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A Nonparametric Statistical Methodology for the Design and Analysis of Final Status Decommissioning Surveys

Draft Report for Comment

U.S. Nuclear Regulatory Commission

Office of Nuclear Regulatory Research

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ABSTRACT

This report describes a nonparametric statistical methodology for the design and analysis of final status decommissioning surveys in support of the proposed rulemaking on decommissioning. The techniques described are alternatives to the existing parametric statistical methodology contained in the U.S. Nuclear Regulatory Commission (NRC) draft report NUREG/CR-5849, entitled, "Manual for Conducting Radiological Surveys in Support of License Termination." Proposed nonparametric statistical methods for testing compliance with decommissioning criteria are provided for radionuclides which occur in natural background and for those that do not occur in natural background. The tests considered applicable are the Wilcoxon Signed Ranks Test, Sign Test, and Quantile Test for the analysis of a single data set, and the Wilcoxon Rank Sum Test and a Quantile Test for comparing two independent data sets. An Elevated Measurement Comparison is also described to deal with any unusually high observations that might occur. This report contains information on the Data Quality Objectives process as it relates to the planning and analysis of final site surveys. The proposed process includes methods for determining the number of samples needed to obtain statistically valid comparisons with decommissioning criteria and the methods for conducting the statistical tests with the resulting sample data.

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ABBREVIATIONS

ALARA	as low as reasonably achievable
CFR	Code of Federal Regulations
DOE	U.S. Department of Energy
DQA	data quality assessment
DQO	data quality objective
EPA	U.S. Environmental Protection Agency
MCA	multichannel analyzer
MDC	minimum detectable concentration
NIST	National Institute for Standards and Technology
NRC	U.S. Nuclear Regulatory Commission
PC	personal computer
PDL	predicted dose level
PIC	pressurized ionization chamber
Q	Quantile Test (two-sample)
Q1	Quantile Test (one-sample)
TEDE	total effective dose equivalent
WRS	Wilcoxon Rank Sum Test
WSR	Wilcoxon Signed Ranks Test

FOREWORD

The NRC is amending its regulations to establish residual radioactivity criteria for decommissioning of licensed nuclear facilities. As part of this initiative, the NRC staff is evaluating the application of nonparametric statistical methods as an alternative to the parametric statistical approach described in the U.S. Nuclear Regulatory Commission (NRC) draft report NUREG/CR-5849, entitled, "Manual for Conducting Radiological Surveys in Support of License Termination." The nonparametric statistical approach described in this report is expected to be simpler and more cost-effective for the design and analysis of final status decommissioning surveys when radiological criteria for decommissioning approach background radiation levels. This report also shows the advantages of using the Data Quality Objectives process as it relates to the planning and analysis of final site surveys. The application of the proposed DQO process includes methods for determining the number of samples needed to obtain statistically valid comparisons with decommissioning criteria and the methods for conducting the statistical tests with the resulting sample data.

This draft report introduces new concepts that are being considered for determining compliance with proposed radiological criteria for decommissioning. The results, approaches and/or methods described herein are provided for information only.

Written comments should be addressed to: Chief, Rules Review and Directives Branch, Division of Freedom of Information and Publications Service, Office of Administration, U.S. Nuclear Regulatory Commission, Washington, DC 20555-0001. Hand deliver comments to: 11545 Rockville Pike, Rockville, Maryland, between 7:15 a.m. and 4:30 p.m. on Federal workdays.

Comments may be submitted electronically, in either ASCII text or WordPerfect format, by calling the NRC Enhanced Participatory Rulemaking on Radiological Criteria for Decommissioning Electronic Bulletin Board, 1-800-880-6091 (see *Federal Register* Vol.58, No.132, July 13, 1993). The bulletin board may be accessed using a personal computer, a modem, and most commonly available communications software packages. Communication software parameters should be set as follows: parity to none, data bits to 8, and stop bits to 1 (N,8,1). Use ANSI or VT-100 terminal emulation. Background documents on the rulemaking are also available for downloading and viewing on the bulletin board. For more information call Ms.Christine Daily, U.S. Nuclear Regulatory Commission, Washington, DC 20555-0001, phone (301) 415-6026; FAX (301) 415-5385.

Comments are sought specifically on the application of nonparametric statistics and the Data Quality Objectives process. Comments on this draft report will be most useful if received 60 days from its publication, but comments received after that time will also be considered.

John E. Glenn

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1 INTRODUCTION

1.1 Overview of NRC Site Decommissioning

At sites and facilities licensed by the Nuclear Regulatory Commission (NRC), the formal decommissioning process begins when a licensee decides to terminate licensed activities. The majority of licenses terminated each year by NRC involve little or no site remediation, and therefore, present no complex decommissioning problems from residual radioactivity. However, license termination at a small number of sites is far more complex because contamination may be spread into various areas within the facility and surrounding areas by the movement of materials and equipment, by activation, and by the dispersion of air, water, or other fluids through or along piping, equipment, walls, floors, and drains. Removal of contamination is expected at nuclear power plants, non-power (research and test) reactors, fuel fabrication plants, uranium hexafluoride production plants, and independent spent fuel storage installations. A small number of universities, medical institutions, radioactive source manufacturers, and companies that use radioisotopes for industrial purposes may also contain radioactive contamination that requires remediation.

NRC regulations in 10 CFR 30.36, 40.42, 50.82, 70.38, and 72.54 require licensees to remove their facilities safely from service and reduce residual radioactivity to a level that permits release of the property for unrestricted use and termination of the license. As part of the decommissioning process, licensees are required to demonstrate that residual radioactivity in facilities and environmental media has been reduced to acceptable levels. Typically, licensees demonstrate compliance with radiological criteria for decommissioning by conducting final status surveys of the site or facility and reporting the survey results to NRC for evaluation. Where appropriate, the NRC staff conducts confirmatory surveys to verify that lands and structures have been adequately remediated.

Existing radiological criteria that are used by NRC to evaluate compliance with decommissioning requirements are a patchwork of applicable regulations, guidance, and practices that were developed independently over a number of years. These criteria are usually well above background radiation levels, which results in most NRC sites being released at predicted dose levels that are small fractions of the public dose limit given in 10 CFR Part 20.

Currently, NRC is amending the regulations in 10 CFR Part 20 to include explicit radiological criteria for decommissioning. On August 22, 1994, proposed radiological criteria were published in the *Federal Register* which specify that radioactivity from licensed operations be reduced to a level as low as reasonably achievable (ALARA) below the level that would result in a 15-mrem-per-year dose to the average individual in the critical group.

To implement this criterion, final status surveys and verification surveys must be capable of detecting very low levels of residual radioactivity in the presence of background at a variety of NRC-licensed facilities and sites. An essential component of such surveys is a statistical methodology that is appropriate for radiological data at or near background levels. This document presents such a methodology.

1.2 Need for This Report

At present, the NRC staff uses guidance for conducting confirmatory radiological surveys that is contained in draft report NUREG/CR-5849, entitled, "Manual for Conducting Radiological Surveys in Support of License Termination." The statistical approach contained in the draft report NUREG/CR-5849 is based on the Student's *t*-test, which is a parametric statistical test that requires survey data to fit either a normal or log-normal distribution. Past survey experience has shown that radiological data at or near background may not meet this assumption.

Thus, an alternative statistical approach is being considered for conducting radiological surveys at or near background. The nonparametric statistical techniques described in this draft report do not require the data to be normally or log-normally distributed and are, therefore, expected to be more appropriate for determining the number of samples required for radiological surveys and analyzing data collected at or near background levels. These tests perform almost as well as parametric tests even when the data are normally distributed, and handle "non-detects" in a better way.

1.3 Objective of This Report

The objective of this draft report is to describe a proposed nonparametric statistical methodology that the NRC staff is evaluating for demonstrating compliance with the proposed radiological criteria for decommissioning. This draft report also describes the Data Quality Objectives (DQO) process as it relates to the planning and analysis of final site surveys. The alternative statistical approach described in this report is expected to be a resource-efficient solution for the design and analysis of final status decommissioning surveys when radiological criteria for decommissioning approach background levels. The proposed process includes methods for determining the number of samples needed to obtain statistically valid comparisons with decommissioning criteria and the methods for conducting the statistical tests with the resulting sample data. An additional objective is to enumerate open issues that require resolution in proposed future research related to the further development of a comprehensive statistical and survey methodology.

This report builds upon information contained in previously published documents (see Section 8). In preparing this draft report, it is assumed that readers possess a basic understanding of statistics and radiological survey procedures, and that implementation of the basic statistical methodology described in this document will be accompanied by sound professional judgment according to the principles of the Data Quality Objectives (EPA QA/G-4) and Data Quality Assessment (EPA QA/G-9) processes.

1.4 Structure of This Report

This report is divided into nine sections, each building on information contained in the previous section(s), and three appendices. This first section is an introduction, and Section 2 is an overview of the statistical concepts used in this report. Section 3 contains a discussion of the

Data Quality Objectives process and how it pertains to planning final status surveys. Section 4 is an overview of the particular survey instruments and methods that can be used in implementing surveys in support of decommissioning.

A detailed explanation of the statistical methods to be used in evaluating a site relative to the proposed decommissioning criteria is contained in Sections 5 and 6. Section 5 addresses tests to be used when the radionuclide in question also appears as part of background, or when non-radionuclide-specific measurements, such as total alpha, beta, or gamma count rates or total exposure rate, are made. Section 6 addresses tests to be used when the radionuclide in question does *not* appear as part of background and radionuclide-specific measurements are made.

Section 7 summarizes key information from previous sections and contains recommendations for implementing NRC requirements on the residual radiological criterion for decommissioning.

Section 8 provides a bibliography of related reference literature from a variety of sources and Section 9 contains a glossary of terms.

Appendix A contains the statistical tables needed to perform the analyses described in this report, Appendix B contains a checklist for conducting final status surveys, and Appendix C contains tables of area factors that can be used to conduct the elevated measurement comparison described in this report.

2 OVERVIEW OF THE STATISTICAL APPROACH

2.1 Introduction

It is recognized that demonstrating that residual concentrations of radioactivity at a site are at very low levels in the presence of background is a complex task involving sophisticated sampling, measurement, and statistical analysis techniques. The difficulty of the task can vary substantially depending on a number of factors, including the radionuclides in question, the background level for those and other radionuclides at the site, and the temporal and spatial variations in background at or near the site. The nonparametric statistical approach described in this report requires that sufficient radiological data must be collected to characterize both the residual radioactivity at the site and the background radioactivity levels in the vicinity of the site. The number of measurements required to accomplish this task will be determined on a site-specific basis and will depend upon the nature of the facility, its size, the selection of the statistical tests used, and certain statistical parameter values that influence how compliance with radiological criteria is determined.

2.1.1 Radionuclides Occurring as Part of Background

For radionuclides that occur as part of background, it is necessary to establish what the background activity concentrations are in the vicinity of the site. This will entail conducting radiological surveys in one or more reference areas to produce sufficient data to determine the radiological characteristics of background.

Criteria for selecting reference areas are discussed in Section 2.3.6. It is recommended that the survey methodology used to characterize background is consistent with the survey methodology used to define radiological conditions at the site, so that site areas and reference areas can be evaluated with the same statistical approach. The selection of the background reference area and the measurement locations within it should also meet strict criteria to minimize biases in the comparison. For example, the same sampling procedure, measurement techniques, and instrumentation should be used at both the remediated area and the reference area.

Following evaluation of the reference area, the site survey is designed to support a comparison of the concentration distribution of the radionuclide(s) at the site to the background concentration distribution for that radionuclide(s) in a reference area. Using the nonparametric statistical techniques described in Section 5, the distributions of background and residual radioactivity levels would then be compared to determine whether the difference between the two distributions is distinguishable. If the concentration distributions meet NRC requirements at acceptable error rates, then the site is acceptable for either unrestricted release or restricted release. The unrestricted release criteria, as defined in proposed 10 CFR 20.1404, is that residual radioactivity that is distinguishable from background radiation results in a total effective dose equivalent (TEDE) to the average member of the critical group that does not exceed 15 mrem per year and that residual radioactivity has been reduced to levels that are as low as reasonably achievable (ALARA). The corresponding dose limits for restricted release are 100 mrem per year and ALARA, as defined in proposed 10 CFR 20.1405.

2.1.2 Radionuclides Not Occurring as Part of Background

A different approach is applied at sites where licensed materials do not occur in background. In such cases, the site survey should be designed so that the dose resulting from a given concentration of the particular radionuclide can be compared to the specific dose limits of 10 CFR 20.1404. The radionuclide concentrations corresponding to those limits can be calculated by applying the default scenarios in NUREG/CR-5512 Volume 1 and determining the concentration of residual radioactivity that would result in a dose to the average member of the critical group of 15 mrem per year. These default calculations have been performed and the results are shown in Tables B-1 and B-2 in Appendix B of NUREG-1500. The nonparametric statistical tests that may be used to compare the concentration of residual radioactivity to a specific limit are described in Section 6.

2.1.3 Radionuclide-Specific Measurements

The discussion in Section 2.1.2 assumes that radionuclide-specific survey methods are used. If other survey methods are used, such as gross activity or exposure rate measurements, then the individual contributions due to background and any residual radioactivity will not be separately identifiable. For example, if Co-60 were the radionuclide of concern, and a survey of total exposure rate was made with an ionization chamber, the contribution to the ionization by Co-60 gamma-rays will not differ in character from the ionization due to gamma-rays from natural radionuclides. If present, the Co-60 would be detectable only as an increase in exposure rate compared to a suitable reference area. Thus, the analysis would have to proceed as if the contamination occurred as part of background using the techniques of Section 5.

Depending on the level of residual activity that it is necessary to detect, many more measurements may be required if gross activity or exposure rate measurements are used than if radionuclide-specific measurements are made. At very low levels, it may be difficult or impossible to distinguish the Co-60 contribution unless radionuclide-specific methods are used.

2.2 Nonparametric Statistics

The basic distinction between parametric and nonparametric statistical techniques is that parametric techniques use specific assumptions about the probability distributions of the radiological data. For parametric statistical techniques, the most common assumption is that the data fit a normal distribution. Additional data and statistical tests would generally be necessary in order to show that this assumption is justified (EPA QA/G-9). Nonparametric techniques (sometimes referred to as distribution-free statistical methods) can be used without regard to the underlying distribution. Thus, nonparametric techniques are appropriate in situations when the probability distribution of the data is either unknown or is some continuous distribution other than the normal distribution.

For survey measurements at or near background, there may be some measurement data which are at or below instrumental detection limits. Such data are not easily treated using parametric methods. Nonparametric techniques are often a better approach to making inferences from such data.

That a statistical approach is nonparametric or distribution free does not imply that it is free of any and all assumptions about the data distribution. Most nonparametric procedures require that measured values be independent and identically distributed. The requisite assumptions for the statistical tests discussed in this report should be carefully checked using the methods of Data Quality Assessment (EPA QA/G-9) before they are applied. Some of these methods are discussed in Section 4.

Many nonparametric techniques are based on ranking the measurement data. The data are ordered from smallest to largest, and assigned the numbers (ranks) 1, 2, 3,... accordingly. The analyses are then performed on the ranks rather than on the original measurement values. The advantage of this approach is that the probability that one measurement is larger than another can be computed exactly by combinatorial (enumeration and counting) methods without reference to a specific probability distribution. Parametric methods rely on assumptions about the data distribution to infer how large the difference between two measurements is expected to be. These methods are better only if the assumptions are true. If the assumptions are not true, the nonparametric methods described in this report will generally produce the correct decision more often than the parametric ones. The proposed nonparametric tests perform nearly as well as the corresponding parametric tests, even when the conditions necessary for applying the parametric tests are fulfilled. Thus, it is possible to apply nonparametric methods in all cases. The relative insensitivity to departures from underlying assumptions of certain statistical methods is called "robustness." This report primarily considers robust nonparametric procedures based on measurement data ranking.

There are many nonparametric techniques that can be used for determining whether residual radioactivity is distinguishable from background. Any one test may perform better or worse than others, depending on the hypotheses to be tested, i.e., the decision that is to be made and the alternative. For example, the Wilcoxon Rank Sum (WRS) test performs well when the decision is whether or not a degree of contamination remains throughout the entire decommissioning site. In comparison, the Quantile test performs well at uncovering smaller areas with somewhat higher contamination concentrations. Thus, in a given area, for a given total excess radioactivity, the WRS test will be better if the excess radioactivity is spread uniformly across the site and the Quantile test will be better when this excess radioactivity is concentrated in a few areas within the site, assuming an adequate number of samples are taken.

Because of the tradeoffs among nonparametric techniques, the NRC staff recommends that two tests and an *elevated measurement comparison* be conducted for each survey unit. The Wilcoxon Rank Sum (or Wilcoxon Signed Ranks) test is selected for its ability to detect uniform failure of remediation activities throughout a survey unit. The Quantile test is chosen to detect when remediation activities have failed in only a few areas within a survey unit. The additional comparison is recommended to determine if there are any individual measurements that exceed a predetermined upper limit. This comparison acts as a "fail-safe" to ensure that any unusually high measurement is investigated further to determine the cause. A brief description of each of these tests is given below. More detailed information on the use of these tests is given in Sections 5 and 6.

2.2.1 Wilcoxon Rank Sum and Signed Ranks Tests

The Wilcoxon Rank Sum (WRS) test and Wilcoxon Signed Ranks (WSR) test are used to detect a uniform shift in the median of a distribution of measurements. The WRS test is a two-sample test that compares the median of a set of measurements in a survey unit to that of a set of measurements in a reference area. The WSR test is a one-sample test that compares the median of a set of measurements in a survey unit to a fixed value, namely the derived concentration limit for a specific radionuclide.

The WRS test, also known as the Mann-Whitney test (Conover), is performed by first listing the *combined* set of site and reference area measurements in increasing numerical order from smallest to largest. The next step is to replace the measurements by their ranks, i.e., their position number in the ordered list. Thus, the ranks are simply integer values from 1 through N, where N is the total number of combined measurements. The rank 1 is assigned to the smallest value, 2 to the second smallest observation, etc. Then, the sum of the ranks of the *survey site* measurements is computed. Because the sum of the combined ranks is a fixed constant equal to $N(N+1)/2$, the sum of the reference area measurement ranks is equal to $N(N+1)/2$ minus the sum of the ranks of the survey site measurements.

If the distribution of radioactivity for the site and background are the same, then any given rank is equally likely to belong to either a reference area measurement or a survey unit measurement. Thus, there is no reason to believe that the average of the survey unit ranks will differ greatly from the average of the reference area ranks. If the site is clean, the probability that the average of the site ranks will be larger than the average of the background ranks is 50 percent by random chance. However, the larger the average of the site ranks, the smaller the probability that it is by chance, and the greater the evidence that the site is contaminated. If the average of the site ranks exceeds a calculated critical value, one can decide that the evidence shows that the site is not clean and does not meet the applicable decommissioning criteria.

The WSR test is performed by first subtracting the derived concentration limit from each observation. The *magnitudes* of the resulting differences are then listed in increasing numerical order, *without regard to sign* (positive or negative). Then the *ranks* of the *positive* differences are summed. Large values of this sum are evidence that the median of the survey unit measurements exceeds the derived concentration guideline.

2.2.2 Quantile Tests

As with the WRS test, the two-sample Quantile test (EPA 230-R-94-004; Johnson et al.) is performed by first listing the *combined* site and background measurements from smallest to largest. However, only the largest measurements in the list are examined. The number of measurements that will be considered in the Quantile test is denoted by "*r*." A count is made of the number of measurements among the largest *r* measurements that are from the site being surveyed for residual radioactivity. This number is denoted by "*k*." If there is no contamination, measurements from the background site and from the survey site might be expected to appear among the *r* largest measurements roughly in proportion to the number of measurements made at each of the sites. If patchy residual contamination exists, then the *r* largest measurements of the combined data sets (reference area and survey unit) are more likely to come from the survey unit.

Suppose there are *m* background measurements and *n* survey site measurements, then *k* should be about *r* times *n/(m+n)*. If the number of measurements from the survey site among the largest *r* is too much larger than this, then there is evidence that the survey unit has not been successfully decontaminated. Gilbert and Simpson have shown that the Quantile test is useful for determining whether any patchy residual contamination exists on the survey site.

Further information on the application of the two-sample Quantile test is given in Section 5.

For the one-sample version of the Quantile test, the number of survey unit measurements exceeding a fixed value is found. The fixed value is a specified percentile for the distribution of survey unit measurements. If the number of measurements exceeding this value is too large, there is evidence that the survey unit has not been adequately decontaminated.

Further information on the application of the one-sample Quantile test is given in Section 6.

2.2.3 Elevated Measurement Comparison

An *elevated measurement comparison* is performed by comparing each measurement from the survey unit to an upper limit residual radioactivity concentration *investigation level* for each radionuclide of concern. A measurement that equals or exceeds this level is an indication that a survey unit may contain residual radioactivity greater than 15 mrem over background levels. If a measurement exceeds the investigation level, additional investigation is required to determine if the decommissioning criteria have been met, regardless of the results of the Wilcoxon test and the Quantile test. A measurement that exceeds the elevated residual radioactivity concentration investigation level is considered an *elevated measurement*.

The elevated measurement comparison is sometimes called a "hot spot test." The latter term may be misleading because it is not a formal statistical test, but a simple comparison of measured values against a limit. Also, there is not a commonly accepted definition of what constitutes a *hot spot* in either area or magnitude of residual radioactivity, yet this term may imply some degree of radiological hazard.

There are several levels of residual radioactivity concentration heterogeneity that may occur in a survey unit:

Uniform Residual Radioactivity - Since residual radioactivity levels are characterized by a distribution around a mean, even in areas of relatively uniform residual radioactivity some measurements will necessarily exceed the mean. These random fluctuations are of no concern provided the mean residual radioactivity level satisfies the Wilcoxon tests for meeting the decommissioning criteria.

Moderately Non-Uniform Residual Radioactivity - Moderate departures from uniformity in residual radioactivity concentrations may exist following remediation. One portion of a measurement area may have virtually no residual radioactivity, while another portion does contain some residual radioactivity. There may be several portions of one type or another in an area, resulting in a patchy contamination pattern. The existence of such a residual radioactivity pattern does not necessarily imply that remediation has been unsuccessful. The Quantile tests are designed to detect this type of residual radioactivity if it would result in the decommissioning dose criteria being exceeded.

Non-Uniform Residual Radioactivity - In this draft report, the term "area of elevated residual radioactivity" is used to describe a limited area of residual activity that may cause the decommissioning dose criteria to be exceeded. It is only these areas that might be considered *hot spots*. For planning purposes, the potential extent of an "area of elevated residual radioactivity" is based on the distance between sampling points in the survey sampling grid (see Section 5). An upper limit value is calculated so that even if all the residual radioactivity in a survey unit were located in this single area between sampling points, the dose criterion for decommissioning would still be met. Following a final survey, individual elevated measurements are flagged by an *investigation level* in order to assure that the upper limit value is not exceeded..

It should be noted that a single large measurement may occur by chance and, in some cases, both the Wilcoxon and Quantile tests may indicate that there is not sufficient evidence that decommissioning criteria have not been met. Such large measurements must be scrutinized since they may indicate very localized areas of residual contamination. The elevated measurement comparison uses an investigation level as a method designed to flag these high measurements for further study. When a measurement is flagged using this method, it should first be determined that it is not due to sampling or analysis error. Such a determination may include resampling the area at which the measurement was originally taken and, if the elevated measurement is confirmed, it would be necessary to review the history of the site and its remediation to see if other such elevated areas may exist. If the elevated measurement is confirmed, then the extent of the area of elevated residual radioactivity and the average concentration within it must be determined in order to evaluate the resulting dose. On the basis of this information, further remediation may be required, followed by an additional survey to ensure compliance with decommissioning criteria. Further information on the elevated measurement comparison and the method for determining investigation levels is discussed in Section 5.

2.3 Terminology and Statistical Concepts

This section discusses the main terms and statistical concepts that are used throughout this report. Further discussion of these concepts is provided in subsequent sections and additional statistical terms are defined in Section 5 of this report.

2.3.1 Data Quality Objectives

An essential consideration in designing survey plans for site decommissioning is that the radiological data that are collected and analyzed are sufficient and of adequate quality for decision-making purposes. It is imperative that the type and quality of radiological data that will be needed to support license termination be considered early in the decommissioning process.

Before commencement of survey work, it is essential that a survey plan be developed that is based on the data needed for decision making and the level of quality needed to support the decision. Such a plan should specify what samples need to be obtained, how and where they will be collected and analyzed, what quality assurance procedures will be used, the method of comparing site areas to reference areas, and what level of decision errors will be considered acceptable.

These decisions become paramount for determining compliance with very low decommissioning criteria because the analytical and statistical requirements are more complex and extensive than for existing radiological criteria for decommissioning. Further information on the DQO process is in Section 3.

2.3.2 Affected Area

Affected areas are areas that have potential radioactive contamination (based on plant operating history) or known radioactive contamination (based on past or preliminary radiological surveillance). This would normally include areas in which radioactive materials were used and stored, in which records indicate spills or other unusual occurrences that could have resulted in spread of contamination, and in which radioactive materials were buried. Areas immediately surrounding or adjacent to locations in which radioactive materials were used or stored, spilled, or buried are included in this classification because of the potential for inadvertent spread of contamination. The use of this term in this report is consistent with the draft report NUREG/CR-5849.

Affected areas are further divided into (1) those that are considered to have a potential for containing small areas of elevated residual activity in excess of guideline levels and (2) those in which such areas of elevated activity would not be anticipated. An area that has the potential for such a spotty residual radioactivity pattern is referred to as (1) **Affected/Non-Uniform - affected areas with potential for non-uniform residual radioactivity** or as (2) **Affected/Uniform - affected areas with little or no potential for non-uniform residual radioactivity**. *Any area that has been remediated is designated affected/non-uniform.* In general, *all* areas are treated as affected/non-uniform until substantial bases are provided to reclassify them to either affected/ uniform, unaffected areas, or areas that have no potential for residual contamination (non-impacted areas).

2.3.3 Unaffected Areas

Unaffected areas are those areas that are not expected to contain any residual radioactivity, based on a knowledge of site history and previous survey information. The criteria used for this segregation need not be as strict as those used in the final status survey, but if there is any reason to believe that there is contamination in an area, it should be designated affected. It should be recognized that as the decommissioning process progresses, an area's classification may require changing, based on accumulated survey data. However, if this reclassification becomes necessary during the final status survey, substantial revisions of the final status survey plan may be required. Thus, if there is any doubt, it is probably more cost effective in the long run to designate an area as affected.

2.3.4 Background Radiation

According to proposed 10 CFR 20.1003, **background radiation** means radiation from cosmic sources, naturally occurring radioactive material, including radon (except as a decay product of source or special nuclear material), and global fallout as it exists in the environment from the testing of nuclear explosive devices or from nuclear accidents like Chernobyl which contribute to background radiation and are not under the control of the licensee. Background radiation does not include radiation from source, byproduct, or special nuclear materials regulated by the Commission.

2.3.5 Indistinguishable From Background

According to proposed 10 CFR 20.1003, the term **indistinguishable from background** has been used to describe a level of residual radioactivity which cannot be distinguished from the background radiation present at a facility, using existing survey methods. Amounts of material that are predicted to result in a dose less than 3 mrem per year are, by the provisions of 10 CFR 20.1404, acceptable for meeting the reduced documentation requirements for demonstrating ALARA.

To apply the dose criteria of the proposed rule, the concentrations of individual radionuclides comprising the residual radioactivity at a site are compared to the concentrations of those same radionuclides present in local background areas that have been matched to the site in terms of geological, chemical, and biological attributes, but which have not been affected by site operations. This comparison establishes a site-specific criterion for individual radionuclides that is dependent on the local variability of background. The distribution of residual radioactivity that is measured in affected areas on site is compared to the distribution of background radionuclides measured in reference areas. Compliance depends on the distributions being statistically indistinguishable at the concentration level corresponding to the dose criterion of 15 mrem per year above background. The implementation of this criterion will vary depending on the background level for all radionuclides at the site, the temporal and spatial variations in background at the site, and the radionuclides under investigation.

2.3.6 Reference Area

A **reference area** (or background area) is a geographical area from which representative samples of background will be selected for comparison with samples collected in specific survey units at the remediated site. The reference area should have similar physical, chemical, radiological, and biological characteristics to the site area being remediated, but should not have been contaminated by site activities. The reference area is where background would be measured and defined for the purpose of decommissioning. The distribution of background radiation and radioactivity in the reference area should be the same as that which would be expected on the site if that site had never been contaminated. It may be necessary to select more than one reference area for a specific site, if the site includes so much physical, chemical, radiological, or biological variability that it cannot be represented by a single reference background area.

2.3.7 Survey Unit

A **survey unit** (or cleanup unit) is an area of specified size and shape at a site for which a separate decision will be made as to whether decontamination has been sufficient for decommissioning. Following remediation, the site will be segregated into areas that are affected/non-uniform, affected/uniform, or unaffected. The affected areas of the remediated site will be divided, when necessary, into survey units. For radionuclides that occur as part of background, statistical tests are applied to compare each survey unit with an appropriately chosen, site-specific reference area. Reference areas will be chosen on the basis of their similarity to given survey units in all respects other than having been contaminated. For radionuclides that do not occur as part of background, the comparison is made directly to a radionuclide concentration or dose limit that has been established for the site.

To facilitate survey design and assure that the number of survey data points for a specific site is relatively uniformly distributed among areas of similar contamination potential, the site is divided into survey units which have common history or other common characteristics or are naturally distinguishable from other portions of the site. Such survey units may combine contiguous rooms or land areas having the same contamination potential. A single survey unit cannot contain both affected and unaffected areas, nor may it consist of affected areas of differing potential for containing elevated measurement areas. Indoor survey units that are affected/non-uniform will generally consist of a single room.

The size of a survey unit is based on its contamination potential, as shown in Table 2.1. The unaffected areas of a licensed facility may consist of a single survey unit of unlimited size.

Table 2.1 Typical Survey Unit Sizes for Affected Areas

Affected Area	Survey Unit Sizes (m^2)			
	Outdoor		Indoor	
	Typical Maximum	Typical Minimum	Typical Maximum	Typical Minimum
Non-Uniform	2000	100	100	10
Uniform	2000-10000	100	100-1000	10

2.3.8 Null and Alternative Hypotheses

The decisions necessary to determine compliance with the criteria for license termination are formulated into precise statistical statements called hypotheses. The truth of these hypotheses can be tested with the survey data. The state that is presumed to exist in reality is expressed as the null hypothesis (denoted by $\mathbf{H_o}$). For a given null hypothesis, there may be specified many alternative hypotheses (denoted as $\mathbf{H_a}$), which are expressions of what is believed to be the possible states of reality if the null hypothesis is not true.

For the purposes of this report, the important decision is whether or not a site meets the applicable decommissioning criteria. This decision will be supported by the individual decisions on whether each survey unit meets the applicable decommissioning critieria. In this report, the null hypothesis, H_o, is that the survey unit meets the applicable decommissioning criteria. The reasons for this choice are discussed in Section 3.6.

The alternative hypothesis is that the survey unit does *not* meet the applicable decommissioning criteria. This means that there is evidence in the data that the survey unit does not meet the criteria outlined in Section 2.3.5. The specific alternative hypothesis is constructed by choosing that dose distinguishable from background which is important to detect.

The precise formulation of null and alternative statistical hypotheses is discussed further in the following sections.

2.3.9 Decision Errors

Errors can be made when making site remediation decisions. The use of statistical methods allows for controlling the probability of making decision errors. When designing a statistical test, acceptable error rates for incorrectly determining that a site meets or does not meet the applicable decommissioning criteria must be specified. In determining these error rates, consideration should be given to the number of sample data points that are necessary to achieve them. Lower error rates (or greater levels of confidence and statistical power of the test) require more measurements. More information on the specification of error rates is given in Section 3.6.

2.3.9.1 Type I Errors

There are two types of decision errors that can be made when performing the statistical tests described in this draft report. The first type of decision error, called a Type I error, occurs when the null hypothesis is rejected when it is actually true. A Type I error is sometimes called a "false positive." This error would occur if it were concluded from survey data that the survey unit had not been successfully remediated when it actually had been. The probability of a Type I error is usually denoted by α. The Type I error rate is often referred to as the significance level or size of the test.

2.3.9.2 Type II Errors

The second type of decision error, called a Type II error, occurs when the null hypothesis is not rejected when it is actually false. A Type II error is sometimes called a "false negative." This error would occur if it were concluded from survey data that the survey unit had been successfully decontaminated when it actually had not been.The probability of a Type II error is usually denoted by β.

The Type II error rate of a test can only be calculated once the hypothetical distribution of survey data under the alternative hypothesis has been completely specified. For the Wilcoxon Rank Sum test, the distribution of the survey data under the alternative hypothesis consists of the background distribution of the radioactivity plus a constant added amount of radioactivity that corresponds to a dose of 15 mrem per year. For the Quantile test, the distribution under the alternative hypothesis is a mixture of the background distribution over most of the survey unit combined with a residual radioactivity distribution over a smaller area sufficient to deliver 15 mrem per year. Because of the different alternatives specified, the WRS test is better able to detect the presence of uniform residual radioactivity, while the Quantile test is better able to detect patchy contamination.

2.3.9.3 Confidence Interval

Previous guidance (NUREG/CR-5849) used the concept of a confidence interval for determining compliance with decommissioning criteria. The hypothesis tests described in this report provide equivalent results. However, the hypothesis testing framework is more flexible because both Type I and Type II error rates can be controlled. In constructing a confidence interval, only one of these errors is controlled.

A hypothesis test is always based on the value of a test statistic, i.e., some function of the observed data. For any test, a confidence interval for the true value of the parameter being estimated by the test statistic can be constructed from all of the values of the test statistic that would not result in a rejection of the null hypothesis. If the Type I error rate of the test is α, the probability that the value of the parameter specified in the null hypothesis of the test lies in the confidence interval is $1-\alpha$. In that case the confidence level of the confidence interval is $1-\alpha$. For this reason, $1-\alpha$ is sometimes mistakenly referred to as the confidence level of the test.

Conversely, a confidence interval may be used to construct a hypothesis test. For example, in NUREG/CR-5849 (Section 8, "Interpretation of Survey Results"), a 95-percent confidence interval for the mean of (assumed) normally-distributed survey measurements is constructed using tabulated values of Student's *t* statistic. The upper end point of this interval is compared to guideline value. This procedure is equivalent to conducting a one-sided Student's *t*-test with $\alpha=0.05$.

2.3.9.4 Power

The power of a statistical test is defined as the probability of rejecting the null hypotheses when it is false. It is numerically equal to $1-\beta$, where β is the Type II error rate. More simply, it is the ability of the test to detect when a survey unit does not meet the decommissioning criteria. Therefore, it is desirable for a test to have high power. The power of the statistical tests described in this report will tend to increase as the amount of residual radioactivity in a survey unit increases. The concepts discussed above are summarized in Table 2.2.

Table 2.2 Summary of Types of Decision Errors

	True Condition	
Decision Based on Sample Data	**Standard Achieved**	**Standard Not Achieved**
Standard Achieved	Correct Decision (Probability = $1-\alpha$)	Type II Error (Probability = β)
Standard Not Achieved	Type I Error (Probability = α)	Correct Decision (Power = $1-\beta$)

2.3.9.5 Example: Detection Limits

The following example illustrates the use of the concepts discussed above as currently used in the determination of detection limits for radioactivity measurements. This calculation, which is generally familiar to radiation protection professionals, also involves hypothesis testing (HPSR/EPA 520/1-80-012; NUREG/CR-4007; Currie 1968). In this situation, there is a measurement error, often taken to be the Poisson counting error, σ, equal to the square root of the number of counts. There is a background counting rate, and any additional radioactivity in a sample must be distinguishable above that. Generally it is assumed that the number of counts is sufficiently large so that a normal appproximation to the Poisson distribution of counts is appropriate.

For this calculation,

<u>Null Hypothesis</u>
H_0: The sample contains no radioactivity above background.

<u>Alternative Hypothesis</u>
H_a: The sample contains added radioactivity at or above the detection limit.

The count obtained from the sample measurement is the test statistic, and it has a different probability distribution under the null and alternative hypothesis (see Figure 2.1). If a sample that contains no radioactivity above background is declared to contain radioactivity above the detection limit, a Type I error is made. Conversely, if a sample that contains radioactivity above the limit is declared to contain no radioactivity above background, a Type II error is made.

The Type I error rate, α, depends on the variability of background, i.e., it is controlled by requiring that the net counts exceed a certain multiple of the measurement standard deviation. Under the null hypothesis, namely when there is no radioactivity above background, the n counts have mean $0 = B - B$.

The standard deviation is

$$\sigma_{B-B} = \sqrt{B + B} = \sqrt{\sigma^2 + \sigma^2} = \sqrt{2}\,\sigma \qquad (2\text{-}1)$$

where B is the background count, and $\sigma = \sqrt{B}$ is its standard deviation. The normal distribution is used to approximate the Poisson distribution of the background counts. This determines the critical level

$$L_C = Z_{1-\alpha}\,\sigma_{B-B} = Z_{1-\alpha}\sqrt{2}\,\sigma \qquad (2\text{-}2)$$

$Z_{1-\alpha}$ is the 1-α percentile of a standard normal distribution, e.g. if $\alpha = 0.05$, then $Z_{1-\alpha} = 1.645$. Note that the distribution of background counts (lefthand curve in Figure 2.1) is used for this calculation.

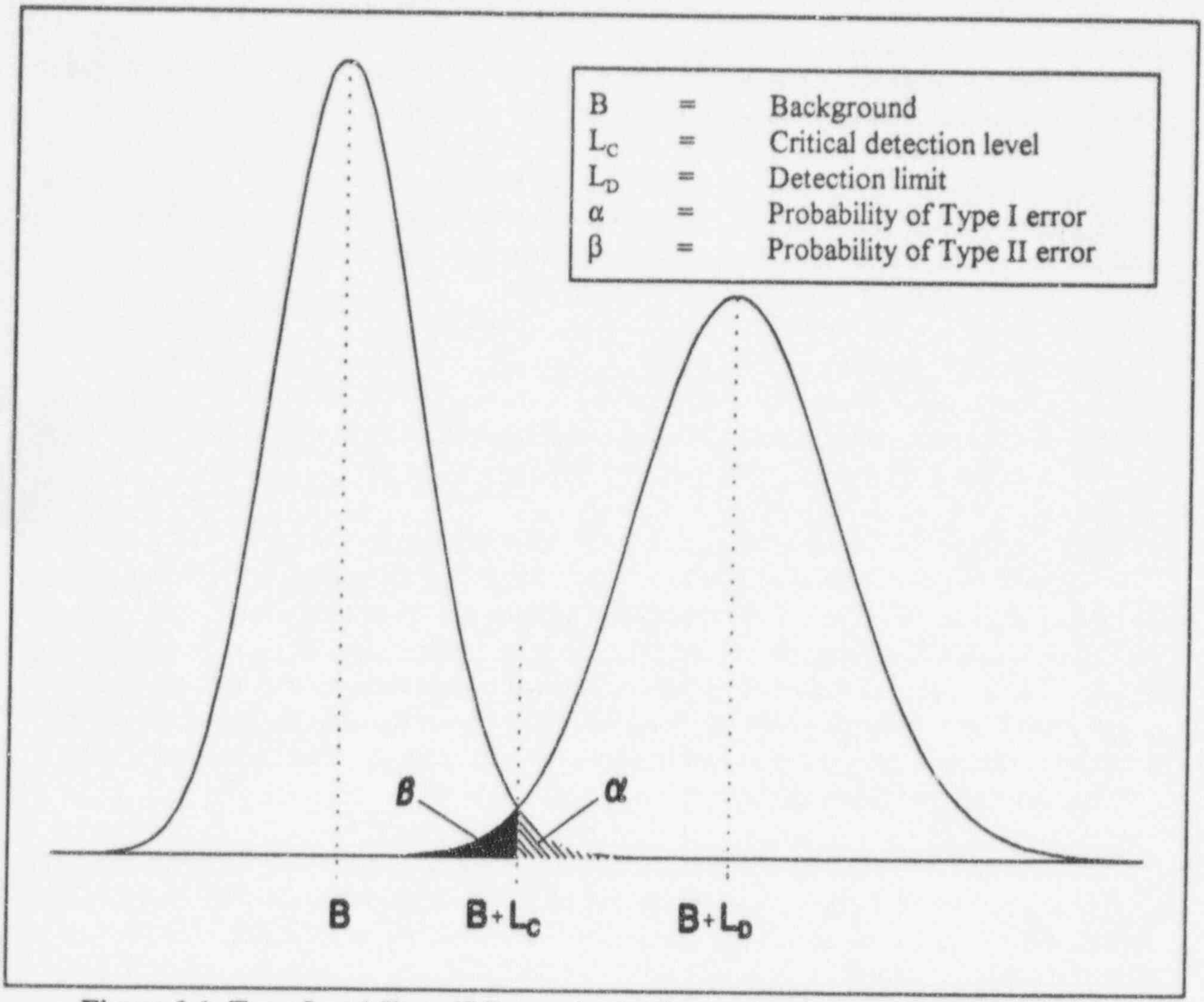


Figure 2.1 Type I and Type II Errors in the Determination of a Detection Limit

The Type II error rate, β, depends on the variability of the added radioactivity and is controlled by requiring that the total counts exceed a certain number of standard deviations above the critical level.

$$L_D = L_C + Z_{1-\beta}\,\sigma_{L_D} = Z_{1-\alpha}\sqrt{2}\,\sigma + Z_{1-\beta}\,\sigma_{L_D} = Z_{1-\alpha}\sqrt{2}\,\sigma + Z_{1-\beta}\sqrt{(L_D+(L_C/Z_{1-\alpha})^2} \qquad (2\text{-}3)$$

The distribution of counts under the alternative hypothesis (right hand curve in Figure 2.1) is used to derive Equation 2-3. If the Type II error is set the same as the Type I error, then $Z_{1-\alpha} = Z_{1-\beta} = k$. Then solving Equation 2-3 for L_D, the count detection limit is found to be

$$L_D = k^2 + 2k\sqrt{2}\,\sigma = k^2 + 2\,L_C \qquad (2\text{-}4)$$

The power 1 - β, is the probability that the measurement will indicate the presence of additional radioactivity in the sample, when the sample actually contains additional activity in the amount necessary to produce an average of L_D counts above background during the measurement.

The statistical procedures described in this report have many similarities to the detection limit calculation:

(1) Null Hypothesis
H_0: The sample contains no radioactivity above background becomes
H_0: The site contains no residual radioactivity above the decommisioning criteria (i.e., the site meets the decommissioning criteria).

(2) Alternative Hypothesis
H_a: The sample contains added radioactivity above the detection limit becomes
H_a: The site contains residual radioactivity above the decommisioning criteria (i.e., the site does not meet the decommissioning criteria).

(3) The Type I error rate (false positives) is computed using the distribution of counts under the null hypothesis. Similarly, the Type I error rates for the tests described in this report will be calculated using the distribution of the test statistic under the null hypothesis.

(4) The Type II error rate (false negatives) is computed using the distribution of counts under the alternative hypothesis. Similarly, the Type II error rates for the tests described in this report will be calculated using the distribution of the test statistic under the alternative hypothesis. This also gives the power of the tests.

(5) The variability of the count obtained from the sample, σ, plays a crucial role in determining the value of the detection limit. Similarly, the variability of the radioactivity measurements in the reference areas and survey units play a crucial role in how well the tests described in this report will perform.

(6) The detection limit can usually be made lower by counting for a longer time, thereby reducing the relative measurement error, at additional cost. Similarly, the ability of the tests described in this report to distinguish smaller amounts of residual radioactivity from background more accurately can be improved by taking a greater number of samples, at additional cost.

(7) Usually, a detection limit is calculated given the Type I and Type II error rates and the background variability. However, if a certain detection limit is pre-specified instead, the procedure above shows how to relate it to the Type I and Type II errors, and the measurement variability. Similarly, the procedures of this report will show the interrelationship of the decommisioning criteria (dose above background) the Type I and Type II errors, and the measurement variability.

The Data Quality Objectives (DQO) process described in Section 3 provides a general method for designing surveys so that accurate remediation decisions can be made cost effectively. Sections 5 and 6 describe the mathematical relationships between the error rates, residual radioactivity levels, measurement variability, and the number of samples required for the statistical tests.

3 DATA QUALITY OBJECTIVES FOR FINAL STATUS SURVEYS

3.1 Introduction

The Data Quality Objectives (DQO) process is a series of planning steps based on the scientific method that is designed to ensure that the type, quantity, and quality of environmental data used in decision making are appropriate for the intended application (EPA QA/G-4). DQOs are qualitative and quantitative statements that

- clarify the study objective
- define the most appropriate data to collect
- determine the most appropriate conditions for collecting the data and
- specify acceptable levels of decision errors that will be used as the basis for establishing the quantity and quality of data needed to support the decision.

The DQO process comprises the following steps:

(1) State the problem, i.e., the objective of the sampling effort.
(2) Identify the decision, i.e., the decision to be made that requires new data
(3) Identify inputs to the decision, i.e., the reasons the new radiological data are needed and how they will be used to support the decision.
(4) Define the study boundaries, i.e., the spatial and temporal aspects of the environmental media that the radiological data represent.
(5) Develop a decision rule, i.e., an "if...then" statement that defines the conditions for choice among alternative actions.
(6) Specify limits on decision errors.
(7) Optimize the design for obtaining data, i.e., the most time- and resource-effective sampling and analysis plan.

All of the these items should be addressed when planning a sampling program to test for the attainment of decommissioning criteria. For most NRC licensees, the objective of the decommissioning process is to remove their facilities safely from service and reduce residual radioactivity to a level that permits release of the property for unrestricted use and termination of the license. The data that will be needed to support this objective will demonstrate that any residual radioactivity remaining on the site results in a dose that does not exceed 15 mrem per year above background. It is important to specify the type and quality of radiological data that will be needed for final status surveys early in the decommissioning process. This process entails early specification of sample collection and analysis procedures, the method of comparing site areas to reference areas, the null and alternative hypotheses, Type I and Type II error rates, quality assurance procedures, and other parameters.

In the following sections, each of the seven steps in the DQO process is discussed as it pertains to the decommissioning process in general, and the planning, design, and performance of the final status survey in particular. Recommendations for measurement methods for radiological surveys in support of decommissioning are developed in a companion report (NUREG-1506.)

3.2 Stating the Problem

The initial step in the decommissioning process is a preliminary assessment of the radiological status of the site. This assessment consists of identifying potential residual radioactive materials, establishing the applicable release criteria, or, if default criteria apply (cf. NUREG-1500), determining the general locations and extent of residual radioactivity, and estimating the levels of residual radioactivity. Information from this assessment is the basis for the licensee's decommissioning plan and the design for subsequent radiological surveys. In the following sections, the specific requirements of a final status survey will be addressed.

The product of this step in the DQO process should be a fairly complete description of the decommissioning problem and should include a summary of historical data, a site conceptual map, identification of the critical group, and an estimation of the resources that will be used for radiological surveys. The information gathered to this point may also be used to support a decision on whether or not to attempt to have the site released for unrestricted versus restricted release. Information from scoping surveys (see below) and the results of preliminary dose assessments should also be used to develop a description of the radiological conditions of the site or for decision-making purposes. The following sections describe some of the activities involved in the first step of the DQO process.

3.2.1 Gather General Site Information

Use should be made of all data that may be available, provided there is evidence of reliability, i.e., that the data quality "can be documented, evaluated and believed" (Taylor). Sources of information may include license operating records, documentation supporting license amendment applications, interviews with employees and others who may be familiar with past operations, radionuclides used or produced on site, radionuclides that could be site contaminants, site environmental data/reports, incidents or unusual occurrence reports; locations of likely residual activity; and past and present results of radiological modeling. It may be useful to summarize this information in an overview report.

3.2.2 Develop a Conceptual Site Model

A site diagram should be developed locating where contamination exists, type of radionuclides in the affected areas, concentrations of radionuclides in the affected areas, potentially contaminated media and migration pathways, and locations of potential reference (background) areas.

3.2.3 Use of Dose Assessment Models

Licensees should consider the entire applicable source term and all credible dose pathways for determining compliance with decommissioning criteria. Actual site survey measurements are preferred over modeling for determining the amount and concentration of residual radioactivity remaining at the site. To calculate the total effective dose equivalent (TEDE) from the source term for an average member of the critical group, licensees should determine the appropriate modeling approach for their site based on information contained in NUREG-1500 and NUREG/CR-5512, Volume 1.

For many sites, the first-level modeling (or "screening") described in NUREG-1500 may be applicable, in which case the default residual radioactivity concentrations listed in Tables B-1 and B-2 in Appendix B of NUREG-1500 can be used, provided that the modeling assumptions are appropriate for their site. A second, more complex, screening level may be applicable when the site being analyzed does not meet the requirements of the first level of screening. The third analysis level described in NUREG-1500 is site-specific modeling. Thus, it is useful to have prior knowledge of site characteristics to select the applicable dose assessment model.

Upon selection of the applicable modeling approach, a residual radioactivity limit is determined for the site. A comparison is then made between the residual radioactivity limit and the site survey measurements of residual radioactivity concentrations using the nonparametric statistical methodology described in this draft report or the parametric statistical methodology in the draft report NUREG/CR-5849.

3.2.4 Specify the Available Resources

Time and budgetary considerations for the decommissioning process should anticipate the number of samples that may be required for the final status survey, and the types of equipment and analyses that will be used. Such information should contain estimates of sample counting times and the time required for the receipt of analytical results and for preparation of reports. Some of the actions appropriate to consider in this activity are discussed in the draft report NUREG/CR-5849.

3.2.5 Example

As an example of the type of information to be gathered at this point in the DQO process, consider the description in Appendix D of the draft report NUREG/CR-5489, excerpted below.

3.2.5.1 Background Information

The Reference Uranium Fuel Fabrication Plant (RFF) in Yorktown, Pennsylvania was built between 1960 and 1964 and was operated from 1964 until mid 1985 by the General Nuclear Corporation. Operating under NRC license, the plant converted natural and enriched uranium hexafluoride (UF_6) to uranium oxide (UO_2), formed the UO_2 into pellets, and incorporated pellets into fuel rods and bundles. Auxiliary facilities were used to recover uranium from scrap and waste materials. Two processes were used for the UF_6 to UO_2 conversion. The primary method involved the hydrolysis of UF_6 to ammonium diuranate (ADU), which was then reduced and calcined to produce dry UO_2 powder; the secondary process was the conversion of UF_6 to U_3O_8 in a flame conversion reactor, followed by reduction to UO_2 powder in a reduction-calciner.

In 1985 the plant was shut down and nuclear materials were removed and shipped to Department of Energy facilities in Idaho Falls, Idaho. The plant remained in the shutdown state until 1986, when decommissioning efforts were initiated. Process equipment, fixtures, piping, etc., were removed and disposed of as radioactive waste. Buildings and adjacent grounds were characterized and those areas exceeding NRC guidelines for license termination were decontaminated; these efforts were completed in late 1990. This document describes the plan for conducting the final status survey of the site. Supporting information is presented in the Site Decommissioning Plan, prepared and submitted to the NRC in May 1986, and in the Characterization Survey Report, submitted in February 1988.

3.2.5.2 Site Description

The Reference Uranium Fuel Fabrication Plant is located on a total land area of approximately 470 hectares (1,160 acres); a moderate size stream (Wandering River) runs through one corner of the site (Figure 3.1). Actual plant processing facilities were on a much smaller, restricted, fenced-in area of approximately 30,000 m^2 (3 hectares). The plant area occupies a low bluff that forms a bank of the river, and several flat alluvial terraces are the main topographical features of the property. These terraces lie at average elevations of 280 to 284 meters above sea level and slope away from the river at grades of 2 to 3 percent. The river was used for disposal of acceptable liquid effluents from the onsite liquid waste systems.

The major structures in the formerly restricted processing area are the main building (with interconnected chemical/metal laboratory and uranium scrap recovery and powder warehouse rooms), an incinerator building, a maintenance building, and a filter house. Auxiliary facilities, which are located outside the fenced area, include a boiler house, a fluoride and nitrate waste treatment plant and associated lagoons, liquid chemical waste treatment lagoons, a sewage treatment plant and sanitary lagoon, and concrete uranium storage pads. The auxiliary facilities were used to recover uranium from scrap and waste materials and to recover valuable chemicals from gaseous and liquid wastes. A map of the site is shown in Figure 3.1.

During the plant's 21 years of operation, an estimated total of 0.2 Ci of radioactivity was released into the atmosphere and subsequently deposited on the site. The property also contained one small, shallow, land burial area for low-level radioactive waste. This area was operated in accordance with 10 CFR 20.304 between 1966 and 1970, receiving an estimated total activity of 0.3 Ci of uranium.

On the basis of what is known about site operations, the significant radiological contaminant is expected to be uranium on storage pads.

3.3 Identify the Decision

A number of decisions will have to be made during the decommissioning process. The general decision flow for decommissioning for unrestricted release is described in NUREG-1500. In this draft report, the flow chart illustrating the process is shown in Figure 3.2 and Figure 3.3.

The objective of the decommissioning process, as discussed in the proposed rule, is to remove a facility or site safely from service and reduce residual radioactivity to a level that permits either (1) release of the property for unrestricted use and termination of the license or (2) release of the property under restricted conditions and termination of the license. For the examples given in this report, the performance objective for the final status survey is to demonstrate that the dose due to residual contamination is less than 15 mrem per year distinguishable from background. This is

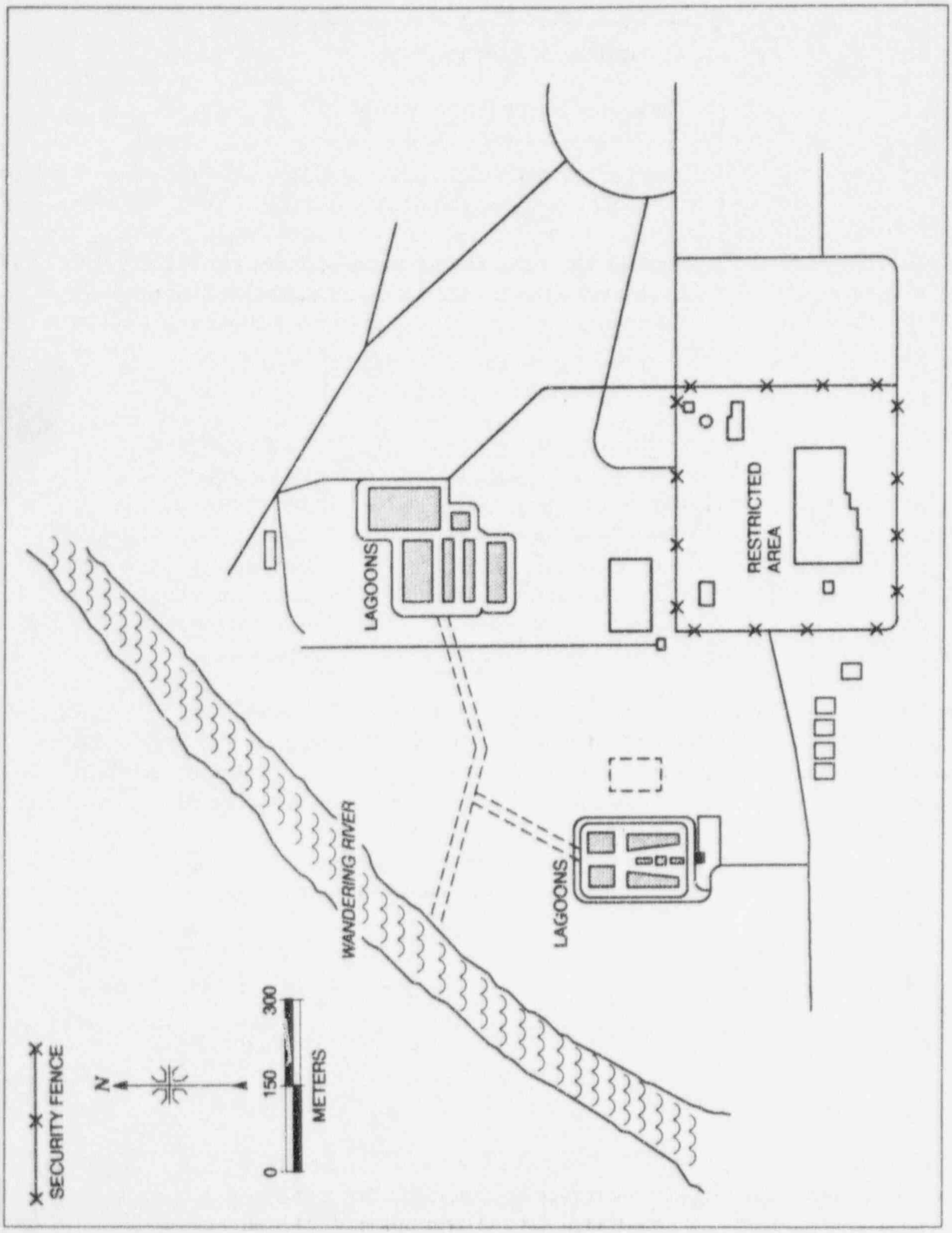


Figure 3.1 Reference Uranium Fuel Fabrication Plant Site

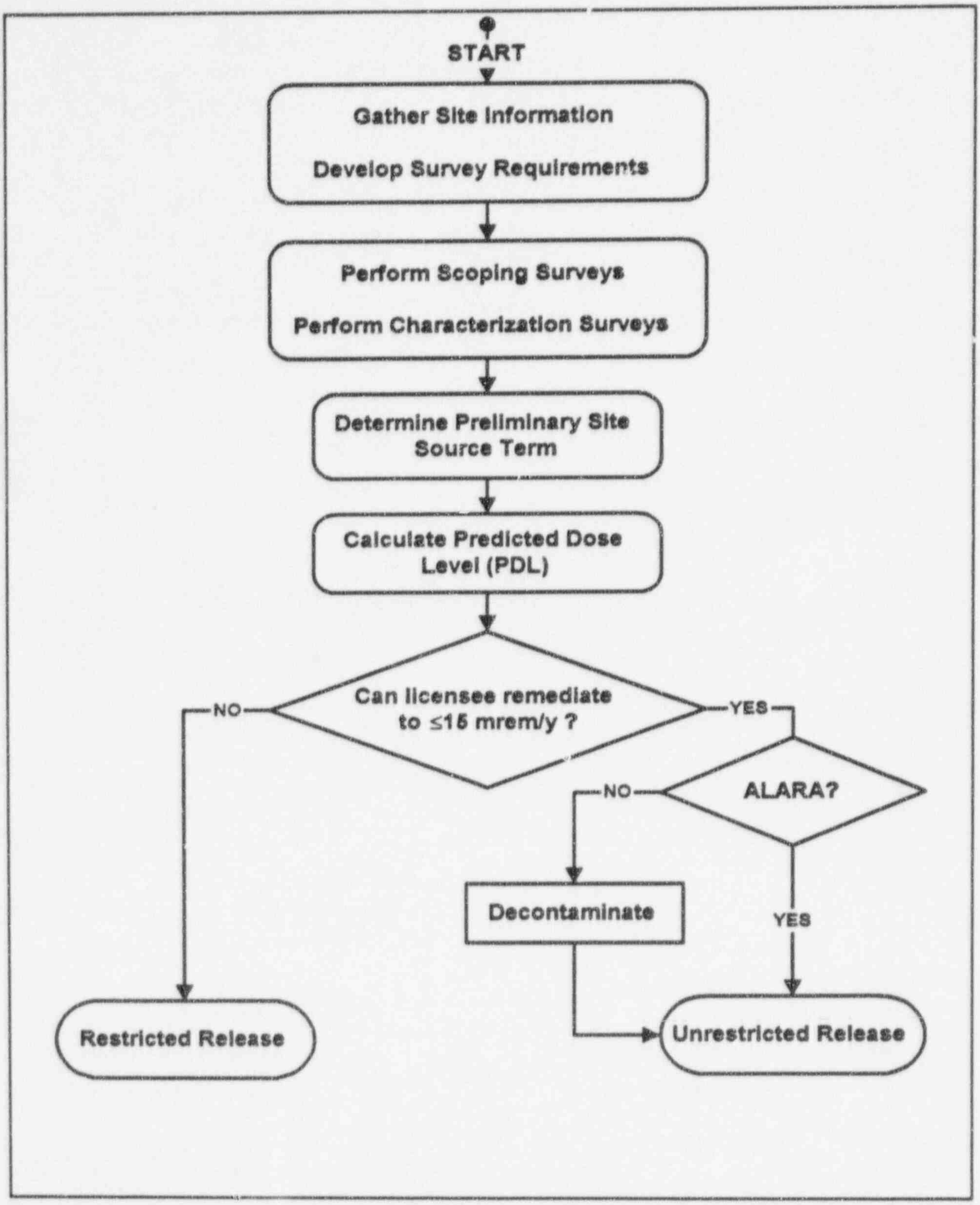


Figure 3.2 Decision Chart for Choosing Unrestricted or Restricted Release of Facilities

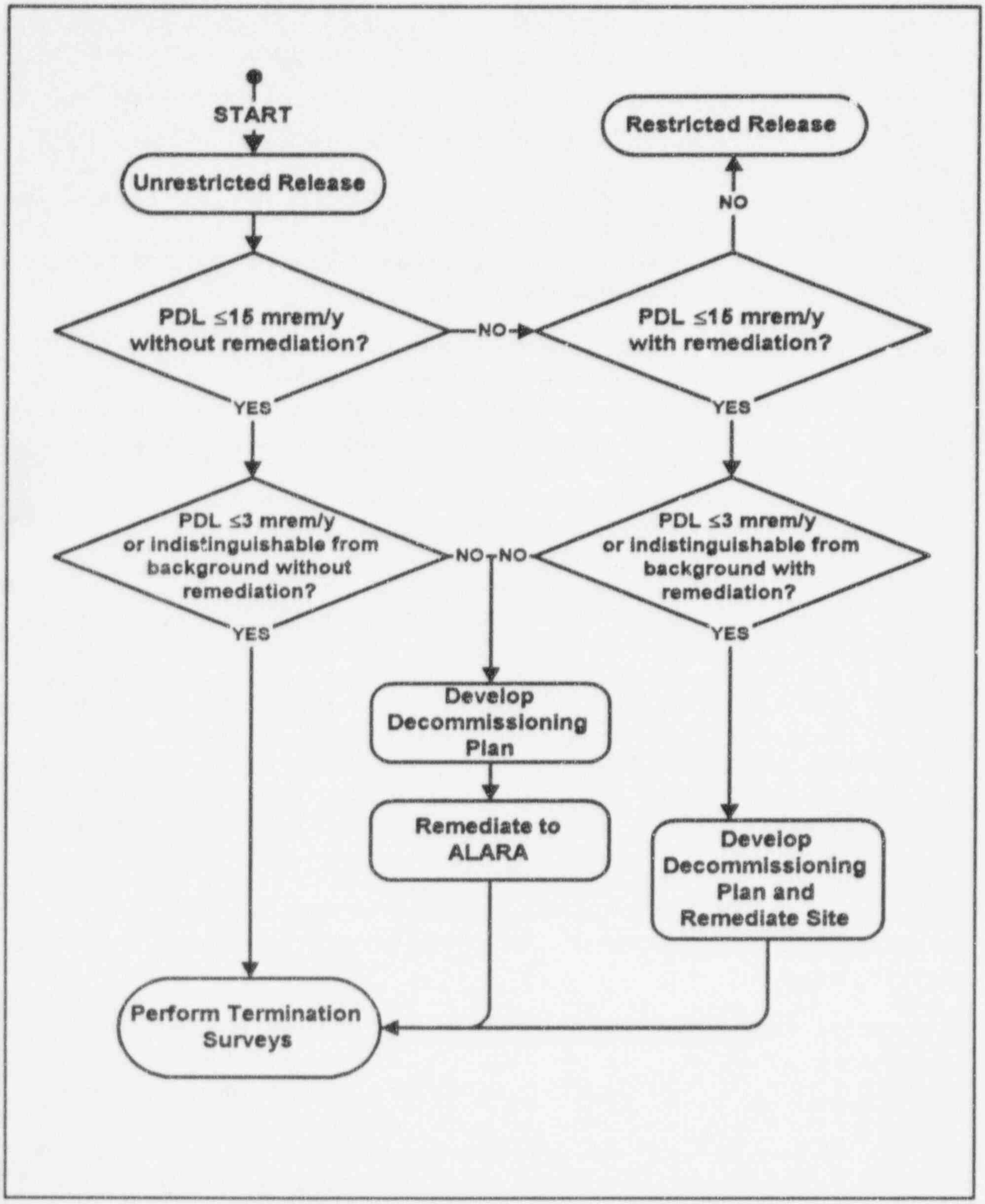


Figure 3.3 Decision Chart for Unrestricted Release

accomplished by demonstrating that the concentrations of residual radioactivity are so distributed that the resulting dose will not exceed 15 mrem per year to the average member of the critical group.

This draft report is applicable for determining whether the decommissioning was successful in meeting the applicable decommissioning criteria. The flow chart for this process is shown in Figure 3.4. *The essential decision is whether the decommissioning criteria have been met.* The decision will be based on statistical tests of radiological data collected in a survey designed for this purpose. Procedures for the design of the final status survey and for the statistical analysis of the results are the primary focus of this report.

3.4 Identify Inputs to the Decision

Although the final status survey is performed near the end of the decommissioning process, it may be possible to produce a more efficient survey design if the requirements of this survey are identified early in the decommissioning planning. By knowing in advance the type, quantity, and quality of data that are needed in the final status survey, information obtained from earlier decommissioning surveys may be used to support the final status survey.

For example, an estimate of the expected variability of the data is needed to determine the size of the sample that is necessary to meet the established error rates. For the final status survey, this estimate can be based on information obtained during earlier steps in the decommissioning process. In particular, data from scoping, characterization, and remediation control surveys might be used to estimate the expected mean and standard deviation of background radionuclides in one or more reference areas. Information on the expected variability of radionuclide concentrations that may remain in the affected areas will also be valuable in planning final status surveys. If these data are not available, a separate scoping survey may be required. In the absence of any data, expert opinion and best judgment would have to be used to estimate the expected variance or coefficient of variation (the mean divided by the standard deviation) of the data.

As discussed previously in Section 3.2.3 of this report, knowledge of the appropriate dose assessment models and applicable residual radioactivity limits are essential for planning the final status survey.

3.4.1 Collection of Survey Data

Surveys performed earlier in the decommissioning process may provide valuable information for designing the final status survey. Decommissioning surveys will typically require the collection of two types of radiological data: (1) direct (*in situ*) field measurements using portable instruments and (2) sample analyses using fixed laboratory equipment or systems. The techniques used may be radionuclide specific or for total (gross) radioactivity. The selection and proper use of appropriate instruments and techniques will be critical factors in assuring that the survey accurately determines the radiological status of the site (see NUREG-1507). Surveys should be conducted in accordance with documented plans and procedures. Recommendations for appropriate instruments and procedures to be used in final status surveys are discussed in Section 4 of this report.

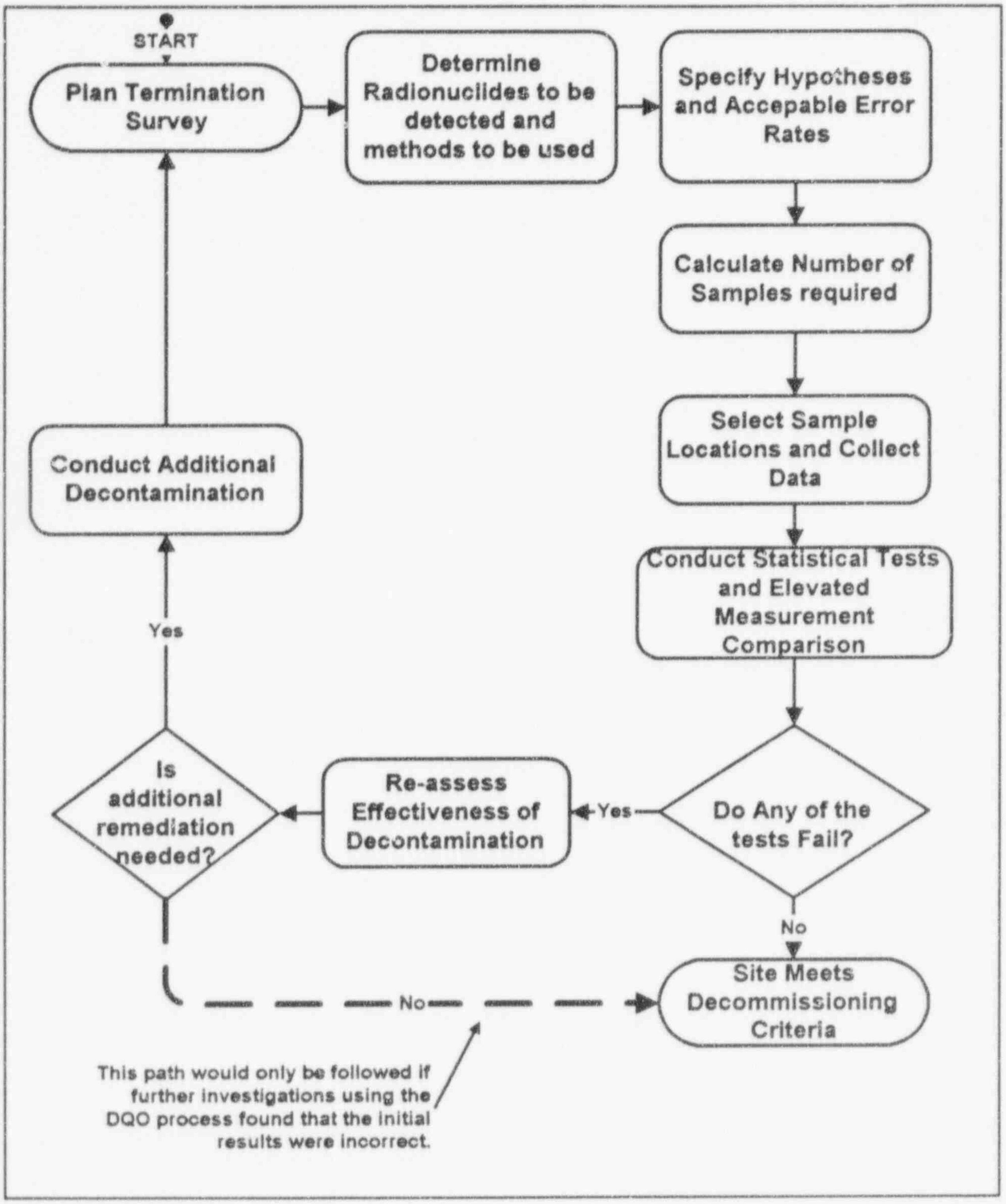


Figure 3.4 Decision Chart for Conducting Final Status Surveys

The different types of surveys that may be performed during the decommissioning process are scoping, site characterization, remediation control, and final status surveys. More information on these surveys is given in Section 4.

3.4.2 Dose Estimates

The criteria in 10 CFR Part 20, Subpart E would be difficult and expensive to verify with environmental samples alone. The low concentration levels, extended time periods for analysis, and multiple pathways of concern make model calculations the most defensible and cost-effective approach.

The NRC has developed models to provide generic dose conversion factors for residual radioactivity that can be applied within a hierarchy of modeling approaches. The models provide a mechanism for translating the residual radioactivity at a site into dose using the site-specific source term and varying levels of related site information. The modeling description and calculational methodology are described in NUREG/CR-5512, Volume 1, and are endorsed in NUREG-1500 as an acceptable methodology.

