ADDITION OF ANS 1979 STANDARD DECAY HEAT TO RELAP5

R. J. Wagner

CONTENTS

ADDITION OF ANS 1979 STANDARD DECAY HEAT TO RELAP5

1. INTRODUCTION

The reactor kinetics model of RELAP5 computes both the immediate fission power and the power from decay of fission fragments using a space independent or point kinetics model. The immediate power is that released at the time of fission and includes power from fission fragment kinetic energy and neutron moderation. Decay power is generated as the fission products undergo radioactive decay. The original decay power model used an ANS standard[1] proposed in 1973. This model has been maintained and in addition, the user can select the fission product model from the 1979 ANS Standard for Decay Heat Power in Light Water Reactors.[2]

The actinide calculation (production of U^{239} by neutron absorption in U^{238} and subsequent two stage beta decay to Pu^{239}) has been changed to the 1979 standard. The new actinide calculation is used regardless of whether the 1973 or 1979 standard is used for fission products.

Reactor kinetics input defaults to the 1973 standard. Input changes are required if user specified actinide data are entered. Current input decks will get identical results if no actinides are used. Slightly different results will be obtained if default actinide data are specified. A new card selects the 1979 standard and enters additional data needed for some options with the 1979 standard.

2. FISSION FRAGMENT DECAY MODEL

The 1979 standard expresses the power $p_\alpha(t)$ in Mev/s as a function of time t resulting from one fission of isotope α at $t = 0$ as

$$p_\alpha(t) = \sum_{j=1}^{N_\alpha} a_{\alpha j} \exp(-\lambda_{\alpha j} t) \quad . \tag{1}$$

Data are presented for three isotopes, U^{235}, U^{238}, and Pu^{239}. The parameters a and λ were obtained by fitting to fission decay power data. The fitting for each isotope used 23 groups ($N_\alpha = 23$). The above expression is an impulse response to one fission and can be extended to an arbitrary fission rate $\psi_\alpha(t)$ through the convolution integral

$$p_\alpha(t) = \sum_{j}^{N_\alpha} a_{\alpha j} \exp(-\lambda_{\alpha j} t) * \psi_\alpha(t) \tag{2}$$

where the convolution operation is defined by

$$A(t)*B(t) = \int_0^\tau A(t - \tau)\, B(\tau)\, d\tau = \int_0^\tau A(\tau)\, B(t - \tau)\, d\tau \quad . \tag{3}$$

Since numerical evaluation of convolution integrals is cumbersome, a set of differential equations equivalent to the convolution integral is derived.

Assume that the power from each group is from radioactive decay of a fission fragment i. Then

$$p_{\alpha j}(t) = \lambda_{\alpha j}\, \gamma_{\alpha j} = a_{\alpha j} \exp(-\lambda_{\alpha j} t) \quad . \tag{4}$$

For simplification in the following derivation, the α and j subscripts are dropped and the following expressions represent an equation for one group for one isotope. From Equation (4),

$$\gamma(t) = \frac{a}{\lambda} \exp(-\lambda t) \quad . \tag{5}$$

Laplace transforming Equation (5),

$$\gamma(s) = \frac{a}{\lambda(s + \lambda)} \quad . \tag{6}$$

Rearranging

$$s\gamma(s) = \frac{a}{\lambda} - \lambda\gamma(s) \quad . \tag{7}$$

Transforming to real time

$$\frac{d\gamma(t)}{dt} = \frac{a}{\lambda} \delta(o) - \lambda\gamma(t) \tag{8}$$

where $\delta(o)$ is the impulse function. Applying a time dependent fission rate $\Psi(t)$ in place of the single fission (impulse response), Equation (7) and (8) become

$$s\ \gamma(s) = \frac{a}{\lambda} \Psi(s) - \lambda\ \gamma(s) \quad . \tag{9}$$

$$\frac{d\gamma(t)}{dt} = \frac{a}{\lambda} \Psi(t) - \lambda\ \gamma(t) \quad . \tag{10}$$

Solution of Equation (9) or (10) (and remembering that $p = \lambda\ \gamma$) for an impulse yields Equation (1) and a similar expression in the standard. Solution of Equation (9) or (10) for an arbitrary fission source yields Equation (2). When specifying

$$\begin{aligned} \Psi(t) &= 1 \quad 0 \le t \le T \\ &= 0 \quad t \ge T \end{aligned} \qquad (11)$$

Equations (9) and (10) yield another solution given in the standard. (Note that the standard defines t as starting at 0 after fissioning for T sec).

