PDR CONTRACT
NRC-02-81-065

FINAL REPORT

TRAINING COURSE ON THE USE OF U.S. GEOLOGICAL SURVEY GROUND-WATER FLOW MODELS

PREPARED FOR
U.S. NUCLEAR REGULATORY COMMISSION
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REPORT

Introduction

This report provides a summary and documentation of a training course conducted for the Nuclear Regulatory Commission (NRC) by S. S. Papadopulos and Associates, Inc. The course was designed to provide NRC scientists with an introduction in the use of two-dimensional and three-dimensional ground-water flow simulation programs published by the U. S. Geological Survey. These computer programs are used extensively by hydrogeologists in both the public and private sector as a powerful tool in the analysis of ground-water systems.

Course Objectives and Scope

The primary objective of the course was to present the basic operating procedures for the two programs. However, effective use of these programs requires a fundamental knowledge of the physical and mathematical principles that form the basis of these programs. Therefore, in addition to basic program operation, other topics discussed during the course included:

* Historical evolution and use of models in hydrogeology.
* Development sequence of numerical models.
* Mathematical description of physical processes.
* Introduction to finite-difference methods.
* Theoretical background on USGS programs.
* Practical considerations in the use of numerical models in ground-water system analysis.

Course Materials

The topics in the previous paragraphs were presented in a 3 day course held at NRC offices in Silver Spring, Maryland, August 11-13, 1981. Materials (notes, problems, etc.) that were distributed to participants are included in the appendices to this report. The USGS reports that pro 'de the basic program documentation were distributed to participa. but are not included in this report. Appendix A of this report includes the technical notes that accompany the lecture presentation. Appendix B provides a glossary of commonly used terms. Appendix C provides notes on matrix algebra that are useful when studying the USGS reports. Appendix D includes problems that were assigned to participants along with appropriate program input and results for solving the basic parts of the problems.

Participant Reaction

The reaction of the course participants to the material presented was very favorable. The only comments were that the discussions of program output and iteration parameters were not sufficiently detailed. This was primarily caused by difficulties with NRC computer facilities which occurred on day 2 of the course. As a result of these difficulties, participants were unable to obtain the results of their problems in a timely manner, thus limiting the time available for discussion. In response to these comments, the problem results presented in Appendix D have been annotated to describe the program output, and a short discussion of iteration parameters is presented below.

The iterative methods employed by the program to solve simultaneous equations use iteration parameters that are defined, in part, by input from the program user. Optimum values of the user defined variables are generally determined by trial in which the "best" value(s) produces the least number of iterations to reach the solution. The following table summarizes the commonly used values for the user defined variables in the two-dimensional program.

Solution procedure	Variables	Values
Line-successive overelaxation	LENGTH	5 - 10[1]
	HMAX	1.6 - 1.9
Iterative alternating direction implicit procedure	LENGTH	6 - 10
	HMAX	1 or 2[2]
Strongly implicit procedure	LENGTH	10[3]
	HMAX	1[4]

[1] Use only if convergence is slow, otherwise LENGTH > ITMAX.
[2] Use 2 for highly anisotropic problems, otherwise use 1.
[3] Always use 10.
[4] Values other than 1 are sometimes useful for "difficult" problems. Two such problems are discussed in the program documentation, pp. 27-29.

In the three-dimensional program, the strongly implicit procedure is the only available method. The number of iteration parameters (variable LENGTH) will normally be 5 to 7 with 5 being most common.

Recommendations

The sequence and depth of the material presented appeared to be adequately suited to the background of most participants. As previously mentioned, difficulties with the NRC computer system precluded a thorough discussion of program output. To circumvent this problem in future courses, NRC should consider a backup computer system, such as one provided by the contractor, or adding one day to the length of the course.

APPENDIX A

APPENDIX A
COURSE NOTES

I. Introduction

Objective:

1) To gain an understanding of the process of modeling ground-water flow systems.

2) To provide a formal introduction to the use of the USGS finite difference computer programs for modeling 2-D and 3-D ground-water flow.

3) To gain an appreciation of the capabilities and limitations of these models.

Purpose of models:

1) Predict the response of the system to stress.

2) Quantify water availability.

3) Quantify ground-water velocity (transport processes).

4) Investigate alternative system characteristics.

5) Guide data collection.

6) Provide input to management or decision making models.

Model types:

1) Statistical or probabilistic - does not consider physical processes, requires sufficient observation of input-output characteristics of the system.

2) Deterministic - considers physical processes of the system.

Types of deterministic models:

1) Mathematical - use equations to describe physical processes.

2) Physical - use an alternate system that has physical processes that are analogous to those of the real system.

3) Analog - use an alternate system (R-C network) to simulate a mathematical approximation to the real system.

Examples of ground-water models:

1) Physical models

   a. Sand box

   b. Hele-shaw (viscous flow)

   c. Stretched membrane

   d. Electrolytic tank or conducting paper.

2) Analog models - resistor/capacitor network

3) Mathematical models

   a. Analytical - Theim, Theis, Hantush - Jacob, etc.

   b. Numerical - finite difference, finite element, polygonal, etc.

Numerical models of ground-water systems:

Physical system

↓

Mathematical expression of system processes by
partial differential equation(s) (P.D.E.)

↓

Approximation of P.D.E.
(finite differences, finite elements, etc.)

↓

Systems of simultaneous algebraic equations

↓

Solution of algebraic equations
(ADI, LSOR, SIP, Gaussian elimination)

↓

Approximate values of dependent variable
(head) at discrete points in time and space

II. Mathematical Fundamentals and Background

Basic equations of ground-water flow

Concept - unsteady flow in a saturated porous medium

Describes - conservation of mass

- Darcy's law (empirical)

Darcy's law

$$v = -k \frac{\Delta h}{\Delta \ell}$$

or

$$v = -k \frac{dh}{d\ell}$$

where h is hydraulic head, ℓ is distance along a flow path and k is the constant of proportionality known as hydraulic conductivity. Law is valid for velocities that produce insignificant inertial effects.

