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BALLISTIC ANALYSIS OF FREE-FALLING PAT PACKAGES

December 13, 1989

J. H. VanSant

Prepared for
U.S. Nuclear Regulatory Commission

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ABSTRACT

This report contains the results of a study of the ballistic characteristics of free-falling plutonium air transport (PAT) packages. The study provides information to support the criteria for a PAT package drop test that are specified in Ref. 1 and a target accuracy analysis presented in Ref. 2. Should the postulated drop test be performed, a package will be released from an aircraft and allowed to fall freely to the ground. The principal influences on package ballistics are the ballistic number, air density, and wind velocity. The ballistic values were calculated using a finite-difference computing method. The results obtained by this method are in good agreement with the values obtained from closed-form equations. Characteristic curves are presented for the example package (2.4 m x 1.2 m diameter cylinder weighing 2.6 Mg). A general correlation for the sea-level-impact velocity of objects with respect to a characteristic parameter, defined as the ballistic number and the package ballistics for a suggested drop method are presented. The method suggested makes use of drag parachutes to provide a low horizontal package velocity as the package is released from the drop aircraft. Only a portion of the data generated for this study is presented to show examples of package ballistics. Packages to be drop-tested may have characteristics that are different from those presented in this report.

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NOMENCLATURE

Symbols:

A	=	aerodynamic profile area, m^2
Bn	=	ballistic number, $W/A\text{-}C_d$, Mg/m^2
C	=	coefficient
ρ	=	density, kg/m^3
d	=	derivative
F	=	force, N
g	=	gravitational acceleration, m/s^2
t	=	time, s
V	=	velocity, m/s
W	=	weight, kg
x	=	horizontal displacement, m
y	=	altitude, m

Subscripts:

a	=	acceleration
d	=	drag
g	=	gravity
M	=	Mach number
s	=	sonic
R	=	Reynolds number
w	=	wind
x	=	horizontal direction
y	=	vertical direction

1. INTRODUCTION

The criteria specified in Ref. 1 for drop-testing a plutonium air transport (PAT) package are based, in part, on ballistic analyses of candidate packages. The method and results of these supporting analyses are presented in this report.

The criteria in Ref. 1 support the drop test requirements defined in Section 5062 of Public Law 100-203, which specifies that PAT packages to be certified by the Nuclear Regulatory Commission (NRC) must be dropped from the maximum cruising altitude of the designated cargo aircraft. Reference 1 allows the test package to be dropped from a lower altitude, provided the resulting ground impact velocity is not less than from the maximum cruising altitude. Ballistic analysis indicates this allowance is probably applicable to most PAT packages that are to be transported in jet cargo aircraft. The maximum cruising altitude of these aircraft is usually very high (e.g., 10 km), and the packages can reach their maximum velocity before impacting the ground. The analyses that were performed considered altitudes to 12.2 km (40 kft).

In this study ballistic analysis utilizes a numerical computing method that determines package velocity, altitude, and horizontal displacement after release at a specified altitude, velocity, and direction. The analysis includes the effects of altitude-dependent air density, wind velocity, and compressibility, and side and axial fall orientations of the package.

The results of the analyses are given for an example package, one that resembles an expected drop test package. The example package is a right-circular cylinder, with a diameter of 1.2 m, a length of 2.4 m, and a weight of 2.6 Mg. Its surface is assumed to be smooth and without protrusions or irregularities. On the other hand, a test package would probably have different surface characteristics that would cause a higher aerodynamic drag, so the test package's drag characteristics may have to be measured in an independent test. (Published drag coefficients are usually only for ideal geometries having smooth surfaces.)

Based on the analysis, packages released from high-altitude aircraft can have a large horizontal displacement at the ground impact point. This displacement adversely influences the target accuracy. Target accuracy can be improved with a modified drop method — one that uses drag parachutes to reduce the package velocity before it is released for free-fall. A ballistic analysis of this drop method is included.

2. CALCULATION METHODOLOGY

2.1 Equations of Motion

The time-dependent values of velocity and altitude are calculated by means of a numerical finite-difference computing method. The applicable equations, obtained from a balance of forces on a free-falling package, are:

$$F_g = F_{ay} + F_{dy} \tag{1}$$

where:

F_g = gravity force = W,
F_{ay} = vertical acceleration force = $(W/g)\, dV_y/dt$,
F_{dy} = vertical drag force = $C_{dy} A_y \rho V_y^2/(2g)$,
A_y = vertical direction profile area,
C_{dy} = vertical direction drag coefficient,
V_y = vertical direction velocity.

The force balance given by Eq. (1) yields expressions for finite velocity change and the velocity at the end of a time step, Δt.

$$\Delta V_y = \Delta t\,[g - C_{dy} A_y \rho V_y^2/(2w)]. \tag{2}$$

$$V_{y2} = V_{y1} + \Delta V_y. \tag{3}$$

The velocity term, V_y, in the right side of Eq. (2) is the average velocity during the time step. Its value can be determined by an iterative computing method. This entails computing V_{y2} several times during each time step and updating V_y each time.

The corresponding altitude change is determined by using the average velocity during each time step Δt.

$$\Delta y = \Delta t\,(V_{y1} + \Delta V_y/2). \tag{4}$$

$$y_2 = y_1 - \Delta y. \tag{5}$$

The horizontal velocity and displacement is likewise derived from a balance of forces in the horizontal direction.

$$F_{ax} = -F_{dx} \tag{6}$$

where:

F_{ax} = horizontal acceleration force = $(W/g)\, dV_x/dt$,

F_{dx} = horizontal drag force = $C_{dx} A_x \rho V_y^2/(2g)$,

A_x = horizontal direction profile area,

C_{dx} = horizontal direction drag coefficient,

V_x = horizontal direction velocity.

Horizontal velocity and displacement are calculated by the same method used to determine vertical velocity and altitude (Eqs. [2], [3], [4], and [5]).

$$\Delta Vx = \Delta t\,[C_{dx} A_x \rho V_x^2/(2w)]. \tag{7}$$

$$V_{x2} = V_{x1} + \Delta V_x. \tag{8}$$

$$\Delta x = \Delta t\,(V_{x1} + \Delta V_x/2). \tag{9}$$

$$x_2 = x_1 + \Delta x. \tag{10}$$

The numerical process is initiated with prescribed initial conditions such as package release orientation, velocities, and altitude. Next, the numerical process executes Eqs. (3), (5), (8), and (10) to find velocity and position values at successive time values, until the altitude is less than zero.

The composite velocity and fall angle of the package is determined from the directional velocities:

$$V = (V_x^2 + V_y^2)^{1/2}. \tag{11}$$

$$\text{angle} = \arctan(V_y/V_x). \tag{12}$$

2.2 Influence of Winds

Packages released from high altitudes can be subjected to winds that influence the package's horizontal displacement. This influence is incorporated into the horizontal velocity Eq. (7) by replacing the velocity term V_x in the right side of the equation with a relative velocity term.

$$V_x^2 = (V_x - V_w)\ \text{abs}\ (V_x - V_w) \tag{13}$$

where:

V_w = wind velocity,

abs = absolute value function.

Reference 3 contains synthetic wind velocity profiles for modeling of wind and altitude relationships. The models show a jet stream occurring at 10.7 km (35 kft) altitude and the wind speed decreasing rapidly when moving either up or down from this altitude. Reference 3 gives an example jet stream velocity of 45 m/s (100 mph) occurring at 50% frequency. This example wind is used in the free-fall calculations to determine a relative effect on package trajectory. During a package drop test program, wind profile data shouid be obtained at the test area and used in ballistics calculations to estimate a more reliable trajectory.

