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**DIRECT GENERATION METHODS OF IN-STRUCTURE SPECTRA**

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## ABSTRACT

This report presents the results and conclusions of a systematic appraisal of existing direct generation methods which are currently employed by the nuclear industry for the development of seismic floor response spectra. The application of direct generation methods in nuclear plant seismic evaluations has been increased over the last several years. Areas of application include: qualification of equipment and pipe supports, reduction of snubber supports, and seismic margin studies. The present study focuses on the theoretical bases, assumptions, and limitations inherent in available direct generation methods. Both deterministic and probabilistic approaches are considered. In the deterministic area, the methods by Biggs-Roesset and Kapur-Shao are considered. The particular methods considered in the probabilistic area are those by Vanmarke-Gasparini, Kaul, Singh, Unruh-Kana, Sundararajan and Shinozuka-Kennedy. These methods are representative of the state-of-the-art on the subject of direct generation.

The conclusions and recommendations to the Nuclear Regulatory Commission (NRC) which are given in this report are based on a qualitative investigation of existing direct generation methods. The intent is to assist the NRC staff with the process of understanding the appropriateness of the use of these methodologies in nuclear plant seismic evaluations. Quantitative studies are underway which are expected to complement the results of the present work.



## EXECUTIVE SUMMARY

The development of in-structure or floor response spectra which are typically used in seismic evaluations of equipment and components of nuclear facilities has traditionally been done through the time history approach. Guidelines and acceptance criteria for generating floor response spectra using the time history approach are given in the Standard Review Plan (SRP) Sections 3.7.1 and 3.7.2. Over the years, however, several algorithms have been developed that allow the computation of floor spectra directly from the input free-field spectra without time-history analysis. Such methods are generally referred to as direct generation methods and their use by the nuclear industry is increasing (e.g., Catawa, Calhoun). Direct generation methods are used in the qualification of equipment and pipe supports, the reduction of snubber supports and seismic margin studies. The SRP acceptance criteria regarding direct generation methods require that such methods be accepted on a case-by-case basis (SRP, Section 3.7.2). Detailed guidelines for direct generation of floor response spectra are not available in the current version of the SRP.

Recognizing the need to develop a capability with direct generation methods to be used in licensing reviews, the Nuclear Regulatory Commission, Office of Nuclear Reactor Regulations (NRC/NRR) contacted Brookhaven National Laboratory (BNL) to carry out the DIGES Project (Direct Generation of Spectra) under FIN L-1227. The BNL Principal Investigator of this Project is A.J. Philippacopoulos (Department of Nuclear Energy). The NRC Project Manager is B. Grenier (NRR/PMSB) and the NRC Lead Engineers are H. Ashar (NRR/ESGB) and N. Thompson (NRR/ESGB). The main objective of the DIGES Project is to provide technical expertise to the NRC staff in evaluating the methods proposed by licensees for the development of floor response spectra from a given ground motion through direct generation.

This report presents the results and conclusions of Task 1 of the DIGES Project. Under this task, a qualitative review of existing methodologies on the subject of direct generation was performed. The objective was to draw conclusions regarding the soundness of the theoretical bases, assumptions and limitations inherent in existing methods. The remaining tasks of the DIGES Project concentrate on computer methods and algorithms related to the direct generation approach. They are currently under development and pertinent results are expected to become available at the end of FY 1991.

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## 1.0 INTRODUCTION

The development of in-structure or floor response spectra is typically performed through the time-history analysis. The term "time-history analysis" is understood to be associated with the input rather than the method of dynamic analysis used to solve the equations of motion of a dynamic model. Over the years, the time history analysis method has been criticized by analysts of nuclear plant seismic response. The main argument has been that it adds an artificial conservatism to the floor spectra. Such conservatism is primarily introduced through the derivation of acceleration time histories which are required to envelope the ground design spectra for which a nuclear facility is designed. From the regulatory perspective, the enveloping requirement assures that a nuclear facility is analyzed for a prescribed set of design ground spectra of site-specific or generic character. It is known, however, that developing a time history given its response spectra is not a unique problem. Several techniques have been proposed in the past to tackle this problem and this subject is still being investigated.

Despite reported shortcomings of the time history method, it is the primary method of generating floor response spectra for nuclear facilities. Recently, however, a different approach, referred to as direct generation method, is being increasingly used by the nuclear industry for the development of floor response spectra. Direct generation methods, are a class of methods by which floor response spectra are generated directly from the ground response spectra without the use of time-history analysis. The earlier type of direct generation methods are deterministic, in that, they employ techniques of deterministic structural dynamics for computing the floor spectral accelerations. Around the mid-seventies, direct generation methods based on a probabilistic representation of the seismic input and random vibration analysis techniques to compute floor response spectra began to be used. Probabilistic direct generation methods are primarily the methods used by the nuclear industry. The early deterministic approach did not attract the interest of nuclear plant analysts, due to the fact that reported results showed the methods to produce floor spectra which are typically higher than those produced by time-history analysis. It is the understanding of the authors, that direct generation methods are favored by the nuclear industry because they are claimed to have the following advantages:

- Floor spectra produced by direct generation methods can be smooth in contrast to the local fluctuations in spectral acceleration obtained from time-history analysis.
- Direct generation methods are more economical than time-history method.
- Direct generation methods allow for a statistical interpretation of the in-structure motion which is physically a meaningful way to treat structural responses to earthquake motion.

There are no reported apparent disadvantages of the direct generation approach. This is possibly due to the lack of detailed variation of parameter studies and comprehensive benchmarking of the method. At any rate, current criteria of the Standard Review Plan (SRP) regarding direct generation methods require acceptance on a case-by-case basis (SRP Section 3.7.2). Typically, comparisons with time-history analysis results have been used with the objective to prove the adequacy of a particular direct generation method. Such comparisons are meaningful when several time histories are used. Theoretically, a series of time history analysis results should be statistically comparable to the results obtained by the use of a direct generation method.

Motivated by the increasing usage of direct generation methods to nuclear plant seismic evaluations, the Nuclear Regulatory Commission (NRC) decided to investigate their adequacy. For this purpose, Brookhaven National Laboratory (BNL) was contacted to carry out the DIGES Project (Direct Generation of Spectra). At this stage, Task 1 of this project has been completed and the results obtained are described in this report. The objective of Task 1 was to perform a qualitative review of existing direct generation methods by focusing on their theoretical bases, assumptions and limitations. To have independent assessments of existing direct generation methods, Professors Y.K. Lin (Florida Atlantic University) and P.D. Spanos (Rice University) worked on Task 1 of the DIGES Project as consultants to BNL. The remaining tasks of the DIGES Project are currently under development and pertinent results are expected to be reported at their completion (estimated completion date: September 1991). These tasks are primarily concerned with computer methods and algorithms related to the direct generation approach. Their overall objective is to provide the NRR with a capability to perform comparable solutions, the results of which can be used as an input to the process of reviewing licensing issues related to the direct generation method.

Section 2 of the report presents aspects related to the definition of design input for seismic response analysis, both from a deterministic as well as a probabilistic perspective. An overview of floor response spectra generation methods is given in Section 3. Existing deterministic and probabilistic direct generation methods are discussed in Sections 4 and 5 respectively. Section 6 deals with the subject of the applicability of the direct generation approach to other aspects of the seismic evaluation process of nuclear facilities. This is followed by a set of conclusions and recommendations.



## 2.0 DETERMINISTIC/PROBABILISTIC DESIGN INPUTS

The idea of representing earthquake records by using the concepts of design spectrum has proved fruitful for about half a century. These concepts have been discussed in many technical papers and books. In fact it has become a routine task to generate the response spectrum of a given earthquake record. In this context, it is logical to examine statistically the response spectra of a collection of earthquakes which are similar in nature, and based on the statistical examination, to develop a design spectrum which can be used in analyzing major buildings and industrial facilities. Clearly, the response spectrum corresponding to a particular accelerogram exhibits significant local fluctuations in the frequency domain. However, the spectra which correspond to an ensemble of accelerograms produced by ground shaking, real or expected, of sites with similar geological and seismological characteristics, can be generally smooth and exhibit statistical trends that characterize the site collectively. In this regard, it is noted that the band of dominant frequencies of accelerograms can be identified on a statistical basis. Thus, when an approach based on the concept of a response spectrum is adopted for a seismic design of important structures like nuclear reactors, it is logical to seek a representation of seismic input based on recorded and expected strong ground shaking at a certain location by using a smooth spectrum. The smooth design spectrum is, on one hand, insensitive to the chaotic fluctuations of any particular response spectrum and on the other hand, it reflects the repetitive characteristics of the ensemble of the spectra. Clearly, the selection of a proper design spectrum for a given site is not a trivial task. It involves the incorporation of historical data, available and extrapolated theoretical results, and a significant element of engineering judgment.

