005740

TEST REPORT

MASONRY WALL IMPACT TESTS

FOR

DRESDEN NUCLEAR POWER STATION

UNITS 2 AND 3

COMMONWEALTH EDISON COMPANY

Prepared By
Bechtel Power Corporation
October 1982

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APPENDIXES

1.0 INTRODUCTION

A portion of the masonry sampling and testing program performed by Wiss, Janney, and Elstner (WJE), under Contract Package 13524-082-CP-08(Q), was to determine the types of mortar used in the masonry work at Dresden. WJE used petrographic analyses to examine the mortar to identify the mortar constituents and their proportions in the mortar mix. Using this information, the mortar was then classified according to the types identified in ASTM C 270, Mortar for Unit Masonry.

For walls 25, 81, and 108, additional information on the mortar was required to make the classification. To obtain additional data, a series of impact tests were performed on these walls. The approximate compressive strength of the mortar as it exists in the masonry wall was identified through the impact tests. This data, in conjunction with the petrographic analysis, may then be used to clarify the mortar classification.

Two additional walls, 33 and 50, were also tested. Based on preliminary information, the mortar type for these walls was specifically identified. Therefore, the results of the impact tests on walls 33 and 50 are to be used for comparison with the results of the tests on walls 25, 81,and 108.

Impact tests were also performed on the masonry block for each of the above five walls. The purpose of these tests was to show a correlation between the compressive strength of the block as determined by the impact tests and the strength of the block obtained by WJE.

2.0 TEST LOCATIONS AND SURFACE PREPARATION

A smooth, uniform surface was required for the impact tests. For walls 33, 50, 81, and 108, WJE had removed sections of masonry during the performance of the testing program. This left exposed mortar joints that were smooth and flush with the masonry block. A portion of the tests was then performed on these surfaces. The remainder of the test was performed on the face of the masonry walls. The mortar joints on the face of the wall were tooled concave; therefore, grinding was required to produce a smooth, flat surface.

For all the walls, the test locations were selected such that as many of the tests as possible were performed adjacent to the area that WJE had selected for its sample.

3.0 TEST PROCEDURE

3.1 EQUIPMENT

The impact tests were performed on the masonry walls using a concrete test hammer (Schmidt hammer). The particular instrument used for the tests at the Dresden Station was produced by Soiltest, Inc., Serial 5940.

The Schmidt hammer is a spring-loaded instrument that measures the surface hardness of a specimen by recording the rebound distance of a steel hammer as it is bounced off the test surface. A relationship exists between the hardness and the compressive strength of hardened concrete materials. Appendix A includes the calibration curves that correlate the rebound value with compressive strength. Figure 1 of Appendix A provides three curves for impact in a vertically upward direction, a horizontal direction, and a vertically downward direction. For impact on an inclined surface, a correction factor is applied to the rebound reading to account for the inclination angle. Figure 3 identifies the proper correction factor; then with the adjusted rebound reading, the appropriate cylinder compressive strength is read from Figure 2. A curve for the cube compressive strength is also provided.

The calibration of the Schmidt hammer was based on actual comparison tests of rebound number and compressive strength of concrete test cylinders. In general, tests that are performed on smooth, uniform surfaces will estimate the compressive strength within a 15 to 20% accuracy. However, areas exhibiting honeycombing, scaling, rough texture, or high porosity have been found to produce inaccurate results.

3.2 OPERATION

After the test locations were selected (and ground smooth if required), the plunger of the hammer was released from its locked position and placed against the mortar or block surface. While the hammer was held perpendicular to the test surface, the plunger was depressed by the application of a gradual increase in pressure until the spring was released and the hammer impacted. The rebound reading was then taken from the scale on the side of the hammer while it was held firmly against the wall.

4.0 TEST RESULTS

A minimum of 10 impact readings were taken on both the mortar and masonry block for each of the five walls. The appropriate compressive strength was then determined for each rebound reading. The distribution of the compressive strengths was plotted and the mean determined from the most representative values. A test was considered as nonrepresentative if its result fell well out of the range of the majority of the test results. Appendix B tabulates the test data, shows the distribution of the results, and identifies those tests that were not considered in the calculation of the mean compressive strength.

4.1 WALL 25

4.1.1 Type of Block — Solid

4.1.2 Mortar Tests

Number of tests	12
Nonrepresentative tests	Tests 3 and 11
Mean compressive strength of the representative tests, psi	4,220

4.2 WALL 81

4.2.1 Type of Block — Solid

4.2.2 Mortar Tests

Number of tests	15
Nonrepresentative tests	Test 10
Mean compressive strength of the representative tests, psi	3,821

4.3 WALL 108

4.3.1 Type of Block — Hollow

4.3.2 Mortar Tests

Number of tests	12
Nonrepresentative tests	Test 1
Mean compressive strength of the representative tests, psi	1,282

4.4 WALL 33

4.4.1 Type of Block — Solid

4.4.2 Mortar Tests

Number of tests	10
Nonrepresentative tests	Test 7
Mean compressive strength of the representative tests, psi	4,200

4.5 WALL 50

4.5.1 Type of Block — Hollow

4.5.2 Mortar Tests

Number of tests	11
Nonrepresentative tests	Tests 2, 3, and 10
Mean compressive strength of the representative tests, psi	1,350

5.0 SUMMARY

Impact tests were performed on five masonry walls. Three of the walls (25, 81, and 108) required the additional data to assist in determining their mortar type. The mortar of the two remaining walls (33 and 50) was identified by WJE and the results of the impact tests on these walls are used for comparison purposes. The in situ compression strength of the mortar was obtained from the calibration curves that were developed for the Schmidt hammer. A judgment on the type of mortar is then based on the minimum compressive strengths specified for the various types of mortar in ASTM C 270.

5.1 HOLLOW MASONRY

Walls 50 and 108 were the two hollow masonry walls tested. The resulting compressive strengths of the mortar are 1,350 and 1,282 psi. In their mortar analysis, WJE found that all of the walls tested consisted of Type N mortar with the exception of wall 108, which was considered to have Type O or K mortar. However, the minimum compression strength specified for Type N mortar is 750 psi; therefore, the impact tests indicate that wall 108 was also constructed using Type N mortar.

5.2 SOLID MASONRY

The solid masonry walls tested were walls 25, 33, and 81. The impact tests show the in situ compressive strength of the mortar to be 4,220, 4,200, and 3,821 psi, respectively. Of the normal weight solid masonry walls tested by WJE, the walls were all found to have a Type N mortar except walls 25 and 81, which were identified as either Type N or O. The impact tests indicate that for the three solid walls tested, the mortar has a compressive strength compatible with Type M mortar which has a minimum specified compressive strength of 2,500 psi. Therefore, based on the Petrographic analyses and impact tests, a reasonable and conservative assumption is to consider Type N mortar for the normal weight solid masonry.

5.3 MASONRY BLOCK TESTS

In addition to the tests performed on the mortar, the masonry blocks of the five walls were also tested. The purpose of these tests was to establish the reliability of the impact tests by comparing those results with the results obtained by WJE. The following shows this comparison.

Wall	Compressive Strength (psi)	
	WJE	Impact Test
25	3,527	3,575
33	3,191	3,555
50	2,020	1,220*
81	3,997	3,244
108	2,623	2,600