3.4.2.1 Initial Compliance Screening

For those sites at which (1) only sealed sources were used (and there is no history of leaking sources) and (2) the licensee can show that no radioactive material has been buried at the site, and there has been no seepage of radioactive material into the soil or groundwater (e.g., from settling ponds or tailings piles or spills of radioactive material), the licensee may perform a simple survey and provide supporting documentation regarding possession history and results of leak tests as a basis for demonstrating compliance with the regulations. This survey would consist of an unaffected area survey as described in Section 4, together with scans of areas that would have accumulated radioactivity had any leaks occurred. Other similar sites, such as those in which only small quantities of short-lived materials were handled, will be evaluated by NRC staff on a case-by-case basis.

3.4.2.2 Source Term

The provisions of 10 CFR Part 20, Subpart E require that a licensee consider the entire applicable source term and all credible dose pathways when determining whether residual radioactivity is less than 15 mrem per year above background or calculating TEDE, or both. The source term consists of all residual radioactivity remaining at the site, including material released during normal operations or during inadvertent releases or accidents, and radioactive materials that may have been buried at the site in accordance with 10 CFR Part 20.

Wherever possible, the licensee should use actual measurements, rather than modeling, when determining the source term (i.e., residual radioactivity remaining at the site) upon which the calculated TEDE will be based.

3.4.2.3 Predicted Dose Level

The site source term is used to estimate a predicted dose level (PDL) for the site. The PDL should be used as part of the process of determining if the site can be released for unrestricted use. The PDL is an estimator used at an early stage of the decommissioning process to support preliminary decisions regarding whether the site can meet the unrestricted release limit as described in Section IV.C.3. of NUREG-1500. It is considered a generic estimate of the potential dose level associated with the site under unrestricted use conditions. Once remediation is complete and the final status survey for the site has been conducted, licensees will calculate the TEDE associated with their sites. The TEDE is based on detailed site information, as described in Section IV.I of NUREG-1500, and is the component used to demonstrate compliance with 10 CFR Part 20, Subpart E.

3.4.2.4 Total Effective Dose Equivalent (TEDE)

When using modeling to estimate TEDE from residual radioactivity remaining at the site, the licensee may use site-specific parameters or may apply generic parameters specified for the first level of screening discussed in Section IV.I.1 of NUREG-1500. In the absence of site-specific information, the licensee should use parameters that provide a sufficient margin of safety, so that the Commission can find reasonable assurance that the TEDE criteria in 10 CFR Part 20, Subpart E will be met.

3.5 Define the Study Boundaries

Defining the spatial and temporal boundaries will help ensure that the samples taken in the survey are representative of the survey unit for which the decommissioning decision will be made. Spatial boundaries describe what measurements or samples should be taken and in what areas. Temporal boundaries describe when the measurements or samples should be taken, and any time constraints on the data collection and analysis. The selection of measurement and sampling points must ensure that the sample is representative of the site category under investigation. Atypical situations, which themselves may require study, should be avoided in attempting to group like areas. Uniformity over a given area should be checked wherever possible. This can be done by inspecting the site and knowing its history from data collected earlier in the decommissioning process, or by scanning measurements. As has been discussed in Sections 2.3.6 and 2.3.7, reference areas and survey units should be as similar as possible with regard to their background characteristics. As discussed in Section 1.3, some estimate of the variability of the data is needed for a good survey design. It follows that the smaller the variability within each reference area or survey unit, the smaller the number of samples that will be needed to achieve the specified Type I and Type II error rates for the test. Thus, it is advantageous to identify survey units that are relatively homogeneous in radiological character.

Considering the variability in collected data that is expected in any environmental sampling program, accurate interpretation of the results is essential. For instance, the presence of Cs-137 in soil, and the observation that it is not at the same level from place to place, does not necessarily indicate a local facility contribution. Such variations may have resulted from disturbance to the site through either natural or human action, which led to removal or addition of material containing fallout from atmospheric nuclear weapons tests, as well as differences in the spatial distribution of the original deposition. Issues of background spatial and temporal variability are discussed briefly below. More information is available in NUREG-1501.

3.5.1 Spatially Representative Sampling

The spatial variation of external radiation is largely related to the makeup of the soil in a locale. The greatest spatial variation in background arises from the differences in levels of radon gas, which can vary from one-tenth the national average to more than ten times the average because of differences in the radium concentration in soil. On a nationwide scale, outdoor gamma radiation levels vary by a factor ten and indoor gamma radiation levels vary by about 50 percent because of the use of different construction materials. A significant source of uncertainty in deciphering changes in radiation levels and radionuclide concentrations is likely to arise from their spatial variability in the environment. In the case of natural radionuclides, local geological features and patterns of soil type result in gradients in their concentrations. Micrometeorological effects and erosion that produces runoff and accumulation cause man-made radionuclide concentrations to exhibit potentially significant variations. For both naturally occurring and man-made radionuclides, human activities, such as soil excavation, must be considered. Thus, measurements within the same region, and even those only meters apart, must be carefully interpreted. Differences of more than 100 percent would not be unusual in certain situations.

Perhaps most significant in spatial extrapolation of radionuclide data is the site selection process. For example, it would be inappropriate to compare uranium concentrations in soils collected from two sites of different geology, such as a sandy beach area and an inland region with heavy clay soil. In the case of the fallout radionuclide Cs-137, concentrations in surface soils could only be extrapolated to other local plots of land that have received the same deposition (rainfall) and have the same history (for example, plowed agricultural land, forest, or undisturbed lawn).

3.5.2 Temporally Representative Sampling

The changes in background radioactivity concentrations and radiation levels that are associated with various physical phenomena occur on time scales ranging from short duration (hours to days) to medium duration (months and years) to long duration (centuries or more). Temporal variability of background is affected by seasonal changes in soil moisture and snow cover, which typically lead to changes in external radiation levels of 10 to 50 percent. To a lesser extent, cosmic radiation and the production rate of cosmogenic radionuclides vary up to 10 percent throughout the course of the solar cycle. However, abrupt changes in background can occur from the input of manmade radionuclides from fallout after a nuclear weapon test or distant reactor accident, which can increase background levels for a few months to a few decades.

Data collected over a limited period may not provide a true average of radiation and radioactivity levels. However, extrapolation of a measurement to longer time intervals involves uncertainties. These uncertainties may only be a few percent in some cases, but a factor of two or more in others. If an external radiation reading is taken at a soil-covered outdoor site and periods of rain and snow cover are avoided, one could expect to be within 10 to 20 percent of the annual average, given the typical degree of temporal variation. In very dry climates, where there is little variation in soil moisture, this might be reduced to between 5 and 10 percent. Barring any unusual physical disturbance to the site, extrapolation of an annual average to periods of a few decades would likely have an uncertainty of between 5 and 10 percent.

Changes in soil moisture content cause changes in *in situ* measurements of radiation levels (i.e., exposure rate and/or flux) because of the effect of soil moisture on the soil density. This will in turn be reflected in the soil concentrations of the radionuclides inferred from *in situ* measurements. Samples that are collected and then dried, processed, and analyzed in the laboratory will have concentrations reported on a dry-soil basis. These differences are important, and must be accounted for in comparing data obtained by the two methods. Thus, the wet weight of soil samples must be obtained before they are dried and processed.

Variability in collected data can be explained by referencing other data, such as weather and geological data. At the same time, it must be understood that these other data have their own sources of uncertainty. In addition, these supplemental data can sometimes lack the spatial or temporal detail needed to correlate with radiation and radioactivity data collected in a survey.

However, it is best to avoid temporal variability to whatever extent possible, since this will contribute to the overall uncertainty of comparisons of survey units and reference areas. This might be accomplished by collecting data from areas to be compared over as short a time interval as possible, and avoiding circumstances known to cause short-term background variations. There may be reasons why samples cannot be taken in certain places or at certain times. These constraints should be identified so that they can be accounted for in the planning process. As part of this step in the DQO process, a site diagram should be prepared showing each potential survey unit and the reference area to which it will be compared. For each unit, the types of samples that will be taken, the analyses needed, and a schedule for sampling and analysis should be listed. The details for laying out sampling grids within survey units is discussed in detail in Section 5.

3.6 Develop a Decision Rule

The primary activity in this step of the DQO process is to describe how the final status survey will be conducted, how the data will be analyzed, and the decisions that will be made based on the outcome of the statistical analyses. The recommended procedure for final status surveys is outlined in Figure 3.5.

3.6.1 Decision Rules for Nonparametric Tests

The nonparametric statistical tests shown in Figure 3.5 are conducted using the null and alternative hypotheses previously discussed in Sections 2.3.8 and 2.3.9.

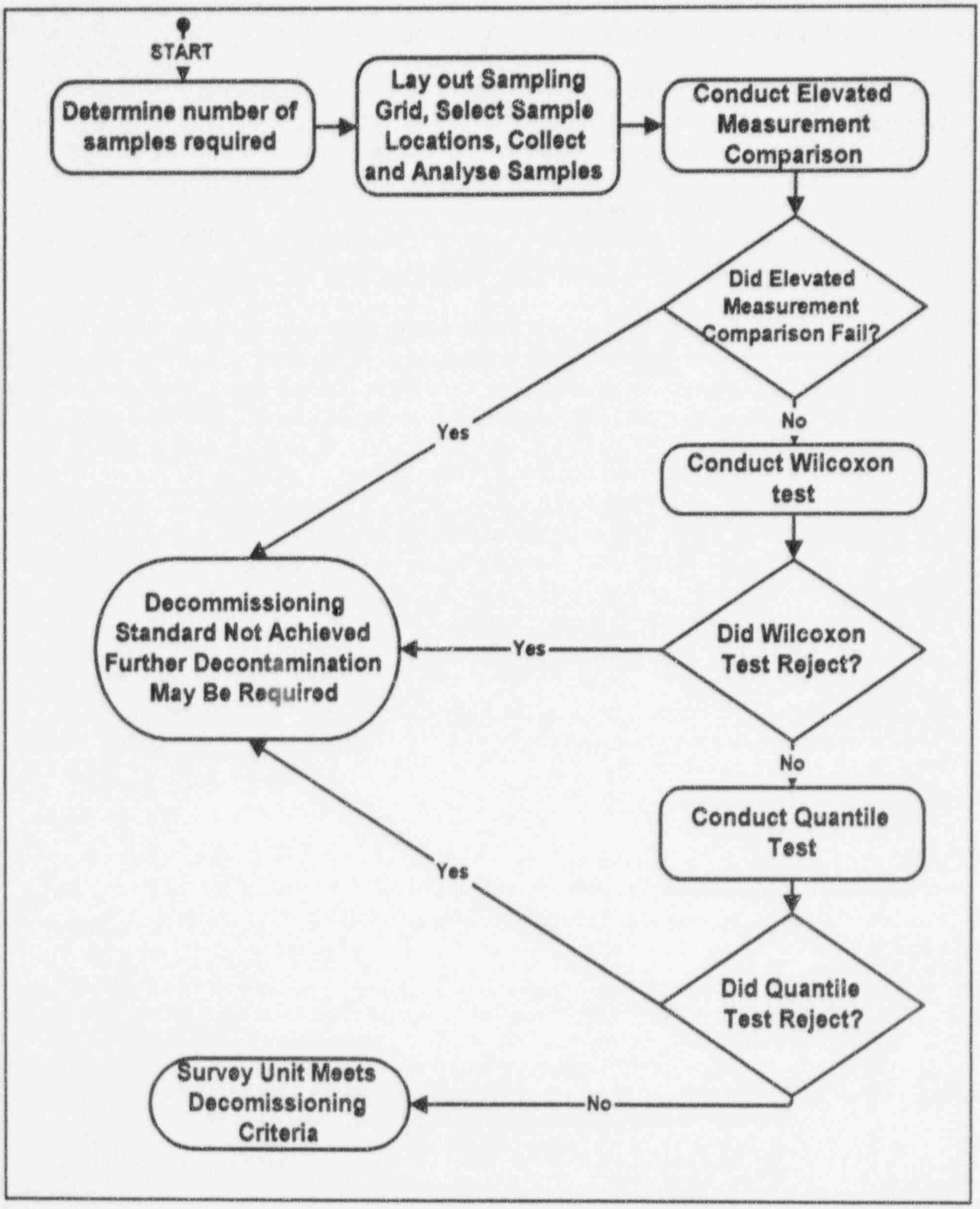


Figure 3.5 Process for Determining if a Survey Site Meets Decommissioning Criteria

Null Hypothesis
H_0: Decommissioning criteria attained
versus
Alternative Hypothesis
H_a: Decommissioning criteria not attained.

These may be restated as
Null Hypothesis
H_0: The site contains no residual radioactivity above the decommisioning criteria
versus
Alternative Hypothesis
H_a: The site contains residual radioactivity above the decommisioning criteria.

The decommissioning criteria for unrestricted release are as stated in § 20.1402 of the proposed rule:

> § 20.1402 Concepts.
>
> (a) The objective of decommissioning is to reduce the residual radioactivity in structures, materials, soils, groundwater, and other media at the site so that the concentration of each radionuclide that could contribute to residual radioactivity is indistinguishable from the background radiation concentration for that radionuclide. The Commission realizes that, as a practical matter, it would be extremely difficult to demonstrate that such an objective has been met. Therefore, the Commission has established a site release limit and is requiring that licensees demonstrate that the residual radioactivity at a site is as far below this limit as reasonably achievable.
>
> (b) The limit for release of a site is 15 mrem per year (0.15 mSv/y) total effective dose equivalent (TEDE) to an average member of the critical group for residual radioactivity distinguishable from background. If doses from residual radioactivity are less than 15 mrem per year TEDE, the Commission will terminate the license and authorize release of the site for unrestricted use following the licensee's demonstration that the residual radioactivity at the site has been reduced to as low as reasonably achievable (ALARA).
>
> (c) ALARA considerations must include all significant risks to humans and the environment resulting from the decommissioning process. Licensees shall demonstrate why further reductions below the limit are not reasonably achievable. Depending on the site-specific ALARA analysis, any dose level less than or equal to 15 mrem per year may be considered ALARA. However, in many situations, licensees may have little or no site contamination and should be able to readily achieve the overall objective for decommissioning (e.g., licensees that use only sealed sources or short-lived radioisotopes).

In order to incorporate these concepts explicitly into the decision-making process, the null and alternative hypotheses may again be re-stated as:

Null Hypothesis
H_0: The site contains no residual radioactivity distinguishable from background.
versus
Alternative Hypothesis
H_a: The site contains residual radioactivity resulting in a dose of 15 mrem/year or more above background.

Recalling the example of Section 2.9.3, it may be seen that this statement of the null hypothesis allows an exact calculation to be made of the Type I error that would correspond to requiring a site indistinguishable from background to be remediated. The Type II error that a site that does not meet the criterion can be also calculated, given some assumptions concerning the distribution of the residual radioactivity. In addition, it is possible to similarly calculate the probability that the null hypothesis will be rejected (i.e., the power of the test) at any specific dose level above background, again given some assumptions concerning the distribution of the residual radioactivity. In Section 3.7 on specifying the limits on decision errors, it will be seen that this allows the ALARA concept to be explicity incorporated into the decision-making process.

3.6.2 Decision Rules for Elevated Measurements

The elevated measurement comparison was discussed in Section 2.2.3 If a measurement exceeds an investigation level, further information will be required to determine if the decommissioning dose criteria have been achieved. The determination of an appropriate investigation level depends on the number of samples taken to perform the nonparametric tests, since the sampling grid will limit the potential area of elevated residual radioactivity and any resulting dose. Methods for determining investigation levels for the elevated measurement comparison are discussed further in Sections 5 and 6.

3.7 Specify Limits on Decision Errors

This step in the DQO process is crucial. It is at this point that the limits on the decision errors rates are developed in order to establish appropriate goals for limiting uncertainty in the data. This is done by establishing the goals for the Type I error rate and the Type II error rate. The procedure for doing this follows.

3.7.1 Determine the Possible Range of the Parameters of Interest

For unrestricted release, the proposed rule, 10 CFR 20.1404, states that if the site were released for unrestricted use, residual radioactivity at the site would not cause the TEDE to an average member of the critical group to exceed 15 mrem per year above background.

The proposed decommissioning rule also states that the licensee must demonstrate that the dose is ALARA. Compliance with the proposed ALARA requirement can be demonstrated by determining that the TEDE to the average member of the critical group from all radionuclides that are distinguishable from background does not exceed a site-specific value such as 3 mrem (0.03 mSv) per year above background. The 3-mrem-per-year value functions only to define the types of analyses and level of detail necessary to demonstrate a site-specific compliance with the

ALARA requirement. Values may vary and may also be considered to be ALARA if properly supported by an analysis of significant risks and efforts required to further reduce those risks.

The proposed dose limits define a central region of interest in terms of TEDE of between 3 and 15 mrem per year over which decisions will be made. For the analysis of final status survey data, which will consist of radionuclide activity concentration measurements, these dose limits must be converted to appropriate radionuclide activity concentrations. This, in many cases, can be done by using the tables in Appendix B of NUREG-1500. Although these tables list the default concentration values equivalent to 3 and 15 mrem per year for four exposure scenarios, other values may be determined by linear interpolation or extrapolation.

If the site contains residual radioactivity from only one radionuclide, the estimated average concentration for the site equivalent to 3 to 15 mrem per year defines the central region of interest for that radionuclide. For sites at which more than one radionuclide remains at a concentration that is distinguishable from background, the values in Appendix B of NUREG-1500 cannot be used directly. However, the mixture of radionuclides can be compared against the default concentrations by applying the mixture rule. This is done by determining the ratio between the concentration of each radionuclide in the mixture and the concentration for that radionuclide listed in the appropriate table in Appendix B of NUREG-1500. The sum of the ratios for all radionuclides in the mixture should not exceed 1.

For example, if radionuclides A, B, and C are detected in concentrations C_A, C_B, and C_C, and if the applicable NUREG-1500 Table B-2 values are T_A, T_B, and T_C, then the following relationship exists when the site meets the 15-mrem-per-year criterion.

$$\frac{C_A}{T_A} + \frac{C_B}{T_B} + \frac{C_c}{T_C} < 1 \tag{3-1}$$

Thus, the concentration range of interest for a particular radionuclide should be modified according to the proportion that it might be expected to contribute to the predicted dose level.

3.7.2 Define Both Types of Decision Errors and Their Consequences

The Type I error rate for final status surveys establishes the acceptable probability of labeling a site that actually meets the reference radiological criterion as being contaminated above background. An error of this type would result in a licensee unnecessarily remediating background. Since the null hypothesis is stated in terms of residual radioactivity being indistinguishable from background, there is the question of what dose level is considered indistinguishable from background. This issue is considered further in the next section.

The Type II error rate establishes the acceptable probability of incorrectly labeling a site that contains residual radioactivity as being indistinguishable from background. An error of this type would result in a site being released for unrestricted use at some level over 15 mrem per year above background because, based on the outcome of the statistical tests, the licensee was not required to perform additional site remediation. The Type II error rate directly affects the total number of NRC sites that may be released above background, which could potentially impact public health and safety and the environment. The Type II error rate should be set at a level which ensures that doses from residual radioactivity do not exceed 15 mrem per year above background for most decommissioning actions. Because the Type II error rate can potentially affect public health and safety and the environment from excessive residual radioactivity and the Type I error would not, there is less tolerance for Type II errors than for Type I errors.

3.7.3 Specify a Range of Possible Radionuclide Concentrations for Which the Consequences of Decision Errors are Relatively Minor

The Type II error rate decreases as the residual radiation level increases. At a level of 15 mrem per year or more above background, the Type II error rate should be low in order to be adequately protective of public health. The Type I error rate, however, should be low whenever the dose due to residual radioactivity is 3 mrem per year or less above background in order to avoid unnecessary remediation costs. In the region between 3 and 15 mrem per year, there are generally no significant health risks in consequence of Type II errors and there is little economic risk in Type I errors. Thus, this region defines a gray area in which the consequences of decision errors are relatively minor. In some cases, ALARA considerations may dictate controlling Type II errors at a level less than 15 mrem per year above background, and site-specific decontamination economics may require controlling Type I errors above 3 mrem per year above background.

3.7.4 Assign Probability Values Above and Below the Gray Area That Reflect the Acceptable Decision Error Rates

According to EPA report QA/G-4, 0.01 is the most stringent limit on decision error rates that is typically encountered for environmental data, but EPA warns that this value should not be considered prescriptive. In many environmental applications, the NRC staff considers the 95-percent confidence level appropriate for assessing radiological data. As discussed in Section 2.3.9, this is equivalent to a Type I error rate of 0.05. The choice of the specific Type I and Type II error rates involves a number of important considerations which are discussed in detail in Section 7.

3.7.5 Construct the Desired Power Curve for the Test That Will Support the Decision

Using the information from the earlier activities in this step, a chart of acceptable error rates for the desired statistical test can be constructed. The horizontal axis covers the concentration (or dose) range of interest. The vertical axis shows the error rate that would be acceptable for each possible value of the true concentration (or dose). To begin, error rates that would seem tolerable when a given dose rate actually exists are plotted on a chart. This has been called a "discomfort curve" by Ryti and Neptune and is used to illustrate the relationship between error rates and a decision maker's discomfort with those error rates.

When low levels of residual radioactivity exist, discomfort is measured by the false positive (Type I) error rate because these errors will cause unnecessary remediations. As the residual radioactivity level increases, more false positives may be tolerated because the contamination levels are higher and further decontamination will result in a health benefit. When the residual radioactivity level reaches the applicable decommissioning criteria, the number of false negatives should be controlled to reduce the possibility of releasing a site that contains residual radioactivity above the limit. Thus, the tolerance for false negatives (Type II errors) decreases as the residual radioactivity levels increases.

To illustrate this concept, an example of a "discomfort curve" is shown in Figure 3.6. In the region between 3 and 15 mrem per year above background, there are generally no significant health risks associated with Type II errors and there is little economic risk in Type I errors. This region defines a "gray area" in which the consequences of decision errors are relatively minor and so there is little if any real discomfort with decision errors. It should be noted that the Type I error rate need not be the same as the Type II error rate and, in this example, the Type II error rate is smaller than the Type I error rate. The example curve assumed that a decision maker has more discomfort from a decision error that would result in the release of a site above the applicable decommissioning criteria than discomfort from a decision that would result in unnecessary remediation.

A discomfort curve can be refined into a chart of acceptable error. The area in which the discomfort level that can be tolerated is fairly high remains a gray area because it is acknowledged that errors of either type are not very serious in terms of their consequences. To the left of the gray area are plotted the acceptable error rates for false positives. Clearly, these should become smaller as the concentration (or dose) that actually exists becomes lower. To the right of the gray area are plotted the acceptable error rates for false negatives when the concentration (or dose) is above the dose limit. As discussed previously, error rates for false negatives should be smaller as the residual radioactivity increases. An example chart of acceptable error is shown in Figure 3.7.

As discussed in Section 2.3.9.4, the power of a statistical test is one minus the Type II or false negative error rate (1-β). For the purpose of this draft report, the power of a statistical test should increase as the concentration (or dose) that actually exists increases. In other words, the greater the contamination, the more easily (and accurately) the residual radioactivity should be detected. To compare the desired results to what is possible to achieve with a statistical test, a power chart for the desired statistical test can be constructed. A chart of acceptable error can be converted into a power chart by replacing β with (1-β). For example, the chart of acceptable error in Figure 3.7 is shown as a power chart in Figure 3.8.

In a power chart, the horizontal axis covers the concentration (or dose) range of interest and the vertical axis gives the probability of rejecting the hypothesis that the survey unit meets the applicable decommissioning criteria. The power chart constructed in the DQO planning process is a discrete approximation to the desired power curve for a statistical test, which is generally a continuous function. Figure 3.8 is an example of a power chart. The power curve is preferred by statisticians because the vertical axis always refers to the same decision, namely the null hypothesis is rejected. In this manner, different statistical tests, e.g., the WRS, Quantile, and Student's *t*-tests, can be compared by their power curves.

The information in a power chart may also be summarized in a table such as Table 3.1. The power chart is the desired power curve for the statistical test that will be selected. The *actual* power curve for a specific test is determined by fixing false positive error rate, α, and the number of samples, *n*, for a given level of variability expected in the data. For given values of α and *n*, the actual power curve may lie above or below the chart in Figure 3.8. The desired power is attained

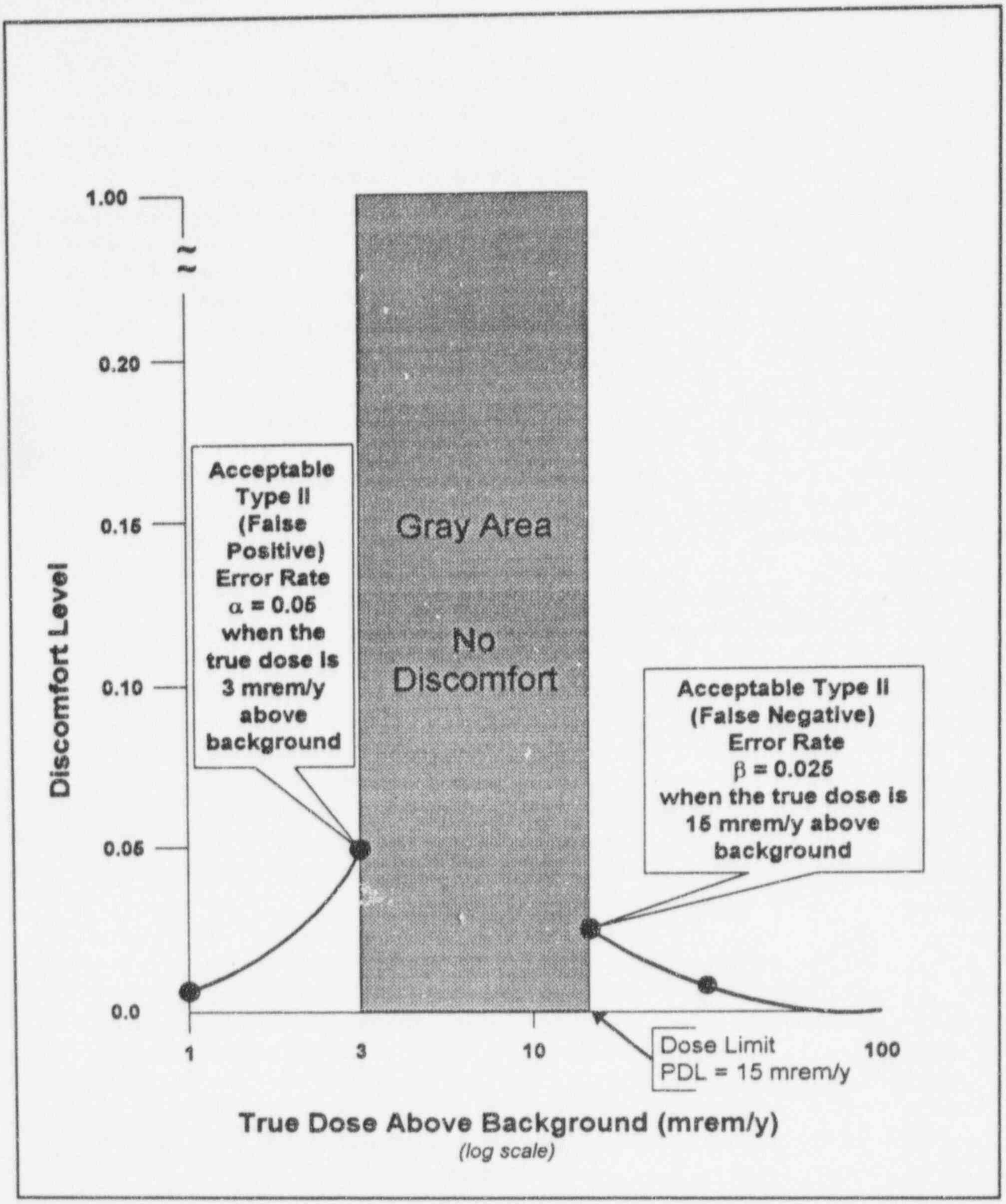


Figure 3.6 Example Decision Maker's "Discomfort Curve"

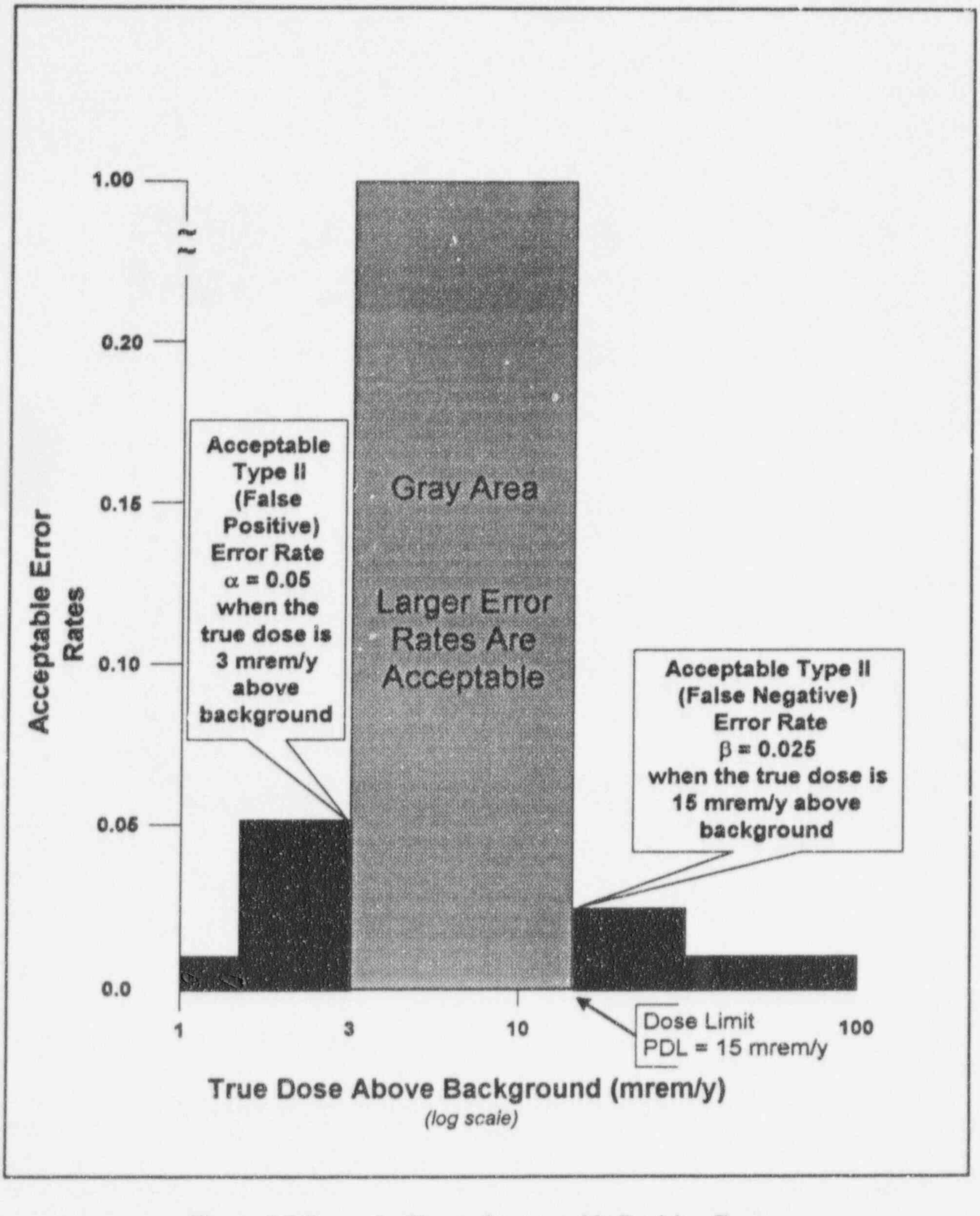


Figure 3.7 Example Chart of Acceptable Decision Errors

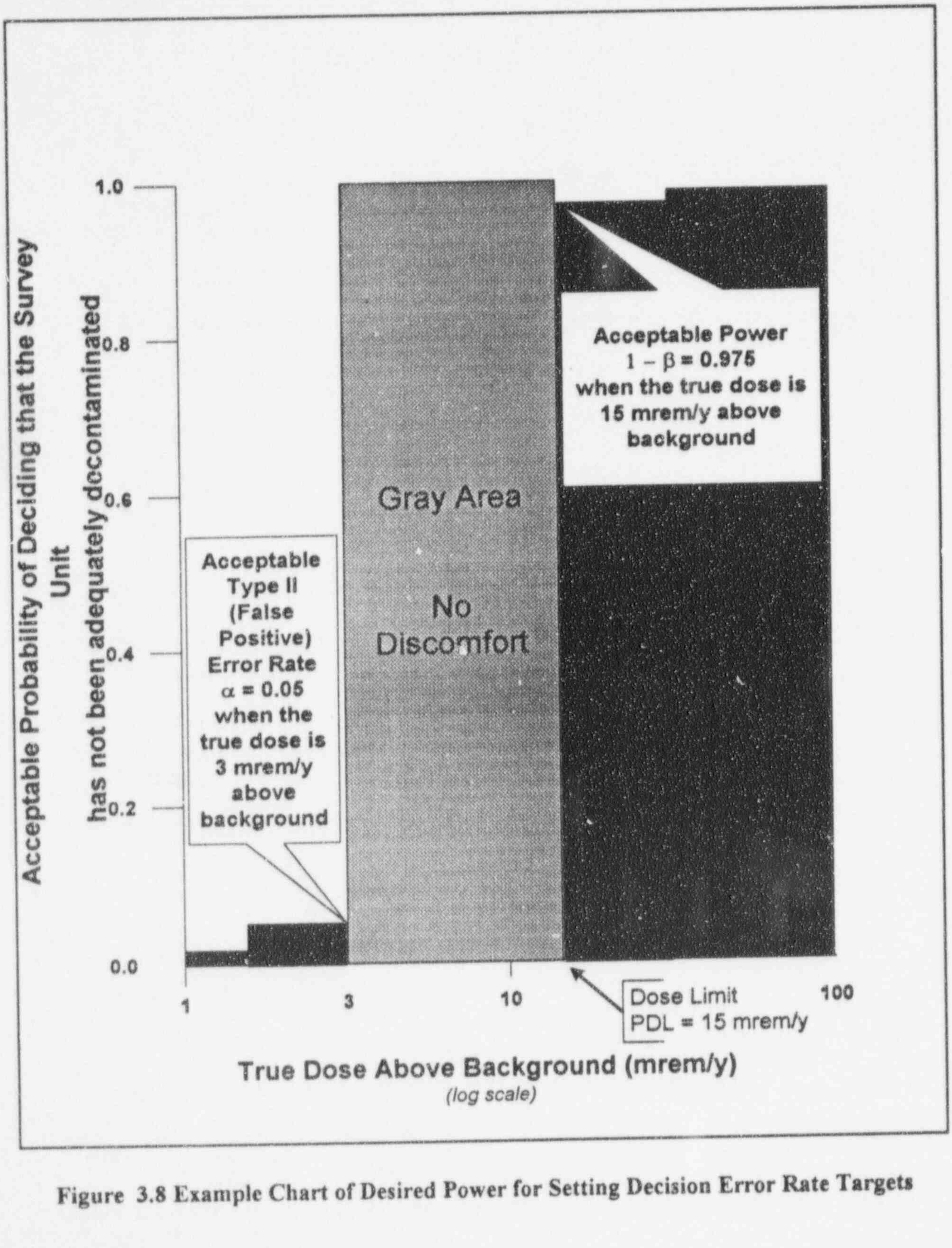


Figure 3.8 Example Chart of Desired Power for Setting Decision Error Rate Targets

when the actual power curve lies *below* the specified values at doses less than 3 mrem per year above background, and *above* the specified values for doses greater than 15 mrem per year above background. The procedure for comparing the desired power to the actual power for a specific test and sample size is discussed in detail in Section 5. Because of mathematical constraints, it is not always possible to match every possible discomfort level with a corresponding power curve. In those cases, one must either chose the sample size that yields the power chart closest to that desired, or perhaps re-assess and modify the relative discomfort levels that give rise to the choices of α and β.

Table 3.1 Decision Error Table Corresponding to Example Power Chart

True Dose Above Background (mrem/yr)	Correct Decision	Acceptable Decision Error
< 1	Meets Criteria	$\alpha \leq 1\%$
1 to 3	Meets Criteria	$\alpha = 5\%$
3 to 15	Meets Criteria	Gray area: errors not critical
15 to 20	Fails Criteria	$\beta = 2.5\%$
> 20	Fails Criteria	$\beta \leq 1\%$

3.8 Optimize the Design

Although the first six steps in this process are usually sequential, some of the activities involved may be taking place concurrently. The process should not be viewed as static, wherein each step is visited only once. At any stage in the process, new information may be available that should then be incorporated into the planning. This is especially true when it comes to planning the final status surveys.

When the criteria for limiting decision error rates from Section 3.7 are incorporated into the statistical design procedures of Section 5, it is possible to compare the power of the tests with the "discomfort curve." A smaller number of samples may still result in acceptable error rates. The specification of survey units, and the variability of the data will also have an effect on the survey design. The advantage of the process is that several alternatives can be explored on paper before time and resources are committed. More information on this process is given in Section 5 for the case of radionuclides that occur as part of background, and in Section 6 for the case of radionuclides that do not occur as part of background.

4 RADIOLOGICAL SURVEYS SUPPORTING DECOMMISSIONING

4.1 Introduction

Current methodology for surveying sites for residual radioactive contamination (see NUREG/CR-5849) was developed for distinguishing levels that are elevated when considered in comparison to natural background radiation. For example, survey meter measurements can be used for gamma dose rates approaching 100 mrem per year, given that typical background levels are on the order of 40 mrem per year. It is generally more difficult to measure radiation and radioactivity at some fraction of background levels because of the variable nature of background radiation. Survey methods will require a new approach in some instances not only for the statistical methods which are described in this report but also for the type of measurements employed. NRC is currently developing new sampling and measurement approaches. To some degree, an integration of both the statistical and the measurement methods is desirable to achieve optimum performance and sensitivity.

Relevant information on the properties of natural background radiation and its variability can be found in the report NUREG-1501. That information forms the basis with which to apply future decommissioning criteria involving radiation levels near background. That report contains a complete summary of the sources of background radiation and their contributions to dose to humans. Causes are given for the variability in background radiation, and for the degree of spatial and temporal variability for each component. General countrywide, regional, and local variability is addressed and averages and ranges of doses for both external and internal radiation are estimated in comparison to worldwide averages and ranges.

The report also gives information on data requirements, measurement techniques, and uncertainties associated with the determination of natural background radiation levels. This includes estimates of the degree of effort and costs for such background determinations as well as those associated with deciphering doses from nuclear facility components at specific levels above background. Instrumentation and methodologies, including spectrometry, that can be used for assessing the various natural background and facility components are categorized. It must be understood that different types of surveys are performed in the various stages of the decommissioning process. Early on, and where known contamination exists, the simplest approach can be used to document the need for a specific building or parcel of land to be remediated. The more sensitive methods will be required for the final status survey and whenever measurements are to be performed in unaffected areas in which there is no expected contamination.

4.2 Types of Radiological Surveys

Throughout the decommissioning process, survey data of various types will be needed to support the remediation decisions that are made, up to and including the final status survey for unrestricted release. Although the primary focus of this report is the final status survey, it is important to note that the data quality objectives process can be effectively applied to all surveys. The information gathered in every survey has potential use in the design of later surveys.

4.2.1 Scoping Surveys

The objective of the scoping survey is to augment historical site assessment findings for sites with potential residual contamination and to identify and classify survey units as either (1) unaffected areas or (2) areas having contamination present at the site that requires additional characterization. Therefore, once a review of pertinent site history indicates that a potential for residual contamination exists, the minimum survey coverage at the site will include an unaffected area final status survey before the site can be released from further consideration. For scoping surveys with this objective, it is necessary to identify default guidelines so that the instrumentation and procedures selected have the necessary detection sensitivities to demonstrate compliance with the guidelines.

If the historical site assessment indicates that contamination is likely, the scoping survey is conducted at the beginning of the decommissioning process to obtain sufficient radiological information to identify the location and quantity of residual radioactivity throughout the site, and to provide for initial estimates of the level of effort required for decontamination. Radiological information obtained from scoping surveys is used to plan the more comprehensive site characterization survey discussed below. This survey does not require that all radiological parameters be assessed when planning for additional characterization. That is, total surface activity or limited sample collection may be sufficient to meet the objectives of the scoping survey.

Scoping surveys are used to identify the potential radionuclide contaminants at the site; the relative ratios of these radionuclides and the general extent of contamination, both in residual radioactivity levels and affected area or volume. This survey provides a preliminary assessment of site conditions, and enables classification of the site into areas that are not impacted, unaffected, affected/uniform, or affected/non-uniform. Some of the data, particularly data from locations not affected by site operations, may be used to supplement the characterization or final status survey results or both. Similar measuring and sampling techniques as used for those categories of surveys may, therefore, be warranted. In particular, an estimate of the variability (i.e., standard deviation) in the distribution of background and residual radioactivity will be needed to properly plan the final status survey. Thus, opportunities for obtaining this information during other surveys should be vigorously pursued.

4.2.2 Characterization Surveys

These surveys are used to more precisely define the quantities and spatial distribution of residual radioactivity. The extent of the survey depends on how the survey information will be used. For example, if site records or the scoping survey show that the survey area is contaminated, the characterization survey may only be designed to define the boundaries of contamination in support of planning associated with decontamination activities. Alternatively, if the survey area is expected to be uncontaminated, the survey may be more detailed so that the information can be used to support the final status survey.

Characterization surveys are meant to define the extent and magnitude of contamination in sufficient detail to produce data for planning the decontamination effort. The type of information obtained is often limited to that necessary to differentiate a surface or area as contaminated or not contaminated. A high degree of accuracy may not be required for such a decision, when the data indicate levels well above the applicable decommissioning criteria. On the other hand, when data are near the limit, a higher degree of accuracy is usually necessary to assure the appropriate decision regarding the true radiological conditions. Further information on characterization surveys can be obtained from the 1994 draft NRC Branch Technical Position on "Site Characterization for Decommissioning" (NRC 1994).

4.2.3 Remediation Control Surveys

Remediation control surveys are used to monitor the effectiveness of decontamination efforts in reducing residual radioactivity to acceptable levels and to guide the cleanup in a real-time mode. Such a survey is intended for expediency and does not produce thorough or accurate data describing the final radiological status of the site.

4.2.4 Final Status Surveys

This survey type is the focus of this draft report. It is this survey, performed after decontamination activities (if any were required) are complete, which produces data to demonstrate that all radiological parameters (total surface activity, removable surface activity, exposure rate, and radionuclide concentrations in soil and other bulk materials) satisfy the applicable decommissioning criteria.

Not all areas of the site will have the same potential for residual contamination and, therefore, not all areas require the same level of survey coverage to achieve an acceptable level of confidence that the site satisfies the established release criteria. By designing the survey so that areas with higher potential for contamination receive a higher degree of survey effort, the process will be both effective and efficient.

Areas that have no potential for residual contamination and, therefore, do not require any level of survey coverage are referred to as **non-impacted** areas. These areas are typically located off site, and may include areas that are used for background reference.

There are three types of final status surveys, which depend on the classification of the survey unit as **unaffected**, **affected/uniform** or **affected/non-uniform**. These classifications were discussed in Section 2. The survey design appropriate for each class of survey unit is discussed further in Section 4.6.

4.2.5 Confirmatory Surveys

After the licensee's termination survey report is accepted, the NRC may perform (or arrange for its agent to perform) a confirmatory survey. The scope of a confirmatory survey is typically limited to less than ten percent of the site, but such surveys are used to verify the radiological status of the site that is reported in a licensee's final status survey. Confirmatory surveys obtain radiological data about the site that are similar to data presented by the licensee and may include independent statistical evaluations of reported data.

4.3 Survey Planning

In keeping with the DQO process outlined in Section 3, a survey plan should be developed in the early stages that incorporates specific measurement techniques based on a number of input factors. These would include such site characteristics as the land area, building, water body, and subsurface contamination. The critical radionuclides can be identified and the concentration or surface activity limits established for various post-remediation land use scenarios. At this point, both the measurement and statistical methods that will be needed to meet release criteria can be established. This will likely be done within the limitations of a license termination budget. An important consideration is balancing the use of rapid field screening techniques against more time consuming laboratory analyses of collected samples.

The nonparametric statistical tests of Sections 5 and 6 of this report are known as *two-sample* and *one-sample* tests, respectively. Their application will depend upon the specific radionuclides under consideration, the concentration or surface activity limits for these radionuclides, and the comparison to background levels in the surrounding environment. Application of these techniques will also depend upon whether a gross dose or count rate survey is employed instead of spectrometric measurements for individual nuclides. The one-sample tests are appropriate when there is no need to compare the survey unit with a reference area. This will be the case when the *radionuclide of concern does not appear in background and radionuclide-specific* measurement methods are used. The two-sample tests are appropriate in all other cases.

4.4 Instrumentation

Among the measurements that will typically be made during radiological surveys are total surface activities, removable surface activities, exposure rates, and radionuclide concentrations in various environmental media (e.g., soil, water, air). It may be necessary to take field measurements and perform laboratory analyses to make these determinations. For certain radionuclides or radionuclide mixtures, alpha, beta, and gamma radiations may all have to be measured. In addition to assessing average radiation levels, small areas with elevated levels of residual contamination must be identified and their extent and activities determined. With so many different applications, it is highly unlikely that any single instrument (detector and readout combination) will be capable of adequately measuring all of the radiological parameters required to demonstrate that criteria for unrestricted release have been satisfied.

In this report, three basic types of measurements are considered:

(1) scanning
(2) direct field measurements
(3) sampling and analysis

Scanning is the process by which the surveyor moves a portable radiation detection instrument over a surface (i.e., ground, wall, floor, equipment) to detect the presence of radiation. A scan is performed to locate radiation anomalies that might indicate elevated areas of residual activity that will require further investigation or action. If scan survey results exceed a scanning action level determined on the basis of the potential contaminant and the detector and survey parameters, the location is noted for further action (direct measurement or sampling).

Direct field measurements are those made at a fixed location using portable instruments (e.g., survey meter, pressurized ionization chamber (PIC), *in situ* spectrometer). The result of a direct measurement, as opposed to a scan, is a quantitative measure of the radioactivity present at the location measured.

Sampling, with subsequent analyses conducted in a laboratory, will be required for certain radionuclides and radiations that cannot be adequately detected using direct measurements. For some nuclides or environmental media, this may be the only realistic technique to employ.

The survey designs with which these measurements are made fall into two categories:

(1) authoritative (judgment) sampling
(2) probability sampling

Authoritative or *judgment sampling* occurs when measurements are made or samples are collected at locations where anomalous radiation levels are observed or suspected. The term "biased sampling" is sometimes used to indicate that the sample locations are not chosen on a random or systematic basis. Biased radiological measurements and samples also may be taken to further define the areal extent of potential contamination and to determine maximum radiation levels within an area.

When data quality objectives involve statistical estimation or hypothesis testing, some form of *probability sampling* is required. The type of probability sampling recommended for use in final status surveys is either simple random sampling (for unaffected areas) or systematic sampling on a triangular grid with a random start (for affected areas).

Of the three measurement types, only the results of direct measurements and sampling are used in conducting the nonparametric statistical tests. All three types of measurement result are subject to an elevated measurement comparison against an upper limit value.

The type of instrumentation or sampling and analysis methodologies or both used for final status surveys will influence the number of samples or direct measurements, or both, that are required for the appropriate statistical analysis of the data. The information necessary to calculate the required number of samples, given the expected variability of the data, is discussed in Sections 5 and 6.

The most obvious of these influences concerns the measurement precision. As a rule, the less precise the measurement, the greater the number of measurements that will be required for the statistical tests to achieve the desired level of uncertainty. The selection of survey instruments may involve a cost analysis of whether it is better to use a more precise (and more expensive) measurement method with correspondingly fewer measurements, or to use a less precise (and perhaps less costly) method that would require the collection of more measurements.

Similar considerations are involved in the choice of making radionuclide-specific measurements versus total alpha, beta, or gamma activity or total exposure rate measurements or both. If total (gross) methods are used, the results will include the variability of natural background. This additional variability will not only require more measurements to overcome but will also necessitate comparison with a reference area using the two-sample techniques of Section 5 rather than the one-sample techniques of Section 6.

If the radionuclide of concern appears as part of background, there is no alternative to a survey unit comparison to a reference area; however, the measurement precision will still affect the number of samples required. Radionuclide-specific methods should be considered in this case as well, since the variability of the total activity present will be greater than that due to any particular radionuclide or series alone.

Instrumentation can be selected using guidelines by comparing its performance capabilities to the applicable decommissioning criteria. Consideration should given to the characteristics of the type of detector, in particular, the minimum detectable concentration (MDC) for the radionuclide under investigation. Appropriate instruments are selected based on the nuclide's principal manner of detection, i.e., via alpha, beta, or photon emissions (x or gamma rays). Though all detectors respond to particle or photon fluence rate or both, readout or raw data conversion is generally performed to yield a quantity that is either a unit of radiation or radioactivity. Conversions of raw data to units of concentration should contain the unit "picocuries per gram" or "dpm per 100 cm^2," as appropriate, to facilitate the use of the dose conversion factors developed in NUREG/CR-5512, Volume 1. The simplest of devices, survey meters, may be appropriate for hand scanning of building surfaces for certain nuclides at certain activity levels. Fixed-place detectors at grid points can be used in other situations. In some situations, the sensitivity needed at background levels will require that measurements be nuclide specific, thereby requiring spectrometric techniques.

Consideration must also be given to new technology that may be developed if it can be satisfactorily demonstrated to be effective to the intended use. Further information on the selection and use of environmental radiation survey instrumentation may be found in NUREG/CR-5849, NUREG-1506, and NUREG-1507. Cox and Guenther have surveyed the industry regarding the sensitivity of such instrumentation.

4.5 Quality Assurance

The quality of data is critical to the successful execution of a survey. Statistical testing of a cleanup unit against that of a reference area requires a certain degree of accuracy and precision in measurements. Poorly calibrated instruments could lead to either improperly labeling an area as still contaminated or releasing it when, in fact, it is above the guidelines. For this reason, calibrations must be performed regularly with traceable standards; the inherent precision of the survey instrument must be evaluated to determine if it meets the need of the survey plan. Energy responses of instruments must be known so that appropriate applications are made to different radiation fields. Replicate, reference, and blank measurements are also an integral part of the survey methodology. Comparisons of field measurement results to those of laboratory sample analyses forms an important quality control check.

Bounds on measurement uncertainties should be established in the planning process and regularly assessed throughout the measurement program. Uncertainties in the measurements add to the variance in distribution of data sets and should be taken into consideration when selecting parameters for the statistical tests and in the interpretation of results of these tests. Failure to adequately consider the effect of measurement errors could result in the added expense of additional measurements. In the worst case, inadequate control of the Type I and Type II statistical errors as determined from a retrospective power calculation, could invalidate the final survey results and require a re-survey.

The occurrence of missing or unusable data can similarly impact the Type I and Type II error rates. A reasonable allowance for such occurrences should be built into the planning process as discussed in Section 5.5.4 and Section 6.4.4.

4.6 Survey Designs

Survey designs will vary according to whether they are performed for scoping, characterization, or final status purposes. A grid area layout for final status survey measurements in affected areas must be constructed using the procedures of Section 5. Proper selection of a reference area will be important for the two-sample tests. It may be necessary in some cases to survey a number of potential reference areas to establish some confidence in the representative nature of the sites. A variety of area types may be encountered, such as open undisturbed land, naturally eroded areas, grounds disturbed by such human activity as plowing or construction, manmade surfaces, and interiors and exteriors of buildings. It is important in each case to establish the appropriateness of the radiation detector/sample location with respect to the potential radionuclide source distribution.

For situations in which comparisons are being made to radionuclides already present in background, the temporal variations of gamma radiation levels due to changes in soil moisture through its effect on soil density should be taken into account, since the concentrations in soil inferred from *in situ* measurements will depend on the soil density assumed for the instrument calibrations. For similar reasons, *in situ* survey measurements should not be made in the presence of snow cover. Spatial variations in the soil composition may have to be taken into account for low-energy photon measurements. Vegetation and ground surface roughness are important considerations for alpha and beta measurements.

An important step in survey design is to integrate the survey techniques (Section 4.4) with the data quality objectives requirements determined earlier (Section 3) and the guidance on statistical tests (Sections 5 and 6) to produce an overall strategy for performing the survey.

Following remediation, areas of highly elevated residual activity will typically represent a very small portion of a site. Random or systematic measurements (or sampling) on a grid have a very low probability of identifying such small areas unless the number of measurements or samples is very large. For this reason, scanning is used in conjunction with direct measurements or samples taken on a grid or both, to locate potential areas of elevated residual radioactivity. Scans are conducted for all radiations potentially present (alpha, beta, low-energy x, and gamma radiations) based on the operational history and surfaces to be surveyed. The scanning technique should employ the most sensitive instrumentation that is suitable for field use. In general, the use of a more sensitive scanning method will mean that fewer direct measurements or samples would be required.

4.6.1 Final Surveys for Unaffected Areas

Scans of unaffected area interior surfaces and open land area should be performed for all radiations which might be emitted from the potential radionuclide contaminants. Between 1 percent and 10 percent of the surface area should be covered by scans in unaffected areas. Generally, a grid is not prepared for unaffected areas.

Direct measurement or sampling is performed at randomly selected locations, the number of which is determined by the requirements of the statistical tests (see Sections 5 and 6). For interior surfaces, a smear for removable contamination should also be made at these locations. Supplemental measurements by *in situ* gamma spectrometry are recommended at a few locations in each unaffected area to demonstrate the absence of photopeaks which would be indicative of residual radioactivity

4.6.2 Final Surveys for Affected Areas With Relatively Uniform Residual Radioactivity

Interior surfaces of affected areas and open land should be scanned for all radiations that might be emitted from the potential radionuclide contaminants. Between 10 percent and 100 percent of the surface area should be covered by scans in affected areas that do not have a high potential for severe inhomogeneities of residual radionulide concentrations.

Direct measurements or samples or both are taken on a systematic grid pattern as described in Sections 5 and 6. The number of measurement locations is determined by the requirements of the nonparametric statistical tests (see Sections 5 and 6). For interior surfaces, a smear for removable contamination should also be made at these locations.

4.6.3 Final Surveys for Affected Areas With Potential for Non-Uniform Residual Radioactivity

The survey for affected areas with potential non-uniform residual radioactivity (affected/non-uniform) is similar to that for affected areas without such potential. However, a 100-percent scan of all interior surfaces and open land area is required. Generally, these areas will also require a more closely spaced measurement pattern. The number of direct measurement or sampling locations or both is determined by the requirements of the nonparametric statistical tests and by the need to determine whether small areas of elevated residual contamination remain (see Sections 5 and 6). A smear to check for removable contamination on interior surfaces should also be made at these locations.

4.7 Data Quality Assessment

The Data Quality Objectives (DQO) process discussed in Section 3 is the first part of a Data Life Cycle (DLC), which consists of planning, implementation, assessment, and decision making. Aspects of implementation were discussed in Sections 4.1 to 4.6. This section discusses data assessment; Sections 5 and 6 discuss the details of the statistical tests used in the decision- making process.

Data Quality Assessment (DQA) is the scientific and statistical evaluation of data to determine if the data are of the right type, quality, and quantity to support their intended use (EPA QA/G-9). There are five steps in the DQA process:

(1) Review the Data Quality Objectives (DQOs) and sampling design.
(2) Conduct a preliminary data review.
(3) Select the statistical test.
(4) Verify the assumptions of the statistical test.
(5) Perform the statistical test.

4.7.1 Review the Data Quality Objectives (DQOs) and Sampling Design

Review the DQO outputs to ensure that they are still applicable. For example, if it were determined from scanning data that a survey unit had been misclassified as "unaffected," but it should have been classified as an "affected" area, the original DQOs for that survey unit would have to be re-developed for its new classification.

Review the sampling design and data collection documentation for consistency with the DQOs. For example, check that the appropriate number of samples were taken in the correct locations, and that they were analyzed with methods of appropriate sensitivity.

A sample checklist is in Appendix B.

4.7.2 Conduct a Preliminary Data Review

Review quality assurance (QA) reports, calculate basic statistical quantities, and prepare graphs of the data. Use this information to learn about the structure of the data and to identify patterns, relationships, or potential anomalies.

At a minimum, the graphical data review should consist of a posting plot, a histogram of the data, and a quantile plot. Basic statistical quantities that should be calculated for the data set are the mean, standard deviation, median, maximum, minimum, and range.

Large differences between the mean and the median would be an early indication of skewness in the data. This would also be evident in a histogram of the data. The construction of a "stem and leaf" plot is a simple way to generate a crude histogram of the data quickly. A histogram with two peaks may indicate residual radioactivity of the type that the Quantile test is designed to detect.

Example:

Suppose the following 20 data points were obtained in a survey unit with a known mean dose rate of 84 mrem per year and standard deviation of 8 mrem per year:

90.7, 83.5, 86.4, 88.5, 84.4, 74.2, 84.1, 87.6, 78.2, 77.6,
86.4, 76.3, 86.5, 77.4, 90.3, 90.1, 79.1, 92.4, 75.5, 80.5.

An initial exploration of these data might be the construction of a stem and leaf display. The "stems" of such a display are the most significant digits of the data. Here the data span three decades. Three is too few stems, so divide each stem into two parts. This results in the six stems 70, 75, 80, 85, 90, 95. The leaves are the least significant digits, so 90.7 has the stem 90 and the leaf 0.7; 77.4 has the stem 75 and the leaf 7.4. As shown in Figure 4.1, simply arrange the leaves of the data into rows, one stem per row.

```
70   4.2
75   8.2, 7.6, 6.3, 7.4, 9.1, 5.5
80   3.5, 4.4, 4.1, 0.5
85   6.4, 8.5, 7.6, 6.4, 6.5
90   0.7, 0.3, 0.1, 2.4
95
```

Figure 4.1 Example of a Stem and Leaf Display

The result is a quick histogram of the data, from which it is easy to pick out the minimum (74.2), the maximum (92.4), and the median (between 84.1 and 84.4).

Next, calculate the average of the data (83.5) and the sample standard deviation (5.7).

A posting plot, which is simply a map of the survey unit with the data values entered at the measurement locations, will reveal potential heterogeneities in the data, especially possible patches of elevated residual radioactivity. Even in a reference area, a posting plot can reveal spatial trends in background data that might affect the results of the two-sample statistical tests.

If the data above had been taken on a triangular grid in a rectangular survey unit, the posting plot might resemble the display in Figure 4.2. Figure 4.2 shows no unusual patterns in the data.

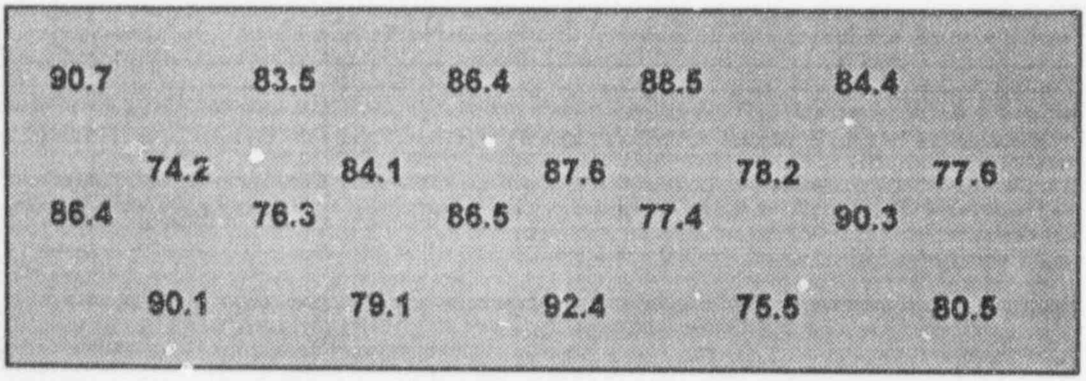


Figure 4.2 Example of a Posting Plot

A Quantile plot is constructed by ranking the data from smallest to largest, and simply plotting the data against the quantity: (rank-0.5)/(number of data points).

Sorting the data is easy once the stem and leaf display has been constructed:

Data :	74.2	75.5	76.3	77.4	77.6	78.2	79.1	80.5	83.5	84.1
Rank:	1	2	3	4	5	6	7	8	9	10
Data :	84.4	86.4	86.4	86.5	87.6	88.5	90.1	90.3	90.7	92.4
Rank	11	12.5	12.5	14	15	16	17	18	19	20

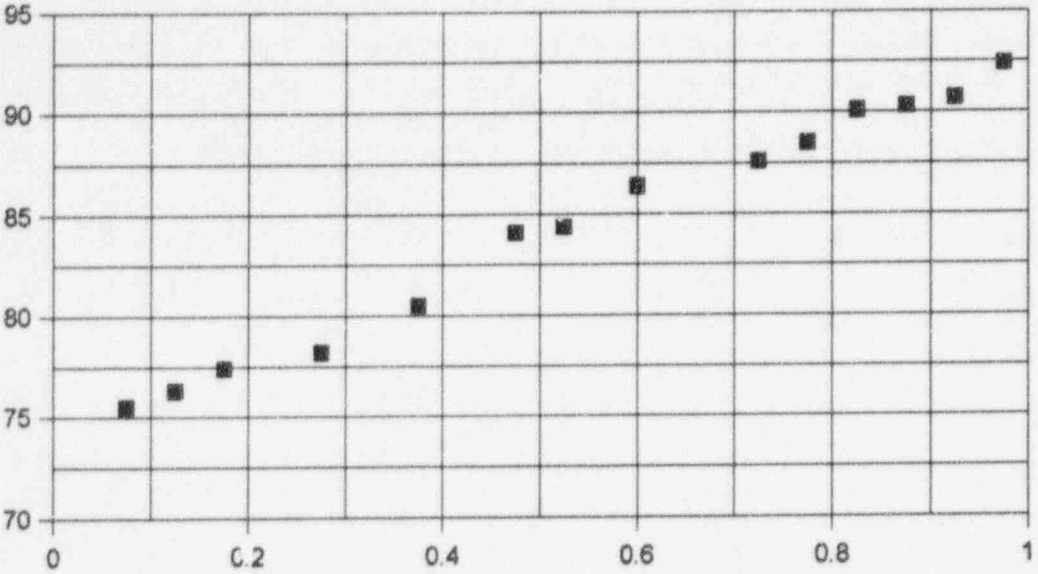


Figure 4.3 Example of a Quantile Plot

The slope of the curve in the Quantile plot is an indication of the amount of data in a given range of values. A small amount of data in a range will result in a large slope. A large amount of data in a range of values will result in a flatter slope. A sharp rise near the bottom or the top is an indication of asymmetry. There are no unusual features in the Quantile plot shown in Figure 4.3.

A Quantile-Quantile plot is useful for comparing two sets of data.

Suppose the following 17 data points were obtained in a reference area with a mean dose rate of 80 mrem per year and a standard deviation of 8 mrem per year:

92.1, 83.2, 81.7, 81.8, 88.5, 82.8, 81.5, 69.7, 82.4, 89.7,
81.4, 79.4, 82.0, 79.9, 81.1, 59.4, 75.3.

A Quantile-Quantile plot can be constructed to compare the distribution of the survey unit data, Y_j, j=1,...n, with the distribution of the reference area data X_i, i=1,... m. (If the reference area data set were the larger, the roles of X and Y would be reversed.) The data from each set are ranked separately from smallest to largest. This has already been done for the survey unit data. For the reference area data, we obtain the following:

Data :	59.4	69.7	75.3	79.4	79.9	81.1	81.4	81.5	81.7	81.8
Rank:	1	2	3	4	5	6	7	8	9	10
Data :	82.0	82.4	82.8	83.2	88.5	89.7	92.1			
Rank	11	12.5	12.5	14	15	16	17			

The median is 81.7, the sample mean is 80.7, and the sample standard deviation is 7.5.

For the larger data set, the data must be interpolated to match the number of points in the smaller data set. This is done by computing $v_1 = 0.5(n/m) + 0.5$ and $v_{i+1} = v_i + (n/m)$ for $i=2,...m$, where m is the number of points in the smaller data set and n is the number of points in the larger data set. For each of the ranks, i, in the smaller data set, a corresponding value in the larger data set is found by first decomposing v_i into its integer part, j, and its fractional part, g. Then the interpolated values are computed from the relationship $Z_i = (1-g)Y_j + gY_{j+1}$.

Finally, Z_i is plotted against X_i , to obtain the Quantile-Quantile plot. An example is shown in Figure 4.4 .

Rank (i)	1	2	3	4	5	6	7	8	9	10
v_i	1.0	2.26	3.44	4.62	5.79	6.97	8.15	9.33	10.50	11.68
Z_i	74.	75.7	76.8	77.5	78.1	79.1	80.9	83.7	84.3	85.8
Rank (i)	11	12.5	12.5	14	15	16	17			
v_i	12.	14.03	15.21	16.38	17.56	18.74	19.91			
Z_i	86.	86.5	87.8	89.1	90.2	90.6	92.3			

The middle data point plots the median of Y against the median of X. That this point lies above the line $Y=X$, shows that the median of Y is larger than the median of X. Indeed, the cluster of points above the line $Y = X$ in the region of the plot where the data points are dense, is an indication that the central portion of the survey unit distribution is shifted toward higher values than the reference area distribution.

Other useful techniques for exploratory data analysis are given in EPA QA/G-9.

4.7.3 Select the Statistical Test

Select the most appropriate procedure for summarizing and analyzing the data, based on the preliminary data review. For final status surveys, the two-sample statistical tests of Section 5 should be used when the radionuclide of concern appears in background, or if measurements are used that are not radionuclide-specific. The one-sample statistical tests of Section 6 should be

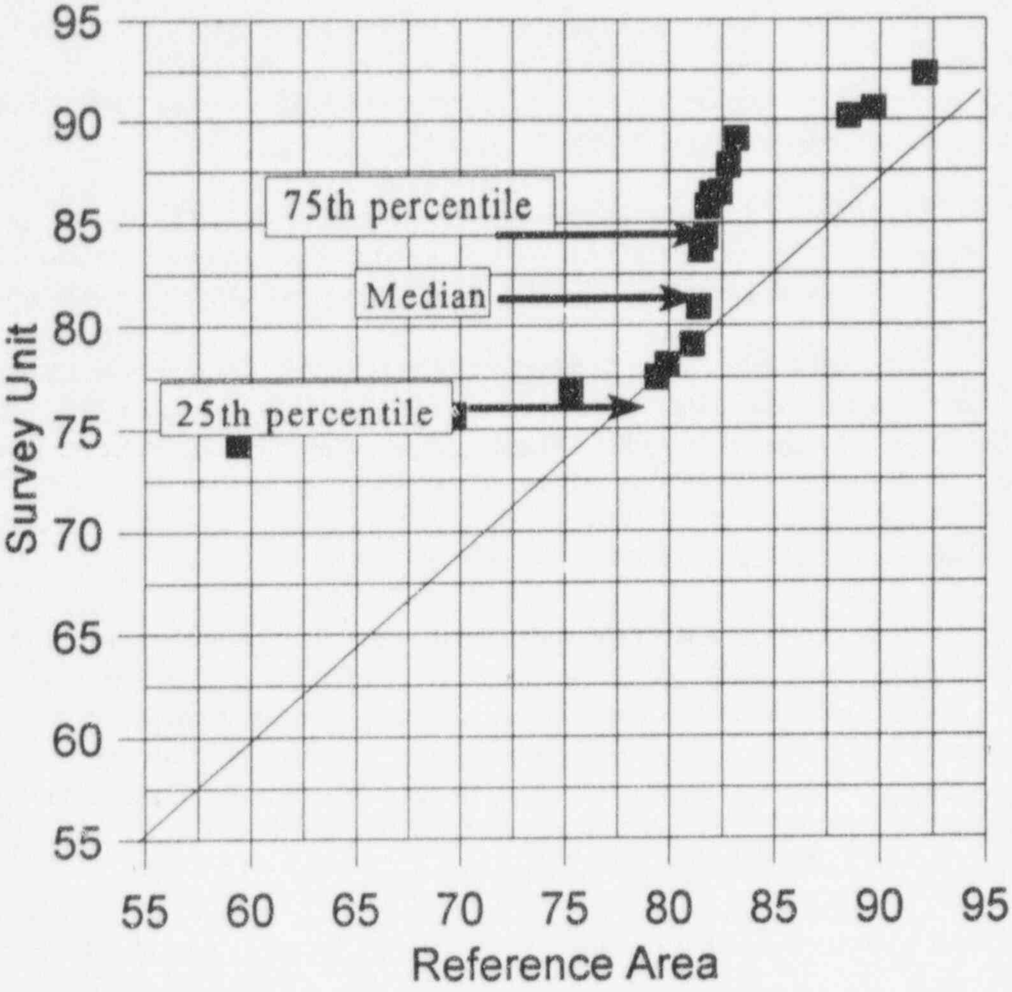


Figure 4.4 Example Q-Q Plot

used only when radionuclide-specific measurements are made of a radionuclide that does not appear in background.

Identify the key underlying assumptions that must hold for the statistical procedures to be valid. The nonparametric tests of Sections 5 and 6 require that the data from each reference area or survey unit consist of independent samples from the same distribution. The Wilcoxon Signed Ranks test (Section 6) assumes that the data are from a symmetric distribution. If the data distribution is symmetric, the median and the mean are the same. If the data are skewed, the Sign test (Section 6) should be used instead of the Wilcoxon Signed Rank test in the one-sample case. The Wilcoxon Rank Sum test assumes that the reference area and survey unit data distributions are the same except for a possible shift in the mean.

4.7.4 Verify the Assumptions of the Statistical Test

Evaluate whether the underlying assumptions hold, or whether departures are acceptable, given the actual data and other information about the study.

Spatial dependencies that could affect the assumptions of independent data can be assessed using the posting plots. More sophisticated tools for determining the extent of spatial dependencies are also available (e.g., EPA/QA/G-9; Cressie). These methods tend to be complex, and are best used with guidance from a professional statistician.

Asymmetry in the data can be diagnosed with a stem and leaf display, a histogram, or a Quantile plot. However, Hardin and Gilbert (PNL-8989) have shown that the Wilcoxon Rank Sum test and Quantile test perform well even with skewed distributions such as the log-normal or Weibull.

One of the primary advantages of the nonparametric tests used in this report is that they require fewer assumptions about the data than their parametric counterparts. If parametric tests are used, (e.g., Student's *t*-test), then these additional assumptions need to be verified (e.g., testing for normality). These issues are discussed in detail in EPA QA/G-9.

4.7.5 Perform the Statistical Test

Perform the calculations required for the statistical tests and document the inferences drawn as a result of these calculations. The specific details for conducting the statistical tests are given in Sections 5 and 6. It is an important part of this procedure, however, to evaluate the power of the tests retrospectively if the null hypothesis is not rejected. If the hypothesis that the site meets the decommissioning criteria is accepted, there must be reasonable assurance that the test was adequate to detect residual contamination in excess of the guidelines, had it existed. For this reason, it is better to plan the surveys cautiously:

- It is better to overestimate the potential data variability than to underestimate it.

- It is better to take too many samples than too few.

- It is better to overestimate minimum detectable concentrations (MDCs) than to underestimate them.

In the worst case, that the DQOs cannot be shown to have been met with reasonable assurance, a re-survey could be required.

5 PLANNING AND DESIGNING THE FINAL STATUS SURVEY WHEN COMPARING A SURVEY UNIT WITH A REFERENCE AREA

5.1 Design Considerations

For the purpose of survey design, the site is segregated into areas that are unaffected, affected/uniform, and affected/non-uniform. Areas that have been remediated are always classified as affected/non-uniform. There may be areas that were classified as affected but that did not require remediation. The affected (uniform or non-uniform, remediated or unremediated) areas of a remediated site should be divided into one or more survey units. The statistical tests discussed in this section will be used to compare each survey unit with an appropriately chosen, site-specific reference area. A reference area will be chosen on the basis of its similarity to the affected area. Each survey unit is a geographical area of specified size and shape for which a separate decision will be made on whether the unit attains the decommissioning criteria. Reference areas are geographical areas from which representative reference samples will be selected for comparison with samples collected in specific survey units at the remediated site.

