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A Varying Elasticity Model of Electricity Demand with Given Appliance Saturation

Wen S. Chern
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Performed for the
U.S. Nuclear Regulatory Commission
Under Interagency Agreement DOE 40-550-75

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A VARYING ELASTICITY MODEL OF ELECTRICITY DEMAND
WITH GIVEN APPLIANCE SATURATION

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ABSTRACT

This report presents the third version of the State-Level Electricity Demand (SLED) Model developed at the Oak Ridge National Laboratory for the Nuclear Regulatory Commission. Specific improvements over previous versions of the SLED model are as follows. (1) A theoretical framework for estimating electric appliance choices and utilization of those appliances at the aggregate level is developed. These refinements enable the model to capture the detailed underlying behavior of electricity consumers and to deal with the effect of market penetration of energy saving technologies on electricity demand. (2) The linkage between average price and marginal price is instituted. Thus the model as estimated can be interpreted using either average or marginal price. The marginal price elasticities are derived from the average price elasticities and presented in the report. (3) Important determinants of price elasticities have been identified and the elasticities of demand are specified to be variable, rather than constant, among states in a region as well as over time. The formulation of variable elasticities permits the estimation of demand coefficients for a wide range of circumstances, such as in utility service areas.

The structural coefficients are estimated by nonlinear three-stage least squares, using annual state data for 1955-1976. Regression results show that the variation of demand elasticities is indeed explainable in the model. For example, the price elasticity of residential demand for electricity is dependent upon the levels of price and income, and the saturation levels of major electric appliances. These results imply that each end-use of electricity has distinctive impacts on price elasticity. The comparative analysis reveals that space heating and air conditioning have greater effect on price elasticities in absolute value than water heating and clothes drying. The estimated demand elasticities also show substantial variation among states. Although differences exist between average price and marginal price elasticities, these differences are in general, not dramatic.

I. INTRODUCTION

Energy demand forecasts are critical inputs to decisions on capacity planning by electric utilities. The recent study of the Energy Modeling Forum on "Electric Load Forecasting" pointed out:

"Forecasts of peak load (kilowatts) and electricity consumption (kilowatt hours) are the starting points in the electric utility planning cycle. As the lead times required to add new generation capacity have lengthened, the costs of new capacity have risen, the importance of forecasting has increased substantially. At the same time, the growth of electricity consumption has broken with past trends, and the uncertainty of forecasting has widened."¹

In the public sector, energy demand forecasts are also critically needed by regulatory agencies such as the Nuclear Regulatory Commission (NRC), the Economic Regulatory Administration, and state energy commissions as bases for making public policies which affect the utility's capacity expansion.

In the case of NRC, the agency has a responsibility for licensing proposed nuclear power plants. As required by the National Environmental Policy Act of 1969, NRC prepares an environmental impact statement before a decision can be made on whether or not to grant the license for either construction or operation of nuclear power plants. One important area in the environmental impact analysis addresses the need for power issue. Specifically, Chapter 8 of the NRC statement is entitled "The Need for the Plant." According to *Environmental Standard Review Plans* published by the NRC in May 1979, this section of the statement includes a detailed analysis and evaluation of the applicant's treatment of these projections, and an independent assessment of forecasts of the service-area growth, electricity consumption, and peak load demand.

The NRC needs a sound analytical modeling tool for its independent assessment because of the growth sophistication that applicants employ in making their forecasts, and also because of the more frequent contentions from intervenors who question the validity of the forecasts. In almost all cases, the need for power is the first issue to be examined. Unless there is a demonstrated need, the plant may simply not be licensed. Thus the NRC staff must defend their independent assessment in all cases. To

build the capability for conducting these independent assessments, the NRC has funded the Oak Ridge National Laboratory (ORNL) to develop various electricity demand forecasting models.

In order to address issues related to capacity expansion by utilities, electricity demand forecasts must have regional detail. The need for regional forecasts has stemmed from the fact that historical growth patterns in electricity demand vary widely from region to region, and this variation is expected to continue in the future. To accomplish the goal of providing detailed regional forecasts to NRC, ORNL first developed the state-level electricity demand (SLED) forecasting model. Version I and Version II of the SLED model were documented in Chern and Just,² and Chern et al.^{3,4} The SLED model forecasts state-level electricity demand in kWh and electricity prices by sector. Efforts were also made at ORNL to disaggregate the SLED model to utility service areas and to expand the forecasting capability to deal with the peak demand in kW and load distribution. The methodologies for these latter extensions are discussed elsewhere.^{5,6}

This study represents one of our continuing efforts to improve the SLED model. Specific improvements include the following. First, a theoretical framework for estimating electric appliance choices and the utilization of those appliances at the aggregate level is developed. One fundamental reason for decomposing the aggregate sectoral demand into a short-run model for estimating the utilization of electric durables and a long-run model for electric durable choice is because such an approach can more explicitly capture the detailed underlying behavior of electricity consumers. Also, during the sample period of 1955-1976, the saturation of some electricity intensive durables reached a very high level. These durables include refrigerators, televisions, clothes washers, and clothes dryers. In fact, the saturation levels for refrigerators and televisions had reached nearly 100% by the end of the sample period.

In some states, the list can be extended to include electric water heaters, air conditioners, and electric ranges. For example, in Texas, the saturation level of air conditioning increased rapidly from 12% in 1955 to 88% in 1976; thus, the potential for further increase is limited. If the future trend of appliance saturation levels is much different from

the historical trend, the standard Koyck-lag used in the Version I and Version II SLED model may not be appropriate for forecasting because it would tend to overestimate electricity demand when important electric appliance holdings reach saturation. A model which explicitly includes appliance saturations can, of course, appropriately reflect this reality.

Furthermore, apparent potential of new energy saving innovations such as solar heating technology must be considered. The penetration of these new technologies would, of course, affect the demand for electricity. The standard Koyck-type distributed lag model such as the one used in Version I and Version II of the SLED model cannot handle the effects of this potentially important penetration. A more sophisticated approach is proposed in this study to deal with the effect of the market penetration of energy saving technologies on electricity demand. This refinement should thus enhance the applicability of the model for not only forecasting but also evaluating policy issues. Note, however, that this refinement applies only to the residential sector, and to a lesser extent, the commercial sector. The standard lag model is retained for the industrial sector. Also, note that this study only reports the first part of this particular refinement effort which deals with estimation of electricity use given appliance saturation. A later study will deal with saturation forecasting.

The second area of improvement in this study deals with issues related to rate structure. Since there are strong possibilities of altering the declining block rate schedule currently used, one must question how a model using average electricity price should be interpreted when a different rate structure is instituted. In this study, the linkage between average price and marginal price is established. Thus the model as estimated can be interpreted using either average price or marginal price. Furthermore, the marginal price elasticities can be derived from the average price elasticities and both sets of elasticity estimates are presented in the report.

The third area of improvement deals with the issue of variable elasticity specification. As detailed in Sect. II, evidence obtained from previous research shows a wide variation of price elasticity among regions. Attempts are made in this study to identify important determinants of the

price elasticity of electricity demand. For example, the price elasticity for residential demand is specified as a function of electricity price, income, and the saturation levels of various electric appliances. The model thus allows the price elasticity to vary from state to state. Specific elasticities at the state level are computable from the estimated structural coefficients. This varying elasticity formulation permits the estimation of demand coefficients for a wide range of circumstances such as in utility service areas.

II. REVIEW OF THE SLED MODEL

For the previous versions of the State-Level Electricity Demand (SLED) model developed by Chern et al.,^{3,4} the basic structure of the model (ignoring additive disturbances) was

$$\ln x_t = \alpha_0 + \alpha_1 \ln x_{t-1} + \alpha_2 \ln p_t + \alpha_3 \ln y_t + \alpha_4 \ln z_t \quad (1)$$

$$p_t - c_t = \beta_0 + \beta_1 (x_t/n_t) + \beta_2 (x_t/n_t)^2 + \beta_3 r_t \quad (2)$$

where

x_t = quantity of electricity consumed (by residential, commercial, or industrial sectors),

p_t = average electricity price (by residential, commercial, or industrial sectors),

y_t = real per capita personal income (or value added in the case of the industrial sector),

z_t = other determinants of demand,

n_t = number of electricity customers (residential, commercial, or industrial),

c_t = average cost of generating and distributing electricity,

r_t = other determinants of price,

t = time period.

Assuming the same elasticities over the states within each of the 9 regions in the United States (dummy variables were used to represent shifts in constant terms from state to state), results of the earlier studies show that price and income elasticities of demand vary considerably among regions. For example, in the residential sector, the short-run price elasticity varies from $-.08$ in the Pacific region to $-.39$ in the West South Central region while long-run price elasticities vary from $-.38$ in the Mountain region to -1.15 in New England.⁷ Short-run income elasticities range between 0.0004 in the West North Central region to 0.462 in the Mountain region.

With this variation in elasticities among regions, several important questions arise: (1) If elasticities vary among regions, then surely they vary among states within regions and, if so, what are the effects of ignoring these differences both on the results of estimation and on forecasts and need-for-facility assessments which require local detail? (2) What explains the variation of elasticities among states and regions? (3) Are the factors which affect these variations in elasticities among regions likely to change within regions and thus become important in forecasting future electricity demand?

Although the earlier study was also concerned with these issues, data were not sufficient within individual states to estimate the model of Eqs. (1) and (2) with an acceptable degree of precision. The objective of the present study, on the other hand, is to specify more formally a non-constant elasticity model corresponding to Eqs. (1) and (2) which can address the above three questions.

Another issue which deserves further attention is the dichotomous nature of decisions to purchase electric durables as opposed to the decision of how intensively to use the existing stock of electric durables. The previous model outlined above represents a reduced form of demand for electricity which implicitly includes both of these underlying processes. This study, on the other hand, explicitly considers these two processes separately by focusing on the intensity-of-use decisions at the aggregate state level given the existing stock of appliances. A later study will then focus on the change in stock of electricity-using durables.

III. TREATMENT OF ELASTICITY VARIATION

Consider first the problem of capturing the variation in elasticities among states. A simple and straight forward approach in studying variation in elasticities is to specify a model of elasticity variation much like a price or demand equation and then consider the corresponding estimation problem. Suppose, for example, that the price elasticity of demand in Eq. (1) follows the equation

$$\alpha_{2i} = a_0 + a_1 \ln p_{ti} + a_2 \ln y_{ti} + a_3 \ln w_{ti} \quad (3)$$

where w_{ti} represents other exogenous factors which may or may not be represented in z_t above. Note that i subscripts have been added to denote states.

