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Semiscale Uncertainty Report: Methodology

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ABSTRACT

Definitions of statistical terms and methods used to derive uncertainty estimates for experimental measurements at the Semiscale Test Facility are presented in this report. Error propagation equations are developed to aid in determining uncertainties for complex calculations. Uncertainty estimates of the data system are presented, as well as the other types of errors, which are analyzed in more detail in subsequent volumes that will cover each of the various types of measurements.

SUMMARY

Uncertainty estimates for Semiscale experimental test data are calculated by the Measurement systems Engineering section. The purpose of this report is to provide a referencable document describing the error sources considered and the methods used to calculate uncertainty estimates for Semiscale test data. Uncertainty estimates at a 95% confidence level are required by the Nuclear Regulatory Commission in all reports based on experimental test data from the Idaho National Engineering Laboratory.

The International Standards Organization's draft standard, "Fluid Flow Measurement Uncertainty," (ISO TC30 SC9) is the basis of the method used at Semiscale. This method has been accepted by several technical societies such as the American Society of Mechanical Engineers and the Instrument Society of America.

All measurements are subject to two types of errors, i.e. bias (systematic) and precision (random). Calibration data (transducer, special, and others) are used as the primary sources of estimates of the various errors. The root-sum-square method is used to accumulate the bias and precision errors separately. The root-sum-square method is used to combine the accumulated bias and precision errors for the uncertainty estimate.

All uncertainty estimates of Semiscale experimental test data are at the 95% confidence level. This represents the uncertainty band outside of which only one data value out of twenty will fall, on the average.

This document, including subsequent volumes, represents the pretest uncertainty analysis. Factors that are expected to be reviewed during a posttest analysis are discussed in the volumes to be issued subsequent to this report. Results of posttest analysis will need to be addressed in the experimental data reports.

FOREWORD

This document (NUREG/CR-2459, EGG-2142) reports the methods used for uncertainty analyses of Semiscale test measurements. Measurement uncertainty analyses are performed to evaluate the anticipated performance for each experimental measurement in the Semiscale system. Results of these analyses are reported in both experimental data reports and topical reports that cover the measurements made for a given test. The subsequent volumes of this report will provide more detail on the analysis of each type of measurement. Measurement uncertainty is required by the Nuclear Regulatory Commission in all reports using experimental data.

ACKNOWLEDGMENTS

CONTENTS

NOMENCLATURE

Accuracy

The closeness or agreement between a measured value and a standard or true value; uncertainty as used herein, is the maximum inaccuracy or error that may reasonably be expected. (See measurement error and total estimation error.)

Bias (β)

The difference between the average of all possible measured values and the true or standard value. The fixed or systematic error that characterizes every member of a set of measurements. The sign may or may not be known on the bias value.

Calibration Hierarchy

The chain of calibrations that link or trace a measuring instrument to the National Bureau of Standards.

Confidence

The value of $1 - \alpha$ of the probability associated with a confidence interval or a statistical tolerance interval. The percentage frequency that an interval estimate of a parameter contains the true value. Ninety-five percent confidence is used in this report, implying that one out of twenty values may lie outside the stated interval.

Degrees of Freedom (ν)

A sample of N values is said to have M degrees of freedom, where $M = N - k$, and where k functions of the sample values are held constant. For example, using a least-square regression curve fit of a fourth-order, $k = 5$; this is because five parameters are held constant (A_0 to A_4).

Elemental Error

The bias and/or precision error associated with a sample source or process in a chain of sources or processes. Lower case symbols (s, b) are used to represent elemental precision or bias errors, respectively.

Estimate

A value calculated from a sample of data as a substitute for an unknown population constant. For example, the sample standard deviation (s) is the estimate that describes the population standard deviation (σ).

Laboratory Standard

An instrument that is calibrated periodically at the National Bureau of Standards (NBS). The laboratory standard may also be called an interlab standard.

Measurement Error

The collective term meaning the difference between the true value and the measured value. Includes both bias and precision error; see accuracy and uncertainty. Accuracy implies small measurement error and small uncertainty.

Precision

The closeness of agreement between the results obtained by applying the experimental procedure several times under prescribed conditions. The smaller the random part of the experimental errors that affect the results, the more precise the procedure.

Preliminary Data

Recorded data that have not been evaluated and determined to be true representative of the condition that existed during the test within defined error bands.

Precision Error (S)

The random error observed in a set of repeated measurements. This error is the result of a large number of small effects—the known repeatability error and sampling error. The precision index, S, defined herein is the computed standard deviation of the measurements.

Qualified Data	Information that has been evaluated and verified to truly represent the associated measure and information desired within defined and defendable uncertainty limits.
Qualified Test	A test that, after review of the preliminary data available, can be determined to have met the requirements and intent (within physically reliable limits) as to "contracted" information to be supplied to the requesting source.
Range	The difference between the greatest and the smallest observed values of a quantitative characteristic. Two ranges are used in this report; the mechanical range that is given by the manufacturer's specification, and the electronic full-scale, which is the applied signal that will deliver 10 V to the data system with appropriate amplifier gain.
Standard Deviation (σ)	The most widely used measure of dispersion of a frequency distribution. It is the precision index and the square root of the variance: s is an estimate of a σ calculated from a sample of data.
Standard Error	The standard deviation of an estimator; the standard error provides an estimate of the random part of the total estimation error involved in estimating a population parameter from a sample.
Standard Error of Estimate (S_{ee})	The measure of dispersion of the dependent variable (output) about the least-squares line in curve fitting or regression analysis. It is the precision index of the output for any fixed level of the independent variable (input). Also called the Residual Standard Deviation, the formula for calculating this is

$$S_{ee} = \left[\sum_{i=1}^{n} \frac{(Yobs - Ycal)^2}{N - k} \right]^{1/2}$$

	for a curve fit of N data points in which k constants are estimated for the curve.
Student-t Distribution	The ratio of the difference between the population mean and the sample mean to a sample standard deviation (multiplied by a constant) in a sample from a normal population. It is used to set confidence limits for the population mean, and it is obtained from tables entered with degrees of freedom and risk level. For a 95% confidence level, a 5% two-tailed risk level is used, and if the degrees of freedom are greater than 30, the value of 2 is used as an estimate of t.
Total Estimation Error	In the estimation of the parameter, the difference between the calculated value of the estimator and the true value of this parameter.
Traceability	The ability to trace the calibration of a measuring device through a chain of calibration to the NBS.
Transfer Standard	A laboratory instrument that is used to calibrate working standards and that is periodically calibrated against the laboratory standard.

Uncertainty Interval (U)   The maximum error reasonably expected for the defined measurement process.

$$U_{95} = \pm \left\{ B^2 + [t_{95}(\nu)\ S]^2 \right\}^{1/2}$$

Welch-Satterthwaite   A formula that is used to estimate the degrees of freedom (ν) when combin-
Formula   ing precision errors of different degrees of freedom. The degrees of freedom for the measurement precision index, S, is

$$\nu = \frac{\left(\sum_i S_i^2 \right)^2}{\sum_i S_i^4 / \nu_i}$$

where ν_i is the degrees of freedom and S_i is the sample standard deviation for error component i. The equation is a function of the degrees of freedom and magnitude of each elemental precision index.

SEMISCALE UNCERTAINTY REPORT: METHODOLOGY

1. INTRODUCTION

This document defines the techniques and the data sources used to estimate uncertainty values of measurements in the Semiscale system.[1,2] The measurements considered here are only those used for experiments, and those operational measurements that have been reviewed and "qualified"[a] for reports such as the Semiscale Experimental Data Reports.

1.1 Objective

The objective is to present the method used for treating measurement error or uncertainty for measurements of Semiscale experimental data. The need for a common method is obvious to those who have reviewed the numerous methods currently used. This report attempts to use the method outlined in International Standards Organization (ISO) Report ISO TC30 SC9 "Fluid Flow Measurement Uncertainty,"[3] the technical contents of which were approved by ISO member nations at a meeting in Leningrad, USSR, in June 1982.

1.2 Scope

This report presents a working outline detailing and illustrating the techniques used for estimating measurement uncertainty. Each type of measurement provides an example of the parameters and techniques used to calculate measurement uncertainty. Additional volumes will be published to cover particular measurement types; temperature, pressure, etc.

An effort has been made to use simple prose with a minimum of jargon. The notation and definitions given are consistent with ISO 3534, "Statistics—Vocabulary and Symbols."

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a. Measurements are qualified according to the requirements of an internal working document of the Semiscale Program.

2. UNCERTAINTY ESTIMATES

Estimates of uncertainty must be provided for all experimental results reported by organizations within the Nuclear Technology Department of EG&G Idaho; the Semiscale Program is one such organization.

Experimental data are used extensively to check calculations and to support decisions affecting nuclear system safety. Users of these data may not be associated with the experimental program; therefore, the group performing an experiment has an obligation to supply the users of the data with uncertainty estimates for all reported quantities.

Uncertainties fall into two categories; the first is the statistical and metrological uncertainties, and the second is the mistakes or "blunders," and failures. Statistical and metrological uncertainties are expected and ideally will be estimated in advance. Mistakes or blunders and failures are not expected, but if detected must be addressed if reported.

Instrument failures or mistakes, if detected, are addressed in the Experimental Data Reports (EDRs) in a table with a label of data presentation. Any subsequent technical report will address these problems in the body of the text, only if the measurement is reported. Measurements with failures or mistakes will not be published, unless the measurement contains some significant information. If a measurement has any known problem, it will not have statistical uncertainty published with it. Because failures or mistakes are not known in advance, they are beyond the scope of this report.

This report will concern i self with the statistical and metrological type of uncertainties.

Uncertainty is an estimate of the test measurement which, in most cases, would not be exceeded. Measurement error, δ, has two components; a fixed error, β, and a random error, ϵ.

To decide if a given elemental source contributes to bias, precision, or both, we adopt the recommendation:[4] "The uncertainty of a measurement should be put into one of two categories depending on how the uncertainty is derived. A random uncertainty derived by a statistical analysis of repeated measurements, while a systematic uncertainty is estimated by nonstatistical methods." This recommendation avoids a complex decision and keeps the statistical estimates separate from the judgment estimates as long as possible.

In making uncertainty analyses, definition of the measurement process is of utmost importance. Uncertainty statements must be based on a well-defined measurement process. The uncertainty of a measurement process will contain errors due to variations between calibrations, test facilities, and measurement instruments. The uncertainty analysis for a comparative, back-to-back test, to compare the effects of changed variables or conditions, will be different from the uncertainty analysis for a single test. Bias may be ignored in comparative testing if the same equipment is used for all testing, and biases do not affect the comparison of one test with another.

At the Semiscale Test Facility, there has been the tendency over several years to install the same instrument on the same data channel to make the same measurement. This practice would tend to reduce the effects of the instrument and data channel biases for a test-to-test comparison, but would tend to make it more difficult to determine values of biases for a single test. Uncertainty values given are on a single test basis. If the tests are being used for a test-to-test comparison, the uncertainties could be reduced slightly. However, some instruments undoubtedly have been changed or replaced, so this reduction of uncertainty should be done only after verifying that the instrumentation has not been changed.

2.1 Precision (Random Error)

Measurements do not, and are not expected to, agree exactly. There are numerous small effects that can cause disagreements, or random error (ϵ). The variation between repeated measurements is called precision error. The standard deviation (σ) is used as a measure of the precision error, ϵ. A large standard deviation means large scatter in the measurements. The statistic (s) is calculated to estimate the standard deviation and is called the precision index

$$s = \left[\frac{\sum_{i=1}^{n} (X_i - \bar{X})^2}{n - 1} \right]^{1/2} = \left[\frac{\sum_{i=1}^{n} X_i^2 - \left(\frac{\sum_{i=1}^{n} X_i}{n} \right)^2}{n - 1} \right]^{1/2} \tag{1}$$

where n is the number of measurements made and $\tilde{X}$ is the average value of individual measurements X_i.

The precision error of the measurement can often be reduced by taking several repeated or simultaneous observations and averaging. The distribution of the averages will have a smaller precision index

$$\sigma_{average} = \frac{\sigma_{individuals}}{\sqrt{n}} \quad \text{and} \quad S_{\bar{X}} \approx \frac{S}{\sqrt{n}} \quad .$$

Throughout this document, the precision index is the sample standard deviation of the measurement, whether it is a single reading or the average of several readings.

The precision index (s) is calculated in many ways:

1. If the variable to be measured can be held constant, a number of repeated measurements can be used to evaluate Equation (1) as an estimate of the precision index.

2. If there are k redundant instruments and the variable to be measured can be held constant to take i repeated readings, the following pooled estimate of the precision index should be used:

$$s = \left[\frac{\sum_{n=1}^{k} \sum_{m=1}^{i} (Xmn - \bar{X}n)^2}{(k * i) - k} \right]^{1/2} \tag{2}$$

3. If an instrument is calibrated and the regression (least-squares) curve fit is used to estimate the applied signal (inverse estimation), then the standard error of estimate multiplied by the regression factor (R) (see Section 4.3, "Transducer Errors") is used to estimate the precision index.

$$s = R \, S_{ee} = R \left[\frac{\sum_{i=1}^{n} (X_{applied} - X_{calculated})^2}{n - k - 1} \right]^{1/2} \quad . \tag{3}$$

where n is the number of calibration points and k is the order of the curve fit.

4. If a pair of instruments is used to measure a variable that is not constant with time, the difference between the readings may be used to estimate the precision of the individual instruments as follows: let $\theta_i = X_{1i} - X_{2i}$

$$s = \frac{S\theta}{\sqrt{2}} = \left[\frac{\sum_{j=1}^{i} \theta_j - \bar{\theta}}{2(i - 1)} \right]^{1/2} \quad . \tag{4}$$

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5. For sample sizes of 10 or less, the range (largest minus smallest) may be used to estimate the precision index.[5] There is loss of degrees of freedom with this technique, and the estimate given by this method is less precise than those above, but it is less complex than Equation (1). The procedure is to estimate s by:

$$s = \frac{\bar{R}}{d_2^*} \quad .$$

Values of d_2^* and degrees of freedom, ν, are taken from Table 1, and $\bar{R}$ is the average range based on (g) samples of size (m).

In general, the degrees of freedom will be given approximately by the reciprocal of $(\pm 2\sqrt{1 + 2 (c.v.)^2/g})$, where c.v. is the coefficient of variation (d_3/d_2) of the range and g is the number of subgroups. Values for d_2 and d_3 are given in Table 2. Both d_2 and d_3 are taken as belonging to a normal universe w, where w equals R/σ'. By definition, d_2 equals the mean value of w, and d_3 equals the reciprocal of σ_w (σ_w is the standard deviation of w). Also, d_2^* is given approximately by d_2 (i.e., the infinity value of d_2) times $(1 + 1/4 \nu)$. Values of ν are also readily built up from the constant differences. Table 1 is a basic table that may be used whenever the average range is used in lieu of s.

2.2 Bias (Fixed Error)

Bias, β, is the fixed or constant components, also called systematic error. A bias error is relatively fixed for the duration of a test. For repeated measurements, each one has the same bias. The bias cannot be determined unless the measurements are compared with the true value of the quantity measured.

Bias is categorized into five classes: (a) known biases calibrated out; (b) known biases ignored; (c) unknown biases eliminated by control of the measurement process; (d) small unknown biases that may have an unknown sign ($\pm$); and (e) small unknown biases with known sign and which contribute to the uncertainty.

2.2.1 Known Biases—Calibrated Out. Known biases are eliminated by comparing the instrument with a standard instrument and obtaining a correction. This process is called calibration or in-place calibration, which diminishes the bias of the measurement and introduces a random uncertainty that will be discussed later.

2.2.2 Known Biases—Ignored. If known biases are considered to be negligible relative to the test objective, they may be ignored.

2.2.3 Unknown Biases Eliminated by Control of the Measurement Process. Unknown biases are not correctable although they may exist. Every effort is made to eliminate all significant biases in order to secure a properly controlled measurement process.

To ensure control, all measurements are monitored with statistical quality control chart methods.[a] Drifts, trends, and movements leading to out-of-control situations should be identified and investigated. Histories of data from calibrations are maintained in an active file while an instrument exists. Historical calibration data are transferred to an inactive file for several years after an instrument is disposed of. These precautions are observed and all measurement data are reviewed and "qualified" before the uncertainty values are considered valid.

Data are qualified in two stages. The first stage is test qualification, in which the preliminary data are reviewed to determine if the test requirements have been met. All measurements are reviewed and compared to other measurements for consistency. Any measurement that is slightly suspicious is flagged for further review. The second stage is data qualification. After all corrections have been applied, all measurements are evaluated and verified to truly represent the associated measurand information desired within defined and defendable uncertainty limits.

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a. These requirements are set forth in an internal working document of the EG&G Idaho Nuclear Technology Department.

