# ESTIMATION, UNCERTAINTY ANALYSIS, AND SENSITIVITY ANALYSIS: <br> DIRECTIONS FOR RMIEP 

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January 4, 1985

\section*{APPENDIX D}

\section*{Use of the Maximus Methodology for}

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Confidence Bound Calculations in Fault Trees--Trial Problem
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\section*{Introduction}

To demonstrate the use of the Maximus Methodology [3] for confidence bound calculations in fault trees, a dominant accident sequence from the Interim Reliability Evaluation Program: Analysis of the Arkansas Nuclear One-Unit 1 Nuclear Power Plant [1] was chosen for analyses. The sequence chosen was the \(B(1,2) D_{1} \mathrm{C}\) sequence, which denotes a reactor coolant pump seal rupture or a rupture in the RCS piping in the range of . \(38^{\prime \prime}\) to \(1.2^{\prime \prime}\) ( \(8(1.2)\) ) followed by failure of the high pressure injection system ( \(D_{1}\) ) and reactor building spray injection system (C).

The Maximus Methodology was developed for system reliabilities modeled by block diagrams. Block diagrams are generally not as extensive as fault tree models for nuclear plant accident sequences. This trial problem was initiated to answer the question--Can Maximus still be used and if so with what modifications?

In this paper, the calculation of confidence bounds in several cases will be considered. The cases illustrate the distinction between data-based and data-free estimates as outlined in the guidelines [2] for the PRA Methods Develoment Program. In case 1, the estimates given for each event are treated as being data-based and recovery is not considered. Case 2 is like case 1 in the treatment of event data, but the probability of recovery (as subjectively determined) is added. In case 3, the probability of the accident sequence is considered as being estimated by both data-based and subjectively-based estimates with recovery probabilities also considered as subjectively determined. The consideration of recovery is an explicit recognition that even though a particular accident sequence may occur it will not necessarily lead to core melt. Human intervention may restore things if done correctly and in a timely manner. The recovery action, however, takes place after the accident sequence has occurred.

Case 3 reflects the most realistic situation for accident sequences in that some of the basic event probability estimates are data based, some are subjectively determined, and recovery is included. However, the other cases are worth considering as they may be applied at intermediate steps, and it is the first case that is comparable to the uncertainty analysis done in reference 1. For all the cases, the information available was in the form of point estimates and error factors, as well as the associations of events whose probabilities were considered as being estimated from the same data base. For the example problems considered here, those estimates considered as data based are translated into pseudo-data by finding the occurrences in demands (or operating time) that gives the same point estimate and gives the error factor times the point estimate as a \(95 \%\) upper statistical confidence bound. If the probability of the event is considered to be subjectively estimated, the interval \(\ell, u\), where = (point estimate/error factor) and \(u=\) (point estimate error factor) is taken as the subjective interval
and the point estimate is taken as the nominal value in carrying out the uncertainty analysis as described in Reference 2. The above procedure of converting to pseudo-data is not being recommended. It is used here to obtain "data" for the sake of illustration.

In the accident sequence considered, \(\mathrm{B}(1.2)\) is the initiating event and \(D_{1} C\) represents the hardware and system fallures that are modeled in the fault tree. The event \(B(1.2)\) has an estimated occurrence rate of, \(02 /\) reactor year. For illustration purposes, we will derive the overall uncertainties in each case by considering the failure rate of \(B(1.2)\) as a constant and also considering it as having been estimated by 2 occurrences in 100 reactor years.

Case 1. All probabilities considered as data based--no recovery
This problem was originally approached by considering the dominant 500 cut sets for the sequence of reference 1 . The estimated occurrence rate from the 500 cut sets is approximately \(98 \%\) of the estimate that would result considering the top 1,355 cut sets. The 500 dominant cut sets are comprised of 135 different basic events. In order to represent DiC in a series-parallel arrangement, the 500 cut sets were examined in a factored form. The seriesparallel arrangement derived from this factored form is given in Figure 1. The numbers inside the boxes are the number of serial basic events that comprise that segment of the sequence. Although constructed from considering the dominant 500 cut sets, the system of Figure 1 has 1,289 cut sets. This is because the representation of the system is block form introduced cut sets not in the original 500 . These additional cút sets were then verified to be actual cut sets of the system.


Figure 1. A series-parallel representation of the dominant cut sets of \(\mathrm{B}(1.2) D_{1} \mathrm{C}\). The A and \(B\) terms are repetitions of the same group of components \(m^{i}\) th the same structure.

In Appendix \(C\) of reference 1 , dominant minimal cut sets in terms of independent subtrees were given. The series-parallel arrangement implied by the configuration given in Appendix \(C\) is consistent with that shown in Figure 1 , except that the parallel arrangements of Figure 1 contain single events that were not included in reference 1. By considering both the independent subtrees given in reference 1 and the elements included in the top 500 cut sets, the representation of Figure 2 is obtained. In Figure 2, each block is one or more basic events in series and those blocks labeled the same are repeats of the same chain of events. The blocks labeled \(P, Q, a\), and \(b\) represent events not listed in reference 1 but contained in the top 500 cut sets. As the total contribution of these were small and they had very little effect on the uncertainty calculations, they are left out of the present analysis.

The events contained in each block are enumerated in the Appendix in Table A2. Those blocks (A through 0) that were derived from reference 1 are documented by inclusion in the Appendix of the appropriate table from that reference. Also added to the tables are identifiers for the population type. Those events whose probabilities are estimated from the same data sources have the same population type identifier. In order not to double or multiple the same data in the overall uncertainty estimate, the available data is divided among those events to which the data apply (see Reference 3). In this example, pseudo-data are constructed by finding the number of occurrences in time that would give the same point estimate and for which the \(95 \%\) upper confidence bound equals the point estimate times the error factor. The intent is to illustrate the analysis with statistical data that correspond, at least roughly, to the subjective estimates and uncertainty assessments in Reference 1 . This gives rise to the following two equations (for the Poisson-type data, these are exact; for binomial-type data, these are based on very good approximations):
\[
\begin{array}{r}
f / T=\hat{p} \\
\frac{x^{2}(2 f+2 ; .95)}{2 T}
\end{array}
\]

Here, \(x^{2}(d f ; a)\) denotes the a percentile of the chi-square distribution with df degrees of freedom. The values \(f\) and \(T\) are the pseudo data of \(f\) occurreices in T time (or demands) and D and EF are the given point estimate and error factor.

By substituting the first equation into the second, the \(T\) values cancel and \(f\) is the solution of:
\[
x^{2}(2 f+2 ; .95) / f=2 \cdot E F .
\]

The solution of the above equation when \(E F=3\) is \(f=2.20\) and when \(E F=10\), \(f\) is .37. The denominator (demand or time) is calculated in each case by dividing \(f\) by the point estimate.

\(-b-\)


Figure 2. Series-parallel arrangement for \(\mathrm{B}(1.2) \mathrm{D}_{1} \mathrm{C}\). Each block is one or more basic events in series. Those blocks labeled the same represent the same event.

The various population types and derived pseudo-data are given in Table A3. Some of the population types have events that have different point estimates. This situation is taken to reflect the case where a rate \(\lambda\) is estimated for all the events of interest, but the actual rate for a particular event \(i\) is \(\lambda t_{i}\). In the Poisson case, if \(\lambda\) is estimated by \(f\) occurrences in \(T\) time, then the estimate of \(\lambda t\) is equivalent to \(f\) occurrences in time \(T / t\). To handle those population types that had different point estimates within them, the largest point estimate is taken as the \(\lambda\) estimate and smaller point estimates have associated with them a time factor for adjustment. For example, consider that two event probabilities, one estimated at \(1.1(-3)\) and one at \(3.3(-3)\), are considered to be from the same population type. Both have error factors of 3 so that we take \(f=2.2\). Using the larger of the two as reflecting the \(\lambda\) to be estimated, we take \(T=2.2 / 3.3(-3)=667\) as the applicable data. If \(3.3(-3)\) is the estimate for \(\lambda\), then \(1.1(-3)\) must correspond to an estimate of \(\lambda / 3\). Therefore, if we divide the applicable data between the two events, giving 1.1 fallures in 333.3 time units for estimating each \(\lambda\) independently, this is equivalent to using 1.1 failures in 1,000 time units for estimating \(\lambda / 3\). And, thus, the time factor of the second event would be given as 3 . The various factors by which times are adjusted are given in Table A4 in the Appendix.

The Maximus method for calculating confidence bounds was applied to the system of Figure 2. The effective number of tests was calculated and combined with the total fallure estimate to calculate the effective number of faflures. The last parallel arrangement (Branches II and IV in Figure 2) was not originally considered in deriving the effective number of tests because it does not represent an independent subsystem but rather is included in the system to represent an additional cut set not present in the parallel-series arrangement. The effective tests for the two branches (II and IV) derived from the first part of the system when combined in a parallel arrangement exceed that originally calculated for the system. Therefore, this cut set does not affect the effective fallure number calculation.

\section*{Computer Program}

There currently exists a Fortran program that calculates effective data for series-parallel systems given component data and using the Maximus methodology. Figure 3 is an example output of this program for the system under consideration here. The inputs to the program are the system description and the component data. In this example, each of the components (events) from the same population type are labeled with the same alphabetic character. Differences in the numeric value following the alphabetic character are needed because of the potentially different test quantities to be assigned in the unpooling process.

