TECHNICAL EVALUATION REPORT

CONTAINMENT LEAKAGE RATE TESTING

(A-04)

YANKEE ATOMIC ELECTRIC COMPANY
MAINE YANKEE ATOMIC POWER STATION

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FOREWORD

This Technical Evaluation Report was prepared by Franklin Research Center under a contract with the U.S. Nuclear Regulatory Commission (Office of Nuclear Reactor Regulation, Division of Operating Reactors) for technical assistance in support of NRC operating reactor licensing actions. The technical evaluation was conducted in accordance with criteria established by the NRC.

Mr. T. J. DelGaizo contributed to the technical preparation of this report through a subcontract with WESTEC Services, Inc.

1. BACKGROUND

On August 4, 1975 [1], the NRC requested that the Yankee Atomic Electric Company (YAEC) review the containment leakage testing program at the Maine Yankee Atomic Power Station and provide a plan for achieving full compliance with 10CFR50, Appendix J, Containment Leakage Testing, including appropriate design modifications, changes to technical specifications, or requests for exemption from the requirements pursuant to 10CFR50.12, where necessary.

On September 3, 1975 [2], Maine Yankee Atomic Power Company (MYAPC) responded to the NRC's request stating that the containment leakage testing program at the Maine Yankee plant was in full compliance with all provisions of 10CFR50, Appendix J. On February 17, 1976 [3], MYAPC reported that the results of a recent inspection by NRC Region I personnel had questioned certain Type C testing practices at the Maine Yankee plant. Accordingly, MYAPC requested an exemption from the requirements of Appendix J to Type C test with air or nitrogen as a medium for certain containment isolation valves.

On March 1, 1976 [4], MYAPC submitted a proposed revision to Technical Specification 4.4 (Containment Testing) to correct certain inconsistencies between Appendix J and the testing practices at the Maine Yankee plant, which had also been discovered during the NRC Region I inspection.

Subsequently, in a letter dated March 29, 1976 [5], MYAPC forwarded information to the NRC in support of the proposed exemption requests to permit testing of certain isolation valves with water as a medium in lieu of air or nitrogen. The NRC staff reviewed this submittal and determined that insufficient justification had been provided to support the proposed correlation between the leakage measurements for the hydraulic and pneumatic tests. Following additional testing and analysis of data, MYAPC submitted a second proposed correlation with supporting data in a letter dated July 28, 1977 [6].

The purpose of this report is to provide technical evaluations of the outstanding licensing submittals related to the implementation of 10CFR50, Appendix J, at the Maine Yankee plant. Consequently, an evaluation of all the test data provided by MYAPC relative to the proposed correlation for the

relationship between hydraulic testing and pneumatic testing is included. In addition, technical evaluations of the proposed technical specification changes [4] are provided.

2. EVALUATION CRITERIA

Code of Federal Regulations, Title 10, Part 50 (10CFR50), Appendix J, Containment Leakage Testing, contains the criteria used for evaluation of the exemption requests. Where applied to the evaluations, the criteria are either referenced or briefly stated, where necessary, to support the results. Furthermore, in recognition of plant-specific conditions which could lead to requests for exemption not explicity covered by the regulations, the NRC directed that the technical review constantly emphasize the basic intent of Appendix J, that potential containment atmospheric leakage paths be identified, monitored, and maintained below established limits.

3. TECHNICAL EVALUATION

3.1 REQUESTS FOR EXEMPTION FROM THE REQUIREMENTS OF APPENDIX J

3.1.1 Type C Testing with Water in Lieu of Air or Nitrogen as a Medium

In Reference 3, MYAPC requested an exemption from the requirements of Appendix J to permit testing of certain containment isolation valves with water as a test medium in lieu of air or nitrogen. MYAPC proposed to convert the liquid leakage measurements to equivalent air leakage by means of a water-to-air correlation. MYAPC stated that to fully drain the lines of the liquid-filled systems in question in order to perform air or nitrogen testing as required by Section III.C.2 of Appendix J could cause to following to occur: (a) plant personnel would be subjected to unwarranted exposure and unnecessary liquid waste would be generated, (b) the evolution of volatile and explosive gases would result, (c) systems required for plant safety would be removed temporarily from service, and (d) air would be injected into the primary system. Furthermore, MYAPC stated that in certain cases, system design either prevented isolation or drainage or resulted in penetrations being sealed with water under accident conditions.

In order to justify the proposed water-to-air correlation, MYAPC submitted test data for the NRC's review in Reference 5. Following indication from the NRC staff that insufficient justification had been provided to verify the correlation, MYAPC forwarded Reference 6 in which a revised correlation was proposed along with additional test data in support of the revised correlation. In Reference 6, MYAPC proposed to convert liquid leakage measurements to equivalent air leakage by the formula: $y = 0.1919x$, where y = air leakage rate in lbm/day, and x = water leak rate in cc/min.

Evaluation

Section III.C.2 of Appendix J requires that Type C testing of containment isolation valves be performed with air or nitrogen as a medium because the objective of the requirements of Appendix J is to test the ability of the containment boundary to prevent the leakage of containment atmosphere.

Consequently, testing with air or nitrogen as a medium is desirable since these media most closely represent the actual post-accident containment atmosphere. System design and other considerations, however, often make it preferable to test with water rather than with air or nitrogen. In this case, it is theoretically possible to measure liquid leakage and to convert this leakage to an equivalent air leakage. However, due to the unknown characteristics of the leakage path and the very low leakage rates involved, no licensee to date has shown the NRC an acceptable method for converting leakage. Consequently, the NRC has approved the substitution of hydraulic testing methods for the required pneumatic testing of Appendix J only where the liquid test is used to demonstrate that post-accident air leakage is precluded by an effective water seal at the containment boundary throughout the post-accident period, such as by meeting the requirements for a water seal system of Section III.C.3 of Appendix J.

An evaluation of MYAPC's proposed correlation is contained in Addendum 1 to this report. In summary, it is shown that MYAPC's proposed correlation of y = 0.1919x does not sufficiently simulate the types of possible leakage path characteristics of the isolation valves and may provide unconservative results. Since MYAPC used a micrometer handle globe valve in the performance of these correlation tests, the data essentially represent an orifice-like leakage path. Actual valve leakage, caused by inadequate contact of mating surfaces, results in some unpredictable combination of orifice-like and capillary-like flow. Should the actual leakage path in a particular situation cause a pure capillary-like flow, the MYAPC correlation would be unconservative by a factor of approximately 2.6.

