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FEM-Based Random Vibration Analysis
of Nuclear Structures under Seismic Loading

by

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Figure Captions

1. Limit State Condition in the {v} Space

2. System Limit State Conditions

Summary

This paper outlines an analytical and numerical procedure developed for the frequency domain finite element analysis of category I nuclear structures, in particular, of reinforced concrete containment shell structures, subjected to earthquake ground acceleration. Such a procedure can be conveniently used in the analytical evaluation of the overall seismic safety of nuclear power plant structures.

Emphasized in this presentation, however, is the analytical procedure associated with the random vibration analysis. The earthquake ground acceleration is assumed to be a Gaussian vector process characterized by a zero mean and three-by-three cross-spectral density matrix of its three components. A SAP V finite shell element code is used to evaluate the natural frequencies and modes that are significantly participating. The nodal displacement vector $\{u\}$ is then expressed in terms of a (truncated) modal expansion involving only those significant modes. The frequency domain modal analysis then produces a cross-spectral density matrix of the generalized coordinate vector $\{q\}$, from which the corresponding covariance matrix can be easily derived. The Cholesky decomposition of the covariance matrix leads to the transformation of $\{q\} = \{q(t)\}$ into a vector $\{v\} = \{v(t)\}$ whose covariance matrix $[V_{vv}]$ is the identity matrix. Furthermore, the covariance matrix $[V_{v\dot{v}}]$ between $\{v\} = \{v(t)\}$ and $\{\dot{v}\} = \{\dot{v}(t)\}$ and the covariance matrix $[V_{\dot{v}\dot{v}}]$ of $\{\dot{v}\} = \{\dot{v}(t)\}$ can also be easily derived from the cross-spectral density matrix of $\{q\}$. Since the stress vector $\{\tau\}$ is a function of $\{q\}$, the vector $\{\tau\}$ consisting of three membrane and three moment components can also be expressed in terms of $\{v\}$. Therefore, failure conditions (or limit state conditions) written in terms of the stress vector can in turn be written in terms of $\{v\}$ and these conditions form closed curves when plotted in the $\{v\}$ plane. The present paper then suggests methods of estimating the expected rate at which the vector process $\{v(t)\}$ outcrosses the failure condition for each element. Such an expected rate is then used to estimate the failure probability of each finite element. The problems that arise in estimating the failure probability of the containment structure as a whole are also discussed.

1. Introduction

The use of more sophisticated load and structural models is increasingly demanded in order to reflect the effects of load-structure interaction more realistically in structural reliability analysis, particularly when dealing with such risk-sensitive systems as nuclear power plant structures. To this end, loads are often idealized in terms of the random functions of temporal and/or spatial variables, and the structural responses are evaluated by means of advanced numerical techniques such as finite element methods. This paper outlines an analytical and numerical procedure developed for the frequency domain finite element analysis of category I nuclear structures, in particular, of reinforced concrete containment shell structures, subjected to earthquake ground acceleration. Such a procedure can be conveniently used in the analytical evaluation of the overall seismic safety of nuclear power plant structures.

Emphasized in this paper, however, is the analytical procedure associated with the random process analysis involving the earthquake ground acceleration which is assumed to be a Gaussian vector process characterized by a zero mean and three-by-three cross-spectral density matrix of its three components. A SAP V finite shell element code is used to evaluate the natural frequencies and modes that are significantly participating.

2. Modal Analysis

The equation of motion for an n-degrees-of-freedom system within the framework of a linear elastic analysis is written as

$$[M]\{\ddot{u}\} + [C]\{\dot{u}\} + [K]\{u\} = - [M][\hat{I}]\{\ddot{z}_g\} \tag{1}$$

where $[M]$, $[C]$ and $[K]$ are the nxn mass, damping and stiffness matrix, respectively, and $\{u\}$ is the nodal displacement, $[\hat{I}]$ is the modified identity matrix given by

$$[\hat{I}] = [\hat{I}_x \quad \hat{I}_y \quad \hat{I}_z] \tag{1a}$$

and the vector $\{\ddot{z}_g\}$ consists of the three elements representing the ground acceleration in the x, y and z directions, respectively;

$$\{\ddot{z}_g\} = [\ddot{z}_{gx} \quad \ddot{z}_{gy} \quad \ddot{z}_{gz}]^T \tag{2}$$