The following expressions approximate the example wind profile shown in Fig. 1. They are used to compute a wind velocity for the package motion Eq. (11).

$$V_w = 7.62 \exp(-1.51 \times 10^{-4} y), \text{ m/s}, \qquad 0 < y < 10.7 \text{ km},$$

$$V_w = 1.02 \times 10^{12} y^{-2.57}, \text{ m/s}, \qquad y > 10.7 \text{ km}. \tag{14}$$

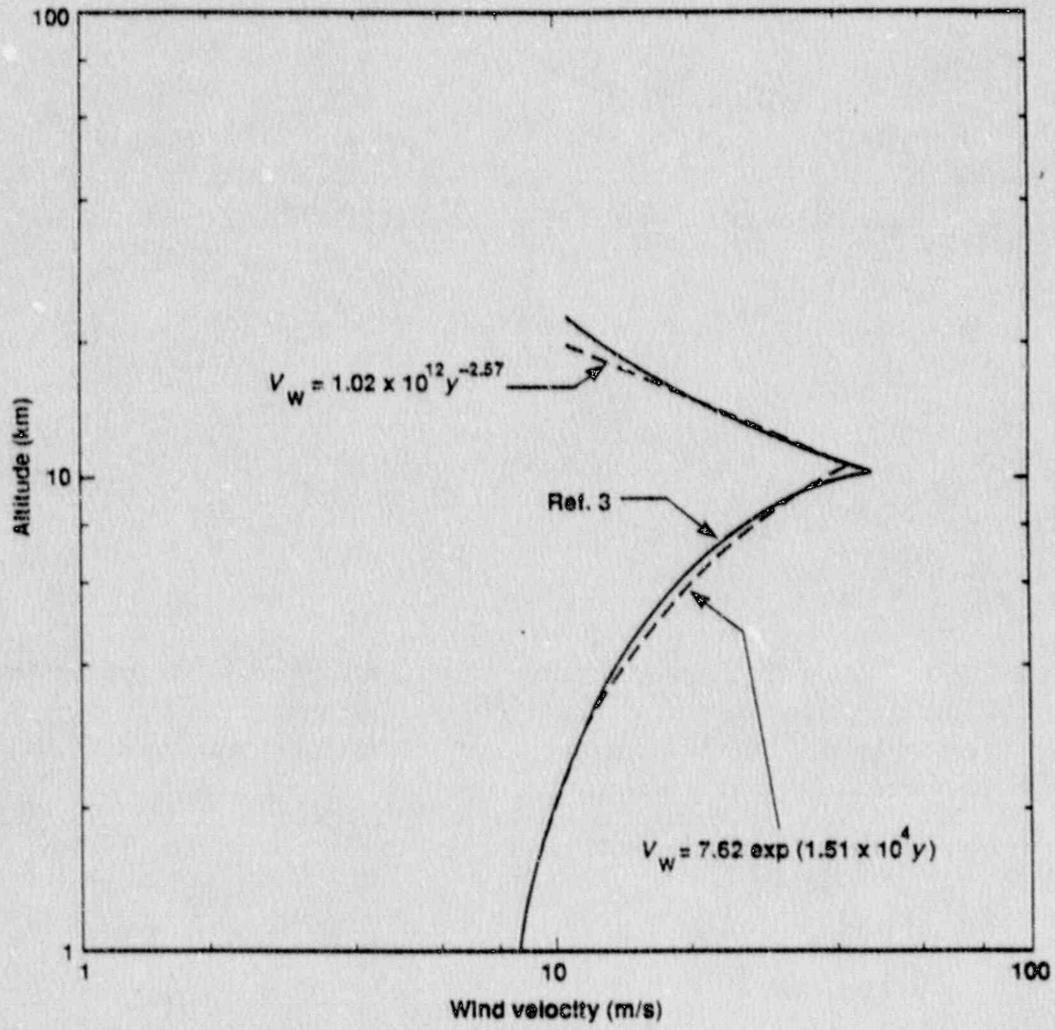

Fig. 1. An example wind velocity profile.

2.3 Drag Coefficients

The drag coefficient of objects moving in a fluid is related to two principal dimensionless parameters — the Reynolds and Mach numbers. The first relates to momentum and viscous forces exerted by the fluid, and the second relates to adiabatic compression forces of the fluid. A drag coefficient value is determined from the product of these two parameter effects.

$$C_d = C_M C_R \tag{15}$$

where:

C_d = value used in Eqs. (2) and (7),

C_M = Mach number correction factor,

C_R = Reynolds number drag coefficient.

Appropriate values of the drag coefficient may be found in the literature (Refs. 4-6) for smooth cylinders for Reynolds numbers up to approximately 2×10^6. Reynolds numbers for the example package free-fall velocities are greater than 10^7. But, the consulted references indicate the drag coefficient changes relatively little when the Reynolds number is greater than about 10^6. Therefore, the selected Reynolds drag coefficient, C_R, for the example package is 0.85 for air flow parallel to the cylinder axis (axial) and 0.35 for air flow normal to the axis (side).

References 4-6 do not provide data on drag coefficients for finite cylinders at orientations other than side or axial. Therefore, only side and axial orientations are considered in the calculations. Coefficient values for other orientations are expected to be between the side and axial coefficients.

The drag coefficient for cylinders is also dependent on their aspect ratio (length/diameter). Data given in Ref. 4 indicate that, when the flow is to the cylinder side and the aspect ratio is less than 3, the drag coefficient is nearly constant. But, when flow is axial, the drag coefficient varies more strongly with aspect ratio. Figure 2 illustrates this effect: it shows curves representing data from two sources (Refs. 4 and 6). Note that there is a marked difference in the two curves. The reason that one curve has a minimum is not understood; nonetheless, the drag behavior indicated by the second curve (Ref. 4) is more plausible. Therefore, the curve from Ref. 4 was used to modify the Reynolds drag coefficient, C_R, with respect to aspect ratio.

The drag coefficient at low subsonic velocities is little affected by the Mach number. However, because package velocities are sometimes in the transonic range, the Mach number influence is included in the free-fall calculations. Its effect is accounted for by applying a correction factor, as in Eq. (15). Reference 6 gives values of C_M only for axial flow. Side flow values are assumed to be approximately the same as for axial flow. The curve shown in Fig. 3 represents the data from Ref. 6.

