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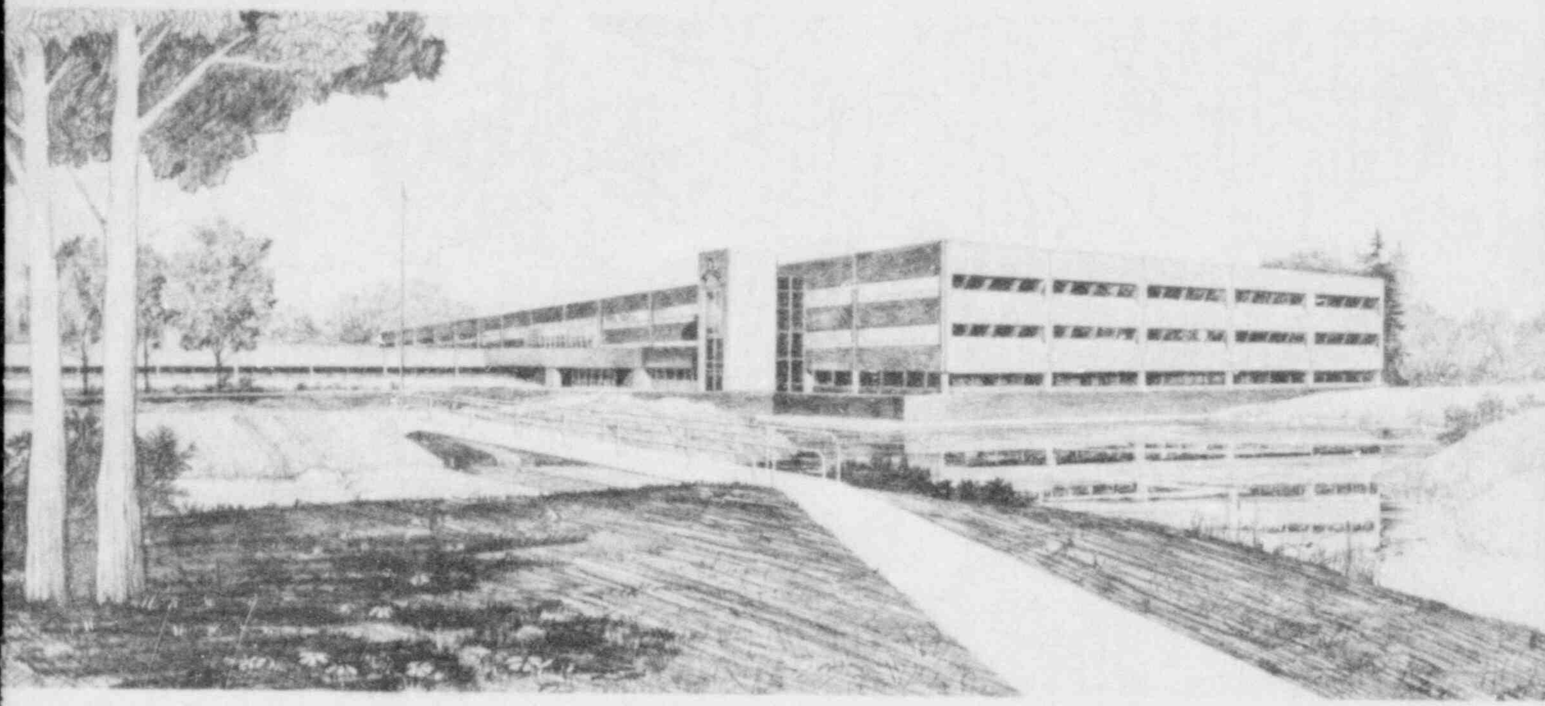

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The Application of Stein and Related Parametric Empirical Bayes Estimators to the Nuclear Plant Reliability Data System

J. R. Hill
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THE APPLICATION OF STEIN AND RELATED PARAMETRIC EMPIRICAL BAYES ESTIMATORS TO THE NUCLEAR PLANT RELIABILITY DATA SYSTEM

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University of Texas, Austin

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ABSTRACT

This report is the result of a preliminary feasibility study of the applicability of Stein and related parametric empirical Bayes (PEB) estimators to the Nuclear Plant Reliability Data System (NPRDS). A new estimator is derived for the means of several independent Poisson distributions with different sampling times. This estimator is applied to data from NPRDS in an attempt to improve failure rate estimation. Theoretical and Monte Carlo results indicate that the new PEB estimator can perform significantly better than the standard maximum likelihood estimator if the estimation of the individual means can be combined through the loss function or through a parametric class of prior distributions.

FIN No. A6358—Applied James Stein Estimation

SUMMARY

This report is the result of a preliminary feasibility study of the applicability of Stein and related estimators to the Nuclear Plant Reliability Data System (NPRDS). The work has been done for the United States Nuclear Regulatory Commission on project contract K-7720 between EG&G Idaho, Inc. and the University of Texas at Austin. The four objectives of the project were:

1. Evaluate the effect of using different distributions in Stein estimation
2. Evaluate the improvements of Stein estimation over other techniques
3. Develop confidence bounds for Stein estimators
4. Use Monte Carlo simulation to compare estimators of component failure rates.

Actually, Stein's estimator is not appropriate for the estimation of nuclear component failure rates since the nuclear data are not normally distributed with equal sampling variances. However, using parametric empirical Bayes (PEB) theory, generalizations of Stein's estimator can be derived for other situations, including estimators needed for nuclear reliability. The highlights of the project include:

1. Definition of the unequal variance function Poisson problem
2. Derivation of a PEB estimator for the unequal variance function Poisson problem
3. Application of the PEB estimator to NPRDS globe valve data
4. Comparison of the maximum likelihood estimator (MLE) and the PEB estimator using Monte Carlo
5. Application of normal theory PEB confidence intervals by making normal approximations to the Poisson sampling distributions.

Theoretical and Monte Carlo results indicate that the new PEB estimator can perform significantly better than the standard MLE if the estimation of the individual failure rates can be combined through the loss function or through a parametric class of prior distributions. More work needs to be done which allows shrinkage toward log-linear models and which makes more accurate calculation of PEB confidence intervals for Poisson problems. In short, preliminary results are very promising, but further work must be done before PEB can be regularly applied in NPRDS failure rate estimation.

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ABBREVIATIONS AND NOTATIONS

NPRDS	The Nuclear Plant Reliability Data System
MLE	Maximum likelihood estimator
PEB	Parametric empirical Bayes
JS	James-Stein
SL	Stein-Lindley
ANOVA	Analysis of variance
$X\|\mu\sim$	X given ($\|$) μ is distributed as ($\sim$)
ind	Independent
iid	Independent identically distributed
$N(\mu, \sigma^2)$	Normal distribution with mean μ, variance σ^2
$Poiss(\lambda)$	Poisson distribution with mean and variance λ
Gam(m, A)	Gamma distribution with mean m, variance A
NB(m, V)	Negative binomial distribution with mean m, variance V
$E(\cdot)$	Expectation operator
$Var(\cdot)$	Variance operator

THE APPLICATION OF STEIN AND RELATED PARAMETRIC EMPIRICAL BAYES ESTIMATORS TO THE NUCLEAR PLANT RELIABILITY DATA SYSTEM

1. INTRODUCTION

This report is the result of a preliminary feasibility study of the applicability of Stein and related estimators to the Nuclear Plant Reliability Data System (NPRDS). The work has been done on project contract K-7720 between EG&G Idaho and the University of Texas, Austin. The four objectives of the project are:

1. Evaluate the effect of using different distributions in Stein estimation

2. Evaluate the improvements of Stein estimation over other techniques

3. Develop confidence bounds for Stein estimators

4. Use Monte Carlo simulation to compare estimators of component failure rates.

Many of our results on these four objectives appear in the theses of Joe R. Hill (1982) and A. Sharif Heger (1983), as well as in Hill, Homayoun, and Koen (1982).

In the reliability context, the basic idea is to estimate the failure rate of one kind of component, by using data not only from that one kind of component but also from other moderately similar components. The data from the different components are combined in two steps: The failure rate is thought of as varying among the possible kinds of components, and all the data are used to estimate a distribution for the failure rate. Then, this distribution is used as a Bayesian prior distribution, and is combined with the data from the kind of component of interest, to produce a posterior distribution for the failure rate of the one kind of component. This posterior distribution may be called the parametric empirical Bayes (PEB) distribution for the kind of component.

One use of the PEB distribution is for the propagation of uncertainties in a probabilistic risk assessment (PRA). The technique is standard in PRAs, but the distributions used have often been either subjective, based wholly on expert opinion, or partly subjective, based on a subjectively chosen prior plus some data. The PEB distribution is more directly based on real data. A minor refinement would be to widen the PEB distribution somewhat, to account for the uncertainty in estimating the prior distribution. This might be important when the data set is small. The amount of widening could be determined by matching (say) the 95% point of the distribution to the upper end of the 90% interval given in Section 3. Use of the PEB distribution for propagating uncertainties is mentioned here, but will not be considered in the rest of this report.

A second use of the PEB distribution is simply to estimate the component's failure rate, by both a point estimator and an interval estimator. These estimators are developed in this report. The point estimators are then compared to the usual maximum likelihood estimators (MLEs), using data sets that simulate NPRDS data. The PEB point estimates are often close to the MLEs, but their overall statistical properties are generally somewhat better than those of the MLE. This favorable conclusion provides indirect support for using the PEB method for other purposes, such as the above-mentioned propagation of uncertainties in a PRA.

We now give, in detail, the background and definitions needed to understand the results of this report. Stein's work (1955, 1961) dealt only with normal random variables having a common variance. Parametric empirical Bayes methods have been used to generalize Stein's rule in three directions. Efron-Morris (1973, 1975) and Morris (1983b) use PEB methods to derive estimators for normal unequal variance problems. Morris (1983c) derives PEB rules for nonnormal but equal variance function problems. Morris (1983a,b) suggests PEB confidence intervals for normal equal and unequal variance problems. We review these generalizations to help motivate the PEB estimators we derive for nuclear data.

The Efron-Morris PEB derivation of normal unequal variance rules begins by assuming

$$X_i|\mu_i \overset{ind}{\sim} N(\mu_i, V_i),\ i = 1,\ldots,k . \tag{1}$$

This might result, for example, from a one-way analysis of variance (ANOVA) with n_i observations for treatment i; i.e., if

$$X_{ij}|\mu_i \overset{ind}{\sim} N(\mu_i, \sigma^2),\ j = 1,\ldots,n_i,$$

$$i = 1,\ldots,k \tag{2}$$

then the MLE for m_i,

$$X_i = \frac{1}{n_i}\sum_{j=1}^{n_i} X_{ij},$$

has Distribution (1) with

$V_i = \sigma^2/n_i$. To motivate the PEB estimates, the μ_i are assumed to have prior distributions

$$\mu_i|m_i, A \overset{ind}{\sim} N(m_i, A),\ i = 1,\ldots,k . \tag{3}$$

The marginal distributions of the X_i are independent with

$$X_i|m_i, A \sim N(m_i, V_i + A),\ i = 1,\ldots,k . \tag{4}$$

Furthermore, the μ_i have posterior distributions

$$\mu_i|\underline{X}, m_i, A \sim N[\mu_i^*, (1 - B_i)V_i] \tag{5}$$

where

$$\mu_i^* = (1 - B_i)X_i + B_i m_i \tag{6}$$

and

$$B_i = V_i/(V_i + A) . \tag{7}$$

The posterior mean, μ_i^*, is a compromise between the prior mean, m_i, and the observed MLE, X_i. The amount of compromise, or shrinking factor, B_i, is determined by the ratio of the sampling variance, V_i, to the marginal variance, $V_i + A$. Small V_i give small B_i, which causes μ_i^* to be close to X_i. Large V_i make B_i close to 1, which forces μ_i^* close to m_i. Also, a large prior variance causes all the B_i to be near zero, so more weight is given to X_i. On the other hand, a small A forces all the B_i to be near 1 and all the μ_i^* to be near m_i. In other words, μ_i^* weights m_i and X_i according to the relative precision of each.