Under the assumption of the existence of normal modes, the modal displacement vector $\{u\}$ is expanded into those modes for which the modal participation is significant.

$$\{u\} = [\phi]\{q\} \tag{3}$$

where

$$[\phi] = [\phi_1 \quad \phi_2 \quad \cdots \quad \phi_m] = \text{truncated normalized modal matrix} \tag{4}$$

$$\{q\} = \text{generalized coordinates} = [q_1 \quad q_2 \quad \cdots \quad q_m]^T \tag{5}$$

$$\{\phi_j\} = \text{j-th normalized modal vector} = [\phi_{1j} \quad \phi_{2j} \quad \cdots \quad \phi_{nj}]^T \tag{6}$$

The modal vectors $\{\phi_1\}$, $\{\phi_2\}$, ..., $\{\phi_m\}$ do not necessarily represent the first m modes. Furthermore, they do not usually indicate a sequence of successive m modes, but rather indicate those significant modes, arranged in increasing order in terms of their corresponding frequencies.

Substituting eq. (3) into eq. (1) and premultiplying by $[\phi]^T$, one obtains

$$\{\ddot{q}\} + [\Delta]\{\dot{q}\} + [\Omega^2]\{q\} = - \{F_x\} \ddot{z}_{gx} - \{F_y\} \ddot{z}_{gy} - \{F_z\} \ddot{z}_{gz} \tag{7}$$

where $[\Delta]$ and $[\Omega^2]$ are the (mxm diagonal) modal damping and frequency matrices; $[\Delta]$ = diago-

nal $[2\zeta_1\omega_1, 2\zeta_2\omega_2, ..., 2\zeta_m\omega_m]$ and $[\Omega^2]$ = diagonal $[\omega_1^2, \omega_2^2, ..., \omega_m^2]$, respectively, and $\{F_x\}$, $\{F_y\}$ and $\{F_z\}$ are the modal participation vectors associated with $\ddot{z}_{gx}$, $\ddot{z}_{gy}$ and $\ddot{z}_{gz}$, respectively;

$$\{F_a\} = [\phi]^T[M]\{\hat{I}_a\} = [f_{1a} \quad f_{2a} \quad .... \quad f_{ma}]^T \qquad a=x,y,z \qquad (8)$$

The solution $\{q\}$ results from eq. (7) as

$$\{q\} = -\sum_a \int_{-\infty}^{t} \ddot{z}_{ga}(\tau) [h(t-\tau)] \{F_a\} d\tau \qquad (9)$$

In eq. (9), the summation is with respect to a = x,y and z and $[h(t)]$ is the modal impulse response function matrix = diagonal $[h_1(t), h_2(t), ..., h_m(t)]$, where $h_j(t) = (1/\omega_j')$ × $\exp(-\zeta_j\omega_j t)\sin\omega_j' t \, H(t)$ with ω_j' = the damped modal frequency and $H(t)$ = the Heaviside unit step function.

The stress vector for element (e), $\{\tau^{(e)}\}$, consisting of p components can then be written as

$$\{\tau^{(e)}\} = [B^{(e)}] \{u^{(e)}\} = [B^{(e)}] [\phi^{(e)}] \{q\} \qquad (10)$$

where $[B^{(e)}]$ is the pxn' matrix that converts the nodal displacement vector $\{u^{(e)}\}$ of element (e) consisting of n' components into the stress vector and $[\phi^{(e)}]$ represents the n'xm matrix obtained from $[\phi]$ deleting all the rows except for those corresponding to the n' rows associated with $\{u^{(e)}\}$ so that

$$\{u^{(e)}\} = [\phi^{(e)}] \{q\} \qquad (11)$$

3. Random Vibration Analysis

If the ground acceleration vector $\{\ddot{z}_g\}$ is a zero-mean Gaussian random vector, so is the generalized coordinate vector $\{q\}$ by virtue of eq. (9). The density function of $\{q\}$ is given by

$$f_{\{q\}}(\{q\}) = \frac{1}{(2\pi)^{m/2} |[V_{qq}]|^{\frac{1}{2}}} \exp \left[-\tfrac{1}{2}\{q\}^T[V_{qq}]^{-1}\{q\}\right] \qquad (12)$$

where $[V_{qq}]$ is the covariance matrix of $\{q\}$. Since the covariance matrix is positive definite, it can be written in the following form by means of the Cholesky decomposition;

$$[V_{qq}] = [L_q][L_q]^T \qquad \text{or} \qquad [V_{qq}]^{-1} = ([L_q]^T)^{-1} [L_q]^{-1} \qquad (13)$$

in which $[L_q]$ is the lower triangular matrix. With the aid of the matrix $[L_q]$, the generalized coordinate vector $\{q\}$ is transformed into $\{v\}$ so that

$$\{v\} = [L_q]^{-1}\{q\} \qquad \text{or} \qquad \{q\} = [L_q]\{v\} \qquad (14)$$