In addition to concluding that the MYAPC correlation is not sufficiently conservative, Addendum 1 provides the Licensee with several recommendations for determining a more conservative correlation. It should be emphasized that these recommendations are provided as a result of the analysis of the MYAPC data, which was necessary in order to evaluate the proposed correlation. These recommendations have not been endorsed by the NRC staff, nor is there any guarantee that the NRC would accept a revised proposal based on one of these recommendations. In view of the number of years that have passed since

the issuance of Appendix J and the number of attempts by licensees to develop a water-to-air conversion acceptable to the NRC, it is likely that the NRC will continue to approve a substitution of hydraulic testing for pneumatic testing only where the water test is used to demonstrate a water seal.

3.2 PROPOSED TECHNICAL SPECIFICATION CHANGES

3.2.1 Containment Testing

In Reference 4, MYAPCO proposed to delete the existing Section 4.4 (Containment Testing) and replace it with a revised Section 4.4. MYAPCO's basis for the revised specifications was to bring the technical specifications at the Maine Yankee plant into closer conformance with Appendix J. The following sections provide technical evaluations of the proposed changes.

3.2.1.1 Type A Integrated Containment Leakage Rate Tests (Specification 4.4.I

Revised Specification 4.4.I requires that Type A tests be performed in accordance with 10CFR50.54 (o), Appendix J. It also requires that the absolute method of calculating the mass of air within the containment be used. Finally, it provides values for the parameters of Pa, Pt, La, and Lt. MYAPC states that the leakage rate limit of La = 0.15 weight percent per 24 hours will, under the most adverse accident conditions, maintain public exposure well below 10CFR100 values in the event of the hypothetical accident.

Evaluation

The proposed revision conforms to the requirements of Appendix J. The actual values of parameters Pa, Pt, La, and Lt are beyond the scope of this review and therefore are not evaluated.

3.2.1.2 Type B and C Containment Leakage Rate Tests (Specification 4.4.II)

Revised Specification 4.4.II requires that Type B and Type C tests be performed in accordance with 10CFR50.54 (o), Appendix J, with the exception of certain liquid-filled systems and lines that will be liquid leakage rate tested, with the results being converted to equivalent air leakage.

Evaluation

The proposed revision conforms to the requirements conforms to the requirements of Appendix J with the exception of the provision for testing for liquid leakage with a conversion to equivalent air leakage. In Section 3.1.1 of this report, MYAPC's proposed correlation was determined not sufficiently conservative for use in converting water leakage to equivalent air leakage. This specification should require pneumatic testing of isolaton valves, unless a hydraulic test is used to demonstrate an effective water seal at the isolation valve throughout the post-accident period.

4. CONCLUSIONS

This report provides technical evaluations of all outstanding issues relative to the implementation of 10CFR50, Appendix J, Containment Leakage Testing, at Maine Yankee Atomic Power Station. The evaluations include a request for exemption from the requirements of Appendix J as well as proposed revisions to the technical specifications at the Maine Yankee plant regarding the containment leakage testing program. The following conclusions are provided:

Requests for Exemption

MYAPC's proposed correlation for the conversion of liquid leakage test measurements to equivalent air leakage rates is not sufficiently conservative when the actual leakage is predominantly capillary-like rather than orifice-like. Hydraulic leakage testing of isolation valves should not be performed unless these tests are being used to demonstrate an effective water seal at the valves throughout the post-accident period.

Proposed Technical Specification Changes

Proposed revisions to Specification 4.4 (Containment Testing) conform to the requirements of Appendix J with the exception of the correlation of liquid leakage to equivalent air leakage.

5. REFERENCES

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   Letter to YAEC
   August 4, 1975

2. J. L. French (MYAPC)
   Letter to NRC
   September 3, 1975

3. J. L. French (MYAPC)
   Letter to NRC
   February 17, 1976

4. W. P. Johnson (MYAPC)
   Letter to NRC
   March 1, 1976

5. W. P. Johnson (MYAPC)
   Letter to NRC
   March 29, 1976

6. E. W. Jackson (MYAPC)
   Letter to NRC
   July 28, 1977

- ADDENDUM 1

MAINE YANKEE PROPOSED CORRELATIO.J FOR WATER TEST TO AIR LEAKAGE RATE CONVERSION

JULY 14, 1980

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MAINE YANKEE PROPOSED CORRELATION FOR WATER TEST TO AIR LEAKAGE RATE CONVERSION

1. INTRODUCTION

Where pneumatic Type C test of a valve is called for by Appendix J but is deemed to be impractical, Maine Yankee Atomic Power Company has recommended that a hydraulic test of leakage be performed instead. A correlation has been proposed by Maine Yankee to enable the air leakage rate that would have been obtained in a pneumatic test to be inferred from the water leakage rate measured in a hydraulic test.

The present discussion briefly describes the experimental measurements on which Maine Yankee bases the proposed correlation. Comments are made concerning the experimental procedure, the conceptual basis for applying the correlation to valves undergoing leakage testing, and the validity of the correlation. Finally, recommended alternative procedures are presented.

2. MAINE YANKEE TESTS FOR HYDRAULIC TEST CORRELATION

Maine Yankee Atomic Power Company performed tests in which water leakage in hydraulic tests was compared with air leakage in pneumatic tests. Test procedure is described in Reference 1, which also presents some test data. Additional test data are given by Maine Yankee in Reference 2, along with Maine Yankee's proposed correlation. In the pneumatic and hydraulic tests, a metering valve with a micrometer handle was used to represent various leakage flow resistances by using various micrometer settings. Additional tests are reported in Reference 1 using a 3/4 inch single seated globe valve. This type of valve was selected because many containment isolation valves are single seated glove valves.

Leakage flow was from a source pressure, ΔP psig, to atmospheric pressure. In the pneumatic tests, the source was a vessel of volume V, pressurized initially to $\Delta P + 5$ psig. The leakage decreased the pressure to $\Delta P - 5$ psig during a measured time interval. The change in air mass inside volume V was calculated by assuming that temperature T was constant inside volume V, and then applying a correction for the small

observed variation in T. The average leakage mass flow rate, $\dot{m}_{av}$, is the change in air mass inside V divided by the measured time interval. (Reference 1 uses a different notation, in which P is the gage pressure inside V, and ΔP represents the decrease in P during the time interval.) $\dot{m}_{av}$ was interpreted as the instantaneous leakage mass flow rate for source pressure ΔP psig.

For the water leakage flow measurements, a constant air pressure was maintained on the free surface of the water in the source vessel, and flow into a graduated receiver was timed.