PEB estimators for the μ_i are constructed by using Distribution (4) to obtain estimates (m_i, B_i) and then substituting for $(\hat{m}_i, \hat{B}_i)$ in Equation (6). The following iterative procedure (Morris, 1983b) defines one way to estimate $\underline{m} = (m_1,\ldots,m_k)'$ and A. Given $\hat{A}$, calculate

1. the empirical Bayes weights

$$w_i = 1/(V_i + \hat{A}) \tag{8}$$

2. the weighted regression estimate of $\underline{m}$

$$\hat{\underline{m}} = Z(Z'WZ)^{-1} Z'WX \tag{9}$$

where the weight matrix $W = \text{diag}(w_i)$, Z is a known (k x r) design matrix, and $\underline{X} = (X_1,\ldots,X_k)'$

3. the weighted average of squared residuals

$$S = \sum w_i(X_i - \hat{m}_i)^2/\sum w_i \tag{10}$$

4. the weighted average of sampling variances

$$\bar{V} = \sum w_i V_i/\sum w_i \tag{11}$$

5. the unbiased estimate of A

$$A = \frac{\sum w_i \frac{k}{k-r}(X_i - \hat{m}_i)^2 - V_i}{\sum w_i}$$

$$= \frac{k}{k-r} S - \bar{V} . \tag{12}$$

Repeat steps 1 through 5 until $\hat{A}$ converges. If $\hat{A} < 0$, replace it by $\hat{A} = 0$. Now define the PEB shrinking factors,

$$\hat{B}_i = \frac{k - r - 2}{k - r} \frac{V_i}{V_i + \hat{A}} \tag{13}$$

and the PEB estimator

$$\hat{\mu}_i = (1 - \hat{B}_i)X_i + \hat{B}_i\hat{m}_i . \tag{14}$$

This formulation includes several special cases. When the sampling variances are equal, i.e., $V_i = V$ or $n_i = n$ for the ANOVA problem, then the problem is called an "equal variance" problem. There are three types of equal variance problems corresponding to differing knowledge of the prior mean. If the prior mean $\underline{m}$ is known, then it does not need to be estimated, so $r = 0$, $\hat{\underline{m}} = \underline{m}$, and

$$\hat{B}^{JS} = \frac{(k - 2)V}{\sum (X_i - m_i)^2} \tag{15}$$

is the James-Stein (1961) shrinking factor. We are not forcing $\hat{A} \geq 0$ here. If $\underline{m}$ is unknown, but the m_i are equal, $m_i = m$, then $r = 1$, $\hat{m}_i = \bar{X}$, and

$$\hat{B}^{SL} = \frac{(k - 3)V}{\sum (X_i - \bar{X})^2} \tag{16}$$

which is Lindley's (1962) modification of Stein's rule. Finally, if $\underline{m}$ is unknown and $m_i \neq m$, but the m_i fit some regression pattern, $\underline{m} = Z\beta$, where Z is a known (k X r) design matrix and $\underline{\beta}$ is an unknown (r X 1) vector of regression coefficients, then β can be estimated using ordinary least squares, i.e., $\hat{\underline{\beta}} = (Z'Z)^{-1} Z'X$, to give $\hat{\underline{m}} = Z\hat{\underline{\beta}}$ as in Equation (9).

These equal variance rules dominate the MLE in the following decision theoretic sense. If the loss for estimating $\underline{\mu} = (\mu_1,\ldots,\mu_k)'$ by $\underline{d}(\underline{X}) = (d_1(\underline{X}),\ldots,d_k(\underline{X}))'$ is given by

$$L[\underline{\mu},\underline{d}(X)] = \sum_{i=1}^{k} \frac{[\mu_i - d_i(\underline{X})]^2}{V} \tag{17}$$

then the risk, i.e., the expected loss, for Stein's rule $\hat{\underline{\mu}}$ is

$$R(\underline{\mu},\hat{\underline{\mu}}) = k - (k - r - 2)E_{\underline{\mu}}\hat{B} . \tag{18}$$

If $k > r + 2$, this is less than the risk of the MLE, $\underline{d}^\circ(\underline{X}) \equiv \underline{X}$, since

$$R(\underline{\mu},\underline{d}^\circ) = k . \tag{19}$$

Stein's (1955) well-known theorem on the inadmissibility of the MLE when $k > r + 2$ follows from Equations (18) and (19).

When the sampling variances are not equal, i.e., $V_i \neq V$ or $n_i \neq n$, this problem is called an "unequal variance" problem. Paralleling the equal variance case, differing knowledge of the prior mean results in three types of unequal variance problems.

The six normal cases are given in Table 1. In all cases, σ^2 and the n_i (hence the V_i) are assumed known and A is unknown. Cases A, B, and C are the "equal variance" problems. Cases D, E, and F are the unequal variance problems.

There are three differences between the equal and unequal variance rules. The main difference is the use of weighted averages, with weights proportional to the inverse of the marginal variances, for the unequal variance estimators in place of the usual averages used for the equal variance rules. Secondly, since the weights depend on the unknown parameter A, the unequal variance rules are derived

Table 1. Six cases in the estimation of k normal means

A. $V_i = V$ ($n_i = n$), m_i known ($r = 0$)

B. $V_i = V$ ($n_i = n$), $m_i = m$ unknown ($r = 1$)

C. $V_i = V$ ($n_i = n$), $m_i \neq m$ unknown ($r > 1$)

D. $V_i \neq V$ ($n_i \neq n$), m_i known ($r = 0$)

E. $V_i \neq V$ ($n_i \neq n$), $m_i = m$ unknown ($r = 1$)

F. $V_i \neq V$ ($n_i \neq n$), $m_i \neq m$ unknown ($r > 1$)

through iteration in contrast to direct computation for the equal variance estimators. Finally, unequal variances cause unequal shrinking factors unlike the constant shrinking factor in the equal variance case.

Although the unequal variance problems are more difficult, the advantage of the PEB viewpoint is that it suggests more general "Stein-like" estimators, for example as given by Equations (8) through (14). The nuclear reliability data we will analyze are unequal variance data; i.e., the times on test for different classes of components are not equal. The data also are not normal so we should not even apply the unequal variance rules just suggested. Instead, we will assume the data are Poisson and use PEB methods to construct appropriate estimators.

The nuclear data to be analyzed are those found in the Nuclear Plant Reliability Data System (NPRDS) (1979). The NPRDS data consist of the number of failures, f_i, and the time on test, t_i, for each of k classes of components. Class i has r_i positions in which components are placed. The NPRDS data collection procedure suggests that each t_i is fixed and each f_i is the realization of a random variable, F_i. In other words, the data are a type I censored sample. The data result from testing components in the jth position ($j = 1,\ldots,r_i$) of class i ($i = 1,\ldots,k$) for a fixed time t_{ij} and observing the number of failures, F_{ij}. This testing takes place with replacement; i.e., when the component in a given location fails it is replaced with a new component. We assume that each component in the ith class has the same unknown constant failure rate, λ_i, and that the a priori distributions of the λ_i are independent and identical.

Assuming the number of failures for position j of class i is Poisson with mean $\lambda_i t_{ij}$, i.e., that

$$F_{ij}|\lambda_i \overset{\text{ind}}{\sim} \text{Poiss}(\lambda_i t_{ij}) \tag{20}$$

for $j = 1,\ldots,r_i$ and $i = 1,\ldots k$, then the MLE for the ith failure rate, λ_i, is $X_i = F_i/t_i$ where $t_i = \sum_{j=1}^{r_i} t_{ij}$ is the total time on test for the ith class and $Fi = \sum_{j=1}^{r_i} F_{ij}$ is the total number of failures in class i. The distributions of the X_i given λ_i are

$$X_i|\lambda_i \overset{\text{ind}}{\sim} \frac{1}{t_i}\ \text{Poiss}(\lambda_i t_i),\ i = 1,\ldots,k \tag{21}$$

so

$$E(X_i|\lambda_i) = \lambda_i \text{ and} \tag{22}$$

$$\text{Var}(X_i|\lambda_i) = \lambda_i/t_i\ . \tag{23}$$

Morris (1982) defines the variance function of a natural exponential family (NEF) to be the function that expresses the variance in terms of the mean. For the equal variance normal case, the sampling variance functions are all the same constant function; $V(\mu_i) = V = \sigma^2/n$. The unequal variance normal problem is now seen to imply that these sampling variance functions are different; i.e., $V_i(\mu_i) = V_i = \sigma^2/n_i$. In both normal cases the variance function is independent of μ_i. Now for the Poisson problems defined above, if $t_i = t$ all i, then the variance functions are all the same; i.e., $V(\lambda_i) = \lambda_i/t$ even though the λ_i differ. However, if the t_i differ, then the $V_i(\lambda_i) = \lambda_i/t_i$ are not the same. This problem will be called "the unequal variance function Poisson problem."

Morris (1983c) proposes PEB estimators for the equal variance function Poisson case, i.e., for the case $t_i = t$ for all i. The X_i given λ_i have means λ_i and common variance function $V(\lambda_i) = \lambda_i/t$. Specifically

$$X_i|\lambda_i \overset{\text{ind}}{\sim} \frac{1}{t}\ \text{Poiss}(\lambda_i t),\ i = 1,\ldots,k\ . \tag{24}$$

The PEB rule is constructed by assuming the λ_i have prior distributions

$$\lambda_i|\mu, A \overset{\text{iid}}{\sim} \text{Gam}(m, A),\ i = 1,\ldots,k \tag{25}$$

where here Gam(m, A) signifies a gamma distributed random variable with mean m and variance A.

The marginal and posterior distributions are determined by Distributions (24) and (25). It follows that the marginal distributions of the X_i are independent negative binomial distributions each with mean m and variance $m/t + A$; that is

$$X_i\ |m, A \overset{\text{ind}}{\sim} \text{NB}(m, m/t + A)\ . \tag{26}$$

The posterior distributions of the λ_i are independent gamma distributions

$$\lambda_i|\underline{X}, m, A \sim \text{Gam}(\lambda_i^*, A_i^*) \tag{27}$$

with posterior means

$$\lambda_i^* = E(\lambda_i | \underline{X}, m, A) = (1 - B)X_i + Bm \qquad (28)$$

where

$$B = \frac{m/t}{m/t + A} \qquad (29)$$

If $\bar{X}$ and S are defined as

$$\bar{X} = \frac{1}{k}\sum_{i=1}^{k} X_i \qquad (30)$$

and

$$S = \frac{1}{k}\sum_{i=1}^{k} (X_i - \bar{X})^2 \qquad (31)$$

then Morris' PEB rule is

$$\hat{\lambda}_i = (1 - \hat{B})X_i + \hat{B}\bar{X} \qquad (32)$$

where

$$\hat{B} = \frac{k - 3}{k}\frac{V(\bar{X})}{S} \qquad (33)$$

$$= \frac{(k - 3)\bar{X}/t}{\sum(X_i - \bar{X})^2} \quad . \qquad (34)$$

This rule looks similar to Stein's rule with the Lindley modification given in Equations (14) and (16) for the equal variance normal problem (Case B, Table 1). The difference is that Equation (34) takes into account the functional dependence of the variance on the mean for Poisson variables.

If the NPRDS data could be modeled as equal variance Poisson data, then we could use Morris' rule. However, the NPRDS data do not come from a designed experiment with equal t_i, but from operating power plants; hence, the times on test, t_i, are unequal. The NPRDS problem, therefore, requires the development of an unequal variance function Poisson estimator.

In Section 2 we derive and evaluate an estimator for the unequal variance function Poisson problem using parametric empirical Bayes methods. In Section 3 we report work of Morris (1983a,b) on PEB confidence intervals for normal problems and suggest how his intervals might be used for Poisson problems. In Section 4 we describe and report the results of Monte Carlo experiments used to compare the MLE and the PEB estimators. Section 5 contains our conclusions and suggestions for further work. The Appendix contains the results of several simulations which illustrate the effect of PEB estimation on individual component estimates.

