

Non-Proprietary Version

Westinghouse SG-95-03-010

*ENCLOSURE 2
NON-PROPRIETARY*

**Burst Pressure Correlation
for Steam Generator Tubes
with Throughwall Axial Cracks**

Draft EPRI Report TR-105505

February, 1995

*NON-PROPRIETARY
VERSION*

Prepared by:

Westinghouse Electric Corporation
Nuclear Services Division

3 COPIES

Principal Investigators:

R. F. Keating
Westinghouse Electric Corporation

P. Hernalsteen
Laborelec

J. A. Begley
Packer Engineering, Inc.

Prepared for
Electric Power Research Institute

EPRI Project Manager
D. A. Steininger

Table of Contents

Section	Title	Page
1.0	Burst Pressure of Throughwall, Axially Cracked Tubes	1
1.1	Introduction	1
1.2	Burst Characterization of Tubes	1
1.3	Tube Burst Testing and the Analysis Database	2
1.4	Regression Analysis of Burst Pressure vs. Crack Length	5
	1.4.1 Analysis of Regression Residuals	7
	1.4.2 Crack Length versus Burst Pressure	8
1.5	Confidence and Prediction Bounds	8
	1.5.1 Burst Pressure versus Crack Length	8
	1.5.2 Crack Length versus Burst Pressure	9
1.6	Results and Discussion	9
1.7	Probability of Burst	10
	1.7.1 Monte Carlo Simulation of the Probability of Burst	10
	1.7.2 Deterministic Estimate of the Probability of Burst	12
	1.7.3 Comparison of Monte Carlo and Deterministic Estimates of the Probability of Burst	13
	Report References	15
	Database References	17

List of Tables

Number	Title	Page
1	Tube Material Properties for Burst Pressure Predictions (W)	19
2	Regression Parameters for the Normalized Burst Pressure as a Function of the Normalized Crack Length	20
3	Regression Parameters for the Normalized Crack Length as a Function of the Normalized Burst Pressure	20
4	Values of the $[F^T F]^{-1}$ Matrix for P_N as a Function of λ	21
5	Values of the $[F^T F]^{-1}$ Matrix for λ as a Function of P_N	21
6	I-600 Through-Wall Burst Test Results (Axial Slits,Bladder NOT Reinforced)	22
7	Undefected Tube Burst Pressures 3/4", 7/8", & 11/16" OD Alloy 600 Tubes	25
8	Axial Through Wall Cracks Database	27

List of Figures

Number	Title	Page
1	Normalized Burst Pressure (P_N) vs. Normalized Crack Length (λ) Comparison of Schelle Data to Non-Reinforced Bladder Burst Curve	33
2	Normalized Burst Pressure (P_N) vs. Normalized Crack Length (λ) Alloy 600 SG Tubes, Final Database	33
3	Normalized Burst Pressure (P_N) vs. Normalized Crack Length (λ) Comparison of Schelle Data to New Prediction Curve	34
4	Normalized Burst Pressure vs. Normalized Crack Length Alloy 600 MA Steam Generator Tubes	34
5	Normalized Burst Pressure vs. Normalized Crack Length Comparison of Predictive Equations	35
6	Residual vs. Predicted Normalized Burst Pressure Alloy 600 MA Steam Generator Tubes	35
7	Distribution of Expected vs. Actual Residuals Normalized Burst Pressure vs. Normalized Crack Length	36
8	Actual vs. Expected Cumulative Probability Normalized Burst Pressure vs. Normalized Crack Length	36
9	Actual vs. Calculated Normalized Burst Pressure Alloy 600 MA Steam Generator Tubes	37
10	Normalized Crack Length vs. Normalized Burst Pressure Alloy 600 MA Steam Generator Tubes	37
11	Burst Pressure vs. Crack Length 3/4" x 0.043", Alloy 600 MA SG Tubes, $\sigma_f = 71.6$ ksi	38
12	Burst Pressure vs. Crack Length 7/8" x 0.050", Alloy 600 MA SG Tubes, $\sigma_f = 68.8$ ksi	38
13	Probability of Burst for Single Axial Throughwall Cracks Alloy 600 MA, SG Tubes at 650°F, $\Delta P = 2560$ psi	39
14	Example Burst Pressure Distribution 3/4" x 0.043", Alloy 600 MA SG Tube, $\alpha = 0.6$ " (1,000,000 Simulations)	39

1.0 Burst Pressure of Throughwall, Axially Cracked Tubes

1.1 Introduction

The purpose of this report is to document the development of a burst pressure to axial crack length correlation for Alloy 600 Steam Generator (SG) tubes. The results are based on a general development using testing results from a variety of tube diameters and thicknesses. The final correlation utilizes a 3-parameter exponential type of equation relating the non-dimensionalized burst pressure to the non-dimensionalized crack length (referred to as the normalized burst pressure and normalized crack length respectively). By suitably adjusting the coefficients of the equation, the final results are presented graphically for tubes with a nominal outside diameter (OD) of 0.875" and a thickness of 0.050", and for tubes with a nominal OD of 0.750" by 0.043" thick. In addition, equations for obtaining burst pressure as a function of crack length and critical crack length as a function of pressure are provided in Section 1.6. Finally, evaluation of the probability of burst as a function of pressure and crack length is provided in Section 1.7.

1.2 Burst Characterization of Tubes

Much of the theory of the burst behavior of tubes with cracks is based on extending the theory of linear elastic fracture mechanics (LEFM) for flat plates containing through-thickness cracks to a cylindrical geometry^[1,2,3,4,5]¹. In general, the elastic fracture behavior of a cylinder with an axial crack can be analyzed using flat plate solutions with an appropriate stress magnification factor, usually referred to as the "shell curvature correction factor" or "bulging factor", to account for bulging along the crack flanks due to the internal pressure. Additional theoretical solutions appear in the literature in the form of plots of the results of finite element solutions, or the numerical solution of the singular integral equations for a cylinder with an axial crack^[6,7].

A frequent way of presenting solutions for the burst pressure is in the form of a relation between a normalized burst pressure, herein referred to as P_N , and a normalized crack length, λ . The normalized burst pressure is simply the actual burst pressure non-dimensionalized by the flow stress of the material and adjusted for the size of the tubing by the ratio of the mean radius to the thickness. This provides a ratio of a membrane stress in the tube to the strength of the material and allows for the correlation to be applicable to multiple tube sizes. The flow stress of the material is usually taken as a linear function of the yield stress, σ_Y ,

¹ Numbers in square brackets refer to references listed as "Report References" starting on page 15 of this report. References in the tabular data refer to those listed as "Database References" starting on page 17.

and the ultimate tensile stress, σ_U , of the material. Acceptable correlations for Alloy 600 tubes have been obtained using one-half of the sum of the two properties as the flow stress.

For a tube with a mean radius of r_m and a thickness t , the normalized burst pressure as a function of the actual burst pressure, P_B , is defined as

$$P_N = \frac{P_B r_m}{(S_Y + S_U) t} \quad (1)$$

Thus, P_N is the ratio of the maximum Tresca stress intensity, taking the average compressive stress in the tube to be $P_B/2$, to twice the flow strength of the material.² The normalizing parameter, λ , for the crack length, a , is defined as

$$\lambda = \frac{a}{\sqrt{r_m t}} \quad (2)$$

a form which arises in theoretical solutions to the burst problem. The burst pressure as a function of axial crack length for a specific tube size is then easily obtained from the non-dimensionalized relationship.

1.3 Tube Burst Testing and the Analysis Database

Historically, the relationships presented for correlating the burst pressure to axial crack length for Alloy 600 tubing are based on empirical data. Until recently, one common method of testing^[8,9] consisted of internally pressurizing an axially cracked (or slitted by electrical discharge machining) tube that had been lined with a flexible neoprene or tygon tube, i.e., a bladder, until a burst occurred.³ Burst is considered to have occurred when the crack opens to the extent that the bladder extrudes, and may rupture, accompanied by ductile (plastic) tearing of the tube material at the ends of the crack. If the bladder has ruptured and tearing of the crack has not occurred, the test specimen is not considered to have truly ruptured. This simply means that the opening of the flanks of the crack was sufficient to

² It is noted that some authors^[1,7] use the mean radius in equation (1), and others^[3] use the inside radius; however, this difference in usage is not significant for thin-walled tubes.

³ Since typical test facilities do not provide for an essentially unlimited water supply, nor do they have pumping capability to maintain pressure if the specimen is leaking significantly, the purpose of the lining is to prevent leakage until a rupture of the tube occurs.

permit extrusion of the bladder, and that the actual, or *true*, burst pressure was not achieved during the test.

Test specimens may consist of tubes with cracks that have been extended by high cycle fatigue from a starting notch, either part way or all the way through the thickness, or which have very narrow axial slits machined in them. Typical slit widths are in the range of 6 to 10 mils. The accepted method of creating the starting notch or the slit is by electrical discharge machining (EDM). Testing has demonstrated that both types of specimens behave similarly, thus the added expense of fatigue extension of the EDM slit is generally not justified. In addition, testing is usually conducted at room temperature, with the results adjusted to operating temperature via the change in the flow stress of the material.

In contrast to the testing previously described, tube burst testing in Belgium and France typically included a thin foil shim on the outside of the bladder at the location of the crack or slot. The purpose of the shim was to provide a small reinforcement to prevent extrusion and rupture of the bladder before rupture of the tube. Shim dimensions are usually $\sim 1/2$ " wide by ~ 6 mils thick with the length chosen to extend $\sim 1/4$ " beyond each end of the slit. The shim material was typically brass, although stainless steel has also been used. Burst pressure results from those tests were typically higher than results obtained from similar tests with the bladder not reinforced. Hence, technical exchange discussions usually included consideration of which methodology was appropriate for characterizing the burst strength of SG tubes.

To determine which methodology produced results more representative of burst pressures which might be expected in operating SGs, Hernalsteen^[10] performed several burst tests at the Schelle fossil plant in Belgium. These tests utilized the large water supply and large pumping capacity of the plant to maintain and increase the pressure during the tests. Burst pressure data was obtained for thirteen $7/8$ " OD by 0.050" thick and two $3/4$ " OD by 0.043" thick Alloy 600 tube specimens with a variety of slot lengths without employing a bladder. The data obtained, referred to hereinafter as the Schelle data, are depicted on Figure 1 relative to a correlation curve between P_N and λ based on a regression analysis of the non-reinforced bladder data contained in References 8 and 9 (a subset⁴ of the data contained in Tables 6 and 7). The results clearly demonstrate burst strengths exceeding the results obtained with non-reinforced bladders. Furthermore, Westinghouse analysis of the Schelle data indicated the results to be consistent with those obtained from tests performed by Westinghouse using foil reinforced bladders. However, in Reference 11 and a subsequent publication, Reference 12, Hernalsteen reported that the presence of the foil could increase the measured burst pressure by

⁴ Belgian data which became available after the publication of Reference 8 were not included in the correlation.

about 10% relative to that obtained from tests without bladders. Thus, the validity of performing burst tests on pulled tube specimens was still in need of additional verification. In addition, given the small number of data points available from the Schelle testing programs, it was apparent that the accuracy of performing burst tests under laboratory conditions needed to be established if statistical inference relative to the results was to be performed. While the results obtained from non-reinforced bladder tests could be counted on to be conservative, continued use could lead to removing tubes from service unnecessarily, as could the use of a rupture equation based on the simulation tests performed utilizing non-reinforced bladders.

To resolve the remaining issue, Hernalsteen noted that the Schelle data could be correlated well using a collapse load theory expression originally published by Erdogan^[6], i.e.,

$$P_N = \frac{P_0}{0.614 + 0.386e^{-1.125\lambda} + 0.433\lambda}, \quad (3)$$

where P_0 was calculated to provide the best average fit to the data. A similar fit of equation (3) to the Westinghouse foil data resulted in a calculated P_0 approximately 4% larger than that found for the Schelle data. For the Belgian foil test results, the value of P_0 found to best fit the data was about 8% greater than that obtained using only the Schelle data. In the Reference 13 meeting, comparisons of the results of the Westinghouse data and the data from the individual programs comprising the Belgian data were made with the Schelle data. It was concluded that the results from Belgian program A4, see Table 8, which employed stainless steel foil, were most influential in determining the value of P_0 found for the Belgian foil tests. The data from the other programs were consistent with the results obtained from the Westinghouse programs. It was thus judged that a 5% reduction in burst pressure would be applied to all test results in which foil reinforced bladders had been used.

Figure 2 depicts the Schelle data along with data obtained from testing performed by Laborelec and Westinghouse with a bladder reinforcing shim present. A summary of the data are included in Table 8. It is to be noted that the reported data reflect a 5% reduction from the measured and/or reported data.⁵ Figure 3 illustrates a comparison of the Schelle data to a regression curve (discussed later) obtained using the data of Figure 2. These figures illustrate that the adjusted results coincide well with the results from the program performed at Schelle.

⁵ Specimens designated with a suffix of "-N" included an additional thickness of nickel plating. An additional 5% reduction in strength was also applied to these specimens based on the thickness of the nickel plate.

It is noted that eight of the test results shown on Figure 2 for short slits, $\lambda < 1.5$, are from tests performed with no bladder reinforcement (the first eight results listed in Table 6). The rationale for the inclusion of this data is that the effect of using a shim should diminish with decreasing crack size, and below a length of, say, 0.25" would be expected to show no effect. Foil reinforced test results for short cracks, and the trend of the Schelle data in this range, indicate this to be the case. Figure 2 also depicts the results for all of the Table 7 test results for tubes with no slits. Some consideration was given to omitting the non-cracked specimens from the database on the grounds that the governing material parameter for the cracked specimens is the flow stress while the governing parameter for the uncracked specimens may be only the ultimate tensile stress. This option was rejected based on the observation that the trend of the burst data for very short cracks converges to the average burst value obtained from the uncracked specimens. Hence, if there is a transition in the governing material parameter, it is apparently gradual instead of dramatic.

The data illustrated on Figure 2 represent the final database selected for the development of the burst pressure to crack length correlation. The database consists of a total of 227 data pairs representing a combination of the previously discussed data as follows:

1. The first eight data pairs from Table 6 (not foil reinforced).
2. All of the data listed in Table 7 (no cracks/slits).
3. All of the data listed in Table 8.

It is noted that the Reference 9 data listed in Table 7 were adjusted to reflect results expected if the tests were performed at ambient temperature, since the burst pressure results reported therein were for tests performed at elevated temperature and the material properties were reported only for ambient conditions. The adjustment was based on temperature scaling factors derived using Reference 8 information.

1.4 Regression Analysis of Burst Pressure vs. Crack Length

For the regression analysis, an equation of the type used by Erdogan^[6] to fit numerical results for the shell curvature correction factor was investigated first. As previously noted, Hernalsteen^[10] compared calculated burst pressures using Erdogan's expression to the Schelle data with relatively good results by finding the best value of P_0 for equation (3), with the coefficients in the denominator being those originally determined by Erdogan. Here, P_0 represents the value of P_N corresponding to no slot or crack being present, i.e., $\lambda = 0$. Although the Schelle burst results were found to match reasonably well with the predictions, the resulting curve did not fit well for very small crack lengths nor for specimens without cracks.

Equation (3) is characterized as a single-peak type of function in that it possesses a single mode with the tails asymptotically approaching zero as λ approaches either plus or minus infinity. In this sense it is similar in shape to a normal probability density function. Thus, the slope for a crack length of zero could be positive, zero, slightly negative, or significantly negative depending on the position of the mode along the abscissa. This type of function was needed by Erdogan in order to fit his numerical results which indicated a slope of zero at the intersection of the ordinate and the abscissa. Examination of the normalized burst pressure data base indicated that alternate expressions could also be used, and that a peaking property might not be necessary. A series of regression analyses were performed considering a variety of linear and non-linear functions, including fitting all four of the coefficients in equation (3),⁶ based on minimization of the residual sum of squares. The actual fitting was performed using a generalized reduced gradient algorithm, and checked against the results from a commercially available code which used a Levenberg-Marquardt algorithm. The rationale for the investigation was to let the data determine the appropriate choice of equation form to be used. Several functions were found which provided similar goodness of fit as measured by the index of determination. An exponential function, i.e.,

$$P_N = b_1 + b_2 e^{b_3 \lambda}, \quad (4)$$

was finally selected based on the combination of maximizing the goodness of fit, minimizing the number of coefficients in the function, and the hypothesis that the burst pressure should be a monotonic decreasing function of the crack length.⁷ For the data of Figure 2, the coefficients of equation (4) were found to be (see Table 2),

(5)

The index of determination for the fit was 99.1%, with a standard error of the estimate of 0.0172. The F distribution statistic for the regression, the ratio of the mean square due to the regression to the mean square due to the residuals, was >11000. The p values for all of the coefficients were less than $1 \cdot 10^{-5}$. Thus, the fit of the equation to the data is judged to be excellent, i.e., the data exhibit insuffi-

⁶ For a regression analysis involving all of the coefficients, it is noted that there are only three independent coefficients in the denominator since the sum of the first and second coefficients is unity.

⁷ Equation (4) is also advantageous in that it can easily be inverted to yield λ as a function of P_N .

cient evidence to reject the proposed model.⁸ Figure 4 depicts the results of the regression analysis, i.e., predictions using equation (5), relative to the database.

Based on the exponential equation form, three regression analyses were performed that,

1. considered omission of the undegraded tubes data,
2. omission of the non-reinforced bladder data, and
3. omission of both sets of data.