Since the elasticity parameters, α_{2i} , are neither known or directly observable, two approaches to estimation of Eq. (3) are possible. One approach is to estimate the model in Eq. (3) where the α_{2i} are replaced by estimates from Eq. (1). To exemplify this approach, assume for the moment that p_t can be treated exogenously in Eq. (1). Then under appropriate stochastic assumptions, Eq. (1) can be estimated by ordinary least squares in the random coefficients regression framework of C. R. Rao⁸ to find consistent but inefficient estimates of $\bar{\alpha}_{2i}$, defined by

$$\bar{\alpha}_{2i} = a_0 + a_1 \overline{\ln p_i} + a_2 \overline{\ln y_i} + a_3 \overline{\ln w_i}, \quad (4)$$

where bars denote sample means. Then the resulting estimates of $\bar{\alpha}_{2i}$ may be used to estimate a_0 , a_1 , a_2 , and a_3 utilizing Eq. (4). Since estimates of $\bar{\alpha}_{2i}$ are consistent, the resulting estimates of Eq. (4) can also be shown to be consistent under usual assumptions according to the methods of Hildreth and Houck,⁹ Just and Pope,¹⁰ and others.

An important problem with the above approach, aside from simultaneity of p_t and x_t , is that efficiency is lost in Eq. (4) since observations with non-perfect correlations are average together yielding fewer data points than are actually available. Of course, this loss of efficiency

would not occur if elasticities were constant over states within a region (at $\bar{\alpha}_{2i}$) and simply followed Eq. (4). A second loss of efficiency occurs because, with the above approach, Eqs. (1) and (4) are estimated independently when the disturbances are, in fact, correlated [because errors in estimates of $\bar{\alpha}_{2i}$ and thus disturbances in (4) depend on disturbances in Eq. (1)]. The corresponding possibilities for efficiency gain are somewhat similar to those of Zellner's seemingly unrelated regressions.

In this situation, efficient estimation of a pair of equations such as (1) and (3) can be handled most easily if the two equations are combined to produce estimates of both equations at once. With the model outlined above, this can be accomplished by substituting (3) into (1) to obtain the equation

$$\begin{aligned} \ln x_t = & \gamma_0 + \gamma_1 \ln x_{t-1} + \gamma_2 \ln p_t + \gamma_3 (\ln p_t)^2 \\ & + \gamma_4 (\ln p_t)(\ln y_t) + \gamma_5 (\ln p_t)(\ln w_t) \\ & + \gamma_6 \ln y_t + \gamma_7 \ln z_t \end{aligned} \quad (5)$$

where

$$\begin{aligned} \gamma_0 = \alpha_0, & \quad \gamma_1 = \alpha_1, & \quad \gamma_2 = \alpha_0 d_2, \\ \gamma_3 = \alpha_1 d_2, & \quad \gamma_4 = \alpha_2 d_2, & \quad \gamma_5 = \alpha_3 d_2, \\ \gamma_6 = \alpha_3, & \quad \gamma_7 = \alpha_4, \end{aligned}$$

and the i subscripts are again dropped for simplicity. Equation (5), however, possesses well known estimation properties in the case where p_t is exogenous; and when Eqs. (2) and (5) are considered together, one obtains a simple nonlinear simultaneous equations model for which well known estimation methods also exist. Furthermore, efficient estimation of Eq. (5) is a simple matter by ordinary least squares for the case where p_t is exogenous (under ordinary stochastic assumptions) and, hence, by analogy, a nonlinear simultaneous equations approach to estimation of (2) and (5) appears to gain more efficiency than nonlinear estimation of (1) and (2) with subsequent regressions according to (4).

Of course, Eq. (5) is simply a special case of a translog demand equation in which some quadratic log terms are ignored.¹¹ An abbreviated form, however, is necessary since the number of variables used in the previous study and considered further in this study is too extensive to make a complete translog form practical. Equation (5) thus serves to indicate which quadratic-log terms are important in explaining the elasticities of interest. All the terms involving $\ln p_t$, of course, contribute to explaining variation in the price elasticity of demand. Similarly, the terms involving $\ln y_t$ contribute to explaining variation in income elasticity.

The approach proposed above provides a basis for improving the earlier versions of the SLED model to allow variation of elasticities among states. However, for the residential and commercial sectors, the basic model structure, as discussed in the following section, is further refined to deal with the short-run usage and the long-run appliance choices separately. Therefore, the approach developed here will be combined with the basic structural equation developed in the following section to formulate the empirical model used for estimating residential and commercial demand equations. For example, the lag term in Eq. (5) will not appear in the short-run usage equations for these two sectors. For the industrial sector, however, Eq. (5) is the general formulation used in this study without further modification.

IV. THE UNDERLYING THEORETICAL MODEL OF CONSUMER ELECTRICITY DEMAND

This section develops the underlying theoretical model for constructing the econometric model of electricity demand by residential customers. Before proceeding to develop the theoretical model appropriate for this study, it is useful to review some fundamental issues related to energy demand modeling.

According to a recent study by Jerry A. Hausman,¹² "energy demand may be viewed usefully as part of a "household" production process in which the services of a long-lived consumer durable good are combined with

energy inputs to produce household services" (p. 33). In this context, consumer energy demand can be broken into two components where the consumer first chooses a durable which determines the amount of energy input required to produce a unit of the associated household service. Then after the durable is in place, the consumer determines the amount of energy consumption through his demand for the household service and the input-output efficiency of the durable. At each stage of the decision process, substitution and adjustment can take place. Of course, substitution possibilities may be greater with some services such as space heating and water heating than with such services as refrigeration. But generally substitution associated with the durable choice is longer run in nature than the durable use decision.

Again according to Hausman, "if an econometric model of energy demand is to be successful, it must allow for the different nature of the adjustment of the two components of household energy demand. Econometric models which do not differentiate the capital-stock decision from the utilization decision cannot capture the interplay of technological change and consumer choice in determining final energy demand" (p. 34).

Basically, the household production function approach referred to by Hausman departs from the assumption that consumers derive utility from goods and services purchased in the market place *per se*; instead utility is derived from commodities which are produced by the family with goods purchased in the market. For example, using a furnace and some natural gas, a family is able to produce the heat from which it derives utility on a cold winter day.

Note also that with household energy using durables, substitution among fuels is possible in an *ex ante* sense before a capital or durable good is purchased. However, the choice of durable generally determines the form of fuel to be used and further dictates a fixed input-output relationship. For example, once a particular type of space heating device is installed, the input-output ratio (BTU's per unit of energy used) is fixed.

Using the household production function approach together with the latter characteristics which are associated with putty-clay production,

the essential features of energy demand described by Hausman can be captured.

IV.A. A General Two Stage Formulation of Electricity Use Decisions

Following the above arguments, suppose a consumer (household) has utility function

$$U(q_0, q_1, \dots, q_J)$$

where q_j represents the quantity of household commodity j consumed. For simplicity, assume that commodities possibly produced using electricity within the household are separable from all others so that q_1, \dots, q_J can represent quantities of commodities which may embody household electricity while q_0 represents a composite of all other commodities. Following Gorman^{13,14} and Green¹⁵ (p. 22), the concept of separability that is employed throughout this study is that which validates a two (or more) stage budgeting procedure, i.e., weak separability together with any of the following: (1) only two groups of goods, (2) strong separability, (3) weak homogeneity, or (4) appropriate combinations of (2) and (3) within exhaustive sets of groups of goods. In the above case, weak separability is sufficient since only two groups of goods need be considered separable: q_0 and q_1, \dots, q_J . In some further cases below, however, q_0, \dots, q_J will be considered as $J+1$ separable groups.¹⁶

Next, following household production theory in the context of putty-clay technology, suppose consumers possess possibilities for substitution among energy sources in producing household commodities in an *ex ante* sense. However, once a durable is purchased, the energy source used in producing the associated household commodity is determined until the durable is replaced. Let short-run production functions associated with given appliance portfolios be represented by

$$q_j = q_j(i_j, x_i^j, z_j)$$

where i_j is an integer index indicating which durable is owned (or which energy source is used) in producing household commodity q_j ; x_i^j is the quantity of energy source i used in producing household commodity q_j ; and z_j is an exogenous variable which reflects household need for use of commodity q_j , such as cooling degree days, heating degree days, etc.

Finally, suppose the annualized fixed cost of ownership and maintenance associated with a durable which uses energy source i to produce household commodity q_j is represented by K_i^j , the price of energy source i is p_i , the price of the composite commodity is p_0 , and household disposable income is y . The household utility maximization problem is thus

$$\begin{aligned} & \max_{x_i^j, q_j, i_j} && U(q_0, q_1, \dots, q_J) \\ & i=1, \dots, I \\ & j=1, \dots, J \\ & \text{subject to} && q_j = q_j(i_j, x_i^j, z_j), \quad j=1, \dots, J \end{aligned}$$

$$\sum_{j=1}^J \sum_{i=1}^I p_i x_i^j + p_0 q_0 + \sum_{j=1}^J \sum_{i=1}^I K_i^j \leq y$$

$$x_j \geq 0, \quad 1 \leq i_j \leq I, \quad (6)$$

where i_j is an integer, and I is the number of energy sources available.

From (6), a general form for the durable decision equation is

$$\begin{aligned} i_j = i_j & (p_0, p_1, \dots, p_I, y, K_1^1, \dots, K_I^1, K_1^2, \dots, K_I^2, \dots, K_1^J, \dots, \\ & K_I^J, z_1, \dots, z_J). \end{aligned} \quad (7)$$

while Eq. (7) entails a fair degree of generality and detail, it may contain too many variables to be empirically tractable in many cases. Further simplification is possible by making stronger separability assumptions. If in fact q_0, \dots, q_J constitutes $J+1$ separable groups in demand, then Eq. (7) can be replaced by

$$i_j = i_j(p_1, \dots, p_I, \hat{p}_0^j, y, K_1^j, \dots, K_I^j, z_1, \dots, z_J) \quad (8)$$

where \hat{p}_0^j is a composite price index for commodities $q_0, q_1, \dots, q_{j-1}, q_{j+1}, \dots, q_J$ (inclusive of durable costs incurred in household production). Here $z_1, \dots, z_{j-1}, z_{j+1}, \dots, z_J$ can also possibly be aggregated into composite need-for-use variables associated with the group of all other commodities (except q_j).

Turning to the energy use decision, consider breaking the overall decision problem in (6) into two steps where in one step optimal quantities of household commodities are chosen for consumption subject to a given set of durables (or given appliance portfolio). The other step is to choose the optimal durable set. The first step problem for consumer good j is given explicitly by

$$\max_{q_j, x_i^j} v(\hat{q}_0^j, q_j) \quad (9)$$

$$\text{s.t.} \quad q_j = q_j(i_j, x_i^j, z_j)$$

$$p_{i_j} x_i^j + \hat{p}_0^j q_0^j \leq y^*$$

$$x_i^j \geq 0$$

where

$$\begin{aligned} y^* &= y - K_i^j \\ x_i^j &\equiv 0, \quad i \neq i_j \end{aligned} \quad (10)$$

and \hat{q}_0^j is the composite quantity index for commodities $q_0, q_1, \dots, q_{j-1}, q_{j+1}, \dots, q_J$. The decision functions in (10) follow trivially since no durables are available to make use of the associated energy in those cases once the durable set is chosen. From (9), a general representation of the resulting additional energy use equation is

$$x_i^j = x_i^j(\hat{p}_0^j, p_{i_j}, y^*, z_j | i=i_j) \quad (11)$$

which contains only 4 right-hand-side variables.