*Variance attributed to a high degree of surface porosity

APPENDIX A

SCHMIDT HAMMER CALIBRATION CURVES

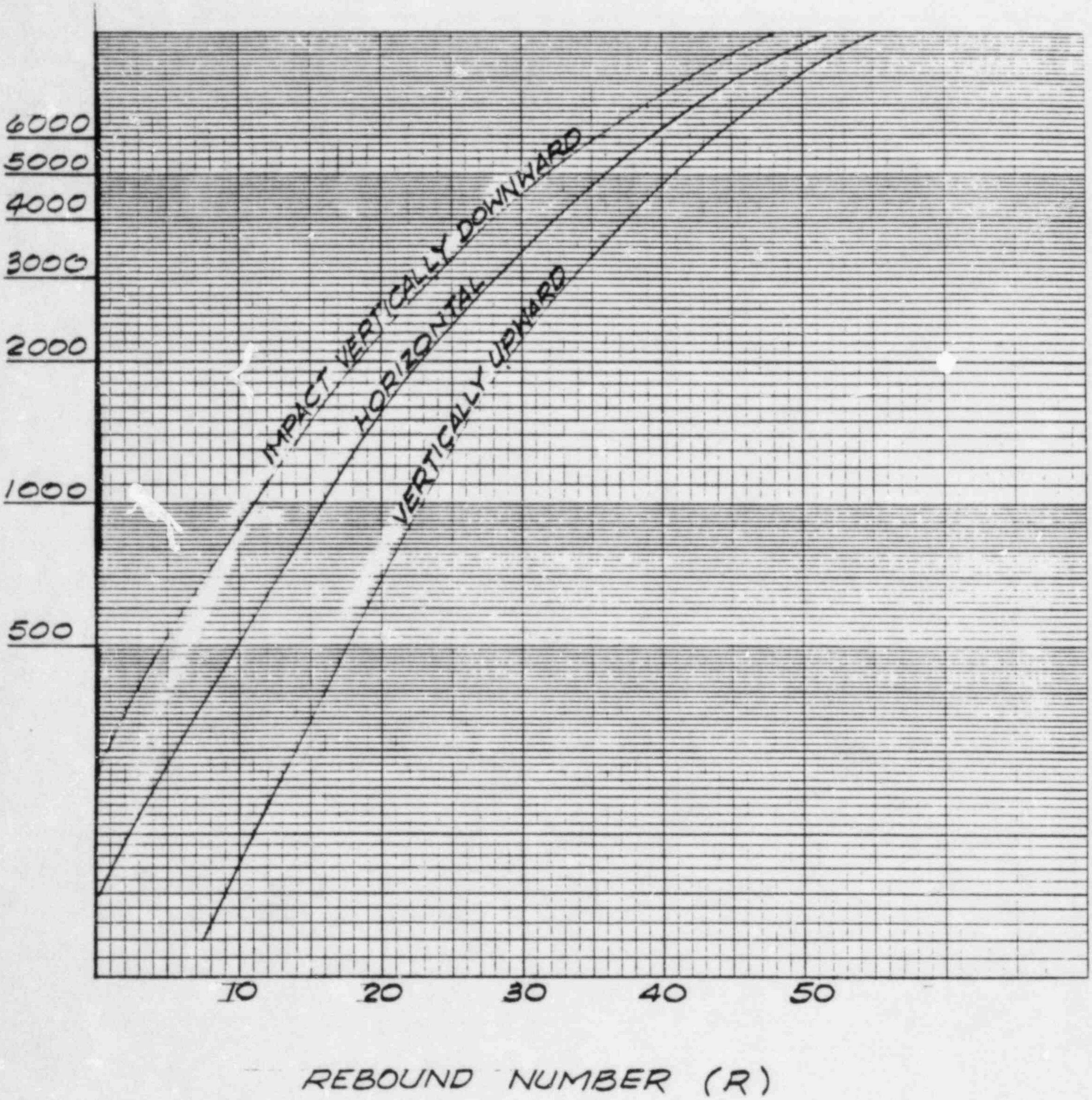
CYLINDER COMPRESSIVE STRENGTH (PSI)
6000
5000
4000
3000
2000
1000
500
IMPACT VERTICALLY DOWNWARD
HORIZONTAL
VERTICALLY UPWARD
10
20
30
40
50
REBOUND NUMBER (R)

FIGURE 1

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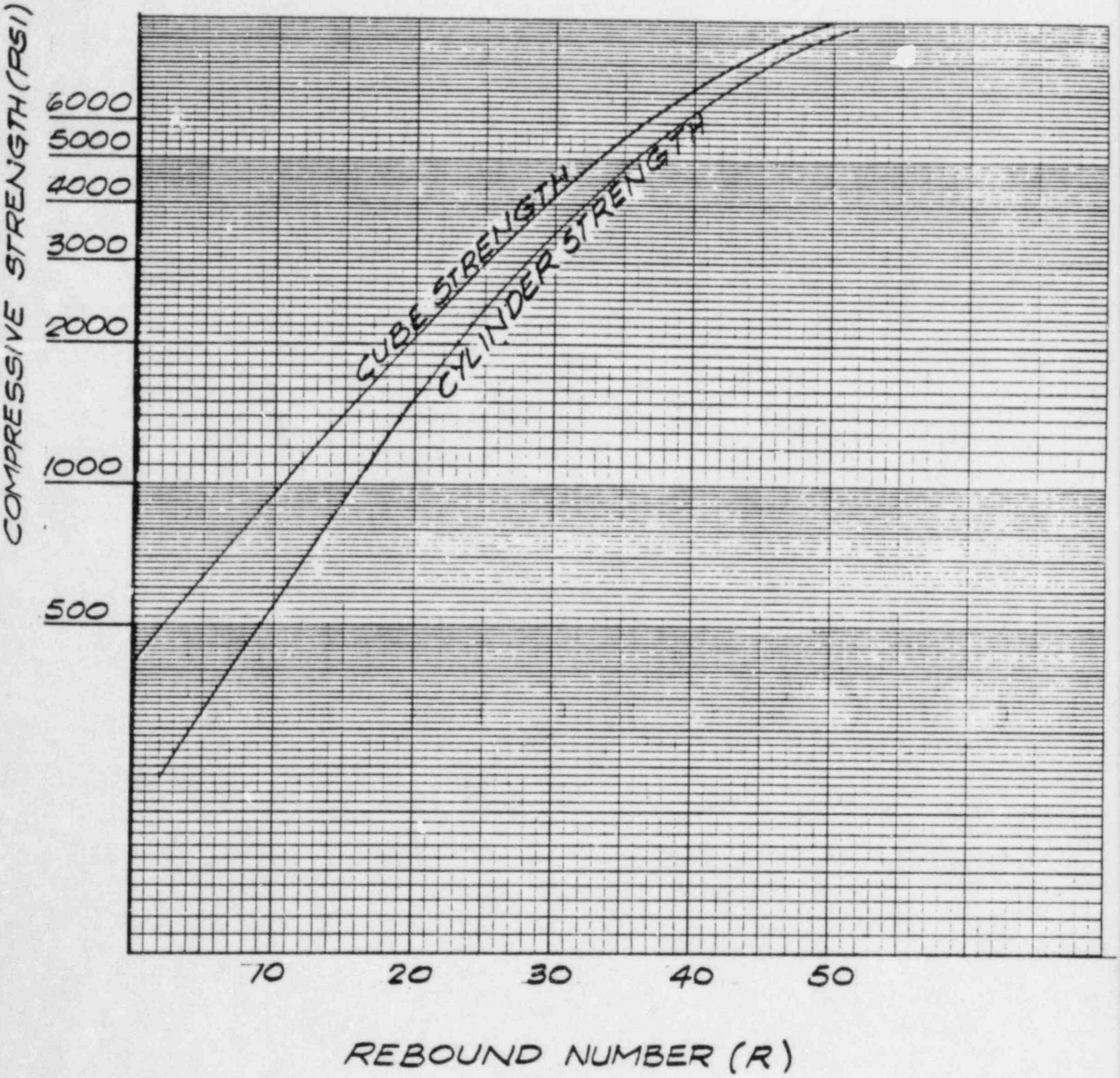

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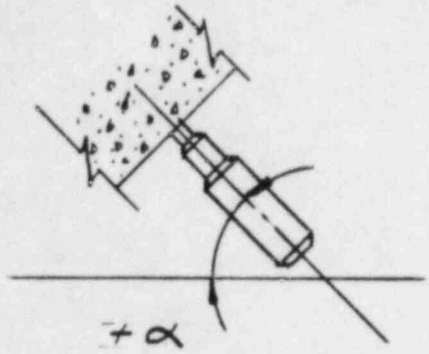
FIGURE 2