Background information on approaches for determining expected ground shaking at given site can be found in references such as Werner (Reference 22) and Lomnitz and Rosenblueth (Reference 10). The design spectrum can be specified on either a deterministic or a probabilistic (stochastic) basis. A deterministic design spectrum is developed by using smoothing procedures to eliminate negligible local abrupt changes in the response spectra of individual recorded accelerograms and by using known and extrapolated theoretical results. A stochastic design spectrum reflects the interpretation of any recorded or expected ground shaking record at the given location as an individual realization of time series. In any case, a probabilistic design spectrum can be generated by specifying, with a selected confidence level, the expected maximum response of a single-degree-of-freedom linear structure as a function of its natural frequency and the ratio of critical damping. Clearly for this purpose, it is important to specify the probability distribution of the maximum of the structural response. This problem is equivalent to the first passage problem of the theory of random vibration for which only approximate analytical results exist (see e.g., Lin (Reference 9) and Spanos (Reference 18)). A probabilistic design spectrum is often constructed numerically and only its mean value and standard deviation are specified.

Collectively, the probabilistic and the deterministic design spectra are, of course, related from a philosophical point of view. Specifically, smoothing of the deterministic spectrum which is obtained from a time history, requires



a rational approach in maintaining conservatism in the design process. Therefore, a statistical approach must be used to introduce in the finally selected design spectrum, adequate safety.

Irrespective of the manner in which the design spectrum has been specified, probabilistic or deterministic, a critical issue has developed over a period of years. Specifically, how one could use the design spectrum to conduct nonlinear analysis especially since the nonlinear behavior of structures is invoked to ensure its capacity to withstand loads without a catastrophe during a major earthquake. The second aspect involves the need to be able to use the design spectrum at the base of a structure and to develop design response spectra at different locations of the structure (in-structure or floor response spectra). The idea behind this approach is that by having a method to generate floor spectra for the primary structure, analysis of secondary systems and equipment can be readily conducted. A logical approach for both problems, the problem of analyzing the nonlinear behavior of a structure and the problem of developing floor response spectra, can be established by generating a time history which is compatible with the original design spectrum and doing a time domain integration of the nonlinear equations of motion or of the linear equations of motion to get the floor spectra. This approach has been used repeatedly. However, several issues have been raised regarding the uniqueness of the time history which is generated to be compatible with a given design spectrum. The efficiency of this approach with regard to the development of floor spectra has been questioned. In any case, time history analysis using an input accelerogram which is compatible with a specified design spectrum is currently used extensively.

### 3.0 OVERVIEW OF METHODS FOR FLOOR SPECTRA GENERATION

#### 3.1 Time History Method

The process of time history analysis starts with the development of an acceleration time history consistent with specified design ground spectra which are considered appropriate for a particular nuclear plant site. Having established an input time history, the remaining activity is to utilize time or frequency domain methods of dynamic analysis to compute structural responses one form of which is the in-structure or floor response spectra. This process has been repeatedly used for the development of floor response spectra which subsequently represent the input to secondary systems assuming that decoupling is valid. The key to the above process is the derivation of the input time history. Theoretically, the determination of an acceleration time history given its response spectrum is not a unique problem in the sense that several time histories can be candidates for the solution. In practice, the solution is handled by numerical manipulations. All the analyst has to do is to get as reasonably close to the design response spectrum so that the acceptance criteria per SRP Section 3.7.1 are satisfied. Since there is not really a precise way to proceed with this problem (solution convergence not well understood), it leads to situations where the time history produces response spectra which in some frequencies are much above the design spectra. Trying to make these differences smaller, it usually causes a mismatch at other frequencies. More recent methods seem to handle the convergence to the design ground spectrum more efficiently than earlier ones in terms of uniformity over the frequency range of interest. The fluctuations of the response spectrum of the generated input time histories are not realistic and basically they reflect the fact that one has tried to force a time history to behave like the smoothed response of a set of earthquake time histories. These fluctuations over frequency are sometimes called by analysts as input imperfections. On several occasions, they are recalled to argue that conservatism has been built into an analysis particularly when the input time history used in the analysis produces response spectra which are higher than the design spectra at the frequencies of the primary system.

Despite several criticisms typical of which is the artificial conservatism in floor response spectra and the artificial look of the synthetic accelerograms, the time history approach has been the main tool for constructing floor response spectra. After many years of research on this subject (References 4, 5, 7, 8, 12, 13, 20, 21, 24, 25, 38), there are methods available today which produce accelerograms having response spectra that satisfy NRC's enveloping requirement by getting comfortably close to the target design spectra. In addition, it has been a consensus that by modifying real accelerograms one can get design inputs which have more realistic appearance than artificial ones.

Recently, the NRC was concerned with the energy distribution of spectrum consistent acceleration time histories over the frequency range of interest to the seismic analysis of nuclear plant structures and components. This led to the development of a power spectral density requirement which is considered as secondary in that the response spectrum enveloping requirement remains the primary acceptance criterion (Reference 40). To prevent possible energy deficiencies when single inputs are used in the analysis, the SRP has been updated to include a power spectral density requirement (SRP, Rev. 2, section 3.7.1).

In view of the non-unique character of the problem, the most reasonable way to proceed with the development of floor response spectra through a time history analysis is to use a set of input time histories. A multiple time history analysis offers a sensible approach to address the problem of non-uniqueness. In this case the target spectrum enveloping requirement can be satisfied in a statistical sense. Reported opinion by nuclear plant analysis, however, is strongly opposing this option due to economical disadvantages (Reference 40).

### 3.2 Direct Generation Methods

Motivated by the disadvantages of the conventional time history method, alternative solutions were formulated and applied in the computation of in-structure spectra of nuclear facilities. A class of such methods are commonly referred to as direct generation methods and their objective is to compute floor spectra without employing time history analysis. The early developments with direct generation methods are deterministic. They utilize spectrum superposition techniques of structural dynamics to construct floor response spectra from given ground design spectra. The main objective is to develop amplification factors for the frequency range of interest which by "multiplication" with the input spectral values yield the corresponding spectral values of the in-structure spectra.

This concept was worked out first by a two degree-of-freedom model and then it was extended to the multi degree-of-freedom case. Typical examples are the works by Biggs-Roesset (Reference 36) and Kapur-Shao (Reference 37). Their approach is attractive due to its simplicity. It is to be noted that in this case there is no need to deal with the target spectrum enveloping requirement since the target spectrum itself is used in the analysis. Application of such earlier direct generation methods to seismic models of nuclear plants, however, have lead to higher floor response spectra than those produced by the time history approach. Accordingly, there has been no use of the above methods in real applications to the authors' knowledge. It should be recognized that a pure deterministic formulation of this problem is really difficult since the input (free-field spectra) as well as the output (floor spectra) are defined in terms of maxima. Thus, the estimation of the maximum response of the oscillator at the floor based on the maxima of the oscillator at the free-field, does not lead itself readily to a rigorous treatment. At some stages of the analysis, certain assumptions have to be made so that the problem becomes tractable. The key issue then becomes the level of conservatism which is produced by adopting any logical scheme to go deterministically from the input design spectrum to the floor response spectrum. This issue needs to be adequately addressed by appropriate refinements, so that such schemes could become attractive to the nuclear industry.

After the mid-seventies, the problem of going from a design spectrum to the floor response spectrum is basically handled probabilistically. The main task of the probabilistic approach is the interpretation of the design input spectrum in terms of a stochastic process. Having established this task, the remaining computational effort is based on established methods of random vibration analysis. Philosophically, one could question why a random vibration approach would make the problem more tractable mathematically. In this regard,

it is noted that random vibration can incorporate issues like stationarity and correlation of processes in a very systematic way. For example, in conducting a random vibratory analysis to determine a floor spectrum from the design spectrum, the orthogonality of the displacement and the velocity of a stationary process can be invoked to simplify the mathematical labor. In any case, a random vibration approach simply involves the derivation of a power spectrum from the design spectrum to characterize the ground shaking. Then, one can use random vibration analysis to calculate the power spectrum of the response on the floor of a structure and then proceed from the power spectrum of the response at a given floor to define the response spectrum at that particular floor. Clearly, this is an interesting concept which hinges upon the methods for determining the equivalence of a design spectrum to a power spectrum.

Typical studies dealing with the probabilistic approach are those by Singh (References 28-30), Kaul (Reference 27), Vanmarke-Gasparini (References 25-26), Unruh-Kana (Reference 31), Sundararajan (Reference 32) and Shinozuka-Kennedy (Reference 35). Strictly speaking, Singh's work deals primarily with the direct generation of floor response spectra. The remaining of the above studies are concerned with the subject of spectrum consistent power spectral density functions which is either implicitly or explicitly used in direct generation methods. Thus, the above studies are strongly interrelated. Finally, it may be noted that although the term "direct" is suitable when referring to the deterministic approach, it appears to be rather loose when referring to the probabilistic approach. Its use is maintained here only for convenience.



