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THEORETICAL BASIS OF DIGES

N. SIMOS, A.J. PHILIPPACOPOULOS

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ABSTRACT

DIGES is a computer code that treats the response of structural and foundation systems due to dynamic loadings. The building foundation configuration and its response is computed by solving the pertinent equations of motion. A variety of structural and foundation systems are considered which are typical to those encountered in today's engineering problems.

This report describes the theoretical basis of DIGES. The primary goal of the report is to present pertinent information required to initiate the process of having individuals from industry, regulatory agencies, and academic institutions as users of the code.

EXECUTIVE SUMMARY

In recent years, more analysts are using direct generation methods in studies of dynamic response of structures as well as mechanical/electrical systems and components. The use of direct generation methods seems to be predominant in re-analysis or margin type assessments of existing structures to updated design criteria. The realization of this trend from a regulatory perspective, reflected the need for a computational tool that can be used to benchmark the results obtained through direct generation. This apparent need lead to the development of the DIGES code.

During the development of DIGES, a systematic effort was made to give a generic character to the code by restricting the amount of limitations imposed on its theoretical basis. We believe that we accomplished this objective. This is reflected by the following basic features of the code:

- DIGES has both deterministic and probabilistic response analysis capabilities. Accordingly, one can use a single time history representing the dynamic input and perform a deterministic dynamic analysis using DIGES. The output computed by DIGES consists of time histories associated with the system response or transformed forms of it, which are essentially Fourier records or conventional response spectra. On the other hand, one can use a power spectal density function or a cross-spectal density matrix representing the dynamic input and perform a probabilistic dynamic analysis. The output computed by DIGES consists, in this case, by power spectral density functions or cross-spectal density matrices and associated correlation functions of the system response. DIGES subsequently converts the response power spectral densities to corresponding response spectra. A third option of analysis by DIGES involves simulation. Briefly, in simulation the dynamic input consists of a set of time histories, each of them deterministically filtered by the system thus producing a set of response time histories and associated response spectra. Representative responses can be then computed statistically from the latter set.

- DIGES is capable of performing response calculations for the two fundamental categories of dynamic input considered in dynamic analysis. Specifically, the user of DIGES has two options for defining the input for response calculations: a) input in the form of applied dynamic loads and b) input in the form of excitation. Therefore, we did not restrict the code only in the seismic area but we broaden its computational capabilities to include general types of dynamic loads imposed on the superstructure. Such capability is required to tackle engineering problems in which the computation of the dynamic response is required for cases involving vibration tests, impact, wind and general type of lateral dynamic loadings.

- DIGES considers both structural as well as soil dynamics in computing responses due to ground motions or due to dynamic loads applied at the superstructure. This is done through a detailed soil-structure interaction formulation which allows for such computations to be carried out. We concentrated on the development of a comprehensive set of transfer functions relating fundamental system response parameters to the ground motion or to the applied loads. This reflects our belief that direct generation, essentially a probabilistic dynamic analysis, should include all system characteristics that are conventionally considered in deterministic response analysis.

- DIGES is capable of handling alternate ways of defining the seismic input. Specifically, the latter can be defined by: ground acceleration time histories, ground response spectra, Fourier amplitude spectra or power spectral densities. Furthermore, the seismic input is defined as either the excitation which is directly applied at the foundation of a structure or the ground motion of a site at a given point. In the latter case, the motion is "transferred" to the foundation through convolution/deconvolution or generally through kinematic interaction. Consequently, we believe that DIGES has a broad capability in the area of seismic input definition.

Another point of interest which is worthwhile to briefly discuss here, is that from published studies on the subject of direct generation it can be seen that the way we define the design seismic input for earthquake response analysis still remains an issue of concern (Refs. 9, 10). While the convenience of defining the seismic input by design ground response spectra has been well understood, the difficulties in using response spectra as input for dynamic analysis continues to be a source of controversies. In simple terms, a response spectrum of an earthquake acceleration record is what the record looks like after subjected to some level of filtering by a single degree-of-freedom system. Theoretically, we cannot uniquely recover an earthquake acceleration record from its response spectrum. One, however, can uniquely recover an earthquake record through transformations, e.g., from its Fourier transform or perhaps from other type of transforms. Fourier transforms, however, have not been found yet to be a convenient way for defining a design seismic input. Therefore, when an analyst is given a response spectrum to perform a dynamic analysis of a given structure, what he/she is asked to do is to compute the response of the structure using as input the response of single degree-of-freedom systems to "some" earthquake. Although spectrum superposition techniques based on modal analysis offer some approximate solutions which could be acceptable in certain applications, a rigorous dynamic analysis (linear or nonlinear) cannot be carried out under these circumstances. These are known facts to which we are simply reminded of. Consequently, it seems that the practicality of defining the seismic input for dynamic analysis by design spectra is still questionable.

The main procedure followed in existing direct generation studies consists of generating PSD's compatible with given ground design spectra. This procedure has similar short-comings to that of generating spectrum compatible times histories. As a result of a recent review of published studies in the area of direct generation (Refs. 9, 10), NRC was advised to undertake a systematic effort leading to the development of design seismic inputs for dynamic analysis using purely probablistic approach. By doing so, the clear advantage is that the resulting design seismic input would describe the design ground motion alone (not the response of the design ground motion to single degree-of-freedom systems). The development of generic type PSD's has also considerable appeal. Such development can be similar to that lead to the Newmark-Kapur generic spectra. The appealing factors of such PSD can be understood by considering the following facts: a) time-histories can be readily synthesized with a given PSD and b) reliable estimates of the statistics of elastic and inelastic nonlinear random seismic response can be efficiently computed by using well established techniques.

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1.0 INTRODUCTION

1.1 Problem Definition

The need to develop acceptable floor response spectra has been an ongoing process. Such spectra are affiliated with dynamic loads that the structure is subjected to and they represent the prediction of the responses at various elevations within the structure that in turn can be utilized to predict the response of equipment and components located at a particular elevation. The seismic loads are conventionally expressed in the form of *design response spectra*.

Consequently, the development of computational schemes which can incorporate the information, or assessment, pertaining to the seismic input and, in conjunction with the dynamic characteristics of the structure, predict the spectral responses at various elevations, has been the focus of earthquake response prediction. The definition of the seismic load, which determines of course the theoretical basis of the link between excitation and response, has been deduced from both deterministic as well as stochastic models.

On one hand, the deterministic approaches seek to assess the in-structure response due to a prescribed ground excitation or a prescribed dynamic load imposed on the structure itself. On the other hand the stochastic approaches attempt to define the in-structure response to an anticipated ground excitation that belongs to a family of earthquakes which in turn is described by *target* response or power spectra.

Within the stochastic processes, however, the statistics that accompany the definition of the ground excitation are *usually* carried over to the floor response with an ensemble of realizations of the stochastic process that defines the ground excitation. This simulation of earthquakes procedure that attempts to *match* the statistics of the target spectrum has been used extensively both by *directly* linking the target response spectrum to an artificial earthquake excitation or by implementing the constraint of the power spectral density function of the ground motion. The latter earthquake simulation process, more sophisticated in nature, matches some of the statistics of the target response spectrum with the sample earthquakes which are deduced from the power spectrum of the stochastic process.

The direct link between a stochastic characterization of the ground excitation and the stochastic in-structure response has received less attention than the time history approach. Through this process, the statistical properties of an anticipated family of earthquakes, expressed by its power spectrum, are transferred to the structure of deterministically defined dynamic properties.

1.2 DIGES Profile

The present effort has been undertaken so that an efficient theoretical/computational tool can be devised such that seismic problems of concern to regulatory agencies can be effectively treated. In this study, the *direct* link between the input excitation and the output response in the stochastic sense is explored (from which DIGES is deduced). This aspect of the seismic analysis, along with the earthquake simulation procedures and the deterministic seismic and dymanic response of the structure, defines the DIGES computational domain.

An overall description of DIGES can been seen in Figure 1.2-1 where its general capabilities are listed. According to Figure 1.2-1, analyses of both stochastic and deterministic nature can be undertaken. While in the deterministic analysis the consideration of dynamic superstructure loads has been implemented (an important element of dynamic analysis) alongside with the classical treatment of defined ground motion, the stochastic analysis mode incorporates both the earthquake simulation and the direct transferring of stochastic properties.

The relationships that connect the dynamic input to the system response are schematically shown in Figures 1.2-2 and 1.2-3 (stochastic and deterministic modes respectively). In both modes of analysis the *link* is the transfer function $H(\omega)$ of the system which identifies the superstructure/foundation/soil medium.

The stochastic mode of Figure 1.2-2, which implements both the simulation and the direct generation, seeks to evaluate the response spectra induced by ground excitations that can be defined by either target response spectra or cross-spectral densities of the stochastic process describing the excitation.

The *direct* stochastic mode determines the cross-spectral density matrix of the response $\Phi_Y(\omega)$ for a stochastic process with cross-spectral density $\Phi_X(\omega)$. For a stochastic process that defines the free-field in terms of *target* response spectra a consistent cross-spectral matrix is formed and eventually transferred to the elevation. The simulation seeks the floor response spectra by utilizing statistical properties of the responses at the same elevation due to an ensemble of ground accelerations whose response spectra that *match* the target spectrum over some of its statistic properties. As shown in Figure 1.2-2, both simulation procedures are implemented (one leads to ground motions from a response spectrum through its power spectrum and the other to ground motions directly from the response spectrum).

Considering that structures are dynamically treated mostly as *linear* systems, so all of the above listed alternative procedures can be utilized, room must be left to explore *non-linear* behavior. The *direct* mode of spectra generation seems to work well for linear systems and for stationary stochastic processes. However, work needs to be done to generalize its applicability. This gap can be filled with the help of simulated earthquakes, which as an ensemble can equivalently represent the stochastic process, by exciting the non-linear system in the time space rather than frequency.

The *deterministic* mode of Figure 1.2-3 utilizes the system transfer function $H(\omega)$ to evaluate the response spectrum at specified elevations due to a defined ground acceleration. This process requires that the input ground motion be expressed in terms of the Fourier expansion of the record. The evaluation of the response of the superstructure as well as the response of the foundation due to applied dynamic loads is also incorporated. Wind loads, impact loads and floor dynamic loads due to equipment can be treated by utilizing appropriate system transfer functions which relate the dynamic superstructure loads to the motion of the building-foundation system.

1.3 Future Work

While the fundamental steps in evaluating in-structure responses are both theoretically and computationally implemented in this study and the fundamental objectives have been met, much work needs to be done in certain areas of the analysis in order to more realistically define both the ground excitation and the behavior of the system. Specifically:

a. The definition of the free field excitation and the realistic correlation of the three components of the motion of the control point, both deterministically and stochastically, require further attention. Indeed, the restriction of stationarity of the stochastic processes which are used for earthquake representation in the present format of the program, must be removed. It will be more appropriate to introduce stochastic models and methods of analysis which account for the variation of intensity of the seismic motion versus time. The area of nonstationary random vibration has become quite mature. This potential aspect of an improved code will give to users the option of assessing the severity of ground motions as they are affected by local geological conditions.

b. The treatment of non-linear aspects in the behavior of the structure and of the foundation is a critical necessity which should be incorporated into the code in the immediate future. Currently, techniques like statistical linearization are reliable tools for incorporating in random vibration analysis both elastic (geometrical or material), and inelastic behavior of structural systems. This option can be a desirable feature of the code which can be used to assess the significance of hysteresis and other nonlinearities in realistic models of combined structural-foundation systems exposed to stochastic excitation.

c. Further work must be done to lead to the definition of foundation input motion, as well as soil medium impedances in a more accessive manner. This can be accomplished by utilizing more sophisticated computational schemes such as boundary integral methods.

d. It is believed that the time has come for reassessing the usefulness of specifying seismic motion exclusively in terms of design spectra. It is clear that for linear analysis, design spectra can expedite the response calculation. However, proceeding to nonlinear analysis introduces difficulties which lead to the necessity of generating spectrum compatible time histories. Alternatively, a systematic effort can be undertaken to generate a power response spectrum similar to the Newmark-Kapur Design Spectrum. The development of a power spectrum of this nature would be readily applicable for linear analysis of structural-foundation systems. Further, by adopting readily available techniques of nonlinear random vibration, it can be used to conduct nonlinear analysis of structures without facing the need of developing spectrum compatible time histories. Finally, it will make available to the community versatile techniques like the Monte Carlo simulation method for assessing a variety of issues involving physical parameters of the problem.

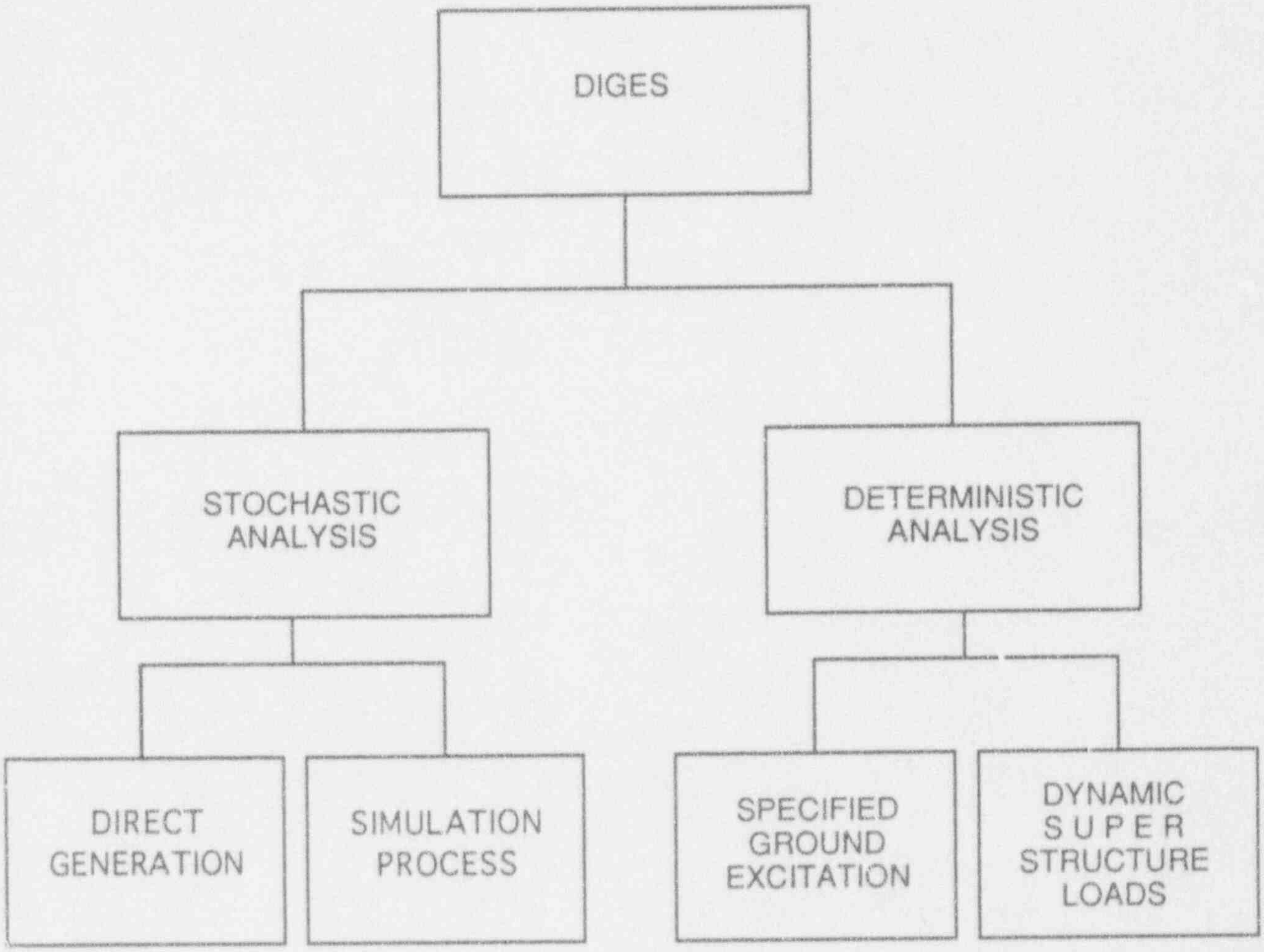

DIGES
STOCHASTIC ANALYSIS
DETERMINISTIC ANALYSIS
DIRECT GENERATION
SIMULATION PROCESS
SPECIFIED GROUND EXCITATION
DYNAMIC SUPER STRUCTURE LOADS

Figure 1.2-1

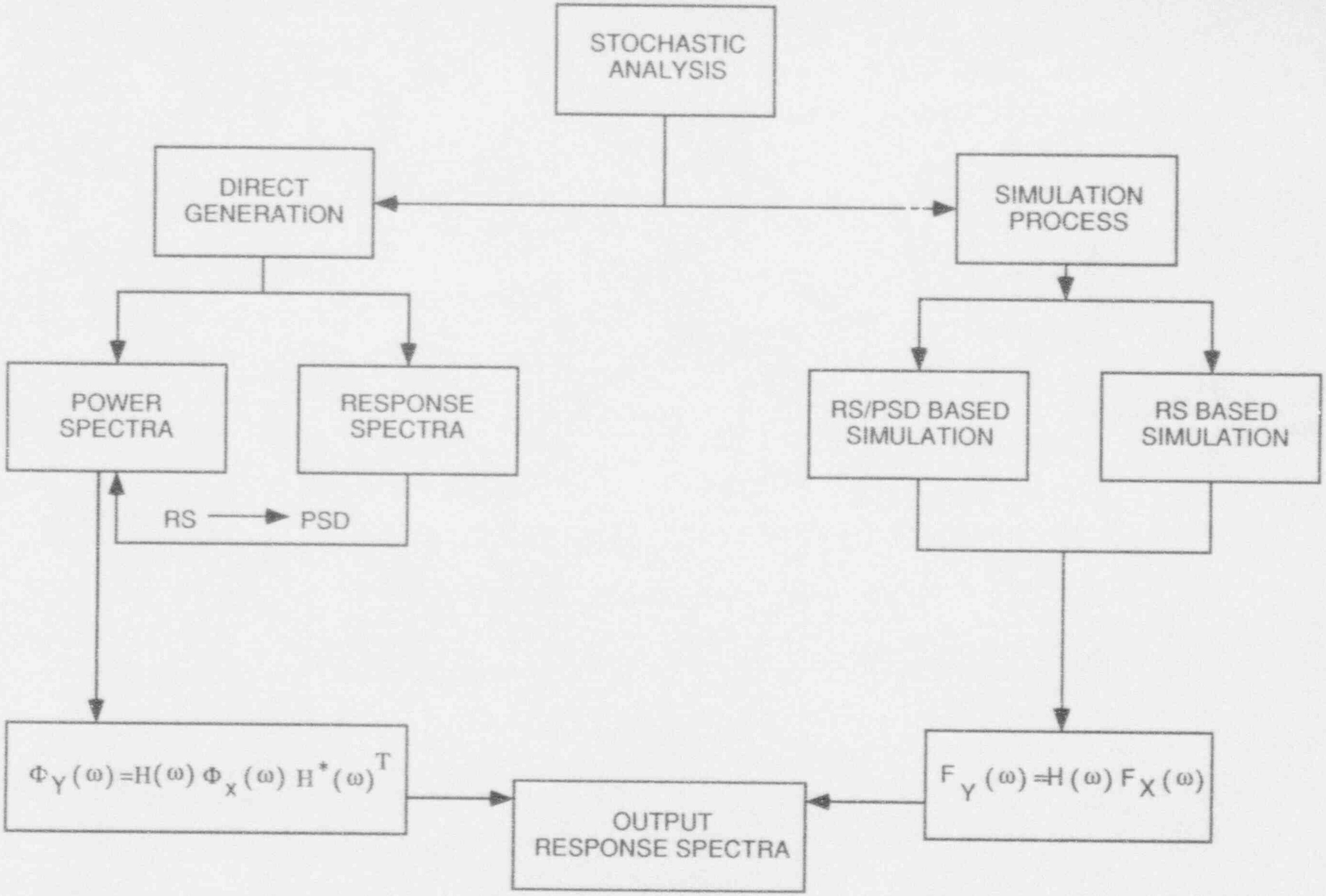

STOCHASTIC ANALYSIS
DIRECT GENERATION
SIMULATION PROCESS
POWER SPECTRA
RESPONSE SPECTRA
RS/PSD BASED SIMULATION
RS BASED SIMULATION
RS → PSD
$\Phi_Y(\omega) = H(\omega)\,\Phi_X(\omega)\,H^*(\omega)^T$
$F_Y(\omega) = H(\omega)\,F_X(\omega)$
OUTPUT RESPONSE SPECTRA

Figure 1.2-2

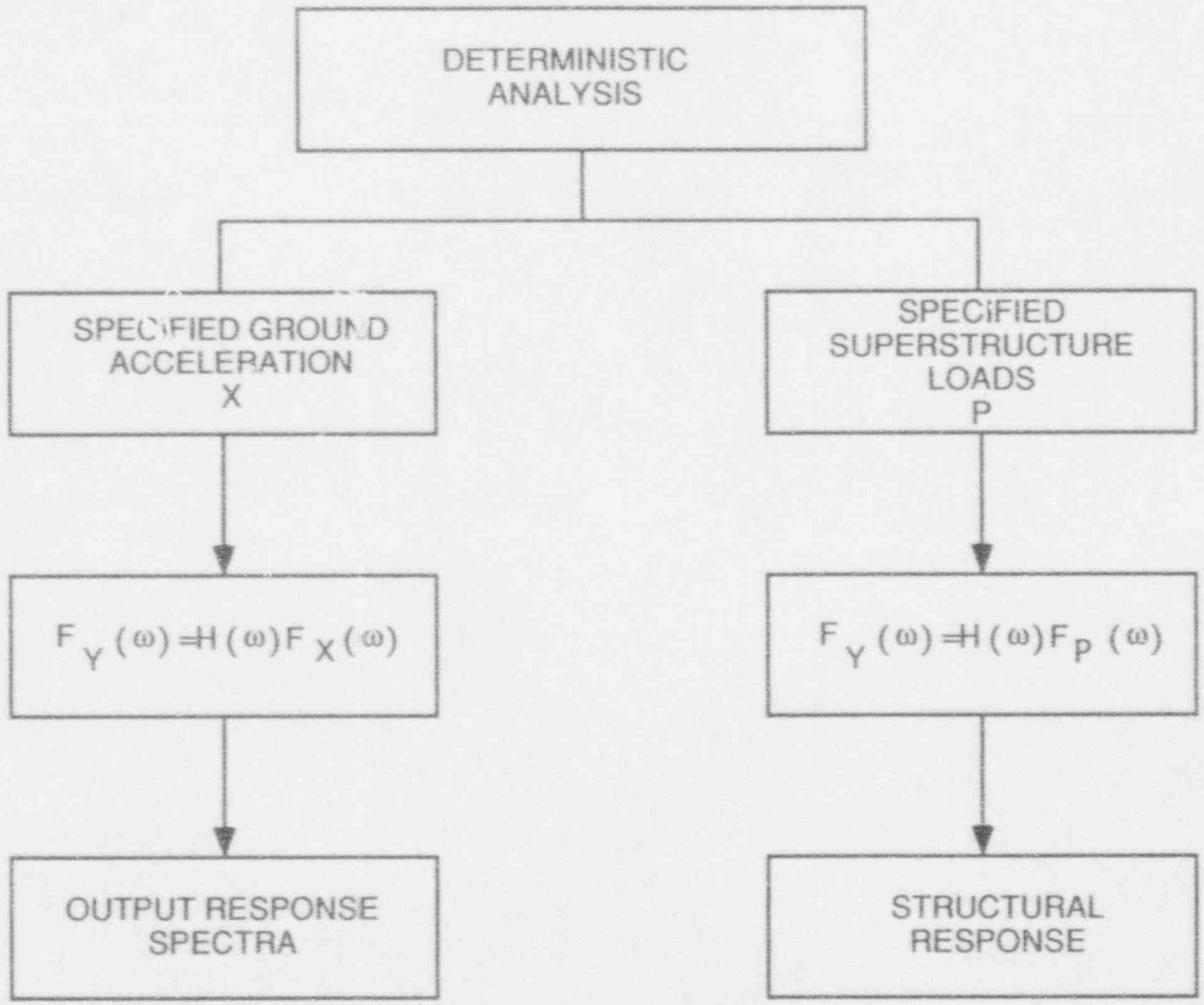

DETERMINISTIC ANALYSIS
SPECIFIED GROUND ACCELERATION X
SPECIFIED SUPERSTRUCTURE LOADS P
$F_Y(\omega) = H(\omega) F_X(\omega)$
$F_Y(\omega) = H(\omega) F_P(\omega)$
OUTPUT RESPONSE SPECTRA
STRUCTURAL RESPONSE

Figure 1.2-3

2.0 THEORETICAL BASIS

DIGES incorporates all necessary components that allow the determination of the response (output) to a given dynamical load (input). This determination assumes that the properties of the system are known in the deterministic sense or that the system properties have uncertainties associated with them. In the latter case, relevant variations in the system response can be treated with DIGES through variation-of-parameter studies. This approach represents standard practice and it is the primary tool for assessing potential impact of system uncertainties. For each parametric solution the system properties are fixed and the output is accordingly determined.

This chapter describes the primary ingredients of the theoretical basis of the DIGES and discusses various aspects related to the particular algorithms implemented into it. In particular, the following topics are presented:

- Definition of the overall problem treated by DIGES
- Solution procedures
- Determination of system transfer functions
- Input definition
- Response characteristics
- Simplifying solutions

2.1 Definition of Overall Problem

The overall problem considered by DIGES is a dynamical problem in which the structure, as well as, the associated foundation are modeled together as a multi-degree-of-freedom system. The superstructure is represented by appropriate stick models idealizing the inertia and flexibility characteristics of the buildings. All stick models necessary for modeling the substructure must be founded on the same foundation mat. Implicitly, this reflects the fact that the current version of DIGES does no perform structure-to-structure interaction. Accordingly, the assumption of common foundation mat must be maintained in modeling of the superstructure. Energy dissipation due to structural motion is taken into account through modal damping. This subject will be discussed in detail in the next Sections where, in addition, the similar subject of energy dissipation due to the foundation motion in terms of geometric or radiation and material damping is discussed. The parameters considered in modeling the foundation are its mass and its flexibility. The latter is represented by a generally complex compliance matrix or through its associated impedance matrix, the elements of which depend on geometric characteristics of the mat and the underlying medium, as well as, on pertinent soil properties (e.g., shear modulus, Poisson's ratio, damping and soil mass densities).

With respect to the types of input handled by DIGES, they can be classified under two main categories as follows:

- input in the form of ground excitation
- input in the form of applied dynamic loads at the superstructure

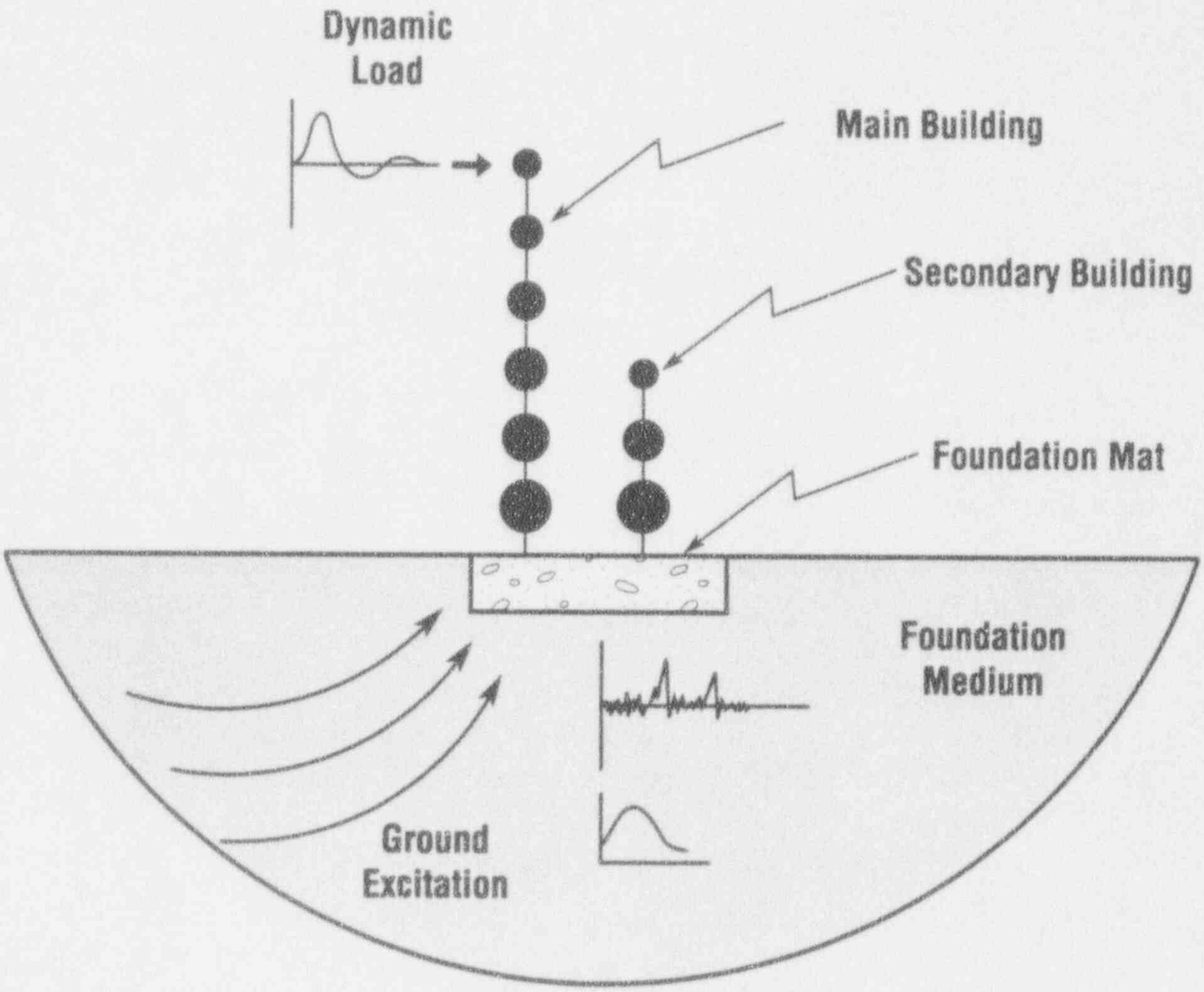
The former is representative of earthquake engineering applications while the latter is associated with general structural dynamics applications. Main emphasis is given to descriptions of relevant information of how DIGES performs solutions to problems of the first of the above categories which is admittedly the more complex. The option of performing dynamic response analysis of building-foundation systems for dynamic loads imposed on the superstructure has also significant applications, e.g., forced vibration test verifications, impact and general lateral dynamic loadings. Figure 2.1-1 shows the two general categories of dynamic input.