Conservation of mass

Mass balance statement -

(mass in - mass out) = (final mass - initial mass)

or mathematically

$$-\nabla \cdot (\rho v) = \frac{\partial m}{\partial t}$$

where m represents fluid mass.

Ground-water flow equation

Combining Darcy's law with the statement of mass conservation yields

$$-\nabla\cdot(-\rho k\nabla h) = \frac{\partial m}{\partial t}$$

By assuming that compression of the aquifer skeleton is elastic, that overburden load remains constant, and that the volume of solids within the aquifer matrix is constant, it can be shown that

$$\frac{\partial m}{\partial t} = \rho Ss \frac{\partial h}{\partial t}$$

where Ss is the specific storage coefficient defined as,

$$Ss = \rho g(\alpha + \theta\beta)$$

where α is the compressibility of the aquifer skeletr θ is the porosity, and β is the fluid compressibility.

Combining these equations yields,

$$\nabla\cdot(\rho k\nabla h) = \rho Ss \frac{\partial h}{\partial t}$$

or assuming ρ is a constant

$$\nabla\cdot(k\cdot\nabla h) = Ss \frac{\partial h}{\partial t}$$

Commonly used variations:

a. Homogeneous and isotropic medium,

$$k\left[\frac{\partial^2 h}{\partial x^2} + \frac{\partial^2 h}{\partial y^2} + \frac{\partial^2 h}{\partial z^2}\right] = Ss \frac{\partial h}{\partial t}$$

b. Vertically averaged flow, homogeneous and isotropic medium,

$$T\left[\frac{\partial^2\bar{h}}{\partial x^2} + \frac{\partial^2\bar{h}}{\partial y^2}\right] = S\,\frac{\partial\bar{h}}{\partial t}$$

where T is transmissivity (product of k and aquifer thickness), S is storage coefficient (product of Ss and aquifer thickness), and $\bar{h}$ is defined as

$$\bar{h} = \frac{1}{(Z_2 - Z_1)}\int_{Z_1}^{Z_2} h\,dz$$

where Z_1 and Z_2 represent the bottom and top of the aquifer.

c. Non-homogeneous and anisotropic medium,

$$\frac{\partial}{\partial x}\left[k_{xx}\frac{\partial h}{\partial x}\right] + \frac{\partial}{\partial y}\left[k_{yy}\frac{\partial h}{\partial y}\right] + \frac{\partial}{\partial z}\left[k_{zz}\frac{\partial h}{\partial z}\right] = Ss\,\frac{\partial h}{\partial t}$$

d. Vertically averaged flow, non-homogeneous and anisotropic medium,

$$\frac{\partial}{\partial x}\left[T_{xx}\frac{\partial\bar{h}}{\partial x}\right] + \frac{\partial}{\partial y}\left[T_{yy}\frac{\partial\bar{h}}{\partial y}\right] = S\,\frac{\partial\bar{h}}{\partial t}$$

e. Unconfined flow (free surface), vertically averaged flow (Dupuit approximation), non-homogeneous and isotropic medium,

$$\frac{\partial}{\partial x}\left[kb\,\frac{\partial\bar{h}}{\partial x}\right] + \frac{\partial}{\partial y}\left[kb\,\frac{\partial\bar{h}}{\partial y}\right] = S_y\,\frac{\partial\bar{h}}{\partial t}$$

Sy is known as specific yield or effective porosity and b is the saturated thickness of the unconfined aquifer.

Assumptions that are most frequently employed are:

1) Compressible fluid in an elastic compressible aquifer.

2) Spatial variations in fluid density are insignificant.

3) Coordinate axes are aligned with principle components of hydraulic conductivity or transmissivity tensor.

4) Areal (vertically averaged) flow within aquifers.

Boundary conditions

The conditions along the perimeter of the system must be defined. Possibilities include

1) No-flow (impermeable)

$$-k \frac{\partial h}{\partial n} = 0$$

where n is the direction normal to the boundary

2) Specified flow

$$-k \frac{\partial h}{\partial n} = C \quad \text{(known constant)}$$

3) Head dependent flow

$$-k \frac{\partial h}{\partial n} = f(h)$$

where f is the functional form describing the relationship between flow rate and head.

4) Specified head

$$h = C \quad \text{(known constant)}$$

Initial conditions

The dependent variable h is a function of x,y,z and t. To completely specify the mathematical problem, $h(x,y,z,0)$ must be known.
In some cases, the method of superposition can be used to simplify the specification of initial conditions.

Analytic solutions

Given a P.D.E., boundary conditions, and initial conditions, "closed-form" analytic solutions can be found via the calculus for many problems of practical interest. Some of the classical solutions include the Theis equation for infinite, homogeneous, isotropic aquifers,

$$s\ (r,t) = \frac{Q}{4\pi T} W(u), \qquad u = \frac{r^2 S}{4Tt}$$

where s is water-level decline caused by pumping rate, Q. The variable r is the radial distance from the pumping well at which the decline is measured. W(u) is known as the well function. Another commonly used equation is one attributed to Hantush and Jacob to analyze "leaky" aquifers.

$$s\ (r,t) = \frac{Q}{4\pi T} W(u, r/B)$$

where $B = [T/L]^{\frac{1}{2}}$ and L is the vertical hydraulic conductivity of an overlying or underlying confining bed divided by its thickness.

Numerical solution of P.D.E.'s

Concept: values of the dependent variable will be calculated at specific points in space and time rather than the "continuous" function obtained from an analytic solution.

Methods: Finite difference, finite element, polygonal (integrated finite difference), boundary integral.

Finite difference methods

Advantages:

a. Theory is intuitively straightforward.

b. Programming is relatively simple.

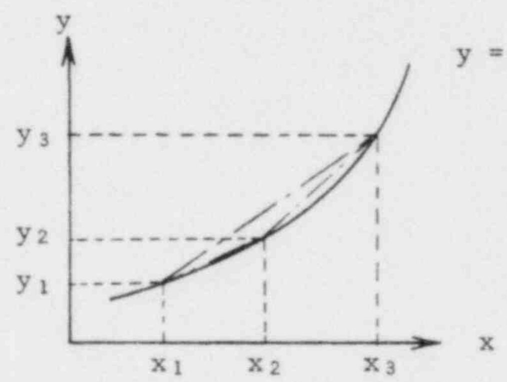
c. Structure of difference equations can be used to advantage with certain solution procedures.