The density function of $f_{\{v\}}(\{v\})$ of the transformed generalized coordinate vector $\{v\}$ is then given by

$$f_{\{v\}}(\{v\}) = \frac{1}{(2\pi)^{m/2}} \exp \left[-\tfrac{1}{2}\{v\}^T\{v\}\right] \qquad (15)$$

Eq. 15 suggests that the components $v_i(t)$ of $\{v\}$ are N(0,1) or a Gaussian variate with mean zero and unit variance and are independent of $v_j(t)$ (i≠j), and that the expected value vectors E{q} and E{v} of $\{q\}$ and $\{v\}$ are zero. Under the further assumption that $\{\ddot{z}_g\}$ and hence $\{v\}$ are stationary, the cross-correlation matrix $[R_{qq}(t_0)]$ of $\{q\}$ is given by $[R_{qq}(t_0)]$ = $E[\{q(t)\}\{q(t+t_0)\}^T]$. It is well known that the Wiener-Khintchine (W-K) transform of $[R_{qq}(t_0)]$ is the cross-spectral density matrix $[S_{qq}(\omega)]$ of $\{q(t)\}$. Hence, the following W-K

transform pair is obtained;

$$[S_{qq}(\omega)] = \frac{1}{2\pi} \int_{-\infty}^{\infty} [R_{qq}(t_0)] e^{-i\omega t_0} dt_0; \qquad [R_{qq}(t_0)] = \int_{-\infty}^{\infty} [S_{qq}(\omega)] e^{i\omega t_0} d\omega \qquad (16)$$

The cross-correlation matrix $[R_{vv}(t_0)]$ and the cross-spectral density matrix $[S_{vv}(\omega)]$ can be written in terms of $[R_{qq}(t_0)]$ and $[S_{qq}(\omega)]$ respectively;

$$[R_{vv}(t_0)] = [L_q]^{-1}[R_{qq}(t_0)]([L_q]^{-1})^T; \qquad [S_{vv}(\omega)] = [L_q]^{-1}[S_{qq}(\omega)]([L_q]^{-1})^T \qquad (17)$$

These quantities are also related through the W-K transform pair.

With the aid of eq. (16) and from the stochastic process theory, one can show that

$$[V_{qq}] = \int_{-\infty}^{\infty} [S_{qq}(\omega)]d\omega; \qquad [V_{\ddot{q}\ddot{q}}] = \int_{-\infty}^{\infty} \omega^2[S_{qq}(\omega)]d\omega; \qquad [V_{q\dot{q}}] = \int_{-\infty}^{\infty} i\omega[S_{qq}(\omega)]d\omega \qquad (18)$$

For the evaluation of these covariances, the following expression for $[S_{qq}(\omega)]$ which results from eq. (7) must be used.

$$[S_{qq}(\omega)] = [H(\omega)][\phi]^T[M][\hat{i}][S_{gg}(\omega)][\hat{i}]^T[M]^T[\phi][H^*(\omega)] \qquad (19)$$

where $[H(\omega)]$ is the modal frequency response function matrix = diagonal $[H_1(\omega), H_2(\omega), \ldots, H_m(\omega)]$ with $H_j(\omega) = 1/\{(\omega_j^2 - \omega^2) + 2i\zeta_j\omega_j\omega\}$ and $[H^*(\omega)]$ indicating the complex conjugate of $[H(\omega)]$. In eq. (19), the matrix $[S_{gg}(\omega)]$ represents the cross-spectral density function matrix of $\{\ddot{z}_g\}$ or

$$[S_{gg}(\omega)] = \begin{bmatrix} S_{ggxx}(\omega) & S_{ggxy}(\omega) & S_{ggxz}(\omega) \\ S_{ggyx}(\omega) & S_{ggyy}(\omega) & S_{ggyz}(\omega) \\ S_{ggzx}(\omega) & S_{ggzy}(\omega) & S_{ggzz}(\omega) \end{bmatrix} \qquad (20)$$