The fundamental approach used by DIGES in computing dynamic responses through deterministic, simulation or probablistic calculations is based on substructure analysis. The specific substructuring followed is similar to that in Refs. 1 through 4. Accordingly, the dynamic equations of motion of the superstructure are employed to determine the corresponding forces exerted on the foundation. The latter forces together with those exerted by the soil on the foundation are then used to balance the inertia forces of the foundation mat. The solution of the resulting equilibrium equations of the foundation produces the total motion of the foundation from which all required response parameters can be computed by back substitution. This concept is shown in Figure 2.1-2.

The above approach has, in our opinion, several advantages. Among them:

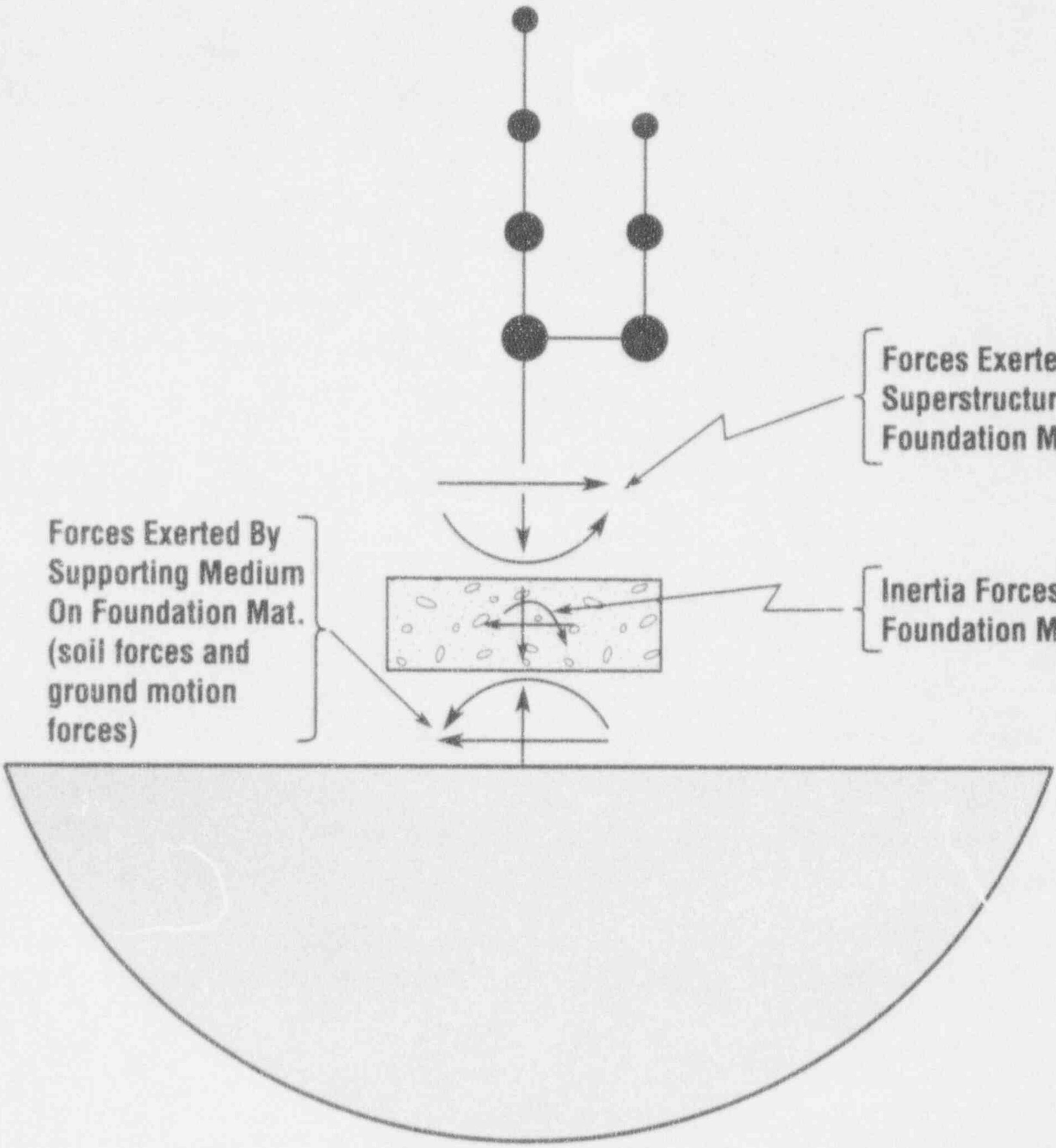
- reasonably follows the mechanics of the problem
- handles energy dissipation consistently; structural, foundation geometric and soil damping are treated separately without the need of introducing composite or other type of damping solutions.
- allows for computational efficiency in the implementation process.
- provides flexibility for obtaining intermediate results throughout the computation.
- easily amended to extensions and/or modifications to incorporate new features resulting from ongoing research.
- performs very well in parametric variation studies or simulation problems.

The primary limitation of this approach is that it is not generally suitable for applications to nonlinear problems. In practice, however, some nonlinear effects are studied by using linear codes and treating the problem through parametric variations in which the primary parameters are stiffness and damping. The results of such studies are frequency shifts and amplitude changes of the response of interest.



Overall System

Figure 2.1-1



Compatibility Scheme

Figure 2.1-2

2.1.1 Computation of Responses

DIGES considers the structure-foundation configuration as a multidegree-of-freedom linear system subjected to dynamic forces that are either applied at the superstructure or resulting from the scattering of the seismic waves by the foundation mat. The applied forces or the foundation input motion can be described a) deterministically, i.e., forcing function or acceleration time histories and b) probabilistically, i.e., cross-spectral densities of the force or excitation. The computation of the relevant response of the structure-foundation system, either response time histories or cross-spectral densities of the response, is carried out in the frequency domain. For this purpose a two-step approach is followed:

Step 1: Computation of appropriate system transfer functions $H(\omega)$ by considering steady-state analysis.

Step 2: Computation of the response to transient loads using Fourier analysis.

The term transfer function is used in this report as synonymous to the term frequency response function. It signifies the steady-state response due to harmonic input. The transfer functions of the system relating the amplitudes of the dynamic input to the response amplitudes are expressed in terms of frequency-dependent complex matrices $H(\omega)$. Their evaluation is based on the equations of motion of the system. Typically, the type of transfer functions which are evaluated correlate:

- the input with the relative structural response with respect to foundation
- the input with the total structural response
- the input with the total forces exerted by the superstructure on the foundation

The analytical modeling of the system is adequately flexible so that other transfer functions of interest can be reasonably implemented.

Having established the system transfer functions, DIGES proceeds with the computation of the response. Deterministic responses are computed by Fourier analysis (Refs. 5, 6):

$$
\left.
\begin{aligned}
y(t) &= \frac{1}{2\pi}\int_{-\infty}^{+\infty} H(\omega)F_x(\omega)e^{i\omega t}d\omega \\
F_x(\omega) &= \int_{-\infty}^{+\infty} x(t)e^{-i\omega t}dt
\end{aligned}
\right\} \qquad 2.2.1\text{-}1
$$

$F_x(\omega)$: Fourier transform of input

$y(t)$ = System response

$H(\omega)$ = System transfer function

Probabilistic responses are computed in terms of cross-spectral densities of the response (Refs. 5, 6):

$$
\begin{aligned}
\Phi_y(\omega) &= H(\omega)\,\Phi_x(\omega)\,H^*(\omega)^T \\
R_y(\tau) &= \int_{-\infty}^{+\infty} H(\omega)\,\Phi_x(\omega)H^*(\omega)^T e^{i\omega\tau}d\omega
\end{aligned}
\qquad 2.1.1\text{-}2
$$

where

$\Phi_x(\omega)$: cross-spectral density of input

$\Phi_y(\omega)$: cross-spectral density of the response

$R_y(\tau)$: correlation matrix of the system response

$H(\omega)$: system transfer function

$H^*(\omega)^T$: transpose of conjugate

2.1.2 Description of Dynamic Input

On the basis of the fundamental acceptance of the concept that the motion at a point on the free-field, for any given seismic event, is a complex superposition of motions induced by different waves impinging on the free surface, the free-field control motion is formulated. Free-field motion at a given site is of course the response of the undisturbed soil to the incident seismic wave (prior to the interference induced by the presence of the superstructure).

For any given seismic input, the free-field motion at a reference point can be seen in the vector form,

$$U_G^o = \left\{U_{G_x}^o, U_{G_y}^o, U_{G_x}^o\right\}^T \qquad 2.1.2\text{-}1$$

where the three components of the vector are generally dependent both deterministically and statistically.

The problem to be addressed is the transferring of the 3-D motion of Eq. 2.1.2-1 from the free-field to the structure, given of course the property profile of the soil medium, the properties of the foundation and the dynamic properties of the structure. The most fundamental transferring of such motion can be seen through the relation,

$$F_y(\omega) = H(\omega) F_x(\omega) \qquad 2.1.2\text{-}2$$

where $\mathbf{F}_x(\omega)$ represents the Fourier transform of the input, $\mathbf{H}(\omega)$ represents the transfer function of the system and $\mathbf{F}_y(\omega)$ represents the Fourier transform of the response.

This simple but powerful relationship between the input excitation and the response is to become the basis for far more complex analyses in terms of both the input excitation and system characteristics.

Consider that the free-field excitation is described by earthquake accelerograms, or by a stochastic process $\alpha(t)$ which is defined by a cross-spectral density matrix. Each of the above carry along information pertaining to a single given event or to a family of events. The transferring of that information provided in any of the above forms constitutes the basic branching of DIGES.

Four basic approaches make up the generic capability of the program. The level of branching can be seen in Figure 1.2-2. Specifically, the first option enables the *direct* transferring of the free-field motion that is expressed in terms of a power density function or ground response spectra into the structure. The cross-spectral density matrix of the free-field is formed and then transferred by utilizing the following relationship which links the output with the corresponding input:

$$\boldsymbol{\Phi}_y(\omega) = \boldsymbol{H}(\omega)\boldsymbol{\Phi}_x(\omega)\boldsymbol{H}^*(\omega)^T \qquad 2.1.2\text{-}3$$

where the asterisk indicates complex conjugate.

Through an interactive scheme consistent spectral densities are calculated to equivalently represent the stochastic process that is in the form of response spectra. The procedure for this step is presented in the next section. It should be mentioned that the above process also renders the name DIGES (Direct Generation of Spectra) to the program.

The second option utilizes *earthquake simulation* procedures according to which artificial earthquake records are generated reflecting the stochastic characteristics of a family of earthquakes that each one is a member of. Such earthquake family can be equally represented by a power or a response spectrum. Each generated earthquake, expressed in its Fourier expansion, is transferred to the structure via the transfer function H(ω) of the system. The response of the structure at any of its degrees-of-freedom is consequently formed on the basis of the family of the responses associated with each member of the input.

The third option reflects the *deterministic* solution capability of the DIGES (see Figure 1.2-3). Specifically, the response of the system to a deterministically defined earthquake in the free-field is calculated utilizing the system transfer function and the Fourier expansion of the earthquake accelerogram. It resembles the previous approach except that the input time history is not artificially generated and the response is the result of a single cycle.

The fourth and last option (see Figure 1.2-3) is the one associated with the response of the system when subjected to *dynamic forces on the superstructure*. This is made possible by incorporating such effects appropriately into the transfer function of the system which links the dynamic load with the motion of the building-foundation system.

2.1.2.1 Direct Generation of Spectra

The first option of the first level of branching shown in Figure 1.2-2 is discussed in this Section of the report. This option enables the stochastic evaluation of the response of the structure which results from a free-field motion also defined stochastically. Given that the stationary stochastic process g(t) representing the free-field motion satisfies a zero mean

$$E[g(t)] = 0 \qquad 2.1.2.1\text{-}1$$

its cross-spectral density matrix is defined by:

$$\Phi_X(\omega) = \begin{bmatrix} \Phi_{xx}(\omega) & \Phi_{xy}(\omega) & \Phi_{xz}(\omega) \\ \Phi_{yz}(\omega) & \Phi_{yy}(\omega) & \Phi_{yz}(\omega) \\ \Phi_{zx}(\omega) & \Phi_{zy}(\omega) & \Phi_{zz}(\omega) \end{bmatrix} \qquad 2.1.2.1\text{-}2$$

On the other hand, the transfer function of the multi-degree-of-freedom system can be expressed by

$$H(\omega) = \begin{bmatrix} H_{1x}(\omega) & H_{1y}(\omega) & H_{1z}(\omega) \\ H_{2x}(\omega) & H_{2y}(\omega) & H_{2z}(\omega) \\ H_{3x}(\omega) & H_{3y}(\omega) & H_{3z}(\omega) \\ \bullet & \bullet & \bullet \\ H_{6Nx}(\omega) & H_{6Ny}(\omega) & H_{6Nz}(\omega) \end{bmatrix} \qquad 2.1.2.1\text{-}3$$

Accordingly, then the cross-spectral density matrix of the response of the system can be expressed by

$$\Phi_Y(\omega) = H(\omega)\Phi_X(\omega)H^*(\omega)^T \qquad 2.1.2.1\text{-}4$$

$\Phi_Y(\omega)$ of size 6N $\times$ 6N (where 6N is the total number of d.o.f. of the system) cross correlates the responses of all the d.o.f. in the system. The diagonal terms of this matrix represent the power-spectral density of the response at the particular degree of freedom.

The completion of the goal which is the generation of response spectra will require to express the resulted stochastic process in a final form consisting of response spectra. These spectra should equivalently represent the response process that is now in the form of power spectra. An outline of the analytical procedure used to transform the power-spectral density of the response to a response spectrum is presented below.

Power spectra to Response spectra

For a *weakly* stationary process g(t) exciting a simple oscillator of natural frequency ω_0 and damping ratio ξ the power spectral density of the response $\Phi_Y(\omega)$ can be related to the excitation as follows:

$$\Phi_Y(\omega) = |H(\omega)|^2\,\Phi_X(\omega) \qquad 2.1.2.1\text{-}5$$

where $H(\omega)$ and $\Phi_X(\omega)$ are the transfer function of the oscillator and the *psd* of the excitation respectively.

With the statistical properties of the excitation process

$$\sigma_x^2 = \int_0^\infty \Phi_X(\omega)d\omega$$

$$\lambda_i = \int_0^\infty \omega^i\Phi_X(\omega)d\omega, \qquad i = 1,2,3,.... \qquad 2.1.2.1\text{-}6$$

the dispersion of the *psd* function about its center frequency is seen through the shape factor δ

$$\delta = \sqrt{1 - \frac{\lambda_1^2}{\lambda_0 \lambda_2}} \qquad 2.1.2.1\text{-}7$$

With a cumulative distribution of the maxima of the response process y(t) given by (Refs. 7, 8)

$$F_{y_m}(r) = \left[1 - e^{\left(-\frac{\alpha^2}{2}\right)}\right] e^{\left[-\nu_0 T \frac{1 - e^{\left(-\frac{\sqrt{\pi}}{2}\alpha\delta_e\right)}}{e^{\frac{\alpha^2}{2}} - 1}\right]} ; \quad r > 0 \qquad 2.1.2.1\text{-}8a$$

where $\delta_e = \delta^{1.2}$, $\alpha = r/\sigma_y$ and

$$\nu_0 = \frac{1}{\pi}\sqrt{\frac{\lambda_2}{\lambda_0}} \qquad 2.1.2.1\text{-}8b$$

the *mean* displacement response spectrum which represents the mean of maximum absolute value of the response process y(t) is

$$RS_d(\omega_0, \xi) = p_m \sigma_x \qquad 2.1.2.1\text{-}9a$$

where the *peak factor* p_m is given by

$$p_m = \sqrt{2\, ln(\nu_e T)} + \frac{0.5772}{\sqrt{2\, ln(\nu_e T)}} \qquad 2.1.22.1\text{-}9b$$

and

$$\begin{aligned} \nu_e T &= max(2.1,\ 2\delta\nu_0 T) &&; \quad 0 < \delta \le 0.1 \\ \nu_e T &= (1.63\delta^{0.45} - 0.38)\nu_0 T &&; \quad 0.1 < \delta < 0.69 \\ \nu_e T &= \nu_0 T &&; \quad 0.69 \le \delta < 1.0 \end{aligned} \qquad 2.1.2.1\text{-}9c$$

Given the displacement response spectrum, the acceleration spectrum will simply be

$$RS_a(\omega_0, \xi) = \omega_0^2 RS_d(\omega_0, \xi) \qquad 2.1.2.1\text{-}10$$

Response spectra to power spectra

Returning to the free-field motion, one can assume that the family of earthquakes which form the 3-D stochastic process can also be represented by response spectra rather than power spectra. The second level of branching in Figure 1.2-2 shows the various forms that the free-field input motion that can be assumed.

Since the transferring of the stochastic process to the structure takes place over the cross spectral density matrix Φ_X of the process (Eq. 2.1.2.1-4), it is necessary to generate power spectra consistent with the given response spectra that characterize the process. It is desirable to make the conversion to a power spectrum using analytic forms which in turn can be utilized in parametric studies. The process is described next.

The response spectrum characterizing the free-field motion $RS_t(\omega,\xi)$ is known for the frequency range of interest. This spectrum could also be called *target* response spectrum. Assume that the power spectrum consistent with the target response spectrum is $\Phi_t(\omega,\lambda)$ where λ is a vector of parameters that are specific of the power spectrum. These parameters define the shape of the analytical expression of the *psd* and they are unknown until the consistency between the power and the response spectra is achieved.

In order to begin the iterative process, an analytic expression for the *psd* is chosen and initial values of parameter vector λ are assumed. Over the years, several closed form expressions that can describe the power spectrum of earthquake ground accelerations have been proposed. The expressions that the DIGES program utilizes are the following:

i. **Kanai-Tajimi form**

$$S_x(\omega,\boldsymbol{\lambda}) = S_0 \frac{1 + 4\zeta_g^2\left(\frac{\omega}{\omega_g}\right)^2}{\left[1 - \left(\frac{\omega}{\omega_g}\right)^2\right]^2 + 4\zeta_g^2\left(\frac{\omega}{\omega_g}\right)^2}$$

$$\boldsymbol{\lambda} = [\zeta_g, \omega_g, S_0]^T \qquad 2.1.2.1\text{-}11$$

ii. **Ruiz-Penzien form**

$$S_x(\omega,\boldsymbol{\lambda}) = S_0 \frac{1 + 4\zeta_g^2\left(\frac{\omega}{\omega_g}\right)^2}{\left[1 - \left(\frac{\omega}{\omega_g}\right)^2\right]^2 + 4\zeta_g^2\left(\frac{\omega}{\omega_g}\right)^2} \frac{\left(\frac{\omega}{\omega_p}\right)^4}{\left[1 - \left(\frac{\omega}{\omega_p}\right)^2\right]^2 + 4\zeta_p^2\left(\frac{\omega}{\omega_p}\right)^2}$$

$$\boldsymbol{\lambda} = \left[\zeta_g, \omega_g, S_0, \zeta_p, \omega_p\right]^T \qquad 2.1.2.1\text{-}12$$

iii. **Superposition form**

$$S_x(\omega,\boldsymbol{\lambda}) = S_0 \sum_{k=1}^{2} P_k \frac{\left(1 + 4\zeta_k^2\right)\left[1 - e^{-\frac{\omega^4}{\omega_k^4}}\right]}{\left[1 - \left(\frac{\omega}{\omega_k}\right)^2\right]^2 + 4\zeta_k^2\left(\frac{\omega}{\omega_k}\right)^2}$$

$$\boldsymbol{\lambda} = \left[\zeta_1, \omega_1, S_1, \zeta_2, \omega_2, S_2\right]^T \qquad 2.1.2.1\text{-}13$$

When any of the above expressions is multiplied by the filter

$$\frac{1}{1 + \alpha^2\omega^2}$$

a new form of analytic *psd* is deduced.

The resulting power spectral density is used to generate a consistent response spectrum according to the procedure already described above. In that process a duration of the stationary part of the seismic event has to be assumed since the target response spectrum does not provide such information. The calculated response spectrum $RS_{calc}(\omega,\xi)$ and the target response spectrum $RS_t(\omega,\xi)$ are then compared. This comparison is taking place over a finite number of frequencies within the range and the criterion of convergence is the minimization of the square of the difference

$$X^2(\boldsymbol{\lambda}) = \left[RS_{calc}(\omega_i,\xi) - RS_t(\omega_i,\xi)\right]^2 \qquad 2.1.2.1\text{-}14$$

Apparently, in this relation one is seeking the values of the vector λ that will minimize χ^2. This non-linear least square fitting is performed by utilizing the *Levenberg-Marquardt method*. The background and the pertinent routines of the method can be found in *Numerical Recipes in Fortran* (Ref. 15).

When minimization of χ^2 has been successful, the final vector λ is returned. Consequently, the power spectral density of the input can be evaluated for the entire frequency range that the target response spectrum $RS_t(\omega,\xi)$ is defined. One should keep in mind that the response/power spectrum consistency has been enforced on a set of frequencies over the range of the analysis and these frequencies are not necessarily the same with those associated with the transfer of motion from the free field to the structure.

Upon completion of the response-to-power spectrum conversion, the free-field cross spectral density matrix is transferred to the structure according to Eq. 2.1.2.1-4. Again a power-to-response spectrum procedure will provide response spectra of the output for any d.o.f. of the system.

2.1.2.2 Earthquake Simulation

The second option of the first level of branching shown in Figure 1.2-2, is associated with the transferring of the stochastic free-field process $\alpha(t)$. What one can achieve through this approach is that as size of the ensemble gets larger, the ensemble of the output responses will retain the stochastic characteristics of the input. Consequently, the generated input ground accelerations, which are members of the family that the stochastic process $\alpha(t)$ represents, are deterministically transferred to the structure by utilizing the transfer function of the system $\mathbf{H}(\omega)$ according to the relationship

$$F_y(\omega) = H(\omega)F_x(\omega) \qquad 2.1.2.2\text{-}1$$

where $F_x(\omega)$ and $F_y(\omega)$ are the Fourier expansions of the input and output respectively. The response of the system can subsequently be expressed in time by means of the Fourier synthesis, i.e.,

$$y(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} F_y(\omega)e^{i\omega t}dt \qquad 2.1.2.2\text{-}2$$

When the complete ensemble of generated earthquakes has been transferred to the structure, the response of the system at any d.o.f. can be then seen as a single response spectrum which is deduced from the ensemble of response spectra, i.e.,

$$\overline{RS}(\omega_i,\xi) = \frac{\sum_{j=1}^{n} RS_j(\omega_i,\xi)}{n} \qquad 2.1.2.2\text{-}3$$

along with the statistical properties of the ensemble of amplifications at every specified frequency ω_i, i.e., $[\mu+\sigma]*RS(\omega_i)$. This process will also include a non-exceedence probability requirement.

Given that the free-field stochastic process can be equally represented by its response or power spectra, simulation of ground accelerations should be possible from both representations.

2.1.2.2.1 *PSD* Based Ground Acceleration Simulation

The ground motion during an earthquake event can be characterized as a three-dimensional process. It is also expected that the motion of a control point in any of the directions is not independent but rather dependent to the other two components. Thus, accepting that such correlation exists, the most appropriate way to stochastically describe the process is through its cross-spectral density matrix. The evaluation of the latter matrix is rather difficult and thus the alternative description of three independent components of ground motion appears to be more accessible. An outline of these two cases will be presented below.

Consider the 1-D stationary stochastic process $\alpha(t)$ of zero mean, autocorrelation and power spectral density functions given by the relations,

$$E[\alpha(t)] = 0$$

$$E[\alpha(t + \tau)\alpha(t)] = R_{\alpha\alpha}(\tau)$$

$$\Phi_\alpha(\omega) = \frac{1}{2\pi} \int_{-\infty}^{\infty} R_{\alpha\alpha}(\tau)e^{-i\omega\tau}d\tau \qquad 2.1.2.2.1\text{-}1$$

$$R_{\alpha\alpha}(\tau) = \int_{-\infty}^{\infty} \Phi_\alpha(\omega)e^{i\omega\tau}d\omega$$

A time history g(t) of an artificial acceleration can be generated from the form (Ref. 16):

$$g(t) = 2\sum_{i=1}^{N} \sqrt{\Phi_\alpha(\omega_i)\Delta\omega}\cos(\omega_i t + \phi_i) \qquad 2.1.2.2.1\text{-}2$$

$$w_i = i\Delta\omega \qquad \Delta\omega = \frac{\omega_u}{N}$$

where ω_u is a cutoff frequency. The generated time history g(t) can be assumed to represent $\alpha(t)$ as $N \to \infty$. In Eq. 2.1.2.2.1-2 ϕ is a vector of random phase angles uniformly distributed between 0 and 2π; $\Phi_\alpha(\omega)$ is the power spectral density of the process. Different choices of the vector of random phase angles will lead to a different simulated process that has both the mean and the autocorrelation of the stochastic process $\alpha(t)$.

By taking into consideration the fact that in an actual earthquake only the strong motion part of the excitation is stationary, so the stochastic relations hold only then, one needs to assume the strong motion duration of the simulated event. Consequently, it can be stated that the simulated process g(t) is periodic with a period.

$$T_0 = \frac{2\pi}{\Delta\omega} \qquad 2.1.2.2.1\text{-}3$$

Apparently the above relation can determine the duration of the stochastic process.

It should be again emphasized that the artificial earthquakes generated are stationary. This implies that the build-up and the die-down portions of an actual earthquake record cannot be seen in any artificial earthquakes of this kind unless the nonstationarity is introduced through a modulating function $\zeta(t)$, shown in Figure 2.1.2.2.1-1 such that

$$g'(t) = \zeta(t)g(t) \qquad 2.1.2.2.1\text{-}4$$

In order to incorporate the stochastic correlation between the components of ground motion, the theoretical model of Ref. 17 has been adopted. According to this model, a stochastic process $g_i^0(t); i = 1,2,3$ that satisfies

$$E\left[g_i^0(t)\right] = 0 \qquad i = 1,2,3 \qquad 2.1.2.2.1\text{-}5$$

is described by the cross-spectral density matrix

$$\Phi^0(\omega) = \left[\Phi_{ij}^0(\omega)\right] \quad ; \quad i,j = 1,2,3 \qquad 2.1.2.2.1\text{-}6$$

$\Phi_{ij}(\omega)$ is the transform of the cross-correlation function $R_{ij}(\tau)$ or autocorrelation if i = j.

In order to simulate a stationary stochastic process $g_i(t)$ (i = 1,2,3) the cross-spectral density matrix is decomposed as:

$$\Phi^0(\omega) = \Psi(\omega)\Psi^*(\omega)^T \qquad 2.1.2.2.1\text{-}7$$

so a $g_i(t)$ can be deduced from the series below (as $N \rightarrow \infty$)

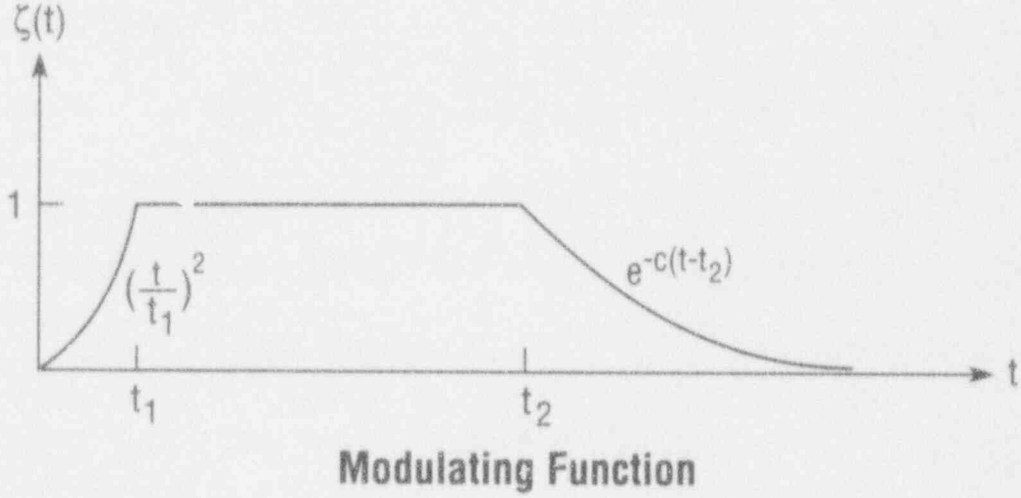


Figure 2.1.2.2.1-1

$$g_i(t) = 2 \sum_{m=1}^{3} \sum_{l=1}^{N} \left|H_{im}(\omega_l)\right| \sqrt{\Delta\omega} \cos\left[\omega_l t + \theta_{im}(\omega_l) + \phi_{ml}\right] \quad ; \quad i = 1,2,3$$

2.1.2.2.1-8a

where:

$$\omega_l = l\Delta\omega \quad ; \quad l = 1,2,3$$

$$\Delta\omega = \frac{\omega_u}{N}$$

2.1.2.2.1-8b

$$\theta_{im} = \tan^{-1} \frac{Im\left[\Psi_{im}(\omega_l)\right]}{Re\left[\Psi_{im}(\omega_l)\right]}$$

In the above ω_u represents an upper cut-off frequency beyond which the *psd* is assumed to vanish while each column of the 3 × N matrix $[\phi_{ml}]$ is a vector of independent random phase angles uniformly distributed in the interval $[0,2\pi]$.

While, by adopting this mathematical model the expected value and the cross-correlation functions of the simulated stochastic process $g_i(t)$ are those of the target process $g_i^0(t)$

$$E\left[g_i^0(t)\right] = E\left[g_i(t)\right] = 0$$

$$R_{ik}^0(\tau) = R_{ik}(\tau)$$

2.1.2.2.1-9

the simulated process $g_i(t)$ is not ergodic. Since, however, the stationary character of the simulated process is still in place, a modulating function $\varsigma(t)$ (e.g., Figure 2.1.2.2-1) is also utilized in order to provide a more realistic ground motion.