The system equation is recursive, where each set of parentheses encloses a subsystem which may contain other subsystems. For example, in the system description of Figure 3, subsystem 1, which is represented as ( \(1 * a 2 b 2 c 2 c 2 g 2 j 1\) ), is a series (denoted by the "*") subsystem representing the independent subtree labeled LPI1408B-VCC-LF in the ANO analysis. Subsystem 1 is itself an element of the subsystem labeled 16 in the description. Subsystem 16 combines
subsystem 1 in series with subsystem 14 , which is the subsystem that combines the parallel arrangement of \(M+N\) and 0 of Figure 2.

From Figure 3, it is seen that the overall effective data are roughly 1.2 failures in 1,430 tests. This analysis does no: include the additional cut sets represented by the parallel arrangement of II with IV appended to the system in Figure 2. If we combine those systems irom Figure 3 that make up the added cut sets (subsystems 16 and 3 ), the effective \(n\) far exceeds 1,430 . Therefore, 1,430 is used as the overall system effective test size and the overall system point estimate of \(9.2(-4)\) gives the effective data of roughly 1.3 fallures in 1,430 tests. The upper \(95 \%\) confidence 1 imit on the sequence occurrence rate, based on 1.3 failures in 1,430 tests is \(3.7(-3)\).

If the point estimate divided into the upper \(95 \%\) bound is taken to be the erior factor, then the error factor from this data would be 4.0. Contrast this with the error factor of 3 that is given in the ANO report. However, note that the lower \(95 \%\) bound on 1.3 failures in 1,430 tests is given by \(8.3(-5)\) and if the point estimate divided by this lower limit is taken to be the error factor, then \(11(=9.2 * .83)\) would be taken as the error factor.

In the methodology used for the ANO report, the distribution on the top event would not have a lognormal distribtuion, and, therefore, the error factor determination could suffer from inconsistencies similar to those discussed above. It would make more sense to compare the results of these two methods by looking directly at the uncertainty intervals. Uncertainty intervals from the ANO report method are not directly available but from the values given in Table 8-4 of reference 1, we can infer that the median of the derived distribution was \(1.25(-3)\). With this value and an error factor of 3 , the upper 95 th percentile must have been approximately \(3.75(-3)\) as compared to \(3.7(-3)\) derived from the Maximus methodology. Thus, the two methods produce upper uncertainty bounds that are virtually the same in this particular example. However, there is a vast difference in the interpretations from the methods. By use of the Maximus methodology, statistical confidence bounds are stressed. That is, one is asking how high the probability of the sequence \(D_{1} \mathrm{C}\) might be and still be consistent with the available data on the individual events. The degree of "consistency" is determined by the confidence level. On the other hand, a Monte Carlo method such as used in ANO, requires the placement of distribution functions on each of the individual event probabilities. These distribution functions do not correspond to anything that we have specifically modeled, and therefore, they reflect an added mathematical level that is often referred to as the "analyst degree-of-betief."

The above analysis reflects only the \(D_{1} C\) portion of the sequence. If the \(.02 /\) reactor year occurrence rate for \(B(1.2)\) is considered as constant, then the overall uncertainty analysis would correspond to that of 1.3 failures in 71,400 reactor years. The lower and upper \(95 \%\) bounds are then given by 1.7(-6) and 7.3(-5), respectively.

If the \(.02 /\) reactor year rate is considered as coming from 2 occurrences in 100 years, the effective overall data is .61 occurrences in 33,200 reactor years (see Reference 3 for combining algorithm) and the lower 95\% confidence limit is \(1.9(-7)\) and the upper \(95 \%\)-confidence limit is \(1-9(-7)\) and the upper \(95 \%\) confidence limit is \(1.2(-4)\).
\begin{tabular}{|c|c|c|c|}
\hline SUBSVSTEM E & EQUIVALENT FAILURES & EQUIVALENT TESTS & MLE OF FELIAEILITY \\
\hline A 1 & 1.3678 & 163.30 & 0.9916 \\
\hline Mun 12 & 0.4837 & 14.60 & 0.9669 \\
\hline 013 & 0.2489 & 17.80 & 0.9860 \\
\hline 14 & 0.0786 & 169.72 & 0.9995 \\
\hline 16 II & 1.4428 & 163.30 & 0.9912 \\
\hline 82 & 0.8107 & 35.70 & 0.9773 \\
\hline \(F 6\) & 0.3735 & 35.70 & 0.9895 \\
\hline G 7 & 0.3699 & 35.70 & 0.9896 \\
\hline 18 & 0.2782 & 55.76 & 0.9950 \\
\hline J 9 & 0.1760 & 35.70 & 0.9951 \\
\hline 17 III & 1.8718 & 35.70 & 0.9476 \\
\hline 15 IT Pew \({ }^{\text {III }}\) & I 0.6608 & 1426.54 & 0.9995 \\
\hline c 3 IV & 1.1048 & 131.90 & 0.9916 \\
\hline 14 & Ø. 8084 & 33.30 & 0.9757 \\
\hline K 10 & 0.2987 & 59.66 & 0.9950 \\
\hline L 11 & 0.1642 & 33.30 & 0.9951 \\
\hline 19 I & 1.1706 & 33.30 & 0.9660 \\
\hline 18 Wemy & - 0.4062 & 1428.38 & 0.9997 \\
\hline \(\theta\) & 1.20139 & 1426.54 & 0.9992 \\
\hline
\end{tabular}

Current system description is:
```

(0*e1(15+(16*(1*a2b2c2c2g2j1)(14+(12*aSaSabb4b4b4b4ce2e2e2f1+1%5 4j41402ptu1
(13*a7a7b5b5c3e3e3e3+2g4j5j515o3)))(17*(2*a1b1deeeeeeeefg1g:.11ow,
(b*ablclg1j)(7*ab1g1j)(8*a3b1jl101)(9*a1bleeejk)))(18*(3*abu3c4c4g3j3)
(19*(4*agbdle4e4e4e4e4e4e4e4f3ggj21204u2)(10*a4bj21305)(11*a9be4e4e4j2k))))

```
\begin{tabular}{|c|c|c|c|}
\hline COMFDNENT & FAILURES & TESTS & Test factor - see Table A4 \\
\hline a & 0.3406 & 103.20 & \\
\hline a 1 & 0.1135 & 103.20 & 3 \\
\hline 22 & 0.4653 & 423.00 & 3 \\
\hline a3 & (1) 0106 & 105.60 & 33 \\
\hline a 4 & (1.)10 \({ }^{\text {(1) }}\) & 105.60 & 33 \\
\hline as & 0.0317 & 28.80 & 3 \\
\hline ab & 0.0950 & 28.80 & \\
\hline a) & 0. 3409 & 37.20 & 3 \\
\hline 28 & 0.3340 & 303.60 & 3 \\
\hline 99 & 0.1162 & 105.60 & 3 \\
\hline \(b\) & 0.1192 & 59.60 & \\
\hline b1 & 0.1114 & 55.70 & \\
\hline b2 & 0.5990 & 299.50 & \\
\hline b3 & 0.4024 & 201.20 & \\
\hline 64 & 0.0348 & 17.40 & \\
\hline bs & 0.0726 & 36.30 & \\
\hline c & 0.0022 & 21.80 & \\
\hline c1 & 0.0176 & 175.70 & \\
\hline c2 & 0.0278 & 278.10 & \\
\hline c3 & 0.0027 & 27.20 & \\
\hline
\end{tabular}

Figure 3. Output from Maximus Method Code for B(1.2)0, C Sequence
\begin{tabular}{lrr}
\(c 4\) & 1.0609 & 10609.00 \\
\(d\) & 0.1928 & 35.70 \\
\(d 1\) & 0.1798 & 33.30 \\
\(e\) & 0.0036 & 35.70 \\
\(e 1\) & 2.1133 & 21133.00 \\
\(e 2\) & 0.0016 & 16.00 \\
\(e 3\) & 0.0020 & 20.00 \\
\(e 4\) & 0.0033 & 33.30 \\
\(f\) & 2.1672 & 21671.50 \\
\(f 1\) & 0.0016 & 15.90 \\
\(f 2\) & 0.0019 & 19.20 \\
\(f 3\) & 0.1378 & 277.50 \\
9 & 0.1464 & 33.30 \\
91 & 0.6695 & 163.70 \\
92 & 0.5408 & 171.90 \\
03 & 0.0730 & 17.80 \\
\(q 4\) & 0.0599 & 14.60 \\
95 & 0.1189 & 118.90 \\
\(j\) & 0.6434 & 643.40
\end{tabular}
*ress return to continue
\begin{tabular}{lrr}
\(j 2\) & 0.1223 & 122.30 \\
\(j 3\) & 0.4191 & 419.10 \\
14 & 0.0378 & 37.80 \\
15 & 0.0520 & 50.20 \\
\(k\) & 1.1000 & 2037.00 \\
1 & 0.9675 & 261.50 \\
11 & 0.4396 & 261.36 \\
12 & 0.5206 & 140.70 \\
13 & 0.2394 & 140.80 \\
14 & 0.0167 & 16.65 \\
15 & 0.0203 & 20.35 \\
0 & 1.3370 & 3109.40 \\
01 & 0.6856 & 3109.30 \\
02 & 0.0041 & 16.53 \\
03 & 0.0045 & 22.50 \\
04 & 0.1115 & 259.20 \\
05 & 0.0571 & 259.20 \\
\(p\) & 2.2000 & 2200.00 \\
\(t\) & 0.3680 & 46.00 \\
1 & 0.0435 & 217.40
\end{tabular}
2.2
2.2
3.7
3.7
1.25
\(13.53 \quad 1.15\)
\(22.53 \quad 2.15\)
259.20
1.45
46.00

9
ress return to continue
\begin{tabular}{lrrr}
41 & 0.0036 & 18.00 & 9 \\
42 & 0.3229 & 179.40 & \\
\(v\) & 2.1490 & 733.00 &
\end{tabular}

Figure 3 (Continued)

The data for each of the components (events) in Figure 3 result from the unpooling process. The algorithm used for unpooling is presented in the Appendix.