Demand equations for household commodities can be determined from (8) and (11) using the production functions in (6) and (9). Generally, data on household commodity demands (e.g., heat produced, hot water produced, etc.) are not available so the primary focus of this study does not involve household production. In fact, the present study focuses specifically on the use equation whereas the durable choice equation will be studied in a subsequent report.

IV.B. Aggregation and Aggregate Use Equation Specification

To consider the prospects for aggregation, suppose individual use equations follow the simple form in (11). Also, assume individual use equations satisfy homogeneity of degree zero in prices and income so that (11) can be rewritten as

$$x_i^j = x_i^j(\tilde{p}_{i_j}, \tilde{y}^*, z_j | i=i_j)$$

where

$$\tilde{p}_{i_j} = p_{i_j} / \hat{p}_0^j$$

$$\tilde{y}^* = y^* / \hat{p}_0^j$$

are deflated electricity price and deflated disposable income (after annualized fixed costs of durable ownership). Of course,

$$x_i^j = x_i^j(\tilde{p}_{i_j}, \tilde{y}^*, z_j | i \neq i_j) = 0$$

since, for example, no electricity would be used to produce household commodity j if the associated durable owned by the household were not electric.

In this framework, if n_1^j households own an electric durable for the purpose of producing household commodity j , then the total amount of electricity used to produce commodity j is

$$x_E^j = n_1^j \bar{x}_E^j (p_E, \bar{y}^*, z_j | i=E)$$

where $i=E$ denotes electricity. Summing over all end uses of electricity in households thus obtains total household use of electricity,

$$x_E = \sum_j n_1^j \bar{x}_E^j (p_E, \bar{y}^*, z_j | i=E) .$$

Average electricity consumption per household is thus

$$\bar{x}_E = x_E/n = \sum_j \phi_j \bar{x}_E^j (p_E, \bar{y}^*, z_j | i=E) \quad (12)$$

where n is the total number of households as before and $\phi_j \equiv n_1^j/n$ is the saturation of electricity using durables in end use j .

The aggregate equation for electricity use in the empirical part of this study for the residential and commercial sectors follows Eq. (12) where $\bar{x}_E^j(\cdot)$ is further specified as linear in logarithms to facilitate discussion of elasticities. That is, the estimated aggregate equation is of the general form

$$\ln \bar{x}_E = \sum_j \phi_j (\gamma_{0j} + \gamma_{1j} \ln p_E + \gamma_{2j} \ln y + \gamma_{3j} \ln z_j) \quad (13)$$

where additional cross products are also considered as suggested in Sect. III. While this equation does not satisfy exact aggregation, the approximating properties should be satisfactory.

V. MODEL SPECIFICATION

This section presents the econometric specification of sectoral electricity demand and prices. For the residential and commercial sectors, the present study only attempts to estimate the short-run usage equation with given saturation levels of electric appliances. Equation (13) is used

along with Eq. (5) without the lag term as the basis for econometric formulation. The industrial demand specification follows Eq. (5) directly.

Since the number of cross-product terms suggested in Eqs. (13) and (5) is large, the inclusion of all cross-product terms is not feasible because of problems of interpretation as well as multicollinearity. Thus, a preliminary analysis was conducted to determine the feasible set of cross-product terms on the basis of (1) the stability of the estimated coefficients and (2) the theoretical expectations that electricity price should have a negative effect; income should have a positive effect; and the saturation levels of appliances should have a positive effect on electricity demand.

Several formulations of the price equation are examined in light of the relationship between average price and marginal price elasticities as discussed later. As discussed in Chern et al.,³ the relationship between aggregate average price P_h and the aggregate average quantity per customer X_h for each sector h is characterized by

$$P_h = f_h(X_h, C) , \quad (14)$$

where C = average cost of producing and distributing electricity.

The particular specification of Eq. (2) used in Version II was based on the assumption that the utility company sets rate schedules such that, based on their expectations, total revenue will exceed costs by some set rate of profit per unit of electricity which they have negotiated with, or believe will satisfy, utility regulatory commissions. An alternative and plausible assumption can also be made, i.e., total revenue grows at a rate just equal to the growth rate of costs. Employing the latter assumption and letting \hat{P}_h represent the utility's expected average price for sector h based on a particular rate schedule, the utility then attempts to set rates such that the expected growth rate in average price \hat{P} over all sectors just keeps pace with the growth rate of average cost.

Given this rationale, the functional form in Eq. (14) can be further deduced as follows. If \hat{X}_h represents the utility's expectation for consumption per customer in sector h , given a particular rate schedule,

then their expectation for average sector price can be determined using Eq. (14):

$$\hat{P}_h = f_h(\hat{X}_h, C) . \quad (15)$$

Thus, by defining $S_h = N_h \hat{X}_h / \sum_i N_i \hat{X}_i$, where N_h is the number of customers in sector h , the overall average price \hat{P} may be written as:

$$\hat{P} = \sum_h \hat{S}_h \hat{P}_h = \sum_h \hat{S}_h f_h(\hat{X}_h, C) \quad (16)$$

Now suppose average cost increases by a factor α . Then overall average price increases by the same factor α (i.e., at the same rate) if and only if

$$\alpha \hat{P} = \sum_h \hat{S}_h f_h(\hat{X}_h, \alpha C) \quad (17)$$

But if Eq. (17) holds for all possible \hat{S}_h (note that $\sum_h \hat{S}_h = 1$), then every f_h function must be homogeneous of degree 1 in C . Since, this must be true for all possible values of \hat{X}_h , one finds that C must appear multiplicatively in Eqs. (14) and (15),

$$\hat{P}_h = f(\hat{X}_h, C) = g(\hat{X}_h) \cdot C . \quad (18)$$

That is, changes in cost are passed on to all sectors to maintain profit rates.

Now suppose X_h is used in place of or as a proxy for \hat{X}_h . Given Eq. (18), it remains to specify an estimable form for $g(X_h)$. One possibility is to specify $g(X_h)$ linearly. However, this specification poses a severe problem of convergence in forecasting as demonstrated by Chern et al.³ For Version II of the SLED model, the following quadratic specification of $g(X_h)$ was used:

$$g(X_h) = \beta_0 + \beta_1 X_h + \beta_2 X_h^2 .$$

Hence, the price relationship in Eq. (18) becomes

$$P_h = (\beta_0 + \beta_1 X_h + \beta_2 X_h^2) C. \quad (19)$$

The g_h functions can thus reflect the fact that costs specific to a particular sector may be passed on to that sector more than to other sectors even though the restriction in Eq. (18) is effective.

As another alternative, the following exponential function can be used for $g(X_h)$ in Eq. (18):

$$g(X_h) = e^{\beta_0 + \beta_1 X_h}$$

Hence the price relationship in Eq. (18) becomes

$$P_h = (e^{\beta_0 + \beta_1 X_h}) C \quad (20)$$

or

$$\ln \frac{P_h}{C} = \beta_0 + \beta_1 \ln X_h. \quad (20)$$

Thus, Eqs. (2), (19) and (20) represent three alternative specifications of the sectoral price equation. Eq. (20) is used in this study because the marginal price elasticities computed from the model are more plausible when this specification is used than with the other two alternatives (the other two led to several marginal elasticities with implausible signs). Consequently, the following model specification is adopted.

V.A. Residential Sector Submodel

Residential demand equation: The residential demand equation is

$$\begin{aligned} \ln E_{it}^R &= \alpha_0 + \alpha_1 \ln P_{it}^R + \alpha_2 (\ln P_{it}^R)^2 + \alpha_3 \ln P_{it}^R \cdot \ln Y_{it} \\ &+ \sum_{j=4}^7 \alpha_j \ln P_{it}^R \cdot S_{j-3, it} + \alpha_8 S_{1 it} \cdot HDD_{it} \end{aligned}$$

$$\begin{aligned}
& + \alpha_9 S_{2it} \cdot CDD_{it} + \sum_{k=10}^{11} \alpha_k S_{k-7,it} + \alpha_{12} \ln Y_{it} \\
& + \alpha_{13} \ln CR_{it} + \sum_{l=14}^{16} \alpha_l W_{l-13} + \alpha_{17} DLP \cdot \ln P_{it}^R \cdot \ln Y_{it} \\
& + \sum_{m=18}^{64} \alpha_m D_{m-17} + u_{it}
\end{aligned} \tag{21}$$

where

- i = state,
- t = time period,
- R = residential sector,
- E = quantity of residential sales of electricity,
- P = average price of electricity deflated by the cost of living index (CLI),
- Y = per capita personal income deflated by the cost of living index (CLI),
- HDD = heating degree-days,
- CDD = cooling degree-days,
- S_1 = saturation level (%) of electric space heating equipment,
- S_2 = saturation level of air conditioners,
- S_3 = saturation level of electric water heaters,
- S_4 = saturation level of electric clothes dryers,
- CR = number of residential customers,
- W = dummy variables for reclassification of customers and other shifts in historical trends of residential sales,
- D = state dummy variables,
- DLP = a dummy variable for five states (Tennessee, Idaho, Nevada, Washington, Oregon and California) where electricity prices were relatively low in the sample period,
- u = error term,
- α = parameters to be estimated.

There are three customers reclassification dummies and 47 state dummy variables. The reclassification dummies were defined previously in Chern et al.³ The dummy variable *DLP* is set to one for the five states with relatively low electricity price and zero for other states.

Under the specification of Eq. (21), the average short-run price elasticity is determined by

$$\eta_{it}^R = \frac{\partial \ln E_{it}^R}{\partial \ln P_{it}^R} = \hat{\alpha}_1 + 2\hat{\alpha}_2 \ln P_{it}^R + \hat{\alpha}_3 \ln Y_{it} + \sum_{j=4}^7 \hat{\alpha}_j S_{j-3,it} + \hat{\alpha}_{17} DLP \cdot \ln Y_{it} \quad (22)$$

and the income elasticity can be determined by

$$\epsilon_{it}^R = \frac{\partial \ln E_{it}^R}{\partial \ln Y_{it}} = \hat{\alpha}_{12} + \hat{\alpha}_3 \ln P_{it}^R + \hat{\alpha}_{17} DLP \cdot \ln P_{it}^R \quad (23)$$

where carets denote estimated coefficients.

