6,714	
	3	1	6	704	704	704	992	1,028	1,028	1,076	
	3		7				48	48	48	48	
	2	8				1,872				1,392	
	2	7	1		3,240	12,696		2,016	2,016	9,600	
	2	6	2	7,236	17,334	35,118	4,128	11,184	11,184	27,144	
	2	5	3	24,696	35,724	51,132	14,976	24,240	24,240	40,728	
	2	4	4	31,068	35,982	42,030	20,592	26,232	26,232	34,972	
	2	3	5	17,712	18,440	19,272	13,392	14,960	14,960	17,184	
	2	2	6	4,464	4,464	4,464	4,152	4,308	4,308	4,516	

TABLE A-6. NUMERATOR TERMS OF TRANSFORMED EQUATION SOLUTIONS (CONTD)

NUMERATOR TERMS FOR X14 SOLUTION (CONTINUED)								PAGE 10 OF 24			
POWERS				COEFFICIENTS							
W	B	C	D	K1	K2	K3	K4	K5	K6	K7	
1	2	1	7	384	384	384	528	528	528	528	
	2		8				16	16	16	16	
	1	9				640				448	
	1	8	1		1,296	5,040		768	768	3,552	
	1	7	2	3,504	8,232	16,520	1,920	4,992	4,992	11,744	
	1	6	3	14,472	20,664	29,304	8,256	12,928	12,928	21,072	
	1	5	4	22,896	26,328	30,552	13,824	17,184	17,184	22,320	
	1	4	5	17,496	18,168	18,936	11,424	12,560	12,560	14,160	
	1	3	6	6,656	6,656	6,656	4,832	4,976	4,976	5,168	
	1	2	7	1,152	1,152	1,152	960	960	960	960	
	1	1	8	64	64	64	64	64	64	64	
		10									
		9	1								
		8	2								
		7	3								
		6	4								
		5	5								
		4	6								
		3	7								
		2	8								
*	*	*	*								
-	6	5				8				8	
	6	4	1		12	36		8	8	36	
	6	3	2	28	40	56	16	28	28	56	
	6	2	3	36	36	36	24	28	28	36	
	6	1	4	8	8	8	8	8	8	8	
	5	6				72				64	
	5	5	1		108	372		72	72	336	

TABLE A-6. NUMERATOR TERMS OF TRANSFORMED EQUATION SOLUTIONS (CONTD)

NUMERATOR TERMS FOR X14 SOLUTION (CONTINUED)								PAGE 11 OF 24			
POWERS				COEFFICIENTS							
W	B	C	D	K1	K2	K3	K4	K5	K6	K7	
-	5	4	2	252	432	720	144	300	300	664	
	5	3	3	492	564	660	312	420	420	624	
	5	2	4	288	288	288	216	240	240	280	
	5	1	5	48	48	48	48	48	48	48	
	4	7				248				200	
	4	6	1		396	1,476		256	256	1,212	
	4	5	2	924	1,860	3,460	528	1,248	1,248	2,908	
	4	4	3	2,448	3,144	4,104	1,512	2,232	2,232	3,552	
	4	3	4	2,248	2,404	2,596	1,552	1,852	1,852	2,332	
	4	2	5	828	828	828	672	716	716	780	
	4	1	6	104	104	104	104	104	104	104	
	3	8				408				304	
	3	7	1		724	2,796		448	448	2,112	
	3	6	2	1,764	3,960	7,800	992	2,528	2,528	5,996	
	3	5	3	5,964	8,260	11,460	3,552	5,512	5,512	9,012	
	3	4	4	7,440	8,376	9,528	4,808	5,948	5,948	7,724	
	3	3	5	4,200	4,312	4,440	3,048	3,340	3,340	3,756	
	3	2	6	1,056	1,056	1,056	896	920	920	952	
	3	1	7	96	96	96	96	96	96	96	
	2	9				320				224	
	2	8	1		648	2,520		384	384	1,776	
	2	7	2	1,752	4,116	8,260	960	2,496	2,496	5,872	
	2	6	3	7,236	10,332	14,652	4,128	6,464	6,464	10,536	
	2	5	4	11,448	13,164	15,276	6,912	8,592	8,592	11,160	
	2	4	5	8,748	9,084	9,468	5,712	6,280	6,280	7,080	
	2	3	6	3,328	3,328	3,328	2,416	2,488	2,488	2,584	
	2	2	7	576	576	576	480	480	480	480	
	2	1	8	32	32	32	32	32	32	32	