A physical model can be attached to the terms in Equation (10). The first term on the right represents production of the isotope during fission; the last term is the loss of the isotope due to decay. A more mechanistic model would provide for production of one isotope due to the decay of another. (See actinide model.)

As shown above, the 1979 ANS standard for decay heat can be implemented by advancing the following set of differential equations,

$$\frac{d\gamma_{\alpha j}(t)}{dt} = \frac{F_\gamma a_{\alpha j}}{\lambda_{\alpha j}} F_\alpha \Psi(t) - \lambda_{\alpha j} \gamma_{\alpha j}(t)$$

$$j = 1, 2, \ldots N_\alpha$$

$$\alpha = 1, 2, 3 \quad . \qquad (12)$$

$$P_\gamma(t) = \sum_{\alpha=1}^{3} \sum_{j=1}^{N_\alpha} \lambda_{\alpha j} \gamma_{\alpha j}(t) \qquad (13)$$

where Ψ is the fission rate from all isotopes, F_α is the fraction of fissions from isotope α, and P_γ is the decay power. Summation of F_α over α is 1.0. F_γ is a input factor to allow easy specification of a conservative calculation. It is usually 1.0 for best estimate calculations and 1.2 was recommended for a conservative calculation with the 1973 data. The 1979 data should allow consistent use of 1.0 for F_γ.

The 1973 proposed standard as implemented in RELAP5 used one isotope and prescribed data for 11 groups. The 1979 standard lists data for three isotopes, U^{235}, U^{238}, and Pu^{239}, and uses 23 groups for each isotope. A user option also allows only the 1979 standard data for U^{235} to be used. The data for both standards are built into the code as default data, but the user may enter different data.

3. ACTINIDE DECAY MODEL

Previously the actinide model was simply the optional selection of another isotope and would be identical to using two isotopes. The default data used two groups. The new capability uses

$$\frac{d\gamma_u(t)}{dt} = F_u \, \Psi(t) - \lambda_u \, \gamma_u(t) \tag{14}$$

$$\frac{d\gamma_N(t)}{dt} = \lambda_u \, \gamma_u(t) - \lambda_N \, \gamma_N(t) \tag{15}$$

$$P_\alpha = n_u \, \lambda_u \, \gamma_u + n_N \, \lambda_N \, \gamma_N \quad . \tag{16}$$

The quantity F_u is user input and is the number of atoms of U^{239} produced by neutron capture in U^{238} per fission from all isotopes. A conservative factor if desired should be factored into F_u. The λ and n valves can be user input or default values equal to those stated in the standard can be used.

The first equation describes the rate of change of atoms of U^{239}. The first term on the right represents the production of U^{239}; the last term is the loss of U^{239} due to beta decay. The second equation describes the rate of change of N_p^{239}. The production of N_p is from the beta decay of U^{239} and and Pu^{239} is formed from the decay of N_p^{239}. Solution of the actinide equations, Equations (14) and (15), for the fission source given in Equation (11) yields the result quoted in the 1979 standard. (Note the standard has a typographical error in that the argument of the last exponential factor should be $-\lambda_1 t$, not $-\lambda_1 T$.)