Figure 3 also shows the following approximating expressions used to compute values of C_M for Eq. (15).

$$C_M = 1 + 1.54\,M^4, \qquad 0.3 < M < 0.5,$$
$$C_M = 2 - \exp[-2.4\,(M - 0.5)], \qquad 0.5 < M < 1.0. \tag{16}$$

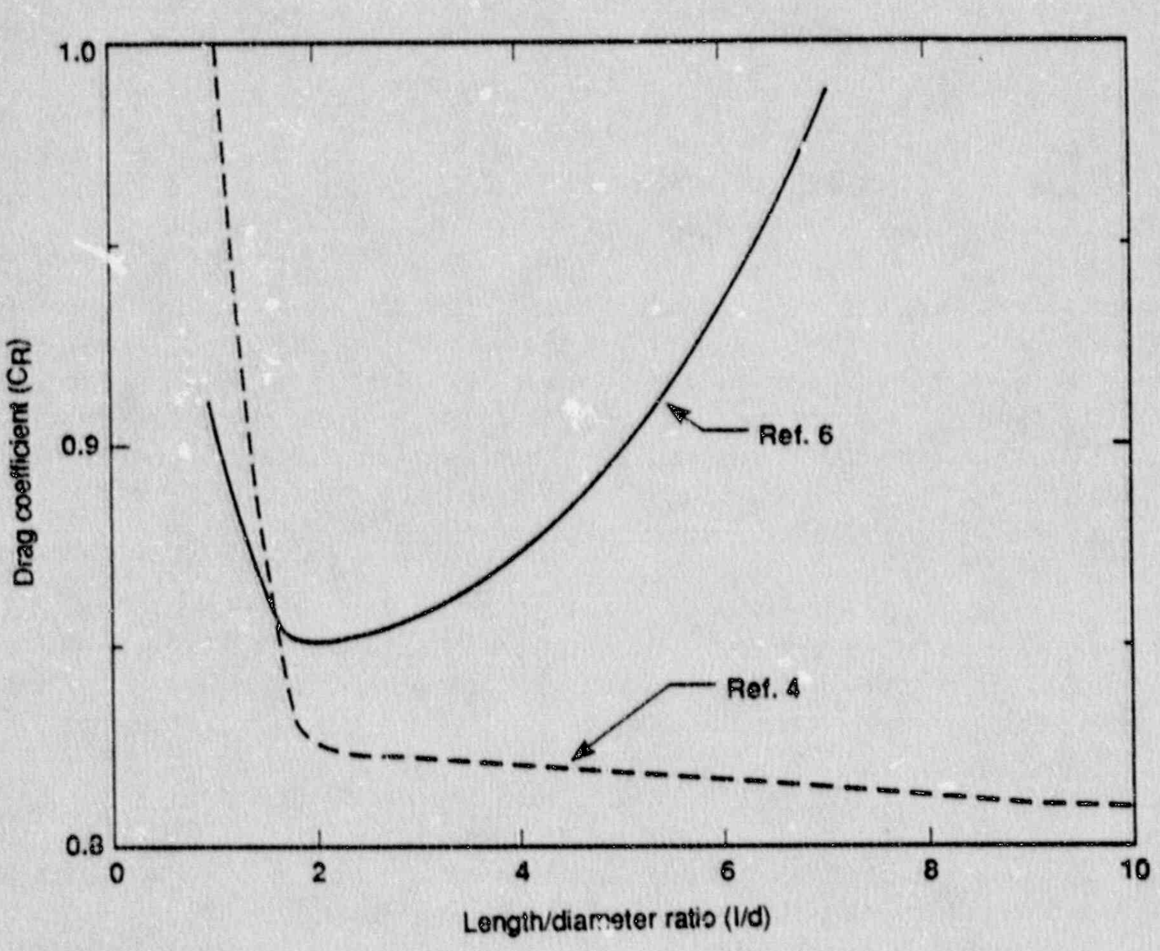

Fig. 2. Aspect ratio effect on the drag coefficient for finite cylinders in axial flow.

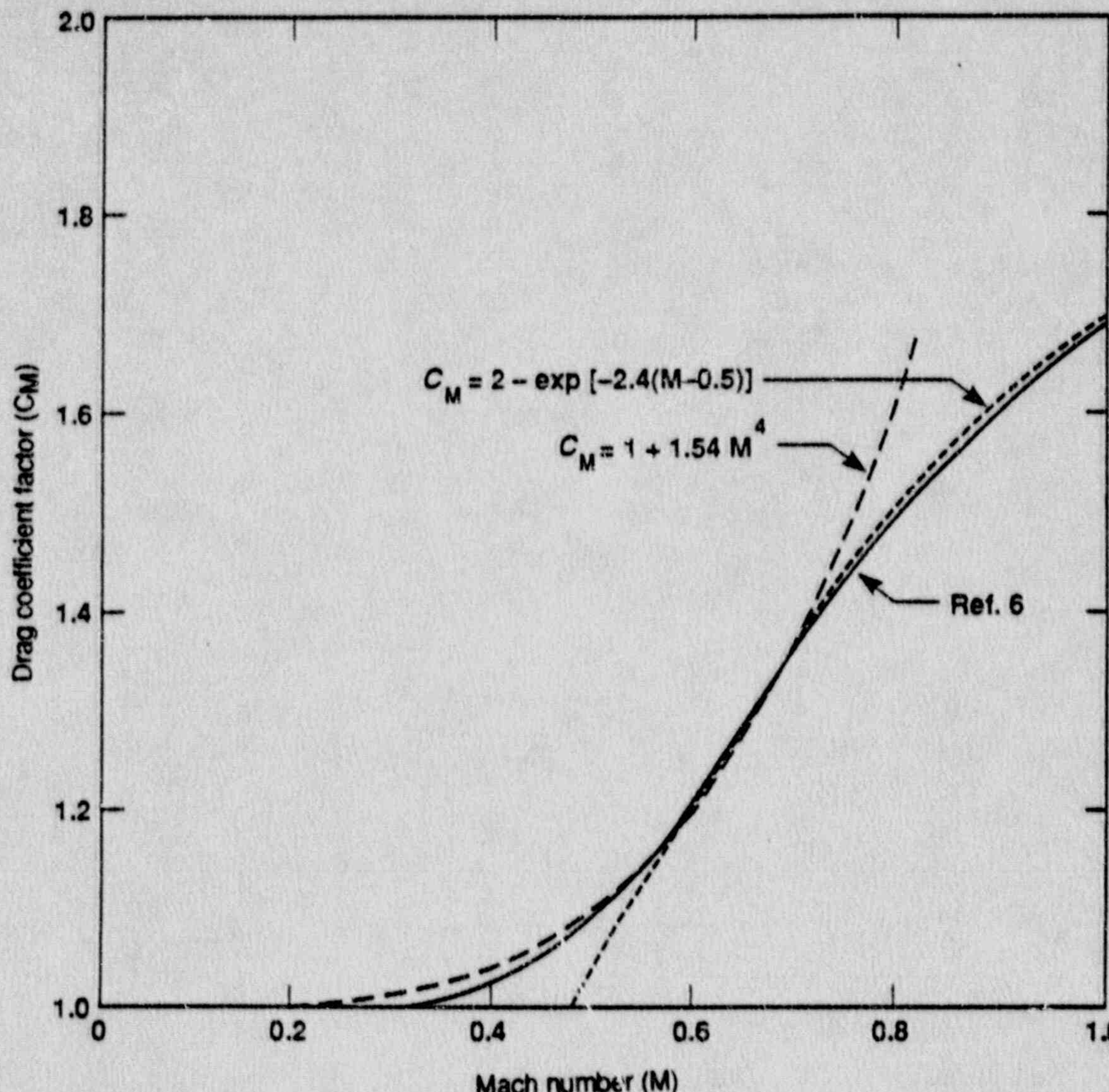

Fig. 3. Drag coefficient correction factor for Mach number effect.