For all three cases the resulting coefficients were similar to those reported in equation (5), thus verifying the initial judgement to include those data in the analysis data base.

A final regression analysis was performed to determine the best value of P_0 for the Erdogan equation, equation (3), for all of the data. A plot of the resulting equation is shown on Figure 5 for comparison with the results using equation (5). Both expressions yield similar results over the range of λ from about 2 to 4. The upper bound of this range corresponds to crack lengths of about 0.5" for 3/4" OD tubes and 0.6" for 7/8" OD tubes. For information, the EPRI equation per Reference 14 is also depicted on Figure 5. While this equation reasonably estimates a lower bound prediction relative to the data, it was not developed using the experimental database.

1.4.1 Analysis of the Regression Residuals

An analysis of the regression residuals was performed by making a scatter plot of the residual normalized burst values versus the predicted normalized burst values, Figure 6, and by plotting the cumulative probability of the residuals relative to the sorted residual values, Figure 7. As an alternate view of Figure 7, the actual cumulative probabilities of the residuals were plotted against the expected cumulative probabilities of the residuals, Figure 8. The scatter plot indicates that no significant correlation exists between the residuals and the predicted values, that the variance of the residuals is approximately uniform, and that no apparent systematic departure from the regression curve is present. The cumulative probability plot indicates that the distribution of the residuals about the regression curve of equation (4) is approximately normal with a mean of zero. Thus, the model is considered to be adequate for describing the burst behavior of Alloy 600 SG tubes with axial cracks. An evaluation of the use of the standard error of the residuals is presented in Section 1.7, Probability of Burst.

⁸ This does not mean that equation (4) is the true form of a functional relationship between the two variables, only that it provides an excellent description of the relationship.

1.4.2 Crack Length versus Burst Pressure

In some situations it is desirable to relate a critical crack length to a specified burst pressure, e.g., the crack length that would correspond to tube burst at an applied pressure of three times normal operating differential pressure. Equation (4) can be rearranged to yield the inverse relation

$$\lambda = -a_1 + a_2 \ln(P_N - a_3), \quad (6)$$

for the normalized crack length as a function of normalized burst pressure. For scoping work, equation (5) could be rearranged to yield an approximate set of coefficients for equation (6), however, these are not the best coefficients to be used for the inverse relationship (in a least squares error sense) and they do not afford the development of confidence and prediction bounds on the crack length as a function of burst pressure. The appropriate coefficients result from performing a non-linear regression fit of the same data as used for the burst pressure equation. This results in the empirical relation (see Table 3),

(7)

The index of determination for the regression of λ on P_N was 98.9%. This is approximately the same as that for the regression of the burst pressure on the normalized crack length.⁹ The F-statistic for the significance of the index of determination was calculated to be about 9700. The p values for all of the coefficients were less than $1 \cdot 10^{-5}$. A plot of the values from equation (7) relative to the data is illustrated on Figure 10.

1.5 Confidence and Prediction Bounds

1.5.1 Burst Pressure versus Crack Length

Two-sided $(1-\alpha) \cdot 100\%$ confidence bounds for the mean value of the normalized burst pressure as a function of the normalized crack length can be found as

$$P_N = P_N^0 \pm t_{(v, 1-\alpha/2)} s \sqrt{\{f_0\}^T [F^T F]^{-1} \{f_0\}}, \quad (8)$$

where P_N^0 is obtained from equation (5), t is a Student's-t variate for regression degrees of freedom, s is the standard deviation of the regression residuals, $\{f_0\}$ is a

⁹ If the regression analyses were linear the indices would be the same, they are different here because of the use of non-linear relations.

vector of the partial derivatives of equation (5) relative to each of the coefficients evaluated at the λ_i value of interest, i.e.,

$$\{f_0\} = \left[1 \quad e^{b_3 \lambda_i} \quad b_2 \lambda_i e^{b_3 \lambda_i} \right]^T, \quad (9)$$

and the values of the normalized covariance matrix, $[F^T F]^{-1}$, are given in Table 4. To facilitate computations, if the elements of the equation (9) vector are designated as f_1 , etc., and the elements of the normalized covariance matrix are designated as R_{11} , R_{12} , etc., the expression inside the radical of equation (8) becomes

$$\{f_0\}^T [F^T F]^{-1} \{f_0\} = R_{11} + f_2^2 R_{22} + f_3^2 R_{33} + 2(f_2 R_{12} + f_3 R_{13} + f_2 f_3 R_{23}). \quad (10)$$

A two-sided $100 \cdot (1-\alpha)\%$ prediction band for the value of the normalized burst pressure as a function of a future value of the normalized crack length can be found as,

$$P_N = P_N^0 \pm t_{(v, 1-\alpha/2)} s \sqrt{1 + \{f_0\}^T [F^T F]^{-1} \{f_0\}}. \quad (11)$$

1.5.2 Crack Length versus Burst Pressure

Two-sided confidence and prediction bounds on the critical normalized crack length as a function of the normalized burst pressure can be found as for the normalized burst pressure as a function of the normalized crack length. For the inverse relation the derivative vector is given by,

$$\{f_0\} = \left[1 \quad \ln(P_{N_i} - a_3) \quad \frac{-a_2}{P_{N_i} - a_3} \right]^T. \quad (12)$$

The standard error of the residuals is given in Table 3 and values of the normalized covariance matrix for λ as a function of P_N are provided in Table 5.

1.6 Results and Discussion

A comparison of the measured normalized burst values to the predicted normalized burst values is provided on Figure 9. Examination of the figure shows that most of the results are enveloped within $\pm 10\%$ of the prediction line. Scatter outside of this region is generally restricted to those burst pressures for which the density of the data is highest, i.e., where more variation would be expected to be displayed.

To illustrate the results for specific tube sizes, the data were adjusted to correspond to depict actual burst pressures versus crack lengths for 3/4" and 7/8" nominal OD tubes with thicknesses of 0.043" and 0.050" respectively. These results are depicted on Figures 11 and 12 for nominal flow stresses of 71.6 and 68.8 ksi respectively (650°F mean values from the fabrication database of Reference 8). In addition to the regression curves, the expected burst curves corresponding to the 95%/95% LTL flow stress of the tube materials at a temperature of 650°F are shown (see Table 1 for a complete listing of the material properties used in this report). The critical crack lengths for postulated ΔP_{SLB} of 2.560 ksi are 0.75" and 0.84" respectively for 3/4" and 7/8" diameter tubes with LTL material properties at 650°F. For the Regulatory Guide 1.121 limit of $1.43 \cdot \Delta P_{SLB}$ (3.657 ksi) the corresponding critical crack lengths are 0.51" and 0.57" for tubes with LTL material properties at 650°F.

The results reported herein are based on performing an analysis specific to the data from the Schelle testing programs, data based on the use of foil reinforced bladders, data for tests conducted on specimens with very short crack lengths, and data for the burst of tubes without cracks. The calculation of coefficients specific to the data available is necessary for the establishment of inference bounds. As reported in the previous paragraph, the critical crack length for burst at SLB conditions based on LTL material properties is still greater than or equal to the thickness of the tube support plates.

1.7 Probability of Burst

The purpose of this section is to evaluate the probability of burst (PoB) at SLB conditions as a function of the crack length. This process is the inverse of establishing a statistical inference prediction bound. Two methods were employed to obtain estimates of the probability of burst as a function of λ , Monte Carlo simulation, and deterministic modelling. Descriptions of the analyses and discussions of the results are provided in the following paragraphs. In summary, conservative estimates may be easily obtained using the deterministic model, however, the level of conservatism increases with decreasing probability of burst. For very low PoBs, e.g., on the order of 10^{-7} , the deterministic model may overestimate the PoB by an order of magnitude. Hence, if the crack length is short and the estimate of the PoB must not include excessive conservatism, Monte Carlo simulation is recommended.

1.7.1 Monte Carlo Simulation for Probability of Burst

Monte Carlo simulations were performed based on sampling the residuals from the regression analysis and sampling the material properties for both 3/4" and 7/8" nominal diameter tubes. The results of the Monte Carlo analyses were verified by estimating the probability of burst utilizing a deterministic combination of the variation of the residuals and material properties.

The analyses utilized the burst curve developed herein coupled with the material properties reported in Reference 8 for a temperature of 650°F. Based on the correlation of normalized burst pressure, P_N , to normalized crack length, λ , the expected burst pressure is calculated as

$$P_B = \frac{2t}{R_m} P_N \sigma_f, \quad (13)$$

where t is the thickness of the tube wall, R_m is the mean radius of the tube, and σ_f is the flow stress of the material, taken as $\frac{1}{2}(S_Y + S_U)$. Thus, the values of P_N used in the correlation included variation of the material properties about the reported value, and variation in the thickness and mean radius of the test specimens. The residuals from the regression analysis of P_N on λ were shown to follow a normal distribution. Likewise, the distribution of the flow stress at operating temperature, as presented in Reference 8, also appears to follow a normal distribution. Upon close examination of the histogram of the flow stress distribution it might be judged that the distribution is slightly skewed left (meaning the mean of the distribution is shifted to the left relative to the median and mode), however, evaluations based on using a normal distribution are then conservative since the peak of the distribution is shifted toward the higher values, thus, the probability of calculating a low flow stress is exaggerated.

For the Monte Carlo analyses, the residual distribution about the burst curve and the material strength properties were independently sampled to calculate randomly distributed burst pressures per equation (13) for each of several selected crack lengths, ranging from 0.5" to 0.75". The number of simulations performed was a function of the probability of burst occurring. For high probabilities of burst, i.e., long crack lengths, 100,000 simulations would likely be sufficient. For low probabilities of burst, i.e., short crack lengths, the number of simulations could reach 200 million if sample biasing techniques are not used (for this report, no simulations were performed for very short crack lengths since a trend relative to deterministic estimates was established for longer length cracks). The sampling process consisted of randomly generating a Student's t distribution variate, multiplying by the standard deviation from the data, and adding the resulting value to the expected value to obtain a random value for the parameter. The normalized burst pressure and the material flow stress were sampled independently. The fraction of burst pressures found to be less than or equal to the SLB differential pressure is then the probability of experiencing a burst for any individual tube. For example, the estimated probability of burst for a 3/4" nominal OD (0.043" thick) tube with a free-span through-wall crack length of 0.50" during a postulated SLB event with a ΔP of 2560 psi is on the order of $3.8 \cdot 10^{-6}$.

A one-sided $100 \cdot (1-\alpha)\%$ upper confidence bound for a Monte Carlo result can be found using the following equation (Reference 18):

$$Pr_U = \frac{1}{\frac{N-n}{(n+1)F_{1-\alpha, 2(n+1), 2(N-n)}} + 1} \quad (14)$$

where N is the total number of Monte Carlo trials, n is the number of observed successes, e.g., $P_B \leq P_{SLB}$, and F is from the F-distribution for the specified number of degrees of freedom for the numerator and denominator respectively. For zero successes in the Monte Carlo simulation, equation (14) can still be used to find an upper confidence bound on the probability. For the above example, the 95% one-sided upper confidence bound on the PoB is $5.0 \cdot 10^{-6}$.

1.7.2 Deterministic Estimate of the Probability of Burst

To check the results of the Monte Carlo analyses, the parameters of the burst pressure were estimated directly from the parameters of the P_N and S_f distributions. The expected value of the burst pressure is obtained using the mean of P_N and S_f respectively, i.e.,

$$P_B = \frac{2t}{R_m} P_N S_f \quad (15)$$

The variance of P_B is found from,

$$V(P_B) = \left(\frac{2t}{R_m} \right)^2 \left[P_N^2 V(S_f) + S_f^2 V(P_N) + V(S_f) V(P_N) \right], \quad (16)$$

where V represents variance. The expression given in equation (16) is a biased estimate of the variance (Reference 17) and its use will slightly over predict the probability of burst.¹⁰ The variance of the normalized burst pressure, $V(P_N)$, about

¹⁰ An unbiased estimate is obtained by reversing the sign for the product of variances term. For this analysis, the choice of biased or unbiased is not significant.

the regression curve for a specific value of the normalized crack length, λ_i , is taken as

$$V(P_N) = s^2 \left(1 + \{f_0\}^T [F^T F]^{-1} \{f_0\} \right), \quad (17)$$

where s is the estimated standard error of the residuals, and N is the number of data pairs used in the analysis.

The results of the deterministic estimates for the for mean and standard deviation (taken as the square root of the variance) of the burst pressure distributions for all of the crack lengths compared, agreed with the results from the Monte Carlo simulations to within 1% of the simulation values. To estimate the probability of burst for a specific crack length, it is assumed that the variable,

$$t = \frac{P_B - P_{SLB}}{s_e}, \quad (18)$$

follows a Student's t distribution, where s_e is found as the square root of the variance from equation (17), and P_{SLB} is the steam line break pressure. The PoB is then calculated as the probability of occurrence of t . Implicit in this calculation is the assumption that the product of the two population normal distributions (for the normalized burst pressure and the flow stress) is also nearly normal. The third moment of the burst pressure distribution, M_3 , is related to the means and variances of the normalized burst pressure and the flow stress distributions as,

$$M_3(P_B) \propto P_N^2 S_f^2 V(P_N) V(S_f). \quad (19)$$

Since each of the terms in equation (19) is positive, M_3 will also be positive. Hence, the distribution of the product of the normalized burst pressure and the flow stress will be skewed right, i.e., with a higher tail for the larger burst pressures. Therefore, the prediction of burst probabilities based on equation (18) would be expected to be conservative. In addition, the degree of conservatism would be expected to increase with decreasing probability of burst, i.e., for shorter crack lengths.

1.7.3 Comparison of Monte Carlo and Deterministic Estimates of the Probability of Burst

Probability of burst curves as a function of crack length were developed for each tube size and are presented on Figure 13. Examination of Figure 13 indicates that the deterministic estimate of the PoB for cracks shorter than 0.65" in 3/4" tubes and 0.7" in 7/8" tubes is conservative relative to the simulation results, and converges to the simulation estimate as the crack length increases. Furthermore,

the deterministic estimate of the PoB for a tube with a 0.50" long free-span crack in a 3/4" nominal OD tube at 650°F is about $1.7 \cdot 10^{-5}$, or three times the simulation result. A similar trend is apparent for tubes with a 7/8" nominal diameter. The magnitude of the deterministic estimate of the PoB relative to the simulation estimate increases as the PoB decreases. In addition, an examination of the distribution of burst pressures from the Monte Carlo simulations verified them to be non-normal and skewed right. An example of the distribution of burst pressures for a crack length of 0.6" in a 3/4" diameter tube is illustrated on Figure 14. The effect of skewing the distribution to the right increases the mean value to above both the median and the mode of the distribution. The area in the tail beyond two standard deviations above the mean is visibly greater than the area in the tail below two standard deviations below the mean. In summary, the relative behavior of the results of simulated and deterministic estimates of the PoB are in accord with the expectations discussed above, the results of the two analyses are considered to verify each other, and for low probabilities of burst the deterministic estimate will be conservative.

Report References:

1. Folias, E. S., *An Axial Crack in a Pressurized Cylindrical Shell*, International Journal of Fracture Mechanics, Vol. 1, pp. 104-113 (1965).
2. Hernalsteen, P., *Evaluation of Critical Sizes for Defects in Small Diameter Tubing*, Structural Mechanics in Reactor Technology (SMiRT), 7th International Conference, paper G/F 4/3, pp 283-288 (1983).
3. Kiefner, J. F., Maxey, W. A., Eiber, R. J., and Duffy, A. R., *Failure Stress Levels of Flaws in Pressurized Cylinders*, ASTM STP-536, American Society of Testing and Materials, pp 461-481, (1973).
4. Hahn, G. T., Sarratte, M., and Rosenfield, A. R., *Criteria for Crack Extension in Cylindrical Pressure Vessels*, International Journal of Fracture Mechanics, Vol. 5, No. 3, pp. 187-210 (1969).
5. NP-6626-SD (Special Distribution Document), "Belgian Approach to Steam Generator Tube Plugging for Primary Water Stress Corrosion Cracking," Electric Power Research Institute, Palo Alto, California, USA (1990).
6. Erdogan, F., and Kibler, J. J., *Cylindrical and Spherical Shells with Cracks*, International Journal of Fracture Mechanics, Vol. 5, No. 3, pp 229-237 (1969).
7. Erdogan, F., Irwin, G. R., and Ratwani, M., *Ductile Fracture of Cylindrical Vessels Containing a Large Flaw*, ASTM STP-601, American Society for Testing and Materials, pp 191-208 (1976)
8. WCAP-12522, "Inconel Alloy 600 Tubing-Material Burst and Strength Properties," Westinghouse Electric Corporation (1990).
9. NUREG/CR-0718, "Steam Generator Tube Integrity Program Phase I Report," Prepared for the United States Nuclear Regulatory Commission by Battelle Pacific Northwest Laboratories (1979).
10. Hernalsteen, P., *The Influence of Testing Conditions on Burst Pressure Assessment for Inconel Tubing*, Structural Mechanics in Reactor Technology (SMiRT), 11th International Conference, Tokyo, Japan, SMiRT Transaction Paper F08/2 (1991).
11. Hernalsteen, P., *Evaluation of Critical Lengths for Through Thickness Axial Cracks in Steam Generator Tubing*, Paper F 7/6 presented at the 6th SMiRT Conference, Paris (1981).
12. Hernalsteen, P., *The Influence of Testing Conditions on Burst-Pressure Assessment for Inconel Tubing*, International Journal of Pressure Vessels & Piping, Vol. 52, pp 41-57 (1992).