## 4.0 DIRECT GENERATION METHODS: DETERMINISTIC APPROACH

### 4.1 Biggs & Roesset Method

This method was originally developed at MIT and an updated version of it appeared at the 1st SMiRT Conference (Reference 36). Typically, the method produces conservative results since it was intended to produce envelope type floor response spectra. Basic ideas are formulated considering a two degree-of-freedom system representing the simplest form of an equipment-structure configuration. The method is then generalized through the use of the modal parameters of the primary system. An intelligent use of limiting cases is made (flexible versus rigid equipment) in an effort to estimate the response behavior for the cases in between. The resulting amplification factors are similar to those of the one degree-of-freedom system. Two types of amplification curves are obtained. The first represent the ratio of the peak equipment acceleration to the acceleration of the equipment if it was ground supported (Figure 1). The second type of amplification curves give the ratio of the equipment peak acceleration to the peak floor acceleration (Figure 2). A step-by-step procedure is outlined in Reference 11 where the above amplification curves are used in conjunction with the modal properties of the structure to generate floor response spectra. The overall approach resembles the spectrum superposition method of dynamic analysis.

The Biggs-Roesset direct method for generating in-structure spectra is a simple, yet intelligent approach to the problem in hand especially when one considers the state-of-the-art in that time frame (around 1968). It has definitely influenced the latter developments on this subject. The main drawback, however, is its repeated use of conservative assumptions throughout the formulation to account for various uncertainties. It is realized that this was the motivation during the development of the method. Refinements, however, are necessary to avoid compound conservatism and to make the method more realistic.

## 4.2 Kapur & Shao Method

This method (Reference 37) is clearly motivated by the Biggs & Roesset method discussed in the previous section. In fact, it was proposed by its authors as a modification of the Biggs & Roesset method. The main development is centered on the premise that the latter method employs empirical elements and considerable engineering judgement. An attempt was made to provide a more rigorous mathematical treatment of the problem.

Again, the basic approach is to utilize the standard two degree-of-freedom equipment-structure model and then extending the results to multi degree-of-freedom systems. It is assumed that the equipment is excited by single harmonic inputs at the level of attachment with the primary structure. This allows them to utilize standard amplification functions of one degree-of-freedom structural systems (Figure 3). Amplification at resonance is computed through several time history analyses by varying the structural and equipment damping. For this purpose, the two degree-of-freedom model was employed in conjunction with a spectrum consistent synthetic time history.

The primary contribution of the Kapur & Shao method is probably due to the analytical forms of the amplification functions used in the direct generation of floor response spectra. The corresponding amplifications proposed by Biggs & Roesset were derived by utilizing a set of earthquakes while their similarity to the amplifications of single oscillators was already pointed out.

In view of the repeated use of conservative assumptions throughout the analysis, it is not clear whether the extension of Biggs & Roesset method offered by Kapur & Shao represents an essential improvement. Furthermore, numerical examples given by the authors are limited, thus not allowing for a detailed assessment.

The greatest problem with these early deterministic developments is the compound conservatism which was built into them. This imposes difficulties if one attempts to quantify what would be the overall conservatism in this case. The consensus is, however, that both approaches are likely to produce higher floor spectra than the time history analysis. Accordingly, any advantages over the time history method (e.g., simplicity, computational efficiency) becomes a secondary issue.

## 5.0 DIRECT GENERATION METHODS: PROBABILISTIC APPROACH

In the probabilistic approach, a future earthquake is treated as a realization of a stochastic process. A structure in an earthquake prone zone or a secondary system supported by such a structure must then be designed to survive all possible realizations with a high probability. In recent years numerous publications on seismic design have appeared in the literature based on established probabilistic analyses. They may be classified into the following two broad categories:

(1) In the first category, the traditional random vibration theory is used in the analyses. The strong motion portion of an earthquake is modeled as a stationary stochastic process, or the entire earthquake is modeled as a stationary stochastic process multiplied by a deterministic envelope function. The underlying stationary process can be characterized statistically by a power spectral density (PSD) function. The additional assumption is usually made that the stochastic process is Gaussian. Several statistical and probabilistic properties of the response of a linear oscillator can then be calculated, such as the variance, the probability distribution of maxima (or peaks), etc.

The present NRC regulation, however, requires that seismic inputs be defined in terms of design response spectra, e.g., RG 1.60, which is obtained from statistical analyses of some existing earthquake records. Thus, a connection must be established between the deterministically based design response spectrum and some probabilistically based property of the response obtained from a random vibration analysis. The works by Vanmarke and Gasparini, Kaul, Singh, Unruh and Kana, and Sundararajan belong to this category. A common objective in these works is to find a PSD function for the ground motion which is compatible with the design response spectrum. Since this PSD, which is the very unknown to be determined, is also required to compute the probabilistic properties of the system response, an iterative procedure is often required, beginning with an assumed PSD. Two questions may be raised. First, whether or not such an iteration process converges. Second, if it does converge, then whether or not each and every sample function generated from the "final" PSD satisfies the "mandated" design response spectrum. The answer to the second question is clearly negative.

(2) In the second category, a PSD requirement is considered complementary to the mandated design response spectrum. Its use is to prevent possible energy deficiency in the seismic excitation model. A recommended PSD is provided from which artificial seismic excitations are generated. These artificial excitations are further screened to retain only those satisfying the design response spectrum. Some of the generated time histories may have to be clipped or fractionally folded to satisfy the peak acceleration limit, which must be equal to the response spectrum at high frequencies. Finally, the selected excitation time histories are used in the time-domain design analysis. This approach guarantees that RG 1.60 is satisfied. The recommended PSD is used solely for the purpose of generating artificial ground motions, not for random vibration analysis. The works by Shinozuka et al., and Kennedy and Shinozuka (Reference 35) belong in this second category.



The relative merits of the classical random vibration approach and the simulation approach depend on the intended use of the PSD for the ground motion so obtained. If such a PSD can be used in lieu of the design response spectrum, then approach (1) may have some advantage, although that advantage is based on the assumption that the systems involved are linear, or an equivalent linearization analysis is appropriate. The second approach has no limitation on linearity. If a PSD is used only to complement the design response spectrum requirement, and the artificial earthquakes generated are used for Monte Carlo calculations, then clipping or fractional folding is considered a suitable means to avoid conflicting with the peak acceleration requirement.

The question of correlation and verification of these methods with available observed data has not been addressed by any of the above authors. At the present time, any attempt in this direction may be restricted to qualitative comparison between recorded and simulated sample functions.

It should be mentioned that, while Singh's works belong to category (1), his main objective is to generate a "floor" PSD to be used for the subsequent analysis of a secondary systems.

Brief descriptions of some published works in the above two categories are given below. For the ease of comparing with the original publications, the original symbols used in each publication are kept whenever possible. Thus different symbols representing the same physical quantity may appear in the following discussion of different publications.

## 5.1 Random Vibration Approach

### 5.1.1 Vanmarke & Gasparini Method

The method (References 25-26) consists of two parts, but they are not entirely consistent. In the first part, the following rather simple formula is derived to convert the specified pseudo-velocity response spectrum  $S_v(\omega_n, \zeta)$  to the power spectrum  $G(\omega_n)$ :

$$G(\omega_n) \approx \frac{1}{\omega_n \left( \frac{\pi}{4\zeta_s} - 1 \right)} \left\{ \frac{\omega_n^2 [S_v(\omega_n, \zeta)]^2}{r^2} - \int_0^{\omega_n} G(\omega) d\omega \right\} \quad (1)$$

where  $\omega_n$  = the natural frequency,  $r$  = peak factor,  $\zeta$  = damping ratio, and  $\zeta_s$  = a fictitious damping ratio, depending on the duration  $s$  of strong ground motion. Eq. 1 is obtained using several approximations:

(1)  $S_v(\omega_n, \zeta) = r\sigma_v$  where  $\sigma_v^2$  is the variance of random velocity response.

(2)  $\sigma_v^2$  is calculated using the stationary input-output formula of random vibration theory, but with the fictitious damping ratio, obtained as

$$\zeta_s = \zeta(1 - e^{-\zeta\omega_n s})^{-1} \quad (2)$$

This fictitious damping accounts for the transient response effect of a suddenly applied white noise.

(3) The peak factor  $r$  is computed from an upper bound approximation

$$r = \sqrt{2 \ln(2n)} \quad (3)$$

where  $n = 1.4(s\omega_n/2\pi)$  based on a 50% probability of exceedance.

In the second part, artificial earthquakes are generated using

$$z(t) = I(t) \sum_j A_j \sin(\omega_j t + \phi_j) \quad (4)$$

where  $I(t)$  is an envelope function and is taken to be of a "trapezoidal", "exponential", or "compound" shape. Response spectra are calculated for these sample functions, and iterations are carried out to adjust  $G(\omega)$  as follows

$$G(\omega)_{i+1} = G(\omega)_i [S_V(\omega)/S_V^{(i)}(\omega)]^2 \quad (5)$$

where subscripts  $i$  and  $i+1$  indicate the  $i$ th and the  $(i+1)$ th iterations, and  $S_V(\omega)$  is the target response spectrum.