Table 1. Values associated with the distribution of the average range[5]

	Number of Observations Per Sample																	
	2		3		4		5		6		7		8		9		10	
Number of Samples	v	d_2^*	v	d_2^*	v	d_2^*	v	d_2^*	v	d_2^*	v	d_2^*	v	d_2^*	v	d_2^*	v	d_2^*
1	1.0	1.41	2.0	1.91	2.9	2.24	3.8	2.48	4.7	2.67	5.5	2.83	6.3	2.96	7.0	3.08	7.7	3.18
2	1.9	1.28	3.8	1.81	5.7	2.15	7.5	2.40	9.2	2.60	10.8	2.77	12.3	2.91	13.8	3.02	15.1	3.13
3	2.8	1.23	5.7	1.77	8.4	2.12	11.1	2.38	13.6	2.58	16.0	2.75	18.3	2.89	20.5	3.01	22.6	3.11
4	3.7	1.21	7.5	1.75	11.2	2.11	14.7	2.37	12.3	2.57	21.3	2.74	24.4	2.88	27.3	3.00	30.1	3.10
5	4.6	1.19	9.3	1.74	13.9	2.10	18.4	2.36	22.6	2.56	26.6	2.73	30.4	2.87	34.0	2.99	37.5	3.10
6	5.5	1.18	11.1	1.73	16.6	2.09	22.0	2.35	27.1	2.56	31.8	2.73	36.4	2.87	40.8	2.99	45.0	3.10
7	6.4	1.17	12.9	1.73	19.4	2.09	25.6	2.35	31.5	2.55	37.1	2.72	42.5	2.87	47.5	2.99	52.4	3.10
8	7.2	1.17	14.8	1.72	22.1	2.08	29.3	2.35	36.0	2.55	42.4	2.72	48.5	2.87	54.3	2.98	59.9	3.09
9	8.1	1.16	16.6	1.72	24.8	2.08	32.9	2.34	40.5	2.55	47.7	2.72	54.5	2.86	61.0	2.98	67.3	3.09
10	9.0	1.16	18.4	1.72	27.6	2.08	36.5	2.34	44.9	2.55	52.9	2.72	60.6	2.86	67.8	2.98	74.8	3.09
11	9.9	1.16	20.2	1.71	30.3	2.08	40.1	2.34	49.4	2.55	58.2	2.72	66.6	2.86	74.6	2.98	82.3	3.09
12	10.8	1.15	22.0	1.71	33.0	2.07	43.7	2.34	53.9	2.55	63.5	2.72	72.7	2.85	81.3	2.98	89.7	3.09
13	11.6	1.15	23.9	1.71	35.7	2.07	47.4	2.34	58.4	2.55	68.4	2.71	78.7	2.85	88.1	2.98	97.2	3.09
14	12.5	1.15	25.7	1.71	38.5	2.07	51.0	2.34	62.8	2.54	74.0	2.71	84.7	2.85	94.8	2.98	104.6	3.08
15	13.4	1.15	27.5	1.71	41.2	2.07	54.6	2.34	67.3	2.54	79.3	2.71	90.8	2.85	101.6	2.98	112.1	3.08
d_2		1.13		1.69		2.06		2.33		2.53		2.70		2.85		2.97		3.0?
c.d.[a]	0.88		1.82		2.74		3.62		4.47		5.27		6.03		6.76		7.4?	

a. c.d. = constant difference

Table 2. Percentage points of the distribution of the relative range $\omega = R/\sigma'$, normal universe[5]

η	Mean ω or d_2	σ'_ω or d_3	Probability That ω is Less than or Equal to Tabular Entry								
			0.001	0.005	0.01	0.050	0.950	0.975	0.990	0.995	0.999
2	1.128	0.8525	0.00	0.01	0.02	0.09	2.77	3.17	3.64	3.97	4.65
3	1.693	0.8884	0.06	0.13	0.19	0.43	3.31	3.68	4.12	4.42	5.06
4	2.059	0.8798	0.20	0.34	0.43	0.76	3.63	3.98	4.40	4.69	5.31
5	2.326	0.8641	0.37	0.55	0.66	1.03	3.86	4.20	4.60	4.89	5.48
6	2.534	0.8480	0.54	0.75	0.87	1.25	4.03	4.36	4.76	5.03	5.62
7	2.704	0.833	0.69	0.92	1.05	1.44	4.17	4.49	4.88	5.15	5.73
8	2.847	0.820	0.83	1.08	1.20	1.60	4.29	4.61	4.99	5.26	5.82
9	2.970	0.808	0.96	1.21	1.34	1.74	4.39	4.70	5.08	5.34	5.90
10	3.078	0.797	1.08	1.33	1.47	1.86	4.47	4.79	5.16	5.42	5.97
11	3.173	0.787	1.20	1.45	1.58	1.97	4.55	4.86	5.23	5.49	6.04
12	3.258	0.778	1.30	1.55	1.68	2.07	4.62	4.92	5.29	5.54	6.09

Data that do not meet all requirements of qualification are either deleted or have notations identifying the restrictions.

2.2.4 Remaining Biases, Unknown Signs, and/or Unknown Magnitudes—Contribution to Uncertainty. In most cases, the bias error is equally likely to be plus or minus about the measurement. That is, it is not known if the bias error is positive or negative, and the bias limit reflects this. The bias limit B, is estimated as an upper limit on the fixed error, β.[3]

It is both difficult and frustrating to estimate the limit of an unknown bias. To determine the exact bias in a measurement, it would be necessary to compare the true value with the measurements. This is almost always impossible. An effort must be made to obtain special tests or data that will provide bias information. The following examples are in order of performance:

1. Interlab, interfacility, independent tests on measurement devices. (See proposed ISO Draft 5725 "Precision of Test Methods—Determination of Repeatability and Reproducibility.") With these data it is possible to obtain the distribution of bias errors between facilities.

2. Special comparisons of standards with instruments in the actual test environment; "in-place calibration." All instrument calibration would ideally be in an actual or similar test environment. This is almost always impossible.

3. Ancillary or concomitant functions that provide the same performance parameter; in a water turbine meter test, flow may be measured with (a) a drag screen device, (b) an orifice, (c) pump speed characteristics, (d) line resistance pressure drops, and (e) weigh tanks.

4. When it is known that a bias results from a particular cause, special calibrations (separate effects tests) may be performed, allowing the cause to perturbate through its complete range to determine the range of bias.

5. When the cause of a bias is known, analysis of the physics of the measurement may be performed, allowing an estimate of the range to be calculated based on measurements of the secondary parameters.

6. If there is no source of data for bias, the estimate must be based on judgment. An estimate of an upper limit on the largest possible bias error is needed. (Largest is intended to imply the equivalent of a 95% chance that the measurement will fall within the estimate.) When requesting a judgment of the bias estimate from a person with a nonstatistical background, ask for a bias judgment that will not be exceeded 99% of the time. The resulting value will be approximately a 95% value; this technique is based on the recommendation of R. B. Abernethy. It is the author's observation that after checking

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this technique against some known bias values, an actual 95% value will be realized about 70% of the time. This technique should also be used when dealing with a "pride of measurement" problem, as it will allow for some personal bias. Instrumentation manufacturer reports and other references may provide information.

2.2.5 Remaining Biases, Known Sign, and Unknown Magnitude—Nonsymmetrical. Sometimes the physics of the measurement system provides knowledge of the sign, but not the magnitude of the bias. For exmaple, hot fluid thermocouples radiate and conduct energy to cooler pipe walls to indicate lower temperatures, or cool fluid thermocouples receive energy radiated and conducted from hotter pipe walls to indicate higher temperatures. During the course of an experiment, the above example may cover both conditions with a known sign change occurring. The bias limits that result may be nonsymmetrical; i.e., not of the form $\pm B$. The are of the form $+B - C$, where both limits may be positive, negative, or the limits may be of mixed sign as indicated.

Estimates of bias with unknown magnitude are derived in the manner discussed in Subsection 2.2.4.

2.3 Combining Precision Errors

The root-sum-squares (RSS) method is used at the Semiscale facility. The precision index (s) is the root-sum-square of the elemental precision indices from all sources

$$S = \left(\sum_j \sum_i s_{ij}^2 \right)^{1/2} . \tag{5}$$

where j defines the subprocesses (a) calibration, (b) data acquisition, and (c) data recording and i defines the sources within the subprocess. For example, the precision index for the calibration process is the RSS of the elemental precision indices

$$S_c = S_{cal} = \left(s_{1c}^2 + s_{2c}^2 + s_{3c}^2 \right)^{1/2} .$$

The precision index for the data acquisition process is the RSS of the elemental precision indices

$$S_d = S_{data\ acquisition} = \left(s_{1d}^2 + s_{2d}^2 + s_{3d}^2 + s_{4d}^2 + s_{5d}^2 + s_{6d}^2 + s_{7d}^2 \right)^{1/2} .$$

The precision index for the transducer is the RSS of the elemental precision indices.

$$S_t = S_{transducer} = \left(s_{1t}^2 + s_{2t}^2 \right)^{1/2} .$$

The precision index for the system is the RSS of the elemental precision indices.

$$S_s = S_{system} = \left(s_{1s}^2 + s_{2s}^2 + s_{3s}^2 \right)^{1/2} .$$

The basic measurement precision index is the RSS of all the elemental precision indices in the measurement system

$$S = \left(s_1^2 + s_2^2 + s_3^2 \right)^{1/2}$$

$$S = \left(s_{1c}^2 + s_{2c}^2 + s_{3c}^2 + s_{4c}^2 + s_{1d}^2 + s_{2d}^2 + s_{3d}^2 + s_{4d}^2 + s_{5d}^2 + s_{6d}^2 + s_{7d}^2 \right.$$

$$\left. + s_{1t}^2 + s_{2t}^2 + s_{1s}^2 + s_{2s}^2 \right)^{1/2} . \tag{6}$$

Precision errors from the calibration process merit special consideration. There are four cases to consider:

1. If the test period is sufficiently long that instrumentation may be calibrated more than once, the precision errors in the calibration hierarchy should be treated as contributing to the overall precision index.

2. For a single set of instrumentation, calibrated only once during the test, all the calibration error is frozen or fossilized into bias. The uncertainty of the calibration process is all bias.

3. For comparative, back-to-back, development tests in which the test objective is the difference between two successive tests, all the calibration error (bias plus precision) is a constant in both tests and is cancelled by taking the difference. Trending errors are an exception, as described next.

4. Elemental calibration errors that trend with time merit special attention. If the calibration data show some trending characteristics, every effort should be made to remove or reduce the trending. If the test process is long, including many calibrations, this error is a precision error (see Paragraph 1).

   On the other hand, if the test is short, an argument can be made that this error is fixed, a bias. We believe this argument is weak, too complex, and may lead to optimistic uncertainty estimates. We therefore recommend trending errors always be treated as precision errors. In back-to-back, comparative tests, trending errors should be carefully evaluated since they may introduce large errors.

   In summary, trending errors are (a) treated as precision—a sample standard deviation can be calculated from the calibration history, (b) never fossilized into bias, and (c) always included in all uncertainty estimates. In other words, a trending error will be the exception to both Paragraphs 2 and 3 above, and will always contribute to the precision term of the uncertainty estimate.

At the Semiscale Test Facility, calibration is on either an annual or semiannual basis. Testing occurs at approximately two week intervals during a given test series; thus, the measurements fit in Category 2—instrumentation calibrated only once for a test.

Trend errors are treated as precision errors, if the trending error is larger than a 2% slope change, it will be recalibrated (see Section 4.1).

An exception to the root-sum-square method of combining precision errors is when the engineering unit conversion curve is derived from calibration data that were not processed through our data system. In that case, the precision error of the data system is between the transducer and the engineering unit conversion curve and is multiplicative, and the precision errors are added (see "Propagation of Measurement Errors," Section 3).

2.4 Combining Bias Errors

Bias error or systematic errors, as described in Section 2.2, are combined using the root-sum-square method of combining. Bias errors are the estimate of the amount that the mean value would differ from the "true value."

If there were only a few sources of elemental bias errors, it would be reasonable to add them together to obtain the overall bias limits. For example, if there were three sources, the probability that they would all be

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plus (or minus) would be one-half raised to the third power or one-eighth. In actual practice, most measurements will have 10, 20, or more sources of bias ('eference 3, p. 10). The probability that they would all be plus or minus is extremely small; therefore, it is more appropriate to combine them by root-sum-square (Reference 3, p. 19).

If a measurement uncertainty analysis identifies four or less sources of bias, there should be concern that some sources have been overlooked. The analysis should be redone and expert help should be recruited to examine the calibration hierarchy, the data acquisition process, and the data reduction procedure for additional sources.

Therefore, the bias limit will be used herein as the root-sum-square of the elemental errors from all sources.

$$B = \left(\sum_j \sum_i B_{ij}^2 \right)^{1/2} \quad .$$

For example: The bias limit for the calibration hierarchy is

$$B_c = B_{Cal} = \left(b_{1c}^2 + b_{2c}^2 + b_{3c}^2 + b_{4c}^2 \right)^{1/2} \quad . \tag{7}$$

The bias limit for the data acquisition process is

$$B_d = B_{data\ acquisition} = \left(b_{1d}^2 + b_{2d}^2 + b_{3d}^2 + b_{4d}^2 + b_{5d}^2 + b_{6d}^2 + b_{7d}^2 \right)^{1/2} . \tag{8}$$

The bias limit for the transducer error is

$$B_t = B_{transducer} = \left(b_{1t}^2 + b_{2t}^2 \right)^{1/2} \quad . \tag{9}$$

The bias limit for the basic measurement process is

$$B = \left(B_c^2 + B_d^2 + B_t^2 \right)^{1/2}$$

$$B = \left(b_{1c}^2 + b_{2c}^2 + b_{3c}^2 + b_{4c}^2 + b_{1d}^2 + b_{2d}^2 + b_{3d}^2 + b_{4d}^2 + b_{5d}^2 + b_{6d}^2 + b_{7d}^2 \right.$$

$$\left. + b_{1t}^2 + b_{2t}^2 \right)^{1/2} \quad . \tag{10}$$

If any of the elemental bias limits are nonsymmetrical, separate root-sum-squares are used to obtain B^+ and B^-. For example, assume b_{2c} and b_{2t} are asymmetrical, i.e., b_{2c}^+, b_{2c}^-, b_{2t}^+, b_{2t}^- are available. Then

$$B^+ = \left[b_{1c}^2 + \left(b_{2c}^+ \right)^2 + b_{3c}^2 + b_{4c}^2 + B_d^2 + b_{1t}^2 + \left(b_{2t}^+ \right)^2 \right]^{1/2}$$

$$B^- = \left[b_{1c}^2 + \left(b_{2c}^- \right)^2 + b_{3c}^2 + b_{4c}^2 + B_d^2 + b_{1t}^2 + \left(b_{2t}^- \right)^2 \right]^{1/2} \quad .$$

2.5 Combining Degrees of Freedom

Estimates of the number of degrees of freedom (ν) is needed to estimate the value of the Student-t statistic. The Welch-Satterthwaite formula, which is a function of the degrees of freedom and magnitude of each elemental precision index, is used to approximate the degrees of freedom. Being weighted by the precision indices, the larger precision errors carry a larger weighting. There are exact methods of calculating the combined degrees of freedom; however, due to ease of calculation, the Welch-Satterthwaite formula is used. Also, by keeping the sample sizes large (greater than 30), particularly the ones with the larger elemental precision errors, the degrees of freedom will always be greater than 30, and the value of 2 can be used as an estimate for the 95% Student-t statistic.

The effect of degrees of freedom is shown in Figure 1. Notice how a few degrees of freedom changes the shape of the curve. As the degrees of freedom approaches infinity the t-distribution approaches the normal Gaussian distribution.
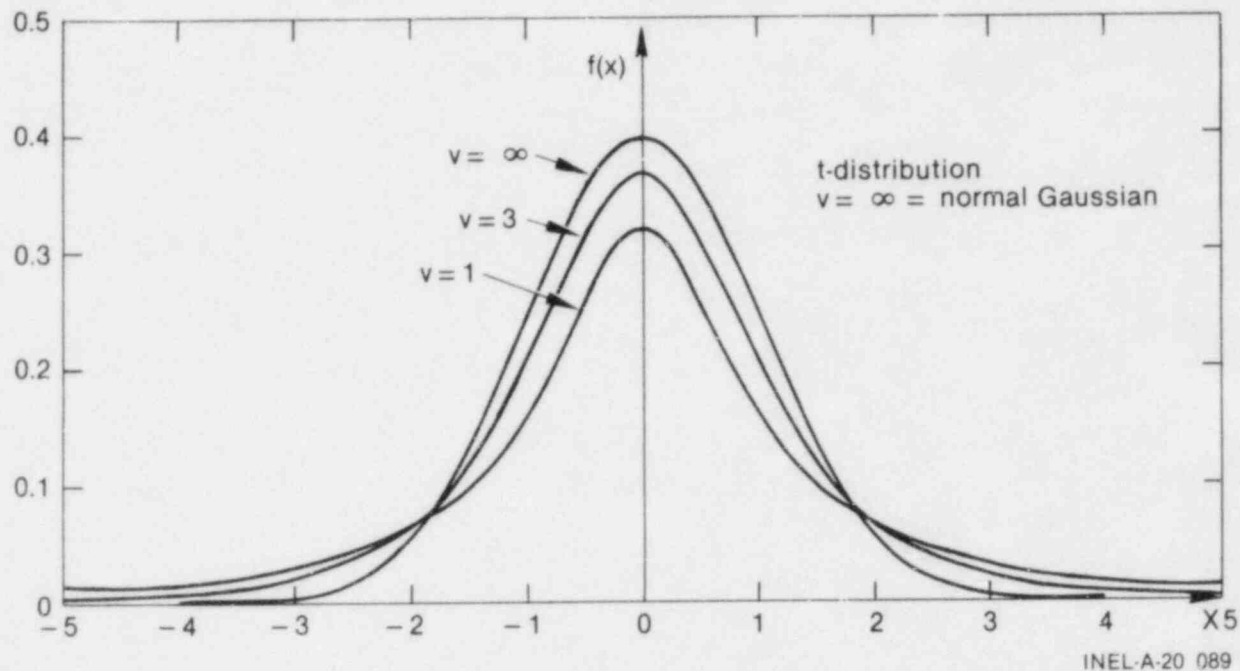

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Figure 1.   Student-t distribution for various degrees of freedom.