\section*{Case 2. Recovery Added}

Some of the falfure events in \(D_{1} C\) can be "recovered," or corrected, thus preventing the sequence from progressing to core melt. Thus, it is more "realistic" to incorporate recovery events and their probabilities into the models.

Of more interest than whether a given sequence, such as \(B(1.2) D_{1} C\), occurs is the case that it occurs and is not recovered from, thus leading to a severe consequence such as core melt. Case 2 considers the event of the accident sequence occurring and no recovery taking place. In probabilistic notation, the parameter of interest is written as follows:
\[
\operatorname{Pr}\left(B(1.2) D_{1} C \text { and no recovery }\right)=\operatorname{Pr}\left(\text { no recovery } \mid B(1.2) D_{1} C\right) \cdot \operatorname{Pr}\left(B(1.2) D_{1} C\right) \text {, }
\]
where \(\operatorname{Pr}(A \mid B)\) denotes the conditional probability of \(A\) when \(B\) is known to have occurred. For uncertainty analysis, if \(\operatorname{Pr}\left(\right.\) no recovery \(\left.\mid B(1.2) D_{1} \mathrm{C}\right)\) was considered to be a known constant, then the effective number of tests (or effective time) derived for the uncertainty analysis of \(\operatorname{Pr}\left(B(1.2) D_{1} C\right)\) would be divided by the value of \(\operatorname{Pr}\left(\right.\) no recovery \(\left.\mid B(1.2) D_{1} C\right)\) to give the effective test size for the estimate of \(\operatorname{Pr}\left(B(1.2) D_{1} C\right.\) and no recovery). Because estimated recovery probabilities will most likely be subjective in nature (i.e., not directly data based) and the uncertainty in recovery factors will be treated by an interval analysis, \(\operatorname{Pr}\left(\right.\) no recovery \(\left.\mid B(1,2) D_{1} \mathrm{C}\right)\) is treated as being constant. Its value is calculated by the ratio, \(\operatorname{Pr}\left(B(1.2) D_{1} C\right.\) and no recovery) \(/ \operatorname{Pr}\left(B(1.2) D_{1} C\right)\).

The conditional probability of no-recovery for \(\mathrm{B}(1.2) D_{1} \mathrm{C}\) in reference 1 was calculated to be .22 . This value was arrived at by calculating the probability of nonrecovery for each subtree and then taking the probability of nonrecovery for a cut set to be the minimum probability of nonrecovery amongst the subtrees represented in the cut set. This is the procedure that would be followed on the original fault tree instead of on the cut sets from the independent subtrees.

Using the value of .22 for the probability of nonrecovery and the effective data of 1.31 failures in 1,430 tests from case 1 , we get that the uncertainty bounds for the estimate of \(D_{1} C\), considering recovery would be based on 1.31 fallures in \(1430 / .22 \approx 6500\) tests. If the initiating event rate is included in the analysis as having a value of .02 , then the uncertainty bounds are based on 1.3 failures in 324,000 reactor years. In this case, the lower and upper 95\% confidence bounds are given by \(3.7(-7)\) and \(1.6(-5)\), respectively.

If the initiating event rate is considered as 2 occurrences in 100 reactor years, then the uncertainty bounds are based on .61 failures in 151,000 reactor
years and the confidence 1 imits are given by \(4.1(-8)\) for the lower 95\% limit and \(2.7(-5)\) for the upper \(95 \%\) confidence 1 imit.

The recovery model is such that for any given minimal cut set the probability of nonrecovery is the minimum of the probabilities of nonrecovery amongst the individual terms of the cut set. For the \(D_{1} C\) sequence, as approximated by the system of Figure 2, the nonrecovery for subtrees \(A, C\), and \(E\) are 1. Therefore, a very good approximation to the probability of \(D_{1} C\) including recovery is obtained by modeling each of the basic events of subsystems III and V from Figure 2 as a parallel arrangement of the basic event with the event of no recovery for that basic event. The uncertainty analysis in this case is easily accomplished by altering the test quantities for those events in III and \(V\) by dividing the old test quantities by the probability of nonrecovery for that event. This was done with the data in Figure 3. The effective test quantity for the parallel arrangement of II with 111 (from Figure 2) was 7690 and that from the parallel arrangement of IV with \(V\) was 5790 . These were derived without re-unpooling the data for the new system. If the unpooling algorithm was followed specific to the new model, the effective test quantity would be greater than 5790 but less than 7690 . The suggested method that gives an effective test quantity of 6500 is roughly in the range that would be obtained if the Maximus methodology was rerun for the parallel-series system discussed above that closely approximates the model with recovery.

\section*{\(\frac{\text { Case 3. Overall uncertainty analysis amongst subjective- and data-based }}{\text { estimates }}\)}

Cases 1 and 2 provide the bases for calculating uncertainty intervals when some of the estimates are subjective and some are data based. In this section, they are combined to demonstrate a complete analysis using the Maximus methodology combined with other features of the guidelines (Reference 2). For this example, five of the population types from the analysis of case 1 were chosen randomly to be considered as subjective estimates. The data types chosen to be subjective were those labeled \(a, f, j\), , and o in Table A3.

The set up and recommended display for uncertainty analysis contained in the Guidelines (Reference 2) is briefly reviewed. Assume the parameter of interest, \(\operatorname{Prob}\left(B(1.2) D_{1} C\right.\) and no recovery), is expressed as a function, \(f(\theta, \omega)\), where \(\omega\) is a vector of parameters subjectively estimated and \(\theta\) is a vector of parameters for which data are available for estination purposes. In the present example, \(\underline{\omega}\) contains not only the parameters from population types labeled a, \(f, j, \ell\), änd 0 , but also all recovery factors.

We define \(n_{l}(\omega)\) and \(n_{U}(\omega)\) to be the lower and upper \(95 \%\) statistical confidence limits based on \(\theta\) evaluated at a specific \(\omega\). With this notation, the quantities of interest \({ }^{-}\)for an uncertainty display are the overall extremes,
\[
L=\min _{\underline{\omega}} n_{l}(\underline{\omega}), \quad U=\max _{\underline{\omega}} n_{u}(\underline{\omega}),
\]
the differences in point estimates over the range of subjectively determined estimates,
\(\min _{\underline{\omega}} f\left(\underline{\theta}^{\star}, \underline{\omega}\right.\) and \(\max _{\underline{\omega}} f\left(\underline{\theta}^{*}, \omega\right)\),
where a* \(^{\text {* }}\) represents the point estimates from the data. Also of interest are the data uncertainty interval at the nominal subjective points,
\[
n_{l}\left(\omega^{\star}\right) \quad \text { and } \quad n_{u}\left(\omega^{\star}\right) \text {, }
\]
and, of course, the overall nominal assessment, \(f\left(\theta^{*}, \hat{u}^{\star}\right)\).
The basis for calculating the lower and upper bounds using the Maximus methodology has been given in cases 1 and 2. For the purposes of this example, the probability of nonrecovery factors ( \(n\) ) are taken to range over ( \(n / 2,2 \cdot n\) ) unless \(2 n>1\) in which case the upper limit is 1 . The other subjectively determined types are assumed to range over ( \(\mathrm{p} / \mathrm{EF}, \mathrm{p}\). EF), where p is the nominal point estimate and EF is the error factor given for that population type. The recovery factors are given in Table A2, taken from the ANO report (Reference 1).

Since \(f(\underline{\theta}, \underline{\omega})\) is an increasing function with respect to each component of the vector \(\omega\), the minimum and maximum of \(f\left(\theta^{*}, \underline{\omega}\right.\) is easily calculated by substituting the minimums for all the component \(\bar{s}\) of \(w\). ihus, all the events of types \(a, f, j, \ell\), and \(o\) are evaluated at their point estimate divided by the error factor and all the probabilities of nonrecovery are halved. For example, consider the subtree labeled A. Subtree A has 6 events (See Table A2) of which the events LPI6164-B00-LF and 61648-CBL-LF are considered as subjectively determined, and, therefore, lower estimates for them are taken to be \((1 E-3) / 3\) and \((1.1 E-3) / 3\). The lower estimate for subtree \(A\) then becomes \(7.0(-3)\). The probability of nonrecovery is taken to be .5 for the lower bound analysis since the original probability of nonrecovery was taken to be 1 (see footnote in Table A2).