A separate set of measurements or samples or both is collected and measured in each survey unit for comparison with the same type of samples and measurements from the applicable reference area. The remediated site may have one, a few, or many survey units. The number, location, size, and shape of survey units will vary depending on the size and topography of the site, the type of remedial action that was used, the expected patterns of residual contamination that might remain after remedial action, the radionuclides to be measured, the estimated level of residual radioactivity that may remain, and finally cost, convenience, and scheduling factors.

The concentrations measured during the final status survey are related to the dose guidelines through dose assessment models, as discussed in Section 3.2.3. Often there are assumptions in these models concerning the distribution and extent of the residual radioactivity. Survey units should be chosen in a manner that is consistent with the model assumptions.

The number of survey samples required for the nonparametric statistical tests does not directly depend on the survey unit size. Thus, the distances between samples in the field may be very different if the survey unit areas are of substantially different size. This would introduce another potential source of variability in the data, which should be avoided when possible. Therefore, the survey units should be approximately the same size. For similar reasons, it is desirable for the reference area to be approximately the same size as the applicable survey unit. However, the reference area should be large enough to encompass the full range of background conditions encountered in the survey units to which it is compared. To reduce variability in the background data, it may be better to choose several different reference areas for comparison with survey units that have very different background characteristics. These reference areas are collectively referred to as the "reference region." As shown later in this section, the number of samples required depends in part on the anticipated standard deviation of the measurements. If the reference region comprises few homogeneous areas, the standard deviation in each of those areas will tend to be less than the standard deviation in the reference region taken as a whole, so that fewer samples are required overall.

Neither the reference region nor the remediation site need be one contiguous area. At some decommissioning sites, a single reference area (perhaps the entire reference region) may be appropriate for all survey units. At other sites, the physical, chemical, or biological characteristics of different cleanup units may differ enough to warrant matching each survey unit with its own unique reference area within the reference region.

In some situations, reference areas that are not impacted but that are close to the survey unit may be preferred, assuming spatial proximity implies similarity of background radiological conditions. If concentrations differ systematically within the reference region, the individual reference areas may contain very different concentration levels. Under such conditions, reference areas and survey units should be matched carefully for similar radiological background characteristics. The conceptual site model developed during the Data Quality Objectives (DQO) process will be useful when choosing reference areas. Consideration may be given to using the entire reference region as the reference area for all survey units. However, since this will tend to increase the inherent variability of the background measurements, more samples may be needed to maintain the error rates established for the statistical tests. The complexity introduced by using more than one reference area should be balanced against the potential for minimizing the number of samples required. Using estimates of the reference area variability, it is possible to examine the alternatives during the survey *planning* stage to choose the most efficient method. However, reference areas should be chosen *before* sample sizes for the statistical tests are calculated. Reference areas should not be chosen on the basis of the resulting sample sizes, as this would tend to bias the results of the tests.

That an area is off site does not necessarily imply it is unaffected. In some cases, a region around the site may be established as a distinct survey unit (or units) from which samples are collected and evaluated for attainment of the applicable decommissioning criteria. However, this region may also be an area that became contaminated as a result of decommissioning activities or environmental transport mechanisms or both, rendering it inappropriate as an unaffected area.

5.2 Criteria for Selecting Reference Areas

The reference region and reference area(s) should be free of contamination from the site. Ideally, the distribution of radioactivity in the applicable reference area should be the same as the distribution of concentrations that would be present in the survey unit if that unit had never become contaminated by onsite activities. A reference area selected for comparison with a given survey unit or set of survey units should not differ from those survey units in physical, chemical, or biological characteristics that might cause measurements in the reference area and the survey unit to differ. Some of the considerations for selecting a reference area include past and present land use (an irrigated lawn *versus* an uncultivated plot) geological character (an area with numerous rock outcroppings *versus* a smooth soil surface), topography (a hill should not be compared with a gully in which runoff collects).

Radionuclide concentrations in the reference area and in survey units should not change after samples are collected in these areas. As discussed in Section 3.5, radionuclide concentrations in the reference area and at the remediated site will be subject to the short-term and long-term variability associated with background. The reference area and the survey unit should be sampled during the same or similar time periods to eliminate or reduce these temporal effects.

Measurements in both the reference area and survey unit should not be spatially correlated. Such correlations violate the assumptions of independence underlying the statistical tests discussed in the remainder of this section. Spatial correlations may occur if there are systematic variations in geological or other site characteristics that cause the level of radioactivity to increase or decrease in certain directions across either the reference or survey areas. Choosing these areas to be as internally homogeneous as possible will minimize the impact of spatial correlations. This will also reduce the random variability contributing to survey uncertainties. The presence of spatial correlations in the data will cause the Type I and Type II error rates predicted under the test assumptions to be incorrect, although how much and in which direction are likely to be very dependent on the site-specific nature of the correlations. In many cases, however, simply plotting the survey data on top of a site map will reveal if there is a potential problem. In Section 4.7, it was recommended that plotting of survey data be a routine part of the data analysis for both survey unit and reference area measurements before any formal statistical tests are performed. Selecting reference areas and survey units that satisfy the criteria given above will require professional judgment supported by historical or new (or both) measurements of samples, and will be aided by the DQO process outlined in Section 3.

To establish reference (background) areas for building interiors, onsite buildings of similar construction, but with no history of licensed operations, can be used. Reference areas and the survey units to which they are compared should have similar age, construction, and material. In general, the same criteria should be used in selecting interior sampling areas as were outlined above for selecting external sampling areas.

5.3 Statistical Tests

The comparison of measurements in the reference area and survey unit is made using two nonparametric statistical tests: the Wilcoxon Rank Sum (WRS) test (also called the Mann-Whitney test) and the Quantile test. In addition, an elevated measurement comparison is made against each measurement to assure that it does not exceed a specified investigation level.

The concept of the statistical power of a test was discussed in Section 2.3.9. The WRS test has more power than the Quantile test to detect uniform failure of remedial action throughout the survey unit. The Quantile test has more power than the WRS test to detect failure of remedial action in only a few areas within the survey unit. The advantage of these tests is that they do not require that the data be normally or log-normally distributed.

The WRS and Quantile tests also allow for "less than" measurements to be present in the reference area and the survey units. Frequently, measurements of radioactivity in soil and solid media will be reported by an analytical laboratory as being less than the analytical limit of detection. This results in a censored data set which generally is more difficult to analyze using parametric statistical tests. For example, the Student's *t*-test is sometimes implemented by replacing all "less than" data with the value of the lower detection limit when calculating averages. This method results in overestimates of the average that may be quite significant when compared to background. In contrast, as a general rule, the WRS test can be used with up to 40 percent "less than" measurements in either the reference area or the survey unit. The Quantile test can be used even when more than 50 percent of the measurements are below the limit of detection.

Both the WRS and Quantile tests should be conducted for each survey unit because the tests detect different types of residual contamination patterns in the survey units. In addition, an elevated measurement comparison is conducted. This consists of determining if any measurements in the remediated survey unit exceed a specified investigation level. If so, then additional investigation is required, at least locally, regardless of the outcome of the WRS and Quantile tests. The hypotheses tested by the WRS and Quantile tests are:

Null Hypothesis
H_0: Decommissioning criteria attained
versus
Alternative Hypothesis
H_a: Decommissioning criteria not attained

The null hypothesis is assumed to be true unless either statistical test indicates that it should be rejected in favor of the alternative.

When applying statistical tests, it should be understood that the use of these hypotheses will occasionally allow some survey unit measurements to be larger than some reference area measurements without rejecting the null hypotheses. The central issue addressed by these statistical tests, is whether the site measurements are sufficiently larger to be considered significantly (statistically) different from reference area measurements. Therefore, to apply these tests, what is meant by "larger" must be defined. This is one of the purposes of constructing the desired power curve as a function of residual radioactivity described in Section 3.7.

Statistical tests are constructed assuming specific alternative hypotheses, and the performance of the test is determined for those alternatives. In practice, it is not always certain what alternative might be most applicable, i.e., the actual pattern of residual radioactivity (if present) is unknown. This is the reason that both the Wilcoxon Rank Sum and Quantile tests are performed.

5.3.1 Wilcoxon Rank Sum Test

Formally, the hypotheses tested by the Wilcoxon Rank Sum test are (Conover):

Null Hypothesis
H_0: $F(x) = G(x)$ for all x
versus
Alternative Hypothesis
H_a: $F(x) > G(x)$ for some x

where
$F(x)$ is the cumulative probability distribution function of measurements in the reference area and $G(x)$ is the cumulative probability distribution function of measurements in the survey unit.

The assumptions are that the samples from the reference area and the survey unit are independent random samples from $F(x)$ and $G(x)$, respectively, and that each measurement is independent of every other measurement, regardless of the set of samples from which it came.

For practical purposes, any difference between $F(x)$ and $G(x)$ will result in a situation where the probability that a random measurement Y, from the survey unit is greater than a random measurement from the reference area X, is no longer equal to ½. If this probability is denoted by $P(Y>X) \equiv P_r$, then the hypotheses may be restated as follows:

Null Hypothesis
H_0: $P_r = ½$
versus
Alternative Hypothesis
H_a: $P_r > ½$

Another way of stating this is:

Null Hypothesis
H_0: the median concentration in the survey unit is the same as that in the reference area.
versus
Alternative Hypothesis
H_a: the median concentration in the survey unit is higher than that in the reference area.

If, in addition, it is assumed that any difference between $F(x)$ and $G(x)$ is due to a shift in the survey unit to higher values, i.e., $F(x) = G(x+\Delta)$, $\Delta > 0$, then the hypotheses can be re-stated as :

Null Hypothesis
H_0: the mean concentration in the survey unit is the same as that in the reference area
versus
Alternative Hypothesis
H_a: the mean concentration in the survey unit is higher than that in the reference area

In particular, if the distribution of measurements is symmetric, the mean and the median of the measurements are the same. Recent studies have shown that the WRS test is relatively insensitive to moderate departures from symmetry when testing hypotheses about the mean (PNL-8989). Thus, the results of applying the WRS test to hypotheses about the mean rather than the median will not be invalidated by measurement distributions that are moderately asymmetric. In Section 5.4, the method for determining P_r and Δ is developed in detail.

5.3.2 Quantile Test

The specific hypothesis tested by the Quantile test (see Johnson et al. and Gilbert & Simpson) is:

Null Hypothesis
H_0: $F(x) = G(x)$ for all x
versus

Alternative Hypothesis
H_a: $G(x) = (1-\epsilon) F(x) + \epsilon F(x - \Delta')$

where
$F(x)$ is the cumulative probability distribution function of measurements in the reference area and
$G(x)$ is the cumulative probability distribution function of measurements in the survey unit.

The Quantile test was specifically deveoped to detect differences between the survey unit and the reference area that consist of a shift by Δ' to higher values in a portion $0<\epsilon<1$ of the survey unit. It should be noted that, in general, Δ' is not the same as the Δ used in the WRS test.

The Quantile test hypotheses can be restated as:

Null Hypothesis
H_0: $\epsilon = 0$ and $\Delta' = 0$
versus
Alternative Hypothesis
H_a: $\epsilon>0$ and $\Delta' > 0$.

Simply put, the null hypothesis is that there is no residual radioactivity in any part of the survey unit. The Quantile test is better at detecting alternatives where only a portion, ϵ, of the survey unit contains excess residual radioactivity. The WRS test is better at detecting alternatives where any excess residual radioactivity is uniform across the entire survey unit. In Section 5.4, the methods for determining appropriate values for ϵ and Δ' are developed.

5.3.3 Elevated Measurement Comparison

The statistical tests discussed previously are designed to evaluate whether the residual radioactivity in an area satisfies the guidelines for contamination conditions that include both a uniform distribution and a "patchy" distribution of contamination. However, neither the Wilcoxon Rank Sum (WRS) test nor the Quantile test can be used to demonstrate that there are not potential elevated areas with residual radioactivity concentration that would result in a dose above the guidelines, if those areas were located entirely between the measurement locations used for those tests. Instead, measurements and sampling on a specified grid size, in conjunction with surface scanning, are used to ensure that any small area of elevated radioactivity that might remain would result in a dose no larger than the guidelines. This procedure is applicable for all radionuclides, regardless of whether or not they are present in background.

As mentioned in Section 5.1, the number of survey data points needed to apply the nonparametric tests does not directly depend on the size of the survey unit. However, once the number of data points is determined (Section 5.5), the spacing of these data points on the sampling grid (Section 5.6) can be determined. The grid area that is bounded by these survey locations represents the largest circular area of residual radioactivity that might exist and not be sampled. The amount of residual radioactivity, in an area of that size, that could result in a dose above the guideline (i.e., H_m) determines the necessary minimum detectable concentration (MDC) of the scan procedure. A method for calculating H_m is given in Section 5.4.

The actual MDCs of scanning techniques are then determined for the available instrumentation (see Section 4.4). The actual MDC of the selected scanning technique is compared to the required scan MDC. If the actual scan MDC is less than the required scan MDC, no additional sampling points are necessary for detecting potential small elevated areas. The scan survey will have adequate sensitivity to detect them. If, however, the actual scan MDC is greater than the required scan MDC, then it is necessary to increase the number of data points on the sampling grid until the area between data points is small enough to detect by scanning, or until the minimum grid spacing of 5 meters outdoors or 1 meter indoors is reached. The procedure for making this adjustment is discussed in Section 5.5.

Each area identified as elevated by the scan survey will be marked for further investigation, which may include additional measurements and sampling to determine the nature and extent of the residual radioactivity, and to determine whether the dose guidelines are actually exceeded by the radioactivity in that area.

5.4 Specification of the Applicable Decommissioning Criteria

The issue of how the test criteria should be developed has been addressed in Section 3.7. The present section will show how the results of the DQO process in Section 3.7 can be developed into formal criteria for statistical testing.

For the WRS test, the specification of the decommissioning criteria is made in terms of the amount of shift, Δ, toward higher values in the survey unit that is important to detect relative to the reference distribution. Δ is the dose limit (15 mrem per year or ALARA) expressed in terms of the corresponding radionuclide concentrations from NUREG-1500, NUREG/CR-5512 (Volume 1), or site-specific analysis.

If σ is the standard deviation of the measurements in the reference area, then Δ/σ, expresses this shift as the number of standard deviations toward higher values that would be considered "large" for the distribution of measurements in the survey unit. The shift Δ is a fixed value depending on the applicable decommissioning criteria. However, the ease or difficulty of detecting this shift statistically depends on the variability in the data, expressed by σ. Therefore, the statistical hypotheses must depend not solely on the absolute shift Δ, but on the relative shift Δ/σ, which expresses the shift relative to the variability in the measurements at a given site. As is generally the case, it is not possible to estimate the number of samples required in the survey units without some information about the variability of the data. As discussed in Section 3.4, some estimate for σ is needed, based on either prior sampling or other information.

The Quantile test also uses an amount of shift, Δ', and that is considered "large" as the specification of the decommissioning criterion. As mentioned previously, the Quantile test is meant to uncover "spotty" residual contamination, so the amount of shift specified for the Quantile test need not be the same as that used for the WRS test. However, it is necessary for the Quantile test to specify, in addition to Δ', and the proportion of the survey unit, ϵ, that is affected by that amount of shift. Because the Quantile test applies to a smaller area of the survey unit, a higher shift may be acceptable.

If a shift of Δ is specified for the WRS test, then an inventory less than $\Delta \bullet$(area) is implied to be "not large." If a shift of Δ' for a proportion ϵ of the measurements is specified for the Quantile test, then an inventory less than $\epsilon\Delta' \bullet$(area) is also implied to be "not large." This suggests that if a proportion of survey area ϵ is of concern, then a shift of Δ' as high as Δ/ϵ might reasonably be specified. Alternatively, if ϵ and Δ' are specified for the Quantile test, a shift of $\epsilon\Delta'$ might be considered for the WRS test. There is, however, no statistical reason that the criteria must be linked.

These ideas are illustrated in Figure 5.1. In each panel of this figure, the solid line is a normal probability density function with mean $\mu = 80$ mrem per year and standard deviation $\sigma = 8$. (The normal distribution is used here to represent the background dose in mrem per year in a reference area for illustrative purposes only.) The dashed lines in each panel represent a possible distribution of dose rates from background plus residual radioactivity in a survey unit that is to be compared with the reference area. In this example, the shift Δ varies from 3 to 50 mrem per year and the proportion of the survey unit with residual contamination, ε, varies from 0.3 to 1.0.

In the panels in which ϵ is near 1.0, the WRS test is more likely than the Quantile test to pick up the difference. When ϵ is small, the Quantile test is more likely than the WRS test to pick up the difference. If Δ is small, as in the top two panels, any method will have difficulty detecting the difference. Note that the increments of $\epsilon \bullet \Delta$ chosen for this example correspond to *average* increases in dose over background in the survey unit of either 3 or 15 mrem per year when $\epsilon=1$. As the amount of shift becomes larger, and the proportion of the survey area becomes smaller ($\epsilon<1$), the issue becomes less one of whether the entire survey area meets the criterion and more an issue of whether there are highly localized areas of contamination remaining within the survey area.

There is a level for which there will be concern if *any* measurement in the survey area exceeds the level. This level, denoted H_m, is related to the area associated with the sample grid spacing. If the spacing between grid points is G, this area will be approximately G^2, depending on the style of grid. On a triangular grid this area is $0.866G^2$. For outdoor areas if $0.866G^2$ is less than about 2,000 m^2 , the dose due to residual radioactivity must be adjusted by an area factor, A_m. Then $H_m = \Delta \cdot$(area factor). Tables of area factors computed using RESRAD 5.6 (ANL/EAD/LD-2) are given in Appendix C. For indoor areas, a similar adjustment must be made. The indoor area factors depend on the size of the room and the dose scenario as well as the spacing between grid points. Again, $H_m = \Delta \cdot$(area factor). Tables of indoor area factors computed using RESRAD BUILD 1.5 (ANL/EAD/LD-3) for a 36 m^2 room are given in Appendix C.

The elevated measurement comparison is intended to flag potential failures in the decommissioning process, and should not be considered the primary means to identify whether or not a site meets decommissioning criteria.

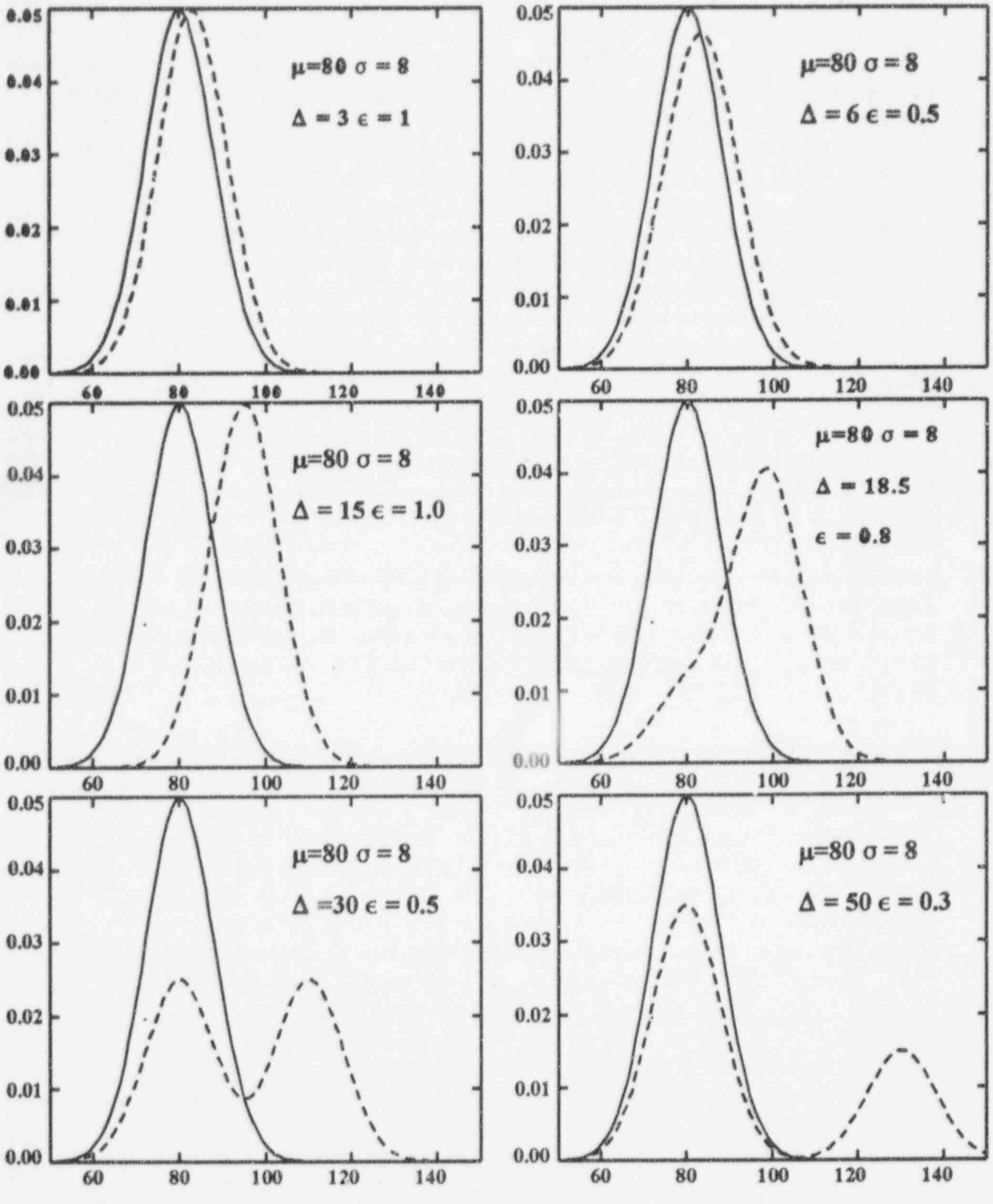


Figure 5.1 Hypothetical Reference Area (Solid) and Survey Unit (Dashed) Annual Dose Distribution.

5.5 Number of Samples

To determine the number of samples to collect, acceptable values of the Type I error rate (α) and Type II error rate (β) must be specified as part of the statistical test. The process for doing this was discussed in Section 3.7. If there are many survey units and each unit requires a separate decision, even if H_0 is true approximately 100α% of the times the test is conducted, the null hypothesis will be incorrectly rejected and the survey unit incorrectly declared to not meet the standard. If a larger value of α is used, the number of times this can be expected to happen increases proportionately. This could lead to many unnecessary additional remediations of survey units that actually meet the standard. On the other hand, larger values of α will reduce the number of samples initially required from each survey unit.

The power (1 - β) is the ability of a statistical test to detect when a survey unit does not meet the applicable decommissioning criteria. A test should have high power, i.e., small β, but smaller specified values of α and β require a larger number of measurements.

The number of samples depends not only on α and β, but also on the size of the shift that is important to detect. In general, the number of samples required for the WRS test and the Quantile test will differ even if α and β are the same.

Throughout the following procedure for determining the number of samples to collect, it must be emphasized that relatively little effort is required to perform the suggested sample size determinations compared to the time and expense involved in collecting and analyzing samples. This is a key advantage to using the DQO process to determine sample sizes. The recommmended steps follow.

(1) Select the overall Type I error rate, α, desired for both tests combined according to the procedures in Section 3.7. Then divide this overall error level by 2 and use this smaller value (α/2) to determine the number of samples. That is, if we denote the Type I error level set for the WRS test by α_W and that for the Quantile test by α_Q, then, because we are using both tests we set $\alpha_W = \alpha_Q = \alpha/2$. Note that the value of β is not affected by the use of the two tests because the power is specified independently for each test.

(2) The number of samples should first be determined using the procedures for the WRS test, given in Section 5.5.1, which assume that residual radioactivity concentrations in the survey unit will likely be uniform in value over space.

(3) Using these values of the sample size, find the power of the WRS and Quantile tests for various alternatives in the tables in Appendix A. The details of this procedure are given in Sections 5.5.1 and 5.5.2.

(4) Compare the power of the WRS test and Quantile test to the desired power curve developed using the process developed in Section 3.7.

(5) If the power of the tests at the computed sample size is too low, increase the number of samples and repeat the comparison until a satisfactory power curve is obtained.

(6) If the computed power of the tests is too high, decrease the number of samples, then repeat the power comparison with the new sample size to ensure that the power is still adequate.

(7) If a very large number of samples is required to achieve the desired Type I (α) and Type II (β) error rates, the error rates may have been selected at a lower level than was necessary or appropriate. This possibility should be examined and, if possible, the error rates may need to be adjusted upward to result in a more realistic number of samples.

(8) The adequacy of the sample size is then examined for detecting an area of elevated activity ($> H_m$) of a given size, and adjusted upward if necessary, according to the procedures of Section 5.5.3.

This iterative procedure is recommended because the sample size determinations for the WRS test are relatively straightforward, whereas the Quantile test requires an estimate of ϵ in order to calculate the sample size. Without a great deal of prior information, this estimate of ϵ is likely to be very speculative. After determining the sample size based on the WRS test, it is possible to see if that sample size results in adequate power for the Quantile test over the range of values of ϵ that is considered important for the survey unit.

Similarly, calculating the required sample size for detecting an area of elevated activity requires specifying the size and shape of the area of concern. Again, the sample size determined for the WRS test can be examined to see if there is adequate coverage of the survey unit to detect the desired range of possible areas of elevated residual radioactivity. In either case, the sample size can be adjusted to suit the purpose.

After the number of samples required to meet the decision requirements is determined, another 10 to 20 percent should be added to allow for the possibility of sample loss during transportation or analysis. This will help to ensure that there are an adequate number of samples to achieve the specified power of the test. In addition, planning should allow for the collection, preparation, and analysis of separate quality control samples.

5.5.1 Determining the Number of Samples for the WRS Test

For the Wilcoxon Rank Sum test, the *total* number of required samples from the reference area and survey unit combined is

$$N = \frac{(Z_{1-\alpha/2} + Z_{1-\beta})^2}{12c(1-c)(P-0.5)^2} \tag{5-1}$$

where:

α = specified Type I error rate
β = specified Type II error rate
$Z_{1-\alpha/2}$ = 100(1-α/2) percentile of the standard normal distribution function
$Z_{1-\beta}$ = 100(1-β) percentile of the standard normal distribution function
c = proportion of samples to be collected in the reference area

P_r = specified probability required to detect that a random measurement from the survey unit is larger than a random measurement from the reference area. P_r is greater than 0.5 whenever $\Delta/\sigma > 0$. If $P_r = 0.5$, then $\Delta/\sigma = 0$, and there is no residual radioactivity to detect.

Table 5.1 gives commonly used values of α (or β), namely, 0.01 , 0.025, 0.05, and 0.10, and the corresponding values of $Z_{1-\alpha/2}$ (or $Z_{1-\beta}$).

Table 5.1 Values of $Z_{1-\alpha/2}$ and $Z_{1-\beta}$ Used To Calculate the Sample Size for the WRS Test

α/2 (or β)	$Z_{1-\alpha/2}$ (or $Z_{1-\beta}$)
0.005	2.576
0.01	2.326
0.0125	2.241
0.025	1.960
0.05	1.645
0.10	1.282

The parameter P_r is determined using the specified shift Δ/σ that must be detected with power 1 - β. Values of P_r, computed for a normal distribution from the equation

$$P_r = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\frac{\Delta}{\sqrt{2}\cdot\sigma}} e^{-x^2/2} dx = \Phi\left(\frac{\Delta}{\sqrt{2}\cdot\sigma}\right) \tag{5-2}$$

can be found from Table 5.2. The normal distribution is used here only to facilitate the conversion of the values of Δ/σ to values of P_r in order to calculate the appropriate sample sizes and error rates. The normal distribution is not used to actually conduct the test.

Values of P_r for other distributions can be calculated from the equation

$$P_r = \text{Probability}(Y - X > 0) = \text{Probability}(X - Y < 0) = \int_{-\infty}^{0}\left[\int_{-\infty}^{\infty} f_X(u + y) f_Y(y) dy\right] du \tag{5-3}$$

where Y is a random measurement from the survey unit with density f_Y and X is a random measurement from the reference area with density f_X. This will generally not be necessary, as Hardin and Gilbert (PNL-8989) have found that using the values of P_r from Equation 5-2 yields good results even when the distributions being tested are lognormal or Weibull. They found that

Table 5.2 Values of P_r for a Given Shift Δ/σ.

Δ/σ	P_r	Δ/σ	P_r
0	0.500	2	0.921
0.0625	0.518	2.0625	0.928
0.125	0.535	2.125	0.933
0.1875	0.553	2.1875	0.939
0.25	0.570	2.25	0.944
0.3125	0.587	2.3125	0.949
0.375	0.605	2.375	0.953
0.4375	0.621	2.4375	0.958
0.5	0.638	2.5	0.961
0.5625	0.655	2.5625	0.965
0.625	0.671	2.625	0.968
0.6875	0.687	2.6875	0.971
0.75	0.702	2.75	0.974
0.8125	0.717	2.8125	0.977
0.875	0.732	2.875	0.979
0.9375	0.746	2.9375	0.981
1	0.760	3	0.983
1.0625	0.774	3.0625	0.985
1.125	0.787	3.125	0.986
1.1875	0.799	3.1875	0.988
1.25	0.812	3.25	0.989
1.3125	0.823	3.3125	0.990
1.375	0.835	3.375	0.991
1.4375	0.845	3.4375	0.992
1.5	0.856	3.5	0.993
1.5625	0.865	3.5625	0.994
1.625	0.875	3.625	0.995
1.6875	0.884	3.6875	0.995
1.75	0.892	3.75	0.996
1.8125	0.900	3.8125	0.996
1.875	0.908	3.875	0.997
1.9375	0.915	3.9375	0.997

the WRS test is insensitive to the distribution type and shape when the power is expressed as a function of P_r.

The proportion, c, of measurements to be taken from the reference area is determined from Hochberg and Tamhane, p. 202 as follows:

$$c = \frac{\nu^2 h^{1/2}}{\nu^2 h^{1/2} + 1} \tag{5-4}$$

where

h = the number of survey units being compared to the given reference area and

$\nu = \sigma_{reference} / \sigma_{survey}$, the ratio of the standard deviation of the measurements in the reference area to the standard deviation of the measurements in the survey units.

The value of h will be known, but the value of v will have to be estimated on the basis of previous samples or expert opinion. This is another case in which an early estimate of the expected data variability is important (see the discussion in Section 3.4). However, if the reference area and the survey units are comparable (as they *should* be if remediation was successful), it is not unreasonable to assume that $v=1$ in the absence of any information to the contrary. In that case, Equation 5-4 simplifies to

$$c = \frac{h^{1/2}}{h^{1/2}+1} \tag{5-5}$$

Table 5.3 gives values of c computed using Equation 5-5.

Table 5.3 Proportion of Samples, c, To Be Taken in the Reference Area When Comparing to h Survey Units When the Measurement Standard Deviations Are the Same for Both.

h	$h^{1/2}$	c
1	1	0.50
2	1.414	0.59
4	2	0.67
5	2.237	0.69
10	3.162	0.76
20	4.472	0.82

Once N is calculated, then $m = c \cdot N$ samples should be taken from the reference area, and $n = (1 - c) \cdot N$ samples should be taken from *each* of the survey units being compared with it.

5.5.1.1 Example

To illustrate the process described above, consider the example given in Section 3.7. Here the reference area is assumed to have a distribution of background dose rate measurements with a mean of 80 mrem per year and a standard deviation of 8 mrem per year. To ensure that there is no residual contamination in the survey unit over 15 mrem per year above background, i.e., that the *total* average dose rate in the survey unit is less than 95 mrem per year, set $\Delta/\sigma = 15/8 = 1.875$. If the survey unit has been adequately remediated, then the standard deviation of the measurements from the reference area and the survey unit should be about the same, so

$$v = \sigma_{reference} / \sigma_{survey} = 1.$$

Since only one survey unit is being compared with this reference area, $h = 1$. Therefore,

$$c = \frac{h^{1/2}}{h^{1/2}+1} = 1/2$$

The number of samples required now depends on the power curve constructed during the DQO process. Table 5.4 provides data to illustrate how the number of samples depends on the Type I error rate (α) and the power ($1-\beta$). This table was constructed using Equation 5-1 and inserting the appropriate value of P_r from Table 5.2 :

$$N = \frac{(Z_{1-\alpha/2} + Z_{1-\beta})^2}{12c(1-c)(P_r - 0.5)^2}$$

$$= \frac{(Z_{1-\alpha/2} + Z_{1-\beta})^2}{12(0.5)(1-0.5)(0.908-0.5)^2}$$

$$= \frac{(Z_{1-\alpha/2} + Z_{1-\beta})^2}{3(0.408)^2} \tag{5-6}$$

Note that since both the WRS and Quantile tests will be used, $\alpha_W = \alpha/2$ is used in Equation 5-1. In general, the number of samples should be *rounded up* to the next integer. An additional 20 percent should be added to ensure that the power will not be underestimated. (The allowance for missing or unusable data, and any quality assurance/quality control (QA/QC) samples are to be added to this larger number.)

Table 5.4 Number of Samples Required for WRS Test With $\Delta/\sigma = 15/8 = 1.875$ and P_r=0.908

α	$\alpha_W=\alpha/2$	$1-\beta$	$Z_{(1-\alpha/2)}$	$Z_{(1-\beta)}$	N	1.16 N_t
0.010	0.0050	0.990	2.576	2.326	49	36
0.010	0.0050	0.975	2.576	1.960	42	32
0.010	0.0050	0.950	2.576	1.645	36	28
0.010	0.0050	0.900	2.576	1.282	30	24
0.025	0.0125	0.990	2.241	2.326	42	31
0.025	0.0125	0.975	2.241	1.960	36	27
0.025	0.0125	0.950	2.241	1.645	31	23
0.025	0.0125	0.900	2.241	1.282	25	20
0.050	0.0250	0.990	1.960	2.326	37	27
0.050	0.0250	0.975	1.960	1.960	31	23
0.050	0.0250	0.950	1.960	1.645	27	20
0.050	0.0250	0.900	1.960	1.282	22	17
0.100	0.0500	0.990	1.645	2.326	32	23
0.100	0.0500	0.975	1.645	1.960	27	19
0.100	0.0500	0.950	1.645	1.645	22	16
0.100	0.0500	0.900	1.645	1.282	18	13

An alternative method for determining the sample size is suggested by an Environmental

Protection Agency report, EPA QA/G-9. The WRS test has a Pitman efficiency of greater than 0.86 relative to the Student's *t*-test for *any* residual radioactivity distribution (Lehmann and D'Abrera, p. 377). This means that the WRS should not require more than about 1/(0.86) = 1.16 times the number of samples required by the Student's *t*-test to achieve the same power. (This result is exact only for very large sample sizes, but can be expected to be a reasonable approximation for most cases.) The sample size required for the *t*-test can be calculated from

$$N_t = 4\frac{\sigma^2}{\Delta^2}(Z_{1-\alpha/2} + Z_{1-\beta})^2 + 0.5(Z_{1-\alpha/2})^2$$

$$= 1.138(Z_{1-\alpha/2} + Z_{1-\beta})^2 + 0.5(Z_{1-\alpha/2})^2 \qquad (5\text{-}7)$$

The values of $1.16N_t$ are shown in the last column of Table 5.4. It is prudent to use the larger sample size calculated from Equation 5-6. The larger sample size will result in higher power, and the consequences of underestimating the power can be severe if the DQOs are not met. Nevertheless, the use of Equation 5-7 provides a useful check.

The number of samples calculated from Equation 5-4 vary from 18 to 48, depending on the values of α and β. The number of samples required is 27 for a Type I and Type II error rate of 5 percent. Adding an additional 20 percent gives (1.2)(27) = 32.4. This means that 17 (16.2 rounded up) measurements each in the reference area and the survey unit are required. Again, this is done to assure that the power of the test will not be underestimated.

5.5.2 Checking the Power of the WRS Test

The power tables in Appendix A.2 were obtained by Gilbert and Simpson (PNL-7409) using computer simulations. They assumed that the reference area and survey unit measurements were normally distributed, and that the survey unit contained randomly distributed residual contamination. In practice, the measurements are often not normally distributed, and so the power results must be viewed as being approximations. Hardin and Gilbert (PNL-8989) performed similar calculations for background data assumed to be distributed according to log-normal and Weibull distributions. They found that the WRS test is insensitive to the distribution type and shape when the power is expressed as a function of P_r.

From the tabulated values of the power of the WRS test in Appendix A.2, for $\alpha/2 = 0.025$, $\epsilon = 1$ and with $\sigma = 8$ we find the following:

Table 5.5 Power of WRS Test for Example Problem

WRS Test	Δ/σ							
α/2 = 0.025	0.5	1.0	1.5	2.0	2.5	3.0	3.5	4.0
m = *n* = 15	0.25	0.73	0.96	1	1	1	1	1
m = *n* = 20	0.32	0.85	0.99	1	1	1	1	1

There is no entry for $m = n = 17$, but the power for these sample sizes will fall between the power for $m = n = 15$ and the power for $m = n = 20$.

Recall from Section 5.4 that ϵ is the proportion of the survey unit that is affected by the amount of shift, Δ. For the WRS test, it is assumed that $\epsilon = 1$. The power for other values of ϵ shown in Table A-2 may be used for comparing the power of the WRS test to that of the Quantile test (Table A-3).

The data in Table 5.5 are plotted in Figure 5.2 and compared against the DQOs, which for this example are $\alpha = 0.05$ ($\alpha/2 = 0.025$) and $\beta = 0.05$. (Note that these DQOs are slightly different from those illustrated in Section 3.7. In Section 3.7, we had $\beta = 0.025$. Again, there is no statistical requirement that $\alpha = \beta$, it just happens that this is the case for this example.)

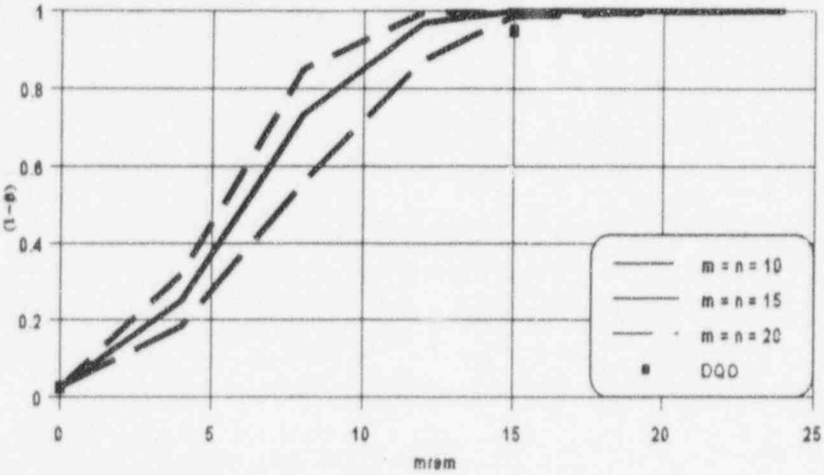


Figure 5.2 Power of WRS Test for the Example Problem

The figure shows that the design objectives are very closely matched by the power curve. Note that the false positive error rate, α, is fixed at zero mrem per year above background (no residual radioactivity). The rate at which the null hypothesis will be rejected at 3 mrem per year above background is less than 20 percent. The power at 15 mrem per year above background appears to be about as required, or perhaps a little higher. As discussed in Section 3.7.5, it is not always possible to design the test so that the error rates are exactly as specified. For the final status survey, priority is given to satisfying the DQO for the power $1 - \beta$ (where β is a false negative error rate). This is because the consequence of false negative of errors would impact human health, whereas the consequences of false positive errors would be primarily economic.

To illustrate the effect of increased variability in the background on required sample sizes, the calculations leading to Table 5.4 were repeated assuming a standard deviation of 16 rather than 8 mrem. As can be seen in Table 5.6, the number of samples required has almost tripled.

Table 5.6 Number of Samples Required for WRS Test With $\Delta/\sigma = 15/16 = 0.9375$ and $P_r = 0.746$

α	$\alpha_W = \alpha/2$	$1-\beta$	$Z_{(1-\alpha/2)}$	$Z_{(1-\beta)}$	N
0.010	0.0050	0.990	2.576	2.326	133
0.010	0.0050	0.975	2.576	1.960	114
0.010	0.0050	0.950	2.576	1.645	98
0.010	0.0050	0.900	2.576	1.282	82
0.025	0.0125	0.990	2.241	2.326	115
0.025	0.0125	0.975	2.241	1.960	98
0.025	0.0125	0.950	2.241	1.645	84
0.025	0.0125	0.900	2.241	1.282	69
0.050	0.0250	0.990	1.960	2.326	101
0.050	0.0250	0.975	1.960	1.960	85
0.050	0.0250	0.950	1.960	1.645	72
0.050	0.0250	0.900	1.960	1.282	58
0.100	0.0500	0.990	1.645	2.326	87
0.100	0.0500	0.975	1.645	1.960	72
0.100	0.0500	0.950	1.645	1.645	60
0.100	0.0500	0.900	1.645	1.282	48

Finally, consider the case in which the difference between the background area and the survey unit that is important to detect is set at 3 mrem rather than 15 mrem, with the standard deviation set at 8 mrem. Table 5.7 shows nearly 20 times the number of samples is required to detect a difference of 3 mrem, compared to the number required to detect a difference of 15 mrem.

Table 5.7 Number of Samples Required for WRS Test With $\Delta/\sigma = 3/8 = 0.375$ and $P_r = 0.605$

α	$\alpha_W = \alpha/2$	$1-\beta$	$Z_{(1-\alpha/2)}$	$Z_{(1-\beta)}$	N
0.010	0.0050	0.990	2.576	2.326	727
0.010	0.0050	0.975	2.576	1.960	623
0.010	0.0050	0.950	2.576	1.645	539
0.010	0.0050	0.900	2.576	1.282	451
0.025	0.0125	0.990	2.241	2.326	631
0.025	0.0125	0.975	2.241	1.960	534
0.025	0.0125	0.950	2.241	1.645	457
0.025	0.0125	0.900	2.241	1.282	376
0.050	0.0250	0.990	1.960	2.326	556
0.050	0.0250	0.975	1.960	1.960	465
0.050	0.0250	0.950	1.960	1.645	393
0.050	0.0250	0.900	1.960	1.282	318
0.100	0.0500	0.990	1.645	2.326	477
0.100	0.0500	0.975	1.645	1.960	393
0.100	0.0500	0.950	1.645	1.645	328
0.100	0.0500	0.900	1.645	1.282	260

The power of the WRS test can be checked in two additional ways. They both involve approximations to the power function, but are derived in different ways (Lehmann and D'Abrera, Chapter 2, Section 3, pp. 69-75).

The first method involves approximating the distribution of the Mann-Whitney form of the WRS test statistic, $W_r + 0.5n(n+1)$, by a normal distribution to compute the probability that the null hypothesis will be rejected when the alternative is true. For this, the mean and variance of $W_r + 0.5n(n+1)$ when the alternative is true must be calculated. When the alternative consists of a shift in the mean of Δ in the survey unit over the reference area the mean

$$E(W_r) = mnp_1$$

and the variance

$$\mathrm{Var}(W_r) = mnp_1(1-p_1) + mn(n-1)(p_2-p_1^2) + nm(m-1)(p_3-p_1^2).$$

p_1 is the probability that a random survey unit measurement is greater than a random reference area measurement. When these measurement distributions are normal, and differ only by a shift, Δ, in the mean, $p_1 = P_r$, and can be calculated from Equation 5-2. p_2 is the probability that two random measurements from the survey unit will each be greater than a single random measurement from the reference area; and p_3 is the probability that two random measurements from the reference area unit will each be less than a single random measurement from the reference area. If the measurement distributions are symmetric, then $p_2 = p_3$. If the measurement distributions are normal, then p_2 is equal to the probability that two correlated standard (i.e., mean = 0 and variance = 1) normal random variables, with correlation coefficient 0.5, are both less than $\Delta/(\sigma\sqrt{2})$. Values of p_1, p_2, and p_3 as a function of Δ/σ are given in Table 5.8.

The power of the WRS test is then computed from

$$Power = 1 - \Phi\left[\frac{W_c - 0.5 - 0.5n(n+1) - E(W_r)}{\sqrt{Var(W_r)}}\right]$$

where W_c is the crtical value found in Table A-1 for the appropriate vales of α, n and m. Values of $\Phi(z)$, the standard normal cumulative distribution function, are given in Table A-7. Using this equation for the example problem $\alpha_W = \alpha/2 = 0.025$, n=17, m=17, Δ/σ=1.875, $p_1 = 0.553$, $p_2 = p_3 = 0.844$, and $W_c = 354$, the approximate power is 0.9999, in agreement with the simulation results in Table 5.5. Comparisons of the result of using this equation with the power tables in Appendix A.2 show that the results are generally accurate enough to be used to determine compliance of the sample design with DQOs.

Table 5.8 Values of p_1 and p_2 for Computing the Mean and Variance of W_r - 0.5$n(n+1)$

Δ/σ	p_1	$p_2 = p_3$	Δ/σ	p_1	$p_2 = p_3$
0.5	0.638163	0.482593	2.1	0.931218	0.881527
0.6	0.664313	0.513387	2.2	0.940103	0.895917
0.7	0.689691	0.544073	2.3	0.948062	0.908982
0.8	0.714196	0.574469	2.4	0.955157	0.920777
0.9	0.737741	0.604402	2.5	0.961450	0.931365
1.0	0.760250	0.633702	2.6	0.967004	0.940817
1.1	0.781662	0.662216	2.7	0.971881	0.949208
1.2	0.801928	0.689800	2.8	0.976143	0.956616
1.3	0.821015	0.716331	2.9	0.979848	0.963118
1.4	0.838901	0.741698	3.0	0.983053	0.968795
1.5	0.855578	0.765812	3.1	0.985811	0.973725
1.6	0.871050	0.788602	3.2	0.988174	0.977981
1.7	0.885334	0.810016	3.3	0.990188	0.981636
1.8	0.898454	0.830022	3.4	0.991895	0.984758
1.9	0.910445	0.848605	3.5	0.993336	0.987410
2.0	0.921350	0.865767			

The second approximation suggested by Lehmann and D'Abrera is useful if the reference area and the survey unit measurement distributions are not normal. This approximation is made assuming that the difference, Δ, in the means between the survey unit and the reference area is small. In this case,

$$Power \approx \Phi\left[\sqrt{\frac{12mn}{n+m+1}}\, f^*(0)\Delta - Z_{1-\alpha/2}\right]$$

Here, $f^*(0)$ is the probability density of the difference of two random variables with the same cumulative distribution, evaluated at zero. For two normally distributed random variables,

$$f^*(0) = \frac{1}{2\sigma\sqrt{\pi}}$$

Using this approximation for the example problem yields a power estimate of 0.9995.

The above power approximation may be inverted to give estimates of the sample size needed to achieve a desired power, namely,

$$n = m \approx \frac{(Z_{1-\alpha/2} + Z_{1-\beta})^2}{6\Delta^2 f^{*2}(0)}$$

If the measurement distributions are normal, this becomes

$$n = m \approx \frac{2\pi\sigma^2(Z_{1-\alpha/2} + Z_{1-\beta})^2}{3\Delta^2}$$

5.5.3 Checking the Power of the Quantile Test

Using the tables in Appendix A.3, the approximate power of the Quantile test for the example discussed in Section 5.5.1 can be checked. As with the WRS test, the entries for $m = n = 15$ and $m = n = 20$ are used, since the power for the sample sizes $m = n = 17$ will fall between these. The shaded areas of the tables below show those combinations of ϵ and Δ for which the Quantile test has a power of 0.95 or greater. Note that in these tables, the value of $\alpha_Q \equiv \alpha/2$ is not exactly 0.025. For $m = n = 15$, α_Q is 0.021; and for $m = n = 20$, α_Q is 0.0[illegible]. This happens because the parameters r and k used in the Quantile test must be integers, and there may often be no combination of two integers that will yield exactly the desired value of $\alpha/2$. In practice, the differences are small and it suffices to use a value of $\alpha/2$ that is close to that desired. Recall from Section 2.2.2 that the Quantile test looks at the r highest measurements of the total of $n+m$ measurements, and that the null hypothesis is rejected if k or more of them are from the survey unit.

From Table 5.9, it may be seen that the Quantile test in this example has reasonably high power even when as little as 60 percent of the site is above 16 mrem. More extensive power results are contained in Hardin and Gilbert (PNL-8989)which may be consulted if more detail is necessary. Power tables for Weibull and log-normal distributions are given in DOE/RL/94/72.

If the Quantile test is not considered to possess sufficient power using the sample sizes determined for the WRS test, then more samples would have to be taken. Of course, this will affect both types of error rates for both tests, and that would also have to be taken into consideration. At this point in the procedure, the concern is with assuring that sufficient samples are taken to conduct the test. How the test is actually applied is discussed further below.

5.5.4 Probability of Detecting an Area of Elevated Activity

As discussed in Section 5.3.3, there should be reasonable assurance that very small areas of elevated residual radioactivity are not missed during the final status survey by sampling on a random start triangular grid. The procedures described in this section are intended to provide that assurance.

Thus far, the determination of sample sizes did not explicitly take into account the actual surface area of the survey unit. When the concern is finding areas of elevated activity, the area of the survey unit must be explicitly taken into account.

Table 5.9 Example Power Tables for the Quantile Test

	Δ (mrem) $n = m = 20$							
ϵ	**4**	**8**	**12**	**16**	**20**	**24**	**28**	**32**
0.1	0.031	0.043	0.063	0.084	0.114	0.138	0.143	0.031
0.2	0.038	0.072	0.127	0.217	0.309	0.402	0.462	0.495
0.3	0.046	0.110	0.225	0.381	0.555	0.687	0.760	0.813
0.4	0.059	0.150	0.318	0.538	0.723	0.868	0.925	0.954
0.5	0.075	0.202	0.414	0.669	0.854	0.941	0.979	0.993
0.6	0.088	0.251	0.512	0.761	0.907	0.976	0.995	0.998
0.7	0.105	0.303	0.600	0.827	0.945	0.987	0.998	1
0.8	0.112	0.346	0.645	0.868	0.966	0.991	0.998	1
0.9	0.129	0.394	0.708	0.898	0.977	0.994	1	1
1	0.155	0.431	0.743	0.923	0.980	0.997	1	1

	Δ (mrem) $n = m = 15$							
ϵ	**4**	**8**	**12**	**16**	**20**	**24**	**28**	**32**
0.1	0.025	0.036	0.046	0.063	0.086	0.085	0.092	0.096
0.2	0.034	0.06	0.094	0.151	0.201	0.25	0.291	0.3
0.3	0.044	0.09	0.162	0.277	0.396	0.489	0.553	0.596
0.4	0.052	0.123	0.244	0.411	0.584	0.723	0.789	0.829
0.5	0.066	0.156	0.329	0.556	0.739	0.858	0.923	0.948
0.6	0.073	0.213	0.421	0.658	0.842	0.931	0.975	0.989
0.7	0.086	0.25	0.498	0.743	0.903	0.973	0.992	0.998
0.8	0.097	0.297	0.561	0.812	0.936	0.986	0.997	1
0.9	0.11	0.331	0.632	0.856	0.961	0.99	0.998	1
1	0.122	0.372	0.684	0.889	0.969	0.994	0.999	1

Gilbert (1987) has described a procedure to determine the number of samples required to find an elliptical area of size L and shape S, where L = half the length of the long axis of the elliptical area and, if the area is circular, then L is simply the radius and $S = 1$.

The number of sampling points, n, is related to the distance between samples, G, and the area of the survey unit, A_S. For a square sampling grid this relationship is $n = A_S/G^2$, and for a triangular grid $n = A_S/(0.866G^2)$. Substituting the known area of the survey unit for A_S, and the number of samples required for the WRS test for n, the corresponding distance between samples is $G = \sqrt{A_s/n}$ for a square grid and $G = \sqrt{A_s/(0.866n)}$ for a triangular grid.

It is important to note that the area of the ellipse being sought is $A_E = \pi S L^2$, so that for a given value of L, an ellipse with shape $S = 0.2$ has only one-fifth the area of an ellipse with $S = 1.0$. Figure 5.3 shows an example of a circular ($S = 1.0$) area with $L = G/2$ and an elliptical ($S = 0.2$) area with $L = G$.

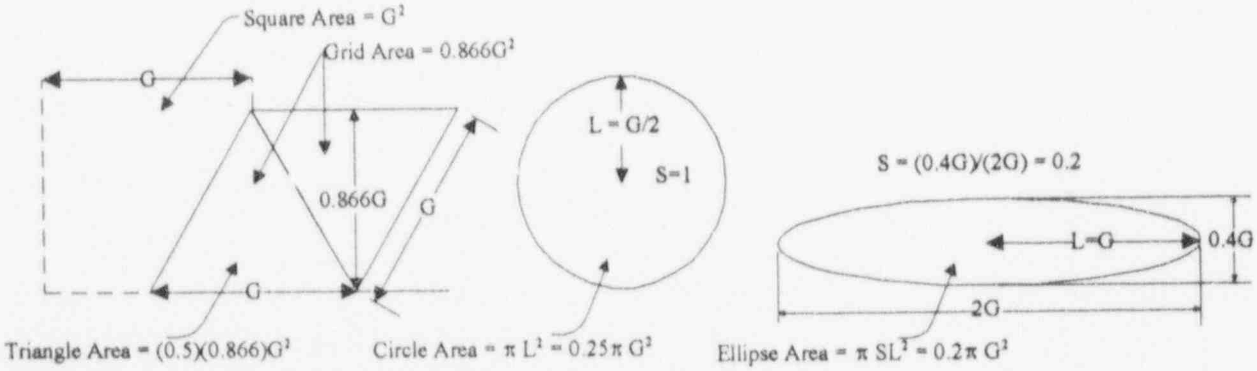


Figure 5.3 Circular and Elliptical Areas Relative to the Sample Grid.

First it is necessary to know how large an elevated area could conceivably be missed by sampling on a triangular grid. A methodology for determining the probability that an elliptically shaped elevated area would be missed by a triangular sampling grid was developed by Singer (1972). Singer's computer code, ELIPGRID, has been improved and modified for use on personal computers by Davidson (ORNL/TM-12774). This code, ELIPGRID-PC, was used to generate the data for Figure 5.4. In this figure, the horizontal axis is the semi-major axis lengh of the area expressed in units of grid spacings, L/G. The left vertical axis shows the probability of *not* finding an elevated area of that size. The different curves correspond to different shape parameters S. The white square in Figure 5.4 corresponds to the elliptical area shown in Figure 5.3 ($S = 0.2$, $L/G = 1$). The probability is about 40 percent that it would go undetected. In contrast, the black square in Figure 5.4 corresponds to the circular area shown in Figure 5.3 ($S = 1.0$, $L/G = 0.5$). The probability is less than 10 percent that it would go undetected, even though its area ($0.785\ G^2$) is only slightly larger than the area of the ellipse ($0.628\ G^2$).

The data used to construct Figure 5.4 are given in Table A-5, (Appendix A.5). In the table of values presented there, the probability of not detecting an elevated area of size πSL^2 with semi-major axis L/G and shape parameter S is listed. The size of the elevated area relative to the area defined by a triangular grid, ($0.866G^2$, see Figure 5.3) is also given in Table A-5. It is apparent from that table, that when the size of an elevated area is close to or greater than the grid area, the probability of missing it is rather low unless the shape parameter is also very low.

It can be concluded that, in most cases, an elevated area of the same size as, or larger than, that defined by the sampling grid would be discovered during the final status survey. However, this does not provide assurance that the guideline dose would not be exceeded by elevated residual radioactivity contained in a smaller area. Since the Wilcoxon Rank Sum (WRS) test and the Quantile test both use the data from the sampling grid, they cannot be used to demonstrate that such small potential elevated areas of contamination do not exist. Instead, measurements and sampling on a specified grid size, in conjunction with surface scanning, are used together to obtain an adequate assurance that any small locations of elevated radioactivity that might exist are still within the dose guidelines.

The second step is to determine the amount of residual radioactivity, H_m, that would have to be contained in an area of size $0.866\ G^2$ in order to exceed the guideline dose. H_m can be expressed as a multiple, A_m, of the guideline residual radioactivity concentration, Δ. Values for the area factor, A_m, can be determined by comparing the dose conversion factor (DCF) obtained from the results of a pathway analysis under the scenario that a unit activity concentration of a given radionuclide is distributed uniformly across the survey unit to the DCF obtained when a unit concentration of that radionuclide is confined to a smaller area. For this draft, these calculations were performed using RESRAD 5.6 (ANL/EAD/LD-2) for outdoor areas, and using RESRAD BUILD 1.5 (ANL/EAD/LD-3) for indoor areas. The results, consisting of tables of area factors for each radionuclide modelled by RESRAD, are given in Appendix C.

The third step is to ensure that the scanning procedure used for the survey unit has a minimum detectable concentration (MDC) which is no greater than the residual radioactivity concentration, $H_m = A_m\Delta$, it is required to detect. The MDCs of various scanning techniques have been investigated, and the results are reported in NUREG-1507. Once a scanning technique is selected, the actual MDC is compared to the required scan MDC. If the actual scan MDC is less than the required scan MDC, no additional sampling points are needed for assessing potential elevated

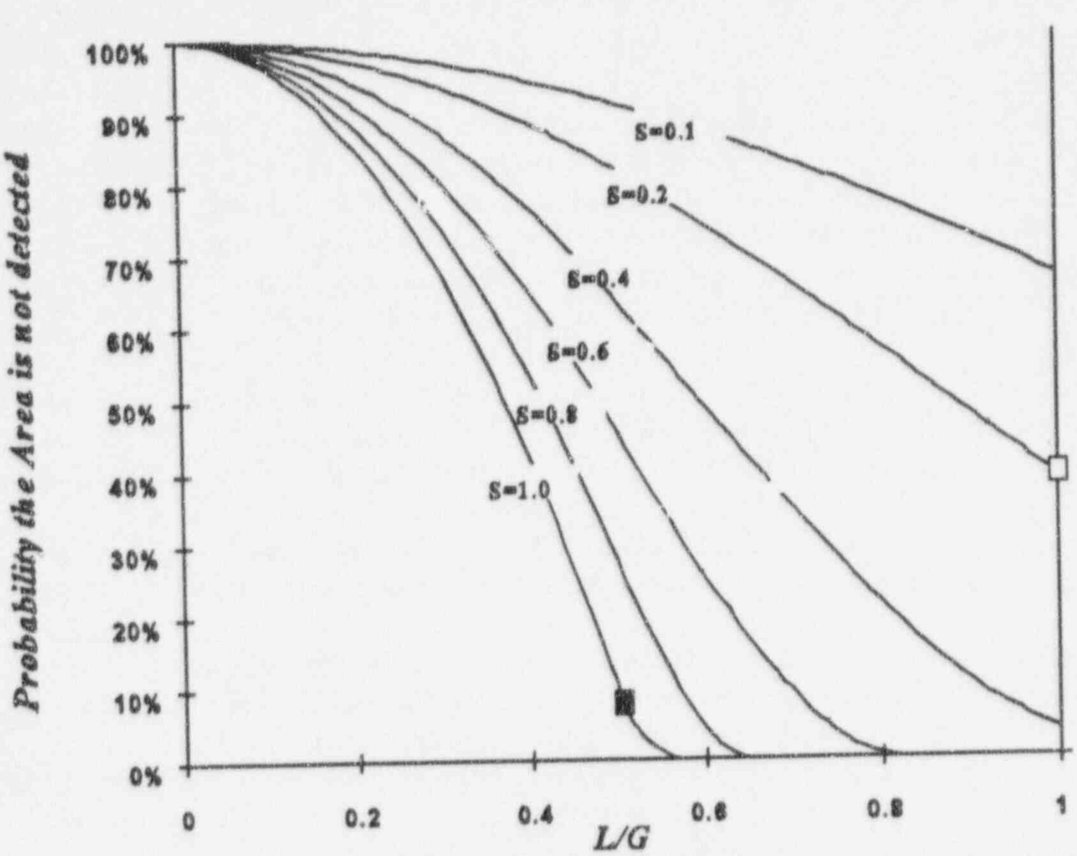


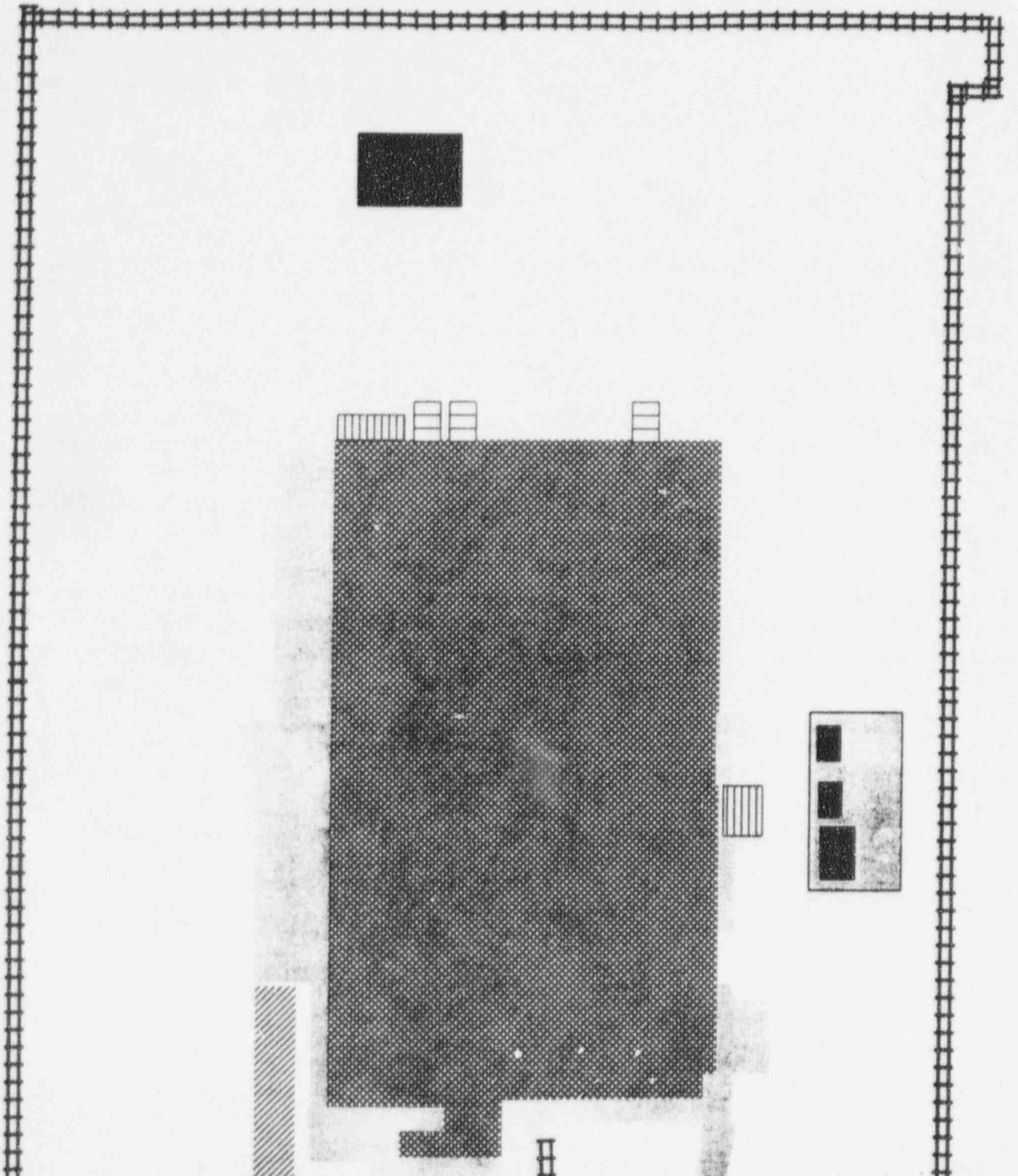
Figure 5.4 Probability That an Area of Size ($\pi SL^2/G^2$) and Semi-Major Axis Length L/G Will Not Be Found With a Triangular Sampling Grid

areas. That is, the scanning survey exhibits adequate sensitivity to detect any elevated areas of concern.

However, if the actual scan MDC is greater than the required scan MDC, then it will be necessary to decrease the sampling grid area by adding additional sampling locations beyond those required for the WRS and Quantile tests. The number of additional sampling locations is found by determining the area factor that corresponds to the actual scan MDC: (area factor) = (MDC)/Δ. The sampling grid area that corresponds to this area factor is found in Appendix C. This area ($0.866G^2_{MDC}$) will be smaller than the sampling grid area ($0.866G^2$) that resulted from the original triangular grid of the survey points needed for the statistical tests, and defines a new grid spacing, G_{MDC}. However, the minimum value for G_{MDC} that should be used is 5 meters outdoors or 1 meter indoors. Dividing the survey unit area, A_S, by the new area sampling grid area, $0.866G^2_{MDC}$, yields the new required survey unit sample size, $n_{MDC} = A_S / (0.866G^2_{MDC})$.

5.5.4.1 Example

Figure 5.5 shows the restricted area of the Reference Uranium Fuel Fabrication Plant used as an example in the draft report NUREG/CR-5849.



1 Figure 5.5 Restricted Area of Reference Uranium Fuel Fabrication Facility

It was determined that the entire soil area within the fenced restricted area is an affected area. It will be treated as one survey unit and compared to a reference area of similar character elsewhere on the site. The total area of the restricted area is about 9,000 m^2. Only about half of this, about 4,500 m^2, is exposed soil. If 17 samples are taken over a triangular grid, then the grid spacing is

$$G = \sqrt{4500/[(.866)(17)]} = \sqrt{305.7} = 17.48 \approx 17$$

where G is rounded down to the nearest meter. Referring to Table A-5, we can construct a table of probabilities for detecting elevated areas of a given size and shape. Recall that the area of an elliptical elevated measurement with shape S and semi-major axis L is $A = \pi(S\,L)L = \pi SL^2$.

Table 5.10 shows that in this example, where the grid area ($0.866G^2 = 0.866(17)^2$) is about 250 m^2, any elevated area of that size or larger will generally be detected. Depending on prior information that is available on the size of potential leaks, etc., one may or may not be able to set an *a priori* size for an elevated area of concern. In the absence of such prior information, this analysis provides an indication of the largest such area of a given shape that might reasonably exist without being detected on the triangular sampling grid.

For smaller areas to be of concern, the residual radioactivity would have to exceed the guideline concentration times the area factor. For example, for U-238 the area factor for 250 m^2, $A_m = 4.7$, is found from Appendix C by interpolating logarithmically between 6.7 at 100 m^2 and 4.4 at 300 m^2. For uniformly distributed contamination, 19.7 pCi/g of U-238 corresponds to the dose guideline of 15 mrem per year (cf. NUREG-1500, Table B). Therefore $H_m \approx A_m \bullet (19.7 \text{ pCi/g}) = 92.6$ pCi/g. The scanning MDC would thus be required to be 90 pCi/g or less. Otherwise, the number of samples taken on the grid would have to be increased.

Table 5.10 Probability That an Area of a Given Size and Shape Will Be Missed in the Example Survey Unit When 17 Samples Are Taken

L (m)	L/G	S	Area (m^2)	Area (% of survey unit)	Probability of non-detection
5	0.29	1	79	1.7	0.69
10	0.59	1	314	7	0.0
20	1.18	1	1257	28	0.0
5	0.29	0.5	39	0.9	0.85
10	0.59	0.5	157	3.5	0.39
20	1.18	0.5	628	14	0.0

L = Length of elevated area semi-major axis

L/G = Length of elevated area semi-major axis, L, relative to the sample grid spacing, G

S = (shape parameter, ratio of length of elevated area minor axis to length of major axis)

5.5.5 Allowance for QA Samples, and Missing or Unusable Data

In any sampling program, a certain percentage of samples should be taken for quality assurance purposes. Allowance for this must be made during the planning stages of the sampling program, and the number of samples taken increased accordingly. As a rule, a minimum of 10 percent of the total number of samples should be earmarked for QA. Thus, whatever sample size is determined to be appropriate, following the analyses described earlier in this section, should be increased by at least 10 percent. The QA samples will not normally be included in the sample grid as separate sampling points. Rather these will consist of blanks, spikes, or duplicate samples.

Missing data or unusable data or both can also occur with any sampling program. Samples can be mislabeled, lost, or fail to meet quality control standards. The pattern of missing data should be examined to determine if there are particular circumstances in common, e.g., sampling method or radionuclide. To account for missing or unusable data, it is prudent to increase the number of samples that would otherwise be collected. By applying the survey planning recommendations in this draft report, a significant effort is made to ensure that the proper number of samples is collected to guide the decisions to be made. This planning effort, however, should account for missing or unusable data to maintain the desired power of the statistical tests.

One approach for determining the number of QA samples is described for consideration. Let n be the number of samples that would be collected if no missing or unusable data are expected (this is the total of the samples needed for the statistical analysis not including those required for QA). Let R be the expected rate of missing or unusable data based on past experience. Then the total number of samples to collect ,n_f, is $n_f = n / (1 - R)$.

The use of this correction will give some assurance that enough samples will be collected to meet the specified Type I and Type II error-rate requirements.

5.6 Sampling Locations

For each survey unit, it is recommended that samples be collected on a random-start equilateral triangular grid. The measurements for a given radionuclide in the survey unit are compared with measurements obtained using a triangular-grid in the reference area. The triangular pattern has the following advantages (PNL-7409):

- It is relatively easy to use.

- It provides a uniform coverage of the area being sampled, whereas simple random or stratified random sampling can leave sub-areas that are not sampled.

- The probability of hitting an elevated area of specified elliptical shape one or more times is almost always greater using a triangular grid than using a square grid when the density of sample points is the same for both types of grids for the areas being investigated (Gilbert).

- Samples collected on a triangular grid are well suited for using geostatistical methods to estimating any spatial correlation structure suspected to exist.

The grid points (sampling locations) must not correspond to patterns of high or low concentrations. If such a correspondence exists, the measurements and statistical test results could be misleading. Normally, correlations among the data in the reference area and in the survey unit would be avoided in the sample planning stage. Recall that both reference areas and survey units are chosen to be as homogeneous as possible. Nonetheless, some simple screening procedures for detecting correlations in the data should be performed. At a minimum, data should be plotted on a site map, and visually examined for any unusual patterns. However, correlations may be unavoidable. In those instances, geostatistical methods, such as kriging, may be necessary to properly evaluate the data, but the occurrence of cases where geostatistical methods are necessary is expected to be rare.

The sampling grid constructed using the procedure in Section 5.7 gives approximate sampling points in the field. There may be small errors in the locations because the sampling coordinates are rounded to distances that are easy to measure and the distance measurement itself has some inaccuracies. However, the sample must be taken as closely as possible to the designated location in order to preserve the randomness of the tests. It is better that small random errors be made in locating the sample point than to allow any systematic bias to occur. There should be no judgment on the part of the field staff in locating the exact sample point. The exact sample collection point must be located without any subjective bias factors such as "difficulty in collecting a sample, the presence of vegetation, or the color of the soil." Any exceptions to this procedure must be documented in the sample log.

5.7 Determining Sampling Points in an Equilateral Triangular Grid Pattern

The essential procedure for determining where samples should be taken in either reference areas or survey units is the same. On a site map, a reproducible coordinate system should be laid out with enough detail to locate positions with an error that will be small compared to the distance between samples. Based on the total number of samples to be taken, a triangular sampling grid is superimposed on the coordinate system. The sampling positions are then located in the field.

The eight steps in the procedure for a triangular grid are as follows (from EPA 230/02-89-042):

(1) Draw a map of the area to be sampled and determine its size, A (e.g., m^2).

(2) Draw a rectangle that encloses the area to be sampled.

(3) Define a coordinate system for locating points (X,Y) within the rectangle, e.g., the number of meters east, X, and the number of meters north, Y, from the southwest corner (0,0) of the rectangle. The northeast corner will then have coordinates (X_{max}, Y_{max}). Note that the local coordinate system need not line up with the principal compass points. It may be convenient to align one of the axes with a site boundary or other local feature.