13. Personal discussions with P. Hernalsteen (LABORELEC) and J. Begley (Packer Engineering), October 21, 1993.
14. NP-6864-L (DRAFT), "PWR Steam Generator Tube Repair Limits: Technical Support Document for Expansion Zone PWSCC in Roll Transitions - Rev. 2," Electric Power Research Institute, Palo Alto, California (June 1993).
15. "User's Guide Microsoft Excel," Version 5.0, Microsoft Corporation, Redmond, Washington, USA (1993).
16. "Tablecurve 2D, Automated Curve Fitting Software for Windows," Version 2, Jandel Scientific, San Rafael, California (1994).
17. Goodman, L. A., *On the Exact Variance of Products*, American Statistical Association Journal, Vol. 55, pp. 708-713 (1960).
18. Hald, A., "Statistical Theory with Engineering Applications," John Wiley & Sons, New York, New York, USA (1952).

Database References:

The following document list is with specific regard to the data portrayed in the attached tables. If one of these documents is referred to within the body of the report it is relative to the number associated with the Report References listed previously.

1. WCAP-12522, "Inconel Alloy 300 Tubing-Material Burst and Strength Properties," Westinghouse Electric Corporation (1990).
2. NUREG/CR-0718, "Steam Generator Tube Integrity Program Phase I Report," Prepared for the United States Nuclear Regulatory Commission by Battelle Pacific Northwest Laboratories (1979).
3. Report No. 68065, "Through-Wall EDM-Flawed Specimens Simulating Axial Cracks in Steam Generator Tubes Near Support Plates," Westinghouse Science & Technology Center (1995)
4. NP-6626-SD (Special Distribution Document), "Belgian Approach to Steam Generator Tube Plugging for Primary Water Stress Corrosion Cracking," Electric Power Research Institute, Palo Alto, California, USA (1990).
5. Hernalsteen, P., *Evaluation of Critical Lengths for Through Thickness Axial Cracks in Steam Generator Tubing*, Paper F 7/6 presented at the 6th SMiRT Conference, Paris (1981).
6. Hernalsteen, P., *The Influence of Testing Conditions on Burst-Pressure Assessment for Inconel Tubing*, International Journal of Pressure Vessels & Piping, Vol. 52, pp 41-57 (1992).
7. SG-92-12-027, "McGuire Unit No. 1 Steam Generator Tube Examination," Westinghouse Electric Corporation (under contract to Duke Power Company), May 1, 1993.
8. RDD:90:5459-01:01, "Laboratory Examination of Pulled RGS Tube Section from McGuire Nuclear Station," Babcock & Wilcox (under contract to Duke Power Company), August 28, 1989.
9. SG-93-06-005, "Examination of Farley Unit 1 Hot Leg Steam Generator Tubes," Westinghouse Electric Corporation (under contract to Alabama Power Company), May 28, 1993.
10. SG-93-02-004, "Examination of D. C. Cook Unit 1 Hot Leg Steam Generator Tubes R11-C60, R12-C29, R18-C16, and R18-C21," Westinghouse Electric Corporation (under contract to the American Electric Power Company), September 23, 1992.

11. NP-7480-L, Volume 1, Revision 1, "Steam Generator Tubing Outside Diameter Stress Corrosion Cracking at Tube Support Plates - Database for Alternate Repair Criteria, Volume 1: 7/8 Inch Diameter Tubing," Electric Power Research Institute, December, 1993.
12. NP-7480-L, Volume 2, "Steam Generator Tubing Outside Diameter Stress Corrosion Cracking at Tube Support Plates - Database for Alternate Repair Criteria, Volume 2: 3/4 Inch Diameter Tubing," Electric Power Research Institute, October, 1993.
13. SG-93-09-012, "Examination of V. C. Summer Unit 1 Hot Leg Steam Generator Tubes," Westinghouse Electric Corporation (under contract to the South Carolina Electric & Gas Company), August 31, 1993.
14. Report No. 68066, "Burst Test Results for SG Tubes in Quatrefoil & Eggcrate Supports," Westinghouse Science & Technology Center (1995).

Table 1: Tube Material Properties for Burst Pressure Predictions (W)		
Property	Value at RT	Value at 650°F
Alloy 600 Mill Annealed 3/4" x 0.043" SG Tubes		
Sample Size	635	627
Yield Strength Mean	53.05	45.78
Yield Strength St. Dev.	4.8602	3.9081
Tensile Strength Mean	101.29	97.35
Tensile Strength St. Dev.	4.2173	3.9676
Flow Stress Mean	77.17	71.57
Flow Stress St. Dev.	4.1422	3.5668
95%/95% LTL Flow	69.925	65.325
Alloy 600 Mill Annealed 7/8" x 0.050" SG Tubes		
Sample Size	361	360
Yield Strength Mean	50.98	41.89
Yield Strength St. Dev.	4.2068	3.5856
Tensile Strength Mean	99.96	95.67
Tensile Strength St. Dev.	3.6123	3.4196
Flow Stress Mean	75.47	68.78
Flow Stress St. Dev.	3.5002	3.1725
95%/95% LTL Flow	69.225	63.115

Table 2: Regression Parameters for the Normalized Burst Pressure as a Function of the Normalized Crack Length

$$P_N = b_1 + b_2 e^{b_3 \lambda}$$

Parameter	Value	Standard Error
b_1		
b_2		
b_3		
dof		
Standard Error		
r^2		

Table 3: Regression Parameters for the Normalized Crack Length as a Function of the Normalized Burst Pressure

$$\lambda = -a_1 + a_2 \ln(P_N - a_3)$$

Parameter	Value	Standard Error
a_1		
a_2		
a_3		
dof		
Standard Error		
r^2		

Table 4: Values of the $[F^T F]^{-1}$ Matrix for P_N as a Function of λ
(Symmetric Matrix)

Table 5: Values of the $[F^T F]^{-1}$ Matrix for λ as a Function of P_N
(Symmetric Matrix)

**Table 6: I-600 Through-Wall Burst Test Results
(Axial Slits, Bladder NOT Reinforced)**

Ref.	Material ID	Tube O.D. (in.)	Thickness t (in.)	$S_Y + S_U$ (ksi)	Crack Length a (in.)	Burst Pressure P_b (ksi)	Normalized Crack Length λ	Normalized Burst Pressure P_N

**Table 7: Undefected Tube Burst Pressures
3/4", 7/8" & 1 1/16" OD Alloy 600 Tubes**

Ref. ⁽¹⁾	Tube Identification	Tube Heat	Tube OD (in.)	Tube Thickness t (in.)	$S_Y + S_U$ (RT, ksi)	Burst Pressure P_B (RT, ksi)	Normalized Burst Pressure P_N
---------------------	---------------------	-----------	---------------	--------------------------	-----------------------	--------------------------------	---------------------------------

**Table 7: Undefected Tube Burst Pressures
3/4", 7/8" & 1 1/16" OD Alloy 600 Tubes**

Ref. ⁽¹⁾	Tube Identification	Tube Heat	Tube OD (in.)	Tube Thickness t (in.)	$S_Y + S_U$ (RT, ksi)	Burst Pressure P_B (RT, ksi)	Normalized Burst Pressure P_N
---------------------	---------------------	-----------	---------------	--------------------------	-----------------------	--------------------------------	---------------------------------

Table 8: Axial Through Wall Cracks Database

Ref. ⁽¹⁾	Reference Program	Material ID	OD (in.)	Tube Thick. t (in.)	$S_Y + S_U$ (ksi)	Crack Length a (in.)	Burst Pressure P_B (ksi)	Normal. Crack Length λ	Normal. Burst ⁽²⁾ Pressure P_N
---------------------	-------------------	-------------	----------	-----------------------	-------------------	------------------------	----------------------------	--------------------------------	---

Table 8: Axial Through Wall Cracks Database

Ref. ⁽¹⁾	Reference Program	Material ID	OD (in.)	Tube Thick. t (in.)	$S_Y + S_U$ (ksi)	Crack Length a (in.)	Burst Pressure P_B (ksi)	Normal. Crack Length λ	Normal. Burst ⁽²⁾ Pressure P_N
---------------------	-------------------	-------------	----------	-----------------------	-------------------	------------------------	----------------------------	--------------------------------	---

Table 8: Axial Through Wall Cracks Database

Ref. ⁽¹⁾	Reference Program	Material ID	OD (in.)	Tube Thick. t (in.)	$S_Y + S_U$ (ksi)	Crack Length a (in.)	Burst Pressure P_B (ksi)	Normal. Crack Length λ	Normal. Burst ⁽²⁾ Pressure P_s
---------------------	-------------------	-------------	----------	-----------------------	-------------------	------------------------	----------------------------	--------------------------------	---

Table 8: Axial Through Wall Cracks Database

Ref. ⁽¹⁾	Reference Program	Material ID	OD (in.)	Tube Thick. t (in.)	$S_Y + S_U$ (ksi)	Crack Length a (in.)	Burst Pressure P_B (ksi)	Normal. Crack Length λ	Normal. Burst ⁽²⁾ Pressure P_N
---------------------	-------------------	-------------	----------	-----------------------	-------------------	------------------------	----------------------------	--------------------------------	---

Table 8: Axial Through Wall Cracks Database

Ref. ⁽¹⁾	Reference Program	Material ID	OD (in.)	Tube Thick. t (in.)	$S_y + S_U$ (ksi)	Crack Length a (in.)	Burst Pressure P_B (ksi)	Normal. Crack Length λ	Normal. Burst ⁽²⁾ Pressure P_N
---------------------	-------------------	-------------	----------	-----------------------	-------------------	------------------------	----------------------------	--------------------------------	---

Table 8: Axial Through Wall Cracks Database

Ref. ⁽¹⁾	Reference Program	Material ID	OD (in.)	Tube Thick. t (in.)	$S_Y + S_U$ (ksi)	Crack Length a (in.)	Burst Pressure P_B (ksi)	Normal. Crack Length λ	Normal. Burst ⁽²⁾ Pressure P_N
---------------------	-------------------	-------------	----------	-----------------------	-------------------	------------------------	----------------------------	--------------------------------	---

Figure 1: Normalized Burst Pressure (P_N) vs. Normalized Crack Length (λ)
Comparison of Schelle Data to Non-Reinforced Bladder Burst Curve

Figure 2: Normalized Burst Pressure (P_N) vs. Normalized Crack Length (λ)
Alloy 600 SG Tubes, Final Database

Figure 3: Normalized Burst Pressure (P_N) vs. Normalized Crack Length (λ)
Comparison of Schelle Data to New Prediction Curve

Figure 4: Normalized Burst Pressure vs Normalized Crack Length
Alloy 600 MA Steam Generator Tubes

Figure 5: Normalized Burst Pressure vs Normalized Crack Length
Comparison of Predictive Equations

Figure 6: Residual vs Predicted Normalized Burst Pressure
Alloy 600 MA Steam Generator Tubes

Figure 7: Distribution of Expected vs Actual Residuals
Normalized Burst Pressure vs Normalized Crack Length

Figure 8: Actual vs Expected Cumulative Probability
Normalized Burst Pressure vs Normalized Crack Length

Figure 9: Actual vs Calculated Normalized Burst Pressure
Alloy 600 MA Steam Generator Tubes

Calculated Normalized Burst Pressure

Figure 10: Normalized Critical Crack Length vs Normalized Burst Pressure
Alloy 600 MA Steam Generator Tubes

Figure 11: Burst Pressure vs. Crack Length
3/4" x 0.043", Alloy 600 MA SG Tubes, $\sigma_y = 71.6$ ksi

Figure 12: Burst Pressure vs. Crack Length
7/8" x 0.050", Alloy 600 MA SG Tubes, $\sigma_y = 68.8$ ksi

Non-Proprietary Version

Westinghouse SG-95-03-010

**Burst Pressure Correlation
for Steam Generator Tubes
with Throughwall Axial Cracks**

Draft EPRI Report TR-105505

February, 1995

Prepared by:

**Westinghouse Electric Corporation
Nuclear Services Division**

Principal Investigators:

R. F. Keating
Westinghouse Electric Corporation

P. Hernalsteen
Laborelec

J. A. Begley
Packer Engineering, Inc.

Prepared for
Electric Power Research Institute

EPRI Project Manager
D. A. Steininger

Table of Contents

Section	Title	Page
1.0	Burst Pressure of Throughwall, Axially Cracked Tubes	1
1.1	Introduction	1
1.2	Burst Characterization of Tubes	1
1.3	Tube Burst Testing and the Analysis Database	2
1.4	Regression Analysis of Burst Pressure vs. Crack Length	5
	1.4.1 Analysis of Regression Residuals	7
	1.4.2 Crack Length versus Burst Pressure	8
1.5	Confidence and Prediction Bounds	8
	1.5.1 Burst Pressure versus Crack Length	8
	1.5.2 Crack Length versus Burst Pressure	9
1.6	Results and Discussion	9
1.7	Probability of Burst	10
	1.7.1 Monte Carlo Simulation of the Probability of Burst	10
	1.7.2 Deterministic Estimate of the Probability of Burst	12
	1.7.3 Comparison of Monte Carlo and Deterministic Estimates of the Probability of Burst	13
	Report References	15
	Database References	17

List of Tables

Number	Title	Page
1	Tube Material Properties for Burst Pressure Predictions (W)	19
2	Regression Parameters for the Normalized Burst Pressure as a Function of the Normalized Crack Length	20
3	Regression Parameters for the Normalized Crack Length as a Function of the Normalized Burst Pressure	20
4	Values of the $[F^T F]^{-1}$ Matrix for P_N as a Function of λ	21
5	Values of the $[F^T F]^{-1}$ Matrix for λ as a Function of P_N	21
6	I-600 Through-Wall Burst Test Results (Axial Slits,Bladder NOT Reinforced)	22
7	Undefected Tube Burst Pressures 3/4", 7/8", & 1 1/16" OD Alloy 600 Tubes	25
8	Axial Through Wall Cracks Database	27

List of Figures

Number	Title	Page
1	Normalized Burst Pressure (P_N) vs. Normalized Crack Length (λ) Comparison of Schelle Data to Non-Reinforced Bladder Burst Curve	33
2	Normalized Burst Pressure (P_N) vs. Normalized Crack Length (λ) Alloy 600 SG Tubes, Final Database	33
3	Normalized Burst Pressure (P_N) vs. Normalized Crack Length (λ) Comparison of Schelle Data to New Prediction Curve	34
4	Normalized Burst Pressure vs. Normalized Crack Length Alloy 600 MA Steam Generator Tubes	34
5	Normalized Burst Pressure vs. Normalized Crack Length Comparison of Predictive Equations	35
6	Residual vs. Predicted Normalized Burst Pressure Alloy 600 MA Steam Generator Tubes	35
7	Distribution of Expected vs. Actual Residuals Normalized Burst Pressure vs. Normalized Crack Length	36
8	Actual vs. Expected Cumulative Probability Normalized Burst Pressure vs. Normalized Crack Length	36
9	Actual vs. Calculated Normalized Burst Pressure Alloy 600 MA Steam Generator Tubes	37
10	Normalized Crack Length vs. Normalized Burst Pressure Alloy 600 MA Steam Generator Tubes	37
11	Burst Pressure vs. Crack Length 3/4" x 0.043", Alloy 600 MA SG Tubes, $\sigma_f = 71.6$ ksi	38
12	Burst Pressure vs. Crack Length 7/8" x 0.050", Alloy 600 MA SG Tubes, $\sigma_f = 68.8$ ksi	38
13	Probability of Burst for Single Axial Throughwall Cracks Alloy 600 MA, SG Tubes at 650°F, $\Delta P = 2560$ psi	39
14	Example Burst Pressure Distribution 3/4" x 0.043", Alloy 600 MA SG Tube, $\alpha = 0.6$ " (1,000,000 Simulations)	39

1.0 Burst Pressure of Throughwall, Axially Cracked Tubes

1.1 Introduction

The purpose of this report is to document the development of a burst pressure to axial crack length correlation for Alloy 600 Steam Generator (SG) tubes. The results are based on a general development using testing results from a variety of tube diameters and thicknesses. The final correlation utilizes a 3-parameter exponential type of equation relating the non-dimensionalized burst pressure to the non-dimensionalized crack length (referred to as the normalized burst pressure and normalized crack length respectively). By suitably adjusting the coefficients of the equation, the final results are presented graphically for tubes with a nominal outside diameter (OD) of 0.875" and a thickness of 0.050", and for tubes with a nominal OD of 0.750" by 0.043" thick. In addition, equations for obtaining burst pressure as a function of crack length and critical crack length as a function of pressure are provided in Section 1.6. Finally, evaluation of the probability of burst as a function of pressure and crack length is provided in Section 1.7.

1.2 Burst Characterization of Tubes

Much of the theory of the burst behavior of tubes with cracks is based on extending the theory of linear elastic fracture mechanics (LEFM) for flat plates containing through-thickness cracks to a cylindrical geometry^[1,2,3,4,5]¹. In general, the elastic fracture behavior of a cylinder with an axial crack can be analyzed using flat plate solutions with an appropriate stress magnification factor, usually referred to as the "shell curvature correction factor" or "bulging factor", to account for bulging along the crack flanks due to the internal pressure. Additional theoretical solutions appear in the literature in the form of plots of the results of finite element solutions, or the numerical solution of the singular integral equations for a cylinder with an axial crack^[6,7].