When the Maximus methodology is applied in order to calculate min \(n_{\ell}(\underline{\omega})\), the approach of cases 1 and 2 are used where the subjectively determined estimates have been evaluated at their lower points. Thus, subtree A would be modeled as having 4 events for which data are avallable, but the point estimate for the subtree would be taken to be \(7(-3)\), thus reflecting the impact of the subjectively determined estimates. This can be done because the LindstromMadden method for determining effective test quantities depends only on the number of tests in the components. The effective fallures is then determined by the point estimate times the effective test quantity.

Table 1 presents the results of such an analysis. These results are also graphically presented in Figure 4. It is wortnwhile here to discuss the interpretation of the display in Figure 4. The nominal point estimate is represented by the slash in the box. The overall uncertainty (allowing the subjectively based paramaeter estimates to be anywhere in their range, combined with \(95 \%\) statistical confidence bounds on the data-based estimates) is represented by the end marks. If the uncertainty surrounding the data-based estimates were eliminated, the total uncertainty interval would shrink down to the endpoints of the box. If the ranges (uncertainty) around the subjectively determined estimates were eliminated, leaving only the data-based uncertainty, the interval would be given by the "*s".

The incorporation of the estimate for the initiating rate in the uncertainty analysis is just as it was in the previous cases. Table 2 and Figure 5 reflect the total uncertainty on the estimate of the occurrence rate for \(B(1.2) D_{1} C\) including recovery.

\section*{Summary}

The purpose of this exercise was to demonstrate the feasibility of using the Maximus methodology for calculating statistical confidence bounds for fault tree sequences. The analysis was done incorporating all the factors that will be present in applying the methodology to the La Salle PRA. These factors include a mixture of subjectively-based and data-based estimates and recovery factors, including uncertainty in the recovery factors. When compared to the uncertainty interval generated by placing distributions on all parameters and performing a Monte Carlo analysis, the Maximus methodolngy produced an upper \(95 \%\) confidence 1 imit that was in the same range (perhaps a little smaller). An exact comparison is difficult because of the practice of converting uncertainty analysis results to error factors.

In the process of applying the Maximus methodology, an algorithm was developed for the unpooling of data used to estimate several parameters. The unpooling of the data is accomplished in a manner as not to be overly conservative. The algorithm is presented in the appendix. The existing Maximus code was altered during this exercise so there would be no absolute constraints on the size of the system or the number of components (units) that could be input to the Maximus method program.

The Maximus methodology applies to parallel-series configurations. For systems that are more general than parallel-series, the Maximus methodology can be used with some modifications. However, the closer the configuration from the fault tree is to a parallel-series arrangment, the easier it is to implement the Maximus method. For this reason, the expression of the sequences in terms of independent subtrees greatly facilitates the implementation.

> Table 1. Combination of Subjectiveand Data-Based Uncertainties for Estimate of Probability of \(D_{1} C\)

Without Recovery Prob. of Nonrecovery With Recovery
\begin{tabular}{|c|c|c|c|}
\hline Nominal point & \(9.2(-4)\) & . 22 & \(2.0(-4)\) \\
\hline \(\min _{\omega} f(\underline{\theta}, \underline{\omega}\) & \(6.1(-4)\) & . 14 & \(8.5(-5)\) \\
\hline \(\max f\left(\underline{\theta}^{*}, \omega\right)\) & 2.3(-3) & . 22 & \(5.2(-4)\) \\
\hline & \(8.3(-5)\) & & \\
\hline \[
\begin{aligned}
& n_{\ell}\left(\omega^{*}\right) \\
& n_{u}\left(\omega^{*}\right)
\end{aligned}
\] & \(3.7(-3)\) & . 22 & \[
8.1(-4)
\] \\
\hline based on & 1.3 failures/ 1420 tests & & 1.3 failures/ 6450 tests \\
\hline based on & \begin{tabular}{l}
3.5(-5) \\
1.04 failures/ 1690 tests
\end{tabular} & . 14 & \[
\begin{aligned}
& 4.8(-6) \\
& 1.04 \text { failures/ } \\
& 12200 \text { tests }
\end{aligned}
\] \\
\hline U & 6.9(-3) & . 22 & 1.5(-3) \\
\hline based on & 2.3 failures/ 980 tests & & 2.3 failures/ 4450 tests \\
\hline
\end{tabular}

Dǎ̌a
Table Al is representation of \(\mathrm{B}(1.2) \mathrm{D}_{1} \mathrm{C}\) in terms of independent subtrses that is given in reference 1 . The subtrees are labeled A-0 to correspond with the labeling in this paper. Table A2 contains the individual elements that comprise the independent subtrees. Added to the tables are small letter desfgnators (e.g., a, b, v, etc.) for population types. Thus, all events labeled a are considered to be estimated from the same data. In Table A3, the population types are enumerated, with the assumed data also given.

Those population types marked with ' \(\mathrm{k}^{\prime}\) in Table A3 contain events with different point estimates. A listing of the different point estimates and the resulting \(T\) factors are given in Table A4. The \(T\) factors are necessary to adjust the equivalent test quantity in the unpooling process. For example, the 206 tests on population type \(u\) would be used to estimate a , but in two cases, the parameter applied in the model is \(/ 9\). When the 206 tests are apportioned between the occurrence of \(\lambda\) and the two occurrences of \(\lambda / 9\), those quantities used for estimating \(\lambda / 9\) are increased by a factor of 9 . This adjustment properly accounts for the data being used to estimate \(\mathrm{N} / 9\) rather than \(\lambda\).

Table A1. LOCA Accident Sequence Cut Sets or B(1.2)D, \(C\)
Initiating Event: \(B(1.2) \quad\) Initiating Event Frequency: \(02 / y r\)
Sequence Identifier: \(B(1.2) D_{1} C\) (Sequence 26 on \(B(1.2 ;\) Even: Tree, Figure \(A-1)\)
Total Sequence: \(B(1.2) \widehat{K} 5_{1} \bar{Y} C\)
\begin{tabular}{llll} 
& Unavallability & & Frequency \\
& & & \\
Sequence (without recovery) & \(1 . \mathrm{E}-3\) & & \(2 . \mathrm{E}-5 / \mathrm{yr}\) \\
Sequence (with recovery) & \(2.2 \mathrm{E}-4\) & & \(4.4 \mathrm{E}-6\)
\end{tabular}
\begin{tabular}{|c|c|c|}
\hline Dominant Minimal Cut Sets Unavallablilty \(\begin{gathered}\text { W/o Recovery }\end{gathered}\) & Probability of Non-Recovery & Unavailablilty w/Recovery \\
\hline (A) LPT1408B-VCC-LF LF \(^{\text {( }}\)-SWS-VCH4B (B) \(1.9 \mathrm{E}-4\) & . 01 & \(1.9 \mathrm{E}-6\) \\
\hline (C) LPI1407A-VCC-LF*LF-SWS-VCH4A (D) \(1.9 \mathrm{E}-4\) & . 01 & \(1.9 \mathrm{E}-6\) \\
\hline (E) LF-LPI-L. 25 1E-4 & 1. & 1E-4 \\
\hline (A) LPI1408B-VCC-LF*LF-SWS-S14 (F) 8.2E-5 & . 01 & \(8.2 \mathrm{E}-7\) \\
\hline (A) LPI1408B-VCC-LF*LF-SWS-S5 (G) 8.2E-5 & . 01 & \(8.2 \mathrm{E}-7\) \\
\hline (C) LPI1407A \(\pi\) NC-LF*LPI1408B-VCC LF (A) 6.7E-5 & 1. & \(6.7 \mathrm{E}-5\) \\
\hline (A) LPI1408B -VCC-LF *LF-SWS-S2 (I) 4.1E-5 & . 05 & 2.1E-6 \\
\hline (A) LPI \(1408 \mathrm{~B}-\mathrm{VCC}-\mathrm{LF} *\) LF-ECS-ROOM100(J) 4.1E-5 & . 01 & 4.1E-7 \\
\hline (C) LPI1407A-VCC-LF*LF-SWS-S1 (K) 4.1E-5 & . 4 & \(1.6 \mathrm{E}-5\) \\
\hline (C)LPI1407A-VCC-LF*LF-ECS-ROOM99 (L) 4.1E-5 & . 01 & 4.1E-7 \\
\hline \((\mathrm{M}+\mathrm{N})(\mathrm{LF}-\mathrm{RBI}-\mathrm{B} 1+\mathrm{LF}-\mathrm{RBI}-\mathrm{B9}) *\) *F-HPI-H14(0) & & \\
\hline *LF-SWS-VCH 4 B ( 5 ) 1.1E-5 & . 01 & 1.1E-7 \\
\hline
\end{tabular}

Table A2



Table A2
(Cont inuer)
\begin{tabular}{ll} 
Pipe (or Wire) Segment Local Fault: LF-SWS-vCH4B (B) & System: Emergency Cooling \\
Sequence Considered: All denoting fault & Critical Time: \(>70\) minutes \\
Unavallability w/o Recovery: \(\quad 2.3 \mathrm{E}-2\) & Unavailability w/Recovery: \(2.3 \mathrm{E}-4\)
\end{tabular}