In a sample, the number of degrees of freedom is the size of the sample. When a statistic is calculated from the sample, the degrees of freedom associated with the statistic are reduced by one for every estimated parameter used in calculating the statistic. For example, from a sample of size n, $\bar{X}$ is calculated as

$$\bar{X} = \sum_{i=1}^{n} X_i / n$$

which has n degrees of freedom and

$$S = \left[\frac{\sum_{i=1}^{n} (X_i - \bar{X})^2}{n - 1} \right]^{1/2}$$

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which has n - 1 degrees of freedom because X (based on the same sample of data) is used to calculate S. In calculating other statistics, more than one degree of freedom may be lost. For example, in calculating the standard error of a curve fit, the number of degrees of freedom that are lost is equal to the number of estimated coefficients for the curve.

For example; The degrees of freedom for the calibration precision index (S_{Cal}) are

$$\nu_c = \frac{\left(\sum_i^n S_{1c}^2\right)^2}{\sum_i^n \dfrac{S_{1c}^4}{\nu_{1c}}} \ .$$

where ν_{1c} is the degree of freedom of each elemental precision index.

$$\nu_c = \frac{\left(S_{1c}^2 + S_{2c}^2 + S_{3c}^2 + S_{4c}^2\right)^2}{\dfrac{S_{1c}^4}{\nu_{1c}} + \dfrac{S_{2c}^4}{\nu_{2c}} + \dfrac{S_{3c}^4}{\nu_{3c}} + \dfrac{S_{4c}^4}{\nu_{4c}}} \ .$$

The degrees of freedom for the measurement precision index, S, is

$$\nu = \frac{\left(S_{1c}^2 + S_{2c}^2 + S_{3c}^2 + S_{4c}^2 + S_{1d}^2 + S_{2d}^2 + S_{3d}^2 + S_{4d}^2 + S_{5d}^2 + S_{6d}^2 + S_{7d}^2 + S_{1t}^2 + S_{2t}^2\right)^2}{\dfrac{S_{1c}^4}{\nu_{1c}} + \dfrac{S_{2c}^4}{\nu_{2c}} + \dfrac{S_{3c}^4}{\nu_{3c}} + \dfrac{S_{4c}^4}{\nu_{4c}} + \dfrac{S_{1d}^4}{\nu_{1d}} + \dfrac{S_{2d}^4}{\nu_{2d}} + \dfrac{S_{3d}^4}{\nu_{3d}} + \dfrac{S_{4d}^4}{\nu_{4d}} + \dfrac{S_{5d}^4}{\nu_{5d}} + \dfrac{S_{6d}^4}{\nu_{6d}} + \dfrac{S_{7d}^4}{\nu_{7d}} + \dfrac{S_{1t}^4}{\nu_{1t}} + \dfrac{S_{2t}^4}{\nu_{2t}}}$$

## 2.6 Combining Bias and Precision Errors

Two methods of combining bias and precision errors are approved in the draft ISO standard "Fluid Flow Measurement Uncertainty."[3] The first is linear addition [Equation (A)] called U_{99}, and the second is the root-sum-square method [Equation (B)] called U_{95}.

$$U_{99} = (B + t_{95}S) \tag{A}$$

$$U_{95} = \left[B^2 + (t_{95}S)^2\right]^{1/2} \ . \tag{B}$$

The terminology U_{99} and U_{95} are consistent with usage in the ISO standard.[3] In the above equations the symbol B is the bias limit, S is the precision index, and t_{95} is the 95th percentile point for the two-tailed Student-t distribution. The t-value is a function of the number of degrees of freedom (ν) used in calculating S (see Table 3). For small samples, t will be larger, and for larger samples t will be smaller, approaching 1.96 as a lower limit. The use of the t inflates the limit U to reduce the risk of underestimating S when a small sample is used to calculate S. Since 30 degrees of freedom (ν) yield a t of 2.04 and infinite degrees of freedom yield a t of 1.96, a selection of $t = 2$ for values of ν from 30 to infinity was made, i.e., $U_{99} = (B + 2S)$, when $\nu \geq 30$.

Table 3. Two-tailed student-t distribution
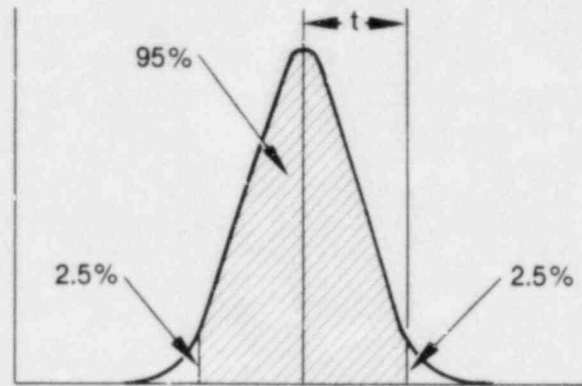
Degrees of Freedom	t	Degrees of Freedom	t
1	12.706	17	2.110
2	4.303	18	2.101
3	3.182	19	2.093
4	2.776	20	2.086
5	2.571	21	2.080
6	2.447	22	2.074
7	2.365	23	2.069
8	2.306	24	2.064
9	2.262	25	2.060
10	2.228	26	2.056
11	2.201	27	2.052
12	2.179	28	2.048
13	2.160	29	2.045
14	2.145	30	2.042
15	2.131	31 or more use 2.0	
16	2.120		

The National Bureau of Standards accepts this standard, provided: the uncertainty interval selected [Equation (A) or (B)] is provided in the presentation; and components (bias, precision, degrees of freedom) are available in an appendix or in supporting documentation.

These three components may be required to (a) substantiate and explain the uncertainty value, (b) provide a sound technical base for improved measurements, and (c) propagate the uncertainty from measured parameters to fluid flow parameters, and from fluid flow parameters to other, more complex parameters (i.e., drag force to mass flow rate, etc.)

A measurement uncertainty analysis is largely completed when:

1. All the elemental sources of error have been identified and categorized into bias limits and precision indices

2. These errors have been propagated to errors in the test results, keeping bias and precision separate

3. An estimate of the degrees of freedom of the precision index of the test results has been calculated from the Welch-Satterthwaite formula, if less than 30.

However, for simplicity of presentation, a single number (some combination of bias and precision) is needed to express a reasonable limit for error. The single number must have a simple interpretation (such as the largest error reasonably expected) and be useful without complex explanation. It is impossible to define a single rigorous statistic because the bias is an upper limit based on judgment or analysis, which has unknown characteristics. Any function of these two numbers must be a hybrid combination of an unknown quantity (bias) and a statistic (precision). If both numbers were statistics, a confidence interval would be recommended. Ninety-five percent confidence levels would be available at the discretion of the analyst.

2.6.1 Uncertainty Interval Coverage. A requirement for a 95% confidence interval[a] is the reason that the root-sum-square method of combining the bias and precision errors is used. This represents approximately one data value out of twenty lying outside the uncertainty bandwidth. A 95% confidence level is also approximately the value that an experienced measurement engineer is comfortable with (neither too large nor too small).

Dr. R. B. Abernethy performed Monte Carlo simulations of the U_{99} and U_{95} methods and presented the following conclusions:[3]

"A rigorous calculation of confidence level or the coverage of the true value by the interval is not possible because the distributions of bias errors and limits, based on judgment, cannot be rigorously defined. Monte Carlo simulation of the intervals can provide approximate coverage on the basis of assuming various bias error distributions and bias limits. As the actual bias error and bias limit distributions will probably never be known, the simulation studies were based on a range of assumptions. The results of these studies comparing the two intervals are:

1. U_{99} averages approximately 99.1% coverage, whereas U_{95} provides 95% based on bias limits assumed to be 95%. For 99.7% bias limits, U_{99} averages 99.7% coverage, and U_{95} averages 97.5% coverage.

2. The ratio of the average U_{99} interval size to U_{95} interval size is 1.35:1.

3. If the bias error is negligible, both intervals provide a 95% statistical confidence (coverage).

4. If the precision error is negligible, both intervals provide 95 or 99.7% confidence, depending on the assumed bias limit size."

The simulation cases considered were:

1. From 3 to 19 error sources, both bias and precision

2. Bias errors distributed both normally and rectangularly

3. Precision distributed normally

4. Bias limits at both 95 and 99.7% for both the normal and the rectangular

5. Precision indices based on sample sizes from 3 to 30

6. Ratio of precision to bias errors at 0.5, 1.0, and 2.0."

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a. As set forth in an internal working document of the EG&G Idaho Nuclear Technology Department.

2.6.2 Symmetrical Interval. Uncertainty (Figure 2) for the symmetrical bias case (zero mean value) is centered about the measurement. The uncertainty interval is given by

$$U_{95}^{-} = -\left[(B)^2 + (t_{95}S)^2 \right]^{1/2} \quad \text{to} \quad U_{95}^{+} = +\left[(B)^2 + (t_{95}S)^2 \right]^{1/2} \quad .$$
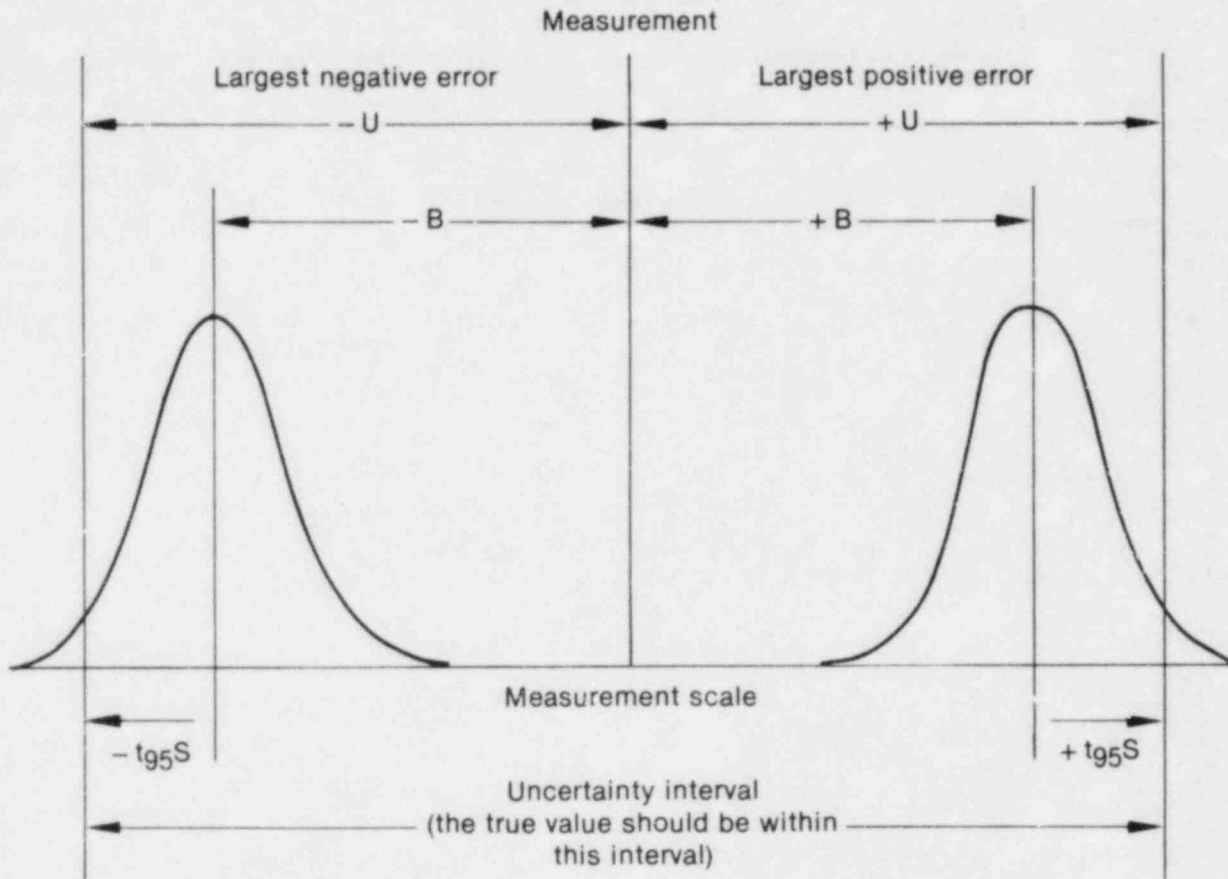
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Figure 2.   Measurement uncertainty; symmetrical bias.

2.6.3 Nonsymmetrical Interval. Uncertainty (Figure 3) for the nonsymmetrical bias case (nonzero mean value) is not centered about the measurement. The upper limit of the interval is defined by the upper limit of the bias interval (B^+); the lower limit is defined by the lower limit of the bias interval (B^-).

The uncertainty interval is given by

$$U_{95}^{-} = \left[(B^-)^2 + (t_{95}S)^2 \right]^{1/2} \quad \text{to} \quad U_{95}^{+} = \left[(B^+)^2 + (t_{95}S)^2 \right]^{1/2} \quad . \tag{11}$$

2.6.4 How to Interpret Uncertainty. Uncertainty is a function of the measurement process. It provides an estimate of the largest error that may reasonably be expected for that measurement process.
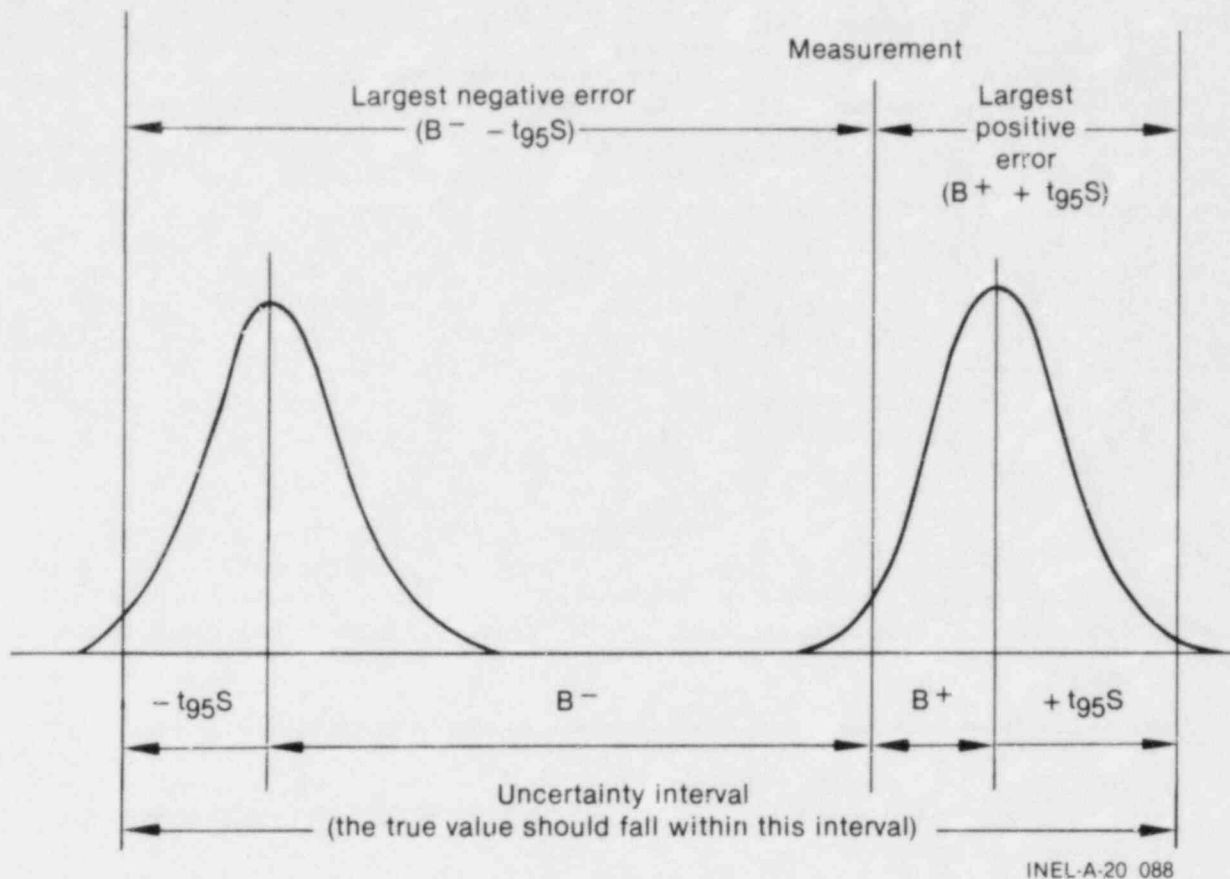
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Figure 3. Measurement uncertainty; nonsymmetrical bias.