In Reference 1, the source pressure was ΔP = 55 psig. Air leakage was measured at a temperature of approximately 86°F. Water leakage was measured at 96°F. In Reference 2, data are presented for source pressures of 55, 30, and 10 psig. These data are at 80°F. In addition, Reference 2 presents data for 55 psig at 120°F for both water and air, so that an indication of the effect of temperature is obtained.

In Reference 1, the correlation obtained with the metering valve is expressed graphically. (No analytical expression is given.) An analytical expression that fits the straight line plot of log (air leakage) vs log (water leak rate) is:

$$y = ax^b \qquad (1)$$

where

y = air leakage rate, lbm/day

x = water leak rate, cc/min

a = 0.6500

b = 0.6922

Data obtained with the globe valve agreed fairly well with this correlation.

In Reference 2, the correlation was obtained in the form

$$y = bx \qquad (2)$$

where the coefficient, b, is a linear function of temperature.

3. CRITIQUE OF MAINE YANKEE TESTS

The internal consistency of each set of test data indicates that the measurements were made with care. The test procedure appears to be

sound. Accordingly, the raw data are reliable. The small correction for temperature variation during the air leakage tests is also correct.

There is, however, the possibility that the data for air leakage at 10 psig have an error of several percent, since $\dot{m}_{av}$, the average leakage rate as the source pressure falls from 15 to 5 psig, does not accurately represent the flow rate at 10 psig. The discrepancy depends on the type of leakage (i.e., viscous, orifice-like, or something intermediate between these). This is discussed in detail in Appendix A.

When ΔP = 10 psig, if the source pressure fell from $\Delta P + \delta P$ to $\Delta P - \delta P$, where δP is 1 psi instead of 5 psi, the discrepancy would decrease from as much as 11.5 percent to less than 0.5 percent. (See Table 4, Appendix A, where for capillary-type viscous flow, $\dot{m}_{av}/\dot{m}_{mid}$ = 0.8852 when δP = 5 psi and 0.9956 when δP = 1 psi.) If the test data were taken with δP = 5 psi, the values of $\dot{m}_{av}$ used in developing the correlation in Reference 2 are too low for ΔP = 10 psig.

For ΔP = 55 psig and T = 80°F, the correlation given by Reference 2 is y = bx where y = air leakage (lbm/day), x = water leakage (cc/min) and b = 0.1919. This correlation is compared with that given in Reference 1 (i.e., Equ. (1) in Table 1.

Table 1. Comparison of Maine Yankee Correlations

Water Leakage (cc/min)	Air Leakage Calculated from Correlation (lbm/day)		
	Ref. 1	Ref. 2	Ratio
1.0	0.65	0.19	3.4
2.0	1.05	0.38	2.7
3.0	1.39	0.58	2.4
4.0	1.70	0.77	2.2
5.0	1.98	0.96	2.1
10.0	3.20	1.92	1.7
20.0	5.17	3.84	1.3
30.0	6.85	5.76	1.2

Not surprisingly, the two correlations do not agree, since one is nonlinear (i.e., Reference 1 correlation is linear in the logarithms of x and y) and the other is linear. The discrepancy is especially pronounced at the low flow rates. No reason is given for discarding the Reference 1 correlation.

For extrapolation to flow rates below the lowest flows for which data were taken, the form of the correlation is particularly important. Examination of Figure 1 of Reference 1, which is a plot of air leakage versus water leakage, shows that the straight line of the form $y = bx$ falls close to the data points. This indicates the good consistency of the data. However, for large values of x (water leakage) the points fall slightly below the line (i.e., y = measured water leakage is slightly less than that given by the line) and for small values of x, y falls slightly above the line. This observation indicates that the linear correlation given in Reference 2 is not reliable for much lower flow rates than the lowest flow rate for which data were taken.

Both correlations (References 1 and 2) are entirely empirical. To be applied to valves that are tested for leakage, it is necessary that the type of leakage path be the same. No reason is evident, however, why valves that are closed should have the same type of leakage path as the partially opened valves used in References 1 and 2.

When a valve is closed, leakage is due to the fact that mating surfaces do not completely obstruct the flow. For example, a surface may have been scratched, with the result that capillary-like grooves are present when the mating surfaces touch or particulate matter or small surface protuberances may keep the mating surfaces very slightly separated. If the leakage is small, the flow regime will be primarily laminar.

A comparable flow through a small valve requires that the valve be substantially opened. Therefore, the flow resistance in the Maine Yankee tests is predominantly orifice-like. The data of Reference 2 are analyzed in Appendix B, where it is shown that the flow was in fact predominantly viscous in all the water test measurements, except those at 55 psig, 120°F. It is also shown that the flow was predominantly orifice-like for all the air test measurements except those at 10 psig.

Since it is possible for the actual leakage path to consist of parallel paths, all of them viscous, a conservative water-to-air correlation formula must be based on the assumption that all flow is viscous. That is, in Equation (B-1) of Appendix B, B is large so that $B\mu >> F$.

Since the orifice-like pressure drop is negligible, $P_1 = P_a$ in Equation (B-5). Also, Equations (B-1) and (B-11) lead to

$$C = 0.06875 \left(\frac{\mu F}{\Delta P}\right)_{water}$$

Equation (B-5) then becomes

$$\dot{m} = 0.06875 \frac{\mu_{water}}{\mu_{air}} \left(\frac{F}{\Delta P}\right)_{water} \frac{P_s^2 - P_a^2}{T} \qquad (3)$$

In Equation (3), F is the water leakage rate (cc/min) for test pressure difference ΔP psi, $\dot{m}$ is air leakage rate (lb/day). P_s and P_a are source and atmospheric pressures (psia) in the air test for which $\dot{m}$ is being calculated, and T(°R) is the air temperature. ($\dot{m}$ has the same meaning as y in Equation (2).)

Discussions leading to $\dot{m}$ proportional to difference of pressure squared and inversely proportional to absolute temperature, as in Equation (3) are given in References 3, 4, and 5.

For tests at 55 psig, 80°F, Equations (2) and (3) yield substantially different results. Reference 2 gives b = 0.1919 in Equation (2) for this pressure and temperature. For F cc/min measured water leakage rate, Equation (2) predicts y = 0.1919 F lb/day air leakage rate. Equation (3) predicts

$$\dot{m} = 0.06875 \times \frac{2.08}{0.0447} \left(\frac{F}{55}\right) \frac{(55+15)^2 - 15^2}{540(°R)}$$

$$= 0.5036 \text{ F lb/day}$$

The ratio $\dot{m}$/y is 2.6. That is, the Maine Yankee correlation, Equation (2) can be unconservative by a factor of 2.6 if the leakage path has entirely viscous flow.