CORRECTION FACTORS FOR INCLINED TESTS



R = SCALE READINGS FOR INCLINED TEST DIRECTION

Ro = CORRESPONDING REBOUND NUMBER FOR HORIZONTAL TEST DIRECTION

Ro = R + ΔR

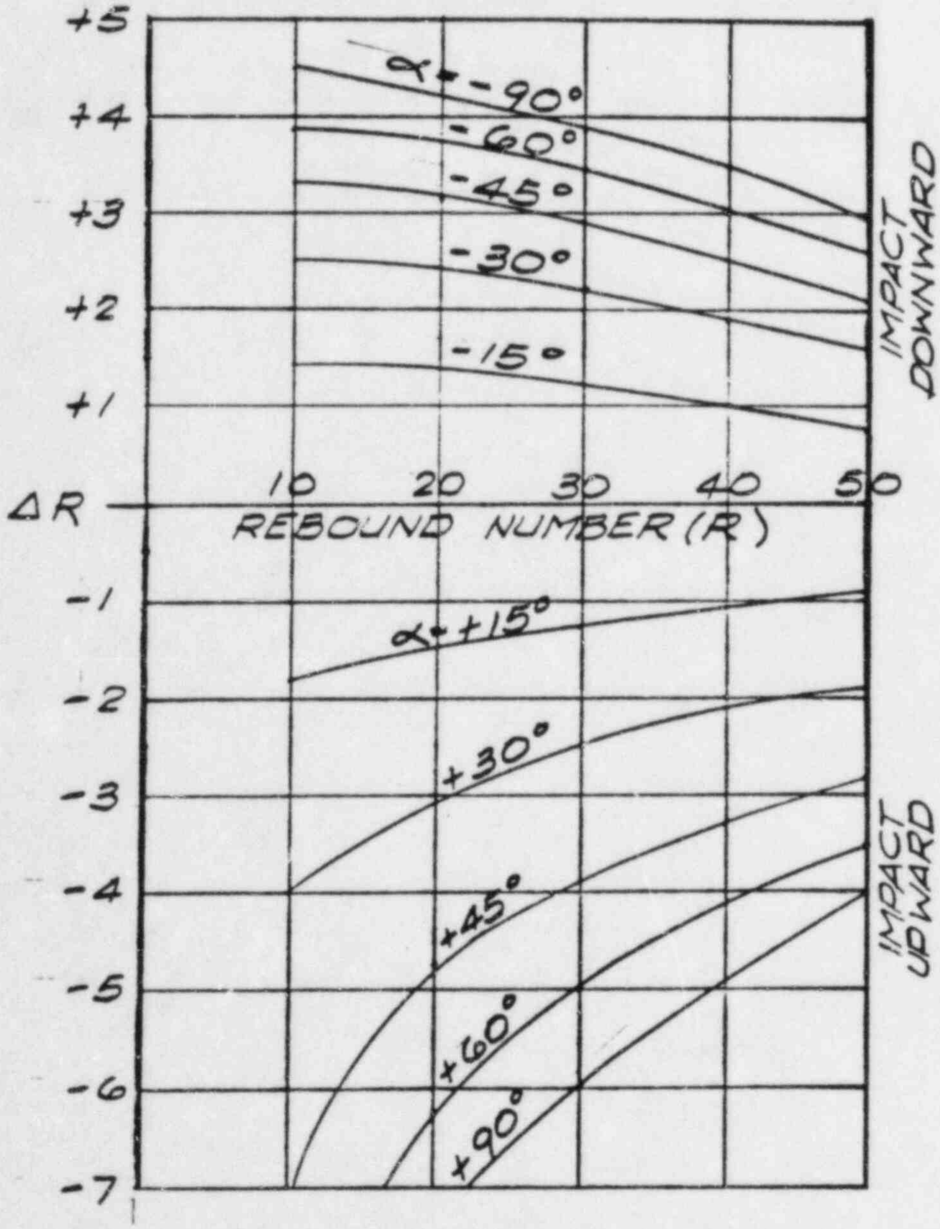


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FIGURE 3

APPENDIX B

TEST DATA - TABULATION AND ANALYSIS

WALL 25

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TYPE OF BLOCK - SOLID

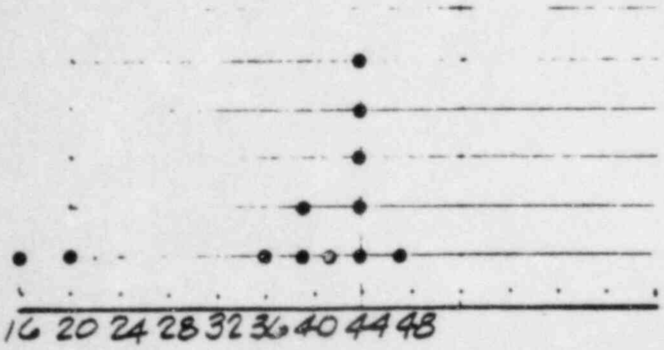
MORTAR TEST			
test no.	rebound reading	direction of impact	compress. strength
1	34	HORIZ.	4400
2	32	"	3900
3	24	"	* 2200
4	34	"	4400
5	33	"	4100
6	34	"	4400
7	34	"	4400
8	34	"	4400
9	31	"	3600
10	32	"	3900
11	20	"	* 1600
12	35	"	4700
Mean of the representative test values			4220

MASONRY BLOCK TEST			
test no.	rebound reading	direction of impact	compress strength
1	30	HORIZ	3400
2	30	"	3400
3	31	"	3600
4	35	"	* 4700
5	32	"	3900
6	30	"	3400
7	33	"	4100
8	30	"	3400
9	35	"	* 4700
10	30	"	3400
Mean of the representative test values			3575

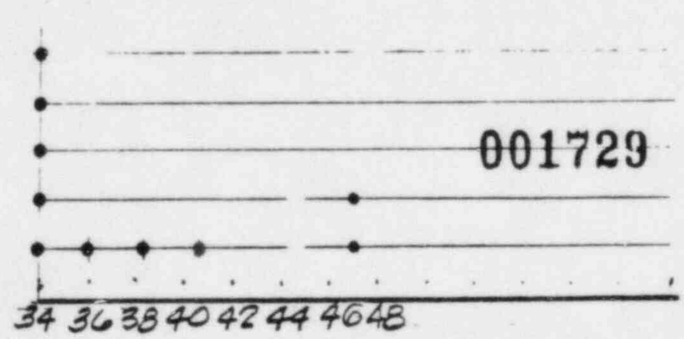
* THESE VALUES NOT INCLUDED IN THE CALCULATION OF THE MEAN COMPRESSIVE STRENGTH.

DISTRIBUTION OF RESULTS

MORTAR



MASONRY BLOCK



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COMPRESSIVE STRENGTH x 100 (PSI)

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WALL 33

TYPE OF BLOCK- SOLID

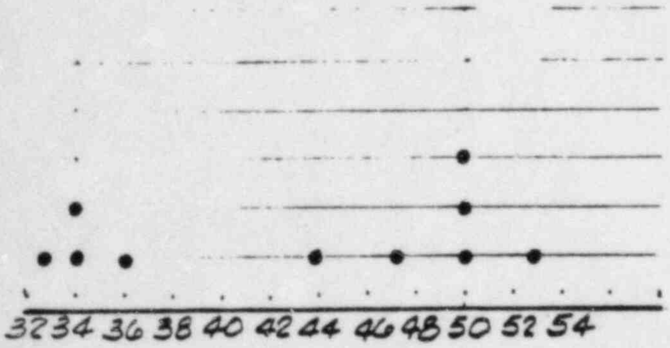
MORTAR TEST			
test no.	rebound reading	direction of impact	compress. strength
1	35	HORIZ.	4700
2	30	"	3400
3	36	"	5000
4	30	"	3400
5	31	"	3600
6	36	"	5000
7	37	"	* 5300
8	36	"	5000
9	35	VERT. UP	3300
10	39	"	4400
Mean of the representative test values			4200

MASONRY BLOCK TEST			
test no.	rebound reading	direction of impact	compress strength
1	33	HORIZ.	4100
2	34	"	4400
3	28	"	3000
4	32	"	3900
5	29	"	3200
6	30	"	3400
7	28	"	3000
8	35	"	* 4700
9	30	"	3400
10	31	"	3600
Mean of the representative test values			3555

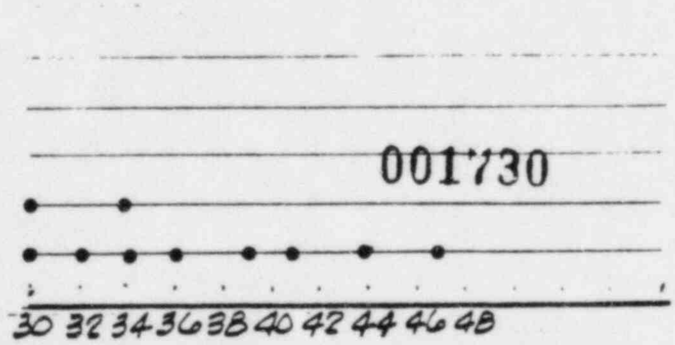
* THESE VALUES NOT INCLUDED IN THE CALCULATION OF THE MEAN COMPRESSIVE STRENGTH.