It is also expected that the following properties hold in Eq. (21):

$$\frac{\partial \ln E}{\partial \ln P} < 0$$

$$\frac{\partial \ln E}{\partial \ln Y} > 0$$

$$\frac{\partial \ln E}{\partial S_j} > 0 \text{ for } j=1, \dots, 4$$

$$\frac{\partial \ln E}{\partial HDD} > 0$$

$$\frac{\partial \ln E}{\partial CDD} > 0$$

In Eq. (21), only four major electric appliances are included. The saturation levels of electric clothes washers, food freezers, ranges, and dishwashers were examined but later excluded because of implausible (though insignificant) results. The problem associated with dishwashers is partially due to the lack of time-series data (the state-level census data are available only for 1970).

Residential price equation: The residential price equation is

$$\begin{aligned} \ln(P_{it}^R / TOC_{it}) = & \beta_0 + \beta_1 \ln(E_{it}^R / CR_{it}) + \beta_2 \ln CR_{it} + \beta_3 \ln HY_{it} \\ & + \beta_4 \ln INV_{it} + \beta_5 \ln IND_{it} + \beta_6 \ln CU_{it} \\ & + \sum_{j=7}^9 \beta_j W_{j-6} + \sum_{k=10}^{56} \beta_k D_{k-9} + v_{it} \end{aligned} \quad (24)$$

where

- P = average electricity price (in nominal terms),
- TOC = average total cost of generating, transmitting, and distributing electricity,
- HY = percentage of total generation by hydropower,
- INV = percentage of total generation by investor-owned utilities,
- IND = percentage of total sales in the industrial sector,
- CU = capacity utilization (%),
- v = error term,
- β = parameters to be estimated,

with other variables as defined previously.

The price equation specification in Eq. (24) is similar to that of the earlier study except that the dependent variable is expressed in terms of the difference between the log of price and the log of TOC , and several variables have been added to explain variations in pricing among states — namely, capacity utilization, the percentage of electricity provided for end use by private (rather than public) utilities, the percentage of

electricity used by (more efficient) industrial users. Normally, one expects that low capacity utilization leads to higher price because of reduced spreading of fixed costs, but very high capacity utilization can also lead to higher price if prices are used for rationing or to finance needed expansions. For example, since residential customers are relatively irregular users with more inelastic demand than commercial or industrial customers, the need for rationing and expansionary funds could cause higher residential prices while some of the fixed cost spreading may continue to be passed on to commercial and industrial users in the form of lower prices.

When private utilities handle a greater share of the market, one could expect either lower prices associated with more efficient management and operation or higher prices because of greater incentive for monopoly pricing. It might also be noted, however, that public utilities may have greater incentives to offer lower prices to residential and possibly commercial users who represent the bulk of potential lobbying interests.

Because hydroelectric generation is generally cheaper than other forms of electricity generation, one would normally expect relatively greater hydroelectric generation shares to lead to lower prices.

Finally, the percentage of industrial use is intended to explain the fact that industrial users are generally given price breaks because of their relatively large and stable individual usages. Such price breaks, however, can only come at the expense of other electricity users — the residential and commercial sectors. Hence, a greater share of use in the industrial sector should likely lead to higher prices in the residential and commercial sectors.

V.B. Commercial Sector Submodel

The end-uses in the commercial sector are similar to those in the residential sector except there are fewer well-defined categories in the commercial sector. Jackson and Johnson¹⁷ defined the major end-uses in the commercial sector as space heating, cooling, water heating, lighting, and other (including cooking and electro mechanical uses). They estimated that the total electricity use in the commercial sector was 5.05×10^{15} Btu in 1975, of which 6.5% was used for space heating,

36.2% was used for cooling, 0.8% was used for water heating, 41.4% was used for lighting, and 15% was used for other uses. As one can see, the water heating use has been rather unimportant. Even though lighting is the most important end-use, its saturation level is virtually 100%. Also, substitution possibilities appear to be rather limited in the "other" category. Thus, the only two important end-uses which appear to require explicit consideration in modeling the saturation level are space heating and cooling. Unfortunately, data on the saturation level for these end uses are not available. Thus, the following simplifying assumption is made:

$$\phi_c = \gamma \phi_r$$

where

- ϕ_r is residential sector saturation,
- ϕ_c is commercial sector saturation,
- γ is a constant parameter.

With this assumption, the saturation levels of electric heating and air conditioning in the residential sector are used as the explanatory variables in the commercial sector.

Commercial demand equation: The commercial demand equation is

$$\begin{aligned} \ln E_{it}^C = & \alpha_0 + \alpha_1 \ln P_{it}^C + \alpha_2 (\ln P_{it}^C)^2 + \alpha_3 \ln P_{it}^C \cdot \ln Y_{it} \\ & + \alpha_4 \ln P_{it}^C \cdot S_{1it} \cdot HDD_{it} + \alpha_5 \ln P_{it}^C \cdot S_{2it} \cdot CDD_{it} \\ & + \alpha_6 S_{1it} \cdot HDD + \alpha_7 S_{2it} \cdot CDD + \alpha_8 S_{1it} + \alpha_9 S_{2it} \\ & + \alpha_{10} \ln Y_{it} + \alpha_{11} \ln POP_{it} + \alpha_{12} \ln URB_{it} \\ & + \sum_{j=13}^{34} \alpha_j W_{j-12} + \sum_{k=35}^{81} \alpha_k D_{k-34} + u_{it} \end{aligned} \quad (25)$$

where

C = commercial sector,

E = quantity of commercial sales of electricity,

P = average price of electricity in the commercial sector
deflated by CLI,

POP = population,

URB = percentage of population in urban areas,

W = dummy variables for reclassification of customers,

u = error term,

α = parameters to be estimated,

with other variables as defined previously. There are 19 reclassification dummies in addition to the three identified in the residential submodel. These reclassification dummy variables were identified and defined in Chern et al.³

Under the specification in Eq. (25), the average short-run price elasticity is determined by

$$\begin{aligned} \eta_{it}^C &= \frac{\partial \ln E_{it}^C}{\partial \ln P_{it}^C} = \hat{\alpha}_1 + 2\hat{\alpha}_2 \ln P_{it}^C + \hat{\alpha}_3 \ln Y_{it} \\ &+ \hat{\alpha}_4 S_{1it} \cdot HDD_{it} + \hat{\alpha}_5 S_{2it} \cdot CDD_{it} \end{aligned} \quad (26)$$

and the income elasticity can be calculated by

$$\epsilon_{it}^C = \frac{\partial \ln E_{it}^C}{\partial \ln Y_{it}} = \hat{\alpha}_3 \ln P_{it}^C + \hat{\alpha}_{10} \quad (27)$$

where the $\hat{\alpha}_j$'s are estimated coefficients. The following properties should hold in Eq. (25):

$$\frac{\partial \ln E}{\partial \ln P} < 0$$

$$\frac{\partial \ln E}{\partial S_j} > 0 \text{ for } j=1,2$$

$$\frac{\partial \ln E}{\partial HDD} > 0$$

$$\frac{\partial \ln E}{\partial CDD} > 0$$

Commercial price equation: The commercial price equation is

$$\begin{aligned} \ln(P_{it}^C / TOC_{it}) = & \beta_0 + \beta_1 \ln(E_{it}^C / CC_{it}) + \beta_2 \ln CC_{it} \\ & + \beta_3 \ln HY_{it} + \beta_4 \ln INV_{it} + \beta_5 \ln IND_{it} \\ & + \beta_6 \ln CU_{it} + \sum_{j=7}^{30} \beta_j W_{j-6} + \sum_{k=31}^{78} \beta_k D_{k-31} \\ & + v_{it} \end{aligned} \quad (28)$$

where

P = average electricity price (in nominal terms),

CC = number of commercial customers,

v = error term,

β = parameters to be estimated,

with other variables as defined previously.

V.C. The Industrial Sector Submodel

Since there is not a simple way to deal explicitly with the choices of electric equipment in the industrial sector, the Koyck model is used to capture both the short-run and long-run demand responses as done previously in Chern et al.^{3,4}

Industrial demand equation: The industrial demand equation is

$$\begin{aligned} \ln E_{it}^I &= \alpha_0 + \alpha_1 \ln E_{it-1}^I + \alpha_2 \ln P_{it}^I + \alpha_3 (\ln P_{it}^I)^2 \\ &+ \alpha_4 \ln P_{it}^I \cdot \ln VA_{it} + \alpha_5 \ln VA_{it} + \alpha_6 \ln PC \\ &+ \sum_{j=7}^{29} \alpha_j W_{j-6} + \sum_{k=30}^{76} \alpha_k D_{k-29} + u_{it} \end{aligned} \quad (29)$$

where

- I = industrial sector,
- E = quantity of industrial sales of electricity,
- P = average price of electricity in the industrial sector deflated by the wholesale price index of intermediate supplies (WPI),
- VA = value added in manufacturing deflated by the wholesale price index of manufacturing output (WPM),
- PC = wholesale price of coal deflated by WPI,
- α = parameters to be estimated,
- u = error term,

with other variables as defined previously.

Under the specification of Eq. (29), the short-run average price elasticity is determined by

$$\eta_{it}^I = \frac{\ln E_{it}^I}{\ln P_{it}^I} = \hat{\alpha}_2 + 2\hat{\alpha}_3 \ln P_{it}^I + \hat{\alpha}_4 \ln VA_{it} \quad (30)$$

and the long-run average price elasticity is determined by

$$\delta_{it}^I = \frac{\eta_{it}^I}{\hat{\alpha}_1} \quad (31)$$

where $\hat{\alpha}_1, \hat{\alpha}_2, \hat{\alpha}_3$ are estimated coefficients.

Industrial price equation: The industrial price equation is

$$\begin{aligned}
 \ln(P_{it}^I/TOC_{it}) = & \beta_0 + \beta_1 \ln(E_{it}^I/CI_{it}) + \beta_2 \ln CI_{it} \\
 & + \beta_3 \ln HY_{it} + \beta_4 \ln INV_{it} + \beta_5 \ln IND_{it} \\
 & + \beta_6 \ln CU_{it} + \sum_{j=7}^{28} \beta_j W_{j-6} + \sum_{k=29}^{75} \beta_k D_{k-28} \\
 & + v_{it}
 \end{aligned} \tag{32}$$

where

P = average electricity price (in nominal terms),

v = error term,

β = parameters to be estimated,

with other variables as defined previously.