A frequent way of presenting solutions for the burst pressure is in the form of a relation between a normalized burst pressure, herein referred to as P_N , and a normalized crack length, λ . The normalized burst pressure is simply the actual burst pressure non-dimensionalized by the flow stress of the material and adjusted for the size of the tubing by the ratio of the mean radius to the thickness. This provides a ratio of a membrane stress in the tube to the strength of the material and allows for the correlation to be applicable to multiple tube sizes. The flow stress of the material is usually taken as a linear function of the yield stress, σ_Y ,

¹ Numbers in square brackets refer to references listed as "Report References" starting on page 15 of this report. References in the tabular data refer to those listed as "Database References" starting on page 17.

and the ultimate tensile stress, σ_U , of the material. Acceptable correlations for Alloy 600 tubes have been obtained using one-half of the sum of the two properties as the flow stress.

For a tube with a mean radius of r_m and a thickness t , the normalized burst pressure as a function of the actual burst pressure, P_B , is defined as

$$P_N = \frac{P_B r_m}{(S_Y + S_U) t} \quad (1)$$

Thus, P_N is the ratio of the maximum Tresca stress intensity, taking the average compressive stress in the tube to be $P_B/2$, to twice the flow strength of the material.² The normalizing parameter, λ , for the crack length, a , is defined as

$$\lambda = \frac{a}{\sqrt{r_m t}} \quad (2)$$

a form which arises in theoretical solutions to the burst problem. The burst pressure as a function of axial crack length for a specific tube size is then easily obtained from the non-dimensionalized relationship.

1.3 Tube Burst Testing and the Analysis Database

Historically, the relationships presented for correlating the burst pressure to axial crack length for Alloy 600 tubing are based on empirical data. Until recently, one common method of testing^[8,9] consisted of internally pressurizing an axially cracked (or slitted by electrical discharge machining) tube that had been lined with a flexible neoprene or tygon tube, i.e., a bladder, until a burst occurred.³ Burst is considered to have occurred when the crack opens to the extent that the bladder extrudes, and may rupture, accompanied by ductile (plastic) tearing of the tube material at the ends of the crack. If the bladder has ruptured and tearing of the crack has not occurred, the test specimen is not considered to have truly ruptured. This simply means that the opening of the flanks of the crack was sufficient to

² It is noted that some authors^[1,7] use the mean radius in equation (1), and others^[3] use the inside radius; however, this difference in usage is not significant for thin-walled tubes.

³ Since typical test facilities do not provide for an essentially unlimited water supply, nor do they have pumping capability to maintain pressure if the specimen is leaking significantly, the purpose of the lining is to prevent leakage until a rupture of the tube occurs.

permit extrusion of the bladder, and that the actual, or *true*, burst pressure was not achieved during the test.

Test specimens may consist of tubes with cracks that have been extended by high cycle fatigue from a starting notch, either part way or all the way through the thickness, or which have very narrow axial slits machined in them. Typical slit widths are in the range of 6 to 10 mils. The accepted method of creating the starting notch or the slit is by electrical discharge machining (EDM). Testing has demonstrated that both types of specimens behave similarly, thus the added expense of fatigue extension of the EDM slit is generally not justified. In addition, testing is usually conducted at room temperature, with the results adjusted to operating temperature via the change in the flow stress of the material.

In contrast to the testing previously described, tube burst testing in Belgium and France typically included a thin foil shim on the outside of the bladder at the location of the crack or slot. The purpose of the shim was to provide a small reinforcement to prevent extrusion and rupture of the bladder before rupture of the tube. Shim dimensions are usually $\sim 1/2$ " wide by ~ 6 mils thick with the length chosen to extend $\sim 1/4$ " beyond each end of the slit. The shim material was typically brass, although stainless steel has also been used. Burst pressure results from those tests were typically higher than results obtained from similar tests with the bladder not reinforced. Hence, technical exchange discussions usually included consideration of which methodology was appropriate for characterizing the burst strength of SG tubes.

To determine which methodology produced results more representative of burst pressures which might be expected in operating SGs, Hernalsteen^[10] performed several burst tests at the Schelle fossil plant in Belgium. These tests utilized the large water supply and large pumping capacity of the plant to maintain and increase the pressure during the tests. Burst pressure data was obtained for thirteen $7/8$ " OD by 0.050" thick and two $3/4$ " OD by 0.043" thick Alloy 600 tube specimens with a variety of slot lengths without employing a bladder. The data obtained, referred to hereinafter as the Schelle data, are depicted on Figure 1 relative to a correlation curve between P_N and λ based on a regression analysis of the non-reinforced bladder data contained in References 8 and 9 (a subset⁴ of the data contained in Tables 6 and 7). The results clearly demonstrate burst strengths exceeding the results obtained with non-reinforced bladders. Furthermore, Westinghouse analysis of the Schelle data indicated the results to be consistent with those obtained from tests performed by Westinghouse using foil reinforced bladders. However, in Reference 11 and a subsequent publication, Reference 12, Hernalsteen reported that the presence of the foil could increase the measured burst pressure by

⁴ Belgian data which became available after the publication of Reference 8 were not included in the correlation.

about 10% relative to that obtained from tests without bladders. Thus, the validity of performing burst tests on pulled tube specimens was still in need of additional verification. In addition, given the small number of data points available from the Schelle testing programs, it was apparent that the accuracy of performing burst tests under laboratory conditions needed to be established if statistical inference relative to the results was to be performed. While the results obtained from non-reinforced bladder tests could be counted on to be conservative, continued use could lead to removing tubes from service unnecessarily, as could the use of a rupture equation based on the simulation tests performed utilizing non-reinforced bladders.

To resolve the remaining issue, Hernalsteen noted that the Schelle data could be correlated well using a collapse load theory expression originally published by Erdogan ^[6], i.e.,

$$P_N = \frac{P_0}{0.614 + 0.386e^{-1.125\lambda} + 0.433\lambda}, \quad (3)$$

where P_0 was calculated to provide the best average fit to the data. A similar fit of equation (3) to the Westinghouse foil data resulted in a calculated P_0 approximately 4% larger than that found for the Schelle data. For the Belgian foil test results, the value of P_0 found to best fit the data was about 8% greater than that obtained using only the Schelle data. In the Reference 13 meeting, comparisons of the results of the Westinghouse data and the data from the individual programs comprising the Belgian data were made with the Schelle data. It was concluded that the results from Belgian program A4, see Table 8, which employed stainless steel foil, were most influential in determining the value of P_0 found for the Belgian foil tests. The data from the other programs were consistent with the results obtained from the Westinghouse programs. It was thus judged that a 5% reduction in burst pressure would be applied to all test results in which foil reinforced bladders had been used.

Figure 2 depicts the Schelle data along with data obtained from testing performed by Laborelec and Westinghouse with a bladder reinforcing shim present. A summary of the data are included in Table 8. It is to be noted that the reported data reflect a 5% reduction from the measured and/or reported data.⁵ Figure 3 illustrates a comparison of the Schelle data to a regression curve (discussed later) obtained using the data of Figure 2. These figures illustrate that the adjusted results coincide well with the results from the program performed at Schelle.

⁵ Specimens designated with a suffix of "-N" included an additional thickness of nickel plating. An additional 5% reduction in strength was also applied to these specimens based on the thickness of the nickel plate.

It is noted that eight of the test results shown on Figure 2 for short slits, $\lambda < 1.5$, are from tests performed with no bladder reinforcement (the first eight results listed in Table 6). The rationale for the inclusion of this data is that the effect of using a shim should diminish with decreasing crack size, and below a length of, say, 0.25" would be expected to show no effect. Foil reinforced test results for short cracks, and the trend of the Schelle data in this range, indicate this to be the case. Figure 2 also depicts the results for all of the Table 7 test results for tubes with no slits. Some consideration was given to omitting the non-cracked specimens from the database on the grounds that the governing material parameter for the cracked specimens is the flow stress while the governing parameter for the uncracked specimens may be only the ultimate tensile stress. This option was rejected based on the observation that the trend of the burst data for very short cracks converges to the average burst value obtained from the uncracked specimens. Hence, if there is a transition in the governing material parameter, it is apparently gradual instead of dramatic.

The data illustrated on Figure 2 represent the final database selected for the development of the burst pressure to crack length correlation. The database consists of a total of 227 data pairs representing a combination of the previously discussed data as follows:

1. The first eight data pairs from Table 6 (not foil reinforced).
2. All of the data listed in Table 7 (no cracks/slits).
3. All of the data listed in Table 8.

It is noted that the Reference 9 data listed in Table 7 were adjusted to reflect results expected if the tests were performed at ambient temperature, since the burst pressure results reported therein were for tests performed at elevated temperature and the material properties were reported only for ambient conditions. The adjustment was based on temperature scaling factors derived using Reference 8 information.

1.4 Regression Analysis of Burst Pressure vs. Crack Length

For the regression analysis, an equation of the type used by Erdogan^[6] to fit numerical results for the shell curvature correction factor was investigated first. As previously noted, Hernalsteen^[10] compared calculated burst pressures using Erdogan's expression to the Schelle data with relatively good results by finding the best value of P_0 for equation (3), with the coefficients in the denominator being those originally determined by Erdogan. Here, P_0 represents the value of P_N corresponding to no slot or crack being present, i.e., $\lambda = 0$. Although the Schelle burst results were found to match reasonably well with the predictions, the resulting curve did not fit well for very small crack lengths nor for specimens without cracks.

Equation (3) is characterized as a single-peak type of function in that it possesses a single mode with the tails asymptotically approaching zero as λ approaches either plus or minus infinity. In this sense it is similar in shape to a normal probability density function. Thus, the slope for a crack length of zero could be positive, zero, slightly negative, or significantly negative depending on the position of the mode along the abscissa. This type of function was needed by Erdogan in order to fit his numerical results which indicated a slope of zero at the intersection of the ordinate and the abscissa. Examination of the normalized burst pressure data base indicated that alternate expressions could also be used, and that a peaking property might not be necessary. A series of regression analyses were performed considering a variety of linear and non-linear functions, including fitting all four of the coefficients in equation (3),⁶ based on minimization of the residual sum of squares. The actual fitting was performed using a generalized reduced gradient algorithm, and checked against the results from a commercially available code which used a Levenberg-Marquardt algorithm. The rationale for the investigation was to let the data determine the appropriate choice of equation form to be used. Several functions were found which provided similar goodness of fit as measured by the index of determination. An exponential function, i.e.,

$$P_N = b_1 + b_2 e^{b_3 \lambda}, \quad (4)$$

was finally selected based on the combination of maximizing the goodness of fit, minimizing the number of coefficients in the function, and the hypothesis that the burst pressure should be a monotonic decreasing function of the crack length.⁷ For the data of Figure 2, the coefficients of equation (4) were found to be (see Table 2),

(5)

The index of determination for the fit was 99.1%, with a standard error of the estimate of 0.0172. The F distribution statistic for the regression, the ratio of the mean square due to the regression to the mean square due to the residuals, was >11000. The p values for all of the coefficients were less than $1 \cdot 10^{-5}$. Thus, the fit of the equation to the data is judged to be excellent, i.e., the data exhibit insuffi-

⁶ For a regression analysis involving all of the coefficients, it is noted that there are only three independent coefficients in the denominator since the sum of the first and second coefficients is unity.

⁷ Equation (4) is also advantageous in that it can easily be inverted to yield λ as a function of P_N .

cient evidence to reject the proposed model.⁸ Figure 4 depicts the results of the regression analysis, i.e., predictions using equation (5), relative to the database.

Based on the exponential equation form, three regression analyses were performed that,

1. considered omission of the undegraded tubes data,
2. omission of the non-reinforced bladder data, and
3. omission of both sets of data.

For all three cases the resulting coefficients were similar to those reported in equation (5), thus verifying the initial judgement to include those data in the analysis data base.

A final regression analysis was performed to determine the best value of P_0 for the Erdogan equation, equation (3), for all of the data. A plot of the resulting equation is shown on Figure 5 for comparison with the results using equation (5). Both expressions yield similar results over the range of λ from about 2 to 4. The upper bound of this range corresponds to crack lengths of about 0.5" for 3/4" OD tubes and 0.6" for 7/8" OD tubes. For information, the EPRI equation per Reference 14 is also depicted on Figure 5. While this equation reasonably estimates a lower bound prediction relative to the data, it was not developed using the experimental database.

1.4.1 Analysis of the Regression Residuals

An analysis of the regression residuals was performed by making a scatter plot of the residual normalized burst values versus the predicted normalized burst values, Figure 6, and by plotting the cumulative probability of the residuals relative to the sorted residual values, Figure 7. As an alternate view of Figure 7, the actual cumulative probabilities of the residuals were plotted against the expected cumulative probabilities of the residuals, Figure 8. The scatter plot indicates that no significant correlation exists between the residuals and the predicted values, that the variance of the residuals is approximately uniform, and that no apparent systematic departure from the regression curve is present. The cumulative probability plot indicates that the distribution of the residuals about the regression curve of equation (4) is approximately normal with a mean of zero. Thus, the model is considered to be adequate for describing the burst behavior of Alloy 600 SG tubes with axial cracks. An evaluation of the use of the standard error of the residuals is presented in Section 1.7, Probability of Burst.

⁸ This does not mean that equation (4) is the true form of a functional relationship between the two variables, only that it provides an excellent description of the relationship.

1.4.2 Crack Length versus Burst Pressure

In some situations it is desirable to relate a critical crack length to a specified burst pressure, e.g., the crack length that would correspond to tube burst at an applied pressure of three times normal operating differential pressure. Equation (4) can be rearranged to yield the inverse relation

$$\lambda = -a_1 + a_2 \ln(P_N - a_3), \quad (6)$$

for the normalized crack length as a function of normalized burst pressure. For scoping work, equation (5) could be rearranged to yield an approximate set of coefficients for equation (6), however, these are not the best coefficients to be used for the inverse relationship (in a least squares error sense) and they do not afford the development of confidence and prediction bounds on the crack length as a function of burst pressure. The appropriate coefficients result from performing a non-linear regression fit of the same data as used for the burst pressure equation. This results in the empirical relation (see Table 3),

(7)

The index of determination for the regression of λ on P_N was 98.9%. This is approximately the same as that for the regression of the burst pressure on the normalized crack length.⁹ The F-statistic for the significance of the index of determination was calculated to be about 9700. The p values for all of the coefficients were less than $1 \cdot 10^{-5}$. A plot of the values from equation (7) relative to the data is illustrated on Figure 10.

1.5 Confidence and Prediction Bounds

1.5.1 Burst Pressure versus Crack Length

Two-sided $(1-\alpha) \cdot 100\%$ confidence bounds for the mean value of the normalized burst pressure as a function of the normalized crack length can be found as

$$P_N = P_N^0 \pm t_{(v, 1-\alpha/2)} s \sqrt{\{f_0\}^T [F^T F]^{-1} \{f_0\}}, \quad (8)$$

where P_N^0 is obtained from equation (5), t is a Student's-t variate for regression degrees of freedom, s is the standard deviation of the regression residuals, $\{f_0\}$ is a

⁹ If the regression analyses were linear the indices would be the same, they are different here because of the use of non-linear relations.

vector of the partial derivatives of equation (5) relative to each of the coefficients evaluated at the λ_i value of interest, i.e.,

$$\{f_0\} = \left[1 \quad e^{b_3 \lambda_i} \quad b_2 \lambda_i e^{b_3 \lambda_i} \right]^T, \quad (9)$$

and the values of the normalized covariance matrix, $[F^T F]^{-1}$, are given in Table 4. To facilitate computations, if the elements of the equation (9) vector are designated as f_1 , etc., and the elements of the normalized covariance matrix are designated as R_{11} , R_{12} , etc., the expression inside the radical of equation (8) becomes

$$\{f_0\}^T [F^T F]^{-1} \{f_0\} = R_{11} + f_2^2 R_{22} + f_3^2 R_{33} + 2(f_2 R_{12} + f_3 R_{13} + f_2 f_3 R_{23}). \quad (10)$$

A two-sided $100 \cdot (1-\alpha)\%$ prediction band for the value of the normalized burst pressure as a function of a future value of the normalized crack length can be found as,

$$P_N = P_N^0 \pm t_{(v, 1-\alpha/2)} s \sqrt{1 + \{f_0\}^T [F^T F]^{-1} \{f_0\}}. \quad (11)$$

1.5.2 Crack Length versus Burst Pressure

Two-sided confidence and prediction bounds on the critical normalized crack length as a function of the normalized burst pressure can be found as for the normalized burst pressure as a function of the normalized crack length. For the inverse relation the derivative vector is given by,

$$\{f_0\} = \left[1 \quad \ln(P_{N_i} - a_3) \quad \frac{-a_2}{P_{N_i} - a_3} \right]^T. \quad (12)$$

The standard error of the residuals is given in Table 3 and values of the normalized covariance matrix for λ as a function of P_N are provided in Table 5.

1.6 Results and Discussion

A comparison of the measured normalized burst values to the predicted normalized burst values is provided on Figure 9. Examination of the figure shows that most of the results are enveloped within $\pm 10\%$ of the prediction line. Scatter outside of this region is generally restricted to those burst pressures for which the density of the data is highest, i.e., where more variation would be expected to be displayed.