DISTRIBUTION OF RESULTS

MORTAR



MASONRY BLOCK



COMPRESSIVE STRENGTH x 100 (PSI)

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WALL 50

TYPE OF BLOCK- HOLLOW

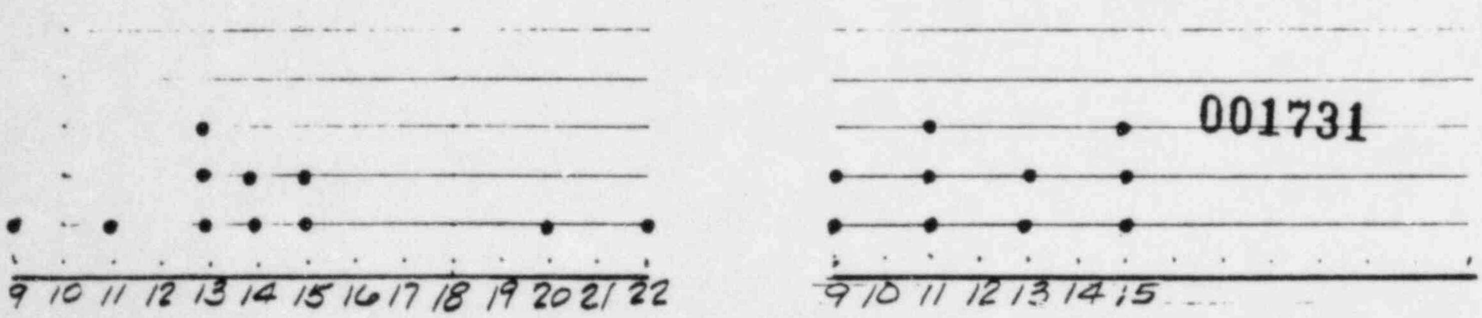
MORTAR TEST			
test no.	rebound reading	direction of impact	compress. strength
1	27	VERT. UP	1500
2	26	30° ↘	*2000
3	22	30° ↗	*2200
4	19	HORIZ.	1400
5	19	"	1400
6	18	"	1300
7	18	"	1300
8	18	"	1300
9	17	"	1100
10	15	"	* 900
11	20	"	1500
Mean of the representative test values			1350

MASONRY BLOCK TEST			
test no.	rebound reading	direction of impact	compress strength
1	18	HORIZ.	1300
2	17	"	1100
3	20	"	1500
4	15	"	900
5	17	"	1100
6	17	"	1100
7	15	"	900
8	18	"	1300
9	20	"	1500
10	20	"	1500
Mean of the representative test values			1220

* THESE VALUES NOT INCLUDED IN THE CALCULATION OF THE MEAN COMPRESSIVE STRENGTH.

DISTRIBUTION OF RESULTS

MORTAR MASONRY BLOCK



COMPRESSIVE STRENGTH x 100 (PSI)

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WALL 81

TYPE OF BLOCK - SOLID

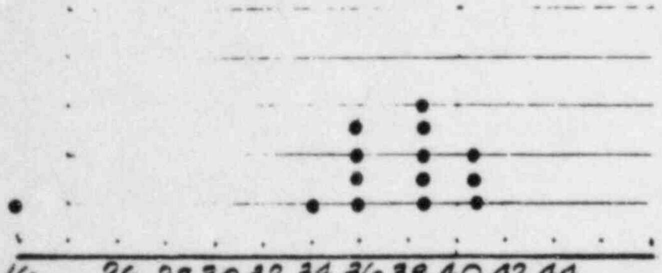
MORTAR TEST			
test no.	rebound reading	direction of impact	compress. strength
1	33	HORIZ.	4100
2	32	"	3900
3	32	"	3900
4	31	"	3600
5	32	"	3900
6	31	"	3600
7	32	"	3900
8	31	"	3600
9	38	VERT. UP	4100
10	20	HORIZ.	* 1600
11	32	"	3900
12	30	"	3400
13	31	"	3600
14	32	"	3900
15	33	"	4100
Mean of the representative test values			3821

MASONRY BLOCK TEST			
test no.	rebound reading	direction of impact	compress strength
1	28	HORIZ.	3000
2	35	"	* 4700
3	30	"	3400
4	29	"	3200
5	29	"	3200
6	29	"	3200
7	30	"	3400
8	26	"	* 2600
9	30	"	3400
10	30	"	3400
11	28	"	3000
Mean of the representative test values			3244

* THESE VALUES NOT INCLUDED IN THE CALCULATION OF THE MEAN COMPRESSIVE STRENGTH.

DISTRIBUTION OF RESULTS

MORTAR



MASONRY BLOCK

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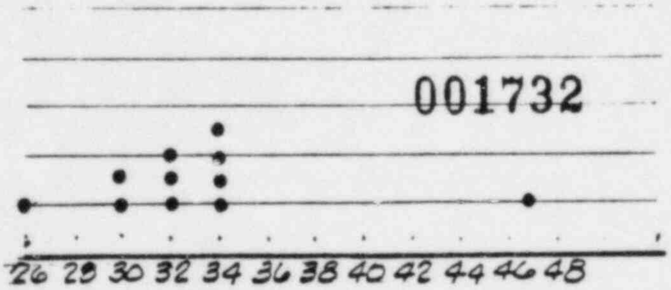

COMPRESSIVE STRENGTH x 100 (PSI)

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WALL 108

TYPE OF BLOCK-HOLLOW

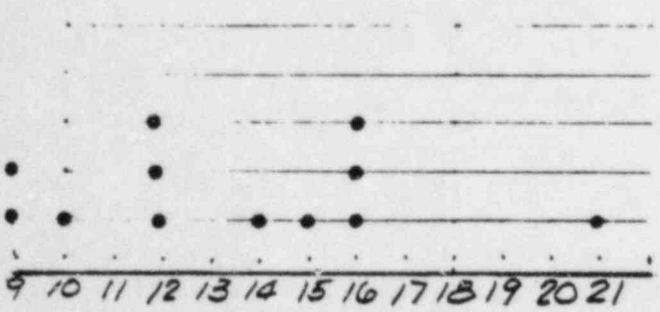
MORTAR TEST			
test no.	rebound reading	direction of impact	compress. strength
1	21	30°	* 2100
2	21	30°	1200
3	18	30°	1600
4	22	VERT. UP	900
5	20	30°	1200
6	19	30°	1000
7	13	VERT. DN.	1200
8	22	30°	1400
9	22	VERT. UP	900
10	18	30°	1600
11	18	30°	1600
12	20	HORIZ.	1500
Mean of the representative test values			1282

MASONRY BLOCK TEST			
test no.	rebound reading	direction of impact	compress strength
1	28	HORIZ.	3000
2	26	"	2600
3	24	"	2200
4	27	"	2800
5	25	"	2400
6	29	"	3200
7	24	"	2200
8	25	"	2400
9	28	"	3000
10	24	"	2200
Mean of the representative test values			2600

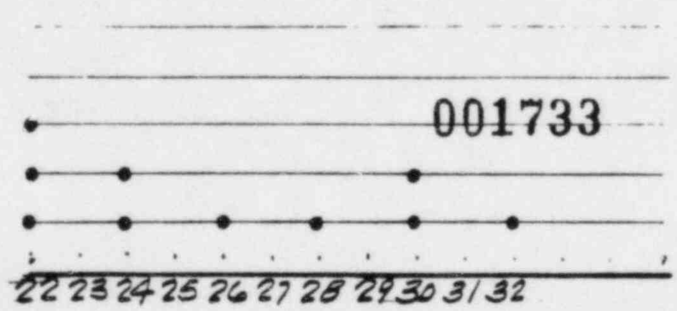
* THESE VALUES NOT INCLUDED IN THE CALCULATION OF THE MEAN COMPRESSIVE STRENGTH.

DISTRIBUTION OF RESULTS

MORTAR



MASONRY BLOCK



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COMPRESSIVE STRENGTH x 100 (PSI)

ATTACHMENT 11-1

TO

RESPONSE TO QUESTION 11

EARTHQUAKE ROCKING RESPONSE OF RIGID BODIES

JOURNAL OF THE
STRUCTURAL DIVISION

EARTHQUAKE ROCKING RESPONSE OF RIGID BODIES

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INTRODUCTION

The rocking response and the possibility of overturning of rigid bodies in earthquakes are central considerations in seismic safety problems. While the present investigation is directed to large concrete blocks, any massive equipment such as heavy electrical or mechanical machinery presents a similar problem to the structural engineer.

A rigid rectangular block resting on a plane surface and responding in the rocking mode has a load-displacement characteristic that is completely different from the more common structural system where seismic response is based on the concepts of flexibility and ductility. Thus, the large body of research associated with the seismic behavior of structural systems cannot be applied directly to the safety of rigid systems subject to overturning. An elastic system has a positive load-deflection characteristic and a set of natural frequencies. In contrast, a rocking block has a load-deflection characteristic that is negative from overturning with a large discontinuity in the zero position, and no discrete natural frequencies. The basic difference between the two systems can scarcely be overstated. In this study, the block is considered as completely rigid and may either be vertically prestressed to the floor or unconnected. The results are equally applicable to systems that can be considered as "stiff" in terms of ground motion, i.e., their natural frequencies are high enough to be out of range of the ground frequencies generally associated with the damaging effects of seismic events.

This study is part of an investigation into the earthquake response of radiation

Note.—Discussion open until July 1, 1980. To extend the closing date one month, a written request must be filed with the Manager of Technical and Professional Publications, ASCE. This paper is part of the copyrighted Journal of the Structural Division, Proceedings of the American Society of Civil Engineers, Vol. 106, No. ST2, February, 1980. Manuscript was submitted for review for possible publication on December 26, 1978.