2. PARAMETRIC EMPIRICAL BAYES POINT ESTIMATORS FOR COMPONENT FAILURE RATES

This section is divided into three subsections. In Subsection 2.1 we derive a parametric empirical Bayes estimator for the unequal variance function Poisson problem. In Subsection 2.2 we compare the new PEB estimator to the MLE using posterior component risk and posterior component relative savings loss. In Subsection 2.3, we apply the estimator to globe valve leak data.

2.1 A Parametric Empirical Bayes Estimator for the Unequal Variance Function Poisson Problem

The parametric empirical Bayes (PEB) methods of Efron and Morris (1973, 1975) have their background in the work of Stein (1955, with James, 1961) on the estimation of the mean of a multivariate normal. In justifying Stein's estimator, Efron and Morris view it as a parametric equivalent of Robbins' (1955) nonparametric empirical Bayes (NPEB) (see Morris, 1983a,b). This viewpoint helps them derive a more general theory that can be used in cases other than that of Stein. For example, Section 1 recounts the Efron-Morris PEB generalizations of Stein's rule to unequal variance normal problems and Morris' application to the equal variance Poisson case. In this section, we use similar techniques to construct a PEB estimator for the unequal variance function Poisson problem defined by Distribution (21).

In particular, we suppose that

$$X_i|\lambda_i \overset{\text{ind}}{\backsim} \frac{1}{t_i}\,\text{Poiss}(\lambda_i t_i) \qquad (35)$$

for $i = 1,\ldots,k$ where the t_i are known and the λ_i are to be estimated. The mean and variance of X_i given λ_i are $E(X_i|\lambda_i) = \lambda_i$ and $V_i(\lambda_i) \equiv \text{Var}(X_i|\lambda_i) = \lambda_i/t_i$, respectively. The conjugate priors for the λ_i are

$$\lambda_i\,|m, A \overset{\text{iid}}{\backsim} \text{Gam}(m, A),\ i = 1,\ldots,k \qquad (36)$$

where Gam(m, A) continues to refer to a gamma distributed random variable with mean m and variance A.

It follows from Distributions (35) and (36) that the posterior distributions of the λ_i are

$$\lambda_i|\underline{X}, m, A \overset{\text{ind}}{\backsim} \text{Gam}(\lambda_i^*, A_i^*) \qquad (37)$$

where

$$\lambda_i^* = E(\lambda_i|\underline{X}, m, A) = (1 - B_i)X_i + B_i m \qquad (38)$$

and

$$A_i^* = \text{Var}(\lambda_i|\underline{X}, m, A) = (1 - B_i)\lambda_i^*/t_i$$

with

$$B_i = \frac{m/t_i}{m/t_i + A} \ . \qquad (39)$$

The marginal distributions of the X_i are

$$X_i|m, A \overset{\text{ind}}{\backsim} \text{NB}(m, m/t_i + A) \qquad (40)$$

where $\text{NB}(m, m/t_i + A)$ signifies a random variable which has a negative binomial distribution with mean m and variance $m/t_i + A$.

Paralleling the normal case with $r = 1$, we construct PEB estimators of λ_i by using Distribution (40) to obtain estimates $(\hat{m}, \hat{B}_i)$ and then substituting for (m, B_i) in Equation (38). The following iterative procedure defines estimates of m and A. Given $\hat{m}$ and $\hat{A}$, calculate

1. The empirical Bayes weights evaluated at $(\hat{m}, \hat{A})$

$$w_i = w_i(\hat{m}, \hat{A}) = 1/(\hat{m}/t_i + \hat{A}) \qquad (41)$$

2. The optimal linear estimate of m

$$\hat{m} \equiv \sum w_i X_i / \sum w_i \qquad (42)$$

3. The weighted average of squared residuals

$$S = \sum w_i (X_i - \hat{m})^2 / \sum w_i \qquad (43)$$

4. The weighted average of sampling variance functions evaluated at $\hat{m}$, with $V_i(m) = m/t_i$

$$\bar{V}(\hat{m}) \equiv \sum w_i V_i(\hat{m}) / \sum w_i \tag{44}$$

5. The unbiased estimate of A

$$\hat{A} \equiv \frac{\sum w_i \frac{1}{k-1} (X_i - \hat{m})^2 - V_i(\hat{m})}{\sum w_i}$$

$$= \frac{1}{k-1} S - \bar{V}(\hat{m}) \; . \tag{45}$$

Steps 1 to 5 are repeated until $(\hat{m}, \hat{A})$ converges. Then, the PEB shrinking factors are calculated as

$$\hat{B}_i = \frac{k-3}{k-1} \frac{V_i(\hat{m})}{V_i(\hat{m}) + \hat{A}}$$

$$= \frac{k-3}{k-1} \frac{\hat{m}/t_i}{\hat{m}/t_i + \hat{A}} \; . \tag{46}$$

Finally, the PEB estimates of the posterior means are

$$\hat{\lambda}_i = (1 - \hat{B}_i) X_i + \hat{B}_i \hat{m}, \; i = 1, \ldots, k \; . \tag{47}$$

In order to prove the following lemmas, we assume the weights are evaluated at the true (m, A) instead of $(\hat{m}, \hat{A})$. This allows us to view the w_i as constants when taking expectations given m and A.

Lemma 1: $\sum w_i = k/(\bar{V}(m) + A)$.

Proof:

$$k = \sum \frac{V_i(m) + A}{V_i(m) + A} = \bar{V}(m) \sum w_i + A \sum w_i$$

$$= (\bar{V}(m) + A) \sum w_i \; . \quad \text{QED}$$

Lemma 2: The estimator $\hat{m}$ has mean m and variance $1/\sum w_i = (\bar{V} + A)/k$. The estimator $\hat{m}$ is the optimal linear estimate of m.

Proof: Each X_i is an unbiased estimate of m; hence so is $\sum c_i X_i$ for all $\{c_i\}$ such that $\sum c_i = 1$. Let

$$Q = \sum c_i^2 \, \mathrm{Var}(X_i) - \theta(\sum c_i - 1)$$

$$= \sum c_i^2 \, [V_i(m) + A] - \theta(\sum c_i - 1) \; .$$

Then

$$\frac{\partial Q}{\partial c_j} = 2c_j [V_j(m) + A] - \theta$$

which implies that

$$c_j^* = \theta^* / 2[V_j(m) + A] \; .$$

But

$$1 = \sum c_j^* = \frac{\theta^*}{2} \sum 1/[V_j(m) + A]$$

implies that

$$\theta^* = 2 / \sum 1/[V_j(m) + A],$$

so

$$c_j^* = \frac{1/[V_j(m) + A]}{\sum 1/[V_i(m) + A]} = \frac{w_j}{\sum w_i} \; .$$

Hence $\hat{m}$ has minimum variance among linear unbiased estimates of m. Also

$$\mathrm{Var}(\hat{m}) = \sum w_i^2 \, \mathrm{Var}(X_i) / (\sum w_i)^2$$

$$= \sum w_i^2 (1/w_i) / (\sum w_i)^2$$

$$= 1/\sum w_i . \quad \text{QED}$$

Lemma 3: $E V_i(\hat{m}) = V_i(m)$ and $E\bar{V}(\hat{m}) = \bar{V}(m)$.

Proof:

$$E V_i(\hat{m}) = E\hat{m}/t_i = m/t_i = V_i(m)$$

$$E\bar{V}(\hat{m}) = \sum w_i E V_i(\hat{m}) / \sum w_i = \sum w_i V_i(m) / \sum w_i$$

$$= \bar{V}(m) \quad . \quad \text{QED}$$

Lemma 4: $ES = \frac{k-1}{k} [\bar{V}(m) + A]$.

Proof:

$$ES = \frac{\sum w_i E(X_i - \hat{m})^2}{\sum w_i} = \frac{\sum w_i (1/w_i - 1/\sum w_j)}{\sum w_i}$$

$$= \frac{k-1}{\sum w_i} = \frac{k-1}{k} (\bar{V}(m) + A) \quad . \quad \text{QED}$$

Lemma 5: $E\hat{A} = A$.

Proof:

$$E\hat{A} = \frac{k}{k-1} ES - E\bar{V}(\hat{m})$$

$$= \frac{k}{k-1} \frac{k-1}{k} (\bar{V} + A) - \bar{V} = A \quad . \quad \text{QED}$$

Last, we show that the PEB rule given in Equations (41) through (47) reduces to Morris' rule [Equations (30) through (34)] when $t_i = t$ for all i.

Lemma 6: If $t_i = t$ for all i, then

(i) $V_i = V$

(ii) $w_i = w = 1/(V + A)$

(iii) $\hat{m} = \bar{X}$

(iv) $S = \frac{1}{k} \sum (\bar{X}_i - X)^2$

(v) $\hat{B}_i = \frac{k-3}{k} \frac{V(\bar{X})}{S} = \hat{B}$.

Proof: (i) to (iv) are obvious and (v) follows since

$$\hat{B}_i = \frac{k-3}{k-1} \frac{V(\bar{X})}{V(\bar{X}) + \hat{A}}$$

$$= \frac{k-3}{k-1} \frac{V(\bar{X})}{V(\bar{X}) + \frac{k}{k-1} S - V(\bar{X})}$$

$$= \frac{k-3}{k} \frac{V(\bar{X})}{S} = \hat{B} \quad . \quad \text{QED}$$

In practice, the estimator is improved by forcing $\hat{A}$ to be nonnegative, i.e., by setting $\hat{A} = 0$ if the calculated $\hat{A} < 0$.

The unequal variance function Poisson estimator defined here is similar in certain respects to both the Efron-Morris normal unequal variance rule with $r = 1$, and the Morris equal variance function Poisson rule. Its derivation almost exactly parallels that of the unequal variance normal rules, and it reduces to Morris' rule when the variance functions are equal.

On the other hand, it also differs significantly from either of them. It differs from Morris' equal variance rule since it allows the t_i to differ. It differs from the Efron-Morris normal rules since it accounts for functional dependence of the variance on the mean; i.e., it assumes Poisson as opposed to normal sampling distributions.

2.2 Evaluation of the Parametric Empirical Bayes Estimator—Comparing the PEB Estimator to the MLE

In order to compare the unequal variance function Poisson estimator to the MLE, we define the posterior component risk of an estimator $\underline{\tilde{\lambda}}(\underline{X})$ as

$$R_i(\underline{\tilde{\lambda}}) = R_i\left[(\underline{X}, m, A), \tilde{\lambda}\right]$$

$$= E\,[(\lambda_i - \tilde{\lambda}_i)^2 \mid (\underline{X}, m, A)] \, .$$

We also define the posterior component relative savings loss of $\underset{\sim}{\tilde{\lambda}}$ as

$$RSL_i(\tilde{\underline{\lambda}}) = RSL_i\left[(\underline{X}, m, A), \tilde{\underline{\lambda}}\right]$$

$$= \frac{R_i(\tilde{\underline{\lambda}}) - R_i(\underline{\lambda}^*)}{R_i(\underline{\lambda}^\circ) - R_i(\underline{\lambda}^*)}$$

where $\underline{\lambda}^\circ(\underline{X}) = \underline{X}$ and $\underline{\lambda}^*$ has components λ_i^*.

The following facts are easily shown:

1. $R_i(\underline{\lambda}^*) = A_i^* = (1 - B_i)^2 \frac{X_i}{t_i} + B_i(1 - B_i) \frac{m}{t_i}$

2. $R_i(\underline{\lambda}^\circ) = A_i^* + B_i^2 (m - X_i)^2$

3. $R_i(\hat{\underline{\lambda}}) = A_i^* + [B_i(m - X_i) - \hat{B}_i(\hat{m} - X_i)]^2$

4. $RSL_i(\hat{\underline{\lambda}}) = \left[1 - \frac{\hat{B}_i(\hat{m} - X_i)}{B_i(m - X_i)}\right]^2 .$

These calculations are similar to results of Efron-Morris (1973).