VI. EMPIRICAL ESTIMATION

VIA. Application in a Time Series-Cross Section Context

As experienced in the earlier studies by Chern et al.,^{3,4} when a reasonable variety of exogeneous variables are considered in Eqs. (1) and (2), the number of available observations is not sufficient to permit much precision in estimation when separate structural parameters are estimated for each state. This problem is only accentuated by adding the cross-product terms in Eqs. (5) and (13). To deal with this problem, the earlier study assumed the same elasticities across groups of states and simply included shift terms for individual states. With the flexibility of Eqs. (5) and (13), however, it is possible to assume the same parameters across states while still allowing flexibility with respect to elasticities. Such an econometric approach is implemented in the model specified in Sect. V. The regression model presented in Sect. V is based on Eqs. (2),

(5), and (13) with parameters constant across states except that constant terms (by inclusion of state dummy terms) vary from state to state.

To investigate the validity of the assumption that parameters are the same across states (aside from the constant terms), however, formal hypothesis tests were performed for each of the three sectors – residential, commercial, and industrial. The model was estimated by state and then over all 48 states. Using the sum of squared residuals from demand in each case, asymptotic F test statistics did not lead to rejection of the hypothesis of constant parameters over states at standard significance levels. Thus, the generality of the regression model apparently provides an adequate explanation of variation in behavior and, thus, of variation in price and income elasticities of demand among states.

VI.B. Estimation Method

As Halvorsen¹⁸ argued previously, when average rather than marginal electricity price is used, the price variable should be treated as an endogenous variable and thus a simultaneous equation method should be used. Since the structural equations of (21), (24), (25), (28), (29), and (32) form a nonlinear simultaneous equation system, they can be estimated by non-linear two-stage least squares (2SLS) and three-stage least squares (3SLS). An additional complication which must be considered in estimation relates to heteroscedasticity due to the pooling of time series and cross section data. Namely, where observations are pooled from different states, heuristic reasoning would suggest that all disturbances would not have identical variances. For example, suppose Eqs. (21) and (24) can be rewritten as

$$\ln E_{it}^R = \alpha_0 + \sum_{j=1}^{67} \alpha_j X_{ijt} + u_{it}$$

$$\ln(P_{it}^R/TOC_{it}) = \beta_0 + \sum_{k=1}^{57} \beta_k Z_{ikt} + v_{it}$$

where u_{it} and v_{it} are explicit stochastic disturbances with $E(u_{it}) = E(v_{it}) = 0$, $\text{Var}(u_{it}) = \sigma_i^2$, $\text{Var}(v_{it}) = \delta_i^2$. The standard application of 2SLS and 3SLS assumes that $\sigma_i = \sigma_j$ and $\delta_i = \delta_j$ for all i and j .¹⁹

In pooling data from different states, however, one would expect different variances of disturbances because of relative differences in size even though other parameters are the same. In this case, the standard error estimates of both disturbances and coefficient estimates obtained from either 2SLS or 3SLS would not be strictly valid. Since ordinary instrumental variables estimates are still consistent in this case, however, the corresponding sample standard errors, $\hat{\sigma}_i$ and $\hat{\delta}_i$, can serve as needed estimates of heteroscedasticity in the pooled data. Thus, the model can be transformed to one of asymptotic homoscedasticity by dividing the data for each state by $\hat{\sigma}_i$ to estimate the demand equation and by $\hat{\delta}_i$ to estimate the price equation.

To determine the extent to which this type of heteroscedasticity affects standard error estimates of coefficients, the corresponding estimates of the model were developed in several preliminary runs for each of the respective sectors. The comparison of these results, however, show that the effects of heteroscedasticity are not substantial. That is, neither coefficient estimates nor their standard error estimates differ substantially from the case where heteroscedasticity is not considered. Thus, the final estimates reported here are obtained from the usual nonlinear 2SLS and 3SLS to avoid the computational complexity of deriving the weighted estimates.

VI.C. Data

The data for most of the variables are taken from the previous studies by Chern et al.^{3,4} The data sources and units of measurement are discussed in detail in Chern et al.³ The sample period covers 1955-1976.

A major set of new variables used in this study is related to the saturation level of electric appliances. The saturation level is defined as the percentage of all occupied housing units using a particular electric appliance. State-level data on saturation are available for

the two census years of 1960 and 1970.^{20,21} The eight major electric appliances included in the census of housing are electric heaters, air conditioners, water heaters, electric ranges, clothes washers, clothes dryers, dishwashers, and food freezers. The 1960 census did not cover dishwashers so that time-series data could not be developed for this appliance. To develop time-series data for econometric analysis, logistic curves representing saturation growth were developed using 1960 and 1970 data according to the following equation:

$$\phi_{rt} = \frac{1}{1 + \exp(c_0 + c_1 t)} \quad (33)$$

Since Eq. (33) has two parameters (c_0 and c_1) they can be estimated with two data points. These parameters are estimated for each state for the seven appliances. Equation (33) is then used to estimate the saturation level for the rest of the years covering 1955-1976 for all states and all seven appliances.

In addition to the census data for 1960 and 1970, there are regional saturation data available for electric heating, air conditioning, and electric cooking for the years of 1973-1976.²² These regional data are used to adjust the saturation level estimated by Eq. (33) for 1973-76 by first computing

$$d = \frac{\sum_{i=1}^I \hat{\phi}_{rti} N_i}{\phi_t \sum_{i=1}^I N_i}$$

where

N_i is the number of residential customers in state i ,

I is the number of states in the region,

$\hat{\phi}_{rti}$ is fitted saturation data from Eq. (33),

ϕ_t is the regional saturation level,

and then computing the adjusted saturation level by

$$\hat{\phi}_{rti} = \frac{1}{d} \hat{\phi}_{rti} \text{ for } t = 73, \dots, 76 .$$

Admittedly, the above method for developing the time-series data on appliance saturation is an *ad hoc* procedure. Data Resources Inc. (DRI)²³ also developed state-level time-series data for major appliances for the period of 1960-74 for the Electric Power Research Institute. In addition to the census data, DRI also incorporated information from the magazine *Merchandising Week*. The data series developed for this study were compared with the DRI series by computing the means and correlation coefficients for the periods for which DRI data are available. The results, as presented in Table 1, show that the two data series are very similar except for clothes washing. A careful examination of the two data series for clothes washing revealed that the DRI data show unusually high saturation levels for almost all states and that variation during this study period is very small. Aside from this discrepancy, the data series match fairly well; correlation coefficients are all greater than 0.96.

Although the above procedure for developing the saturation data may introduce some bias into the results, the bias arises essentially in an errors-in-variables context. Hence, if variation in true end uses is more substantial between states than within states, the variation of errors in variables is small relative to variation in the variables. Under these conditions, it has been shown that the bias associated with errors in variables is small. Furthermore, it seems that such conditions are likely to hold with respect to most end uses. That is, air conditioning seems to vary widely from state to state because of weather differences while one would expect variation within state to be fairly stable and only indicate a mild trend. Even more so, space heating, water heating, and cooking uses vary substantially among states presumably because of wide variation in electricity prices and availability of alternative fuels while the long term nature of the associated durables would suggest much less variation within states. Thus, the only variables for which errors in variables may be a problem appear to be for the more minor end uses in clothes washing, dishwashing, and television; but even

Table 1. Comparison of the ORNL and DRI data series on appliance saturation level

End-Use Category	Sample Period	ORNL Data Series Mean (%)	DRI Data Series Mean (%)	Correlation Coefficient
Electric Heating	1960-73	5.23	6.09	0.968
Air Conditioning	1960-74	27.5	33.1	0.980
Water Heating	1960-74	29.7	29.7	0.997
Electric Cooking	1960-74	45.1	45.1	0.997
Clothes Washing	1960-74	63.7	74.4	0.195
Clothes Drying	1960-74	27.7	27.6	0.978
Food Freezing	1960-74	27.9	30.0	0.967

in these cases, one would expect the logistic saturation curve to provide a good approximation.

In addition to the saturation variables, there are five more new variables used in this study. Data for the urbanization variable (URB), defined as the percentage of population in urban areas, are taken from the Bureau of the Census.²⁴ For the generation proportions by hydropower (HY), investor-owned utilities (INV), and the proportion of sales in the industrial sector (IND), data are taken from the Edison Electric Institute.²⁵ The capacity utilization refers to actual kWh generated as a percentage of the kWh which could have been generated if total installed name plate generating capacity has been fully utilized throughout the year. Data on capacity utilization are taken from Federal Power Commission.^{26,27,28} The average of the beginning and end of year generating capacity is used to approximate the capacity available throughout the year.

VI.D. Empirical Results

Using the model specified in Sect. V and the data described in Sect. VI.C, nonlinear three-stage least squares (3SLS) estimation was used to obtain the results in Tables 2 through 7. Consider first the estimated results for residential demand (Table 2). The estimated coefficient for the cross-product term, $\ln P^R \cdot \ln P^R$ has a positive sign, indicating that higher electricity prices would lead to a lower electricity price elasticity. This result is somewhat surprising because one would expect that higher prices could lead to more consumer sensitivity and, hence, a greater price elasticity. However, these results reveal the same phenomena obtained when Version I of the SLED model was updated with additional data for 1975-76 in the previous study by Chern et al.⁴

One explanation may be that the reversal from a declining trend of real electricity prices to an increasing trend in the early 1970's may result in a change in the relationship between marginal and average price elasticities. That is, average price elasticities tend to be larger than the marginal price elasticities as will be shown later. And as shown in a recent study by Houthakker,²⁹ the differences between average price and marginal price have been much smaller since electricity

Table 2. Three-stage least squares estimates of residential demand, 1955-1976^aNormalized Variable: $\ln E^R$

Variable	Estimated Coefficient	Estimated Standard Error	Asymptotic t-Ratio
Constant	0.394	1.18	0.33
$\ln F^R$	-4.707	0.46	-10.23
$\ln F^R \cdot \ln F^R$	0.487	0.057	8.50
$\ln F^R \cdot \ln Y$	0.369	0.071	5.16
$\ln F^R \cdot S_1$	-0.00268	0.00059	-4.56
$\ln F^R \cdot S_2$	0.00118	0.00014	8.38
$\ln F^R \cdot S_3$	0.00754	0.00076	9.92
$\ln F^R \cdot S_4$	0.0134	0.0012	10.81
$S_1 \cdot HDD$	0.00000232	0.00000026	8.84
$S_2 \cdot CDD$	0.000000919	0.00000027	3.44
S_3	-0.0177	0.0025	-7.18
S_4	-0.0382	0.0039	-9.81
$\ln Y$	-1.073	0.26	-4.14
$\ln CR$	1.252	0.033	37.65
R^2	0.994		

^aEstimated coefficients for the three reclassification dummies and 47 state dummies are not reported. R^2 is the weighted R^2 for the system that corresponds to the approximate F-test on all non-intercept parameters in the system.