Disadvantages:

a. Some geometric properties may be difficult to represent adequately.

b. Number of resulting algebraic equations can become very large.

Concept: Approximate differential expressions using discrete functional values.



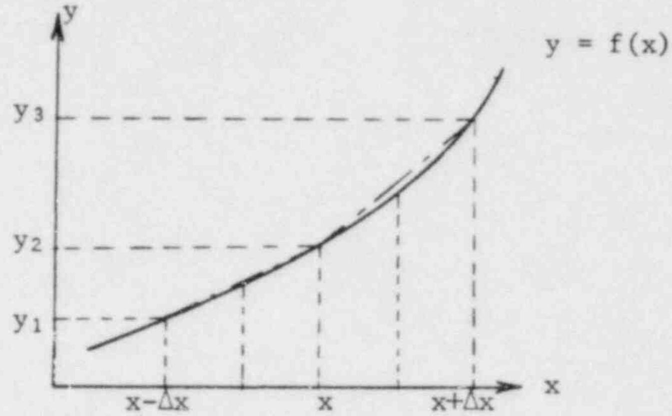
Difference approximations:

a. Backward, $f'(x_2) \approx \dfrac{y_2 - y_1}{x_2 - x_1}$

b. Forward, $f'(x_2) \approx \dfrac{y_3 - y_2}{x_3 - x_2}$

c. Central, $f'(x_2) \approx \dfrac{y_3 - y_1}{x_3 - x_1}$

Second order derivatives:



$$f''(x) \approx \frac{f'(x + \frac{\Delta x}{2}) - f'(x - \frac{\Delta x}{2})}{\Delta x}$$

$$f''(x) \approx \frac{\frac{y_3 - y_2}{\Delta x} - \frac{y_2 - y_1}{\Delta x}}{\Delta x}$$

$$f''(x) \approx \frac{y_3 - 2y_2 + y_1}{\Delta x^2}$$

Considerations:

Truncation error - a measure of the discrepancy between the derivative and the approximation. This error can be examined using a Taylor series expansion. The results of this examination are usually expressed in terms of the order of the function for which the approximation is correct. That is, forward and backward difference approximations are first-order correct $[O(\Delta x)]$ and thus are exactly correct for linear functions but will have truncation error for higher order functions. Central difference approximations are second-order correct. To illustrate this consider:

$$y = f(x) = ax^2 + bx + c$$

$$f'(x) = 2ax + b$$

$$f''(x) = 2a$$

Forward difference:

$$f'(x) \approx \frac{f(x + \Delta x) - f(x)}{\Delta x}$$

$$= \frac{a(x + \Delta x)^2 + b(x + \Delta x) + c - ax^2 - bx - c}{\Delta x}$$

$$= \frac{ax^2 + 2ax\Delta x + a\Delta x^2 + bx + b\Delta x + c - ax^2 - bx - c}{\Delta x}$$

$$= 2ax + b + a\Delta x$$

Central difference:

$$f'(x) \approx \frac{f'(x + \Delta x) - f'(x - \Delta x)}{2\Delta x}$$

$$= \frac{a(x + \Delta x)^2 + b(x + \Delta x) + c - a(x - \Delta x)^2 - b(x - \Delta x) - c}{2\Delta x}$$

$$= (ax^2 + 2ax\Delta x + a\Delta x^2 + bx + b\Delta x + c - ax^2 + 2ax\Delta x$$

$$- a\Delta x^2 - bx + b\Delta x - c) / 2\Delta x$$

$$= (4ax\Delta x + 2b\Delta x) / 2\Delta x$$

$$= 2ax + b$$

Also, $$f''(x) \approx \frac{f(x + \Delta x) - 2f(x) + f(x - \Delta x)}{\Delta x^2}$$

$$= a(x + \Delta x)^2 + b(x + \Delta x) + c - 2(ax^2 + bx + c) +$$

$$a(x - \Delta x)^2 + b(x - \Delta x) + c \quad / \Delta x^2$$

$$= (ax^2 + 2ax\Delta x + a\Delta x^2 + bx + b\Delta x + c - 2ax^2 - 2bx$$

$$- 2c + ax^2 - 2ax\Delta x + a\Delta x^2 + bx - b\Delta x + c) / \Delta x^2$$

$$= (2a\Delta x^2) / \Delta x^2$$

$$= 2a$$

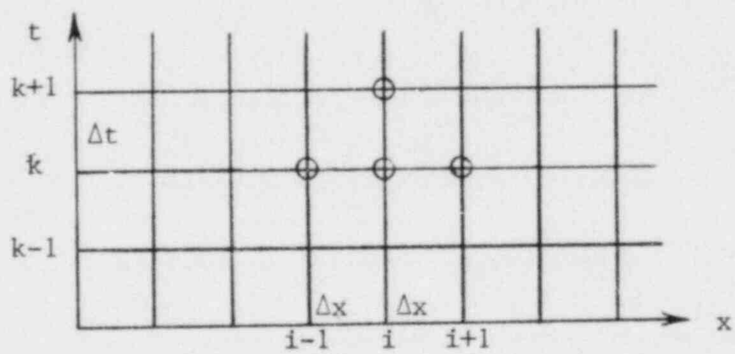
Application to solving P.D.E.'s

$$\frac{\partial^2 h}{\partial x^2} = \frac{\partial h}{\partial t}$$

$$\frac{h_{i+1,k} - 2h_{i,k} + h_{i-1,k}}{\Delta x^2} = \frac{h_{i,k+1} - h_{i,k}}{\Delta t}$$

or $h_{i,k+1} = rh_{i+1,k} + (1 - 2r)h_{i,k} + rh_{i-1,k}$

where $r = \Delta t / \Delta x^2$



This is termed an explicit scheme. It does not involve simultaneous equations but the solution will be unstable for $r < ½$ or $\Delta t > \frac{\Delta x^2}{2}$. Instability refers to the propagation of truncation errors at successive time steps. If the errors accumulate, the scheme is unstable.