TABLE A-6. NUMERATOR TERMS OF TRANSFORMED EQUATION SOLUTIONS (CONT'D)

NUMERATOR TERMS FOR X14 SOLUTION (CONTINUED)					PAGE 13 OF 24						
POWERS				COEFFICIENTS							
W	B	C	D	K8	K9	K10	K11	K12	K13	K14	
11										1	
*	*	*	*								
10	1									6	
		1				2				17	
			1					1	1	12	
*	*	*	*								
9	2									15	
	1	1				12				93	
	1		1					6	6	66	
		2				30				130	
		1	1		4	24		16	16	186	
			2				2	11	11	62	
*	*	*	*								
8	3									20	
	2	1				30				210	
	2		1					15	15	150	
	1	2				162				642	
	1	1	1		24	132		87	87	924	
	1		2				12	60	60	310	
		3				200				590	
		2	1			54	324	114	114	1,284	
		1	2		12	42	124	30	159	159	868
			3					18	51	51	180
*	*	*	*								
7	4									15	
	3	1				40				250	
	3		1					20	20	180	
	2	2				360				1,293	

TABLE A-6. NUMERATOR TERMS OF TRANSFORMED EQUATION SOLUTIONS (CONT'D)

NUMERATOR TERMS FOR X14 SOLUTION (CONTINUED)								PAGE 14 OF 24			
POWERS				COEFFICIENTS							
W	B	C	D	K8	K9	K10	K11	K12	K13	K14	
7	2	1	1		60	300		195	195	1,875	
	2		2				30	135	135	633	
	1	3				960				2,598	
	1	2	1		288	1,584		555	555	5,691	
	1	1	2	72	228	620	162	777	777	3,875	
	1		3				96	250	250	810	
		4				780				1,765	
		3	1		320	1,920		476	476	5,196	
		2	2	144	504	1,488	198	1,011	1,011	5,345	
		1	3	96	184	360	240	658	658	2,250	
			4				66	129	129	321	
*	*	*	*								
6	5									6	
	4	1				30				165	
	4		1					15	15	120	
	3	2				420				1,352	
	3	1	1		80	360		230	230	1,980	
	3		2				40	160	160	672	
	2	3				1,866				4,593	
	2	2	1		630	3,150		1,098	1,098	10,149	
	2	1	2	180	510	1,266	360	1,545	1,545	6,963	
	2		3				210	498	498	1,464	
	1	4				3,276				6,822	
	1	3	1		1,494	8,214		2,043	2,043	20,229	
	1	2	2	756	2,394	6,510	948	4,359	4,359	20,977	
	1	1	3	504	892	1,620	1,134	2,848	2,848	8,910	
	1		4				308	560	560	1,284	
		5				1,970				3,653	

TABLE A-6. NUMERATOR TERMS OF TRANSFORMED EQUATION SOLUTIONS (CONTD)

NUMERATOR TERMS FOR X14 SOLUTION (CONTINUED)								PAGE 15 OF 24			
POWERS				COEFFICIENTS							
W	B	C	D	K8	K9	K10	K11	K12	K13	K14	
6		4	1		1,092	6,552		1,289	1,289	13,644	
		3	2	748	2,614	7,714	754	3,709	3,709	18,991	
		2	3	1,008	1,932	3,780	1,386	3,676	3,676	12,174	
		1	4	308	434	642	770	1,463	1,463	3,531	
			5				126	192	192	360	
*	*	*	*								
5	6									1	
	5	1				12				57	
	5		1					6	6	42	
	4	2				270				768	
	4	1	1		60	240		150	150	1,140	
	4		2				30	105	105	388	
	3	3				1,864				4,129	
	3	2	1		720	3,240		1,122	1,122	9,234	
	3	1	2	240	600	1,344	420	1,590	1,590	6,384	
	3		3				240	512	512	1,344	
	2	4				5,454				10,368	
	2	3	1		2,802	13,998		3,495	3,495	31,053	
	2	2	2	1,620	4,590	11,394	1,836	7,506	7,506	32,493	
	2	1	3	1,080	1,752	2,928	2,160	4,920	4,920	13,908	
	2		4				576	966	966	2,014	
	1	5				7,092				12,141	
	1	4	1		4,374	24,030		4,779	4,779	45,711	
	1	3	2	3,372	10,650	28,934	3,138	13,827	13,827	64,199	
	1	2	3	4,536	8,028	14,580	5,688	13,770	13,770	41,574	
	1	1	4	1,392	1,848	2,568	3,120	5,502	5,502	12,198	
	1		5				504	724	724	1,260	
		6				3,366				5,336	