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shielding systems used in particle accelerator laboratories. These shields typically consist of massive concrete blocks stacked in various configurations, individual block sizes commonly being 3 ft × 4 ft × 5 ft (0.9 m × 1.2 m × 1.5 m) and weighing 7 tons (heavy concrete), or 5 ft × 5 ft × 5 ft (1.5 m × 1.5 m × 1.5 m) weighing 10 tons (ordinary concrete). The blocks stacks may be as high as 20 ft (6.1 m). A typical concrete shield is shown in Fig. 1.

FIG. 1.—Typical Radiation Shielding System (Patient Positioner at Medical Care of 184-in. Synchrocyclotron at Lawrence Berkeley Laboratory)

There are two response modes that should be considered in designing such a system:

1. If the stack is allowed to slide freely, this, in effect, uncouples or partially uncouples the block from the horizontal component of ground motion. The control quantity in this case for purposes of design is the value of the base friction coefficient, μ. The response of a block under these conditions has been reported (1).
2. If the aspect ratio, H/B, of the block is greater than $1/\mu$, it will not slide under the action of ground motion; depending on the intensity of motion it will rock and possibly overturn if not adequately anchored to the ground. A simultaneous vertical component of ground motion alters the critical value of aspect ratio.

This paper deals with the two-dimensional rocking problem. It considers the case of a rigid rectangular block under the action of an in-plane horizontal component of arbitrary ground motion together with a vertical component. It is assumed that if a shielding system consists of a stack of blocks as indicated in Fig. 1, they are tied together in such a way that the system rocks as a unit from the base.

A computer program was written to solve numerically the equation of motion of the block, with the option of including vertical prestressing to increase the stability of the system. The loss of energy due to impact is represented by a simple coefficient of restitution. Tests were conducted on a shaking table using concrete blocks subjected to harmonic as well as to simulated earthquake motions.

After establishing the reliability of the analytical model, some parametric studies were made on the rocking and overturning of rigid blocks of varying

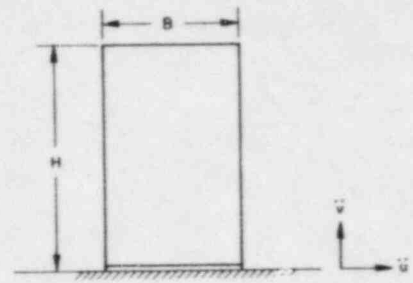


FIG. 2.—Rigid Block under Ground Accelerations

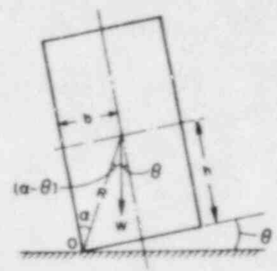


FIG. 3.—Freely Rocking Block

sizes and aspect ratios, and different values of coefficient of restitution, under selected strong motion earthquakes. The effect of a vertical prestressing force was also studied. Based on these data, some general observations are presented.

Analysis

Boundary between Rocking and Sliding.—Consider the block shown in Fig. 2 having width and height dimensions of B and H, respectively, and subjected to simultaneous horizontal and vertical accelerations $\ddot{u}(t)$ and $\ddot{v}(t)$. If sliding is prevented, the block will rock if

$$M\ddot{u}\left(\frac{H}{2}\right) > W\left(1+\frac{\ddot{v}}{g}\right)\frac{B}{2} \qquad (1a)$$

or $$\ddot{u} > g\left(1+\frac{\ddot{v}}{g}\right)\frac{B}{H} \qquad (1b)$$

in which M = mass of the block; W = weight of the block; and g = acceleration of gravity. If sliding and rocking are both possible, then it can be shown (1) that the block will start rocking only if

$$\mu_s > \frac{B}{H} \quad \text{. . . . .} \quad (2)$$

in which μ_s = static coefficient of friction. However, if $\mu_s < B/H$, the block will slide.

Free Vibrations.—The rigid block shown in Fig. 3 will oscillate about the edges when it is given an initial angular displacement θ_0. The equation of motion for the free rocking block has been given by Housner (2) as follows:

$$I_0\ddot{\theta} = -WR\sin(\alpha - \theta) \quad \text{. . . . .} \quad (3)$$

in which I_0 = mass moment of inertia about edge 0, $R = (1/2)(\sqrt{B^2 + H^2})$; and α = angle of block shown in Fig. 3.

For tall slender blocks ($\sin\alpha \approx \alpha$), Eq. 3 may be written in the following form:

$$\ddot{\theta} - p^2\theta = -p^2\alpha \quad \text{. . . . .} \quad (4)$$

in which $p = \sqrt{3g/(4R)}$. Eq. 4 is independent of the density of the block material. If the block is given an initial displacement θ_0, the solution of Eq. 4 is given by

$$\theta = \alpha - (\alpha - \theta_0)\cosh(pt) \quad \text{. . . . .} \quad (5)$$

It can be shown (2) that the natural period of vibration T of a slender block can be approximated by the following equation:

$$T = \frac{4}{p}\cosh^{-1}\left(\frac{1}{1 - \dfrac{\theta_0}{\alpha}}\right) \quad \text{. . . . .} \quad (6)$$

Eq. 6 gives the period, T, in terms of θ_0/α. Fig. 4 shows that the period is strongly dependent on the amplitude ratio θ_0/α, indicating the highly nonlinear nature of the rocking problem.

Coefficient of Restitution.—During the rocking of the block, there is some dissipation of energy at each impact. Under free rocking, this results in the period of each half-cycle being shorter than that which immediately preceded it. The coefficient of restitution ν is defined as

$$\nu = \sqrt{\frac{I_0\dot{\theta}_{i+1}^2}{I_0\dot{\theta}_i^2}} = \frac{\dot{\theta}_{i+1}}{\dot{\theta}_i} \quad \text{. . . . .} \quad (7)$$

in which $\dot{\theta}_i$ = angular velocity before impact; and $\dot{\theta}_{i+1}$ = angular velocity after impact.

The value of ν will in general be dependent on θ_i and the material properties.

Rocking due to Half Sine-Wave Pulse.—To gain some general insight into overturning, Housner (2) considered the stability of a slender block subjected to a half sine-wave acceleration ground pulse. For a pulse period T_s, amplitude a, and for $\omega/p > 3$ (in which $\omega = 2\pi/T_s$ and $p = \sqrt{3g/4R}$), the block will overturn if

$$\frac{aT_s}{2} > 2\pi\alpha\sqrt{\frac{Rg}{3}} \quad \text{. . . . .} \quad (8)$$

The quantity aT_s is simply the product of the amplitude of the pulse and its duration. Also, the block will overturn only if $a/g > B/H$. From Eq. 8 the following observations can be made:

1. For a given value of α (i.e., for geometrically similar blocks) the product of pulse amplitude and duration must increase proportionately with $\sqrt{R}$ to overturn the block. Stability increases with size.
2. For a given value of R, aT_s must increase proportionately with α to overturn the block. For a given size, the stability of a block increases with reduction in aspect ratio.

It should be noted that although these are true for a half sine-wave input, they are not strictly true for all earthquake ground motions, though the general behavior is similar.

Rocking under Earthquake Ground Motions.—The acceleration pulses in an earthquake accelerogram are randomly distributed, and once a block starts rocking

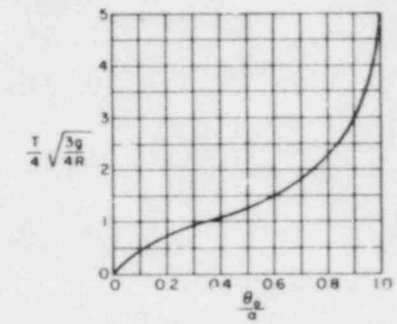


FIG. 4.—Period T of Block Rocking with Amplitude θ_0 (2)

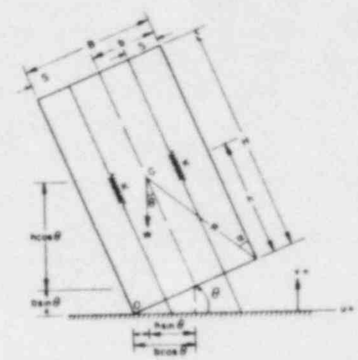


FIG. 5.—Rocking of Block under Ground Accelerations

in an earthquake there is an energy build-up in the system as the block is subjected to successive pulses. In this situation the block can overturn at much smaller peak accelerations than those predicted by a single half-sine pulse of given duration. Thus, the single pulse solution is of limited value when considering the rocking and overturning response of blocks to arbitrary ground motions. The following analysis is quite general in that it treats any ground motion input and imposes no restriction on the geometry of the block.