2.4 Air Properties

The ballistic calculations use altitude dependent air properties for standard atmospheric conditions. The only properties needed were density and sonic velocity and these were taken from Ref. 6. Equations (17) and (18) are valid in the altitude range of interest and were used to calculate these values for the analyses. Figure 4 shows air density curves from Eq. (17) and Ref. 6.

$$\rho = 1.225\,(1 - .000448\, y^{.794}),\ \text{kg/m}^3. \qquad (17)$$

$$V_s = 40 - .0041\, y,\ \text{m/s}. \qquad (18)$$

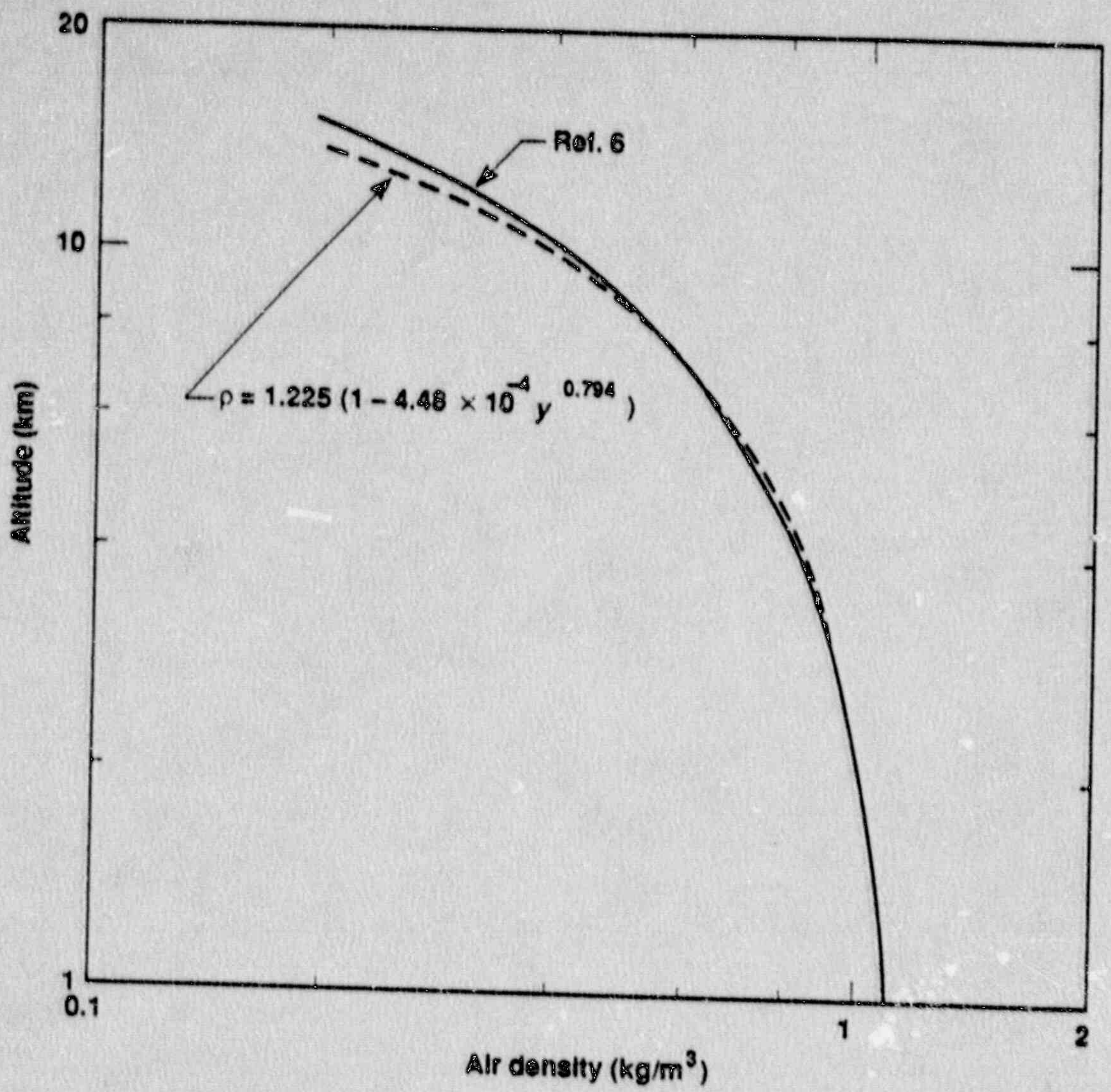

Fig. 4. Air-density-to-altitude relationship.

2.5 Verification

The reliability of the described computational method can be demonstrated by comparing calculated velocities for selected conditions to velocities from corresponding theoretical closed-form solutions derived from Eq. (1). Three conditions were compared. The first is the free-fall of an object in vacuum; it has the following theoretical solution.

$$V = g\,t = (2\,g\,\Delta y)^{1/2}. \tag{19}$$

The second condition is the terminal velocity of an object reached in a constant density fluid; this has the following theoretical velocity solution.

$$V = [(2\,W\,g)\;/\;(\rho\,A\,C_d)]^{1/2}. \tag{20}$$

The third is the free-fall of an object in a constant density fluid. It has the following time-dependent velocity solution.

$$V = (g/K)\,[1 - \exp(-2\,K\,t)] \,/\, [1 + \exp(-2\,K\,t)] \qquad (21)$$

where:

$K \quad = \quad [\rho\, g\, A\, C_d/(2\,W)]^{1/2}$.

A comparison of velocities determined from the numerical method to velocities from the above theoretical solutions are essentially equal.

2.6 Parameter Values

The following parameter values represent the example PAT package chosen for performing the ballistic calculations:

geometry = right circular cylinder,
diameter = 1.2 m,
length = 2.4 m,
weight = 2.6 Mg,
Reynolds drag coefficient for axial fall = 0.85,
Reynolds drag coefficient for side fall = 0.35.

Also, for ballistic calculations of a pallet/package assembly drop with attached drag parachutes, the following parameter values were chosen:

pallet/package weight = 2.8 Mg,
parachute(s) diameter = 5 m,
parachute drag coefficient = 1.35.

3. RESULTS

Free-fall ballistic calculations using a computer code that calculates package velocity, altitude, and horizontal displacement according to the equations given in Section 2 were performed. Input data to the code include package dimensions, weight, orientation, and Reynolds drag coefficient. The wind jet-stream velocity is also input to the code for deriving a wind profile expression in the form of Eq. (14). Samples of data generated by the code are presented in the following subsections.

3.1 Example Package

Figure 5 shows the velocity-to-altitude relationship for several release altitudes of the example PAT package. The release velocity is zero and the influence of the example wind profile (Eq. 14) are included. The cylinder axis orientation is vertical (axial), except for the two noted curves for side orientation. Note that there is little difference in velocity for the two orientations. This occurs because the product of profile area and drag coefficient (A C_d) is nearly equal for the two orientations of the example package.