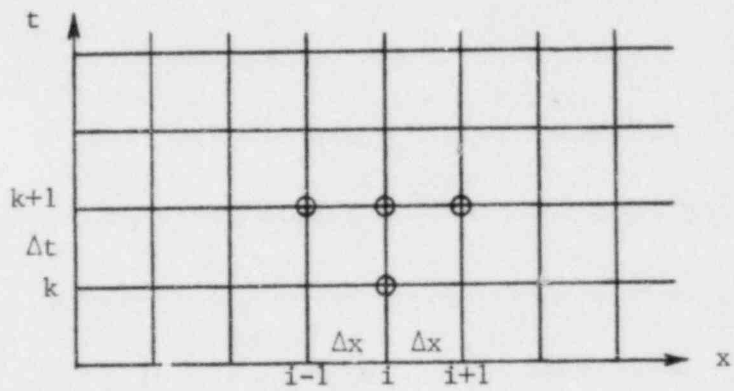
Alternatively, an implicit scheme can be used.

$$\frac{\partial^2 h}{\partial x^2} = \frac{\partial h}{\partial t}$$

$$\frac{h_{i+1,k+1} - 2h_{i,k+1} + h_{i-1,k+1}}{\Delta x^2} = \frac{h_{i,k+1} - h_{i,k}}{\Delta t}$$

or $$rh_{i+1,k+1} - (2r+1)h_{i,k+1} + rh_{i-1,k-1} = -h_{i,k}$$

where $$r = \Delta t / \Delta x^2$$



This scheme can be shown to be unconditionally stable. Truncation errors are not eliminated, but they do not accumulate at successive time levels. However, a set of simultaneous equations must be solved at each time level.

Solving simultaneous algebraic equations

Two approaches:

1) "Direct"methods - a solution is obtained after a fixed number of operations. The Thomas method is a direct method designed for tridiagonal equation systems. Most methods are various forms of Gaussian elimination.

2) "Iterative" methods - a sequence of estimates is generated that (hopefully) tend to the solution.

Advantages of direct methods:

1) The solution (within computer round-off error) is obtained after a fixed number of operations.
2) An initial estimate of the solution is not required.
3) Iteration parameters and convergence criteria are not required.

Disadvantage of direct methods:

As the number of equations increases, the number of computer operations and the amount of computer storage locations may become excessive.

Advantages of iterative methods:

1) If the initial estimate of the solution is "good", process can be very efficient.
2) Computer storage requirements are less severe.

Disadvantages of iterative methods:

1) If the initial estimate of the solution is not "good", the process may require many iterations.
2) Most methods require iteration parameters.
3) The criteria for deciding when to terminate the iteration sequence can be ambiguous.

Examples:

Direct methods

- Thomas algorithmn
- Gauss-Doolittle method
- Cholewsky-Crout elimination

Iterative methods:

- Point-successive over-relaxation
- Line-successive over-relaxation
- Slice-successive over-relaxation
- Alternating direction implicit procedure
- Iterative alternating direction implicit procedure
- Strongly implicit procedure
- Conjugate gradient procedure
- Modified conjugate gradient procedure

III. Considerations for designing generalized 2-D and 3-D programs

Requirements:

- Flexibility
- Reliability
- Documentation

Numerical procedures:

- Approximating equations
- Solution of simultaneous equations

IV. 2-D and 3-D USGS programs

Capabilities

2-D:

- Single aquifer with leakage from adjacent strata
- Arbitrary distribution of recharge and/or pumpage
- Confined or unconfined conditions or both
- Anisotropic permeability or transmissivity

. Evapotranspiration function

. Approximation for transient behavior of confining bed

. Arbitrary boundary conditions

. Several equation solving schemes

3-D:

. Multi-aquifer or "true" 3-D

. Unconfined conditions in uppermost layer

. Arbitrary boundary conditions

. Anisotropic permeability or transmissivity

<u>Mathematical formulation, "numerical" procedures, and program operation</u>

2-D:

General equation -

$$\frac{\partial}{\partial x}\left[T_{xx}\frac{\partial h}{\partial x}\right] + \frac{\partial}{\partial y}\left[T_{yy}\frac{\partial h}{\partial y}\right] = S\frac{\partial h}{\partial t} + W + F(h)$$

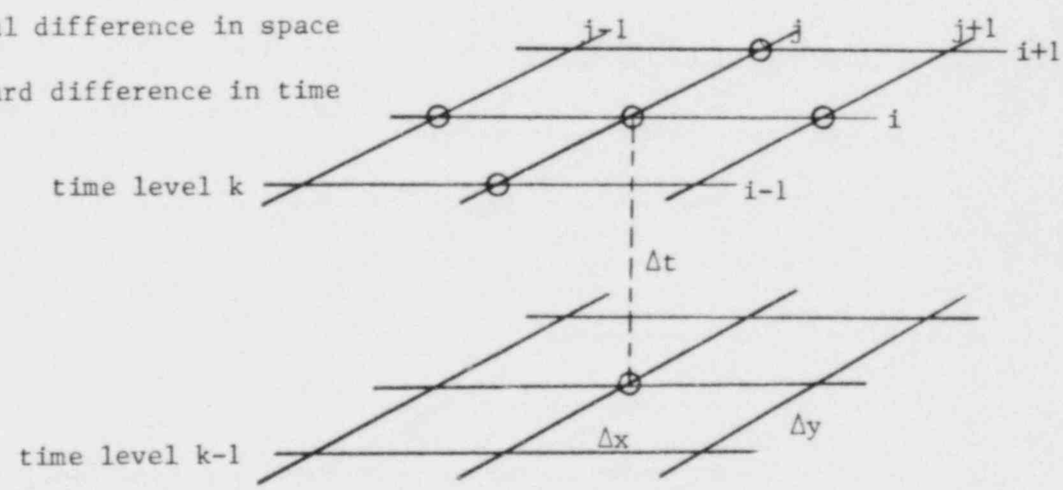
or

$$\frac{\partial}{\partial x}\left[k_{xx}b\frac{\partial h}{\partial x}\right] + \frac{\partial}{\partial y}\left[k_{yy}b\frac{\partial h}{\partial y}\right] = S_y\frac{\partial h}{\partial t} + W + F(h)$$