It is suggested, however, not to introduce the non-stationary form of the simulated earthquakes if direct comparison of the output response spectra (average of spectra from all simulated earthquakes) is to take place with spectra that are generated by utilizing the *direct* method. That is of course due to the fact that the direction generation,

$$\Phi_{out}(\omega) = H(\omega)\Phi_{inp}(\omega)H^*(\omega)^T$$

is only valid for stationary input and output stochastic process.

As mentioned earlier, the Fourier transform of each simulated earthquake will be transferred to a given d.o.f. on the structure according to Eq. 2.1.2.2-1. The inverse Fourier transform of the response $F_y(\omega)$ will represent the time history of the response. Subsequently, with the time history of the system variable (e.g. acceleration) at the d.o.f. and an assumed percentage of damping the response spectrum is calculated.

Lastly, the ensemble of response spectra (one for each simulated earthquake) will be statistically processed to obtain a representative single response spectrum at the degree-of-freedom on the superstructure.

2.1.2.2.2 Simulation Based on Response Spectra

The random process that represents the earthquake ground motions at a particular site could be also defined in the form of a response spectrum. Simulated earthquakes that belong to the family of the target response spectrum can assume the form,

$$g(t) = \zeta(t) \sum_{n=1}^{N} C_n(\omega)\sin(\omega_n t + \phi_n) \qquad 2.1.2.2.2\text{-}1$$

where $C_n(\omega)$ is the amplitude of the n-th contributing sinusoid and ϕ_n is its phase angle. $\zeta(t)$ is a deterministic envelope function as described in the previous section.

Any choice of the vector ϕ which contains the uniformly distributed in the interval $[0,2\pi]$ non-correlated phase angles and a set of amplitudes $C(\omega)$ will define a ground motion.

In order for the generated ground motion to be consistent with the target response spectrum, the amplitudes $C(\omega)$ have to be adjusted so that the difference between the target response spectrum, which characterizes the random process, and the generated one from Eq. 2.1.2.2.2-1 is minimized over the control frequencies. This is done iteratively and it consists of the following steps:

a. A random vector of phase angles is chosen.

b. An initial set of amplitudes $C_{initial}(\omega)$ is chosen. While it is desirable to minimize the iterations by starting at the *best guess* for the vector $C_{initial}(\omega)$, that can be achieved by assigning the amplitudes of the Fourier expansion the values of the zero-damping target spectrum, theoretically any choice should work. In DIGES, the entire initial vector could be set equal to one.

c. With the two vectors available, the response spectrum of the generated time history is calculated for a given duration and damping.

d. The iteration procedure is implemented. For each iteration cycle i the generated response spectrum is compared to the target spectrum and the initially chosen amplitudes $C_{initial}(\omega)$ are adjusted according to the relation

$$C_{i+1}(\omega) = C_i(\omega)\left[\frac{RS_i(\omega)}{RS_{target}(\omega)}\right] \qquad 2.1.2.2.2\text{-}2$$

While exact convergence criteria for the above iterative process are not available, it has seen that agreement of the generated to the target spectrum can be achieved after a few iterations. The simulated earthquake is of course the time history that results from Eq. 2.1.2.2.2-1 after the response spectra have been matched.

Similarly, the ensemble of ground motions is transferred to the structure by utilizing the transfer function of the system (Eq. 2.1.2.2-1). Each response, a time history of the variable of interest at a given d.o.f., will be again represented by its response spectrum and finally an ensemble spectrum will be deduced.

2.1.2.3 Deterministic Input Analysis

As can be seen from Figure 1.2-3, DIGES has the option of performing deterministic analysis for cases involving seismic excitation as well as dynamic loads imposed on the superstructure. In both cases, the response is based on the motion of the building-foundation system. This implies that the analysis incorporates SSI effects for both seismic as well as superstructure loads. We felt that it is important to have DIGES performing deterministic analysis in addition to its capability of stochastic analysis.

When DIGES performs deterministic analysis, the seismic excitation a(t) or the dynamic load imposed on the superstructure P(t) are transformed into the frequency domain by Fourier transform, i.e.,

$$F_a(\omega) = \int_{-\infty}^{+\infty} a(t)e^{-i\omega t}dt \qquad 2.1.2.3\text{-}1$$

and

$$F_p(\omega) = \int_{-\infty}^{+\infty} P(t)e^{-i\omega t}dt \qquad 2.1.2.3\text{-}2$$

respectively.

Accordingly, the response of the building-foundation system is computed by

$$y(t) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} H_a(\omega)F_\alpha(\omega)e^{i\omega t}d\omega \qquad 2.1.2.3\text{-}3$$

or

$$y(t) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} H_p(\omega)F_p(\omega)e^{i\omega t}d\omega \qquad 2.1.2.3\text{-}4$$

for seismic excitation or dynamic load respectively. In Eqs. 2.1.2.3-3 and 4 $H_a(\omega)$ and $H_p(\omega)$ are the transfer functions of the building-foundation system relating its steady-state response to a unit excitation or to a unit dynamic load respectively.

2.2 Description of System Motion

2.2.1 Equations of Motion of Superstructure

The dynamic characteristics of the superstructure are obtained by DIGES using a three-dimensional idealization of its inertia and stiffness properties. This is done through stick model type of representation of the structure in which lumped masses are interconnected with 3D beam elements. Each mass is associated with 6 degrees-of-freedom. Let N be the total number of lumped masses representing the superstructure. Then the equations of motion for the 6xN degrees-of-freedom system are:

$$M\ddot{u}_t + C\dot{u} + Ku = 0 \qquad 2.2.1\text{-}1$$

where:

M = 6Nx6N mass matrix

C = 6Nx6N damping matrix

K = 6Nx6N stiffness matrix

U_t = 6N displacement vector representing the total motion of the superstructure

U = 6N displacement vector representing the relative motion of the superstructure with respect to the foundation.

The relationship between total and relative motions of the system is discussed in the next section.

2.2.1.1 Flexural and Rigid-Body Motion

The total motion of the system consists of two parts: flexural motions due to the deformation of the superstructure relative to the foundation and rigid body motions with respect to a fixed system. In the later case, the structure-foundation system behaves as a rigid body. Based on this, the total displacement vector $\mathbf{u}_t$ at the superstructure can be decomposed into two parts: the relative displacement vector u plus a rigid displacement vector $\mathbf{u}_r$. Therefore, we can write:

$$u_t = u + u_r \qquad 2.2.1.1\text{-}1$$

All vectors in Eq. 2.2.1.1-1 are 6N-vectors. The instantaneous motion of the structure foundation system acting as a rigid body is completely defined by its velocity of translation plus an angular velocity. The velocity of the translation is equal to that of some arbitrary point of the structure-foundation system while the axis of angular velocity passes through the same point. Based on principles of rigid body kinematics, the angular velocity of the system is independent of the choice of the base point in terms of both magnitude and direction. The velocity of translation, however, depends on the choice of the base point but it can be determined by knowing the corresponding velocity at any other point on the structure-foundation system and the angular velocity vector characterizing the motion. In view of these kinematic principles, computationally convenient choices of a base point for referring the rigid body motion of the system are: a) center of gravity of the building foundation system, b) center of gravity of the foundation itself, c) a geometrically convenient point at the rigid foundation. In general, any of these choices can be followed. The main reason for amplifying this issue is to emphasize that the freedom of choosing a reference point in DIGES must be complemented with consistency in defining pertinent parameters of the system, e.g., compliance or impedance functions of the foundation, driving forces related to the solution of the scattering of seismic waves by the foundation, etc.

In DIGES, the rigid displacement vector, $\mathbf{u}_r$, representing the total displacement at any point of the superstructure due to rigid body motion is calculated from the total motion at a reference point O_F in the rigid foundation. In order to maintain consistency throughout the discussion in this report, the latter will be termed as "foundation reference point." Let $\mathbf{u}_o$ denote the 6 vector of the total foundation motion which is the total motion of the foundation reference point, i.e.,

$$u_o = \left\{ \frac{\delta \boldsymbol{r}_F}{\delta \boldsymbol{\theta}} \right\} = \left\{ \begin{matrix} \Delta_x^o \\ \Delta_y^o \\ \Delta_z^o \\ \hline \theta_x^o \\ \theta_y^o \\ \theta_z^o \end{matrix} \right\} \qquad 2.2.1.1\text{-}2$$

The 6 vector $\mathbf{u}_o$ is a key parameter in our analyses. Knowing $\mathbf{u}_o$, the rigid body motion at any point of interest which could be located either at the superstructure or at the foundation itself, could then be completely defined.

Writing Eq. 2.2.1.1-1 for the i-th node of the superstructure we have,

$$u_t^{(i)} = u^{(i)} + u_r^{(i)} \qquad 2.2.1.1\text{-}3a$$

or in terms of their components,

$$\begin{Bmatrix} \Delta_{x,t}^{(i)} \\ \Delta_{y,t}^{(i)} \\ \Delta_{z,t}^{(i)} \\ \theta_{x,t}^{(i)} \\ \theta_{y,t}^{(i)} \\ \theta_{z,t}^{(i)} \end{Bmatrix} = \begin{Bmatrix} \Delta_{x}^{(i)} \\ \Delta_{y}^{(i)} \\ \Delta_{z}^{(i)} \\ \theta_{x}^{(i)} \\ \theta_{y}^{(i)} \\ \theta_{z}^{(i)} \end{Bmatrix} + \begin{Bmatrix} \Delta_{x,r}^{(i)} \\ \Delta_{y,r}^{(i)} \\ \Delta_{z,r}^{(i)} \\ \theta_{x,r}^{(i)} \\ \theta_{y,r}^{(i)} \\ \theta_{z,r}^{(i)} \end{Bmatrix} \qquad 2.2.1.1\text{-}3b$$

(subscripts t and r denote "total" and "rigid" respectively).

The relative motion $\mathbf{u}^{(i)}$ of the i-th node is referred to the foundation reference point, i.e.,

$$\boldsymbol{\delta r}_o^{(i)} = \begin{Bmatrix} \Delta_x^{(i)} \\ \Delta_y^{(i)} \\ \Delta_x^{(i)} \end{Bmatrix} \qquad 2.2.1.1\text{-}3c$$

The corresponding rigid body motion $\mathbf{u}_r^{(i)}$ of the i-th node is computed from the motion of the foundation reference point as follows:

With respect to Figure 2.2.1.1-1 the total rigid displacement of the i-th node of the superstructure consists of a translational displacement (equal and parallel to that of the reference point) plus a small rotation about the axis of rotation, i.e.,

$$\delta \boldsymbol{r} = \delta \boldsymbol{r}_F + \delta \boldsymbol{\theta} \times \boldsymbol{r}_o \qquad 2.2.1.1\text{-}4$$

where δ is used to signify small motions.

Using the relevant components and by carrying out the cross product, Eq. 2.2.1.1-4 can be written as:

$$\begin{Bmatrix} \Delta_{x,r}^{(i)} \\ \Delta_{y,r}^{(i)} \\ \Delta_{z,r}^{(i)} \end{Bmatrix} = \begin{Bmatrix} \Delta_x^o \\ \Delta_y^o \\ \Delta_z^o \end{Bmatrix} + \begin{bmatrix} 0 & r_{o,z} & -r_{o,y} \\ -r_{o,z} & 0 & r_{o,x} \\ r_{o,y} & -r_{o,x} & 0 \end{bmatrix} \begin{Bmatrix} \theta_x^o \\ \theta_y^o \\ \theta_z^o \end{Bmatrix} \qquad 2.2.1.1\text{-}5$$

where $\mathbf{r}_o$ is the position vector of the i-th node with respect to the reference point 0_F, i.e. $\mathbf{r}_o = r_{o,x}\,\mathbf{i} + r_{o,y}\,\mathbf{j} + r_{o,z}\,\mathbf{k}$, and $\delta\boldsymbol{\theta}$ is the angular displacement associated with the rigid body motion and has components $\left\{\theta_x^o\ \theta_y^o\ \theta_z^o\right\}$. Accordingly,

$$\begin{Bmatrix} \theta_{x,r}^{(i)} \\ \theta_{y,r}^{(i)} \\ \theta_{z,r}^{(i)} \end{Bmatrix} = \begin{Bmatrix} \theta_x^o \\ \theta_y^o \\ \theta_z^o \end{Bmatrix} \qquad 2.2.1.1\text{-}6$$

In view of Eqs. 2.2.1.1-5 and 6 the rigid body motion $\mathbf{u}_r^{(i)}$ of the i-th node of the superstructure can be written as:

$$u_r^{(i)} = A^{(i)} u_o \qquad 2.2.1.1\text{-}8a$$

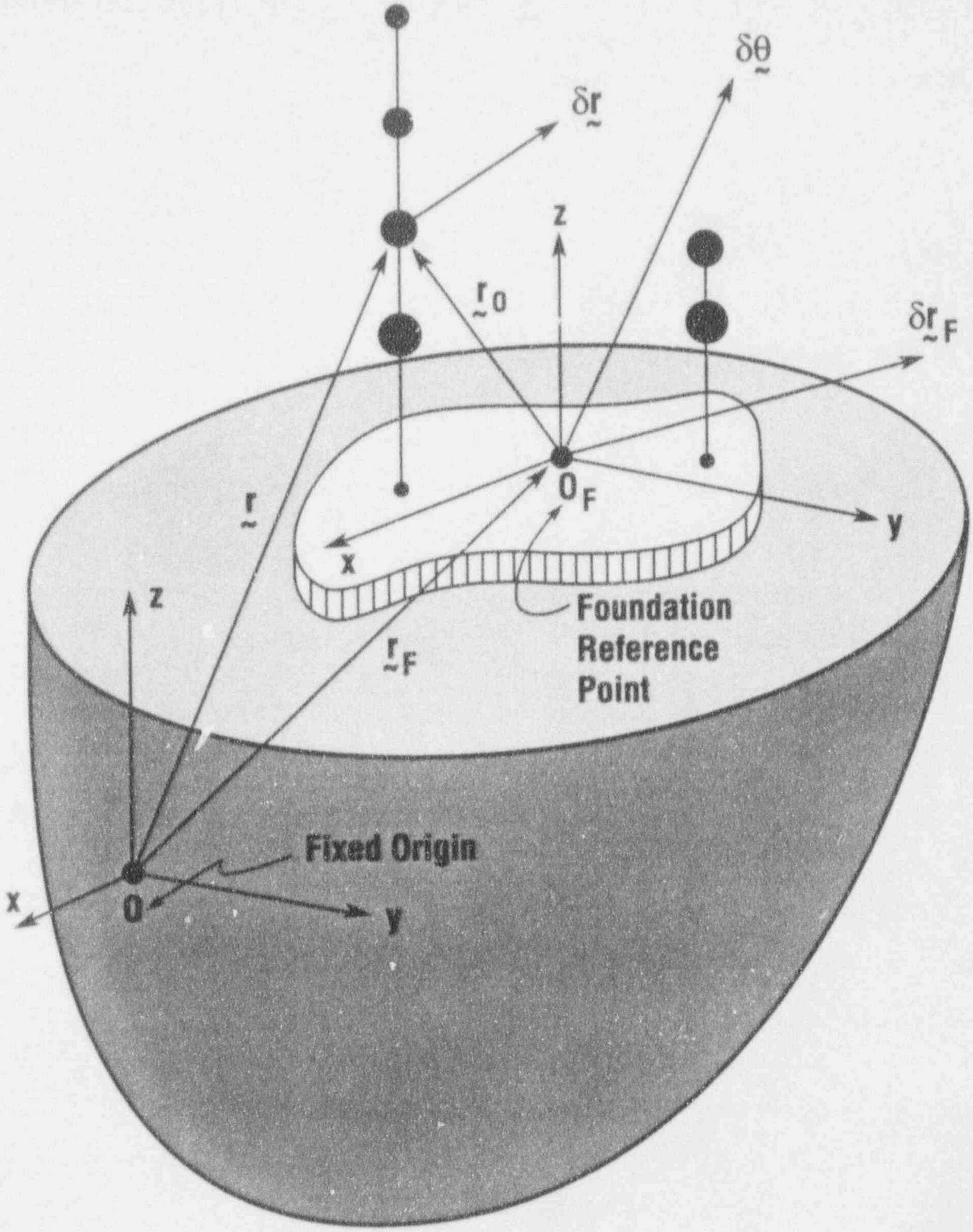
where

$$A^{(i)} = \begin{bmatrix} I & R^{(i)} \\ O & I \end{bmatrix} \qquad 2.2.1.1\text{-}8b$$

and

$$R^{(i)} = \begin{bmatrix} o & z_i - z_o & -(y_i - y_o) \\ -(z_i - z_o) & o & x_i - x_o \\ y_i - y_o & -(x_i - x_o) & o \end{bmatrix} \qquad 2.2\ 1.1\text{-}8c$$

- **I** and **O** are 3x3 unit and null matrices respectively.
- (x_i, y_i, z_i) and (x_o, y_o, z_o) are rectangular coordinates of the i-th node and the foundation reference point respectively.



Structure Foundation Kinematics

Figure 2.2.1.1-1

Substituting Eq. 2.2.1.1-8 into Eq. 2.2.1.1-1 for all nodes of the superstructure we obtain:

$$u_t = u + A\,u_o \qquad 2.2.1.1\text{-}9$$

where **A** is a 6Nx6 matrix assembled in DIGES by using Eq. 2.2.1.1-8b for all nodes of the superstructure, while $\mathbf{u}_t$ and **u** are 6N vectors representing the total and relative motion of the superstructure.

In view of Eq. 2.2.1.1-9, the total motion of the superstructure (motion with respect to a fixed system) is equal to a rigid body motion plus a flexural motion relative to the foundation reference point.

2.2.1.2 Transfer Functions Between Superstructure Motion and Total Foundation Motion

DIGES employs modal decomposition of the superstructure for determining relative motions of the superstructure with respect to the foundation. The key parameters of the superstructure which are required for this purpose are its mass and stiffness. The latter are assembled by direct finite element approach in terms of the 6Nx6N M and K mass and stiffness matrices respectively. The mass matrix M consists of element mass matrices plus lumped masses at various nodes as specified by the user. The element stiffness matrix is three dimensional which allows for 6 DOF's per node. Finally, relevant dissipation due to structural motion is computed in terms of modal damping using the fixed-base modes of the structure.

Substitution of Eq. 2.2.1.1-9 into Eq. 2.2.1-1 gives equations of motion of the superstructure in terms of its relative motion **u** with respection to the foundation:

$$M\ddot{u} + C\dot{u} + Ku = -M\,A\,\ddot{u}_o \qquad 2.2.1.2\text{-}1$$

Since the transfer function between the relative motion of the superstructure and the total foundation motion is of interest, we are considering steady-state harmonic motion at frequency ω. Accordingly, the displacement vectors u and u_o can be described as follows:

$$u = u(t) = U(\omega)e^{i\omega t}$$
$$u_o = u_o(t) = U_o(\omega)e^{i\omega t} \qquad 2.2.1.2\text{-}2$$

Substitution of Eq. 2.2.1.2-2 into Eq. 2.2.1.2-1 gives:

$$(-\omega^2 \boldsymbol{M} + i\omega\ \boldsymbol{C} + \boldsymbol{K})\ \boldsymbol{U} = \omega^2 \boldsymbol{M A U}_o$$

from which,

$$\begin{cases} \boldsymbol{U} = \boldsymbol{H}_1 \boldsymbol{U}_o \\ \boldsymbol{H}_1 = \boldsymbol{H}_1(\omega) = \omega^2[-\omega^2 \boldsymbol{M} + i\ \omega\ \boldsymbol{C} + \boldsymbol{K}]^{-1}\ \boldsymbol{M}\ \boldsymbol{A} \end{cases} \qquad 2.2.1.2\text{-}3$$

where the 6Nx6 frequency dependent matrix $\mathbf{H}_1$ represents the transfer functions between the total foundation motion and the relative motion of the superstructure. Considering Eqs. 2.2.1.1-1, 2.2.1.1-9, 2.2.1.2-2, and 2.2.1.2-3 we can write:

$$\begin{cases} \boldsymbol{U}_t = \boldsymbol{H}_2 \boldsymbol{U}_o \\ \boldsymbol{H}_2 = \boldsymbol{H}_2(\omega) = \boldsymbol{H}_1(\omega) + \boldsymbol{A} \end{cases} \qquad 2.2.1.2\text{-}4$$

where the 6Nx6 frequency-dependent matrix $\mathbf{H}_2$ represents the transfer function between the total foundation motion and the total motion of the superstructure.

In view of Eqs. 2.2.1.1-9 and 2.2.1.2-4, the total motion of the superstructure (required for computing in-structure responses) consists of flexural and rigid body motions and its amplitudes at frequency ω are completely defined by the corresponding amplitudes of the total foundation motion $\mathbf{U}_o$ and the transfer function $\mathbf{H}_2$ given by Eq. 2.2.1.2-4.

From Eqs. 2.2.1.2-3 and 4 it is concluded that the computation of the transfer functions, $\mathrm{H}_1(\omega)$ and $\mathrm{H}_2(\omega)$ requires the inversion of the 6Nx6N complex frequency-dependent matrix $[-\omega^2 \mathbf{M} + i\omega\ \mathbf{C} + \mathbf{K}]$ assuming that **M**, **C**, and **K** are appropriately defined. If the superstructure itself, considered as a conventional fixed-base system cannot be decomposed into classical modes, then the computation of the transfer functions $\mathbf{H}_1(\omega)$ and $\mathbf{H}_2(\omega)$ can be done by numerical inversion of the matrix $[-\omega^2 \mathbf{M} + i\ \omega\ \mathbf{C} + \mathbf{K}]$ at each frequency of interest. If fixed-base modes exist in the classical sense, then DIGES computes the above transfer functions using the results of the modal analysis. In the latter case, DIGES performs modal analysis by the solution of the following equation:

$$M\ddot{u} + Ku = 0 \qquad 2.2.1.2\text{-}5$$

from which the 6Nx6N modal matrix Φ containing the fixed-base modal shapes of the superstructure is obtained. By employing the modal matrix and the resulting modal equations DIGES computes $\mathbf{H}_1(\omega)$ and $\mathbf{H}_2(\omega)$ on the basis of the following formulation.

By employing the fixed-based modal matrix Φ, the amplitudes of the relative motion of the superstructure can be written as:

$$U = \Phi\, \eta \qquad 2.2.1.2\text{-}6$$

where η_j is the amplitude of the modal displacement of the j-th mode. Accordingly, we can write:

$$\eta_j = H_o^{(j)}\, \Gamma_j\, U_o \qquad 2.2.1.2\text{-}7$$

where $H_o^{(j)}$ is the modal transfer function of the j-th mode given by the relation:

$$\left. \begin{aligned} H_o^{(j)} &= \frac{\Omega_j^2}{1 + 2i\, \xi_j \Omega_j - \Omega_j^2} \\ \Omega_j &= \frac{\omega}{\omega_j} \end{aligned} \right\} \qquad 2.2.1.2\text{-}8$$

ω_j = frequency of j-th mode;

ξ_j = modal damping of j-th mode;

ϕ_j = 6N modal shape vector of j-th mode;

Γ_j = $\dfrac{\phi_j^T MA}{\phi_j^T M \phi_j}$ modal participation 6-vector of j-th mode;

U_o = 6-vector of amplitudes of the total foundation motion, and;

$$i = \sqrt{-1}$$

DIGES normalizes the modal matrix Φ with respect to the mass matrix. Therefore, $\Phi^T M \Phi = I$.

Substitution of Eqs. 2.2.1.2-7 and 8 into Eq. 2.2.1.2-6 yields the relation:

$$\begin{cases} U = H_1 U_o \\ H_1 = H_1(\omega) = \Phi H_o \Gamma^T \end{cases} \qquad 2.2.1.2\text{-}9$$

where

$$\Gamma = A^T M \Phi \qquad 2.2.1.2\text{-}10$$

and H_o is a diagonal frequency-dependent matrix with elements $H_o^{(i)}$ given by Eq. 2.2.1.2-8. The corresponding transfer function for the total motion of the superstructure is obtained by substitution of $H_1(\omega)$ from Eq. 2.2.1.2-9 into Eq. 2.2.1.2-4, i.e.,

$$H_2 = H_2(\omega) = H_2 + A = \Phi H_0 \Gamma^T + A \qquad 2.2.1.2\text{-}11$$

For every frequency ω, DIGES computes $H_o(\omega)$ using the corresponding modal frequency ω_i and damping ξ_i. Then, the 6Nx6 transfer matrix $H_1(\omega)$ is calculated from Eq. 2.2.1.2-9 while the corresponding 6Nx6 transfer matrix $H_2(\omega)$ is calculated from Eq. 2.2.1.2-4.

2.2.1.3 Transfer Functions Between Superstructure Forces and Total Foundation Motion

In the previous section, the transfer functions relating the motion of the superstructure and the total foundation motion were discussed. The associated transfer matrices H_1 and H_2 are employed in this section to compute the forces exerted by the superstructure on the foundation. Let H_3 represent the transfer matrix which at each frequency ω relates the amplitude of the total foundation motion U_o to the amplitudes of the forces F_{ss} exerted by the superstructure on the foundation. Then,

$$F_{ss} = H_3 U_o \qquad 2.2.1.3\text{-}1$$

where F_{ss}, U_o are 6-vectors and $H_3 = H_3(\omega)$ is a 6x6 frequency-dependent complex matrix. The vector F_{ss} is referred to the same point to that of the total foundation motion U_o, that is, the foundation reference point 0_F. The pertinent transfer functions can be then derived by computing the resultant of the inertial forces of the superstructure with respect to the foundation reference point. Let $\mathbf{F}_{ss}^{i}$ be the 6-vector of the inertia forces at the i-th node of the superstructure due to the total motion of the foundation, i.e.,

$$F_{ss}^{(i)} = \begin{Bmatrix} F_x^{(i)} \\ F_y^{(i)} \\ F_z^{(i)} \\ M_x^{(i)} \\ M_y^{(i)} \\ M_z^{(i)} \end{Bmatrix} \qquad 2.2.1.3\text{-}2$$

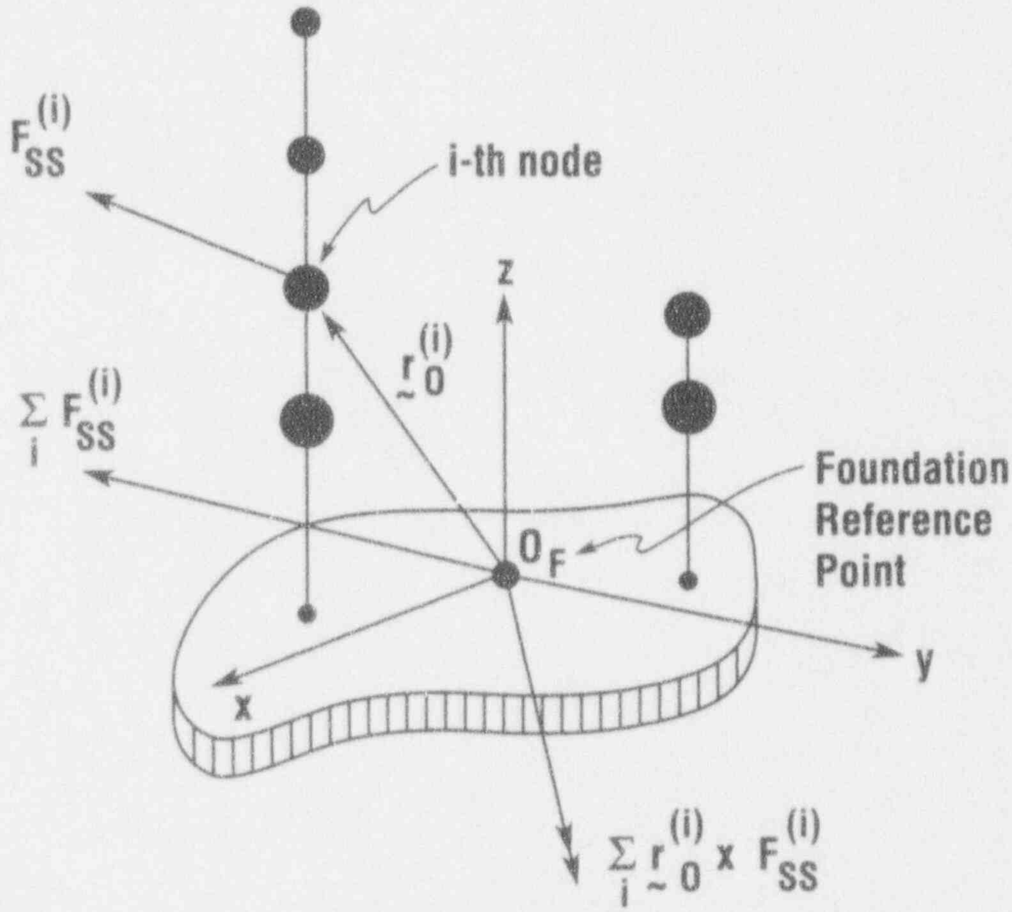
The resultant of all these forces and moments (over all the nodes of the superstructure), with respect to the foundation reference point, represents the vector $\mathbf{F}_{ss}$, i.e.,

$$F_{ss} = \begin{Bmatrix} F_x^{o} \\ F_y^{o} \\ F_z^{o} \\ M_x^{o} \\ M_y^{o} \\ M_z^{o} \end{Bmatrix} \qquad 2.2.1.3\text{-}3$$

Consequently, F_{ss} can be determined by considering the contributions (See Figure 2.2.1.3-1).

$$\sum_i F_{ss}^{(i)} \;\; ; \;\; \sum_i r_o^{(i)} \times F_{ss}^{(i)} \qquad 2.2.1.3\text{-}4$$

expressing summations of nodal inertia forces and moments with respect to foundation reference point O_F.