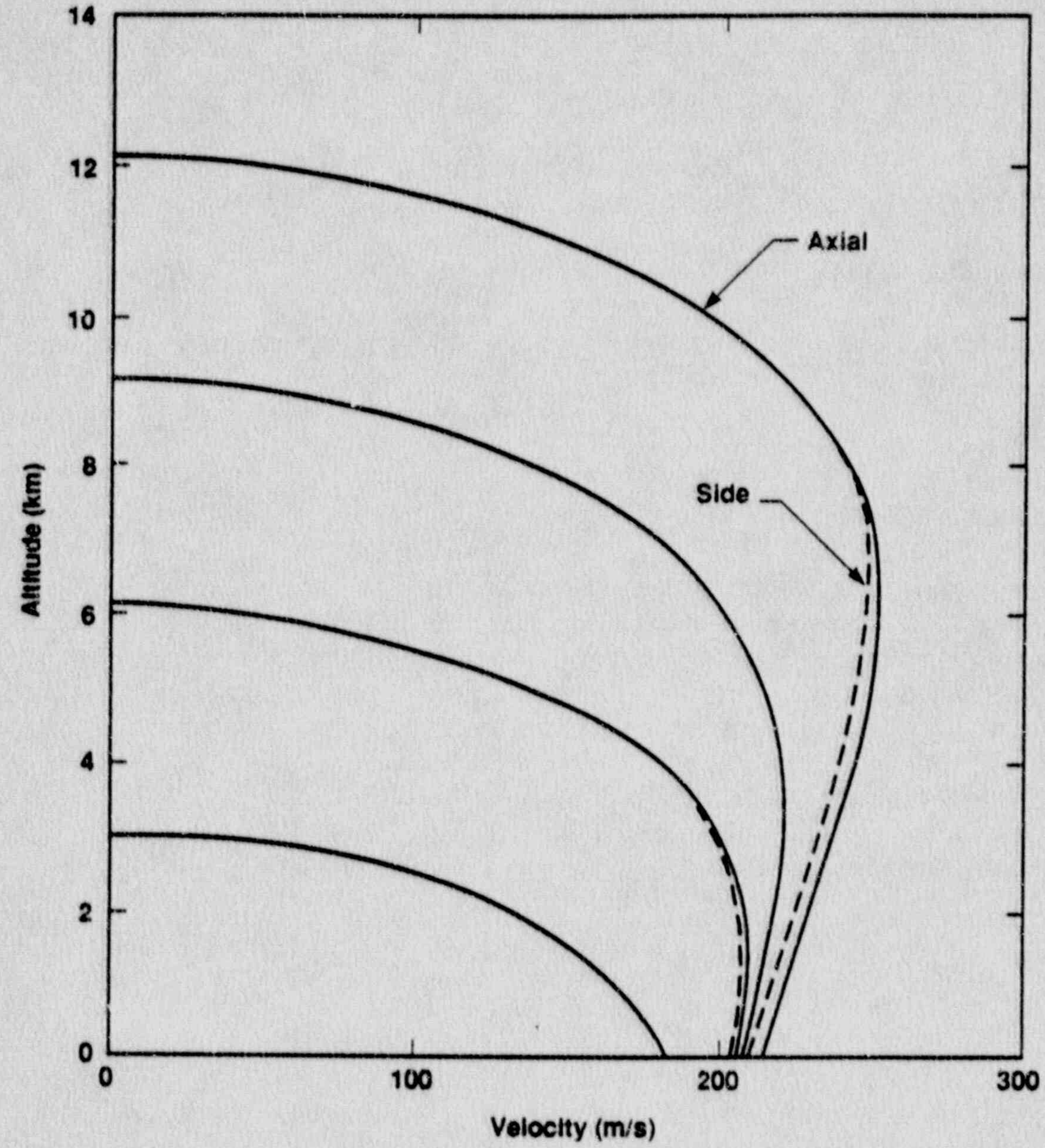

Fig. 5. Free-fall velocity of the example PAT package release from selected altitudes. Release velocity = 0.

Also, note that for release altitudes greater than 6 km the impact velocity is nearly constant because the object achieves its maximum velocity before impact. A velocity decrease after the maximum velocity results from increasing air density as altitude decreases.

When a release velocity is imposed on the package, (e.g., the velocity of the cargo aircraft), the package's initial velocity is a horizontal component. Figure 6 illustrates the trajectory resulting from the horizontal displacement and altitude relationship for a package released at 12.2 km (40 kft) and at several different release velocities. The release velocity is in the same direction as the wind. Note that when the release velocity is 250 m/s (example of a jet transport aircraft velocity), the package's horizontal displacement at sea level is more than 10 km from its release position. Also, the impact velocity is approximately 15 m/s more than when the release velocity is zero. These results suggest that when performing a package drop test, it would be preferable to utilize a lower release altitude and a minimum release velocity.

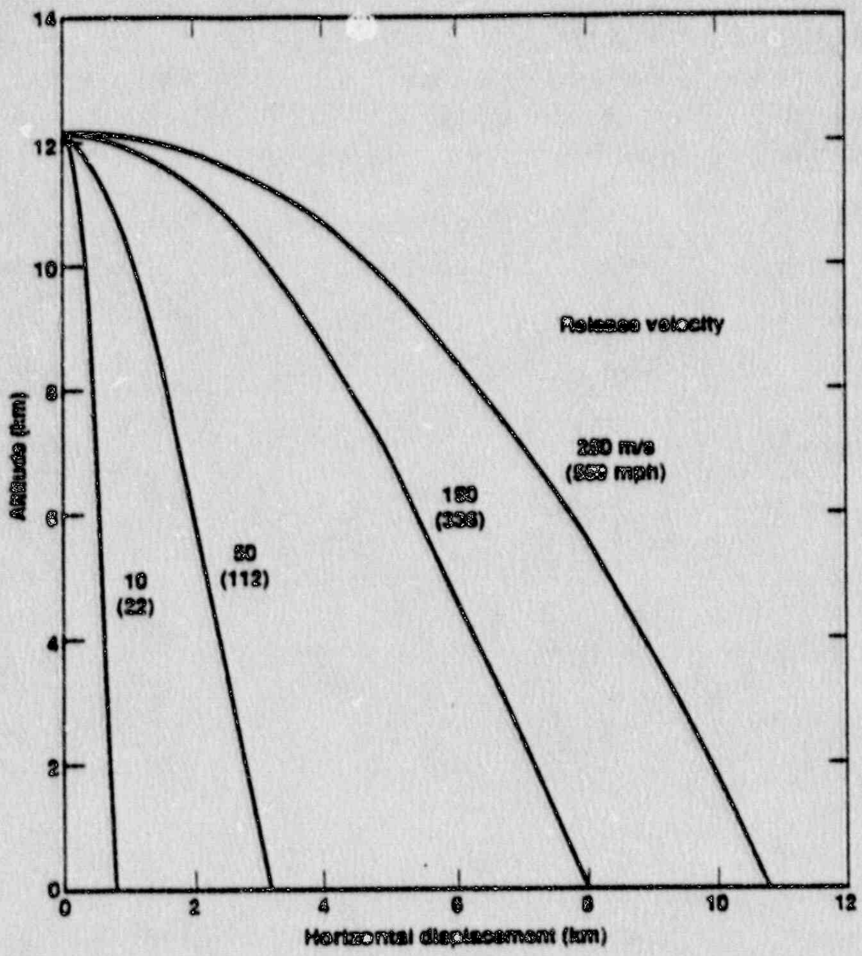

Fig. 6. Horizontal displacement of the example PAT package released at 12.2 km (40 kft) and at selected release velocities.

Figure 6 does not show a curve for zero-release velocity. For this case, the horizontal displacement at sea level is approximately 200 m. This indicates that the example wind imparts only approximately 200 m horizontal displacement when the object is released at 12 km. Packages released at lower altitudes would have a smaller displacement caused by the wind.

To further develop an understanding of the example package free-fall behavior, Figure 7 shows two velocity curves. They show the relationship between the release altitude of the package and its maximum and sea level velocities. The applied conditions are a zero-release velocity, the example wind profile, and an axial orientation. Note that the sea level velocity is nearly constant for release altitudes above 6 km and that the sea level and maximum velocities are approximately equal for release altitudes below 6 km.