Errors larger than the uncertainty should rarely occur. On repeated runs within a given measurement process, the variation in test results should be within the uncertainty value. These differences might look like Figure 4.

In summary, measurements are subject to two types of errors, i.e., bias and precision. One sample standard deviation is used as the precision index, s. The fixed error, β, is estimated and used as the bias limit, B. An accurate measurement is one that has small error, both bias and precision. The bias and precision errors are combined using the root-sum-square method, with the precision errors being multiplied by the 95% t-value.
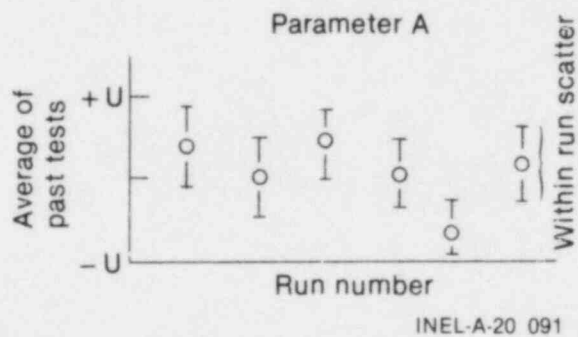

Figure 4. Run-to-run difference.

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2.6.5 Pretest Versus Posttest Measurement Uncertainty Analysis. The accuracy of the test is often part of the test requirements. Such requirements are defined by pretest uncertainty analysis, which allows for corrective action before the test to improve the uncertainties when they are too large. The pretest analysis is based on data and information that exist before the test, such as calibration histories, previous tests with similar instrumentation, prior measurement uncertainty analysis, and expert opinions. For complex tests there are often alternatives to evaluate, such as different test designs, instrumentation layouts, alternate calculation procedures, concomitant variables, etc. Pretest analysis will identify the most accurate test method.

If the difference to be detected in an experiment is of the same size or smaller than the projected uncertainty, corrective action should be taken to reduce the uncertainty. Therefore, a measurement uncertainty analysis should always be done before the test or experiment. The corrective action to reduce the uncertainty may involve (a) improvements or additions to the instrumentation, (b) selection of a different function to obtain the parameter of interest, and/or (c) repeated testing. Cost and time will dictate the choice. If corrective action cannot be taken, the test should be cancelled, since there is a high risk that the real differences will be lost in the uncertainty interval (undetected).

Posttest analysis is required to confirm the pretest estimates or to identify problems. Comparison of test results with the pretest analysis is an excellent data validity check. The precision of the repeated points or redundant instruments should not be significantly larger than the pretest estimates. When redundant instrumentation or calculation methods are available, the individual averages should be within the pretest uncertainty interval (for individuals). The final uncertainty intervals should be based on posttest analysis.

At the Semiscale Test Facility, in-place calibrations for some primary and secondary effects are performed prior to testing. Comparisons of test data with known data values that should be obtained—both before and after all tests—allow (a) data corrections such as zero offsets to be determined and (b) verification of test data and uncertainty values. Data corrections are made if the correction moves the data to within the uncertainty bands. When data corrections cannot move the data to within uncertainty bands, further analysis is performed to either reject the data or increase the uncertainty bands.

Uncertainty bands should be increased only after a thorough analysis of the available information concerning that measurement. The method used to increase the size of the uncertainty band uses Table 1 and the average range of the deviations to estimate the equivalent of a standard deviation. Bias from some known or unknown source is usually the cause and can easily be root-sum-squared into the pretest uncertainty. (Square the old uncertainty value and add the bias squared and take the square root.) If the source is precision, one must go back to the original components of bias and precision to add in the new precision component.

3. PROPAGATION OF MEASUREMENT ERRORS (Taylor's Series)

The "law of propagation of error" is a tool that physical scientists have conveniently and frequently used in their work for many years.[6] The derivation of the Taylor's series will be given first (Section 3.1), followed by an example (Section 3.2).

Rarely are fluid flow parameters measured directly; usually, more basic quantities such as temperature and pressure are measured and the fluid flow parameter is calculated as a function of the measurements. Error in the measurements is propagated to the parameter through the function. The effect of the propagation may be approximated with the Taylor's series methods. It is convenient to introduce the concept of the sensitivity of a result to a subsidiary quantity as the error propagated to the result due to unit error in the measurement of the component quantity. The "sensitivity coefficient" of each subsidiary quantity is most easily obtained in one of two ways.

1. Analytically: When a known mathematical relationship exists between the result, R, and subsidiary quantities, $Y_1, Y_2, \ldots, Y_k$, the dimensional sensitivity coefficient, θ_i, of the quantity Y_i is obtained by partial differentiation.

    Thus, if $R = f(Y_1, Y_2, \ldots, Y_k)$, then

    $$\theta_i = \frac{\partial R}{\partial Y_i} \quad .$$

2. Numerically: When no mathematical relationship is available or when differentiation is difficult, finite increments may be used to evaluate θ_i.

    Here θ_i, is given by

    $$\theta_i = \frac{\Delta R}{\Delta Y_i} \quad .$$

    The result is calculated using Y_i, to obtain R, and then recalculated using $(Y_i + \Delta Y_i)$ to obtain $(R + \Delta R)$. The value of ΔY_i used should be as small as practicable.

Care should be taken to ensure that the errors are independent. With complex parameters, the same measurement may be used more than once in the formula. This may increase or decrease the error, depending on whether the sign of the measurement is the same or opposite. If the Taylor's series relates the most elementary measurements to the ultimate parameter or result, these "linked" relationships will be properly accounted for.

This subject is discussed further in the following section, in which examples are given.

3.1 Derivation of Propagation of Errors by Taylor's Series

The proofs in this section are shown for two- and three-variable functions. These proofs can be easily extended to functions with more variables, although, because of its length, the general case is not shown here.[3]

3.1.1 Two Independent Variables. The terms x and y are independent; therefore, cross product terms will not appear. If x and y are not independent, linked terms can be treated as a third variable. If it is assumed that response Z is defined as a function of measured variables (x and y), the two restrictions that must be considered are:

1. Z is continuous in the neighborhood of the point (μ_x, μ_y). Both x and y will have error distributions about this point and the notation (μ_x and μ_y) indicates the mean values of these distributions.

2. Z has continuous partial derivatives in a neighborhood of the point (μ_x, μ_y).

These conditions are satisfied if the functions to be considered are restricted to smooth curves in a neighborhood of the point with no discontinuities (jumps or breaks in the curve). The Taylor's series expansion for Z is

$$Z = Z(\mu_x, \mu_y) + \frac{\partial Z}{\partial x}(x - \mu_x) + \frac{\partial Z}{\partial y}(y - \mu_y) + R_2 \tag{12}$$

where $\frac{\partial Z}{\partial x}$ and $\frac{\partial Z}{\partial y}$ are evaluated at the point (μ_x, μ_y).

$$R_2 \leq 1/2 \left[\frac{\partial^2 Z}{\partial x^2}(x - \mu_x)^2 + \frac{\partial^2 Z}{\partial y^2}(y - \mu_y)^2 \right] \tag{13}$$

where $\frac{\partial^2 Z}{\partial x^2}$ and $\frac{\partial^2 Z}{\partial y^2}$ are evaluated at (θ_1, θ_2), with θ_1 between x and μ_x, and θ_2 between y and μ_y.

The quantity R_2, the remainder after two terms, is not significant if either:

1. $(x - \mu_x)$ and $(y - \mu_y)$ are small.

2. The second partials $\frac{\partial^2 Z}{\partial x^2}$ and $\frac{\partial^2 Z}{\partial y^2}$ are small or zero. These partials are zero for linear functions.

By assuming R_2 to be small or zero, Equation (12) becomes

$$Z \approx \mu_z + \frac{\partial Z}{\partial x}(x - \mu_x) + \frac{\partial Z}{\partial y}(y - \mu_y) \quad . \tag{14}$$

By defining μ_z as the average value of the distribution of Z, the difference $(Z - \mu_z)$ is the difference of Z about its average value. This difference may be approximated by

$$Z - \mu_z \approx \frac{\partial Z}{\partial x}(x - \mu_x) + \frac{\partial Z}{\partial y}(y - \mu_y) \quad . \tag{15}$$

where the partials are evaluated at the point (μ_x, μ_y).

The variation in Z is defined by

$$\sigma_z^2 \equiv \int (Z - \mu_z)^2 \, p_z \, dZ$$

where p_z is the probability density function of Z. Therefore,

$$\sigma_z^2 \approx \iint \left[\frac{\partial Z}{\partial x}(x - \mu_x) + \frac{\partial Z}{\partial y}(y - \mu_y) \right]^2 p_{xy} \, dx \, dy \tag{16}$$

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$$\approx \iint \left[\frac{\partial Z}{\partial x} (x - \mu_x)\right]^2 p_{xy} \, dydx + \iint \left[\frac{\partial Z}{\partial y} (y - \mu_y)\right]^2 p_{xy} \, dxdy$$

$$+ 2 \iint \left[\frac{\partial Z}{\partial x} (x - \mu_x)\right] \left[\frac{\partial Z}{\partial y} (y - \mu_y)\right] p_{xy} \, dxdy \tag{17}$$

where p_{xy} is the joint distribution function of x and y. Integrating the first term of Equation (17) with respect to y and the second term of the equation with respect to x gives

$$\sigma_Z^2 \approx \int \frac{\partial Z}{\partial x}^2 (x - \mu_x)^2 \, p_x \, dx + \frac{\partial Z}{\partial y}^2 (y - \mu_y)^2 \, p_y \, dy$$

$$+ 2 \iint \frac{\partial Z}{\partial x} (x - \mu_x) \frac{\partial Z}{\partial y} (y - \mu_y) \, p_{xy} \, dxdy \quad . \tag{18}$$

If μ_x and μ_y are the means of the distributions of x and y, then define the following:

$$\sigma_x^2 = \int (x - \mu_x)^2 \, p_x \, dx \tag{19}$$

$$\sigma_y^2 = \int (y - \mu_y)^2 \, p_y \, dy \tag{20}$$

$$\rho_{xy} \, \sigma_x \, \sigma_y = \iint (x - \mu_x)(y - \mu_y) \, p_{xy} \, dxdy \tag{21}$$

where ρ_{xy} is the coefficient of correlation between x and y. Combining the definitions and Equation (19) gives

$$\sigma_z^2 = \left(\frac{\partial z}{\partial x}\right)^2 \sigma_x^2 + \left(\frac{\partial z}{\partial y}\right)^2 \sigma_y^2 + 2 \frac{\partial z}{\partial y} \frac{\partial z}{\partial x} \rho_{xy} \, \sigma_x \, \sigma_y \quad . \tag{22}$$

If x and y are independent variables, then $\rho_{xy} = 0$ and

$$\sigma_z^2 = \left(\frac{\partial z}{\partial x}\right)^2 \sigma_x^2 + \left(\frac{\partial z}{\partial y}\right)^2 \sigma_y^2 \quad . \tag{23}$$

3.1.2 Three Independent Variables. The terms x, y, and w are independent; therefore, cross product terms will not appear. If x, y, and w are not independent, linked terms can be treated as a third variable. If it is assumed that Z is a function of variables x, y, and w, two restrictions must be considered:

1. Z is continuous in a neighborhood of the point (μ_x, μ_y, μ_w)

2. Z has continuous partial derivatives in a neighborhood of (μ_x, μ_y, μ_w).

If these restrictions are satisfied, the Taylor's series expansion for Z in the vicinity of (μ_x, μ_y, μ_w) is

$$Z = \mu_Z + \frac{\partial Z}{\partial x} (x - \mu_x) + \frac{\partial Z}{\partial y} (y - \mu_y) + \frac{\partial Z}{\partial w} (w - \mu_w) + R_2 \tag{24}$$

where

$\frac{\partial Z}{\partial x}$, $\frac{\partial Z}{\partial y}$, and $\frac{\partial Z}{\partial w}$ are evaluated at (μ_x, μ_y, μ_w),

$$R_2 \leq 1/2 \left[\frac{\partial^2 Z}{\partial x^2} (x - \mu_x)^2 + \frac{\partial^2 Z}{\partial y^2} (y - \mu_y)^2 + \frac{\partial^2 Z}{\partial w^2} (w - \mu_w)^2 \right] \tag{25}$$

These second partials are evaluated at a point θ_1, θ_2, θ_3 defined so that θ_1 is between μ_x and x, θ_2 is between μ_y and y, and θ_3 is between μ_w and w. The same restrictions apply to R_2 as defined for two-variable functions.

By assuming R_2 to be small or zero, Equation (24) becomes

$$Z - \mu_Z \approx \frac{\partial Z}{\partial z} (x - \mu_x) + \frac{\partial Z}{\partial y} (y - \mu_y) + \frac{\partial Z}{\partial w} (w - \mu_w) \tag{26}$$

where the partials are evaluated at the point (μ_x, μ_y, μ_w).

The variation in Z is defined by

$$\sigma_z^2 \approx \int (Z - \mu_z)^2 \, p_Z \, dZ \tag{27}$$

where p_Z is the probability density function of Z. Therefore,

$$\sigma_z^2 \approx \iiint \left[\frac{\partial Z}{\partial x} (x - \mu_x) + \frac{\partial Z}{\partial y} (y - \mu_y) + \frac{\partial Z}{\partial w} (w - \mu_w) \right]^2 p_{x,y,w} \, dxdydw$$

$$\approx \iiint \left[\frac{\partial Z}{\partial x} (x - \mu_x) \right]^2 p_{x,y,w} \, dwdydx + \dots \tag{28}$$

$$+ 2 \iiint \frac{\partial Z}{\partial x} \frac{\partial Z}{\partial y} (x - \mu_x)(y - \mu_y) \, p_{x,y,w} \, dwdxdy + \dots \tag{29}$$

where $p_{x,y,w}$ is the joint distribution function of x, y, and w. Integrating in the proper order produces

$$\sigma_Z^2 = \int \left(\frac{\partial Z}{\partial x} \right)^2 (x - \mu_x)^2 \, p_x \, dx$$

$$+ \dots + 2 \iint \frac{\partial Z}{\partial x} \frac{\partial Z}{\partial y} (x - \mu_x)(y - \mu_y) \, p_{xy} \, dxdy \; . \tag{30}$$

Therefore,

$$\sigma_Z^2 = \left(\frac{\partial Z}{\partial x} \right)^2 \sigma_x^2 + \left(\frac{\partial Z}{\partial y} \right)^2 \sigma_y^2 \left(\frac{\partial Z}{\partial w} \right)^2 \sigma_w^2 + 2 \frac{\partial Z}{\partial x} \frac{\partial Z}{\partial y} \rho_{xy} \sigma_x \sigma_y$$

$$+ 2 \frac{\partial Z}{\partial x} \frac{\partial Z}{\partial w} \rho_{xw} \sigma_x \sigma_w + 2 \frac{\partial Z}{\partial y} \frac{\partial Z}{\partial w} \rho_{yw} \sigma_y \sigma_w \; . \tag{31}$$

If x, y, and w are independent variables, then $p_{xy} = p_{xw} = p_{yw} = 0$ and

$$\sigma_Z^2 = \left(\frac{\partial Z}{\partial x}\right)^2 \sigma_x^2 + \left(\frac{\partial Z}{\partial y}\right)^2 \sigma_y^2 + \left(\frac{\partial Z}{\partial w}\right)^2 \sigma_w^2 \quad . \tag{32}$$

3.1.3 Monte Carlo Simulation. To determine the restrictions that must be placed on applications of the method of partial derivatives, Dr. R. B. Abernethy performed a Monte Carlo simulation to check for the computation of various functions. Comparative results are listed in Tables 4 and 5.[a]

Table 4. Results of Monte Carlo simulation for theoretical input σ_x^2 , μ_x, σ_y^2 , μ_y

Function	Simulation Run Number	Theoretical Input σ_x^2	μ_x	σ_y^2	μ_y	Method of Partials Estimated Variance (theoretical)	Method of Partials Estimated Variance (actual input)	Input Variance Corrected for Nonindependence (method of partials)	Observed Variance (simulator results)
x + y	1	1.0	10	4.0	20	5.0	4.9477	4.8496	4.8567
	2	1.0	10	4.0	20	5.0	4.9186	4.8435	4.8506
	3	1.0	10	4.0	20	5.0	5.0786	4.9493	4.9564
	4	1.0	10	4.0	20	5.0	5.1639	5.2444	5.2515
x - y	1	1.0	10	4.0	20	5.0	4.9477	5.0358	5.0410
	2	1.0	10	4.0	20	5.0	4.9186	4.9937	4.9885
	3	1.0	10	4.0	20	5.0	5.0786	5.2079	5.2028
	4	1.0	10	4.0	20	5.0	5.1639	5.0834	5.0782
(x)(y)	1	1.0	10	4.0	20	800.0	792.81	773.27	768.63
	2	1.0	10	4.0	20	800.0	794.33	779.29	797.48
	3	1.0	10	4.0	20	800.0	802.28	776.41	775.78
	4	1.0	10	4.0	20	800.0	867.67	885.85	883.38
x/y	1	1.0	10	4.0	20	0.005	0.0050	0.0051	0.0054
	2	1.0	10	4.0	20	0.005	0.0050	0.0051	0.0054
	3	1.0	10	4.0	20	0.005	0.0050	0.0052	0.0055
	4	1.0	10	4.0	20	0.005	0.0054	0.0053	0.0057

Table 5. Results of Monte Carlo simulation for theoretical input μ_{x_i}, $\sigma_{x_i}^2$

Function, z	Number of Simulations	Theoretical Input μ_{x_i}	$\sigma_{x_i}^2$	Estimated Parameters (method of partials) μ_Z	σ_Z^2	Simulation Results μ_Z	σ_Z^2
$(x_1 x_2)/x_3$	2	20	1.0	20	3.00	20.2 20.6	2.56 3.24
$(x_1 x_2)/(x_3 x_4 x_5)$	2	20	1.0	0.05	3.12×10^{-5}	0.0505	3.6×10^{-5}
$(x_1 x_2 x_3 x_4)/(x_5 x_6 x_7)$	2	20	1.0	20	7.00 20.25	20.04 8.41	8.41
$(x_1 x_2 x_3) \overset{9}{\underset{i=4}{N}} x_i$	1	20	1.0	1.25×10^{-4}	3.52×10^{-10}	1.29×10^{-4}	4.0×10^{-10}
$\overset{6}{\underset{i=1}{N}} x_i \; (x_7 x_8 x_9)$	2	20	1.0	8000	1.44×10^6	8150 8300	1.69×10^6 1.82×10^6

a. Taken from Reference 3, p. 22.