Probabllity of Non-Recovery: 0.01
\begin{tabular}{|c|c|c|c|c|c|c|}
\hline Sub-Event Name (See Append1x B) & Is it Recoverable? & Location of Recovery Action & g. w/o Rec. & P(NR) & 9, of Rec. & Comments \\
\hline ECSCH4BA-CWV-LF & Y & Local & \(\ell \quad 3.7 \mathrm{E}-3\) & . 01 & 3.7E-5 & For all, recovery \\
\hline 5254A-CBL-LF & Y & Local & a 1.1E-3 & . 01 & \(1.1 E-5\) & action is to \\
\hline ECS5254A-B00-LF & Y & Local & j \(1 E-3\) & . 01 & 1E-5 & manually start \\
\hline ECS5254A-B00-CC & \(\mathbf{Y}\) & Local & b \(2 E-3\) & . 01 & 2E-5 & portable fans. \\
\hline ECS5254A-B-AASF & Y & Local & d \(5.4 \mathrm{E}-3\) & . 01 & \(5.4 \mathrm{E}-5\) & \\
\hline A-ECS-2 & \(Y\) & Local & - 4.3E-284 & . 01 & 4.3E-86 & \\
\hline A-ECS-15 & - & --- & \(\epsilon\) & --- & \(\epsilon\) & \\
\hline SWS608BX-XOC-LF & Y & Local & e \(1 \mathrm{E}-4\) & . 01 & 1E-6 & \\
\hline SWS3900X-XOC-LF & \(\mathbf{Y}\) & Local & e \(1 \mathrm{E}-4\) & . 01 & 1E-6 & \\
\hline SWS606BX-XOC-LF & \(\mathbf{Y}\) & Local & e \(18-4\) & . 01 & 1E-6 & \\
\hline SWS 3902X-XOC-LF & Y & Local & e 18-4 & . 01 & 1E-6 & \\
\hline ECS602BX-XOC-LF & Y & Local & e 18-4 & . 01 & \(1 \mathrm{E}-6\) & \\
\hline ECS604BX-ROC-LF & \(Y\) & Loca 1 & e 1E-4 & . 01 & 18-6 & \\
\hline ECS601BX-XOC-LF & \(\mathbf{Y}\) & Local & \(f 1 E-4\) & . 01 & \(1 \mathrm{E}-6\) & \\
\hline \begin{tabular}{l}
ECS6036A-DPC-LF \\
ECS6-36A-BR-L5 \\
ECS600BX-XOC-LF
\end{tabular} & \(\mathbf{Y}\) & Local & \[
g \begin{aligned}
& 4.1 E-3 \\
& 4.1 E-3
\end{aligned}
\] & . 01 & 4.1E-5 & \\
\hline ECS600BX-XOC-LF & \(Y\) & Local & \[
\text { e } 1 \mathrm{E}-4
\] & . 01 & \(1 \mathrm{E}-6\) & \\
\hline R-HCP-vCH48-2 & Y & Local & 4. 2E-4 & . 01 & 2E-6 & \\
\hline ECS200BX-XOC-LF & Y & Local & e \(1 \mathrm{E}-4\) & . 01 & 1E-6 & \\
\hline
\end{tabular}

Table A2
(Cont inued)



Table A2
(Cont inued)
\begin{tabular}{ll} 
Pipe (or Wire) Segment Local Fault: LF-SWS-VCH4A (D) & System: Emergency Cooling \\
Sequence Considered: All denoting fault & Critical Tise: \(>70\) minutes \\
Unavallability w/o Recovery: \(2.5 E-2\) & Unavailability w/Recovery: \(2.5 \mathrm{E}-4\)
\end{tabular}

Probability of Non-Recovery: 0.01
\begin{tabular}{|c|c|c|c|c|c|c|}
\hline \[
\begin{aligned}
& \text { Sub-Event Name } \\
& \text { (See Appendix B) }
\end{aligned}
\] & Is it Recoverable? & Location of Recovery Action & 9. w/o Rec. & \(P(N R)\) & q, \%/Rec. & Comments \\
\hline ECSCH4AB-CWU-LF & Y & Local & \& \(3.7 \mathrm{E}-3\) & . 01 & 3.7E-5 & For all, recovery \\
\hline 62543 -CBL-LF & \(Y\) & Local & a 1.1E-3 & . 01 & 1.1E-5 & action is to \\
\hline ECS6254B-B-AASE & \(Y\) & Local & d \(5.4 \mathrm{E}-3\) & . 01 & \(5.4 \mathrm{E}-5\) & manually atart \\
\hline ECS6254B-B00-LF & \(Y\) & Local & j \(1 \mathrm{E}-3\) & . 01 & -1E-5 & portable fans. \\
\hline ECs6254B-BOO-CC & \(Y\) & Local & b \(2 \mathrm{E}-3\) & . 01 & \(2 \mathrm{E}-5\) & \\
\hline A-ECS -3 & \(\mathbf{Y}\) & Local & - \(4.3 \mathrm{E}-4\) & . 01 & 4. 3E-6 & \\
\hline \(\mathrm{R}-\mathrm{HCP}-\mathrm{VCH} 4 \mathrm{~A}-3\) & \(Y\) & Local & 4. \(1.8 \mathrm{E}-3\) & . 01 & \(1.8 \mathrm{E}-5\) & \\
\hline ECS602 \(6 \mathrm{X}-\mathrm{XOC}-\mathrm{F}\) & \(Y\) & Local & e \(1 \mathrm{E}-4\) & . 01 & 1E-6 & \\
\hline ECS604AX-CCC-L. & \(\mathbf{Y}\) & Local & f \(1 \mathrm{E}-4\) & . 01 & 1E-6 & \\
\hline ECS601AX-XOC-LF & \(\boldsymbol{\gamma}\) & Local & e. \(1 \mathrm{E}-4\) & . 01 & 1E-6 & \\
\hline ECS60343-DPC-LF & \(Y\) & Local & g 4.1E-3 & . 01 & 4.1E-5 & \\
\hline A-ECS-14 & - & --- & ¢ & -- & € & \\
\hline ECS600AX-XOC-LF & \(\mathbf{Y}\) & Loca: & e \(1 \mathrm{E}-4\) & . 01 & 18-6 & \\
\hline ECS6034B-DPC-LF & \(\mathbf{Y}\) & Local & f \(4.1 \mathrm{E}-3\) & . 01 & 4.1E-5 & \\
\hline ECS200AX-XOC-LF & \(\mathbf{Y}\) & Local & e 1E-4 & .01 & 1E-6 & \\
\hline SWS608AX-XOC-LF & \(\boldsymbol{Y}\) & Local & e 1E-4 & . 01 & 1E-6 & \\
\hline SWS3903X-X0C-LF & \(\mathbf{Y}\) & Local & e \(18-4\) & . 01 & 1E-6 & \\
\hline SWS606AX-XOC-LF & \(Y\) & Local & e 1E-4 & .01 & 1E-6 & \\
\hline SWS 3905 X -XOC-LF & \(\boldsymbol{Y}\) & Local & e. \(18-4\) & . 01 & 1E-6 & \\
\hline
\end{tabular}

\begin{tabular}{|c|c|c|c|c|c|c|}
\hline Sub-Event Name & Is it & Location of & & & & \\
\hline (See Appendix B) & Recoverable? & Recovery Action & 9. w/o Rec. & P (NR) & q, w/ Rec. & Comments \\
\hline LPIOBW1X-XOC-LF & N & -- & 1E-4 & 1 & \(1 E-4\) & \\
\hline
\end{tabular}

Table A2
(Cont inued)
Pipe (or Wire) Segment Local Fault: LF-SWS-S14 (F) System: Service Water
Sequence Considered: All LOSP
\begin{tabular}{ll} 
Unavailability w/o Recovery: \(1 \mathrm{E}-2\) & Critical Time: 30 min \\
& Probability of Non-Recovery: 0.09
\end{tabular}
\begin{tabular}{|c|c|c|c|c|c|c|}
\hline Sub-Event Name (See Appendix B) & Is it Recoverable? & Location of Recovery Action & q, w/o Rec. & \(P\) (NR) & g. w/ Rec. & Comments \\
\hline SWS \(3820 \mathrm{~A}-\mathrm{VOO}-\mathrm{LF}\) & \(Y\) & Local & \(94 \mathrm{E}-3\) & . 1 & \(4 \mathrm{E}-4\) & \\
\hline 5181A-CBL-LF & Y & Local & a \(3.3 \mathrm{E}-3\) & . 1 & 3. 3E-4 & \\
\hline SWS - 5181A-B00-LF & * & Local & 1 \(1 E-3\) & . 1 & 1E-4 & \\
\hline SWS-5181A-BOO-CC & Y & Control Room & b \(2 \mathrm{E}-3\) & . 05 & 1E-4 & \\
\hline ESFU113-UCT-LF & \(\mathbf{Y}\) & Control Room & c \(1 \mathrm{E}-4\) & . 05 & SE-6 & \\
\hline
\end{tabular}
\[
\begin{aligned}
& \text { 1.1) } 0 \text { • } \\
& \begin{array}{ll}
4.4 \\
3
\end{array} \\
& \text { "15 } \\
& \text { 人.. } \\
& \text { C }+\mathrm{H}_{8} \mathrm{H}
\end{aligned}
\]
\[
\begin{gathered}
(+1) \\
1=0
\end{gathered}
\]

Table A2
(Cont inued)
```

Pipe (or Wire) Segment Local Fault: LF-SHS-SS (G)
Sequence Considered: A1: LOSP
Unavailability w/o Recovery: 1E-2
System: Service Water
Critical Time: 30 min
Unavallability w/Recovery: 9.3E-4