It follows that if

$$0 \le \hat{B}_i \frac{(\hat{m} - X_i)}{(m - X_i)} \le 2B_i \qquad (48)$$

then the PEB estimator $\hat{\lambda}_i$ will have smaller posterior component risk, and relative savings loss, than the MLE, X_i. Since $\hat{m}$ is unbiased for m and $\hat{B}_i$ is approximately unbiased for B_i, Inequality (48) should hold quite often.

Results of Monte Carlo experiments comparing the PEB estimator and the MLE are given in Section 4.

2.3. Application to the Nuclear Plant Reliability Data System—Globe Valve Leaks

The PEB rule is now applied to NPRDS (1979) data for globe valves. The failure mode of interest is "leaks." Classes are distinguished by their operator types. The data are given in Table 2. The MLE and PEB estimates are given in Table 3.

The class variance functions are very unequal because the times on test range from 0.552×10^6 to 235.902×10^6 calendar hours (approximately 63 to 27,000 years). Since the t_i vary significantly so do the shrinking factors, $\hat{B}_i$. The shrinking pattern

Table 2. Globe valve (leaks) — data from NPRDS (1979) represent cumulative totals through 1979.

Category i	Operator Type	Number of Components $= r_i$	Number of Leaks $= f_i$	Times on Test $= t_i$ (10^6 cal. hrs.)
1	Manual	5706	31	236.902
2	Pneumatic/diaphragm/cylinder	2764	157	115.944
3	Electric motor/servo	961	30	36.812
4	Mechanical	188	13	7.597
5	Solenoid	174	7	5.466
6	Hydraulic	43	7	1.689
7	None/other/undefined	33	0	1.123
8	Explosive/squib	13	0	0.552

Table 3. MLE and PEB leak rate estimates for globe valves $\hat{m} = 1.20265$, $\hat{A} = 1.16918$, $k = 8$, $(\hat{m},\hat{A})$ converged in 5 iterations

Category i	MLE $X_i = f_i/t_i$	PEB $\hat{\lambda}_i = (1 - \hat{B}_i)X_i + \hat{B}_i\hat{m}$	Shrinking Factor $\hat{B}_i = \frac{k-3}{k-1} \frac{\hat{m}/t_i}{\hat{m}/t_i + \hat{A}}$
1	0.1309	0.1342	0.003088
2	1.3541	1.3532	0.006281
3	0.8150	0.8225	0.019416
4	1.7112	1.6679	0.085180
5	1.2806	1.2718	0.113129
6	4.1445	3.3491	0.270358
7	0.0000	0.4107	0.341477
8	0.0000	0.5590	0.464836

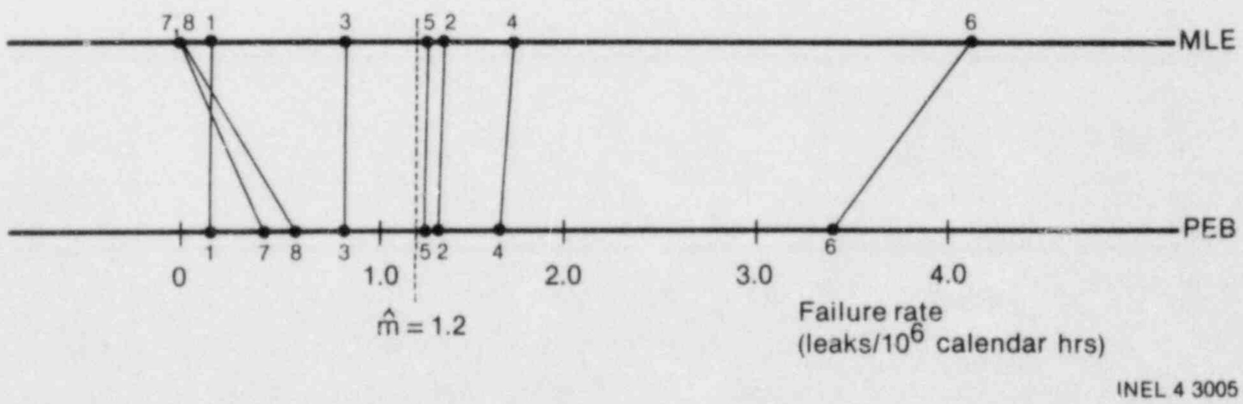

Figure 1. Shrinking pattern for globe valve leaks.

is illustrated in Figure 1. Only the classes with small t_i have significant shrinkage. A crossover can be seen, i.e., a reordering of the estimated failure rates. The PEB estimates suggest that Class 1 has lower failure rate than either Class 7 or 8. This contrasts with the reverse ordering for the MLE.

Classes 7 and 8 have $X_i = 0$ but $\hat{\lambda}_i > 0$. Surely (?) the failure rates for these operator types are not zero. A more plausible explanation for the lack of observed failures for these component types is the small times for their testing. In fact, assuming $\hat{\lambda}_8$ to be correct, if Class 8 is used for 0.2×10^6 calendar hours per year, then it will take at least 9 years before components in Class 8 expect one or more failures. Similarly, it will take 5 years at 0.5×10^6 calendar hours per year for Class 7 to expect one or more failures.

Actually λ_8 does equal zero, since the explosive-operated valves will not leak by virtue of their design. This observation highlights the need for caution when applying any type of estimator: applicable engineering information about components should be incorporated in any statistical analysis.

3. PARAMETRIC EMPIRICAL BAYES INTERVAL ESTIMATES FOR COMPONENT FAILURE RATES

This objective is easily the most ambitious of the project. Empirical Bayes confidence interval theory is in its infancy. Morris has only recently (1983a,b) defined interval estimates for the equal and unequal variance normal problems. Though these intervals represent a major breakthrough, there still is much work to be done in this area. Here we will report on Morris' advances and then make some comments as to how his intervals might be used for calculating confidence bounds for component failure rates. We then illustrate one of our suggestions on the globe valve data analyzed in Section 2.3.

For the situation described by Distributions (1) through (5), Morris (1983b) defines an approximate empirical Bayes confidence interval given by

$$\hat{\mu}_i \pm z\, s_i \tag{49}$$

where $\hat{\mu}_i$ is calculated using Equations (8) through (14), and

$$s_i^2 = V_i\left(1 - \frac{k - \hat{r}_i}{k}\hat{B}_i\right) + v_i(X_i - \hat{m}_i)^2$$

with

$$\hat{r}_i = k\, w_i[Z(Z'WZ)^{-1}Z']_{ii}$$

($\hat{r}_i$ estimates the effective values of r for component i)

and

$$v_i = \frac{2}{k - r - 2}\hat{B}_i^2\left(\frac{V + \hat{A}}{V_i + \hat{A}}\right)$$

(v_i approximates the variance of $\hat{B}_i$).

The Interval (49) is claimed to contain μ_i with probability $1 - \alpha = 2\Phi(z) - 1$.

The posterior variance of μ_i, given X_i, m_i, and A, i.e. $V_i(1\text{-}B_i)$, could be estimated by $V_i(1\text{-}\hat{B}_i)$ and used to create an interval estimate of μ_i. The Interval (49) is wider than the resulting interval because s_i^2 accounts for the additional variance contributions that result from estimating m_i and A. In short, s_i^2 is the appropriate measure of variance associated with the point estimate $\hat{\mu}_i$.

If component failure rates can be described, as in Section 1, by the Poisson model

$$X_i|\lambda_i \overset{\text{ind}}{\sim} \frac{1}{t_i}\text{Poiss}(\lambda_i t_i),\ i = 1,\ldots,k$$

then one of several normal approximations could be used in order to take advantage of Morris' intervals. Normality might be assumed for the mean, variance stabilized, skewness stabilized, or natural parameter scale, each being justified by the central limit theorem applied to maximum likelihood estimates.

For example, if we use the variance stabilizing transformation, $Y_i = \sqrt{X_i}$, $\mu_i = \sqrt{\lambda_i}$, and $V_i = 1/4t_i$, then approximately,

$$Y_i\ \mu_i \mid \overset{\text{ind}}{\sim} N(\mu_i, V_i),\ i = 1,\ldots,k$$

which is exactly the assumption needed to apply Morris' theory to find EB confidence intervals for the μ_i. Approximate 95% PEB confidence intervals for the λ_i are found by squaring the upper and lower bounds of Interval (49). The results of this procedure are given in Table 4.

For discussion purposes we focus on upper confidence bounds for Categories 7 and 8. Approximate 95% PEB upper confidence bounds are calculated as:

Category 7

$$\begin{aligned}(\hat{\mu}_7 &+ z_{0.95}\, s_7)^2 \\ &= [0.27099 + (1.645)(0.430175)]^2 \\ &= (0.9786)^2 = 0.9577\end{aligned}$$

Category 8

$$\begin{aligned}(\hat{\mu}_8 &+ z_{0.95}\, s_8)^2 \\ &= [0.38733 + (1.645)(0.553867)]^2 \\ &= (1.2984)^2 = 1.6859.\end{aligned}$$

The corresponding standard upper 95% confidence bounds (Johnson and Kotz (1969), p. 96) are:

Table 4. PEB 95% confidence intervals for globe valves
$k = 8$, $\hat{m} = 0.926936$, $\hat{A} = 0.321288$, convergence in 10 iterations

f_i	t_i	$X_i = f_i/t_i$	$Y_i = \sqrt{X_i}$	$V_i = \frac{1}{4t_i}$	$\hat{B}_i = \frac{k-3}{k-1}\frac{V_i}{V_i + \hat{A}}$	v_i	$\hat{r}_i$
31	236.902	0.13086	0.36174	0.001055	0.002338	0.0000027	1.23776
157	115.944	1.35410	1.16366	0.002156	0.004762	0.0000112	1.23354
30	36.812	0.81495	0.90275	0.006791	0.014786	0.0001063	1.21612
13	7.597	1.71120	1.30813	0.032908	0.066363	0.0019844	1.12645
7	5.466	1.28064	1.13166	0.045737	0.089012	0.0034452	1.08707
7	1.689	4.14446	2.03580	0.148017	0.225283	0.0172590	0.85016
0	1.123	0.00000	0.00000	0.222618	0.292354	0.0250788	0.73355
0	0.552	0.00000	0.00000	0.452899	0.417857	0.0359934	0.51536

s_i^2	s_i	μ_i^L	$\hat{\mu}_i = (1 - \hat{B}_i)Y_i + \hat{B}_i\hat{m}$	μ_i^U	$\lambda_i^L = (\mu_i^L)^2$	$\hat{\lambda}_i = (\hat{\mu}_i)^2$	$\lambda_i^U = (\mu_i^U)2$
0.001054	0.032466	0.29943	0.36306	0.42670	0.08966	0.13181	0.18207
0.002148	0.046348	1.07169	1.16253	1.25337	1.14852	1.35148	1.57095
0.006706	0.081891	0.74260	0.90310	1.06361	0.55145	0.81560	1.13127
0.031320	0.176974	0.93596	1.28283	1.62970	0.87603	1.64566	2.65592
0.042364	0.205825	0.71002	1.11343	1.51685	0.50412	1.23973	2.30083
0.139436	0.373411	1.05410	1.78599	2.51787	1.11113	3.18976	6.33969
0.185051	0.430175	0[a]	0.27099	1.11414	0	0.07344	1.24130
0.306769	0.553867	0[a]	0.38733	1.47291	0	0.15002	2.16945
					95% lower limit for λ_i		95% upper limit for λ_i

a. Calculated values of μ_7^L and μ_8^L were negative.

Category 7

$$\frac{1}{2t_7}\chi^2_{2,0.95} = 3.00/1.123 = 2.6714$$

Category 8

$$\frac{1}{2t_8}\chi^2_{2,0.95} = 3.00/0.552 = 5.4348 \quad .$$

The PEB upper confidence bounds are significantly less than the standard upper confidence bounds. The confidence levels of the two bounds do not represent the same probability calculation. The standard confidence bound is based on the probability that the random upper bound exceeds the true failure rate if sampling of the data is repeated with fixed failure rates. The PEB confidence bound is based on the probability that the random upper bound exceeds the random true failure rate if sampling of data and parameters is repeated.