Table 4 contrasts the results of the Monte Carlo simulation of the tabulated functions, Column 7, with the estimates using partial derivatives, Column 6. One thousand functional values were obtained in each simulation. Column 1 identifies the function simulated and Column 2 gives the number of the simulation run. Column 3, Theoretical Input, includes the parameters of the populations from which the random numbers were drawn. Column 4 lists the method of partials estimates of variance for the function based on the theoretical input (Column 3). Column 5 lists the estimates of variance for the function calculated using the method of partial derivatives from the observed variation of the variables x and y. Column 6 gives Column 5 corrected for the observed correlation between the pairs of x, y input values. The correction factor is

$$2\rho_{xy} \; \sigma_x^2 \; \sigma_y^2 \; \frac{\partial Z}{\partial x} \; \frac{\partial Z}{\partial y}$$

where ρ is the observed correlation between paired values of x and y, σ_x^2 and σ_y^2 are the observed variances of x and y, and $\frac{\partial Z}{\partial x}$ and $\frac{\partial Z}{\partial y}$ are the partial derivatives of the function Z. Column 7 lists the simulator results for the function (Column 1) for 1000 data points.

Columns 1 through 3 of Table 5 present the input to the Monte Carlo simulator. The theoretical input, Column 3, shows the parameters of the population of random numbers that were used to produce the functional values. Column 5 summarizes the results of the simulation; these results may be compared with the estimates from the method of partials, Column 4.

Simulation results have shown that the method of partial derivatives is most accurate for functions involving sums and differencs of the observed variables. For these functions, if the variables are mutually independent, the Taylor's series is exact for any magnitude of error in the measured parameters. If the variables are not mutually independent, a correction factor can be computed that will ensure exactitude of the method. The correction factor $\left(2\rho_{xy} \; \sigma_x \; \sigma_y \frac{\partial Z}{\partial x} \frac{\partial Z}{\partial y} \right)$ is the third term in Equation (22). If ρ_{xy} is not zero, this term should be included in estimating σ_Z^2. From data, ρ_{xy} may be estimated with

$$r = \frac{S_{xy}}{S_x S_y} = \frac{\sum (x_i - \bar{x})(y_i - \bar{y})}{\left[\sum (x_i - \bar{x})^2 \; \sum (y_i - \bar{y})^2 \right]^{1/2}} \tag{33}$$

where n pairs of observations are available, and x and y are the average of the x_i and y_i values, respectively.

Close approximations can be made for errors that exist in functions involving products and quotients of independently varying observed values if the ratio of measured errors to their respective nominal values is small (<0.1). The approximation improves as measured errors decrease in relation to their nominals. For all of the functions examined involving two or more independent variables, the approximation is within 10% of the true error. The simulation results are summarized in Tables 4 and 5.

Table 6 shows the Taylor's formula for several functions. The Taylor's formula for the coefficient of variation is also listed. The coefficient of variation is easily converted to a percentage variation by multiplying by 100.

3.2 Airflow Example

In this example, airfow is determined by the use of a choked venturi and measurements of upstream stagnation temperature and stagnation pressure (Figure 5).

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Table 6. Error propagation formulas

Function	Taylor's Formula	Coefficient of Variation
$w = f(x,y)$	$S_w = \left[\left(\frac{\partial w}{\partial x} S_x \right)^2 + \left(\frac{\partial w}{\partial y} S_y \right)^2 \right]^{1/2}$	
$\omega = Ax + By$	$S_w = \left(A^2 S_x^2 + B^2 S_y^2 \right)^{1/2}$	$V_w^2 = \dfrac{A^2 x^2 V_x^2 + B^2 y^2 V_y^2}{(Ax + By)^2}$
$w = \frac{1}{y}$	$S_w = \left(\dfrac{S_y^2}{y^4} \right)^{1/2}$	$V_w^2 - V_y^2$
$w = \frac{x}{x + y}$	$S_w = \left\{ \left[\dfrac{y\, S_x}{(x + y)^2} \right]^2 + \left[\dfrac{x\, S_y}{(x + y)^2} \right]^2 \right\}^{1/2}$	$V_w^2 = y^2(V_x^2 + V_y^2)/(x + y)^2$
$w = \frac{x}{1 + x}$	$S_w = \left[\dfrac{y\, S_x}{(1 + x)^4} \right]^{1/2}$	$V_w^2 = \dfrac{V_x^2}{(1 + x)^2}$
$w = xy$	$S_w = \left[(y S_x)^2 + (x S_y)^2 \right]^{1/2}$	$V_w^2 = V_x^2 + V_y^2$
$w = x^2$	$S_w = \left(4x^2 S_x^2 \right)^{1/2}$	$V_w^2 = 4V_x^2$
$w = x^{1/2}$	$S_w = \left(\dfrac{S_x^2}{4x} \right)^{1/2}$	$V_w^2 = \dfrac{V_x^2}{4}$
$w = \ln x$	$S_w = \left(\dfrac{S_x^2}{x^2} \right)^{1/2}$	$V_w^2 = \left(\dfrac{V_x}{\ln x} \right)^2$
$s = kx^a y^b$	$S_w = \left[\left(aky\, x^{a-1} S_x \right) + \left(bkx^a y^{b-1} S_y \right)^2 \right]^{1/2}$	$V_w^2 = \left(aV_x \right)^2 + \left(bV_y \right)^2$ where:

$$V_x = \frac{S_x}{\bar{x}}$$

$$V_y = \frac{S_y}{\bar{y}}$$

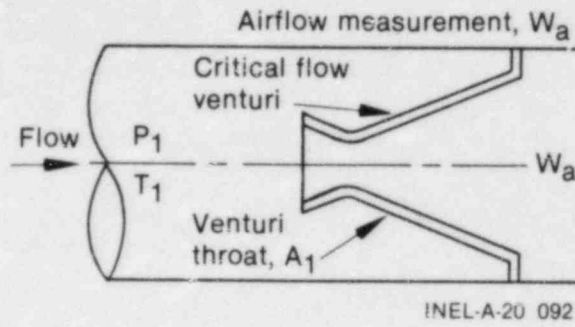$$V_w \quad \frac{S_w}{\bar{w}}; \quad \bar{w} = f(\bar{x}, \bar{y})$$

Figure 5.   Flow through a choked venturi.

The flow is calculated from

$$\dot{m} = C a F a \phi^* \frac{P1t}{\sqrt{T1t}}$$ (34)

where

$\dot{m}$ = mass flow rate of air

Fa = factor to account for thermal expansion of the venturi

a = venturi throat area

P1t = total (stagnation) pressure upstream

T1t = total temperature upstream

ϕ^* = factor to account for the properties of the air (critical flow constant)

C = discharge coefficient

A_{eff} = Ca (may be determined from calibration).

The precision index for the flow (S_m) is calculated using the Taylor's series expansion (this method is derived in Subsection 3.2.1):

$$S_{\dot{m}} = \left[\left(\frac{\partial \dot{m}}{\partial Fa} S_{Fa} \right)^2 + \left(\frac{\partial \dot{m}}{\partial \phi^*} S_\phi^* \right)^2 + \left(\frac{\partial \dot{m}}{\partial a} S_a \right)^2 + \left(\frac{\partial \dot{m}}{\partial P1t} S_{P1t} \right)^2 \right.$$

$$\left. + \left(\frac{\partial \dot{m}}{\partial T1t} S_{T1t} \right)^2 \right]^{1/2}$$ (35)

where

$\frac{\partial \dot{m}}{\partial Fa}$ denotes the partial derivative of $\dot{m}$ with respect to Fa.

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Taking the necessary partial derivatives and assuming $C = 1$ and has negligible error,

$$S_m = C \left[\left(\frac{\phi^* aP1t}{\sqrt{T1t}} S_{Fa} \right)^2 + \left(\frac{Fa\ aP1t}{\sqrt{T1t}} S_\phi^* \right)^2 + \left(\frac{Fa\phi^* P1t}{\sqrt{T1t}} S_a \right)^2 \right.$$

$$\left. + \left(\frac{Fad^* a}{\sqrt{T1t}} S_{P1t} \right)^2 \left(\frac{Fad^* aP1t}{-2\sqrt{T1t}^3} S_{T1t} \right)^2 \right]^{1/2} . \tag{36}$$

By inserting the values and precision errors from Table 7 into Equation (36), the precision index of 0.17 kg/s for airflow is obtained.

Table 7. Flow data

Parameter	Units	Nominal Value	Precision Index	Bias Limit
F_a	—	1.00	0	0.001
Φ^*	$\dfrac{kg \cdot K}{N \cdot s}$	0.0404	0.0	4.04×10^{-5}
a	m^2 0.191	0.148	9.55×10^{-5}	3.82×10^{-4}
P_{11}	Pa	2.54×10^5	345.0	345.0
T_{11}	K	303.0	0.17	0.17
m	kg/s	112.64	0.17	0.32

The bias limit in the flow calculation is propagated from the bias limits of the measured variables. Using the Taylor's series formula gives

$$B_f = \left[\left(\frac{\partial f}{\partial X_1} B_{X1} \right)^2 + \left(\frac{\partial f}{\partial X_2} B_{X2} \right)^2 + \left(\frac{\partial f}{\partial X_3} B_{X3} \right)^2 + \cdots + \left(\frac{\partial f}{\partial X_n} B_{Xn} \right)^2 \right]^{1/2} . \tag{37}$$

For this example, where $m = Fa\phi^* CaP1t/T1t$:

$$B_m = \left[\left(\frac{\partial m}{\partial F_a} B_{Fa} \right)^2 + \left(\frac{\partial m}{\partial \phi^*} B_\phi^* \right)^2 + \left(\frac{\partial m}{\partial a} B_a \right)^2 + \left(\frac{\partial m}{\partial P1t} B_{P1t} \right)^2 \right.$$

$$\left. + \left(\frac{\partial m}{T1t} B_{T1t} \right)^2 \right]^{1/2} . \tag{38}$$

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Taking the necessary partial derivatives gives

$$
B_m = C\left[\left(\frac{\phi^* aP1t}{\sqrt{T1t}} B_{Fa}\right)^2 + \left(\frac{Fa\ aP1t}{\sqrt{T1t}} B_{\phi^*}\right)^2 + \left(\frac{Fa\phi^* P1t}{\sqrt{T1t}} B_a\right)^2 + \left(\frac{Fa\phi^* aP1t}{\sqrt{T1t}} B_{P1t}\right)^2\right.
$$

$$
\left. + \left(\frac{Fa\phi^* aP1t}{-2\sqrt{T1t}^3} B_{T1t}\right)^2\right]^{1/2}
\tag{39}
$$

By inserting the values and bias limits of the measured parameters from Table 7 into Equation (39), a bias limit of 0.32 kg/s is obtained for a nominal airflow of 112.64 kg/s.

Table 7 contains a summary of the measurement uncertainty analysis for this flow measurement. It should be noted that the listed errors apply only to the nominal values.

3.3 Brief Check of Error Propagation

The method used to calculate the uncertainty values for Semiscale test data has been presented. To verify that error propagation equations work, they have been used for estimating errors on calculations such as "R - Prime" (fluid flow line resistance).

$$
R' = \frac{\rho \Delta P}{\dot{m}^2} = \frac{\Delta P}{\rho Q^2} \quad .
$$

Time-averaged steps were used—one out of twenty data values should fall outside of the error-propagated uncertainty bands as 95% uncertainty bands are used. Results obtained from three examples are as follows: none out of 17 data points fell outside, one out of 27 data points, and one out of 32 data points. This was an indication that (a) uncertainty estimates were correct, and (b) results of error propagation appear correct.

4. MEASUREMENT ERROR

All measurements have errors. These errors are the difference between the measurements and the true value. The problem is that the true value is usually unknown. The errors may be positive or negative and may be of a variable magnitude. Many errors vary with time or other parameters. The actual errors are rarely known; however, uncertainty intervals can be estimated or inferred as upper bounds on the errors. The problem is to construct an uncertainty interval that models these errors.

Two simple examples to help clarify the description of errors are demonstrated below.

A marksman in the field of shooting is concerned with uncertainty. The marksman uses different terminology to describe the uncertainty. When a marksman discusses the "size of group," he is referring to the diameter of a circle that will cover the scatter of the shots fired. When we discuss "precision" we are referring to a dimension that is expected to contain a given percentage of the scatter of a measurement. A marksman will refer to the group as being "a certain number of inches to the right or to the left." We would refer to the measurement as having a "bias" of a certain value, positive or negative if known. The marksman may refer to a "five-shot group." We would say "the sample size is five."

The number of shots in a group is a measure of significance, i.e., a 10-shot group being more significant than a three-shot group. Likewise, 10 degrees of freedom carries more significance than 3 degrees of freedom.

In both of the following examples there are 20 data points or "shots."

Example 1

The marksman would refer to Figure 6 as having a 4.4-unit group that is centered 2 units right and 2 units up. A measurement specialist would say that Figure 2 has a standard deviation of one unit in both x and y axes, with a positive bias of two units in both the x and y axes. Instead of saying it is a 20-shot group, the specialist would say the sample size is 20 and there are 19 degrees of freedom (1 degree of freedom was used to estimate the standard deviation). To estimate the total uncertainty (see Section 3.3), the following equation is used:

$$U_{95} = \pm\left[(\text{Bias})^2 + (t_{95}\ \text{Precision})^2\right]^{1/2}$$

where

t_{95} = t-value for 95% confidence level and 19 degrees of freedom is 2.093

U_{95} = uncertainty at 95% level is from -0.9 units to 4.9 units in both the x and y axis.

At the bottom of Figure 6 is a plot of relative expected frequency in the x axis. The total area under this curve from plus to minus infinity is 1 or 100%.
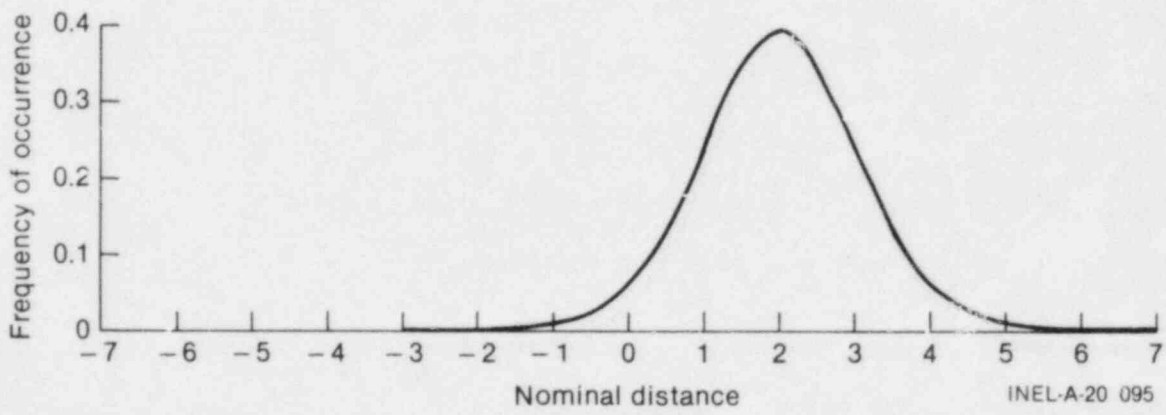
Example 2

A marksman would refer to Figure 7 as having a 12-unit group that is centered "dead on." The measurement specialist would say that Figure 7 has a standard deviation of 3 units in both x and y axes, with no bias. Total uncertainty at 95% confidence level, using the same equation as Example 1, is equal to ±6.3 units in both x and y axes.