TABLE A-6. NUMERATOR TERMS OF TRANSFORMED EQUATION SOLUTIONS (CONTD)

NUMERATOR TERMS FOR X14 SOLUTION (CONTINUED)								PAGE 16 OF 24			
POWERS				COEFFICIENTS							
W	B	C	D	K8	K9	K10	K11	K12	K13	K14	
5		5	1		2,364	14,184		2,364	2,364	24,282	
		4	2	2,196	7,650	22,554	1,824	8,646	8,646	42,885	
		3	3	4,488	8,588	16,788	4,524	11,606	11,606	37,218	
		2	4	2,772	3,906	5,778	3,810	7,035	7,035	16,459	
		1	5	504	592	720	1,260	1,876	1,876	3,420	
			6				132	168	168	248	
*	*	*	*								
4	6	1				2				8	
	6		1					1	1	6	
	5	2				90				222	
	5	1	1		24	84		51	51	336	
	5		2				12	36	36	114	
	4	3				996				1,959	
	4	2	1		450	1,800		618	618	4,458	
	4	1	2	180	390	776	270	885	885	3,104	
	4		3				150	283	283	648	
	3	4				4,530				7,784	
	3	3	1		2,668	11,988		3,007	3,007	23,652	
	3	2	2	1,800	4,500	10,080	1,824	6,522	6,522	25,002	
	3	1	3	1,200	1,768	2,688	2,100	4,282	4,282	10,752	
	3		4				544	832	832	1,548	
	2	5				9,828				15,417	
	2	4	1		6,840	34,110		6,873	6,873	58,713	
	2	3	2	6,036	17,028	42,198	5,154	20,052	20,052	83,337	
	2	2	3	8,100	13,140	21,960	9,180	20,067	20,067	54,492	
	2	1	4	2,496	3,108	4,028	4,944	8,022	8,022	16,112	
	2		5				780	1,048	1,048	1,668	
	1	6				10,098				14,826	

TABLE A-6. NUMERATOR TERMS OF TRANSFORMED EQUATION SOLUTIONS (CONT'D)

NUMERATOR TERMS FOR X14 SOLUTION (CONTINUED)								PAGE 17 OF 24			
POWERS				COEFFICIENTS							
W	B	C	D	K8	K9	K10	K11	K12	K13	K14	
4	1	5	1		7,902	43,362		7,362	7,362	68,067	
	1	4	2	8,280	26,010	70,530	6,420	27,105	27,105	121,429	
	1	3	3	16,860	29,768	53,988	15,690	36,602	36,602	106,602	
	1	2	4	10,440	13,860	19,260	14,304	24,002	24,002	47,772	
	1	1	5	1,920	2,168	2,520	3,000	4,272	4,272	10,080	
	1		6				440	536	536	744	
		7				3,940				5,500	
		6	1		3,366	20,196		2,972	2,972	29,652	
		5	2	3,984	13,812	40,662	2,904	13,272	13,272	63,801	
		4	3	10,980	20,940	40,860	9,120	22,633	22,633	70,278	
		3	4	10,276	14,458	21,362	10,354	18,577	18,577	42,113	
		2	5	3,780	4,440	5,400	5,190	7,548	7,548	13,368	
		1	6	440	464	496	1,100	1,376	1,376	1,984	
			7				72	80	80	96	
*	*	*	*								
3	6	2				12				25	
	6	1	1		4	12		7	7	39	
	6		2				2	5	5	13	
	5	3				264				453	
	5	2	1		144	504		171	171	1,059	
	5	1	2	72	132	228	90	249	249	741	
	5		3				48	78	78	150	
	4	4				1,926				2,955	
	4	3	1		1,332	5,316		1,341	1,341	3,177	
	4	2	2	1,080	2,340	4,656	966	2,955	2,955	9,814	
	4	1	3	720	952	1,296	1,080	1,936	1,936	4,212	
	4		4				266	365	365	589	
	3	5				6,508				9,271	