This simple example illustrates the potential improvement of the PEB upper confidence bounds over the standard upper bounds. We need to check coverage probabilities using simulation. Also, we recommend that confidence bounds be developed more specifically for the Poisson problems. Though this objective is the most difficult, it could also be the most productive and useful to component failure rate estimation. We hope to continue work in this important field.

4. MONTE CARLO COMPARISONS OF MAXIMUM LIKELIHOOD AND PARAMETRIC EMPIRICAL BAYES ESTIMATES OF COMPONENT FAILURE RATES

This section is divided into two subsections. In the first subsection we give an extended discussion of a short simulation study in order to highlight the situations in which we can or cannot expect to improve upon the MLE. In the second subsection we give the results of several Monte Carlo studies.

4.1. A Short Simulation Study

Using the NPRDS data for globe valves recorded in Table 5, a short simulation study was conducted. The basic procedure for the study was as follows:

1. True failure rates were generated using $\lambda_i^S \underset{\sim}{\text{ind}}$ Gamma $(m = 1.026,\ A = 1.024)$, $i = 1,\ldots,k = 8$

2. Observed failure rates were simulated using $X_i^S \underset{\sim}{\text{ind}} (1/t_i)\text{Poisson}(\lambda_i^S t_i)$, $i = 1,\ldots,8$

3. Using the simulated data $(X_i^S,\ldots,X_8^S)$, PEB estimates $(\hat{\lambda}_i^S,\ldots,\hat{\lambda}_8^S)$ were calculated according to the procedure outlined in Equations (41) to (47)

Table 5. Data for globe valves (mode of failure: leak)

Category i	Operator Type	Observation Times[a] $= t_i$	Number of Components $= n_i$	Number of Failures $= f_i$	Failure Rates[b] $= X_i$
1	Manual	293.017	5,922	33	0.133
2	Electric motor/servo	45.616	999	33	0.923
3	Hydraulic	2.066	43	7	3.388
4	Pneumatic/diaphragm	141.339	2,811	171	1.210
5	Solenoid	7.154	183	9	1.258
6	Explosive/squib	0.665	13	0	0.000
7	Mechanical	9.240	188	14	1.515
8	Other[c]	1.716	42	0	0.000

a. 10^6 calendar hours.

b. Failures per million calendar hours.

c. Other, none, and undefined combined.

$$\bar{X} = \frac{1}{8}\sum_1^k X_i = 1.026,\quad s^2 = \frac{1}{7}\sum_1^k (X_i - \bar{X})^2 = 1.024$$

4. Component losses for the MLE, X_i^S, and the PEB, $\hat{\lambda}_i^S$, were calculated using the squared difference from λ_i^S.

Step 1 was repeated four times, hence the 32 experiments (4 repetitions times k = 8 λ_i's per repetition) listed in Table 6. Each time Step 1 was done, Steps 2 to 4 were repeated eight times and the component losses for these eight repetitions were averaged. These average losses are given in Table 6. As an example of the calculations involved, the eight repetitions of Steps 2 to 4 are given in Table 7 for a case where $\lambda_2^S = 1.40652$, Experiment 23 in Table 6.

It has been pointed out that PEB can do much worse than the MLE; for example, Experiment 2 has MLE average loss of 0.00152 and PEB average loss of 0.15808. Naturally, this causes concern for anyone interested in applying PEB. We offer several comments in an attempt to allay this concern.

Firstly, it should be noted that the given average losses are estimates of the frequency risk; that is, they estimate the average loss when sampling of the data is repeated for fixed values of the parameters. It is known that the MLE will perform well for such frequency averages of component loss, so examples of the MLE beating PEB in this sense should not be surprising.

Secondly, for the example given above, the Bayes estimate, (which uses the actual values of m and A), will also do badly. The reason lies in the fact that t_6 was very small, which implies that both Bayes and PEB estimates shrink heavily toward m and $\hat{m}$, respectively, but, since $\hat{m}$ and m are much larger than λ_6, this takes the estimates away from the small λ_6. Efron and Morris (1972) have suggested "Limited Translation Rules" (LTR) to protect against overshrinkage. Similar ideas could be applied in the Poisson problem.

Thirdly, Experiment 2 was part of a larger experiment in which there were k = 8 component categories. Actually, Experiment 2 was associated with Experiments 1, 10, 13, 21, 23, 27, and 31. If the average component losses for these eight experiments are added together, then the PEB estimator does about as well as the MLE; in particular, total average PEB loss was 1.93188 compared with total average MLE loss which was 1.92995. Comparing the sums of component losses

Table 6. Summary of average losses for MLE and PEB estimators for each failure rate (globe valves data, leak mode of failure)

Experiment	Simulated Failure Rates[a]	Average Losses[b] (MLE)	Average Losses[b] (PEB)
1	0.00077	0.00144	0.00651
2	0.03904	0.00152	0.15808
3	0.04916	0.53078	1.34727
4	0.06172	0.00438	0.02680
5	0.21169	0.18776	0.13098
6	0.23712	0.00102	0.00112
7	0.26121	0.06823	0.01986
8	0.33046	0.08895	0.08870
9	0.33387	0.00073	0.00081
10	0.44217	0.03626	0.02884
11	0.54723	0.18763	0.12505
12	0.65556	0.00539	0.00519
13	0.70959	0.44694	0.23141
14	0.79063	0.07844	0.07530
15	0.79812	0.09058	0.08126
16	0.91587	1.22050	0.97810
17	0.94388	0.03447	0.02960
18	1.00920	0.05343	0.05272
19	1.08046	1.55688	0.81674
20	1.08920	0.27821	0.22475
21	1.29132	0.00407	0.00406
22	1.34945	0.02777	0.02754
23	1.40652	0.02210	0.02175
24	1.43794	0.05440	0.05578
25	1.85540	0.74838	0.45302
26	1.96735	0.46672	0.83273
27	2.25489	0.01090	0.01079
28	2.26682	0.25199	0.20477
29	3.40901	0.01183	0.01192
30	4.39659	0.05400	0.05306
31	4.83535	1.40672	1.47044
32	5.33254	0.00883	0.00899

a. Failures per million calendar hours.

b. Using squared error loss.

Table 7. Example of calculations for Experiment 23 in Table 6; $\lambda_2^S = 1.40652 \times 10^{-6}$

Simulated Failure Rate	MLE Estimates	MLE Losses	PEB Estimates	PEB Losses
1.407×10^{-6}	1.096×10^{-6}	0.0964	1.097×10^{-6}	0.0959
1.407×10^{-6}	1.491×10^{-6}	0.0071	1.489×10^{-6}	0.0067
1.407×10^{-6}	1.184×10^{-6}	0.0496	1.338×10^{-6}	0.0491
1.407×10^{-6}	1.337×10^{-6}	0.0048	1.185×10^{-6}	0.0047
1.407×10^{-6}	1.513×10^{-6}	0.0113	1.505×10^{-6}	0.0098
1.407×10^{-6}	1.337×10^{-6}	0.0048	1.337×10^{-6}	0.0048
1.407×10^{-6}	1.381×10^{-6}	0.0006	1.382×10^{-6}	0.0006
1.407×10^{-6}	1.359×10^{-6}	0.0022	1.357×10^{-6}	0.0024
Average	--------------------	0.0221	--------------------	0.0218

has the effect of making the difference between MLE and PEB component losses more important than their ratio; specifically, for example, the fact that the PEB average loss for Experiment 2 was 0.15656 larger than the MLE average loss is more important than the fact that the PEB average loss is a hundred times as large. In general, when average component losses are summed, the MLE can be dominated using PEB. Generally, this domination will become apparent only if Steps 2 to 4 are repeated a large number of times (more than the eight times done here). Preliminary Monte Carlo studies support this conclusion.

Fourthly, Bayesian evaluation of estimators involves averaging the loss over the posterior distribution of the parameters given the data and the hyperparameters m and A. Calculations in Subsection 2.2 suggest we could hope to dominate the MLE componentwise in terms of posterior risk, for (m,A) values of high likelihood. This type of data analytic evaluation of Stein-like estimators can be found in Efron-Morris (1975).

Finally, PEB evaluation of an estimator involves averaging the loss over the distribution of both the data and the parameters (see Morris, 1983a,b). This averaging process could be estimated by repeating Steps 1 to 4 a large number of times and then averaging the component losses for these repetitions (see Subsection 4.2.2). Morris' results for the normal case (1983b) suggest we could hope to improve upon the MLE componentwise when taking this PEB average. Of course, componentwise domination implies domination for the sum of component risks.

This discussion is summarized in Table 8. For each method of evaluating estimators and for component and total losses, answers are given for the question: Is it possible to improve upon the MLE?

In all cases when the MLE can be dominated additional information is needed which connects in some way the independent sampling problems assumed for Case (A). The MLE can be improved for Case (B) only because the loss structure, sum of component losses, connects the individual estimation problems. For Cases (C) and (E) additional information relating the components comes in the assumption of a parametric class of prior distributions. Cases (D) and (F) relate the

Table 8. Is it possible to improve upon the MLE?

Method of Evaluation (Averaging Distribution) \ Type of Loss	Component Losses	Total Loss
Frequency (Sampling)	A No	B Yes
Bayes (Posterior)	C Yes (in regions of high likelihood)	D Yes (in regions of high likelihood)
Empirical Bayes (Joint)	E Yes	F Yes

components in both ways by assuming total loss and prior structure. In short, the MLE can be improved upon only when one is willing to make additional assumptions. However, if you are willing to make the additional assumptions, the possible improvement can be substantial, as illustrated by the Monte Carlo results given in the next subsection.

4.2. Two Monte Carlo Experiments With Data Simulated to Look Like NPRDS Globe Valve Data

In this subsection, we give the results of two Monte Carlo experiments. Experiment 1 compares MLE and PEB estimates using frequency averages for sums of component squared error losses. Experiment 2 compares the two estimators using PEB averages of individual component squared error losses. As predicted in the discussion in Subsection 4.1, the PEB estimator significantly outperforms the MLE.

The basic algorithm we used to generate data that looked like NPRDS data was as follows:

1. Obtain the observed failure rates and times on test (X_i, t_i), $i = 1,\ldots,k$, from NPRDS

2. Calculate $(\hat{m}, \hat{A})$ using Equations (41) to (45)

3. Calculate for $i = 1,\ldots,k$

$$\hat{B}_i = \frac{k-3}{k-1}\,\frac{\hat{m}/t_i}{\hat{m}/t_i + \hat{A}}$$

$$\hat{\lambda}_i = (1 - \hat{B}_i)X_i + \hat{B}_i\hat{m}$$

$$\hat{A}_i = (1 - \hat{B}_i)\hat{\lambda}_i/t_i$$

4. Simulate true failure rates using

$$\lambda_i^S \overset{ind}{\sim} \text{Gamma}\,(\hat{\lambda}_i, \hat{A}_i),\ i = 1,\ldots,k$$

5. Simulate observed failure rates using

$$X_i^S \overset{ind}{\sim} (1/t_i)\text{Poisson}(\lambda_i^S t_i),\ i = 1,\ldots,k\ .$$

The simulated (λ_i^S, X_i^S) should be similar to the actual NPRDS data (λ_i, X_i). Steps 2 and 3 were applied to the simulated data $(\underline{X}^S, \underline{t})$ in order to calculate $\hat{\lambda}_i^S$; then X_i^S and $\hat{\lambda}_i^S$ were compared as estimates of λ_i^S.

Actually, both experiments used the NPRDS globe valve data recorded in Table 2. Results of Steps 2 and 3 are given in Table 3, except for the $\hat{A}_i$ which are easily calculated from the $\hat{\lambda}_i$, the $\hat{B}_i$, and the t_i.

It will help us to explain the experiments if we define a few terms. Step 4 will be repeated NL times with index $n = 1,\ldots,NL$. For each n, Step 5 will be repeated NX times with index $j = 1,\ldots,NX$. Furthermore we define $L_{ijn}(MLE)$ to be the ith component squared error loss for the jth set of X's for the nth set of λ's when using the MLE. We define $L_{ijn}(PEB)$ analogously.