Forces Exerted By Superstructure On Foundation

Figure 2.2.1.3-1

The resultant forces F_x^o, F_y^o, and F_z^o are the sums of the corresponding forces $F_x^{(i)}$, $F_y^{(i)}$, and $F_z^{(i)}$ respectively. The moments M_x^o, M_y^o, and M_z^o are the sums of the corresponding moments, $M_x^{(i)}$, $M_y^{(i)}$, and $M_z^{(i)}$, plus the resulting moments of the forces $F_x^{(i)}$, $F_y^{(i)}$, and $F_z^{(i)}$ with respect to the foundation reference point. The summation is over all nodes of the superstructure. Based on this, the forces exerted on the foundation due to the inertia forces of the i-th node of the superstructure are:

$$\begin{Bmatrix} F_x^o \\ F_y^o \\ F_z^o \end{Bmatrix}_{(i)} = \begin{Bmatrix} F_x^{(i)} \\ F_y^{(i)} \\ F_z^{(i)} \end{Bmatrix} \qquad 2.2.1.3\text{-}5a$$

and

$$\begin{Bmatrix} M_x^o \\ M_y^o \\ M_z^o \end{Bmatrix}_{(i)} = \begin{Bmatrix} M_x^{(i)} \\ M_y^{(i)} \\ M_z^{(i)} \end{Bmatrix} + \begin{bmatrix} o & -r_{o,z}^{(i)} & r_{o,y}^{(i)} \\ r_{o,z}^{(i)} & o & -r_{o,x}^{(i)} \\ -r_{o,y}^{(i)} & r_{o,x}^{(i)} & o \end{bmatrix} \begin{Bmatrix} F_x^{(i)} \\ F_y^{(i)} \\ F_z^{(i)} \end{Bmatrix} \qquad 2.2.1.3\text{-}5b$$

where $\mathbf{r}_o^{(i)}$ is the position vector of the i-th node with respect to the foundation reference point. Eqs. 2.2.1.3-2 through 5 can be written in the following compact form:

$$\begin{Bmatrix} F_x^o \\ F_y^o \\ F_z^o \\ M_x^o \\ M_y^o \\ M_z^o \end{Bmatrix}_{(i)} = \begin{bmatrix} I & O \\ Q^{(i)} & I \end{bmatrix} \begin{Bmatrix} F_x^{(i)} \\ F_y^{(i)} \\ F_z^{(i)} \\ M_x^{(i)} \\ M_y^{(i)} \\ M_z^{(i)} \end{Bmatrix} \qquad 2.2.1.3\text{-}6a$$

where:

$$Q^{(i)} = \begin{bmatrix} 0 & -(z_i - z_o) & (y_i - y_o) \\ z_i - z_o & 0 & -(x_i - x_o) \\ -(y_i - y_o) & (x_i - x_o) & 0 \end{bmatrix} \qquad 2.2.1.3\text{-}6b$$

By comparing Eqs. 2.2.1.3-6b and 2.2.1.1-8c, it is concluded that $\mathbf{Q}^{(i)}$ is the transpose of $\mathbf{R}^{(i)}$. Thus, in view of Eqs. 2.2.1.3-6a and 2.2.1.1-8, b, $\mathbf{E}^{(i)}$ is the transpose of $\mathbf{A}^{(i)}$. This is a consistent result since in view of Eq. 2.2.1.1-8a, $\mathbf{A}^{(i)}$ is a 6x6 transformation matrix.

By carrying out the summation over the N nodes of the superstructure and by taking into account that each element of $F^{(i)}$ in Eq. 2.2.1.3-6, is equal to the corresponding row of the mass matrix $\mathbf{M}$ times the vector of the total motion of the superstructure, we obtain:

$$F_{ss} = \omega^2 A^T M U_t \qquad 2.2.1.3\text{-}7$$

Substituting U_t from Eqs. 2.2.1.2-11 we can write:

$$\left.\begin{aligned} F_{ss} &= H_3 U_o \\ H_3 = H_3(\omega) &= \omega^2 A^T M H_2 \\ &= \omega^2 A^T M(H_1 + A) \\ &= \omega^2 (A^T M A + \Gamma H_o \Gamma^T) \end{aligned}\right\} \qquad 2.2.1.3\text{-}8$$

2.2.1.4 Summary of Superstructure Transfer Functions

The following is a summary of transfer functions associated with the motion of the superstructure as computed by DIGES (see Figure 2.2.1.4-1).

A. Between total foundation motion and superstructure relative motion:

$$\left.\begin{aligned} U &= H_1 U_o \\ H_1 = H_1(\omega) &= \Phi H_o \Gamma^T \end{aligned}\right\} \qquad 2.2.1.2\text{-}9$$

H_o = diagonal matrix with j-th element:

$$\left.\begin{aligned} H_o^{(j)} &= \frac{\Omega_j^2}{1 + 2i\ \xi_j\ \ \Omega_j\ -\ \Omega_j^2} \\ \Omega_j &= \frac{\omega}{\omega_j}\ ;\ (\omega_j,\ \xi_j)\ =\ modal\ (frequency,\ damping) \end{aligned}\right\} \qquad 2.2.1.2\text{-}8$$

A = transformation matrix consisting of 6x6 submatrices

$$\begin{bmatrix} I & R^{(i)} \\ 0 & I \end{bmatrix}$$

$$R^{(i)} = \begin{bmatrix} 0 & z_i - z_o & -(y_i - y_o) \\ -(z_i - z_o) & 0 & x_i - x_o \\ y_i - y_o & -(x_i - x_o) & 0 \end{bmatrix} \qquad 2.2.1.1\text{-}8$$

I = 3x3 unit matrix

O = 3x3 zero matrix

(x_o, y_o, z_o) = coordinates of reference point

(x_i, y_i, z_i) = coordinates of i-th node

B. Between total foundation motion and total superstructure motion

$$\left.\begin{aligned} U_t &= H_2\ U_o \\ H_2 &= H_2(\omega)\ =\ H_1\ +\ A \\ &= \Phi H_o \Gamma^T\ +\ A \end{aligned}\right\} \qquad 2.2.1.2\text{-}11$$

C. Between total foundation motion and forces exerted by superstructure on the foundation:

$$\left.\begin{aligned} F_{ss} &= H_3\ U_o \\ H_3 &= H_3(\omega) = \omega^2 A^T\ M\ H_2 \\ &= \omega^2 (A^T M A\ +\ \Gamma H_o \Gamma^T) \end{aligned}\right\} \qquad 2.2.1.3\text{-}8$$

The computational steps required for defining the transfer functions relating:

- flexural motions of superstructure
- total motions of superstructure
- forces exerted by superstructure on foundation

} with foundation motion

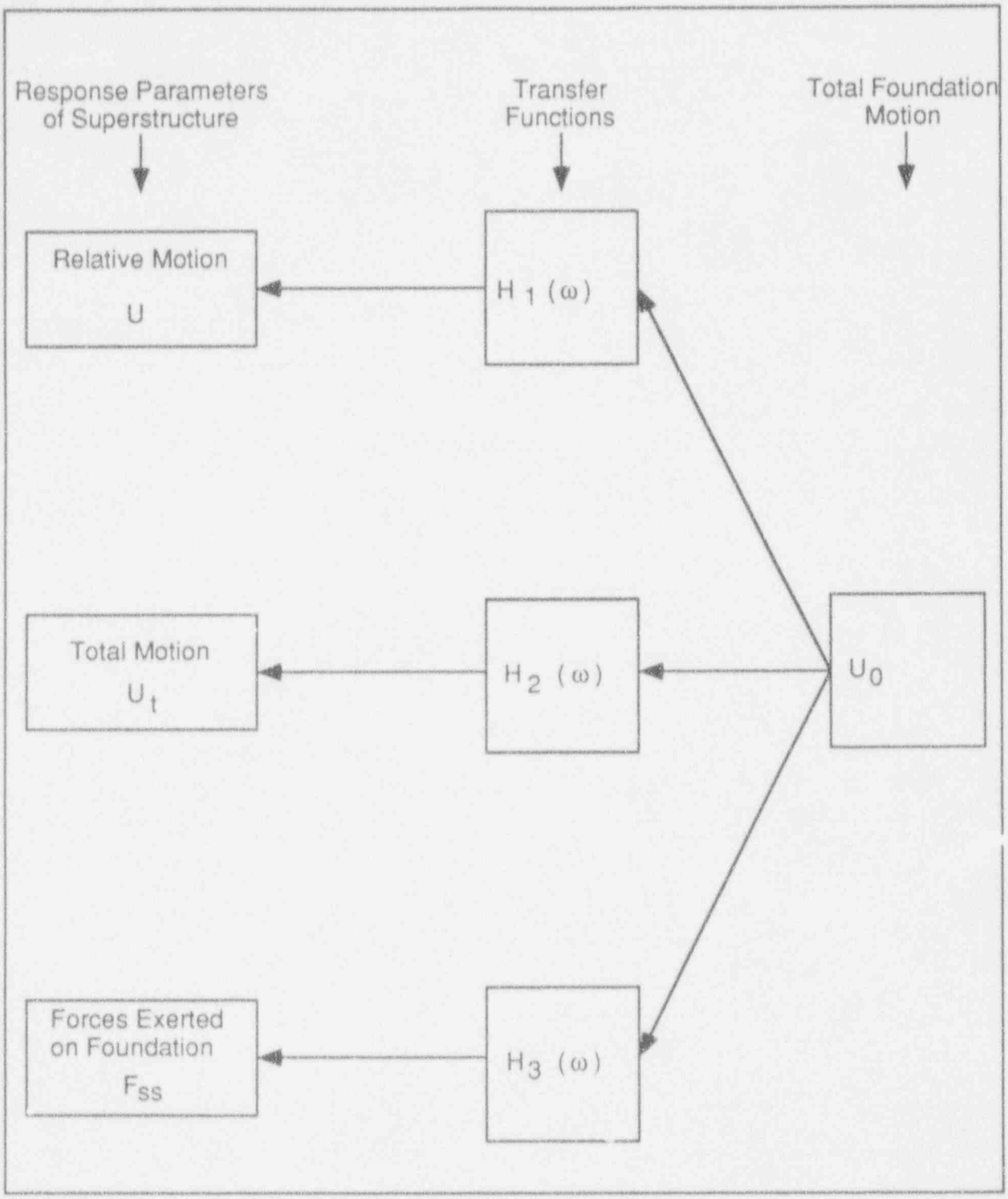
are summarized as follows:

- Compute stiffness and mass matrices **M** and **K** respectively,
- Compute transformation matrix **A**
- Perform fixed-base modal analysis. Compute modal shape matrix $\mathbf{\Phi}$ and modal frequencies ω_j. Compute participation matrix $\mathbf{\Gamma}$ from Eq. 2.2.1.2-10.
- At each frequencies ω of the analysis compute:
  - diagonal matrix $\mathbf{H}_o$ using the modal frequency ω_j and modal damping ξ_j of the j-th mode.
  - compute $\mathbf{H}_1(\omega)$ from Eq. 2.2.1.2-9
  - compute $\mathbf{H}_2(\omega)$ from Eq. 2.2.1.2-11
  - compute $\mathbf{H}_3(\omega)$ from Eq. 2.2.1.3-8
- Repeat these calculations for all frequencies of interest, thus building up function $\mathbf{H}_1$, $\mathbf{H}_2$, and $\mathbf{H}_3$ in terms of frequency ω.

2.2.2 Dynamic Response of Foundation

2.2.2.1 Equations of Motion of Foundation

The motion of the foundation is completely defined by the motion of the foundation reference point O_F, that is, the total foundation motion. With reference to Figure 2.2.2.1-1, the latter consists of three displacements Δ_x^o, Δ_y^o and Δ_z^o represented by the vector $\delta \mathbf{r}_F$ and three rotations θ_x^o, θ_y^o and θ_z^o represented by the vector $\delta \boldsymbol{\theta}$, i.e.,



Transfer Functions of Superstructure

Figure 2.2.1.4-1

$$U_o = \begin{Bmatrix} \delta r_F \\ \delta\theta \end{Bmatrix} = \begin{Bmatrix} \Delta_x^o \\ \Delta_y^o \\ \Delta_z^o \\ \theta_x^o \\ \theta_y^o \\ \theta_z^o \end{Bmatrix} \qquad 2.2.2.1\text{-}1$$

The total external forces and moments applied at the foundation are represented by the 6-vector $\mathbf{F}_o$, i.e.,

$$F_o = \begin{Bmatrix} F^o \\ M^o \end{Bmatrix} = \begin{Bmatrix} F_x^o \\ F_y^o \\ F_z^o \\ M_x^o \\ M_y^o \\ M_z^o \end{Bmatrix} \qquad 2.2.2.1\text{-}2$$

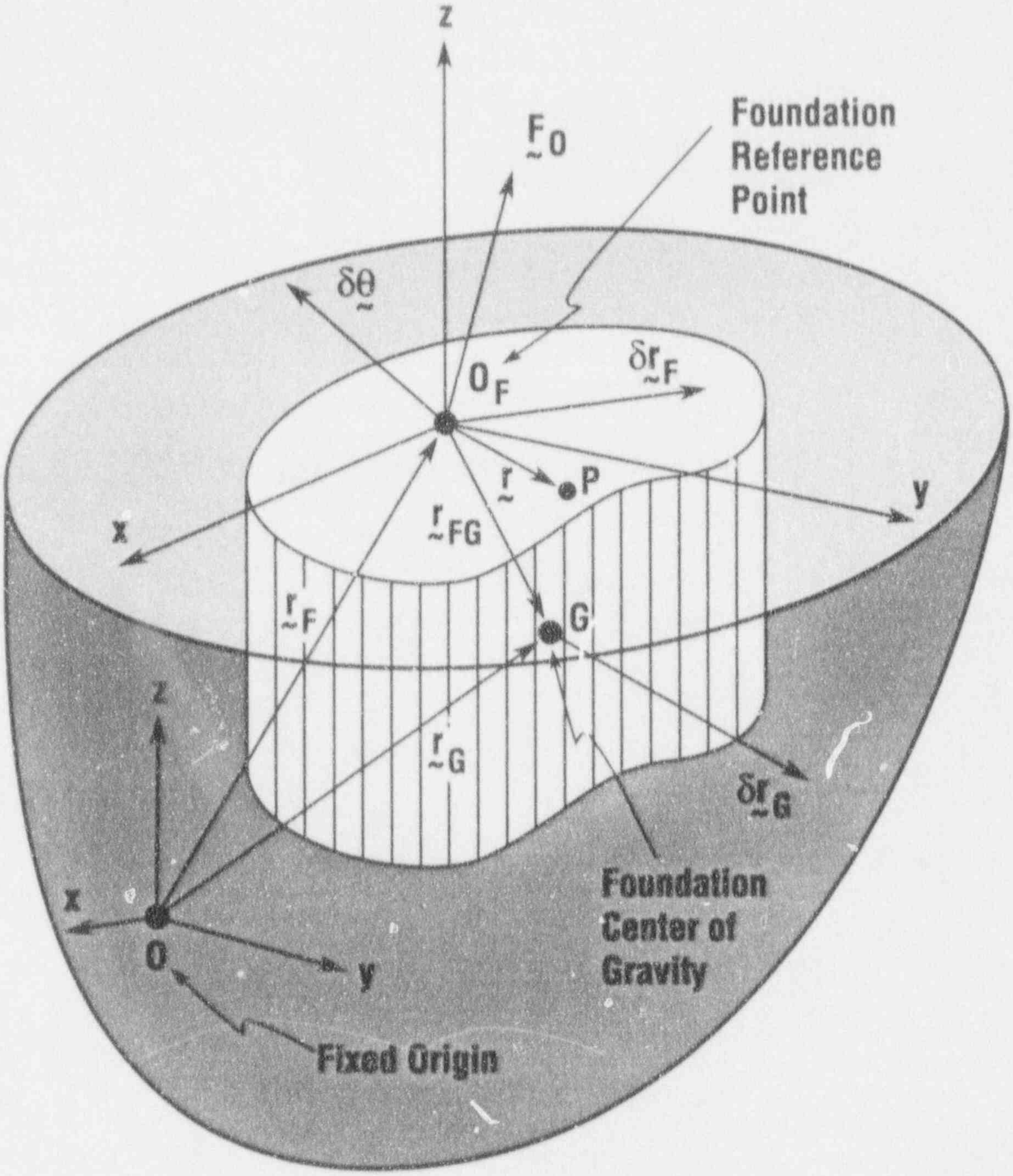
Note that $\mathbf{F}_o$ is defined with respect to the foundation reference point O_F in consistency with the vector $\mathbf{U}_o$. The displacement at any point P of the rigid foundation can be found from the total foundation motion by,

$$\delta \boldsymbol{r} = \delta \boldsymbol{r}_F + \delta \boldsymbol{\theta} \times \boldsymbol{r} \qquad 2.2.2.1\text{-}3$$

where $\delta \mathbf{r}_F$ and $\delta\theta$ are defined by Eq. 2.2.2.1-1 and $\mathbf{r}$ is the position vector of P relative to the foundation reference point (See Figure 2.2.2.2-1). In particular, the displacement of the center of gravity of the foundation is,

$$\delta \boldsymbol{r}_G = \delta \boldsymbol{r}_F + \delta \boldsymbol{\theta} \times \boldsymbol{r}_G \qquad 2.2.2.1\text{-}4$$

The equations of motion of the rigid foundation can be obtained by balancing its linear and angular momenta to the external forces $\mathbf{F}_o$ as follows:



Motion of Foundation

Figure 2.2.2.2-1

$$\frac{dL}{dt} = F^o \qquad 2.2.2.1\text{-}5a$$

$$\frac{dH}{dt} = M^o \qquad 2.2.2.1\text{-}5b$$

where **L** is the linear momentum of the foundation and H is its angular momentum with respect to reference point O_F.

The linear momentum of the foundation is:

$$L = m v_G \qquad 2.2.2.1\text{-}6$$

Where m is the mass of the foundation and $\mathbf{v}_G$ the velocity of motion at its center of gravity, i.e., $\mathbf{v}_G = d\mathbf{r}_G/dt$. By taking the time derivative of Eq. 2.2.2.1-4, substituting the result into Eq. 2.2.2.1-6 and then taking the time derivative of the linear momentum, Eq. 2.2.2.1-5a can be written in terms of the total foundation motion as follows:

$$-\omega^2 m \left(\begin{Bmatrix} \Delta_x^o \\ \Delta_y^o \\ \Delta_z^o \end{Bmatrix} + \begin{bmatrix} 0 & z_G & -y_G \\ -z_G & 0 & x_G \\ y_G & -x_G & 0 \end{bmatrix} \right) \begin{Bmatrix} \theta_x^o \\ \theta_y^o \\ \theta_z^o \end{Bmatrix} = \begin{Bmatrix} F_x^o \\ F_y^o \\ F_z^o \end{Bmatrix} \qquad 2.2.2.1\text{-}7$$

Where x_G, y_G and z_G are the coordinates of the center of gravity of the foundation with respect to $O_F XYZ$.

Next, the angular momentum of the foundation with respect to the foundation reference point O_F is:

$$H = \int_V \rho \boldsymbol{r} \times \boldsymbol{v} \, dV \qquad 2.2.2.1\text{-}8$$

where **v** is the velocity at any point P of the foundation, whose position vector with respect to the foundation reference point is **r**, i.e., $\mathbf{v} = d\mathbf{r}/dt$ and the integration is over the volume V of the foundation. By calculating the velocity with the aid of Eq. 2.2.2.1-3 and substituting the result into Eq. 2.2.2.1-8,

$$\frac{dH}{dt} = -\omega^2 \int_V \rho(\Lambda_1 i + \Lambda_2 j + \Lambda_3 k) dV \qquad 2.2.2.1\text{-}9a$$

where

$$\begin{Bmatrix} \Lambda_1 \\ \Lambda_2 \\ \Lambda_3 \end{Bmatrix} = \begin{bmatrix} 0 & z & y \\ z & 0 & -x \\ -y & x & 0 \end{bmatrix} \begin{Bmatrix} \Delta_x^o \\ \Delta_y^o \\ \Delta_z^o \end{Bmatrix} + \begin{bmatrix} y^2+z^2 & -xy & -xz \\ -xy & x^2+z^2 & -yz \\ -xz & -yz & x^2+y^2 \end{bmatrix} \begin{Bmatrix} \theta_x^o \\ \theta_y^o \\ \theta_z^o \end{Bmatrix} \qquad 2.2.2.1\text{-}9b$$

ρ = mass density of foundation

x_G, y_G, z_G = coordinates of the center of gravity of the foundation.

The above volume integrals become the mass moments and mass products of inertia of the foundation about O_FXYZ. Substituting Eq. 2.2.2.1-9b into Eq. 2.2.2.1-5b:

$$-\omega^2 m \left(\begin{bmatrix} 0 & z_G & y_G \\ z_G & 0 & -x_G \\ -y_G & x_G & 0 \end{bmatrix} \begin{Bmatrix} \Delta_x^o \\ \Delta_y^o \\ \Delta_z^o \end{Bmatrix} + \frac{1}{m} \begin{bmatrix} I_{xx} & I_{xy} & I_{xz} \\ I_{yx} & I_{yy} & I_{yz} \\ I_{zx} & I_{zy} & I_{zz} \end{bmatrix} \begin{Bmatrix} \theta_x^o \\ \theta_y^0 \\ \theta_z^o \end{Bmatrix} \right) = \begin{Bmatrix} M_x^o \\ M_y^o \\ M_z^o \end{Bmatrix}$$

2.2.2.1-10

Combination of Eqs. 2.2.2.1-7 and 2.2.2.1-10 gives the final form of the equations of motion of the foundation (in terms of the total foundation motion $\mathbf{U}_o$ and the resultant forces $\mathbf{F}_o$) with respect to the foundation reference point O_F.

$$-\omega^2 \boldsymbol{M}_o \boldsymbol{U}_o = \boldsymbol{F}_o \qquad 2.2.2.1\text{-}11a$$

where,

$$M_o = m \begin{bmatrix} I & A_G \\ A_G^T & \frac{1}{m} I_o \end{bmatrix} \qquad 2.2.2.1\text{-}11b$$

is the mass matrix of the foundation, and,

$$A_G = \begin{bmatrix} 0 & z_G & -y_G \\ -z_G & 0 & x_G \\ y_G & -x_G & 0 \end{bmatrix} \qquad 2.2.2.1\text{-}11c$$

m = mass of the foundation

I = 3x3 unit matrix

I_o = 3x3 moment of inertia matrix about O_FXYZ

x_G, y_G, z_G = coordinates of the center of gravity with respect to O_FXYZ.

In applying Eq. 2.2.2.1-11, it should be kept in mind that ALL external forces should be referred to the foundation reference system O_FXYZ which is common with that of reference of the mass matrix of the system. If it is decided to use the center of gravity of the foundation as a reference point (take G as O_F), then again both the mass matrix and the resultant of the external forces must be referred to the center of gravity.

2.2.2.2 Foundation Forces

The forces associated with the motion of the foundation are due to:

a. forces exerted by the superstructure on the foundation $\mathbf{F}_{ss}$
b. forces due to interactions between the surrounding soil and the foundation $\mathbf{F}_s$
c. forces due to ground motion that result in from seismic waves impinging on the foundation $\mathbf{F}_G$

The general equations of motion of the foundation expressed by Eqs. 2.2.2.1-11 can be written as:

$$-\omega^2 M_o U_o + F_s = F_{ss} \qquad 2.2.2.2\text{-}1$$

The total motion of the foundation U_o consists of its relative motion with respect to the soil expressed by the 6-vector U_s plus the foundation input motion due to seismic waves which is expressed by the 6-vector U_G (i.e., $U_o = U_s + U_G$). The foundation forces due to the interaction with the soil in the absence of seismic excitation are:

$$F_s = K_s U_s \qquad 2.2.2.2\text{-}2$$

where K_s is the impedance matrix of the foundation. The foundation forces associated with the seismic wave incidence on the foundation can be written in a similar form;

$$F_G = K_s U_G \qquad 2.2.2.2\text{-}3$$

In DIGES, two loading cases are considered:

Case 1: The motion of the building-foundation system is due to dynamic loads imposed on the superstructure only.

Case 2: The motion of the building-foundation system is due to seismic waves incidence on the foundation only.

In the first case, the total motion of the foundation U_o represents its relative motion with respect to the soil (i.e., $U_o = U_S$). In this case, Eq. 2.2.2.2-1 becomes:

$$(-\omega^2 M_o + K_S) U_s = F_{ss} \qquad 2.2.2.2\text{-}4$$

When seismic excitation only is of interest (i.e., Case 2), then $U_o = U_S + U_G$ and Eq. 2.2.2.2-1 becomes:

$$(-\omega^2 M_o + K_s) U_o = F_{ss} + F_G \qquad 2.2.2.2\text{-}5$$

The solutions of Eqs. 2.2.2.2-4 and 5 are given in the next two sections of the report. Note that the case of simultaneous application of dynamic loads on the superstructure and seismic loads would require the knowledge of the phasing between the two loads.

2.2.2.3 Dynamic Loads on Superstructure

The dynamic loads imposed on the superstructure are considered to be applied at its nodes in the global sense, and thus are associated with the 6N degrees-of-freedom of the system. Distributed loads are ultimately converted into nodal loads. Let **P** be the 6N vector representing these loads, i.e.,

$$P = \begin{Bmatrix} P_1 \\ \cdot \\ \cdot \\ P_i \\ \cdot \\ \cdot \\ P_N \end{Bmatrix}_{6N\times 1} \quad ; P_i = \begin{Bmatrix} F_x^{(i)} \\ F_y^{(i)} \\ F_z^{(i)} \\ M_x^{(i)} \\ M_y^{(i)} \\ M_z^{(i)} \end{Bmatrix}_{6\times 1} \qquad 2.2.2.3\text{-}1$$

The equations of motion of the foundation are given by Eq. 2.2.2.2-4 in which the total foundation motion $\mathbf{U}_o$ is essentially the relative motion of the foundation with respect to the soil, i.e., $\mathbf{U}_t = \mathbf{U}_S$.

The transfer function $\mathbf{H}_i$ for the relative displacements of the superstructure can be computed from its fixed-base modal analysis (modal shapes Φ, modal frequencies and damping) by considering that in this case the amplitudes of the modal displacements become:

$$\eta_j = H_o^{(j)} \left(\frac{\phi_j^T MA}{\phi_j^T M \phi_j} U_o + \frac{1}{\omega} \frac{\phi_j}{\phi_j^T M \phi_j} P \right) \qquad 2.2.2.3\text{-}2$$

where $H_o^{(j)}$ is given by Eq. 2.2.1.2-8.