The assumption of suddenly applied white noise in the first part implies that  $I(t)$  is a unit step function; thus, it is inconsistent with the second part in which  $I(t)$  can be slowly varying.

Generalization of artificial earthquakes and iteration of the spectral density function are carried out on a computer program SIMQKE. Figure 4a shows simulated and target response spectra for the case where no iterations were performed. Results after four iterations are shown in Figure 4b. However, the authors indicated that the iterations were not expected to converge to all control frequencies, and that it was not productive to iterate for more than 4 cycles. Furthermore, the derived spectral density function will decrease when the chosen strong motion duration  $s$  increases, and the simulated accelerograms are to be scaled to the maximum acceleration indicated in the target response spectrum  $S_V$ . These statements raise additional doubt about the practicality of the approach.

### 5.1.2 Kaul's Method

In Kaul's method (Reference 27), the design response spectrum  $R(\omega)$  is interpreted as the magnitude which would be exceeded by the maximum response of a simple oscillator only 15.9% of the time. This definition is attributed to Newmark, Blume and Kapur (Reference 11), which is the same basis for NRC Regulatory Guide 1.60. As such, the response spectrum can be related directly to power spectrum  $f(p)$ . (Note that in Kaul's notation  $\omega$  = natural frequency of the oscillator, and  $p$  = frequency in rad/s.)

Both approximate and "exact" procedures are given to convert  $R(\omega)$  to  $f(p)$  based on the following simplified assumptions:

- (1) the excitation and the acceleration response are both stationary Gaussian random processes, and both are examined for a duration of  $T$ .
- (2) The peaks in the response process are independent; thus, no allowance is given for the "clumping" effect of neighboring peaks.
- (3) The Longuet-Higgins parameter  $\epsilon$  is small, where

$$\epsilon = \{[1 - m_2^2 / (m_0 m_4)]\}^{1/2} \quad (6)$$

and where the  $m_j$  are the one-sided spectral moments of the acceleration response.

- (4) The number of peaks are large within  $T$ .

Assumptions (3) and (4) are used to justify certain asymptotic expansions from which the following relation is obtained between the response spectrum and the response spectral moments:

$$R(\omega) = \left\{ -2m_0 \ln \left[ -\frac{\pi}{T} (m_0/m_2)^{1/2} \ln(1-r) \right] \right\}^{1/2} \quad (7)$$

where  $r$  is the probability of exceedance. Since the response spectral moments are not known without the knowledge of the response power spectrum  $f(p)$ , Eq. 7 cannot be used to determine  $f(p)$  from  $R(\omega)$  directly. Instead, an iteration process is required. The so-called "exact" solution is such a process.

Assuming that  $f(p)$  can be expressed as

$$f(p) = \sum_{i=1}^n b_i F(\omega_i, p) \quad (8)$$

where  $F(\omega_i, p)$  are suitably selected functions which decay to zero at large  $p$  at least as rapidly as  $p^{-4}$ . The spectral moments can be computed from Eq. 8, and the results are functions of the coefficients  $b_i$  and parameters  $\omega_i$ . Substituting these spectral moments into Eq. 7, and evaluating this equation at  $i = 1, 2, \dots, n$ , we obtain  $n$  algebraic equations for the coefficients  $b_i$ . These are complicated nonlinear equations, for which the author proposes an iterative procedure for the solution.

The approximate procedure is based on the additional stipulations that the acceleration response is a narrow-band process such that the following approximations are valid:

$$m_0 \approx (\pi\omega/2\xi)f(\omega) \quad (9)$$

$$m_2 \approx (\pi\omega^3/2\xi)f(\omega) \quad (10)$$

where  $\xi$  is the damping ratio of the structure. Substitution of Eqs. 9 and 10 into Eq. 7 yields

$$f(\omega) = \frac{2\xi}{\pi\omega} R^2(\omega) / \left\{ -2 \ln \left[ \frac{-\pi}{\omega T} \ln(1-r) \right] \right\} \quad (11)$$

permitting the determination of  $f(p)$  from  $R(\omega)$  at the natural frequency  $p = \omega$ .

In the low frequency range when the assumption of response stationarity is not justified, the author suggests a correction using an equivalent damping:

$$\xi_e = \xi + \frac{2}{\omega T} \quad (12)$$

The idea was due originally to Rosenblueth and Elorduy.

The greatest problem with Kaul's method is the implication of a narrow-band response. This may be acceptable for the displacement response, but is questionable for the acceleration response. It is also of interest to note that Kaul deals with the absolute acceleration, rather than the acceleration relative to the ground which is the concern of some other authors.

### 5.1.3 Singh's Method

Interpreting each response spectrum  $R$  as the standard deviation  $\sigma$  of the corresponding response variable multiplied by a suitable peak factor  $C$ , Singh and his co-workers (References 28-30) have developed algorithms to compute the floor response spectrum for a secondary system supported by a primary structure. The given input is the response spectrum  $\Phi_g(\omega)$  of the ground motion assumed to be obtainable from the power spectral density of the underlying random earthquake process. Singh's works may be divided into three stages: (1) classically damped structure plus an "untuned" secondary system, (2) classically damped structure plus a secondary system tuned to one of the structural frequencies, (3) non-classically damped structure plus a secondary system tuned to one of the structural frequencies. The traditional linear random vibration theory is used in the analyses.

If the primary structure is classically damped, then the structural response can be expressed in terms of the normal modes. Let  $\omega_j$  be the natural frequencies of these modes and  $\beta_j$  be their corresponding damping ratios. The squared floor spectrum for a secondary system, modeled as a simple linear oscillator with a natural frequency  $\omega_0$  and a damping ratio  $\beta_0$  may be written as

$$\begin{aligned}
 R_u^2(\omega_0) = & C^2 \sum_{j=1}^N \gamma_j^2 \psi_j^2(u) \int_{-\infty}^{\infty} \Phi_g(\omega) \frac{(\omega_j^4 + 4\beta_j^2 \omega_j^2 \omega^2)}{(\omega_0^4 + 4\beta_0^2 \omega_0^2 \omega^2)} |H_j(\omega)|^2 |H_0(\omega)|^2 d\omega \\
 & + 2 C^2 \sum_{j=1}^N \sum_{k=j+1}^N \gamma_j \gamma_k \psi_j(u) \psi_k(u) \int_{-\infty}^{\infty} \Phi_g(\omega) H_j(\omega) H_k^*(\omega) |H_0(\omega)|^2 (\omega_j^2 \omega_k^2 + 4\beta_j \beta_k \omega_j \omega_k \omega^2) \\
 & (\omega_0^4 + 4\beta_0^2 \omega_0^2 \omega^2) d\omega
 \end{aligned} \tag{13}$$



in which  $\Psi_j(u)$  = modal displacement at floor  $u$ ,  $\gamma_j$  = participation factor of mode  $j$ ,  $H_j(\omega)$  = frequency response function of mode  $j$ , and  $H_0(\omega)$  = frequency response function of the secondary system. Since each frequency response function is given by

$$H_{\ell}(\omega) = \frac{1}{(\omega_{\ell}^2 - \omega^2 + 2i\beta_{\ell}\omega_{\ell}\omega)} \quad (14)$$

the integrations in Eq. 13 can be carried out approximately under the assumption that the ground acceleration spectral density  $\Phi_g(\omega)$  is slowly varying near each of these natural frequencies. Singh's works for the first two stages are accounts of such integrations.

In particular, attention is focused on the following two integrals

$$I_1(\omega_j) = C^2 \int \omega_j^4 \Phi_g(\omega) |H_j(\omega)|^2 d\omega \quad (15)$$

$$I_2(\omega_j) = C^2 \int \omega_j^2 \omega^2 \Phi_g(\omega) |H_j(\omega)|^2 d\omega \quad (16)$$

These are essentially the pseudo acceleration spectrum and velocity response spectrum, respectively. In order to permit additional approximations, Singh argues that

$$I_2(\omega_j) \approx C^2 \omega_j^4 \Phi_g(\omega_j) \int |H_j(\omega)|^2 d\omega \quad (17)$$

and

$$I_1(\omega_j) \approx I_2(\omega_j) + I_b(\omega_j) \quad (18)$$

where  $I_b(\omega_j)$  represents the "background noise" contribution in  $(0, \omega_j)$ .

The implications and accuracies of various approximations are difficult to assess without actually carrying out the computations and comparing with the exact results for different cases. The numerical examples given in Singh's papers do show good agreements with the simulations (see e.g., Figure 5).