Table 3. Three-stage least squares estimates of residential price, 1955-1976^a

Normalized Variable: $\ln(P^R/TOC)$

Variable	Estimated Coefficient	Estimated Standard Error	Asymptotic t-Ratio
Constant	-2.330	0.79	-2.95
$\ln(E^R/CR)$	-0.212	0.022	-9.52
$\ln CR$	0.057	0.044	1.31
$\ln HY$	0.00207	0.0064	0.33
$\ln INV$	-0.026	0.017	-1.54
$\ln IND$	0.0588	0.022	2.69
$\ln CU$	0.185	0.017	10.64
R^2	0.994		

^aEstimated coefficients for the three reclassification dummies and 47 state dummies are not reported. R^2 is the weighted R^2 for the system that corresponds to the approximate F-test on all non-intercept parameters in the system.

Table 4. Three-stage least squares estimates of commercial demand, 1955-1976^aNormalized Variable: $\ln E^C$

Variable	Estimated Coefficient	Estimated Standard Error	Asymptotic t-Ratio
Constant	-0.407	1.72	-0.24
$\ln P^C$	-3.541	0.67	-5.29
$\ln P^C \cdot \ln P^C$	0.301	0.08	3.77
$\ln P^C \cdot \ln Y$	0.704	0.14	5.06
$\ln P^C \cdot S_1 \cdot HDD$	0.0000028	0.00000057	4.89
$\ln P^C \cdot S_2 \cdot CDD$	0.0000034	0.00000059	5.79
$S_1 \cdot HDD$	-0.0000064	0.0000017	-3.82
$S_2 \cdot CDD$	-0.0000124	0.000017	-7.25
S_1	-0.00528	0.0033	-1.58
S_2	0.00798	0.00090	8.86
$\ln Y$	-1.355	0.506	-2.68
$\ln POP$	1.777	0.065	27.52
$\ln URB$	-0.0214	0.094	-0.23
R^2	0.984		

^aEstimated coefficients for the 22 reclassification dummies and 47 state dummies are not reported. R^2 is the weighted R^2 for the system that corresponds to the approximate F-test on all non-intercept parameters in the system.

Table 5. Three-stage least squares estimates of commercial price, 1955-76^aNormalized Variable: $\ln(P^C/TOC)$

Variable	Estimated Coefficient	Estimated Standard Error	Asymptotic t-Ratio
Constant	1.872	0.53	3.51
$\ln(E^C/CC)$	-0.0982	0.014	-7.03
$\ln CC$	-0.183	0.036	-5.11
$\ln HY$	0.00468	0.0063	0.74
$\ln INV$	0.0547	0.016	3.33
$\ln IND$	-0.00186	0.024	-0.08
$\ln CU$	0.108	0.017	6.36
R^2	0.984		

^aEstimated coefficients for the 22 reclassification dummies and 47 state dummies are not reported. R^2 is the weighted R^2 for the system that corresponds to the approximate F-test on all non-intercept parameters in the system.

Table 6. Three-stage least squares estimates of industrial demand, 1955-1976^aNormalized Variable: $\ln E^I$

Variable	Estimated Coefficient	Estimated Standard Error	Asymptotic t-Ratio
Constant	3.409	0.473	7.21
$\ln E^I_{t-1}$	0.560	0.017	32.22
$\ln P^I$	-1.276	0.267	-4.78
$\ln P^I \cdot \ln P^I$	0.0556	0.040	1.40
$\ln P^I \cdot \ln VA$	0.0489	0.016	3.15
$\ln VA$	0.182	0.037	4.87
$\ln CI$	0.0870	0.011	8.20
$\ln PC$	0.0964	0.011	8.98
R^2	0.991		

^aEstimated coefficients for the 22 reclassification dummies and 47 state dummies are not reported. R^2 is the weighted R^2 for the system that corresponds to the approximate F-test on all non-intercept parameters in the system.

Table 7. Three-stage least squares estimates of industrial price, 1955-1976^aNormalized Variable: $\ln(P^I/TOC)$

Variable	Estimated Coefficient	Estimated Standard Error	Asymptotic t-Ratio
Constant	-1.134	0.17	-6.49
$\ln(E^I/CI)$	0.0729	0.0091	7.97
$\ln CI$	0.109	0.01	10.67
$\ln HY$	-0.00182	0.0063	-0.29
$\ln INV$	0.0406	0.016	2.58
$\ln IND$	-0.180	0.024	-7.42
$\ln CU$	0.067	0.018	3.82
R^2	0.991		

^aEstimated coefficients for the 22 reclassification dummies and 47 state dummies are not reported. R^2 is the weighted R^2 for the system that corresponds to the approximate F-test on all non-intercept parameters in the system.

prices have drastically increased in recent years. Also, it seems that people now are quite sensitive to the large positive changes in price that rate increases are providing. But perhaps their needs are so increasingly inflexible that the statistical finding obtained in this study is, indeed, consistent. The same results with regards to the cross-product term are also obtained for both the commercial and industrial sectors.

Consider next the cross-effects between price and income. One would expect that higher income would decrease consumer sensitivity and, thus, imply less price elasticity. Similarly, a higher price should imply greater response to changes in income (at low prices, consumption would take place with little consideration of income) and should, thus, cause higher income elasticity. The estimated coefficient for the cross product term, $\ln P^R \cdot \ln Y$ confirms these expectations. The same results also hold in the commercial sector.

The effects of appliance saturations on the price elasticity are much less obvious because the end-uses which have nearly 100% saturations (such as lighting and television) and those for which adequate data are not available (such as dishwashing) are not explicitly included in the model. Thus, the average impacts of these excluded end uses are reflected by the three price variables ($\ln P^R$, $\ln P^R \cdot \ln P^R$, and $\ln P^R \cdot \ln Y$) and the constant and dummy terms. In the context of the model estimated in Table 2, the effects of various included appliance saturations can only be interpreted in a relative sense by comparison with the mean effects of the end-uses for which saturations are not included. The results show that a greater use of electricity for space heating (as reflected by S_1) leads to a higher price elasticity of demand than the excluded end uses. On the other hand, a greater use of electricity for air conditioning (S_2), water heating (S_3), and clothes drying (S_4) leads to a lower price elasticity of demand.

Further analysis of the relative effects of various end-uses on price elasticity is possible by considering decomposition of the price elasticity with respect to end-uses included in the model. Using the sample means of the appliance saturation levels (in percent), Table 8 is developed for the comparison. The last column of Table 8 shows the extent to which the average price elasticity is adjusted by various end-use saturations. Since the average price elasticity has a negative sign, the

Table 8. Comparison of the effects on price elasticity of appliance saturation level among end-uses, residential sector

End-Use	Estimated Coefficient of $\ln P^R \cdot S_j$ (1)	Sample Mean of S_j (2)	Effect of Appliance Saturation (1) x (2)
Space Heating	-0.00268	4.90	-0.012
Air Conditioning	0.00118	24.23	0.029
Water Heating	0.00754	28.85	0.218
Clothes Drying	0.0134	24.77	0.332

results show that space heating uses of electricity would increase the price elasticity in absolute value, while water heating and clothes drying uses decrease the price elasticity. Even though air conditioning usage decreases the price elasticity, the effect is much smaller than water heating and clothes drying. Although the price elasticity associated with each of the end-uses cannot be computed from the model, these relative impacts clearly show that the price responsiveness is greater for space heating and air conditioning than for clothes drying and water heating. Given that in the recent energy crunch much of consumers' adjustments seemed to be focused on "adjusting the thermostat," one would indeed expect that regions with greater use of electricity for space heating and air conditioning would have higher price elasticity of demand. Similar arguments would imply that greater saturation in less adjustable uses, such as water heating and clothes drying, should imply less price elasticity of demand. Thus, the estimated effects of saturation on price elasticity appear reasonable.

Another important check on the reasonableness of the estimation results is to see whether or not the important comparative static assumptions as discussed in Sect. V hold. Using the sample mean values, these properties hold — the electricity price has a negative impact on demand; and income, heating and cooling degree days, and all saturation variables have positive impacts on demand.

Turning to the commercial sector, it seems that the same qualitative relationships should likely hold between price and income, but that the role of end uses may be somewhat different. Namely, a comfortable environment may be regarded as a necessity; in which case greater use of electricity for space heating and air conditioning would imply less price elasticity of demand. As one can see, the results in Table 4 correspond to these expectations.

Consider the results of the industrial sector in Table 6. The estimated coefficient for the square term $\ln P^I \cdot \ln P^I$ is positive, but its numerical value is relatively small and also insignificant statistically. This result indicates that price elasticity of demand for electricity of the industrial sector does not vary significantly as electricity price changes. The estimated coefficient for the cross-product term $\ln P^I \cdot \ln VA$ is positive and significant at the 0.01 level,

indicating that, as value added increases, the price elasticity of demand for electricity is decreasing. Such a result is consistent with theoretical implications for use of electricity as a productive input (e.g., for the operation of machinery and application in various chemical processes). That is, demands for productive inputs are derived demands. The level of output is the dominant factor determining the quantity of inputs demanded; when the price of productive input is small, the output quantity decision becomes of overriding importance. Consequently, the price elasticities of demand for inputs such as electricity, become relatively smaller.

Since the industrial model is specified as a dynamic model which captures both short-run and long-run responses, the cross-price variables are also included. However, as it turns out, only the price of coal has a coefficient with the expected sign and a high t-ratio.

Turning now to the estimated price equations (Tables 3, 5, 7), the quantity variable (average usage) has a very high t-ratio. The results also show that higher shares of hydropower have an insignificant effect on the prices of electricity in all three sectors. The share of generation by investor-owned utilities has a significant positive effect on the electricity prices of the commercial and industrial sectors; its effect on the electricity price of the residential sector, on the other hand, is statistically insignificant. The share of industrial sales has a positive coefficient with a t-ratio of 2.69 in the residential sector and negative coefficients with t-ratios of -0.08 and -7.42 in the commercial and industrial sectors, respectively. These results imply that price breaks for the industrial customers are perhaps made at the expense of residential customers. Finally, the results show that capacity utilization has a positive and significant impact on the electricity prices in all three sectors.