The amplitudes of the relative displacement **U** are:

$$U = \Phi\eta = \Phi H_o \Gamma^T U_s + \frac{1}{\omega^2} \Phi H_o \Phi^T P \qquad 2.2.2.3\text{-}3$$

The total motion of the superstructure (relative to the soil) is:

$$U_t = U + A U_s \qquad 2.2.2.3\text{-}4$$

The forces exerted by the superstructure on the foundation is the sum of the corresponding inertia forces plus those associated with the applied forces on the superstructure. The inertia forces are given by Eq. 2.2.1.3-7. The equivalent forces exerted on the foundation due to P can be obtained in a similar fashion by using Eq. 2.2.1.3-6. Based on the above, the total force exerted by the superstructure on the foundation is:

$$F_{ss} = \omega^2 A^T M U_t + A^T P \qquad 2.2.2.3\text{-}5a$$

Using Eqs. 2.2.2.3-3 and 4, $\mathbf{F}_{ss}$ takes the form:

$$\begin{aligned} F_{ss} &= \omega^2 (A^T MA + \Gamma H_o \Gamma^T) U_s \\ &+ (A^T + \Gamma H_o \Phi^T) P \end{aligned} \qquad 2.2.2.3\text{-}5b$$

Substitution of $\mathbf{F}_{ss}$ into Eq. 2.2.2.2-4 yields the transfer function between the motion of the foundation with respect to the soil and the force **P** imposed at the superstructure:

$$\begin{cases} U_s = H_4(\omega) P \\ H_4 = H_4(\omega) = \left[K_s - \omega^2 (M_o + A^T MA + \Gamma H_o \Gamma^T)\right]^{-1} (\Gamma H_o \Phi^T + A^T) \end{cases} \qquad 2.2.2.3\text{-}6$$

Note that the computation of $\mathbf{H}_4$ requires the inversion of a 6x6 complex matrix. The transfer function relating the response of the superstructure to the applied loads can then be obtained by back-substitution into Eqs. 2.2.2.3-3, 4, and 5:

- Transfer function between applied force and structural displacements relative to the soil.

$$U = H_1 P$$

$$H_1 = H_1(\omega) = \Phi H_o \left(\Gamma^T H_4 + \frac{1}{\omega^2} \Phi^T \right) \qquad 2.2.2.3\text{-}7$$

- Transfer function between applied force and total structural displacements,

$$U_t = H_2 P$$

$$H_2 = H_2(\omega) = \Phi H_o \left(\Gamma^T H_4 + \frac{1}{\omega^2} \Phi^T \right) + A H_4 \qquad 2.2.2.3\text{-}8$$

- Transfer function between applied force and forces exerted on the foundation,

$$F_{ss} = H_3 P$$

$$H_3 = H_3(\omega) = \omega^2 \left(A^T M A + \Gamma H_o \Gamma^T \right) H_4 + A^T + \Gamma H_o \Phi^T \qquad 2.2.2.3\text{-}9$$

2.2.2.4 Ground Excitation

For the case of seismic excitation, the forces $\mathbf{F}_{ss}$ exerted by the superstructure on the foundation are the inertia forces given by Eq. 2.2.1.3-8. The transfer function between the total response of the foundation $\mathbf{U}_o$ and the foundation input motion $\mathbf{U}_G$ can be computed by substitution of Eq. 2.2.2.1.3-8 into the right side of Eq. 2.2.2.2-5:

$$\begin{cases} U_o = H_4(\omega) U_G \\ H_4 = H_4(\omega) = \left[K_s - \omega^2 \left(M_o + A^T M A + \Gamma H_o \Gamma^T \right) \right]^{-1} K_s \end{cases} \qquad 2.2.2.4\text{-}1$$

According to Eq. 2.2.2.4-1, the solution for the transfer function, $\mathbf{H}_4$ requires the inversion of a 6x6 matrix. Note, that this matrix is the same as in Eq. 2.2.2.3-6. Having computed $\mathbf{H}_4$ from Eq. 2.2.2.4-1 the transfer functions for the superstructure are obtained as follows:

- Transfer function between structural displacements relative to the foundation and the foundation input motion:

$$U = H_1 U_G$$

$$H_1 = H_1(\omega) = \Phi H_o \Gamma^T H_4 \qquad 2.2.2.4\text{-}2$$

- Transfer function between total structural displacements and the foundation input motion:

$$U_t = H_2 U_G$$
$$H_2 = H_2(\omega) = \left(\Phi H_o \Gamma^T + A\right) H_4 \qquad 2.2.2.4\text{-}3$$

- Transfer function between the forces exerted by the superstructure and the foundation input motion

$$F_{ss} = H_3 U_G$$
$$H_3 = H_3(\omega) = \omega^2 \left(A^T M A + \Gamma H_o \Gamma^T\right) H_4(\omega) \qquad 2.2.2.4\text{-}4$$

In view of Eq. 2.2.2.4-1, $\mathbf{H}_4(\omega)$ relates the total motion of the foundation $\mathbf{U}_o$ to the foundation input motion $\mathbf{U}_G$. The latter can be further represented in terms of the free-field motion $\mathbf{U}_G^o$, i.e.,

$$U_G = H_5(\omega)\ U_G^o \qquad 2.2.2.4\text{-}5$$

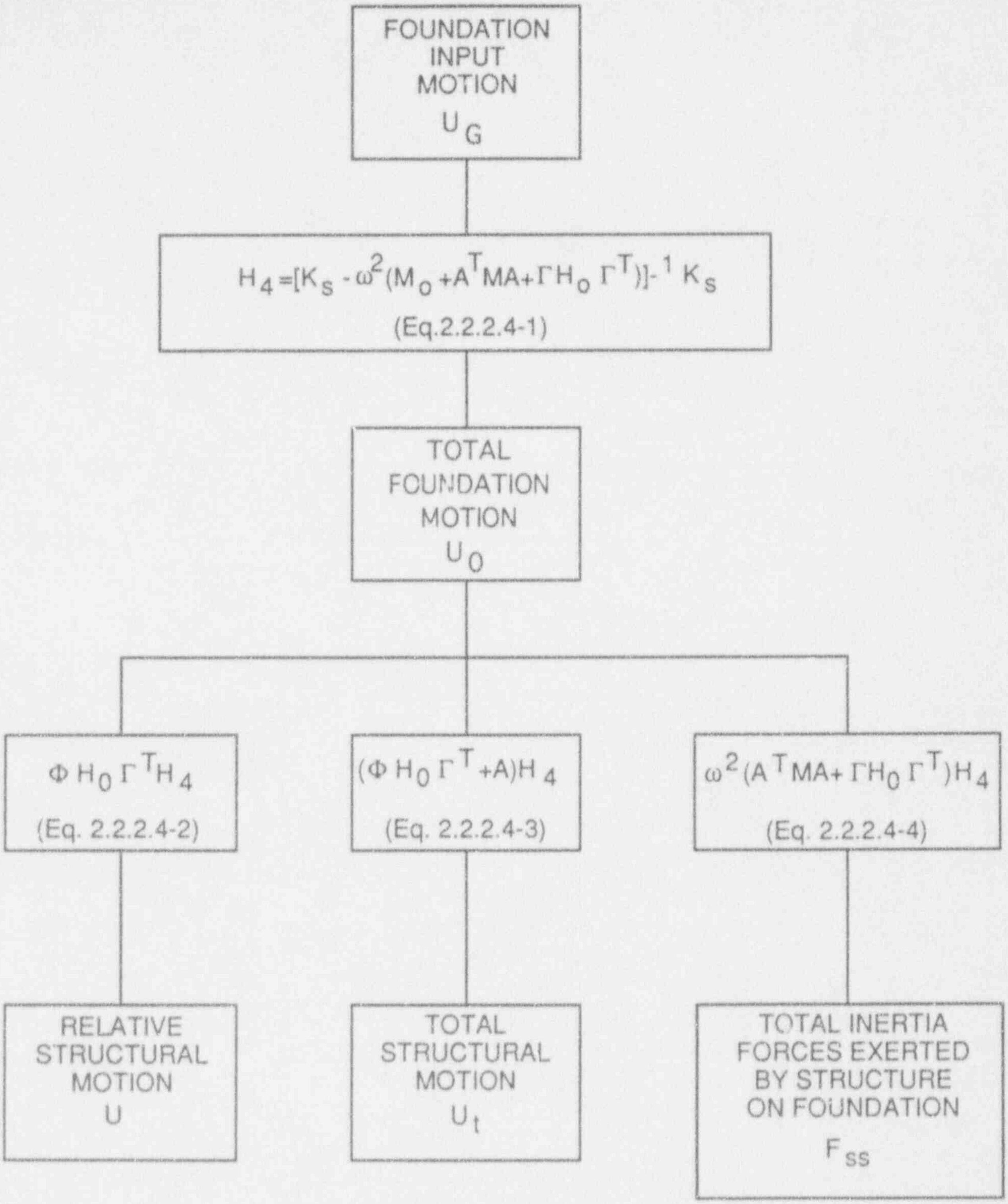
where $\mathbf{H}_5(\omega)$ contains the scattering coefficients associated with the scattering of the seismic waves by the foundation. By incorporating Eq. 2.2.2.4-5 into Eqs. 2.2.2.4-1 through 4, the response is directly related to the free-field.

2.2.3 Summary of Building-Foundation System Transfer Functions

For convenience, the transfer functions of the building-foundation system are summarized in Figures 2.2.3-1 and 2.2.3-2 for the cases of ground excitation and dynamic loads applied at the superstructure respectively. The relevant transfer functions for the case of ground excitation are relating the response of the superstructure to the *foundation input motion*. Similarly, the transfer functions for the case of dynamic loads imposed on the superstructure are relating the response of the superstructure to the *applied loads*. In both loading cases, the response of the superstructure involved:

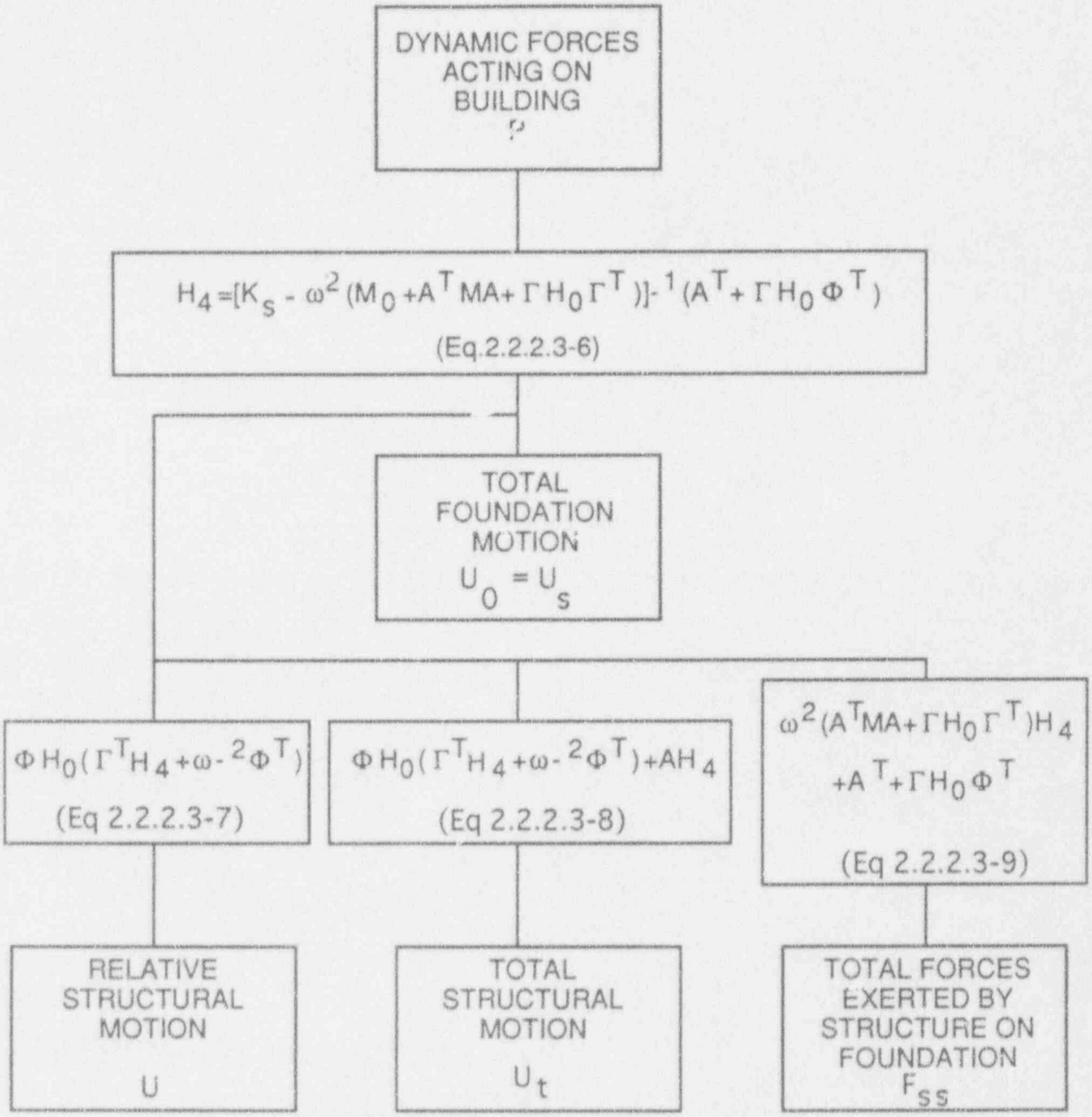
- Flexural motions of the superstructure with respect to the foundation reference point (i.e., relative displacements)

- Total motion of the superstructure with respect to the foundation input motion or with respect to the soil.
- Forces exerted by the superstructure on the foundation at the foundation reference point.



SUMMARY OF TRANSFER FUNCTIONS
(CASE: GROUND EXCITIATION)

Figure 2.2.3-1



SUMMARY OF TRANSFER FUNCTIONS
(CASE : SUPERSTRUCTURE DYNAMIC LOADS)

Figure 2.2.3-2

3.0 DIGES Algorithm

The implementation of the theoretical background, pertaining to the transferring of the free-field motion onto the superstructure, into a computational tool has been achieved with the DIGES computer program which utilizes the FORTRAN 77 computer language. The program itself resides in the a SPARC 1+ Workstation with open window capabilities. The workstation is part of an *ethernet* network that allows login from a remote host. This gives it the capability of window sharing between two remote hosts for on-line communication.

The program itself reflects the capabilities already described in the main text, such as *direct generation of spectra, earthquake simulation, deterministic earthquake solutions and system response under dynamic superstructure loads.*

In the following the main programming features of DIGES will be outlined as well as the sequence of processes that are incorporated for a complete solution of any of the four major modules of analysis. Figure 3-1 outlines the basic solution flow chart while Figures 3-2a through d describe in more detail the operations executed in each solution module.

Main Program

The main program of DIGES plays the role of the driver of the program. It identifies the module of analysis, the form of free-field input, the frequency content of the input along with the size of the dynamic problem, the geometric/dynamic properties of the soil/structure system and the locations where the response is to be evaluated. With this provided information (both interactively and externally) the program has been designed to allocate memory space exactly equal to what the solution needs. This is course has the advantage of utilizing all the working memory the host system may allow instead of confining the computer code to a specified problem size. Further, the ability of swapping memory space while the program is in execution allows even further flexibility.

The sequence of operations are dictated by the main subroutine of the program which calls the following main families of subroutines:

a. The modal properties of the superstructure are evaluated by successively invoking the subroutines that (1) identify the nodal locations with the degree of freedom, (2) calculate the stiffness of the finite element system representing the superstructure (3) evaluate the bandwidth and the global stiffness and mass matrices and (4) call the eigensolution subroutine that returns the eigenvalues and the eigenvectors of the superstructure.

b. In order for the equivalent mass matrix of the superstructure to be calculated which implies the transferring of superstructure information on the foundation the system invokes the subroutines that (1) calculate the rigid displacement matrix (2) the modal participation matrix (3) mass matrix of the superstructure for displacement about a reference point on the foundation and (4) the mass matrix of the rigid foundation.

To this point the calculated information is independent of the analysis frequency. From this point on the evaluation of system properties takes place at every selected frequency value associated with the specified frequency range of the solution.

c. The **total transfer function** $H(\omega)$ is established for the complete frequency range. $H(\omega)$ is the product of (1) a transfer function which transfers the total foundation input characteristics to any d.o.f. of the superstructure, (2) a transfer function relating the foundation input motion to the total foundation motion and (3) the scattering matrix that incorporates the modification of the free-field motion due to the presence of the massless foundation.

Specifically, the program invokes the modal amplification matrix routine, the frequency dependent equivalent mass matrix routine and, on the basis of the type of foundation, the impedance (or compliance) matrices. Finally the scattering matrix is called and the total transfer matrix of the system (complex) is calculated for every frequency of the analysis.

For the case of system response due to dynamic loads on the superstructure the first of the above transfer functions is appropriately modified.

d. The **free-field input** is established in the frequency domain. If (1) the *direct* transfer option has been chosen and the control motion is in the form of a power spectral density, the cross-spectral matrix over the frequency range is formed. When the control motion is in response spectra form, the consistent cross-spectral matrix is evaluated for the solution frequency range by invoking the subroutines that generate *psd* functions consistent with the free-field response spectrum. If (2) the *simulation* process is chosen then, depending on the spectra type (power or response) chosen as representative of the stochastic process, the Fourier coefficients of the simulated earthquakes are calculated by going through the process as many times as the number of simulations chosen. If (3) the deterministic analysis for an actual earthquake is to take place, the Fourier coefficients of the record are established. This of course is similar to the previous process for a single simulation except that the time history is not generated. Finally, (4) if the system is excited by a dynamic load in the superstructure, the Fourier coefficients of the dynamic load are established by invoking the appropriate Fourier Transform subroutine.

e. On the basis of the final form of the free-field motion (*psd* or Fourier expansion coefficients) the routines that calculate either the cross-spectral density matrix of the output or the Fourier coefficients of the time history of the output are called appropriately. If the output response is in a *psd* form, its response spectrum is calculated directly from the process described in Section 2.1.2. If the response is in the form of a time history (or the Fourier coeff.), the corresponding response spectra are calculated.

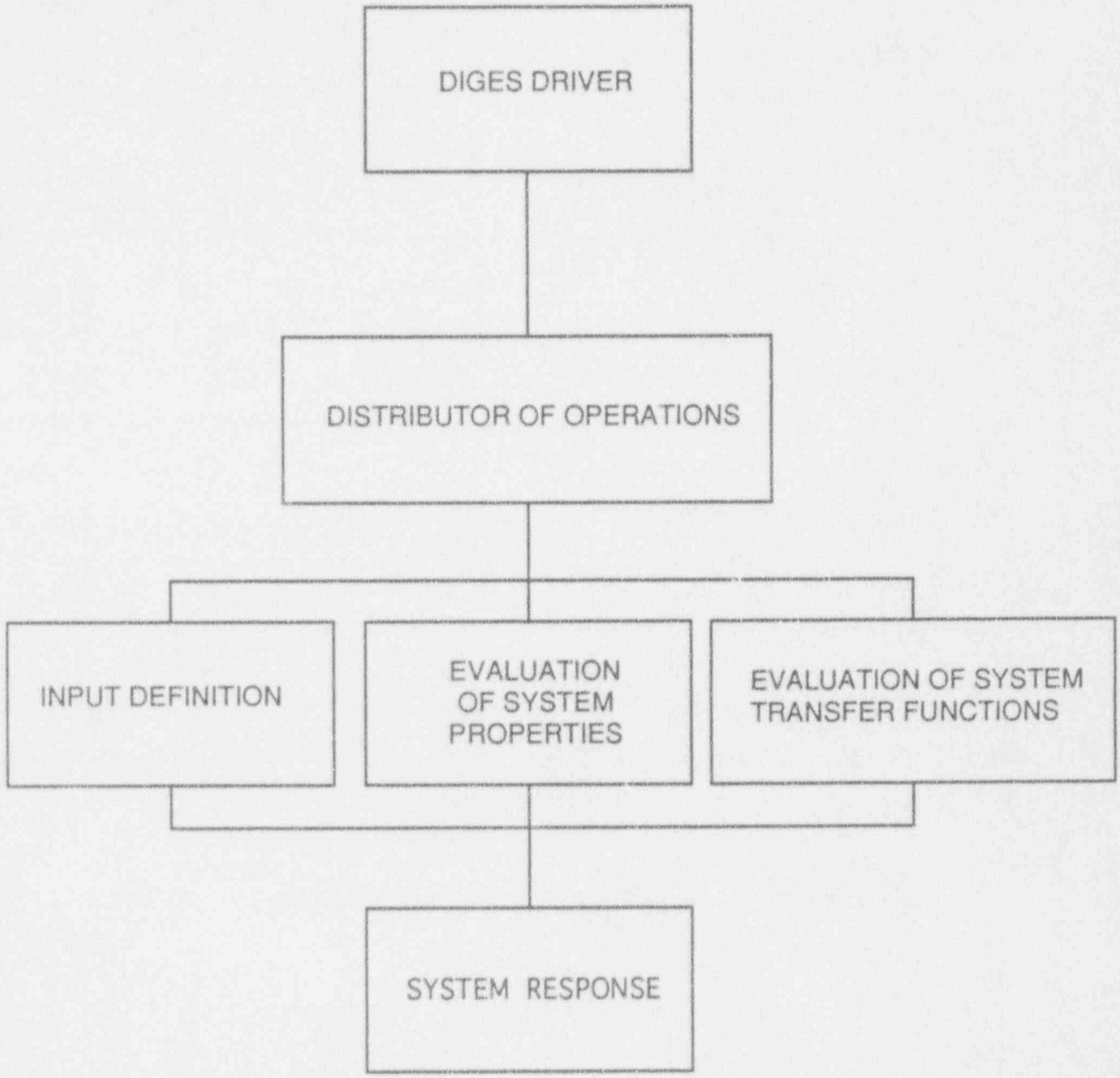

DIGES DRIVER
DISTRIBUTOR OF OPERATIONS
INPUT DEFINITION
EVALUATION OF SYSTEM PROPERTIES
EVALUATION OF SYSTEM TRANSFER FUNCTIONS
SYSTEM RESPONSE

Figure 3-1

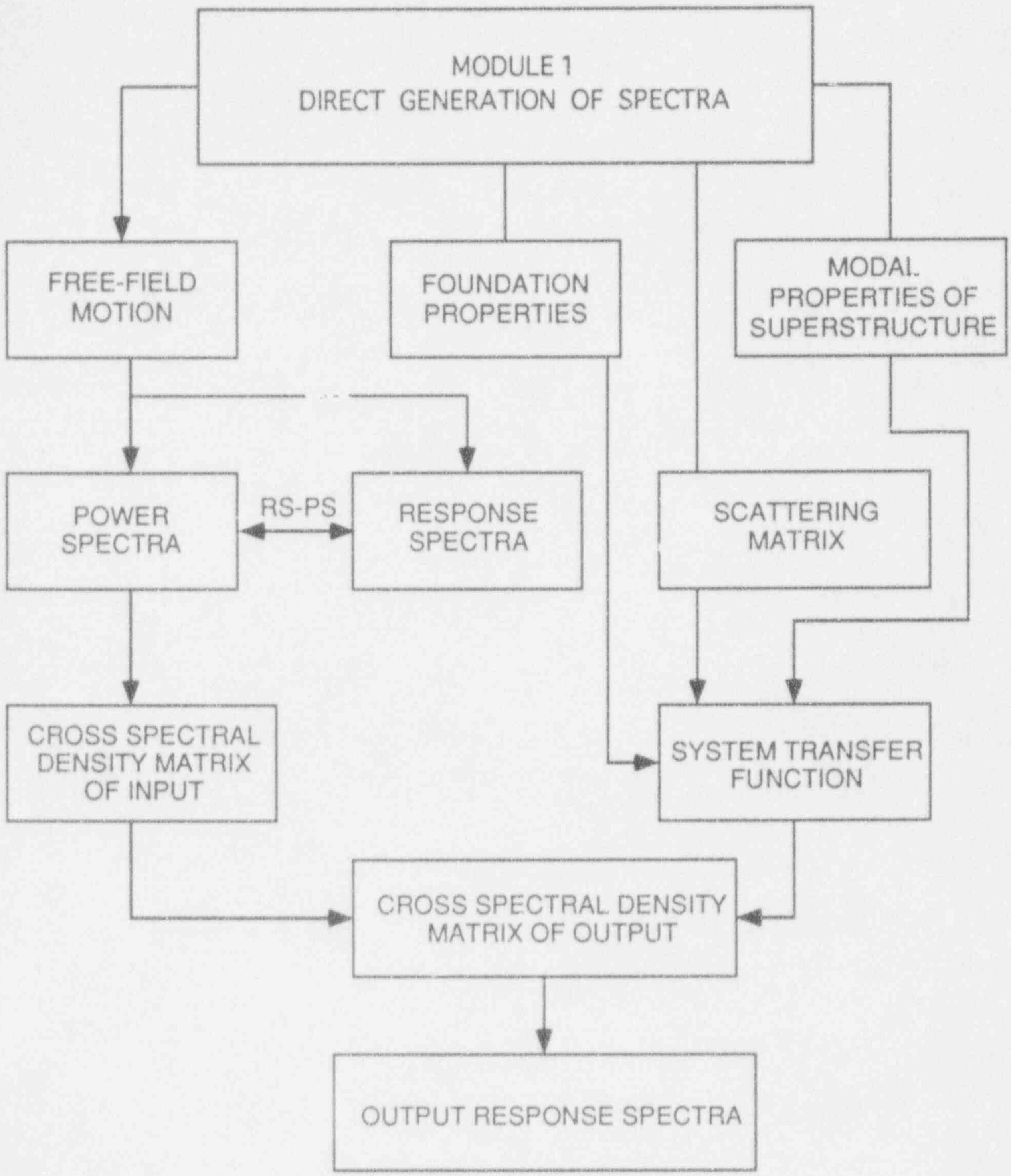

MODULE 1
DIRECT GENERATION OF SPECTRA
FREE-FIELD MOTION
FOUNDATION PROPERTIES
MODAL PROPERTIES OF SUPERSTRUCTURE
POWER SPECTRA
RS-PS
RESPONSE SPECTRA
SCATTERING MATRIX
CROSS SPECTRAL DENSITY MATRIX OF INPUT
SYSTEM TRANSFER FUNCTION
CROSS SPECTRAL DENSITY MATRIX OF OUTPUT
OUTPUT RESPONSE SPECTRA

Figure 3-2a

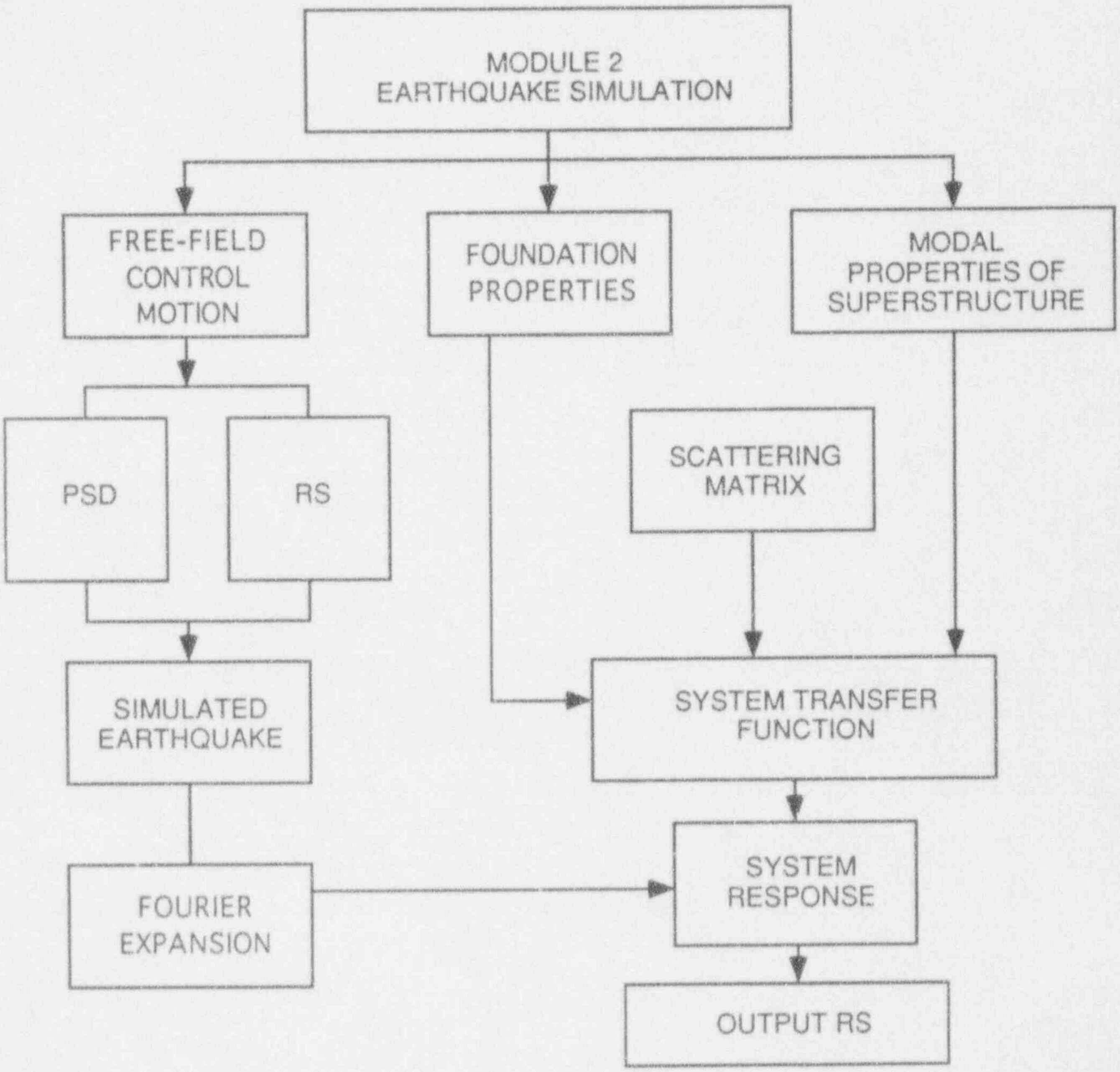

MODULE 2
EARTHQUAKE SIMULATION
FREE-FIELD
CONTROL
MOTION
FOUNDATION
PROPERTIES
MODAL
PROPERTIES OF
SUPERSTRUCTURE
PSD
RS
SCATTERING
MATRIX
SIMULATED
EARTHQUAKE
SYSTEM TRANSFER
FUNCTION
SYSTEM
RESPONSE
FOURIER
EXPANSION
OUTPUT RS

Figure 3-2b

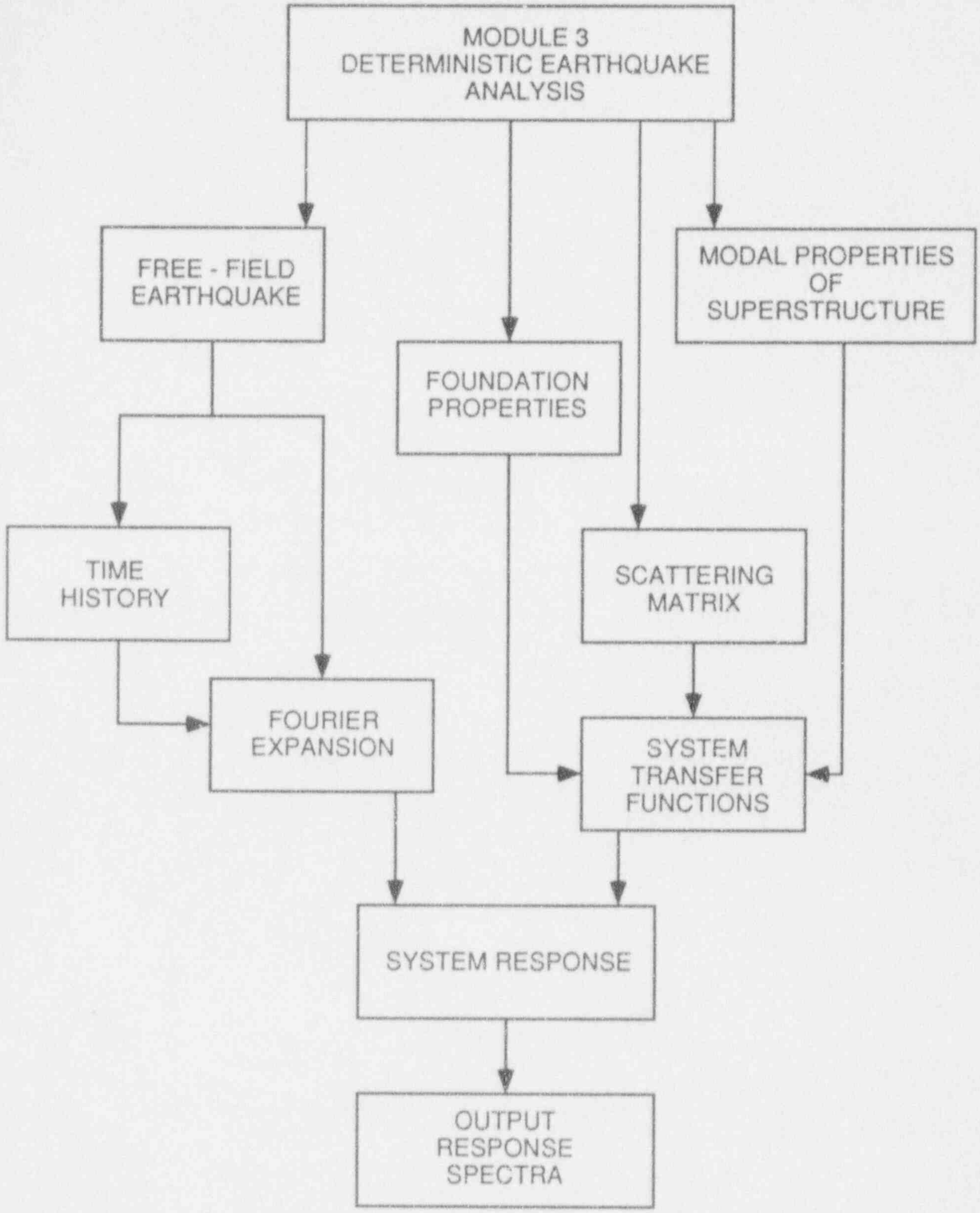

MODULE 3
DETERMINISTIC EARTHQUAKE
ANALYSIS
FREE - FIELD
EARTHQUAKE
MODAL PROPERTIES
OF
SUPERSTRUCTURE
FOUNDATION
PROPERTIES
TIME
HISTORY
SCATTERING
MATRIX
FOURIER
EXPANSION
SYSTEM
TRANSFER
FUNCTIONS
SYSTEM RESPONSE
OUTPUT
RESPONSE
SPECTRA