The case of non-classical damping is treated in the usual manner in which the equations of motion are expressed in the first-order matrix form, and complex eigenvalues and eigenvectors are obtained for subsequent complex modal analysis.

#### 5.1.4 Unruh & Kana's Method

Theoretically, this method (Reference 31) is based on the analysis of Kaul (Reference 27); thus, it has the same shortcomings, including the questionable implications of narrow-band acceleration response and independent peaks. However, the paper differs from that of Kaul in that a different iterative scheme is suggested which is described below using Kaul's original notation.

An initial approximation for the spectral density  $f(\omega)$  is obtained from Eq. 11 using the target response spectrum  $R(\omega)$ ; namely,

$$f_1(\omega) = \frac{2\xi}{\pi\omega} R^2(\omega) / \{-2 \ln\left[\frac{-\pi}{T} \ln(1-r)\right]\} \quad (11)$$

This spectral density is then used to compute the associated response spectrum

$$R_1(\omega) = \left\{ -2m_0 \ln\left[-\frac{\pi}{T}(m_0/m_2)^{1/2} \ln(1-r)\right] \right\}^{1/2} \quad (7)$$

The result is denoted by  $R_1(\omega)$  and is generally different from the given  $R(\omega)$  since the spectral moments  $m_0$  and  $m_2$  on the right hand side are associated with the approximate  $f_1(\omega)$ . The second approximation for  $f(\omega)$  is obtained from

$$f_2(\omega) = f_1(\omega) [R(\omega)/R_1(\omega)]^2 \quad (19)$$

The iteration process can be repeated according to

$$f_{i+1}(\omega) = f_i(\omega) [R(\omega)/R_i(\omega)]^2 \quad (20)$$

until a desired convergence accuracy is reached. This iteration procedure appears to be more convenient than that suggested by Kaul. Note also that the scaling procedure indicated in Eq. 20 is the same as that employed by Vanmarcke

and Gasparini, but the way to obtain the initial estimate for the spectral density  $f_1(\omega)$  is different.

Numerical examples given by the authors show that the initial estimate obtained from Eq. 11 is mostly unconservative. However, the converged results are slightly conservative, except in the high frequency range, where they are overly conservative by as much as 40% when comparing with the prescribed peak ground acceleration.

It is of interest to note that the authors are also interested in other types of dynamic excitations, and the response spectrum/spectral density conversion procedure has been applied to instrument qualification testing with the duration of dynamic loads as short as two seconds.

### 5.1.5 Sundararajan's Method

This is another perturbation scheme (Reference 32) to find a response-consistent power spectrum by iteration, with an additional assumption that the spectral density may be approximated by straight-line segments between specified frequency points  $\omega_i$ . The basic relationships used are again

$$R_v(\omega_i, \beta) = v_{\max}(\omega_i, \beta) = \omega_i d_{\max}(\omega_i, \beta) \quad (21)$$

and

$$d_{\max}(\omega_i, \beta) = k(\omega_i) \sigma_d(\omega_i, \beta) \quad (22)$$

where  $R_v(\omega_i, \beta)$  is the velocity response spectrum at frequency  $\omega_i$  corresponding to a damping ratio  $\beta$ ,  $d_{\max}$  and  $v_{\max}$  are the maximum displacement and velocity response, respectively and  $k(\omega_i)$  is the peak factor computed from either the Davenport's formula

$$k = \frac{\{2 \ln(\frac{T \sigma_v}{\pi \sigma_d}) + 0.577\}^{1/2}}{[2 \ln(\frac{T \sigma_v}{\pi \sigma_d})]^{1/2}} \quad (23)$$

or the Amin-Gungor formula

$$k = 2 \ln \left[ - \frac{T \sigma_v}{\pi \sigma_d} \frac{1}{\ln(1 - P_e)} \right]^{1/2} \quad (24)$$

In Eqs. 23 and 24,  $T$  = duration of strong motion earthquake,  $\sigma_d$  = standard deviation of displacement response,  $\sigma_v$  = standard deviation of velocity

response, and  $P_e$  = probability of exceedance. Eq. 24 is equivalent to Kaul's formula, Eq. 7. The authors believe that the choice of either Eq. 23 or Eq. 24 does not influence the results significantly as long as the choice is consistent, quoting a statement attributable to Singh and Chu. However, Eq. 24 has the flexibility of adjusting to a required probability of exceedance  $P_e$ .

The iteration begins with a white noise assumption from which the initial estimates of  $\sigma_d$ ,  $\sigma_v$  and  $k$  can be computed. These are used to obtain the starting solution for the spectral density  $S^{(0)}(\omega_i)$ ; namely,

$$S^{(0)}(\omega_i) = \frac{4B\omega_i}{\pi k^2} R_v^2(\omega_i, \beta); \quad i = 1, \dots, N \quad (25)$$

This spectral density is used to compute  $V_{max}$  according to Eqs. 21 and 22, and the percent error is evaluated by comparing  $V_{max}$  with the target  $R_v$ . If the error is greater than the required tolerance, a new spectral density is obtained from

$$S^{(1)}(\omega_i) = C_1 S^{(0)}(\omega_i) = \left(\frac{R_v}{V_{max}}\right)^2 S^{(0)}(\omega_i); \quad i = 1, \dots, N \quad (26)$$

Nonstationarity in the ground motion process is accounted for by the use of an effective damping for the oscillator

$$\beta_e = \beta + (2/\omega T)$$

which is a formula attributable to Rosenblueth and Elorduy.

This paper is similar to the paper of Unruh and Kana (Reference 31) in that both are based on the analysis of Kaul (Reference 27), but the perturbation schemes are different.



## 5.2 Simulation Approach

### 5.2.1 Shinozuka's Method

Artificial earthquakes are generated in three steps (Reference 33). First, the Kanai-Tajimi spectrum, enhanced in the higher frequency-range, is used to generate sample functions of a stationary random process according to

$$\zeta_0(t) = \sqrt{2} \sum_{k=1}^N \sqrt{S_0^*(\omega_k) \Delta\omega} \cos(\omega_k t + \phi_k) \quad (27)$$

where  $\phi_k$  are independent realizations of the random variable  $\phi$  uniformly distributed in  $(0, 2\pi]$ , and where

$$S_0^*(\omega) = S_0 \frac{1 + 4\zeta_g^2(\omega/\omega_g)^2}{[1 - (\omega/\omega_g)^2]^2 + 4\zeta_g^2(\omega/\omega_g)^2} F(\omega) \quad (28)$$

$$F(\omega) = [1 + p(\frac{\omega}{\omega_c})^2] / [1 + q(\frac{\omega}{\omega_c})^2], \quad p > q > 0 \quad (29)$$

Next, these sample functions are modified by "fractional folding" beyond the  $\pm 1g$  level to obtain  $\zeta_2(t)$  (Figure 6). Finally, artificial earthquakes are obtained by multiplying  $\zeta_2(t)$  by an envelope  $g(t)$  (Figure 7), namely,

$$z_2(t) = g(t)\zeta_2(t) \quad (30)$$

If a generated  $z_2(t)$  does not satisfy NRC RG 1.60, another will be generated using a different set of  $\phi_k$ . This is repeated until the desired number of artificial earthquakes satisfying the requirement are obtained.

This approach differs from the others in that no attempt is made to match the response spectrum with  $k\sigma$  where  $\sigma$  is the r.m.s. response and  $k$  is an appropriate peak factor. Instead, the NRC RG 1.60 requirement is satisfied (perhaps conservatively) on the basis of sample functions. Fractional folding of sample functions to satisfy the 1.0g peak acceleration limit will have some consequences. For example, the underlying random process is no longer Gaussian, and the spectral density is no longer  $S^*_o(\omega)$ . However, the authors are not concerned with these properties since the generated artificial earthquakes are used for Monte Carlo calculations. The use of a smooth spectral density, such as Eq. 28, is to avoid possible deficiency of energy in any frequency window. The spectrum enhancement is implemented to raise the level in the higher frequency range to clearly above a target PSD, since the computed PSD based on a single artificial earthquake is found to be highly oscillatory in this range.

The above algorithm is the basis for a proposed NRC revision to the Standard Review Plan (Reference 34).