```

Probability of Non-Recovery: 0.09
\begin{tabular}{|c|c|c|c|c|c|c|}
\hline Sub-Event Name (See Appendix B) & Is \(1 t\) Recoverable? & Location of Recovery Action & g, w/o Rec. & \(\mathrm{P}(\mathrm{NR})\) & q, w/ Rec. & Comments \\
\hline SWS 3643A-VOO-LF & \(\boldsymbol{Y}\) & Local & \(94 \mathrm{E}-3\) & .1 & \(4 \mathrm{E}-4\) & \\
\hline 5653A-CBL-LF & Y & Local & a 3.3E-3 & . 1 & 3. 3E-4 & \\
\hline SWS5653A-B00-LF & \(Y\) & Local & \(j\) 12-3 & . 1 & \(18-4\) & \\
\hline SWS5653A-B00-CC & Y & Contral Room & b \(2 \mathrm{E}-3\) & . 05 & \(1 E-4\) & \\
\hline A-SWS-14 & - & --- & \(\epsilon\) & --- & \(\epsilon\) & \\
\hline
\end{tabular}

Table A2 (Cont Inued)
\begin{tabular}{ll} 
Pipe (or Wire) Segment Local Fault: LF-SWS-S2 (I) & System: Service Water \\
Sequence Considered: All LOSP & Critical Tiwe: 30 minutes \\
Unavailability w/o Recovery: \(5 E-3\) & Unavallability w/Recovery: \(4.6 E-4\)
\end{tabular}
\begin{tabular}{|c|c|c|c|c|c|c|}
\hline Sub-Event Name (See Append Ix B) & Is it Recoverable? & Location of Recovery Action & 9, w/o Rec. & P (NR) & q. w/ Rec. & Comments \\
\hline SuS0018X-COC-LF & -- & -- & \(\epsilon\) & --- & - \(\epsilon\) & \\
\hline SWS002BX-COC-LF & -- & --- & \(\epsilon\) & --- & \(\epsilon\) & \\
\hline A-SWS-3 & N & --- & - 2.2E-4 & 1 & 2. \(2 \mathrm{E}-4\) & \\
\hline SWSOP4BA-PMD-LF & \(Y\) & Control Room & \& \(1.7 \mathrm{E}-3\) & . 05 & 8. \(5 \mathrm{E}-5\) & Start standby pump is recovery action \\
\hline 0303-CBL-LF & \(Y\) & Contral Room & a \(1 \mathrm{E}-4\) & . 05 & SE-6 & \\
\hline SWS0303A-B00-LF & Y & Control Room & d \(\mathrm{IE}-3\) & . 05 & SE-5 & \\
\hline SWS0303A-B00-CC & \(Y\) & Control Room & b \(2 E-3\) & . 05 & \(1 E-4\) & \\
\hline
\end{tabular}
\begin{tabular}{ll} 
Pipe (or Wire) Segment Local Fault: LF-ECS-ROOM \(100(J)\) & System: Emergency Cooling \\
Sequence Considered: Ali denoting fault & Critical Time: \(>70\) minutes \\
Unavailability w/o Recovery: \(4.9 \mathrm{E}-3\) & Unavailability w/Recovery: \(4.9 \mathrm{E}-5\)
\end{tabular}
\begin{tabular}{|c|c|c|c|c|c|c|}
\hline \begin{tabular}{l}
Sub-Event Name \\
(See Appendix B)
\end{tabular} & Is it Recoverable? & Location of Recovery Action & 9, w/o Rec. & P(NR) & g. w/Rec. & Comments \\
\hline ECSVC2BA-FAN-LF & Y & Local & \(k \quad 5.4 E-4\) & . 01 & 5.4E-6 & For all, recovery action is to \\
\hline 5246A-CBL-LF & \(Y\) & Local & a \(1.1 \mathrm{E}-3\) & . 01 & 1.1E-5 & manually start portable fans. \\
\hline ECS5246A-B00-LF & \(Y\) & Local & j \(1 E-3\) & . 01 & \(1 E-5\) & \\
\hline ECS \(5246 \mathrm{~A}-\mathrm{BOO}-\mathrm{CC}\) & \(Y\) & Local & b \(2 \mathrm{E}-3\) & . 01 & \(2 \mathrm{E}-5\) & \\
\hline A-ECS-11 & - & --- & \(\epsilon\) & --- & \(\epsilon\) & \\
\hline ECSC418X-XOC-LF & Y & Local & e \(1 \mathrm{E}-4\) & . 01 & \(1 \varepsilon-6\) & \\
\hline ECSC448X-XOC-LF & \(\mathbf{Y}\) & Local & e \(1 \mathrm{E}-4\) & . 01 & \(18-6\) & \\
\hline \(\operatorname{ECSC} 45 \mathrm{BX}-\mathrm{XOC}-\mathrm{LF}\) & \(Y\) & Local & e. 18-4 & . 01 & 1E-6 & \\
\hline
\end{tabular}
\begin{tabular}{ll} 
Pipe (or Wire) Segment Local Fault: LF-SWS-SI (K) & System: Service Water \\
Sequence Considered: All LOSP & Critical Time: 30 winutes \\
Unavallability w/o Recovery: \(5 \mathrm{E}-3\) & Unavallability w/Recovery: \(2.2 \mathrm{E}-3\)
\end{tabular}
\begin{tabular}{|c|c|c|c|c|c|c|}
\hline Sub-Event Name
(See Appendix B) & \begin{tabular}{l}
Is it \\
Recoverable?
\end{tabular} & Location of Recovery Action & 9, v/0 Rec. & P(NR) & 9, w/ Rec. & Comments \\
\hline SHSOO1CX-COC-LF & -- & --- & \(\epsilon\) & - & \(\epsilon\) & \\
\hline SWS002CX-COC-LF & -- & --- & \(\epsilon\) & -- & \(\epsilon\) & \\
\hline A-SWS-1 & N & -- & - 2.2E-4 & 1 & 2.2E-4 & \\
\hline SHSOP4CB-PMD-LF & N & --- & f \(1.7 \mathrm{E}-3\) & 1 & 1. \(7 \mathrm{E}-3\) & \\
\hline 0402-CBL-LF & N & --- & a \({ }^{\text {P }} \mathrm{E}-4\) & 1 & 18-4 & \\
\hline SWS0402B-B00-LF & \(Y\) & Local & j \(1 E-1\) & . 1 & 1E-4 & \\
\hline SWS0402B-B00-CC & \(\mathbf{Y}\) & Control Room & b \(2 \mathrm{E}-3\) & . 05 & \(1 E-4\) & \\
\hline
\end{tabular}

Failure Probabilities, With Recovery, of Support System Faults
\begin{tabular}{ll} 
Pipe (or Wire) Segment Local Fault: \(1 F-E C S-R O O M\) & \(99(L)\) \\
Sequence Considered: All denoting fault & Systea: Emergency Cooling \\
Unavallability w/o Recovery: \(4.9 \mathrm{E}-3\) & Critical Time: \(>70\) minutes
\end{tabular}
\[
\text { Probabillty of Non-Recovery: } 0.01
\]
\begin{tabular}{|c|c|c|c|c|c|c|c|}
\hline Sub-Event Name (See Appendix B) & \begin{tabular}{l}
Is it \\
Recoverable?
\end{tabular} & Location of Recovery Action & & , w/o Rec. & P (NR) & 9, w/ Rec. & Comments \\
\hline ECSC2DB-FAN-LF & \(\mathbf{Y}\) & Local & \(k\) & \(5.4 \mathrm{E}-4\) & . 01 & \(5.4 \mathrm{E}-6\) & All subfaulta \\
\hline \(6246 \mathrm{~B}-\mathrm{CBL}-\mathrm{LF}\) & Y & Local & \(a\) & \(1.18-3\) & . 01 & \(1.1 \mathrm{E}-5\) & are recoverable by the use of \\
\hline ECS \(6246 \mathrm{~B}-\mathrm{BOO}-\mathrm{LF}\) & \(\mathbf{Y}\) & Local & \(j\) & \(1 \mathrm{E}-3\) & . 01 & \(1 E-5\) & portable fans. \\
\hline ECS6246B-B00-CC & \(\mathbf{Y}\) & Local & \(b\) & \(2 \mathrm{E}-3\) & . 01 & \(2 \mathrm{E}-5\) & \\
\hline A-ECS-8 & - & --- & & \(\epsilon\) & -- & \(\epsilon\) & \\
\hline ECSC410x-xoc-LF & Y & Local & \(e\) & 1E-4 & . 01 & \(1 \mathrm{E}-6\) & \\
\hline Ecsc \(440 \mathrm{D}-\mathrm{xOC}-\mathrm{LF}\) & \(\mathbf{Y}\) & Local & \(e\) & 1E-4 & . 01 & \(1 \mathrm{E}-6\) & \\
\hline ECSC45Dx-xOC-LF & \(Y\) & Local & e & 18-4 & . 01 & 18-6 & \\
\hline
\end{tabular}