Figure 3-2c

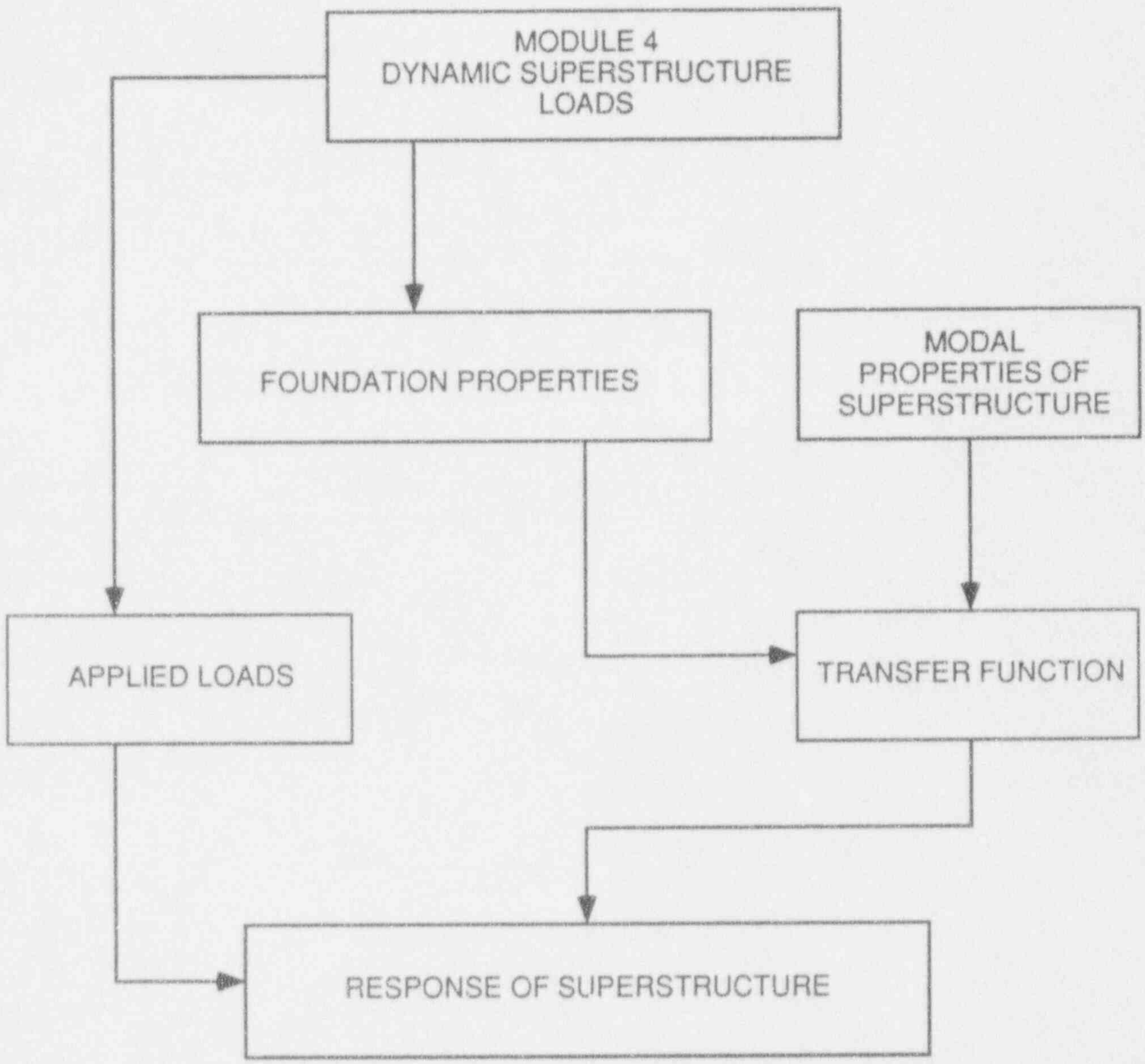

MODULE 4
DYNAMIC SUPERSTRUCTURE
LOADS
FOUNDATION PROPERTIES
MODAL
PROPERTIES OF
SUPERSTRUCTURE
APPLIED LOADS
TRANSFER FUNCTION
RESPONSE OF SUPERSTRUCTURE

Figure 3-2d

4.0 SUPERSTRUCTURE MODELING

The DIGES formulation idealizes the superstructure with a stick model consisting of flexible members with equivalent stiffness and mass matrices. The finite element representation utilizes 3-D elastic beam properties which allows for six degrees of freedom in each nodal location (Refs. 18, 19). The special feature that allows rigid links between nodal locations in the finite element idealization has been incorporated. The presence of such links will allow for a **master/slave** relationship between nodes and of course enable the evaluation of the response of a superstructure with such particularities.

While a *consistent* stiffness matrix is always used, the mass matrix that accounts for the equivalent mass can be both *consistent* or lumped (diagonal matrix).

The orientation of a single finite 3-D beam element connecting two nodes (I and J) is shown in Figure 4-1. Figure 4-2 shows the order of the degrees of freedom in element coordinates. The stiffness matrix in element coordinates is given by Eq. 4-1 and the mass matrix by Eq. 4-2 respectively:

$$
K_{loc} = \begin{bmatrix}
\frac{AE}{L} \\
0 & \frac{12EI_y}{L^3(1+\Phi_y)} \\
0 & 0 & \frac{12EI_y}{L^3(1+\Phi_x)} \\
0 & 0 & 0 & \frac{GJ}{L} \\
0 & 0 & \frac{-6EI_y}{L^3(1+\Phi_x)} & 0 & \frac{(4+\Phi_z)EI_z}{L(1+\Phi_z)} \\
0 & \frac{6EI_x}{L^3(1+\Phi_y)} & 0 & 0 & 0 & \frac{(4+\Phi_y)EI_x}{L(1+\Phi_y)} \\
\frac{-AB}{L} & 0 & 0 & 0 & 0 & 0 & \frac{AE}{L} \\
0 & \frac{-12EI_z}{L^3(1+\Phi_z)} & 0 & 0 & 0 & \frac{-6EI_x}{L^3(1+\Phi_y)} & 0 & \frac{12EI_x}{L^3(1+\Phi_y)} \\
0 & 0 & \frac{-12EI_y}{L^3(1+\Phi_x)} & 0 & \frac{6EI_y}{L^3(1+\Phi_x)} & 0 & 0 & 0 & \frac{12EI_y}{L^3(1+\Phi_x)} \\
0 & 0 & 0 & \frac{-GL}{L} & 0 & 0 & 0 & 0 & 0 & \frac{GL}{L} \\
0 & 0 & \frac{-6EI_y}{L^3(1+\Phi_x)} & 0 & \frac{(2-\Phi_x)EI_y}{L(1+\Phi_x)} & 0 & 0 & 0 & \frac{6EI_y}{L^3(1+\Phi_x)} & 0 & \frac{(4+\Phi_x)EI_y}{L(1+\Phi_x)} \\
0 & \frac{-6EI_y}{L^3(1+\Phi_z)} & 0 & 0 & 0 & \frac{(2-\Phi_y)EI_x}{L(1+\Phi_y)} & 0 & \frac{-6EI_x}{L^3(1+\Phi_y)} & 0 & 0 & 0 & \frac{(4+\Phi_y)EI_z}{L(1+\Phi_y)}
\end{bmatrix} \qquad 4\text{-}1
$$

$$
M_{loc} = \rho A L \begin{bmatrix}
\frac{1}{3} \\
0 & \frac{13}{35} + \frac{6Iz}{5AL^2} \\
0 & 0 & \frac{13}{35} + \frac{6I_y}{5AL^2} \\
0 & 0 & 0 & \frac{J_x}{3A} \\
0 & 0 & -\frac{11L}{210} - \frac{I_y}{10AL} & 0 & \frac{l^2}{105} + \frac{2I_y}{15A} \\
0 & \frac{11L}{210} + \frac{I_x}{10AL} & 0 & 0 & 0 & \frac{L^2}{105} + \frac{2I_x}{15A} \\
\frac{1}{6} & 0 & 0 & 0 & 0 & 0 & \frac{1}{3} \\
0 & \frac{9}{70} - \frac{6I_x}{5al^2} & 0 & 0 & 0 & \frac{13L}{430} - \frac{I_x}{10AL} & 0 & \frac{13}{35} + \frac{6I_x}{5AL^2} \\
0 & 0 & \frac{9}{70} - \frac{6I_z}{5AL^2} & 0 & -\frac{13L}{420} + \frac{I_y}{10AL} & 0 & 0 & 0 & \frac{13}{25} + \frac{6I_y}{5AL^2} \\
0 & 0 & 0 & \frac{J_x}{6A} & 0 & 0 & 0 & 0 & 0 & \frac{J_x}{3A} \\
0 & 0 & \frac{13L}{420} - \frac{I_y}{10AL} & 0 & -\frac{L^2}{140} - \frac{I_y}{30A} & 0 & 0 & 0 & \frac{11L}{210} + \frac{I_y}{10AL} & 0 & \frac{L^2}{105} + \frac{2I_y}{15A} \\
0 & -\frac{13L}{420} + \frac{I_x}{10AL} & 0 & 0 & 0 & -\frac{L^2}{140} - \frac{I_x}{10AL} & 0 & 0 & 0 & \frac{L^2}{105} + \frac{2I_x}{15A}
\end{bmatrix} \qquad 4\text{-}2
$$

where:

$$\phi_y = \frac{12EI_x}{GA_{s_z}L^2} \quad ; \quad \phi_z = \frac{12EI_y}{GA_{s_y}L^2}$$

E	=	Young's modulus
I_i	=	moment of inertia normal to direction i
G	=	Shear modulus = $\frac{E}{2(1 + \nu)}$
A	=	Cross sectional area
A_{si}	=	shear area normal to direction i
L	=	length of element connecting nodes I and J.
ν	=	Poisson's ratio
J	=	tortional moment of inertia ($= J_2$ if $I_x = 0$, $= I_x$ otherwise)
J_x	=	polar moment of inertia = $I_y + I_z$
ρ	=	density

The global solution, however, must be expressed in the global rather than local coordinates since the superstructure degrees of freedom are expressed in these coordinates. This, in order to form the final system matrices in the global coordinates, specified as X, Y, and Z on Figure 4-1, a transformation matrix T_r is utilized such that,

$$K_{gl} = T_r^T K_{loc} T_r \qquad 4\text{-}3$$

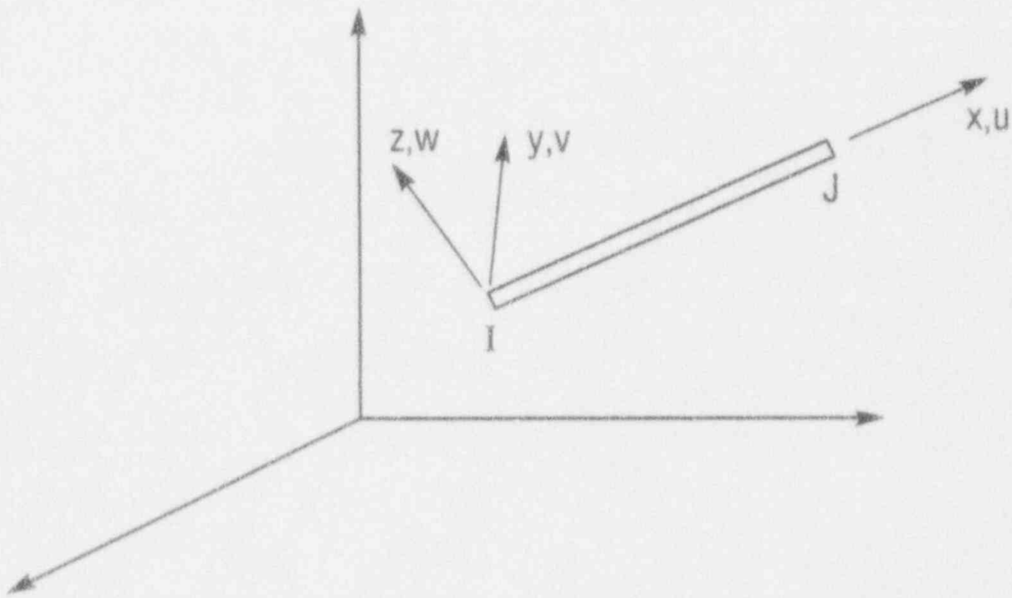
and similarly,

$$M_{gl} = T_r^T M_{loc} T_r \qquad 4\text{-}4$$

T_r relates the vector of displacements in the element Cartesian coordinates to the Global Cartesian coordinates through the relation

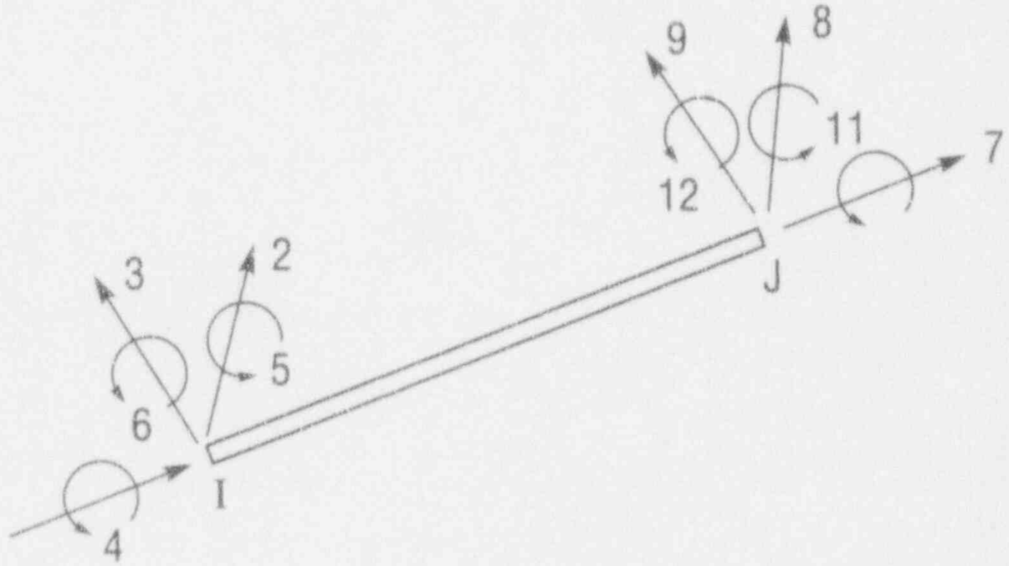
$$u_l = T_r u \qquad 4\text{-}5$$

The formation of the global stiffness and mass matrices **K** and **M** will involve the assemblage of the individual stiffness and mass matrices given by Eqs. 4-3 and 4 respectively.



Beam Element

Figure 4-1



Beam Element: Degrees-Of-Freedom

Figure 4-2

5.0 IMPEDANCES AND FOUNDATION INPUT MOTIONS

5.1 Foundation Impedances

The relationship of harmonic generalized external forces and moments exciting a rigid foundation and the response of such foundation is expressed in terms of the 6 × 6 frequency dependent impedance matrix $\mathbf{K}_s$. In addition, this matrix depends on the geometry of the foundation as well as the properties of the underlying soil medium. The complexity that accompanies the exact description of the interaction between the foundation and the soil has limited the number of generic analytic solutions (e.g., Refs. 11, 13). To circumvent this difficulty studies of parametric nature have been conducted and *approximate* analytical solutions have been deduced for simple geometry foundations such as circular and rectangular (Ref. 12). The problem, even for the simple geometries, gets further complicated for foundations that are embedded into the soil.

Several sets of approximate impedance formulae have been implemented into the DIGES computational process. Each of the elements of $\mathbf{K}_s$ reflect both the stiffness and the damping contribution according to the relation $K = k + ia_0c$ are where k and c normalized stiffness and damping coefficients and a_0 is a dimensionless frequency. (See DIGES User's Manual.) In addition, DIGES provides the option of user-supplied impedance data.

5.2 Foundation Input Motion

According to Equation 2.2.2.4-5, the foundation input motion $\mathbf{U}_G$ is related to the free-field motion U_G^0 through $\mathbf{H}_s(\omega)$. DIGES distinguishes three general cases relating the free-field motion with the foundation input motion:

Case 1: Free-field directly applied as input motion

According to this case, schematically shown in Figure 5.2-1, the foundation input motion is equal to the free-field motion $\left(i.e.,\ U_G = U_G^0\right)$.. This case represents early stages of seismic analyses of building-foundation systems according to which the criteria motion was directly applied at the bottom of the soil springs. This reflects primarily cases involving surface foundations. Since the free-field is applied directly as the excitation of the building foundation system the 6 × 3 matrix $\mathbf{H}_5(\omega)$ takes the form

$$H_5(\omega) = \begin{bmatrix} I \\ 0 \end{bmatrix} \qquad 5.2\text{-}1$$

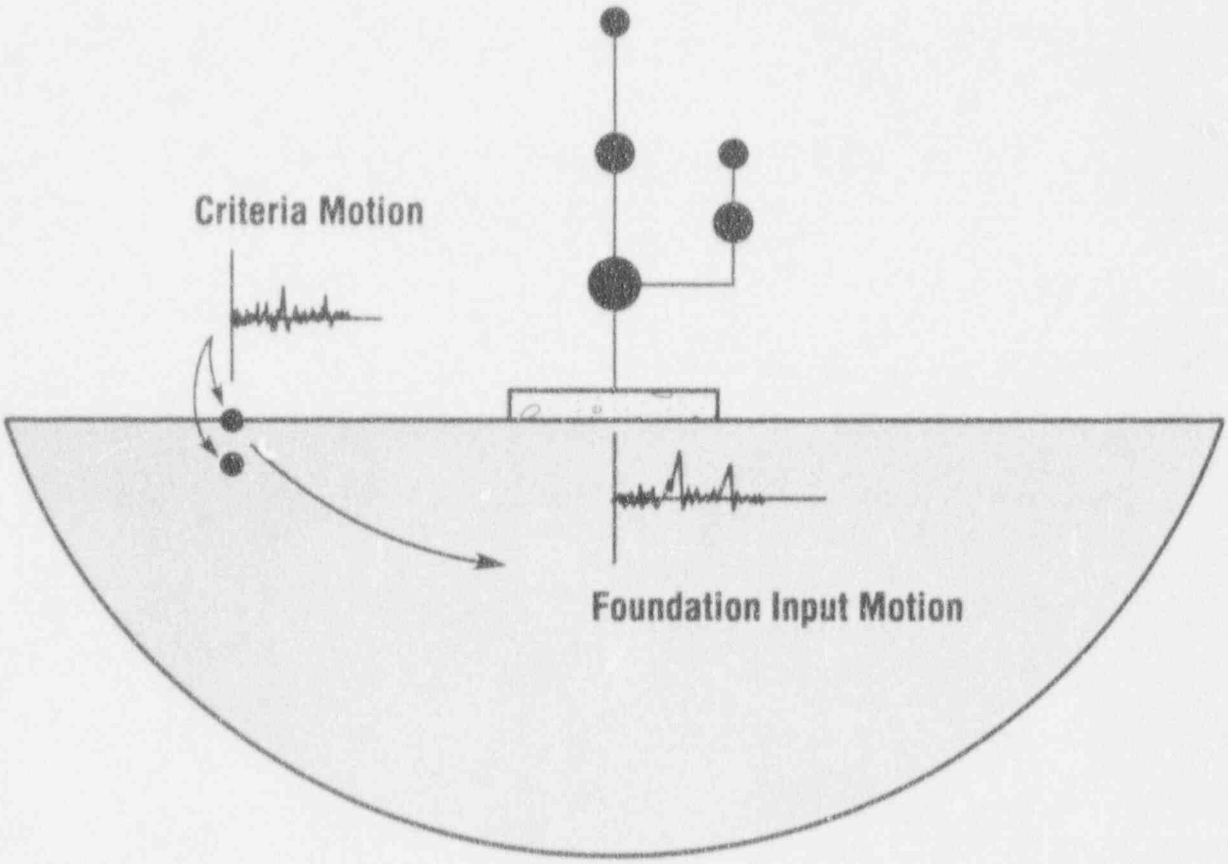
where **I** and **0** are 3 × 3 unit and null matrices respectively.

Case 2: Convolution/Deconvolution

In this case the foundation input motion is the free-field motion at some depth, depending on the embeddment depth of the foundation (Figure 5.2-2). The free-field motion at a given depth is obtained through convolution or deconvolution depending on whether the criteria motion is treated as an outcrop motion or as a surface (or near surface for very soft top layers) motion, respectively. In both cases, the transfer matrix $\mathbf{H}_5(\omega)$ has the following form:

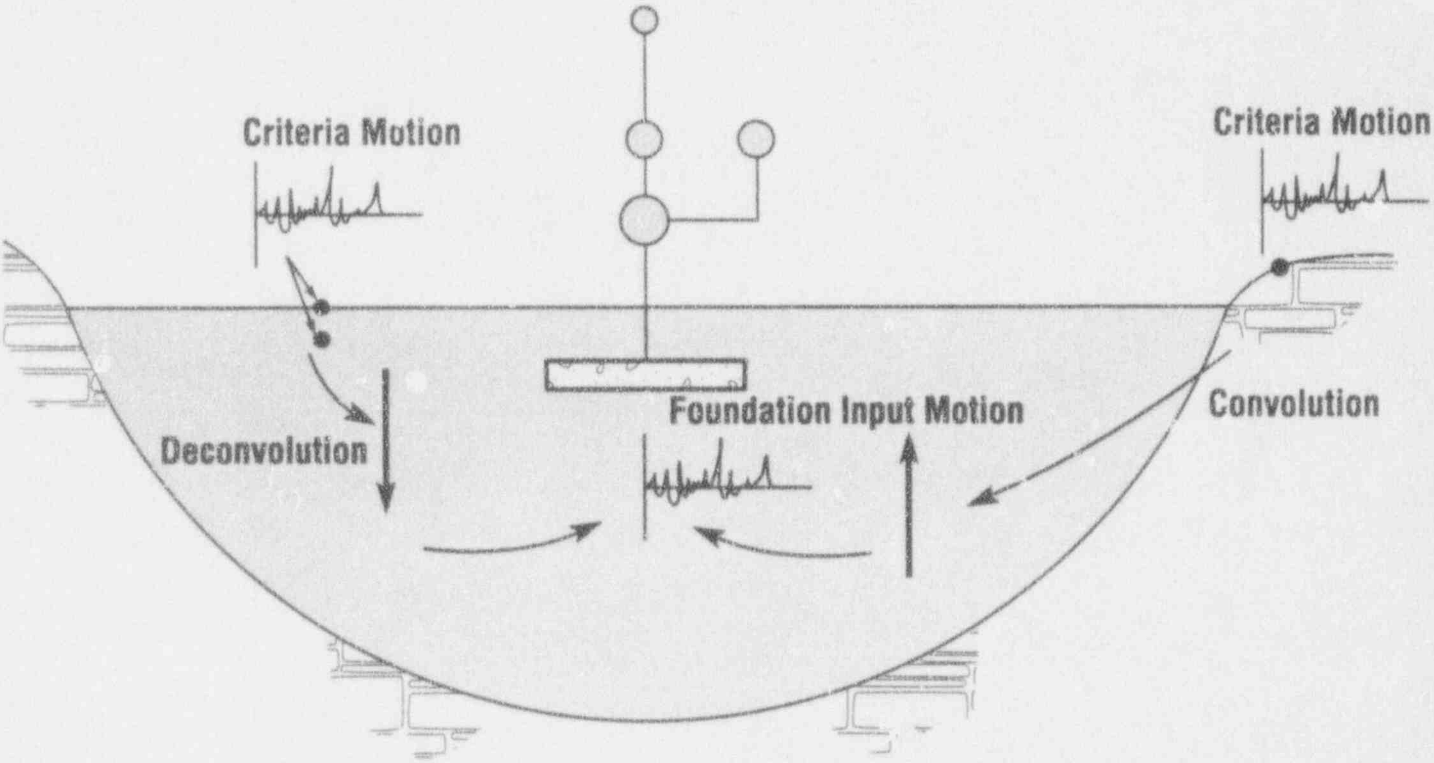
$$H_5(\omega) = \begin{bmatrix} H(\omega) \\ 0 \end{bmatrix} \qquad 5.2\text{-}2$$

where **0** is a 3 × 3 null matrix and the 3 × 3 frequency dependent submatrix $\mathbf{H}(\omega)$ contains the transfer functions for convolution/deconvolution. When one dimensional propagation of shear and dilatational waves is assumed, the $\mathbf{H}(\omega)$ is a diagonal matrix. Otherwise $H(\omega)$ has off-diagonal turns representing coupling between horizontal and vertical components of motion, e.g., cases involving inclined waves. (See Section 6.3.3.1.) DIGES has the option that allows the user to input convolution/deconvolution data from external sources (e.g, CARES, SHAKE). A set of models is also available which are presented in Section 6.3 of this report.



Case 1: Free-Field Directly Applied To Building-Foundation System

Figure 5.2-1



Case 2: Convolution/Deconvolution

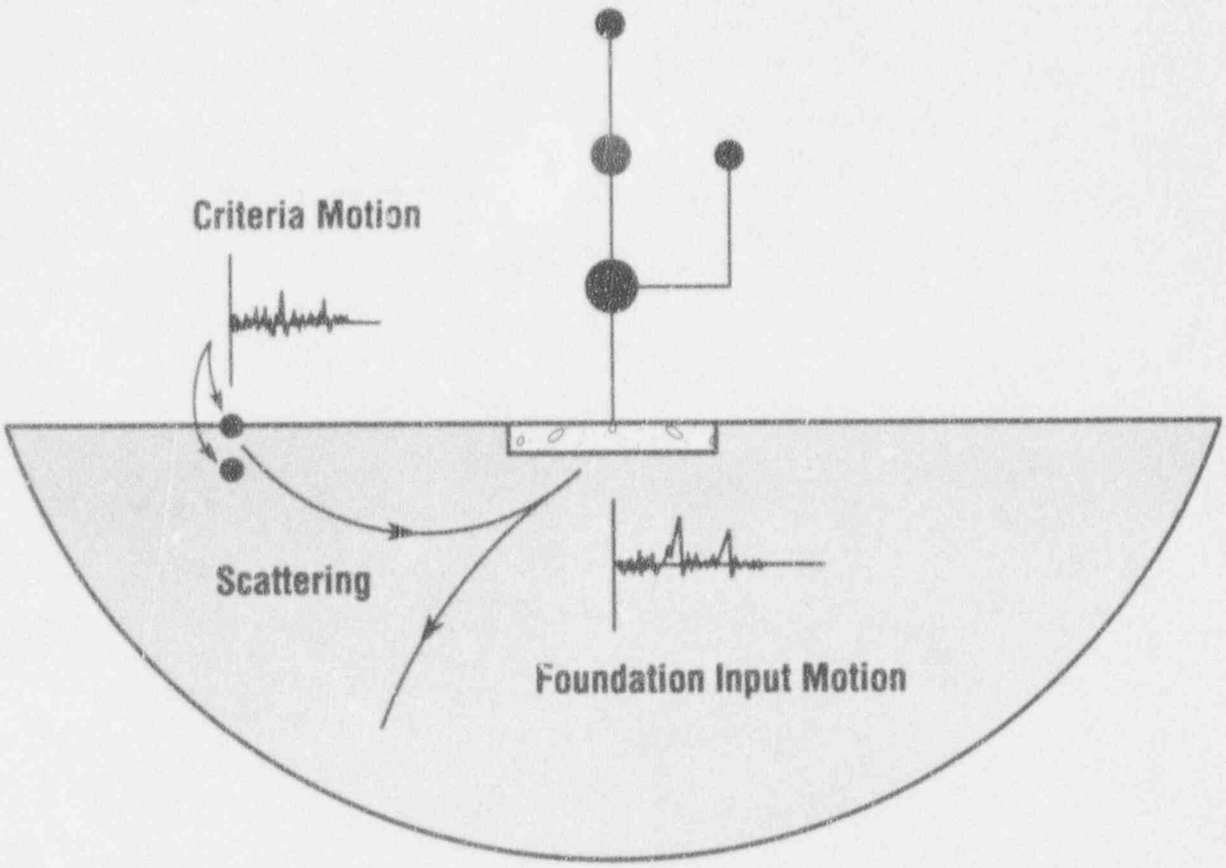
Figur 2-2

Case 3: Kinematic Interaction

In the case of foundation input motion incorporating kinematic interaction effects due to the scattering of the seismic waves by the rigid foundation (Figure 5.2-3) $\mathbf{H}_5(\omega)$ is a 6 × 3 frequency dependent matrix containing the scattering coefficients which depend on the types of seismic waves considered, the properties of the underlying medium and the geometry of the foundation itself. When kinematic interaction is considered, the lower 3 × 3 part of the $\mathbf{H}_5(\omega)$ matrix is no longer zero (as opposed to Cases 1 and 2). Specifically, the lower 3 × 3 portion of the $\mathbf{H}_5(\omega)$ contains scattering coefficients relating the rocking and torsional motion of the foundation due to the horizontal and vertical components of the free-field motion U_G^0. Consequently, the transfer matrix $\mathbf{H}_5(\omega)$ has the form:

$$H_5(\omega) = \begin{bmatrix} H_U(\omega) \\ H_L(\omega) \end{bmatrix} \qquad 5.2\text{-}3$$

where $\mathbf{H}_U(\omega)$, $\mathbf{H}_L(\omega)$ are generally full 3×3 frequency-dependent submatrices. DIGES has the option that allows the user to input the relevant coefficients of $\mathbf{H}_5(\omega)$ from available sources (e.g., Refs. 11, 14). It is recommended that a database be implemented into DIGES so it can be used with minimum input data.