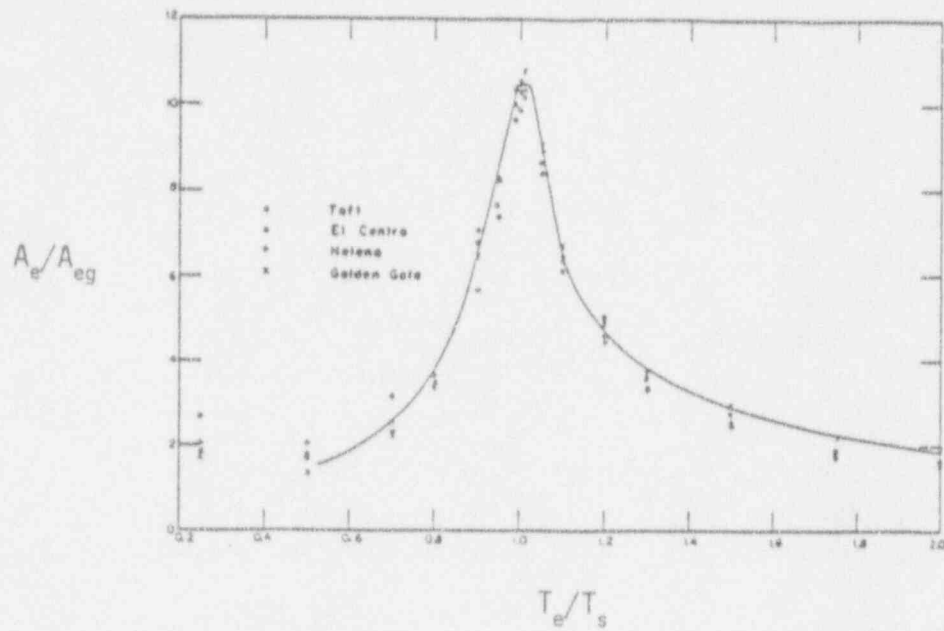
### 5.2.2 Kennedy & Shinozuka's Method

The method (Reference 35) is based on a premise that the design response spectrum is the primary requirement, and that the PSD requirement is used to prevent a severe deficiency of power over any frequency range, not to add additional conservatism beyond that contained in the design response spectrum. Thus, a minimum PSD requirement compatible with RG 1.60 is recommended. Furthermore, the frequencies at which 10%, 50% and 90% of the cumulative power occurs are believed to be important descriptors of the ground motion.

Based on these observations, the following PSD requirement (called PSD 3) is recommended.

$$S_o(\omega) = \begin{cases} 650 \text{ in}^2/\text{s}^3 & (f/2.5)^{0.2} & f \leq 2.5 \\ 650 \text{ in}^2/\text{s}^3 & (2.5/f)^{1.8}, & 2.5 \leq f \leq 9 \\ 64.8 \text{ in}^2/\text{s}^3 & (9/f)^3, & 9 \leq f \leq 16 \\ 11.5 \text{ in}^2/\text{s}^3 & (16/f)^8, & f \geq 16 \end{cases} \quad (31)$$

where  $f = \omega/2\pi$ . A peak acceleration of 1.0g is assumed, and it should be scaled for other peak accelerations. A minimum requirement is set at 80% of PSD 3, and a conservative envelope at 130% of PSD 3. The authors claimed that time histories generated by PSD 3 and clipped at  $\pm 1.0g$  fits the 2% damped, 1.0g, RG 1.60 Response Spectrum accurately at all frequencies above 0.25 Hz.



Structural Damping 0.04

Equipment Damping 0.005

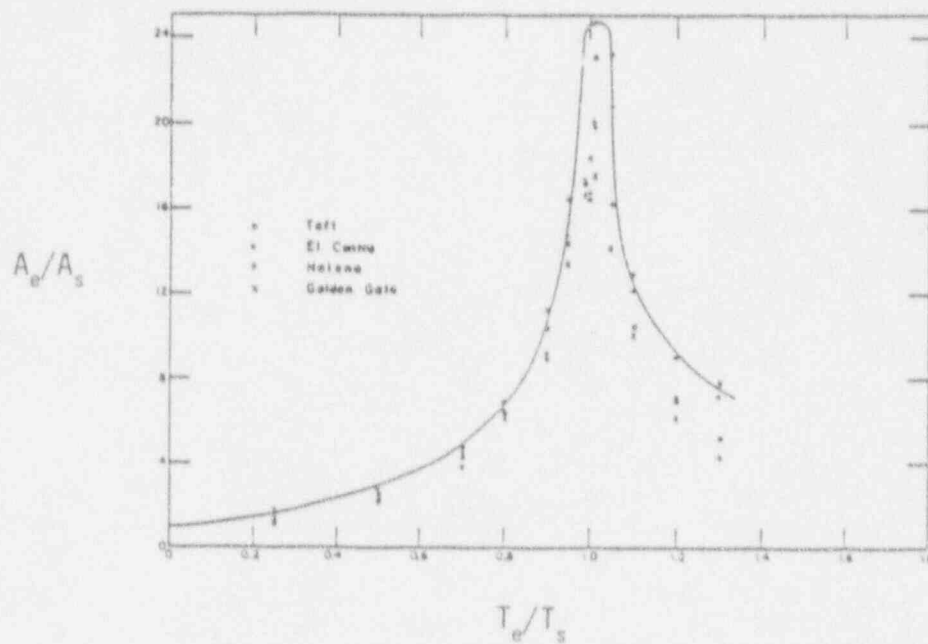
$A_e$ : Peak Equipment Acceleration

$A_{eg}$ : Acceleration of Equipment if Ground Supported

$T_e$ : Equipment Period

$T_s$ : Structural Period

Figure 1: Amplification of Ground Motion  
(Biggs-Roesset Direct Deterministic Method)



Structural Damping 0.04

Equipment Damping 0.005

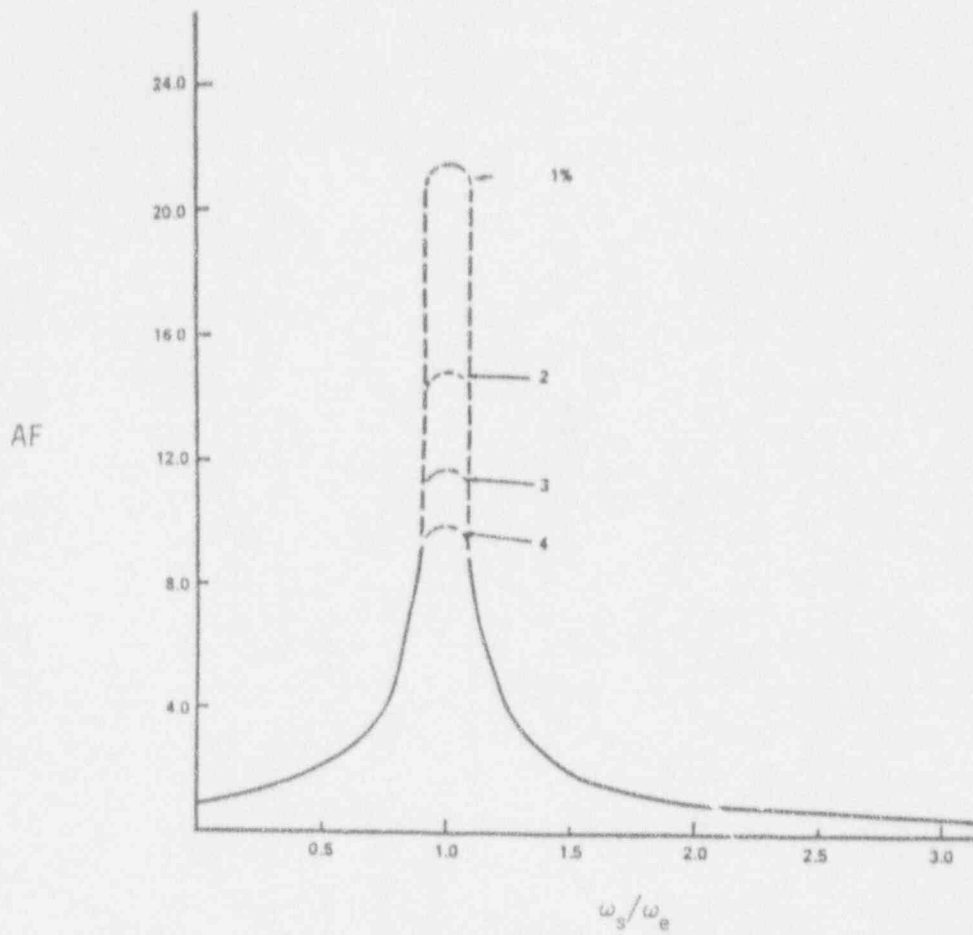
$A_e$ : Peak Equipment Acceleration

$A_s$ : Maximum Floor Acceleration

$T_e$ : Equipment Period

$T_s$ : Structural Period

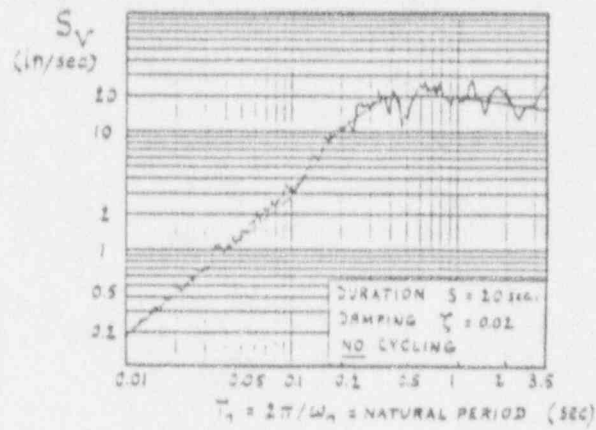
Figure 2: Amplification of Structural Motion  
(Biggs-Roesset Direct Deterministic Method)