4. RECOMMENDED PROCEDURE

Any one of the following procedures, A, B, or C should be adopted.

A. Additional Test Data

Additional test data should be taken, with test procedure similar to that used by Maine Yankee in References 1 and 2, but subject to the following comments:

First, the procedure for measuring $\dot{m}$ should be modified for low pressure, as discussed in Appendix A. The modification consists of raising the initial and final source pressures slightly, to obtain a more accurate value for $\dot{m}$ at low pressure.

Second, many valves should be tested, covering the full range of types and sizes that are candidates for water-to-air conversion. These valves should all be closed when tested. They should be subjected to various kinds of wear or simulated wear (i.e., scratched, distorted, etc.) but not be partially open. The range of leakage flows from typical small acceptable air leakage rates to maximum permissible air leakage rate should be represented fully. The analysis of the data should develop a correlation that is a conservative envelope.

B. Conservative Theoretical Correlation

Use Equation (3) as the correlation formula.

C. Measure Valve Leakage Characteristic

With the test setup installed for measuring water leakage flow of the actual valve to be Type C tested at function differential pressure, make additional water flow rate measurements at several smaller pressures, so that the coefficients R and B in Equation (B-1), Appendix B, can be determined. Compute C and D from Equations (B-11) and (B-8). Then, with μ and T equal to air viscosity and absolute temperature, use Equations (B-5) and (B-6) to calculate $\dot{m}$ when P_s is function pressure (psia). (A simple procedure is to try several values of P_1, selecting the one for which Equations (B-5) and (B-6) yield the same value of $\dot{m}$.) Perform a statistical analysis of the data to determine the accuracy with which R, B, C, D, and $\dot{m}$ have been determined. Select a value for $\dot{m}$ such that one may assert, at 95% confidence level, that $\dot{m}$ does not exceed this value.

REFERENCES

1. LaRhette, R.P., "Water Leakage to Air Leakage Correlation," Maine Yankee Atomic Power Company (not dated).

2. Jackson, E.W., "Containment Isolation Valve Leakage —— Liquid-to-Air Conversion," Maine Yankee Atomic Power Company, July 28, 1977. Letter to USNRC with Attachment, "Water to Air Conversion Factor".

3. Maccary, R.R., DiNunno, J.J., Holt, A.E., and Arlotto, G.A., "Leakage Characteristics of Steel Containment Vessels and the Analysis of Leakage Rate Determinations," May, 1964, Division of Safety Standards, AEC, TID-20583.

4. Cottrell, Wm. B., and Savolainen, A.W., editors, "U.S. Reactor Containment Technology," ORNL-NSIC-5, Aug. 1965. Chapter 10, "Performance Tests," by R.F. Griffin and G.H. Dyer. Sections 10.4.5 and 10.4.6 adapted from Reference 3.

5. Amesz, J., "Conversion of Leak Flow-Rates for Various Fluids and Different Pressure Conditions," 1966, EUR 2982.e, ORGEL Program, Ispra Establishment, Italy.

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APPENDIX A.

Derivation of $\dot{m}_{av}/\dot{m}_{mid}$

The outlet pressure for the leakage path is atmospheric pressure, P_{atm}. Consequently ΔP across the leakage path is equal to the inlet gage pressure. This has initial and final values ΔP_o and ΔP_f.

Assuming isothermal expansion of the air in the source chamber of volume V, the density of the air in the source chamber is

$$\rho = \rho_o \frac{\Delta P + P_{atm}}{\Delta P_o + P_{atm}} = \rho_o \frac{x + 1}{x_o + 1}$$

where $x = \Delta P/P_{atm}$. If ΔP falls from ΔP_o to ΔP_f in time t_f, the average mass flow rate through the leakage path is

$$\dot{m}_{av} = \frac{(\rho_o - \rho_f)V}{t_f}$$

$$\dot{m}_{av} = \frac{\rho_o V(x_o - x_f)}{t_f(x_o + 1)} \qquad \text{(A-1)}$$

The mass flow rate through the leakage path is a function of pressure, and may be expressed as

$$\dot{m} = f(x) \qquad \text{(A-2)}$$

The absolute pressure is proportional to the mass of air inside the source chamber. Consequently,

$$\dot{x} = -C\dot{m}$$

where C is a positive coefficient. That is,

$$\dot{x} = -Cf(x)$$

The time required for the pressure to fall from P_o to P_f is found by integration:

$$t_f = -\frac{1}{C}\int_{x_o}^{x_f}\frac{dx}{f(x)} = \frac{1}{C}\int_{x_f}^{x_o}\frac{dx}{f(x)}$$

This leads to

$$\dot{m}_{av} = \frac{C\rho_o V(x_o - x_f)}{(x_o + 1)\int_{x_f}^{x_o}\frac{dx}{f(x)}} \qquad \text{(A-3)}$$

At the average pressure, the mass flow rate is

$$\dot{m}_{mid} = f\left(\frac{x_o + x_f}{2}\right) \qquad \text{(A-4)}$$

The mass of air inside the source chamber is

$$M = \rho V = \rho_o V\frac{x + 1}{x_o + 1}$$

Therefore, the outflow rate, $\dot{m} = -\dot{M}$, is

$$\dot{m} = -\frac{\rho_o V}{x_o + 1}\dot{x}$$

Thus, $C = \frac{x_o + 1}{\rho_o V}$, so that Equ. (A-3) becomes

$$\dot{m}_{av} = \frac{x_o - x_f}{\int_{x_f}^{x_o}\frac{dx}{f(x)}}$$

and

$$\frac{\dot{m}_{av}}{\dot{m}_{mid}} = \frac{x_o - x_f}{f\left(\frac{x_o + x_f}{2}\right)\int_{x_f}^{x_o}\frac{dx}{f(x)}} \qquad \text{(A-5)}$$

Three cases will be considered for possible leakage path characteristics: Case 1, orifice-like flow; case 2, intermediate type of flow; and case 3, capillary-type viscous flow. Case 2 is intermediate between cases 1 and 3, and is mathematically simple. However, it does not represent an actual physical flow regime.

Case 1, Orifice-like flow. The velocity v through the orifice satisfies the relation

$$\frac{\rho v^2}{2g} = K\Delta P = KP_{atm}x \qquad \text{(A-6)}$$

where K is a coefficient.