\section*{Table A2}
(ContInued)

\begin{tabular}{|c|c|c|c|c|c|c|}
\hline \begin{tabular}{l}
Sub-Event Name \\
(See Appendix B)
\end{tabular} & Is it Recoverable? & Location of Recovery Action & 9, w/0 Rec. & P (NR) & 9,w/Rec. & Comments \\
\hline 2400B-vCC-LF & Y & Local & 4.18-3 & . 1 & 5E-4 & \[
\begin{aligned}
& 1 \text { Time }= \\
& -3 \text { recove } \\
& -4 \text { is not }
\end{aligned}
\] \\
\hline
\end{tabular}
\begin{tabular}{|c|c|c|c|c|c|c|}
\hline \(6171 \mathrm{X}-\mathrm{CBL}-\mathrm{LF}\) & Y & Local & & 3. \(3 \mathrm{E}-3\) & . 1 & 3. \(3 \mathrm{E}-4\) \\
\hline \(6171 \mathrm{X}-\mathrm{B00}-\mathrm{LF}\) & \(\mathbf{Y}\) & t.ocs 1 & & \(1 E-3\) & . 1 & 1E-4 \\
\hline 6171x-800-CC & \(Y\) & Control Room & & \(2 \mathrm{E}-3\) & . 05 & \(1 \mathrm{E}-4\) \\
\hline A-RBI-5 & -- & -- & & \(\epsilon\) & -- & \(\epsilon\) \\
\hline A-1104.05-0 & \(\boldsymbol{Y}\) & Local & P & \(1 E-3\) & . 03 & \(3 \mathrm{E}-5\) \\
\hline
\end{tabular}
\begin{tabular}{ll} 
Pipe (or Wire) Segment Local Fault: \(L P-R B I-B 1+L F-R B I-B 9\) (Cont.) & System: \\
Sequence Considered: & Critical Time: \\
Unavailability w/o Recovery: & Unavailability w/ Recovery:
\end{tabular}

Probability of Non-Recovery:


\section*{Table A2}
- (Cont inued)
\begin{tabular}{ll} 
Pipe (or Wire) Segment Local Fault: LF-RBI-B1 + LF-RBI-B9 (Cont.) & System: \\
Sequence Considered: & \((M+N)\) \\
Unavallability \(w / 0\) Recovery: & Critical Time: \\
Unavallability w/Recovery:
\end{tabular}

Probability of Non-Recovery:
\begin{tabular}{|c|c|c|c|c|c|c|}
\hline Sub-Event Name (See Appendix B) & Is it Recoverable? & Location of Recovery Action & q, w/o Rec. & P(NR) & g, w/ Rec. & Comments \\
\hline 3805B-NCC-CC & \(Y\) & Control Room & b \(2 \mathrm{E}-3\) & . 01 & 2E-5 & Crit Time > 70 min \\
\hline 0404B-B00-LF & \(Y\) & Local & f \(1 \mathrm{E}-3\) & . 03 & 3E-5 & \\
\hline 04048-B00-CC & Y & Control Room & b \(2 \mathrm{E}-3\) & . 01 & 2E-5 & \\
\hline U239-UCT-LF & Y & Control Room & c 1E-4 & . 01 & 1E-6 & \\
\hline R-HCP-021B-8 & Y & Local & - \(2 \mathrm{E}-4\) & . 01 & 2E-6 & Crit Time \(>70 \mathrm{~min}\) \\
\hline 8-110405-5-21B & Y & Local & t \(\mathrm{BE}-3\) & . 01 & \(8 \mathrm{E}-5\) & Crit \(\mathrm{Tlme}>70 \mathrm{~min}\) \\
\hline 2B32B-CBL-LF & Y & Local & a \(1.1 \mathrm{E}-3\) & . 03 & 3. 3E-5 & \\
\hline SWS2B32B-B00-CC & Y & Control Room & b \(2 \mathrm{E}-3\) & .01 & \(2 \mathrm{E}-5\) & \\
\hline Y \(02-120-\mathrm{LF}\) & N & -- & & -- & & \\
\hline IEAO6BB-TEM-LF & N & - & & -- & & \\
\hline
\end{tabular}

Table A2
(Continued)
\begin{tabular}{ll} 
Pipe (or Wire) Segment Local Fault: \(\mathrm{LF}-\mathrm{RBI}-\mathrm{Bl}+\mathrm{LF}-\mathrm{RBI}-\mathrm{B9}\) (Cont.) & System: \\
Sequence Considered: & Critical Time: \\
Unavailability w/o Recovery: & Unavailability w/ Recovery: \\
& Probability of Non-Recovery:
\end{tabular}
\begin{tabular}{|c|c|c|c|c|c|c|}
\hline Sub-Event Name (See Appendix B) & Is it Recoverable? & Location of Recovery Action & 4, w/o Rec. & \(P(N R)\) & 9, w/Rec. & Comments \\
\hline IEA61438B-CBL-LF & N & -- & \(\epsilon\) & - & \(\epsilon\) & \\
\hline TEAOS2BB-BCO-LF & \(Y\) & -- & \(\epsilon\) & -- & \(\epsilon\) & \\
\hline IEA61938B-BCO-LF & \(Y\) & - & \(\epsilon\) & -- & \(\epsilon\) & \\
\hline
\end{tabular}
(Cont inued)

\begin{tabular}{|c|c|c|c|c|c|c|}
\hline Sub-Event Name (See Appendix B) & \begin{tabular}{l}
Is it \\
Recoverable?
\end{tabular} & Location of Recovery Action & 9. W/o Rec. & P (NR) & q. w/ Rec. & Comments \\
\hline HPIV19CX-CCC-LF & 4 & -- & \(1 \mathrm{E}-4\) & 1 & 1E-4 & \\
\hline HPIV2OCX-XOC & N & -- & \(1 \mathrm{E}-4\) & 1 & 1E-4 & \\
\hline A-HPI-4 & N & -- & \(2 \mathrm{E}-4\) & 1 & \(2 \mathrm{~B}-4\) & \\
\hline A-HPI-5 & -- & -- & \(\epsilon\) & -- & € & \\
\hline A-HPI-6 & -- & -- & \(\epsilon\) & -- & \(\epsilon\) & \\
\hline HPIV18CX-XOC-LF & N & -- & 1E-4 & 1 & 18-4 & \\
\hline HP1P36CB-PMD-LF & N & -- & 12-3 & 1 & 1E-3 & \\
\hline A4068-CBL-LF & N & -- & 1.1E-3 & 1 & \(1.18-3\) & \\
\hline HPIA4068-B00-LF & \(\mathbf{Y}\) & Local & 1E-3 & . 05 & \(5 \mathrm{E}-5\) & \\
\hline HP1A406B-B00-CC & \(Y\) & Contral Room & 2E-3 & . 03 & \(6 \mathrm{E}-5\) & \\
\hline
\end{tabular}
Pipe (or Hire) Segment Local Fault: LF-HPI-B14 (Cont.)
\begin{tabular}{lll} 
Sequence Considered: & (O) & System: \\
Unavailability w/o Recovery: & Critical Time: \\
& Probability of Non-Recovery:
\end{tabular}


Table A3. "Population Type" Data
\begin{tabular}{|c|c|c|c|c|c|c|c|}
\hline \begin{tabular}{l}
Pop. \\
Type
\end{tabular} & \[
\begin{aligned}
& \text { File } \\
& \text { No. }
\end{aligned}
\] & Point Estimate & Error Factor & \begin{tabular}{l}
Equiv. \\
Failures
\end{tabular} & \begin{tabular}{l}
Equiv. \\
Tests
\end{tabular} & Total Number of Occurrences & Blocks ( Occurrences) \\
\hline *a & 1 & 3.3(-3) & 3 & 2.20 & 667 & 15 & \(A, B, C, D, F, G, 1, J, K, L, M+N(3), D(2)\) \\
\hline b & 2 & \(2.0(-3)\) & 3 & 2.20 & 1100 & 16 & \(A, B, C, D, F, G, I, J, K, L, M+N(4), C(2)\) \\
\hline \(c\) & 3 & 1.0(-4) & 3 & 2.20 & 22000 & 7 & \(A(2), C(2), F, 4+N, 0\) \\
\hline d & 4 & \(5.4(-3)\) & 10 & . 37 & 69 & 2 & B, \({ }^{\text {d }}\) \\
\hline e & 5 & \(1.0(-4)\) & 3 & 2.20 & 22000 & 29 & \(B(8), D(8), E, J(3), L(3), M+N(3), O(3)\) \\
\hline f & 6 & 1.0(-t) & 3 & 2.20 & 22000 & 5 & B, \(, \mathrm{M}, \mathrm{N}(2), 0\) \\
\hline 9 & 8 & 4.1(-3) & 3 & 2.20 & 537 & 10 & \(A, B(2), C, D(2), F, G, M+N, O\) \\
\hline \(j\) & 11 & 1.0(-3) & 3 & 2.20 & 2200 & 14 & \(A, B, C, D, F, G, I, J, K, L, M * N(2), O(2)\) \\
\hline k & 12 & \(5.4(-4)\) & 3 & 2.20 & 4074 & 2 & J,L \\
\hline *1 & 13 & 3.7(-3) & 3 & 2.20 & 595 & 6 & B, D, I, K, M + , 0 \\
\hline * 0 & 26 & 4.3(-4) & 3 & 2.20 & 5116 & 6 & \(B, D, I, K, M+N, 0\) \\
\hline \(p\) & 28 & \(1.0(-3)\) & 3 & 2.20 & 2200 & 1 & \(\mathrm{M}+\mathrm{N}\) \\
\hline t & 40 & \(8.0(-3)\) & 10 & . 37 & 46 & 1 & \(\mathrm{M}+\mathrm{N}\) \\
\hline *u & 41 & \(1.8(-3)\) & 10 & . 37 & 206 & 3 & 8, \(\mathrm{D}, \mathrm{M}+\mathrm{N}\) \\
\hline \(v\) & 47 & \(3.0(-3)\) & 3 & 2.20 & 733 & 1 & \(\mathrm{m}+\mathrm{N}\) \\
\hline
\end{tabular}
* These types have individual point estimates within the type that differ. See Table A4 for applicable adjustment factors.