4. REACTOR KINETICS EQUATIONS

The point kinetics equations are

$$\frac{d\phi(t)}{dt} = \frac{[\rho(t) - \beta]\ \phi(t)}{\Lambda} + \sum_{i=1}^{N} \lambda_i\ C_i(t) + S \tag{17}$$

$$\frac{dC_i(t)}{dt} = \frac{\beta\ f_i}{\Lambda}\phi(t) - \lambda_i\ C_i(t) \quad i = 1, 2, \ldots N \quad , \tag{18}$$

$$\Psi(t) = \Sigma_f\ \phi(t) \tag{19}$$

$$P_f(t) = Q_f\ \Psi(t) \tag{20}$$

where

ϕ = neutron flux

C_i = the number of delayed neutron precursors of group i

β = the effective delayed neutron fraction

Λ = the prompt neutron generation time

ρ = the reactivity

f_i = the fraction of delayed neutrons of group i

λ_i = the decay constant of group i

S = the source

Σ_f = the fission cross section

P_f = the immediate fission power in MeV/s

Q_f = the immediate fission energy per fission in MeV.

5. TRANSFORMATION OF EQUATIONS FOR SOLUTION

The differential equations to be advanced in time are Equation (17), (18), (12), (14), and (15). (The equations are ordered in storage as listed for programming convenience and to enhance vectorization.) Multiplying by Σ_f and X, the conversion from MeV/s to watts, as needed, the equations become,

$$\frac{d}{dt}[X\ \Psi(t)] = \frac{[\rho(t) - \beta]\ x\ \Psi(t)}{\Lambda} + \sum_{i=1}^{N} \lambda_i\ X\ \Sigma_f\ C_i(t) \qquad (21)$$

$$\frac{d}{dt}[X\ \Sigma_f\ C_i(t)] = \frac{\beta f_i\ X\ \Psi(t)}{\Lambda} - \lambda_i\ X\ \Sigma_f\ C_i(t)$$

$$i = 1, 2, \ldots, N \qquad (22)$$

$$\frac{d}{dt}[X\gamma_{\alpha j}(t)] = \frac{F_\gamma\ a_{\alpha j}\ F_\alpha\ X}{\lambda_{\alpha j}}\Psi(t) - \lambda_{\alpha j}\ X\ \gamma_{\alpha j}(t)$$

$$j = 1, 2, \ldots, N_\alpha$$

$$\alpha = 1, 2, 3 \qquad (23)$$

$$\frac{d}{dt}[X\ \gamma_U(t)] = F_U\ X\ \Psi(t) - \lambda_U\ X\ \gamma_U(t) \qquad (24)$$

$$\frac{d}{dt}[X\ \gamma_N(t)] = \lambda_U\ X\ \gamma_U(t) - \lambda_N\ X\ \gamma_N(t)\quad . \qquad (25)$$

The total power P_T is the sum of immediate fission power, fission product decay, and actinide decay, and now in units of watts is

$$P_T(t) = Q_F\ X\ \Psi(t) + \sum_{\alpha=1}^{3}\sum_{j=1}^{N_\alpha} \lambda_{\alpha j}\ X\ \gamma_{\alpha j}(t) + \eta_u\ \lambda_u\ X\ \gamma_u(t)$$

$$+ \eta_N\ \lambda_N\ X\ \gamma_N(t) \quad . \tag{26}$$

For solution convenience, the following substitutions are made,

$$\rho(t) = \beta\ r(t) \tag{27}$$

$$X\ \Psi(t) = \Psi'(t) \tag{28}$$

$$X\ \frac{\Lambda S}{\beta} = S' \tag{29}$$

$$X\ \Sigma_f\ C_i(t) = \frac{\beta\ f_i\ W_i(t)}{\Lambda\ \lambda_i} \tag{30}$$

$$X\ \gamma_{\alpha j}(t) = \frac{F_\gamma\ a_{\alpha j}\ F_\alpha}{\lambda_{\alpha j}^2}\ Z_{\alpha j}(t) \tag{31}$$

$$X\ \gamma_u(t) = \frac{F_u}{\lambda_u}\ Z_u(t) \tag{32}$$

$$X\ \gamma_N(t) = Z_N(t) \quad . \tag{33}$$

The equations to be advanced are now,

$$\frac{d\Psi'}{dt}(t) = \frac{\beta}{\Lambda}\left[[r(t) - 1]\ \Psi'(t) + \sum_{i=1}^{N} f_i\ W_i(t) + S' \right] \tag{34}$$