TABLE A-6. NUMERATOR TERMS OF TRANSFORMED EQUATION SOLUTIONS (CONT'D)

NUMERATOR TERMS FOR X14 SOLUTION (CONTINUED)								PAGE 18 OF 24			
POWERS				COEFFICIENTS							
W	B	C	D	K8	K9	K10	K11	K12	K13	K14	
3	3	4	1		5,208	23,328		4,777	4,777	35,913	
	3	3	2	5,384	13,364	29,844	4,190	14,123	14,123	51,633	
	3	2	3	7,200	10,608	16,128	7,296	14,198	14,198	34,038	
	3	1	4	2,224	2,584	3,096	3,820	5,638	5,638	10,062	
	3		5				576	716	716	1,020	
	2	6				11,178				15,096	
	2	5	1		9,894	49,206		8,544	8,544	70,212	
	2	4	2	11,916	33,318	82,278	8,592	31,788	31,788	126,801	
	2	3	3	24,144	39,024	65,064	20,616	43,218	43,218	112,638	
	2	2	4	14,976	18,648	24,168	16,830	26,403	26,403	51,019	
	2	1	5	2,784	3,016	3,336	5,376	7,042	7,042	10,842	
	2		6				536	620	620	796	
	1	7				9,456				12,264	
	1	6	1		9,018	49,410		7,464	7,464	66,768	
	1	5	2	12,084	37,650	101,798	8,304	33,000	33,600	145,286	
	1	4	3	33,120	58,188	105,228	25,680	57,722	57,722	162,138	
	1	3	4	31,024	41,104	57,024	28,780	47,698	47,698	98,658	
	1	2	5	11,520	13,008	15,120	14,256	19,498	19,498	31,890	
	1	1	6	1,376	1,424	1,488	2,984	3,572	3,572	4,836	
	1		7				192	208	208	240	
		8				3,120				3,920	
		7	1		3,152	18,912		2,528	2,528	24,528	
		6	2	4,572	15,750	46,278	3,040	13,408	13,408	62,512	
		5	3	15,936	30,236	58,836	11,616	27,896	27,896	83,892	
		4	4	20,076	28,158	41,502	16,672	29,068	29,068	63,836	
		3	5	11,184	13,120	15,936	11,256	15,988	15,988	27,492	
		2	6	2,640	2,784	2,976	3,616	4,444	4,444	6,252	
		1	7	192	192	192	480	528	528	624	

TABLE A-6. NUMERATOR TERMS OF TRANSFORMED EQUATION SOLUTIONS (CONT'D)

NUMERATOR TERMS FOR X14 SOLUTION (CONTINUED)								PAGE 19 OF 24			
POWERS				COEFFICIENTS							
W	B	C	D	K8	K9	K10	K11	K12	K13	K14	
3			8				16	16	16	16	
*	*	*	*								
2	6	3				26				38	
	6	2	1		18	54		18	18	93	
	6	1	2	12	18	26	12	27	27	65	
	6		3				6	8	8	12	
	5	4				378				510	
	5	3	1		318	1,110		282	282	1,641	
	5	2	2	324	594	1,026	252	639	639	1,779	
	5	1	3	216	252	300	270	414	414	750	
	5		4				60	72	72	96	
	4	5				2,058				2,634	
	4	4	1		1,944	7,722		1,614	1,614	10,479	
	4	3	2	2,436	5,214	10,316	1,716	4,881	4,881	15,314	
	4	2	3	3,240	4,284	5,832	2,898	4,923	4,923	10,140	
	4	1	4	996	1,074	1,178	1,446	1,911	1,911	2,945	
	4		5				198	224	224	276	
	3	6				5,526				6,810	
	3	5	1		5,646	25,170		4,494	4,494	32,307	
	3	4	2	8,028	19,638	43,578	5,364	17,013	17,013	59,263	
	3	3	3	16,152	23,660	35,820	12,570	23,312	23,312	53,262	
	3	2	4	10,008	11,628	13,932	9,984	14,202	14,202	24,250	
	3	1	5	1,872	1,944	2,040	3,060	3,708	3,708	5,100	
	3		6				280	304	304	352	
	2	7				7,836				9,384	
	2	6	1		8,478	42,012		6,552	6,552	51,840	
	2	5	2	13,140	36,252	89,046	8,496	29,880	29,880	114,399	
	2	4	3	35,748	57,348	95,148	25,776	51,795	51,795	129,474	