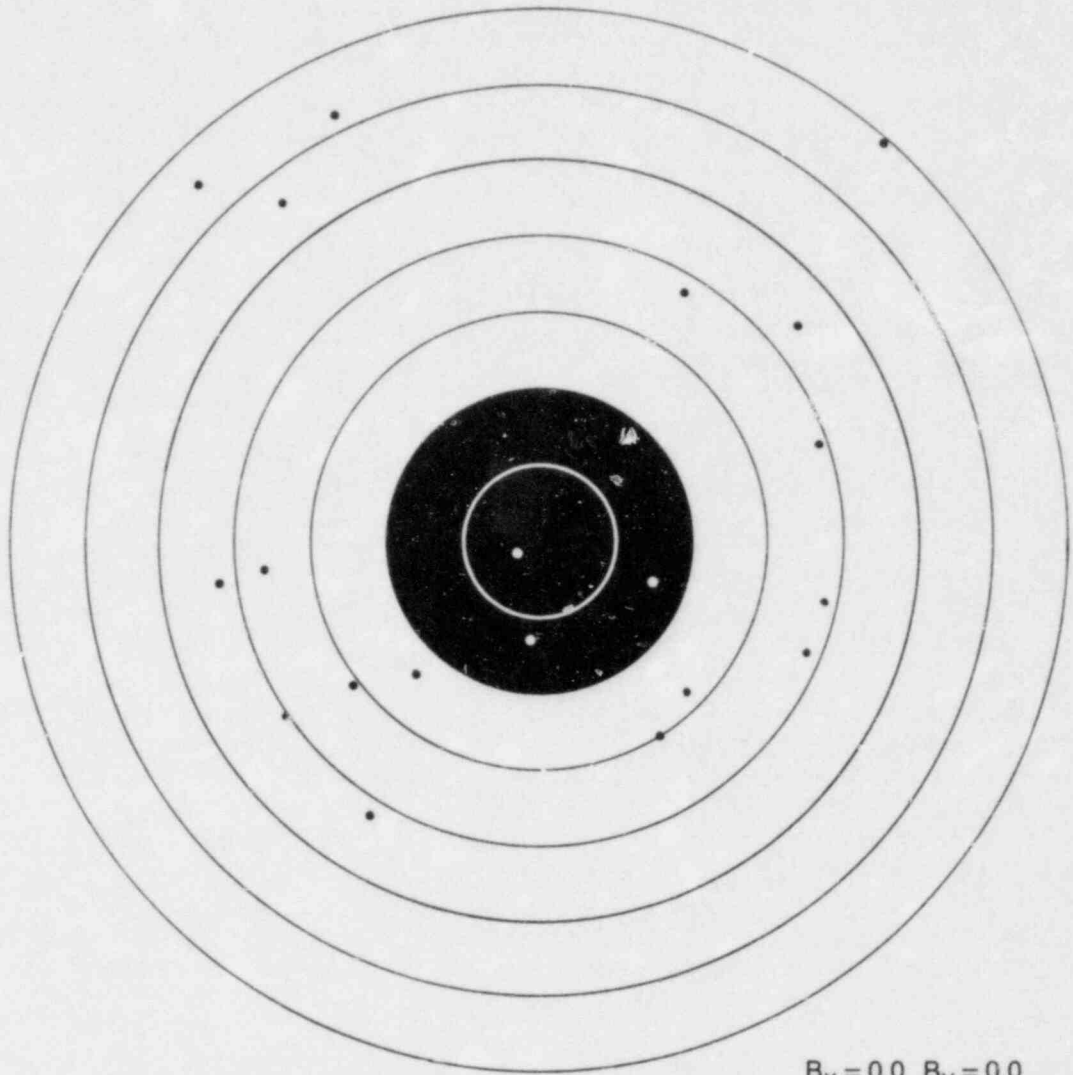At the bottom of Figure 7 is a plot of relative expected frequency in the x axis.
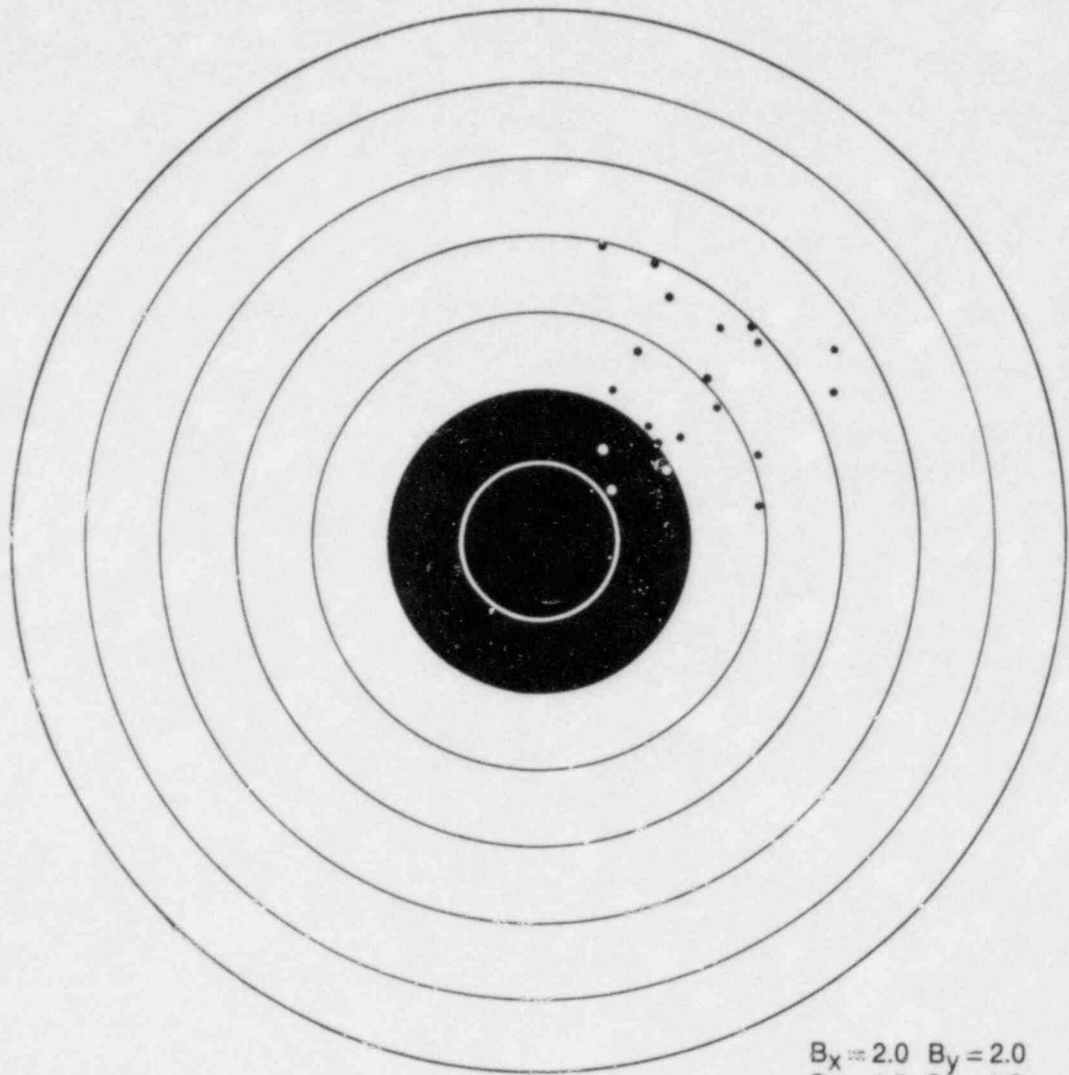
$B_x = 2.0 \quad B_y = 2.0$
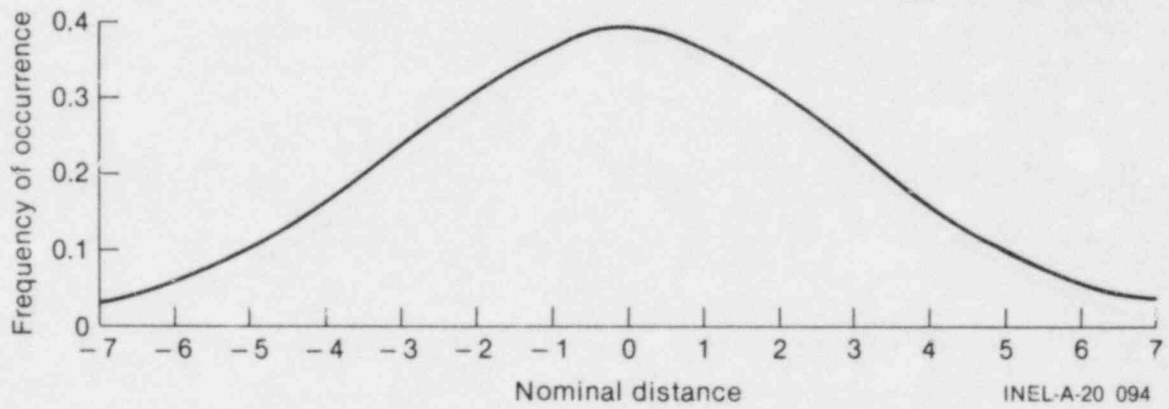$S_x = 1.0 \quad S_y = 1.0$

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Figure 6.   Example 1.

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$B_x = 0.0$ $B_y = 0.0$
$S_x = 3.0$ $S_y = 3.0$

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Figure 7.   Example 2.

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Examples given cover (Example 1) large bias with small precision errors, and (Example 2) small bias with large precision errors. Two additional possibilities exist: (a) large bias with large precision errors usually called inaccurate, and (b) small bias with small precision errors usually called accurate.

In target shooting, the process is similar to calibration. Calibration involves an assumed true value—in this case the bullseye, and the actual data—in this case the bullet holes. Comparisons and decisions based on these comparisons are made, such as adjustment of the sights or other corrective action such as more practice, or the decision may be that no action is needed.

An actual measurement in contrast to calibration does not have the true value assessable. This would be similar to examining the back side of the target. The game is to look at the data—the pattern of the shots—and, using past information such as the performance of the marksman, rifle, shells, and environmental effects, guess the true value or where the bullseye was.

Associated with the environmental effects are several bias factors that raise questions in the posttest analysis: (a) what was the temperature and how much effect does it have on the powderburn rate i.e., bullet velocity, (b) how much wind (most ranges have flags to estimate the magnitude and direction) and what was the effect, and (c) was compensation made by the marksman for any of these effects?

For ease of estimating where the true value is, you may break the system into parts that can be analyzed separately. For example, you might choose to analyze the following items separately: (a) the marksman, (b) the rifle, (c) the cartridges, and (d) the environment.

Semiscale also breaks the measurement system (see Figure 8) into parts that can be analyzed separately. Some of these parts will not change, and thus not need to be reanalyzed for other measurements. The measurement system is broken down into subsections and analyzed for the following errors:

1. Calibration Errors—Those errors that occur in the calibration hierarchy between the National Bureau of Standards and the measurement of the instrument.

2. Data System Errors—Those errors that occur in the data acquisition system. Included are the excitation voltage source, signal conditioning, amplifier, multiplexing, analog-to-digital conversion, and round-off errors within the computer.
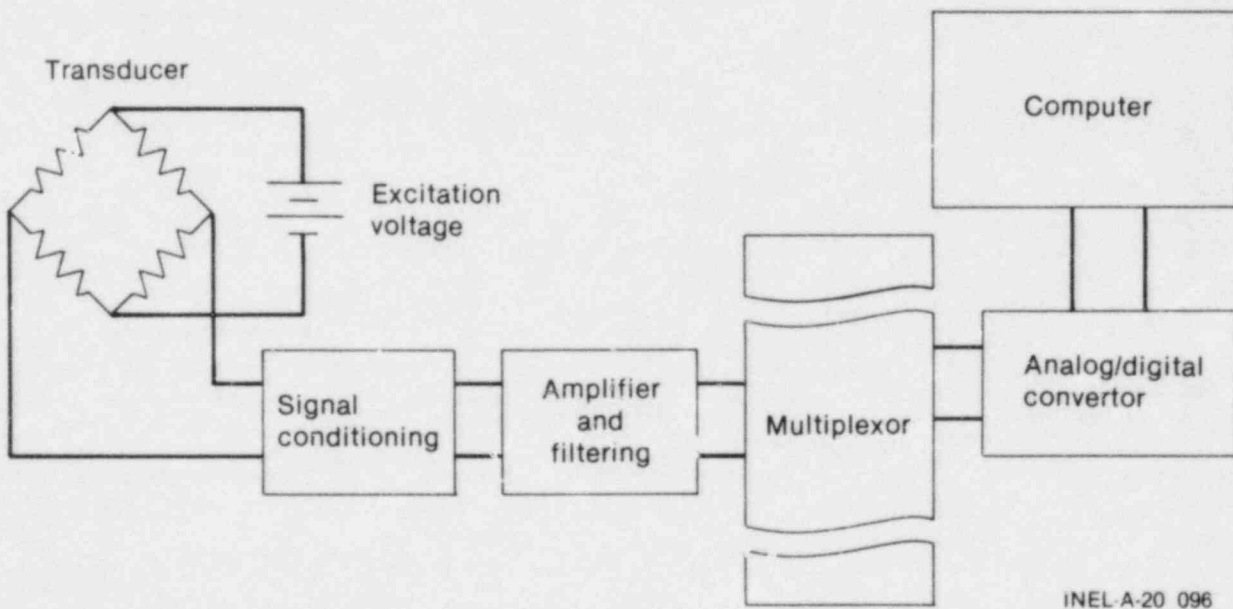

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Figure 8.   Data acquisition system.

3. Transducer Error—Those errors that occur at the transducer and the errors in the transducer's engineering unit conversion curve (usually derived from calibration data), and zeroing of the transducer.

4. System Errors—Those errors that occur between the system being measured and the transducer doing the measuring. Included are the errors that occur in the interface, such as in the tubing between a pressure probe and the pressure transducer, and secondary effects.

4.1 Calibration

In discussing calibration errors, there are two types; those associated with calibration of an instrument, referred to as "transducer error," and those associated with the calibration hierarchy and errors with the standards, referred to as "calibration errors."

The measurement error of the test result may be increased or reduced by calibration of the measurement instrument. The objective of this section is to estimate the error contribution of the calibration process to the test result. Both precision and bias errors may be involved.

There are a number of issues involved. To illustrate, we will consider an example first, followed by a discussion of several types of calibration (Subsection 2.2.4). Section 2.3 relates the defined measurement process to calibration errors and calibration errors that shift, jump, or trend with time.

Example 3—A Calibration Constant

Assume a test meter is to be compared or calibrated with a master meter at one flow level. The objective is to determine a correction, called a calibration constant, that will be added to the test meter observations when it is installed for test. This calibration constant correction will make the test meter "read like" the master meter. During the calibration, the master meter is used to set the flow level, since it is usually more accurate than the test meter. To reduce the calibration precision error, n comparisons will be made and averaged. If the data were plotted, they might look like this:

Master Meter		Test Meter
Δ		
Δ		
Δ		
Δ		
Δ		
Δ		
Δ		
Δ		
Δ		
Δ		x
Δ		xx
Δ		xxx x
Δ		x xxxxx
90	Flow	91.4

If the master meter bias limit from its own calibration is judged to be no larger than B_m, what will the test meter uncertainty be after calibration?

Define Δ_i, = Master Meter Reading$_i$ minus Test Meter Reading$_i$. Calibration constant equals the average Δ.

$$k = \frac{\sum\limits_{i-1}^{N} \Delta_i}{N} \ .$$

The sample standard deviation of the calibration data is

$$S_\Delta = \left[\frac{\sum\limits_{i=1}^{N} (\Delta_i - k)^2}{N - 1} \right]^{1/2} \ .$$

The standard deviation of k is

$$S_k = \frac{S_\Delta}{\sqrt{N}} \ .$$

The test meter is later installed in a test spool piece. Each observation made on the test meter is corrected by adding k. By this process the error in k from the calibration process is propagated to the corrected data from the spool piece.