The covariance matrices associated with $\{v\}$ and $\{\dot{v}\}$ can be derived with the aid of those associated with $\{q\}$ and $\{\dot{q}\}$;

$$[V_{vv}] = [L_q]^{-1}[V_{qq}]([L_q]^{-1})^T = [I_m] = m \times m \text{ identity matrix} \qquad (21)$$

$$[V_{\ddot{v}\ddot{v}}] = [L_q]^{-1}[V_{\ddot{q}\ddot{q}}]([L_q]^{-1})^T; \qquad [V_{v\dot{v}}] = [L_q]^{-1}[V_{q\dot{q}}]([L_q]^{-1})^T \qquad (22)$$

With the aid of eqs. (10) and (14), the stress vector for element (e) can be written in terms of $\{v\}$ as

$$\{\tau^{(e)}\} = [B^{(e)}][\phi^{(e)}]\{q\} = [B^{(e)}][\phi^{(e)}][L_q]\{v\} \qquad (23)$$

This expression for the element stress vector plays an important role in evaluating the element limit state probability and eventually the system limit state probability (the limit state probability for the entire structure). The expression is essential for the purpose of evaluating the latter in particular, since it expresses the element stress in terms of $\{q\}$ which is common to all the finite elements within the structure and hence is the source of the statistical dependence between the stress vectors in the different finite elements; such a statistical dependence must be known for the evaluation of the system limit state probability.

In concluding this section, it is pointed out that, in view of the positive definiteness of $[V_{vv}]$ and $[V_{\ddot{v}\ddot{v}}]$, a linear transformation $\{w\} = [\bar{L}]\{q\}$ exists such that $[V_{ww}] = [I_m]$ and at the same time $[V_{\ddot{w}\ddot{w}}]$ becomes diagonal. Such a transformation can of course be used in the present study. The advantage of doing so is not overwhelming, however, since one still has to deal with $[V_{w\dot{w}}]$ which in general cannot be diagonalized simultaneously.

4. Limit State Probabilities

In this study, it is assumed that the system limit state is defined in terms of the state of stress in such a way that the limit state is reached if the following inequation is satisfied anywhere in the structure;

$$g(\{\tau\}) \leq 0 \tag{24}$$

where $g(\{\tau\})$ is an abbreviated form indicating a function of the stress components. The general nature of the function $g(\{\tau\})$ is such that the so-called limit state equation $g(\{\tau\}) = 0$ usually represents a closed (hyper-) surface known as the limit state surface and that $g(\{\tau\}) < 0$ indicates the domain outside the limit state surface. As a possible form of $g(\cdot)$, consider the limit state condition specified by the domain in which at least one of the following set of k inequalities, each representing a hyper-plane, is satisfied;

$$g_j(\{\tau\}) = R_j - \{A_j\}^T\{\tau\} \leq 0 \qquad (j=1,2,\ldots,k) \tag{25}$$

where $\{A_j\}$ are constant vectors. The corresponding limit state equation represents a closed hyper-surface constructed out of appropriate parts of the k hyper-planes in eq. (25) so as to enclose the smallest hyper-volume D_τ containing the origin $\{\tau\} = \{0\}$ in the stress component space. For example, if eq. (25) involves only two components, say τ_1 and τ_2, of the stress vector for all values of j, then the corresponding limit state equation represents the smallest closed area that can be constructed out of the k straight lines $R_j - \{A_j\}^T\{\tau\} = 0$ (j=1, 2,...,k). It is pointed out that the hyper-polyhedral limit state surface associated with eq. (25) can be used with relative ease to approximate a limit state surface of any shape either by inscribing or by circumscribing the surface.