VII. ESTIMATED ELECTRICITY DEMAND ELASTICITIES

VII.A. Average Price Versus Marginal Price Elasticities

In this study, the use of average rather than marginal electricity price is based on a similar, but modified, version of arguments by

Halvorsen. That is, while the theoretical advantages of using marginal price and an intrarate income effect are well understood for representing the effects of declining block rates for electricity consumption, the lack of reliable and widely applicable marginal price information necessitates the use of average price. Nevertheless, a theoretically satisfying justification is possible because of the relationship that must hold between average and marginal prices for a given rate schedule. That is, suppose marginal price is given by $p^m(x)$ where p^m falls according to a declining block rate schedule as use x increases. Then where p^a is average price one finds

$$p^a \cdot x = \int_0^x p_m(s) ds .$$

Differentiating both sides of this equation with respect to x and substituting Eq. (20) yields an explicit relationship between p^a and p^m .

$$p^a + x \partial p^a / \partial x = p^m ,$$

i.e.,

$$p^a = p^m - \beta_1 e^{\beta_0 x} \beta_1 e \quad (34)$$

where p^a and x are the same as p_h and x_h , respectively, in Eq. (20). The relationship in (34) thus permits the estimated model to be interpreted in terms of marginal electricity price as well as average electricity price. Furthermore, it suggests an additional advantage of the model specification used here over the one employed by Halvorsen¹⁸ because the relationship between marginal and average price turns out to depend on quantity as one would expect. The only necessary additional consideration is that a simultaneous equations procedure must be employed in estimation since, with a declining block rate, the average price is influenced by quantity even at the level of an individual consumer (see Halvorsen). This was, of course, implemented in the estimation presented in the preceding section.

Based on the estimated equations, another important question can be answered in this study. That is, to determine whether and the extent to which elasticities of demand vary among states, the estimated parameters in Table 2 can be substituted into Eq. (22) to estimate average (short-run) price elasticities by state for the residential sector. Similarly, the results in Table 4 can be used in the context of Eq. (26) for the commercial sector. The results in Table 6 can be used in the context Eqs. (30) and (31) for computing, respectively, short-run and long-run price elasticities for the industrial sector.

The relationship in Eq. (34) can then be used in estimating a more useful concept of (short-run) marginal price elasticity of demand. The marginal price elasticity is often more useful since an average price elasticity cannot be used to determine the effects of new developments except where the rate schedule remains fixed (in real terms). The assumption of fixed rate schedules, might be a reasonable approximation during the sample period, but, because of new possibilities in rate schedule design, it may be far from applicable for post-sample forecasting.

For example, if utilities switch from declining block rates to flat rate schedules, then the relationship in Eq. (34) would become simply $p^a = p^m$ and estimated equations could be interpreted correctly only in terms of marginal price, i.e., the demand equation in (5),

$$\begin{aligned} \ln x_t = & \gamma_0 + \gamma_1 \ln x_{t-1} + [\gamma_2 + \gamma_3 \ln (p_t^m - \beta_1 e^{\beta_0} x_t^{\beta_1} c_t) \\ & + \gamma_4 \ln y_t + \gamma_5 \ln w_t] \ln (p_t^m - \beta_1 e^{\beta_0} x_t^{\beta_1} c_t) \\ & + \gamma_6 \ln y_t + \gamma_7 \ln z_t, \end{aligned} \quad (35)$$

would become

$$\begin{aligned} \ln x_t = & \gamma_0 + \gamma_1 \ln x_{t-1} + [\gamma_2 + \gamma_3 \ln (\tilde{p}_t^a - \beta_1 e^{\beta_0} x_t^{\beta_1} c_t) \\ & + \gamma_4 \ln y_t + \gamma_5 \ln w_t] \ln (\tilde{p}_t^a - \beta_1 e^{\beta_0} x_t^{\beta_1} c_t) \\ & + \gamma_6 \ln \tilde{y}_t + \gamma_7 \ln z_t, \end{aligned}$$

where \tilde{p}_t^a is the new flat rate price. In addition, one should properly consider the intra-block income effect

$$(p_t^a - p_t^m)x_t = -\beta_1 x_t e^{\beta_0 x_t \beta_1 c_t}$$

by modifying the income term from y_t to \tilde{y}_t in transforming the demand equation from a declining block rate to a flat rate schedule.

To consider marginal price elasticities for the estimated average-price demand equations, substitute (34) into (5) to obtain (35). For notational simplicity ignore the subscript t except where $t-1$ appears. Exponentiation of Eq. (35) implies that

$$x - A (p^m - \beta_1 e^{\beta_0 x \beta_1 c}) e^{\bar{\alpha}} + \gamma_3 \ln^2(p^m - \beta_1 e^{\beta_0 x \beta_1 c}) = 0$$

where

$$A = e^{\gamma_0} x_{t-1}^{\gamma_1} y^{\gamma_6} w^{\gamma_7}$$

$$\bar{\alpha} = \gamma_2 + \gamma_4 \ln y + \gamma_5 \ln w.$$

From implicit differentiation,

$$\begin{aligned} \frac{dx}{dp^m} &= \frac{\bar{\alpha}(x/p^a) + 2\gamma_3(x/p^a) \ln p^a}{1 + \bar{\alpha}(x/p^a) \beta_1^2 (p^a/x) + 2\gamma_3(x/p^a)(\ln p^a) \beta_1^2 (p^a/x)} \\ &= \frac{(\bar{\alpha} + 2\gamma_3 \ln p^a)(x/p^a)}{1 + \beta_1^2 (\bar{\alpha} + 2\gamma_3 \ln p^a)} \end{aligned}$$

so that the marginal price elasticity is given by

$$E_{p^m} = \frac{dx}{dp^m} \frac{p^m}{x} = \frac{1 + \beta_1}{\beta_1^2 + 1/(\bar{\alpha} + 2\gamma_3 \ln p^a)} \quad (36)$$

The term $\bar{\alpha} + 2\gamma_3 \ln P^{\alpha}$ is the average price elasticity as more completely expressed in Eqs. (22), (26), and (30). Eq. (36) is applicable for calculating the short-run marginal price elasticity for all three consuming sectors. Note that the lag variable $\ln x_{t-1}$ in Eq. (35) does not appear in the empirical model used for the residential and commercial sectors. Thus, no long-run marginal price elasticities are derived for these two sectors.

VII.B. Estimates of State-Level Electricity Demand Elasticities

As shown in the preceding section, both average price and marginal price elasticities can be computed in the model developed in this study. In addition to these price elasticities, state-level income elasticities can be computed using Eqs. (23) and (27), respectively, for the residential and commercial sectors. To compute these elasticities, the time series data are averaged as in Eq. (4), so the estimates represent elasticities for the individual states at average data points for the sample period, 1970-1980. These elasticities for the U.S. are computed using the sample means for all 48 states. The estimated elasticities of demand are reported in Tables 9, 10, and 11.

Consider the estimated demand elasticities for the residential sector (Table 9). Results indicate that the variation in elasticities among states is apparently substantial. The short-run average price elasticity of demand varies from -0.04 in North Dakota to -0.85 in Mississippi. The estimated marginal price elasticity is smaller in absolute value than the average price elasticity. The estimated income elasticity ranges from 0.04 in Alabama to 0.51 in Idaho. For the U.S. as a whole, the estimated short-run average price elasticity is -0.48, the short-run marginal price elasticity is -0.39 and the short-run income elasticity is 0.15. These estimated elasticities are all plausible.

Turning to the estimates of demand elasticities for the commercial sector, results show that the estimated average price elasticities all have correct signs, but as with those of the residential sector, they vary substantially among states. The average price elasticity of demand ranges from -0.03 in Florida to -0.94 in Idaho, with the mean for the

Table 9. Estimates of short-run price and income elasticities by state, residential sector

		AVERAGE PRICE	MARGINAL PRICE	INCOME
1	MAINE	-0.3494	-0.2797	0.2148
2	NEW HAMPSHIRE	-0.2512	-0.2002	0.2288
3	VERMONT	-0.3407	-0.2726	0.1851
4	MASSACHUSETTS	-0.4711	-0.3792	0.2200
5	RHODE ISLAND	-0.4629	-0.3725	0.2339
6	CONNECTICUT	-0.3497	-0.2799	0.1757
7	NEW YORK	-0.5329	-0.4302	0.2148
8	NEW JERSEY	-0.5310	-0.4286	0.1993
9	PENNSYLVANIA	-0.4190	-0.3365	0.1966
10	OHIO	-0.4008	-0.3216	0.1525
11	INDIANA	-0.4404	-0.3540	0.1329
12	ILLINOIS	-0.4865	-0.3918	0.2033
13	MICHIGAN	-0.3893	-0.3122	0.1648
14	WISCONSIN	-0.3970	-0.3185	0.1252
15	MINNESOTA	-0.3483	-0.2788	0.1759
16	IOWA	-0.3062	-0.2446	0.2010
17	MISSOURI	-0.5227	-0.4218	0.1966
18	NORTH DAKOTA	-0.0406	-0.0321	0.1968
19	SOUTH DAKOTA	-0.1581	-0.1255	0.2018
20	NEBRASKA	-0.4547	-0.3657	0.1280
21	KANSAS	-0.4935	-0.3977	0.1797
22	DELAWARE	-0.1940	-0.1542	0.2088
23	MARYLAND AND D C	-0.4770	-0.3841	0.1822
24	VIRGINIA	-0.5643	-0.4561	0.1196
25	WEST VIRGINIA	-0.4616	-0.3714	0.1773
26	NORTH CAROLINA	-0.5092	-0.4106	0.1042
27	SOUTH CAROLINA	-0.5149	-0.4153	0.1294
28	GEORGIA	-0.5913	-0.4786	0.1124
29	FLORIDA	-0.1622	-0.1288	0.1972
30	KENTUCKY	-0.6968	-0.5667	0.1249
31	TENNESSEE	-0.7894	-0.6448	0.3706
32	ALABAMA	-0.8436	-0.6908	0.0404
33	MISSISSIPPI	-0.8465	-0.6933	0.1224
34	ARKANSAS	-0.6556	-0.5323	0.2113
35	LOUISIANA	-0.6730	-0.5469	0.1969
36	OKLAHOMA	-0.4701	-0.3784	0.2212
37	TEXAS	-0.6003	-0.4861	0.1815
38	MONTANA	-0.3979	-0.3192	0.1043
39	IDAHO	-0.1092	-0.0854	0.5096
40	WYOMING	-0.4466	-0.3591	0.1551
41	COLORADO	-0.4271	-0.3431	0.1999
42	NEW MEXICO	-0.5157	-0.4160	0.2226
43	ARIZONA	-0.7307	-0.5952	0.1617
44	UTAH	-0.5441	-0.4394	0.1255
45	NEVADA	-0.4713	-0.3794	0.5044
46	WASHINGTON	-0.3546	-0.2939	0.2741
47	OREGON	-0.2513	-0.2002	0.3690
48	CALIFORNIA	-0.6474	-0.5254	0.1472
49	U S	-0.4824	-0.3885	0.1457

Table 10. Estimates of short-run price and income elasticities by state, commercial sector