To illustrate the results for specific tube sizes, the data were adjusted to correspond to depict actual burst pressures versus crack lengths for 3/4" and 7/8" nominal OD tubes with thicknesses of 0.043" and 0.050" respectively. These results are depicted on Figures 11 and 12 for nominal flow stresses of 71.6 and 68.8 ksi respectively (650°F mean values from the fabrication database of Reference 8). In addition to the regression curves, the expected burst curves corresponding to the 95%/95% LTL flow stress of the tube materials at a temperature of 650°F are shown (see Table 1 for a complete listing of the material properties used in this report). The critical crack lengths for postulated ΔP_{SLB} of 2.560 ksi are 0.75" and 0.84" respectively for 3/4" and 7/8" diameter tubes with LTL material properties at 650°F. For the Regulatory Guide 1.121 limit of $1.43 \cdot \Delta P_{SLB}$ (3.657 ksi) the corresponding critical crack lengths are 0.51" and 0.57" for tubes with LTL material properties at 650°F.

The results reported herein are based on performing an analysis specific to the data from the Schelle testing programs, data based on the use of foil reinforced bladders, data for tests conducted on specimens with very short crack lengths, and data for the burst of tubes without cracks. The calculation of coefficients specific to the data available is necessary for the establishment of inference bounds. As reported in the previous paragraph, the critical crack length for burst at SLB conditions based on LTL material properties is still greater than or equal to the thickness of the tube support plates.

1.7 Probability of Burst

The purpose of this section is to evaluate the probability of burst (PoB) at SLB conditions as a function of the crack length. This process is the inverse of establishing a statistical inference prediction bound. Two methods were employed to obtain estimates of the probability of burst as a function of λ , Monte Carlo simulation, and deterministic modelling. Descriptions of the analyses and discussions of the results are provided in the following paragraphs. In summary, conservative estimates may be easily obtained using the deterministic model, however, the level of conservatism increases with decreasing probability of burst. For very low PoBs, e.g., on the order of 10^{-7} , the deterministic model may overestimate the PoB by an order of magnitude. Hence, if the crack length is short and the estimate of the PoB must not include excessive conservatism, Monte Carlo simulation is recommended.

1.7.1 Monte Carlo Simulation for Probability of Burst

Monte Carlo simulations were performed based on sampling the residuals from the regression analysis and sampling the material properties for both 3/4" and 7/8" nominal diameter tubes. The results of the Monte Carlo analyses were verified by estimating the probability of burst utilizing a deterministic combination of the variation of the residuals and material properties.

The analyses utilized the burst curve developed herein coupled with the material properties reported in Reference 8 for a temperature of 650°F. Based on the correlation of normalized burst pressure, P_N , to normalized crack length, λ , the expected burst pressure is calculated as

$$P_B = \frac{2t}{R_m} P_N \sigma_f, \quad (13)$$

where t is the thickness of the tube wall, R_m is the mean radius of the tube, and σ_f is the flow stress of the material, taken as $\frac{1}{2}(S_Y + S_U)$. Thus, the values of P_N used in the correlation included variation of the material properties about the reported value, and variation in the thickness and mean radius of the test specimens. The residuals from the regression analysis of P_N on λ were shown to follow a normal distribution. Likewise, the distribution of the flow stress at operating temperature, as presented in Reference 8, also appears to follow a normal distribution. Upon close examination of the histogram of the flow stress distribution it might be judged that the distribution is slightly skewed left (meaning the mean of the distribution is shifted to the left relative to the median and mode), however, evaluations based on using a normal distribution are then conservative since the peak of the distribution is shifted toward the higher values, thus, the probability of calculating a low flow stress is exaggerated.

For the Monte Carlo analyses, the residual distribution about the burst curve and the material strength properties were independently sampled to calculate randomly distributed burst pressures per equation (13) for each of several selected crack lengths, ranging from 0.5" to 0.75". The number of simulations performed was a function of the probability of burst occurring. For high probabilities of burst, i.e., long crack lengths, 100,000 simulations would likely be sufficient. For low probabilities of burst, i.e., short crack lengths, the number of simulations could reach 200 million if sample biasing techniques are not used (for this report, no simulations were performed for very short crack lengths since a trend relative to deterministic estimates was established for longer length cracks). The sampling process consisted of randomly generating a Student's t distribution variate, multiplying by the standard deviation from the data, and adding the resulting value to the expected value to obtain a random value for the parameter. The normalized burst pressure and the material flow stress were sampled independently. The fraction of burst pressures found to be less than or equal to the SLB differential pressure is then the probability of experiencing a burst for any individual tube. For example, the estimated probability of burst for a 3/4" nominal OD (0.043" thick) tube with a free-span through-wall crack length of 0.50" during a postulated SLB event with a ΔP of 2560 psi is on the order of $3.8 \cdot 10^{-6}$.

A one-sided 100·(1- α)% upper confidence bound for a Monte Carlo result can be found using the following equation (Reference 18):

$$Pr_U = \frac{1}{\frac{N-n}{(n+1)F_{1-\alpha, 2(n+1), 2(N-n)}} + 1} \quad (14)$$

where N is the total number of Monte Carlo trials, n is the number of observed successes, e.g., $P_B \leq P_{SLB}$, and F is from the F-distribution for the specified number of degrees of freedom for the numerator and denominator respectively. For zero successes in the Monte Carlo simulation, equation (14) can still be used to find an upper confidence bound on the probability. For the above example, the 95% one-sided upper confidence bound on the PoB is $5.0 \cdot 10^{-6}$.

1.7.2 Deterministic Estimate of the Probability of Burst

To check the results of the Monte Carlo analyses, the parameters of the burst pressure were estimated directly from the parameters of the P_N and S_f distributions. The expected value of the burst pressure is obtained using the mean of P_N and S_f respectively, i.e.,

$$P_B = \frac{2t}{R_m} P_N S_f \quad (15)$$

The variance of P_B is found from,

$$V(P_B) = \left(\frac{2t}{R_m} \right)^2 \left[P_N^2 V(S_f) + S_f^2 V(P_N) + V(S_f) V(P_N) \right], \quad (16)$$

where V represents variance. The expression given in equation (16) is a biased estimate of the variance (Reference 17) and its use will slightly over predict the probability of burst.¹⁰ The variance of the normalized burst pressure, $V(P_N)$, about

¹⁰ An unbiased estimate is obtained by reversing the sign for the product of variances term. For this analysis, the choice of biased or unbiased is not significant.

the regression curve for a specific value of the normalized crack length, λ_i , is taken as

$$V(P_N) = s^2 \left(1 + \{f_0\}^T [F^T F]^{-1} \{f_0\} \right), \quad (17)$$

where s is the estimated standard error of the residuals, and N is the number of data pairs used in the analysis.

The results of the deterministic estimates for the for mean and standard deviation (taken as the square root of the variance) of the burst pressure distributions for all of the crack lengths compared, agreed with the results from the Monte Carlo simulations to within 1% of the simulation values. To estimate the probability of burst for a specific crack length, it is assumed that the variable,

$$t = \frac{P_B - P_{SLB}}{s_e}, \quad (18)$$

follows a Student's t distribution, where s_e is found as the square root of the variance from equation (17), and P_{SLB} is the steam line break pressure. The PoB is then calculated as the probability of occurrence of t . Implicit in this calculation is the assumption that the product of the two population normal distributions (for the normalized burst pressure and the flow stress) is also nearly normal. The third moment of the burst pressure distribution, M_3 , is related to the means and variances of the normalized burst pressure and the flow stress distributions as,

$$M_3(P_B) \propto P_N^2 S_f^2 V(P_N) V(S_f). \quad (19)$$

Since each of the terms in equation (19) is positive, M_3 will also be positive. Hence, the distribution of the product of the normalized burst pressure and the flow stress will be skewed right, i.e., with a higher tail for the larger burst pressures. Therefore, the prediction of burst probabilities based on equation (18) would be expected to be conservative. In addition, the degree of conservatism would be expected to increase with decreasing probability of burst, i.e., for shorter crack lengths.

1.7.3 Comparison of Monte Carlo and Deterministic Estimates of the Probability of Burst

Probability of burst curves as a function of crack length were developed for each tube size and are presented on Figure 13. Examination of Figure 13 indicates that the deterministic estimate of the PoB for cracks shorter than 0.65" in 3/4" tubes and 0.7" in 7/8" tubes is conservative relative to the simulation results, and converges to the simulation estimate as the crack length increases. Furthermore,

the deterministic estimate of the PoB for a tube with a 0.50" long free-span crack in a 3/4" nominal OD tube at 650°F is about $1.7 \cdot 10^{-5}$, or three times the simulation result. A similar trend is apparent for tubes with a 7/8" nominal diameter. The magnitude of the deterministic estimate of the PoB relative to the simulation estimate increases as the PoB decreases. In addition, an examination of the distribution of burst pressures from the Monte Carlo simulations verified them to be non-normal and skewed right. An example of the distribution of burst pressures for a crack length of 0.6" in a 3/4" diameter tube is illustrated on Figure 14. The effect of skewing the distribution to the right increases the mean value to above both the median and the mode of the distribution. The area in the tail beyond two standard deviations above the mean is visibly greater than the area in the tail below two standard deviations below the mean. In summary, the relative behavior of the results of simulated and deterministic estimates of the PoB are in accord with the expectations discussed above, the results of the two analyses are considered to verify each other, and for low probabilities of burst the deterministic estimate will be conservative.

Report References:

1. Folias, E. S., *An Axial Crack in a Pressurized Cylindrical Shell*, International Journal of Fracture Mechanics, Vol. 1, pp. 104-113 (1965).
2. Hernalsteen, P., *Evaluation of Critical Sizes for Defects in Small Diameter Tubing*, Structural Mechanics in Reactor Technology (SMiRT), 7th International Conference, paper G/F 4/3, pp 283-288 (1983).
3. Kiefner, J. F., Maxey, W. A., Eiber, R. J., and Duffy, A. R., *Failure Stress Levels of Flaws in Pressurized Cylinders*, ASTM STP-536, American Society of Testing and Materials, pp 461-481, (1973).
4. Hahn, G. T., Sarratte, M., and Rosenfield, A. R., *Criteria for Crack Extension in Cylindrical Pressure Vessels*, International Journal of Fracture Mechanics, Vol. 5, No. 3, pp. 187-210 (1969).
5. NP-6626-SD (Special Distribution Document), "Belgian Approach to Steam Generator Tube Plugging for Primary Water Stress Corrosion Cracking," Electric Power Research Institute, Palo Alto, California, USA (1990).
6. Erdogan, F., and Kibler, J. J., *Cylindrical and Spherical Shells with Cracks*, International Journal of Fracture Mechanics, Vol. 5, No. 3, pp 229-237 (1969).
7. Erdogan, F., Irwin, G. R., and Ratwani, M., *Ductile Fracture of Cylindrical Vessels Containing a Large Flaw*, ASTM STP-601, American Society for Testing and Materials, pp 191-208 (1976)
8. WCAP-12522, "Inconel Alloy 600 Tubing-Material Burst and Strength Properties," Westinghouse Electric Corporation (1990).
9. NUREG/CR-0718, "Steam Generator Tube Integrity Program Phase I Report," Prepared for the United States Nuclear Regulatory Commission by Battelle Pacific Northwest Laboratories (1979).
10. Hernalsteen, P., *The Influence of Testing Conditions on Burst Pressure Assessment for Inconel Tubing*, Structural Mechanics in Reactor Technology (SMiRT), 11th International Conference, Tokyo, Japan, SMiRT Transaction Paper F08/2 (1991).
11. Hernalsteen, P., *Evaluation of Critical Lengths for Through Thickness Axial Cracks in Steam Generator Tubing*, Paper F 7/6 presented at the 6th SMiRT Conference, Paris (1981).
12. Hernalsteen, P., *The Influence of Testing Conditions on Burst-Pressure Assessment for Inconel Tubing*, International Journal of Pressure Vessels & Piping, Vol. 52, pp 41-57 (1992).

13. Personal discussions with P. Hernalsteen (LABORELEC) and J. Begley (Packer Engineering), October 21, 1993.
14. NP-6864-L (DRAFT), "PWR Steam Generator Tube Repair Limits: Technical Support Document for Expansion Zone PWSCC in Roll Transitions - Rev. 2," Electric Power Research Institute, Palo Alto, California (June 1993).
15. "User's Guide Microsoft Excel," Version 5.0, Microsoft Corporation, Redmond, Washington, USA (1993).
16. "Tablecurve 2D, Automated Curve Fitting Software for Windows," Version 2, Jandel Scientific, San Rafael, California (1994).
17. Goodman, L. A., *On the Exact Variance of Products*, American Statistical Association Journal, Vol. 55, pp. 708-713 (1960).
18. Hald, A., "Statistical Theory with Engineering Applications," John Wiley & Sons, New York, New York, USA (1952).

Database References:

The following document list is with specific regard to the data portrayed in the attached tables. If one of these documents is referred to within the body of the report it is relative to the number associated with the Report References listed previously.

1. WCAP-12522, "Inconel Alloy 600 Tubing-Material Burst and Strength Properties," Westinghouse Electric Corporation (1990).
2. NUREG/CR-0718, "Steam Generator Tube Integrity Program Phase I Report," Prepared for the United States Nuclear Regulatory Commission by Battelle Pacific Northwest Laboratories (1979).
3. Report No. 68065, "Through-Wall EDM-Flawed Specimens Simulating Axial Cracks in Steam Generator Tubes Near Support Plates," Westinghouse Science & Technology Center (1995)
4. NP-6626-SD (Special Distribution Document), "Belgian Approach to Steam Generator Tube Plugging for Primary Water Stress Corrosion Cracking," Electric Power Research Institute, Palo Alto, California, USA (1990).
5. Hernalsteen, P., *Evaluation of Critical Lengths for Through Thickness Axial Cracks in Steam Generator Tubing*, Paper F 7/6 presented at the 6th SMiRT Conference, Paris (1981).
6. Hernalsteen, P., *The Influence of Testing Conditions on Burst-Pressure Assessment for Inconel Tubing*, International Journal of Pressure Vessels & Piping, Vol. 52, pp 41-57 (1992).
7. SG-92-12-027, "McGuire Unit No. 1 Steam Generator Tube Examination," Westinghouse Electric Corporation (under contract to Duke Power Company), May 1, 1993.
8. RDD:90:5459-01:01, "Laboratory Examination of Pulled RGS Tube Section from McGuire Nuclear Station," Babcock & Wilcox (under contract to Duke Power Company), August 28, 1989.
9. SG-93-06-005, "Examination of Farley Unit 1 Hot Leg Steam Generator Tubes," Westinghouse Electric Corporation (under contract to Alabama Power Company), May 28, 1993.
10. SG-93-02-004, "Examination of D. C. Cook Unit 1 Hot Leg Steam Generator Tubes R11-C60, R12-C29, R18-C16, and R18-C21," Westinghouse Electric Corporation (under contract to the American Electric Power Company), September 23, 1992.

11. NP-7480-L, Volume 1, Revision 1, "Steam Generator Tubing Outside Diameter Stress Corrosion Cracking at Tube Support Plates - Database for Alternate Repair Criteria, Volume 1: 7/8 Inch Diameter Tubing," Electric Power Research Institute, December, 1993.
12. NP-7480-L, Volume 2, "Steam Generator Tubing Outside Diameter Stress Corrosion Cracking at Tube Support Plates - Database for Alternate Repair Criteria, Volume 2: 3/4 Inch Diameter Tubing," Electric Power Research Institute, October, 1993.
13. SG-93-09-012, "Examination of V. C. Summer Unit 1 Hot Leg Steam Generator Tubes," Westinghouse Electric Corporation (under contract to the South Carolina Electric & Gas Company), August 31, 1993.
14. Report No. 68066, "Burst Test Results for SG Tubes in Quatrefoil & Eggcrate Supports," Westinghouse Science & Technology Center (1995).