$$\frac{d}{dt}W_i(t) = \lambda_i \Psi'(t) - \lambda_i W_i(t) \quad i = 1, 2, \ldots, N \tag{35}$$

$$\frac{d}{dt}Z_{\alpha j}(t) = \lambda_j \Psi'(t) - \lambda_j Z_j(t) \quad j = 1, 2, \ldots, N_\alpha$$

$$\alpha = 1, 2, 3 \tag{36}$$

$$\frac{d}{dt}Z_u(t) = \lambda_u \Psi'(t) - \lambda_u Z_u(t) \tag{37}$$

$$\frac{d}{dt}Z_N(t) = F_u Z_u(t) - \lambda_N Z_N(t) \tag{38}$$

$$P_T(t) = Q_f \Psi'(t) + \sum_{\alpha=1}^{3} \sum_{j=1}^{N_\alpha} \frac{F_\gamma a_{\alpha j} F_\alpha Z_{\alpha j}(t)}{\lambda_{\alpha j}}$$

$$+ F_u \eta_u Z_u(t) + \eta_N \lambda_N Z_N(t) \quad . \tag{39}$$

These equations are advanced using the modified Runge-Kutta method described in the RELAP5 Users Manual.[3] Because of the format of the equations, extensive vectorization is possible with suitable organization. Since the default number of equations with the 1979 standard and using three isotopes is 78, a compile time option to use Cyber-176 vector subroutines has been implemented.

6. INITIALIZATION

Two initialization options are provided. In both options, the fission rate and delayed neutrons are in steady state or equilibrium conditions, that is, their time derivitives are zero. With r(o) an input quantity,

$$W_i(o) = \Psi(o) \quad i = 1, 2, \ldots, N \tag{40}$$

$$S' = -r(o)\ \Psi(o) \quad . \tag{41}$$

The first option assumes that the fission product decay and actinides are also in equilibrium. This is equivalent to assuming that the reactor has been operating at a constant total power for an infinite period of time. The initial conditions are

$$Z_{\alpha j}(o) = \Psi(o) \quad j = 1, 2, \ldots, N_\alpha$$

$$\alpha = 1, 2, 3 \tag{42}$$

$$Z_u(o) = \Psi(o) \tag{43}$$

$$Z_N(o) = \frac{F_u}{\lambda_u}\ \Psi(o) \tag{44}$$

$$P_T(o) = Q\ \Psi(o) \tag{45}$$

$$Q = Q_f + \sum_{\alpha=1}^{3} \sum_{j=1}^{N_\alpha} \frac{F_\gamma\ a_{\alpha j}\ F_\alpha}{\lambda_{\alpha j}} + F_u \eta_u + F_u \eta_N \quad . \tag{46}$$

The quantity Q, which is the total energy in MeV generated per fission, is either an input value or can be defaulted to 200 MeV. The quantity Q_f is defined from Equation (46) and the user input or defaulted data even if the second initialization option is used. The total power is an input quantity, and the source Ψ is computed from Equation (45).

The second option uses a power history to determine the initial values of the fission product and actinide quantities. The power history consists of one or more periods of constant total power. For each period, the input consists of the total power, the time duration at that power, and in the case of three isotopes, the fraction of power from each isotope. The fission product and actinide differential equations, Equations (36), (37), and (38) are advanced in time starting with initial values of zero. The fission rate Ψ is defined from Equation (39). The fission rate is reset to zero whenever a negative value is computed. This would occur whenever the user entered total power is less than the current fission product and actinide decay power. Thus for shutdown periods, the user may conveniently enter zero total power even though significant decay power remains. The fission product and actinide values at the end of the power history become the initial values for the transient. The initial fission rate is computed from Equation (39) using the total reactor power at the start of the transient (which may be different from the last power history value). If this fission rate is negative or zero, it is reset such that the immediate fission power is 10^{-12} times the decay power.