The mass flow rate is proportional to ρv. Equ. (A-6) shows that this is proportional to $(\rho x)^{1/2}$. Therefore, f(x) is proportional to $[x(x+1)]^{1/2}$. With Equ. (A-5) this leads to

$$\frac{\dot{m}_{av}}{\dot{m}_{mid}} = \frac{2(x_o - x_f)}{[(x_o + x_f)\ (x_o + x_f + 2)]^{1/2} \int_{x_f}^{x_o} \frac{dx}{[x(x+1)]^{1/2}}} \qquad \text{(A-7)}$$

The integral is

$$\ln\left(\frac{2\sqrt{x_o^2 + x_o} + 2x_o + 1}{2\sqrt{x_f^2 + x_f} + 2x_f + 1}\right)$$

Table A-1 shows $\dot{m}_{av}/\dot{m}_{mid}$ for several pairs of values of ΔP_o and ΔP_f, assuming P_{atm} = 15 psia.

Table A-1. Values of $\dot{m}_{av}/\dot{m}_{mid}$
Case 1, Orifice-like flow

ΔP_o (psig)	ΔP_f (psig)	$\dot{m}_{av}/\dot{m}_{mid}$
60	50	0.9978
35	25	0.9934
15	5	0.9523

Case 2, Intermediate type of flow. In this case, the mathematically simple assumption is made that the mass flow rate is proportional to ΔP. That is, $f(x)$ is proportional to x. With Equ. (A-5), this leads to

$$\frac{\dot{m}_{av}}{\dot{m}_{mid}} = \frac{2(x_o - x_f)}{(x_o + x_f)\ \ln\left(\frac{x_o}{x_f}\right)} \qquad \text{(A-8)}$$

Table A-2 shows values of $\dot{m}_{av}/\dot{m}_{mid}$ for case 2.

Table A-2. Values of $\dot{m}_{av}/\dot{m}_{mid}$ for Case 2, Intermediate type of flow

ΔP_o (psig)	ΔP_f (psig)	$\dot{m}_{av}/\dot{m}_{mid}$
60	50	0.9972
35	25	0.9907
15	5	0.9102

Case 3, Capillary-type viscous flow. In this case, the mass flow rate is proportional to $(\Delta P + P_{atm})^2 - P_{atm}^2$. Therefore, $f(x)$ is proportional to $(x + 1)^2 - 1$, or $x(x + 2)$. With Equ. (A-5), this leads to

$$\frac{\dot{m}_{av}}{\dot{m}_{mid}} = \frac{x_o - x_f}{\left(\frac{x_o + x_f}{2}\right)\left(\frac{x_o + x_f + 4}{2}\right)\int_{x_f}^{x_o} \frac{dx}{x(x + 2)}} \qquad \text{(A-9)}$$

The integral is

$$\frac{1}{2}\ \ln\left(\frac{x_o}{x_f} \cdot \frac{x_f + 2}{x_o + 2}\right)$$

Table A-3 shows values of $\dot{m}_{av}/\dot{m}_{mid}$ for capillary-type viscous flow.

Table A-3. Values of $\dot{m}_{av}/\dot{m}_{mid}$ for Case 3, Capillary-type Viscous Flow

ΔP_o (psig)	ΔP_f (psig)	$\dot{m}_{av}/\dot{m}_{mid}$
60	50	0.9943
35	25	0.9838
15	5	0.8852

Tables A-1, 2, and 3 are combined in Table A-4, which shows $\dot{m}_{av}/\dot{m}_{mid}$ for all three cases for each nominal ΔP (ΔP_{mid}). In Table A-4, various pairs, ΔP_o and ΔP_f, are shown for each value of ΔP_{mid}. In addition, $(\Delta P)_{matched}$ is shown for each case. $(\Delta P)_{matched}$ is defined as the value of ΔP for which $\dot{m}$ is exactly equal to $\dot{m}_{av}$. Since $\dot{m}_{mid}$ is always greater than $\dot{m}_{av}$, $(\Delta P)_{matched}$ is smaller than ΔP_{mid}.

Table A-4. Values of $\dot{m}_{av}/\dot{m}_{mid}$ and $(\Delta P_{matched})$ for all three cases. (See note.)

ΔP_{mid} (psig)	δP (psi)	ΔP_o (psig)	ΔP_f (psig)	$\dot{m}_{av}/\dot{m}_{mid}$ Case 1	Case 2	Case 3	$(\Delta P)_{matched}$ (psig) Case 1	Case 2	Case 3
55	5	60	50	.9978	.9972	.9943	54.9	54.8	54.8
	3	58	52	.9992	.9990	.9980	55.0	54.9	54.9
	1	56	54	.9999	.9999	.9998	55.0	55.0	55.0
30	5	35	25	.9934	.9907	.9838	29.8	29.7	29.7
	3	33	27	.9976	.9967	.9942	29.9	29.9	29.9
	1	31	29	.9997	.9996	.9994	30.0	30.0	30.0
10	5	15	5	.9523	.9102	.8852	9.3	9.1	9.1
	3	13	7	.9836	.9692	.9600	9.8	9.7	9.7
	1	11	9	.9982	.9967	.9956	10.0	10.0	10.0

Note: Case 1, Orifice-like flow.
Case 2, Intermediate.
Case 3, Capillary viscous flow.

Table A-4 shows that the average flow rate ($\dot{m}_{av}$) found by measuring the time required for the gage pressure in the source volume to fall from $\Delta P_o = \Delta P_{mid} + \delta P$ to $\Delta P_f = \Delta P_{mid} - \delta P$, is slightly lower than the flow rate ($\dot{m}_{mid}$) at ΔP_{mid}. In all three cases, for ΔP_{mid} = 55 psig, the error produced by using δP = 5 psi is less than 1 percent. The error is greatly reduced if δP is made smaller.

For δP = 5 psi, the error increases substantially as ΔP_{mid} is reduced, becoming 5% for case 1 and 11.5% for case 3, when ΔP_{mid} is 10 psig. The errors for ΔP_{mid} = 10 psig are all reduced to less than 0.5% by using δP = 1 psi.

When the error is large (i.e., for ΔP_{mid} = 10 psig with δP = 5 psi) it differs markedly from one case to another. Consequently, it is impossible to apply a correction unless the type of leakage path is known.

There is a remedy for this situation, as may be seen by examining the values of $(\Delta P)_{matched}$. For each pair or values of ΔP_{mid} and δP, a value of $(\Delta P)_{matched}$ can be found that agrees with all three cases to within 0.1 psi. For example, for ΔP_{mid} psig and δP = 5 psi, $\Delta P_{matched}$ = 9.2 psig with an error of only ± 0.1 psi in all three cases. $\Delta P_{matched}$ is the source volume pressure for which the leakage rate is $\dot{m}_{av}$.