Table 1: Tube Material Properties for Burst Pressure Predictions (W)		
Property	Value at RT	Value at 650°F
Alloy 600 Mill Annealed 3/4" x 0.043" SG Tubes		
Sample Size	635	627
Yield Strength Mean	53.05	45.78
Yield Strength St. Dev.	4.8602	3.9081
Tensile Strength Mean	101.29	97.35
Tensile Strength St. Dev.	4.2173	3.9676
Flow Stress Mean	77.17	71.57
Flow Stress St. Dev.	4.1422	3.5668
95%/95% LTL Flow	69.925	65.325
Alloy 600 Mill Annealed 7/8" x 0.050" SG Tubes		
Sample Size	361	360
Yield Strength Mean	50.98	41.89
Yield Strength St. Dev.	4.2068	3.5856
Tensile Strength Mean	99.96	95.67
Tensile Strength St. Dev.	3.6123	3.4196
Flow Stress Mean	75.47	68.78
Flow Stress St. Dev.	3.5002	3.1725
95%/95% LTL Flow	69.225	63.115

Table 2: Regression Parameters for the Normalized Burst Pressure as a Function of the Normalized Crack Length

$$P_N = b_1 + b_2 e^{b_3 \lambda}$$

Parameter	Value	Standard Error
b_1		
b_2		
b_3		
dof		
Standard Error		
r^2		

Table 3: Regression Parameters for the Normalized Crack Length as a Function of the Normalized Burst Pressure

$$\lambda = -a_1 + a_2 \ln(P_N - a_3)$$

Parameter	Value	Standard Error
a_1		
a_2		
a_3		
dof		
Standard Error		
r^2		

Table 4: Values of the $[F^T F]^{-1}$ Matrix for P_N as a Function of λ
(Symmetric Matrix)

Table 5: Values of the $[F^T F]^{-1}$ Matrix for λ as a Function of P_N
(Symmetric Matrix)

**Table 6: I-600 Through-Wall Burst Test Results
(Axial Slits, Bladder NOT Reinforced)**

Ref.	Material ID	Tube O.D. (in.)	Thickness t (in.)	$S_Y + S_U$ (ksi)	Crack Length a (in.)	Burst Pressure P_b (ksi)	Normalized Crack Length λ	Normalized Burst Pressure P_N

**Table 7: Undefected Tube Burst Pressures
3/4", 7/8" & 1 1/16" OD Alloy 600 Tubes**

Ref. ⁽¹⁾	Tube Identification	Tube Heat	Tube OD (in.)	Tube Thickness t (in.)	$S_Y + S_U$ (RT, ksi)	Burst Pressure P_B (RT, ksi)	Normalized Burst Pressure P_N
---------------------	---------------------	-----------	---------------	--------------------------	-----------------------	--------------------------------	---------------------------------

Table 8: Axial Through Wall Cracks Database

Ref. ⁽¹⁾	Reference Program	Material ID	OD (in.)	Tube Thick. t (in.)	$S_Y + S_U$ (ksi)	Crack Length a (in.)	Burst Pressure P_B (ksi)	Normal. Crack Length λ	Normal. Burst ⁽²⁾ Pressure P_N
---------------------	-------------------	-------------	----------	-----------------------	-------------------	------------------------	----------------------------	--------------------------------	---

Table 8: Axial Through Wall Cracks Database

Ref. ⁽¹⁾	Reference Program	Material ID	OD (in.)	Tube Thick. t (in.)	$S_y + S_U$ (ksi)	Crack Length a (in.)	Burst Pressure P_B (ksi)	Normal. Crack Length λ	Normal. Burst ⁽²⁾ Pressure P_N
---------------------	-------------------	-------------	----------	-----------------------	-------------------	------------------------	----------------------------	--------------------------------	---

Table 8: Axial Through Wall Cracks Database

Ref. ⁽¹⁾	Reference Program	Material ID	OD (in.)	Tube Thick. t (in.)	$S_Y + S_U$ (ksi)	Crack Length a (in.)	Burst Pressure P_B (ksi)	Normal. Crack Length λ	Normal. Burst ⁽²⁾ Pressure P_{cr}
---------------------	-------------------	-------------	----------	-----------------------	-------------------	------------------------	----------------------------	--------------------------------	--

Table 8: Axial Through Wall Cracks Database

Ref. ⁽¹⁾	Reference Program	Material ID	OD (in.)	Tube Thick. t (in.)	$S_Y + S_U$ (ksi)	Crack Length a (in.)	Burst Pressure P_B (ksi)	Normal. Crack Length λ	Normal. Burst ⁽²⁾ Pressure P_N
---------------------	-------------------	-------------	----------	-----------------------	-------------------	------------------------	----------------------------	--------------------------------	---

Table 8: Axial Through Wall Cracks Database

Ref. ⁽¹⁾	Reference Program	Material ID	OD (in.)	Tube Thick. t (in.)	$S_Y + S_U$ (ksi)	Crack Length a (in.)	Burst Pressure P_B (ksi)	Normal. Crack Length λ	Normal. Burst ⁽²⁾ Pressure P_N
---------------------	-------------------	-------------	----------	-----------------------	-------------------	------------------------	----------------------------	--------------------------------	---

Table 8: Axial Through Wall Cracks Database

Ref. ⁽¹⁾	Reference Program	Material ID	OD (in.)	Tube Thick. t (in.)	$S_y + S_U$ (ksi)	Crack Length a (in.)	Burst Pressure P_B (ksi)	Normal. Crack Length λ	Normal. Burst ⁽²⁾ Pressure P_N
---------------------	-------------------	-------------	----------	-----------------------	-------------------	------------------------	----------------------------	--------------------------------	---

Figure 1: Normalized Burst Pressure (P_N) vs. Normalized Crack Length (λ)
Comparison of Schelle Data to Non-Reinforced Bladder Burst Curve

Figure 2: Normalized Burst Pressure (P_N) vs. Normalized Crack Length (λ)
Alloy 600 SG Tubes, Final Database

Figure 3: Normalized Burst Pressure (P_N) vs. Normalized Crack Length (λ)
Comparison of Schelle Data to New Prediction Curve

Figure 4: Normalized Burst Pressure vs Normalized Crack Length
Alloy 600 MA Steam Generator Tubes

Figure 5: Normalized Burst Pressure vs Normalized Crack Length
Comparison of Predictive Equations

Figure 6: Residual vs Predicted Normalized Burst Pressure
Alloy 600 MA Steam Generator Tubes

Figure 7: Distribution of Expected vs Actual Residuals
Normalized Burst Pressure vs Normalized Crack Length

Figure 8: Actual vs Expected Cumulative Probability
Normalized Burst Pressure vs Normalized Crack Length

Figure 9: Actual vs Calculated Normalized Burst Pressure
Alloy 600 MA Steam Generator Tubes

Calculated Normalized Burst Pressure

Figure 10: Normalized Critical Crack Length vs Normalized Burst Pressure
Alloy 600 MA Steam Generator Tubes

Figure 11: Burst Pressure vs. Crack Length
3/4" x 0.043", Alloy 600 MA SG Tubes, $\sigma_y = 71.6$ ksi

Figure 12: Burst Pressure vs. Crack Length
7/8" x 0.050", Alloy 600 MA SG Tubes, $\sigma_y = 68.8$ ksi

Non-Proprietary Version

Westinghouse SG-95-03-010

**Burst Pressure Correlation
for Steam Generator Tubes
with Throughwall Axial Cracks**

Draft EPRI Report TR-105505

February, 1995

Prepared by:

**Westinghouse Electric Corporation
Nuclear Services Division**

Principal Investigators:

R. F. Keating
Westinghouse Electric Corporation

P. Hernalsteen
Laborelec

J. A. Begley
Packer Engineering, Inc.

**Prepared for
Electric Power Research Institute**

**EPRI Project Manager
D. A. Steininger**

Table of Contents

Section	Title	Page
1.0	Burst Pressure of Throughwall, Axially Cracked Tubes	1
1.1	Introduction	1
1.2	Burst Characterization of Tubes	1
1.3	Tube Burst Testing and the Analysis Database	2
1.4	Regression Analysis of Burst Pressure vs. Crack Length	5
	1.4.1 Analysis of Regression Residuals	7
	1.4.2 Crack Length versus Burst Pressure	8
1.5	Confidence and Prediction Bounds	8
	1.5.1 Burst Pressure versus Crack Length	8
	1.5.2 Crack Length versus Burst Pressure	9
1.6	Results and Discussion	9
1.7	Probability of Burst	10
	1.7.1 Monte Carlo Simulation of the Probability of Burst	10
	1.7.2 Deterministic Estimate of the Probability of Burst	12
	1.7.3 Comparison of Monte Carlo and Deterministic Estimates of the Probability of Burst	13
	Report References	15
	Database References	17

List of Tables

Number	Title	Page
1	Tube Material Properties for Burst Pressure Predictions (W)	19
2	Regression Parameters for the Normalized Burst Pressure as a Function of the Normalized Crack Length	20
3	Regression Parameters for the Normalized Crack Length as a Function of the Normalized Burst Pressure	20
4	Values of the $[F^T F]^{-1}$ Matrix for P_N as a Function of λ	21
5	Values of the $[F^T F]^{-1}$ Matrix for λ as a Function of P_N	21
6	I-600 Through-Wall Burst Test Results (Axial Slits, Bladder NOT Reinforced)	22
7	Undefected Tube Burst Pressures 3/4", 7/8", & 1 1/16" OD Alloy 600 Tubes	25
8	Axial Through Wall Cracks Database	27

List of Figures

Number	Title	Page
1	Normalized Burst Pressure (P_N) vs. Normalized Crack Length (λ) Comparison of Schelle Data to Non-Reinforced Bladder Burst Curve	33
2	Normalized Burst Pressure (P_N) vs. Normalized Crack Length (λ) Alloy 600 SG Tubes, Final Database	33
3	Normalized Burst Pressure (P_N) vs. Normalized Crack Length (λ) Comparison of Schelle Data to New Prediction Curve	34
4	Normalized Burst Pressure vs. Normalized Crack Length Alloy 600 MA Steam Generator Tubes	34
5	Normalized Burst Pressure vs. Normalized Crack Length Comparison of Predictive Equations	35
6	Residual vs. Predicted Normalized Burst Pressure Alloy 600 MA Steam Generator Tubes	35
7	Distribution of Expected vs. Actual Residuals Normalized Burst Pressure vs. Normalized Crack Length	36
8	Actual vs. Expected Cumulative Probability Normalized Burst Pressure vs. Normalized Crack Length	36
9	Actual vs. Calculated Normalized Burst Pressure Alloy 600 MA Steam Generator Tubes	37
10	Normalized Crack Length vs. Normalized Burst Pressure Alloy 600 MA Steam Generator Tubes	37
11	Burst Pressure vs. Crack Length 3/4" x 0.043", Alloy 600 MA SG Tubes, $\sigma_f = 71.6$ ksi	38
12	Burst Pressure vs. Crack Length 7/8" x 0.050", Alloy 600 MA SG Tubes, $\sigma_f = 68.8$ ksi	38
13	Probability of Burst for Single Axial Throughwall Cracks Alloy 600 MA, SG Tubes at 650°F, $\Delta P = 2560$ psi	39
14	Example Burst Pressure Distribution 3/4" x 0.043", Alloy 600 MA SG Tube, $\alpha = 0.6$ (1,000,000 Simulations)	39

1.0 Burst Pressure of Throughwall, Axially Cracked Tubes

1.1 Introduction

The purpose of this report is to document the development of a burst pressure to axial crack length correlation for Alloy 600 Steam Generator (SG) tubes. The results are based on a general development using testing results from a variety of tube diameters and thicknesses. The final correlation utilizes a 3-parameter exponential type of equation relating the non-dimensionalized burst pressure to the non-dimensionalized crack length (referred to as the normalized burst pressure and normalized crack length respectively). By suitably adjusting the coefficients of the equation, the final results are presented graphically for tubes with a nominal outside diameter (OD) of 0.875" and a thickness of 0.050", and for tubes with a nominal OD of 0.750" by 0.043" thick. In addition, equations for obtaining burst pressure as a function of crack length and critical crack length as a function of pressure are provided in Section 1.6. Finally, evaluation of the probability of burst as a function of pressure and crack length is provided in Section 1.7.

1.2 Burst Characterization of Tubes

Much of the theory of the burst behavior of tubes with cracks is based on extending the theory of linear elastic fracture mechanics (LEFM) for flat plates containing through-thickness cracks to a cylindrical geometry^[1,2,3,4,5]¹. In general, the elastic fracture behavior of a cylinder with an axial crack can be analyzed using flat plate solutions with an appropriate stress magnification factor, usually referred to as the "shell curvature correction factor" or "bulging factor", to account for bulging along the crack flanks due to the internal pressure. Additional theoretical solutions appear in the literature in the form of plots of the results of finite element solutions, or the numerical solution of the singular integral equations for a cylinder with an axial crack^[6,7].

A frequent way of presenting solutions for the burst pressure is in the form of a relation between a normalized burst pressure, herein referred to as P_N , and a normalized crack length, λ . The normalized burst pressure is simply the actual burst pressure non-dimensionalized by the flow stress of the material and adjusted for the size of the tubing by the ratio of the mean radius to the thickness. This provides a ratio of a membrane stress in the tube to the strength of the material and allows for the correlation to be applicable to multiple tube sizes. The flow stress of the material is usually taken as a linear function of the yield stress, σ_Y ,

¹ Numbers in square brackets refer to references listed as "Report References" starting on page 15 of this report. References in the tabular data refer to those listed as "Database References" starting on page 17.

and the ultimate tensile stress, σ_U , of the material. Acceptable correlations for Alloy 600 tubes have been obtained using one-half of the sum of the two properties as the flow stress.

For a tube with a mean radius of r_m and a thickness t , the normalized burst pressure as a function of the actual burst pressure, P_B , is defined as

$$P_N = \frac{P_B r_m}{(S_Y + S_U) t} \quad (1)$$

Thus, P_N is the ratio of the maximum Tresca stress intensity, taking the average compressive stress in the tube to be $P_B/2$, to twice the flow strength of the material.² The normalizing parameter, λ , for the crack length, a , is defined as

$$\lambda = \frac{a}{\sqrt{r_m t}}, \quad (2)$$

a form which arises in theoretical solutions to the burst problem. The burst pressure as a function of axial crack length for a specific tube size is then easily obtained from the non-dimensionalized relationship.

1.3 Tube Burst Testing and the Analysis Database

Historically, the relationships presented for correlating the burst pressure to axial crack length for Alloy 600 tubing are based on empirical data. Until recently, one common method of testing^[8,9] consisted of internally pressurizing an axially cracked (or slitted by electrical discharge machining) tube that had been lined with a flexible neoprene or tygon tube, i.e., a bladder, until a burst occurred.³ Burst is considered to have occurred when the crack opens to the extent that the bladder extrudes, and may rupture, accompanied by ductile (plastic) tearing of the tube material at the ends of the crack. If the bladder has ruptured and tearing of the crack has not occurred, the test specimen is not considered to have truly ruptured. This simply means that the opening of the flanks of the crack was sufficient to

² It is noted that some authors^[1,7] use the mean radius in equation (1), and others^[3] use the inside radius; however, this difference in usage is not significant for thin-walled tubes.

³ Since typical test facilities do not provide for an essentially unlimited water supply, nor do they have pumping capability to maintain pressure if the specimen is leaking significantly, the purpose of the lining is to prevent leakage until a rupture of the tube occurs.

permit extrusion of the bladder, and that the actual, or *true*, burst pressure was not achieved during the test.

Test specimens may consist of tubes with cracks that have been extended by high cycle fatigue from a starting notch, either part way or all the way through the thickness, or which have very narrow axial slits machined in them. Typical slit widths are in the range of 6 to 10 mils. The accepted method of creating the starting notch or the slit is by electrical discharge machining (EDM). Testing has demonstrated that both types of specimens behave similarly, thus the added expense of fatigue extension of the EDM slit is generally not justified. In addition, testing is usually conducted at room temperature, with the results adjusted to operating temperature via the change in the flow stress of the material.

In contrast to the testing previously described, tube burst testing in Belgium and France typically included a thin foil shim on the outside of the bladder at the location of the crack or slot. The purpose of the shim was to provide a small reinforcement to prevent extrusion and rupture of the bladder before rupture of the tube. Shim dimensions are usually $\sim 1/2$ " wide by ~ 6 mils thick with the length chosen to extend $\sim 1/4$ " beyond each end of the slit. The shim material was typically brass, although stainless steel has also been used. Burst pressure results from those tests were typically higher than results obtained from similar tests with the bladder not reinforced. Hence, technical exchange discussions usually included consideration of which methodology was appropriate for characterizing the burst strength of SG tubes.

To determine which methodology produced results more representative of burst pressures which might be expected in operating SGs, Hernalsteen^[10] performed several burst tests at the Schelle fossil plant in Belgium. These tests utilized the large water supply and large pumping capacity of the plant to maintain and increase the pressure during the tests. Burst pressure data was obtained for thirteen $7/8$ " OD by 0.050" thick and two $3/4$ " OD by 0.043" thick Alloy 600 tube specimens with a variety of slot lengths without employing a bladder. The data obtained, referred to hereinafter as the Schelle data, are depicted on Figure 1 relative to a correlation curve between P_N and λ based on a regression analysis of the non-reinforced bladder data contained in References 8 and 9 (a subset⁴ of the data contained in Tables 6 and 7). The results clearly demonstrate burst strengths exceeding the results obtained with non-reinforced bladders. Furthermore, Westinghouse analysis of the Schelle data indicated the results to be consistent with those obtained from tests performed by Westinghouse using foil reinforced bladders. However, in Reference 11 and a subsequent publication, Reference 12, Hernalsteen reported that the presence of the foil could increase the measured burst pressure by

⁴ Belgian data which became available after the publication of Reference 8 were not included in the correlation.

about 10% relative to that obtained from tests without bladders. Thus, the validity of performing burst tests on pulled tube specimens was still in need of additional verification. In addition, given the small number of data points available from the Schelle testing programs, it was apparent that the accuracy of performing burst tests under laboratory conditions needed to be established if statistical inference relative to the results was to be performed. While the results obtained from non-reinforced bladder tests could be counted on to be conservative, continued use could lead to removing tubes from service unnecessarily, as could the use of a rupture equation based on the simulation tests performed utilizing non-reinforced bladders.

To resolve the remaining issue, Hernalsteen noted that the Schelle data could be correlated well using a collapse load theory expression originally published by Erdogan^[8], i.e.,

$$P_N = \frac{P_0}{0.614 + 0.386e^{-1.125\lambda} + 0.433\lambda}, \quad (3)$$

where P_0 was calculated to provide the best average fit to the data. A similar fit of equation (3) to the Westinghouse foil data resulted in a calculated P_0 approximately 4% larger than that found for the Schelle data. For the Belgian foil test results, the value of P_0 found to best fit the data was about 8% greater than that obtained using only the Schelle data. In the Reference 13 meeting, comparisons of the results of the Westinghouse data and the data from the individual programs comprising the Belgian data were made with the Schelle data. It was concluded that the results from Belgian program A4, see Table 8, which employed stainless steel foil, were most influential in determining the value of P_0 found for the Belgian foil tests. The data from the other programs were consistent with the results obtained from the Westinghouse programs. It was thus judged that a 5% reduction in burst pressure would be applied to all test results in which foil reinforced bladders had been used.