Figure 5.6 shows how this was done for the restricted area of the Reference Uranium Fuel Fabrication Facility. The coordinate system has been laid out in the north-south and east-west directions. There are 9 ten-meter east-west coordinates, and 11 ten-meter north-south coordinates. The total area is 9,900 m^2, of which approximately 9,000 m^2 is the affected area

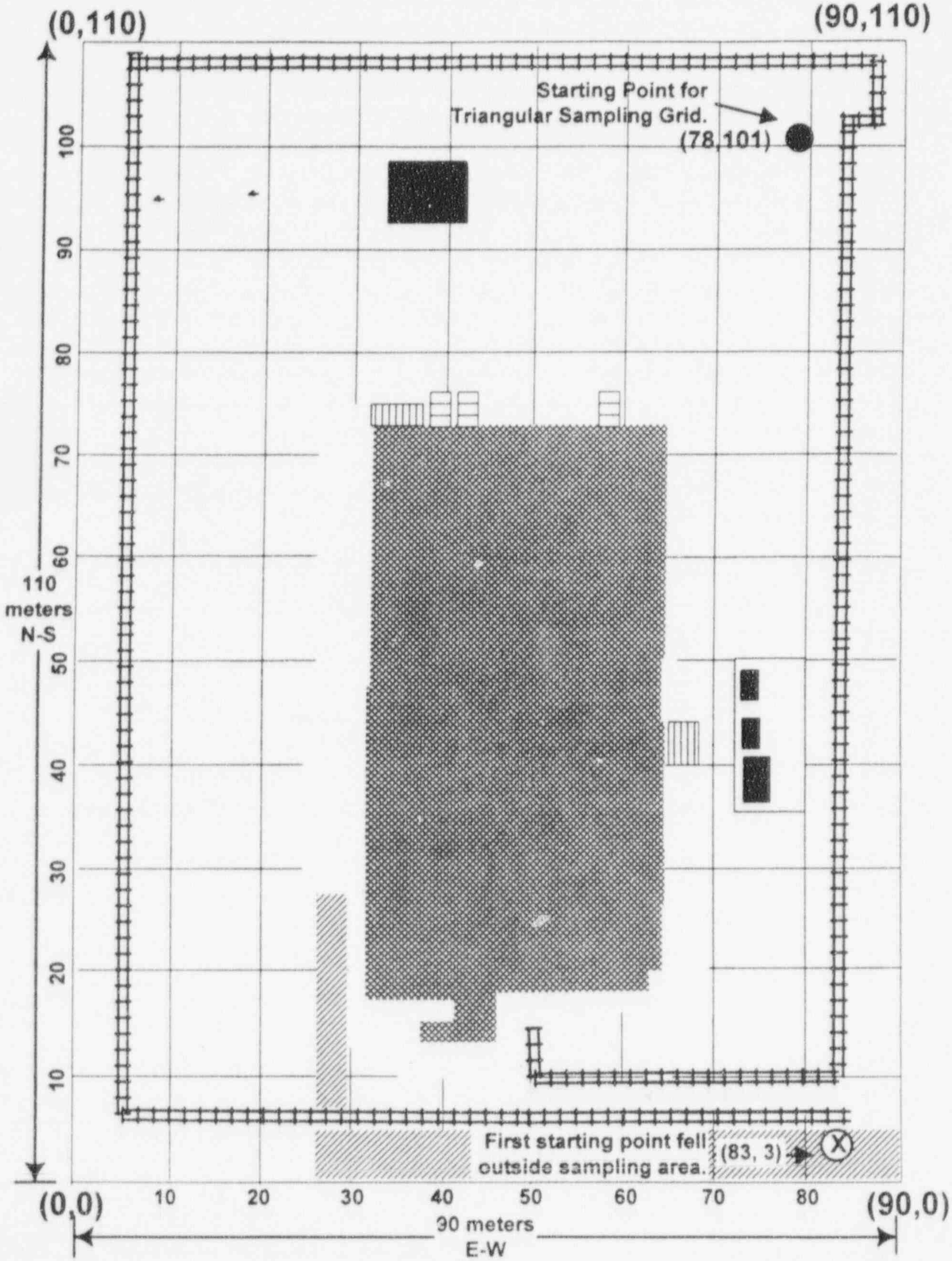


Figure 5.6 Laying Out a Site Coordinate System

within the fence line. The soil area to be surveyed is about 4,500 m^2. The remainder of the area is covered by buildings, walkways, etc., which will be part of other survey units.

(4) Locate a random starting point by drawing two random numbers from a uniform distribution on the interval [0,1]. Random numbers can be generated using the random number function of a spreadsheet or a scientific calculator. Table A-6 contains 1000 random numbers generated using a spreadsheet, and similar tables can be found in many statistics texts. Choose any starting point in the table, and then take numbers consecutively either across rows or down columns. For example, in Table A-6, starting at row 23 in column 2 and working down, the two numbers 0.93062 and 0.029842 are found. Scale the first number by the length of the east-west coordinate axis to get 83.76 = (90)(0.93062). Round the coordinates to the nearest values that can be easily measured in the field (e.g., nearest meter). This gives 84 meters to the nearest meter. Similarly scale the second number by the length of the north-south coordinate axis to get 3.28 = (110)(0.029842) or 3 meters to the nearest meter. This gives (84,3) as the starting coordinate for the sampling grid. Since this does not fall within the area to be sampled (it falls on an area of asphalt), the next two random numbers (0.863244,0.921291) are taken, giving (78, 101). Continue until a point that falls within the sampling area is obtained. In this case (78, 101) does fall in the area to be sampled. The points are shown on Figure 5.6.

(5) Compute the spacing, *G*, of the sampling locations on the triangular grid using the number of sampling locations required (*n*) computed in the previous section, rounded down to the nearest meter.

$$G = \sqrt{\frac{A}{0.866n}} = \sqrt{\frac{4500}{(0.866)(17)}} = 17.5 \text{ meters} \approx 17 \text{ meters}$$

(6) From the starting location, lay out a row of sampling points parallel to the X-axis and distance G apart. This is shown in Figure 5.7.

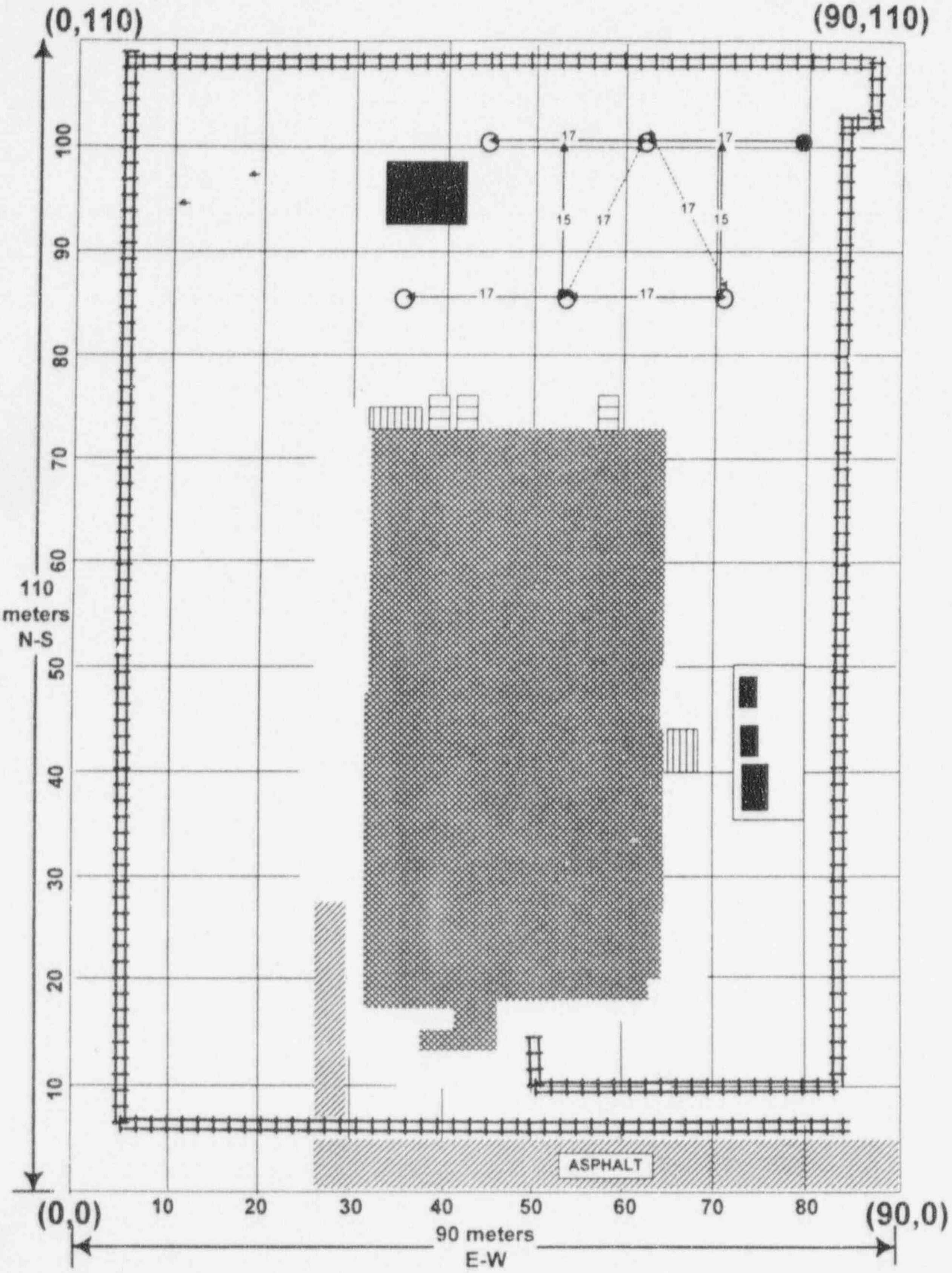
(7) To start additional rows, locate the midpoint between two adjacent sampling locations on the sample row and mark a spot at a distance

$$0.866\sqrt{\frac{A}{0.866n}} = \sqrt{\frac{(0.866)(4500)}{(17)}} = 15.14 \text{ meters} \approx 15 \text{ meters}$$

perpendicular to the row. Again, this number should be rounded *down* if necessary. This is the starting location for the new row. This is also shown in Figure 5.7.

(8) Continue until all grid points within the sampling area have been located. Ignore any sampling locations that fall outside the area to be sampled. The completed sampling grid is shown in Figure 5.8.

Using this procedure, the number of sampling points on the triangular grid within the sampling area may differ from the desired number (*n*) depending on the shape of the area. In this example, because of the very irregular shape of the region caused by its wrapping around the building, 20



1 Figure 5.7 Laying Out a Triangular Grid

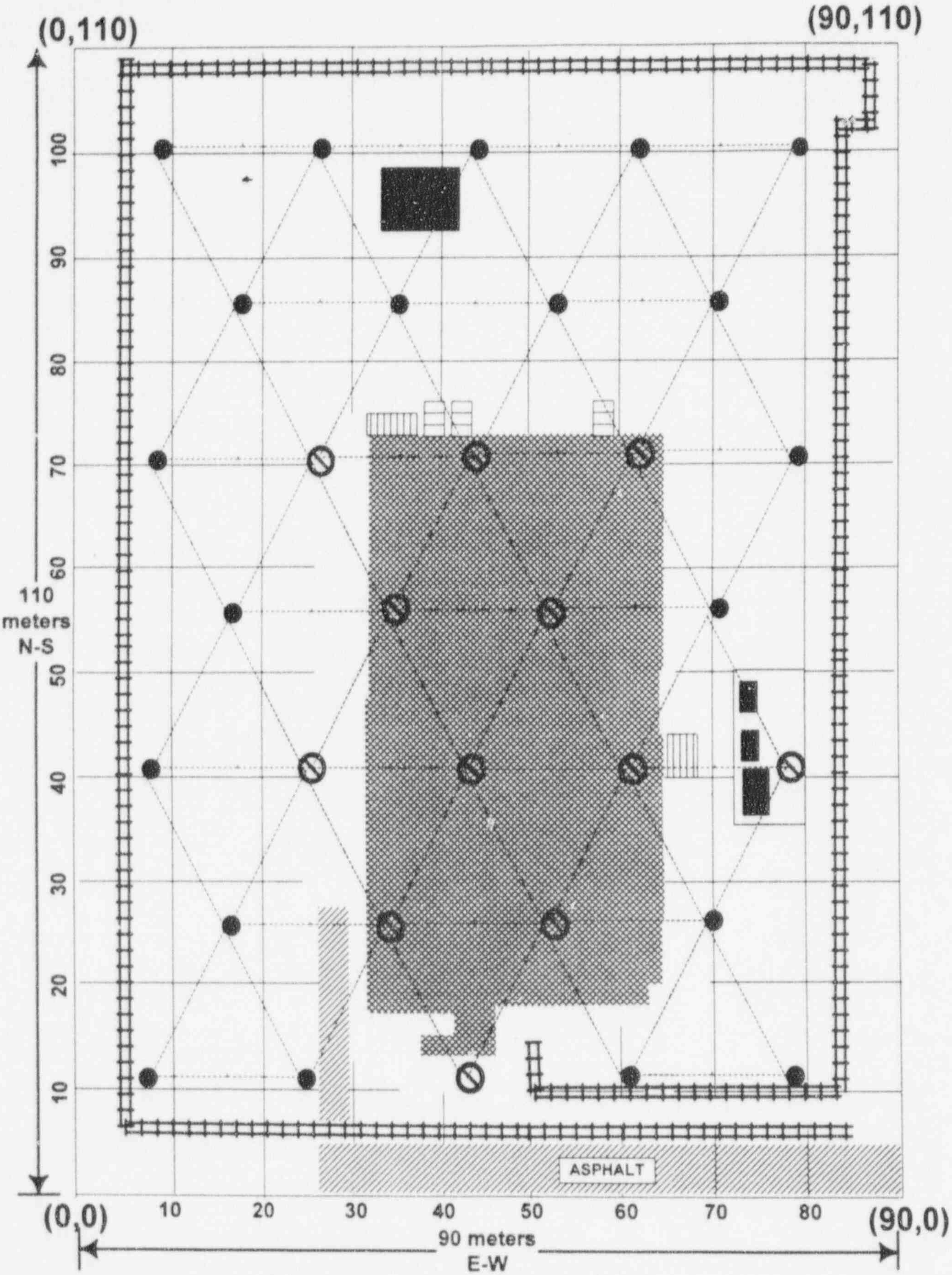


Figure 5.8 Completed Sample Grid

sampling points are found on the grid. If the number of points is greater than the desired number, use all the points.

If the number of points is less than the desired number, the additional points required are determined using the same procedure that was used to determine the grid starting point. These will be at individual random locations within the sampling area, and should be used regardless of where they occur relative to the grid.

Using random survey locations will not affect the WRS or Quantile tests. The probability of detecting an elevated area is based on the relationship between the sample grid spacing and the size of the elevated area, not on the number of samples taken. However, any errors introduced by a loss of sample points at the boundaries will tend to be balanced by the additional random sampling locations in the interior of the sampling grid.

The grid spacing, G, is based on its approximate relationship to the number of samples and the survey site area,

$$G = \sqrt{A/(.866n)}$$

This relationship might not work well for very irregularly shaped survey units, leaving a relatively large number of random sample locations to be found. In such a case, it may be preferable to adjust the grid spacing, G, to a smaller value and recalculate the sample grid.

5.8 Applying the Tests

Both the WRS and Quantile tests are two-sample tests designed for comparing reference areas and survey units. The equivalent one-sample versions (see Section 6) of these tests can be used when there is no background for the radionuclide being considered.

5.8.1 Applying the Wilcoxon Rank Sum Test

The WRS test is applied as follows:

(1) The m sample measurements from the reference area and the n sample measurements from the survey unit are pooled and ranked in order of increasing size from 1 to N, *where* $N = m+n$.

(2) If several measurements are tied (have the same value), they are all assigned the average rank of that group of tied measurements.

(3) If there are T "less than" values, they are all given the average of the ranks from 1 to T. Therefore, they are all assigned the rank $T(T+1)/(2T) = (T+1)/2$, which is the average of the first T integers.

If there is more than one detection limit, all observations below the largest one should be treated as "less than" values. If more than 40 percent of the data from either the reference area or survey unit are 'less than', do not use the WRS test, but still conduct the Quantile test.

(4) Sum the ranks of the measurements from the survey unit, W_s.
Note that since the sum of the first N integers is $N(N+1)/2$, one can equivalently sum the ranks from the reference area, W_r, and compute $W_s = N(N+1)/2 - W_r$.

(5) Compare W_s with the critical value given in Table A-1 for the appropriate values of n, m, and $\alpha_W = \alpha/2$. If W_s is greater than the tabulated value, we reject the hypothesis that the site has been successfully remediated.

5.8.2 WRS Test Example

The example given in the previous section is continued in this section. The Reference Uranium Fuel Fabrication Facility has released U-234, U-235, and U-238 into the environment. Section 6 develops the scenario involving radionuclide specific analyses. For this section, however, it is assumed that the dose in the survey unit is the quantity of concern. We will have measurements of concentration (pCi/g) in a reference area with a mean dose rate of 80 mrem per year and standard deviation 8 mrem per year. For this example, assume that the concentarion values have been converted to the equivalent dose rate. It was calculated in Section 5.5 that 17 measurements in both the reference area and the survey unit were required. In laying out the survey unit sampling grid in Section 5.7, twenty sampling locations were identified. As discussed there, when more sample locations are identified than were calculated to be required, it is necessary to sample all of the identified locations.

Table 5.11 shows the example analysis of the data obtained. The measurements are shown in columns A and F of Table 5.11. In columns B and G we have inserted the code "R" to denote a reference area measurement, and "S" to denote a survey unit measurement. In column A, the data are simply listed as they were obtained. In Column F, the data are sorted in ascending order. The ranks of the data appear in Columns C and H. They range from 1 to 37, since there is a total of 17+20 measurements. Note that there were two cases of measurements tied with the same value, at 86.4 and 88.5. Tied measurements are always each assigned the average of the ranks. Therefore, both measurements at 88.4, are assigned rank (26+27)/2 = 26.5. It should also be noted that the sum of the ranks is still 37(37+1)/2 = 703. It is recommended to check this as a guard against errors in the rankings.

Columns D and I contain only the ranks belonging to the survey unit measurements. The total is 412.5. This is to be compared with the entry in Table A-1 for $\alpha_w = \alpha/2 = 0.025$, with $n = 20$ and $m = 17$. This critical value is 444. Thus, the sum of the survey unit ranks is less than the critical value and the null hypothesis that the survey unit has been successfully remediated is accepted. The calculations for the WRS test are very well suited for calculation on a spreadsheet. This is how the analysis discussed above was done. The Microsoft Excel version 5.0 formula sheet

Table 5.11 Example Analysis Using the WRS Test

	A	B	C	D	E	F	G	H	I
1	**Wilcoxon Rank Sum Test**					**Sorted Data**			
2	**Data**	**Area**	**Ranks**			**Data**	**Area**	**Ranks**	
3	92.1	R	36	0		59.4	R	1	0
4	83.2	R	22	0		69.7	R	2	0
5	81.7	R	17	0		74.2	S	3	3
6	81.8	R	18	0		75.3	R	4	0
7	88.5	R	30.5	0		75.5	S	5	5
8	82.8	R	21	0		76.3	S	6	6
9	81.5	R	16	0		77.4	S	7	7
10	69.7	R	2	0		77.6	S	8	8
11	82.4	R	20	0		78.2	S	9	9
12	89.7	R	32	0		79.1	S	10	10
13	81.4	R	15	0		79.4	R	11	0
14	79.4	R	11	0		79.9	R	12	0
15	82.0	R	19	0		80.5	S	13	13
16	79.9	R	12	0		81.1	R	14	0
17	81.1	R	14	0		81.4	R	15	0
18	59.4	R	1	0		81.5	R	16	0
19	75.3	R	4	0		81.7	R	17	0
20	90.7	S	35	35		81.8	R	18	0
21	83.5	S	23	23		82.0	R	19	0
22	86.4	S	26.5	26.5		82.4	R	20	0
23	88.5	S	30.5	30.5		82.8	R	21	0
24	84.4	S	25	25		83.2	R	22	0
25	74.2	S	3	3		83.5	S	23	23
26	84.1	S	24	24		84.1	S	24	24
27	87.6	S	29	29		84.4	S	25	25
28	78.2	S	9	9		86.4	S	26.5	26.5
29	77.6	S	8	8		86.4	S	26.5	26.5
30	86.4	S	26.5	26.5		86.5	S	28	28
31	76.3	S	6	6		87.6	S	29	29
32	86.5	S	28	28		88.5	R	30.5	0
33	77.4	S	7	7		88.5	S	30.5	30.5
34	90.3	S	34	34		89.7	R	32	0
35	90.1	S	33	33		90.1	S	33	33
36	79.1	S	10	10		90.3	S	34	34
37	92.4	S	37	37		90.7	S	35	35
38	75.5	S	5	5		92.1	R	36	0
39	80.5	S	13	13		92.4	S	37	37
40		Sum	703	412.5				703	412.5
41	Critical Value for a/2=0.025, n=20, m=17 is 444								

corresponding to Table 5.11 is given in Table 5.12. The function in Column C of Table 5.12 calculates the ranks of the data. The RANK function in Excel does not return tied ranks in the way needed for the WRS. The COUNTIF function corrects for this. Column D simply picks out the survey unit ranks from Column C. These are summed in cell D40. No formulas are shown for Columns F through I, since these are simply sorted copies of the values of Columns A through D.

5.8.3 Applying the Quantile Test

The Quantile test is performed after the WRS test, if the null hypothesis for that test has been accepted. For the Quantile test, the appropriate table in Appendix A.4 is selected, according to the value of $\alpha_Q = \alpha/2$ (Table A-4 page 1 for $\alpha_Q = 0.01$, page 2 for $\alpha_Q = 0.025$, page 3 for $\alpha_Q = 0.05$, or page 4 for $\alpha_Q = 0.1$). Find the nearest value of *n* and *m* that is tabulated in the appropriate table. In Table A-4 page 2, $n = m = 20$ are the closest tabulated values to the actual numbers of measurements $n = 20$ and $m = 17$. In this case $r = 5$ and $k = 5$. The $r = 5$ largest measurements in Column F of Table 5.11 are examined. The null hypothesis is rejected only if $k = 5$ of these are from the survey unit.

The Quantile test as applied above gives only an approximate result, since tabulated values were used for *n* and *m* that were close to, but not equal to, those actually used. Therefore, the actual value of α_Q will be different than that listed in the table. Fortunately, it is easy to adjust the test so that the value of α_Q is appropriate to the actual values of *n* and *m*. The number, *k*, out of the *r* largest measurements has what is known as a "hypergeometric distribution" when the null hypothesis is true. This makes it possible to calculate the value of α_Q exactly. These calculations are suitable for a spreadsheet analysis, since many spreadsheets have the hypergeometric function built in.

5.8.4 Quantile Test Example

Table 5.13 shows the calculations for the example continued from Section 5.8.2. Rows 6 through 10 contain the five largest measurements from the data set, and the area that they came from. Rows 1 through 5 simply repeat the information needed for the approximate analysis using the tabulated values of *r* and *k*. Row 18, Columns A, B, and C contain the actual values of *n* and *m*, and the tabulated value for *r*, respectively. Columns D and E, of Row 18 show the theoretical mean and standard deviation of *k* under the null hypothesis (i.e., when the null hypothesis is true). This is the mean and standard deviation of a hypergeometric distribution with the given values of *m*, *n*, and *r*. In Rows 21 to 26, Column A shows the possible values of *k*, Column B shows the hypergeometric probability of obtaining that value of *k*, and Column C shows the value of α_Q that would apply if this value of *k* were used for the test.

From Table 5.13, cell C26 indicates that with $r = 5$ and $k = 5$, $\alpha_Q = 0.0356$, which is larger than the desired value of 0.025. If a combination of *r* and *k* needs to be determined, which yields a value of α_Q nearer to 0.025, *r* should be increased by 1 in the spreadsheet cell C18 and then the resulting values of α as a function of k should be examined. The results of doing this are shown in Table 5.14, which shows that a value of α_Q can be obtained closer to 0.025, namely the value of 0.0167 in Cell C27 for $r = 6$ and $k = 6$.

The spreadsheet formulas used for the example in Table 5.13 are shown in Table 5.15, Rows 17 through 26.

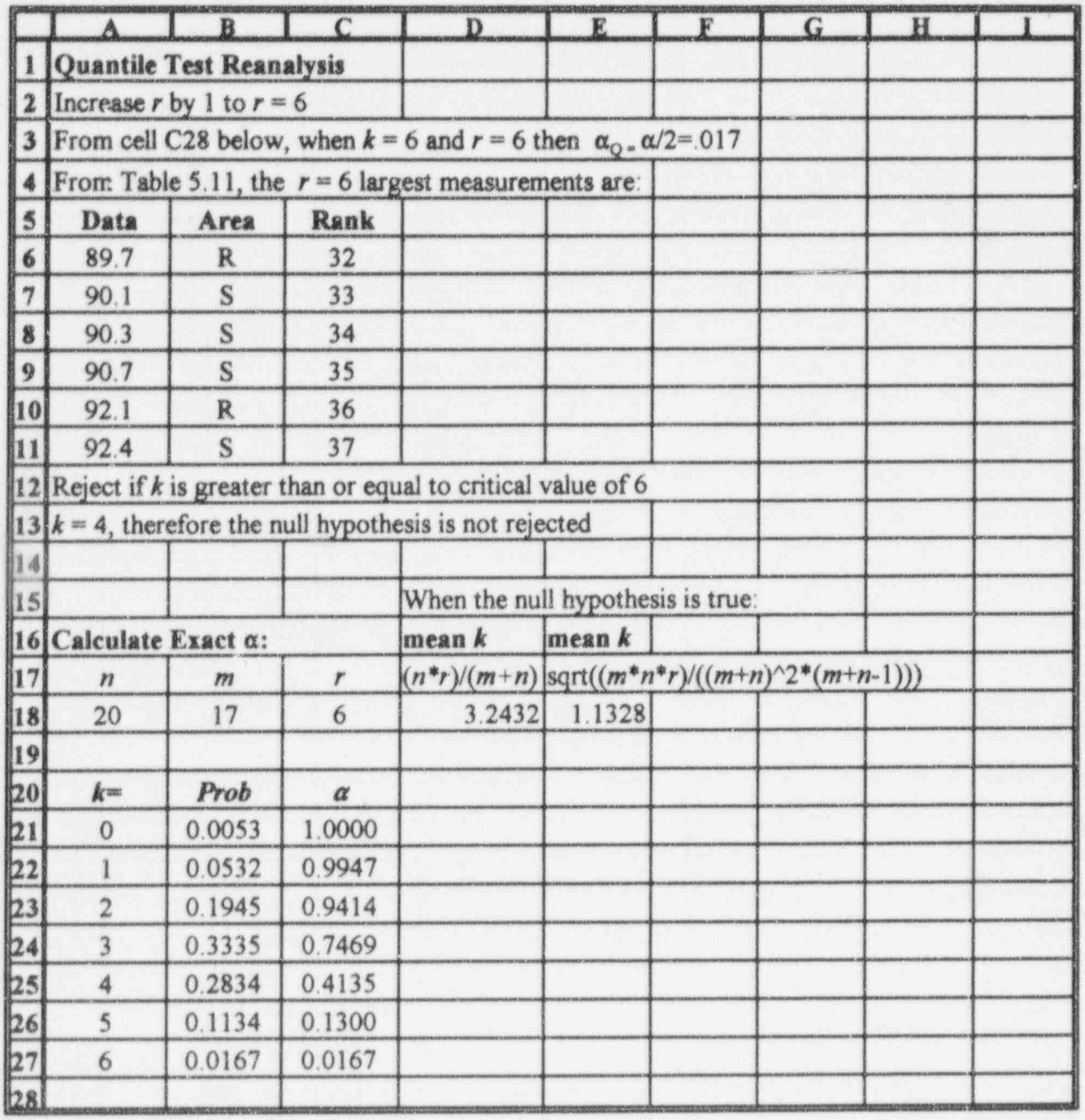
Table 5.12 Spreadsheet Formulas Used in Table 5.11

	A	B	C	D
1	Wilcoxon Rank Sum Test			
2	Data	Area	Ranks	
3	92.1	R	=RANK(A3,$A$3:$A$39,1)+(COUNTIF($A$3:$A$39,A3)-1)/2	=IF(B3="S",C3,0)
4	83.2	R	=RANK(A4,$A$3:$A$39,1)+(COUNTIF($A$3:$A$39,A4)-1)/2	=IF(B4="S",C4,0)
5	81.7	R	=RANK(A5,$A$3:$A$39,1)+(COUNTIF($A$3:$A$39,A5)-1)/2	=IF(B5="S",C5,0)
6	81.8	R	=RANK(A6,$A$3:$A$39,1)+(COUNTIF($A$3:$A$39,A6)-1)/2	=IF(B6="S",C6,0)
7	88.5	R	=RANK(A7,$A$3:$A$39,1)+(COUNTIF($A$3:$A$39,A7)-1)/2	=IF(B7="S",C7,0)
8	82.8	R	=RANK(A8,$A$3:$A$39,1)+(COUNTIF($A$3:$A$39,A8)-1)/2	=IF(B8="S",C8,0)
9	81.5	R	=RANK(A9,$A$3:$A$39,1)+(COUNTIF($A$3:$A$39,A9)-1)/2	=IF(B9="S",C9,0)
10	69.7	R	=RANK(A10,$A$3:$A$39,1)+(COUNTIF($A$3:$A$39,A10)-1)/2	=IF(B10="S",C10,0)
11	82.4	R	=RANK(A11,$A$3:$A$39,1)+(COUNTIF($A$3:$A$39,A11)-1)/2	=IF(B11="S",C11,0)
12	89.7	R	=RANK(A12,$A$3:$A$39,1)+(COUNTIF($A$3:$A$39,A12)-1)/2	=IF(B12="S",C12,0)
13	81.4	R	=RANK(A13,$A$3:$A$39,1)+(COUNTIF($A$3:$A$39,A13)-1)/2	=IF(B13="S",C13,0)
14	79.4	R	=RANK(A14,$A$3:$A$39,1)+(COUNTIF($A$3:$A$39,A14)-1)/2	=IF(B14="S",C14,0)
15	82	R	=RANK(A15,$A$3:$A$39,1)+(COUNTIF($A$3:$A$39,A15)-1)/2	=IF(B15="S",C15,0)
16	79.9	R	=RANK(A16,$A$3:$A$39,1)+(COUNTIF($A$3:$A$39,A16)-1)/2	=IF(B16="S",C16,0)
17	81.1	R	=RANK(A17,$A$3:$A$39,1)+(COUNTIF($A$3:$A$39,A17)-1)/2	=IF(B17="S",C17,0)
18	59.4	R	=RANK(A18,$A$3:$A$39,1)+(COUNTIF($A$3:$A$39,A18)-1)/2	=IF(B18="S",C18,0)
19	75.3	R	=RANK(A19,$A$3:$A$39,1)+(COUNTIF($A$3:$A$39,A19)-1)/2	=IF(B19="S",C19,0)
20	90.7	S	=RANK(A20,$A$3:$A$39,1)+(COUNTIF($A$3:$A$39,A20)-1)/2	=IF(B20="S",C20,0)
21	83.5	S	=RANK(A21,$A$3:$A$39,1)+(COUNTIF($A$3:$A$39,A21)-1)/2	=IF(B21="S",C21,0)
22	86.4	S	=RANK(A22,$A$3:$A$39,1)+(COUNTIF($A$3:$A$39,A22)-1)/2	=IF(B22="S",C22,0)
23	88.5	S	=RANK(A23,$A$3:$A$39,1)+(COUNTIF($A$3:$A$39,A23)-1)/2	=IF(B23="S",C23,0)
24	84.4	S	=RANK(A24,$A$3:$A$39,1)+(COUNTIF($A$3:$A$39,A24)-1)/2	=IF(B24="S",C24,0)
25	74.2	S	=RANK(A25,$A$3:$A$39,1)+(COUNTIF($A$3:$A$39,A25)-1)/2	=IF(B25="S",C25,0)
26	84.1	S	=RANK(A26,$A$3:$A$39,1)+(COUNTIF($A$3:$A$39,A26)-1)/2	=IF(B26="S",C26,0)
27	87.6	S	=RANK(A27,$A$3:$A$39,1)+(COUNTIF($A$3:$A$39,A27)-1)/2	=IF(B27="S",C27,0)
28	78.2	S	=RANK(A28,$A$3:$A$39,1)+(COUNTIF($A$3:$A$39,A28)-1)/2	=IF(B28="S",C28,0)
29	77.6	S	=RANK(A29,$A$3:$A$39,1)+(COUNTIF($A$3:$A$39,A29)-1)/2	=IF(B29="S",C29,0)
30	86.4	S	=RANK(A30,$A$3:$A$39,1)+(COUNTIF($A$3:$A$39,A30)-1)/2	=IF(B30="S",C30,0)
31	76.3	S	=RANK(A31,$A$3:$A$39,1)+(COUNTIF($A$3:$A$39,A31)-1)/2	=IF(B31="S",C31,0)
32	86.5	S	=RANK(A32,$A$3:$A$39,1)+(COUNTIF($A$3:$A$39,A32)-1)/2	=IF(B32="S",C32,0)
33	77.4	S	=RANK(A33,$A$3:$A$39,1)+(COUNTIF($A$3:$A$39,A33)-1)/2	=IF(B33="S",C33,0)
34	90.3	S	=RANK(A34,$A$3:$A$39,1)+(COUNTIF($A$3:$A$39,A34)-1)/2	=IF(B34="S",C34,0)
35	90.1	S	=RANK(A35,$A$3:$A$39,1)+(COUNTIF($A$3:$A$39,A35)-1)/2	=IF(B35="S",C35,0)
36	79.1	S	=RANK(A36,$A$3:$A$39,1)+(COUNTIF($A$3:$A$39,A36)-1)/2	=IF(B36="S",C36,0)
37	92.4	S	=RANK(A37,$A$3:$A$39,1)+(COUNTIF($A$3:$A$39,A37)-1)/2	=IF(B37="S",C37,0)
38	75.5	S	=RANK(A38,$A$3:$A$39,1)+(COUNTIF($A$3:$A$39,A38)-1)/2	=IF(B38="S",C38,0)
39	80.5	S	=RANK(A39,$A$3:$A$39,1)+(COUNTIF($A$3:$A$39,A39)-1)/2	=IF(B39="S",C39,0)
40		Sum	=SUM(C3:C39)	=SUM(D3:D39)
41	Critical Value for a/2=0.025, n=20, m=17 is 444			

Table 5.13 Example Analysis Using the Quantile Test

	A	B	C	D	E	F	G	H	I
1	**Quantile Test**								
2	Nearest entry in Table A-4 is for $m = n = 20$								
3	This entry has $k = 5$, $r = 5$ and $\alpha_Q = \alpha/2 = .024$								
4	From Table 5.11, the $r = 5$ largest measurements are:								
5	**Data**	**Area**	**Rank**						
6	90.1	S	33						
7	90.3	S	34						
8	90.7	S	35						
9	92.1	R	36						
10	92.4	S	37						
11	Reject if k is greater than or equal to critical value of 5								
12	$k = 4$, therefore the null hypothesis is not rejected								
13									
14									
15				When the null hypothesis is true:					
16	Calculate exact α:			mean k	std dev k				
17	n	m	r	$(n*r/(m+n)$	$\text{sqrt}((m*n*r)/((m+n)\text{^}2*(m+n-1)))$				
18	20	17	5	2.7027	1.0506				
19									
20	**k =**	**Prob**	**α**						
21	0	0.0142	1.0000						
22	1	0.1092	0.9858						
23	2	0.2964	0.8766						
24	3	0.3557	0.5802						
25	4	0.1890	0.2245						
26	5	0.0356	0.0356						
27									
28									

Table 5.14 Example Re-analysis Using the Quantile Test



	A	B	C	D	E	F	G	H	I
1	**Quantile Test Reanalysis**								
2	Increase r by 1 to $r = 6$								
3	From cell C28 below, when $k = 6$ and $r = 6$ then $\alpha_Q = \alpha/2 = .017$								
4	From Table 5.11, the $r = 6$ largest measurements are:								
5	**Data**	**Area**	**Rank**						
6	89.7	R	32						
7	90.1	S	33						
8	90.3	S	34						
9	90.7	S	35						
10	92.1	R	36						
11	92.4	S	37						
12	Reject if k is greater than or equal to critical value of 6								
13	$k = 4$, therefore the null hypothesis is not rejected								
14									
15				When the null hypothesis is true:					
16	**Calculate Exact α:**			**mean k**	**mean k**				
17	n	m	r	$(n*r)/(m+n)$	sqrt$((m*n*r)/((m+n)^2*(m+n-1)))$				
18	20	17	6	3.2432	1.1328				
19									
20	k=	*Prob*	α						
21	0	0.0053	1.0000						
22	1	0.0532	0.9947						
23	2	0.1945	0.9414						
24	3	0.3335	0.7469						
25	4	0.2834	0.4135						
26	5	0.1134	0.1300						
27	6	0.0167	0.0167						
28									

Table 5.15 Spreadsheet Formulas Used in Table 5.13

	A	B	C	D	E	F	G	H	I
1	**Quantile Test**								
2	Nearest entry in Table A-4 is for m = n= 20								
3	This entry has $k = 5$, $r = 5$ and $\alpha_Q = \alpha/2=.024$								
4	From Table 5.11, the $r = 5$ largest measurements are:								
5	**Data**	**Area**	**Rank**						
6	90.1	S	33						
7	90.3	S	34						
8	90.7	S	35						
9	92.1	R	36						
10	92.4	S	37						
11	Reject if *k* is greater than or equal to critical value of 5								
12	*k* = 4, therefore the null hypothesis is not rejected								
13									
14									
15				When the null hypothesis is true:					
16	Calculate exact α:			*mean k*	*std dev k*				
17	*n*	*m*	*r*	`(n*r)/(m+n)`	`sqrt((m*n*r)/((m+n)^2*(m+n-1)))`				
18	cell named *n*	cell named *m*	cell named *r*_	`=(n*r_)/(m+n)`	`=SQRT ( (m*n*r_*(n+m-r_) ) / ( (m+n)^2 * (m+n-1) ))`				
19									
20	**k=**	**Prob**				α			
21	0	`=HYPGEOMDIST(A21,n,r_,n+m)`				1			
22	1	`=HYPGEOMDIST(A22,n,r_,n+m)`				`=1-SUM($B$21)`			
23	2	`=HYPGEOMDIST(A23,n,r_,n+m)`				`=1-SUM($B$21:B22)`			
24	3	`=HYPGEOMDIST(A24,n,r_,n+m)`				`=1-SUM($B$21:B23)`			
25	4	`=HYPGEOMDIST(A25,n,r_,n+m)`				`=1-SUM($B$21:B24)`			
26	5	`=HYPGEOMDIST(A26,n,r_,n+m)`				`=1-SUM($B$21:B25)`			
27									
28									

Table 5.16 shows some other possible values of r, k, and α_Q for this example. This process can be continued as necessary. Unfortunately, there is no simple way to determine the effect of these changes on the power of the test. Since it is the power of the test which determines the false negative error rate, it is preferable to use the tabulated values of r and k unless α_Q is intolerably high. This should be determined as part of the DQO process described in Section 3.7. It is during that process that the optimal values for m and n as well as r and k should be determined.

Table 5.16 Values of α_Q as a Function of r and k for the Quantile Test With n=20 and m=17

r	k	α_Q
4	4	0.0734
5	5	0.0356
6	5	0.1300
6	6	0.0167
7	6	0.0715
7	7	0.0075

5.8.5 Elevated Measurement Comparison

The elevated measurement comparison consists of comparing each measurement from the survey unit with the concentration value H_m discussed in Sections 5.3.3, 5.4, and 5.5.4. Any measurement from the survey unit that is equal to or greater than H_m indicates an area of relatively high concentrations that must be investigated, regardless of the outcome of the WRS or Quantile tests. The elevated measurement comparison is used in conjunction with the WRS and Quantile tests because the latter two tests can fail to reject H_0 when only a very few high measurements in the survey unit are obtained. The use of the elevated measurement comparison against the value H_m may be viewed as insurance that unusually large measurements will receive proper attention regardless of the outcome of the WRS and Quantile tests. The elevated measurement comparison is intended to flag potential failures in the decommissioning process, and should not be considered the primary means to identify whether or not a site meets decommissioning criteria.

The elevated measurement comparison value is $H_m = A_m \Delta$, where A_m is the area factor and Δ is the radionuclide concentration corresponding to the guideline dose. The area factor depends on the sampling grid area, $0.866 \cdot G^2$ where G is the distance between sampling points. Tables of area factors for outdoor survey units computed using RESRAD 5.6 (ANL/EAD/LD-2), and of indoor area factors computed using RESRAD BUILD 1.5 (ANL/EAD/LD-3) are given in Appendix C. That a given measurement exceeds H_m is not enough by itself to determine if the dose guideline has been exceeded. The dose also depends on the area over which the elevated residual activity exists. Therefore, each measurement identified as elevated will be marked for further investigation, which may include additional measurements and sampling to determine the nature and extent of the residual radioactivity, and whether the dose guidelines are actually exceeded by the radioactivity in that area.

The elevated measurement comparison is performed for both measurements obtained on the grid, and for scanning measurements.

Unusual readings should also be flagged for further investigation. Any measurement that exceeds 3 standard deviations above the mean, and also exceeds the guideline, should be investigated further.

The smear samples that are taken at indoor grid locations are an indication of removable surface activity. The average surface activity in a survey unit should not exceed 10 percent of the guideline value. This is the amount of removable activity that was used in the RESRAD BUILD calculations for the indoor area factors. No individual smear should exceed 50 percent of the guideline value.

6 PLANNING AND DESIGNING THE FINAL STATUS SURVEY WITH NO REFERENCE AREA

6.1 Design Considerations

The statistical tests discussed in this section will be used to compare each survey unit directly with the applicable decommissioning criteria. The methods of this section may only be used if there is no background concentration of the residual radioactivity being measured. This will be the case only when the radionuclide of concern does not occur in natural background, and radionuclide-specific measurements are made to determine its concentrations. Otherwise, the methods of Section 5 must be used.

Because there is no background concentration of residual radioactivity being considered, there are no reference areas required and, therefore, no reference area samples. Because of this, the statistical tests in this section are called "one-sample tests." The survey site need not be one contiguous area, but the statistical tests should be applied to individual contiguous survey units separately.

Throughout this section, a familiarity with the contents of Section 5 is assumed.

6.2 One-Sample Statistical Tests

The comparison of measurements in the survey unit to the decommissioning criteria is made using two nonparametric statistical tests: either the Wilcoxon Signed Ranks (WSR) test or Sign test and a Quantile (Q1) test. These tests are one-sample analogues of the Wilcoxon Rank Sum (WRS) and Quantile (Q) tests discussed in Section 5. The choice of using the Wilcoxon Signed Ranks test or the Sign test depends on whether the distribution of radioactivity is assumed to be symmetric (like a normal distribution) or skewed (like a log-normal distribution). One or the other of the Wilcoxon Signed Rank or Sign test may be used for a given survey unit, but not both. In addition, the elevated measurement comparison discussed in Section 5 is also made against each measurement to ensure that it does not exceed a specified upper limit.

Like the WRS test, the WSR test (or Sign test) is designed to detect uniform failure of remedial action throughout the survey unit. Like the two-sample Quantile test, the one-sample Quantile (Q1) test is designed to detect failure of remedial action in only a few areas within the survey unit. As with the WRS and Quantile tests discussed in Section 5, the advantage of the WSR, Sign, and Q1 tests is that they do not require the assumption that the data follow any particular distribution, such as normal or log-normal.

Similarly, the WSR, Sign, and Q1 tests also allow for "less than" measurements to be present in the survey unit data. As with the two-sample tests, both the WSR (or Sign) and Q1 tests should be conducted for each survey unit because the tests will detect different types of residual contamination patterns in the survey units. The Elevated-Measurement Comparison is conducted to determine if any measurements in the remediated survey unit exceed a specified upper limit value, H_m. If so, then additional investigation is required, at least locally, regardless of the outcome of the WSR and Q1 tests.

The hypotheses tested by the WSR and one-sample Quantile tests are:

<u>Null Hypothesis</u>
H_0: Decommissioning criteria attained.
versus
<u>Alternative Hypothesis</u>
H_a: Decommissioning criteria not attained.

The null hypothesis is assumed to be true unless either statistical test indicates that it should be rejected in favor of the alternative.

Again, it should be understood that the use of statistical tests will occasionally allow some survey unit measurements to exceed a derived concentration standard without rejecting the null hypothesis that the decommissioning criteria have been attained.

6.2.1 Wilcoxon Signed Ranks Test

Formally, the specific hypothesis tested by the Wilcoxon Signed Ranks Test is as follows (Conover):

<u>Null Hypothesis</u>
H_0: The median of D_i is less than or equal to zero.
versus
<u>Alternative Hypothesis</u>
H_a: The median of D_i is greater than zero.

where
$D_i = X_i - \Delta$, the X_i are the survey unit measurements, and Δ is the derived concentration limit for the radionuclide, calculated as indicated in Section 3.7.1

The assumptions are that the survey unit measurements are independent random samples from a symmetric distribution. The Wilcoxon Signed Ranks test may be more robust (less sensitive to departures from symmetry) than the Student's t-test. However, in cases where asymmetry is expected, the Sign test (Section 6.2.3) may be more appropriate since it requires no assumption of symmetry (Sprent).

For practical purposes, H_a means the probability that a random measurement X_i from the survey unit is larger than Δ is greater than 1/2, i.e., $P(X_i > \Delta) \equiv p > 1/2$. Thus, the hypotheses may be restated as :

Null Hypothesis
$H_0: p \leq 1/2$
versus
Alternative Hypothesis
$H_a: p > 1/2$

6.2.2 One-Sample Quantile Test

A number x_q is called the qth *quantile* of the distribution of a random variable X if the probability that $X < x_q$ is less than or equal to q **and** the probability that $X > x_q$ is less than or equal to 1-q, i.e., if $P(X < x_q) \leq q$ and $P(X > x_q) \leq 1 - q$. For example, the 0.5 quantile is the median.

The specific hypothesis tested by the one-sample Quantile test is as follows (Conover):

Null Hypothesis
$H_0: P(X \leq \Delta') \geq F(\Delta') = q$
versus
Alternative Hypothesis
$H_a: P(X \leq \Delta') < q$

where Δ' is the value below which a proportion q of the survey unit measurement is specified to lie, i.e., Δ' is the qth quantile of the measurement distribution when the null hypothesis is true. If the proportion of measurements larger than Δ' is too high, the null hypothesis will be rejected.

$$F(\Delta') = \int_{-\infty}^{\Delta'} f(x)dx$$

is the probability that a random measurement from the survey unit is less than Δ', when the null hypothesis is true. f is the probability density of the measurements when the null hypothesis is true. If, for example, the distribution of measurements when no contamination is present is normal with mean 0 and standard deviation σ, then

$$F(\Delta') = \int_{-\infty}^{\Delta'} \frac{1}{\sqrt{2\pi}\sigma}\exp(-x^2/2\sigma^2)dx = \Phi(\Delta'/\sigma)$$

where $\Phi(\cdot)$ represents the cumulative normal distribution function (see Table A-7 in Appendix A). As with the two-sample Quantile test, the alternatives considered are measurement probability distributions of the form $G(x) = (1-\epsilon)\,F(x) + \epsilon\,F(x - \Delta')$, i.e., under the alternative, a proportion ϵ of the survey unit contains a mean residual radioactivity concentration of Δ'.

Methods for determining appropriate values for ϵ and Δ' are analogous to those given in Section 5.4 for the two-sample Quantile test.

6.2.3 Sign Test

The specific hypotheses tested by the Sign test are the same as that for the WSR test:

Null Hypothesis
$H_0: p \le 1/2$
versus
Alternative Hypothesis
$H_a: p > 1/2$

The Sign test is also a special case of the one-sample Quantile test, namely, when $q = 0.5$.

6.3 Specification of the Applicable Decommissioning Criteria

For the WSR test (or Sign test) the specification of the decommissioning criteria is made in terms of the amount of shift, Δ, above zero residual radioactivity in the survey unit that is important to detect. If σ is the standard deviation of the measurements in the survey unit , then Δ/σ, expresses this shift as the number of standard deviations to the right that would be considered "large" for the distribution of measurements in the survey unit. The procedure for determining Δ/σ is the same as that already given in Section 5.4.

The one-sample Quantile test (Q1), like the two-sample Quantile test (Q), uses the specification a shift of Δ' above zero for a proportion ϵ of the measurements. The amount of shift specified for the Q1 test need not be the same as that used for the WSR (or Sign) test. Methods for determining appropriate values for Δ' and ϵ have been discussed in Section 5.4.

The level H_m, used for the elevated measurement comparison, is also determined in the same manner as described in Section 5.4.

6.4 Number of Samples

The number of samples required for the survey unit in the present case is determined using considerations and procedures very similar to those already discussed in Section 5.5. Throughout this process, it must again be emphasized that relatively little effort is required to perform the suggested sample size determinations compared to the time and expense involved in collecting and analyzing samples. Therefore, designs with different specified error rates, and values of Δ, Δ', σ, ϵ, and H_m can be examined to find the most efficient methods for attaining the required objectives.

The following procedure is recommended for determining the number of samples to collect in a particular survey design: First, the overall Type I error level desired for both tests combined is divided by 2, because two tests are being used. The value $\alpha/2$ is used to determine the number of samples to be taken. We denote the Type I error level set for the WSR test by α_W and that for the Q1 test by α_Q. Then $\alpha_W = \alpha_Q = \alpha/2$. Second, the number of samples is determined using the procedures for the WSR test, or if the data are anticipated to come from a skewed distribution, the Sign test. Only one of these two tests may be used in a given survey unit. Third, the adequacy of the sample size determined from the above process for detecting an area of elevated activity (greater than H_m) of a given size is then examined, and adjusted if necessary as discussed in Section 5.5.4.

After different designs have been considered, and the number of samples required to meet the decision requirements is determined, another 10 percent or so should be added to allow for the possibility of sample loss during transportation or analysis. In addition, planning should allow for the collection, preparation, and analysis of separate quality control samples.

6.4.1 Determining the Number of Samples for the WSR Test

For the Wilcoxon Signed Ranks test, the number of samples required from the survey unit can be approximated as follows (Noether):

$$N=\frac{(Z_{1-\alpha/2}+Z_{1-\beta})^2}{3(p'-0.5)^2} \qquad \text{(6-1a)}$$

For the Sign test this number is

$$N=\frac{(Z_{1-\alpha/2}+Z_{1-\beta})^2}{4(p-0.5)^2} \qquad \text{(6-1b)}$$

where:

α = specified Type I error rate

β = specified Type II error rate

$Z_{1-\alpha/2}$ = 100(1- α/2) percentile of the normal distribution

$Z_{1-\beta}$ = 100(1- β) percentile of the normal distribution

p = probability that a random measurement from the survey unit is less than Δ

p' = probability that the sum of two independent random measurements from the survey unit is less than 2Δ

Commonly used values of α (and β) are 0.01 , 0.025, 0.05, and 0.10 for which the corresponding values of $Z_{1-\alpha/2}$ (or $Z_{1-\beta}$) may be found from Table 5.1.

The parameter p' (or p) is determined using the specified shift Δ/σ. If the data are normally distributed

$$p = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\frac{\Delta}{\sigma}} e^{-x^2/2}dx = \Phi\left(\frac{\Delta}{\sigma}\right) \qquad \text{(6-2a)}$$

Values of p as a function of Δ/σ, computed from Equation 6-2a, can be found in Table 6.1. Values of p for other probability distributions with density function $f(x)$ can be computed from

$$p = \int_{-\infty}^{\Delta} f(x)dx = F(\Delta). \tag{6-2b}$$

If the data are normally distributed

$$p' = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\frac{\sqrt{2}\cdot\Delta}{\sigma}} e^{-x^2/2}dx = \Phi\left(\frac{\sqrt{2}\cdot\Delta}{\sigma}\right) \tag{6-2c}$$

Values of p' as a function of Δ/σ, computed from Equation 6-2c, can be found in Table 6.1. Values of p' for other probability distributions with density function $f(x)$ can be computed from

$$p' = Probability(X + Y < 2\Delta) = \int_{-\infty}^{2\Delta}\left[\int_{-\infty}^{\infty} f(u-y)f(y)dy\right]du \tag{6-2d}$$

To illustrate the process described above, consider the following example: A site that had been contaminated with Co-60 has been remediated. A radionuclide-specific method (e.g., gamma-ray spectrometry) will be used to determine the residual contamination in soil samples.

From Appendix B in NUREG-1500, the most restrictive default concentration to achieve 15 mrem per year is 2.97 pCi/g in the residential scenario. This scales to 0.593 pCi/g to achieve 3 mrem per year. Suppose that a combination of the random residual activity and measurement uncertainty results in an estimate for the total variability (1 standard deviation) in the measurements of about 1 pCi/g. Then Δ/σ is about 3. From Table 6.1, we find that $p' = 1.0$.

The number of samples required now depends on the power curve constructed during the DQO process. How the number of samples depends on the Type I error rate α and the power ($1-\beta$) is shown in Table 6.2 which was constructed using Equation 6-1 with $p' = 1.0$, i.e.,

$$N = \frac{(Z_{1-\alpha/2}+Z_{1-\beta})^2}{3(p'-0.5)^2} = \frac{(Z_{1-\alpha/2}+Z_{1-\beta})^2}{3(1-0.5)^2} = \frac{(Z_{1-\alpha/2}+Z_{1-\beta})^2}{3(0.5)^2} \tag{6-3}$$

Note that since both the WSR and Q1 tests will be used, $\alpha_W = \alpha/2$ is used in Equation 6-1 and Equation 6-3. The number of samples obtained from Equation 6-3 should always be rounded up to the next integer.

The number of samples required for the WSR test varies from 12 to 33, depending on the values of α and β. For $\alpha = 0.05$ and $\beta = 0.05$, $N = 17$. Also shown in the last column for comparison, is the estimated sample size required for the Sign test, using Equation 6-1b. In this example it would appear that the Sign test actually requires about 25 percent fewer samples.

The Sign test will require fewer samples whenever $|p' - 0.5| / |p - 0.5| < 1.15$ (Noether). However, it is always prudent to use the larger number. The sample size obtained from Equation 6-1 and Equation 6-3 should also be increased by an additional 20 percent, so that there will be little chance that the estimated power will underestimate the actual power specified in the DQOs. This results in a sample size of $(1.2)(17) = 20.4$, or 21 samples to be taken in the survey unit.

Table 6.1 Values of p' and p for Given Values of the Shift Δ/σ

Δ/σ	WSR p'	SIGN p	Δ/σ	WSR p'	SIGN p
0.00	0.500	0.500	1.5	0.983	0.933
0.0625	0.535	0.525	1.5625	0.986	0.941
0.125	0.570	0.550	1.625	0.989	0.948
0.1875	0.605	0.574	1.6875	0.991	0.954
0.25	0.638	0.599	1.75	0.993	0.960
0.3125	0.671	0.623	1.8125	0.995	0.965
0.375	0.702	0.646	1.875	0.996	0.970
0.4375	0.732	0.669	1.9375	0.997	0.974
0.5	0.760	0.691	2	0.998	0.977
0.5625	0.787	0.713	2.0625	0.998	0.980
0.625	0.812	0.734	2.125	0.999	0.983
0.6875	0.835	0.754	2.1875	0.999	0.986
0.75	0.856	0.773	2.25	0.999	0.988
0.8125	0.875	0.792	2.3125	0.999	0.990
0.875	0.892	0.809	2.375	1.000	0.991
0.9375	0.908	0.826	2.4375	1.000	0.993
1	0.921	0.841	2.5	1.000	0.994
1.0625	0.934	0.856	2.5625	1.000	0.995
1.125	0.944	0.870	2.625	1.000	0.996
1.1875	0.953	0.882	2.6875	1.000	0.996
1.25	0.961	0.894	2.75	1.000	0.997
1.3125	0.968	0.905	2.8125	1.000	0.998
1.375	0.974	0.915	2.875	1.000	0.998
1.4375	0.979	0.925	2.9375	1.000	0.998

The effect of increased variability in the measurement data will be an increase in the required sample sizes. As Δ/σ becomes smaller, p' (or p) also becomes smaller. This decreases the denominator of Equation 6-1a and 6-1b, increasing the sample size N accordingly.

An alternative method for determining the sample size is suggested by EPA (QA/G-9). The WSR test has a Pitman efficiency of greater than 0.86 relative to the Student's t-test for *any* residual radioactivity distribution (Lehmann and D'Abrera, p. 379). This means that the WSR should not require more than about $1/(0.86) = 1.16$ times the number of samples required by the one-sample Student's t-test to achieve the same power. (This result is exact only for very large sample sizes, but can be expected to be a reasonable approximation for other cases.) The sample size required for the *t*-test can be calculated from:

$$N_t = \frac{\sigma^2}{\Delta^2}(Z_{1-\alpha/2} + Z_{1-\beta})^2 + 0.5(Z_{1-\alpha/2})^2 \tag{6-4}$$

Table 6.2 Number of Samples Required for WSR Test With $\Delta/\sigma = 3$ and $p' = 1.0$

α	$\alpha_W = \alpha/2$	$1-\beta$	$Z_{(1-\alpha/2)}$	$Z_{(1-\beta)}$	N: WSR	N: Sign	$1.16N_t$
0.010	0.0050	0.990	2.576	2.326	33	25	7
0.010	0.0050	0.975	2.576	1.960	28	21	7
0.010	0.0050	0.950	2.576	1.645	24	18	7
0.010	0.0050	0.900	2.576	1.282	20	15	6
0.025	0.0125	0.990	2.241	2.326	28	21	6
0.025	0.0125	0.975	2.241	1.960	24	18	6
0.025	0.0125	0.950	2.241	1.645	21	16	5
0.025	0.0125	0.900	2.241	1.282	17	13	5
0.050	0.0250	0.990	1.960	2.326	25	18	5
0.050	0.0250	0.975	1.960	1.960	21	16	5
0.050	0.0250	0.950	1.960	1.645	18	13	4
0.050	0.0250	0.900	1.960	1.282	15	11	4
0.100	0.0500	0.990	1.645	2.326	22	16	4
0.100	0.0500	0.975	1.645	1.960	18	13	4
0.100	0.0500	0.950	1.645	1.645	15	11	3
0.100	0.0500	0.900	1.645	1.282	12	9	3

The values of $1.16N_t$ are shown in the last column of Table 6.2. It is prudent to use the larger sample size calculated from Equation 6-3. The larger sample size will result in higher power, and the consequences of underestimating the power can be severe if the DQOs are not met. Nevertheless, the use of Equation 6-4 provides a useful check.

6.4.2 Checking the Power of the WSR Test

To estimate an approximate power curve for this test, we can invert Equation 6-1a and solve for $1 - \beta$ given different values of Δ/σ, using Table 6.1 :

$$Z_{1-\beta} = \sqrt{3N}\,(p'-0.5) - Z_{1-\alpha_W} = \sqrt{3(17)}\,(p'-0.5) - 1.96 = 7.141(p'-0.5) - 1.96 \tag{6-5}$$

Values of 1-β corresponding to values of $Z_{(1-\beta)}$ may be found in Table A-7 (Normal Distribution Function in Appendix A). The results are shown in Table 6.3. Similarly, the approximate power curve for the Sign test can be found by substituting $N = (1.2)(13) = 15.6$, i.e.,16, and p into Equation 6-1b. These results are also shown in Table 6.3.

Table 6.3 Approximate Power of the WSR Test With $\alpha_W = \alpha/2$, $Z_{(1-\alpha/2)} = 1.96$, $\sigma = 1$, $N_{WSR} = 21$, and $N_{sign} = 16$

Δ	Δ/σ	WSR Test			Sign Test		
(pCi/g)		p'	$Z_{(1-\beta)}$	1-β	p	$Z_{(1-\beta)}$	1-β
0.00	0.00	0.500	-1.960	0.025	0.500	-1.960	0.025
0.5	0.5	0.760	0.106	0.542	0.691	-0.428	0.334
1.0	1.0	0.921	1.384	0.917	0.841	0.771	0.780
1.5	1.5	0.983	1.874	0.970	0.933	1.506	0.934
2.0	2.0	0.998	1.990	0.977	0.977	1.858	0.968
2.5	2.5	1.000	2.007	0.978	0.994	1.990	0.977
3.0	3.0	1.000	2.009	0.978	0.999	2.029	0.979

The data of Table 6.3 are plotted in Figure 6.1, which shows that the design objectives are reasonably well matched by the power curve. Note that the false positive error rate, $\alpha_W = \alpha/2$, is fixed at 0.025 for zero mrem per year (no residual radioactivity). The rate at which the null hypothesis will be rejected at 3 mrem per year (0.6 pCi/g) above background is about 65 percent for the WSR test and about 50 percent for the Sign test. The power at 15 mrem per year (3 pCi/g) above background is above that required for both tests. As discussed in Section 3.7.5, it is not always possible to design the test so that the error rates are exactly as specified. For the final termination survey, priority is given to satisfying the DQO for the power 1 - β (where β is the false negative error rate). This is because the consequence of false negative errors would be an impact on human health, whereas the consequences of false positive errors are primarily economic.

There are two additional ways that the power of the WSR test can be checked. They both involve approximations to the power function, but are derived in different ways (Lehmann and D'Abrera, Chapter 4, Section 3, pp. 69-75).

The first method involves approximating the distribution of the WSR test statistic, W_r, by a normal distribution to compute the probability that the null hypothesis will be rejected when the

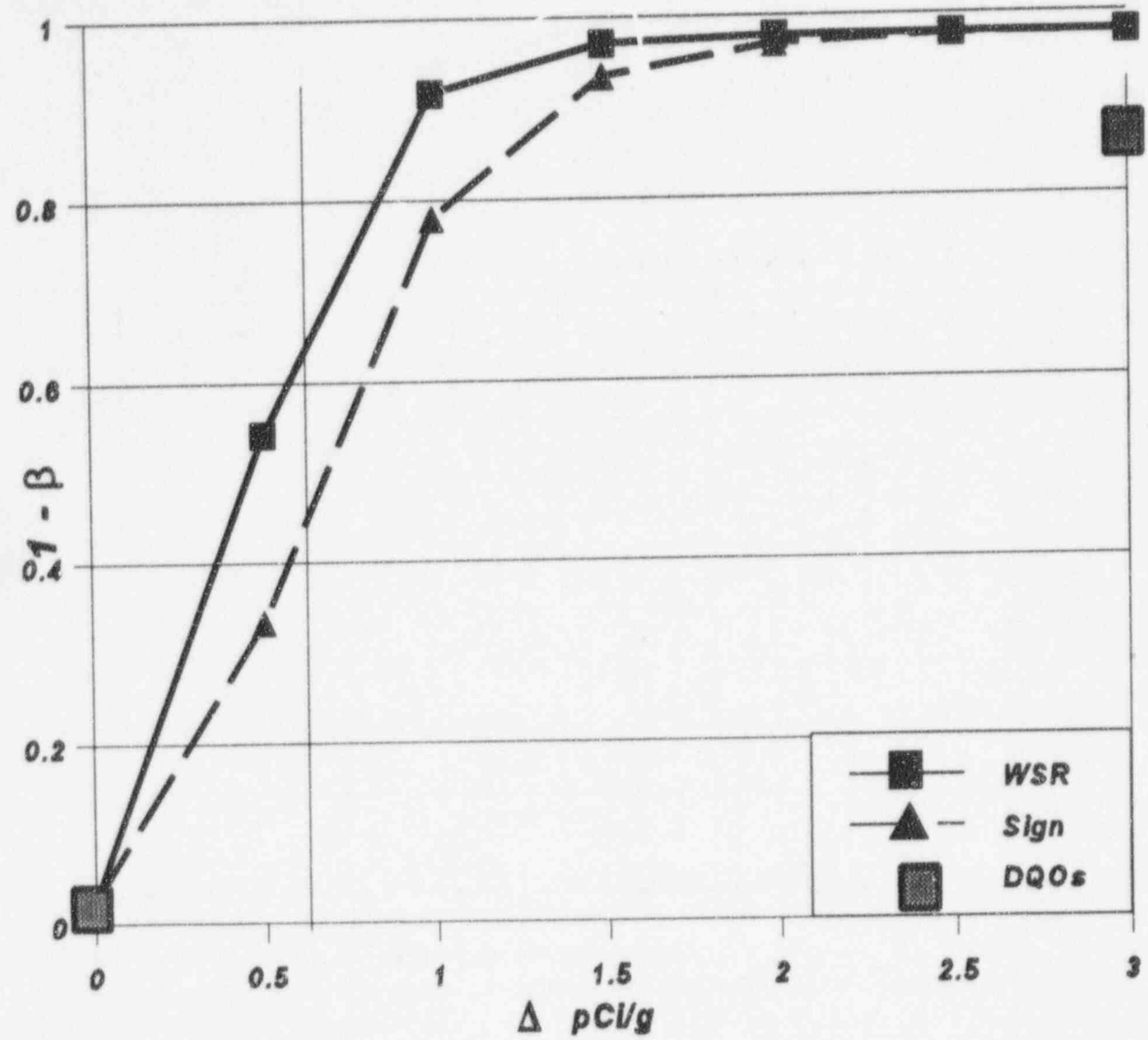


Figure 6.1 Power of the WSR and Sign Tests for the Example Problem

alternative is true. For this, the mean and variance of W_s when the alternative is true must be calculated. The mean is

$$E(W_s) = 0.5N(N-1)p' + Np$$

where p and p' are as defined following Equation 6-1. Values of p and p' can be calculated from Equation 6-2. For the case of a normal distribution, their values can be found in Table 6.1. The variance

$$\mathrm{Var}(W_s) = N(N-1)(N-2)(p_2' - (p')^2) + 0.5N(N-1)[2(p-p')^2 + 3p'(1-p')] + Np(1-p)$$

p_2' is the probability that the sum of one random survey unit measurement and a second random survey unit measurement is less than 2Δ, *and* the sum of the first random survey unit measurement and a third random survey unit measurement is also less than 2Δ. If the measurement distributions are normal, then p_2' is equal to the probability that two correlated standard (i.e., mean = 0 and variance = 1) normal random variables, with correlation coefficient 0.5, are both less than $(\sqrt{2}\ \Delta)/\sigma$. Values of a p_2' function of Δ/σ are given in Table 6.4.

Table 6.4 Values of p_1 and p_2' for Computing the Variance of W_s

Δ/σ	p_2'	Δ/σ	p_2'
0.5	0.633702	1.8	0.98965
0.6	0.689800	1.9	0.993107
0.7	0.741698	2.0	0.995497
0.8	0.788602	2.1	0.997099
0.9	0.830022	2.2	0.998186
1.0	0.865767	2.3	0.998882
1.1	0.895917	2.4	0.999324
1.2	0.920777	2.5	0.999599
1.3	0.940817	2.6	0.999767
1.4	0.956616	2.7	0.999867
1.5	0.968795	2.8	0.999926
1.6	0.977981	2.9	0.999959
1.7	0.984758	3.0	0.999978

The power of the WSR test is then computed from

$$\text{Power} = 1 - \Phi\left[\frac{W_c - 0.5 - E(W_s)}{\sqrt{Var(W_s)}}\right]$$

where W_c is the critical value found in Table A-1 for the appropriate vales of α, and N. Values of $\Phi(z)$, the standard normal cumulative distribution function, are given in Table A-7. Using this equation for the example problem confirms that the power is near 1.

The second approximation suggested by Lehmann and D'Abrera is useful if the survey unit measurement distribution is not normal. This approximation is made assuming that Δ is small. In that case

$$Power \approx \Phi\left[\frac{N(N-1)e^*(0) + Ne(0)}{\sqrt{N(N+1)(2N+1)/24}}\,\Delta - Z_{1-\alpha/2}\right]$$

Here, $e(0)$ is the probability density of the survey unit measurements, evaluated at zero; and $e^*(0)$ is the probability density of the sum of two independent measurements with the same density, $e(x)$, also evaluated at zero. For two normally distributed random variables, with mean 0 and variance σ^2

$$e(0) = \frac{1}{\sigma\sqrt{2\pi}} \quad \text{and} \quad e^*(0) = \frac{1}{2\sigma\sqrt{\pi}}$$

Using this approximation for the example problem yields a power estimate near 1.

The preceding power approximation may be inverted to give estimates of the sample size needed to achieve a desired power, namely

$$N \approx \frac{(Z_{1-\alpha/2} + Z_{1-\beta})^2}{12\Delta^2 e^{*2}(0)}$$

If the measurement distribution is normal, this becomes

$$N \approx \frac{\pi\sigma^2(Z_{1-\alpha/2} + Z_{1-\beta})^2}{3\Delta^2}$$

For the example problem, the estimated sample size is 4. This is smaller than the values in Table 6.2, and the larger values in that table are the ones that should be used.

6.4.3 Checking the Power of the One-Sample Quantile Test and the Sign Test

Once the WSR test has been performed, if the null hypothesis has been accepted, the one-sample Quantile test is performed. The test is that at least $100q\%$ of the concentrations in the survey unit are less than Δ'. Δ' might be determined as in Section 5.4 as equal to Δ/ϵ, where Δ is the decommissioning limit used for the WSR or Sign test. The essential purpose is to see that the measurement distribution does not have an unusual skew toward higher values. Values of ϵ that are important to detect should be determined during the DQO process.

The procedure for conducting the one-sample Quantile test is simply to count the number, k, of measurements that are greater than Δ'. If the null hypothesis is true, then the probability that more than k measurements are greater than Δ' can be described by a binomial distribution

$$\sum_{i=k+1}^{N} \binom{N}{i} [1-q]^i \cdot [q]^{N-i} = \alpha_Q \tag{6-6}$$

where $q = F(\Delta')$, as defined in Section 6.2.2, and

$$\binom{N}{i} = \frac{N!}{(N-i)!\ i!}$$

is called a binomial coefficient. The symbol $i!$, called i factorial, is the product of the first i integers, $i! = i(i-1)(i-2)...(3)(2)(1)$. $0!$ is defined to be equal to 1.

Note that

$$\sum_{i=k+1}^{N} \binom{N}{i} [1-q]^{i} \cdot [q]^{N-i} + \sum_{i=0}^{k} \binom{N}{i} [1-q]^{i} \cdot [q]^{N-i} = \alpha_Q + (1-\alpha_Q) = 1$$

since the sums cover all possible numbers of measurements from zero to N.

Values of the sum

$$\sum_{i=0}^{k} \binom{N}{i} [1-q]^{i} \cdot [q]^{N-i} = (1-\alpha_Q)$$

are given in Table A-9 in Appendix A.9 for sample sizes, N, up to 20.

For values of $N > 20$, the following approximation is used:

$$1-\alpha_Q = \sum_{i=0}^{k} \binom{N}{i} [1-q]^{i} \cdot [q]^{N-i} \approx \Phi\left(\frac{k-N(1-q)}{\sqrt{Nq(1-q)}} \right) \qquad (6\text{-}7)$$

where $\Phi(\cdot)$ represents the cumulative normal distribution function (Table A-7).

Using Table A-7 or Equation 6-7, together with the desired values of q and α determined during the DQO process, the value of k for the test is found.