As k is constant, this error must be a constant or bias error. It includes the bias error in the master meter plus the precision error in the calibration process. We can estimate an upper limit for this bias as

$$B_k = B_m + t_{95} S_k \ .$$

This calibration bias limit would be combined with bias limits from data acquisition and data reduction to obtain the measurement bias limit. There would also be precision error from these processes.

If the uncalibrated test meter had a bias limit judged to be B_T, the calibration process improves the test accuracy if B_k is less than B_T. Note that the calibration process does not change the test meter precision error (repeatability), which is included in the data acquisition precision. However, the test meter precision contributes to the calibration precision S_k. This contribution is reduced by averaging the calibration data.

4.1.1 Why Calibrate. There are at least three reasons for subjecting a test instrument to a special calibration. If none of these are required or justified, the instrument may be used without calibration.

1. Improve Accuracy—If the test meter is calibrated against a master meter of much better accuracy, the measurement accuracy of the test meter may be significantly improved. As indicated in Example 3, a measurement uncertainty analysis, with and without calibration, may be used to quantify the improvement.

2. Provide Traceability—In recent years, the demanding requirements of military and commercial contracts have led to the establishment of extensive hierarchies of laboratories within industry. In this country, a national standards laboratory is at the apex of these hierarchies, providing the ultimate reference for each standards laboratory. It has become commonplace for Government contracting agencies (including the Department of Energy) to require contractors to establish and prove traceability of their measurements to this laboratory. This requirement has created extensive hierarchies of standards within the individual standards laboratories.

Each calibration in the hierarchy constitutes an error source. Figure 9 presents a typical transducer calibration hierarchy. Associated with each comparison in the calibration hierarchy is a pair of
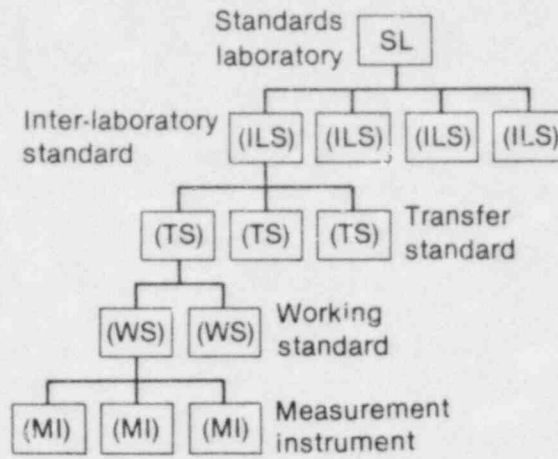
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Figure 9. Basic measurement calibration hierarchy.

elemental errors. These errors are the known bias and the precision index in each process. Note that these elemental errors are not cumulative, e.g., B_{2c} is not a function of B_{1c}. The error sources are listed in Table 8. Calibration to establish traceability may not improve accuracy, but is required.

3. Functional Checkout—Sometimes instruments are compared to master meters to check compliance with a purchase specification. A calibration curve or constant is not generated. If the test meter appears to satisfy the specification, it is accepted; if not, it will be rejected and returned to the vendor. Functional checkouts of instrumentation do not affect the measurement uncertainty analysis quantitatively; rather, they are accept-reject tests.

4.1.2 Calibration. Calibrations at the Semiscale Test Facility fit into each of the categories listed in the preceding subsection, and each of the various types of measurements have different calibration errors. Therefore, the actual analysis will be reported in the appropriate volume to be issued for the various measurement types.

Calibration errors are, in reality, calibration hierarchy errors. The National Bureau of Standards is at the pinnacle of the calibration hierarchy and is considered as having the true value. Error values for calibration are derived from estimates provided by whoever does our calibration. This includes the Standards and Calibration Laboratory, outside contractors, or ourselves.

Table 8. Calibration hierarchy error sources

Calibration	Bias Limit	Precision Index	Degrees of Freedom
SL—ILS	B_1	S_1	df_1
ILS—TS	B_2	S_2	df_2
TS—WS	B_3	S_3	df_3
WS—MI	B_4	S_4	df_4

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4.2 Data System

As discussed in Section 4, "Measurement Error," the data system consists of a signal conditioner, amplifier, multiplexer, analog-to-digital converter, and roundoff error on the engineering unit conversion coefficients. Components of the data system will be described and analysed for errors in the following subsections. Error estimates will be summarized in a table combined into bias and precision error estimates for the data system.

4.2.1 Signal Conditioners and Amplifiers. The signal conditioner and amplifier subsystem adapts the signals from the detectors to the recording subsystem. The data system uses a universal signal conditioner (Bay Laboratories, Inc., Model 7442) through which most data signals are processed. A thermocouple mode card is plugged into the signal conditioner for a temperature measurement and a full bridge mode card for a strain gauge type of measurement such as pressure. The mode cards have the capability of providing calibration signals to the amplifiers. Mode card calibration accuracy is 0.1% (0.05% at the 75% step) in 25% steps, and is used as a check on amplifier performance.

A Bay Laboratories, Inc., Model 5204 amplifier is set juxtaposed with the 7442 signal conditioner. Manufacturer specifications for the amplifier accuracy is ±0.1% of reading, stability is ±0.2% of reading per six months and linearity is ±0.005% of full-scale. The amplifiers are calibrated once a year with a precision voltage source. Any amplifier that is not consistent within the ±0.1% of reading using a precision voltage source is removed and repaired so that it meets the requirement. Voltage substitutions using mode cards and signal conditioners are used as a check to comply with the requirement of calibrating amplifers at least once every 180 days;[a] additional checks may be performed if there are any suspicions about an amplifier.

Specifications for the signal conditioner, which is a passive device, pertain to the power supply and will be covered as excitation voltage. Estimates of the precision error of the signal conditioner and amplifier pair is ±0.1% of the full-scale value.

4.2.2 Multiplexer and Analog-to-Digital Converter. The multiplexer and analog-to-digital converter are within the same case. Data Technology is the manufacturer of both units used at the test facility. One unit has a 12-bit analog-to-digital converter, the other has 16 bits connected to use only 12 bits. The manufacturer's specifications for the units are as follows: accuracy is stated as ±0.065% of reading with an offset of ±0.5 the least significant bit, plus a drift of 0.1% of reading per 30 days on the 12-bit unit. Accuracy is stated as ±0.023% ±0.5 the least significant bit on the 16-bit unit.

During tests, each of the units have a precision voltage source on one channel and a short for a zero reading on another channel. Statistical data on both channels are very similar. Precision errors, as estimated from the reference channels, are ±0.02% of the full-scale value. Bias errors, as estimated from the shorted channels, are ±0.024% of the full-scale value.

4.2.3 Roundoff Errors. Coefficients for the engineering unit conversion curves (except for thermocouples) are symmetrically rounded at four significant digits. Estimated errors from this process could have been handled with transducer errors, Section 4.3, but will be handled in this section. Maximum error would be ±0.05%, and this error will be handled as if it were a precision error. Precision error, as estimated, has an expected value of ±0.022% of the full-scale value. Bias error is estimated as negligible.

4.2.4 Excitation Voltage. Excitation voltage is provided by a power supply within each signal conditioner. Noise and ripple is specified as ±0.004% of reading, plus 150 μV when monitored. The noise and ripple is essentially nonexistent. When checked, the 5-V value is normally within ±3 mV. Precision error is estimated as nonexistent. Power supplies are estimated as having a value of 0.05% of the full-scale value bias error.

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a. As stated in an EG&G Idaho Quality manual.

4.2.5 Summary of Data System. The results of the data system errors are summarized in Table 9. At the bottom of the table is the root-sum-square of the precision and the bias errors. These results will be used in the subsequent volumes that will cover the various measurement types.

Table 9. Data system error summary

Error Source	Precision, S	Bias, B	Degrees of Freedom, ν
Signal conditioner and amplifier	0.1% of full scale	0.02% of full scale	600
Multiplexer and analog-digital converter	0.02% of full scale	0.024% of full scale	1838
Roundoff	0.0022% of full scale	—	100
Excitation voltage	—	0.05% of full scale	—
Root-sum-square	0.104% of full scale	0.059% of full scale	649

4.3 Transducer Errors

Transducer error as defined here includes the errors due to the engineering unit conversion curve (based on calibration data) as well as errors associated with the transducer (secondary effects).

Secondary effects are those parameters other than the primary parameter of interest that have an effect on the transducer. An example of this would be the pressure sensitivity of a differential pressure transducer. Most of the secondary effects values are derived from special or in-place calibrations that are used specifically to remove or correct measurement data for these effects. Frequent use of the word calibration may give the feeling that this section should be renamed "Calibration Errors." If all of the calibrations were made in-place with the same exact environmental conditions as during an experiment, the secondary effects would be cancelled out. Since it is impossible to establish the exact environmental conditions, we are dependent on the special calibration to qualify or quantify these effects. The calibrations used and typical values will be listed in the subsequent volumes that will cover the various types of measurements.

4.3.1 Transducer Calibration. Calibration, in this subsection, is the primary calibration of an instrument as required by EG&G Idaho Quality standards. Calibration requirements of instruments are usually for a one-year interval; however, if the experimental environment is severe, such as during large-scale breaks, transducers are normally calibrated on a six-month interval.

Calibration consists of recording the output response of a test instrument with respect to an applied signal referenced to a master instrument. One of the instruments, usually the master since it is more accurate, is used to set the measurement parameter in incremental steps (uniform or variable) over the range of the test instrument. A curve fit of this relationship is needed for use in the data reduction process. This will allow measurements from the test instrument to be corrected to those of the master instrument to improve accuracy and provide traceability. Errors in the calibration process contribute to errors in the test measurement and are included in the measurement uncertainty (see Section 4.2).

For data reduction at the Semiscale facility, the engineering unit conversion curves can be up to a fourth order polynominal curve fit. Least squares, also called regression analysis, is the method used to estimate the coefficients in the engineering unit conversion curve of the form

$$Y = \alpha_0 + \alpha_1 X + \alpha_2 X^2 + \alpha_3 X^3 + \alpha_4 X^4 .$$

The objective of regression analysis is to define the constants, $\alpha_0, \alpha_1, \ldots, \alpha_x$ of the curve, such that the sum of the squares of the vertical deviations of each data point from the curve is minimized, i.e., least sum of squared deviations

$$Q = \Sigma (Y - \hat{Y})^2 = \Sigma [Y - (\alpha_0 + \alpha_1 X + \ldots \alpha_k X)^k]^2$$

where $\hat{Y}$ is the calculated or estimated value of Y.

This theory can be restated using matrix notation as follows:[7]

$$\text{Let } Y = \begin{array}{c} y_1 \\ y_2 \\ \cdot \\ \cdot \\ \cdot \\ y_n \end{array}, \quad X = \begin{array}{ccccc} x_1 & x^2 & \ldots & x^\kappa \\ x_2 & x_2^2 & & x_2^\kappa \\ \cdot & \cdot & & \cdot \\ \cdot & \cdot & & \cdot \\ \cdot & \cdot & & \cdot \\ x_n & x_n^2 & & x_h^\kappa \end{array}, \quad \text{and } A = \begin{array}{c} \alpha_0 \\ \alpha_1 \\ \cdot \\ \cdot \\ \cdot \\ \alpha_\kappa \end{array} .$$

The expected value of the Y's, then, is expressed as

$E(Y) = XA$ and the condition of independence and common variance.

Under these conditions, the minimum variance unbiased estimates $\hat{A}$ of A are given by the solution of the normal equations.

$$X^t X \hat{A} = X^t Y$$

where the superscript t means transposed.

A solution for the A_i's can be arrived at without explicitly computing the C_{ij}'s, of course, but is needed in the following equations.

In matrix notation, this step is given by computing the inverse of the matrix of normal equations, i.e.,

$$(X^t X)^{-1} = \begin{array}{cccc} c_{00} & c_{01} & \cdots & c_{0\kappa} \\ c_{10} & c_{11} & \cdots & c_{1\kappa} \\ \cdot & \cdot & & \cdot \\ \cdot & \cdot & & \cdot \\ \cdot & \cdot & & \cdot \\ c_{\kappa 0} & c_{\kappa 1} & \cdots & c_{\kappa\kappa} \end{array} = C$$

$$(X^t X)^{-1} (X^t X \hat{A}) = CX^t Y = I\hat{A} \qquad .$$

The trace of IA yields the coefficients $\alpha_0, \ldots, \alpha_k$.

Standard error of estimate or standard deviation about the curve (see Figure 10) is calculated from calibration data using the following equation:

$$(S_{ee})^2 = \sum_{i=1}^{n} \frac{(Y_i - \hat{Y}_i)^2}{n - k - 1} = \sum_{i=1}^{n} \frac{(Y_i - \hat{A}X_i)^2}{n - k - 1}$$

where

Y = the known applied signal

$\hat{Y}$ = the estimated or calculated applied signal

$\hat{A}$ = the coefficient matrix

X = the measured transducer output signal

n = the number of calibration data points

k = the order or degree of curve fit.
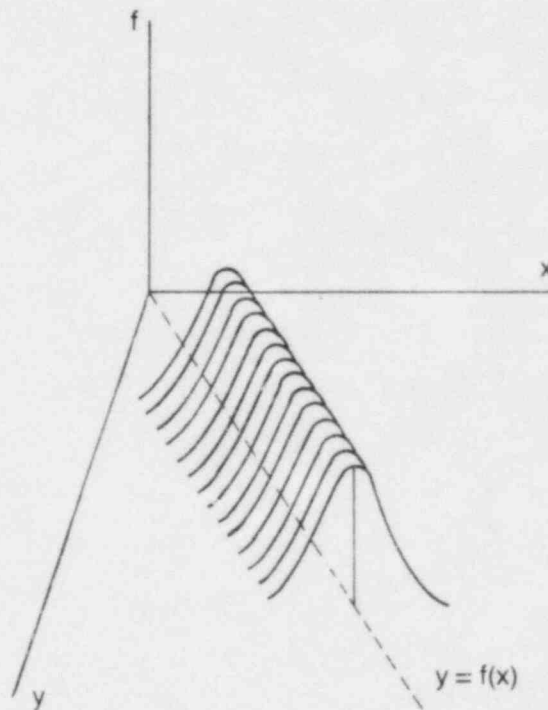
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Figure 10.   Normal distribution about a regression curve.

4.3.2 Inverse Estimation. The purpose of a calibration curve is to estimate master instrument readings (Y's) from observed output values of the test instrument (X's). Initially, the Y's were known with small error. In proceeding from observed X's to estimated $\hat{Y}$'s, the errors of observation and calibration will be carried along.

For example, if the relationship is a straight line and determined by least squares, the estimates of the parameters, α_0 and α_1, will have errors. In the form $Y = b_0 + b_1 (X - \bar{X})$, b_0 and b_1 will be independent of each other and each will be independent of the residual error $(S_{ee})^2$

$$\text{for } b_0, \quad S_{b_0}^2 = \frac{(S_{ee})^2}{n}$$

$$\text{for } b_1, \quad S_{b_1}^2 = \frac{(S_{ee})^2}{\sum (X - \bar{X})^2}$$

An inversely estimated $\hat{Y}_0$ for given output X becomes

$$\hat{Y}_0 = \bar{Y} + \frac{X - b_0}{b_1}$$

which is now a function of three estimated quantities: X, b_0, and b_1. The concept of using inverse estimation where error is present in both the X and Y variables was first introduced by Berkson.[8] The inverse estimate $\hat{Y}_0$ will be approximately normally distributed about its unknown true value (neglecting bias for the moment). From the propagation of errors, the variance of the estimated $\hat{Y}_0$ is approximately

$$S_{\hat{Y}_0}^2 = \left(\frac{\partial \hat{Y}_0}{\partial X_0} \right)^2 S_X^2 + \left(\frac{\partial \hat{Y}_0}{\partial b_0} \right)^2 S_{b_0}^2 + \left(\frac{\partial \hat{Y}_0}{\partial b_1} \right)^2 S_{b_1}^2 \quad .$$

Evaluating the partial derivatives

$$\frac{\partial \hat{Y}_0}{\partial b_0} = \frac{1}{b_1}$$

$$\frac{\partial \hat{Y}_0}{\partial b_0} = \frac{1}{b_1}$$

$$\frac{\partial \hat{Y}_0}{\partial b_1} = - \frac{X - b_0}{b_1^2}$$

$$S_{\hat{Y}_0}^2 = \frac{(S_{ee})^2}{(b_1)^2} \left[1 + \frac{1}{n} + \frac{(X - b_0)^2}{b_1^2 \sum (X - \bar{X})^2} \right] \quad .$$

Note that both $\Sigma (X - \bar{X})_2$ and n increase with n (the number of calibration points). Therefore, as the pedigree of the calibration is improved with more data, $S_{\hat{Y}}$ will approach

$$S_{\hat{Y}_0} \simeq \frac{S}{b_1 \sqrt{n}} \quad .$$

Another method of accounting for the inverse estimation[9] is multiply the standard error of estimate by a regression factor, R, where

$$R = 1 + 1/n + \left[\frac{(X - \bar{X})^2}{\frac{(n \sum X)^2 - (\sum X)^2}{n}} \right]^{1/2}$$

for the linear case using partial regression form. Partial regression is formed by subtracting the mean value from the original form.

$$Y = \alpha_0 + \alpha_1 X + \ . \ . \ . + \alpha_\kappa X^\kappa$$

$$\bar{Y} = \alpha_0 + \alpha_1 \bar{X} + \ . \ . \ . \ \alpha_\kappa \bar{X^\kappa} \quad .$$

This leads to the paritial regression form

$$Y - \bar{Y} = A \ X - A \ \bar{X} = \alpha_1 \ (X - \bar{X}) + \ . \ . \ . \ \alpha_\kappa \ (X - \bar{X^\kappa})^\kappa = A(X - \bar{X}) \quad .$$

Note that the α_0 term subtracts out and the matrix calculation is one order lower; this will improve the accuracy of the matrix inversion routine for higher order curve fits.

$$R = [1 + 1/n + (X - \bar{X})^t \ C(X - \bar{X})]^{1/2}$$

where C is the variance/covariance matrix in the partial regression form.