Difference approximation -

. Central difference in space

. Backward difference in time

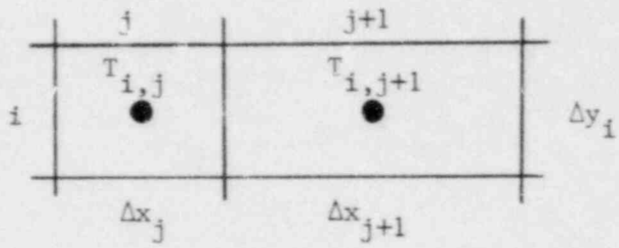


$$\frac{T_{xx}}{\Delta x^2}\left(h_{i,j+1,k} - 2h_{i,j,k} + h_{i,j-1,k}\right)$$

$$+ \frac{T_{yy}}{\Delta y^2}\left(h_{i+1,j,k} - 2h_{i,j,k} + h_{i-1,j,k}\right) - \frac{S_{i,j}}{\Delta t_k} h_{i,j,k}$$

$$- F(h_{i,j,k}) = -\frac{S_{i,j}}{\Delta t_k} h_{i,j,k-1} + W_{i,j,k}$$

Effects of non-homogeneous aquifer properties and/or non-uniform grid:



Block centered grid - assuming properties are constant within grid blocks, the coefficient that describes the correct head dissapation between node i,j and i,j + 1 is a weighted harmonic mean of the form

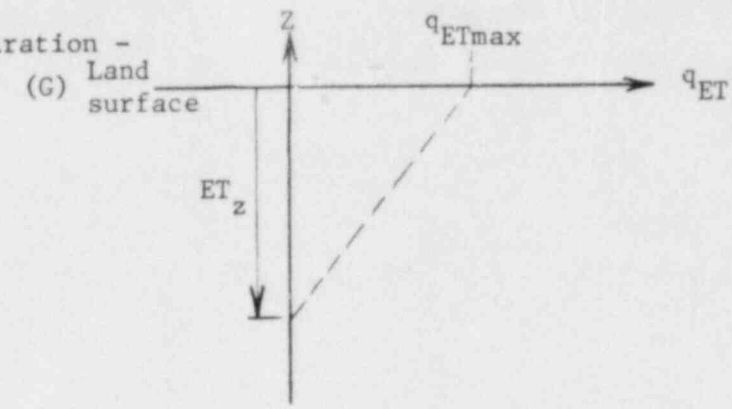
$$F_{i,j} = \left[\frac{T}{\Delta x^2}\right]_{i,j+1/2} = \left[\frac{2T_{i,j}T_{i,j+1}}{T_{i,j}\Delta x_{j+1} + T_{i,j+1}\Delta x_j}\right] / \Delta x_j$$

A non-uniform grid increases the truncation error of the difference approximation. However, if the change in the grid spacing (Δx) or in the transmissivity is less than about 50% from one node to the next, the additional error is insignificant.

Source/sink terms -

Recharge and wells - specify values at each node

Evapotranspiration -



$$q_{ETmax} = \begin{cases} q_{ETmax} & h_{i,j,k} \geq G_{i,j} \\ q_{ETmax}\left[1 - \dfrac{(G_{i,j} - h_{i,j,k})}{ET_z}\right] & (G_{i,j} - h_{i,j,k}) < h_{i,j,k} < G_{i,j} \\ 0 & h_{i,j,k} \leq (G_{i,j} - ET_z) \end{cases}$$

Leakage - steady and non-steady

a. Steady
$$q_L = \frac{k'_{i,j}}{m_{i,j}} (\hat{h}_{i,j} - h_{i,j,k})$$

Assumes that <u>storage</u> effects in the confining strata dissipate at a relatively rapid rate and contribute an insignificant quantity of water.

b. Non-steady approximation

$$q_L \simeq (h_{i,j,o} - h_{i,j,k}) \frac{3k_{i,j}}{(\pi t')^{\frac{1}{2}} m_{i,j}} \left\{1 + 2\sum_{n=1}^{\infty} \exp\left[\frac{-3n^2}{t'}\right]\right\}$$

$$+ \frac{k'_{i,j}}{m_{i,j}} (\hat{h}_{i,j} - h_{i,j,o})$$

where $t' = \dfrac{k'_{i,j}\, t}{m^2_{i,j} S'_s}$ is a dimensionless time factor

For $t' > 5 - 10$, storage is insignificant. The above approximation is adequate for most cases of practical interest in which the storage effects cannot be neglected.

Other factors -

a. Unconfined aquifers:

$$T_{i,j} = k_{i,j} b_{i,j,k}$$

Where $$b_{i,j,k} = h_{i,j,k} - z_{B(i,j)}$$

Storage properties defined by specific yield, S_y.

b. Computing water levels in pumping (or injection) well:

Confined - $$h_w = h_{i,j,k} - \frac{Q_{w(i,j,k)}}{2\pi T_{i,j}} \ln (r_e / r_w)$$

Unconfined - $$H_w = \left[H^2_{i,j,k} - \frac{Q_{w(i,j,k)}}{\pi k_{i,j}} \ln (r_e / r_w) \right]^{\frac{1}{2}}$$

c. Aquifers which are both confined and unconfined - specify the elevation of the top of the aquifer at each node. If $h_{i,j,k} > z_{t(i,j)}$, the confined equation applies. If $h_{i,j,k} < z_{t(i,j)}$, the unconfined equation applies.

Solution of simultaneous equations -

- Line-successive over-relaxation with option for two-dimensional correction procedure
- Iterative alternating direction implicit procedure
- Strongly implicit procedure
- Direct solution (Gauss-Doolittle decomposition using an alternate diagonal ordering scheme) Sec USGS Open-file report 79-202.

Program operation -

Input data:

Group I

- Card 3 - left justify options within the 5 column field; specify option by punching the underlined character string.
- Card 4 - right justify integer variables.