4.2.1. Evaluation With Frequency Averages of Sums of Component Squared Error Losses. For this experiment we averaged the sums of component losses over repeated sampling given by Step 5. In particular, for each set of λ's we averaged the sum of component losses for NX = 500 repetitions of Step 5. These averages estimate the frequency risk of the estimators for sum of squared error loss.

In the notation given above, the sum of component losses for the jth set of X's for the nth set of λ's when using the MLE is

$$L_{.jn}(MLE) = \sum_{i=1}^{k} L_{ijn}(MLE)$$

and similarly when using the PEB estimate. The frequency average of the sum of squared losses for the nth set of λ's when using the MLE is

$$\bar{L}_{..n}(MLE) = \frac{1}{NX} \sum_{j=1}^{NX} L_{.jn}(MLE)$$

with that for the PEB estimate defined likewise. The efficiency of the PEB estimate relative to the MLE will be defined as

Efficiency = MLE average loss/PEB average loss.

Table 9 contains the results of NL = 10 repetitions of the above Monte Carlo routine. The table also includes the observed mean and variance of the λ's. The PEB estimator clearly outperformed the MLE. Only in two cases did the MLE have smaller average loss, but the percentages of lost efficiency for these cases were only 12.6 and 2.3. On the other hand, for the other eight cases the PEB significantly improved upon the MLE with percent efficiency

Table 9. Average total loss comparison of MLE and PEB estimates

Random No. Seed = 4863.81030
No. of Component Categories = 8
No. of Repetitions of λ = 10
No. of Repetitions per λ of X = 500

Trial	Average Total Losses				
n	MLE	PEB	% Efficiency	Lambda-Mean	Variance
1	2.92636	2.59611	112.7	1.10305	1.01093
2	4.35840	4.32205	100.8	1.33015	2.03527
3	5.68135	3.72526	152.5	1.46098	1.17801
4	3.40159	1.65010	206.1	0.95750	0.39820
5	6.67345	4.64720	143.6	1.43675	1.38681
6	7.12965	3.59787	198.2	1.40127	0.58544
7	3.02403	3.46026	87.4	1.22517	1.74693
8	5.27728	4.21066	125.3	1.36065	1.72773
9	2.74367	2.80839	97.7	1.13025	1.12605
10	2.26390	1.41756	159.7	0.94183	0.41307
Total	43.57967	32.43547	134.0	12.34760	11.60845

ranging from 100.8 to 206.1. It should be noted that the cases for which the PEB improves the most are those for which the observed variances of the λ's were small. These are the cases when shrinking is most desirable.

4.2.2 Evaluation with PEB Averages of Individual Component Squared Error Losses. For this experiment we averaged individual component losses over repetitions of both Steps 4 and 5. Specifically, we averaged component losses for NL = 1000 repetitions of Step 4, with NX = 1 repetition of Step 5 for each. These averages estimate the PEB risk of the estimators.

Notationally, the PEB average of the ith component squared error loss when using the MLE is

$$\bar{L}_{i1.}(\text{MLE}) = \frac{1}{NL}\sum_{n=1}^{NL} L_{i1n}(\text{MLE})$$

and analogously for the PEB estimate.

Tables 10 and 11 report the results of this experiment for two different initial seeds. Again, the PEB estimators are significantly better than the MLE. The PEB has smaller average loss for seven of the eight categories in both trials. The percent efficiency of the PEB relative to the MLE is approximately a decreasing function of the time on test for the category. This is not surprising since it is those categories with low times on test which gain the most from the shrinkage toward a global mean value.

The results of Experiments 1 and 2 strongly favor PEB methods. If you are willing to combine the estimation of true failure rates in either the loss function or the prior, then you should use PEB, not MLE. If, however, you insist that neither of these assumptions is valid, then you should continue to use the MLE.

Table 10. Average category loss comparison of PEB and MLE estimates

Random No. Seed = 4863.81030
No. of Component Categories = 8
No. of Repetitions of λ = 1000
No. of Repetitions per λ of X = 1

Category i	Average Category Losses		% Efficiency	Times
	MLE	PEB		
1	0.00052	0.00056	92.9	236.902
2	0.01176	0.01155	101.8	115.944
3	0.02223	0.02105	105.6	36.812
4	0.21707	0.18993	114.2	7.597
5	0.22784	0.18530	123.0	5.466
6	1.88552	1.85733	101.5	1.689
7	0.36259	0.26254	138.1	1.123
8	1.01472	0.54380	186.6	0.552
Total	3.74224	3.07206	121.8	406.085

Total number of PEB estimates that failed to converge = 0

Table 11. Average category loss comparison of PEB and MLE estimates

Random No. Seed = 715.5800
No. of Component Categories = 8
No. of Repetitions of λ = 1000
No. of Repetitions per λ of X = 1

Category i	Average Category Losses: MLE	PEB	% Efficiency	Times
1	0.00052	0.00057	91.2	236.902
2	0.01168	0.01155	101.1	115.944
3	0.02221	0.02124	104.6	36.812
4	0.21081	0.18664	113.0	7.597
5	0.22888	0.18064	126.7	5.466
6	2.08642	1.98046	105.4	1.689
7	0.36257	0.27187	133.4	1.123
8	0.89729	0.49624	180.8	0.552
Total	3.82039	3.14921	121.3	406.085

Total number of PEB estimates that failed to converge = 0

5. CONCLUSIONS AND SUGGESTIONS FOR FURTHER WORK

Application of Stein and related estimators to NPRDS requires development of estimators that account for various deviations from Stein's normal equal variance problem. Estimators must accommodate:

1. Poisson sampling distributions
2. Unequal times on test
3. Confidence intervals
4. Shrinkage toward regression surfaces.

For any one of the above deviations from Stein's problem, good PEB estimators have been proposed. Morris has proposed PEB estimators for the normal problem with added features 2, 3, and 4. Results of this project have dealt with deviations 1, 2, and using normal approximations, 3. Further work needs to be done in which all of 1 through 4 are simultaneously allowed. In particular, we need to develop estimators and confidence intervals for the unequal variance function Poisson problem with shrinkage toward a log-linear model.

In this direction, we suggest the following framework. For i = 1,...,k, assume the distribution of the observed failure rate, X_i, given the true failure rate, λ_i, is

$$X_i|\lambda_i \overset{\text{ind}}{\sim} \frac{1}{t_i} \text{Poiss}\,(\lambda_i t_i)$$

and assume the true failure rate has prior distribution

$$\lambda_i|\underline{\beta},\alpha \overset{\text{ind}}{\sim} \text{Gam}(m_i, \alpha m_i)$$

where α is a "lack of fit" parameter and $m_i = \exp(\underline{z}_i'\underline{\beta})$ is a log-linear model with $\underline{z}_i = (z_{i1},\dots z_{ir})$, an r-dimensional vector of explanatory variables, and $\underline{\beta} = (\beta_1,\dots,\beta_r)'$, an r-vector of regression coefficients. These distributional assumptions imply that the posterior distributions are

$$\lambda_i|\underline{X}, \underline{\beta}, \alpha \overset{\text{ind}}{\sim} \text{Gam}\left[m_i^*, (1 - B_i)\frac{m_i^*}{t_i}\right] \qquad (50)$$

where the ith posterior mean is

$$m_i^* = (1 - B_i)X_i + B_i m_i \qquad (51)$$

a weighted average of the observed failure rate, X_i, and log-linear mean, m_i, with weight

$$B_i = \frac{1}{1 + \alpha t_i}\,. \qquad (52)$$

The marginal distributions are

$$X_i|\underline{\beta}, \alpha \overset{\text{ind}}{\sim} \text{NB}[m_i, (\frac{1}{t_i} + \alpha)m_i]\,. \qquad (53)$$

These distributions look like Poissons with log-linear means and enlarged variance factors. The vector of regression coefficients, $\underline{\beta}$, and the lack of fit parameter, α, can both be estimated using Distribution (53). The estimates $(\hat{\underline{\beta}},\hat{\alpha})$ can then be used to define PEB point and interval estimates. These and other ideas need further investigation to be useful for estimating component failure rates.

To conclude, we review our progress on the four objectives of the project. Objective 1 required derivation of Stein-like estimators for distributional assumptions other than Stein's normal equal variance case. We successfully derived estimators for the unequal variance function Poisson case with shrinkage toward an unknown constant. For the new theory to be useful, we still need to work out the details of shrinkage toward a log-linear model. Objective 2 called for the evaluation of the new estimators. We found some theoretical success by looking at posterior risk functions. Our simulation results were also optimistic. More work on this objective should be directed at specific results for NPRDS data sets. Objective 3 concerned development of PEB confidence bounds. Unfortunately, theoretical results for this area are very difficult. We reported the current state of the art, which is restricted to normal problems, and suggested several ways in which we could make normal approximations. We hope work on this objective can be continued. Objective 4 called for Monte Carlo computer simulations to help evaluate the new estimator. Computer programs were developed to simulate unequal variance Poisson data with failure rates that come from a gamma distribution. Results for the cases studied were very promising. Further simulation work should continue to center on data generated using NPRDS data sets.

In short, our preliminary feasibility study indicates the potential advantages in applying Stein and related PEB estimators to NPRDS. We feel, however, that some more basic statistical analysis of NPRDS data sets needs to be done. Specifically, we feel that more complicated models appropriate for NPRDS data, such as the log-linear models suggested above, must be studied. Also, whatever statistical models and estimators are used, care must be exercised to account for existing engineering information.

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APPENDIX A
SIMULATED ILLUSTRATIONS

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SIMULATED ILLUSTRATIONS

A computer code has been developed to assist us in comparing the parametric empirical Bayes (PEB) estimator and the maximum likelihood estimator (MLE) for estimating Nuclear Plant Reliability Data System (NPRDS) component failure rates. The program was written in FORTRAN-IV on The University of Texas CDC 64/6600 computer system. The code, which uses IMSL (1982) random deviate generators GGAMR and GGPON, is described in detail by Heger (1983).

The data obtained for the analyses were extracted from NPRDS 1980 and 1981 Annual Reports (1980, 1981). The NPRDS produced failure statistics on selected systems and components related to nuclear safety in nuclear power plants. The systems and components included in the NPRDS data base are, in general, those classified as Safety Class 1 and 2 in ANSI 18.3 and ANSI N212, excluding spent fuel storage, passive reactor vessel structural internals, PWR piping 1 inch and below, and BWR piping 1-1/2 inches and below. Approximately 3,500 components within 25 to 35 systems are included in the reportable scope for each unit. The participating utility supplies engineering data for each reportable component and system. These engineering data, in coded form, are entered in the data base and are available as parameters on which to sort and select components for specific analyses. In order to compute the component or system service hours, the engineering data report includes estimates of service time as related to reactor service hours at three different conditions: (a) critical, (b) standby, and (c) shutdown. It is assumed that these three service conditions will account for 100% of total service time. Actual service times as reported on the Quarterly Operating Reports are used to compute the estimated service hours for each system and component in the NPRDS data base. Each participating utility reports in considerable detail all equipment related failures of a reportable system or component.

Four simulation experiments were conducted in which data from several sources were simulated and analyzed. The data analyzed were on nuclear components as documented in the NPRDS 1980 and 1981 Annual Reports (1980, 1981). For one mode of failure of the component under study, population size, failure frequencies and observation periods were extracted. The analyses were applied to data for globe valves, gate valves, pumps and internal combustion engines. Dimensions were established by operator types for valves and by subclassifications for pumps and engines. Section A-1 discusses the application of globe valves data for failure mode of "leak" to the simulation experiment. Gate valves data for failure mode of "leak" were applied in the next experiment; this is discussed in Section A-2. Pumps data for "leak" failure mode and engines data for "won't start/move" mode of failure were used in other simulation experiments. The results are given in Sections A-3 and A-4, respectively.

A-1. Globe Valves Experiment

Data. Population size, frequency of failure, and experiment periods were recorded for globe valves for the failure mode of *leak*. The source of data was the NPRDS 1980 Annual Reports (1980). Dimensions were distinguished by their operator types. Ten operator types were noted and are listed in Table 5. However, due to the small number of valves observed for *other*, *none*, and *undefined* operator types, the data from these groups were lumped together and listed under *other*. Therefore, eight dimensions were yielded for the analyses.