Figure 2 depicts the Schelle data along with data obtained from testing performed by Laborelec and Westinghouse with a bladder reinforcing shim present. A summary of the data are included in Table 8. It is to be noted that the reported data reflect a 5% reduction from the measured and/or reported data.⁵ Figure 3 illustrates a comparison of the Schelle data to a regression curve (discussed later) obtained using the data of Figure 2. These figures illustrate that the adjusted results coincide well with the results from the program performed at Schelle.

⁵ Specimens designated with a suffix of "-N" included an additional thickness of nickel plating. An additional 5% reduction in strength was also applied to these specimens based on the thickness of the nickel plate.

It is noted that eight of the test results shown on Figure 2 for short slits, $\lambda < 1.5$, are from tests performed with no bladder reinforcement (the first eight results listed in Table 6). The rationale for the inclusion of this data is that the effect of using a shim should diminish with decreasing crack size, and below a length of, say, 0.25" would be expected to show no effect. Foil reinforced test results for short cracks, and the trend of the Schelle data in this range, indicate this to be the case. Figure 2 also depicts the results for all of the Table 7 test results for tubes with no slits. Some consideration was given to omitting the non-cracked specimens from the database on the grounds that the governing material parameter for the cracked specimens is the flow stress while the governing parameter for the uncracked specimens may be only the ultimate tensile stress. This option was rejected based on the observation that the trend of the burst data for very short cracks converges to the average burst value obtained from the uncracked specimens. Hence, if there is a transition in the governing material parameter, it is apparently gradual instead of dramatic.

The data illustrated on Figure 2 represent the final database selected for the development of the burst pressure to crack length correlation. The database consists of a total of 227 data pairs representing a combination of the previously discussed data as follows:

1. The first eight data pairs from Table 6 (not foil reinforced).
2. All of the data listed in Table 7 (no cracks/slits).
3. All of the data listed in Table 8.

It is noted that the Reference 9 data listed in Table 7 were adjusted to reflect results expected if the tests were performed at ambient temperature, since the burst pressure results reported therein were for tests performed at elevated temperature and the material properties were reported only for ambient conditions. The adjustment was based on temperature scaling factors derived using Reference 8 information.

1.4 Regression Analysis of Burst Pressure vs. Crack Length

For the regression analysis, an equation of the type used by Erdogan^[6] to fit numerical results for the shell curvature correction factor was investigated first. As previously noted, Hernalsteen^[10] compared calculated burst pressures using Erdogan's expression to the Schelle data with relatively good results by finding the best value of P_0 for equation (3), with the coefficients in the denominator being those originally determined by Erdogan. Here, P_0 represents the value of P_N corresponding to no slot or crack being present, i.e., $\lambda = 0$. Although the Schelle burst results were found to match reasonably well with the predictions, the resulting curve did not fit well for very small crack lengths nor for specimens without cracks.

Equation (3) is characterized as a single-peak type of function in that it possesses a single mode with the tails asymptotically approaching zero as λ approaches either plus or minus infinity. In this sense it is similar in shape to a normal probability density function. Thus, the slope for a crack length of zero could be positive, zero, slightly negative, or significantly negative depending on the position of the mode along the abscissa. This type of function was needed by Erdogan in order to fit his numerical results which indicated a slope of zero at the intersection of the ordinate and the abscissa. Examination of the normalized burst pressure data base indicated that alternate expressions could also be used, and that a peaking property might not be necessary. A series of regression analyses were performed considering a variety of linear and non-linear functions, including fitting all four of the coefficients in equation (3),⁶ based on minimization of the residual sum of squares. The actual fitting was performed using a generalized reduced gradient algorithm, and checked against the results from a commercially available code which used a Levenberg-Marquardt algorithm. The rationale for the investigation was to let the data determine the appropriate choice of equation form to be used. Several functions were found which provided similar goodness of fit as measured by the index of determination. An exponential function, i.e.,

$$P_N = b_1 + b_2 e^{b_3 \lambda}, \quad (4)$$

was finally selected based on the combination of maximizing the goodness of fit, minimizing the number of coefficients in the function, and the hypothesis that the burst pressure should be a monotonic decreasing function of the crack length.⁷ For the data of Figure 2, the coefficients of equation (4) were found to be (see Table 2),

(5)

The index of determination for the fit was 99.1%, with a standard error of the estimate of 0.0172. The F distribution statistic for the regression, the ratio of the mean square due to the regression to the mean square due to the residuals, was >11000. The p values for all of the coefficients were less than $1 \cdot 10^{-5}$. Thus, the fit of the equation to the data is judged to be excellent, i.e., the data exhibit insuffi-

⁶ For a regression analysis involving all of the coefficients, it is noted that there are only three independent coefficients in the denominator since the sum of the first and second coefficients is unity.

⁷ Equation (4) is also advantageous in that it can easily be inverted to yield λ as a function of P_N .

cient evidence to reject the proposed model.⁸ Figure 4 depicts the results of the regression analysis, i.e., predictions using equation (5), relative to the database.

Based on the exponential equation form, three regression analyses were performed that,

1. considered omission of the undegraded tubes data,
2. omission of the non-reinforced bladder data, and
3. omission of both sets of data.

For all three cases the resulting coefficients were similar to those reported in equation (5), thus verifying the initial judgement to include those data in the analysis data base.

A final regression analysis was performed to determine the best value of P_0 for the Erdogan equation, equation (3), for all of the data. A plot of the resulting equation is shown on Figure 5 for comparison with the results using equation (5). Both expressions yield similar results over the range of λ from about 2 to 4. The upper bound of this range corresponds to crack lengths of about 0.5" for 3/4" OD tubes and 0.6" for 7/8" OD tubes. For information, the EPRI equation per Reference 14 is also depicted on Figure 5. While this equation reasonably estimates a lower bound prediction relative to the data, it was not developed using the experimental database.

1.4.1 Analysis of the Regression Residuals

An analysis of the regression residuals was performed by making a scatter plot of the residual normalized burst values versus the predicted normalized burst values, Figure 6, and by plotting the cumulative probability of the residuals relative to the sorted residual values, Figure 7. As an alternate view of Figure 7, the actual cumulative probabilities of the residuals were plotted against the expected cumulative probabilities of the residuals, Figure 8. The scatter plot indicates that no significant correlation exists between the residuals and the predicted values, that the variance of the residuals is approximately uniform, and that no apparent systematic departure from the regression curve is present. The cumulative probability plot indicates that the distribution of the residuals about the regression curve of equation (4) is approximately normal with a mean of zero. Thus, the model is considered to be adequate for describing the burst behavior of Alloy 600 SG tubes with axial cracks. An evaluation of the use of the standard error of the residuals is presented in Section 1.7, Probability of Burst.

⁸ This does not mean that equation (4) is the true form of a functional relationship between the two variables, only that it provides an excellent description of the relationship.

1.4.2 Crack Length versus Burst Pressure

In some situations it is desirable to relate a critical crack length to a specified burst pressure, e.g., the crack length that would correspond to tube burst at an applied pressure of three times normal operating differential pressure. Equation (4) can be rearranged to yield the inverse relation

$$\lambda = -a_1 + a_2 \ln(P_N - a_3), \quad (6)$$

for the normalized crack length as a function of normalized burst pressure. For scoping work, equation (5) could be rearranged to yield an approximate set of coefficients for equation (6), however, these are not the best coefficients to be used for the inverse relationship (in a least squares error sense) and they do not afford the development of confidence and prediction bounds on the crack length as a function of burst pressure. The appropriate coefficients result from performing a non-linear regression fit of the same data as used for the burst pressure equation. This results in the empirical relation (see Table 3),

(7)

The index of determination for the regression of λ on P_N was 98.9%. This is approximately the same as that for the regression of the burst pressure on the normalized crack length.⁹ The F-statistic for the significance of the index of determination was calculated to be about 9700. The p values for all of the coefficients were less than $1 \cdot 10^{-5}$. A plot of the values from equation (7) relative to the data is illustrated on Figure 10.

1.5 Confidence and Prediction Bounds

1.5.1 Burst Pressure versus Crack Length

Two-sided $(1-\alpha) \cdot 100\%$ confidence bounds for the mean value of the normalized burst pressure as a function of the normalized crack length can be found as

$$P_N = P_N^0 \pm t_{(v, 1-\alpha/2)} s \sqrt{\{f_0\}^T [F^T F]^{-1} \{f_0\}}, \quad (8)$$

where P_N^0 is obtained from equation (5), t is a Student's-t variate for regression degrees of freedom, s is the standard deviation of the regression residuals, $\{f_0\}$ is a

⁹ If the regression analyses were linear the indices would be the same, they are different here because of the use of non-linear relations.

vector of the partial derivatives of equation (5) relative to each of the coefficients evaluated at the λ_i value of interest, i.e.,

$$\{f_0\} = \left[1 \quad e^{b_3 \lambda_i} \quad b_2 \lambda_i e^{b_3 \lambda_i} \right]^T, \quad (9)$$

and the values of the normalized covariance matrix, $[F^T F]^{-1}$, are given in Table 4. To facilitate computations, if the elements of the equation (9) vector are designated as f_1 , etc., and the elements of the normalized covariance matrix are designated as R_{11} , R_{12} , etc., the expression inside the radical of equation (8) becomes

$$\{f_0\}^T [F^T F]^{-1} \{f_0\} = R_{11} + f_2^2 R_{22} + f_3^2 R_{33} + 2(f_2 R_{12} + f_3 R_{13} + f_2 f_3 R_{23}). \quad (10)$$

A two-sided $100 \cdot (1-\alpha)\%$ prediction band for the value of the normalized burst pressure as a function of a future value of the normalized crack length can be found as,

$$P_N = P_N^0 \pm t_{(v, 1-\alpha/2)} s \sqrt{1 + \{f_0\}^T [F^T F]^{-1} \{f_0\}}. \quad (11)$$

1.5.2 Crack Length versus Burst Pressure

Two-sided confidence and prediction bounds on the critical normalized crack length as a function of the normalized burst pressure can be found as for the normalized burst pressure as a function of the normalized crack length. For the inverse relation the derivative vector is given by,

$$\{f_0\} = \left[1 \quad \ln(P_{N_i} - a_3) \quad \frac{-a_2}{P_{N_i} - a_3} \right]^T. \quad (12)$$

The standard error of the residuals is given in Table 3 and values of the normalized covariance matrix for λ as a function of P_N are provided in Table 5.

1.6 Results and Discussion

A comparison of the measured normalized burst values to the predicted normalized burst values is provided on Figure 9. Examination of the figure shows that most of the results are enveloped within $\pm 10\%$ of the prediction line. Scatter outside of this region is generally restricted to those burst pressures for which the density of the data is highest, i.e., where more variation would be expected to be displayed.

To illustrate the results for specific tube sizes, the data were adjusted to correspond to depict actual burst pressures versus crack lengths for 3/4" and 7/8" nominal OD tubes with thicknesses of 0.043" and 0.050" respectively. These results are depicted on Figures 11 and 12 for nominal flow stresses of 71.6 and 68.8 ksi respectively (650°F mean values from the fabrication database of Reference 8). In addition to the regression curves, the expected burst curves corresponding to the 95%/95% LTL flow stress of the tube materials at a temperature of 650°F are shown (see Table 1 for a complete listing of the material properties used in this report). The critical crack lengths for postulated ΔP_{SLB} of 2.560 ksi are 0.75" and 0.84" respectively for 3/4" and 7/8" diameter tubes with LTL material properties at 650°F. For the Regulatory Guide 1.121 limit of $1.43 \cdot \Delta P_{SLB}$ (3.657 ksi) the corresponding critical crack lengths are 0.51" and 0.57" for tubes with LTL material properties at 650°F.

The results reported herein are based on performing an analysis specific to the data from the Schelle testing programs, data based on the use of foil reinforced bladders, data for tests conducted on specimens with very short crack lengths, and data for the burst of tubes without cracks. The calculation of coefficients specific to the data available is necessary for the establishment of inference bounds. As reported in the previous paragraph, the critical crack length for burst at SLB conditions based on LTL material properties is still greater than or equal to the thickness of the tube support plates.

1.7 Probability of Burst

The purpose of this section is to evaluate the probability of burst (PoB) at SLB conditions as a function of the crack length. This process is the inverse of establishing a statistical inference prediction bound. Two methods were employed to obtain estimates of the probability of burst as a function of λ , Monte Carlo simulation, and deterministic modelling. Descriptions of the analyses and discussions of the results are provided in the following paragraphs. In summary, conservative estimates may be easily obtained using the deterministic model, however, the level of conservatism increases with decreasing probability of burst. For very low PoBs, e.g., on the order of 10^{-7} , the deterministic model may overestimate the PoB by an order of magnitude. Hence, if the crack length is short and the estimate of the PoB must not include excessive conservatism, Monte Carlo simulation is recommended.

1.7.1 Monte Carlo Simulation for Probability of Burst

Monte Carlo simulations were performed based on sampling the residuals from the regression analysis and sampling the material properties for both 3/4" and 7/8" nominal diameter tubes. The results of the Monte Carlo analyses were verified by estimating the probability of burst utilizing a deterministic combination of the variation of the residuals and material properties.

The analyses utilized the burst curve developed herein coupled with the material properties reported in Reference 8 for a temperature of 650°F. Based on the correlation of normalized burst pressure, P_N , to normalized crack length, λ , the expected burst pressure is calculated as

$$P_B = \frac{2t}{R_m} P_N \sigma_f, \quad (13)$$

where t is the thickness of the tube wall, R_m is the mean radius of the tube, and σ_f is the flow stress of the material, taken as $\frac{1}{2}(S_Y + S_U)$. Thus, the values of P_N used in the correlation included variation of the material properties about the reported value, and variation in the thickness and mean radius of the test specimens. The residuals from the regression analysis of P_N on λ were shown to follow a normal distribution. Likewise, the distribution of the flow stress at operating temperature, as presented in Reference 8, also appears to follow a normal distribution. Upon close examination of the histogram of the flow stress distribution it might be judged that the distribution is slightly skewed left (meaning the mean of the distribution is shifted to the left relative to the median and mode), however, evaluations based on using a normal distribution are then conservative since the peak of the distribution is shifted toward the higher values, thus, the probability of calculating a low flow stress is exaggerated.

For the Monte Carlo analyses, the residual distribution about the burst curve and the material strength properties were independently sampled to calculate randomly distributed burst pressures per equation (13) for each of several selected crack lengths, ranging from 0.5" to 0.75". The number of simulations performed was a function of the probability of burst occurring. For high probabilities of burst, i.e., long crack lengths, 100,000 simulations would likely be sufficient. For low probabilities of burst, i.e., short crack lengths, the number of simulations could reach 200 million if sample biasing techniques are not used (for this report, no simulations were performed for very short crack lengths since a trend relative to deterministic estimates was established for longer length cracks). The sampling process consisted of randomly generating a Student's t distribution variate, multiplying by the standard deviation from the data, and adding the resulting value to the expected value to obtain a random value for the parameter. The normalized burst pressure and the material flow stress were sampled independently. The fraction of burst pressures found to be less than or equal to the SLB differential pressure is then the probability of experiencing a burst for any individual tube. For example, the estimated probability of burst for a 3/4" nominal OD (0.043" thick) tube with a free-span through-wall crack length of 0.50" during a postulated SLB event with a ΔP of 2560 psi is on the order of $3.8 \cdot 10^{-6}$.

A one-sided 100-(1- α)% upper confidence bound for a Monte Carlo result can be found using the following equation (Reference 18):

$$Pr_U = \frac{1}{\frac{N-n}{(n+1)F_{1-\alpha, 2(n+1), 2(N-n)}} + 1} \quad (14)$$

where N is the total number of Monte Carlo trials, n is the number of observed successes, e.g., $P_B \leq P_{SLB}$, and F is from the F-distribution for the specified number of degrees of freedom for the numerator and denominator respectively. For zero successes in the Monte Carlo simulation, equation (14) can still be used to find an upper confidence bound on the probability. For the above example, the 95% one-sided upper confidence bound on the PoB is $5.0 \cdot 10^{-6}$.

1.7.2 Deterministic Estimate of the Probability of Burst

To check the results of the Monte Carlo analyses, the parameters of the burst pressure were estimated directly from the parameters of the P_N and S_f distributions. The expected value of the burst pressure is obtained using the mean of P_N and S_f respectively, i.e.,

$$P_B = \frac{2t}{R_m} P_N S_f \quad (15)$$

The variance of P_B is found from,

$$V(P_B) = \left(\frac{2t}{R_m} \right)^2 \left[P_N^2 V(S_f) + S_f^2 V(P_N) + V(S_f) V(P_N) \right], \quad (16)$$

where V represents variance. The expression given in equation (16) is a biased estimate of the variance (Reference 17) and its use will slightly over predict the probability of burst.¹⁰ The variance of the normalized burst pressure, $V(P_N)$, about

¹⁰ An unbiased estimate is obtained by reversing the sign for the product of variances term. For this analysis, the choice of biased or unbiased is not significant.

the regression curve for a specific value of the normalized crack length, λ_1 , is taken as

$$V(P_N) = s^2 \left(1 + \{f_0\}^T [F^T F]^{-1} \{f_0\} \right), \quad (17)$$

where s is the estimated standard error of the residuals, and N is the number of data pairs used in the analysis.