Group II

- Card 1 - the inverse of FACT1 or FACT2 is the contour interval.
- Card 2 - NPER, KTH and LENGTH are integer variables and should be right justified; LENGTH must be 10 for SIP.

Group III

- Each data set consists of one card that indicates whether the parameter or property is uniform or non-uniform. If non-uniform values are indicated (IVAR = 1), the card <u>must</u> be followed by a series of cards that describe the variation of the values throughout the grid. Values are input by <u>rows</u> according to the appropriate format. Begin successive rows on a new card.
- Right justify when using an E format specification or punch the decimal point to avoid problems.
- Do not include any cards for variables related to options that have not been selected.
- Only <u>one</u> row (or column) of values is required for variables DELX and DELY.

Group IV

. Variables KP,KPM1, NWEL, and NUMT are integer and must be right justified.

. There will be NPER sets of group IV data.

. TMAX, DELT, NUMT and CDLT are related by the following equation:

$$TMAX = \frac{CDLT\ (CDLT^{NUMT} - 1)}{(CDLT - 1)} DELT$$

. Pumping rate is minus for withdrawal.

. RADIUS should be zero or blank if extrapolation is not required.

Output:

. Check words of Y vector used.

. Check input data values.

. Check mass balance.

3-D:

General equations -

a. "True" 3-D

$$\frac{\partial}{\partial x}\left[k_{xx}\frac{\partial h}{\partial x}\right] + \frac{\partial}{\partial y}\left[k_{yy}\frac{\partial h}{\partial y}\right] + \frac{\partial}{\partial z}\left[k_{zz}\frac{\partial h}{\partial z}\right] = Ss\frac{\partial h}{\partial t} + W$$

b. Multi-aquifer; aquifer k

$$\frac{\partial}{\partial x}\left[T_{xx}\frac{\partial h}{\partial x}\right] + \frac{\partial}{\partial y}\left[T_{yy}\frac{\partial h}{\partial y}\right] + L_{k+1}(h) + L_{k-1}(h)$$

$$= S\frac{\partial h}{\partial t} + W$$

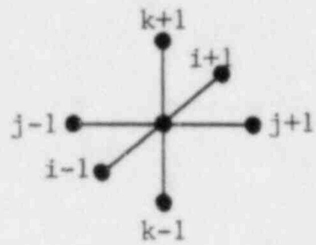
Where $L_{k+1}(h) = \left[\frac{k'}{m}\right]_{k,k+1} (h_{k+1} - h)$

Note index k represents the vertical dimension or an aquifer unit.

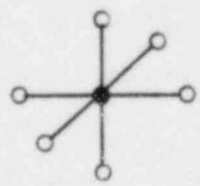
Difference approximation -

. Central difference in space

. Backward difference in time



time level $t+\Delta t$



time level t

. Block centered grid

. Weighted harmonic mean coefficients

Source/sink terms -

. Well discharge (or recharge) can be specified at each node.

. Distributed (areal) recharge can be specified for nodes in the uppermost layer only.

Unconfined conditions (upper aquifer only) -

$$T_{xx(i,j,k)} = k_{xx(i,j,k)} \, b_{i,j,k}$$

Where $b_{i,j,k} = h_{i,j,k} - z_{b(i,j,k)}$

Solution of simultaneous equations -

- Strongly implicit procedure

Program operation -

Input data:

Group I

- Card 3 - specifying NCH smaller than the actual number of constant head nodes causes problems that are sometimes difficult to detect. Overestimating NCH does not affect program operation.
- Card 4 - left justify options within the 5 column field; specify option by punching the underlined character string.

Group II

- Card 1 - NPER, KTH and LENGTH are integer variables and must be right justified.
- Card 2 - layer numbers should be specified for variables LEVEL1 and LEVEL2. For example, if maps are required for the first, second, and fourth layers, the number 124000000 should be coded.

Group III

- Each data set describing a particular parameter or property is composed of a subset for each layer. Each subset consists of one card that indicates whether the parameter or property for that layer is uniform or non-uniform. If non-uniform values are indicated (IVAR = 1), the first card must be followed by a series of cards that describe the variation of values within the particular layer. Note that with this format, values are not required for each node if the non-uniform values are limited to one or two layers. Values are input by rows according to the appropriate format. Begin successive rows on a new card. Note that the initial card for each subset that describe transmissivity (or hydraulic conductivity) values has three additional parameters that describe directional properties of T or k.

- Do not include any cards for variables related to options that have not been selected (data sets 5, 6, 7, and 8).
- Only IO, JO and/or KO values are required to define non-uniform grid characteristics (data sets 8, 9, and 10).
- Parameter TK, which describes the leakance of confining beds in multi-aquifer problems, consists of KO-1 layers (1 confining bed between each aquifer).

Group IV

- Variables KP, KPM1, NWEL, and NUMT are integer and must be right justified.
- There will be NPER sets of group IV data.
- Time step variables are related by the equation.

$$TMAX = \frac{CDLT\ (CDLT^{NUMT} - 1)}{(CDLT - 1)} DELT$$

Definition of input variables and units for 3-D versus multi-aquifer simulations.

- Multi-aquifer

T - transmissivity, $L^2 T^{-1}$

S - storage coefficient, dimensionless

TK - leakage coefficient (or leakance) for confining layers that are not treated as explicit model layers, T^{-1}. If no confining layer exists between two aquifers TK should be

$$\frac{2\ k_{z1}\ k_{z2}}{k_{z1}\ b_2 + k_{z2}\ b_1}$$

QRE - recharge rate (per unit area), LT^{-1}

WELL - well discharge (or recharge), L^3T^{-1}

. 3-D

T - hydraulic conductivity, LT^{-1}

S - specific storage, L^{-1}

TK - weighted harmonic mean coefficient, T^{-1}

QRE - recharge rate per unit <u>volume</u>, T^{-1}

WELL - well discharge (or recharge) per unit thickness of the layer in which the well is placed, L^2T^{-1}

(for example, $WELL_{i,j,k} = Q_{i,j,k} / \Delta z_k$, where $Q_{i,j,k}$ is the flow rate in L^3T^{-1})

Output:

. Check words of Y vector used

. Check input data values

. Check mass balance

V. "Model" development considerations

<u>"Conceptual" model</u>

. Provides the framework for the design and operation of the model.