The differential equations for the power history calculation are advanced using the same numerical technique as for the transient advancement except for a simplified time step control. Time step control consists of starting the advancement of each history period with a time step of 1 s. The time step is doubled after each advancement. When the next advancement would exceed the time duration, the last advancement is with the remaining time. This scheme was selected since with each different power value, the solution moves toward a new equilibrium condition asymptotically and the most rapid change is at the beginning of a power change.

7. OUTPUT QUANTITIES

The following quantities are edited in major edits and are available for minor edits. Symbols in parentheses are the minor edit request names.

1. Total power (RKTPOW), $P_T(t)$
2. Immediate fission power (RKFIPOW), $Q_f \Psi'(t)$
3. Power from fission product and actinide decay (RKGAPOW), $P_T(t) - Q_f\Psi'(t)$
4. Reactivity (RKREAC), $r(t)$
5. Average reciprical period ω (major edit only) defined each time step by $\Psi(t + \Delta t) = \Psi(t) \exp(\omega\Delta t)$.

The edits of input data show all reactor kinetics data whether entered by the user or defaulted. The data entered and edited are: for delayed neutron data, f_i, λ_i; for fission product decay, a_j/λ_j, λ_j; and for actinides, η_u, λ_u, η_N, λ_N. The use of a_j/λ_j is a carryover from the 1973 standard and will be changed to a_j as time permits. This will not affect current use since the default data is recommended.

8. TESTING

Testing consisted of qualitative checking and some quantitative checking. The qualitative checks began by comparing results from the new and prior versions where identical results are expected. Then, a series of runs with no reactor feedback and specifying the ANS73, ANS79-1 and ANS79-3 default data with and without actinides were made. Reactor power histories with constant power and varying time durations were entered. Input editing includes the power from immediate fission power and decay power for an infinite operating time. The power history results should approach the infinite results for increasing time durations. With no feedback, the reactor power should be constant during transient advancement. The results of the qualitative testing were satisfactory.

The quantiative test used the ANS79-3 option with user supplied data for fission fragment decay. Only one group for each isotope was used to simplify hand calculations. The power values for infinite time duration were hand checked. The transient response to a constant reactivity insertion was checked. The asymptotic response to constant, positive reactivity is an exponential rise in immediate fission and decay powers with the reciprical period ω given by

$$r = \omega \left[\frac{\Lambda}{\beta} + \sum_{i=1}^{I} \frac{f_i}{\omega + \lambda_i} \right] .$$

The program generated the correct exponential rise. Qualitatively, the program showed the expected initial jump in immediate fission power and transition to exponential rise.

The reactor kinetics coding consists of two R level input subroutines, an I level input subroutine, and a transient subroutine. The majority of the changes were in the R level subroutines. The I level subroutine was unchanged. The transient subroutine was changed only to reflect actinide changes. Additional coding was added to the transient subroutine to use vector subroutines. Either vector or standard DO loops are selectable by a compile time option.

The above testing is considered adequate when considering where the significant modifications were made.

REFERENCES

1. American Nuclear Society Proposed Standard, ANS 5.1 "Decay Energy Release Rates Following Shutdown of Uranium-Fueled Thermal Reactors," October 1971, revised October 1973.

2. American National Standard for Decay Heat Power in Light Water Reactors, ANSI/ANS-5.1-1979.

3. V. H. Ransom et al., "RELAP5/MOD1 Code Manual Volume 1: System Models and Numerical Methods," NUREG/CR-1826, EGG-2070, March 1982.

EG&G Idaho, Inc.

PDR

Report No. WR-NSMD-83-003

Date: January 1983

NRC Research and/or Technical Assistance Rept

INTERNAL TECHNICAL REPORT

Title: ADDITION OF ANS 1979 STANDARD DECAY HEAT TO RELAP5

Organization: NUCLEAR SAFETY METHODS DIVISION

Author: R. J. Wagner

Checked By: VHRansom

Approved By: J. M. Howe

~~THIS DOCUMENT HAS NOT RECEIVED PATENT CLEARANCE AND IS NOT TO BE TRANSMITTED TO THE PUBLIC DOMAIN~~