		AVERAGE PRICE	MARGINAL PRICE	INCOME
1	MAINE	-0.6403	-0.5810	1.1128
2	NEW HAMPSHIRE	-0.4991	-0.4522	1.1828
3	VERMONT	-0.6744	-0.6121	1.0398
4	MASSACHUSETTS	-0.5664	-0.5136	1.0902
5	RHODE ISLAND	-0.5702	-0.5171	1.1029
6	CONNECTICUT	-0.4817	-0.4364	1.0192
7	NEW YORK	-0.5148	-0.4665	1.0369
8	NEW JERSEY	-0.4691	-0.4249	1.0232
9	PENNSYLVANIA	-0.5607	-0.5084	0.9820
10	OHIO	-0.6117	-0.5549	0.9479
11	INDIANA	-0.5993	-0.5436	0.9493
12	ILLINOIS	-0.4384	-0.3970	0.9991
13	MICHIGAN	-0.5095	-0.4617	1.0212
14	WISCONSIN	-0.6668	-0.6052	0.9857
15	MINNESOTA	-0.5633	-0.5107	1.0986
16	IOWA	-0.5076	-0.4600	1.0854
17	MISSOURI	-0.4780	-0.4331	1.0199
18	NORTH DAKOTA	-0.6033	-0.5472	1.0981
19	SOUTH DAKOTA	-0.5913	-0.5363	1.0917
20	NEBRASKA	-0.6743	-0.6120	0.8148
21	KANSAS	-0.4925	-0.4463	0.9201
22	DELAWARE	-0.4133	-0.3742	0.9624
23	MARYLAND AND D C	-0.3962	-0.3586	1.0057
24	VIRGINIA	-0.7175	-0.6515	0.8407
25	WEST VIRGINIA	-0.7695	-0.6991	0.9616
26	NORTH CAROLINA	-0.7919	-0.7196	0.8239
27	SOUTH CAROLINA	-0.7566	-0.6873	0.8869
28	GEORGIA	-0.5072	-0.4596	0.9955
29	FLORIDA	-0.0332	-0.0300	1.1201
30	KENTUCKY	-0.7406	-0.6727	0.8818
31	TENNESSEE	-0.5155	-0.4672	0.7068
32	ALABAMA	-0.6438	-0.5842	0.8763
33	MISSISSIPPI	-0.6765	-0.6141	0.9484
34	ARKANSAS	-0.6557	-0.5950	1.0049
35	LOUISIANA	-0.3831	-0.3467	0.9986
36	OKLAHOMA	-0.3879	-0.3511	0.9891
37	TEXAS	-0.2884	-0.2608	0.9062
38	MONTANA	-0.8304	-0.7549	0.8422
39	IDAHO	-0.9432	-0.8583	0.6126
40	WYOMING	-0.8706	-0.7918	0.7315
41	COLORADO	-0.6663	-0.6052	0.9243
42	NEW MEXICO	-0.6804	-0.6177	0.9543
43	ARIZONA	-0.4377	-0.3954	0.8753
44	UTAH	-0.8039	-0.7306	0.8853
45	NEVADA	-0.2410	-0.2178	0.7250
46	WASHINGTON	-0.7012	-0.6366	0.5050
47	OREGON	-0.7956	-0.7230	0.4813
48	CALIFORNIA	-0.5919	-0.5368	0.3549
49	U S	-0.5823	-0.5291	0.9264

Table 11. Estimates of price elasticities by State,
industrial sector

		AVERAGE PRICE		MARGINAL PRICE	
		SHORT-RUN	LONG-RUN	SHORT-RUN	LONG-RUN
1	MAINE	-0.6640	-1.5090	-0.7148	-1.6245
2	NEW HAMPSHIRE	-0.6536	-1.4855	-0.7037	-1.5921
3	VERMONT	-0.6828	-1.5518	-0.7352	-1.6708
4	MASSACHUSETTS	-0.5233	-1.1892	-0.5629	-1.2793
5	RHODE ISLAND	-0.6139	-1.3952	-0.6607	-1.5016
6	CONNECTICUT	-0.5547	-1.2605	-0.5958	-1.3563
7	NEW YORK	-0.5139	-1.1679	-0.5538	-1.2564
8	NEW JERSEY	-0.5317	-1.2083	-0.5720	-1.2999
9	PENNSYLVANIA	-0.5240	-1.1908	-0.5627	-1.2810
10	OHIO	-0.5405	-1.2284	-0.5815	-1.3216
11	INDIANA	-0.5595	-1.2716	-0.6021	-1.3683
12	ILLINOIS	-0.5239	-1.1907	-0.5626	-1.2809
13	MICHIGAN	-0.5280	-1.2000	-0.5681	-1.2910
14	WISCONSIN	-0.5619	-1.2769	-0.6086	-1.3740
15	MINNESOTA	-0.5825	-1.3238	-0.6268	-1.4245
16	IOWA	-0.6073	-1.3802	-0.6536	-1.4854
17	MISSOURI	-0.5778	-1.3132	-0.6218	-1.4131
18	NORTH DAKOTA	-0.7184	-1.6326	-0.7736	-1.7582
19	SOUTH DAKOTA	-0.7321	-1.6638	-0.7884	-1.7918
20	NEBRASKA	-0.6776	-1.5400	-0.7296	-1.6580
21	KANSAS	-0.6427	-1.4606	-0.6919	-1.5723
22	DELAWARE	-0.6788	-1.5427	-0.7308	-1.6609
23	MARYLAND AND D C	-0.5880	-1.3364	-0.6328	-1.4382
24	VIRGINIA	-0.6137	-1.3947	-0.6605	-1.5011
25	WEST VIRGINIA	-0.6642	-1.5094	-0.7150	-1.6250
26	NORTH CAROLINA	-0.6068	-1.3790	-0.6530	-1.4841
27	SOUTH CAROLINA	-0.6590	-1.4977	-0.7095	-1.6124
28	GEORGIA	-0.6181	-1.4046	-0.6652	-1.5118
29	FLORIDA	-0.6061	-1.3775	-0.6523	-1.4825
30	KENTUCKY	-0.6432	-1.4617	-0.6924	-1.5735
31	TENNESSEE	-0.6607	-1.5106	-0.7156	-1.6263
32	ALABAMA	-0.6672	-1.5163	-0.7183	-1.6324
33	MISSISSIPPI	-0.6688	-1.5200	-0.7200	-1.6364
34	ARKANSAS	-0.6840	-1.5544	-0.7364	-1.6736
35	LOUISIANA	-0.6587	-1.4969	-0.7091	-1.6115
36	OKLAHOMA	-0.6717	-1.5265	-0.7221	-1.6434
37	TEXAS	-0.5874	-1.3348	-0.6321	-1.4365
38	MONTANA	-0.8405	-1.9102	-0.9057	-2.0584
39	IDAHO	-0.7851	-1.7843	-0.8458	-1.9222
40	WYOMING	-0.7922	-1.8003	-0.8534	-1.9395
41	COLORADO	-0.6479	-1.4725	-0.6975	-1.5851
42	NEW MEXICO	-0.7509	-1.7066	-0.8089	-1.9391
43	ARIZONA	-0.6793	-1.5437	-0.7314	-1.6621
44	UTAH	-0.6813	-1.5482	-0.7335	-1.6669
45	NEVADA	-0.8452	-1.9208	-0.9108	-2.0699
46	WASHINGTON	-0.7578	-1.7222	-0.8162	-1.9550
47	OREGON	-0.7600	-1.7271	-0.8186	-1.9603
48	CALIFORNIA	-0.5213	-1.2074	-0.5715	-1.2999
49	U S	-0.6430	-1.4614	-0.6922	-1.5731

U.S. as a whole equal to -0.58. Similar to the residential sector, the marginal price elasticities of demand are all less than corresponding average price elasticities for the commercial sector. The average income elasticities in the commercial sector also vary substantially among states, ranging from 0.51 in Washington to 1.18 in New Hampshire, with a mean of 0.94 for the U.S. as a whole. A comparison of Tables 9 and 10 reveals that commercial users of electricity are generally more responsive to changes in electricity price and income. Price and income elasticities of demand for the commercial sectors are generally higher than those of the residential sector.

Table 11 presents the short-run and long-run price elasticities of demand in the industrial sector. Results show that the estimated short-run average price elasticities all have the correct sign and range from -0.51 in New York to -0.85 in Nevada. The estimated long-run elasticity ranges from -1.17 in New York to -1.92 in Nevada. Comparing the average price and marginal price elasticities, one finds that the latter is larger than the former for all states.

The estimated elasticities presented above appear to be more reasonable than those in the earlier study since they vary more smoothly across states and perhaps cover a more reasonable range. Although, in some cases, the elasticities are similar within regions (in which elasticities were previously assumed constant across states), there are also cases which suggest wide variation in elasticities within regions. For example, the price elasticity of demand in residential use appears to be quite different in Arizona than in the rest of the Mountain region. Similarly, the income elasticity of residential demand seems to differ considerably between Washington and California in the Pacific region.

The results of this section indeed imply substantial and explainable variation in elasticities. Furthermore, the elasticity equations suggest that elasticities will change over time since they depend on price and income as well as durable ownership and use. Since price and income changes can supposedly be predicted for the future, it stands to reason that consideration of the effects of these changes on elasticities should improve accuracy in forecasting electricity demand.

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VIII. CONCLUSIONS

In this study, a varying elasticity model with given appliances saturation is developed for estimating short-run electricity demand in the residential and commercial sector. The dynamic model for estimating industrial electricity demand is specified to allow electricity price elasticity to vary from state to state. The structural coefficients are estimated by nonlinear three-stage least squares using annual state data for 1955-1976. The regression results show that the variation of demand elasticities (such as price and income elasticities) is indeed explainable in the model. For example, the price elasticity of residential demand is dependent upon the levels of price and income, and the saturation levels of the major electric appliances. These results imply that each end-use of electricity has distinctive price elasticity impacts. The comparative analysis shows that space heating and air conditioning have higher price elasticities in absolute value than water heating and clothes drying.

Model estimates are derived using average electricity price in a simultaneous equation framework. One advantage of the model specification used in this study over the one employed by Halvorsen¹⁸ is that the relationship between marginal and average price is allowed to depend on quantity as one would expect. The explicit relationship between marginal and average price, thus, permits the estimated model to be interpreted in terms of marginal electricity price as well as average electricity price. Furthermore, both sets of elasticities can be computed in the model. The estimated demand elasticities show substantial variation among states. Also, there are differences between average price and marginal price elasticities; however, these differences are in general not dramatic.

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$$\begin{aligned} \ln q = & \gamma_0 + \gamma_1 \ln x_1 + \gamma_2 \ln x_2 + \gamma_3 \ln x_3 \\ & + \gamma_4 (\ln x_1)^2 + \gamma_5 (\ln x_2)^2 + \gamma_6 (\ln x_3)^2 \\ & + \gamma_7 (\ln x_1)(\ln x_2) + \gamma_8 (\ln x_1)(\ln x_3) \\ & + \gamma_9 (\ln x_2)(\ln x_3) . \end{aligned}$$

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