In Table 5 Columns 1 and 2 are group number and types or operator, respectively. Columns 3, 4, and 5 are the observed experiment time ($\underline{t}$), population size ($\underline{n}$), and frequency of failure ($\underline{f}$) for each row. Column 6 represents the point estimates of the failure rates ($\underline{X}$) for each row which is the ratio of f_i over t_i. In this study, it is assumed that $\underline{\lambda}$ comes from a Gamma distribution with mean m and variance A. For the simulation study an estimate of mean was calculated by

$$m|\underline{X} = \sum_{i=1}^{k} X_i / k$$

For Table A-1 data where k = 8

$m|\underline{X} = 1.026.$

An estimate of variance is given by

$$A|\underline{X} = \sum_{i=1}^{k} (X_i - m)^2/(k-1).$$

With Table A-1 data

$$A|\underline{X} = 1.024.$$

The mean (m) and variance (A) are treated as the real mean and variance of the prior distribution for the failure rates, and are input to the simulation routine.

Results. Table A-1 tabulates the results of the simulation experiment and estimates of failure rates. For each dimension, simulated failure rates, observation time, simulated frequency of failure, MLE and PEB estimates of the failure rates are given, respectively. It is apparent from the results listed in Table A-1 that in the majority of cases the PEB method leads to an estimate of the failure rates closer to its true value than the MLE estimates. Calculated losses, using the square error loss method, are tabulated in Table A-2, for both methods. It should be noted that the experiment periods vary from 0.665×10^6 to 293.017×10^6 calendar hours; this, leads to a wide variation in the simulated frequency of failure. Failure estimates for both methods are plotted. Failure rate estimates for the MLE method are located on the upper horizontal line. The PEB estimates of the failure rates are plotted on the lower horizontal line. The vertical dotted line represents the mean of the failure rate's prior distribution. In Figure A-1 six of the PEB estimates are closer to the mean than their MLE counterparts. The other estimates are very close in value for both methods. The value of m shown in the figure is the true prior mean used in the simulation. The PEB estimators shrink toward the estimated mean, $\hat{m}$, which is different from m.

Table A-1. Summary of results for globe valves (mode of failure: leak)

Category i	True[a] Failure Rates,[c] λ_i	Observation Times,[b] t_i	Simulated Failure Frequencies, f_i	MLE Estimates,[c] X_i	PEB Estimates,[c] $\hat{\lambda}_i$
1	3.409	293.017	972	3.317	3.314
2	1.438	45.616	55	1.206	1.216
3	1.855	2.066	2	0.968	1.175
4	4.397	141.339	626	4.429	4.418
5	2.267	7.154	19	2.656	2.598
6	0.049	0.665	0	0.000	0.790
7	0.791	9.240	7	0.758	0.831
8	0.547	1.716	0	0.000	0.472

a. Simulated by the program.

b. 10^6 calendar hours.

c. Failures per million calendar hours.

Table A-2. Calculated square error loss (globe valves, leak mode of failure)

Category	Simulated Failure Rates	MLE Loss	PEB Loss
1	3.409×10^{-6}	0.84×10^{-2}	0.90×10^{-2}
2	1.438×10^{-6}	0.54×10^{-2}	0.50×10^{-1}
3	1.855×10^{-6}	0.79	0.46
4	4.397×10^{-6}	0.11×10^{-2}	0.45×10^{-3}
5	2.267×10^{-6}	0.15	0.11
6	0.049×10^{-6}	0.24×10^{-2}	0.55
7	0.791×10^{-6}	0.11×10^{-2}	0.17×10^{-2}
8	0.547×10^{-6}	0.30	0.57×10^{-2}

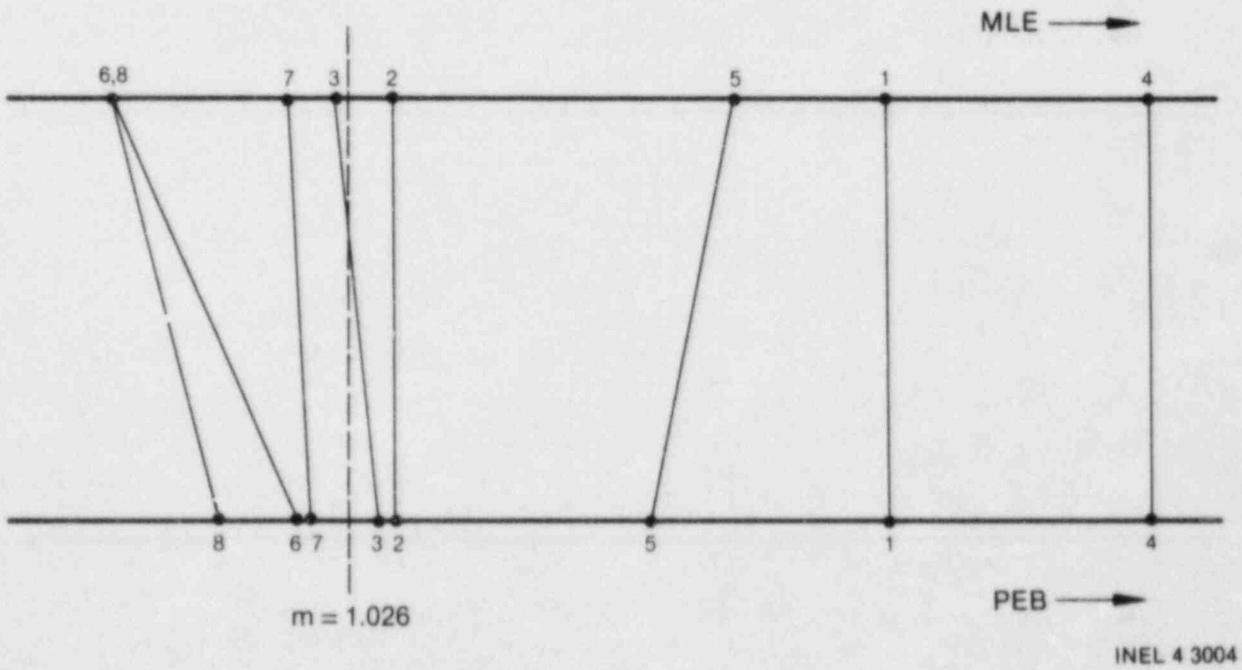

Figure A-1. Comparison of the estimates of the MLE and PEB methods for globe valves for failure mode of leak.

A-2. Gate Valves Experiment

Data. A second Monte Carlo experiment was conducted using data for gate valves. Population size, frequency of failure events, and observation periods were obtained from the NPRDS 1980 Annual Reports (1980). The MLE and PEB estimators were compared using the results of this simulation experiment.

Test data for the gate valves is tabulated in Table A-3. For the data given in Table A-3, the prior mean $(m|\underline{X})$ is 0.70 and the variance of the distribution $(A|\underline{X})$ is 1.22.

Results. Simulated failure rates, and their MLE and PEB estimates are listed in Table A-4. In six of the ten categories considered for the gate valves, the PEB estimates were closer to the simulated failure rates than the MLE estimates. Estimation losses were calculated using the Square Error Loss (SEL) method; those values are listed in Table A-5. Figure A-2 illustrates the estimation results in the same format as was described in Section A-1. For the gate valves data, eight of the ten categories have PEB estimates that are closer to the prior mean than their respective MLE estimates.

A-3. Pumps Experiment

Data. A group of pumps with flow rate capacities of 500 to 2,499 gpm were selected for this experiment. Population size, frequencies of failure events for leaks, and experiment periods were obtained from the 1980 NPRDS Annual Reports (1980). These data were sorted by types of pumps under study; the data are presented in Table A-6. Based on the data collected, the prior distribution mean $(m|\underline{X})$ is 1.90 and the prior variance $(A|\underline{X})$ is 10.76.

Results. Using the simulation program, the failure rates were simulated, followed by calculation of their MLE and PEB estimators. Results are summarized in Tables A-7 and A-8. A graphical presentation of the simulation experiment is given in Figure A-3, where on the upper horizontal line the

Table A-3. Data for gate valves (failure mode: leak)

Category i	Operator Type	Observation Times,[a] t_i	Number of Components, n_i	Number of Failures, f_i	Failure Rates,[b] X_i
1	Manual	317.795	6,641	47	0.148
2	Elec. motor/servo	151.467	3,207	97	0.640
3	Hydraulic	1.385	28	0	0.000
4	Pneumatic/diaphragm	16.689	372	18	1.079
5	Solenoid	3.488	70	4	1.147
6	Float	0.131	2	0	0.000
7	Explosive/squib	0.415	10	0	0.000
8	Mechanical	0.559	14	2	3.578
9	Other[c]	2.453	64	1	0.408
10	Piston	0.166	3	0	0.000

a. 10^6 calendar hours.

b. Failures per million calendar hours.

c. Other, none, and undefined combined.

Table A-4. Summary of results for gate valves (mode of failure: leak)

Category i	True[a] Failure Rates,[c] λ_i	Observation Times,[b] t_i	Simulated Failure Frequencies, f_i	MLE Estimates,[c] X_i	PEB Estimates,[c] $\hat{\lambda}_i$
1	0.001	317.795	0	0.000	0.002
2	0.652	151.467	104	0.687	0.680
3	0.029	1.385	0	0.000	0.132
4	0.043	16.689	0	0.000	0.029
5	0.243	3.488	1	0.287	0.272
6	0.019	0.131	0	0.000	0.184
7	2.161	0.415	0	0.000	0.169
8	2.154	0.559	1	1.789	0.776
9	0.002	2.453	0	0.000	0.106
10	1.771	0.166	0	0.000	0.182

a. Simulated by the program.

b. 10^6 calendar hours.

c. Failures per million calendar hours.

Table A-5. Calculated square error loss (gate valves, leak mode of failure)

Category	Simulated Failure Rates	MLE Loss	PEB Loss
1	0.001×10^{-6}	0.11×10^{-5}	0.59×10^{-6}
2	0.652×10^{-6}	0.12×10^{-2}	0.77×10^{-3}
3	0.029×10^{-6}	0.86×10^{-3}	0.11×10^{-1}
4	0.043×10^{-6}	0.18×10^{-2}	0.18×10^{-3}
5	0.243×10^{-6}	0.19×10^{-2}	0.84×10^{-3}
6	0.019×10^{-6}	0.36×10^{-3}	0.27×10^{-1}
7	2.161×10^{-6}	4.67	3.97
8	2.154×10^{-6}	0.13	1.90
9	0.002×10^{-6}	0.13×10^{-5}	0.11×10^{-1}
10	1.771×10^{-6}	3.14	2.53

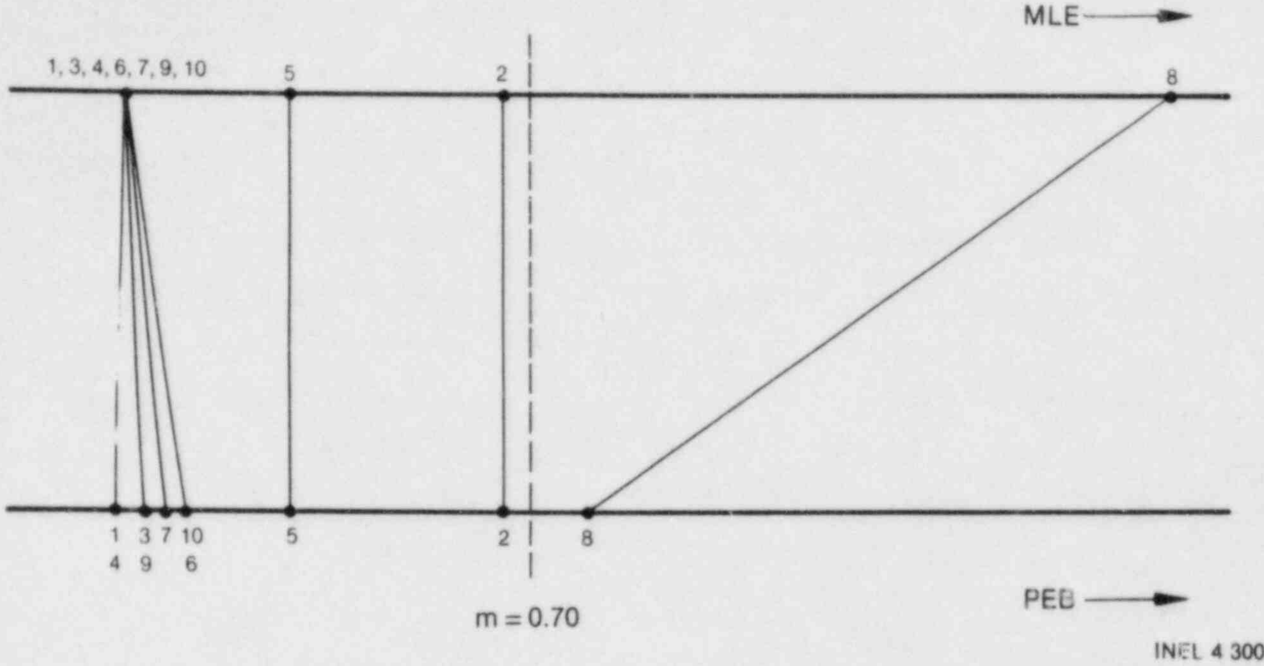

Figure A-2. Comparison of the estimates of the MLE and PEB methods for gate valves for failure mode of leak.