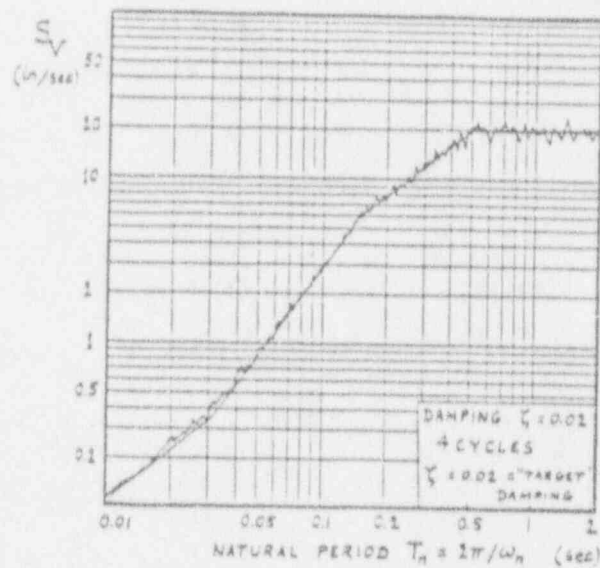
Structural Damping: 2%  
 Equipment Damping: 1, 2, 3 and 4%  
 AF: Amplification Factor  
 $\omega_s$ : Structural Frequency  
 $\omega_e$ : Equipment Frequency

Figure 3: Amplification Factors  
 (Kapur-Shao Direct Deterministic Method)





(a)



(b)

Figure 4: Simulated and Target Response Spectra  
(Vanmarke-Gasparini Method)

(a) No Iterations

(b) After 4 Iterations

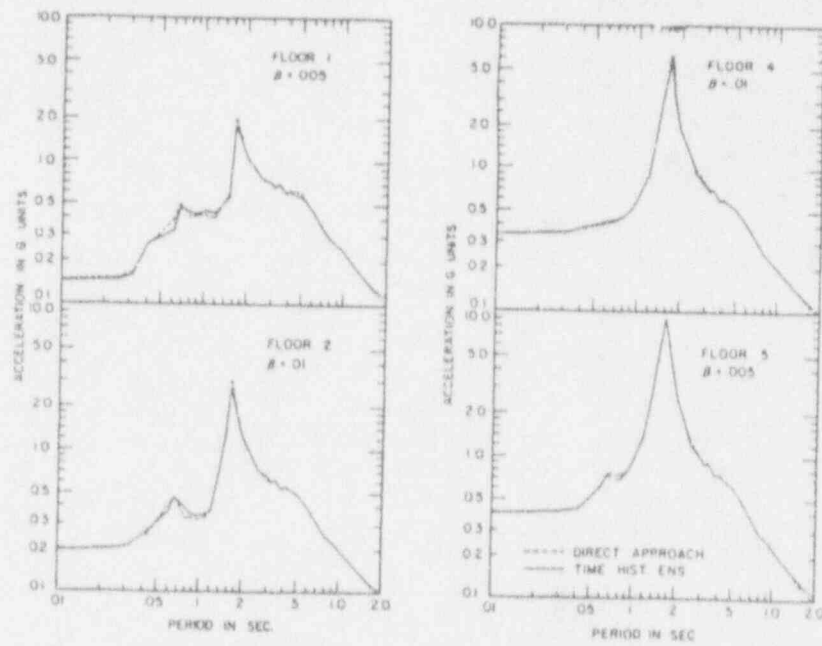


Figure 5: Time History Floor Spectra Versus  
 Direct Generation Floor Spectra  
 (Singh's Method) of a Five-Story Structure

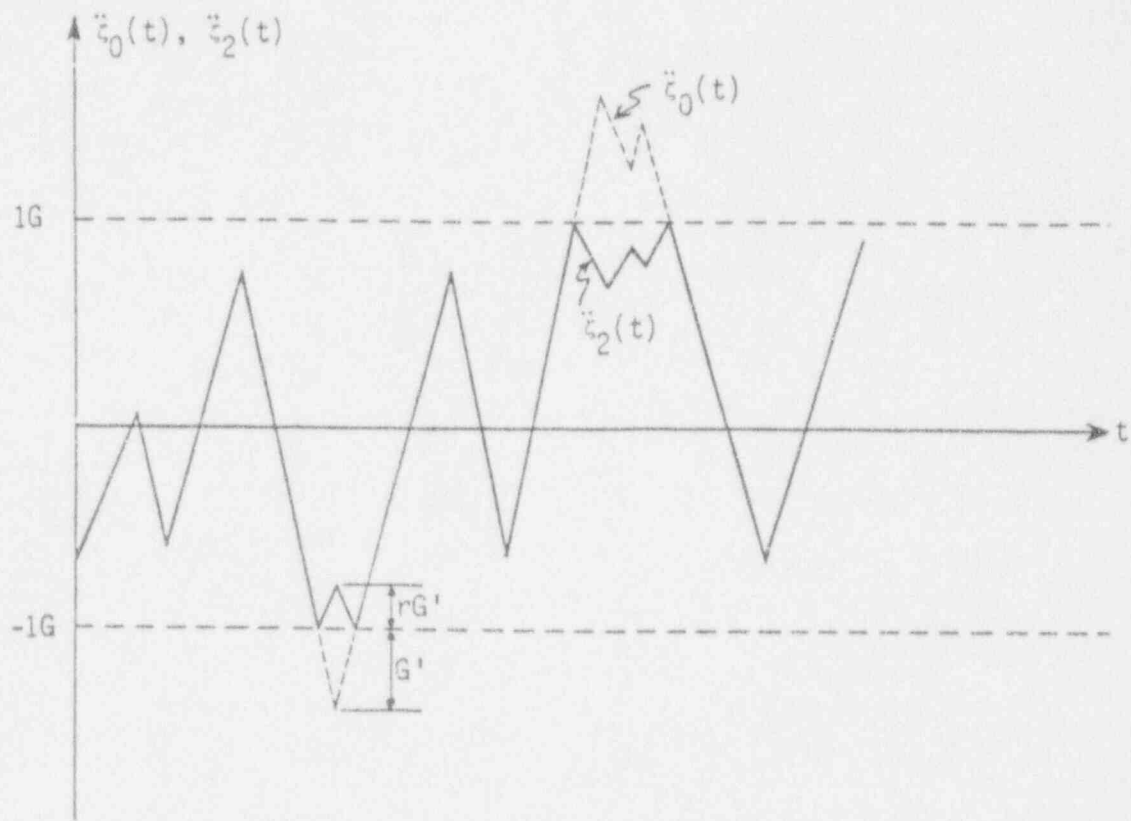


Figure 6: Fractional Folding ( $0 \leq r \leq 1$ )

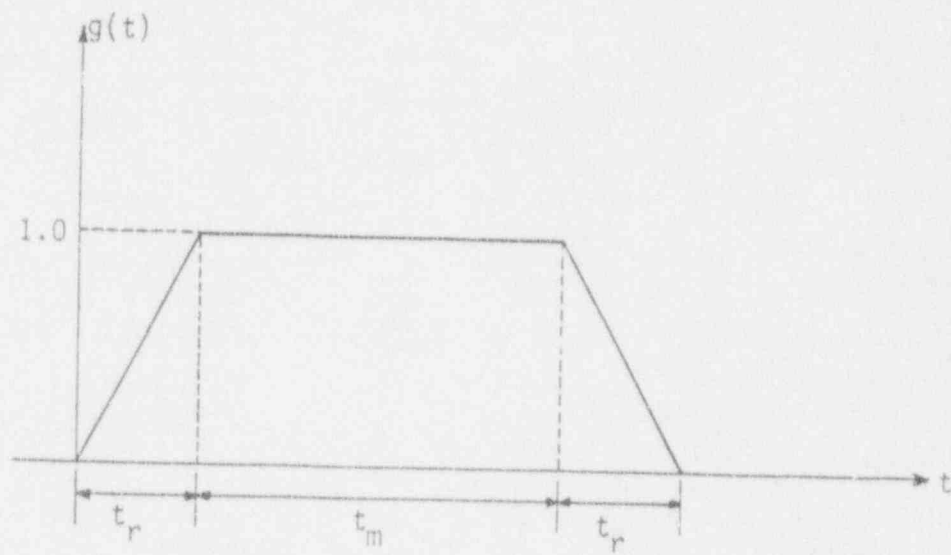


Figure 7: Deterministic Envelope Function

## 6.0 APPLICABILITY AND VERIFICATION OF DIRECT GENERATION METHODS

From our review of existing methods on direct generation, we noted that they primarily concentrate on the response spectrum to spectral density conversion. This problem is admittedly an important ingredient of existing direct generation probabilistic methods and it is expected to continue attracting the interest of researchers and practitioners in the near future. It should be pointed out, however, that some effort should be also spent in future studies, towards addressing the applicability of the method to other related problems of the seismic analysis process of nuclear facilities. Subsections 6.1 and 6.2 discuss briefly the cases of soil-structure interaction (SSI) and nonlinear response respectively. There are, of course, other applications of the method as well, e.g., seismic qualification of equipment attached to secondary systems, piping seismic analysis etc. This subject, however, is beyond the scope of the present study. Another equally important issue is the verification of direct generation methods which is briefly discussed in Subsection 6.3.