Rewriting eq. (25) for element (e),

$$R_j^{(e)} - \{A_j^{(e)}\}^T\{\tau^{(e)}\} \leq 0 \qquad (j=1,2,\ldots,k) \tag{26}$$

and substituting eq. (23) into eq. (26), one obtains

$$R_j^{(e)} - \{\overline{A}_j^{(e)}\}^T\{v\} \leq 0 \qquad (j=1,2,\ldots,k) \tag{27}$$

where

$$\{\overline{A}_j^{(e)}\}^T = \{A_j^{(e)}\}^T [B^{(e)}][\phi^{(e)}][\Sigma_q] \tag{28}$$

If the equality is considered in eq. (28), the equation represents a hyper-plane $H_j^{(e)}$ in the $\{v\}$ space. Note that $\{v\}$ is the transformed generalized coordinate vector whose dimension is equal to the number of significant modes used in the structural analysis and is in general not very large. The transformation of eq. (26) into eq. (27) involves a linear transformation and therefore a hyper-plane in the $\{\tau\}$ space remains a hyper-plane in the $\{v\}$ space. Of crucial importance, however, is the fact that, with eq. (27), the limit state conditions for all the finite elements are specified in the same $\{v\}$ space. Recall the hyper-volume D_τ defined in the preceding paragraph. If $\{v\}$ consists of two components v_1 and v_2, the domain D_τ is mapped into the domain D of quadrilateral shape in the two-dimensional $\{v\}$ space as shown in Fig. 1, assuming that k = 4. The limit state surface in the $\{\tau\}$ space is also transformed into the corresponding limit state surface F_D in the $\{v\}$ space through the transformation.

One can rewrite eq. (27) in the following form.

$$\{n_j^{(e)}\}^T\{v\} = r_j^{(e)} \tag{29}$$

where $\{n_j^{(e)}\}$ is the unit outward vector normal to $H_j^{(e)}$;

$$\{n_j^{(e)}\} = \frac{1}{|\overline{A}_j^{(e)}|} \; \{\overline{A}_j^{(e)}\} \tag{30}$$

and $r_j^{(e)}$ is the shortest distance between the origin $\{v\} = \{0\}$ and $H_j^{(e)}$;

$$r_j^{(e)} = \frac{1}{|\overline{A}_j^{(e)}|} \; R_j^{(e)} \tag{31}$$

It can be shown (Ditlevsen [1], Veneziano, et al [2], Kako, et al [3] and Shinozuka [4]) that the rate $v_D^{(e)}$ for the vector process $\{v\}$ to out-cross the limit state surface $F_D^{(e)}$ associated with element (e) is approximated by

$$v_D^{(e)} \doteq \frac{1}{2\pi} \sum_{j=1}^{k} \sigma_{nnj}^{(e)} \; e^{-\frac{(r_j^{(e)})^2}{2}} \tag{32}$$

in which $\sigma_{nnj}^{(e)}$ is given by

$$\sigma_{nnj}^{(e)} = \sqrt{\sum_{a=1}^{m} \sum_{b=1}^{m} n_{aj}^{(e)} \; n_{bj}^{(e)} \; E[\dot{v}_a \dot{v}_b]} \tag{33}$$

In eq. (33), $n_{aj}^{(e)}$ and $n_{bj}^{(e)}$ are, respectively, the a and b components of $\{n_j^{(e)}\}$. Usually, the summation in eq. (32) is dominated by the one term associated with the hyper-plane with the shortest distance $r_{min}^{(e)}$ ($r_1^{(e)}$ in Fig. 1) to the origin. Furthermore, if one considers the crossing rate out of a hyper-sphere with a center at the origin and with a radius $r_{min}^{(e)}$ ($r_1^{(e)}$ in Fig. 1), this crossing rate may be used as an upper bound for $v_D^{(e)}$. The crossing rate can be shown to be (Kako, et al [3])

$$v_D^{(e)} \leq (r_{min}^{(e)})^{m-1} \sqrt{\frac{1}{n} \sum_{i=1}^{m} \kappa_{ii}^{(e)}} \; e^{-\frac{1}{2}(r_{min}^{(e)})^2} / \{\sqrt{2\pi} \; 2^{\frac{m}{2}-1} \Gamma(\frac{m}{2})\} \tag{34}$$

in which

$$\kappa_{ii}^{(e)} = E[\dot{v}_i^2(t)] - \sum_{j=1}^{m} \{E[v_j(t)\dot{v}_i(t)]\}^2 \tag{35}$$

Note that $E[\dot{v}_i(t)]$ and $E[v_j(t)\dot{v}_i(t)]$ in eq. (35) are the i-i and j-i components of the covariance matrices $[V_{\dot{v}\dot{v}}]$ and $[V_{v\dot{v}}]$, respectively, as described in eq. (22).