Table A-6. Data for 500 - 2,499 GPM pumps (mode of failure: leak)

Category i	Operator Type	Observation times,[a] t_i	Number of Components, n_i	Number of Failures, f_i	Failure Rates,[b] X_i
1	Centrifugal	9.353	170	18	1.925
2	Diaphragm	0.132	2	1	7.576
3	Gear	0.132	2	0	0.000
4	Reciprocating	0.286	5	0	0.000
5	Rotary	0.075	1	0	0.000

a. 10^6 calendar hours.

b. Failures per million calendar hours.

Table A-7. Summary of results for 500 - 2,499 GMP pumps (mode of failure: leak)

Category i	True[a] Failure Rates,[c] λ_i	Observation Times,[b] t_i	Simulated Failure Frequencies, f_i	MLE Estimates,[c] X_i	PEB Estimates,[c] $\hat{\lambda}_i$
1	0.146	9.353	2	0.212	0.226
2	1.459	0.132	1	7.576	5.328
3	0.001	0.132	0	0.000	0.386
4	0.451	0.285	0	0.000	0.285
5	1.638	0.075	0	0.000	0.444

a. Simulated by the program.

b. 10^6 calendar hours.

c. Failures per million calendar hours.

Table A-8. Calculated square error loss (pumps, 500 - 2,499 gpm, leak failure mode)

Category	Simulated Failure Rates	MLE Loss	PEB Loss
1	0.146×10^{-6}	0.44×10^{-2}	0.64×10^{-2}
2	1.495×10^{-6}	36.90	14.70
3	0.001×10^{-6}	0.54×10^{-6}	0.15
4	0.451×10^{-6}	0.20	0.28×10^{-1}
5	1.638×10^{-6}	2.68	1.42

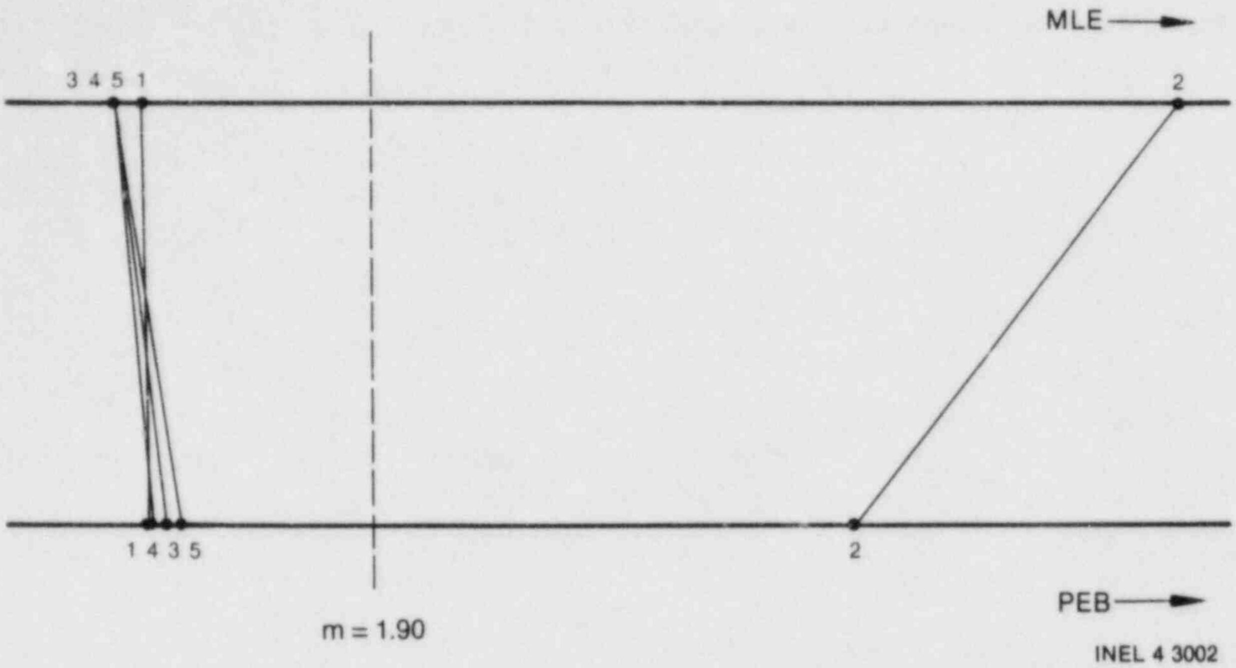

Figure A-3. Comparison of the estimates of the MLE and PEB methods for 500 - 2,499 gpm pumps for failure mode of leak.

MLE estimates are plotted. The PEB estimates are located on the lower line. In Table A-7, three of the five PEB estimates are closer to the simulated values. All of the PEB estimates are closer to the prior mean of their distribution than their MLE counterparts, as shown in Figure A-3.

A-4. Internal Combustion Engines

Data. Data for the internal combustion engines that had failed because they did not move or start were gathered from the 1980 NPRDS Annual Reports (1980). The data listed in Table A-9 are sorted by types of the engines. Based on the collected data, the prior mean for the observed failure rates $(m|\underline{X})$ is 14.26 and the variance $(A|\underline{X})$ is 244.46.

Results. A survey of Table A-10 data reveals that in all cases the PEB estimators are closer to the simulated failure rates than those of MLE. The square error losses are compared in Table A-11. A graphical comparison of the two methods is given in Figure A-4.

Table A-9. Data for internal combustion engines (mode of failure: won't start/move)

Category i	Operator Type	Observation times,[a] t_i	Number of Components, n_i	Number of Failures, f_i	Failure Rates,[b] X_i
1	Two stroke, inline block	1.005	20	1	0.995
2	Two stroke, V-block	1.565	27	22	14.058
3	Four stroke, inline block	0.149	3	6	40.268
4	Four stoke, V-block	2.220	39	7	3.153
5	Other	0.078	2	1	12.821

a. 10^6 calendar hours.

b. Failures per million calendar hours.

Table A-10. Summary of results for internal combustion engines (failure mode: won't start/move)

Category i	True[a] Failure Rates,[c] λ_i	Observation Times,[b] t_i	Simulated Failure Frequencies, f_i	MLE Estimates,[c] X_i	PEB Estimates,[c] $\hat{\lambda}_i$
1	0.557	1.005	0	0.000	0.402
2	12.460	1.565	22	14.060	13.640
3	2.788	0.149	0	0.000	1.479
4	3.347	2.220	6	2.703	2.804
5	7.735	0.078	1	12.820	10.320

a. Simulated by the program.

b. 10^6 calendar hours.

c. Failures per million calendar hours.

Table A-11. Calculated square error loss
(internal combustion engines, mode of failure: won't start/move)

Category	Simulated Failure Rates	MLE Loss	PEB Loss
1	0.557×10^{-6}	0.31	0.24×10^{-1}
2	12.460×10^{-6}	2.57	1.40
3	2.788×10^{-6}	7.77	1.71
4	3.347×10^{-6}	0.42	0.29
5	7.735×10^{-6}	25.90	6.71

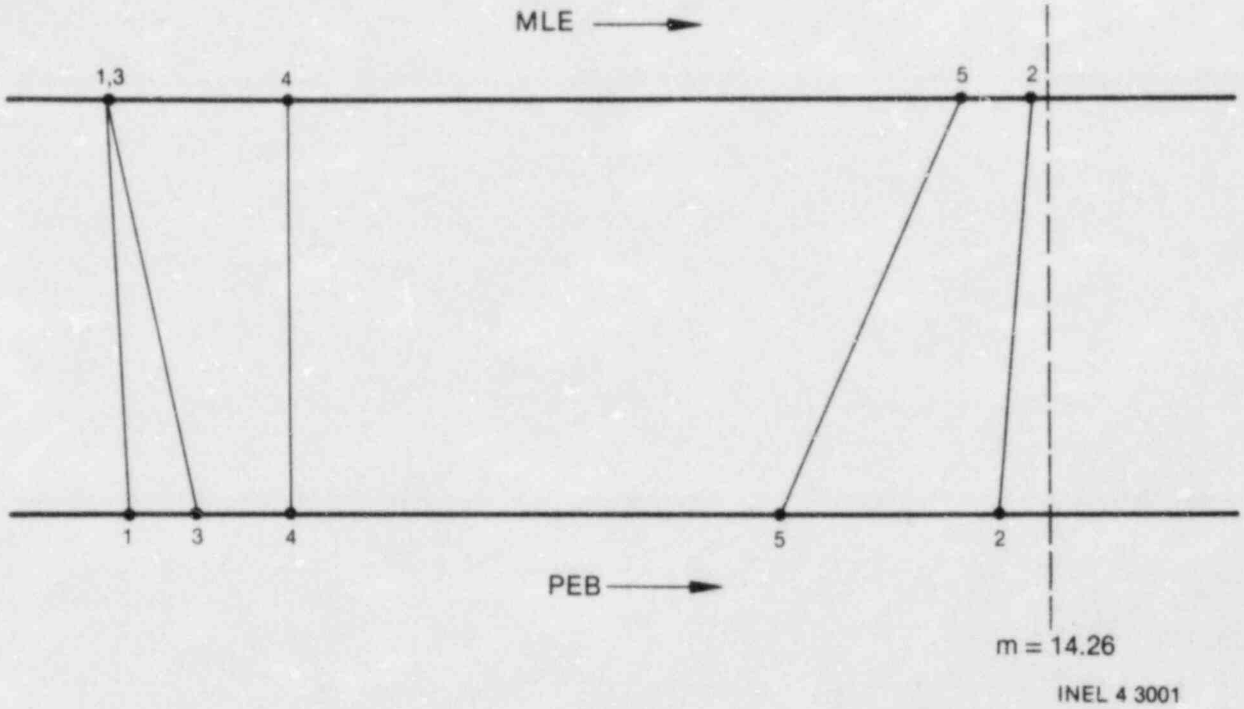

Figure A-4. Comparison of the estimates of the MLE and PEB methods for internal combustion engines for failure mode of won't start/move.

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This report is the result of a preliminary feasibility study of the applicability of Stein and related parametric empirical Bayes (PEB) estimators to the Nuclear Plant Reliability Data System (NPRDS). A new estimator is derived for the means of several independent Poisson distributions with different sampling times. This estimator is applied to data from NPRDS in an attempt to improve failure rate estimation. Theoretical and Monte Carlo results indicate that the new PEB estimator can perform significantly better than the standard maximum likelihood estimator if the estimation of the individual means can be combined through the loss function or through a parametric class of prior distributions.

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