. Is the single most important element in the modeling process.

Model design

Grid characteristics

. Orientation should be accordance with aquifer anisotropy characteristics or other structural features of the system.

. Node spacing should be sufficient to adequately describe non-homogeneities of the system. Use a uniform spacing if it does not produce an unreasonable number of node points.

Time step sequence

. Should be compatible with the time horizon and the areal scale of the problem.

. May have to experiment to insure that the time step sequence does not influence results.

Boundary conditions

. Use hydrogeologic boundaries whenever possible.

. Use constant head boundaries with caution.

. If the exact boundary or boundary condition is uncertain, conduct tests to determine its influence.

. Constant head conditions are imposed at node points, no-flow conditions are imposed at block boundaries.

. Head-dependent flow conditions are useful for representing aquifer conditions beyond the area of primary interest.

Initial conditions (transient simulations)

Three alternatives -

1) Contoured water-levels

2) Computed steady-state water-levels

3) Superposition

. Contoured water-levels

Use with caution. The values are probably not a solution to the equations described by the model because of model error. Head changes will occur that are a response to both the initial conditions and new stresses on the system. After a period of time, the impact of the initial conditions will diminish but the question of "how long?" is difficult to answer.

. Computed steady-state water levels

Usually better than contoured water-levels. Changes in water-level are a response to new stresses only.

. Superposition

Strictly applicable only for linear (confined) systems, but can be used effectively on many non-linear (unconfined) systems if the non-linearity is not too severe. The principle is:

$$T \frac{\partial^2 h}{\partial x^2} = S \frac{\partial h}{\partial t}$$

if $h = h_o - s$

$$T \frac{\partial^2 (h_o - s)}{\partial x^2} = S \frac{\partial (h_o - s)}{\partial t}$$

$$T \frac{\partial^2 h_o}{\partial x^2} - T \frac{\partial^2 s}{\partial x^2} = S \frac{\partial h_o}{\partial t} - S \frac{\partial s}{\partial t}$$

but $T \frac{\partial^2 h_o}{\partial x^2} = S \frac{\partial h_o}{\partial t}$ because h_o is a solution

Thus

$$T \frac{\partial^2 s}{\partial x} = S \frac{\partial s}{\partial t}$$

and s obeys the same differential equation. Boundary conditions should be the same as those for h_o or h. Initial conditions are $s = 0$ everywhere at $t = 0$.

For unconfined systems, an approximate superposition principle can be derived. The result is referred to as "Jacob's correction", which is a method of correcting water-level declines (drawdown) calculated from linear equations. If s' is a solution of

$$\frac{\partial}{\partial x}\left[kb\,\frac{\partial s'}{\partial x}\right] = S_y\,\frac{\partial s'}{\partial t}$$

then s' is an "unadjusted" drawdown and is related to the real drawdown by

$$s' = s - s^2 / 2b$$

where b is the initial saturated thickness of the unconfined aquifer. The approximation is generally valid if drawdowns do not exceed 25% of the initial saturated thickness. For some steady-state simulations, the errors may be small for drawdowns of as much as 50% of the initial saturated thickness.

Model calibration

. Refers to the process of adjusting model parameters to obtain the best simulation of an observed historical response of the system. Usually done by trial and error, although recent research has produced formal procedures. One danger of trial and error procedures is "over-calibration". A quantitative measure of the process should be established such as

- total or mean squared deviations between computed and observed water levels.
- plots of percent of observation points with deviations less than or equal to specified values.
- ratio of a) squared deviation of computed water levels from the mean value of observed levels to b) squared deviation of observed water levels from their mean value.

- Types of calibration simulations
  - Steady-state (conductance parameters, boundary conditions, and source/sink values)
  - Time-averaged "steady-state" (if water-levels have changed, storage terms should be computed and added to the analysis)
  - Transient (includes storage properties implicitly)

Model verification

- Refers to the comparison of the computed system response to an independent set of observations of system response. By "independent", we mean that these observations were not used in the calibration process.
- Results of these tests can be used to measure the reliability of the model. In ground-water modeling, however, we frequently encounter the situation in which the response that is to be simulated is the result of a stress that is much larger than any that have occurred in the past. To evaluate model reliability in this case is more difficult. Techniques such as sensitivity analysis or "Monte Carlo simulation" can be useful.

Typical sequence of events in the development of a ground-water model

Identification of analysis objectives or in other words, 'What role will ground-water models play in the analysis of the problem?'

- An aid in understanding system behavior
- A mechanism for guiding data collection
- To quantify ground-water velocities
- As a predictive tool for analyzing the effects of ground-water development

Development of a conceptual model

- Identification of geometric characteristics (boundaries, thickness, lithology, etc.)
- Estimate system water budget
- Identification of system parameters that will require consideration and preliminary estimates of their values.

Design of the mathematical model

- Estimates of system parameters
- Evaluation of boundary conditions
- Evaluation of grid configuration and time step sequence

Initial data collection

- Geologic data
- Aquifer tests
- Water-level measurements
- Recharge/discharge characteristics (pumpage, stream - ground-water interaction)

Model calibration

- Compute "best" estimates of system parameters

Sensitivity analysis

- Establish the relative importance of the various system parameters
- Establish critical data requirements
- Guide for future data collection efforts
- Develop measures of model reliability through "Monte Carlo simulation"

Model verification

. T... of model against an observed event

Philosophical considerations

. Be aware of the model's conceptual framework when evaluating results -- be sure the model is capable of producing the response you are seeking.

. Be aware of data reliability to prevent "over-calibration" or attempts to calibrate parameters that have no "observabilit " or "visibility" within the available data (no sensitivity at points of observation).

. Be aware of study objectives to insure that the model can answer the appropriate questions.

. Be aware of the deterministic nature (physical basis) of the model.

. Be aware of potential non-uniqueness of the model.

. Be aware of the power of the many available analytical models.