TABLE A-6. NUMERATOR TERMS OF TRANSFORMED EQUATION SOLUTIONS (CONT'D)

NUMERATOR TERMS FOR X14 SOLUTION (CONTINUED)										PAGE 20 OF 24	
POWERS				COEFFICIENTS							
W	B	C	D	K8	K9	K10	K11	K12	K13	K14	
2	2	3	4	33,468	41,550	53,702	28,374	43,017	43,017	79,877	
	2	2	5	12,528	13,572	15,012	13,770	17,574	17,574	26,130	
	2	1	6	1,536	1,560	1,592	2,796	3,180	3,180	3,980	
	2		7				168	176	176	192	
	1	8				5,616				6,576	
	1	7	1		6,348	34,716		4,800	4,800	41,592	
	1	6	2	10,476	32,292	86,976	6,624	25,704	25,704	107,340	
	1	5	3	36,252	63,216	113,820	24,912	53,964	53,964	146,202	
	1	4	4	45,648	60,228	83,268	35,304	56,718	56,718	113,240	
	1	3	5	25,632	28,900	33,540	23,580	31,462	31,462	49,830	
	1	2	6	6,192	6,408	6,696	7,500	8,820	8,820	11,636	
	1	1	7	480	480	480	984	1,056	1,056	1,200	
	1		8				32	32	32	32	
		9				1,600				1,840	
		8	1		1,872	11,232		1,392	1,392	13,152	
		7	2	3,240	11,076	32,468	2,016	8,592	8,592	38,888	
		6	3	13,716	25,848	50,112	9,120	21,208	21,208	61,800	
		5	4	21,792	30,420	44,668	15,888	26,928	26,928	57,292	
		4	5	16,308	19,080	23,112	13,536	18,780	18,780	31,344	
		3	6	5,824	6,136	6,552	5,848	7,060	7,060	9,684	
		2	7	864	864	864	1,176	1,280	1,280	1,488	
		1	8	32	32	32	80	80	80	80	
*	*	*	*								
1	6	4				24				28	
	6	3	1		26	78		20	20	96	
	6	2	2	36	54	78	24	48	48	106	
	6	1	3	24	24	24	24	30	30	42	
	6		4				4	4	4	4	

TABLE A-6. NUMERATOR TERMS OF TRANSFORMED EQUATION SOLUTIONS (CONTD)

NUMERATOR TERMS FOR X14 SOLUTION (CONTINUED)										PAGE 21 OF 24	
POWERS				COEFFICIENTS							
W	B	C	D	K8	K9	K10	K11	K12	K13	K14	
1	5	5				264				300	
	5	4	1		306	1,062		228	228	1,248	
	5	3	2	492	882	1,506	312	720	720	1,866	
	5	2	3	648	756	900	504	726	726	1,230	
	5	1	4	192	192	192	228	264	264	336	
	5		5				24	24	24	24	
	4	6				1,152				1,284	
	4	5	1		1,398	5,514		1,020	1,020	6,288	
	4	4	2	2,448	5,112	9,996	1,512	3,984	3,984	11,782	
	4	3	3	4,872	6,380	8,616	3,432	5,510	5,510	10,704	
	4	2	4	2,988	3,222	3,534	2,604	3,306	3,306	4,840	
	4	1	5	552	552	552	732	810	810	966	
	4		6				52	52	52	52	
	3	7				2,568				2,820	
	3	6	1		3,222	14,274		2,316	2,316	15,936	
	3	5	2	5,964	14,274	31,370	3,624	10,800	10,800	35,822	
	3	4	3	16,056	23,244	34,884	10,728	18,938	18,938	41,166	
	3	3	4	14,968	17,308	20,636	11,500	15,748	15,748	25,664	
	3	2	5	5,616	5,832	6,120	5,376	6,340	6,340	8,388	
	3	1	6	704	704	704	1,016	1,088	1,088	1,232	
	3		7				48	48	48	48	
	2	8				3,096				3,360	
	2	7	1		3,984	19,656		2,832	2,832	21,600	
	2	6	2	7,668	20,790	50,706	4,608	15,408	15,408	56,640	
	2	5	3	26,280	41,688	68,652	16,992	32,724	32,724	78,426	
	2	4	4	33,012	40,734	52,350	23,664	34,662	34,662	61,792	
	2	3	5	18,648	20,156	22,236	15,516	19,286	19,286	27,654	
	2	2	6	4,608	4,680	4,776	4,812	5,376	5,376	6,544	