For the full regression form,[6]

$$R = \left[x^t (X^t X) x \right]^{1/2}$$

where $x^t = x, x^2, \ . \ . \ ., x^\kappa$, x is the value of x at the point of interest, and X is matrix of summation of all the x's.

Because the computer systems at the Standards and Calibration Laboratory are not programmed to calculate R in either form, calculations of R for several instruments were performed by hand. It was decided to provide an estimate of typical R for various orders of curve fits for a purely linear case.

The regression factor R will produce the Scheffe[10] prediction bands, which are of one order higher than the curve fit (see Figure 11), i.e., a first-order linear curve has a second-order uncertainty band. Since the regression factor R is estimated from a purely linear case and the actual value for R is slightly larger, we choose the value for R from the end. This choice will give a slightly conservative value for R. More detail will be provided in a subsequent volume on differential pressure or pressure measurements.
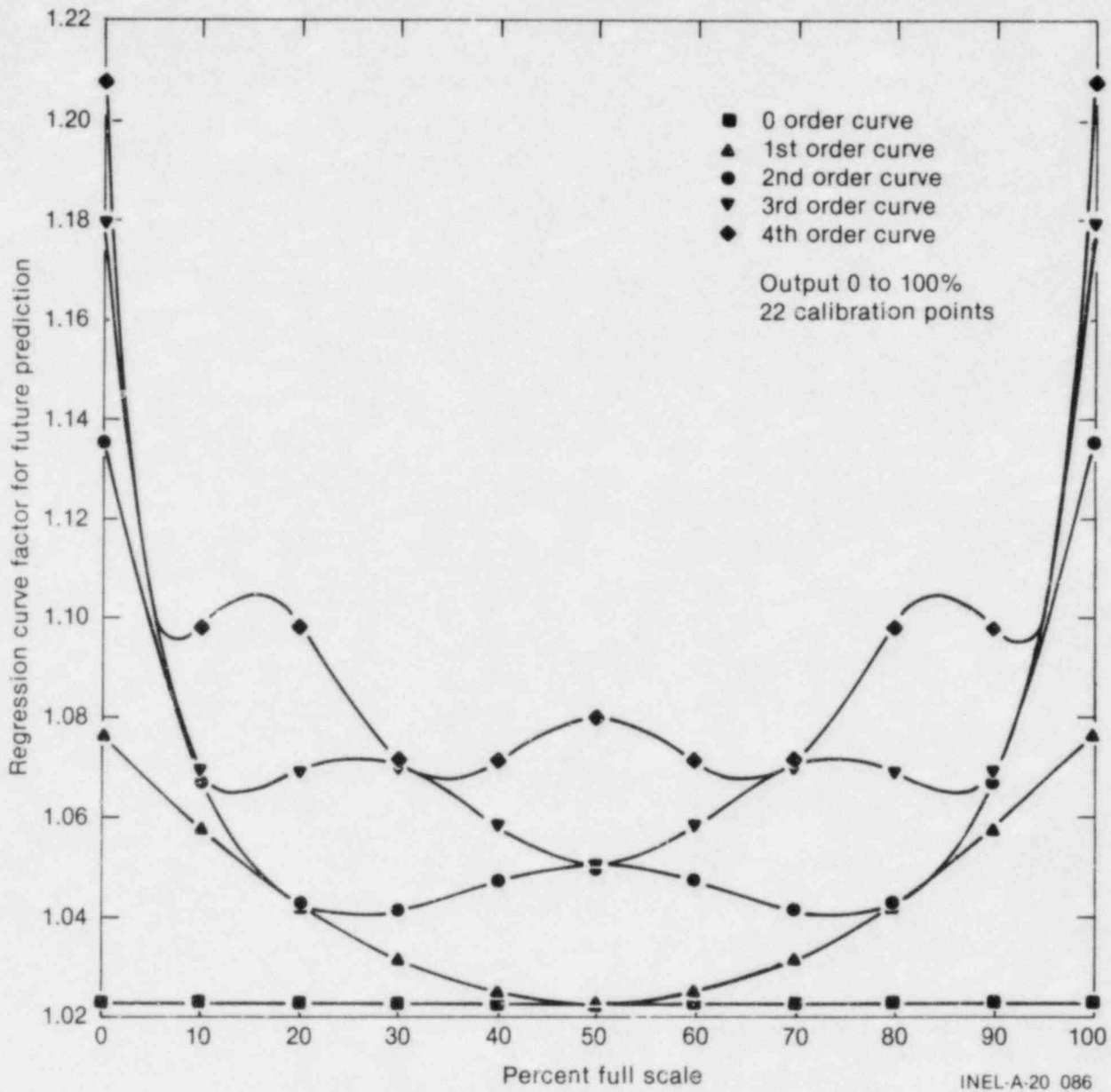
Figure 11. Regression factor for linear data.

4.3.3 Calibration Variation. The standard error of estimate (S_{ee}) is a measure of the precision error about the curve within the calibration data. From calibration-to-calibration, there may be additional variation, which is estimated using calibration histories. At the Semiscale Test Facility we have some calibration histories for transducers in active service that date back nearly 10 years. Calibrating yearly as required by EG&G Idaho Quality standards, and with our own six-month requirement during large break tests, means that the older transducers may have 10 or 12 calibrations on them, and newer transducers one or two.

Transducers of a given measurement type with a slope change of less than 2% have their percent of change on the slope lumped together. Lumping all the percent of slope change together allows the calculation of a statistically significant number. Transducers with more than a 2% change of slope have their calibration histories analyzed for a trend, and may be returned for recalibration. Many of the transducers with more than a 2% change return to their former slopes with future calibration. This tends to indicate a possible systematic error in the calibration procedure, which, fortunately, occurs infrequently and the trend seems to be decreasing.

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For cases in which this calibration-to-calibration history is unavailable, an additional bias error based on judgment is added to the other elemental transducer bias limits to account for calibration-to-calibration error.

4.3.4 Transducer Zeroing. At the Semiscale Test Facility, the nominal conditions existing when transducers have zeros adjusted are: primary system is liquid full, temperature 295 K, pressure 0.43 MPa; and secondary systems are liquid full, temperature 295 K, pressure 0.086 MPa. Amplifiers for thermocouples are zeroed with shorted inputs. Most measurements have their "zero" adjusted at the amplifier, thus removing errors such as bridge zero measurand output, etc. Checks are made before a test to verify consistency, such as comparing temperatures at isothermal conditions with and without flow, and comparing pressure readings with a master pressure measurement while varying the pressure hydrostatically. Amplifier zero adjustments are within 5 mV, with a full-scale of ± 10 V; this amounts to a 0.05% of full-scale bias error, B_t.

4.3.5 Order of Curve Fit. Pressure and differential pressure transducers have the order of curve fit chosen by a program called "minimum-bias."[11] Bias as addressed in this subsection is not bias as defined in this report. Bias, as used here is a systematic discrepancy between a fitted curve and the true equation governing the data, and can arise from either oversmoothing or undersmoothing a curve fitted to experimental data. Bias is almost sure to exist in the representation of data by an arbitrary curve. The minimum bias criterion selects that curve out of several least squares polynomial approximations, beginning with degree zero and of increasing degree of which the ratio of the overall sum of residuals (actual value minus estimated value) squared to the sum of two subset sums of residuals squared is minimum. To restate this, the same order of curve is fitted to data from the first half, data from the second half, and overall data. The bias factor, M_B, is equal to the ratio of residuals over all the data divided by the sum of residuals of the first half of the data plus the residuals of the second half of the data. Mathematically expressed,

$$M_B = \frac{\sum_{1}^{n} (Y - \hat{Y}_t)^2 + C}{\sum_{1}^{n/2} (Y - \hat{Y}_1)^2 + \sum_{n/2+1}^{n} (Y - \hat{Y}_2)^2 + C}$$

where

Y = original data

$\hat{Y}_t$ = estimated data from overall curve

$\hat{Y}_1$ = estimated data from first half curve

$\hat{Y}_2$ = estimated data from second half curve

C = a small constant added to keep equation within bounds if denominator goes to zero

M_B = minimum bias factor.

The object is to find the order that minimizes M_B. This criterion strongly favors curves having high predictive merit for future data. In other words, the curve in the first half predicts the data in the second half.

Curves for thermocouples are all of fourth order, and all other measurements (except pressure and differential pressure, previously discussed, and those with known curves) are linear, first-order curves.

4.3.6 Summary. Various forms of calibration, dependent on the type of measurement, are used, and since calibration forms the basis for the "transducer" errors, the discussion will be provided in subsequent volumes for the various types of measurements.

4.4 System Errors

System errors refer to those errors that are a function of probe design, environmental conditions, and installation effects. System errors are discussed in more detail in the volumes for the various types of measurements.

System errors are among the more difficult to analyze. Installation effects should be determined or estimated by either performing "separate effects" tests, which are special calibrations to determine the measurement characteristics, or by mathematically analyzing the "physics" of the measurement. Another method sometimes used is estimation based on experience; this should be avoided if possible, since most of these errors are bias errors and during replicated tests are not observable.

Environmental errors are those errors that affect the transducer as a function of its environment. An example would be the temperature sensitivity of a differential pressure transducer. Since these types of problems are dependent on the type of measurement they are addressed in the volumes on the various types of measurements. Environmental errors that are not covered with transducer errors (Section 4.3) but are determined by pre- or posttest checks are treated as system errors.

5. REPORTING UNCERTAINTIES

The equation, $U_{95} = \pm \left[B_2 + (t_{95}\ S)^2 \right]^{1/2}$, used for calculating uncertainties is awkward to use when either or both bias and precision errors are in the form of a constant plus a percent of reading. To simplify the form, a linear, first-order equation is fitted to the values at 10% steps.

As a result, for the worst case there is a high value for uncertainty at zero reading, the value is on at approximately 12% of the full-scale reading, it is about 10% low at approximately 30% of the full-scale reading, is on at approximately 80% of reading, and reads 7% high at 100% of the full-scale reading. The form that the uncertainty value then takes is a constant in the engineering units plus a percent of reading. Errors in using this form are less than 10% of the uncertainty value and are much easier to use. Uncertainty values are thus reported in the following form:

$U_{95} = \pm$ (XX engineering units + YY percent of reading)

where XX and YY is a two-digit, unknown number.

Values for uncertainty are carried out to two significant digits, and either the constant portion or the percent of reading may have a zero value. An example would be a temperature reading:

$U_{95} = (2.3\ K + 0.23\%$ of reading).

5.1 Reporting Error Summary

The uncertainty analysis report should include (a) a summary of test result errors and (b) a table showing the contributions of the elementary error sources.

The definition of the components, bias limit, precision index, and the limit (U) suggests a summary format for reporting measurement error. The format will describe the components of error, which are necessary to estimate further propagation of the errors, and a single value (U) that is the largest error expected from the combined errors. Additional information, degrees of freedom for the estimate of S, is required to use the precision index. These summary numbers provide the information necessary to accept or reject the measurement error. The reporting format is:

1. S, the estimate of the precision index, calculated from data.

2. ν, the degrees of freedom associated with the estimate of the precision index (S). The degrees of freedom for small samples (less than 30) is obtained from the Welch-Satterthwaite procedure illustrated in the examples given in Section 2.5. This may be omitted if the alternate model is used and there is no need to further propagate the error.

3. B, the upper limit of the bias error of the measurement process or B^- and B^+ if the bias limit is asymmetrical.

4. The uncertainty interval formula should be stated. $U_{99} = (B + t_{95}\ S)$ or $U_{95} = \left[B_{95}^2 \ (t_{95}\ S)^2 \right]^{1/2}$ the uncertainty limit within which the error should fall. The t-value is the 95th percentile of the two-tailed Student-t distribution and is taken as 2 if the sample size is 30 or greater. If the bias limit is nonsymmetrical, two values will be given for the uncertainty; U_{95}^- and U_{95}^+. No more than two significant places should be reported.

The model components, S, ν, B, and U, are required to report the error of any measurement process. For simplification, the first three components may be relegated to the detailed sections of uncertainty

reports and presentations. The first three components, S, ν, and B, are necessary to (a) indicate corrective action if the uncertainty is unacceptably large before the test, (b) to propagate the uncertainty to more complex parameters, and (c) to substantiate the uncertainty limit.

To support the measurement uncertainty summary, a table detailing the elemental error sources is needed for several purposes. If corrective action is needed to reduce the uncertainty or to identify data validity problems, the elemental contributions are required. Further, if the uncertainty quoted in the summary appears to be optimistically small, the list of sources considered should be reviewed to identify missing sources. For this reason, it is important to list all sources considered, even if negligible.

Note that all errors in Table 9 have been propagated from the basic measurement to the end test result before listing and, therefore, they are expressed in units of the test result.

6. CONCLUSIONS

In summary, measurements are subject to two types of errors, i.e., bias and precision. One sample standard deviation is used as the precision index, s. The fixed error, β, is estimated and used as the bias limit, B. Both bias and precision are combined independently of each other using the root-sum-square method. To combine bias and precision, the total combined precision is first multiplied by the 95% t-value and combined with the bias using the root-sum-square method.

Accuracy of the test may be a part of the test requirements. Pretest uncertainty analysis will allow for corrective action before the test if the uncertainties are too large. Posttest analysis is required to confirm the pretest analysis or to identify problems. Comparison of test results with pretest analysis is an excellent data validity check.

Propagation of errors using the Taylor's series method is derived. Two approaches are used to calculate errors when two or more measurements are used to calculate additional parameters. First is the analytical approach; when a known mathematical relationship exists, partial derivatives are used to determine sensitivities. Second is the numerical approach; when a complex relationship exists, the finite differences are used to determine sensitivities.

Monte Carlo simulations are presented to show restrictions that must be placed on the method of partial derivatives. This is most accurate for functions involving sums and differences of the observed variables. For these functions if the variables are not mutually independent, a correction factor can be computed that will ensure exactitude of the method. Close approximations can be made for errors that exist in functions involving products and quotients of independently varying observed values, if the ratio of measured errors to their respective nominal values is small (less than 10%). For all of the functions examined involving two or more independent variables, the approximation is within 10% of the true value.

All measurements have errors. Calibration has two types of errors, the hierarchical errors referred to as "calibration errors," and transducer response to calibration referred to as "transducers errors." Three reasons are given to calibrate; to improve accuracy, to prove traceability, and as a functional checkout.

Components of the data system are defined, including specifications and test measurements, to identify error parameters. Results of error analysis of the data system are summarized in a tabular form. These results will be used in the subsequent volumes in analyzing uncertainties for each type of measurement.

Transducer errors are defined as those that occur at the transducer, and the values are determined by calibration. The method of least squares is developed, "inverse estimation" is defined, and the effects of regression analysis are described. Effects of calibration-to-calibration variation are discussed. Typical conditions existing when transducer are zeroed in both the primary and secondary systems are mentioned.

Transducers that have unique engineering unit conversion curves on the basis of calibration data have the order of curve fit determined by the "minimum bias" criterion. The minimum bias criterion is described by the type of curves that it prefers.

System errors are defined as errors due to environmental conditions and installation factors. Analysis of system errors are deferred to the subsequent volumes that will cover each type of measurement reported.

The form in which uncertainty will be reported is covered. When the bias or precision is a function of percent of reading, the uncertainty, U_{95}, will be nonlinear because of the square root. This function is linearized to make it easier to use and the effects are described. The error of using this technique will be within $\pm 10\%$ of the true value of U_{95}.

The standard on uncertainty of the International Standards Organization requires that all elemental error sources considered be identified for reference, and that the three components of uncertainty (precision s, bias

B, and degrees of freedom ν) be referenced in some report. The elemental sources considered to be typical values of precision, bias, and degrees of freedom will be provided in the subsequent volumes for a given type of measurement.

Uncertainty values, U_{95}, are calculated and presented in all reports that reference "qualified" experimental data.

7. REFERENCES

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