For the example in Section 6.4.1, the guideline concentration value for Co-60 is $\Delta = 2.97$ pCi/g, and the measurement standard deviation was estimated to be about 1 pCi/g, thus $\sigma = \Delta/3$. If the measurements from the survey unit are expected, under the null hypothesis, to be approximately normal with mean zero and standard deviation σ, then $q = F(\Delta') = \Phi(\Delta'/\sigma)$. Suppose it is desired to test whether less than half the survey unit contains residual radioactivity at 10 percent over the guideline value concentration. Then $\epsilon = 0.5$, and $\Delta' = 1.1$. Thus, $q = \Phi(\Delta'/\sigma) = \Phi(1.1\Delta/0.33\Delta) = \Phi(3.3) = 0.9995$, from Table A-7. For the WSR test, it was determined that $N = 21$ measurements would be made. To calculate the number, k, of measurements above 3.3 pCi/g that would cause rejection of the null hypothesis, Equation 6-7 is used:

$$1-\alpha/2 = 0.975 \approx \Phi(1.96) = \Phi\left(\frac{k-21(1-0.9995)}{\sqrt{(21)(0.9995)(1-0.9995)}} \right)$$

so

$$1.96 = \frac{k-21(1-0.9995)}{\sqrt{(21)(0.9995)(1-0.9995)}} \quad or \quad k \approx 0.2$$

Values of k must be integers, so $k = 0$. Thus, even one measurement exceeding 3.3 pCi/g would cause rejection of the null hypothesis.

The power of the one-sample Quantile test can be calculated exactly. This is the probability that one or more measurements exceeding 3.3 pCi/g would be obtained if half the site had contamination at that average level. Under this alternative hypothesis, the distribution of measurements in the survey unit is described by

$$G(x) = (1-\epsilon)\, F(x) + \epsilon\, F(x - \Delta')$$

where $F(x)$ is the distribution of measurements in the parts of the survey unit without contamination. Then the power of the test to detect an area ϵ contaminated to the level Δ' is the probability that k or more measurements greater than Δ' will be obtained:

$$Power = 1-\beta = \sum_{i=k+1}^{N} \binom{N}{i} [1-q^*]^{i} [q^*]^{N-i} = 1 - \sum_{i=0}^{k} \binom{N}{i} [1-q^*]^{i} [q^*]^{N-i} \qquad (6\text{-}8)$$

where $q^* = G(\Delta') = (1-\epsilon)\, F(\Delta') + \epsilon\, F(\Delta' - \Delta') = (0.5)\, \mathrm{F}(\Delta') + 0.5\, \mathrm{F}(0)$.

If $F(x)$ is normal with mean 0 and standard deviation σ, then

$$q^* = (0.5)\, \Phi(\Delta'/\sigma) + 0.5\, \Phi(0) = (0.5)\, \Phi(\Delta'/\sigma) + 0.25, \text{ since } \Phi(0) = 0.5.$$

$\Phi(\Delta'/\sigma)$ has already been calculated above, $\Phi(\Delta'/\sigma) = \Phi(1.1\Delta/0.33\Delta) = \Phi(3.3) = 0.9995$. Therefore, $q^* = (0.5)(0.9995) + 0.25 = 0.49975 + 0.25 = 0.74975$.

Using the approximation of Equation 6-7 in Equation 6-8, with $k = 0$

$$1-\beta \approx 1-\Phi\left(\frac{0-21(1-0.74975)}{\sqrt{(21)(0.74975)(1-0.74975)}}\right) = 1-\Phi(-2.65) = 1-(1-0.996) = 0.996$$

Thus, if the alternative hypothesis were true, the null hypothesis would be very likely to be rejected.

The power calculations for the Sign test are done in a similar way, as shown in Section 6.6.5.

6.4.4 Probability of Detecting an Area of Elevated Activity

The considerations involved in determining the probability of detecting an area of elevated activity for measurements that do not require a background comparison are the same as already discussed in Sections 5.5.4 and 5.8.5. This is because the "elevated measurement" comparison is done without regard to background variations.

Recall the example site shown in Figure 5.5 for the Reference Uranium Fuel Fabrication Plant. For this example, this site will be considered to have been remediated from Co-60 contamination.

The survey site area is 4,500m². In the present example, with 21 sample points, the sampling grid area can be estimated to be about 4,500/21 ≈ 214 m². H_m ≈ (area factor)•Δ = (1.1)(3 pCi/g) ≈ 3.3 pCi/g using the area factors in Table C-1. From Appendix C of NUREG-1500, the external dose due to Co-60 is about 5 mrem per year per pCi/g. An area elevated by 3.3 pCi/g would result in a local external exposure rate increase of about 2 μR per hour. This could be detected with an *in situ* spectrometer or a PIC measurement within each sampling grid area.

6.4.5 Allowance for QA Samples, and Missing or Unusable Data

As discussed in Section 5.5.4, a minimum of 10 percent of the total number of samples should be earmarked for QA. Thus, whatever sample size is determined to be appropriate following the analyses described earlier in this section, it should be increased by at least 10 percent. The QA samples will not normally be included in the sample grid as separate sampling points. Rather, these will consist of blanks, spikes, or duplicate samples.

Allowance must also be made for potential missing or unusable data. If R is the expected rate of missing or unusable data based on past experience, then the total number of samples to collect, n_f, is

$$n_f = n/(1 - R)$$

The use of this correction will give some assurance that enough samples will be collected to meet the specified Type I and Type II error-rate requirements.

6.5 Sampling Locations

For each survey unit, samples are collected on a random-start equilateral triangular grid. The procedure to be used is the same as that given in Sections 5.6 and 5.7.

6.6 Applying the Tests

The WSR, Sign, and Q1 tests are one-sample tests designed for comparing survey units to decommissioning criteria.

6.6.1 Applying the Wilcoxon Signed Rank Test

The WSR test is applied as follows:

(1) From each survey unit measurement, X_i, $i = 1,...,N$, subtract the derived concentration limit Δ. This results in a set of differences $D_i = X_i - \Delta$.

(2) Next, order the differences according to their magnitudes (i.e., absolute values), $|D_i|$ without regard to sign. However, keep track of the sign associated with each difference. This can be done by coding a magnitude as (-) for negative and (+) for positive. (This idea is similar to the way the reference area measurements were coded as R and the survey unit measurements as S in applying the Wilcoxon Rank Sum test in Section 5.8.)

(3) If any difference is zero, discard it from the analysis, and reduce the sample size, N, by the number of such zero differences.

(4) Sum the ranks of the magnitudes of the *positive* differences (i.e., those coded as +). The result is the test statistic T+.

(5) Large values of T+ indicate that the null hypothesis is false. The value of T+ is compared to the critical values in Table A-8. If T+ is larger than the critical value, $W_{1-\alpha}$, in that table, the null hypothesis is rejected. Otherwise, the null hypothesis is accepted.

6.6.2 WSR Test Example

The example given Section 6.4 is continued. As already calculated, 21 measurements are needed in the survey unit. In laying out the survey unit sampling grid, more than 21 locations may actually be obtained. As discussed earlier, if more sample locations are identified than are calculated to be required, all of the identified locations are still sampled. For this example, it is assumed the number remains 21. The measurements are shown in column A of Table 6.5. (These data were artificially generated from a normal distribution with a mean of 2 pCi/g and a standard deviation of 1 pCi/g.) Notice that one of these measurements is negative (-0.5 in cell A14). This might occur if a result is below the lower limit of detection, or an analysis background (e.g., the Compton continuum under a spectrum peak) is subtracted to obtain the net concentration value. The analysis will not be affected by the presence of such values.

Column B contains the differences $D_i = X_i - \Delta$ (Δ is 2.97 pCi/g for this example), Column C contains the magnitudes of the differences, $|D_i|$, and Column D contains the signs of the differences. Column E contains the ranks of the magnitudes. The sum (231 in cell E24) should always equal $N(N+1)/2$. Finally, Column F contains the ranks of the magnitudes of the positive differences. Cell F24 contains the sum of the ranks of the magnitudes of the positive differences, which is the test statistic T+. The value of T+ is compared to the appropriate critical value in Table A-8. In this case, for N=21 and $1-\alpha = 0.975$, the critical value $W_{1-\alpha} = 172$. Since T+ = 36.5 does not exceed this value, the null hypothesis that the survey unit had been adequately decontaminated is accepted. Table 6.6 shows the spreadsheet functions that were used to create Table 6.5.

6.6.3 Applying the One-Sample Quantile Test

Once the WSR test has been performed, if the null hypothesis has been accepted, the one-sample Quantile test is performed. In order to do this, first the number k is found from Equation 6-7:

$$1-\alpha_Q = \sum_{i=0}^{k} \binom{N}{i} [1-q]^i \cdot [q]^{N-i} \approx \Phi\left(\frac{k-N(1-q)}{\sqrt{Nq(1-q)}} \right)$$

Table 6.5 Example Wilcoxon Signed Ranks Test Analysis

	A	*B*	*C*	*D*	*E*	*F*
1	**Wilcoxon Signed Ranks Test**					**Positive**
2	**Data**	**Data -Δ**	**Magnitude**	**Sign**	**Ranks**	**Ranks**
3	1.75	-1.22	1.22	-1	12	0
4	0.54	-2.43	2.43	-1	20	0
5	1.91	-1.06	1.06	-1	10	0
6	2.82	-0.15	0.15	-1	1.5	0
7	3.12	0.15	0.15	1	1.5	1.5
8	2.08	-0.89	0.89	-1	8	0
9	1.7	-1.27	1.27	-1	13	0
10	1.85	-1.12	1.12	-1	11	0
11	0.78	-2.19	2.19	-1	18	0
12	0.65	-2.32	2.32	-1	19	0
13	3.32	0.35	0.35	1	4	4
14	-0.5	-3.47	3.47	-1	21	0
15	4.47	1.5	1.5	1	14	14
16	0.84	-2.13	2.13	-1	17	0
17	0.88	-2.09	2.09	-1	16	0
18	3.22	0.25	0.25	1	3	3
19	3.47	0.5	0.5	1	5	5
20	2.3	-0.67	0.67	-1	6	0
21	1.43	-1.54	1.54	-1	15	0
22	2.09	-0.88	0.88	-1	7	0
23	3.91	0.94	0.94	1	9	9
24			Sum:	6	**231**	**36.5**
25	*Critical Value from Table A-8 for N=21 and 1-α/2 = 0.975 is 172*					

using $\alpha_Q = \alpha/2$, *and* $q = \Phi(\Delta'/\sigma)$. Table A-9 is used to evaluate the binomial probability if N is 20 or less. The function Φ is evaluated using Table A-7, if N is greater than 20.

The resulting value of k is used to evaluate the power using

$$1 - \beta = 1 - \sum_{i=0}^{k} \binom{N}{i} [1 - q^*]^i \cdot [q^*]^{N-i} \approx 1 - \Phi\left(\frac{k - N(1 - q^*)}{\sqrt{N q^* (1 - q^*)}} \right)$$

with

$$q^* = (1 - \epsilon)\Phi(\Delta'/\sigma) + 0.5\epsilon$$

Table 6.6 Spreadsheet Formulas for Table 6.5

	A	B	C	D	E	F
1	Wilcoxon Signed Ranks Test				Ranks	Positive Ranks
2	Data	Data -Δ	Magnitude	Sign		
3	1.75	=ROUND(A3-2.97,2)	=ABS(B3)	=SIGN(B3)	=RANK(C3,$C$3:$C$23,1)+ (COUNTIF($C$3:$C$23,C3)-1/2	=IF(D3>0,E3,0)
4	0.54	=ROUND(A4-2.97,2)	=ABS(B4)	=SIGN(B4)	=RANK(C4,$C$3:$C$23,1)+ (COUNTIF($C$3:$C$23,C4)-1/2	=IF(D4>0,E4,0)
5	1.91	=ROUND(A5-2.97,2)	=ABS(B5)	=SIGN(B5)	=RANK(C5,$C$3:$C$23,1)+ (COUNTIF($C$3:$C$23,C5)-1/2	=IF(D5>0,E5,0)
6	2.82	=ROUND(A6-2.97,2)	=ABS(B6)	=SIGN(B6)	=RANK(C6,$C$3:$C$23,1)+ (COUNTIF($C$3:$C$23,C6)-1/2	=IF(D6>0,E6,0)
7	3.12	=ROUND(A7-2.97,2)	=ABS(B7)	=SIGN(B7)	=RANK(C7,$C$3:$C$23,1)+ (COUNTIF($C$3:$C$23,C7)-1/2	=IF(D7>0,E7,0)
8	2.08	=ROUND(A8-2.97,2)	=ABS(B8)	=SIGN(B8)	=RANK(C8,$C$3:$C$23,1)+ (COUNTIF($C$3:$C$23,C8)-1/2	=IF(D8>0,E8,0)
9	1.7	=ROUND(A9-2.97,2)	=ABS(B9)	=SIGN(B9)	=RANK(C9,$C$3:$C$23,1)+ (COUNTIF($C$3:$C$23,C9)-1/2	=IF(D9>0,E9,0)
10	1.85	=ROUND(A10-2.97,2)	=ABS(B10)	=SIGN(B10)	=RANK(C10,$C$3:$C$23,1)+ (COUNTIF($C$3:$C$23,C10)-1/2	=IF(D10>0,E10,0)
11	0.78	=ROUND(A11-2.97,2)	=ABS(B11)	=SIGN(B11)	=RANK(C11,$C$3:$C$23,1)+ (COUNTIF($C$3:$C$23,C11)-1/2	=IF(D11>0,E11,0)
12	0.65	=ROUND(A12-2.97,2)	=ABS(B12)	=SIGN(B12)	=RANK(C12,$C$3:$C$23,1)+ (COUNTIF($C$3:$C$23,C12)-1/2	=IF(D12>0,E12,0)
13	3.32	=ROUND(A13-2.97,2)	=ABS(B13)	=SIGN(B13)	=RANK(C13,$C$3:$C$23,1)+ (COUNTIF($C$3:$C$23,C13)-1)/2	=IF(D13>0,E13,0)
14	-0.5	=ROUND(A14-2.97,2)	=ABS(B14)	=SIGN(B14)	=RANK(C14,$C$3:$C$23,1)+ (COUNTIF($C$3:$C$23,C14)-1)/2	=IF(D14>0,E14,0)
15	4.47	=ROUND(A15-2.97,2)	=ABS(B15)	=SIGN(B15)	=RANK(C15,$C$3:$C$23,1)+ (COUNTIF($C$3:$C$23,C15)-1)/2	=IF(D15>0,E15,0)
16	0.84	=ROUND(A16-2.97,2)	=ABS(B16)	=SIGN(B16)	=RANK(C16,$C$3:$C$23,1)+ (COUNTIF($C$3:$C$23,C16)-1)/2	=IF(D16>0,E16,0)
17	0.88	=ROUND(A17-2.97,2)	=ABS(B17)	=SIGN(B17)	=RANK(C17,$C$3:$C$23,1)+ (COUNTIF($C$3:$C$23,C17)-1)/2	=IF(D17>0,E17,0)
18	3.22	=ROUND(A18-2.97,2)	=ABS(B18)	=SIGN(B18)	=RANK(C18,$C$3:$C$23,1)+ (COUNTIF($C$3:$C$23,C18)-1)/2	=IF(D18>0,E18,0)
19	3.47	=ROUND(A19-2.97,2)	=ABS(B19)	=SIGN(B19)	=RANK(C19,$C$3:$C$23,1)+ (COUNTIF($C$3:$C$23,C19)-1)/2	=IF(D19>0,E19,0)
20	2.3	=ROUND(A20-2.97,2)	=ABS(B20)	=SIGN(B20)	=RANK(C20,$C$3:$C$23,1)+ (COUNTIF($C$3:$C$23,C20)-1)/2	=IF(D20>0,E20,0)
21	1.43	=ROUND(A21-2.97,2)	=ABS(B21)	=SIGN(B21)	=RANK(C21,$C$3:$C$23,1)+ (COUNTIF($C$3:$C$23,C21)-1)/2	=IF(D21>0,E21,0)
22	2.09	=ROUND(A22-2.97,2)	=ABS(B22)	=SIGN(B22)	=RANK(C22,$C$3:$C$23,1)+ (COUNTIF($C$3:$C$23,C22)-1)/2	=IF(D22>0,E22,0)
23	3.91	=ROUND(A23-2.97,2)	=ABS(B23)	=SIGN(B23)	=RANK(C23,$C$3:$C$23,1)+ (COUNTIF($C$3:$C$23,C23)-1)/2	=IF(D23>0,E23,0)
24				=COUNTIF (D3:D23,1)	=SUM(E3:E23)	=SUM(F3:F23)
25	Critical Value from Table A-8 for N=21 and 1-α/2 = 0.975 is 172					

The normal distribution is used here in the same way that it was used in Section 5, namely, to provide a convenient method for calculating q and q^* from Δ, Δ', ϵ, and σ.

The methods of Section 6.4.3 can be used to calculate these quantities using other measurement distributions, if necessary.

6.6.4 One-Sample Quantile Test Example

Continuing the example of Section 6.4.3, the analysis above was performed for the choice of $\Delta' = 1.1\Delta = 3.3$ pCi/g and $\epsilon = 0.5$.

To apply the test it is only necessary to observe that four of the measurements in Table 6.5 are above Δ'. Thus, the null hypothesis that less than half the survey unit has residual radioactivity averaging 10 percent over the guideline is rejected.

6.6.5 Sign Test

The Sign test is carried out in a manner very similar to that for the one-sample Quantile test (Q1) given above. In fact, it is only necessary to set $q = 0.5$, $\epsilon = 1$, and count the number of measurements greater than Δ.

However, the value of *k* for the Sign test should be found from Equation 6-8 rather than Equation 6-7, because of the priority given to minimizing Type II errors:

$$1-\beta = 1-\sum_{i=0}^{k}\binom{N}{i}[1-q^*]^i\cdot[q^*]^{N-i} \approx 1-\Phi\left(\frac{k-N(1-q^*)}{\sqrt{Nq^*(1-q^*)}}\right)$$

with $q^* = G(\Delta)$, where $G(x) = (1-\epsilon)F(x) + \epsilon F(x-\Delta') = F(x-\Delta')$, since $\epsilon = 1$. Δ' is the concentration of residual radioactivity actually present, and Δ is the guideline concentration.

If $F(x)$ is normal with mean 0 and standard deviation σ, then $q^* = \Phi((\Delta - \Delta')/\sigma)$. When the actual residual radioactivity concentration is at the guideline, $q^* = 0.5$, then

$$1-\beta = 0.95 = 1-\sum_{i=0}^{k}\binom{21}{i}[1-0.5]^i\cdot[0.5]^{21-i} \approx 1-\Phi\left(\frac{k-(21)(1-0.5)}{\sqrt{(21)(0.5)(1-0.5)}}\right)$$

or

$$0.05 \approx \Phi\left(\frac{k-10.5}{\sqrt{5.25}}\right) \quad so \quad \frac{k-10.5}{\sqrt{5.25}} = -1.645$$

so $k = 6.73$. Taking $k = 6$ will yield higher power $(1 - \Phi(-1.964) = 0.975)$ than taking $k = 7$ $(1 - \Phi(-1.575) = 0.9424)$. If $k = 6$, then $k+1 = 7$ or more measurements above the guideline would have to be observed in order to reject the null hypothesis.

In Table 6.5, there are six measurements above Δ, which does not exceed *k*; therefore the null hypothesis is not rejected.

Using $k = 6$ in Equation 6-8, the power of the Sign test for other values of residual radioactivity, Δ', may be found. For example, if $\Delta' = 0.6$ pCi/g (corresponding to a dose of 3 mrem per year), then

$$1-\beta = 1-\sum_{i=0}^{6}\binom{21}{i}[1-q^*]^i\cdot[q^*]^{21-i} \approx 1-\Phi\left(\frac{6-21(1-q^*)}{\sqrt{21q^*(1-q^*)}}\right)$$

with $q^* = G(\Delta) = F(x - \Delta') = \Phi((\Delta - \Delta')/\sigma) = q^* = \Phi(3.0 - 0.6) = \Phi(2.4) = 0.9918$.
So the power

$$1-\beta \approx 1-\Phi\left(\frac{6-21(1-0.9918)}{\sqrt{21(0.9918)(1-0.9918)}}\right) = 1-\Phi(37.8) \approx 0.$$

A similar calculation can be performed for several values of Δ', using the sample standard deviation, $s = 1.23$, in order to construct a retrospective power curve for the test. This is an important step when the null hypothesis is not rejected, since it demonstrates whether the DQOs have been met. Note that the power is slightly less than anticipated because the sample standard deviation of the measurements (1.23) is larger than that used in the planning (1.0). This illustrates the importance of not underestimating that parameter. Because some conservative choices were made in determining the sample size, the DQOs have still been met. The results of the retrospective power calculations are shown in Table 6.7 and Figure 6.2.

Table 6.7 Retrospective Power of the Sign Test for the Example

Δ'	Δ'/s	Power
0.0	0.00	0.0000
0.5	0.41	0.0000
1.0	0.81	0.0000
1.5	1.22	0.0055
2.0	1.63	0.1905
2.5	2.03	0.7073
3.0	2.44	0.9752
3.5	2.85	0.9998
4.0	3.25	1.0000
4.5	3.66	1.0000

6.6.6 Elevated Measurement Comparison

The elevated measurement comparison consists of comparing each measurement from the survey unit with the concentration value H_m discussed in Sections 5.3.3, 5.4, 5.5.4, and 5.8.5. Any measurement from the survey unit that is equal to or greater than H_m indicates an area of relatively high concentrations that must be investigated, regardless of the outcome of the WSR or one-sample Quantile tests.

The elevated measurement comparison value is $H_m = A_m \Delta$, where A_m is the area factor and Δ is the radionuclide concentration corresponding to the guideline dose. In Section 6.4.4, it was calculated that H_m = 3.3 pCi/g for the grid area of 214 m^2. From Table 6.5, there are four measurements that would require additional investigation.

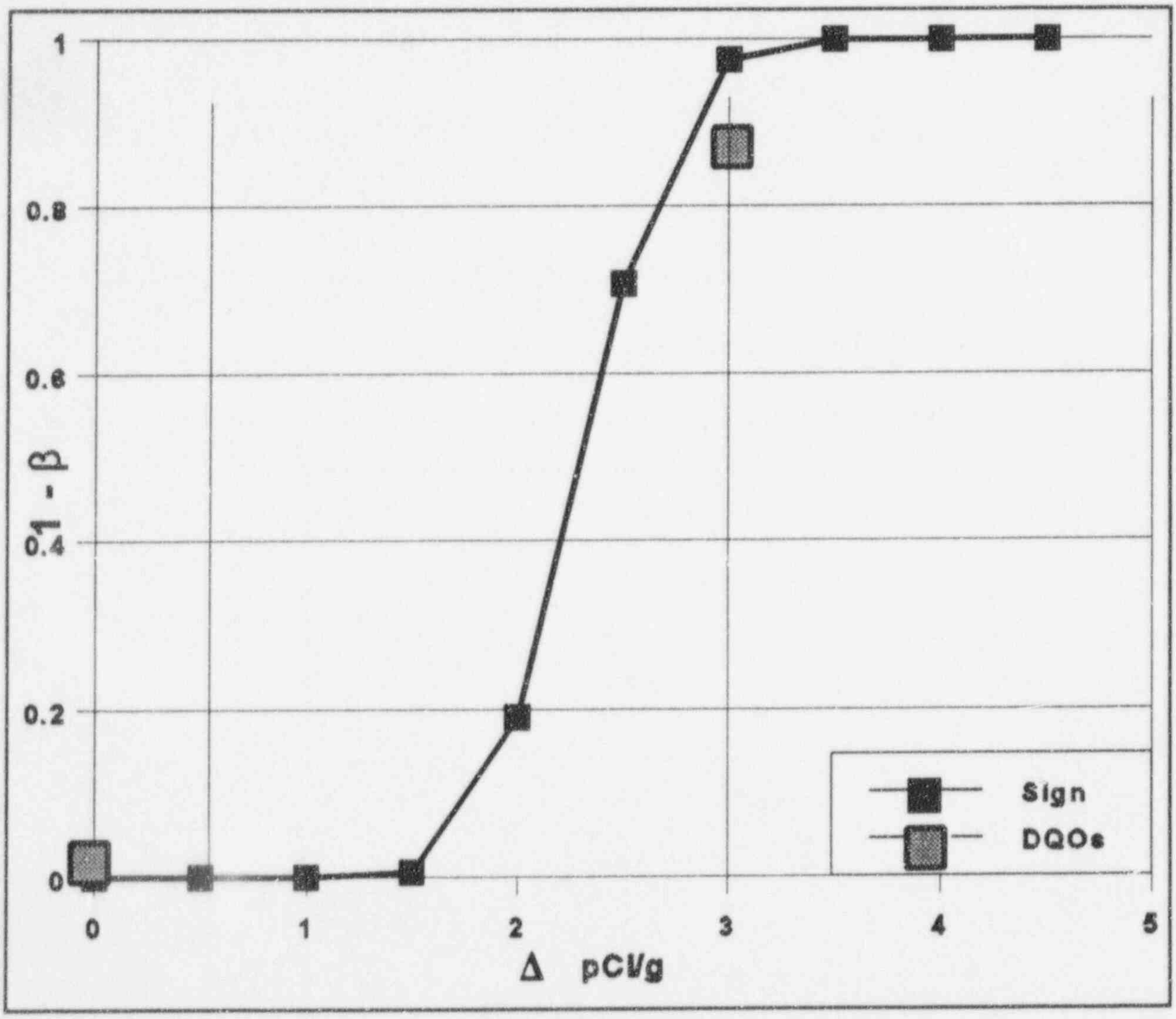


Figure 6.2 Retrospective Power Curve for Sign Test Example

7 SUMMARY AND RECOMMENDATIONS

7.1 Selection of Statistical Parameter Values

7.1.1 Introduction

It cannot be emphasized too strongly that adequate consideration be given to the intended design of the termination survey during the initial planning stages of the decommissioning plan. Successful completion of the final survey is necessary before decommissioning can occur. As discussed above, it is early in the decommissioning process that acceptable error rates should be established for an incorrect determination that a site meets or does not meet the cleanup criteria. Selection of these error rates requires specification of certain parameter values that are components of nonparametric statistical techniques. This section of the report discusses the potential impacts of these decisions and proposes recommendations for selecting parameter values for the Wilcoxon and Quantile tests.

Some choices of decision error rates and test parameters can greatly influence the performance of the statistical tests and their results. These decisions also impact the complexity and cost of final status and confirmatory surveys by requiring greater or lesser amounts of radiological data to support the data requirements of the statistical tests. Because it is so important to select appropriate error rates and test parameter values, the NRC staff is specifically seeking comments on the proposed recommendations so that modifications can be made where appropriate.

7.1.2 Type I Decision Errors

Specification of a Type I error rate for final status and confirmatory surveys establishes the acceptable probability in labeling a site that actually meets the reference radiological criterion as being contaminated above background. An error of this type would result in a licensee performing unnecessary remediations. If compliance with an indistinguishable from background decommissioning criterion is determined on a radionuclide-specific basis, this would most affect those sites that contain residual radioactivity that is also part of background, such as sites that utilize radioactive material in the uranium and thorium decay series and sites that contain Cs-137 and Sr-90 from fallout.

If standard error rates were to be established for all NRC licensees, a high Type I error rate would cause more licensees to perform unnecessary remediations and, conversely, a low Type I error rate would cause fewer licensees to perform unnecessary remediations. Obviously, specification of low Type I error rates, such as 1 percent, are preferred because fewer licensees would perform unnecessary remediations in response to this type of decision error. However, low Type I error rates require a larger number of radiological measurements to satisfy the statistical tests. The number of measurements required is also dependent on the power of the statistical test and the magnitude of the difference from background that is important to detect. Thus, consideration must be given to the number of radiological measurements because of the increased cost and complexity of performing site and reference area surveys. For most decommissioning cases, an optimization of cost versus benefit would provide the basis for site-specific decisions on appropriate Type I and Type II error rates.

7.1.3 Type II Decision Errors

Specification of a Type II error rate establishes the acceptable probability of incorrectly labeling a site that contains residual radioactivity as being indistinguishable from background. An error of this type would result in a site being released for unrestricted use at some level above background because, based on the outcome of the statistical tests, the licensee was not required to perform additional site remediation.

The Type II error rate directly affects the total number of NRC sites that may be released above background, which could potentially impact public health and safety and the environment. Therefore, specification of Type II error rates should consider all significant risks to humans and the environment resulting from the decommissioning process (including transportation and disposal of radioactive wastes generated in the process) and from residual radioactivity remaining at the site following termination of the license. According to recommendations contained in the proposed decommissioning rule, final status surveys and confirmatory surveys should be capable of detecting 15 mrem per year above background with the objective of being able to distinguish residual radioactivity levels at or near background. Thus, the Type II error rate should be set at a level which ensures that doses from residual radioactivity do not exceed 15 mrem per year above background for most decommissioning actions.

As with establishment of Type I error rates, consideration must be given to the number of radiological measurements required by establishing a particular Type II error rate because of the increased cost and complexity of performing site and reference area surveys. If a high Type II error rate is established for all NRC licensees, it might result in erroneously accepting a relatively large number of sites exceeding 15 mrem per year above background. However, the overall radiological impacts would not be great because even these sites would still be released well below the NRC's recommended public dose limit of 100 mrem per year. However, because the Type II error rate can potentially impact public health and safety and the environment from excessive residual radioactivity and the Type I error would not, there is less tolerance for Type II errors than for Type I errors.

7.1.4 Standardized Versus Site-Specific Specification of Test Parameter Values

There are tradeoffs between establishing a standard for all decommissioning sites and allowing for Type I and Type II error rates to be established on a site-specific basis. The blanket specification of a low Type I error rate would seem preferable to minimize the number of licensees that might be required to unnecessarily remediate background at their sites. However, such an approach would also require that *all* licensees make a larger number of measurements to achieve this low error rate, which would increase the cost and complexity of final status and confirmatory surveys at *all* sites. If a low Type I error rate were to be standardized, this would mean that all licensees would spend more resources on such surveys to ensure that a smaller number of licensees did not unnecessarily remediate background. Conversely, standardization of a high Type I error rate would mean that a greater number of licensees would perform unnecessary remediation, but the average number of required survey measurements per site would decrease.

The type and extent of radiological contamination requiring remediation at the time of decommissioning will vary widely at NRC-licensed sites. Because these sites are located throughout the United States, background will also vary widely because of its inherent temporal and spatial variability. By establishing very low error rates for all NRC sites, those sites or facilities that contain widespread or complex patterns of radioactive contamination may be required to make an unwarranted number of radiological measurements. For example, a large uranium or thorium processing site that has a highly variable background level could be required to make more radiological measurements than would be accommodated by a realistic decommissioning budget. Due to the diversity of radiological characteristics at NRC sites, specification of standardized statistical parameter values is difficult to justify because it would severely limit the flexibility to account for site-specific factors.

An alternative to applying an NRC-established error rates would be for licensees and the NRC staff to jointly define acceptable error rates on a site-specific basis. In this manner, local radiological conditions and other modifying factors could be taken into account while ensuring that an appropriate level of confidence in the site decontamination was attained. This report recommends that the Type I and Type II error rates be determined on a site-specific basis in order to allow regulatory flexibility in accounting for local radiological conditions. A framework for determining appropriate error rates is discussed in Section 3.

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9 GLOSSARY

Activity: A measure of the rate at which radioactive material is undergoing radioactive decay, usually given in terms of the number of nuclear disintegrations occurring in a given quantity of material over a unit of time. The unit of activity is the curie (Ci) or the becquerel (Bq). Also, known as radioactivity.

Affected Area: Areas that have potential radioactive contamination (based on plant operating history) or known radioactive contamination (based on past or preliminary radiological surveillance). This would normally include areas in which radioactive materials were used and stored, records indicate spills or other unusual occurrences that could have resulted in spread of contamination, and radioactive materials were buried. Areas immediately surrounding or adjacent to locations in which radioactive materials were used or stored, spilled, or buried are included in this classification because of the potential for inadvertent spread of contamination. Affected areas are further divided into those areas that are considered to have a potential for containing small areas of elevated residual activity in excess of guideline levels and those in which such areas of elevated activity would not be anticipated. An area that has the potential for such a spotty residual radioactivity pattern (i.e., affected with potential for non-uniform residual radioactivity) is referred to as **affected/non-uniform.** An area with little or no potential for non-uniform residual radioactivity is referred to as **affected/uniform**. *Any area that has been remediated is designated affected/non-uniform.* In general, *all* areas are treated as affected/non-uniform until substantial bases are provided to reclassify them to either affected/uniform, unaffected, or non-impacted.

ALARA: The licensee must demonstrate that if the site were released for unrestricted use, residual radioactivity at the site that can be distinguished from background radioactivity would not result in a dose to an average member of the critical group exceeding 15 mrem per year (10 CFR 20.1404). The licensee must also demonstrate that the dose is as low as reasonably achievable (ALARA). The evaluation of ALARA should be based on a multi-variant analysis that considers both onsite and offsite radiological and non-radiological risks and evaluates individual and collective dose for both public and worker populations. An expanded discussion of a suggested approach for performing a site-specific ALARA analysis is in Appendix G of NUREG-1500.

Depending on the site-specific ALARA analysis, any dose level less than or equal to 15 mrem per year may be considered ALARA. In certain cases, the Commission will consider that the licensee has complied with the ALARA requirement if the licensee can demonstrate that the total effective dose equivalent (TEDE) to the average member of the critical group from all radionuclides that are distinguishable from background radiation does not exceed 3 mrem (0.03 mSv) per year. Values greater than 3 mrem (0.03 mSv) will also be considered as ALARA if properly supported by an analysis of significant risks and efforts required to further reduce those risks.

Alpha (α): The specified maximum probability of a Type I error, i.e., the maximum probability of rejecting the null hypothesis when it is true. It is the maximum acceptable probability that a statistical test incorrectly indicates that a survey unit does not attain the cleanup standard. Alpha is also called the size of the test.

Alpha Particle: A positively charged particle emitted by some radioactive materials undergoing radioactive decay. Alpha particles are the least penetrating of the three common forms of radiation (alpha, beta, gamma); they can be stopped by a sheet of paper and cannot penetrate skin.

Alternative Hypothesis: See Hypothesis.

Area Factor: If residual radioactivity exists over an area which is smaller than that assumed in the dose assessment models, the derived concentration guideline must be adjusted by an area factor, A_m. In particular, the level set for the elevated measurement, $H_m = \Delta \cdot (\text{area factor})$. Tables of area factors computed using RESRAD 5.6 (Yu et al., 1993) are given in Appendix C. For indoor areas, a similar adjustment must be made. The indoor area factors depend on the size of the room and the dose scenario as well as the spacing between grid points. Again, $H_m = \Delta \cdot (\text{area factor})$. Tables of indoor area factors computed using RESRAD BUILD 1.5 (Yu et al., 1994) for a 36 m^2 room are given in Appendix C.

Arithmetic Mean: The average value obtained as the sum of individual values divided by the number of values.

Arithmetic Standard Deviation: A statistic used to quantify the variability of a set of data. It is calculated by first subtracting the arithmetic mean from each data value. These differences are squared, the squares are summed, and the sum divided by the number of data values less one. Finally, the square root is taken. The calculation process is summarized in the term *R*oot *M*ean *S*quare *D*eviation.

Attainment Objectives: Specifying the design and scope of the sampling study including the radionuclides to be tested, the cleanup standards to be attained, the measure or parameter to be compared to the cleanup standard, and the Type I and Type II error rates for the selected statistical tests.

Background Radiation: Radiation from cosmic sources; naturally occurring radioactive material, including radon (except as a decay product of source or special nuclear material); and global fallout as it exists in the environment from the testing of nuclear explosive devices or from nuclear accidents like Chernobyl which contribute to background radiation and are not under the control of the licensee. Background radiation does not include radiation from source, byproduct, or special nuclear materials regulated by the Commission.

Becquerel: A unit of activity equal to one disintegration per second. Also see Curie.

Beta (β): The probability of a Type II Error, i.e., the probability of accepting the null hypothesis when it is false. β is the specified, allowable (small) probability that a statistical test incorrectly indicates that a survey unit has been successfully remediated.

Beta Particle: An electron emitted from the nucleus during radioactive decay. Beta particles are easily stopped by a thin sheet of metal or plastic.

Biased Sample (or Measurement): Samples (or measurements) taken from a location where radiation levels or other site characteristics are expected to be unusual. Also called judgment sample or authoritative sample. Samples (or measurements) that are *not* biased are considered representative of the site being studied.

Byproduct Material: Any radioactive material (except special nuclear material) created or made radioactive by exposure to the radiation, incident to the process of producing or utilizing special nuclear material.

c: In this document, the proportion of the total number of samples in the reference area and survey unit that are to be taken in the reference area. c is used with the Wilcoxon Rank Sum (WRS) test.

Characterization Survey: Facility or site sampling, monitoring, and analysis activities to determine the extent and nature of contamination. Characterization provides the basis for acquiring the necessary technical information to develop, analyze, and select appropriate cleanup techniques.

Cleanup: Actions taken to remove a hazardous substance that could affect humans or the environment or both. The term "cleanup" is sometimes used interchangeably with the terms remedial action, remediation, and decontamination.

Cleanup Standard: The cleanup standard for the Wilcoxon tests and for the Quantile tests are specific values of statistical parameters. For the WRS test, the standard is $\Delta/\sigma = 0$. For the Quantile test, the standard is $\epsilon = 0$ and $\Delta/\sigma = 0$. Also see Release Criteria.

Cleanup (Survey) Unit: A geographical area of specified size and shape at a remediated site for which a separate decision will be made whether the unit attains the site-specific reference-based cleanup standard for the designated pollution parameter. See Affected Area, Survey Unit.

Composite Sample: A sample formed by collecting several samples and combining them (or selected portions of them) into a new sample which is then thoroughly mixed.

Confidence Interval: An interval for which there is a specified probability of its containing the true value of an estimated parameter.

Confirmatory Survey: limited independent (third-party) measurements, sampling, and analyses to verify the findings of a final status survey.

Contamination: The presence of residual radioactivity, in excess of levels which are acceptable for release of a site or facility for unrestricted use.

Core Sample: A soil sample taken by core drilling.

Criteria (Release Criteria): Combination of numerical activity guideline levels and conditions for their application. If criteria are satisfied, the site may be released without restrictions. See Release Criteria.

Critical Group: the group of individuals reasonably expected to receive the greatest exposure to residual radioactivity for any applicable set of circumstances

Curie: A measure of the rate of radioactive decay. One curie (Ci) is equal to 37 billion disintegrations per second (3.7×10^{10} dis/s = 3.7×10^{10} Bq), which is approximately equal to the decay of one gram of radium-226. Fractions of a curie, e.g., picocurie (pCi) (or 10^{-12} Ci) and microcurie (μCi) (or 10^{-6} Ci), are levels typically encountered in the decommissioning process.

Decay: The spontaneous radioactive transformation of one nuclide into a different nuclide or into a lower energy state of the same nuclide. Also, known as radioactive decay.

Decommission: To remove a facility or site safely from service and reduce residual radioactivity to a level that permits (1) release of the property for unrestricted use and termination of the license or (2) release of the property under restricted conditions and termination of the license.

Decommissioning: The process of removing a facility from operation, followed by decontamination, and license termination.

The objective of decommissioning is to reduce the residual radioactivity in structures, materials, soils, groundwater, and other media at the site so that the concentration of each radionuclide that could contribute to residual radioactivity is indistinguishable from the background radiation concentration for that radionuclide. The Commission realizes that, as a practical matter, it would be extremely difficult to demonstrate that such an objective has been met. Therefore, the Commission has established a site release limit and is requiring that licensees demonstrate that the residual radioactivity at a site is as far below this limit as reasonably achievable. (10 CFR 20.1402)

Decontamination: The removal of radiological contaminants from, or their neutralization on, a person, object or area to within levels established by governing regulatory agencies. Also, known as remediation, remedial action, and cleanup.

Delta (Δ): The amount that the distribution of measurements for a survey unit is shifted to the right of the distribution of measurements of the reference area. Δ divided by σ, the standard deviation of the measurements, is the shift expressed in multiples of standard deviations.

Derived Guidelines: Levels of radioactivity presented in terms of ambient radiation, surface activity levels, and soil activity concentrations; these levels are derived from activity/dose relationships through various exposure pathway scenarios. Also known as guidelines. Use of such are described in NUREG-1500.

Design Specification Process: The process of determining the sampling and analysis procedures that are needed to demonstrate that the attainment objectives have been achieved.

Detection Sensitivity: The ability to identify the presence of radiation or radioactivity.

Direct Measurement: Radioactivity measurement obtained by placing the detector against the surface or in the media being surveyed. The resulting radioactivity level is read out directly.

Dose Commitment: The dose that an organ or tissue would receive during a specified period of time (e.g., 50 or 70 years) as a result of intake (as by ingestion or inhalation) of one or more radionuclides from a given release.

Dose Equivalent (Dose): A quantity that expresses all radiations on a common scale for calculating the effective absorbed dose. It is the product of absorbed dose (rads) multiplied by a quality factor and any other modifying factors. It is measured in rem (roentgen equivalent man).

DQA (Data Quality Assessment): Data Quality Assessment (DQA) is the scientific and statistical evaluation of data to determine if the data are of the right type, quality, and quantity to support their intended use.

DQOs (Data Quality Objectives): Qualitative and quantitative statements that specify the type, quantity, and quality of data that are required for the specified objective.

Elevated Area: An area over which residual radioactivity exceeds a specified value H_m.

Elevated Measurement: A measurement that exceeds a specified value H_m.

Elevated Measurement Comparison: This comparison is used in conjunction with both the Wilcoxon Rank Sum test and the Quantile test to determine if there are any measurements that exceed a specified value H_m.

Epsilon (ϵ): The proportion of soil in a survey unit that has not been remediated to the reference-based cleanup standard. ϵ is used in the Quantile tests.

Exposure Pathway: The route by which radioactivity travels through the environment to eventually cause a radiation exposure to a person or group.

Exposure Rate: The amount of ionization produced per unit time in air by x-rays or gamma rays. The unit of exposure rate is roentgens per hour (R/h); for decommissioning activities the typical units are microroentgens per hour (μR/h), i.e. 10^{-6} R/h.

External Radiation: Radiation from a source outside the body.

Final Status Survey: Measurements and sampling to describe the radiological conditions of a site, following completion of decontamination activities (if any) and in preparation for unrestricted release.

Gamma Radiation: Penetrating high-energy, short-wavelength electromagnetic radiation (similar to x-rays) emitted during radioactive decay. Gamma rays are very penetrating and require dense materials (such as lead or uranium) for shielding.

Grid: A network of parallel horizontal and vertical lines forming squares on a map that may be overlaid on a property parcel for the purpose of identification of exact locations. Also, known as reference grid system.

Grid Block: A square defined by two adjacent vertical and two adjacent horizontal grid lines.

h: The number of survey units that will be compared to a specified reference area.

Half-Life: The time required for one-half of the atoms present to disintegrate.

H_m: A concentration value such that any measurement from the survey unit that is larger than H_m indicates that an area of residual radioactivity may exist that would result in a dose above guideline levels.

Hot Measurement: See Elevated Measurement.

Hot Spot: See Elevated Area.

Hypothesis: An assumption about a property or characteristic of a population under study. The goal of statistical inference is to decide which of two complementary hypotheses is likely to be true. The null hypothesis is that the survey unit has been successfully remediated and the alternative hypothesis is that the survey unit has not been successfully remediated.

Indistinguishable From Background: The term "indistinguishable from background" means that the detectable concentration distribution of a radionuclide is not statistically different from the background concentration distribution of that radionuclide in the vicinity of the site or, in the case of structures, in similar materials using adequate measurement technology, survey, and statistical techniques.

Indistinguishable From Background Criteria: To apply the "indistinguishable from background" criteria, the concentration of individual radionuclides comprising the residual radioactivity at a site are compared to the concentration of those same radionuclides present in local background areas that have been matched to the site in terms of geological, chemical, and biological attributes, but which have not been affected by site operations. This comparison establishes a site-specific criterion for individual radionuclides that is dependent on the local variability of background. The distribution of residual radioactivity that is measured in affected areas on site is compared to the distribution of background radionuclides measured in unaffected areas (reference areas), with compliance dependent on the distributions being statistically indistinguishable. The implementation of these criteria will vary depending on the background level for all radionuclides at the site, the temporal and spatial variations in background at the site, and the radionuclides under investigation (NUREG-1500).

Inventory: Total residual quantity of formerly licensed radioactive material at a site.

k: When conducting the Quantile test, *k* is the number of measurements from the survey unit that are among the *r* largest measurements of the combined set of reference area and cleanup unit measurements.

Less-Than Data: Measurements that are less than the lower limit of detection.

License: A license issued under the regulations in Parts 30 through 35, 39, 40, 60, 61, 70 or Part 72 of 10 CFR Chapter I. Licensee means the holder of such license.

License Termination: Discontinuation of a license, the eventual conclusion to decommissioning.

Lower Limit of Detection, L_D: The smallest amount of radiation or radioactivity that statistically yields a net result above the method background. The critical detection level, L_C, is the lower bound on the 95-percent detection interval defined for L_D and is the level at which there is a 5-percent chance of calling a background value "greater than background." This value should be used when actually counting samples or making direct radiation measurements. Any response above this level should be considered as above background, i.e, a net positive result. This will ensure 95-percent detection capability for L_D. A 95-percent confidence interval should be calculated for all responses greater than L_C.

m: The number of measurements required from the reference area to conduct a statistical test with specified Type I and Type II error rates.

Minimum Detectable Concentration (MDC):. The minimum detectable concentration (MDC) is the *a priori* activity level that a specific instrument and technique can be expected to detect 95 percent of the time. When stating the detection capability of an instrument, this value should be used. The MDC is the detection limit, L_D, multiplied by an appropriate conversion factor to give units of activity.

Missing or Unusable Data: Data (measurements) that are mislabeled, lost, or do not meet quality control standards. "Less-than" data are not considered to be missing or unusable data. See R.

Multiple Comparison Test: A test constructed so that the Type I error rate for a group of individual tests does not exceed a specific alpha level.

N: N = m + n, is the total number of measurements required from the reference area and a cleanup unit being compared with the reference area. See m and n.

n: The number of measurements required from a survey unit to conduct a statistical test that has specified Type I and Type II error rates.

n_f: The number of samples that should be collected in an area to assure that the required number of measurements from that area for conducting statistical tests is obtained. $n_f = n/(1-R)$.

Naturally Occurring Radionuclides: Radionuclides and their associated progeny produced during the formation of the earth or by interactions of terrestrial matter with cosmic rays.

Nonparametric Test: A test based on relatively few assumptions about the exact form of the underlying probability distributions of the measurements. As a consequence, nonparametric tests are generally valid for a fairly broad class of distributions. The Wilcoxon tests and the Quantile tests are nonparametric tests.

Normal (Gaussian) Distribution: A family of bell-shaped distributions described by the mean and variance.

Outlier: Measurements that are unusually large relative to the bulk of the measurements in the data set.

p: The probability that a random measurement from the survey unit is less than Δ.

p′: The probability that the sum of two independent random measurements from the survey unit is less than 2Δ.

P_r: The probability that a measurement of a sample collected at a random location in the survey unit is greater than a measurement of a sample collected at a random location in the reference area.

Pitman Efficiency: A measure of performance for statistical tests. It is equal to the reciprocal of the ratio of the sample sizes required by each of two tests to achieve the same power, as these sample sizes become large.

Power (1 -β): The probability of rejecting the null hypothesis when it is false. The power is equal to one minus the Type II error rate, i.e. $(1-\beta)$. The power of a test is the probability the test will correctly indicate when a survey unit has not been successfully remediated.

Quality Assurance/Quality Control: A system of procedures, checks, audits, and corrective actions to ensure that design, performance, monitoring and sampling, and other technical and reporting activities are of sufficient quality to satisfy the objective for which they are undertaken.

Quantile Test: A nonparametric test that looks at only the *r* largest measurements of the N combined reference area and survey unit measurements. If a sufficiently large number of these *r* measurements are from the survey unit, then the test indicates the survey unit has not attained the reference-based cleanup standard.

R: In this report, the rate of missing or unusable measurements expected to occur for samples collected in reference areas or survey units. See Missing or Unusable Data. See n_f. (Not to be confused with the symbol for the radiation exposure unit, roentgen.)

Radiological Survey: Measurements of radiation levels associated with a site together with appropriate documentation and data evaluation.

Radionuclide: An unstable nuclide that undergoes radioactive decay.

Readily Removable: Removable using nondestructive, common, housekeeping techniques (e.g., washing with moderate amounts of detergent and water) that do not generate large volumes of radioactive waste requiring subsequent disposal or produce chemical wastes that are expected to adversely affect public health or the environment.

Reference Areas: Geographical areas from which representative reference samples will be selected for comparison with samples collected in specific survey units at the remediated site. A site radiological reference area (background area) is defined as an area that has similar physical, chemical, radiological, and biological characteristics as the site area being remediated, but which has not been contaminated by site activities. The distribution and concentration of background radiation in the reference area should be the same as what would be expected on the site if that site had never been contaminated. It may be necessary to select more than one reference area for a specific site, if the site includes so much physical, chemical, radiological, or biological variability that it cannot be represented by a single reference background area.

Reference Region: The geographical region from which reference areas will be selected for comparison with survey units.

Release Criteria: A site will be considered acceptable for unrestricted use (10 CFR 20.1404) if
(i) the residual radioactivity that is distinguishable from background radiation results in a TEDE to the average member of the critical group that does not exceed 15 mrem (0.15 mSv) per year; and
(ii) the residual radioactivity has been reduced to levels that are as low as reasonably achievable (ALARA).

Release Limit: The *limit* for release of a site is 15 mrem per year (0.15 mSv/y) TEDE for residual radioactivity distinguishable from background. If doses from residual radioactivity are less than 15 mrem per year TEDE, the Commission will terminate the license and authorize release of the site for unrestricted use following the licensee's demonstration that the residual radioactivity at the site is ALARA.

REM (Roentgen Equivalent Man): Unit of dose equivalent; that quantity of type of ionizing radiation that, when absorbed by humans, produces the equivalent specific biological effect to that produced by one rad of 250 keV x-rays.

Remediation: The removal of contamination from a site. Also known as remedial action and decontamination.

Remediation Control Survey: Monitoring the progress of remedial action by real time measurement of areas being decontaminate to determine whether efforts are being effective and to guide further decontamination activities.

Removable Activity: Surface activity that can be removed and collected for measurement by wiping the surface with moderate pressure.

Representative Measurement A measurement that is selected using a procedure in such a way that it, in combination with other representative measurements, will give an accurate representation of the phenomenon being studied.

Residual Radioactivity: Radioactivity in structures, materials, soils, groundwater, and other media at a site resulting from activities under the licensee's control. This includes radioactivity from all licensed and unlicensed sources used by the licensee, but excludes background radiation. It also includes radioactive materials remaining at the site as a result of routine or accidental releases of radioactive material at the site and previous burials at the site, even if those burials were made in accordance with the provisions of 10 CFR Part 20.

Restoration: Actions to return a remediated area to a usable state, following decontamination.

Restricted Use: A designation following remediation requiring radiological controls at a formerly licensed site.

Roentgen: Unit of exposure. One roentgen is the amount of gamma rays or x-rays required to produce one electrostatic unit (esu) of charge of one sign (either positive or negative) in one cubic centimeter of dry air under standard conditions. Equal to 2.58×10^{-4} C/kg of charge in air.

Scanning: An evaluation technique performed by moving a detection device over the surface at some consistent speed and distance above the surface to detect elevated levels of radiation.

Shape Parameter, S (of an Elliptical Hot Spot): The ratio of the semi-minor axis length to the semi-major axis length. For a circle, the shape parameter is one. A small shape parameter corresponds to a flat ellipse.

Shift (Δ): See Delta.

Size of a Test: See Alpha.

Scoping Survey: A survey that is conducted to identify which radionuclides are present as contaminants, the relative ratios in which they occur, and the general levels and extent of the contamination.

Soil Activity (Soil Concentration): The level of radioactivity present in soil and expressed in units of activity per soil mass (typically, picocuries per gram (pCi/g)).

Source Material: Uranium or thorium or both, other than that classified as special nuclear material.

Source Term: The source term consists of all residual radioactivity remaining at the site, including material released during normal operations and during inadvertent releases or accidents;

radioactive materials which may have been buried at the site in accordance with 10 CFR Part 20 are also included.

Special Nuclear Material: Plutonium, U-233, and uranium enriched in U-235. Special nuclear material is generally considered material capable of undergoing a fission reaction.

Standard Normal Distribution: A normal (Gaussian) distribution with mean zero and variance one.

Subsurface Soil Sample: A soil sample taken deeper than 15 cm below the soil surface.

Surface Contamination: Residual radioactivity found on building or equipment surfaces and expressed in units of activity per surface area, typically disintegrations per minute per 100 cm^2.

Surface Soil Sample: A soil sample taken from the first 15 cm of surface soil

Survey: Evaluation of a representative portion of a population to develop conclusions regarding the population as a whole. In the decommissioning process several different types of surveys are conducted, including background, scoping, characterization, remediation control, final status, and confirmatory.

Survey Plan: A plan for determining the radiological characteristics of a site.

Survey Unit: A geographical area of specified size and shape at a remediated site for which a separate decision will be made whether the unit attains the site-specific reference-based cleanup standard for the designated pollution parameter. Survey units are generally formed by grouping contiguous site areas with a similar use history and the same classification of contamination potential. Survey units are established to facilitate the survey process and the statistical analysis of survey data.

Tandem Testing: When two or more statistical tests are conducted using the same data set.

TEDE (Total Effective Dose Equivalent): The effective dose equivalent is the summation of the products of the dose equivalent received by specified tissues of the body and a tissue-specific weighting factor. It is a risk-equivalent value, expressed in rem, that can be used to estimate the health effects on an exposed individual.

When calculating TEDE, the licensee should base estimates on the greatest annual TEDE dose expected within the first 1000 years after decommissioning. Estimates must be substantiated using actual measurements to the maximum extent practical.

Tied Measurements: Two or more measurements that have the same value.

Triangular Sampling Grid: A grid of sampling locations that is arranged in a triangular pattern.

Two-Sample *t*-Test: A test described in most statistics books that may be used in place of the Wilcoxon Rank Sum test if the reference area and cleanup unit measurements are known to be normally (Gaussian) distributed and there are no "less than" measurements in either data set.

Unaffected Area: Any area that is not expected to contain any residual radioactivity, based on a knowledge of site history and previous survey information.

Unrestricted Area: Any area to which access is not controlled by the licensee for purposes of protecting individuals from exposure to radiation and radioactive materials, and any area used for residential quarters.

Unrestricted Release: Use of a former radioactive materials site without requirements for future radiological controls. Also, known as unrestricted use.

Wilcoxon Rank Sum (WRS) Test: The nonparametric test used to detect when the remedial action has failed more or less uniformly throughout the survey unit to achieve the reference-based cleanup standard.

$Z_{1-\phi}$: the value from the standard normal distribution that cuts off 100 ϕ percent of the upper tail of the standard normal distribution. See Standard Normal Distribution.

APPENDIX A: STATISTICAL TABLES

A.1 Critical Values for the WRS Test

Table A-1 Critical Values for the WRS test

	m=	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
n=2	α=0.001	7	9	11	13	15	17	19	21	23	25	27	29	31	33	35	37	39	41	43
	α=0.005	7	9	11	13	15	17	19	21	23	25	27	29	31	33	35	37	39	40	42
	α=0.01	7	9	11	13	15	17	19	21	23	25	27	28	30	32	34	36	38	39	41
	α=0.025	7	9	11	13	15	17	18	20	22	23	25	27	29	31	33	34	36	38	40
	α=0.05	7	9	11	12	14	16	17	19	21	23	24	26	27	29	31	33	34	36	38
	α=0.1	7	8	10	11	13	15	16	18	19	21	22	24	26	27	29	30	32	33	35

	m=	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
n=3	α=0.001	12	15	18	21	24	27	30	33	36	39	42	45	48	51	54	56	59	62	65
	α=0.005	12	15	18	21	24	27	30	32	35	38	40	43	46	48	51	54	57	59	62
	α=0.01	12	15	18	21	24	26	29	31	34	37	39	42	45	47	50	52	55	58	60
	α=0.025	12	15	18	20	22	25	27	30	32	35	37	40	42	45	47	50	52	55	57
	α=0.05	12	14	17	19	21	24	26	28	31	33	36	38	40	43	45	47	50	52	54
	α=0.1	11	13	16	18	20	22	24	27	29	31	33	35	37	40	42	44	46	48	50

	m=	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
n=4	α=0.001	18	22	26	30	34	38	42	46	49	53	57	60	64	68	71	75	78	82	86
	α=0.005	18	22	26	30	33	37	40	44	47	51	54	58	61	64	68	71	75	78	81
	α=0.01	18	22	26	29	32	36	39	42	46	49	52	56	59	62	66	69	72	76	79
	α=0.025	18	22	25	28	31	34	37	41	44	47	50	53	56	59	62	66	69	72	75
	α=0.05	18	21	24	27	30	33	36	39	42	45	48	51	54	57	59	62	65	68	71
	α=0.1	17	20	22	25	28	31	34	36	39	42	45	48	50	53	56	59	61	64	67

	m=	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
n=5	α=0.001	25	30	35	40	45	50	54	58	63	67	72	76	81	85	89	94	98	102	107
	α=0.005	25	30	35	39	43	48	52	56	60	64	68	72	77	81	85	89	93	97	101
	α=0.01	25	30	34	38	42	46	50	54	58	62	66	70	74	78	82	86	90	94	98
	α=0.025	25	29	33	37	41	44	48	52	56	60	63	67	71	75	79	82	86	90	94
	α=0.05	24	28	32	35	39	43	46	50	53	57	61	64	68	71	75	79	82	86	89
	α=0.1	23	27	30	34	37	41	44	47	51	54	57	61	64	67	71	74	77	81	84

	m=	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
n=6	α=0.001	33	39	45	51	57	63	67	72	77	82	88	93	98	103	108	113	118	123	128
	α=0.005	33	39	44	49	54	59	64	69	74	79	83	88	93	98	103	107	112	117	122
	α=0.01	33	39	43	48	53	58	62	67	72	77	81	86	91	95	100	104	109	114	118
	α=0.025	33	37	42	47	51	56	60	64	69	73	78	82	87	91	95	100	104	109	113
	α=0.05	32	36	41	45	49	54	58	62	66	70	75	79	83	87	91	96	100	104	108
	α=0.1	31	35	39	43	47	51	55	59	63	67	71	75	79	83	87	91	94	98	102

Critical Values for the WRS test

	m=	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
n=7	α=0.001	42	49	56	63	69	75	81	87	92	98	104	110	116	122	128	133	139	145	151
	α=0.005	42	49	55	61	66	72	77	83	88	94	99	105	110	116	121	127	132	138	143
	α=0.01	42	48	54	59	65	70	76	81	86	92	97	102	108	113	118	123	129	134	139
	α=0.025	42	47	52	57	63	68	73	78	83	88	93	98	103	108	113	118	123	128	133
	α=0.05	41	46	51	56	61	65	70	75	80	85	90	94	99	104	109	113	118	123	128
	α=0.1	40	44	49	54	58	63	67	72	76	81	85	90	94	99	103	108	112	117	121

	m=	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
n=8	α=0.001	52	60	68	75	82	89	95	102	109	115	122	128	135	141	148	154	161	167	174
	α=0.005	52	60	66	73	79	85	92	98	104	110	116	122	129	135	141	147	153	159	165
	α=0.01	52	59	65	71	77	84	90	96	102	108	114	120	125	131	137	143	149	155	161
	α=0.025	51	57	63	69	75	81	86	92	98	104	109	115	121	126	132	137	143	149	154
	α=0.05	50	56	62	67	73	78	84	89	95	100	105	111	116	122	127	132	138	143	148
	α=0.1	49	54	60	65	70	75	80	85	91	96	101	106	111	116	121	126	131	136	141

	m=	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
n=9	α=0.001	63	72	81	88	96	104	111	118	126	133	140	147	155	162	169	176	183	190	198
	α=0.005	63	71	79	86	93	100	107	114	121	127	134	141	148	155	161	168	175	182	188
	α=0.01	63	70	77	84	91	98	105	111	118	125	131	138	144	151	157	164	170	177	184
	α=0.025	62	69	76	82	88	95	101	108	114	120	126	133	139	145	151	158	164	170	176
	α=0.05	61	67	74	80	86	92	98	104	110	116	122	128	134	140	146	152	158	164	170
	α=0.1	60	66	71	77	83	89	94	100	106	112	117	123	129	134	140	145	151	157	162

	m=	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
n=10	α=0.001	75	85	94	103	111	119	128	136	144	152	160	167	175	183	191	199	207	215	222
	α=0.005	75	84	92	100	108	115	123	131	138	146	153	160	168	175	183	190	197	205	212
	α=0.01	75	83	91	98	106	113	121	128	135	142	150	157	164	171	178	186	193	200	207
	α=0.025	74	81	89	96	103	110	117	124	131	138	145	151	158	165	172	179	186	192	199
	α=0.05	73	80	87	93	100	107	114	120	127	133	140	147	153	160	166	173	179	186	192
	α=0.1	71	78	84	91	97	103	110	116	122	128	135	141	147	153	160	166	172	178	184

	m=	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
n=11	α=0.001	88	99	109	118	127	136	145	154	163	171	180	188	197	206	214	223	231	240	248
	α=0.005	88	98	107	115	124	132	140	148	157	165	173	181	189	197	205	213	221	229	237
	α=0.01	88	97	105	113	122	130	138	146	153	161	169	177	185	193	200	208	216	224	232
	α=0.025	87	95	103	111	118	126	134	141	149	156	164	171	179	186	194	201	208	216	223
	α=0.05	86	93	101	108	115	123	130	137	144	152	159	166	173	180	187	195	202	209	216
	α=0.1	84	91	98	105	112	119	126	133	139	146	153	160	167	173	180	187	194	201	207

	m=	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
n=12	α=0.001	102	114	125	135	145	154	164	173	183	192	202	210	220	230	238	247	256	266	275
	α=0.005	102	112	122	131	140	149	158	167	176	185	194	202	211	220	228	237	246	254	263
	α=0.01	102	111	120	129	138	147	156	164	173	181	190	198	207	215	223	232	240	249	257
	α=0.025	100	109	118	126	135	143	151	159	168	176	184	192	200	208	216	224	232	240	248
	α=0.05	99	108	116	124	132	140	147	155	165	171	179	186	194	202	209	217	225	233	240
	α=0.1	97	105	113	120	128	135	143	150	158	165	172	180	187	194	202	209	216	224	231

	m=	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
n=13	α=0.001	117	130	141	152	163	173	183	193	203	213	223	233	243	253	263	273	282	292	302
	α=0.005	117	128	139	148	158	168	177	187	196	206	215	225	234	243	253	262	271	280	290
	α=0.01	116	127	137	146	156	165	174	184	193	202	211	220	229	238	247	256	265	274	283
	α=0.025	115	125	134	143	152	161	170	179	187	196	205	214	222	231	239	248	257	265	274
	α=0.05	114	123	132	140	149	157	166	174	183	191	199	208	216	224	233	241	249	257	266
	α=0.1	112	120	129	137	145	153	161	169	177	185	193	201	209	217	224	232	240	248	256

	m=	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
n=14	α=0.001	133	147	159	171	182	193	204	215	225	236	247	257	268	278	289	299	310	320	330
	α=0.005	133	145	156	167	177	187	198	208	218	228	238	248	258	268	278	288	298	307	317
	α=0.01	132	144	154	164	175	185	194	204	214	224	234	243	253	263	272	282	291	301	311
	α=0.025	131	141	151	161	171	180	190	199	208	218	227	236	245	255	264	273	282	292	301
	α=0.05	129	139	149	158	167	176	185	194	203	212	221	230	239	248	257	265	274	283	292
	α=0.1	128	136	145	154	163	171	180	189	197	206	214	223	231	240	248	257	265	273	282

	m=	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
n=15	α=0.001	150	165	178	190	202	212	225	237	248	260	271	282	293	304	316	327	338	349	360
	α=0.005	150	162	174	186	197	208	219	230	240	251	262	272	283	293	304	314	325	335	346
	α=0.01	149	161	172	183	194	205	215	226	236	247	257	267	278	288	298	308	319	329	339
	α=0.025	148	159	169	180	190	200	210	220	230	240	250	260	270	280	289	299	309	319	329
	α=0.05	146	157	167	176	186	196	206	215	225	234	244	253	263	272	282	291	301	310	319
	α=0.1	144	154	163	172	182	191	200	209	218	227	236	246	255	264	273	282	291	300	309

	m=	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
n=16	α=0.001	168	184	197	210	223	236	248	260	272	284	296	308	320	332	343	355	367	379	390
	α=0.005	168	181	194	206	218	229	241	252	264	275	286	298	309	320	331	342	353	365	376
	α=0.01	167	180	192	203	215	226	237	248	259	270	281	292	303	314	325	336	347	357	368
	α=0.025	166	177	188	200	210	221	232	242	253	264	274	284	295	305	316	326	337	347	357
	α=0.05	164	175	185	196	206	217	227	237	247	257	267	278	288	298	308	318	328	338	348
	α=0.1	162	172	182	192	202	211	221	231	241	250	260	269	279	289	298	308	317	327	336

	m=	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
n=17	α=0.001	187	203	218	232	245	258	271	284	297	310	322	335	347	360	372	384	397	409	422
	α=0.005	187	201	214	227	239	252	264	276	288	300	312	324	336	347	359	371	383	394	406
	α=0.01	186	199	212	224	236	248	260	272	284	295	307	318	330	341	353	364	376	387	399
	α=0.025	184	197	209	220	232	243	254	266	277	288	299	310	321	332	343	354	365	376	387
	α=0.05	183	194	205	217	228	238	249	260	271	282	292	303	313	324	335	345	356	366	377
	α=0.1	180	191	202	212	223	233	243	253	264	274	284	294	305	315	325	335	345	355	365

	m=	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
n=18	α=0.001	207	224	239	254	268	282	296	309	323	336	349	362	376	389	402	415	428	441	454
	α=0.005	207	222	236	249	262	275	288	301	313	326	339	351	364	376	388	401	413	425	438
	α=0.01	206	220	233	246	259	272	284	296	309	321	333	345	357	370	382	394	406	418	430
	α=0.025	204	217	230	242	254	266	278	290	302	313	325	337	348	360	372	383	395	406	418
	α=0.05	202	215	226	238	250	261	273	284	295	307	318	329	340	352	363	374	385	396	407
	α=0.1	200	211	222	233	244	255	266	277	288	299	309	320	331	342	352	363	374	384	395

Critical Values for the WRS test

	m=	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
n=19	α=0.001	228	246	262	277	292	307	321	335	350	364	377	391	405	419	433	446	460	473	487
	α=0.005	227	243	258	272	286	300	313	327	340	353	366	379	392	405	419	431	444	457	470
	α=0.01	226	242	256	269	283	296	309	322	335	348	361	373	386	399	411	424	437	449	462
	α=0.025	225	239	252	265	278	290	303	315	327	340	352	364	377	389	401	413	425	437	450
	α=0.05	223	236	248	261	273	285	297	309	321	333	345	356	368	380	392	403	415	427	439
	α=0.1	220	232	244	256	267	279	290	302	313	325	336	347	358	370	381	392	403	415	426

	m=	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
n=20	α=0.001	250	269	286	302	317	333	348	363	377	392	407	421	435	450	464	479	493	507	521
	α=0.005	249	266	281	296	311	325	339	353	367	381	395	409	422	436	450	463	477	490	504
	α=0.01	248	264	279	293	307	321	335	349	362	376	389	402	416	429	442	456	469	482	495
	α=0.025	247	261	275	289	302	315	329	341	354	367	380	393	406	419	431	444	457	470	482
	α=0.05	245	258	271	284	297	310	322	335	347	360	372	385	397	409	422	434	446	459	471
	α=0.1	242	254	267	279	291	303	315	327	339	351	363	375	387	399	410	422	434	446	458

Reject the null hypothesis if WRS is greater than the table (critical) value.
For *n* or *m* greater than 20, the table (critical) value can be calculated from:

$$n(n+m+1)/2 \; + \; z\sqrt{nm(n+m+1)/12} \qquad \text{(A-1)}$$

if there are few or no ties, and from

$$n(n+m+1)/2 \; + \; z\sqrt{\frac{nm}{12}\left[(n+m+1) - \sum_{j=1}^{g}\frac{t_j(t_j^2-1)}{(n+m)(n+m-1)}\right]} \qquad \text{(A-2)}$$

if there are many ties, where g is the number of groups of tied measurements and t_j is the number of tied measurements in the jth group. z is the (1-α) percentile of a standard normal distribution, which can be found in the following table:

α	z
0.001	3.09
0.005	2.575
0.01	2.326
0.025	1.960
0.05	1.645
0.1	1.282

A.2 Power of the WRS Test

The table in this section provides values for the approximate power (1-β) of the Wilcoxon Rank Sum test when there are equal numbers of measurements in the reference area (**m**) and in the Survey Unit (**n**). These values correspond to the probability that the WRS Test will correctly reject the null hypothesis that decontamination criteria is met when there is residual contamination Δ/σ above background over 100ϵ percent of the survey unit. The approximate power is given for four values of α (0.01, 0.025, 0.05, and 0.1). This table was constructed from Tables A.2-A.5 in PNL-7409.