TABLE A-6. NUMERATOR TERMS OF TRANSFORMED EQUATION SOLUTIONS (CONTD)

NUMERATOR TERMS FOR X14 SOLUTION (CONTINUED)								PAGE 22 OF 24			
POWERS				COEFFICIENTS							
W	B	C	D	K8	K9	K10	K11	K12	K13	K14	
1	2	1	7	384	384	384	600	624	624	672	
	2		8				16	16	16	16	
	1	9				1,920				2,064	
	1	8	1		2,520	13,752		1,776	1,776	14,928	
	1	7	2	4,992	15,192	40,728	2,976	11,088	11,088	44,760	
	1	6	3	20,952	36,180	64,764	13,248	27,672	27,672	72,336	
	1	5	4	33,240	43,584	59,944	22,800	35,520	35,520	68,452	
	1	4	5	25,056	28,164	32,580	19,248	25,060	25,060	38,424	
	1	3	6	9,152	9,464	9,880	8,264	9,548	9,548	12,268	
	1	2	7	1,440	1,440	1,440	1,656	1,760	1,760	1,968	
	1	1	8	64	64	64	112	112	112	112	
			10			480				512	
		9	1		640	3,840		448	448	4,128	
		8	2	1,296	4,392	12,840	768	3,168	3,168	13,936	
		7	3	6,480	12,112	23,376	4,032	9,088	9,088	25,680	
		6	4	12,456	17,280	25,248	8,288	13,664	13,664	28,176	
		5	5	11,712	13,648	16,464	8,544	11,584	11,584	18,768	
		4	6	5,592	5,880	6,264	4,640	5,504	5,504	7,360	
		3	7	1,248	1,248	1,248	1,248	1,344	1,344	1,536	
		2	8	96	96	96	128	128	128	128	
*	*	*	*								
-	6	5				8				8	
	6	4	1		12	36		8	8	36	
	6	3	2	28	40	56	16	28	28	56	
	6	2	3	36	36	36	24	28	28	36	
	6	1	4	8	8	8	8	8	8	8	
	5	6				72				72	
	5	5	1		108	372		72	72	372	

TABLE A-6. NUMERATOR TERMS OF TRANSFORMED EQUATION SOLUTIONS (CONT'D)

NUMERATOR TERMS FOR X14 SOLUTION (CONTINUED)								PAGE 23 OF 24			
POWERS				COEFFICIENTS							
W	B	C	D	K8	K9	K10	K11	K12	K13	K14	
-	5	4	2	252	432	720	144	300	300	720	
	5	3	3	492	564	660	312	420	420	660	
	5	2	4	288	288	288	216	240	240	288	
	5	1	5	48	48	48	48	48	48	48	
	4	7				264				264	
	4	6	1		396	1,548		264	264	1,548	
	4	5	2	924	1,860	3,572	528	1,284	1,284	3,572	
	4	4	3	2,448	3,144	4,176	1,512	2,288	2,288	4,176	
	4	3	4	2,248	2,404	2,612	1,552	1,888	1,888	2,612	
	4	2	5	828	828	828	672	724	724	828	
	4	1	6	104	104	104	104	104	104	104	
	3	8				504				504	
	3	7	1		756	3,324		504	504	3,324	
	3	6	2	1,764	4,104	8,904	1,008	2,820	2,820	8,904	
	3	5	3	5,964	8,484	12,564	3,624	6,084	6,084	12,564	
	3	4	4	7,440	8,520	10,056	4,920	6,480	6,480	10,056	
	3	3	5	4,200	4,344	4,536	3,120	3,576	3,576	4,536	
	3	2	6	1,056	1,056	1,056	912	960	960	1,056	
	3	1	7	96	96	96	96	96	96	96	
	2	9				528				528	
	2	8	1		792	3,888		528	528	3,888	
	2	7	2	1,848	4,908	11,868	1,056	3,360	3,360	11,868	
	2	6	3	7,668	11,988	19,548	4,608	8,508	8,508	19,548	
	2	5	4	12,120	14,820	18,884	7,800	11,040	11,040	18,884	
	2	4	5	9,180	9,876	10,836	6,480	7,844	7,844	10,836	
	2	3	6	3,424	3,472	3,536	2,728	2,992	2,992	3,536	
	2	2	7	576	576	576	528	544	544	576	
	2	1	8	32	32	32	32	32	32	32	