\title{
Table A4. Population Types With Mixed Point Estimates With Appropriate Test Factors
}

Point Estimates--Blocks of Occurrence
\begin{tabular}{|c|c|c|c|}
\hline Pop. Type & Base & Other & Factor \\
\hline \multirow[t]{2}{*}{a} & 3.3(-3) F,G,M+N & 1.1(-3) A, B, C, D, J, L, M+N(2),O(2) & 3 \\
\hline & & 1(-4) 1, K & 33 \\
\hline \multirow{4}{*}{0} & \(3.7(-3) \quad B, 0\) & 1.7(-3) 1, K & 2.2 \\
\hline & & 1( -3 ) \(M+N, 0\) & 3.7 \\
\hline & 4.3(-4) B, D & 2.2(-4) 1, K, M + N & 1.95 \\
\hline & & \(2(-4) 0\) & 2.15 \\
\hline \(u\) & \(1.8(-3) 0\) & 2(-4) B, M+N & 9 \\
\hline
\end{tabular}

\section*{Unpooling Algorithm}

The proposed unpooling scheme unpools each of the data-type populations, compares the test quantities that result to the existing unpooled types, makes adjustments in the current type if necessary, and then moves to the next data type. The process is elaborated on here and illustrated using the \(D_{1} C\) sequence and data from the main report.

Step 1. The system under consideration is broken into series subsystems for which an equivalent test quantity will be recorded and updated with the addition of each population type. Inftially, the equivalent test quantity for each subsystem is treated as missing or unassigned. Also calculated at this step is the failure probability for each subsystem. Three values will be used for unpooling purposes.

Example. For the \(B(1.2) D_{1} C\) sequence, the system to be considered is given in Figure A1. The blocks are labeled with the leading block label used in the body of the report.


Point Estimates
\(A-8.38(-3)\)
B \(-5.24(-2)\)
C \(-8.38(-3)\)
D \(-3.40(-2)\)
E - \(1.00(-4)\)
\(\mathrm{M}+\mathrm{N}-3.31(-2)\)
\(0-1.39(-2)\)
\(B\) includes B,F,G,I,J from Figure 2
D includes D,K,L from Figure 2

Figure Al. Overall system in terms of branches for which equivalent test quantities are needed.

Step 2. All data types that appear only once in the total system are assigned and the minimum test quantity for each segment is recorded.

Example. Population types \(p, t\), and \(v\) occur singly, all in the \(M+N\) branch. Therefore, branch \(M+N\) now has a minimum test quantity of 46 from the component of type \(t\).

Step 3. All the data types that have more than one occurrence are ordered according to the number of tests divided by the sum of the reciprocals of the T factors for each occurrence. This represents the quantity that will apply to each occurrence of a population type if a split is done to make each occurrence have the same amount of applicable data. This ordering will be used for purposes of unpooling.

Example. For the \(B(1.2) D_{1} C\) sequence, ine ordering is \(d, g, b, a, u, j\), , \(e, 0, k, c\), and \(f\). Population type \(d\) is considered first as there are, in general, less "data" for each of she occurrences \((69 / 2=34.5)\). Population type \(g\) is next with approximately ( \(537 / 10 \Rightarrow 53.7\) test quantities that can be assigned to each occurrence. Notice that for type a, 10 of the unpooled values will be times a factor of 3,2 will be times a factor of 33 , and 3 will be at the base value (facter of 1). Thus, to unpool so that avery occurrence has the same amount of data, the 667 test quantity is divided by \(10 \cdot 1 / 3+2 \cdot 1 / 33+3=6.39\), to give 104.3 tests to each occurrence.

Step 4. The individual population types are unpooled for each population type in the order determined by step 3. The unpooling is done in such a way as to maximize the effective overall cest quantity incorporating the given component with the already unpooled cata and the minimum test quantities that apply to each subsystem. Subsystens that have no minimum test quantities as yet assigned are treated as constants.

Example 1. Population type \(d\) is the first type to be unpooled, as determined from step 3 . Only the subsystem \(M+N\) has a test quantity associated with it from the data types with a single occurrence considered in step 2. Considering the point estimate for subsystem 0 as a constant, the test size of 46 from \(M+N\) is equivalent to a test size of \(3309(=46 / 1.39(-2))\) for the system of \(M+N\) in parallel with 0 . The test size of 3309 would then also apply to the whole subsystem containing \(A, M+N\), and 0 . If \(n\) of the 69 tests on population type \(d\) were assigned to the occurrence in subsystem \(B\), then the equivalent test quantity for that combination would be given by \(8.84(-3) \times 3309=29.3\) failures in 3309 tests combined in parallel with \(5.24(-2)\). n failures in \(n\) tests. The effective test quantity for the other parallel branch is (69-n)/8.38(-3) since subsystem \(C\) is treated as a constant. Since the effective test quantity increases with \(n\) in the first case and decreases with \(n\) in the second, the minimum of the two will be maximized when the two expressions above are equal. This occurs when \(n=35.7\), therefore, population type d is unpooled by considering 35.7 tests in subsystem \(B\) and 33.3 tests in subsystem D. The equivalent test quantities are now 3309 for \(M+N, 35.7\) for B, and 33.3 for D. The rest of the subsystems would still be considered as constants (having no equivalent test quantities).

Example 2. It will be instructive to also consider the next population type \(g\) here at step 4. There are occurrences of population type \(g\) in all but subsystem \(E\). There are single occurrences of type \(g\) in subsystems \(A, M+N, 0\), and \(C\) and there are four occurrences in subsystem \(B\) and two occurrences in subsystem D. Let \(n_{A}, n_{B}, \ldots\) denote the unpooled test quantity for subsystems \(A, B, \ldots\). . The total test quantity is 537 , and thus, we want to assign the
test quantities such that \(n_{A}+n_{M+N}+n_{0}+n_{C}+4 \cdot n_{B}+2 \cdot n_{D}=537\), and the overall equivalent system test size is maximized. At this stage, we are not concerned with the equivalent test quantities that have already been assigned to the subsystems in which population type \(g\) appears. We perform the optimization problem for \(g\) and then compare the equivalent test quantities for g alone to those already assigned and make appropriate adjustments in step 5. The solution of the problem for allocating \(g\) is \(n_{A}=168.4, n_{B}=35.4\), \(n_{C}=109.6, n_{D}=42.6, n_{M+N}=14.5\), and \(n_{0}=17.7\), with an equivalent system test size of 1450 .

A specific method for solving the above problem is not being recommended. The above solution was obtained by programming the Maximus rules for parallel systems on a desk calculator and iterating intelligently to obtain ihe solution.

Notice in the solution for \(g\) that \(n_{0}=42.6\), but from the unpooling of population type \(d\), the equivalent test size for subsystem \(D\) was 33.3. This difference forms the basis for the next step.

Step 5. If, for a specific population type in step 4, any of the equivalent tests for a subsystem exceed the equivalent test quantity already assigned to that subsystem and there is some other subsystem in which the current population type is minimum, then rework step 4 , but first allocating the existing equivalent test size to those subsystems where this value was less than that calculated in step 4.

Example. In step 4 for population type \(g n_{D}=42.6\), which exceeds the existing test size of subsystem D of 33.3 and in all the other subsystems the assignment from type \(g\) is the minimum. Therefore, nD is set to 33.3 and the allocation of the remaining \(537-2(33.3)=470.4\) is done for population type \(g\) as was done originally in step 4. The result of this step is that \(n_{A}=163.3\), \(n_{B}=35.7, n_{C}=131.9, n_{D}=33.3, n_{M+N}=14.6\), and \(n_{0}=17.8\). These are the values used in the overall analysis and are reflected in the allocations of Figure 3.

Step 6. Return to step 4 (and 5) for the next population type.
For all the remaining population types in the example followed, the effective numbers for each branch all exceed that assigned in determining the allocation for type \(g\). Therefore, the combination of data types \(d\) and \(g\) determine effective quantities for each branch.

The unpooling algorithm as presented is meant to give the flavor of a systematic way to look at the unpooling question. The algorithm has not been completely defined in that the method of optimization for steps 4 and 5 is not specified. In practice, a stepwise method may be the easfest to implement. The different population types that determine the equivalent test quantities may interact to such an extent that the whole procedure would have to be reapplied. For example, in the \(D_{1} C\) case considered here, population types d and \(g\) are the determining population types. However, the first time through the algorithm the d population was unpooled assuming some of the sutsystem
branches were constant. Once population type \(g\) was unpooled, one would need to reexamine the unpooling of type \(d\) again, and so on between the two, in order to converge to an "optimal" unpooling.

In the case worked here, an equivalent system test quantity of 1427 was obtained, but it is known from \(g\) alone that 1450 is an upper bound. Thus, the iterations between population types \(d\) and \(g\) seemed unnecessary.

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