Table A-2 Approximate Power of the WRS Test

					Δ/σ				
	ϵ	**0.5**	**1**	**1.5**	**2**	**2.5**	**3**	**3.5**	**4**
m = 10	0.1	0.014	0.016	0.02	0.019	0.02	0.022	0.025	0.019
n = 10	0.2	0.016	0.025	0.03	0.043	0.047	0.05	0.049	0.051
	0.3	0.021	0.037	0.053	0.078	0.093	0.101	0.106	0.107
	0.4	0.026	0.052	0.099	0.132	0.165	0.185	0.197	0.196
α = 0.01	0.5	0.033	0.081	0.152	0.22	0.274	0.316	0.327	0.334
	0.6	0.039	0.118	0.234	0.333	0.438	0.486	0.499	0.514
	0.7	0.052	0.165	0.327	0.505	0.604	0.666	0.691	0.7
	0.8	0.058	0.212	0.458	0.676	0.79	0.835	0.865	0.873
	0.9	0.073	0.28	0.596	0.823	0.926	0.959	0.998	0.973
	1	0.089	0.38	0.751	0.946	0.995	1	1	1

					Δ/σ				
	ϵ	**0.5**	**1**	**1.5**	**2**	**2.5**	**3**	**3.5**	**4**
m = 15	0.1	0.012	0.017	0.021	0.022	0.029	0.027	0.026	0.027
n = 15	0.2	0.016	0.03	0.042	0.056	0.066	0.071	0.072	0.078
	0.3	0.024	0.049	0.089	0.12	0.144	0.158	0.17	0.166
	0.4	0.032	0.08	0.152	0.213	0.274	0.294	0.315	0.321
α = 0.01	0.5	0.042	0.123	0.251	0.356	0.442	0.495	0.514	0.525
	0.6	0.058	0.183	0.374	0.533	0.644	0.703	0.715	0.734
	0.7	0.071	0.258	0.512	0.722	0.825	0.868	0.885	0.9
	0.8	0.091	0.352	0.683	0.878	0.946	0.968	0.975	0.976
	0.9	0.112	0.457	0.821	0.968	0.993	0.998	0.999	1
	1	0.144	0.574	0.924	0.997	1	1	1	1

					Δ/σ				
	ϵ	**0.5**	**1**	**1.5**	**2**	**2.5**	**3**	**3.5**	**4**
m = 20	0.1	0.014	0.017	0.025	0.03	0.032	0.032	0.037	0.037
n = 20	0.2	0.018	0.036	0.055	0.076	0.086	0.096	0.105	0.1
	0.3	0.03	0.065	0.119	0.165	0.204	0.228	0.237	0.248
	0.4	0.04	0.109	0.221	0.314	0.377	0.42	0.432	0.449
α = 0.01	0.5	0.055	0.179	0.357	0.499	0.6	0.646	0.672	0.679
	0.6	0.074	0.259	0.511	0.704	0.802	0.838	0.859	0.867
	0.7	0.094	0.358	0.694	0.871	0.932	0.959	0.962	0.967
	0.8	0.123	0.483	0.838	0.958	0.98.8	0.995	0.996	0.997
	0.9	0.163	0.617	0.937	0.994	1	1	1	1
	1	0.194	0.741	0.983	1	1	1	1	1

Table A.2 (continued) Power of the WRS Test (α=0.01)

					Δ/σ				
	ε	**0.5**	**1**	**1.5**	**2**	**2.5**	**3**	**3.5**	**4**
m = 25	0.1	0.017	0.022	0.028	0.037	0.038	0.037	0.038	0.039
n = 25	0.2	0.022	0.046	0.069	0.096	0.113	0.12	0.129	0.123
	0.3	0.033	0.083	0.15	0.218	0.262	0.297	0.313	0.307
	0.4	0.047	0.138	0.277	0.404	0.481	0.538	0.557	0.559
α = 0.01	0.5	0.069	0.229	0.448	0.62	0.722	0.761	0.791	0.796
	0.6	0.088	0.338	0.639	0.82	0.889	0.923	0.937	0.94
	0.7	0.126	0.469	0.804	0.935	0.976	0.989	0.991	0.991
	0.8	0.153	0.616	0.92	0.99	0.997	0.999	0.999	1
	0.9	0.207	0.738	0.977	0.999	1	1	1	1
	1	0.262	0.841	0.996	1	1	1	1	1
					Δ/σ				
	ε	**0.5**	**1**	**1.5**	**2**	**2.5**	**3**	**3.5**	**4**
m = 30	0.1	0.018	0.022	0.033	0.038	0.038	0.042	0.049	0.045
n = 30	0.2	0.023	0.05	0.075	0.104	0.134	0.143	0.149	0.151
	0.3	0.036	0.097	0.173	0.26	0.32	0.355	0.361	0.362
	0.4	0.054	0.165	0.335	0.476	0.563	0.607	0.637	0.643
α = 0.01	0.5	0.079	0.28	0.527	0.714	0.795	0.836	0.863	0.869
	0.6	0.106	0.401	0.719	0.884	0.948	0.962	0.971	0.971
	0.7	0.145	0.552	0.875	0.973	0.992	0.996	0.998	0.998
	0.8	0.182	0.696	0.962	0.997	0.999	1	1	1
	0.9	0.248	0.822	0.993	1	1	1	1	1
	1	0.31	0.908	1	1	1	1	1	1
					Δ/σ				
	ε	**0.5**	**1**	**1.5**	**2**	**2.5**	**3**	**3.5**	**4**
m = 40	0.1	0.018	0.024	0.037	0.044	0.052	0.058	0.054	0.057
n = 40	0.2	0.029	0.058	0.109	0.147	0.189	0.192	0.21	0.209
	0.3	0.046	0.131	0.255	0.356	0.422	0.474	0.485	0.497
	0.4	0.071	0.24	0.451	0.619	0.718	0.76	0.784	0.787
α = 0.01	0.5	0.101	0.376	0.68	0.853	0.909	0.94	0.95	0.95
	0.6	0.141	0.542	0.858	0.965	0.988	0.994	0.994	0.995
	0.7	0.197	0.693	0.957	0.996	0.999	1	1	1
	0.8	0.262	0.836	0.994	1	1	1	1	1
	0.9	0.335	0.93	1	1	1	1	1	1
	1	0.423	0.975	1	1	1	1	1	1
					Δ/σ				
	ε	**0.5**	**1**	**1.5**	**2**	**2.5**	**3**	**3.5**	**4**
m = 50	0.1	0.018	0.03	0.043	0.051	0.082	0.085	0.068	0.068
n = 50	0.2	0.33	0.73	0.133	0.19	0.229	0.25	0.261	0.261
	0.3	0.053	0.162	0.311	0.44	0.531	0.579	0.595	0.607
	0.4	0.08	0.299	0.566	0.729	0.819	0.861	0.872	0.882
α = 0.01	0.5	0.126	0.458	0.787	0.926	0.963	0.979	0.984	0.985
	0.6	0.18	0.648	0.934	0.988	0.997	0.999	0.999	0.999
	0.7	0.254	0.81	0.986	1	1	1	1	1
	0.8	0.336	0.92	0.998	1	1	1	1	1
	0.9	0.429	0.975	1	1	1	1	1	1
	1	0.521	0.993	1	1	1	1	1	1

Table A.2 (continued) Power of the WRS Test (α=0.01)

					Δ/σ				
	ε	**0.5**	**1**	**1.5**	**2**	**2.5**	**3**	**3.5**	**4**
m = 60	0.1	0.019	0.033	0.048	0.061	0.072	0.074	0.078	0.082
n = 60	0.2	0.032	0.095	0.16	0.234	0.28	0.313	0.328	0.332
	0.3	0.058	0.192	0.382	0.538	0.624	0.669	0.698	0.707
	0.4	0.096	0.365	0.652	0.824	0.892	0.924	0.928	0.936
α = 0.01	0.5	0.149	0.56	0.865	0.966	0.986	0.994	0.993	0.996
	0.8	0.218	0.75	0.973	0.997	0.999	1	1	1
	0.7	0.501	0.888	0.995	1	1	1	1	1
	0.8	0.408	0.96	1	1	1	1	1	1
	0.9	0.515	0.99	1	1	1	1	1	1
	1	0.619	0.998	1	1	1	1	1	1

					Δ/σ				
	ε	**0.5**	**1**	**1.5**	**2**	**2.5**	**3**	**3.5**	**4**
m = 75	0.1	0.02	0.037	0.06	0.076	0.09	0.098	0.1	0.103
n = 75	0.2	0.041	0.11	0.204	0.304	0.355	0.394	0.414	0.411
	0.3	0.07	0.248	0.471	0.647	0.743	0.778	0.806	0.806
	0.4	0.123	0.451	0.763	0.909	0.948	0.969	0.977	0.977
α = 0.01	0.5	0.192	0.671	0.937	0.989	0.997	0.998	0.999	0.999
	0.6	0.285	0.846	0.992	0.999	1	1	1	1
	0.7	0.385	0.95	1	1	1	1	1	1
	0.8	0.51	0.99	1	1	1	1	1	1
	0.9	0.623	0.998	1	1	1	1	1	1
	1	0.728	1	1	1	1	1	1	1

					Δ/σ				
	ε	**0.5**	**1**	**1.5**	**2**	**2.5**	**3**	**3.5**	**4**
m = 100	0.1	0.025	0.048	0.072	0.101	0.112	0.123	0.13	0.134
n = 100	0.2	0.055	0.146	0.272	0.392	0.484	0.509	0.539	0.55
	0.3	0.003	0.332	0.811	0.787	0.862	0.898	0.909	0.914
	0.4	0.168	0.586	0.888	0.971	0.989	0.994	0.997	0.996
α = 0.01	0.5	0.262	0.817	0.982	0.999	1	1	1	1
	0.8	0.377	0.938	0.999	1	1	1	1	1
	0.7	0.521	0.989	1	1	1	1	1	1
	0.8	0.648	0.999	1	1	1	1	1	1
	0.9	0.769	1	1	1	1	1	1	1
	1	0.867	1	1	1	1	1	1	1

Table A.2 (continued) Power of the WRS Test (α=0.025)

					Δ/σ				
	ε	**0.5**	**1**	**1.5**	**2**	**2.5**	**3**	**3.5**	**4**
m = 10	0.1	0.033	0.039	0.048	0.051	0.054	0.055	0.062	0.061
n = 10	0.2	0.043	0.056	0.081	0.095	0.105	0.112	0.115	0.114
	0.3	0.053	0.088	0.124	0.16	0.188	0.198	0.212	0.209
	0.4	0.062	0.125	0.187	0.26	0.3	0.32	0.336	0.352
α = 0.025	0.5	0.075	0.169	0.277	0.379	0.443	0.486	0.499	0.507
	0.6	0.093	0.221	0.388	0.512	0.609	0.656	0.684	0.683
	0.7	0.109	0.292	0.506	0.669	0.772	0.809	0.829	0.844
	0.8	0.132	0.366	0.638	0.819	0.891	0.93	0.934	0.943
	0.9	0.158	0.456	0.77	0.919	0.975	0.989	0.992	0.993
	1	0.184	0.559	0.873	0.986	0.999	1	1	1

					Δ/σ				
	ε	**0.5**	**1**	**1.5**	**2**	**2.5**	**3**	**3.5**	**4**
m = 15	0.1	0.034	0.039	0.05	0.055	0.06	0.085	0.064	0.064
n = 15	0.2	0.044	0.07	0.093	0.12	0.142	0.138	0.149	0.154
	0.3	0.055	0.113	0.163	0.215	0.254	0.275	0.288	0.29
	0.4	0.076	0.163	0.262	0.355	0.42	0.467	0.475	0.472
α = 0.025	0.5	0.092	0.221	0.393	0.513	0.616	0.657	0.669	0.682
	0.6	0.112	0.311	0.539	0.7	0.789	0.829	0.848	0.851
	0.7	0.147	0.407	0.702	0.843	0.915	0.938	0.948	0.952
	0.8	0.167	0.504	0.817	0.941	0.979	0.989	0.992	0.991
	0.9	0.212	0.62	0.907	0.99	0.998	0.999	1	1
	1	0.251	0.733	0.969	1	1	1	1	1

					Δ/σ				
	ε	**0.5**	**1**	**1.5**	**2**	**2.5**	**3**	**3.5**	**4**
m = 20	0.1	0.035	0.047	0.059	0.065	0.065	0.069	0.079	0.074
n = 20	0.2	0.049	0.077	0.114	0.145	0.17	0.177	0.194	0.185
	0.3	0.06	0.131	0.205	0.276	0.322	0.353	0.365	0.377
	0.4	0.082	0.199	0.338	0.453	0.534	0.577	0.591	0.612
α = 0.025	0.5	0.104	0.286	0.501	0.644	0.743	0.781	0.798	0.807
	0.6	0.145	0.391	0.666	0.819	0.885	0.922	0.925	0.931
	0.7	0.179	0.519	0.808	0.936	0.972	0.982	0.987	0.989
	0.8	0.221	0.639	0.915	0.985	0.996	0.998	0.999	0.999
	0.9	0.274	0.751	0.972	0.998	1	1	1	1
	1	0.321	0.85	0.995	1	1	1	1	1

					Δ/σ				
	ε	**0.5**	**1**	**1.5**	**2**	**2.5**	**3**	**3.5**	**4**
m = 25	0.1	0.036	0.051	0.06	0.073	0.082	0.082	0.083	0.086
n = 25	0.2	0.053	0.089	0.132	0.172	0.202	0.205	0.225	0.225
	0.3	0.072	0.153	0.244	0.341	0.391	0.42	0.449	0.444
	0.4	0.101	0.247	0.412	0.555	0.638	0.666	0.693	0.7
α = 0.025	0.5	0.127	0.354	0.599	0.749	0.825	0.855	0.877	0.885
	0.6	0.162	0.484	0.76	0.898	0.945	0.967	0.973	0.972
	0.7	0.217	0.619	0.893	0.974	0.99	0.995	0.997	0.997
	0.8	0.265	0.755	0.962	0.996	1	1	1	1
	0.9	0.335	0.942	0.991	1	1	1	1	1
	1	0.391	0.924	1	1	1	1	1	1

					Δ/σ				
	ε	**0.5**	**1**	**1.5**	**2**	**2.5**	**3**	**3.5**	**4**
m = 30	0.1	0.039	0.052	0.073	0.082	0.089	0.089	0.096	0.094
n = 30	0.2	0.055	0.098	0.16	0.197	0.234	0.25	0.256	0.262
	0.3	0.081	0.18	0.291	0.401	0.462	0.493	0.517	0.521
	0.4	0.112	0.283	0.475	0.628	0.707	0.755	0.769	0.777
α = 0.025	0.5	0.149	0.422	0.679	0.829	0.894	0.921	0.931	0.932
	0.6	0.2	0.552	0.836	0.944	0.978	0.985	0.988	0.988
	0.7	0.25	0.7	0.939	0.991	0.997	0.999	0.999	0.999
	0.8	0.308	0.82	0.986	0.999	1	1	1	1
	0.9	0.387	0.908	0.998	1	1	1	1	1
	1	0.469	0.962	1	1	1	1	1	1

					Δ/σ				
	ε	**0.5**	**1**	**1.5**	**2**	**2.5**	**3**	**3.5**	**4**
m = 40	0.1	0.039	0.059	0.08	0.092	0.11	0.113	0.115	0.117
n = 40	0.2	0.058	0.125	0.199	0.257	0.295	0.322	0.339	0.344
	0.3	0.091	0.232	0.375	0.499	0.579	0.611	0.636	0.641
	0.4	0.142	0.357	0.602	0.757	0.823	0.873	0.881	0.88
α = 0.025	0.5	0.19	0.516	0.808	0.919	0.961	0.972	0.978	0.98
	0.6	0.251	0.69	0.93	0.986	0.995	0.998	0.998	0.999
	0.7	0.317	0.821	0.983	0.999	1	1	1	1
	0.8	0.398	0.915	0.998	1	1	1	1	1
	0.9	0.488	0.97	1	1	1	1	1	1
	1	0.574	0.991	1	1	1	1	1	1

					Δ/σ				
	ε	**0.5**	**1**	**1.5**	**2**	**2.5**	**3**	**3.5**	**4**
m = 50	0.1	0.041	0.066	0.091	0.112	0.121	0.122	0.13	0.133
n = 50	0.2	0.067	0.144	0.234	0.313	0.356	0.38	0.399	0.404
	0.3	0.102	0.274	0.46	0.594	0.677	0.715	0.74	0.743
	0.4	0.148	0.427	0.703	0.342	0.898	0.929	0.94	0.945
α = 0.025	0.5	0.224	0.617	0.879	0.966	0.984	0.991	0.995	0.994
	0.6	0.292	0.785	0.97	0.966	0.999	1	1	1
	0.7	0.388	0.901	0.995	1	1	1	1	
	0.8	0.485	0.966	1	1	1	1	1	1
	0.9	0.589	0.99	1	1	1	1	1	1
	1	0.666	0.998	1	1	1	1	1	1

					Δ/σ				
	ε	**0.5**	**1**	**1.5**	**2**	**2.5**	**3**	**3.5**	**4**
m = 60	0.1	0.046	0.072	0.098	0.123	0.14	0.145	0.146	0.149
n = 60	0.2	0.076	0.163	0.27	0.347	0.414	0.447	0.465	0.475
	0.3	0.117	0.32	0.526	0.671	0.755	0.802	0.807	0.814
	0.4	0.176	0.501	0.779	0.902	0.946	0.963	0.972	0.972
α = 0.025	0.5	0.252	0.705	0.938	0.984	0.995	0.998	0.998	0.998
	0.6	0.344	0.856	0.989	0.999	1	1	1	1
	0.7	0.45	0.949	0.998	1	1	1	1	1
	0.8	0.566	0.982	1	1	1	1	1	1
	0.9	0.653	0.997	1	1	1	1	1	1
	1	0.754	1	1	1	1	1	1	1

Table A.2 (continued) Power of the WRS Test (α=0.025)

					Δ/σ				
	ε	**0.5**	**1**	**1.5**	**2**	**2.5**	**3**	**3.5**	**4**
m = 75	0.1	0.048	0.075	0.113	0.145	0.166	0.175	0.18	0.176
n = 75	0.2	0.086	0.192	0.324	0.439	0.497	0.532	0.556	0.567
	0.3	0.134	0.387	0.621	0.774	0.843	0.877	0.889	0.897
	0.4	0.213	0.603	0.868	0.958	0.981	0.987	0.99	0.991
α = 0.025	0.5	0.313	0.796	0.971	0.997	1	1	1	1
	0.6	0.42	0.923	0.997	1	1	1	1	1
	0.7	0.54	0.977	1	1	1	1	1	1
	0.8	0.654	0.995	1	1	1	1	1	1
	0.9	0.756	1	1	1	1	1	1	1
	1	0.838	1	1	1	1	1	1	1

					Δ/σ				
	ε	**0.5**	**1**	**1.5**	**2**	**2.5**	**3**	**3.5**	**4**
m = 100	0.4	0.055	0.093	0.134	0.176	0.203	0.217	0.215	0.231
n = 100	0.2	0.097	0.241	0.408	0.541	0.623	0.666	0.675	0.678
	0.3	0.173	0.486	0.752	0.875	0.926	0.948	0.958	0.959
	0.4	0.273	0.726	0.946	0.987	0.996	0.998	0.999	0.999
α = 0.025	0.5	0.392	0.9	0.994	1	1	1	1	1
	0.6	0.529	0.976	1	1	1	1	1	1
	0.7	0.665	0.996	1	1	1	1	1	1
	0.8	0.777	1	1	1	1	1	1	1
	0.9	0.875	1	1	1	1	1	1	1
	1	0.933	1	1	1	1	1	1	1

Table A.2 (continued) Power of the WRS Test (α=0.05)

					Δ/σ				
	ε	**0.5**	**1**	**1.5**	**2**	**2.5**	**3**	**3.5**	**4**
m = 10	0.1	0.065	0.076	0.091	0.095	0.101	0.111	0.104	0.101
n = 10	0.2	0.08	0.109	0.138	0.158	0.174	0.182	0.199	0.193
	0.3	0.101	0.149	0.211	0.263	0.294	0.302	0.31	0.309
	0.4	0.11	0.197	0.291	0.376	0.435	0.445	0.469	0.476
α = 0.05	0.5	0.136	0.259	0.404	0.506	0.576	0.619	0.632	0.632
	0.6	0.159	0.33	0.522	0.653	0.731	0.768	0.792	0.795
	0.7	0.194	0.413	0.636	0.785	0.862	0.892	0.899	0.907
	0.8	0.216	0.495	0.751	0.895	0.949	0.966	0.971	0.975
	0.9	0.256	0.587	0.855	0.966	0.989	0.994	0.997	0.998
	1	0.282	0.677	0.939	0.995	1	1	1	1
					Δ/σ				
	ε	**0.5**	**1**	**1.5**	**2**	**2.5**	**3**	**3.5**	**4**
m = 15	0.1	0.072	0.084	0.105	0.109	0.121	0.12	0.126	0.128
n = 15	0.2	0.065	0.132	0.168	0.206	0.229	0.241	0.241	0.245
	0.3	0.11	0.193	0.27	0.338	0.391	0.414	0.415	0.418
	0.4	0.134	0.253	0.385	0.498	0.558	0.593	0.616	0.626
α = 0.05	0.5	0.168	0.347	0.536	0.664	0.738	0.77	0.793	0.791
	0.6	0.2	0.448	0.683	0.804	0.878	0.904	0.916	0.922
	0.7	0.234	0.546	0.802	0.914	0.959	0.972	0.976	0.979
	0.8	0.279	0.654	0.898	0.975	0.992	0.996	0.997	0.998
	0.9	0.33	0.753	0.959	0.997	1	1	1	1
	1	0.369	0.841	0.988	1	1	1	1	1
					Δ/σ				
	ε	**0.5**	**1**	**1.5**	**2**	**2.5**	**3**	**3.5**	**4**
m = 20	0.1	0.066	0.09	0.108	0.122	0.125	0.134	0.134	0.137
n = 20	0.2	0.091	0.145	0.191	0.244	0.262	0.277	0.288	0.291
	0.3	0.122	0.213	0.321	0.406	0.459	0.489	0.489	0.496
	0.4	0.151	0.303	0.461	0.586	0.657	0.699	0.711	0.721
α = 0.05	0.5	0.187	0.407	0.629	0.767	0.836	0.864	0.877	0.883
	0.6	0.232	0.532	0.775	0.893	0.945	0.959	0.965	0.971
	0.7	0.283	0.652	0.896	0.988	0.988	0.994	0.995	0.995
	0.8	0.331	0.758	0.959	0.994	0.999	0.999	1	1
	0.9	0.386	0.849	0.989	0.999	1	1	1	1
	1	0.451	0.917	0.998	1	1	1	1	1
					Δ/σ				
	ε	**0.5**	**1**	**1.5**	**2**	**2.5**	**3**	**3.5**	**4**
m = 25	0.1	0.072	0.092	0.115	0.137	0.15	0.152	0.151	0.152
n = 25	0.2	0.096	0.159	0.229	0.278	0.305	0.333	0.326	0.335
	0.3	0.128	0.243	0.367	0.462	0.536	0.562	0.578	0.587
	0.4	0.169	0.36	0.545	0.685	0.753	0.786	0.802	0.813
α = 0.05	0.5	0.211	0.483	0.727	0.842	0.902	0.928	0.936	0.931
	0.6	0.269	0.614	0.852	0.951	0.973	0.984	0.987	0.987
	0.7	0.325	0.744	0.944	0.99	0.996	0.999	0.999	0.998
	0.8	0.39	0.841	0.983	0.999	1	1	1	1
	0.9	0.465	0.913	0.997	1	1	1	1	1
	1	0.53	0.957	1	1	1	1	1	1

Table A.2 (continued) Power of the WRS Test (α=0.05)

					Δ/σ				
	ε	**0.5**	**1**	**1.5**	**2**	**2.5**	**3**	**3.5**	**4**
m = 30	0.1	0.073	0.097	0.125	0.136	0.147	0.159	0.17	0.162
n = 30	0.2	0.103	0.167	0.241	0.294	0.345	0.364	0.372	0.376
	0.3	0.142	0.265	0.42	0.515	0.581	0.622	0.645	0.646
	0.4	0.178	0.398	0.602	0.743	0.813	0.638	0.856	0.854
α = 0.05	0.5	0.24	0.542	0.787	0.897	0.942	0.952	0.966	0.966
	0.6	0.29	0.679	0.904	0.973	0.991	0.994	0.995	0.996
	0.7	0.353	0.803	0.971	0.996	0.999	1	1	1
	0.8	0.444	0.894	0.994	1	1	1	1	1
	0.9	0.505	0.95	0.999	1	1	1	1	1
	1	0.596	0.98	1	1	1	1	1	1

					Δ/σ				
	ε	**0.5**	**1**	**1.5**	**2**	**2.5**	**3**	**3.5**	**4**
m = 40	0.1	0.077	0.109	0.136	0.164	0.178	0.189	0.189	0.202
n = 40	0.2	0.113	0.198	0.297	0.365	0.498	0.45	0.45	0.47
	0.3	0.166	0.334	0.509	0.626	0.701	0.741	0.744	0.759
	0.4	0.216	0.489	0.718	0.848	0.899	0.925	0.933	0.937
α = 0.05	0.5	0.279	0.655	0.88	0.959	0.98	0.989	0.99	0.993
	0.6	0.36	0.791	0.982	0.993	0.999	0.999	0.999	0.999
	0.7	0.444	0.897	0.994	0.999	1	1	1	1
	0.8	0.519	0.959	0.999	1	1	1	1	1
	0.9	0.617	0.988	1	1	1	1	1	1
	1	0.699	0.996	1	1	1	1	1	1

					Δ/σ				
	ε	**0.5**	**1**	**1.5**	**2**	**2.5**	**3**	**3.5**	**4**
m = 50	0.1	0.083	0.117	0.15	0.183	0.193	0.212	0.213	0.214
n = 50	0.2	0.121	0.224	0.338	0.427	0.487	0.513	0.53	0.541
	0.3	0.177	0.394	0.578	0.711	0.779	0.808	0.635	0.829
	0.4	0.246	0.564	0.803	0.904	0.948	0.955	0.968	0.97
α = 0.05	0.5	0.327	0.735	0.936	0.985	0.993	0.997	0.998	0.997
	0.6	0.41	0.865	0.988	0.999	1	1	1	1
	0.7	0.506	0.949	0.998	1	1	1	1	1
	0.8	0.61	0.984	1	1	1	1	1	1
	0.9	0.704	0.995	1	1	1	1	1	1
	1	0.786	1	1	1	1	1	1	1

					Δ/σ				
	ε	**0.5**	**1**	**1.5**	**2**	**2.5**	**3**	**3.5**	**4**
m = 60	0.1	0.084	0.126	0.171	0.204	0.23	0.237	0.24	0.243
n = 60	0.2	0.129	0.257	0.39	0.475	0.55	0.578	0.596	0.604
	0.3	0.195	0.435	0.655	0.779	0.841	0.872	0.882	0.893
	0.4	0.282	0.632	0.854	0.947	0.973	0.983	0.985	0.987
α = 0.05	0.5	0.366	0.804	0.966	0.993	0.998	0.999	1	1
	0.6	0.467	0.92	0.995	1	1	1	1	1
	0.7	0.583	0.972	0.999	1	1	1	1	1
	0.8	0.675	0.993	1	1	1	1	1	1
	0.9	0.771	0.999	1	1	1	1	1	1
	1	0.947	1	1	1	1	1	1	1

Table A.2 (continued) Power of the WRS Test (α=0.05)

					Δ/σ				
	ε	**0.5**	**1**	**1.5**	**2**	**2.5**	**3**	**3.5**	**4**
m = 75	0.1	0.09	0.135	0.185	0.221	0.258	0.271	0.278	0.274
n = 75	0.2	0.145	0.288	0.443	0.558	0.629	0.661	0.68	0.672
	0.3	0.226	0.509	0.738	0.861	0.906	0.933	0.937	0.942
	0.4	0.314	0.726	0.925	0.977	0.989	0.994	0.995	0.996
α = 0.05	0.5	0.432	0.881	0.989	0.999	1	1	1	1
	0.6	0.556	0.956	0.999	1	1	1	1	1
	0.7	0.664	0.99	1	1	1	1	1	1
	0.8	0.764	0.999	1	1	1	1	1	1
	0.9	0.848	1	1	1	1	1	1	1
	1	0.909	1	1	1	1	1	1	1

					Δ/σ				
	ε	**0.5**	**1**	**1.5**	**2**	**2.5**	**3**	**3.5**	**4**
m = 100	0.1	0.101	0.158	0.22	0.271	0.303	0.314	0.332	0.334
n = 100	0.2	0.175	0.35	0.542	0.659	0.721	0.772	0.792	0.798
	0.3	0.261	0.604	0.835	0.931	0.961	0.975	0.978	0.982
	0.4	0.385	0.821	0.973	0.993	0.998	0.999	0.999	0.999
α = 0.05	0.5	0.515	0.941	0.998	1	1	1	1	1
	0.6	0.847	0.987	1	1	1	1	1	1
	0.7	0.77	0.998	1	1	1	1	1	1
	0.8	0.858	1	1	1	1	1	1	1
	0.9	0.925	1	1	1	1	1	1	1
	1	0.964	1	1	1	1	1	1	1

Table A.2 (continued) Power of the WRS Test (α=0.10)

					Δ/σ				
	ε	**0.5**	**1**	**1.5**	**2**	**2.5**	**3**	**3.5**	**4**
m = 10	0.1	0.131	0.149	0.176	0.173	0.185	0.195	0.202	0.186
n = 10	0.2	0.152	0.203	0.235	0.287	0.299	0.315	0.319	0.324
	0.3	0.181	0.263	0.334	0.392	0.428	0.46	0.466	0.473
	0.4	0.205	0.326	0.449	0.52	0.583	0.608	0.63	0.629
α = 0.1	0.5	0.234	0.402	0.564	0.662	0.731	0.762	0.763	0.765
	0.6	0.268	0.487	0.675	0.788	0.846	0.87	0.884	0.886
	0.7	0.302	0.577	0.776	0.891	0.932	0.95	0.952	0.959
	0.8	0.354	0.659	0.871	0.955	0.979	0.988	0.991	0.992
	0.9	0.396	0.732	0.932	0.986	0.997	0.999	0.999	0.999
	1	0.435	0.609	0.976	0.999	1	1	1	1
					Δ/σ				
	ε	**0.5**	**1**	**1.5**	**2**	**2.5**	**3**	**3.5**	**4**
m = 15	0.1	0.128	0.157	0.18	0.206	0.215	0.215	0.213	0.215
n = 15	0.2	0.163	0.221	0.292	0.342	0.359	0.378	0.375	0.393
	0.3	0.198	0.306	0.418	0.492	0.53	0.56	0.572	0.58
	0.4	0.235	0.407	0.545	0.647	0.704	0.734	0.745	0.757
α = 0.1	0.5	0.282	0.496	0.682	0.802	0.847	0.873	0.889	0.887
	0.6	0.324	0.603	0.814	0.894	0.936	0.954	0.96	0.961
	0.7	0.375	0.696	0.891	0.961	0.983	0.99	0.99	0.992
	0.8	0.425	0.791	0.953	0.991	0.998	0.999	0.999	0.999
	0.9	0.469	0.863	0.984	0.999	1	1	1	1
	1	0.535	0.923	0.997	1	1	1	1	1
					Δ/σ				
	ε	**0.5**	**1**	**1.5**	**2**	**2.5**	**3**	**3.5**	**4**
m = 20	0.1	0.127	0.156	0.183	0.203	0.212	0.224	0.235	0.233
n = 20	0.2	0.164	0.24	0.303	0.358	0.393	0.411	0.424	0.42
	0.3	0.205	0.34	0.454	0.545	0.594	0.624	0.646	0.642
	0.4	0.256	0.44	0.619	0.723	0.781	0.812	0.827	0.823
α = 0.1	0.5	0.292	0.553	0.762	0.868	0.911	0.928	0.935	0.938
	0.6	0.303	0.672	0.872	0.95	0.973	0.979	0.984	0.987
	0.7	0.407	0.772	0.943	0.987	0.995	0.998	0.998	0.998
	0.8	0.47	0.859	0.981	0.998	1	1	1	1
	0.9	0.53	0.925	0.997	1	1	1	1	1
	1	0.602	0.959	0.999	1	1	1	1	1
					Δ/σ				
	ε	**0.5**	**1**	**1.5**	**2**	**2.5**	**3**	**3.5**	**4**
m = 25	0.1	0.132	0.165	0.193	0.227	0.242	0.234	0.248	0.248
n = 25	0.2	0.172	0.254	0.349	0.401	0.445	0.465	0.475	0.48
	0.3	0.215	0.362	0.5439	0.607	0.661	0.687	0.711	0.712
	0.4	0.27	0.506	0.685	0.797	0.854	0.873	0.M0	0.888
α = 0.1	0.5	0.331	0.623	0.852	0.919	0.952	0.968	0.968	0.967
	0.6	0.392	0.746	0.923	0.977	0.992	0.993	0.995	0.996
	0.7	0.458	0.844	0.972	0.994	0.999	0.999	0.999	1
	0.8	0.535	0.915	0.994	1	1	1	1	1
	0.9	0.595	0.957	0.999	1	1	1	1	1
	1	0.669	0.985	1	1	1	1	1	1

Table A.2 (continued) Power of the WRS Test (α=0.10)

					Δ/σ				
	ϵ	**0.5**	**1**	**1.5**	**2**	**2.5**	**3**	**3.5**	**4**
m = 30	0.1	0.138	0.179	0.212	0.239	0.256	0.264	0.269	0.265
n = 30	0.2	0.177	0.279	0.379	0.448	0.483	0.518	0.521	0.526
	0.3	0.241	0.412	0.563	0.665	0.726	0.755	0.762	0.776
	0.4	0.292	0.542	0.741	0.852	0.895	0.921	0.926	0.922
α = 0.1	0.5	0.358	0.685	0.883	0.95	0.974	0.982	0.987	0.987
	0.8	0.44	0.804	0.953	0.989	0.995	0.998	0.998	0.999
	7	0.545	0.893	0.987	0.998	1	1	1	1
	0.8	0.587	0.949	0.998	1	1	1	1	1
	0.9	0.663	0.98	1	1	1	1	1	1
	1	0.73	0.993	1	1	1	1	1	1

					Δ/σ				
	ϵ	**0.5**	**1**	**1.5**	**2**	**2.5**	**3**	**3.5**	**4**
m = 40	0.1	0.139	0.189	0.228	0.264	0.281	0.296	0.301	0.303
n = 40	0.2	0.197	0.31	0.418	0.501	0.56	0.584	0.601	0.6
	0.3	0.298	0.473	0.647	0.761	0.816	0.839	0.848	0.85
	0.4	0.336	0.635	0.832	0.917	0.951	0.963	0.969	0.969
α = 0.1	0.5	0.423	0.768	0.939	0.983	0.993	0.996	0.996	0.997
	0.6	0.5	0.879	0.986	0.998	0.909	0.999	1	1
	0.7	0.591	0.947	0.999	1	1	1	1	1
	0.8	0.672	0.983	1	1	1	1	1	1
	0.9	0.743	0.995	1	1	1	1	1	1
	1	0.818	0.998	1	1	1	1	1	1

					Δ/σ				
	ϵ	**0.5**	**1**	**1.5**	**2**	**2.5**	**3**	**3.5**	**4**
m = 50	0.1	0.145	0.209	0.25	0.289	0.318	0.33	0.34	0.341
n = 50	0.2	0.214	0.348	0.48	0.566	0.633	0.668	0.672	0.681
	0.3	0.283	0.536	0.718	0.824	0.871	0.896	0.908	0.904
	0.4	0.379	0.707	0.885	0.957	0.979	0.987	0.985	0.987
α = 0.1	0.5	0.468	0.838	0.971	0.995	0.998	0.999	0.999	0.999
	0.6	0.554	0.931	0.996	0.999	1	1	1	1
	0.7	0.652	0.978	1	1	1	1	1	1
	0.8	0.741	0.993	1	1	1	1	1	1
	0.9	0.824	0.999	1	1	1	1	1	1
	1	0.877	1	1	1	1	1	1	1

					Δ/σ				
	ϵ	**0.5**	**1**	**1.5**	**2**	**2.5**	**3**	**3.5**	**4**
m = 60	0.1	0.161	0.214	0.274	0.312	0.342	0.359	0.366	0.366
n = 60	0.2	0.223	0.381	0.528	0.628	0.684	0.719	0.727	0.728
	0.3	0.316	0.571	0.773	0.873	0.915	0.933	0.94	0.945
	0.4	0.41	0.753	0.93	0.978	0.99	0.994	0.994	0.995
α = 0.1	0.5	0.504	0.881	0.986	0.999	1	1	1	1
	0.6	0.623	0.959	0.998	1	1	1	1	1
	0.7	0.718	0.99	1	1	1	1	1	1
	0.8	0.798	0.998	1	1	1	1	1	1
	0.9	0.867	1	1	1	1	1	1	1
	1	0.913	1	1	1	1	1	1	1

Table A.2 (continued) Power of the WRS Test (α=0.10)

		Δ/σ							
	ϵ	**0.5**	**1**	**1.5**	**2**	**2.5**	**3**	**3.5**	**4**
m = 75	0.1	0.163	0.237	0.295	0.354	0.377	0.391	0.415	0.412
n = 75	0.2	0.235	0.417	0.585	0.704	0.757	0.779	0.795	0.798
	0.3	0.341	0.646	0.846	0.923	0.954	0.965	0.973	0.975
	0.4	0.464	0.828	0.964	0.991	0.996	0.998	0.998	0.999
α = 0.1	0.5	0.588	0.937	0.996	1	1	1	1	1
	0.6	0.686	0.982	0.999	1	1	1	1	1
	0.7	0.782	0.996	1	1	1	1	1	1
	0.8	0.866	1	1	1	1	1	1	1
	0.9	0.917	1	1	1	1	1	1	1
	1	0.956	1	1	1	1	1	1	1

		Δ/σ							
	ϵ	**0.5**	**1**	**1.5**	**2**	**2.5**	**3**	**3.5**	**4**
m = 100	0.1	0.178	0.258	0.345	0.398	0.442	0.464	0.479	0.483
n = 100	0.2	0.286	0.494	0.681	0.78	0.637	0.861	0.874	0.875
	0.3	0.396	0.737	0.908	0.97	0.984	0.992	0.992	0.993
	0.4	0.53	0.904	0.986	0.998	1	1	1	1
α = 0.1	0.5	0.663	0.975	0.999	1	1	1	1	1
	0.6	0.78	0.998	1	1	1	1	1	1
	0.7	0.864	1	1	1	1	1	1	1
	0.8	0.934	1	1	1	1	1	1	1
	0.9	0.964	1	1	1	1	1	1	1
	1	0.984	1	1	1	1	1	1	1

A.3 Power of the Quantile Test

The table in this section provides values for the approximate power $(1-\beta)$ of the Quantile test when there are equal numbers of measurements in the reference area (**m**) and in the Survey Unit (**n**). These values correspond to the probability that the Quantile test will correctly reject the null hypothesis that decontamination criteria is met when there are residual contamination Δ/σ over background on over100ϵ percent of the survey unit. The approximate power is given for four values of α (0.01, 0.025, 0.05 and 0.10) appear on three sucessive pages each. These tables were constructed from data in PNL-7409.

Table A.3 Approximate Power of the Quantile Test

					Δ/σ				
	ϵ	**0.5**	**1**	**1.5**	**2**	**2.5**	**3**	**3.5**	**4**
m = 10	0.1	0.018	0.025	0.029	0.036	0.038	0.045	0.043	0.05
n = 10	0.2	0.026	0.04	0.058	0.082	0.102	0.108	0.119	0.122
r = 5	0.3	0.032	0.054	0.096	0.146	0.2	0.233	0.264	0.278
k = 5	0.4	0.036	0.078	0.149	0.244	0.333	0.418	0.463	0.49
α = 0.015	0.5	0.043	0.1	0.211	0.349	0.495	0.598	0.663	0.697
	0.6	0.05	0.137	0.283	0.469	0.642	0.761	0.821	0.869
	0.7	0.063	0.169	0.359	0.569	0.75	0.875	0.935	0.955
	0.8	0.079	0.207	0.426	0.662	0.848	0.936	0.976	0.992
	0.9	0.08	0.25	0.5	0.745	0.896	0.97	0.993	0.997
	1	0.09	0.284	0.564	0.806	0.933	0.982	0.997	1

					Δ/σ				
	ϵ	**0.5**	**1**	**1.5**	**2**	**2.5**	**3**	**3.5**	**4**
m = 15	0.1	0.011	0.015	0.021	0.027	0.033	0.037	0.039	0.04
n = 15	0.2	0.015	0.027	0.047	0.074	0.103	0.129	0.147	0.157
r = 6	0.3	0.019	0.043	0.088	0.157	0.237	0.311	0.363	0.393
k = 6	0.4	0.024	0.064	0.146	0.272	0.415	0.54	0.623	0.668
α = 0.008	0.5	0.03	0.09	0.216	0.402	0.594	0.74	0.827	0.869
	0.6	0.036	0.121	0.294	0.527	0.737	0.872	0.938	0.964
	0.7	0.043	0.155	0.374	0.635	0.835	0.939	0 98	0.993
	0.8	0.051	0.193	0.45	0.72	0.894	0.969	0.993	0.999
	0.9	0.06	0.232	0.52	0.784	0.929	0.982	0.997	0.999
	1	0.07	0.272	0.581	831	0.95	0.989	0.998	1

					Δ/σ				
	ϵ	**0.5**	**1**	**1.5**	**2**	**2.5**	**3**	**3.5**	**4**
m = 20	0.1	0.014	0.02	0.03	0.042	0.055	0.065	0.071	0.075
n = 20	0.2	0.018	0.037	0.07	0.122	0.185	0.246	0.291	0.317
r = 6	0.3	0.024	0.059	0.133	0.251	0.392	0.52	0.608	0.658
k = 6	0.4	0.031	0.089	0.213	0.402	0.602	0.755	0.845	0.888
α = 0.01	0.5	0.038	0.124	0.302	0.544	0.759	0.891	0.953	0.976
	0.6	0.047	0.163	0.391	0.66	0.856	0.952	0.986	0.996
	0.7	0.056	0.205	0.474	0.746	0.911	0.976	0.995	0.999
	8	0.066	0.249	0.547	0.808	0.942	0.987	0.998	1
	0.9	0.077	0.292	0.61	0.852	0.96	0.992	0.999	1
	1	0.089	0.335	0.663	0.883	0.971	0.994	0.999	1

Table A.3 (continued) Power of the Quantile Test ($\alpha = 0.01$)

					Δ/σ				
	ε	**0.5**	**1**	**1.5**	**2**	**2.5**	**3**	**3.5**	**4**
m = 25	0.1	0.017	0.025	0.038	0.059	0.079	0.096	0.119	0.12
n = 25	0.2	0.024	0.045	0.091	0.17	0.266	0.368	0.445	0.49
r = 6	0.3	0.029	0.074	0.176	0.332	0.514	0.683	0.776	0.826
k = 6	0.4	0.037	0.107	0.272	0.503	0.723	0.866	0.94	0.97
α = 0.008	0.5	0.044	0.148	0.383	0.647	0.846	0.944	0.983	0.995
	0.6	0.055	0.193	0.453	0.739	0.907	0.978	0.995	0.999
	0.7	0.064	0.24	0.539	0.81	0.942	0.987	0.998	1
	0.8	0.082	0.288	0.609	0.857	0.961	0.992	0.998	1
	0.9	0.091	0.336	0.674	0.892	0.971	0.995	0.999	1
	1	0.105	0.38	0.715	0.909	0.978	0.997	0.999	1
					Δ/σ				
	ε	**0.5**	**1**	**1.5**	**2**	**2.5**	**3**	**3.5**	**4**
m = 30	0.1	0.018	0.024	0.052	0.069	0.108	0.136	0.171	0.187
n = 30	0.2	0.024	0.055	0.115	0.218	0.357	0.494	0.584	0.644
r = 6	0.3	0.028	0.085	0.214	0.41	0.623	0.785	0.881	0.923
k = 6	0.4	0.038	0.134	0.316	0.581	0.808	0.928	0.976	0.991
α = 0.013	0.5	0.051	0.169	0.419	0.702	0.895	0.972	0.993	0.998
	0.6	0.06	0.233	0.521	0.79	0.931	0.984	0.998	0.999
	0.7	0.074	0.279	0.592	0.839	0.959	0.994	0.999	1
	0.8	0.088	0.324	0.659	0.885	0.974	0.996	0.999	1
	0.9	0.102	0.373	0.701	0.906	0.979	0.997	0.999	1
	1	0.117	0.416	0.755	0.923	0.986	0.998	1	1
					Δ/σ				
	ε	**0.5**	**1**	**1.5**	**2**	**2.5**	**3**	**3.5**	**4**
m = 40	0.1	0.016	0.026	0.043	0.062	0.078	0.089	0.094	0.095
n = 40	0.2	0.024	0.059	0.128	0.224	0.318	0.384	0.417	0.43
r = 15	0.3	0.035	0.113	0.277	0.491	0.669	0.769	0.814	0.83
k = 12	0.4	0.049	0.188	0.463	0.744	0.901	0.958	0.975	0.98
α = 0.01	0.5	0.067	0.28	0.541	0.898	0.981	0.996	0.999	0.999
	0.6	0.088	0.382	0.779	0.965	0.997	1	1	1
	0.7	0.112	0.484	0.872	0.989	1	1	1	1
	0.8	0.14	0.579	0.928	0.996	1	1	1	1
	0.9	0.171	0.664	0.96	0.999	1	1	1	1
	1	0.205	0.735	0.978	1	1	1	1	1
					Δ/σ				
	ε	**0.5**	**1**	**1.5**	**2**	**2.5**	**3**	**3.5**	**4**
m = 50	0.1	0.019	0.033	0.059	0.092	0.125	0.149	0.161	0.166
n = 50	0.2	0.029	0.078	0.182	0.335	0.485	0.588	0.641	0.662
r = 15	0.3	0.043	0.149	0.376	0.65	0.837	0.92	0.949	0.959
k = 12	0.4	0.061	0.243	0.583	0.864	0.971	0.994	0.998	0.999
α = 0.011	0.5	0.83	0.352	0.75	0.957	0.996	1	1	1
	0.6	0.108	0.464	0.881	0.987	1	1	1	1
	0.7	0.138	0.568	0.925	0.996	1	1	1	1
	0.8	0.171	0.66	0.96	0.999	1	1	1	1
	0.9	0.207	0.737	0.979	1	1	1	1	1
	1	0.245	0.798	0.988	1	1	1	1	1

Table A.3 (continued) Power of the Quantile Test (α≈ 0.01)

					Δ/σ				
	ε	**0.5**	**1**	**1.5**	**2**	**2.5**	**3**	**3.5**	**4**
m = 60	0.1	0.014	0.028	0.058	0.113	0.189	0.266	0.323	0.364
n = 60	0.2	0.022	0.066	0.188	0.401	0.64	0.808	0.89	0.923
r = 10	0.3	0.032	0.125	0.385	0.687	0.902	0.978	0.995	0.998
k = 9	0.4	0.045	0.201	0.54	0.854	0.976	0.998	1	1
α = 0.008	0.5	0.06	0.285	0.68	0.932	0.993	1	1	1
	0.6	0.078	0.37	0.779	0.966	0.998	1	1	1
	0.7	0.098	0.451	0.647	0.982	0.999	1	1	1
	0.8	0.121	0.525	0.892	0.99	1	1	1	1
	0.9	0.144	0.591	0.923	0.994	1	1	1	1
	1	0.17	0.648	0.943	0.996	1	1	1	1

					Δ/σ				
	ε	**0.5**	**1**	**1.5**	**2**	**2.5**	**3**	**3.5**	**4**
m = 75	0.1	0.015	0.032	0.074	0.157	0.277	0.401	0.492	0.543
n = 75	0.2	0.024	0.08	0.236	0.508	0.771	0.915	0.908	0.984
r = 10	0.3	0.036	0.151	0.44	0.78	0.953	0.994	0.999	1
k = 9	0.4	0.051	0.238	0.618	0.907	0.989	0.999	1	1
α = 0.009	0.5	0.069	0.33	0.745	0.958	0.997	1	1	1
	0.6	0.089	0.42	0.83	0.98	0.999	1	1	1
	0.7	0.112	0.503	0.884	0.989	0.999	1	1	1
	0.8	0.137	0.576	0.92	0.994	1	1	1	1
	0.9	0.163	0.639	0.943	0.996	1	1	1	1
	1	0.191	0.692	0.958	0.998	1	1	1	1

					Δ/σ				
	ε	**0.5**	**1**	**1.5**	**2**	**2.5**	**3**	**3.5**	**4**
m = 100	0.1	0.017	0.039	0.1	0.23	0.421	0.807	0.73	0.792
n = 100	0.2	0.027	0.1	0.31	0.641	0.888	0.978	0.996	0.999
r = 10	0.3	0.041	0.187	0.538	0.866	0.982	0.999	1	1
k = 9	0.4	0.059	0.288	0.704	0.949	0.996	1	1	1
α = 0.009	0.5	0.08	0.389	0.813	0.978	0.999	1	1	1
	0.8	0.103	0.483	0.879	0.989	1	1	1	1
	0.7	0.13	0.565	0.919	0.994	1	1	1	1
	0.8	0.158	0.635	0.945	0.997	1	1	1	1
	0.9	0.187	0.693	0.961	0.998	1	1	1	1
	1	0.217	0.742	0.971	0.999	1	1	1	1

Table A.3 (continued) Power of the Quantile Test (α≈ 0.025)

					Δ/σ				
	ϵ	**0.5**	**1**	**1.5**	**2**	**2.5**	**3**	**3.5**	**4**
m = 10	0.1	0.034	0.042	0.051	0.05.5	0.0568	0.061	0.062	0.063
n = 10	0.2	0.042	0.064	0.083	0.1	0.111	0.117	0.122	0.124
r = 7	0.3	0.049	0.084	0.135	0.176	0.202	0.219	0.23	0.237
k = 6	0.4	0.065	0.124	0.197	0.281	0.333	0.374	0.396	0.409
α = 0.029	0.5	0.076	0.152	0.272	0.398	0.503	0.554	0.582	0.604
	0.6	0.084	0.198	0.37	0.549	0.67	0.736	0.772	0.785
	0.7	0.102	0.249	0.468	0.678	0.809	0.878	0.903	0.921
	0.8	0.116	0.311	0.565	0.787	0.911	0.962	0.98	0.981
	0.9	0.137	0.37	0.658	0.874	0.965	0.991	0.999	0.999
	1	0.15	0.423	0.735	0.927	0.987	0.999	1	1

					Δ/σ				
	ϵ	**0.5**	**1**	**1.5**	**2**	**2.5**	**3**	**3.5**	**4**
m = 15	0.1	0.025	0.036	0.046	0.063	0.086	0.085	0.092	0.096
n = 15	0.2	0.034	0.06	0.094	0.151	0.201	0.25	0.291	0.3
r = 5	0.3	0.044	0.09	0.162	0.277	0.396	0.489	0.553	0.596
k = 5	0.4	0.052	0.123	0.244	0.411	0.584	0.723	0.789	0.829
α = 0.021	0.5	0.066	0.156	0.329	0.556	0.739	0.858	0.923	0.948
	0.6	0.073	0.213	0.421	0.658	0.842	0.931	2.975	0.989
	0.7	0.086	0.25	0.498	0.743	0.903	0.973	0.992	0.998
	0.8	0.097	0.297	0.561	0.812	0.936	0.986	0.997	1
	0.9	0.11	0.331	0.632	0.856	0.961	0.99	0.998	1
	1	0.122	0.372	0.684	0.889	0.969	0.994	0.999	1

					Δ/σ				
	ϵ	**0.5**	**1**	**1.5**	**2**	**2.5**	**3**	**3.5**	**4**
m = 20	0.1	0.031	0.043	0.063	0.084	0.114	0.138	0.143	0.16
n = 20	0.2	0.038	0.072	0.127	0.217	0.309	0.402	0.462	0.495
r = 5	0.3	0.046	0.11	0.225	0.381	0.555	0.687	0.76	0.813
k = 5	0.4	0.059	0.15	0.318	0.538	0.723	0.868	0.925	0.954
α = 0.024	0.5	0.075	0.202	0.414	0.669	0.854	0.941	0.979	0.993
	0.6	0.088	0.251	0.512	0.761	0.907	0.976	0.995	0.998
	0.7	0.105	0.303	0.6	0.827	0.945	0.987	0.998	1
	0.8	0.112	0.346	0.645	0.868	0.966	0.991	0.998	1
	0.9	0.129	0.394	0.708	0.898	0.977	0.994	1	1
	1	0.155	0.431	0.743	0.923	0.98	0.997	1	1

					Δ/σ				
	ϵ	**0.5**	**1**	**1.5**	**2**	**2.5**	**3**	**3.5**	**4**
m = 25	0.1	0.03	0.053	0.081	0.113	0.157	0.188	0.215	0.234
n = 25	0.2	0.051	0.084	0.16	0.275	0.422	0.532	0.616	0.666
r = 5	0.3	0.051	0.128	0.273	0.463	0.662	0.804	0.885	0.918
k = 5	0.4	0.068	0.187	0.388	0.633	0.821	0.927	0.97	0.987
α = 0.025	0.5	0.083	0.233	0.48	0.746	0.901	0.972	0.993	0.998
	0.6	0.095	0.294	0.576	0.818	0.945	0.987	0.997	1
	0.7	0.115	0.346	0.648	0.87	0.994	0.995	0.998	1
	0.8	0.128	0.385	0.708	0.898	0.976	0.995	1	1
	0.9	0.142	0.437	0.744	0.924	0.983	0.997	1	1
	1	0.168	0.468	0.7830.941		0.988	0.998	1	1

Table A.3 (continued) Power of the Quantile Test (α ≈ 0.025)

					Δ/σ				
	ε	**0.5**	**1**	**1.5**	**2**	**2.5**	**3**	**3.5**	**4**
m = 30	0.1	0.037	0.048	0.088	0.137	0.194	0.253	0.295	0.316
n = 30	0.2	0.043	0.098	0.187	0.332	0.495	0.644	0.734	0.795
r = 5	0.3	0.056	0.142	0.306	0.535	0.745	0.88	0.941	0.965
k = 5	0.4	0.074	0.197	0.432	0.691	0.874	0.958	0.988	0.998
α = 0.026	0.5	0.089	0.256	0.536	0.792	0.929	0.981	0.996	1
	0.6	0.107	0.317	0.62	0.853	0.962	0.992	0.999	1
	0.7	0.126	0.368	0.68	0.891	0.975	0.995	0.999	1
	0.8	0.146	0.419	0.737	0.919	0.982	0.997	0.999	1
	0.9	0.16	0.467	0.769	0.935	0.988	0.998	1	1
	1	0.173	0.497	0.807	0.949	0.989	0.998	1	1

					Δ/σ				
	ε	**0.5**	**1**	**1.5**	**2**	**2.5**	**3**	**3.5**	**4**
m = 40	0.1	0.036	0.081	0.11	0.18	0.273	0.371	0.438	0.49
n = 40	0.2	0.058	0.114	0.233	0.43	0.645	0.793	0.887	0.924
r = 5	0.3	0.068	0.166	0.374	0.641	0.841	0.946	0.994	0.996
k = 5	0.4	0.079	0.229	0.507	0.777	0.923	0.994	0.998	1
α = 0.027	0.5	0.102	0.295	0.607	0.941	0.961	0.993	0.999	1
	0.6	0.116	0.36	0.682	0.891	0.977	0.995	0.999	1
	0.7	0.137	0.416	0.735	0.92	0.984	0.998	1	1
	0.8	0.16	0.469	0.79	0.943	0.988	0.999	1	1
	0.9	0.187	0.519	0.822	0.952	0.993	0.999	1	1
	1	0.202	0.556	0.847	0.961	0.993	1	1	1

					Δ/σ				
	ε	**0.5**	**1**	**1.5**	**2**	**2.5**	**3**	**3.5**	**4**
m = 50	0.1	0.037	0.064	0.116	0.176	0.251	0.306	0.339	0.358
n = 50	0.2	0.052	0.138	0.289	0.496	0.685	0.803	0.854	0.876
r = 11	0.3	0.080	0.23	0.512	0.778	0.925	0.975	0.991	0.994
k = 9	0.4	0.105	0.342	0.691	0.918	0.989	0.998	1	1
α = 0.026	0.5	0.134	0.435	0.806	0.972	0.998	1	1	1
	0.6	0.171	0.941	0.894	0.991	1	1	1	1
	0.7	0.199	0.627	0.935	0.996	1	1	1	1
	0.8	0.243	0.708	0.961	0.999	1	1	1	1
	0.9	0.282	0.769	0.978	1	1	1	1	1
	1	0.312	0.818	0.984	1	1	1	1	1

					Δ/σ				
	ε	**0.5**	**1**	**1.5**	**2**	**2.5**	**3**	**3.5**	**4**
m = 60	0.1	0.043	0.078	0.138	0.217	0.329	0.409	0.465	0.48
n = 60	0.2	0.064	0.157	0.344	0.591	0.792	0.897	0.942	0.953
r = 11	0.3	0.084	0.261	0.563	0.85	0.965	0.994	0.998	0.999
k = 9	0.4	0.107	0.374	0.75	0.952	0.995	1	1	1
α = 0.027	0.5	0.141	0.485	0.86	0.986	0.999	1	1	1
	0.6	0.183	0.586	0.917	0.994	1	1	1	1
	0.7	0.221	0.676	0.952	0.998	1	1	1	
	0.8	0.258	0.745	0.974	0.999	1	1	1	1
	0.9	0.301	0.806	0.982	1	1	1	1	1
	1	0.34	0.848	0.991	1	1	1	1	1

Table A.3 (continued) Power of the Quantile Test (α= 0.025)

		Δ/σ							
	ε	**0.5**	**1**	**1.5**	**2**	**2.5**	**3**	**3.5**	**4**
m = 75	0.1	0.036	0.078	0.142	0.242	0.361	0.45	0.507	0.526
n = 75	0.2	0.06	0.166	0.391	0.661	0.857	0.934	0.969	0.975
r = 14	0.3	0.082	0.293	0.644	0.906	0.987	0.999	1	1
k = 11	0.4	0.124	0.429	0.822	0.981	0.999	1	1	1
α = 0.023	0.5	0.159	0.561	0.918	0.996	1	1	1	1
	0.6	0.202	0.671	0.963	0.999	1	1	1	1
	0.7	0.243	0.761	0.982	1	1	1	1	1
	0.8	0.289	0.829	0.991	1	1	1	1	1
	0.9	0.339	0.878	0.995	1	1	1	1	1
	1	0.385	0.91	0.998	1	1	1	1	1

		Δ/σ							
	ε	**0.5**	**1**	**1.5**	**2**	**2.5**	**3**	**3.5**	**4**
m = 100	0.1	0.042	0.09	0.192	0.352	0.537	0.662	0.726	0.771
n = 100	0.2	0.065	0.205	0.497	0.797	0.953	0.991	0.997	0.999
r = 14	0.3	0.099	0.363	0.753	0.964	0.997	1	1	1
k = 11	0.4	0.138	0.509	0.891	0.993	1	1	1	1
α = 0.024	0.5	0.18	0.625	0.953	0.999	1	1	1	1
	0.6	0.234	0.745	0.98	1	1	1	1	1
	0.7	0.274	0.823	0.99	1	1	1	1	1
	0.8	0.333	0.874	0.995	1	1	1	1	1
	0.9	0.378	0.911	0.998	1	1	1	1	1
	1	0.44	0.938	0.999	1	1	1	1	1

Table A.3 (continued) Power of the Quantile Test (α = 0.05)

					Δ/σ				
	ϵ	**0.5**	**1**	**1.5**	**2**	**2.5**	**3**	**3.5**	**4**
m = 10	0.1	0.052	0.065	0.079	0.094	0.105	0.113	0.117	0.119
n = 10	0.2	0.062	0.092	0.132	0.177	0.218	0.25	0.27	0.28
r = 4	0.3	0.074	0.125	0.199	0.287	0.372	0.437	0.479	0.5
k = 4	0.4	0.086	0.162	0.276	0.411	0.536	0.629	0.686	0.714
α = 0.043	0.5	0.098	0.203	0.358	0.533	0.683	0.786	0.843	0.869
	0.6	0.112	0.247	0.439	0.641	0.797	0.89	0.936	0.955
	0.7	0.127	0.291	0.516	0.729	0.874	0.948	0.978	0.989
	0.8	0.142	0.336	0.584	0.796	0.921	0.975	0.993	0.998
	0.9	0.157	0.379	0.644	0.845	0.948	0.986	0.997	0.999
	1	0.173	0.422	0.695	0.88	0.964	0.992	0.998	1

					Δ/σ				
	ϵ	**0.5**	**1**	**1.5**	**2**	**2.5**	**3**	**3.5**	**4**
m = 15	0.1	0.062	0.081	0.106	0.136	0.164	0.186	0.2	0.207
n = 15	0.2	0.075	0.12	0.187	0.273	0.361	0.433	0.481	0.507
r = 4	0.3	0.09	0.165	0.284	0.431	0.572	0.68	0.745	0.779
k = 4	0.4	0.105	0.215	0.384	0.577	0.74	0.847	0.903	0.928
α = 0.05	0.5	0.122	0.267	0.478	0.694	0.85	0.934	0.97	0.983
	0.6	0.139	0.318	0.562	0.78	0.913	0.971	0.991	0.997
	0.7	0.157	0.369	0.633	0.839	0.947	0.986	0.997	0.999
	0.8	0.175	0.417	0.692	0.881	0.965	0.992	0.999	1
	0.9	0.194	0.462	0.739	0.909	0.976	0.995	0.999	1
	1	0.213	0.504	0.778	0.928	0.983	0.997	0.999	1

				Δ/σ					
	ϵ	**0.5**	**1**	**1.5**	**2**	**2.5**	**3**	**3.5**	**4**
m = 20	0.1	0.067	0.091	0.127	0.173	0.22	0.261	0.29	0.306
n = 20	0.2	0.083	0.139	0.232	0.354	0.481	0.586	0.655	0.693
r = 4	0.3	0.099	0.194	0.347	0.535	0.704	0.821	0.885	0.915
k = 4	0.4	0.118	0.252	0.458	0.678	0.842	0.932	0.97	0.984
α = 0.053	0.5	0.136	0.31	0.555	0.779	0.915	0.973	0.992	0.998
	0.6	0.156	0.366	0.634	0.845	0.951	0.988	0.998	1
	0.7	0.176	0.419	0.699	0.888	0.969	0.994	0.999	1
	0.8	0.197	0.468	0.749	0.916	0.979	0.996	0.999	1
	0.9	0.217	0.513	0.789	0.936	0.985	0.997	1	1
	1	0.238	0.554	0.821	0.949	0.989	0.998	1	1

					Δ/σ				
	ϵ	**0.5**	**1**	**1.5**	**2**	**2.5**	**3**	**3.5**	**4**
m = 25	0.1	0.065	0.091	0.127	0.169	0.206	0.233	0.248	0.254
n = 25	0.2	0.083	0.149	0.251	0.375	0.491	0.573	0.618	0.639
r = 7	0.3	0.104	0.219	0.399	0.599	0.755	0.945	0.887	0.903
k = 6	0.4	0.127	0.297	0.544	0.771	0.906	0.982	0.98	0.986
α = 0.049	0.5	0.153	0.377	0.667	0.879	0.968	0.993	0.998	0.999
	0.6	0.179	0.455	0.763	0.937	0.989	0.999	1	1
	0.7	0.207	0.528	0.832	0.987	0.996	1	1	1
	0.8	0.236	0.594	0.881	0.981	0.998	1	1	1
	0.9	0.265	0.652	0.915	0.989	0.999	1	1	1
	1	0.295	0.702	0.938	0.993	1	1	1	1

Table A.3 (continued) Power of the Quantile Test (α≈ 0.05)

					Δ/σ				
	ε	**0.5**	**1**	**1.5**	**2**	**2.5**	**3**	**3.5**	**4**
m = 30	0.1	0.069	0.1	0.146	0.202	0.256	0.297	0.321	0.332
n = 30	0.2	0.06	0.167	0.292	0.449	0.592	0.691	0.745	0.769
r = 7	0.3	0.113	0.246	0.457	0.681	0.84	0.92	0.951	0.963
k = 6	0.4	0.138	0.332	0.607	0.636	0.949	0.986	0.995	0.997
α = 0.051	0.5	0.166	0.417	0.724	0.919	0.985	0.998	1	1
	0.6	0.195	0.498	0.809	0.959	0.995	1	1	1
	0.7	0.225	0.571	0.868	0.979	0.998	1	1	1
	0.8	0.256	0.635	0.908	0.988	0.999	1	1	1
	0.9	0.288	0.69	0.934	0.993	1	1	1	1
	1	0.319	0.737	0.952	0.996	1	1	1	1

					Δ/σ				
	ε	**0.5**	**1**	**1.5**	**2**	**2.5**	**3**	**3.5**	**4**
m = 40	0.1	0.075	0.114	0.178	0.264	0.364	0.426	0.471	0.493
n = 40	0.2	0.099	0.196	0.363	0.568	0.742	0.848	0.899	0.919
r = 7	0.3	0.126	0.29	0.548	0.791	0.929	0.978	0.992	0.996
k = 6	0.4	0.155	0.387	0.695	0.907	0.982	0.998	1	1
α = 0.054	0.5	0.187	0.479	0.798	0.958	0.985	1	1	1
	0.6	0.219	0.561	0.866	0.98	0.998	1	1	1
	0.7	0.253	0.632	0.91	0.989	0.999	1	1	1
	0.8	0.287	0.693	0.938	0.994	1	1	1	1
	0.9	0.321	0.743	0.956	0.996	1	1	1	1
	1	0.354	0.784	0.968	0.998	1	1	1	1

					Δ/σ				
	ε	**0.5**	**1**	**1.5**	**2**	**2.5**	**3**	**3.5**	**4**
m = 50	0.1	0.067	0.108	0.176	0.266	0.356	0.423	0.463	0.48
n = 50	0.2	0.093	0.201	0.39	0.612	0.783	0.876	0.916	0.931
r = 10	0.3	0.123	0.313	0.606	0.85	0.959	0.989	0.996	0.998
k = 8	0.4	0.157	0.43	0.767	0.95	0.994	0.999	1	1
α = 0.046	0.5	0.194	0.54	0.869	0.984	0.999	1	1	1
	0.6	0.234	0.636	0.927	0.995	1	1	1	1
	0.7	0.275	0.715	0.959	0.998	1	1	1	1
	0.8	0.317	0.778	0.976	0.999	1	1	1	1
	0.9	0.359	0.828	0.986	1	1	1	1	1
	1	0.4	0.866	0.991	1	1	1	1	1

					Δ/σ				
	ε	**0.5**	**1**	**1.5**	**2**	**2.5**	**3**	**3.5**	**4**
m = 60	0.1	0.07	0.119	0.203	0.32	0.44	0.532	0.585	0.61
n = 60	0.2	0.099	0.224	0.446	0.696	0.865	0.942	0.969	0.977
r = 10	0.3	0.132	0.348	0.669	0.901	0.982	0.997	0.999	1
k = 8	0.4	0.17	0.472	0.818	0.971	0.998	1	1	1
α = 0.047	0.5	0.21	0.584	0.903	0.991	1	1	1	1
	0.6	0.253	0.678	0.948	0.997	1	1	1	1
	0.7	0.296	0.753	0.971	0.999	1	1	1	1
	0.8	0.34	0.811	0.984	1	1	1	1	1
	0.9	0.384	0.855	0.99	1	1	1	1	1
	1	0.426	0.888	0.994	1	1	1	1	1

Table A.3 (continued) Power of the Quantile Test (α ≈ 0.05)

		Δ/σ							
	ε	**0.5**	**1**	**1.5**	**2**	**2.5**	**3**	**3.5**	**4**
m = 75	0.1	0.075	0.132	0.24	0.394	0.553	0.672	0.739	0.769
n = 75	0.2	0.106	0.254	0.517	0.786	0.934	0.982	0.994	0.996
r = 10	0.3	0.143	0.392	0.738	0.944	0.994	1	1	1
k = 8	0.4	0.185	0.523	0.867	0.986	0.999	1	1	1
α = 0.049	0.5	0.229	0.635	0.933	0.996	1	1	1	1
	0.6	0.275	0.724	0.966	0.999	1	1	1	1
	0.7	0.322	0.793	0.981	0.999	1	1	1	1
	0.8	0.368	0.844	0.99	1	1	1	1	1
	0.9	0.413	0.883	0.994	1	1	1	1	1
	1	0.457	0.911	0.996	1	1	1	1	1

		Δ/σ							
	ε	**0.5**	**1**	**1.5**	**2**	**2.5**	**3**	**3.5**	**4**
m = 100	0.1	0.079	0.15	0.293	0.501	0.703	0.833	0.895	0.921
n = 100	0.2	0.116	0.294	0.606	0.875	0.978	0.997	1	1
r = 10	0.3	0.157	0.448	0.812	0.975	0.999	1	1	1
k = 8	0.4	0.204	0.584	0.914	0.994	1	1	1	1
α = 0.05	0.5	0.253	0.693	0.959	0.998	1	1	1	1
	0.6	0.303	0.776	0.98	0.999	1	1	1	1
	0.7	0.353	0.836	0.989	1	1	1	1	1
	0.8	0.402	0.879	0.994	1	1	1	1	1
	0.9	0.449	0.911	0.997	1	1	1	1	1
	1	0.494	0.933	0.998	1	1	1	1	1

Table A.3 (continued) Power of the Quantile Test (α = 0.10)

					Δ/σ				
	ε	**0.5**	**1**	**1.5**	**2**	**2.5**	**3**	**3.5**	**4**
m = 10	0.1	0.119	0.144	0.174	0.21	0.241	0.249	0.266	0.271
n = 10	0.2	0.138	0.197	0.257	0.336	0.41	0.463	0.496	0.512
r = 3	0.3	0.166	0.242	0.36	0.486	0.594	0.674	0.715	0.738
k = 3	0.4	0.179	0.306	0.457	0.607	0.734	0.822	0.866	0.878
α = 0.105	0.5	0.196	0.351	0.54	0.706	0.836	0.912	0.946	0.96
	0.6	0.227	0.4	0.607	0.789	0.909	0.958	0.983	0.991
	0.7	0.239	0.453	0.683	0.855	0.939	0.983	0.993	0.997
	0.8	0.264	0.491	0.735	0.892	0.963	0.991	0.998	1
	0.9	0.292	0.546	0.773	0.919	0.973	0.995	0.998	1
	1	0.301	0.581	0.803	0.936	0.984	0.998	0.999	1

					Δ/σ				
	ε	**0.5**	**1**	**1.5**	**2**	**2.5**	**3**	**3.5**	**4**
m = 15	0.1	0.131	0.171	0.217	0.262	0.313	0.36	0.386	0.394
n = 15	0.2	0.155	0.226	0.327	0.443	0.557	0.644	0.699	0.727
r = 3	0.3	0.176	0.285	0.443	0.614	0.749	0.847	0.889	0.912
k = 3	0.4	0.208	0.356	0.551	0.741	0.867	0.935	0.967	0.98
α = 0.113	0.5	0.227	0.414	0.644	0.816	0.924	0.975	0.992	0.995
	0.6	0.253	0.472	0.701	0.877	0.961	0.988	0.997	1
	0.7	0.271	0.517	0.758	0.909	0.975	0.993	0.999	1
	0.8	0.301	0.571	0.794	0.934	0.982	0.996	0.999	1
	0.9	0.322	0.603	0.833	0.952	0.988	0.999	1	1
	1	0.347	0.64	0.858	0.956	0.992	0.999	1	1

					Δ/σ				
	ε	**0.5**	**1**	**1.5**	**2**	**2.5**	**3**	**3.5**	**4**
m = 20	0.1	0.115	0.148	0.192	0.23	0.276	0.287	0.308	0.312
n = 20	0.2	0.136	0.219	0.325	0.443	0.54	0.605	0.636	0.653
r = 6	0.3	0.165	0.29	0.465	0.648	0.771	0.843	0.873	0.885
k = 5	0.4	0.19	0.379	0.605	0.793	0.906	0.956	0.972	0.978
α = 0.089	0.5	0.235	0.484	0.714	0.892	0.966	0.992	0.996	0.997
	0.6	0.261	0.522	0.802	0.935	0.988	0.998	1	1
	0.7	0.281	0.589	0.865	0.969	0.996	1	1	1
	0.8	0.319	0.661	0.902	0.983	0.999	1	1	1
	0.9	0.354	0.711	0.931	0.99	0.999	1	1	1
	1	0.38	0.754	0.947	0.994	1	1	1	1

					Δ/σ				
	ε	**0.5**	**1**	**1.5**	**2**	**2.5**	**3**	**3.5**	**4**
m = 25	0.1	0.127	0.167	0.229	0.283	0.333	0.376	0.395	0.403
n = 25	0.2	0.15	0.236	0.375	0.529	0.637	0.733	0.769	0.784
r = 6	0.3	0.177	0.332	0.532	0.742	0.858	0.922	0.947	0.96
k = 5	0.4	0.209	0.42	0.678	0.865	0.955	0.985	0.993	0.996
α = 0.093	0.5	0.238	0.501	0.769	0.934	0.984	0.997	1	1
	0.6	0.274	0.58	0.848	0.965	0.995	1	1	1
	0.7	0.319	0.651	0.895	0.983	0.998	1	1	1
	0.8	0.35	0.703	0.927	0.992	0.999	1	1	1
	0.9	0.375	0.743	0.949	0.994	1	1	1	1
	1	0.403	0.786	0.963	0.997	1	1	1	1

Table A.3 (continued) Power of the Quantile Test (α ≈ 0.10)

					Δ/σ				
	ε	**0.5**	**1**	**1.5**	**2**	**2.5**	**3**	**3.5**	**4**
m = 30	0.1	0.124	0.174	0.246	0.318	0.392	0.446	0.482	0.493
n = 30	0.2	0.156	0.257	0.418	0.601	0.731	0.821	0.861	0.879
r = 6	0.3	0.193	0.357	0.584	0.799	0.912	0.964	0.981	0.984
k = 5	0.4	0.221	0.457	0.718	0.906	0.976	0.995	0.999	1
α = 0.098	0.5	0.251	0.535	0.812	0.956	0.994	0.999	1	1
	0.6	0.293	0.612	0.88	0.979	0.998	1	1	1
	0.7	0.325	0.678	0.919	0.987	1	1	1	1
	0.8	0.36	0.735	0.943	0.994	1	1	1	1
	0.9	0.4	0.777	0.962	0.996	1	1	1	1
	1	0.43	0.824	0.973	0.999	1	1	1	1

					Δ/σ				
	ε	**0.5**	**1**	**1.5**	**2**	**2.5**	**3**	**3.5**	**4**
m = 40	0.1	0.134	0.192	0.278	0.393	0.507	0.582	0.624	0.652
n = 40	0.2	0.168	0.294	0.492	0.694	0.644	0.924	0.954	0.958
r = 6	0.3	0.198	0.403	0.662	0.879	0.966	0.993	0.997	0.999
k = 5	0.4	0.239	0.515	0.79	0.946	0.992	0.999	1	1
α = 0.098	0.5	0.285	0.593	0.874	0.975	0.997	1	1	1
	0.6	0.325	0.665	0.913	0.989	1	1	1	1
	0.7	0.36	0.73	0.943	0.995	1	1	1	1
	0.8	0.391	0.776	0.962	0.997	1	1	1	1
	0.9	0.43	0.811	0.973	0.998	1	1	1	1
	1	0.465	0.848	0.98	0.999	1	1	1	1

					Δ/σ				
	ε	**0.5**	**1**	**1.5**	**2**	**2.5**	**3**	**3.5**	**4**
m = 50	0.1	0.137	0.205	0.31	0.462	0.588	0.694	0.744	0.771
n = 50	0.2	0.179	0.326	0.548	0.768	0.913	0.966	0.987	0.992
r = 6	0.3	0.215	0.44	0.719	0.914	0.985	0.997	1	1
k = 5	0.4	0.256	0.544	0.834	0.966	0.997	1	1	1
α = 0.102	0.5	0.298	0.631	0.897	0.983	0.999	1	1	1
	0.6	0.34	0.707	0.938	0.994	1	1	1	1
	0.7	0.378	0.761	0.957	0.997	1	1	1	
	0.8	0.425	0.804	0.97	0.999	1	1	1	1
	0.9	0.456	0.846	0.98	0.999	1	1	1	1
	1	0.482	0.675	0.986	0.999	1	1	1	1

					Δ/σ				
	ε	**0.5**	**1**	**1.5**	**2**	**2.5**	**3**	**3.5**	**4**
m = 60	0.1	0.143	0.212	0.331	0.504	0.665	0.79	0.839	0.862
n = 60	0.2	0.179	0.345	0.596	0.833	0.945	0.986	0.997	0.998
r = 6	0.3	0.219	0.476	0.76	0.941	0.991	1	1	1
k = 5	0.4	0.268	0.568	0.861	0.977	0.997	1	1	1
α = 0.098	0.5	0.307	0.668	0.916	0.99	0.999	1	1	1
	0.6	0.356	0.734	0.95	0.996	1	1	1	1
	0.7	0.391	0.786	0.968	0.998	1	1	1	1
	0.8	0.427	0.826	0.978	0.998	1	1	1	1
	0.9	0.476	0.856	0.984	0.999	1	1	1	1
	1	0.492	0.889	0.989	1	1	1	1	1

Table A.3 (continued) Power of the Quantile Test (α≈ 0.10)

		Δ/σ							
	ε	**0.5**	**1**	**1.5**	**2**	**2.5**	**3**	**3.5**	**4**
m = 75	0.1	0.142	0.226	0.382	0.577	0.748	0.867	0.917	0.942
n = 75	0.2	0.188	0.37	0.638	0.868	0.975	0.995	0.999	1
r = 6	0.3	0.23	0.504	0.807	0.963	0.997	1	1	1
k = 5	0.4	0.281	0.608	0.893	0.985	0.999	1	1	1
α = 0.102	0.5	0.316	0.699	0.942	0.995	1	1	1	1
	0.6	0.363	0.762	0.963	0.997	1	1	1	1
	0.7	0.406	0.816	0.974	0.998	1	1	1	1
	0.8	0.445	0.844	0.981	1	1	1	1	1
	0.9	0.491	0.88	0.989	1	1	1	1	1
	1	0.536	0.905	0.991	1	1	1	1	1

		Δ/σ							
	ε	**0.5**	**1**	**1.5**	**2**	**2.5**	**3**	**3.5**	**4**
m = 100	0.1	0.145	0.248	0.435	0.665	0.847	0.939	0.975	0.986
n = 100	0.2	0.192	0.402	0.709	0.922	0.988	0.999	1	1
r = 6	0.3	0.232	0.549	0.851	0.979	0.999	1	1	1
k = 5	0.4	0.294	0.658	0.92	0.994	1	1	1	1
α = 0.102	0.5	0.342	0.735	0.954	0.996	1	1	1	1
	0.6	0.389	0.793	0.975	0.998	1	1	1	1
	0.7	0.436	0.845	0.982	0.999	1	1	1	1
	0.8	0.488	0.879	0.988	1	1	1	1	1
	0.9	0.513	0.895	0.992	1	1	1	1	1
	1	0.551	0.919	0.995	1	1	1	1	1

A.4 Values of r and k for the Quantile Test

In a report prepared at Pacific Nortwest Laboratory (PNL-7409), Gilbert and Simpson have calculated values of the parameters **r** and **k** needed for the Quantile Test for certain combinations of **m** and **n** (the number of measurements in the Reference area and the Survey Unit, respectively) when **m** and **n** are not equal. The value of α computed from simulation studies is also given. The following tables list these values for α approximately equal to 0.01, 0.025, 0.05, and 0.10.