To obtain $\Delta P_{matched}$ = 55, 30, or 10 psig, ΔP_{mid} may be increased slightly. The corresponding increased values of ΔP_o and ΔP_f are shown in Table A-5 for δP = 5 psi. $\dot{m}_{av}$ is equal to $\dot{m}$ at $\Delta P_{nominal}$ with an error of less than one percent in all cases (1.2% in Case 1 for $\Delta P_{nominal}$ = 10 psig).

Table A-5. ΔP_o and ΔP_f for $\dot{m}_{av}$ to equal leakage at $\Delta P_{nominal}$

$\Delta P_{nominal}$ (psig)	ΔP_o (psig)	ΔP_f (psig)	$\dot{m}_{av}/\dot{m}$ at $\Delta P_{nominal}$ Case 1	Case 2	Case 3
55	60.2	50.2	1.001	1.001	1.000
30	35.3	25.3	1.002	1.001	0.999
10	15.8	5.8	1.012	0.998	0.992

APPENDIX B

Valve Flow Resistance

In the Maine Yankee tests using a valve with a micrometer handle, the flow resistance may be regarded as a viscous flow resistance in series with an orifice-like resistance. For an incompressible fluid, it is unimportant which resistance is upstream. For a gas, the relation between flow rate and pressure drop depends upon which resistance is upstream. A correlation is developed here on the assumption that the viscous resistance is upstream.

For water, the pressure drop, ΔP psi, is related to the flow rate, F cc/min, as follows:

$$\Delta P = R(B\mu F + F^2) \qquad \text{(B-1)}$$

where the viscosity, μ, affects the viscous resistance term but not the orifice-like quadratic term. The effect of temperature is attributed entirely to its effect on viscosity, because the density of water varies only very slightly with temperature.

For ΔP = 55 psi, the water flow data of Reference 2 are shown in Table B-1. Examination of the ratio of flow measured at 120°F and 80°F, as well as the individual readings, suggests that two readings at 120°F are inconsistent with the rest of the data. Replacement values for these two readings are indicated in Table B-1.

For a given P, since R does not depend on temperature, Equ (B-1) gives

$$B = \frac{F_{120}^2 - F_{80}^2}{\mu_{80}F_{80} - \mu_{120}F_{120}} \qquad \text{(B-2)}$$

where the subscripts 80 and 120 are temperatures (°F). Using the data

Table B-1. Water Flow Data for ΔP = 55 psig

Leakage Setting	Flow (cc/min) 80°F	120°F	Flow Ratio (120°F/80°F)
10	6.511	7.722	1.186
12	11.163	12.121*	1.086
14	16.300	20.033	1.229
16	22.264	26.667	1.198
18	27.522	32.609	1.185
20	33.428	38.961	1.166
22	39.872	42.283**	1.060

*Replace by 13.202
**Replace by 47.22

These replacements make flow ratio the same as flow ratio for Leakage Setting = 10.

in Table B-1 (with the replacement values indicated there), Table B-2 shows the values of B and R calculated from Equations (B-1) and (B-2). The values used for water viscosity are (in units of lb/hr-ft) 2.08 at 80°F and 1.36 at 120°F.

Table B-2. Values of B and R

Leakage Setting	B $\left[\frac{\text{hr-ft-cc}}{\text{lb-min}}\right]$	R $\left[\frac{\text{psi-min}^2}{\text{cc}^2}\right]$
10	5.668	0.462
12	9.437	0.160
14	20.268	0.0575
16	21.454	0.0369
18	23.717	0.0260
20	24.211	0.0196
22	34.195	0.0124

Table B-3 shows the water leakage data of Reference 2 at ΔP = 10 and 30 psig. Temperature is 80°F in both cases. Table B-3 also shows the flow rates calculated by means of Equation (B-1), using the values of B and R given in Table B-2.

Table B-3. Comparison of Measured and Calculated Water Flows for ΔP = 10 and 30 psig. T = 80°F

Leakage Setting	F_{calc}(cc/min)		F_{meas}(cc/min)		$\frac{F_{calc}-F_{meas}}{F_{meas}} \times 100\%$	
	ΔP=10 (psig)	30	10	30	10	30
10	1.615	4.090	-	3.590*	-	1.7
12	2.788	7.032	2.438	6.586	14.4	6.8
14	3.786	10.003	-	10.336	-	-3.2
16	5.416	13.893	5.310	13.100	2.0	6.1
18	6.846	17.313	7.092	16.997	-3.5	1.9
20	8.647	21.346	8.942	21.239	-3.3	0.5
22	9.948	25.134	10.811	24.691	-8.0	1.8

*Replace by 4.020, based on F_{av}/F_{10} being the same for 30 and 55 psig, where F_{10} is F_{meas} for Leakage Setting = 10, and F_{av} is average of F_{12}, F_{14}, F_{16}, F_{18}, F_{20}, and F_{22}.

The agreement between calculated and measured flow rates is fairly good. Of the twelve comparisons, eight show differences of 3.3 percent or less. Consequently; Equation (B-1) is fairly well verified. The ratio of viscous pressure drop to orifice-like pressure drop is B_{μ}/F. Table B-4 shows values of this ratio. Evidently the viscous pressure drop is predominant for all the 80°F tests, and is about equal to the orifice-like pressure drop at 55 psig, 120°F.

Table B-4. B_{μ}/F for Various Values of ΔP and T for Water

Leakage Setting	B_{μ}/F_{meas} at T=80°F			B_{μ}/F_{meas} at T=120°F
	ΔP=10	30	55	55
10	-	2.9*	1.8	1.0
12	8.1	3.0	1.8	1.0*
14	-	4.1	2.6	1.4
16	8.4	3.2	2.0	1.1
18	7.0	2.9	1.8	1.0
20	5.6	2.4	1.5	0.8
22	6.6	2.9	1.8	1.0*

*Replacement value of F_{meas} used.