Consider the block shown in Fig. 5 subjected to arbitrary horizontal and vertical ground accelerations $\ddot{u}(t)$ and $\ddot{v}(t)$, respectively. The distances b and h locate the centroid G from the bottom corner of the block as shown in Fig. 5. Let K and F_0 be the stiffness and initial value of the vertical prestress. Prestressing may or may not be present. Using virtual displacements and taking moments of all forces about the edge of the block 0, the following equation of motion can be derived for rocking

$$I_0 \ddot{\theta} - M\ddot{u}(b \sin\theta + h\cos\theta) + M(\ddot{v} + g)(b\cos\theta - h\sin\theta)$$
$$+ S(F_0 + K\Delta_1)\cos\theta + (B - S)(F_0 + K\Delta_2)\cos\theta = 0 \quad \ldots\ldots\ldots\ldots \quad (9)$$

in which S defines the position of the prestressing force; and Δ_1 and Δ_2 = the extensions in the prestressing rods. In the derivation of the foregoing equation, the prestressing rods are assumed to be linearly elastic and hinged at the floor level, and the expression for the moments due to the rod forces about 0 are sufficiently accurate for practical values of θ.

If $K = 0$ and we substitute for $I_0 = (4/3)\, MR^2$ in Eq. 9,

$$\frac{4}{3} R^2 \ddot{\theta} - \ddot{u}(b\sin\theta + h\cos\theta) + (\ddot{v} + g)(b\cos\theta + h\sin\theta) = 0 \quad \ldots\ldots \quad (10)$$

is obtained. The only other necessary information to solve Eqs. 9 or 10 is the coefficient of restitution. In the absence of prestressing rods the response of the block is a function of the block dimensions and is independent of block mass.

Assumptions in Analytical Model.—The following assumptions were made to solve the equations of motion:

1. The conditions given by Eqs. 1 and 2 are satisfied, i.e., the block responds in the rocking mode without sliding.
2. The coefficient of restitution, ν, is assumed to be constant. This is not strictly necessary, and any relationship between ν and angular velocity at the time of impact could be incorporated into the computer program.
3. The bottom surface of the block is plane or slightly concave so that the block rocks on its edges.
4. One edge of the block is always in contact with the ground. This defines the contact geometry between block and ground, and assumes that the block does not bounce on impact.

Solution of General Equations of Motion.—A computer program BLOKROK was written to solve Eq. 9 using a step-by-step numerical integration procedure based upon a predictor corrector approach. The conditions for initiation of rocking and the energy loss represented by the coefficient of restitution were incorporated. The computer program includes the effects of arbitrary horizontal and vertical ground motions as well as any prestressing forces. Ground motions are read in the form of acceleration-time histories and the results are plotted using the Calcomp plotter.

A typical Calcomp plot of the response of a rigid block 2 ft (0.61 m) wide and 8 ft (3.2 m) high is shown in Fig. 6. The coefficient of restitution ν (COR) is 0.95. There is a single centrally located vertical prestressing rod with an axial stiffness of 0.4 W/in., and an initial prestressing force of 0.4 W. The graphs from top to bottom are the horizontal and vertical earthquake accelerations (San Fernando earthquake), and angular acceleration velocity and displacement of the block. The two parallel lines shown in the displacement plot are drawn at $\theta = \alpha$ and $\theta = -\alpha$.

The total force P in the rods is given by 0.4 $W\Delta$ + 0.4 W in which Δ is

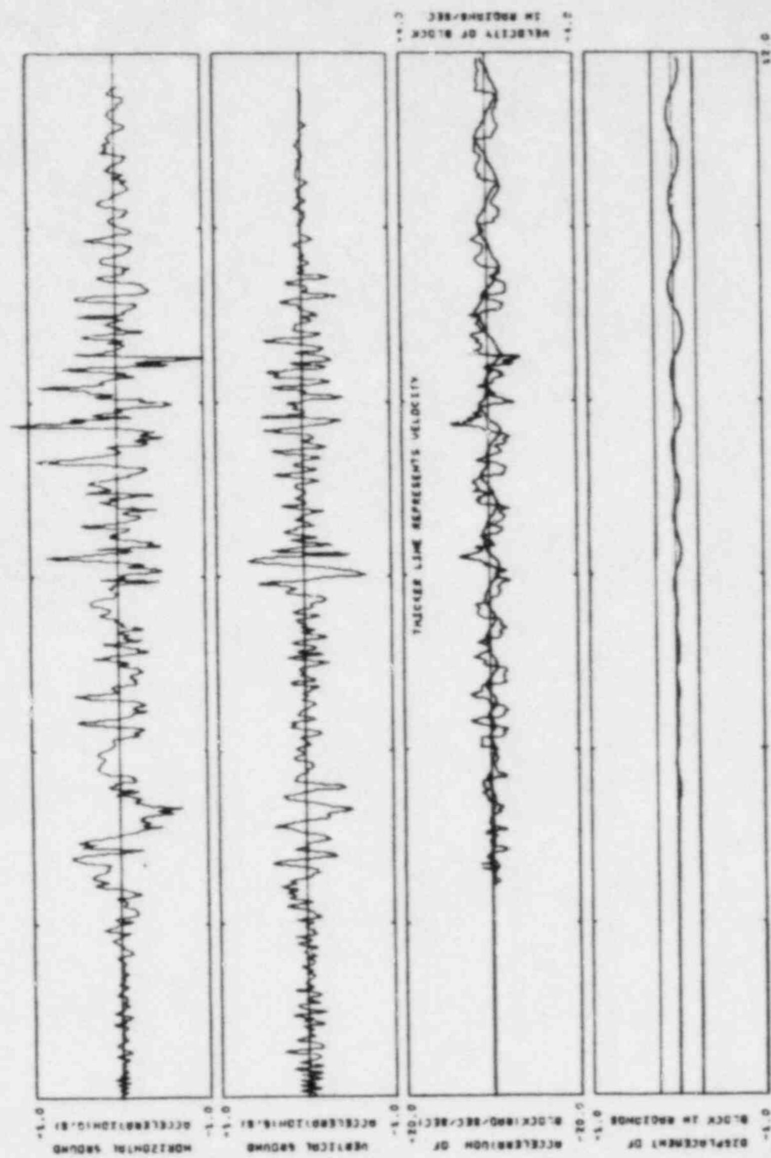


FIG. 6.—Rocking of Block Subjected to San Fernando Earthquake 1971 (Pacoima Dam Record S16E) B = 24 in., H = 96 in., CCR = 0.95, K = 0.4 W/in., Prestressing Force = 0.4 W

the extension of the rod. In this example the block rocks to a maximum value of θ/α = ?.3 and does not overturn. Without the vertical restraint the block does overturn, indicating the effectiveness of vertical prestressing even when the stiffness and initial prestress are both very small.

Experimental Study

Block Design and Instrumentation

To check the accuracy of the analytical model, tests were made on a 6-in. (15.2-cm) wide, 30-in. (76.2-cm) high concrete block (Fig. 7). To achieve the

FIG. 7.—Test Setup of 30-in. × 6-in. Concrete Block Showing Instrumentation

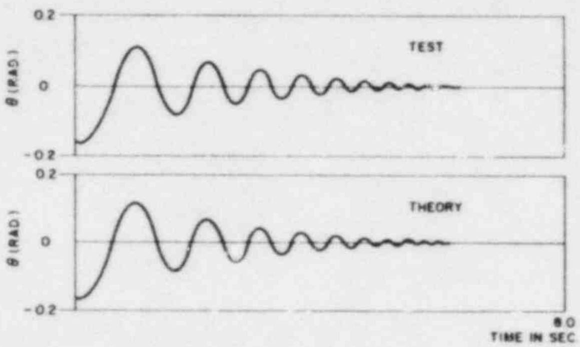


FIG. 8.—Comparison of Angular Displacements of Freely Rocking 30-in. × 6-in. Block (ν = 0.925)

required boundary condition at the base of the block, a 3/8-in. (0.95-cm) thick aluminum plate, slightly concave on the lower surface, was cemented to the block. Also, a plane surface on which the block would rock was provided by a 1-in. (25-mm) thick steel plate hydrostoned and prestressed to the shaking table.

The displacement at the top of the block was measured by means of two lightly spring-loaded potentiometers. The use of two potentiometers was necessary to cancel the effects of the small horizontal forces that each exerted on the block. The potentiometers were mounted on stiff steel posts fixed to the shaking table. Horizontal displacements measured at the top of the block were converted to angular displacement θ. Horizontal cantilever beams on each side of the block and fixed to the steel posts were used as stops to prevent the total overturning of the block and to prevent damage to the potentiometers. The space between the stops and the block faces permitted a ratio $\theta/\alpha > 1.5$, thus ensuring that the block had effectively overturned.

Tests were conducted on the 20-ft × 20-ft (6.1-m × 6.1-m) shaking table at the University of California that is capable of applying both horizontal and vertical ground motions (4). The recorded data included digitized time-histories of the following quantities taken at 50 samples/sec: horizontal and vertical components of table displacement and acceleration, and the horizontal displacement of the block top relative to the table.