Case 3: Kinematic Interaction

Figure 5.2-3

6.0 SOLUTIONS FOR SIMPLE MODELS

The discritization of typical building-foundation models results in a large number of equations thus making it difficult to carry out hand solutions. For simple models, however, such task is manageable and it is pursued in this section of the report. By simple models we mean building-foundation models in which the superstructure portion of the model is defined by a single fixed-base natural frequency. The goal is to verify the solutions given in Section 2.0 of the report by comparing them to analytic solutions obtained by direct equilibrium. For convenience we distinguish two cases of loading i.e., the case of a ground excitation and the case in which dynamic loads are imposed on the superstructure. These cases are analyzed in Sections 6.1 and 6.2.

Finally, a set of simple models are also presented which deal with wave motions in uniform soil deposits or soil deposits overlying a rock medium.

6.1 Solutions for Dynamic Loads Imposed on Superstructure

The model considered is shown in Figure 6.1-1. The superstructure is represented by a mass m which is lumped at height h above the foundation. The mass of the foundation is m_b and the mass moment of inertia of the foundation is I_b. The kinematic parameters of the building-foundation system are:

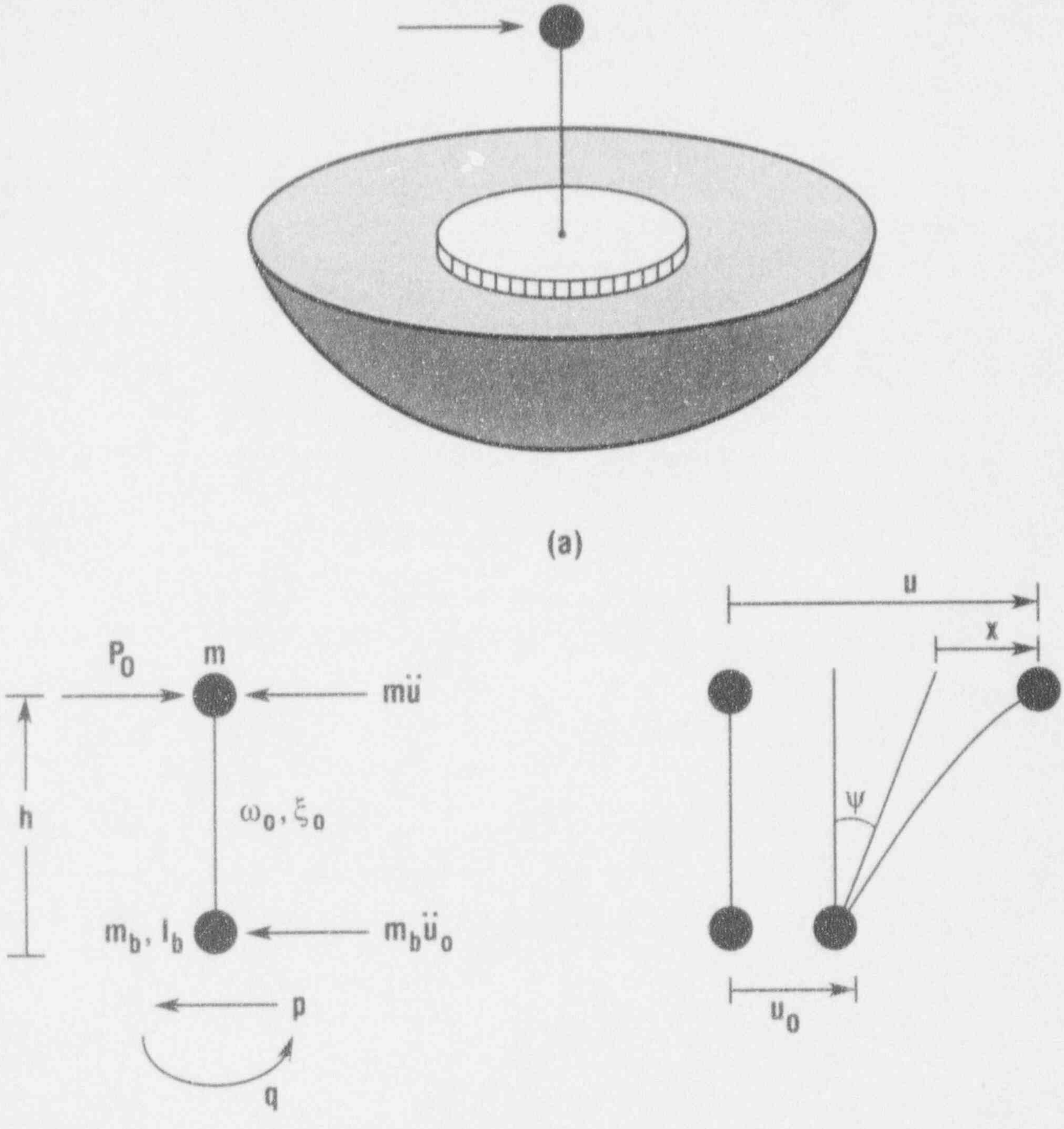
x : structural displacement relative to foundation

x_b : foundation translation relative to the soil

ψ : rigid body rotation

u : total structural displacement

u_o : total foundation translation



Simple Model (Case: Dynamic Load On Superstructure)
(a) Overall Configuration
(b) Forces and Kinematics

Figure 6.1-1

Note that:

$$u = u_o + h\psi + x \qquad 6.1\text{-}1a$$

$$u_o \equiv x_b \qquad 6.1\text{-}1b$$

The dynamic force $P_0(t)$ is imposed on the structural mass m. The foundation forces due to the interaction with the soil are p(t) and q(t). For simplicity the coupling between translation and rocking is assumed to be zero. Accordingly, the force-displacement relation of the foundation can be written in the time domain as

$$\begin{Bmatrix} p \\ q \end{Bmatrix} = \begin{bmatrix} k_x & 0 \\ 0 & k_\psi \end{bmatrix} \begin{Bmatrix} u_o \\ \psi \end{Bmatrix} + \begin{bmatrix} c_x & 0 \\ 0 & c_\psi \end{bmatrix} \begin{Bmatrix} \dot{u}_o \\ \dot{\psi} \end{Bmatrix} \qquad 6.1\text{-}2a$$

or in frequency domain as

$$\begin{Bmatrix} P \\ Q \end{Bmatrix} = \begin{bmatrix} K_x(\omega) + i\omega C_x(\omega) & 0 \\ 0 & K_\psi(\omega) + i\omega C_\psi(\omega) \end{bmatrix} \begin{Bmatrix} U \\ \Psi \end{Bmatrix} \qquad 6.1\text{-}2b$$

where the 2 × 2 matrix in Eq. 6.1-2b is the impedance matrix of the foundation $\mathbf{K}_s(\omega)$. Equilibrium of structural mass m yields:

$$\ddot{u}(t) + 2\xi_o\omega_o\dot{x}(t) + \omega_o^2 x(t) = \frac{P_o(t)}{m} \qquad 6.1\text{-}3$$

where ω_o, ξ_o represent the structural frequency and damping respectively. Consideration of the overall equilibrium of the building-foundation system produces the following two equations in terms of forces and moments respectively:

$$m\ddot{u}(t) + m_b\ddot{u}_o(t) + p(t) = P_o(t) \qquad 6.1\text{-}4$$

$$mh\ddot{u}(t) + I_b\ddot{\psi}(t) + q(t) = hP_o(t) \qquad 6.1\text{-}5$$

Equations 6.1-3, 4 and 5 are the equations of motion of the model shown in Figure 6.1-1. The solution will be formulated in terms of the total foundation motion (i.e., translation u_o and rotation ψ) and the relative structural motion (i.e., x). Accordingly, the relative structural displacement can be obtained from Eq. 6.1-3 in the frequency domain as:

$$X(\omega) = \mathcal{H}_o(\omega)\left[U_o(\omega) + h\Psi(\omega) + \frac{1}{m\omega^2}P_o(\omega)\right]$$
$$\mathcal{H}_o(\omega) = \frac{\Omega^2}{1 + i2\xi_o\Omega - \Omega^2} \quad ; \quad \Omega = \frac{\omega}{\omega_o} \qquad 6.1\text{-}6$$

With the aid of Eqs. 6.1-2b and 6.1-6 we can put Eq. 6.1-4 into the following form:

$$\left[-\omega^2(m + m_b) - \omega^2 m\mathcal{H}_o(\omega) + K_x(\omega) + i\omega C_x(\omega)\right]U_o(\omega)$$
$$- \omega^2 mh\left[1 + \mathcal{H}_o(\omega)\right]\Psi(\omega) = \left[1 + \mathcal{H}_o(\omega)\right]P_o(\omega) \qquad 6.1\text{-}7$$

Similarly, using Eqs. 6.1-2b and 6.1-6 we can put Eq. 6.1-5 into the following form:

$$-\omega^2 mh\left[1 + \mathcal{H}_o(\omega)\right]U_o(\omega) +$$
$$+ \left[-\omega^2 I_b - \omega^2 mh^2\left[1 + \mathcal{H}_o(\omega)\right] + K_\psi(\omega) + i\omega C_\psi(\omega)\right]\Psi(\omega) = \qquad 6.1\text{-}8$$
$$= h\left[1 + \mathcal{H}_o(\omega)\right]P_o(\omega)$$

Equations 6.1-7 and 8 are then solved to obtain the transfer function associated with the translation and rocking of the foundation as follows:

$$\boldsymbol{U}_s = \boldsymbol{H}_4(\omega)\boldsymbol{P} \qquad 6.1\text{-}9a$$

where:

$$\boldsymbol{U}_s = \begin{Bmatrix} U_o(\omega) \\ \Psi(\omega) \end{Bmatrix} ; \; \boldsymbol{P} = \left[1 + \mathcal{H}_o(\omega)\right]\begin{Bmatrix} 1 \\ h \end{Bmatrix} P_o(\omega) \qquad 6.1\text{-}9b$$

and the 2 × 2 complex matrix $H_4(\omega)$ is given by:

$$H_4(\omega) = \begin{bmatrix} -\omega^2(m + m_b) - \omega^2\mathcal{H}_o(\omega) + K_x(\omega) + i\omega C_x(\omega) & -\omega^2 mh\left[1 + \mathcal{H}_o(\omega)\right] \\ -\omega^2 mh\left[1 + \mathcal{H}_o(\omega)\right] & -\omega^2 I_b - \omega^2 mh^2\left[1 + \mathcal{H}_o(\omega)\right] + K_\psi(\omega) + i\omega C_\psi(\omega) \end{bmatrix}^{-1}$$

6.1-9c

We shall show next that Eq. 6.1-9 is identical to Eq. 2.2.2.3-6:

For the model shown in Figure 6.1-1, **A** and **Γ** of Eq. 2.2.2.3-6 become:

$$A = \{1 \quad h\} \quad ; \quad \mathbf{\Gamma} = \sqrt{m} \begin{Bmatrix} 1 \\ h \end{Bmatrix} \qquad 6.1\text{-}10$$

since modes are normalized to the mass matrix (i.e., $m\phi^2 = 1$).

In view of eq. 6.1-10, the matrix products involved in Eq. 2.2.2.3-6 become:

$$\mathbf{A}^T M A = m \begin{bmatrix} 1 & h \\ h & h^2 \end{bmatrix} \qquad 6.1\text{-}11a$$

$$\Gamma H_o \Gamma^T = m \mathcal{H}_o(\omega) \begin{bmatrix} 1 & h \\ h & h^2 \end{bmatrix} \qquad 6.1\text{-}11b$$

$$\Gamma H_o \Phi^T = \mathcal{H}_o(\omega) \begin{Bmatrix} 1 \\ h \end{Bmatrix} \qquad 6.1\text{-}11c$$

Substitution of Eqs. 6.1.-11 as well as Eq. 6.1-2b into Eq. 2.2.2.3-6 yields exactly Eq. 6.1-9.

Next, analytic expressions are defined for the total foundation motion and for the superstructure motion of the model shown in Figure 6.1-1. By carrying out the inversion in Eq. 6.1-9c we obtain:

$$\begin{Bmatrix} U_o(\omega) \\ \Psi(\omega) \end{Bmatrix} = \begin{Bmatrix} -\omega^2 I_b + k_\psi(\omega) + i\omega c_\psi(\omega) \\ -\omega^2 h m_b + h\left[k_x(\omega) + i\omega c_x(\omega)\right] \end{Bmatrix} \frac{1 + \mathcal{H}_o(\omega)}{\Pi(\omega)} P_o(\omega) \qquad 6.1\text{-}12a$$

$$\Pi(\omega) = \left[-\omega^2(m + m_b) - \omega^2 m\mathcal{H}_o(\omega) + k_x(\omega) + i\omega c_x(\omega)\right] \cdot$$

$$\cdot \left\{-\omega^2 I_b - \omega^2 mh^2\left[+ \mathcal{H}_o(\omega)\right] + k_\psi(\omega) + i\omega c_\psi(\omega)\right\} - \qquad 6.1\text{-}12b$$

$$- \left\{\omega^2 mh\left[1 + \mathcal{H}_o(\omega)\right]\right\}^2$$

The relative structural motion is obtained by the relation:

$$x(t) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} H_1(\omega)F_p(\omega)e^{i\omega t}d\omega \qquad 6.1\text{-}13a$$

where $F_p(\omega)$ is the Fourier transform of the applied force at the superstructure and $H_1(\omega)$ is the transfer function between the applied force and the flexural motion. By substitution of Eq. 6.1-12 into Eq. 6.1-6 we obtain:

$$H_1(\omega) = \frac{\mathcal{H}_o(\omega)}{m\omega^2} + \frac{\left[1 + \mathcal{H}_o(\omega)\right]\mathcal{H}_o(\omega)}{\Pi(\omega)}\left[-\omega^2 I_o + i\omega c_o(\omega) + k_o(\omega)\right] \qquad 6.1\text{-}13b$$

$$I_o = m_b h^2 + I_b \qquad 6.1\text{-}13c$$

$$c_o(\omega) = h^2 c_x(\omega) + c_\psi(\omega) \qquad 6.1\text{-}13d$$

$$k_o(\omega) = h^2 k_x(\omega) + k_\psi(\omega) \qquad 6.1\text{-}13e$$

Note that the first term of Eq. 6.1-13b represents the corresponding transfer function for the fixed-base case.

Assuming that both the applied force and the relative structural motion are weakly stationary, then the spectral density of the relative structural motion Φ_{xx} is given by:

$$\Phi_{xx}(\omega) = |H_1(\omega)|^2\Phi_{pp}(\omega) \qquad 6.1\text{-}14a$$

where $\Phi_{pp}(\omega)$ is the power spectral density function of the applied force and $H_1(\omega)$ is the transfer function given by Eq. 6.1-13b.

The mean-square relative structural motion $E[x^2(t)]$ can be obtained by:

$$E[x^2(t)] = \int_{-\infty}^{+\infty} \Phi_{xx}(\omega) d\omega \qquad 6.1\text{-}14b$$

where $\Phi_{xx}(\omega)$ is given by Eq. 6.1-14a.

Finally, using Eqs. 6.1-12, 13 and Eq. 6.1-1a the total structural displacement can be computed deterministically by

$$u(t) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} H_2(\omega) F_p(\omega) e^{i\omega t} d\omega \qquad 6.1\text{-}15a$$

where $F_p(\omega)$ is the Fourier transform of the force imposed on the superstructure and $H_2(\omega)$ is the relevant transfer function given by:

$$H_2(\omega) = \frac{\mathcal{H}_o(\omega)}{m\omega^2} + \frac{1 + \mathcal{H}_o(\omega)}{\Pi(\omega)} \left[-\omega^2 I_o + i\omega c_o(\omega) + k_o(\omega)\right] \qquad 6.1\text{-}15b$$

When both the applied force and the response processes are weakly stationary then the power spectral density function of the total structural displacement $\Phi_{uu}(\omega)$ is given by

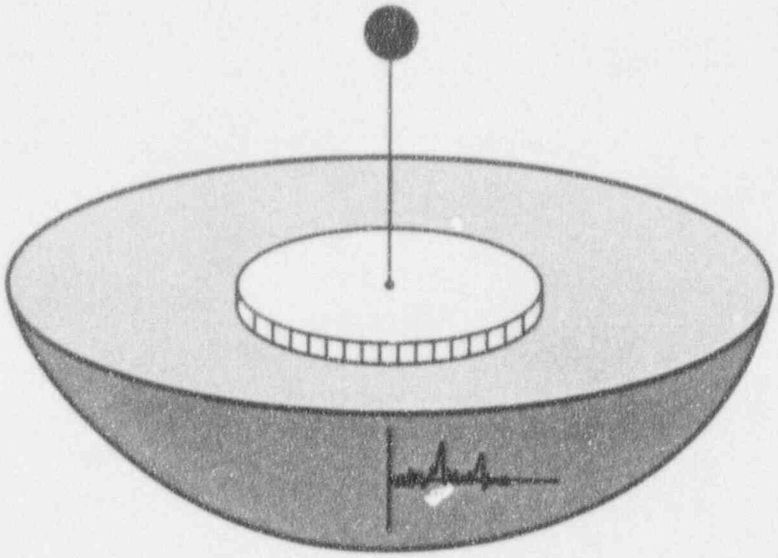
$$\Phi_{uu}(\omega) = |H_2(\omega)|^2 \Phi_{pp}(\omega) \qquad 6.1\text{-}15c$$

where $\Phi_{pp}(\omega)$ is the spectral density of the applied force.

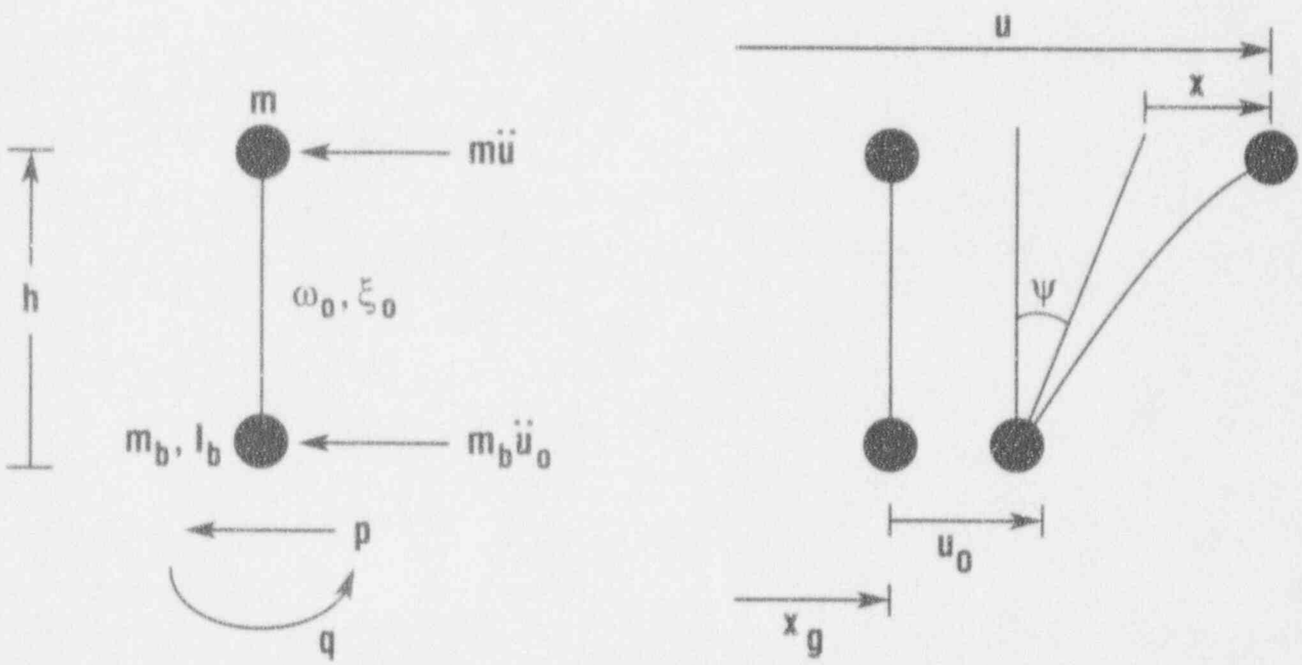
Equations 6.1-13b and 6.1-15b give analytic expressions for the transfer functions associated with the relative and total structural motion of the model shown in Figure 6.1-1.

6.2 Solutions for Ground Excitations

The model considered in this Section is shown in Figure 6.2-1. Essentially, it is similar to that of Figure 6.1-1 except that instead of having a dynamic load applied at the structure we now consider that the model is subjected to a ground excitation $\ddot{x}_g(t)$. The kinematic parameters of the model are the same with those described in Section 6.1 with the exception that the total foundation translation $u_o(t)$ is now equal to the foundation translation relative to the soil $x_h(t)$ plus the ground displacement $x_g(t)$. Accordingly, Eq. 6.1-1b becomes:



(a)



(b)

Simple Model (Case: Ground Excitation)
(a) Overall Configuration
(b) Forces and Kinematics

Figure 6.2.1

$$u_o = x_b + x_g \qquad 6.2\text{-}1$$

while Eq. 6.1-1a is still applicable in this case and therefore the total structural motion is given by:

$$u = u_o + h\psi + x \qquad 6.2\text{-}2$$

The parameters involved in Eqs. 6.2-1 and 2 are shown in Figure 6.2-1.

In view of Eq. 6.2-1, the force-displacement relationship of the foundation (See Eq. 6.1-2) can be written in terms of the total foundation motion as:

$$\begin{Bmatrix} P \\ Q \end{Bmatrix} = \begin{bmatrix} k_x(\omega) + i\omega c_x(\omega) & 0 \\ 0 & k_\psi(\omega) + i\omega c_\psi(\omega) \end{bmatrix} \begin{Bmatrix} U_o - X_g \\ \Psi \end{Bmatrix} \qquad 6.2\text{-}3$$

The equilibrium of the structural mass m is expressed by

$$\ddot{u}(t) + 2\xi_o\omega_o\dot{x}(t) + \omega_o^2(t) = -\ddot{x}_g(t) \qquad 6.2\text{-}4$$

From the global equilibrium of the model in translation and rocking we can write:

$$m\ddot{u}(t) + m_b\ddot{u}_b(t) + p(t) = 0 \qquad 6.2\text{-}5$$

$$mh\ddot{u}(t) + I_b\ddot{\Psi}(t) + q(t) = 0 \qquad 6.2\text{-}6$$

respectively.

Using Eq. 6.2-2, the solution of Eq. 6.2-4 can be written in the frequency-domain as:

$$X(\omega) = \mathcal{H}_o(\omega)\left[U_o(\omega) + h\Psi(\omega)\right] \qquad 6.2\text{-}7a$$

$$\mathcal{H}_o(\omega) = \frac{\Omega^2}{1 + i2\xi_o\Omega - \Omega^2} \; ; \; \Omega = \frac{\omega}{\omega_o} \qquad 6.2\text{-}7b$$

where ω_o, ξ_o are the fixed-base structural frequency and damping respectively.

Substituting Eqs. 6.2-2 and 6.2-7 into Eqs. 6.2-5 and 6 we obtain:

$$-\omega^2\left[m\left[1 + \mathcal{H}_o(\omega)\right] + m_b\right]U_o(\omega) - \omega^2 mh\left[1 + \mathcal{H}_o(\omega)\right]\Psi(\omega) + P(\omega) = 0 \qquad 6.2\text{-}8$$

$$-\omega^2 mh\left[1 + \mathcal{H}_o(\omega)\right]U_o(\omega) - \omega^2\left[I_b + mh^2\left[1 + \mathcal{H}_o(\omega)\right]\right]\Psi(\omega) + Q(\omega) = 0 \qquad 6.2\text{-}9$$

Substitution of Eqs. 6.2-3 into Eqs. 6.2-8 and 9 yields the transfer functions relating the total foundation motion and the free-field.

$$\boldsymbol{U}_o = \boldsymbol{H}_4(\omega)\boldsymbol{U}_G \qquad 6.2\text{-}10a$$

where

$$\boldsymbol{U}_o = \begin{Bmatrix} U_o(\omega) \\ \Psi(\omega) \end{Bmatrix} \; ; \; \boldsymbol{U}_G = \begin{Bmatrix} X_g(\omega) \\ 0 \end{Bmatrix} \qquad 6.2\text{-}10b$$

and the 2 × 2 complex matrix $H_4(\omega)$ is given by:

$$H_4(\omega) = \begin{bmatrix} -\omega^2(m + m_b) - \omega^2 m\mathcal{H}_o(\omega) + k_x(\omega) + i\omega c_x(\omega) & -\omega^2 mh\left[1 + \mathcal{H}_o(\omega)\right] \\ -\omega^2 mh\left[1 + \mathcal{H}_o(\omega)\right] & -\omega^2 I_b - \omega^2 mh^2\left[1 + \mathcal{H}_o(\omega)\right] + k_\psi(\omega) + i\omega c_\psi(\omega) \end{bmatrix}^{-1}$$

$$\cdot \begin{bmatrix} k_x(\omega) + i\omega c_x(\omega) & 0 \\ 0 & k_\psi(\omega) + i\omega c_\psi(\omega) \end{bmatrix}$$

6.2-10c

Using similar procedures as in Section 6.1 it can be shown that Eq. 6.2-10 is identical to Eq. 2.2.2.4-1 for the model considered. Specifically, it can be verified that substitution of Eqs. 6.1-11 into Eq. 2.2.2.4-1 yields Eq. 6.2-10.

Next we shall derive the transfer functions H_1 and H_2 of the model which are associated with the relative and total displacement of the structural mass m respectively.

By carrying out the inversion in Eq. 6.2-10c, the total foundation motion can be written as:

$$\begin{Bmatrix} U_o(\omega) \\ \Psi(\omega) \end{Bmatrix} = \begin{Bmatrix} -\omega^2 I_b - \omega^2 mh^2[1 + \mathcal{H}_o(\omega)] + k_\psi(\omega) + i\omega c_\psi(\omega) \\ \omega^2 mh[1 + \mathcal{H}_o(\omega)] \end{Bmatrix} \frac{k_x(\omega) + i\omega c_x(\omega)}{\Pi(\omega)} X_g(\omega)$$

6.2-11

where $\Pi(\omega)$ is given by Eq. 6.1-12b.

The transfer function $H_1(\omega)$ for the relative displacement of the structural mass m can be obtained by substitution of Eq. 6.2-11 into Eq. 6.2-7. This leads to the following relation:

$$H_1(\omega) = \frac{\mathcal{H}_o(\omega)}{\Pi(\omega)} \left[-\omega^2 I_b + k_\psi(\omega) + i\omega c_\psi(\omega) \right] \left[k_x(\omega) + i\omega c_x(\omega) \right] \qquad 6.2\text{-}12$$

Similarly, the transfer function $H_2(\omega)$ for the total structural displacement can be obtained by substitution of Eqs. 6.2-11 and 12 into Eq. 6.2-2. This operation yields:

$$H_2(\omega) = \frac{1 + \mathcal{H}_o(\omega)}{\Pi(\omega)} \left[-\omega^2 I_b + k_\psi(\omega) + i\omega c_\psi(\omega) \right] \left[k_x(\omega) + i\omega c_x(\omega) \right] \qquad 6.2\text{-}13$$

For probabilistic analysis, the power spectral density of the relative and total motion of the structural mass m are given by:

$$\Phi_{xx}(\omega) = |H_1(\omega)|^2 \Phi_{x_g x_g}(\omega) \qquad 6.2\text{-}15a$$

and

$$\Phi_{uu}(\omega) = |H_2(\omega)|^2 \Phi_{x_g x_g}(\omega) \qquad 6.2\text{-}15b$$

respectively. These equations are valid for weakly stationary ground motion and responses. In Eq. 6.2-15 $\Phi_{x_g x_g}(\omega)$ is the power spectral density function of the ground excitation. The transmittance or system functions $|H_1(\omega)|^2$ and $|H_2(\omega)|^2$ can be obtained from Eqs. 6.2-12 and 6.2-13 respectively.

6.3 Solutions for Convolution/Deconvolution

In Section 5, we distinguished three cases with respect to the transfer functions relating the foundations input motion $\mathbf{U}_G$ to the "free-field motion U_G^0. Here we shall present some solutions related to the second case i.e., convolution /deconvolution. Specifically, we present analytic expressions for the transfer matrix $\mathbf{H}_5(\omega)$ in Eq. 2.2.2.4-5 for cases involving uniform and layered soil deposits. For the latter case, a two-layered configuration representing a soil deposit overlying a uniform rock formation is analyzed.

6.3.1 Uniform Deep Soil Deposits

Soil deposits which can be modeled by uniform half spaces are considered here. We present transfer functions for cases involving inclined SH waves as well as inclined P waves. These two cases are presented in the following two sections.