If the maximum cruising altitude of the designated cargo aircraft for the example PAT package is 12.2 km (40 kft), its sea level impact velocity — according to Fig. 7 — is 205 m/s. The same impact velocity can be achieved if the package is released at lower altitudes, providing its impact elevation is above sea level. Figure 8 shows this relationship. The curve shows, for example, that the 205 m/s impact velocity could be achieved at an elevation of 1.0 km (3.3 kft) if the example package were released at 6.5 km. These conditions would probably be more appropriate for a drop test. Note that the area above the curve represents impact velocities greater than 205 m/s.

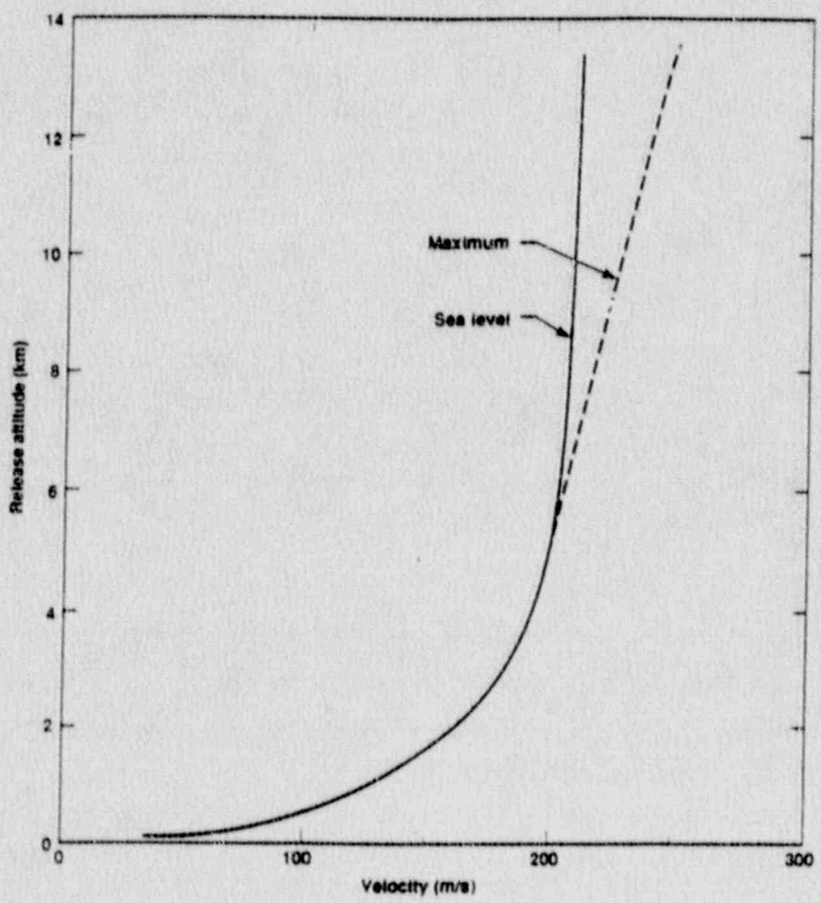

Fig. 7. Maximum and sea level velocities with respect to release altitude for the example PAT package.

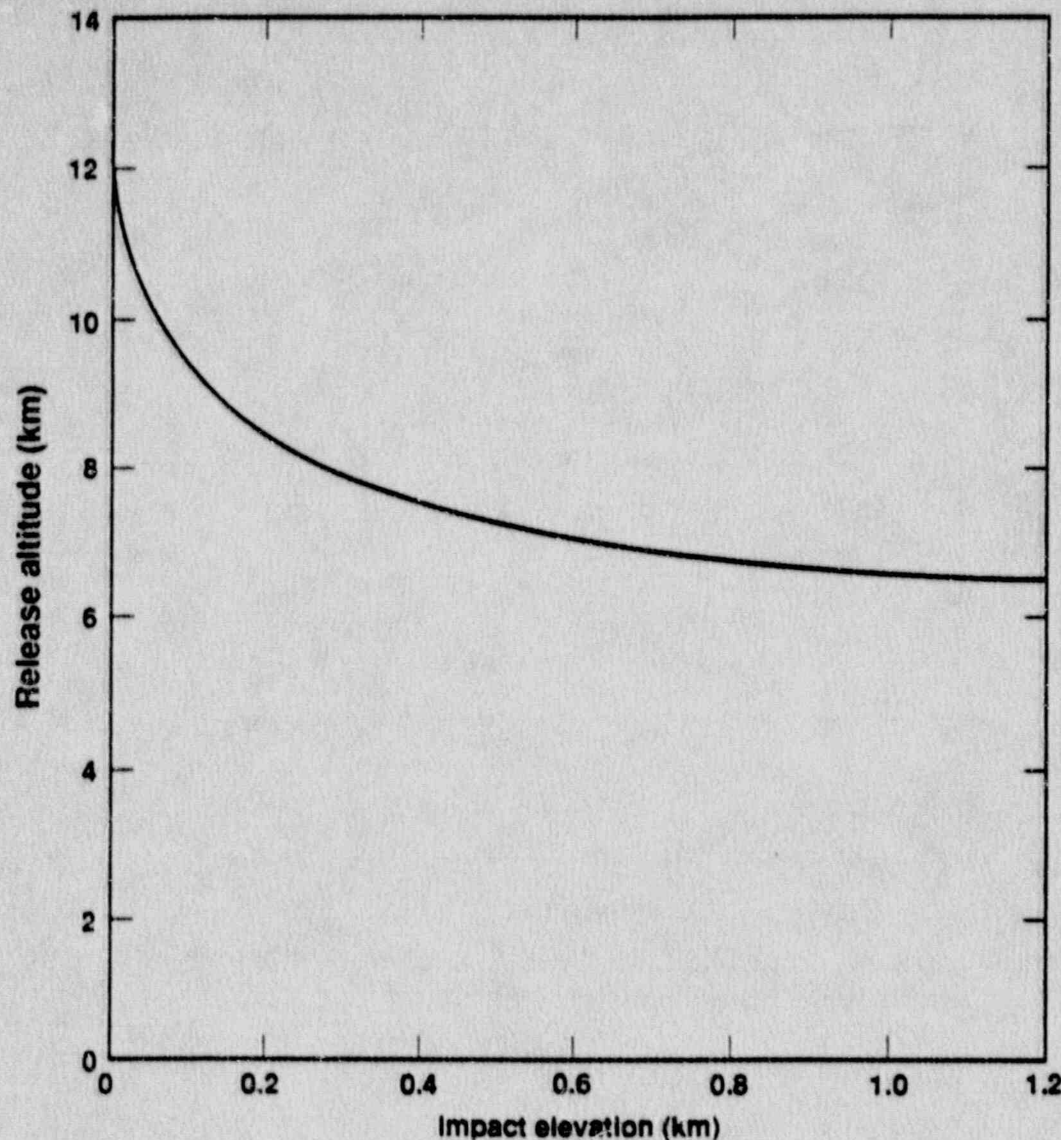

Fig. 8. Release altitude and impact elevation relationship for 205 m/s impact velocity of the example PAT package.

3.2 Other Packages

A descriptive parameter occurring in the velocity Eqs. (2), (7), (20), and (21) is the collective group of terms $W/(A\ C_d)$. This group is given the label "ballistic number", Bn. Its relevance to a package's free-fall velocity is that an increased Bn value results in higher package velocities. Figure 9 illustrates this relationship. There is also a relationship between the sea level velocity and Bn values for packages that have achieved their maximum free-fall velocity, as shown by the curve in Fig. 10. The velocities from the ballistics computer code are, as expected, nearly the same as the velocities determined by the terminal velocity Eq. (20). The small discrepancy is accounted for by noting that the C_d value in Bn and Eq. (20) does not include the Mach number correction, C_M, which is relatively small at sea level.