Coefficient of Restitution ν

The value of ν was determined by free rocking tests on the block shown in Fig. 7. The block was given an initial displacement θ_0 less than the block angle α, and was allowed to rock freely from a zero initial velocity. A continuous record of the angular displacement was digitized and plotted against time as shown in Fig. 8.

Using the computer program BLOKROK an analysis was carried out using different values of ν and initial test displacement θ_0. For each value of ν the analytical response curve of the block was compared with the test result until the two matched as shown in Fig. 8. The value $\nu = 0.925$, which in this case gave the best fit, was taken as the effective value of the coefficient of restitution. The comparison also demonstrated that ν was effectively constant. Tests conducted on a 36-in. × 9-in. (97.4-cm × 22.9-cm) block produced similar results.

Shaking Table Tests

Tests were carried out using harmonic as well as simulated earthquake ground motions (4). All such tests were conducted on the 30-in. × 6-in. (76.2-cm × 15.2-cm) block shown in Fig. 7.

The harmonic tests used a frequency of 2 Hz for both horizontal and vertical motions, and the amplitudes used were such that the block overturned in each case. The experimental data from these were found to be repeatable and thus suitable for comparing with equivalent analytical results. It was found, however, that similar tests using simulated earthquake motions were not exactly repeatable and therefore could not be used for a precise comparison with theory. The reason for the lack of repeatability was attributed to a slight pitching motion in the shaking table and the sensitivity of the rocking response of the block to the precise ground motion.

Comparison of Test and Analytical Results

Free Rocking Tests.—As indicated previously, the free rocking test was conducted for the purpose of determining the value of ν by fitting an analytical solution to the experimental data. This comparison is also given in Fig. 9 where

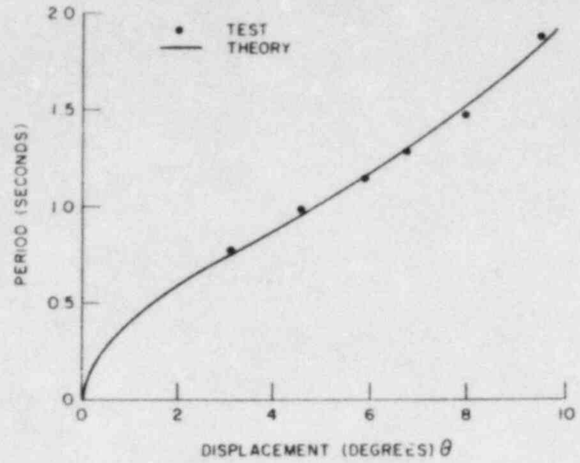


FIG. 9.—Test and Computed Values of Natural Period of Block Rocking with Amplitude θ, Height and Width of the Block are 36 in. and 9 in., Respectively

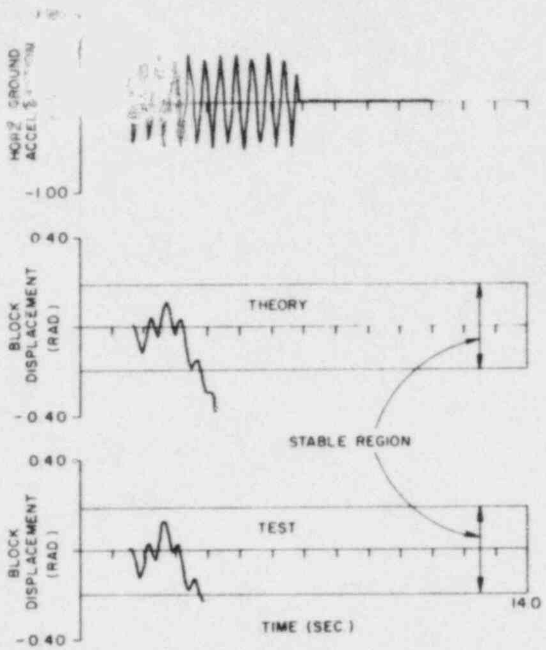


FIG. 10.—Comparison of Test and Theoretical Displacement of 30-in. × 6-in. Rocking Block under Horizontal Ground Acceleration

the period of free rocking is plotted against the angular displacement: this is a highly nonlinear phenomenon with the period of rocking varying from zero to infinity as θ varies from zero to α. This characteristic should be taken into account in selecting a time increment in the analytical solution.

Ground Motion Tests.—Figs. 10 and 11 show a comparison of the measured and predicted angular displacement θ of the 30-in. × 6-in. (76.2-cm × 15.2-cm) block under harmonic ground motions of 2 Hz frequency. The ground acceleration traces shown in these figures indicate the measured shaking table motions for harmonic input. Fig. 10 is the response under horizontal accelerations only,

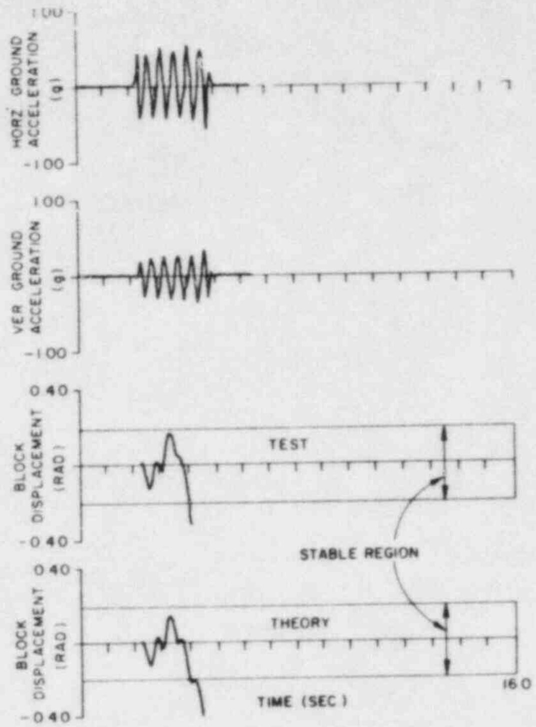


FIG. 11.—Comparison of Test and Theoretical Angular Displacements of 30-in. × 6-in. Block under Horizontal and Vertical Ground Accelerations

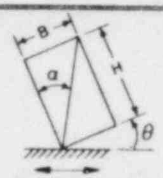
whereas Fig. 11 also includes vertical ground acceleration. The analytical results were obtained by using the measured table motions and a constant value of $\nu = 0.925$, which was obtained from a prior free rocking test. It can be seen in these figures that the measured and predicted results match reasonably well, and the block overturns at approximately the same time and in the same direction in both cases. The stable region in these figures is enveloped by $|\theta| = \alpha$. Comparisons of test data and analytical results were made only for harmonic table motions due to the difficulty of obtaining repeatable test results with simulated earthquake motions as examined previously above.

The step size required for accurate integration in the computer solution is dependent on the size and aspect ratio of the block and on the characteristics of the ground motion. If the response is such t[illegible]t the block immediately starts to rock with a large amplitude, and thus a long period, the time increment is not critical. For example, in the response of the free to long period harmonic motion shown in Fig. 10, the block starts by rocking at a period of approx 0.5 sec without any small amplitude build up. In this case a step size of 0.005 sec is quite adequate. However, if there is an initial small amplitude response, the associated shorter period requires a smaller step size. In studies with the Pacoima Dam record, a 0.001 sec step size was required for satisfactory results, and for the artificial earthquakes A-1 and B-1 an even shorter step size was required. In general, the analysis should be checked using a decreasing step size until satisfactory agreement is attained.

Rocking Response of Rigid Blocks to Earthquake Motions

The rocking response of free rigid blocks under various strong motion earthquakes was studied by computer. Time-history responses of different sized blocks and with varying aspect ratios was carried out and the results plotted. Three different base widths were studied, namely, 1 ft (0.31 m), 2 ft (0.61 m), and 3 ft (0.91 m); and for each of these three, four different aspect ratios, namely, 2/1, 3/1, 4/1, and 5/1, were studied. Results were also obtained for 15 ft × 5 ft (4.58 m × 1.53 m) and 16 ft × 4 ft (4.88 m × 1.22 m) blocks. Each of these 14 blocks was subjected to five different strong motion earthquakes: the S16° E and S74° W components of the Pacoima Dam Record from the San Fernando Earthquake of 1971, the ground motion generated for a study of the Olive View Hospital for the same earthquake, and two further artificially

TABLE 1.—Rocking Response of Rigid Block under Various Strong Motion Accelerograms



F = OVERTURNING

H/B (ft)	COR ν	MAXIMUM θ/α VALUES UNDER EARTHQUAKES: SAN FERNANDO EARTHQUAKE: PACOIMA DAM S16°E	PACOIMA DAM S74°W	OLIVE VIEW HOSPITAL RECORD	ARTIFICIAL EARTHQUAKE A-1	ARTIFICIAL EARTHQUAKE B-1
15/5	1.00	F	0.13	0.15	0.35	0.08
	0.95	0.55	0.10	0.11	0.01	0.01
16/4	1.00	F	0.59	0.32	F	0.67
	0.95	0.82	0.33	0.24	0.60	0.54
15/3	1.00	F	F	0.30	F	F
	0.95	F	0.99	0.34	F	0.41

generated earthquakes A-1 and B-1 representing earthquakes of magnitude 8 and 7, respectively (3). In addition, two values of coefficient of restitution