## 6.1 Applicability of Direct Generation to Soil-Structure Interaction

Theoretically, existing direct generation methods can be easily adopted to dynamical problems involving soil-structure interaction (SSI) effects. Published studies on direct generation are essentially concerned with classical or nonclassical type structural modes which are blended into a probabilistic formulation of structural responses (usually floor acceleration). One could extend the direct generation case dealing with nonclassical modes to include SSI effects. This approach, however, does not seem to be appealing. It would be more appropriate to utilize available SSI methods to perform direct generation of the probabilistic type. Specifically, having established an appropriate free-field spectral density from the corresponding design response spectrum, which defines the control motion, one can proceed with the solution of the free-field and the soil-structure problems to compute output spectral densities. The latter can be associated with the motion of the free-field, foundation and desired structural locations.

In this regard, an effort was made to extend the deterministic SSI methodology of the FLUSH type to a probabilistic one by developing the PLUSH computer code (Reference 41). For this purpose Vanmarke's method was employed (see subsection 5.1.1 of this report). Only limited numerical examples from the application of the PLUSH code are currently available. Consequently, no generic conclusions can be reached. In recent seismic re-evaluations of nuclear facilities some utilities have also used direct generation methods in conjunction with SASSI/CLASSI type SSI methodologies. These studies are currently under NRC's review.

Another related problem is the specification of the input to the soil-structure system in terms of a cross-spectral density matrix rather than a single power spectral density. Preliminary work on this subject, using a multivariate stochastic process idealization of the input seismic motion, was performed by Shinozuka and Deodatis (Reference 42). It appears that this approach offers a logical extension of existing direct generation methods.

Based on our review, we conclude that although direct generation methods are theoretically suitable for SSI problems, there is a need for numerical results which demonstrate the range of applicability of direct generation to SSI. Existing data are rather limited. Additional data can be obtained using a comprehensive set of benchmark problems accounting for both soil and structural parameters.



## 6.2 Applicability of Direct Generation to Nonlinear Response Problems

The time history approach is traditionally considered as the primary tool for conducting nonlinear seismic analysis of nuclear systems and components. Approximate techniques have also been used to account for a limited inelastic energy absorption in seismic response evaluations. Such techniques typically follow Newmark's approximate method for constructing inelastic response spectra and are believed to generally produce conservative results as compared to the time history approach. Inelasticity is usually treated in terms of the ductility factor, e.g., system or member ductility and story drift ductility factor.

The construction of approximate inelastic response spectra from corresponding elastic ones resembles, in a broad sense, a "response spectrum linearization" process. On a different scale, linearization techniques, such as statistical linearization are often used by the nuclear industry to compute the response of nuclear piping systems with energy absorbers. In essence, such techniques (Reference 19) allow for the use of linear methods of analysis in nonlinear problems. The general nonlinear case, however, is typically treated on the basis of the time history approach, by direct integration of the nonlinear equation of motion.

Although the direct generation approach has been used thus far for the basic purpose of constructing floor response spectra, its use can be expanded to address inelastic response problems encountered in nuclear plant seismic evaluations. It is to be noted that the direct approach utilizes the concept of the power spectrum which does not contain any structural filtering effects, as in the case of a response spectrum. Accordingly, it is expected that the applicability of direct generation to inelastic response problems would be more advantageous to response spectrum techniques. It is recommended that a systematic effort be undertaken to address the applicability of direct generation methods to nonlinear seismic problems.

### 6.3 Verification of Direct Generation Methods

Verification efforts, in existing studies of direct generation, are focused on comparisons between the results obtained from the direct generation process and the corresponding results from application of the time history method. The motivation is basically to demonstrate that direct generation produces results which agree well with those of the time history method. It is important, however, that the direct method be verified on the basis of earthquake data. We are aware of the effort by Duke Power Company to correlate the direct generation method with recorded data from the LeRoy Ohio Earthquake using a model of the Perry containment. It appears, however, that while direct generation correlated reasonably well with time-history results, both methods fail to accurately predict the actual recorded data. This was attributed to insufficiently refined structural models utilized in this particular verification study. It is recommended that additional effort be made to enable acceptable correlations between direct generation results and actual seismic data.

## 7.0 CONCLUSIONS

The key issue with the time history analysis method is the derivation of spectrum consistent acceleration time histories. Concerns regarding this issue have been raised but it seems that reliable methods are currently available to treat this problem. One for example can proceed deterministically to derive such time histories using Kauls' method (Reference 38) or its extension to multi-damping case (Reference 39). The statistical approach by Spanos (Reference 20) is equally acceptable for this purpose. Such methods produce time histories which satisfy NRC's enveloping requirements without paying the price of artificial conservatism in the analysis, which has been a major complaint against the time history analysis method. Possible deficiencies in the analysis due to the non-uniqueness of the solution could be further addressed through multiple time history analysis.

Direct generation methods of the early deterministic type do not seem to offer a clear advantage over the time history approach. This conclusion is partially influenced by the fact that insufficient parametric variations are currently available. It is realized that proceeding deterministically from the design spectrum to the floor response spectrum is an appealing idea. Published studies, however, seem to repeatedly use conservative assumptions throughout the analysis which makes their overall assessment a difficult task. From a limited number of comparative numerical results, it has been demonstrated that they produce higher floor response spectra than those produced by time history analysis. It is thus recommended that direct generation methods of the deterministic type receive additional attention towards rectifying existing deficiencies and developing a consistent deterministic approach for going from the design spectrum to the floor response spectrum.

The key issue with the direct generation methods of the probabilistic kind is the interpretation of a given design ground spectrum in terms of a stochastic process. Typically a stationary model of such a process is assumed and subsequently its parameters are evaluated. Conclusions drawn from the review of existing methods described in section 5 of this report are summarized below:

- (1)  $R(\omega, \beta)$  defines a family of admissible functions. These admissible functions constitute a stochastic process. Clearly, such a process is nonstationary in time, and it must satisfy the requirement that the absolute maximum is fixed at the value of the peak acceleration of  $R(\omega, \beta)$ . This process is not easily amenable to analytical treatments. Furthermore, different  $\beta$  values correspond to different stochastic processes.
- (2) For a random vibration analysis to be carried out simply, the assumption of a stationary Gaussian process, with or without a time envelope is usually made. Such a process is not dependent on the structural parameter  $\beta$ , and is clearly not compatible with the stochastic process implied in  $R(\omega, \beta)$ . Thus, a choice between the two types of representations must eventually be made.

- (3) One shortcoming in the Gaussian model is the inability to limit the peak acceleration value. If this deficiency is deemed important, then an entirely new model should be used which has similar analytical advantage as that of a Gaussian model. Possible candidates are the random-pulse-train models developed by Lin and associates.

Time history methods and direct probabilistic methods of floor spectra generation should yield credible results, if properly implemented. Obviously the results from both methods must compare well since they provide solutions to the same problem. In this context, a meaningful comparison between the two methods should not be based on the results from a single time history analysis but rather on the results from a series of time history analyses.

Both the time history analysis method and the direct generation probabilistic method are based on the fundamental premise that the input to the structural response analysis is represented by a ground design response spectrum. Analytically, it is preferable to start the seismic response analysis process by making available to the analyst a set of appropriate time histories or their Fourier spectra. Starting with prescribed design spectra and working backwards to determine other forms of the seismic input suitable to a particular analysis was shown over the years to cause controversies. Their resolution is typically handled by debating over conservatism which in many cases is not well defined. Starting, however, with appropriate time histories or Fourier spectra, a subsequent analysis to determine floor response spectra using the time history method or the probabilistic direct generation method or any other method for that matter can be adequately assessed. It is thus recommended that the specification of the seismic input for structural response analysis in terms of design response spectra be carefully re-examined.

The direct generation probabilistic approach appears to have in many aspects more advantages than the time history approach: has better computational efficiency, offers a rational interpretation of floor response spectra and is suitable for nonlinear analysis, seismic margins and risk assessment studies. It is recommended that future studies concentrate on application of the method to soil-structure interaction and nonlinear response analysis. Furthermore, additional correlations with recorded data are recommended.

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