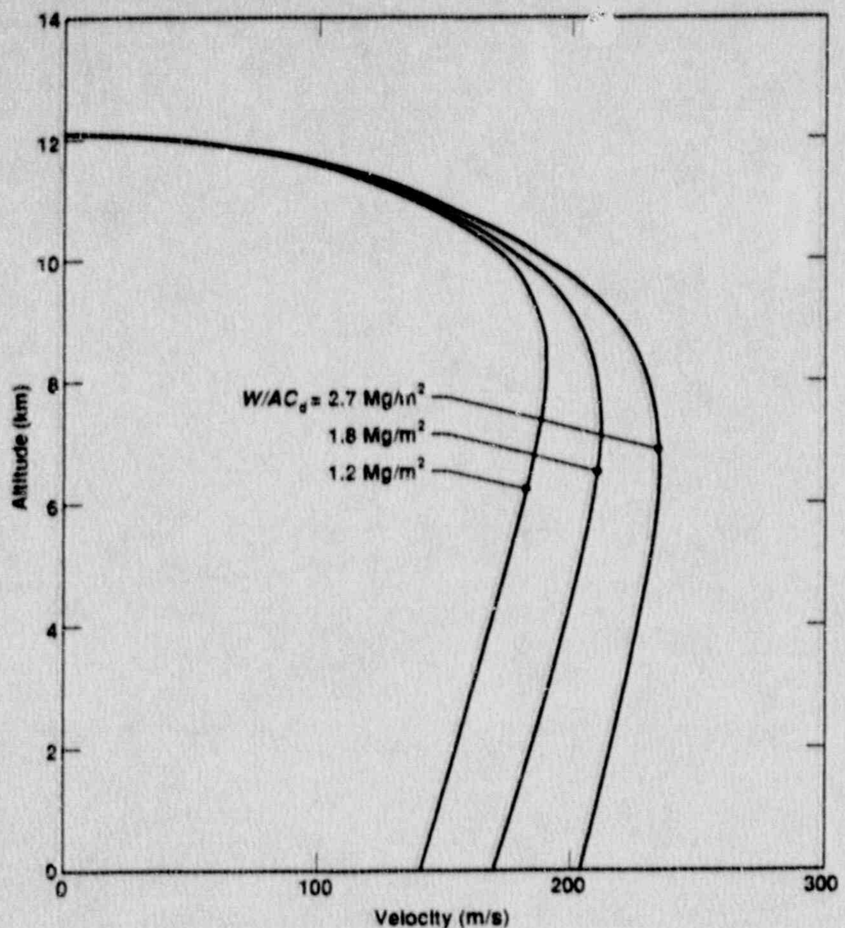

Fig. 9. Free-fall velocity from 12.2 km (40 kft) for three ballistic numbers.

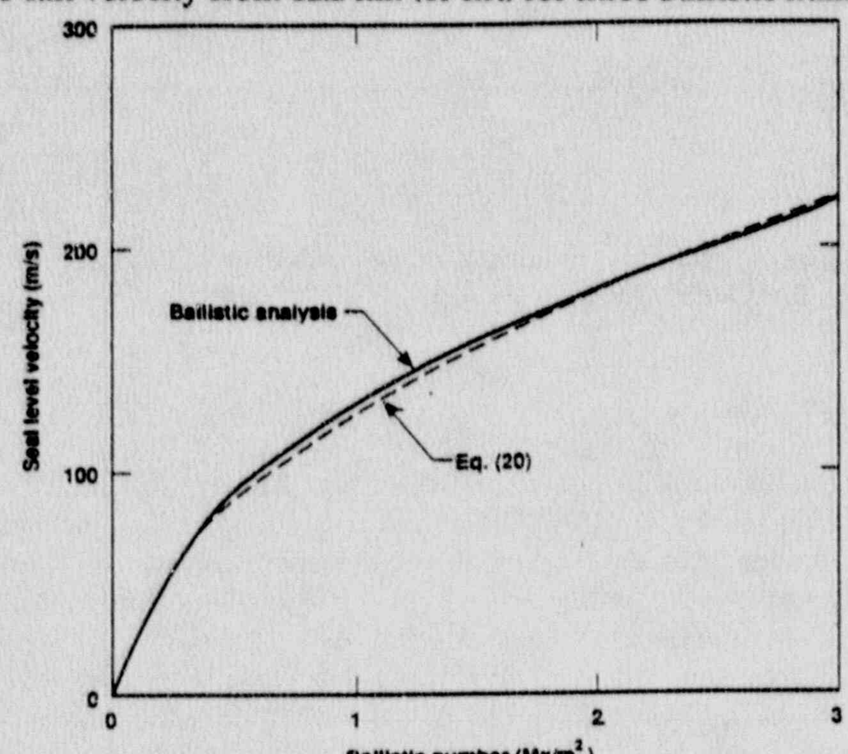

Fig. 10. Sea level velocity and ballistic number relationship for nonaccelerating objects. Release velocity = 0.

3.3 Parach ·e Drop

The results given in Sections 3.1 and 3.2 indicate that, in performing a drop test of the example package, release altitudes as low as 6 km would be acceptable. Also, a low release velocity is desirable to reduce the horizontal displacement of the package. Reference 2 gives a proposed method to accomplish this. The test package would be dropped from an aircraft flying into the wind to achieve a minimum ground velocity. The horizontal velocity of the package would be further reduced by fastening the package to a pallet equipped with drag parachutes. After the pallet/package assembly velocity is sufficiently low, the package would be released from the pallet for free-fall. Shown in Fig. 11 are velocity versus altitude curves for a pallet/package assembly dropped at 7.6 km (25 kft) from an aircraft flying at 100 m/s ground speed. The curves are for one, two, and three 5-m diameter parachutes attached to the assembly and a windless condition. The curves indicate that the pallet/package velocity changes little after descending about 500 m.