TABLE 2.—Rocking Response of Rigid Block under Various Strong Motion Accelerograms (ν = 1.0)

Height/width (H/B), in feet (1)	Maximum θ/α values under earthquakes: SAN FERNANDO EARTHQUAKE: Pacoima Dam Record S16° E (2)	Pacoima Dam Record S74° W (3)	Artificial Earthquake A-1 (4)	Artificial Earthquake B-1 (5)
2/1	F	0.63	0.0	0.0
3/1	F	F	F	F
4/1	F	F	F	F
5/1	F	F	F	F
4/2	F	0.42	0.0	0.0
6/2	F	0.38	F	0.40
8/2	F	0.73	F	F
10/2	F	F	F	F
6/3	F	0.20	0.0	0.0
9/3	F	0.29	F	0.16
12/3	F	0.65	F	0.72
15/3	F	F	F	F

TABLE 3.—Rocking Response of Rigid Block under Various Strong Motion Accelerograms (ν = 0.90)

Height/width, in feet (1)	Maximum θ/α values under earthquakes: SAN FERNANDO EARTHQUAKE: Pacoima Dam Record S16° E (2)	Pacoima Dam Record S75° W (3)	Artificial Earthquake A-1 (4)	Artificial Earthquake B-1 (5)
2/1	F	F	0.00	.000
3/1	F	F	0.005	.002
4/1	F	F	F	F
5/1	F	F	F	F
4/2	F	0.30	0.000	0.000
6/2	F	0.58	0.003	0.001
8/2	F	0.43	0.33	0.62
10/2	F	0.75	F	0.66
6/3	0.38	0.23	0.00	0.00
9/3	0.75	0.22	0.002	0.001
12/3	F	0.28	0.22	0.56
15/3	F	0.43	F	0.37

were used in each case, ν = 1.0 representing no energy loss on impact, and ν = 0.90 or 0.95. The recorded vertical accelerogram at Pacoima Dam was included in the analysis of the first two cases, and in the remaining cases only the horizontal component was used. In all of these cases the blocks were taken as free to rock without vertical tie-down.

The results are presented in Tables 1, 2, and 3, and show the maximum angular displacement θ expressed in terms of the block angle α. A value of F indicates overturning.

General Observations on Rocking Response.—From parametric studies summarized in Tables 1, 2 and 3, the following general observations can be made on the rocking, stability, and overturning behavior or rigid free-standing blocks under earthquake motions:

1. For a given aspect ratio H/B (i.e., for a constant value of α) as the size of the block is increased (i.e., as R is increased) the response given as θ/α under a given ground motion decreases. This is in line with the earlier observation that for a given value of α a block with larger R will be more stable under a half sine-wave pulse ground motion. For example, three blocks with aspect ratio of 2/1 and with base widths of 1 ft (0.31 m), 2 ft (0.61 m) and 3 ft (0.91 m) have angular displacements of θ/α = 0.63, 0.42, and 0.20, respectively (Table 2).

2. For a given base width, the rocking response and danger of overturning generally increases with the height or aspect ratio of the block. That there are also exceptions to this general trend will be observed in the response of the 2-ft (0.61-m) wide block under the Pacoima Dam Record (S74° W) in Table 3. The 6-ft (1.83-m) high block has a higher response than the 8-ft (2.44-m) block.

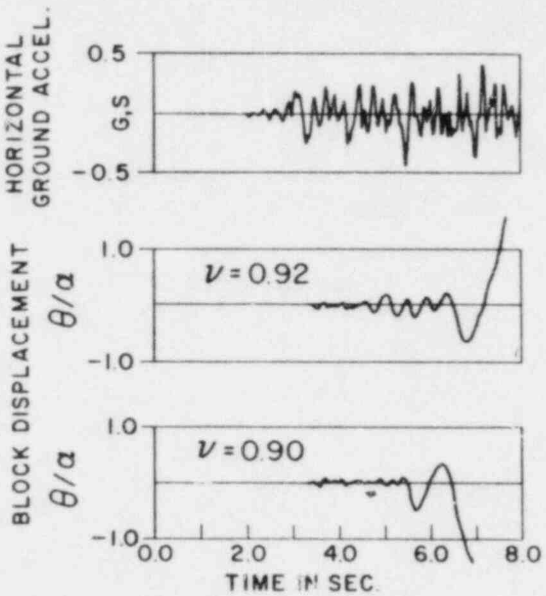


FIG. 12.—Rocking Response of 30-in. × 6-in. Block to Olive View Hospital Ground Motion Showing Sensitivity to Coefficient of Restitution

3. The response of a given block under a given ground motion will generally decrease as the coefficient of restitution is decreased. That this is not always the case, however, may be seen in Table 1 from the response of the 15 ft × 3 ft (4.58 m × 0.92 m) block at ν = 1.0 and ν = 0.95 under the Olive View Hospital record. The response values are θ/α = 0.30 and θ/α = 0.34, respectively. This is due to the highly nonlinear nature of the problem where the period of rocking is amplitude-sensitive and thus differs substantially from a lightly damped linear system where an increase in viscous damping will generally reduce the response.

4. All the free blocks in this study would overturn or approach overturning under one of the five earthquakes considered with the exception of the 15 ft × 5 ft (4.58 m × 1.53 m) (ν = 0.95), and the 6 ft × 3 ft (1.83 m × 0.92 m) (ν = 0.90), and the 9 ft × 3 ft (2.75 m × 0.92 m) (ν = 0.90) blocks. Considering observation No. 1, it would appear that blocks larger than 15 ft × 5 ft (4.58 m × 1.53 m) and 6 ft × 3 ft (1.83 m × 0.92 m) for aspect ratios of 3/1 and 2/1, respectively, would have little probability of overturning in a strong earthquake.

5. Unlike a linear elastic problem, the rocking problem is very sensitive to small changes. This can be seen in Fig. 12 where a small change in the value of ν completely changes the time-history response under the same ground motion, in this example the Olive View Hospital record. The difference in sensitivity between the elastic problem, where a small increase in damping causes a reduction in dynamic response, and the block problem, where a slight change in coefficient of restitution may completely alter the dynamic response, can be seen in this example.

6. The rocking response is extremely sensitive to the boundary condition at the base of the block as already considered. For this reason it seems unlikely that much useful data can be derived regarding the precise strength of an earthquake from a casual listing of the dimensions of solid bodies that overturn and remain standing after an earthquake, unless the rocking surfaces are precisely defined. Any slight convexity in the surface of the block or of the ground invalidates the results given in this paper.

7. Clearly, the addition of a vertical tie-down dues improve rocking stability. The 8 ft × 2 ft (2.44 m × 0.61 m) free block that overturns under the Pacoima S16° E record motion becomes stable with a small central prestressing in Fig. 6. In such a solution, the tensile force produced in the vertical restraint must be considered in the design of the foundation.

Summary and Conclusions

The rocking response of rigid bodies under the action of ground motion is quite different from the typical response associated with a structural system, either elastic or ductile. The block problem is highly nonlinear, its rocking frequency being amplitude-dependent, and the rocking response is very dependent on the boundary condition at its base. The computer program developed for this study gives results that agree closely with shaking table tests conducted with large amplitude low frequency harmonic table motions. Correlation with seismic-type input was not achieved as the experimental response was not found to be repeatable. Parametric studies on block response to various strong motion earthquakes shows the sensitivity of the response to aspect ratio, block size, and coefficient of restitution. In general, stability is greater for lower coefficient of restitution, smaller aspect ratio, and larger blocks, but the computed results show exceptions to all of these general trends.

Acknowledgments

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Appendix I.—References

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Appendix II.—Notation

The following symbols are used in this paper:

a = amplitude of acceleration;
B = width of block;
b = $B/2$;
g = acceleration of gravity;
H = height of block;
h = $H/2$;
K = stiffness;
M = mass of block;
R = $\sqrt{b^2 + h^2}$;
T = period of vibration;
t = time;
$\ddot{u}$ = d^2u/dt^2 = horizontal ground acceleration;
$\ddot{v}$ = d^2v/dt^2 = vertical ground acceleration;
W = weight of block;
α = $\tan^{-1}(B/H)$ = block angle;
θ = angular displacement of block;
$\dot{\theta}$ = $d\theta/dt$ = angular velocity;
$\ddot{\theta}$ = $d^2\theta/dt^2$ = angular acceleration;
μ = coefficient of friction; and
ν = coefficient of restitution.