Table A-4 Values of r and k for the Quantile Test when α is approximately 0.01.

m	Number of Survey Unit Measurements, n																			
	5	10	15	20	25	30	35	40	45	50	55	60	65	70	75	80	85	90	95	100
5	r,k α		11,11 0.008	13,13 0.015	16,16 0.014	19,19 0.013	22,22 0.013	25,25 0.013	28,28 0.012											r,k α
10		6,6 0.005	7,7 0.013	9,9 0.012	11,11 0.011	13,13 0.01	14,14 0.014	16,16 0.013	18,18 0.012	19,19 0.015	21,21 0.014	23,23 0.013	25,25 0.012	26,26 0.015	28,28 0.014	30,30 0.013				
15	3,3 0.009	7,6 0.007	6,6 0.008	7,7 0.011	8,8 0.014	10,10 0.009	11,11 0.011	12,12 0.013	13,13 0.014	15,15 0.011	16,16 0.012	17,17 0.013	18,18 0.014	19,19 0.015	21,21 0.012	22,22 0.013	23,23 0.014	24,24 0.015	26,26 0.013	27,27 0.013
20	6,4 0.005	4,4 0.008	5,5 0.009	6,6 0.01	7,7 0.011	8,8 0.011	9,9 0.011	10,10 0.011	11,11 0.011	12,12 0.011	13,13 0.011	14,14 0.012	15,15 0.012	16,16 0.012	17,17 0.012	18,18 0.012	19,19 0.012	19,19 0.015	20,20 0.015	21,21 0.015
25	4,3 0.009	7,5 0.012	4,4 0.015	5,5 0.013	6,6 0.011	7,7 0.01	8,8 0.009	9,9 0.009	9,9 0.014	10,10 0.012	11,11 0.011	12,12 0.011	12,12 0.015	13,13 0.014	14,14 0.013	15,15 0.012	16,16 0.011	16,16 0.014	17,17 0.014	18,18 0.013
30	4,3 0.006	3,3 0.012	4,4 0.009	5,5 0.007	6,6 0.006	6,6 0.012	7,7 0.01	8,8 0.008	8,8 0.013	9,9 0.011	10,10 0.009	10,10 0.013	11,11 0.011	1211 0.014	12,12 0.013	13,13 0.012	14,14 0.011	14,14 0.014	15,15 0.012	15,15 0.015
35	2,2 0.013	3,3 0.008	4,4 0.006	4,4 0.014	5,5 0.01	6,6 0.007	6,6 0.012	7,7 0.009	7,7 0.014	8,8 0.011	9,9 0.009	9,9 0.013	10,10 0.01	10,10 0.014	11,11 0.011	11,11 0.015	12,12 0.012	13,13 0.011	13,13 0.013	14,14 0.012
40	2,2 0.01	3,3 0.006	7,5 0.013	4,4 0.01	5,5 0.006	5,5 0.012	6,6 0.008	6,6 0.013	7,7 0.009	7,7 0.013	8,8 0.01	8,8 0.014	9,9 0.011	9,9 0.014	10,10 0.011	10,10 0.014	11,11 0.012	11,11 0.014	12,12 0.012	12,12 0.014
45	2,2 0.008	6,4 0.008	3,3 0.013	4,4 0.007	4,4 0.014	5,5 0.008	5,5 0.014	6,6 0.009	6,6 0.013	7,7 0.009	7.7 0.013	8,8 0.009	8,8 0.012	9,9 0.009	9,9 0.012	10,10 0.009	10,10 0.012	10,10 0.015	11,11 0.012	11,11 0.014
50		4,3 0.013	3,3 0.01	4,4 0.005	4,4 0.01	5,5 0.006	5,5 0.01	5,5 0.015	6,6 0.009	6,6 0.013	7,7 0.009	7,7 0.012	8,8 0.009	8,8 0.011	8,8 0.014	9,9 0.011	9,9 0.013	10,10 0.01	10,10 0.012	10,10 0.015
55		4,3 0.01	3,3 0.008	7,5 0.013	4,4 0.008	4,4 0.014	5,5 0.007	5,5 0.011	6,6 0.007	6,6 0.01	6,6 0.014	7,7 0.009	7,7 0.012	8,8 0.008	8,8 0.01	8,8 0.013	9,9 0.009	9,9 0.012	9,9 0.014	10,10 0.011
60		4,3 0.008	3,3 0.007	3,3 0.014	4,4 0.006	4,4 0.011	5,5 0.006	5,5 0.009	5,5 0.013	6,6 0.007	6,6 0.01	6,6 0.014	7,7 0.009	7,7 0.011	7,7 0.014	8,8 0.01	8,8 0.012	8,8 0.015	9,9 0.01	9,9 0.013
65		4,3 0.007	3,3 0.006	3,3 0.012	6,5 0.006	4,4 0.009	4,4 0.013	5,5 0.007	5,5 0.01	5,5 0.014	6,6 0.008	6,6 0.011	6,6 0.014	7,7 0.009	7,7 0.011	7,7 0.014	8,8 0.009	8,8 0.011	8,8 0.014	9,9 0.01
70		2,2 0.014	6,4 0.008	3,3 0.01	7,5 0.013	4,4 0.007	4,4 0.011	5,5 0.005	5,5 0.008	5,5 0.011	5,5 0.015	6,6 0.008	6,6 0.011	6,6 0.014	7,7 0.009	7,7 0.011	7,7 0.013	8,8 0.009	8,8 0.011	8,8 0.013
75		2,2 0.013	4,3 0.014	3,3 0.008	3,3 0.014	4,4 0.006	4,4 0.009	4,4 0.013	5,5 0.006	5,5 0.009	5,5 0.012	6,6 0.007	6,6 0.009	6,6 0.011	6,6 0.014	7,7 0.009	7,7 0.011	7,7 0.013	8,8. 0.008	8,8 0.01
80		2,2 0.011	4,3 0.012	3,3 0.007	3,3 0.012	6,5 0.006	4,4 0.008	4,4 0.011	5,5 0.005	5,5 0.007	5,5 0.01	5,5 0.013	6,6 0.007	6,6 0.009	6,6 0.012	6,6 0.014	7,7 0.009	7,7 0.01	7,7 0.013	7,7 0.015
85		2,2 0.01	4,3 0.01	3,3 0.006	3,3 0.011	7,5 0.013	4,4 0.006	4,4 0.009	4,4 0.013	5,5 0.006	5,5 0.008	5,5 0.011	5,5 0.014	6,6 0.008	6,6 0.01	6,6 0.012	6,6 0.014	7,7 0.008	7,7 0.01	7,7 0.012
90			4,3 0.009	3,3 0.005	3,3 0.009	3,3 0.014	4,4 0.005	4,4 0.008	4,4 0.011	5,5 0.005	5,5 0.007	5,5 0.009	5,5 0.012	5,5 0.015	6,6 0.008	6,6 0.01	6,6 0.012	6,6 0.014	7,7 0.008	7,7 0.019
95			4,3 0.008	6,4 0.008	3,3 0.008	3,3 0.013	6,5 0.005	4,4 0.007	4,4 0.01	4,4 0.013	5,5 0.006	5,5 0.008	5,5 0.01	5,5 0.013	6,6 0.007	6,6 0.008	6,6 0.01	6,6 0.012	6,6 0.014	7,7 0.008
100	r,k α		4,3 0.007	4,3 0.014	3,3 0.007	3,3 0.011	7,5 0.013	4,4 0.006	4,4 0.008	4,4 0.011	4,4 0.015	5,5 0.007	5,5 0.009	5,5 0.011	5,5 0.013	6,6 0.007	6,6 0.008	6,6 0.01	6,6 0.012	6,6 0.014

Table A-4 (continued) Values of r and k for the Quantile Test when α is approximately 0.025.

	Number of Survey Unit Measurements, n																			
m	**5**	**10**	**15**	**20**	**25**	**30**	**35**	**40**	**45**	**50**	**55**	**60**	**65**	**70**	**75**	**80**	**85**	**90**	**95**	**100**
5	r,k α		9,9 0.03	12,12 0.024	15,15 0.021	17,17 0.026	20,20 0.024	22,22 0.028	25,25 0.025											r,k α
10		7,6 0.029	6,6 0.028	8,8 0.022	9,9 0.029	11,11 0.024	12,12 0.029	14,14 0.025	17,17 0.025	18,18 0.029	20,20 0.026	21,21 0.029	23,23 0.026	24,24 0.029	26,26 0.026	27,27 0.029				
15	11,5 0.03	6,5 0.023	5,5 0.021	6,6 0.024	7,7 0.026	8,8 0.027	9,9 0.028	10,10 0.029	11,11 0.03	13,13 0.022	15,15 0.023	14,14 0.023	16,16 0.024	17,17 0.025	18,18 0.025	19,19 0.026	21,21 0.021	21,21 0.027	22,22 0.027	23,23 0.027
20	8,4 0.023	3,3 0.03	4,4 0.026	5,5 0.024	6,6 0.022	7,7 0.02	12,11 0.021	13,12 0.024	9,9 0.028	10,10 0.026	11,11 0.024	12,12 0.023	13,13 0.022	13,13 0.029	14,14 0.027	15,15 0.026	16 16 0.025	17,17 0.024	17,17 0.029	18,18 0.028
25	2,2 0.023	8,5 0.027	6,5 0.021	7,6 6.023	5,5 0.025	6,6 0.02	10,9 0.026	7,7 0.027	8,8 0.023	13,12 0.027	9,9 0.027	10,10 0.024	11,11 0.022	11,11 0.028	12,12 0.025	13,13 0.823	13,13 0.628	14,14 0.025	15,15 0.023	15,15 0.028
30	6,3 0.026	6,4 0.026	9,6 0.026	4,4 0.021	7,6 0.029	5,5 0.026	9,8 0.024	6,6 0.029	7,7 0.023	12,11 0.021	8,8 0.025	9,9 0.021	9,9 0.027	10,10 0.023	10,10 0.029	11,11 0.025	11,11 0.03	12,12 0.026	13,13 0.023	13,13 0.027
35	7,3 0.03	4,3 0.03	3,3 0.023	6,5 0.02	4,4 0.026	10,8 0.022	5,5 0.027	9,8 0.024	6,6 0.027	7,7 0.02	7,7 0.027	8,8 0.021	8,8 0.027	9,9 0.022	9,9 0.027	10,10 0.022	10,10 0.027	11,11 0.022	11,11 0.027	12,12 0.023
40	3,2 0.029	4,3 0.022	8,5 0.028	11,7 0.025	6,5 0.028	4,4 0.03	10,8 0.026	5,5 0.027	9,8 0.023	6,6 0.026	10,9 0.028	7,7 0.024	12,11 0.02	8,8 0.023	8,8 0.029	9,9 0.022	9,9 0.027	10,10 0.021	10,10 0.026	11,11 0.021
45	3,2 0.023	8,4 0.029	6,4 0.036	3,3 0.026	8,6 0.021	4,4 0.023	7,6 0.025	5,5 0.02	5,5 0.028	9,8 0.023	6,6 0.024	10,9 0.026	7,7 0.022	7,7 0.027	8,8 0.02	8,8 0.025	8,8 0.03	9,9 0.023	9,9 0.027	10,10 0.021
50		2,2 0.025	6,4 0.022	3,3 0.021	11,7 0.077	6,5 6.026	4,4 0.026	7,6 0.028	5,5 0.021	5,5 0.028	9,8 0.022	6,6 0.023	6,6 0.029	7,7 0.02	7,7 0.025	12,11 0.02	8,8 0.022	8,8 0.026	13,12 0.027	9,9 0.023
55		2,2 0.022	4,3 0.029	8,5 0.028	3,3 0.028	8,6 0.021	4,4 0.02	4,4 0.029	10,8 0.021	5,5 0.022	5,5 0.028	9,8 0.022	6,6 0.092	6,6 0.028	10,9 0.029	7,7 0.023	7,7 0.027	12,11 0.023	8,8 0.023	8,8 0.027
60		14,5 0.022	4,3 0.024	8,5 0.021	3,3 0.023	11,7 0.029	6,5 0.024	4,4 0.023	7,6 0.023	10,8 0.024	5,5 0.023	5,5 0.029	9,8 0.022	6,6 0.022	6,6 0.027	10,9 0.027	7,7 0.021	7,7 0.025	7,7 0.03	8,8 0.021
65		6,3 0.028	7,4 0.021	6,4 0.025	10,6 0.025	3,3 0.029	8,6 0.021	6,5 0.029	4,4 0.026	7,6 0.026	10,8 0.026	5,5 0.023	5,5 0.029	9,8 0.022	6,6 0.021	6,6 0.026	10,9 0.026	7,7 0.020	7,7 0.024	7,7 0.028
70		6,3 0.024	2,2 0.029	6,4 0.021	8,5 0.028	3,3 0.025	13,8 0.026	6,5 0.023	4,4 0.022	4,4 0.028	7,6 0.028	10,8 0.027	5,5 0.024	5,5 0.029	9,8 0.022	6,6 0.021	6,6 0.025	6,6 0.029	10,9 0.03	7,7 0.022
75		11,4 0.022	2,2 0.026	4,3 0.028	8,5 0.022	3,3 0.022	9,6 0.028	8,6 0.021	6,5 0.027	4,4 0.024	7,6 0.023	7,6 0.03	10,8 0.029	5,5 0.024	5,5 0.029	9,8 0.021	6,6 0.021	6,6 0.024	6,6 0.028	10,9 0.028
80		7,3 0.028	2,2 0.024	4,3 0.024	6,4 0.028	10,6 0.024	3,3 0.027	13,8 0.027	6,5 0.023	4,4 0.02	4,4 0.026	7,6 0.024	10,8 0.023	5,5 0.07	5,5 0.025	5,5 0.029	9,8 0.021	6,6 0.02	6,6 0.024	6,6 0.027
85		3,2 0.029	2,2 0.021	4,3 0.021	6,4 0.023	8,5 0.028	3,3 0.023	9,6 0.03	8,6 0.02	6,5 0.026	4,4 0.022	4,4 0.028	7,6 0.026	10,8 0.024	5,5 0.021	5,5 0.025	5,5 0.029	9,8 0.021	6,6 0.02	6,6 0.023
90			5,3 0.02	11,5 0.027	9,5 0.023	8,5 0.023	3,3 0.021	3,3 0.028	13,8 0.028	6,5 0.022	6,5 0.029	4,4 0.024	4,4 0.029	7,6 0.028	10,8 0.026	5,5 0.022	5,5 0.025	5,5 0.03	9,8 0.021	9,8 0.025
95			10,4 0.029	2,2 0.029	4,3 0.028	6,4 0.029	10,6 0.023	3,3 0.025	11,7 0.026	8,6 0.02	6,5 0.025	4,4 0.021	4,4 0.026	7,6 0.024	7,6 0.029	10,8 0.027	5,5 0.022	5,5 0.026	5,5 0.03	9,8 0.021
100	r,k α		6,3 0.029	2,2 0.027	4,3 0.025	6,4 0.025	8,5 0.028	3,3 0.022	3,3 0.029	13,8 0.028	6,5 0.022	6,5 0.028	4,4 0.023	4,4 0.027	7,6 0.025	10,8 0.022	10,8 0.028	5,5 0.022	5,5 0.026	5,5 0.03

Table A-4 (continued) Values of r and k for the Quantile Test when α is approximately 0.05.

	Number of Survey Unit Measurements, n																			
m	5	10	15	20	25	30	35	40	45	50	55	60	65	70	75	80	85	90	95	100
5	r,k α		8,8 0.051	10,10 0.057	13 13 0.043	15 15 0.048	17,17 0.051	19,19 0.054	21,21 0.056											r,k α
10		4,4 0.043	5,5 0.057	14,12 0.045	8,8 0.046	9,9 0.052	10,10 0.058	12,12 0.046	13,13 0.05	14,14 0.054	15,15 0.057	17,17 0.049	18,18 0.052	19,19 0.055	20,20 0.057	21,21 0.059	23,23 0.053			
15	2,2 0.053	3,3 0.052	4,4 0.05	5,5 0.048	6,6 0.046	7,7 0.045	8,8 0.052	9,9 0.043	9,9 0.06	10,10 0.057	11,11 0.055	12,12 0.054	13,13 0.052	14,14 0.051	15,15 0.05	16,16 0.049	16,16 0.058	17,17 0.057	18,18 0.056	19,19 0.055
20	9,4 0.04	8,5 0.056	6,5 0.04	4,4 0.053	5,5 0.043	9,8 0.052	6,6 0.056	7,7 48	8,8 0.043	8,8 0.057	9,9 0.051	10,10 0.046	10,10 0.057	11,11 0.052	12,12 0.048	12,12 0.057	13,13 0.053	14,14 0.049	14,14 0.057	15,15 0.054
25	6,3 0.041	6,4 0.043	3,3 0.046	6,5 0.052	4,4 0.055	5,5 0.041	5,5 0.059	6,6 0.046	11,10 0.042	7,7 0.05	8,8 0.042	8,8 0.053	9,9 0.045	9,9 0.055	10,10 0.048	11,11 0.042	11,11 0.05	11,11 0.058	12,12 0.052	12,12 0.06
30	3,2 0.047	2,2 0.058	10,6 0.052	3,3 0.058	11,8 0.045	4,4 0.056	8,7 0.044	5,5 0.054	6,6 0.04	6,6 0.053	7,7 0.041	7,7 0.052	8,8 0.042	8,8 0.051	9,9 0.042	9,9 0.05	9,9 0.059	10,10 0.049	10,10 0.057	11,11 0.049
35	8,3 0.046	2,2 0.045	6,4 0.058	3,3 0.043	6,5 0.041	4,4 0.04	4,4 0.057	8,7 0.043	5,5 0.051	9,8 0.052	6,6 0.047	6,6 0.057	7,7 0.043	7,7 0.053	8,8 0.041	8,8 0.049	8,8 0.057	9,9 0.046	9,9 0.053	10,10 0.044
40	4,2 0.055	5,3 0.048	4,3 0.057	10,6 0.059	3,3 0.053	6,5 0.048	4,4 0.043	4,4 0.058	8,7 0.042	5,5 0.048	9,8 0.047	6,6 0.042	6,6 0.051	11,10 0.042	7,7 0.045	7,7 0.053	8,8 0.041	8,8 0.048	8,8 0.055	9,9 0.043
45	4,2 0.045	9,4 0.047	2,2 0.059	8,5 0.052	3,3 0.042	8,6 0.041	6,5 0.054	4,4 0.045	4,4 0.058	8,7 0.041	5,5 0.046	5,5 0.057	9,8 0.056	6,6 0.047	6,6 0.055	11,10 0.046	7,7 0.047	7,7 0.054	8,8 0.041	8,8 0.047
50		6,3 0.051	2,2 0.05	6,4 0.051	12,7 0.05	3,3 0.049	8,6 0.049	6,5 0.059	4,4 0.047	4,4 0.059	8,7 0.041	5,5 0.045	5,5 0.054	9,8 0.051	6,6 0.043	6,6 0.05	6,6 0.058	7,7 0.041	7,7 0.048	7,7 0.054
55		3,2 0.039	2,2 0.043	4,3 0.056	8,5 0.058	3,3 0.041	5,4 0.041	6,5 0.046	9,7 0.042	4,4 0.048	4,4 0.059	8,7 0.04	5,5 0.043	5,5 0.052	9,8 0.048	6,6 0.04	6,6 0.047	6,6 0.054	11,10 0.043	7,7 0.043
60		3,2 0.052	5,3 0.052	4,3 0.046	6,4 0.059	3,3 0.035	3,3 0.047	8,6 0.043	6,5 51	9,7 0.046	4,4 0.049	4,4 0.059	13,10 0.052	5,5 0.042	5,5 0.05	5,5 0.058	9,8 0.054	6,6 0.044	6,6 0.05	6,6 0.056
65		3,2 0.045	5,3 0.043	2,2 0.053	6,4 0.048	10,6 0.05	3,3 0.04	3,3 0.052	6,5 0.041	6,5 0.055	4,4 0.042	4,4 0.05	4,4 0.06	13,10 0.052	5,5 0.041	5,5 0.048	5,5 0.055	9,8 0.051	6,6 0.041	6,6 0.047
70		8,3 0.057	9,4 0.048	2,2 0.047	4,3 0.055	8,5 0.05	5,4 0.041	3,3 0.046	3,3 0.057	6,5 0.045	6,5 0.058	4,4 0.043	4,4 0.051	4.4 0.06	13,10 0.051	5,5 0.041	5,5 0.047	5,5 0.054	9,8 0.048	9,8 0.057
75		8,3 0.049	6,3 0.056	2,2 0.043	4,3 0.047	6,4 0.054	10,6 0.053	3,3 0.04	3,3 0.051	8,6 0.044	6,5 0.049	9,7 0.041	4,4 0.044	4,4 0.052	5,5 0.06	13,10 0.051	8,7 0.047	5,5 0.046	5,5 0.052	5,5 0.058
80		4,2 0.059	6,3 0.048	5,3 0.053	2,2 0.055	6,4 0.046	8,5 0.055	5,4 0.041	3,3 0.045	3,3 0.055	6,5 0.041	6,5 0.052	9,7 0.043	4,4 0.045	4,4 0.053	7,6 0.058	13,10 0.051	8 7 0.046	5,5 0.045	5,5 0.051
85		4,2 0.054	3,2 0.058	5,3 0.047	2,2 0.05	4,3 0.054	4,3 0.048	10,6 0.056	5,4 0.049	3,3 0.049	3,3 0.059	6,5 0.044	6,5 0.055	9,7 0.046	4,4 0.046	4,4 0.053	7,6 0.059	10,8 0.06	8,7 0.045	5,5 0.044
90			3,2 0.053	5,3 0.041	2,2 0.046	6,4 0.059	6,4 0.051	8,5 0.058	5,4 0.042	3,3 0.044	3,3 0.053	8,6 0.045	6,5 0.047	6,5 0.058	4,4 0.041	4,4 0.047	4,4 0.054	7,6 0.059	10,8 0.06	8,7 0.041
95			3,2 0.048	9,4 0.048	2,2 0.042	2,2 0.056	4,3 0.059	8,5 0.05	10,6 0.058	5,4 0.048	3,3 0.048	3,3 0.056	6,5 0.041	6,5 0.05	9,7 0.040	4,4 0.042	4,4 0.048	4,4 0.054	7,6 0.59	10,8 0.059
100	r,k α		3,2 0.044	6,3 0.057	5,3 0.054	2,2 0.052	4,3 0.053	6,4 0.056	10,6 0.049	5,4 0.043	3,3 0.043	3,3 0.051	3,3 0.059	6,5 0.044	6,5 0.053	9,7 0.042	4,4 0.043	4,4 0.049	4,4 0.055	7,6 0.059

Table A-4 (continued) Values of r and k for the Quantile Test when α is approximately 0.10.

	Number of Survey Unit Measurements, n																			
m	**5**	**10**	**15**	**20**	**25**	**30**	**35**	**40**	**45**	**50**	**55**	**60**	**65**	**70**	**75**	**80**	**85**	**90**	**95**	**100**
5	r,k α		7,7 0.083	8,8 0.116	10,10 0.109	12,12 0.104	14,14 0.1	15.15 0.117	17,17 0.112											r,k α
10		3,3 0.105	4,4 0.108	5,5 0.109	6,6 0.109	7,7 0.109	8,8 0.109	9,9 0.109	10,10 0.109	11,11 0.109	12,12 0.109	13,13 0.109	14,14 0.109	15,15 0.109	16.16 0.109	1712 0.109	18,18 0.109			
15	9,4 0.098	10,6 0.106	3,3 0.112	4,4 0.093	5,5 0.081	5,5 0.117	6,6 0.102	7,7 0.092	7,7 0.118	8,8 0.106	9,9 0.098	9,9 0.118	10,10 0.109	11,11 0.101	11,11 0.118	12,12 0.11	13,13 0.104	13,13 0.118	14,14 0.111	15,15 0.106
20	3,2 0.091	2,2 0.103	5,4 0.093	3,3 0.115	4,4 0.085	4,4 0.119	5,5 0.093	10,9 0.084	6,6 0.099	7,7 0.083	7,7 0.102	8,8 0.088	8,8 0.105	9,9 0.092	9,9 0.107	10,10 0.095	10,11 0.108	11,11 0.098	11,11 0.11	12,12 0.1
25	4,2 0.119	7,4 0.084	8,5 0.112	3,3 0.08	3,3 0.117	4,4 0.08	4,4 0.107	8,7 0.108	5,5 0.101	10,9 0.088	6,6 0.096	6,6 0.114	7,7 0.093	7,7 0.108	8,8 0.091	8,8 0.104	8,8 0.117	9,9 0.1	9,9 0.112	10,10 0.098
30	4,2. 0.089	5,3 0.089	2,2 0.106	14,8 0.111	3,3 0.088	3,3 0.119	9,7 0.116	4,4 0.1	8,7 0.093	5,5 0.088	5,5 0.106	6,6 0.08	6,6 0.095	6,6 0.11	7,7 0.087	7,7 0.1	7,7 0.113	8,8 0.092	8,8 0.103	8,8 0.115
35	5,2 0.109	3,2 0.119	2,2 0.086	6,4 0.12	5,4 0.091	3,3 0.093	3,3 0.12	9,7 0.112	4,4 0.094	4,4 0.114	8,7 0.107	5,5 0.094	5,5 0.11	6,6 0.081	6,6 0.094	6,6 0.107	6,6 0.12	7,7 0.094	7,7 0.105	7,7 0.116
40	5,2 0.087	3,2 0.098	5,3 0.119	2,2 0.107	12,7 0.109	5,4 0.102	3,3 0.097	6,5 0.100	9,7 0.109	4,4 0.09	4,4 0.107	8,7 0.097	5,5 0.086	5,5 0.099	5,5 0.112	6,6 0.082	6,6 0.093	6,6 0.104	6,6 0.116	7,7 0.089
45	6,2 0.103	3,2 0.082	5,3 0.094	2,2 0.091	6,4 0.115	7,5 0.086	5,4 0.112	3,3 0.1	6,5 0.101	9,7 0.107	4,4 0.087	4,4 0.102	4,4 0.117	8,7 0.107	5,5 0.091	5,5 0.103	5,5 0.115	6,6 0.083	6,6 0.093	6,6 0.103
50		7,3 0.083	9,4 0.115	7,4 0.097	2,2 0.108	10,6 0.112	5,4 0.09	3,3 0.084	3,3 0.103	6,5 0.102	9,7 0.105	4,4 0.084	4,4 0.098	4,4 0.112	8,7 0.099	5,5 0.084	5,5 0.95	5.5 0.105	5,5 0.116	6,6 0.083
55		4,2 0.109	3,2 0.114	5,3 0.114	2,2 0.095	6,4 0.112	14,8 0.111	5,4 0.098	3,3 0.088	3,3 0.104	6,5 0.103	9,7 0.104	4,4 0.082	4,4 0.095	4,4 0.107	4,4 0.12	8,7 0.107	5,5 0.088	5,5 0.098	5,5 0.108
60		4,2 0.095	3,2 0.1	5,3 0.097	2,2 0.084	2,2 0.109	8,5 0.119	5,4 0.082	5,4 0.105	3,3 0.091	3,3 0.106	6,5 0.103	9,7 0.102	4,4 0.081	4,4 0.092	4,4 0.103	4,4 0.115	8,7 0.1	5,5 0.083	5,5 0.092
65		4,2 0.084	3,2 0.089	5,3 0.082	7,4 0.090	2,2 0.097	6,4 0.11	12,7 0.113	5,4 0.089	5,4 0.111	3,3 0.093	3,3 0.108	6,5 0.104	9,7 0.101	7,6 0.084	4,4 0.09	4,4 0.1	4,4 0.11	8,7 0.094	8,7 0.107
70		5,2 0.115	7,3 0.101	9,4 0.106	5,3 0.112	2,2 0.088	2,2 0.109	8,5 0.114	7,5 0.081	5,4 0.096	3,3 0.083	3,3 0.096	3,3 0.109	6,5 0.104	9,7 0.191	7,6 0.082	4,4 0.088	4,4 0.097	4,4 0.107	4,4 0.117
75		5,2 103	7,3 0.088	3,2 0.111	5,3 0.098	7,4 0.101	2,2 0.099	2,2 0.119	10,6 0.117	5,4 0.083	5,4 0.102	3,3 0.085	3,3 0.098	3,3 0.11	6,5 0.105	9,7 0.1	7,6 0.081	4,4 0.086	4,4 0.095	4,4 0.104
80		5,2 0.093	4,2 0.116	3,2 0.101	5,3 0.086	7,4 0.086	2,2 0.091	2,2 0.109	8,5 0.111	14,8 0.11	5,4 0.089	5,4 0.107	3,3 0.088	3,3 0.099	3,3 0.111	6,5 0.105	6,5 0.12	9,7 0.116	4,4 0.084	4,4 0.093
85		5,2 0.084	4,2 0.106	3,2 0.092	9,4 117	5,3 0.111	2,2 0.083	2,2 0.101	2,2 0.118	10,6 0.112	7,5 0.084	5,4 0.094	5,4 0.111	3,3 [illegible]	3,3 [illegible]	3,3 0.112	6,5 0.105	6,5 0.119	9,7 0.114	4,4 0.083
90			4,2 0.097	3,2 0.085	3,2 0.119	5,3 0.099	7,4 0.095	2,2 0.093	2,2 0.109	8,5 0.108	12,7 0.114	5,4 0.083	5,4 0.099	3,3 0.082	3,3 0.092	3,3 0.102	3,3 0.113	6,5 0.105	6,5 0.119	9,7 0.113
95			4,2 0.089	7,3 100	3,2 0.11	5,3 0.089	7,4 0.084	2,2 0.086	2,2 0.102	2,2 0.117	10,6 0.08	14,8 0.117	5,4 0.088	5,4 0.103	3,3 0.084	3,3 0.094	3,3 0.103	3,3 0.113	6,5 0.106	6,5 0.118
100	r,k α		4,2 0.082	7,3 0.09	3,2 0.102	5,3 0.08	5,3 0.109	2,2 0.08	2,2 0.095	2,2 0.11	6,4 0.118	12,7 0.109	7,5 0.086	5,4 0.093	5,4 0.08	3,3 0.086	3,3 0.095	3,3 0.104	3,3 0.114	6,5 0.106

A.5 Probability of Detecting an Elevated Area

This table provides the risk that an elevated area with length L/G and shape S will not be detected and describe the area (%) of the elevated area relative to a triangular sample grid area $0.866G^2$)

Table A.5 Probability of Detecting an Elevated Area

	Shape Parameter, S																			
	0.10		0.20		0.30		0.40		0.50		0.60		0.70		0.80		0.90		1.00	
L/G	Risk	Area	Risk	Area	Risk	Area	Risk	Area	Risk	Area	Risk	Area	Risk	Area	Risk	Area	Risk	Area	Risk	Area
0.01	1.00	<1%	1.00	<1%	1.00	<1%	1.00	<1%	1.00	<1%	1.00	<1%	1.00	<1%	1.00	<1%	1.00	<1%	1.00	<1%
0.02	1.00	<1%	1.00	<1%	1.00	<1%	1.00	<1%	1.00	<1%	1.00	<1%	1.00	<1%	1.00	<1%	1.00	<1%	1.00	<1%
0.03	1.00	<1%	1.00	<1%	1.00	<1%	1.00	<1%	1.00	<1%	1.00	<1%	1.00	<1%	1.00	<1%	1.00	<1%	1.00	<1%
0.04	1.00	<1%	1.00	<1%	1.00	<1%	1.00	<1%	1.00	<1%	1.00	<1%	1.00	<1%	1.00	<1%	0.99	1%	0.99	1%
0.05	1.00	<1%	1.00	<1%	1.00	<1%	1.00	<1%	1.00	<1%	0.99	1%	0.99	1%	0.99	1%	0.99	1%	0.99	1%
0.06	1.00	<1%	1.00	<1%	1.00	<1%	0.99	<1%	0.99	1%	0.99	1%	0.99	1%	0.99	1%	0.99	1%	0.99	1%
0.07	1.00	<1%	1.00	<1%	0.99	1%	0.99	<1%	0.99	1%	0.99	1%	0.99	1%	0.99	1%	0.98	2%	0.98	2%
0.08	1.00	<1%	1.00	<1%	0.99	1%	0.99	<1%	0.99	1%	0.99	1%	0.98	2%	0.98	2%	0.98	2%	0.98	2%
0.09	1.00	<1%	0.99	1%	0.99	1%	0.99	1%	0.99	1%	0.98	2%	0.98	2%	0.98	2%	0.97	3%	0.97	3%
0.1	1.00	<1%	0.99	1%	0.99	1%	0.99	1%	0.98	2%	0.98	2%	0.97	3%	0.97	3%	0.97	3%	0.96	4%
0.11	1.00	<1%	0.99	1%	0.99	1%	0.98	2%	0.98	2%	0.97	3%	0.97	3%	0.96	4%	0.96	4%	0.96	4%
0.12	0.99	1%	0.99	1%	0.98	2%	0.98	2%	0.97	3%	0.97	3%	0.96	4%	0.96	4%	0.95	5%	0.95	5%
0.13	0.99	1%	0.99	1%	0.98	2%	0.98	2%	0.97	3%	0.96	4%	0.96	4%	0.95	5%	0.94	6%	0.94	6%
0.14	0.99	1%	0.99	1%	0.98	2%	0.97	3%	0.96	4%	0.96	4%	0.95	5%	0.94	6%	0.94	6%	0.93	7%
0.15	0.99	1%	0.98	2%	0.98	2%	0.97	3%	0.96	4%	0.95	5%	0.94	6%	0.93	7%	0.93	7%	0.92	8%
0.16	0.99	1%	0.98	2%	0.97	3%	0.96	4%	0.95	5%	0.94	6%	0.94	7%	0.93	7%	0.92	8%	0.91	9%
0.17	0.99	1%	0.98	2%	0.97	3%	0.96	4%	0.95	5%	0.94	6%	0.93	7%	0.92	8%	0.91	9%	0.90	10%
0.18	0.99	1%	0.98	2%	0.96	4%	0.95	5%	0.94	6%	0.93	7%	0.92	8%	0.91	9%	0.89	11%	0.88	12%
0.19	0.99	1%	0.97	3%	0.96	4%	0.95	5%	0.93	7%	0.92	8%	0.91	9%	0.90	10%	0.88	12%	0.87	13%
0.2	0.99	1%	0.97	3%	0.96	4%	0.94	6%	0.93	7%	0.91	9%	0.90	10%	0.88	12%	0.87	13%	0.85	15%
0.21	0.98	2%	0.97	3%	0.95	5%	0.94	6%	0.92	8%	0.90	10%	0.89	11%	0.87	13%	0.86	14%	0.84	16%
0.22	0.98	2%	0.96	4%	0.95	5%	0.93	7%	0.91	9%	0.89	11%	0.88	12%	0.86	14%	0.84	16%	0.82	18%
0.23	0.98	2%	0.96	4%	0.94	6%	0.92	8%	0.90	10%	0.88	12%	0.87	13%	0.85	15%	0.83	17%	0.81	19%
0.24	0.98	2%	0.96	4%	0.94	6%	0.92	8%	0.90	10%	0.87	13%	0.85	15%	0.83	17%	0.81	19%	0.79	21%
0.25	0.98	2%	0.95	5%	0.93	7%	0.91	9%	0.89	11%	0.86	14%	0.84	16%	0.82	18%	0.80	20%	0.77	23%
0.26	0.98	2%	0.95	5%	0.93	7%	0.90	10%	0.88	12%	0.85	15%	0.83	17%	0.80	20%	0.78	22%	0.75	25%
0.27	0.97	3%	0.95	5%	0.92	8%	0.89	11%	0.87	13%	0.84	16%	0.81	19%	0.79	21%	0.76	24%	0.74	26%
0.28	0.97	3%	0.94	6%	0.91	9%	0.89	11%	0.86	14%	0.83	17%	0.80	20%	0.77	23%	0.74	26%	0.72	28%
0.29	0.97	3%	0.94	6%	0.91	9%	0.88	12%	0.85	15%	0.82	18%	0.79	21%	0.76	24%	0.73	27%	0.69	31%
0.3	0.97	3%	0.93	7%	0.90	10%	0.87	13%	0.84	16%	0.80	20%	0.77	23%	0.74	26%	0.71	29%	0.67	33%

Table A-5 (continued) Risk that an Elevated Area with length L/G and Shape S will not be detected and the Area (%) of the elevated area relative to a triangular sample grid area 0.866G^2)

	Shape Parameter, S																			
	0.10		0.20		0.30		0.40		0.50		0.60		0.70		0.80		0.90		1.00	
L/G	**Risk**	**Area**	**Risk**	**Area**	**Risk**	**Area**	**Risk**	**Area**	**Risk**	**Area**	**Risk**	**Area**	**Risk**	**Area**	**Risk**	**Area**	**Risk**	**Area**	**Risk**	**Area**
0.31	0.97	3%	0.93	7%	0.90	10%	0.86	14%	0.83	17%	0.79	21%	0.76	24%	0.72	28%	0.69	31%	0.65	35%
0.32	0.96	4%	0.93	7%	0.89	11%	0.85	15%	0.81	19%	0.78	22%	0.74	26%	0.70	30%	0.67	33%	0.63	37%
0.33	0.96	4%	0.92	8%	0.88	12%	0.84	16%	0.80	20%	0.76	24%	0.72	28%	0.68	32%	0.64	36%	0.61	40%
0.34	0.96	4%	0.92	8%	0.87	13%	0.83	17%	0.79	21%	0.75	25%	0.71	29%	0.66	34%	0.62	38%	0.58	42%
0.35	0.96	4%	0.91	9%	0.87	13%	0.82	18%	0.78	22%	0.73	27%	0.69	31%	0.64	36%	0.60	40%	0.56	44%
0.36	0.95	5%	0.91	9%	0.86	14%	0.81	19%	0.76	24%	0.72	28%	0.67	33%	0.62	38%	0.58	42%	0.53	47%
0.37	0.95	5%	0.90	10%	0.85	15%	0.80	20%	0.75	25%	0.70	30%	0.65	35%	0.60	40%	0.55	45%	0.50	50%
0.38	0.95	5%	0.90	10%	0.84	16%	0.79	21%	0.74	26%	0.69	31%	0.63	37%	0.58	42%	0.53	47%	0.48	52%
0.39	0.94	6%	0.89	11%	0.83	17%	0.78	22%	0.72	28%	0.67	33%	0.61	39%	0.56	44%	0.50	50%	0.45	55%
0.4	0.94	6%	0.88	12%	0.83	17%	0.77	23%	0.71	29%	0.65	35%	0.59	41%	0.54	46%	0.48	52%	0.42	58%
0.41	0.94	6%	0.88	12%	0.82	18%	0.76	24%	0.70	30%	0.63	37%	0.57	43%	0.51	49%	0.45	55%	0.39	61%
0.42	0.94	6%	0.87	13%	0.81	19%	0.74	26%	0.68	32%	0.62	38%	0.55	45%	0.49	51%	0.42	58%	0.36	64%
0.43	0.93	7%	0.87	13%	0.80	20%	0.73	27%	0.66	34%	0.60	40%	0.53	47%	0.46	54%	0.40	60%	0.33	67%
0.44	0.93	7%	0.86	14%	0.79	21%	0.72	28%	0.65	35%	0.58	42%	0.51	49%	0.44	56%	0.37	63%	0.30	70%
0.45	0.93	7%	0.85	15%	0.78	22%	0.71	29%	0.63	37%	0.56	44%	0.49	51%	0.41	59%	0.34	66%	0.27	73%
0.46	0.92	8%	0.85	15%	0.77	23%	0.69	31%	0.62	38%	0.54	46%	0.46	54%	0.39	61%	0.31	69%	0.23	77%
0.47	0.92	8%	0.84	16%	0.76	24%	0.68	32%	0.60	40%	0.52	48%	0.44	56%	0.36	64%	0.28	72%	0.20	80%
0.48	0.92	8%	0.83	17%	0.75	25%	0.67	33%	0.58	42%	0.50	50%	0.41	59%	0.33	67%	0.25	75%	0.16	84%
0.49	0.91	9%	0.83	17%	0.74	26%	0.65	35%	0.56	44%	0.48	52%	0.39	61%	0.30	70%	0.22	78%	0.13	87%
0.5	0.91	9%	0.82	18%	0.73	27%	0.64	36%	0.55	45%	0.46	54%	0.37	63%	0.27	73%	0.18	82%	0.09	91%
0.51	0.91	9%	0.81	19%	0.72	28%	0.62	38%	0.53	47%	0.43	57%	0.34	66%	0.25	75%	0.15	85%	0.07	94%
0.52	0.90	10%	0.80	20%	0.71	29%	0.61	39%	0.51	49%	0.41	59%	0.32	69%	0.22	78%	0.13	88%	0.05	98%
0.53	0.90	10%	0.80	20%	0.70	31%	0.59	41%	0.49	51%	0.39	61%	0.29	71%	0.19	82%	0.10	92%	0.03	102%
0.54	0.89	11%	0.79	21%	0.68	32%	0.58	42%	0.47	53%	0.37	63%	0.27	74%	0.17	85%	0.08	95%	0.02	106%
0.55	0.89	11%	0.78	22%	0.67	33%	0.56	44%	0.46	55%	0.35	66%	0.24	77%	0.14	88%	0.06	99%	0.01	110%
0.56	0.89	11%	0.77	23%	0.66	34%	0.55	46%	0.44	57%	0.33	68%	0.22	80%	0.12	91%	0.04	102%	0.00	114%
0.57	0.88	12%	0.77	24%	0.65	35%	0.54	47%	0.42	59%	0.31	71%	0.20	83%	0.10	94%	0.02	106%	0.00	118%
0.58	0.88	12%	0.76	24%	0.64	37%	0.52	49%	0.40	61%	0.29	73%	0.18	85%	0.08	98%	0.01	110%	0.00	122%
0.59	0.87	13%	0.75	25%	0.63	38%	0.51	51%	0.39	63%	0.27	76%	0.16	88%	0.06	101%	0.00	114%	0.00	126%
0.6	0.87	13%	0.74	26%	0.62	39%	0.49	52%	0.37	65%	0.25	78%	0.14	91%	0.04	104%	0.00	118%	0.00	131%
0.61	0.87	13%	0.73	27%	0.60	40%	0.48	54%	0.35	67%	0.23	81%	0.12	94%	0.03	108%	0.00	121%	0.00	135%
0.62	0.86	14%	0.73	28%	0.59	42%	0.46	56%	0.34	70%	0.21	84%	0.10	98%	0.02	112%	0.00	126%	0.00	139%
0.63	0.86	14%	0.72	29%	0.58	43%	0.45	58%	0.32	72%	0.20	86%	0.09	101%	0.01	115%	0.00	130%	0.00	144%
0.64	0.85	15%	0.71	30%	0.57	45%	0.43	59%	0.30	74%	0.18	89%	0.07	104%	0.00	119%	0.00	134%	0.00	149%
0.65	0.85	15%	0.70	31%	0.56	46%	0.42	61%	0.29	77%	0.16	92%	0.06	107%	0.00	123%	0.00	138%	0.00	153%
0.66	0.84	16%	0.69	32%	0.55	47%	0.40	63%	0.27	79%	0.15	95%	0.05	111%	0.00	126%	0.00	142%	0.00	158%
0.67	0.84	16%	0.68	33%	0.53	49%	0.39	65%	0.25	81%	0.13	98%	0.03	114%	0.00	130%	0.00	147%	0.00	163%
0.68	0.84	17%	0.68	34%	0.52	50%	0.38	67%	0.24	84%	0.12	101%	0.02	117%	0.00	134%	0.00	151%	0.00	168%
0.69	0.83	17%	0.67	35%	0.51	52%	0.36	69%	0.22	86%	0.10	104%	0.01	121%	0.00	138%	0.00	155%	0.00	173%
0.7	0.83	18%	0.66	36%	0.50	53%	0.35	71%	0.21	89%	0.09	107%	0.01	124%	0.00	142%	0.00	160%	0.00	178%

Table A-5 (continued) Risk that an Elevated Area with length L/G and Shape S will not be detected and the Area (%) of the elevated area relative to a triangular sample grid area $0.866G^2$)

	Shape Parameter, S																			
	0.10		0.20		0.30		0.40		0.50		0.60		0.70		0.80		0.90		1.00	
L/G	**Risk**	**Area**	**Risk**	**Area**	**Risk**	**Area**	**Risk**	**Area**	**Risk**	**Area**	**Risk**	**Area**	**Risk**	**Area**	**Risk**	**Area**	**Risk**	**Area**	**Risk**	**Area**
0.71	0.82	18%	0.65	37%	0.49	55%	0.33	73%	0.20	91%	0.08	110%	0.00	128%	0.00	146%	0.00	165%	0.00	183%
0.72	0.82	19%	0.64	38%	0.48	56%	0.32	75%	0.18	94%	0.07	113%	0.00	132%	0.00	150%	0.00	169%	0.00	188%
0.73	0.81	19%	0.63	39%	0.46	58%	0.31	77%	0.17	97%	0.05	116%	0.00	135%	0.00	155%	0.00	174%	0.00	193%
0.74	0.81	20%	0.62	40%	0.45	60%	0.29	79%	0.15	99%	0.04	119%	0.00	139%	0.00	159%	0.00	179%	0.00	199%
0.75	0.80	20%	0.61	41%	0.44	61%	0.28	82%	0.14	102%	0.04	122%	0.00	143%	0.00	163%	0.00	184%	0.00	204%
0.76	0.80	21%	0.61	42%	0.43	63%	0.27	84%	0.13	105%	0.03	126%	0.00	147%	0.00	168%	0.00	189%	0.00	210%
0.77	0.79	22%	0.60	43%	0.42	65%	0.25	86%	0.12	108%	0.02	129%	0.00	151%	0.00	172%	0.00	194%	0.00	215%
0.78	0.79	22%	0.59	44%	0.40	66%	0.24	88%	0.10	110%	0.01	132%	0.00	154%	0.00	177%	0.00	199%	0.00	221%
0.79	0.78	23%	0.58	45%	0.39	68%	0.23	91%	0.09	113%	0.01	136%	0.00	158%	0.00	181%	0.00	204%	0.00	226%
0.8	0.78	23%	0.57	46%	0.38	70%	0.22	93%	0.08	116%	0.00	139%	0.00	163%	0.00	186%	0.00	209%	0.00	232%
0.81	0.77	24%	0.56	48%	0.37	71%	0.20	95%	0.07	119%	0.00	143%	0.00	167%	0.00	190%	0.00	214%	0.00	238%
0.82	0.77	24%	0.55	49%	0.36	73%	0.19	98%	0.06	122%	0.00	146%	0.00	171%	0.00	195%	0.00	220%	0.00	244%
0.83	0.76	25%	0.54	50%	0.35	75%	0.18	100%	0.05	125%	0.00	150%	0.00	175%	0.00	200%	0.00	225%	0.00	250%
0.84	0.76	26%	0.53	51%	0.33	77%	0.17	102%	0.05	128%	0.00	154%	0.00	179%	0.00	205%	0.00	230%	0.00	256%
0.85	0.75	26%	0.52	52%	0.32	79%	0.16	105%	0.04	131%	0.00	157%	0.00	183%	0.00	210%	0.00	236%	0.00	262%
0.86	0.74	27%	0.51	54%	0.31	80%	0.14	107%	0.03	134%	0.00	161%	0.00	188%	0.00	215%	0.00	241%	0.00	268%
0.87	0.74	27%	0.50	55%	0.30	82%	0.13	110%	0.02	137%	0.00	165%	0.00	192%	0.00	220%	0.00	247%	0.00	275%
0.88	0.73	28%	0.50	56%	0.29	84%	0.12	112%	0.02	140%	0.00	169%	0.00	197%	0.00	225%	0.00	253%	0.00	281%
0.89	0.73	29%	0.49	57%	0.28	86%	0.11	115%	0.01	144%	0.00	172%	0.00	201%	0.00	230%	0.00	259%	0.00	287%
0.9	0.72	29%	0.48	59%	0.27	88%	0.10	118%	0.01	147%	0.00	176%	0.00	206%	0.00	235%	0.00	264%	0.00	294%
0.91	0.72	30%	0.47	60%	0.26	90%	0.10	120%	0.01	150%	0.00	180%	0.00	210%	0.00	240%	0.00	270%	0.00	300%
0.92	0.71	31%	0.46	61%	0.25	92%	0.09	123%	0.00	154%	0.00	184%	0.00	215%	0.00	246%	0.00	276%	0.00	307%
0.93	0.71	31%	0.45	63%	0.24	94%	0.08	126%	0.00	157%	0.00	188%	0.00	220%	0.00	251%	0.00	282%	0.00	314%
0.94	0.70	32%	0.44	64%	0.23	96%	0.07	128%	0.00	160%	0.00	192%	0.00	224%	0.00	256%	0.00	288%	0.00	321%
0.95	0.69	33%	0.43	65%	0.22	98%	0.07	131%	0.00	164%	0.00	196%	0.00	229%	0.00	262%	0.00	295%	0.00	327%
0.96	0.69	33%	0.42	67%	0.21	100%	0.06	134%	0.00	167%	0.00	201%	0.00	234%	0.00	267%	0.00	301%	0.00	334%
0.97	0.68	34%	0.41	68%	0.20	102%	0.05	137%	0.00	171%	0.00	205%	0.00	239%	0.00	273%	0.00	307%	0.00	341%
0.98	0.68	35%	0.40	70%	0.19	105%	0.05	139%	0.00	174%	0.00	209%	0.00	244%	0.00	279%	0.00	314%	0.00	348%
0.99	0.67	36%	0.40	71%	0.18	107%	0.04	142%	0.00	178%	0.00	213%	0.00	249%	0.00	284%	0.00	320%	0.00	356%
1	0.67	36%	0.39	73%	0.17	109%	0.04	145%	0.00	181%	0.00	218%	0.00	254%	0.00	290%	0.00	326%	0.00	363%

A.6 Random Numbers

Table A-6 1000 Random Numbers Uniformly Distributed Between 0 and 1.

0.163601	0.647423	0.555548	0.248859	0.259801	0.718368	0.305020	0.812482	0.601951	0.973160
0.934196	0.951102	0.979831	0.132364	0.157808	0.040605	0.997626	0.896462	0.360578	0.443218
0.054552	0.965257	0.999181	0.172627	0.583713	0.852958	0.116336	0.748483	0.058602	0.738495
0.972409	0.241889	0.799991	0.926726	0.585505	0.453993	0.877990	0.947022	0.910821	0.388081
0.556401	0.621126	0.293328	0.984335	0.366531	0.912588	0.733824	0.092405	0.717362	0.423421
0.625153	0.838711	0.196153	0.630553	0.867808	0.957094	0.830218	0.783518	0.141557	0.444997
0.527330	0.124034	0.351792	0.161947	0.688925	0.140346	0.553577	0.890058	0.470457	0.566196
0.826643	0.673286	0.550827	0.885295	0.690781	0.371540	0.108632	0.090765	0.618443	0.937184
0.296068	0.891272	0.392367	0.649633	0.261410	0.523221	0.769081	0.358794	0.924341	0.167665
0.848882	0.083603	0.274621	0.268003	0.272254	0.017727	0.309463	0.445986	0.244653	0.944564
0.779276	0.484461	0.101393	0.995100	0.085164	0.611426	0.030270	0.494982	0.426236	0.270225
0.095038	0.577943	0.186239	0.267852	0.786070	0.208937	0.184565	0.826397	0.256825	0.489034
0.011672	0.844846	0.443407	0.915087	0.275906	0.883009	0.243728	0.865552	0.796671	0.314429
0.215993	0.476035	0.354717	0.883172	0.840666	0.393867	0.374810	0.222167	0.114691	0.596046
0.982374	0.101973	0.683995	0.730612	0.548200	0.084302	0.145212	0.337680	0.566173	0.592776
0.860868	0.794380	0.819422	0.752871	0.158956	0.317468	0.062387	0.909843	0.779089	0.648967
0.718917	0.696798	0.463655	0.762408	0.823097	0.843209	0.368678	0.996266	0.542048	0.663842
0.800735	0.225556	0.398048	0.437067	0.642698	0.144068	0.104212	0.675095	0.318953	0.648478
0.915538	0.711742	0.232159	0.242961	0.327863	0.156608	0.260175	0.385141	0.681475	0.978186
0.975506	0.652654	0.928348	0.513444	0.744095	0.972031	0.527368	0.494287	0.602829	0.592834
0.435196	0.272807	0.452254	0.793464	0.817291	0.828245	0.407518	0.441518	0.358966	0.619741
0.692512	0.368151	0.821543	0.583707	0.802354	0.133831	0.569521	0.474516	0.437608	0.961559
0.678823	0.930602	0.657348	0.025057	0.294093	0.499623	0.006423	0.290613	0.325204	0.044439
0.642075	0.029842	0.289042	0.891009	0.813844	0.973093	0.952871	0.361623	0.709933	0.466955
0.174285	0.863244	0.133649	0.773819	0.891664	0.246417	0.272407	0.517658	0.132225	0.795514
0.951401	0.921291	0.210993	0.369411	0.196909	0.054389	0.364475	0.716718	0.096843	0.308418
0.186824	0.005407	0.310843	0.998118	0.725887	0.143171	0.293721	0.841304	0.661969	0.409622
0.105673	0.026338	0.878006	0.105936	0.612556	0.124601	0.922558	0.648985	0.896805	0.737256
0.801080	0.619461	0.933720	0.275881	0.637352	0.644996	0.713379	0.302687	0.904515	0.457172
0.101214	0.236405	0.945199	0.005975	0.893786	0.082317	0.648743	0.511871	0.298942	0.121573
0.177754	0.930066	0.390527	0.575622	0.390428	0.600575	0.460949	0.191600	0.910079	0.099444
0.846157	0.322467	0.156607	0.253388	0.739021	0.133498	0.293141	0.144834	0.626600	0.045169
0.812147	0.306383	0.201517	0.306651	0.827112	0.277716	0.660224	0.268538	0.518416	0.579216
0.691055	0.059046	0.104390	0.427038	0.148688	0.480788	0.026511	0.572705	0.745522	0.986078
0.483819	0.797573	0.174899	0.892670	0.118990	0.813221	0.857964	0.279164	0.883509	0.154562
0.165133	0.985134	0.214681	0.595309	0.741697	0.418602	0.301917	0.338913	0.680062	0.097350
0.281668	0.476899	0.839512	0.057760	0.474156	0.898409	0.482638	0.198725	0.888281	0.018872
0.554337	0.350955	0.942401	0.526759	0.509846	0.408165	0.800079	0.789263	0.564192	0.140684
0.873143	0.349662	0.238282	0.383195	0.568383	0.298471	0.490431	0.731405	0.339906	0.431645
0.401675	0.061151	0.771468	0.795760	0.365952	0.221234	0.947374	0.375686	0.828215	0.113060

Table A-6 (continued) 1000 Random Numbers Uniformly Distributed between 0 and 1.

0.574987	0.154831	0.808117	0.723544	0.134014	0.360957	0.166572	0.112314	0.242857	0.309290
0.745415	0.929459	0.425406	0.118845	0.386382	0.867386	0.808757	0.009573	0.229879	0.849242
0.613554	0.926550	0.857632	0.014438	0.004214	0.592513	0.280223	0.283447	0.943793	0.205750
0.880368	0.303741	0.247850	0.341580	0.867155	0.542130	0.473418	0.650251	0.326222	0.036285
0.567556	0.183534	0.696381	0.373333	0.716762	0.526636	0.306862	0.904790	0.151931	0.328792
0.280015	0.237361	0.336240	0.424191	0.192603	0.770194	0.284572	0.992475	0.308979	0.698329
0.502862	0.818555	0.238758	0.057148	0.461531	0.904929	0.521982	0.599127	0.239509	0.424858
0.738375	0.794328	0.305231	0.887161	0.021104	0.469779	0.913966	0.266514	0.647901	0.246223
0.366209	0.749763	0.634971	0.261038	0.869115	0.787951	0.678287	0.667142	0.216531	0.763214
0.739267	0.554299	0.979969	0.489597	0.545130	0.931869	0.096443	0.374089	0.140070	0.840563
0.375690	0.866922	0.256930	0.518074	0.217373	0.027043	0.801938	0.040364	0.624283	0.292810
0.894101	0.178824	0.443631	0.110614	0.556232	0.969563	0.291364	0.695764	0.306903	0.303885
0.668169	0.296926	0.324041	0.616290	0.799426	0.372555	0.070954	0.045748	0.505327	0.027722
0.470107	0.135634	0.271284	0.494071	0.485610	0.382772	0.418470	0.004082	0.298068	0.539847
0.047906	0.694949	0.309033	0.223989	0.008978	0.383695	0.479858	0.894958	0.597796	0.162072
0.917713	0.072793	0.107402	0.007328	0.176598	0.576809	0.052969	0.421803	0.737514	0.340966
0.839439	0.338565	0.254833	0.924413	0.871833	0.480599	0.172846	0.736102	0.471802	0.783451
0.488244	0.260352	0.129716	0.153558	0.305933	0.777100	0.111924	0.412930	0.601453	0.083217
0.488369	0.485094	0.322236	0.894264	0.781546	0.770237	0.707400	0.587451	0.571609	0.981580
0.311380	0.270400	0.807264	0.348433	0.172763	0.914856	0.011893	0.014317	0.820797	0.261767
0.028802	0.072165	0.944160	0.804761	0.770481	0.104256	0.112919	0.184068	0.940946	0.238087
0.466082	0.603884	0.959713	0.547834	0.487552	0.455150	0.240324	0.428921	0.648821	0.277620
0.720229	0.575779	0.939622	0.234554	0.767389	0.735335	0.941002	0.794021	0.291615	0.165732
0.861579	0.778039	0.331677	0.608231	0.646094	0.498720	0.140520	0.259197	0.782477	0.922273
0.849884	0.917789	0.816247	0.572502	0.753757	0.857324	0.988330	0.597085	0.186087	0.771997
0.989999	0.994007	0.349735	0.954437	0.741124	0.791852	0.986074	0.444554	0.177531	0.743725
0.337214	0.987184	0.344245	0.039033	0.549585	0.688526	0.225470	0.556251	0.157058	0.681447
0.706330	0.082994	0.299909	0.613361	0.031334	0.941102	0.772731	0.198070	0.460602	0.778659
0.417239	0.916556	0.707773	0.249767	0.169301	0.914420	0.732687	0.934912	0.985594	0.726957
0.653326	0.529996	0.305465	0.181747	0.153359	0.353168	0.673377	0.448970	0.546347	0.885438
0.099373	0.156385	0.067157	0.755573	0.689979	0.494021	0.996216	0.051811	0.049321	0.595525
0.860299	0.210143	0.026232	0.838499	0.108975	0.455260	0.320633	0.150619	0.445073	0.275619
0.067160	0.791992	0.363875	0.825052	0.047561	0.311194	0.447486	0.971659	0.876616	0.455018
0.944317	0.348844	0.210015	0.769274	0.253032	0.239894	0.208165	0.600014	0.945046	0.505316
0.917419	0.185575	0.743859	0.655124	0.185320	0.237660	0.271534	0.949825	0.441666	0.811135
0.365705	0.800723	0.116707	0.386073	0.837800	0.244896	0.337304	0.869528	0.845737	0.194553
0.911453	0.591254	0.920222	0.707522	0.782902	0.092884	0.426444	0.320336	0.226369	0.377845
0.027171	0.058193	0.726183	0.057705	0.935493	0.688071	0.752543	0.932781	0.048914	0.591035
0.768066	0.387888	0.655990	0.690208	0.746739	0.936409	0.685458	0.090931	0.242120	0.067899
0.052305	0.899285	0.092643	0.058916	0.826653	0.772790	0.785028	0.967761	0.588503	0.896590

Table A-6 (continued) 1000 Random Numbers Uniformly Distributed between 0 and 1.

0.623285	0.492051	0.644294	0.821341	0.600824	0.901289	0.774379	0.391874	0.810022	0.437879
0.624284	0.308522	0.208541	0.297156	0.576129	0.373705	0.370345	0.372748	0.965550	0.874416
0.853117	0.671602	0.018316	0.095780	0.871263	0.885420	0.919787	0.439594	0.460586	0.629443
0.967796	0.933631	0.397054	0.682343	0.505977	0.406611	0.539543	0.066152	0.885414	0.857606
0.759450	0.768853	0.115419	0.744466	0.607572	0.179839	0.413809	0.228607	0.362857	0.826932
0.514703	0.108915	0.864053	0.076280	0.352557	0.674917	0.572689	0.588574	0.596215	0.639101
0.826296	0.264540	0.255775	0.180449	0.405715	0.740170	0.423514	0.537793	0.877436	0.512284
0.354198	0.792775	0.051583	0.806962	0.385851	0.655314	0.046701	0.860466	0.848112	0.515684
0.744807	0.960789	0.123099	0.163569	0.621969	0.571558	0.482449	0.346358	0.795845	0.207558
0.642312	0.356643	0.797708	0.505570	0.418534	0.634642	0.033111	0.393330	0.105093	0.328848
0.824625	0.855876	0.770743	0.678619	0.927298	0.204828	0.831460	0.979875	0.566627	0.056160
0.755877	0.679791	0.442388	0.899944	0.563383	0.197074	0.679568	0.244433	0.786084	0.337991
0.625370	0.967123	0.321605	0.697578	0.122418	0.475395	0.068207	0.070374	0.353248	0.461960
0.124012	0.133851	0.761154	0.501578	0.204221	0.866481	0.925783	0.329001	0.327832	0.844681
0.825392	0.382001	0.847909	0.520741	0.404959	0.308849	0.418976	0.972838	0.452438	0.600528
0.999194	0.297058	0.617183	0.570478	0.875712	0.581618	0.284410	0.405575	0.362205	0.427077
0.536855	0.667083	0.636883	0.043774	0.113509	0.980045	0.237797	0.618925	0.670767	0.814902
0.361632	0.797162	0.136063	0.487575	0.682796	0.952708	0.759989	0.058556	0.292400	0.871674
0.923253	0.479871	0.022855	0.673915	0.733795	0.811955	0.417970	0.095675	0.831670	0.043950
0.845432	0.202336	0.348421	0.05[illegible]1	0.171916	0.600557	0.284838	0.606715	0.758190	0.394811

A.7 Normal Distribution

Approximations for the Normal Distribution Function

Values of the standard normal cumulative distribution funtion are given in Table A.7. In lieu of that table, the following approximations can be used with the aid of a pocket calculator or computer.

The Quantiles $Z_{1-\alpha}$, of the normal distribution are obtained from the equation:

$$1-\alpha = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{Z_{1-\alpha}} e^{-x^2/2} dx = \Phi(Z_{1-\alpha})$$

Approximations to $Z_{1-\alpha}$ may be obtained from:

$$Z_{1-\alpha} \approx \eta - \left(\frac{2.30753 + 0.27061\ \eta}{1 + 0.99229\ \eta + 0.04481\ \eta^2}\right)$$

where

$$\eta = \sqrt{\ln(1/\alpha^2)}$$

for $0 < \alpha \leq 0.5$ (Hastings, 1955).

Values of the cumulative normal distribution function, $\Phi(t)$, may be approximated by

$$\overline{\Phi}(t) = 1 - \Phi(t) \approx \begin{cases} 1/2\exp[-(t^2+1.2t^{0.8})/2] & \textit{for } 0 \leq t \leq 2.7 \\ \phi(t)/t = \frac{1}{\sqrt{2\pi}\cdot t}\exp[-(t^2/2)] & \textit{for } t > 2.7 \end{cases}$$

Table A-7 Cumulative Normal Distribution Function $\Phi(z)$

z	*0.00*	*0.01*	*0.02*	*0.03*	*0.04*	*0.05*	*0.06*	*0.07*	*0.08*	*0.09*
0.00	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.10	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5674	0.5714	0.5753
0.20	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.30	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.40	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.50	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.60	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.70	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.80	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.90	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.6315	0.8340	0.8365	0.8389
1.00	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.10	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.20	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.30	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.40	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.50	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.60	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.70	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.80	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.90	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.00	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.10	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.20	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.30	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.40	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.50	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.60	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.70	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.80	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.90	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.00	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990
3.10	0.9990	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993
3.20	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995
3.30	0.9995	0.9995	0.9995	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9997
3.40	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9998

Negative values of z can be obtained from the relationship $\Phi(-z) = 1 - \Phi(z)$.

A.8 Critical Values for the Wilcoxon Signed Ranks Test

Table A-8 Critical Values for the Wilcoxon Signed Ranks Test, $W_{1-\alpha}$

(adapted from Conover, 1980)

N	1-α						N	1-α					
	0.995	0.99	0.975	0.95	0.90	0.80		0.995	0.99	0.975	0.95	0.90	0.80
4	10	10	10	10	9	7	28	314	304	289	275	260	240
5	15	15	15	14	12	11	29	334	324	308	294	277	257
6	21	21	20	18	17	15	30	355	344	327	313	295	274
7	28	27	25	24	22	19	31	377	365	348	332	314	291
8	35	34	32	30	27	24	32	399	387	368	352	333	309
9	43	41	39	36	34	30	33	422	409	390	373	353	328
10	51	49	46	44	40	36	34	446	432	412	394	373	347
11	60	58	55	52	48	43	35	470	455	434	416	394	367
12	70	68	64	60	56	50	36	494	479	457	438	415	387
13	81	78	73	69	64	58	37	519	504	481	461	437	408
14	92	89	83	79	73	66	38	545	529	505	484	459	429
15	104	100	94	89	83	75	39	572	555	530	508	482	451
16	116	112	106	100	93	85	40	599	581	555	533	506	473
17	129	125	118	111	104	95	41	626	608	581	558	530	496
18	143	138	130	123	115	105	42	655	636	608	583	554	519
19	157	152	143	136	127	116	43	683	664	635	609	580	543
20	172	166	157	149	140	127	44	713	693	662	636	605	568
21	187	181	172	163	153	140	45	743	722	691	663	632	593
22	204	197	186	177	166	153	46	773	752	719	691	658	618
23	221	213	202	192	181	166	47	804	782	749	720	686	644
24	238	230	218	208	195	180	48	836	813	779	748	713	671
25	256	248	235	224	211	194	49	868	844	809	778	742	698
26	275	266	252	240	226	209	50	901	877	840	808	771	725
27	294	284	270	258	243	224							

Reject the null hypothesis if the sum of the ranks of the positive differences is greater than the table (critical) value.

If n is larger than 50, then the critical value can be calculated from

$$W_{1-\alpha} = \left[n(n+1)/4\right] + z_{1-\alpha}\sqrt{n(n+1)(2n+1)/24}$$

where $z_{1-\alpha}$ is the 1- α quantile of the normal distribution given in Table A.7.

Appendix A.9 Tables of the Binomial Distribution

Tabulated values are

$$\sum_{i=0}^{k} \binom{N}{i} p^{i}(1-p)^{n-i}$$

, for N from 1 to 20 and for p from 0.05 to 0.95. The value of p is given in the first row on each page.