The results of the deterministic estimates for the for mean and standard deviation (taken as the square root of the variance) of the burst pressure distributions for all of the crack lengths compared, agreed with the results from the Monte Carlo simulations to within 1% of the simulation values. To estimate the probability of burst for a specific crack length, it is assumed that the variable,

$$t = \frac{P_B - P_{SLB}}{s_e}, \quad (18)$$

follows a Student's t distribution, where s_e is found as the square root of the variance from equation (17), and P_{SLB} is the steam line break pressure. The PoB is then calculated as the probability of occurrence of t . Implicit in this calculation is the assumption that the product of the two population normal distributions (for the normalized burst pressure and the flow stress) is also nearly normal. The third moment of the burst pressure distribution, M_3 , is related to the means and variances of the normalized burst pressure and the flow stress distributions as,

$$M_3(P_B) \propto P_N^2 S_f^2 V(P_N) V(S_f). \quad (19)$$

Since each of the terms in equation (19) is positive, M_3 will also be positive. Hence, the distribution of the product of the normalized burst pressure and the flow stress will be skewed right, i.e., with a higher tail for the larger burst pressures. Therefore, the prediction of burst probabilities based on equation (18) would be expected to be conservative. In addition, the degree of conservatism would be expected to increase with decreasing probability of burst, i.e., for shorter crack lengths.

1.7.3 Comparison of Monte Carlo and Deterministic Estimates of the Probability of Burst

Probability of burst curves as a function of crack length were developed for each tube size and are presented on Figure 13. Examination of Figure 13 indicates that the deterministic estimate of the PoB for cracks shorter than 0.65" in 3/4" tubes and 0.7" in 7/8" tubes is conservative relative to the simulation results, and converges to the simulation estimate as the crack length increases. Furthermore,

the deterministic estimate of the PoB for a tube with a 0.50" long free-span crack in a 3/4" nominal OD tube at 650°F is about $1.7 \cdot 10^{-5}$, or three times the simulation result. A similar trend is apparent for tubes with a 7/8" nominal diameter. The magnitude of the deterministic estimate of the PoB relative to the simulation estimate increases as the PoB decreases. In addition, an examination of the distribution of burst pressures from the Monte Carlo simulations verified them to be non-normal and skewed right. An example of the distribution of burst pressures for a crack length of 0.6" in a 3/4" diameter tube is illustrated on Figure 14. The effect of skewing the distribution to the right increases the mean value to above both the median and the mode of the distribution. The area in the tail beyond two standard deviations above the mean is visibly greater than the area in the tail below two standard deviations below the mean. In summary, the relative behavior of the results of simulated and deterministic estimates of the PoB are in accord with the expectations discussed above, the results of the two analyses are considered to verify each other, and for low probabilities of burst the deterministic estimate will be conservative.

Report References:

1. Folias, E. S., *An Axial Crack in a Pressurized Cylindrical Shell*, International Journal of Fracture Mechanics, Vol. 1, pp. 104-113 (1965).
2. Hernalsteen, P., *Evaluation of Critical Sizes for Defects in Small Diameter Tubing*, Structural Mechanics in Reactor Technology (SMiRT), 7th International Conference, paper G/F 4/3, pp 283-288 (1983).
3. Kiefner, J. F., Maxey, W. A., Eiber, R. J., and Duffy, A. R., *Failure Stress Levels of Flaws in Pressurized Cylinders*, ASTM STP-536, American Society of Testing and Materials, pp 461-481, (1973).
4. Hahn, G. T., Sarratte, M., and Rosenfield, A. R., *Criteria for Crack Extension in Cylindrical Pressure Vessels*, International Journal of Fracture Mechanics, Vol. 5, No. 3, pp. 187-210 (1969).
5. NP-6626-SD (Special Distribution Document), "Belgian Approach to Steam Generator Tube Plugging for Primary Water Stress Corrosion Cracking," Electric Power Research Institute, Palo Alto, California, USA (1990).
6. Erdogan, F., and Kibler, J. J., *Cylindrical and Spherical Shells with Cracks*, International Journal of Fracture Mechanics, Vol. 5, No. 3, pp 229-237 (1969).
7. Erdogan, F., Irwin, G. R., and Ratwani, M., *Ductile Fracture of Cylindrical Vessels Containing a Large Flaw*, ASTM STP-601, American Society for Testing and Materials, pp 191-208 (1976)
8. WCAP-12522, "Inconel Alloy 600 Tubing-Material Burst and Strength Properties," Westinghouse Electric Corporation (1990).
9. NUREG/CR-0718, "Steam Generator Tube Integrity Program Phase I Report," Prepared for the United States Nuclear Regulatory Commission by Battelle Pacific Northwest Laboratories (1979).
10. Hernalsteen, P., *The Influence of Testing Conditions on Burst Pressure Assessment for Inconel Tubing*, Structural Mechanics in Reactor Technology (SMiRT), 11th International Conference, Tokyo, Japan, SMiRT Transaction Paper F08/2 (1991).
11. Hernalsteen, P., *Evaluation of Critical Lengths for Through Thickness Axial Cracks in Steam Generator Tubing*, Paper F 7/6 presented at the 6th SMiRT Conference, Paris (1981).
12. Hernalsteen, P., *The Influence of Testing Conditions on Burst-Pressure Assessment for Inconel Tubing*, International Journal of Pressure Vessels & Piping, Vol. 52, pp 41-57 (1992).

13. Personal discussions with P. Hernalsteen (LABORELEC) and J. Begley (Packer Engineering), October 21, 1993.
14. NP-6864-L (DRAFT), "PWR Steam Generator Tube Repair Limits: Technical Support Document for Expansion Zone PWSCC in Roll Transitions - Rev. 2," Electric Power Research Institute, Palo Alto, California (June 1993).
15. "User's Guide Microsoft Excel," Version 5.0, Microsoft Corporation, Redmond, Washington, USA (1993).
16. "Tablecurve 2D, Automated Curve Fitting Software for Windows," Version 2, Jandel Scientific, San Rafael, California (1994).
17. Goodman, L. A., *On the Exact Variance of Products*, American Statistical Association Journal, Vol. 55, pp. 708-713 (1960).
18. Hald, A., "Statistical Theory with Engineering Applications," John Wiley & Sons, New York, New York, USA (1952).

Database References:

The following document list is with specific regard to the data portrayed in the attached tables. If one of these documents is referred to within the body of the report it is relative to the number associated with the Report References listed previously.

1. WCAP-12522, "Inconel Alloy 600 Tubing-Material Burst and Strength Properties," Westinghouse Electric Corporation (1990).
2. NUREG/CR-0718, "Steam Generator Tube Integrity Program Phase I Report," Prepared for the United States Nuclear Regulatory Commission by Battelle Pacific Northwest Laboratories (1979).
3. Report No. 68065, "Through-Wall EDM-Flawed Specimens Simulating Axial Cracks in Steam Generator Tubes Near Support Plates," Westinghouse Science & Technology Center (1995)
4. NP-6626-SD (Special Distribution Document), "Belgian Approach to Steam Generator Tube Plugging for Primary Water Stress Corrosion Cracking," Electric Power Research Institute, Palo Alto, California, USA (1990).
5. Hernalsteen, P., *Evaluation of Critical Lengths for Through Thickness Axial Cracks in Steam Generator Tubing*, Paper F 7/6 presented at the 6th SMiRT Conference, Paris (1981).
6. Hernalsteen, P., *The Influence of Testing Conditions on Burst-Pressure Assessment for Inconel Tubing*, International Journal of Pressure Vessels & Piping, Vol. 52, pp 41-57 (1992).
7. SG-92-12-027, "McGuire Unit No. 1 Steam Generator Tube Examination," Westinghouse Electric Corporation (under contract to Duke Power Company), May 1, 1993.
8. RDD:90:5459-01:01, "Laboratory Examination of Pulled RGS Tube Section from McGuire Nuclear Station," Babcock & Wilcox (under contract to Duke Power Company), August 28, 1989.
9. SG-93-06-005, "Examination of Farley Unit 1 Hot Leg Steam Generator Tubes," Westinghouse Electric Corporation (under contract to Alabama Power Company), May 28, 1993.
10. SG-93-02-004, "Examination of D. C. Cook Unit 1 Hot Leg Steam Generator Tubes R11-C60, R12-C29, R18-C16, and R18-C21," Westinghouse Electric Corporation (under contract to the American Electric Power Company), September 23, 1992.

11. NP-7480-L, Volume 1, Revision 1, "Steam Generator Tubing Outside Diameter Stress Corrosion Cracking at Tube Support Plates - Database for Alternate Repair Criteria, Volume 1: 7/8 Inch Diameter Tubing," Electric Power Research Institute, December, 1993.
12. NP-7480-L, Volume 2, "Steam Generator Tubing Outside Diameter Stress Corrosion Cracking at Tube Support Plates - Database for Alternate Repair Criteria, Volume 2: 3/4 Inch Diameter Tubing," Electric Power Research Institute, October, 1993.
13. SG-93-09-012, "Examination of V. C. Summer Unit 1 Hot Leg Steam Generator Tubes," Westinghouse Electric Corporation (under contract to the South Carolina Electric & Gas Company), August 31, 1993.
14. Report No. 68066, "Burst Test Results for SG Tubes in Quatrefoil & Eggcrate Supports," Westinghouse Science & Technology Center (1995).

**Table 1: Tube Material Properties
for Burst Pressure Predictions (W)**

Property	Value at RT	Value at 650°F
Alloy 600 Mill Annealed 3/4" x 0.043" SG Tubes		
Sample Size	635	627
Yield Strength Mean	53.05	45.78
Yield Strength St. Dev.	4.8602	3.9081
Tensile Strength Mean	101.29	97.35
Tensile Strength St. Dev.	4.2173	3.9676
Flow Stress Mean	77.17	71.57
Flow Stress St. Dev.	4.1422	3.5668
95%/95% LTL Flow	69.925	65.325
Alloy 600 Mill Annealed 7/8" x 0.050" SG Tubes		
Sample Size	361	360
Yield Strength Mean	50.98	41.89
Yield Strength St. Dev.	4.2068	3.5856
Tensile Strength Mean	99.96	95.67
Tensile Strength St. Dev.	3.6123	3.4196
Flow Stress Mean	75.47	68.78
Flow Stress St. Dev.	3.5002	3.1725
95%/95% LTL Flow	69.225	63.115

Table 2: Regression Parameters for the Normalized Burst Pressure as a Function of the Normalized Crack Length

$$P_N = b_1 + b_2 e^{b_3 \lambda}$$

Parameter	Value	Standard Error
b_1		
b_2		
b_3		
dof		
Standard Error		
r^2		

Table 3: Regression Parameters for the Normalized Crack Length as a Function of the Normalized Burst Pressure

$$\lambda = -a_1 + a_2 \ln(P_N - a_3)$$

Parameter	Value	Standard Error
a_1		
a_2		
a_3		
dof		
Standard Error		
r^2		

Table 4: Values of the $[F^T F]^{-1}$ Matrix for
 P_N as a Function of λ
(Symmetric Matrix)

Table 5: Values of the $[F^T F]^{-1}$ Matrix for
 λ as a Function of P_N
(Symmetric Matrix)

**Table 6: I-600 Through-Wall Burst Test Results
(Axial Slits, Bladder NOT Reinforced)**

Ref.	Material ID	Tube O.D. (in.)	Thickness t (in.)	$S_Y + S_U$ (ksi)	Crack Length a (in.)	Burst Pressure P_b (ksi)	Normalized Crack Length λ	Normalized Burst Pressure P_N
------	-------------	--------------------	------------------------	----------------------	---------------------------	-------------------------------	-----------------------------------	---------------------------------

**Table 7: Undefected Tube Burst Pressures
3/4", 7/8" & 1 1/16" OD Alloy 600 Tubes**

Ref. ⁽¹⁾	Tube Identification	Tube Heat	Tube OD (in.)	Tube Thickness <i>t</i> (in.)	$S_Y + S_U$ (RT, ksi)	Burst Pressure P_B (RT, ksi)	Normalized Burst Pressure P_N
---------------------	---------------------	-----------	---------------	-------------------------------	-----------------------	--------------------------------	---------------------------------

**Table 7: Undefected Tube Burst Pressures
3/4", 7/8" & 1 1/16" OD Alloy 600 Tubes**

Ref. ⁽¹⁾	Tube Identification	Tube Heat	Tube OD (in.)	Tube Thickness t (in.)	$S_Y + S_U$ (RT, ksi)	Burst Pressure P_B (RT, ksi)	Normalized Burst Pressure P_N
---------------------	---------------------	-----------	---------------	--------------------------	-----------------------	--------------------------------	---------------------------------

Table 8: Axial Through Wall Cracks Database

Ref. ⁽¹⁾	Reference Program	Material ID	OD (in.)	Tube Thick. t (in.)	$S_Y + S_U$ (ksi)	Crack Length a (in.)	Burst Pressure P_B (ksi)	Normal. Crack Length λ	Normal. Burst ⁽²⁾ Pressure P_N
---------------------	-------------------	-------------	----------	-----------------------	-------------------	------------------------	----------------------------	--------------------------------	---

Table 8: Axial Through Wall Cracks Database

Ref. ⁽¹⁾	Reference Program	Material ID	OD (in.)	Tube Thick. t (in.)	$S_Y + S_U$ (ksi)	Crack Length a (in.)	Burst Pressure P_B (ksi)	Normal. Crack Length λ	Normal. Burst ⁽²⁾ Pressure P_N
---------------------	-------------------	-------------	----------	-----------------------	-------------------	------------------------	----------------------------	--------------------------------	---

Table 8: Axial Through Wall Cracks Database

Ref. ⁽¹⁾	Reference Program	Material ID	OD (in.)	Tube Thick. t (in.)	$S_y + S_U$ (ksi)	Crack Length a (in.)	Burst Pressure P_B (ksi)	Normal. Crack Length λ	Normal. Burst ⁽²⁾ Pressure $P_{s'}$
---------------------	-------------------	-------------	----------	-----------------------	-------------------	------------------------	----------------------------	--------------------------------	--

Table 8: Axial Through Wall Cracks Database

Ref. ⁽¹⁾	Reference Program	Material ID	OD (in.)	Tube Thick. t (in.)	$S_Y + S_U$ (ksi)	Crack Length a (in.)	Burst Pressure P_B (ksi)	Normal. Crack Length λ	Norm. Burs. Pressure P_N ⁽²⁾
---------------------	-------------------	-------------	----------	-----------------------	-------------------	------------------------	----------------------------	--------------------------------	---

Table 8: Axial Through Wall Cracks Database

Ref. ⁽¹⁾	Reference Program	Material ID	OD (in.)	Tube Thick. t (in.)	$S_y + S_U$ (ksi)	Crack Length a (in.)	Burst Pressure P_B (ksi)	Normal. Crack Length λ	Normal. Burst ⁽²⁾ Pressure P_N
---------------------	-------------------	-------------	----------	-----------------------	-------------------	------------------------	----------------------------	--------------------------------	---

Table 8: Axial Through Wall Cracks Database

Ref. ⁽¹⁾	Reference Program	Material ID	OD (in.)	Tube Thick. t (in.)	$S_Y + S_U$ (ksi)	Crack Length a (in.)	Burst Pressure P_B (ksi)	Normal. Crack Length λ	Normal. Burst ⁽²⁾ Pressure P_N
---------------------	-------------------	-------------	----------	-----------------------	-------------------	------------------------	----------------------------	--------------------------------	---

Figure 1: Normalized Burst Pressure (P_N) vs. Normalized Crack Length (λ)
Comparison of Schelle Data to Non-Reinforced Bladder Burst Curve

Figure 2: Normalized Burst Pressure (P_N) vs. Normalized Crack Length (λ)
Alloy 600 SG Tubes, Final Database

Figure 3: Normalized Burst Pressure (P_N) vs. Normalized Crack Length (λ)
Comparison of Schelle Data to New Prediction Curve

Figure 4: Normalized Burst Pressure vs Normalized Crack Length
Alloy 600 MA Steam Generator Tubes

**Figure 5: Normalized Burst Pressure vs Normalized Crack Length
Comparison of Predictive Equations**

**Figure 6: Residual vs Predicted Normalized Burst Pressure
Alloy 600 MA Steam Generator Tubes**

Figure 7: Distribution of Expected vs Actual Residuals
Normalized Burst Pressure vs Normalized Crack Length

Figure 8: Actual vs Expected Cumulative Probability
Normalized Burst Pressure vs Normalized Crack Length

**Figure 9: Actual vs Calculated Normalized Burst Pressure
Alloy 600 MA Steam Generator Tubes**

Calculated Normalized Burst Pressure

**Figure 10: Normalized Critical Crack Length vs Normalized Burst Pressure
Alloy 600 MA Steam Generator Tubes**

Figure 11: Burst Pressure vs. Crack Length
3/4" x 0.043", Alloy 600 MA SG Tubes, $\sigma_y = 71.6$ ksi

Figure 12: Burst Pressure vs. Crack Length
7/8" x 0.050", Alloy 600 MA SG Tubes, $\sigma_y = 68.8$ ksi