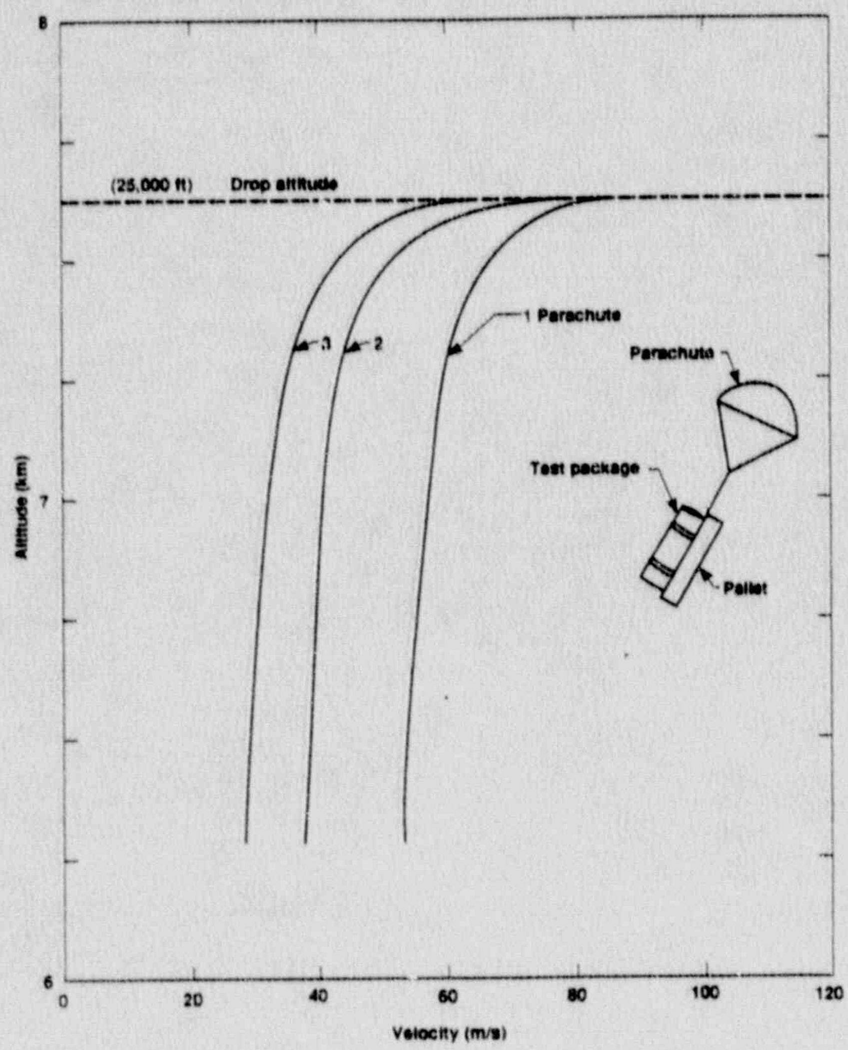

Fig. 11. Velocity of example PAT pallet/package assembly with drag parachutes. Release velocity = 100 m/s.

The case of dropping a pallet/package assembly from an aircraft flying into the example wind at 77.7 m/s ground speed — corresponding to an indicated airspeed of 66.8 m/s. — is used as an example. In this case, the assembly is dropped at 7.6 km with two 5-m-diam parachutes attached, and the package is released at 7.0 km (23 kft). Figure 12 shows the horizontal displacement for this case to be about 1 km. Note that the wind causes the package to be displaced in a direction opposite to the aircraft flight direction. If the package were dropped from the aircraft without the benefit of the drag parachutes, its horizontal displacement would be about 3 km in the opposite direction.

Figure 12 also gives a curve for the same conditions without a wind. The aircraft indicated-airspeed is also the same (66.8 m/s), but its ground velocity is 100 m/s. The horizontal displacement during this condition is in the same direction as the aircraft and much less than when the example wind occurs.

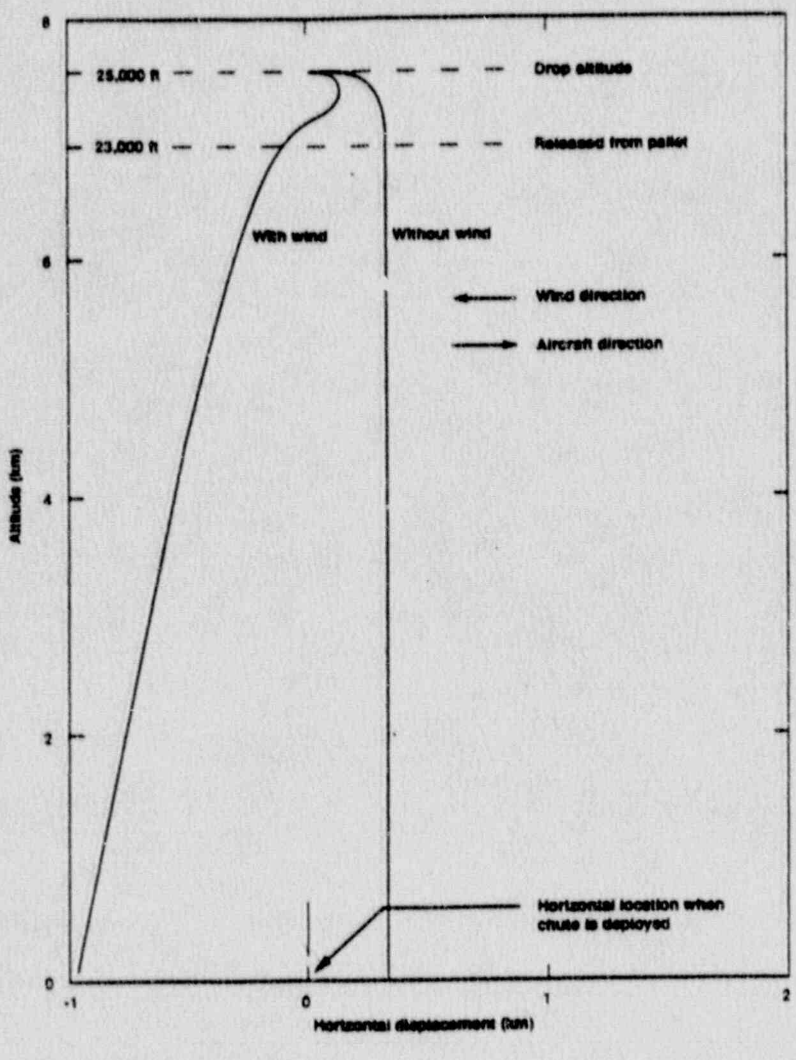

Fig. 12. Horizontal displacement of example PAT package dropped with drag parachutes from an aircraft flying into an example wind. Aircraft ground speed = 77.3 m/s. Package release altitude = 7 km (23 kft).

4. CONCLUSIONS

The ballistic number for a free falling object has a strong influence on its ground impact velocity. Consequently, any modification to the example package that reduces this number will significantly reduce its impact velocity and thereby enhance its probability of survival.

If the impact velocity of a PAT package must be less than 129 m/s (a value specified in Ref. 7), then the ballistic number must be less than 1.0.

The drag coefficient, which is a parameter in the ballistic number, is dependent on geometry and surface irregularities. The drag coefficients used in these analyses are for an ideal shape having smooth surfaces, so they are probably not valid for actual PAT packages. Therefore, coefficients obtained experimentally from prototype packages are needed to better estimate package impact velocities.

An example method for measuring a package's ballistic number (and drag coefficient) is to drop a model package from a sufficiently high altitude during known atmospheric and release conditions and measure its time to impact. For example, a test package could be released from a helicopter at a measured altitude and the resulting time to impact could be determined by using an electronic timer and a high-speed camera. The test conditions and descent time could then be used in a ballistics code to compute a corresponding ballistics number. This method could provide better drag data and be more cost-effective than by obtaining data from wind-tunnel testing.

Impact velocities of the example package released above 6 km (20 kft) are nearly constant. This implies that a drop test could be initiated at much lower altitudes than the cargo aircraft's maximum cruise altitude. If this is the case, greater accuracy in hitting the drop target can be achieved.

Target accuracy during a drop test can be further improved by using drag parachutes to reduce the package's horizontal displacement. This method is described in Section 3.3.

The trajectory and impact velocity of packages can be calculated reliably provided accurate values of the free-fall conditions are available. However, the accuracy of impacting a selected target can be influenced by other factors, as described in Ref. 2.

The aerodynamic characteristics of some packages may result in a tumbling behavior during their free fall. Information on how this behavior affects the package's velocity or trajectory could not be found; aerodynamic characteristics of tumbling may merit further investigation. However, if the package is designed to maintain a constant attitude during free fall, its ballistic characteristics can be more easily measured and predicted.

5. REFERENCES

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