6.3.1.1 Inclined SH-Waves

Consider the incident SH wave shown in Figure 6.3.1.1-1. The displacement is given by:

$$u(r;\omega,t) = Ae^{ik(r\cdot p - c_s t)}d \qquad \text{6.3.1.1-1a}$$

where:

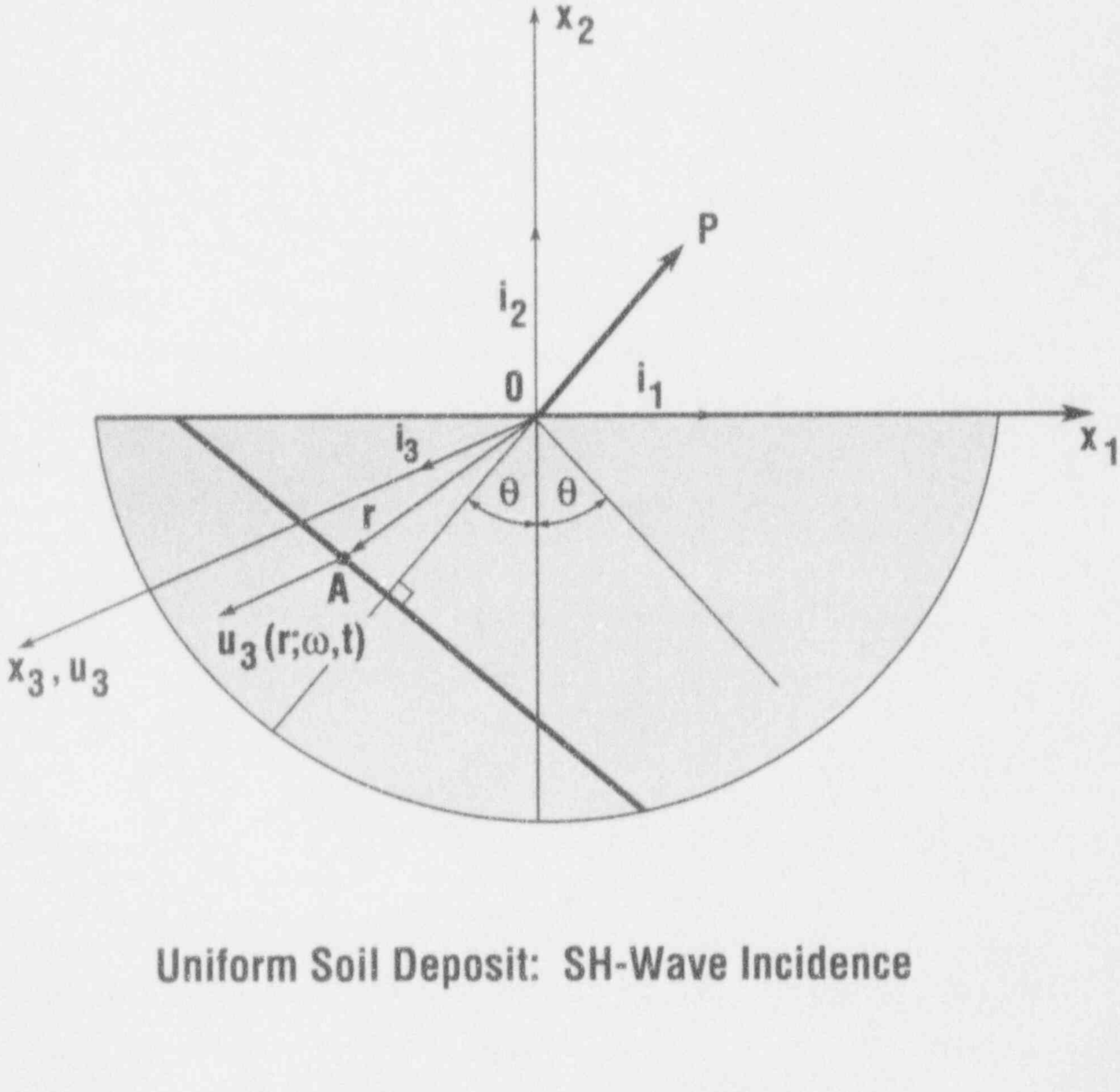
p is the unit propagation vector:

$$p = \sin\theta i_1 + \cos\theta i_2 \qquad \text{6.3.1.1-1b}$$

d is the unit vector defining the direction of motion:

$$d = i_3 \qquad \text{6.3.1.1-1c}$$

r is the position vector:



Uniform Soil Deposit: SH-Wave Incidence

Figure 6.3.1.1-1

$$r = x_1 i_1 + x_2 i_2 \qquad 6.3.1.1\text{-}1d$$

and A, k, c_s are the amplitude, wavenumber and phase velocity respectively. The corresponding apparent wavenumber and apparent phase velocity are:

$$k_A = k\sin\theta \qquad 6.3.1.1\text{-}2a$$

$$c_A = \frac{c_s}{\sin\theta} \qquad 6.3.1.1\text{-}2b$$

respectively.

The requirement that the surface is free of tractions yields that the reflected wave is in phase with the incident wave. The total displacement (due to incident plus reflected waves) is:

$$u_3(x_1, x_2 \; ; \; \omega, t) = 2A\cos a_\theta e^{ik(x_1\sin\theta - c_s t)} \qquad 6.3.1.1\text{-}3a$$

where a_θ is the dimensionless frequency:

$$a_\theta = kx_2\cos\theta = \frac{\omega x_2}{c_s}\cos\theta \qquad 6.3.1.1\text{-}3b$$

The total stress and the total strain due to the incident and reflected SH waves are:

$$\tau_{23}(x_1, x_2 \; ; \; \omega, t) = \tau_{32}(x_1, x_2 \; ; \; \omega, t) = -2k\mu A\cos\theta\sin a_\theta e^{ik(x_1\sin\theta - c_s t)} \qquad 6.3.1.1\text{-}4a$$

and

$$\gamma_{23}(x_1, x_2 \; ; \; \omega, t) = \gamma_{32}(x_1, x_2 \; ; \; \omega, t) = -kA\cos\theta\sin a_\theta e^{ik(x_1\sin\theta - c_s t)} \qquad 6.3.1.1\text{-}4b$$

respectively. In Eqs. 6.3.1.1-4 μ is the shear modulus of the halfspace and a_θ is the dimensionless frequency given by Eq. 6.3.1.1-3b.

For vertical incidence, i.e, $\theta = 0$ we have:

Displacement:

$$u_3(x_1,x_2\ ;\ \omega,t) = 2A\cos a_o e^{-i\omega t} \qquad 6..3.1.1\text{-5a}$$

Stress:

$$\tau_{23}(x_1,x_2\ ;\ \omega,t) = \tau_{32}(x_1,x_2\ ;\ \omega,t) = -2k\mu A\sin a_o e^{-i\omega t} \qquad 6.3.1.1\text{-5b}$$

Strain:

$$\gamma_{23}(x_1,x_2\ ;\ \omega,t) = \gamma_{32}(x_1,x_2\ ;\ \omega,t) = -kA\sin a_o e^{-i\omega t} \qquad 6.3.1.1\text{-5c}$$

where a_o is the dimensionless frequency:

$$a_o = kx_2 = \frac{\omega x_2}{c_s} \qquad 6.3.1.1\text{-5d}$$

In view of Eqs. 6.3.1.1-3, the transfer function between the displacement (or acceleration) at depth x_2 = -h and the displacement (or acceleration) at the surface $x_2 = 0$ is:

$$H(\omega) = \frac{u_3(x_1,-h\ ;\omega,t)}{u_3(x_1,0\ ;\ \omega t)} = \cos a_h \qquad 6.3.1.1\text{-6a}$$

where a_h is the dimensionless frequency:

$$a_h = kh\cos\theta = \frac{\omega h}{c_s}\cos\theta \qquad 6.3.1.1\text{-6b}$$

Note that in Eq. 6.3.1.1-6a the time term was cancelled out since we kept the exponentials in Eq. 6.3.1.1-3 equal.

For vertical incidence, the corresponding expression for $H(\omega)$ and a_h are:

where the dimensionless frequency a_h^o is given by:

$$H_h(\omega) = \frac{u_3(x_1, -h\ ;\ \omega,t)}{u_3(x_1,0\ ;\ \omega,t)} = \cos a_h^o \qquad 6.3.1.1\text{-}7a$$

$$a_h^o = kh = \frac{\omega h}{c_s} \qquad 6.3.1.1\text{-}7b$$

6.3.1.2 Inclined P-Waves

Consider the incidence of an inclined P-wave at angle θ with the vertical axis as shown in Figure 6.3.1.2-1. From the condition that the surface (i.e., plane $x_2 = 0$) is free of tractions ($\tau_{21} = \tau_{22} = 0$) we obtain:

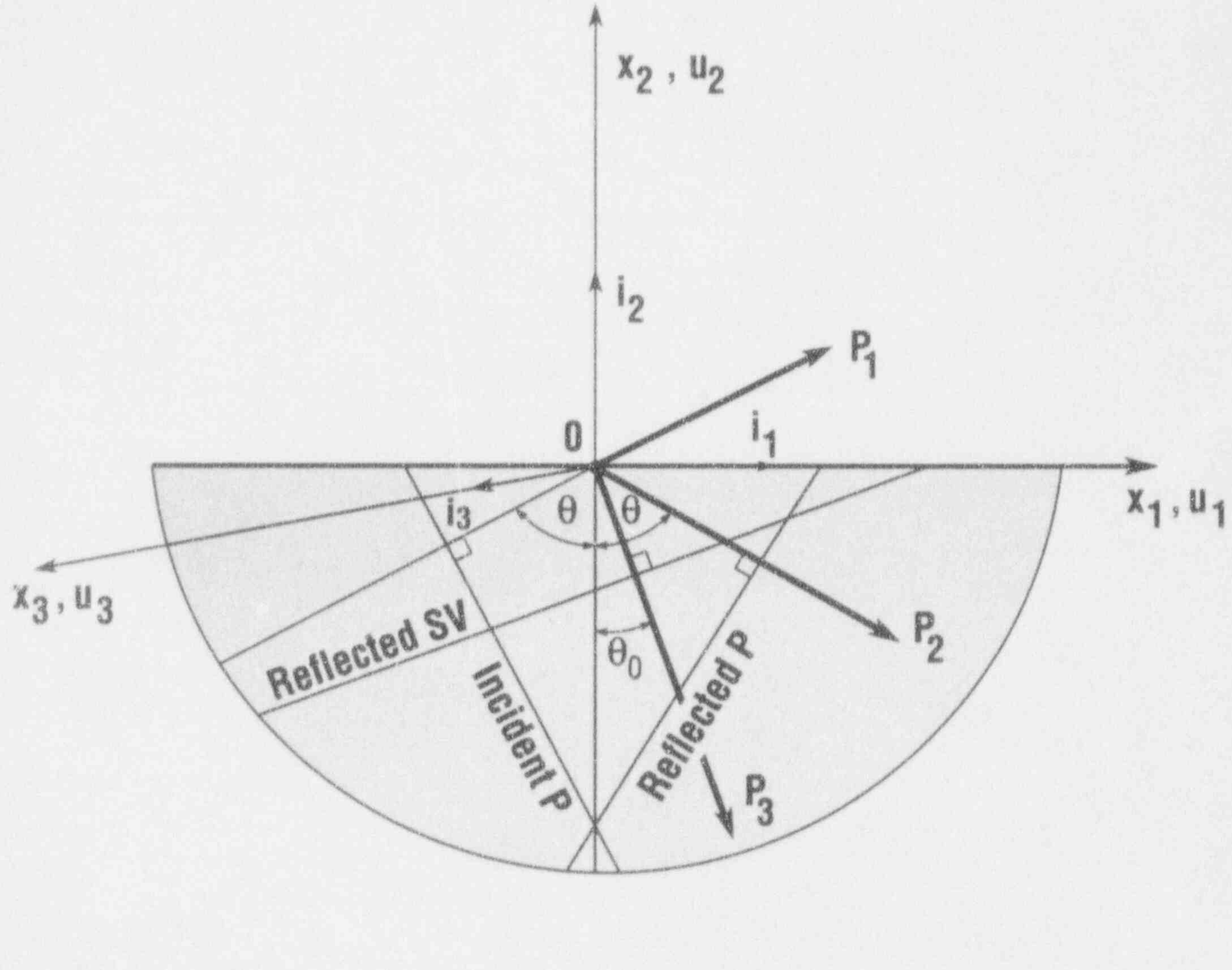
Incident P-wave:

$$\begin{Bmatrix} u_1 \\ u_2 \\ u_3 \end{Bmatrix} = A_1 \begin{Bmatrix} \sin\theta \\ \cos\theta \\ 0 \end{Bmatrix} e^{ik(r \cdot p_1 - c_p t)} \qquad 6.3.1.2\text{-}1$$

Reflected P-wave:

$$\begin{Bmatrix} u_1 \\ u_2 \\ u_3 \end{Bmatrix} = A_2 \begin{Bmatrix} \sin\theta \\ -\cos\theta \\ 0 \end{Bmatrix} e^{ik(r \cdot p_2 - c_p t)} \qquad 6.3.1.2\text{-}2$$

Reflected SV-wave:



Uniform Soil Deposit: P-Wave incidence

Figure 6.3.1.2-1

$$\begin{Bmatrix} u_1 \\ u_2 \\ u_3 \end{Bmatrix} = A_3 \begin{Bmatrix} \cos\theta_o \\ \sin\theta_o \\ 0 \end{Bmatrix} e^{ik(r \cdot p_3 - c_s t)} \qquad 6.3.1.2\text{-}3$$

where:

c_p, c_s	:	P and S wave velocities respectively
A_1, A_2, A_3	:	wave amplitudes for the incident P-wave, reflected P-wave and reflected SV-wave respectively.
$\mathbf{p}_1, \mathbf{p}_2, \mathbf{p}_3$	:	propagation vectors associated with the incident P-wave, reflected P-wave and reflected SV-wave respectively.

Furthermore, the reflection angle θ_o and the wavenumber k_o for the reflected SV-wave are:

$$\sin\theta_o = \frac{\sin\theta}{s} \qquad 6.3.1.2\text{-}4a$$

$$k_o = sk \qquad 6.3.1.2\text{-}4b$$

respectively. In Eqs. 6.3.1.2-4, s represents the ratio of the P to the S wave velocities, i.e,

$$s = \frac{c_p}{c_s} \qquad 6.3.1.2\text{-}5$$

Finally, the amplitudes A_1, A_2 and A_3 satisfy the relation:

$$\begin{bmatrix} \lambda + 2\mu\cos^2\theta & -s\mu\sin 2\theta_o \\ -\mu\sin 2\theta & -s\mu\cos 2\theta_o \end{bmatrix} \begin{Bmatrix} q_1 \\ q_2 \end{Bmatrix} = - \begin{Bmatrix} \lambda + 2\mu\cos^2\theta \\ \mu\sin 2\theta \end{Bmatrix} \qquad 6.3.1.2\text{-}6a$$

where

$$q_1 = \frac{A_2}{A_1} \quad ; \quad q_2 = \frac{A_3}{A_1} \qquad 6.3.1.2\text{-}6b$$

and λ, μ are the Lamé's constants for the material of the halfspace.

Using Eqs. 6.3.1.2-1 through 6.3.1.2-6, the total displacement due to the incident and reflected waves can be written as:

$$\begin{Bmatrix} u_1(x_1,x_2\ ;\ \omega,t) \\ u_2(x_1,x_2\ ;\ \omega,t) \\ u_3(x_1,x_2\ ;\ \omega,t) \end{Bmatrix} = A \begin{Bmatrix} \sin\theta\left(e^{ikx_2\cos\theta} + q_1 e^{-ikx_2\cos\theta}\right) + \cos\theta_o q_2 e^{-ik_o x_2\cos\theta_o} \\ \cos\theta\left(e^{ikx_2\cos\theta} - q_1 e^{-ikx_2\cos\theta}\right) + \sin\theta_o q_2 e^{-ik_o x_2\cos\theta_o} \\ 0 \end{Bmatrix} e^{ik(x_1\sin\theta - c_p t)}$$

6.3.1.2-7

in which we have set A to represent the amplitude of the incident P-wave (i.e, $A = A_1$).

Using Eq. 6.3.1.2-7, several transfer functions can be constructed. Of primary interest is the transfer function between the horizontal and the vertical displacement at the free-surface. Setting $x_2 = 0$ in Eq. 6.3.1.2-7, the transfer function between the horizontal to vertical displacement becomes:

$$\frac{u_1|_{x_2=0}}{u_2|_{x_2=0}} = \frac{(1+q_1)\sin\theta + q_2\cos\theta_o}{(1-q_1)\cos\theta + q_2\sin\theta_o} \qquad 6.3.1.2\text{-}8$$

The transfer function between the vertical displacement at depth h (i.e., $x_2 = -h$) and the vertical displacement at the surface (i.e, $x_2 = 0$) is:

$$\frac{u_2|_{x_2=-h}}{u_2|_{x_2=0}} = \frac{\left(e^{-ia_L} - q_1 e^{ia_L}\right)\cos\theta + q_2 e^{ia_s}\sin\theta_o}{(1-q_1)\cos\theta + q_2\sin\theta_o} \qquad 6.3.1.2\text{-}9$$

Similarly, the transfer function between the horizontal displacement at depth h and the horizontal displacement at the surface is:

$$\frac{u_1|_{x_2=-h}}{u_1|_{x_2=0}} = \frac{\left(e^{-ia_L} + q_1 e^{ia_L}\right)\sin\theta + q_2 e^{ia_s}\cos\theta_o}{(1 + q_1)\sin\theta + q_2\cos\theta_o} \qquad 6.3.1.2\text{-}10$$

In Eqs. 6.3.1.2-9 and 10 we have set:

$$a_L = kh\cos\theta = \frac{\omega h}{c_L}\cos\theta \qquad 6.3.1.2\text{-}11a$$

$$a_s = k_o h\cos\theta_o = \frac{\omega h}{c_s}\cos\theta_o \qquad 6.3.1.2\text{-}11b$$

to represent dimensionless frequencies for P and S waves respectively.

Finally, for vertical incidence ($\theta = 0$), Eq. 6.3.1.2-6 gives: $q_1 = -1$; $q_2 = 0$ and by substitution into Eq. 6.3.1.2-7 we obtain:

$$\begin{Bmatrix} u_1 \\ u_2 \\ u_3 \end{Bmatrix} = A \begin{Bmatrix} 0 \\ e^{ia_L^o} + e^{-ia_L^o} \\ 0 \end{Bmatrix} e^{e^{-i\omega t}} \qquad 6.3.1.2\text{-}12a$$

where the dimensionless frequency a_L^o is given by:

$$a_L^o = \frac{\omega h}{c_L} \qquad 6.3.1.2\text{-}12b$$

Note that Eq. 6.3.1.2-12 is the standard one-dimensional P-wave solution from which the transfer function between the vertical displacement at depth h and the surface displacement is obtained as:

$$\frac{u_2|_{x_2=-h}}{u_2|_{x_2=0}} = \cos a_L^o \qquad 6.3.1.2\text{-}13$$

(Note the similarity between Eqs. 6.3.1.2-13 and 6.3.1.1-7). Equation 6.3.1.2-13 can be also obtained directly from eq. 6.3.1.2-9 by setting $q_1 = -1$, $q_2 = 0$ and $\theta = 0$.

In conclusion, Eqs. 6.3.1.2-8, 9 and 10 are the basic transfer functions which can be used to fill-in the elements of the transfer matrix in Eq. 5.2-2. If vertical incidence is of interest, then the transfer function given by Eq. 6.3.1.2-13 is the only non-zero element in Eq. 5.2.2. For inclined P-wave incidence, however, Eqs. 6.3.1.2-8, 9 and 10 should be used in a consistent manner with Eq. 5.2.2 since both horizontal as well as vertical components are involved in the deconvolution.

6.3.2 Soil Deposit Overlying a Rock Formation

The model considered is shown in Figure 6.3.2-1. The halfspace represents the rock underlying the soil. This case was selected in order to present analytic expressions for transfer functions involving base rock or outcropping motion. The incidence of an SH-wave from the underlying halfspace on the interface between the soil deposit and rock formation is considered. We identify the relevant parameters of .ock and soil by the subscripts R and S respectively.

As shown in Figure 6.3.2-1, the following waves are involved:

Incident SH-wave:

$$u_3^{(1)} = A_1 e^{ik(r \cdot p_1 - c_{S,R}t)} \qquad 6.3.2\text{-1a}$$

Reflected SH-wave at interface:

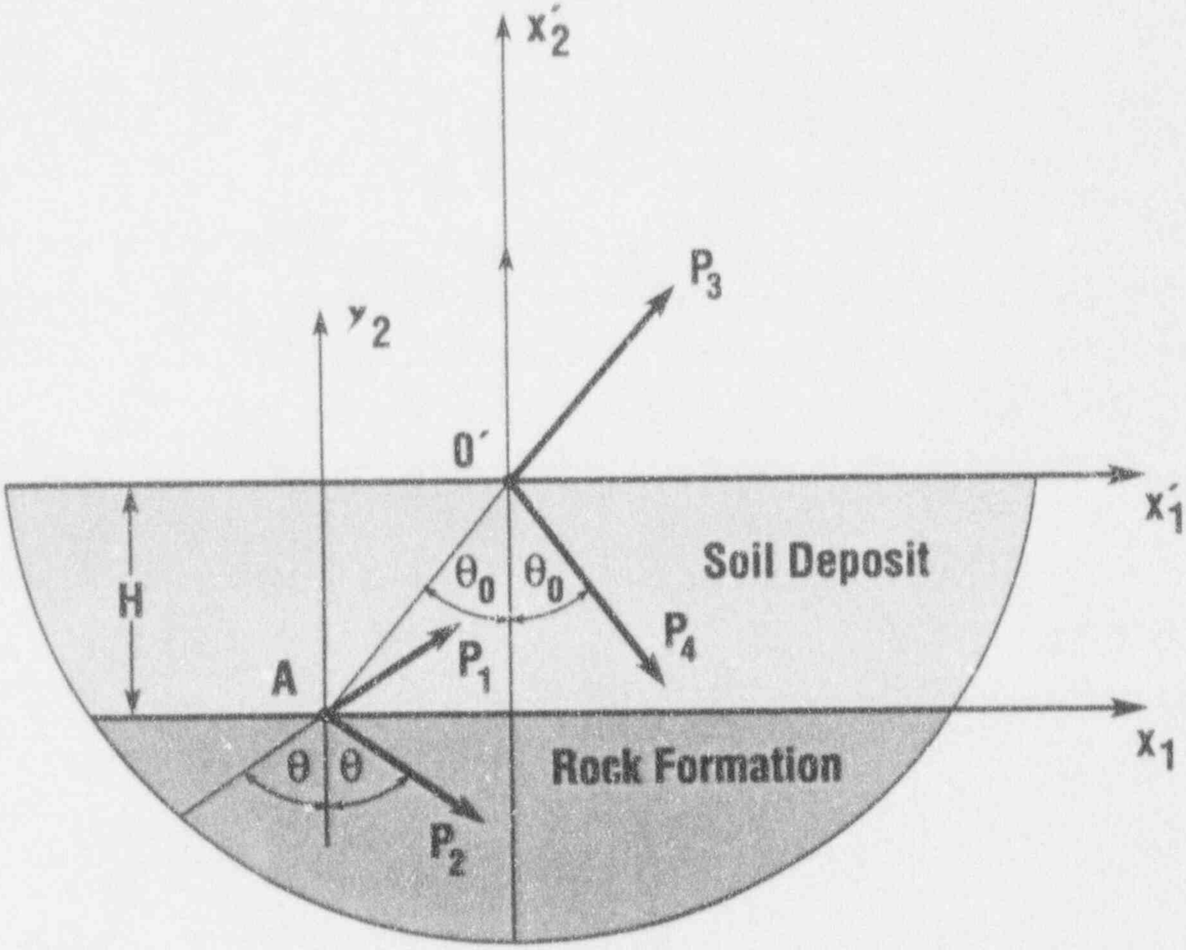
$$u_3^{(2)} = A_2 e^{ik(r \cdot p_2 - c_{S,R}t)} \qquad 6.3.2\text{-1b}$$

Refracted SH-wave across interface:

$$u_3^{(3)} = A_3 e^{ik_{\sigma}(r' \cdot p_3 - c_{S,S}t)} \qquad 6.3.2\text{-1c}$$

Reflected SH-wave at surface

$$u_3^{(4)} = A_3 e^{ik_{\sigma}(r' \cdot p_4 - c_{S,S}t)} \qquad 6.3.2\text{-1d}$$



Soil Deposit Overlying Rock: SH-Waves

Figure 6.3.2-1

where:

$c_{S,R}$, $c_{S,S}$	:	S-wave velocities in the rock and soil respectively
r, r'	:	position vectors with respect to ox,x_2 and o'x', x_2' systems respectively
$\mathbf{p}_1,\ldots,\mathbf{p}_4$	:	propagation vectors
$A_1,\ldots,A_3$	:	wave amplitudes
k, k_o	:	wave numbers
μ_S, μ_R	:	shear modulus of soil and rock respectively

Continuity of total stresses τ_{23} and total displacements u_3 across the interface between rock and soil yields the following relations with respect to the amplitudes A_1, A_2 and A_3:

$$\begin{Bmatrix} A_1 \\ A_2 \end{Bmatrix} = \frac{A_3}{2} \begin{bmatrix} 1-q & 1+q \\ 1+q & 1-q \end{bmatrix} \begin{Bmatrix} e^{ia_H} \\ e^{-ia_H} \end{Bmatrix} \qquad 6.3.2\text{-}2a$$

where

$$q = \frac{c_{S,R}\cos\theta_o \mu_s}{c_{S,S}\cos\theta \mu_R} \qquad 6.3.2\text{-}2b$$

$$a_H = \frac{\omega H}{c_{S,S}} \cos\theta_o \ (\textit{dimensionless frequency}) \qquad 6.3.2\text{-}2c$$

$$\sin\theta_o = \frac{c_{S,S}}{c_{S,R}} \sin\theta \qquad 6.3.2\text{-}2d$$

With the aid of eqs. 6.3.2-1 and 2 the following transfer functions can be defined:

$$H_1(\omega) = \frac{\textit{motion at depth } x_2' = -h}{\textit{motion at surface}} = \frac{1}{2}\left(e^{ia_h} + e^{-ia_h}\right) \qquad 6.3.2\text{-}3a$$

where the dimensionless frequency a_h is given by:

$$a_h = \frac{\omega h}{c_{S,S}} \cos\theta_o \qquad 6.3.2\text{-}3b$$

$$H_2(\omega) = \frac{\textit{motion at depth } x_2' = -h}{\textit{motion at interface } x_2' = -H \textit{ or } x_2 = 0} = \frac{e^{ia_h} + e^{-ia_h}}{e^{ia_H} + e^{-ia_H}} \qquad 6.3.2\text{-}4$$

$$H_3(\omega) = \frac{\textit{motion at depth } x_2' = -h}{\textit{motion at interface without top soil}} = \frac{e^{ia_h} + e^{-ia_h}}{(1 - q)e^{ia_H} + (1 + q)e^{-ia_H}} \qquad 6.3.2\text{-}5$$

Considering that the rock is sufficiently stiff, then Eq. 6.3.2-5 gives the transfer function between the motion at depth h and the outcropping motion.

Using eqs. 6.3.2-3, 4 and 5 it can be found that:

- at the free-surface (h = 0)

$$H_1^o(\omega) \equiv 1 \qquad 6.3.2\text{-}6a$$

$$H_2^o(\omega) = \frac{2}{e^{ia_H} + e^{-ia_H}} \qquad 6.3.2\text{-}6b$$

$$H_3^o = \frac{2}{(1 - q)e^{ia_H} + (1 + q)e^{-ia_H}} \qquad 6.3.2\text{-}6c$$

- at the interface (h ≡ H)

$$H_1^H(\omega) = \frac{e^{ia_H} + e^{-ia_H}}{2} \qquad 6.3.2\text{-}7a$$

$$H_2^H(\omega) = 1 \qquad 6.3.2\text{-}7b$$

$$H_3^H(\omega) = \frac{e^{ia_H} + e^{-ia_H}}{(1 - q)e^{ia_H} + (1 + q)e^{-ia_H}} \qquad 6.3.2\text{-}7c$$

Based on Eqs. 6.3.2-6 and 7 it is concluded that:

$$H_1^o(\omega)H_2^H(\omega) = H_2^o(\omega)H_1^H(\omega) = 1 \qquad 6.3.2\text{-}8a$$

while,

$$H_3^o(\omega)H_3^H(\omega) \neq 1 \qquad 6.3.2\text{-}8b$$

Finally, the amplitudes of the strains and the stresses at depth h in the soil deposit are:

$$\gamma_{23} = \frac{i\omega}{2}\,\frac{A_3\cos\theta_o}{c_{S,S}}\left(e^{ia_h} - e^{-ia_h}\right) \qquad 6.3.2\text{-}9a$$

and

$$\tau_{23} = i\omega\mu_s\,\frac{A_3\cos\theta_o}{c_{S,S}}\left(e^{ia_h} - e^{-ia_h}\right) \qquad 6.3.2\text{-}9b$$

respectively. According to Eq. 6.3.2-2a, the three amplitudes A_1, A_2 and A_3 are related with two equations. The third equation required to completely define them depends on how we define the input to the system of Figure 6.3.2-1. For example, if the outcropping motion $U_g e^{i\omega t}$ is known, then $A_1 = \frac{1}{2} U_g$ and A_2, A_3 can be subsequently obtained from Eq. 6.3.2-2a.

6.3.3 Complex Form of Propagation Parameters

Several parameters from those used in Sections 6.3-1 and 2 are complex due to dissipation in the foundation medium. Conventionally, it is assumed that the latter exhibits viscoslastic behavior which is incorporated into the analysis through the use of complex material constants. For soils, the shear modulus is usually taken as:

$$\mu^* = \mu + i\omega\eta \qquad 6.3.3\text{-}1$$

where η is the soil viscosity which can be related to the soil damping ratio ξ by:

$$\eta = \frac{2\mu\xi}{\omega} \qquad 6.3.3\text{-}2$$

Typically the soil shear modulus μ and soil damping ratio ξ are taken as frequency-independent parameters. From Eqs. 6.3.3-1 and 2 we have the following expression for the complex soil shear modulus:

$$\mu^* = \mu(i + i2\xi) \qquad 6.3.3\text{-}3$$

A similar expression can be written for λ, i.e.,

$$\lambda^* = \lambda(1 + i2\xi) \qquad 6.3.3\text{-}4$$

The complex representation of the Lamé's constants given by eqs. 6.3.3-3 and 4 for viscoelastic soil behavior assumes that the damping ratio is the same for both dilatational as well as distortional motion.

The P-wave and S-wave velocities should also be replaced with the following complex counterparts:

$$c_p^* = \frac{c_p}{1 - i\xi} \qquad 6.3.3\text{-}5a$$

$$c_s^* = \frac{c_s}{1 - i\xi} \qquad 6.3.3\text{-}5b$$

and the corresponding wavenumbers by:

$$k^* = k(1 - i\xi) \qquad 6.6.3.3\text{-}5c$$

Furthermore, in our formulation given in Sections 6.3.1 and 2 we presented relevant transfer functions associated with wave motions as well as stresses and strains in terms of dimensionless frequencies of the general form

$$a = kx = \frac{\omega x}{c} \qquad 6.3.3\text{-}6a$$

The physical significance of such dimensionless frequencies is that they compare a given length x to the wavelength of the waves under consideration. The corresponding complex dimensionless frequencies are:

$$a^* = a(1 - i\xi) \qquad 6.3.3\text{-}6b$$

Equations 6.3.3-3 up to 6 give the complex representation of the parameters required to compute the numerical values of the transfer functions and the other wave response quantities given in Sections 6.3.1 and 2.

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