Now consider air flow from source pressure P_s psia through a capillary-like viscous resistance to an intermediate pressure P_1 psia, and then through an orifice-like resistance to atmospheric pressure, P_a psia. For air, the mass flow per unit orifice area is ρv, where ρ is the density of air in the orifice and v is velocity in the orifice. ρv is given by

$$(\rho v)^2 = \frac{2\gamma g}{\gamma - 1} \frac{P_a^2}{R_o T} \left(\frac{P_1}{P_a} - 1 \right) G \tag{B-3}$$

where $\gamma = 1.4$ is the specific heat ratio for air, $g = 32.2$ ft/sec^2 is the acceleration of gravity, T is source (upstream, at P_1) temperature (°R), P is absolute pressure (psf), $R_o = 53.26$ ft-lb/lb°F is the gas constant for air and G is given by

$$G = \left(\frac{P_e}{P_a} \right)^2 \frac{x^{\frac{\gamma-1}{\gamma}} \left(x^{\frac{\gamma-1}{\gamma}} - 1 \right)}{\left(\frac{P_1}{P_a} - 1 \right)} \tag{B-4}$$

$$x = \frac{P_1}{P_e}$$

$P_e = P_a$ for subsonic flow

$P_e = 0.5283\ P_1$ for choked flow

Choked flow occurs when

$$\frac{P_a}{P_1} \leq \left[\frac{\gamma+1}{2} \right]^{-\frac{\gamma}{\gamma-1}} = 0.5283$$

$\sqrt{G}$ is proportional to $\rho v / \sqrt{P_1 - P_a}$. Values of $\sqrt{G}$ are listed in Table B-5. $\sqrt{G_o}$, the limiting value of $\sqrt{G}$ for small $(P_1 - P_a)$, is $\sqrt{(\gamma-1)/\gamma} = 0.5345$.

Table B-5. $\sqrt{G}$ for Various Values of $P_1 - P_a$ for Orifice. (P_a taken = 15 psia.)

$P_1 - P_a$ (psi)	$\sqrt{G}$	$\sqrt{G} / \sqrt{G_o}$
0.01	0.5345	1.000
1	0.5332	0.998
5	0.5282	0.988
13.3	0.5185	0.970
13.4*	0.5184	0.970
15 *	0.5176	0.968
20 *	0.5230	0.978
25 *	0.5346	1.000
30 *	0.5490	1.027
35 *	0.5648	1.057
40 *	0.5811	1.087
45 *	0.5977	1.118
50 *	0.6143	1.149
55 *	0.6307	1.180
60 *	0.6470	1.210

* Choked flow

Table B-5 shows that for $P_1 \lesssim 50$ psig (i.e. $P_1 - P_a \lesssim 50$ psi), ρv is proportional to $\sqrt{P_1 (\text{psig})}$ to within 15 percent. Accordingly, the approximation will be made that the mass flow rate, $\dot{m}$, is proportional to $\sqrt{P_1 - P_a}$. (In Table B-7, below, all P_1 values are < 50 psig.)

For the capillary-like flow, the mass flow rate is

$$\dot{m} = \frac{C}{T\mu} \left(P_s^{\ 2} - P_1^{\ 2} \right) \qquad \text{(B-5)}$$

where C is a constant that depends only on the geometry, and P_s and P_1 are absolute pressures. Since capillary-like flow is isothermal for an ideal gas, T is the temperature at P_s as well as at P_1. For the two resistances in series, $\dot{m}$ obeys Equation (B-5) and also

$$\dot{m} = D\sqrt{P_1 - P_a} \tag{B-6}$$

P_1 may be found by eliminating $\dot{m}$ between these two equations.

A relation between the coefficients C and D for air flow and the coefficients R and B for water flow can be found by assuming $(P_s - P_a)$ to be small compared to P_a. The air density is then practically constant, so that the air flow is incompressible.

For water and air flow through the same orifice, neglecting the variation of orifice coefficient with Reynolds number, and assuming the same ΔP,

$$\frac{\dot{m}(\text{lb/hr air})}{F(\text{cc/min water})} = \frac{3600 \times 24\ \frac{\text{sec}}{\text{day}}}{(2.54 \times 12)^3 \times 60\ \frac{\text{cc}}{\text{ft}^3}\ \frac{\text{sec}}{\text{min}}} \sqrt{\rho_{air}\ \rho_{water}}$$

With $\rho_{water} = 62.4\ \text{lb/ft}^3$, and

$$\rho_{air} = \frac{15 \times 144\ \text{psf}}{R_o\ T}$$

this becomes (with R_o = 53.26 ft-lb/lb°F)

$$\frac{\dot{m}}{F} = \frac{2.5582}{\sqrt{T(°R)}}\ \frac{\text{lb/day}}{\text{cc/min}} \tag{B-7}$$

Comparison with Equations (B-1) and (B-6) leads to

$$D = \frac{2.5582}{\sqrt{RT}} \tag{B-8}$$

For P_s and P_1 both close to P_a, Equation (B-5) becomes

$$\dot{m} = \frac{2C\,P_a}{T\mu}\,\Delta P \qquad \text{(B-9)}$$

For water and air flow through the same capillary, a similar argument shows that

$$\frac{\dot{m}}{F} = \frac{2.0624}{T(^\circ R)}\,\frac{\mu_{water}}{\mu_{air}} \qquad \text{(B-10)}$$

Comparison with Equations (B-1) and (B-9) leads to

$$C = \frac{2.0624}{2RB\,P_a}$$

$$C = \frac{0.06875}{RB} \qquad \text{(B-11)}$$

Table B-6 shows the air leakage rate calculated from Equations (B-5), (B-6), (B-8), and (B-11). The values of B and R given in Table B-2 were used to calculate C and D. The viscosity of air is 0.0447 lb/hr-ft at 80°F and 0.0473 lb/hr-ft at 120°F.

Table B-6. Calculated Air Flow Rate, Based on Measured Water Flow Rate Data

Leakage Setting	C	D 80°F	D 120°F	$\dot{m}_{calc}$ (lb/day) T = 80°F, P_s = 55 psig	T = 80°F, 30	T = 80°F, 10	120°F, 55
10	.02625	.162	.156	1.114	0.752	0.305	1.064
12	.04553	.275	.266	1.894	1.281	0.524	1.817
14	.05899	.459	.443	3.089	2.030	0.759	2.951
16	.08684	.573	.553	3.79	2.625	1.042	3.749
18	.1115	.683	.659	4.69	3.173	1.292	4.498
20	.1449	.786	.759	5.45	3.726	1.578	5.232
22	.1621	.989	.954	6.80	4.598	1.875	6.514

Table B-7 shows the calculated values of P_1, the pressure at the junction between the upstream capillary-type viscous flow resistance

and the downstream orifice-like resistance. In addition, Table B-7 shows the percentages by which the calculated values of $\dot{m}$ shown in Table B-6 differ from the measured air leakage flow rates in Reference 2.