As indicated earlier, the reliability of a structure as a whole is defined by the probability that the limit state surface is out-crossed somewhere in the structure during the structure's expected service life. In terms of a finite element analysis, this implies that the system reliability is equal to the probability that the out-crossing will occur within at least one finite element among all the elements of the structure. For ease of discussion, consider a structure which is divided into four finite elements (a), (b), (c) and (d) and assume that $\{v\}$ is comprised of two elements. Fig. 2 schematically shows such a case in which a limit state surface of quadrilateral shape is indicated for each of these four elements. Furthermore, Fig. 2 indicates that element (a) is most severely stressed, element (b) to a lesser degree, element (c) to an even lesser degree and element (d) to the least. The limit state condition for the structure as a whole is then defined as the probability that $\{v\}$ will out-cross the system limit surface S which defines the shaded domain D_S. In the domain D_S^* complementary to D_S, the limit state condition is reached in at least one of the finite elements. Using the approximation given in eq. (32) with $r_1^{(a)}$ denoting the distance between the origin and the closest hyper-plane $H_1^{(a)}$ and also using eq. (34), one obtains

$$\nu_{D_S} \doteq \frac{1}{2\pi} \sigma_{nn1}^{(a)} e^{-\frac{(r_1^{(a)})^2}{2}} < (r_1^{(a)})^{m-1} \sqrt{\frac{1}{m} \sum_{i=1}^{m} \kappa_{ii}^{(a)}} \, e^{-\frac{1}{2}(r_1^{(a)})^2} / \{\sqrt{2\pi} \, 2^{\frac{m}{2}-1} \Gamma(\frac{n}{2})\} \quad (36)$$

Let $D*^{(e)}$ and $D^{(e)}$ denote the domains where the limit state condition is respectively satisfied and not satisfied for element (e) in the {v} space. Also, let $r_1^{(a)}$ denote the shortest distance between the origin {v} = {0} and the system limit state surface. Then, eq. (36) is valid for the general case in which the structure consists of an arbitrary number of finite elements. The system limit state surface is here defined as the interface between $D_S = \cap D^{(e)}_e$ and D_S^*.

Referring to Fig. 2, the following observation is in order; First, assume that the system limit state is reached when the states of stress in elements (a), (b) and (c) simultaneously reach the limit state defined in eq. (26). Then, the approximation and upper bound for the ν_{D_S} associated with this system limit state are obtained respectively from eq. (36) by replacing $r_1^{(a)}$ with $r_1^{(c)}$ (see Fig. 2). Obviously, this results in an out-crossing rate ν_{D_S} smaller than that for the system limit state defined earlier in a more conservative sense. The implication of this observation is that, in general, not only the system limit state can be defined in terms of the state of structural response in which a specific set of finite elements will reach the limit state simultaneously thus producing a mode of structural failure, but also the out-crossing rate associated with such a system limit state can be estimated.

Finally, if the earthquake occurs in accordance with a Poisson process with an arrival rate λ and if its duration is μ_D each time it occurs, then the structural reliability L(T) can be evaluated in approximation as

$$L(T) = \exp(-\lambda T \nu_{D_S} \mu_D) \quad (37)$$

where T is the expected service life of the structure. Since $\nu_{D_S}\mu_D + F_0$ is an upper bound for {v} to out-cross the system limit state surface S at least once during the duration μ_D as shown by Shinozuka [5] and Shinozuka and Yao [6], one has the following lower bound for L(T);

$$L(T) \geq 1 - \lambda T(\nu_{D_S}\mu_D + F_0) \quad (38)$$

where F_0 is the probability that the structure is initially in the limit state.

5. Conclusion

A method has been developed for the estimation of the structural reliability when a structure is subjected to loads that can be idealized in terms of a Gaussian random vector process. An earthquake ground motion is taken as a typical example of such a load. The limit state condition considered in this study is that which pertains to a hyper-polyhedral limit state surface. The finite element method has been used for the structural response evaluation within the framework of the modal analysis. Also, observations have been made as to how the probabilities of various modes of structural failure can be estimated in conjunction with the finite element analysis. With the aid of the method developed above, a reliability analysis has indeed been performed on a reinforced concrete containment structure. The results are presented in companion papers by Shinozuka, et al [7], Shinozuka, et al [8] and Chang, et al [9].

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Fig. 1

Fig. 2