Table A.9 Binomial Distribution

n	*k*	***0.0500***	*0.1000*	***0.1500***	***0.2000***	***0.2500***	***0.3000***	***0.3500***	***0.4000***	***0.4500***	***0.5000***	***0.5500***	***0.6000***	***0.6500***	***0.7000***	***0.7500***	***0.8000***	***0.8500***	***0.9000***	***0.9500***
1	*0*	0.9500	0.9000	0.8500	0.8000	0.7500	0.7000	0.6500	0.6000	0.5500	0.5000	0.4500	0.4000	0.3500	0.3000	0.2500	0.2000	0.1500	0.1000	0.0500
	1	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
2	*0*	0.9025	0.8100	0.7225	0.6400	0.5625	0.4900	0.4225	0.3600	0.3025	0.2500	0.2025	0.1600	0.1225	0.0900	0.0625	0.0400	0.0225	0.0100	0.0025
	1	0.9975	0.9900	0.9775	0.9600	0.9375	0.9100	0.8775	0.8400	0.7975	0.7500	0.6975	0.6400	0.5775	0.5100	0.4375	0.3600	0.2775	0.1900	0.0975
	2	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
3	*0*	0.8574	0.7290	0.6141	0.5120	0.4219	0.3430	0.274[illegible]	0.2160	0.1664	0.1250	0.0911	0.0640	0.0429	0.0270	0.0156	0.0080	0.0034	0.0010	0.0001
	1	0.9928	0.9720	0.9393	0.8960	0.8438	0.7840	0.7183	0.6480	0.5748	0.5000	0.4253	0.3520	0.2818	0.2160	0.1563	0.1040	0.0608	0.0280	0.0073
	2	0.9999	0.9990	0.9966	0.9920	0.9844	0.9730	0.9571	0.9360	0.9089	0.8750	0.8336	0.7840	0.7254	0.6570	0.5781	0.4880	0.3859	0.2710	0.1426
	3	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
4	*0*	0.8145	0.6561	0.5220	0.4096	0.3164	0.2401	0.1785	0.1296	0.0915	0.0625	0.0410	0.0256	0.0150	0.0081	0.0039	0.0016	0.0005	0.0001	0.0000
	1	0.9860	0.9477	0.8905	0.8192	0.7383	0.6517	0.5630	0.4752	0.3910	0.3125	0.2415	0.1792	0.1265	0.0837	0.0508	0.0272	0.0120	0.0037	0.0005
	2	0.9995	0.9963	0.9880	0.9728	0.9492	0.9163	0.8735	0.8208	0.7585	0.6875	0.6090	0.5248	0.4370	0.3483	0.2617	0.1808	0.1095	0.0523	0.0140
	3	1.0000	0.9999	0.9995	0.9984	0.9961	0.9919	0.9850	0.9744	0.9590	0.9375	0.9085	0.8704	0.8215	0.7599	0.6836	0.5904	0.4780	0.3439	0.1855
	4	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
5	*0*	0.7738	0.5905	0.4437	0.3277	0.2373	0.1681	0.1160	0.0778	0.0503	0.0313	0.0185	0.0102	0.0053	0.0024	0.0010	0.0003	0.0001	0.0000	0.0000
	1	0.9774	0.9185	0.8352	0.7373	0.6328	0.5282	0.4284	0.3370	0.2562	0.1875	0.1312	0.0870	0.0540	0.0308	0.0156	0.0067	0.0022	0.0005	0.0000
	2	0.9988	0.9914	0.9734	0.9421	0.8965	0.8369	0.7648	0.6826	0.5931	0.5000	0.4069	0.3174	0.2352	0.1631	0.1035	0.0579	0.0266	0.0086	0.0012
	3	1.0000	0.9995	0.9978	0.9933	0.9844	0.9692	0.9460	0.9130	0.8688	0.8125	0.7438	0.6630	0.5716	0.4718	0.3672	0.2627	0.1648	0.0815	0.0226
	4	1.0000	1.0000	0.9999	0.9997	0.9990	0.9976	0.9947	0.9898	0.9815	0.9688	0.9497	0.9222	0.8840	0.8319	0.7627	0.6723	0.5563	0.4095	0.2262
	5	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
6	*0*	0.7351	0.5314	0.3771	0.2621	0.1780	0.1176	0.0754	0.0467	0.0277	0.0156	0.0083	0.0041	0.0018	0.0007	0.0002	0.0001	0.0000	0.0000	0.0000
	1	0.9672	0.8857	0.7765	0.6554	0.5339	0.4202	0.3191	0.2333	0.1636	0.1094	0.0692	0.0410	0.0223	0.0109	0.0046	0.0016	0.0004	0.0001	0.0000
	2	0.9978	0.9842	0.9527	0.9011	0.8306	0.7443	0.6471	0.5443	0.4415	0.3438	0.2553	0.1792	0.1174	0.0705	0.0376	0.0170	0.0059	0.0013	0.0001
	3	0.9999	0.9987	0.9941	0.9830	0.9624	0.9295	0.8826	0.8208	0.7447	0.6563	0.5585	0.4557	0.3529	0.2557	0.1694	0.0989	0.0473	0.0159	0.0022
	4	1.0000	0.9999	0.9996	0.9984	0.9954	0.9891	0.9777	0.9590	0.9308	0.8906	0.8364	0.7667	0.6809	0.5798	0.4661	0.3446	0.2235	0.1143	0.0328
	5	1.0000	1.0000	1.0000	0.9999	0.9998	0.9993	0.9982	0.9959	0.9917	0.9844	0.9723	0.9533	0.9246	0.8824	0.8220	0.7379	0.6229	0.4686	0.2649
	6	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

Table A.9 Binomial Distribution (continued)

n	*k*	*0.0500*	*0.1000*	*0.1500*	*0.2000*	*0.2500*	*0.3000*	*0.3500*	*0.4000*	*0.4500*	*0.5000*	*0.5500*	*0.6000*	*0.6500*	*0.7000*	*0.7500*	*0.8000*	*0.8500*	*0.9000*	*0.9500*
7	*0*	0.6983	0.4783	0.3206	0.2097	0.1335	0.0824	0.0490	0.0280	0.0152	0.0078	0.0037	0.0016	0.0006	0.0002	0.0001	0.0000	0.0000	0.0000	0.0000
	1	0.9556	0.8503	0.7166	0.5767	0.4449	0.3294	0.2338	0.1586	0.1024	0.0625	0.0357	0.0188	0.0090	0.0038	0.0013	0.0004	0.0001	0.0000	0.0000
	2	0.9962	0.9743	0.9262	0.8520	0.7564	0.6471	0.5323	0.4199	0.3164	0.2266	0.1529	0.0963	0.0556	0.0288	0.0129	0.0047	0.0012	0.0002	0.0000
	3	0.9998	0.9973	0.9879	0.9667	0.9294	0.8740	0.8002	0.7102	0.6083	0.5000	0.3917	0.2898	0.1998	0.1260	0.0706	0.0333	0.0121	0.0027	0.0002
	4	1.0000	0.9998	0.9988	0.9953	0.9871	0.9712	0.9444	0.9037	0.8471	0.7734	0.6836	0.5801	0.4677	0.3529	0.2436	0.1480	0.0738	0.0257	0.0038
	5	1.0000	1.0000	0.9999	0.9996	0.9987	0.9962	0.9910	0.9812	0.9643	0.9375	0.8976	0.8414	0.7662	0.6706	0.5551	0.4233	0.2834	0.1497	0.0444
	6	1.0000	1.0000	1.0000	1.0000	0.9999	0.9998	0.9994	0.9984	0.9963	0.9922	0.9848	0.9720	0.9510	0.9176	0.8665	0.7903	0.6794	0.5217	0.3017
	7	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
8	*0*	0.6634	0.4305	0.2725	0.1678	0.1001	0.0576	0.0319	0.0168	0.0084	0.0039	0.0017	0.0007	0.0002	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000
	1	0.9428	0.8131	0.6572	0.5033	0.3671	0.2553	0.1691	0.1064	0.0632	0.0352	0.0181	0.0085	0.0036	0.0013	0.0004	0.0001	0.0000	0.0000	0.0000
	2	0.9942	0.9619	0.8948	0.7969	0.6785	0.5518	0.4278	0.3154	0.2201	0.1445	0.0885	0.0498	0.0253	0.0113	0.0042	0.0012	0.0002	0.0000	0.0000
	3	0.9996	0.9950	0.9786	0.9437	0.8862	0.8059	0.7064	0.5941	0.4770	0.3633	0.2604	0.1737	0.1061	0.0580	0.0273	0.0104	0.0029	0.0004	0.0000
	4	1.0000	0.9996	0.9971	0.9896	0.9727	0.9420	0.8939	0.8263	0.7396	0.6367	0.5230	0.4059	0.2936	0.1941	0.1138	0.0563	0.0214	0.0050	0.0004
	5	1.0000	1.0000	0.9998	0.9988	0.9958	0.9887	0.9747	0.9502	0.9115	0.8555	0.7799	0.6846	0.5722	0.4482	0.3215	0.2031	0.1052	0.0381	0.0058
	6	1.0000	1.0000	1.0000	0.9999	0.9996	0.9987	0.9964	0.9915	0.9819	0.9648	0.9368	0.8936	0.8309	0.7447	0.6329	0.4967	0.3428	0.1869	0.0572
	7	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9998	0.9993	0.9983	0.9961	0.9916	0.9832	0.9681	0.9424	0.8999	0.8322	0.7275	0.5695	0.3366
	8	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
9	*0*	0.6302	0.3874	0.2316	0.1342	0.0751	0.0404	0.0207	0.0101	0.0046	0.0020	0.0008	0.0003	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	1	0.9288	0.7748	0.5995	0.4362	0.3003	0.1960	0.1211	0.0705	0.0385	0.0195	0.0091	0.0038	0.0014	0.0004	0.0001	0.0000	0.0000	0.0000	0.0000
	2	0.9916	0.9470	0.8591	0.7382	0.6007	0.4628	0.3373	0.2318	0.1495	0.0898	0.0498	0.0250	0.0112	0.0043	0.0013	0.0003	0.0000	0.0000	0.0000
	3	0.9994	0.9917	0.9661	0.9144	0.8343	0.7297	0.6089	0.4826	0.3614	0.2539	0.1658	0.0994	0.0536	0.0253	0.0100	0.0031	0.0006	0.0001	0.0000
	4	1.0000	0.9991	0.9944	0.9804	0.9511	0.9012	0.8283	0.7334	0.6214	0.5000	0.3786	0.2666	0.1717	0.0988	0.0489	0.0196	0.0056	0.0009	0.0000
	5	1.0000	0.9999	0.9994	0.9969	0.9900	0.9747	0.9464	0.9006	0.8342	0.7461	0.6386	0.5174	0.3911	0.2703	0.1657	0.0856	0.0339	0.0083	0.0006
	6	1.0000	1.0000	1.0000	0.9997	0.9987	0.9957	0.9888	0.9750	0.9502	0.9102	0.8505	0.7682	0.6627	0.5372	0.3993	0.2618	0.1409	0.0530	0.0084
	7	1.0000	1.0000	1.0000	1.0000	0.9999	0.9996	0.9986	0.9962	0.9909	0.9805	0.9615	0.9295	0.8789	0.8040	0.6997	0.5638	0.4005	0.2252	0.0712
	8	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9997	0.9992	0.9980	0.9954	0.9899	0.9793	0.9596	0.9249	0.8658	0.7684	0.6126	0.3698
	9	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
10	*0*	0.5987	0.3487	0.1969	0.1074	0.0563	0.0282	0.0135	0.0060	0.0025	0.0010	0.0003	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	1	0.9139	0.7361	0.5443	0.3758	0.2440	0.1493	0.0860	0.0464	0.0233	0.0107	0.0045	0.0017	0.0005	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000
	2	0.9885	0.9298	0.8202	0.6778	0.5256	0.3828	0.2616	0.1673	0.0996	0.0547	0.0274	0.0123	0.0048	0.0016	0.0004	0.0001	0.0000	0.0000	0.0000
	3	0.9990	0.9872	0.9500	0.8791	0.7759	0.6496	0.5138	0.3823	0.2660	0.1719	0.1020	0.0548	0.0260	0.0106	0.0035	0.0009	0.0001	0.0000	0.0000
	4	0.9999	0.9984	0.9901	0.9672	0.9219	0.8497	0.7515	0.6331	0.5044	0.3770	0.2616	0.1662	0.0949	0.0473	0.0197	0.0064	0.0014	0.0001	0.0000
	5	1.0000	0.9999	0.9986	0.9936	0.9803	0.9527	0.9051	0.8338	0.7384	0.6230	0.4956	0.3669	0.2485	0.1503	0.0781	0.0328	0.0099	0.0016	0.0001
	6	1.0000	1.0000	0.9999	0.9991	0.9965	0.9894	0.9740	0.9452	0.8980	0.8281	0.7340	0.6177	0.4862	0.3504	0.2241	0.1209	0.0500	0.0128	0.0010
	7	1.0000	1.0000	1.0000	0.9999	0.9996	0.9984	0.9952	0.9877	0.9726	0.9453	0.9004	0.8327	0.7384	0.6172	0.4744	0.3222	0.1798	0.0702	0.0115
	8	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9995	0.9983	0.9955	0.9893	0.9767	0.9536	0.9140	0.8507	0.7560	0.6242	0.4557	0.2639	0.0861
	9	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9997	0.9990	0.9975	0.9940	0.9865	0.9718	0.9437	0.8926	0.8031	0.6513	0.4013
	10	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

Table A.9 Binomial Distribution (continued)

n	*k*	*0.0500*	*0.1000*	*0.1500*	*0.2000*	*0.2500*	*0.3000*	*0.3500*	*0.4000*	*0.4500*	*0.5000*	*0.5500*	*0.6000*	*0.6500*	*0.7000*	*0.7500*	*0.8000*	*0.8500*	*0.9000*	*0.9500*
11	*0*	0.5688	0.3138	0.1673	0.0859	0.0422	0.0198	0.0088	0.0036	0.0014	0.0005	0.0002	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	1	0.8981	0.6974	0.4922	0.3221	0.1971	0.1130	0.0606	0.0302	0.0139	0.0059	0.0022	0.0007	0.0002	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	2	0.9848	0.9104	0.7788	0.6174	0.4552	0.3127	0.2001	0.1189	0.0652	0.0327	0.0148	0.0059	0.0020	0.0006	0.0001	0.0000	0.0000	0.0000	0.0000
	3	0.9984	0.9815	0.9306	0.8389	0.7133	0.5696	0.4256	0.2963	0.1911	0.1133	0.0610	0.0293	0.0122	0.0043	0.0012	0.0002	0.0000	0.0000	0.0000
	4	0.9999	0.9972	0.9841	0.9496	0.8854	0.7897	0.6683	0.5328	0.3971	0.2744	0.1738	0.0994	0.0501	0.0216	0.0076	0.0020	0.0003	0.0000	0.0000
	5	1.0000	0.9997	0.9973	0.9883	0.9657	0.9218	0.8513	0.7535	0.6331	0.5000	0.3669	0.2465	0.1487	0.0782	0.0343	0.0117	0.0027	0.0003	0.0000
	6	1.0000	1.0000	0.9997	0.9980	0.9924	0.9784	0.9499	0.9006	0.8262	0.7256	0.6029	0.4672	0.3317	0.2103	0.1146	0.0504	0.0159	0.0028	0.0001
	7	1.0000	1.0000	1.0000	0.9998	0.9988	0.9957	0.9878	0.9707	0.9390	0.8867	0.8089	0.7037	0.5744	0.4304	0.2867	0.1611	0.0694	0.0185	0.0016
	8	1.0000	1.0000	1.0000	1.0000	0.9999	0.9994	0.9980	0.9941	0.9852	0.9673	0.9348	0.8811	0.7999	0.6873	0.5448	0.3826	0.2212	0.0896	0.0152
	9	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9998	0.9993	0.9978	0.9941	0.9861	0.9698	0.9394	0.8870	0.8029	0.6779	0.5078	0.3026	0.1019
	10	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9998	0.9995	0.9986	0.9964	0.9912	0.9802	0.9578	0.9141	0.8327	0.6862	0.4312
	11	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
12	*0*	0.5404	0.2824	0.1422	0.0687	0.0317	0.0138	0.0057	0.0022	0.0008	0.0002	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	1	0.8816	0.6590	0.4435	0.2749	0.1584	0.0850	0.0424	0.0196	0.0083	0.0032	0.0011	0.0003	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	2	0.9804	0.8891	0.7358	0.5583	0.3907	0.2528	0.1513	0.0834	0.0421	0.0193	0.0079	0.0028	0.0008	0.0002	0.0000	0.0000	0.0000	0.0000	0.0000
	3	0.9978	0.9744	0.9078	0.7946	0.6488	0.4925	0.3467	0.2253	0.1345	0.0730	0.0356	0.0153	0.0056	0.0017	0.0004	0.0001	0.0000	0.0000	0.0000
	4	0.9998	0.9957	0.9761	0.9274	0.8424	0.7237	0.5833	0.4382	0.3044	0.1938	0.1117	0.0573	0.0255	0.0095	0.0028	0.0006	0.0001	0.0000	0.0000
	5	1.0000	0.9995	0.9954	0.9806	0.9456	0.8822	0.7873	0.6652	0.5269	0.3872	0.2607	0.1582	0.0846	0.0386	0.0143	0.0039	0.0007	0.0001	0.0000
	6	1.0000	0.9999	0.9993	0.9961	0.9857	0.9614	0.9154	0.8418	0.7393	0.6128	0.4731	0.3348	0.2127	0.1178	0.0544	0.0194	0.0046	0.0005	0.0000
	7	1.0000	1.0000	0.9999	0.9994	0.9972	0.9905	0.9745	0.9427	0.8883	0.8062	0.6956	0.5618	0.4167	0.2763	0.1576	0.0726	0.0239	0.0043	0.0002
	8	1.0000	1.0000	1.0000	0.9999	0.9996	0.9983	0.9944	0.9847	0.9644	0.9270	0.8655	0.7747	0.6533	0.5075	0.3512	0.2054	0.0922	0.0256	0.0022
	9	1.0000	1.0000	1.0000	1.0000	1.0000	0.9998	0.9992	0.9972	0.9921	0.9807	0.9579	0.9166	0.8487	0.7472	0.6093	0.4417	0.2642	0.1109	0.0196
	10	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9997	0.9989	0.9968	0.9917	0.9804	0.9576	0.9150	0.8416	0.7251	0.5565	0.3410	0.1184
	11	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9998	0.9992	0.9978	0.9943	0.9862	0.9683	0.9313	0.8578	0.7176	0.4596
	12	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

Table A.9 Binomial Distribution (continued)

n	*k*	*0.0500*	*0.1000*	*0.1500*	*0.2000*	*0.2500*	*0.3000*	*0.3500*	*0.4000*	*0.4500*	*0.5000*	*0.5500*	*0.6000*	*0.6500*	*0.7000*	*0.7500*	*0.8000*	*0.8500*	*0.9000*	*0.9500*
13	*0*	0.5133	0.2542	0.1209	0.0550	0.0238	0.0097	0.0037	0.0013	0.0004	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	1	0.8646	0.6213	0.3983	0.2336	0.1267	0.0637	0.0296	0.0126	0.0049	0.0017	0.0005	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	2	0.9755	0.8661	0.6920	0.5017	0.3326	0.2025	0.1132	0.0579	0.0269	0.0112	0.0041	0.0013	0.0003	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000
	3	0.9969	0.9658	0.8820	0.7473	0.5843	0.4206	0.2783	0.1686	0.0929	0.0461	0.0203	0.0078	0.0025	0.0007	0.0001	0.0000	0.0000	0.0000	0.0000
	4	0.9997	0.9935	0.9658	0.9009	0.7940	0.6543	0.5005	0.3530	0.2279	0.1334	0.0698	0.0321	0.0126	0.0040	0.0010	0.0002	0.0000	0.0000	0.0000
	5	1.0000	0.9991	0.9925	0.9700	0.9198	0.8346	0.7159	0.5744	0.4268	0.2905	0.1788	0.0977	0.0462	0.0182	0.0056	0.0012	0.0002	0.0000	0.0000
	6	1.0000	0.9999	0.9987	0.9930	0.9757	0.9376	0.8705	0.7712	0.6437	0.5000	0.3563	0.2288	0.1295	0.0624	0.0243	0.0070	0.0013	0.0001	0.0000
	7	1.0000	1.0000	0.9998	0.9988	0.9944	0.9818	0.9538	0.9023	0.8212	0.7095	0.5732	0.4256	0.2841	0.1654	0.0802	0.0300	0.0075	0.0009	0.0000
	8	1.0000	1.0000	1.0000	0.9998	0.9990	0.9960	0.9874	0.9679	0.9302	0.8666	0.7721	0.6470	0.4995	0.3457	0.2060	0.0991	0.0342	0.0065	0.0003
	9	1.0000	1.0000	1.0000	1.0000	0.9999	0.9993	0.9975	0.9922	0.9797	0.9539	0.9071	0.8314	0.7217	0.5794	0.4157	0.2527	0.1180	0.0342	0.0031
	10	1.0000	1.9000	1.0000	1.0000	1.0000	0.9999	0.9997	0.9987	0.9959	0.9888	0.9731	0.9421	0.8868	0.7975	0.6674	0.4983	0.3080	0.1339	0.0245
	11	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9995	0.9983	0.9951	0.9874	0.9704	0.9363	0.8733	0.7664	0.6017	0.3787	0.1354
	12	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9996	0.9987	0.9963	0.9903	0.9762	0.9450	0.8791	0.7458	0.4867
	13	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
14	*0*	0.4877	0.2288	0.1028	0.0440	0.0178	0.0068	0.0024	0.0008	0.0002	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	1	0.8470	0.5846	0.3567	0.1979	0.1010	0.0475	0.0205	0.0081	0.0029	0.0009	0.0003	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	2	0.9699	0.8416	0.6479	0.4481	0.2811	0.1608	0.0839	0.0398	0.0170	0.0065	0.0022	0.0006	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	3	0.9958	0.9559	0.8535	0.6982	0.5213	0.3552	0.2205	0.1243	0.0632	0.0287	0.0114	0.0039	0.0011	0.0002	0.0000	0.0000	0.0000	0.0000	0.0000
	4	0.9996	0.9908	0.9533	0.8702	0.7415	0.5842	0.4227	0.2793	0.1672	0.0898	0.0426	0.0175	0.0060	0.0017	0.0003	0.0000	0.0000	0.0000	0.0000
	5	1.0000	0.9985	0.9885	0.9561	0.8883	0.7805	0.6405	0.4859	0.3373	0.2120	0.1189	0.0583	0.0243	0.0083	0.0022	0.0004	0.0000	0.0000	0.0000
	6	1.0000	0.9998	0.9978	0.9884	0.9617	0.9067	0.8164	0.6925	0.5461	0.3953	0.2586	0.1501	0.0753	0.0315	0.0103	0.0024	0.0003	0.0000	0.0000
	7	1.0000	1.0000	0.9997	0.9976	0.9897	0.9685	0.9247	0.8499	0.7414	0.6047	0.4539	0.3075	0.1836	0.0933	0.0383	0.0116	0.0022	0.0002	0.0000
	8	1.0000	1.0000	1.0000	0.9996	0.9978	0.9917	0.9757	0.9417	0.8811	0.7880	0.6627	0.5141	0.3595	0.2195	0.1117	0.0439	0.0115	0.0015	0.0000
	9	1.0000	1.0000	1.0000	1.0000	0.9997	0.9983	0.9940	0.9825	0.9574	0.9102	0.8328	0.7207	0.5773	0.4158	0.2585	0.1298	0.0467	0.0092	0.0004
	10	1.0000	1.0000	1.0000	1.0000	1.0000	0.9998	0.9989	0.9961	0.9886	0.9713	0.9368	0.8757	0.7795	0.6448	0.4787	0.3018	0.1465	0.0441	0.0042
	11	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9994	0.9978	0.9935	0.9830	0.9602	0.9161	0.8392	0.7189	0.5519	0.3521	0.1584	0.0301
	12	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9997	0.9991	0.9971	0.9919	0.9795	0.9525	0.8990	0.8021	0.6433	0.4154	0.1530
	13	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9998	0.9992	0.9976	0.9932	0.9822	0.9560	0.8972	0.7712	0.5123
	14	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

Table A.9 Binomial Distribution (continued)

n	*k*	*0.0500*	*0.1000*	*0.1500*	*0.2000*	*0.2500*	*0.3000*	*0.3500*	*0.4000*	*0.4500*	*0.5000*	*0.5500*	*0.6000*	*0.6500*	*0.7000*	*0.7500*	*0.8000*	*0.8500*	*0.9000*	*0.9500*
15	*0*	0.4633	0.2059	0.0874	0.0352	0.0134	0.0047	0.0016	0.0005	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	1	0.8290	0.5490	0.3186	0.1671	0.0802	0.0353	0.0142	0.0052	0.0017	0.0005	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	2	0.9638	0.8159	0.6042	0.3980	0.2361	0.1268	0.0617	0.0271	0.0107	0.0037	0.0011	0.0003	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	3	0.9945	0.9444	0.8227	0.6482	0.4613	0.2969	0.1727	0.0905	0.0424	0.0176	0.0063	0.0019	0.0005	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000
	4	0.9994	0.9873	0.9383	0.8358	0.6865	0.5155	0.3519	0.2173	0.1204	0.0592	0.0255	0.0093	0.0028	0.0007	0.0001	0.0000	0.0000	0.0000	0.0000
	5	0.9999	0.9978	0.9832	0.9389	0.8516	0.7216	0.5643	0.4032	0.2608	0.1509	0.0769	0.0338	0.0124	0.0037	0.0008	0.0001	0.0000	0.0000	0.0000
	6	1.0000	0.9997	0.9964	0.9819	0.9434	0.8689	0.7548	0.6098	0.4522	0.3036	0.1818	0.0950	0.0422	0.0152	0.0042	0.0008	0.0001	0.0000	0.0000
	7	1.0000	1.0000	0.9994	0.9958	0.9827	0.9500	0.8868	0.7869	0.6535	0.5000	0.3465	0.2131	0.1132	0.0500	0.0173	0.0042	0.0006	0.0000	0.0000
	8	1.0000	1.0000	0.9999	0.9992	0.9958	0.9848	0.9578	0.9050	0.8182	0.6964	0.5478	0.3902	0.2452	0.1311	0.0566	0.0181	0.0036	0.0003	0.0000
	9	1.0000	1.0000	1.0000	0.9999	0.9992	0.9963	0.9876	0.9662	0.9231	0.8491	0.7392	0.5968	0.4357	0.2784	0.1484	0.0611	0.0168	0.0022	0.0001
	10	1.0000	1.0000	1.0000	1.0000	0.9999	0.9993	0.9972	0.9907	0.9745	0.9408	0.8796	0.7827	0.6481	0.4845	0.3135	0.1642	0.0617	0.0127	0.0006
	11	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9995	0.9981	0.9937	0.9824	0.9576	0.9095	0.8273	0.7031	0.5387	0.3518	0.1773	0.0556	0.0055
	12	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9997	0.9989	0.9963	0.9893	0.9729	0.9383	0.8732	0.7639	0.6020	0.3958	0.1841	0.0362
	13	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9995	0.9983	0.9948	0.9858	0.9647	0.9198	0.8329	0.6814	0.4510	0.1710
	14	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9995	0.9984	0.9953	0.9866	0.9648	0.9126	0.7941	0.5367
	15	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
16	*0*	0.4401	0.1853	0.0743	0.0281	0.0100	0.0033	0.0010	0.0003	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	1	0.8108	0.5147	0.2839	0.1407	0.0635	0.0261	0.0098	0.0033	0.0010	0.0003	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	2	0.9571	0.7892	0.5614	0.3518	0.1971	0.0994	0.0451	0.0183	0.0066	0.0021	0.0006	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	3	0.9930	0.9316	0.7899	0.5981	0.4050	0.2459	0.1339	0.0651	0.0281	0.0106	0.0035	0.0009	0.0002	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	4	0.9991	0.9830	0.9209	0.7982	0.6302	0.4499	0.2892	0.1666	0.0853	0.0384	0.0149	0.0049	0.0013	0.0003	0.0000	0.0000	0.0000	0.0000	0.0000
	5	0.9999	0.9967	0.9765	0.9183	0.8103	0.6598	0.4900	0.3288	0.1976	0.1051	0.0486	0.0191	0.0062	0.0016	0.0003	0.0000	0.0000	0.0000	0.0000
	6	1.0000	0.9995	0.9944	0.9733	0.9204	0.8247	0.6881	0.5272	0.3660	0.2272	0.1241	0.0583	0.0229	0.0071	0.0016	0.0002	0.0000	0.0000	0.0000
	7	1.0000	0.9999	0.9989	0.9930	0.9729	0.9256	0.8406	0.7161	0.5629	0.4018	0.2559	0.1423	0.0671	0.0257	0.0075	0.0015	0.0002	0.0000	0.0000
	8	1.0000	1.0000	0.9998	0.9985	0.9925	0.9743	0.9329	0.8577	0.7441	0.5982	0.4371	0.2839	0.1594	0.0744	0.0271	0.0070	0.0011	0.0001	0.0000
	9	1.0000	1.0000	1.0000	0.9998	0.9984	0.9929	0.9771	0.9417	0.8759	0.7728	0.6340	0.4728	0.3119	0.1753	0.0796	0.0267	0.0056	0.0005	0.0000
	10	1.0000	1.0000	1.0000	1.0000	0.9997	0.9984	0.9938	0.9809	0.9514	0.8949	0.8024	0.6712	0.5100	0.3402	0.1897	0.0817	0.0235	0.0033	0.0001
	11	1.0000	1.0000	1.0000	1.0000	1.0000	0.9997	0.9987	0.9951	0.9851	0.9616	0.9147	0.8334	0.7108	0.5501	0.3698	0.2018	0.0791	0.0170	0.0009
	12	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9998	0.9991	0.9965	0.9894	0.9719	0.9349	0.8661	0.7541	0.5950	0.4019	0.2101	0.0684	0.0070
	13	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9994	0.9979	0.9934	0.9817	0.9549	0.9006	0.8029	0.6482	0.4386	0.2108	0.0429
	14	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9997	0.9990	0.9967	0.9902	0.9739	0.9365	0.8593	0.7161	0.4853	0.1892
	15	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9997	0.9990	0.9967	0.9900	0.9719	0.9257	0.8147	0.5599
	16	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

Table A.9 Binomial Distribution (continued)

n	k	0.0500	0.1000	0.1500	0.2000	0.2500	0.3000	0.3500	0.4000	0.4500	0.5000	0.5500	0.6000	0.6500	0.7000	0.7500	0.8000	0.8500	0.9000	0.9500
17	0	0.4181	0.1668	0.0631	0.0225	0.0075	0.0023	0.0007	0.0002	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	1	0.7922	0.4818	0.2525	0.1182	0.0501	0.0193	0.0067	0.0021	0.0006	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	2	0.9497	0.7618	0.5198	0.3096	0.1637	0.0774	0.0327	0.0123	0.0041	0.0012	0.0003	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	3	0.9912	0.9174	0.7556	0.5489	0.3530	0.2019	0.1028	0.0464	0.0184	0.0064	0.0019	0.0005	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	4	0.9988	0.9779	0.9013	0.7582	0.5739	0.3887	0.2348	0.1260	0.0596	0.0245	0.0086	0.0025	0.0006	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000
	5	0.9999	0.9953	0.9681	0.8943	0.7653	0.5968	0.4197	0.2639	0.1471	0.0717	0.0301	0.0106	0.0030	0.0007	0.0001	0.0000	0.0000	0.0000	0.0000
	6	1.0000	0.9992	0.9917	0.9623	0.8929	0.7752	0.6188	0.4478	0.2902	0.1662	0.0826	0.0348	0.0120	0.0032	0.0006	0.0001	0.0000	0.0000	0.0000
	7	1.0000	0.9999	0.9983	0.9891	0.9598	0.8954	0.7872	0.6405	0.4743	0.3145	0.1834	0.0919	0.0383	0.0127	0.0031	0.0005	0.0000	0.0000	0.0000
	8	1.0000	1.0000	0.9997	0.9974	0.9876	0.9597	0.9006	0.8011	0.6626	0.5000	0.3374	0.1989	0.0994	0.0403	0.0124	0.0026	0.0003	0.0000	0.0000
	9	1.0000	1.0000	1.0000	0.9995	0.9969	0.9873	0.9617	0.9081	0.8166	0.6855	0.5257	0.3595	0.2128	0.1046	0.0402	0.0109	0.0017	0.0001	0.0000
	10	1.0000	1.0000	1.0000	0.9999	0.9994	0.9968	0.9880	0.9652	0.9174	0.8338	0.7098	0.5522	0.3812	0.2248	0.1071	0.0377	0.0083	0.0008	0.0000
	11	1.0000	1.0000	1.0000	1.0000	0.9999	0.9993	0.9970	0.9894	0.9699	0.9283	0.8529	0.7361	0.5803	0.4032	0.2347	0.1057	0.0319	0.0047	0.0001
	12	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9994	0.9975	0.9914	0.9755	0.9404	0.8740	0.7652	0.6113	0.4261	0.2418	0.0987	0.0221	0.0012
	13	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9995	0.9981	0.9936	0.9816	0.9536	0.8972	0.7981	0.6470	0.4511	0.2444	0.0826	0.0088
	14	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9997	0.9988	0.9959	0.9877	0.9673	0.9226	0.8363	0.6904	0.4802	0.2382	0.0503
	15	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9994	0.9979	0.9933	0.9807	0.9499	0.8818	0.7475	0.5182	0.2078
	16	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9998	0.9993	0.9977	0.9925	0.9775	0.9369	0.8332	0.5819
	17	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
18	0	0.3972	0.1501	0.0536	0.0180	0.0056	0.0016	0.0004	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	1	0.7735	0.4503	0.2241	0.0991	0.0395	0.0142	0.0046	0.0013	0.0003	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	2	0.9419	0.7338	0.4797	0.2713	0.1353	0.0600	0.0236	0.0082	0.0025	0.0007	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	3	0.9891	0.9018	0.7202	0.5010	0.3057	0.1646	0.0783	0.0328	0.0120	0.0038	0.0010	0.0002	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	4	0.9985	0.9718	0.8794	0.7164	0.5187	0.3327	0.1886	0.0942	0.0411	0.0154	0.0049	0.0013	0.0003	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	5	0.9998	0.9936	0.9581	0.8671	0.7175	0.5344	0.3550	0.2088	0.1077	0.0481	0.0183	0.0058	0.0014	0.0003	0.0000	0.0000	0.0000	0.0000	0.0000
	6	1.0000	0.9988	0.9882	0.9487	0.8610	0.7217	0.5491	0.3743	0.2258	0.1189	0.0537	0.0203	0.0062	0.0014	0.0002	0.0000	0.0000	0.0000	0.0000
	7	1.0000	0.9998	0.9973	0.9837	0.9431	0.8593	0.7283	0.5634	0.3915	0.2403	0.1280	0.0576	0.0212	0.0061	0.0012	0.0002	0.0000	0.0000	0.0000
	8	1.0000	1.0000	0.9995	0.9957	0.9807	0.9404	0.8609	0.7368	0.5778	0.4073	0.2527	0.1347	0.0597	0.0210	0.0054	0.0009	0.0001	0.0000	0.0000
	9	1.0000	1.0000	0.9999	0.9991	0.9946	0.9790	0.9403	0.8653	0.7473	0.5927	0.4222	0.2632	0.1391	0.0596	0.0193	0.0043	0.0005	0.0000	0.0000
	10	1.0000	1.0000	1.0000	0.9998	0.9988	0.9939	0.9788	0.9424	0.8720	0.7597	0.6085	0.4366	0.2717	0.1407	0.0569	0.0163	0.0027	0.0002	0.0000
	11	1.0000	1.0000	1.0000	1.0000	0.9998	0.9986	0.9938	0.9797	0.9463	0.8811	0.7742	0.6257	0.4509	0.2783	0.1390	0.0513	0.0118	0.0012	0.0000
	12	1.0000	1.0000	1.0000	1.0000	1.0000	0.9997	0.9986	0.9942	0.9817	0.9519	0.8923	0.7912	0.6450	0.4656	0.2825	0.1329	0.0419	0.0064	0.0002
	13	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9997	0.9987	0.9951	0.9846	0.9589	0.9058	0.8114	0.6673	0.4813	0.2836	0.1206	0.0282	0.0015
	14	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9998	0.9990	0.9962	0.9880	0.9672	0.9217	0.8354	0.6943	0.4990	0.2798	0.0982	0.0109
	15	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9993	0.9975	0.9918	0.9764	0.9400	0.8647	0.7287	0.5203	0.2662	0.0581
	16	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9997	0.9987	0.9954	0.9858	0.9605	0.9009	0.7759	0.5497	0.2265
	17	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9996	0.9984	0.9944	0.9820	0.9464	0.8499	0.6028
	18	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

n	k	0.0500	0.1000	0.1500	0.2000	0.2500	0.3000	0.3500	0.4000	0.4500	0.5000	0.5500	0.6000	0.6500	0.7000	0.7500	0.8000	0.8500	0.9000	0.9500
19	0	0.3774	0.1351	0.0456	0.0144	0.0042	0.0011	0.0003	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	1	0.7547	0.4203	0.1985	0.0829	0.0310	0.0104	0.0031	0.0008	0.0002	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	2	0.9335	0.7054	0.4413	0.2369	0.1113	0.0462	0.0170	0.0055	0.0015	0.0004	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	3	0.9868	0.8850	0.6841	0.4551	0.2631	0.1332	0.0591	0.0230	0.0077	0.0022	0.0005	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	4	0.9980	0.9648	0.8556	0.6733	0.4654	0.2822	0.1500	0.0696	0.0280	0.0096	0.0028	0.0006	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	5	0.9998	0.9914	0.9463	0.8369	0.6678	0.4739	0.2968	0.1629	0.0777	0.0318	0.0109	0.0031	0.0007	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000
	6	1.0000	0.9983	0.9837	0.9324	0.8251	0.6655	0.4812	0.3081	0.1727	0.0835	0.0342	0.0116	0.0031	0.0006	0.0001	0.0000	0.0000	0.0000	0.0000
	7	1.0000	0.9997	0.9959	0.9767	0.9225	0.8180	0.6656	0.4878	0.3169	0.1796	0.0871	0.0352	0.0114	0.0028	0.0005	0.0000	0.0000	0.0000	0.0000
	8	1.0000	1.0000	0.9992	0.9933	0.9713	0.9161	0.8145	0.6675	0.4940	0.3238	0.1841	0.0885	0.0347	0.0105	0.0023	0.0003	0.0000	0.0000	0.0000
	9	1.0000	1.0000	0.9999	0.9984	0.9911	0.9674	0.9125	0.8139	0.6710	0.5000	0.3290	0.1861	0.0875	0.0326	0.0089	0.0016	0.0001	0.0000	0.0000
	10	1.0000	1.0000	1.0000	0.9997	0.9977	0.9895	0.9653	0.9115	0.8159	0.6762	0.5060	0.3325	0.1855	0.0839	0.0287	0.0067	0.0008	0.0000	0.00
	11	1.0000	1.0000	1.0000	1.0000	0.9995	0.9972	0.9886	0.9648	0.9129	0.8204	0.6831	0.5122	0.3344	0.1820	0.0775	0.0233	0.0041	0.0003	0.0000
	12	1.0000	1.0000	1.0000	1.0000	0.9999	0.9994	0.9969	0.9884	0.9658	0.9165	0.8273	0.6919	0.5188	0.3345	0.1749	0.0676	0.0163	0.0017	0.0000
	13	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9993	0.9969	0.9891	0.9682	0.9223	0.8371	0.7032	0.5261	0.3322	0.1631	0.0537	0.0086	0.0002
	14	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9994	0.9972	0.9904	0.9720	0.9304	0.8500	0.7178	0.5346	0.3267	0.1444	0.0352	0.0020
	15	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9995	0.9978	0.9923	0.9770	0.9409	0.8668	0.7369	0.5449	0.3159	0.1150	0.0132
	16	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9996	0.9985	0.9945	0.9830	0.9538	0.8887	0.7631	0.5587	0.2946	0.0665
	17	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9998	0.9992	0.9969	0.9896	0.9690	0.9171	0.8015	0.5797	0.2453
	18	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9997	0.9989	0.9958	0.9856	0.9544	0.8649	0.6226
	19	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

Table A.9 Binomial Distribution (continued)

20	0	0.3585	0.1216	0.0388	0.0115	0.0032	0.0008	0.0002	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	1	0.7358	0.3917	0.1756	0.0692	0.0243	0.0076	0.0021	0.0005	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	2	0.9245	0.6769	0.4049	0.2061	0.0913	0.0355	0.0121	0.0036	0.0009	0.0002	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	3	0.9841	0.8670	0.6477	0.4114	0.2252	0.1071	0.0444	0.0160	0.0049	0.0013	0.0003	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	4	0.9974	0.9568	0.8298	0.6296	0.4148	0.2375	0.1182	0.0510	0.0189	0.0059	0.0015	0.0003	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	5	0.9997	0.9887	0.9327	0.8042	0.6172	0.4164	0.2454	0.1256	0.0553	0.0207	0.0064	0.0016	0.0003	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	6	1.0000	0.9976	0.9781	0.9133	0.7858	0.6080	0.4166	0.2500	0.1299	0.0577	0.0214	0.0065	0.0015	0.0003	0.0000	0.0000	0.0000	0.0000	0.0000
	7	1.0000	0.9996	0.9941	0.9679	0.8982	0.7723	0.6010	0.4159	0.2520	0.1316	0.0580	0.0210	0.0060	0.0013	0.0002	0.0000	0.0000	0.0000	0.0000
	8	1.0000	0.9999	0.9987	0.9900	0.9591	0.8867	0.7624	0.5956	0.4143	0.2517	0.1308	0.0565	0.0196	0.0051	0.0009	0.0001	0.0000	0.0000	0.0000
	9	1.0000	1.0000	0.9998	0.9974	0.9861	0.9520	0.8782	0.7553	0.5914	0.4119	0.2493	0.1275	0.0532	0.0171	0.0039	0.0006	0.0000	0.0000	0.0000
	10	1.0000	1.0000	1.0000	0.9994	0.9961	0.9829	0.9468	0.8725	0.7507	0.5881	0.4086	0.2447	0.1218	0.0480	0.0139	0.0026	0.0002	0.0000	0.0000
	11	1.0000	1.0000	1.0000	0.9999	0.9991	0.9949	0.9804	0.9435	0.8692	0.7483	0.5857	0.4044	0.2376	0.1133	0.0409	0.0100	0.0013	0.0001	0.0000
	12	1.0000	1.0000	1.0000	1.0000	0.9998	0.9987	0.9940	0.9790	0.9420	0.8684	0.7480	0.5841	0.3990	0.2277	0.1018	0.0321	0.0059	0.0004	0.0000
	13	1.0000	1.0000	1.0000	1.0000	1.0000	0.9997	0.9985	0.9935	0.9786	0.9423	0.8701	0.7500	0.5834	0.3920	0.2142	0.0867	0.0219	0.0024	0.0000
	14	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9997	0.9984	0.9936	0.9793	0.9447	0.8744	0.7546	0.5836	0.3828	0.1958	0.0673	0.0113	0.0003
	15	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9997	0.9985	0.9941	0.9811	0.9490	0.8818	0.7625	0.5852	0.3704	0.1702	0.0432	0.0026
	16	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9997	0.9987	0.9951	0.9840	0.9556	0.8929	0.7748	0.5886	0.3523	0.1330	0.0159
	17	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9998	0.9991	0.9964	0.9879	0.9645	0.9087	0.7939	0.5951	0.3231	0.0755
	18	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9995	0.9979	0.9924	0.9757	0.9308	0.8244	0.6083	0.2642
	19	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9998	0.9992	0.9968	0.9885	0.9612	0.8784	0.6415
	20	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

APPENDIX B. FINAL STATUS SURVEY CHECKLIST

—— Establish Data Quality Objectives.

—— Identify the contaminants.

—— Establish that residual radioactivity limits have been determined for the radionuclides present at the site.

—— Segregate the site into affected and unaffected areas, based on contamination potential.

—— Identify survey units.

—— Determine whether the radionuclides of concern exist in background.

If yes, two-sample tests comparing the survey unit to a suitable reference area are required to demonstrate compliance.

If no, determine if radionuclide specific measurements be made.

    If yes, one-sample statistical tests may be used.

    If no, two-sample tests are required .

—— Select representative reference (background) areas for both indoor and outdoor survey areas that require a two-sample test. Reference areas must:

    —— be free of contamination from site operations,

    —— exhibit similar physical, chemical, and biological characteristics to the survey area,

    —— have similar construction, but having no history of radioactive operations.

—— Select survey instrumentation and survey techniques.

    —— Identify any surrogate radionuclides and ratios that may be used for scans

    —— Determine MDCs - the instrumentation selected must be capable of detecting the contamination at the guideline levels.

—— Specify sample collection and analysis procedures.

—— Prepare area if necessary - clear and provide access to areas to be surveyed.

- Establish site coordinate reference system(s)

SURVEY DESIGN

- Construct the desired power curve for the test to support decision to accept or reject the null hypotheses of the WRS and Quantile statistical tests.
- Determine numbers of data points for statistical tests.
    - Specify the number of samples/measurements to be obtained based on the requirements of the statistical tests.
    - Evaluate the power of the statistical tests.
    - Ensure that the sample size is sufficient for detecting areas of elevated activity.
    - Allow for additional samples/measurements for QC
    - Allow for possible sample/measurement errors or losses.
- Specify sampling locations.
- Establish scanning procedures for survey
- Specify methods of data reduction and comparison of survey site areas to reference areas.
- Establish quality control procedures for ensuring validity of survey data:
    - instrumentation calibration protocols,
    - necessary replicate and blank measurements,
    - cross-check field measurements and laboratory sample analyses.

CONDUCTING SURVEYS

- Perform reference (background) area measurements and sampling.

Conduct survey activities:

- Perform surface scans of the affected and unaffected areas.
- Conduct direct measurements and sampling on random start triangular grid.

- Document measurement and sample locations.

EVALUATING SURVEY RESULTS

- Analyze samples.
- Perform data reduction on survey results.
- Compare survey results with regulatory guidelines:
  - Conduct elevated measurement comparison.
  - Conduct Wilcoxon test.
  - Conduct Quantile test.
- If any of the tests fail, revisit DQOs to determine additional remediation/survey needs.
- If all tests pass, prepare final status report.

APPENDIX C: TABLES OF AREA DOSE FACTORS

C.1 Outdoor Area Dose Factors

The outdoor area factors discussed in Section 5.4, are used to determine the elevated measurement comparison value $H_m = \Delta \bullet (\text{area factor})$. The outdoor area factors listed in Table C.1 were calculated using RESRAD 5.6 (ANL/EAD/LD-2). For each radionuclide, all dose pathways were calculated assuming an initial concentration of 1 pCi/g. The area of contamination in RESRAD 5.6 defaults to 10000 m^2. Other than changing this to 1, 3, 10, 30, 100, 300, 1000, or 3000 m^2, the RESRAD default values were not changed. The area factors were then computed by taking the ratio of the dose per unit concentration generated by RESRAD for the default 10000 m^2 to that generated for the other areas listed. Thus, if the guideline limit concentration for residual radioactivity distributed over 10000 m^2 is multiplied by this value, the resulting concentration distributed over the specified smaller area delivers the same average dose.

The area factors for selected radionuclides is plotted in Figure C.1. There it can be seen that radionuclides generally group into three types. Those that deliver dose primarily through internal pathways, those that deliver dose primarily through the external pathway, and a few for which both are important. Generally, the radionuclides that deliver dose via internal pathways (e.g. C-14, H-3) have the highest area factors. These area factors scale with the area in a manner suggesting that it is the total inventory of the radionuclides that is most important. The area factors for radionuclides that deliver dose primarily through external gamma have lower area factors, reflecting the fact that this dose can be delivered at a distance. Thus, in a mixture, it will generally be these radionuclides that will have the limiting area factors. Fortunately, these are also the radionuclides most easily detected using scanning techniques.

Interpolations for areas not listed in Table C.1 should be done logarithmically. For example, if the area factor for Am-241 is needed for 25 m^2, the table lists 96.3 for 10 m^2 and 44.2 for 30 m^2. First convert all these values to logs:

$\log_{10}(10) = 1$, $\log_{10}(30) = 1.477$, $\log_{10}(25) = 1.398$, $\log_{10}(96.3) = 1.984$, and $\log_{10}(44.2) = 1.645$

The interpolation is done using these values:

$$\log_{10}(A_{25}) = \log_{10}(96.3) + [\log_{10}(25) - \log_{10}(10)] \{ [\log_{10}(44.2) - (\log_{10}(96.3)] / [\log_{10}(30) - \log_{10}(10)] \}$$

$$\log_{10}(A_{25}) = 1.984 + [1.398 - 1)] \{ [1.645 - 1.984] / [1.477 - 1] \}$$

$$\log_{10}(A_{25}) = 1.984 + [0.398] \{ [-0.339] / [0.477] \}$$

$$\log_{10}(A_{25}) = 1.701$$

Therefore, $A_{25} = 10^{(1.701)} = 50.2$.

Table C-1 Outdoor Area Dose Factors

	Area Factor								
	1 m^2	3 m^2	10 m^2	30 m^2	100 m^2	300 m^2	1000 m^2	3000 m^2	10000 m^2
Ac-227	6.9	4.5	3.0	2.3	1.9	1.5	1.0	1.0	1.0
Ag-108m	15.6	7.1	3.4	2.5	2.0	1.9	1.8	1.3	1.0
Ag-110m	9.5	4.3	2.1	1.5	1.2	1.1	1.1	1.0	1.0
Al-26	11.8	5.3	2.5	1.8	1.5	1.4	1.3	1.0	1.0
Am-241	208.7	139.7	96.3	44.2	13.4	4.4	1.3	1.0	1.0
Am-243	229.8	131.0	75.2	44.3	13.4	4.4	1.3	1.0	1.0
Au-195	8.5	4.1	2.0	1.5	1.2	1.1	1.1	1.0	1.0
Bi-207	9.4	4.2	2.0	1.5	1.2	1.1	1.1	1.0	1.0
C-14	4053.8	1351.3	405.4	135.0	40.4	13.3	3.7	2.1	1.0
Ca-41	1109.3	370.8	111.3	37.1	11.1	3.7	1.1	1.1	1.0
Cd-109	1224.4	458.1	138.8	46.6	14.0	4.6	1.3	1.0	1.0
Ce-144	9.3	4.2	2.1	1.5	1.2	1.1	1.0	1.0	1.0
Cf-252	8.0	5.5	3.9	3.0	2.3	1.7	1.0	1.0	1.0
Cl-36	2477.7	831.0	251.0	84.1	25.3	8.4	2.5	1.9	1.0
Cm-243	8.7	5.0	2.9	2.1	1.7	1.4	1.0	1.0	1.0
Cm-244	8.8	6.0	4.2	3.2	2.5	1.8	1.0	1.0	1.0
Cm-248	8.9	6.0	4.2	3.3	2.5	1.8	1.0	1.0	1.0
Co-57	8.7	4.1	2.0	1.5	1.2	1.1	1.1	1.0	1.0
Co-60	9.8	4.4	2.1	1.5	1.2	1.1	1.1	1.0	1.0
Cs-134	10.1	4.6	2.2	1.6	1.3	1.2	1.1	1.1	1.0
Cs-135	1036.7	497.9	177.2	62.4	19.1	6.4	1.9	1.6	1.0
Cs-137	11.0	5.0	2.4	1.7	1.4	1.3	1.1	1.1	1.0
Eu-152	9.3	4.2	2.0	1.5	1.2	1.1	1.1	1.0	1.0
Eu-154	9.5	4.2	2.0	1.5	1.2	1.1	1.1	1.0	1.0
Eu-155	7.9	3.8	1.9	1.4	1.2	1.1	1.0	1.0	1.0
Fe-55	483.9	285.2	148.5	71.7	27.2	10.0	3.1	2.1	1.0
Gd-152	4.9	3.3	2.4	1.9	1.5	1.3	1.1	1.0	1.0
Gd-153	7.8	3.8	1.9	1.4	1.2	1.1	1.1	1.0	1.0
Ge-68	9.9	4.5	2.2	1.6	1.3	1.2	1.1	1.1	1.0
H-3	1430.9	491.0	147.2	49.0	14.6	4.8	1.3	1.1	1.0
I-129	1734.9	578.1	173.3	57.7	17.2	5.6	1.6	1.2	1.0
K-40	22.8	10.2	4.9	3.4	2.6	2.1	1.3	1.2	1.0
Mn-54	9.5	4.3	2.1	1.5	1.2	1.1	1.1	1.0	1.0

Table C-1 (continued) Outdoor Area Dose Factors

	Area Factor								
	1 m²	**3 m²**	**10 m²**	**30 m²**	**100 m²**	**300 m²**	**1000 m²**	**3000 m²**	**10000 m²**
Na-22	9.4	4.3	2.1	1.5	1.2	1.1	1.1	1.0	1.0
Nb-94	9.8	4.4	2.1	1.6	1.3	1.2	1.1	1.0	1.0
Ni-59	1115.6	449.9	152.6	53.9	16.6	5.6	1.7	1.5	1.0
Ni-63	1175.2	463.7	154.8	54.2	16.6	5.6	1.7	1.5	1.0
Np-237	50.2	27.8	15.0	9.7	5.6	2.8	1.0	1.0	1.0
Pa-231	147.9	96.1	63.5	43.8	13.4	4.4	1.3	1.0	1.0
Pb-210	601.5	253.9	89.8	32.6	10.2	3.5	1.0	1.0	1.0
Pm-147	31.8	18.7	10.8	7.5	4.8	2.7	1.1	1.1	1.0
Pu-238	8.9	6.0	4.2	3.3	2.5	1.8	1.0	1.0	1.0
Pu-239	8.9	6.1	4.3	3.3	2.5	1.8	1.0	1.0	1.0
Pu-240	8.9	6.1	4.3	3.3	2.5	1.8	1.0	1.0	1.0
Pu-241	267.9	179.9	124.4	44.2	13.4	4.4	1.3	1.0	1.0
Pu-242	8.9	6.0	4.2	3.3	2.5	1.8	1.0	1.0	1.0
Pu-244	9.1	4.4	2.2	1.6	1.3	1.2	1.1	1.0	1.0
Ra-226	54.8	21.3	7.8	3.2	1.1	1.1	1.0	1.0	1.0
Ra-228	16.0	7.3	3.5	2.5	1.9	1.6	1.1	1.0	1.0
Ru-106	34.0	15.5	7.5	5.4	4.4	4.0	1.3	1.0	1.0
Sb-125	9.0	4.1	2.0	1.5	1.2	1.1	1.1	1.0	1.0
Sm-147	7.6	5.2	3.7	2.9	2.3	1.7	1.1	1.1	1.0
Sm-151	1383.6	461.3	138.3	46.0	13.7	4.5	1.3	1.0	1.0
Sr-90	728.8	286.2	98.7	37.2	11.9	4.1	1.2	1.2	1.0
Tc-99	1481.6	494.2	148.1	49.2	14.6	4.7	1.3	1.0	1.0
Th-228	9.8	4.5	2.2	1.6	1.3	1.2	1.1	1.0	1.0
Th-229	5.8	3.6	2.3	1.7	1.4	1.3	1.1	1.0	1.0
Th-230	48.7	21.4	8.5	3.7	1.3	1.2	1.0	1.0	1.0
Th-232	12.5	6.2	3.2	2.3	1.8	1.5	1.1	1.0	1.0
Tl-204	2085.8	697.8	209.9	70.0	20.9	6.9	2.0	1.4	1.0
U-232	9.3	4.5	2.3	1.7	1.3	1.2	1.1	1.0	1.0
U-233	42.2	28.7	20.2	15.8	8.3	3.7	1.3	1.0	1.0
U-234	41.0	27.9	19.7	15.4	10.7	4.1	1.3	1.0	1.0
U-235	58.8	30.2	15.9	11.8	9.6	4.4	1.3	1.0	1.0
U-236	39.7	27.0	19.1	14.9	11.8	4.4	1.3	1.0	1.0
U-238	30.6	18.3	11.1	8.4	6.7	4.4	1.3	1.0	1.0
Zn-65	17.0	7.6	3.7	2.6	2.1	1.8	1.4	1.3	1.0

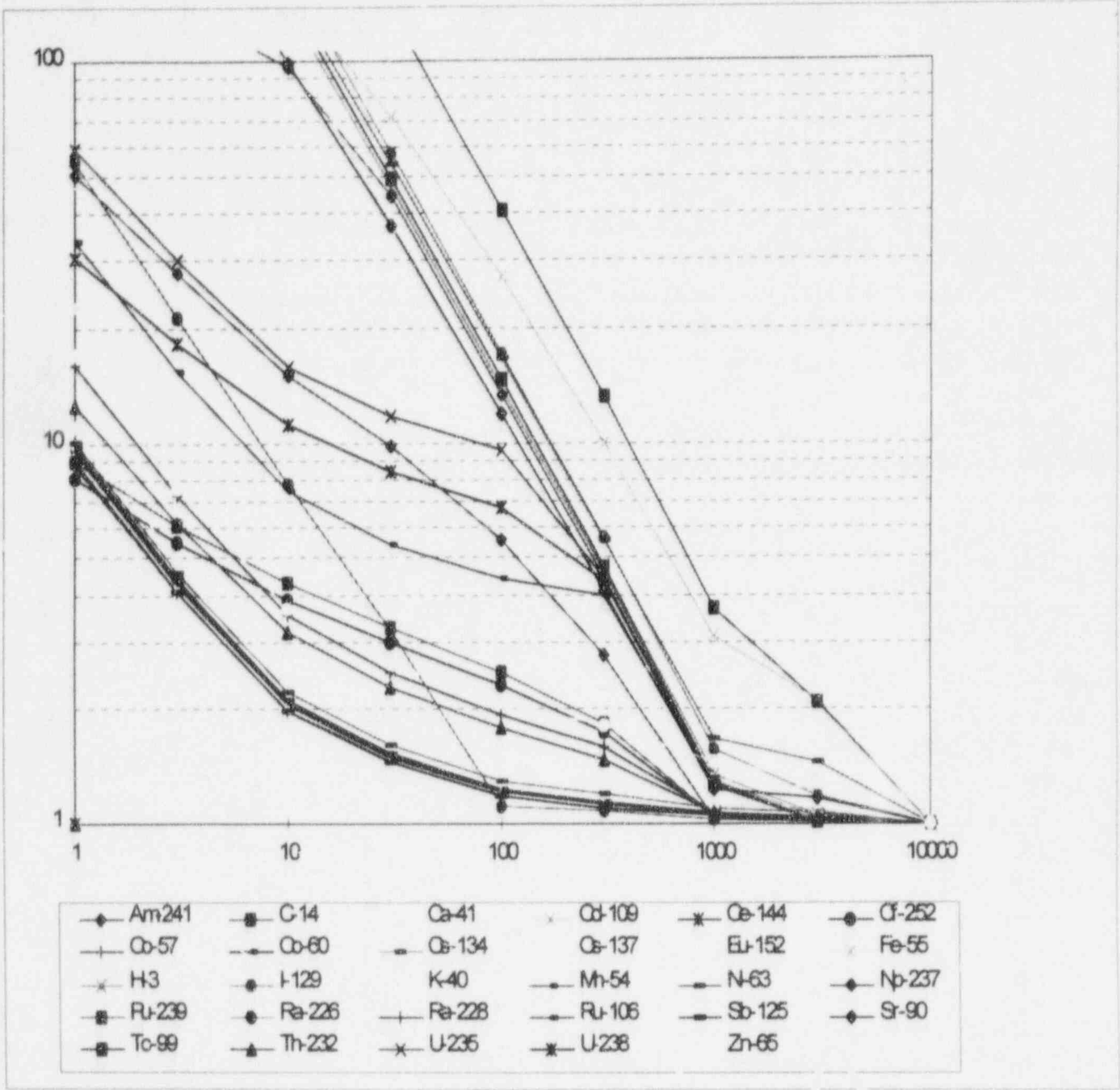

Figure C.1 Outdoor Area Factors for Selected Radionuclides

C.2 Indoor Area Dose Factors

The indoor area factors discussed in Section 5.4, are used to determine the elevated measurement comparison value $H_m = \Delta \cdot$(Area Factor). The indoor area factors listed in Table C.2 were calculated using RESRAD BUILD 1.5 (Yu et al., 1994). For each radionuclide, all dose pathways were calculated assuming an initial concentration of 1 pCi/m^2. The area of contamination in RESRAD BUILD 1.5 defaults to 36 m^2. The other areas compared to this value were 1, 4, 9, 16, or 25 m^2. Removable contamination was assumed to be 10%. No other changes to the RESRAD BUILD default values were made. Dose was computed for one receptor, who spent 100% of time in the contaminated room. The area factors were then computed by taking the ratio of the dose per unit concentration generated by RESRAD for the default 10000 m^2 to that generated for the other areas listed. Thus, if the guideline limit concentration for residual radioactivity distributed over 10000 m^2 is multiplied by this value, the resulting concentration distributed over the specified smaller area delivers the same average dose. There are obviously many other exposure scenarios which may result in different area factors. However, the factors in Table C.2 might be expected to be conservative

The area factors for selected radionuclides are plotted in Figure C.2. As with the outdoor area factors, the radionuclides that deliver dose primarily through internal pathways have higher area factors than those that deliver dose primarily through the external pathway. These area factors scale with the area in a manner suggesting that it is the total inventory of removable fraction of these radionuclides that is most important. The area factors for radionuclides that deliver dose primarily through external gamma have lower area factors, reflecting the fact that this dose can be delivered at a distance. Thus, in a mixture, it will generally be these radionuclides that will ususally have the limiting area factors. Fortunately, these are also the radionuclides most easily detected using scanning techniques.

Interpolations for areas not listed in Table C.2 should be done logarithmically, in the same manner as that described for the outdoor area factors.

Table C-2 Indoor Area Dose Factors

	Area Factor					
Nuclide	**1 m²**	**4 m²**	**9 m²**	**16 m²**	**25 m²**	**36 m²**
Ac-227	36.0	9.0	4.0	2.3	1.4	1.0
Ag-108m	9.2	3.1	1.9	1.4	1.2	1.0
Ag-110m	9.1	3.1	1.9	1.4	1.2	1.0
Al-26	10.2	3.4	2.0	1.5	1.2	1.0
Am-241	36.0	9.0	4.0	2.2	1.4	1.0
Am-243	35.5	8.9	4.0	2.2	1.4	1.0
Au-195	9.1	3.1	1.9	1.4	1.1	1.0
Bi-207	9.2	3.1	1.9	1.4	1.2	1.0
C-14	35.9	9.0	4.0	2.2	1.4	1.0
Ca-41	36.1	9.0	4.0	2.3	1.4	1.0
Cd-109	10.4	3.4	2.0	1.5	1.2	1.0
Ce-144	11.5	3.8	2.2	1.6	1.2	1.0
Cf-252	36.0	9.0	4.0	2.3	1.4	1.0
Cl-36	33.8	8.6	3.9	2.2	1.4	1.0
Cm-243	35.6	8.9	4.0	2.2	1.4	1.0
Cm-244	35.9	9.0	4.0	2.2	1.4	1.0
Cm-248	36.0	9.0	4.0	2.2	1.4	1.0
Co-57	9.6	3.2	2.0	1.4	1.2	1.0
Co-60	9.2	3.1	1.9	1.4	1.2	1.0
Cs-134	9.2	3.1	1.9	1.4	1.2	1.0
Cs-135	36.0	9.0	4.0	2.3	1.4	1.0
Cs-137	9.4	3.2	1.9	1.4	1.2	1.0
Eu-152	9.3	3.1	1.9	1.4	1.2	1.0
Eu-154	9.3	3.1	1.9	1.4	1.2	1.0
Eu-155	9.5	3.2	1.9	1.4	1.2	1.0
Fe-55	36.1	9.0	4.0	2.3	1.4	1.0
Gd-152	36.1	9.0	4.0	2.3	1.4	1.0
Gd-153	9.1	3.1	1.9	1.4	1.1	1.0
Ge-68	6.3	2.3	1.5	1.2	1.1	1.0
H-3	35.9	9.0	4.0	2.3	1.4	1.0
I-129	20.3	6.0	3.1	1.9	1.3	1.0
K-40	12.9	4.1	2.4	1.6	1.3	1.0
Mn-54	9.6	3.2	1.9	1.4	1.2	1.0

Table C-2 (continued) Indoor Area Dose Factors

	Area Factor					
Nuclide	**1 m^2**	**4 m^2**	**9 m^2**	**16 m^2**	**25 m^2**	**36 m^2**
Na-22	9.8	3.3	2.0	1.4	1.2	1.0
Nb-94	9.3	3.1	1.9	1.4	1.2	1.0
Ni-59	36.1	9.0	4.0	2.3	1.4	1.0
Ni-63	36.0	9.0	4.0	2.3	1.4	1.0
Np-237	35.5	8.9	4.0	2.2	1.4	1.0
Pa-231	35.9	9.0	4.0	2.2	1.4	1.0
Pb-210	35.9	9.0	4.0	2.2	1.4	1.0
Pm-147	34.7	8.8	4.0	2.2	1.4	1.0
Pu-238	36.0	9.0	4.0	2.3	1.4	1.0
Pu-239	36.0	9.0	4.0	2.2	1.4	1.0
Pu-240	36.0	9.0	4.0	2.2	1.4	1.0
Pu-241	36.0	9.0	4.0	2.3	1.4	1.0
Pu-242	36.0	9.0	4.0	2.3	1.4	1.0
Pu-244	35.3	8.9	4.0	2.2	1.4	1.0
Ra-226	18.1	5.5	2.9	1.9	1.3	1.0
Ra-228	20.6	6.1	3.1	1.9	1.4	1.0
Ru-106	10.1	3.4	2.0	1.5	1.2	1.0
Sb-125	9.2	3.1	1.9	1.4	1.2	1.0
Sm-147	36.0	9.0	4.0	2.2	1.4	1.0
Sm-151	35.7	8.9	4.0	2.2	1.4	1.0
Sr-90	33.4	8.6	3.9	2.2	1.4	1.0
Tc-99	36.1	9.0	4.0	2.3	1.4	1.0
Th-228	31.5	8.2	3.8	2.2	1.4	1.0
Th-229	36.0	9.0	4.0	2.3	1.4	1.0
Th-230	36.1	9.0	4.0	2.3	1.4	1.0
Th-232	36.0	9.0	4.0	2.2	1.4	1.0
Tl-204	12.4	4.0	2.3	1.6	1.2	1.0
U-232	36.0	9.0	4.0	2.3	1.4	1.0
U-233	36.0	9.0	4.0	2.2	1.4	1.0
U-234	36.0	9.0	4.0	2.2	1.4	1.0
U-235	34.5	8.7	3.9	2.2	1.4	1.0
U-236	35.9	9.0	4.0	2.2	1.4	1.0
U-238	35.7	9.0	4.0	2.2	1.4	1.0
Zn-65	9.2	3.1	1.9	1.4	1.2	1.0

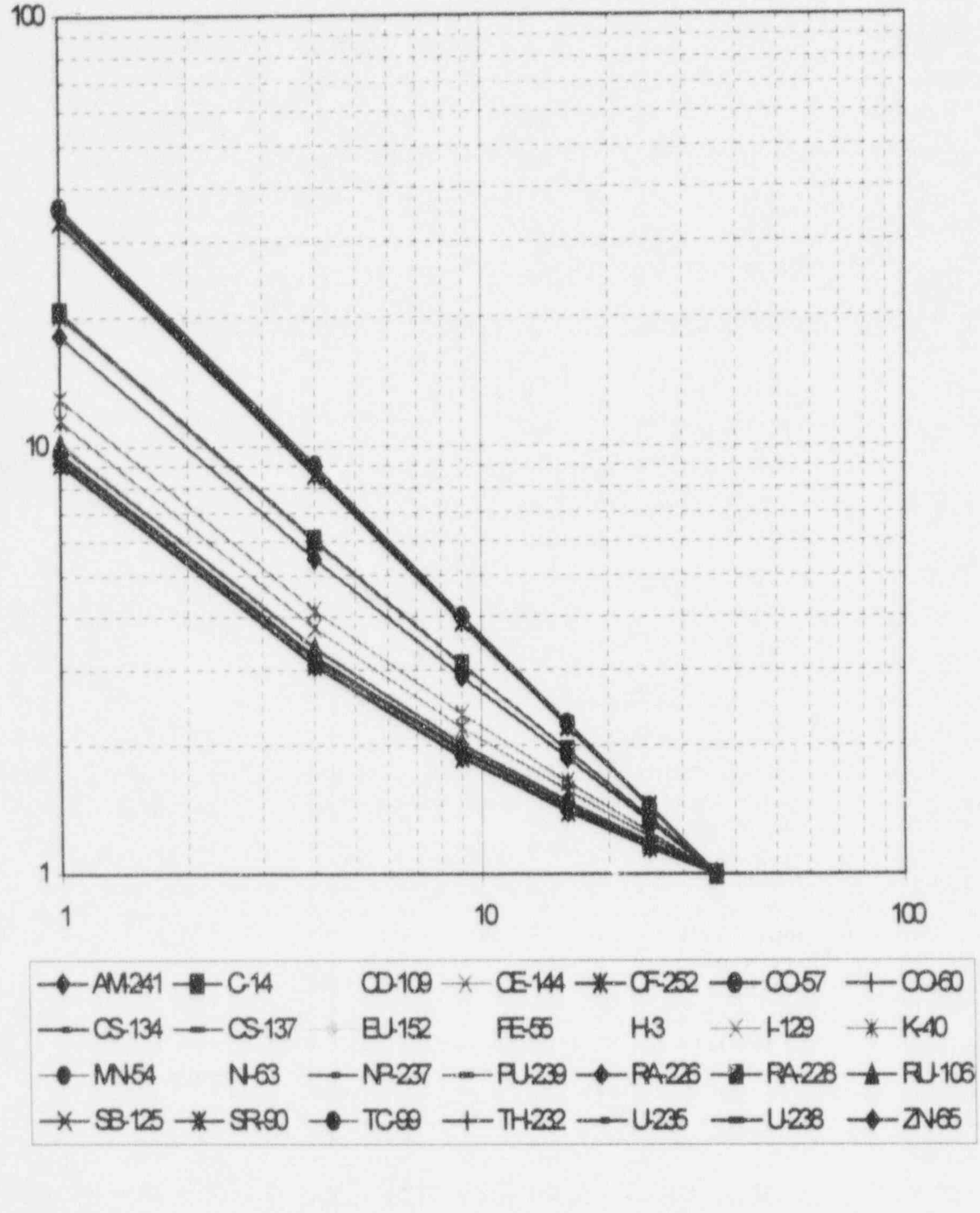

Figure C.2 Indoor Area Factors for Selected Radionuclides

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11. ABSTRACT (200 words or less)

This report describes a nonparametric statistical methodology for the design and analysi of final status decommissioning surveys in support of the proposed rulemaking on decommissioning. The techniques described are alternatives to the exisisting parametric statistical methodology in NRC draft NUREG/CR-5849, "Manual for Conducting Radiological Surveys in Support of License Termination." Proposed nonparametric statistical methods for testing compliance with decommissioning criteria are provided for radionuclides which occur in natural background and for those that do not occur in natural background. The tests considered applicable are the Wilcoxon Signed Ranks test, Sign test, and Quantile test for the analysis of a single data set, and the Wilcoxon Rank Sum test and a Quantile test for comparing two independent data sets. An Elevated Measurement Comparison is also described to deal with any unusually high observations. This report contains information on the Data Quality Objectives process as it relates to the planning and analysis of final site surveys. The proposed process includes methods for determining the number of samples needed to obtain statistically valid comparisons with decommissioning criteria and the methods for conducting the statistical tests with the resulting sample data.

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