TABLE A-7. DENOMINATOR TERMS OF TRANSFORMED EQUATION SOLUTIONS

	W	W	W	W	W	W	W	W	W	W	W	W
	B	B	B	B	B	B	-	-	-	-	-	-
	2C	2C	2C	C	C	C	2C	2C	2C	C	C	C
	2D	D	-	2D	D	-	2D	D	-	2D	D	-
X1												
X2												
X3												
X4												
X5												
X6												
X7												
X8												
X9												
X10												
X11												
X12												
X13												
X14												

TABLE A-8. PRIMARY MODEL EQUATION DEFINITION

PRIMARY MODEL PROBABILITY EQUATIONS																			
P	K1	K2	K3	K4	K5	K6	K7	K8	K9	K10	K11	K12	K13	K14	K15	A	B	C	D
P1	1															1	1	2	2
P2	-2															1	1	2	2
	2	1														1	1	2	1
P3	1															1	1	2	2
	-2	-1														1	1	2	1
	1	1	1													1	1	2	0
P4	-2															1	1	2	2
	2			1												1	1	1	2
P5	2															1	1	2	2
	-2	-1														1	1	2	1
	-2			-1												1	1	1	2
	2	1		1	1											1	1	1	1
P6	2															1	1	2	2
	-2	-1														1	1	2	1
	-2			-1												1	1	1	2
	2	1		1		1										1	1	1	1

TABLE A-8. PRIMARY MODEL EQUATION DEFINITION (CONT'D)

PRIMARY MODEL PROBABILITY EQUATIONS (CONTINUED)																			
P	K1	K2	K3	K4	K5	K6	K7	K8	K9	K10	K11	K12	K13	K14	K15	A	B	C	D
P7	-2															1	1	2	2
	4	2														1	1	2	1
	2			1												1	1	1	2
	-4	-2		-2	-1	-1										1	1	1	1
	-2	-2	-2													1	1	2	0
	2	2	2	1	1	1	1									1	1	1	0
P8	-1															1	1	2	2
	1							1								1	0	2	2
P9	2															1	1	2	2
	-2	-1														1	1	2	1
	-2							-2								1	0	2	2
	2	1						2	1							1	0	2	1
P10	-1															1	1	2	2
	2	1														1	1	2	1
	-1	-1	-1													1	1	2	0
	1							1								1	0	2	2
	-2	-1						-2	-1							1	0	2	1
	1	1	1					1	1	1						1	0	2	0
P11	2															1	1	2	2
	-2			-1												1	1	1	2
	-2							-2								1	0	2	2
	2			1				2			1					1	0	1	2

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13. ABSTRACT (200 words or less) <p>The methodology presented in this report formulates an approach for the optimization of benefits resulting from NRC decision making processes. Recent increases in occupational exposures in nuclear power plants resulting from NRC regulatory practices have led to the questioning by NRC of the overall benefit of specific regulations. The optimization methodology in this report provides a tool for the determination of the cost-benefit of proposed NRC regulations. Detailed methods are presented for the modeling of plant safety systems undergoing inspection, testing, and/or repair. This methodology utilizes dynamic Markov modeling techniques with extensive additional model development associated with operator errors involved in the inspection, test, and repair activities of the plant. Closed form solutions to the Markov models are provided. The report appendix presents the Markov model solution process in detail sufficient for model verification. Other methods necessary for the optimization process are discussed in lesser detail. An application of the methodology dealing with steam generator inspection frequency and steam generator tube rupture events is presented. The example determines the steam generator inspection intervals which minimize expected costs and total expected occupational and public dose.</p>					
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