In view of the fact that the calculated values of $\dot{m}$ for air are based entirely on the measured values of F for water, the percent deviation from the measured values (E in Table B-7) are not very large. The deviations may be attributed to the incorrectness of the assumption, made in the foregoing discussion, that flow resistance is not affected by Reynolds number. The data in Table 1 of Reference 2 show an effect due to Reynolds number, however: For the air leak rate variation with temperature, except for Leakage Setting = 16 (for which the value $\dot{m}$ = 4.376 at 120°F is probably too low) all the air leak rates are higher at 120°F than at 80°F, in spite of the fact that the viscosity of air increases with temperature. In the absence of a Reynolds number effect, Equations (B-5) and (B-11) show that orifice mass flow resistance increases with temperature (i.e., for constant P_s and P_1, $\dot{m}$ decreases). Similarly, Equations (B-6) and (B-8) show that flow resistance for capillary-like viscous flow should also increase with temperature.

Table B-7. Junction Pressure and Percentage Deviations of Calculated Air Flow Rates

P_1 = Junction Pressure (psig)
$E = [(\dot{m}_{calc} - \dot{m}_{meas})/\dot{m}_{meas}] \times 100\%$

Leakage Setting	T = 80°F						T = 120°F	
	P_s = 10 psig		30		55 psig		55	
	P_1	E	P_1	E	P_1	E	P_1	E
10	3.6	-	21.5	4.6	47.3	-20.8	46.6	-28.4
12	3.6	11.0	21.7	8.7	47.4	-18.9	46.7	-28.0
14	2.7	-	19.6	13.9	45.3	- 5.9	44.4	-10.5
16	3.3	7.8	21.0	10.9	47.0	-13.9	46.0	-14.3
18	3.6	6.1	21.6	7.0	47.3	-12.1	46.6	-20.5
20	4.0	7.3	22.5	1.0	48.2	-15.4	47.5	-22.6
22	3.6	10.0	21.6	5.4	47.4	- 7.8	46.6	-17.6
$\frac{(P_1)_{av}\text{(psig)}}{P_s\text{ (psig)}}$	0.30		0.71		0.86		0.84	

Figure B-1, taken from Reference B-1, shows the effect of Reynolds number ($Re = vd/\nu$, where $\nu = \mu/\rho$) on the discharge coefficient, C_D, of an orifice. For very small Re, flow is laminar. Reference B-2 shows that for small Re, $\Delta P = 3\ q\mu/c^3$, where c = radius of orifice. From this it follows that C_D is proportional to $(Re)^{1/2}$ for small values of Re. For very large Re, the discharge coefficient has a limiting value of 0.61. For intermediate values of Re, C_D has a peak value that depends on geometrical factors, but as Figure B-1 shows, the peak can be substantially higher than 0.61. For somewhat higher Re than the value at which the peak occurs, C_D decreases fairly rapidly with increasing Re. Reference B-3 shows plots of K vs d/D for various values of Re. (K differs from C_D only by a geometrical factor.) By reading from these plots, one may

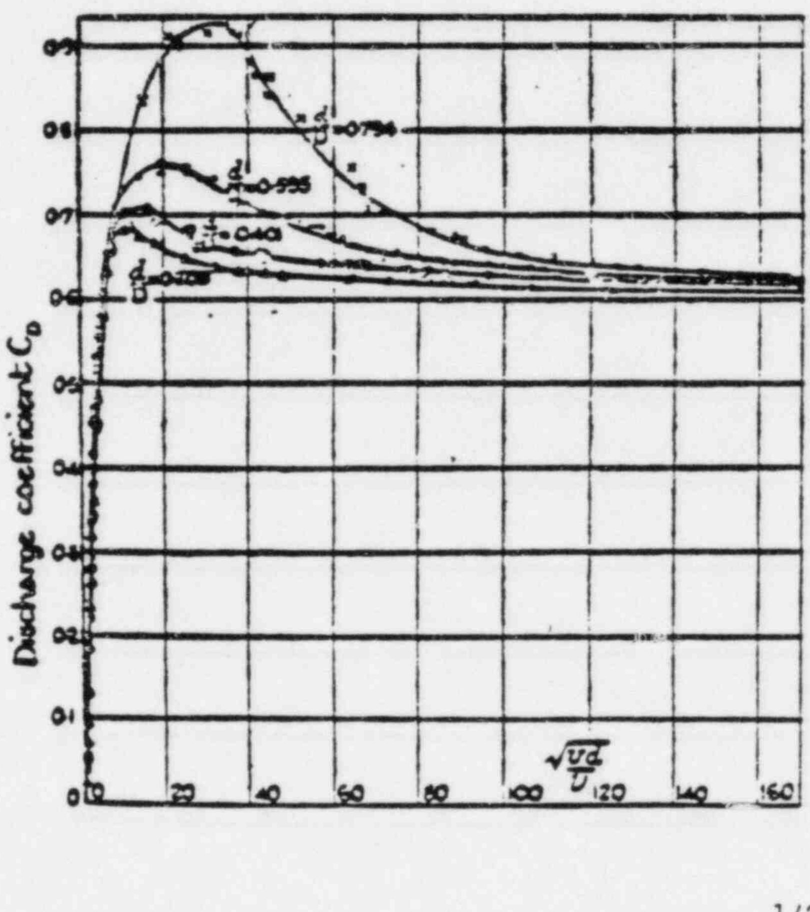

Figure B-1. Discharge Coefficient versus $(Re)^{1/2}$ for Pipe Orifices. d/D = Orifice to Diameter Ratio. From Reference B-1.

obtain plots of K vs $(Re)^{1/2}$. These are generally similar to the plots of C_D vs $(Re)^{1/2}$ in Figure B-1, showing similar peaking.

The anomalous increase in air leak rate with increasing temperature may be explained by the fact that the flow is primarily orifice-like for P_s = 55 psig (P_1/P_s is 0.86 or 0.84 in Table B- , indicating that most of the pressure drop is due to the orifice-like resistance) and by supposing that Re is in the range for which C_D decreases with increasing Re.

The values of $(P_1)_{av}/P_s$ in Table B-7 show that for P_s = 55 or 30 psig, the pressure drop (P_1-P_a) or P_1psig due to the orifice-like flow resistance predominates over that (P_s-P_1) due to the capillary-like viscous flow resistance, but for P_s = 10 psig, the reverse is true: The capillary-like flow resistance is strongly predominant. This shift in the relative importance of the orifice-like and capillary-like flow resistances occurs with a change in $\dot{m}$ by a factor of only about 2.5 between P_s = 10 and 30 psig.

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