APPENDIX B

APPENDIX B

DEFINITIONS OF COMMONLY USED TERMS

Aquifer - a formation, group of formations, or part of a formation that contains sufficient saturated permeable material to yield significant quantities of water to wells or springs.

Confined or artesian aquifer - an aquifer that has wells that contain water levels above the top of the aquifer.

Unconfined aquifer - an aquifer that has a water table.

Hydraulic conductivity - usually denoted as K, is the volume of water at the existing kinematic viscosity that will move in unit time under a unit hydraulic gradient through a unit area measured at right angles to the direction of flow. It is commonly referred to as permeability even though "intrinsic permeability" is the term that is strictly a property of the porous medium only. The terms are related by

$$K = \frac{\rho g k}{\mu} = \frac{g k}{\nu}$$

where k is intrinsic permeability, ρ is fluid density, μ is dynamic viscosity, and ν is kinematic viscosity. Hydraulic conductivity and permeability are used interchangeably to refer to K. Units are LT^{-1}

Transmissivity - a product of K and saturated aquifer thickness. It is the rate at which water of the existing kinematic viscosity is transmitted through a unit width of aquifer under a unit hydraulic gradient. Units are L^2T^{-1}.

Specific storage - the volume of water released from or taken into storage per unit volume of the porous medium per unit change in head. Units are L^{-1}.

Storage coefficient - the volume of water an aquifer releases from or takes into storage per unit surface area of the aquifer per unit change in head. It is a product of specific storage and saturated aquifer thickness and is dimensionless.

Specific yield - the ratio of 1) the volume of water which the porous medium, after being saturated, will yield by gravity to 2) the volume of the porous medium. For practical purposes, it is the storage coefficient for an unconfined aquifer. It has no units.

Water table - the surface of an unconfined water body at which the pressure is atmospheric.

Specific discharge, or specific flux - the rate of ground-water discharge per unit area measured at right angles to the direction of flow. It has the dimensions of velocity (LT^{-1}) but is not the velocity of the water within the pores of the medium. It is also referred to as bulk velocity or Darcy velocity.

Velocity, average interstitial - specific discharge divided by effective porosity.

Confining bed - a body of "less permeable" material stratigraphically adjacent to one or more aquifers. Sometimes referred to as "aquiclude" or "aquitard".

Leakance - a characteristic of a confining bed that describes its ability to conduct water. It is the vertical permeability of the confining bed divided by its thickness. Units are T^{-1}.

Fluid potential, static head, and total head - fluid potential is the mechanical energy per unit mass of a fluid at any given point in space and time with respect to an arbitrary state and datum. It has components related to elevation, pr sure, and velocity. In ground-water movement the velocity related terms are generally negligible and if they are ignored, the static head is a measure of the potential. It is the height above a standard datum of the surface of a column of water (or other fluid) that can be supported by the static pressure at a given point. Total head is essentially the same as static head but includes the velocity related terms. In ground water, most references to head imply static head. For an incompressible fluid whose velocity is negligible, the fluid potential is related to the static head by,

$$\phi = gh$$

Potentiometric surface - a surface which represents the static head. For an aquifer, it represents the levels to which water will rise in tightly cased wells.

Source:

Lohman, S.W., and others, 1972, Definitions of selected ground-water terms - revisions and conceptual refinements: U.S. Geol. Survey Water-Supply Paper 1988, 21p.

APPENDIX C

APPENDIX C

NOTES ON MATRIX ALGEBRA

Consider a set of equations:

$$m_{11}h_1 + m_{12}h_2 + m_{13}h_3 = d_1$$
$$m_{21}h_1 + m_{22}h_2 + m_{23}h_3 = d_2$$
$$m_{31}h_1 + m_{32}h_2 = m_{33}h_3 = d_3$$

Alternative notation

$$[M] = \bar{\bar{M}} = \underset{\sim}{M} = \mathbf{M}$$

This can be written in a shorthand way, called matrix notation

$$\bar{\bar{M}}\bar{h} = \bar{d}$$

where

$$\bar{\bar{M}} = \begin{bmatrix} m_{11} & m_{12} & m_{13} \\ m_{21} & m_{22} & m_{23} \\ m_{31} & m_{32} & m_{34} \end{bmatrix} \qquad \bar{h} = \begin{Bmatrix} h_1 \\ h_2 \\ h_3 \end{Bmatrix} \qquad \bar{d} = \begin{Bmatrix} d_1 \\ d_2 \\ d_3 \end{Bmatrix}$$

Normally, the elements of $\bar{\bar{M}}$(denoted m) are referred to by row and column location, i.e., m_{11}, m_{12}, m_{13}, ... etc.

dimension of a matrix - number of subscripts required to locate an element, $\bar{\bar{M}}$ is 2-dimensional, $\bar{h}$ and $\bar{d}$ are 1-dimensional.

order of a matrix - indicates the size of the matrix, i.e. 3 x 3.

square matrix - number of rows equals number of columns.

Operations

Addition - sum the corresponding elements to produce the elements of the resulting matrix

- only if the matrices are of the same order
- subtraction is addition of negative

Example:

$$\text{if } \bar{\bar{K}} = \begin{bmatrix} k_{11} & k_{12} & k_{13} \\ k_{21} & k_{22} & k_{23} \\ k_{31} & k_{32} & k_{33} \end{bmatrix}$$

then

$$\bar{\bar{K}} + \bar{\bar{M}} = \begin{bmatrix} k_{11}+m_{11} & k_{12}+m_{12} & k_{13}+m_{13} \\ k_{21}+m_{21} & k_{22}+m_{22} & k_{23}+m_{23} \\ k_{31}+m_{31} & k_{32}+m_{32} & k_{33}+m_{33} \end{bmatrix}$$

Thus, the process is commutative and associative

$\bar{\bar{A}} + \bar{\bar{B}} = \bar{\bar{B}} + \bar{\bar{A}}$ and $\bar{\bar{A}} + (\bar{\bar{B}} + \bar{\bar{C}}) = (\bar{\bar{A}} + \bar{\bar{B}}) + \bar{\bar{C}}$

Multiplication of a matrix by a scalar quantity

Each element of the matrix is multiplied by the scalar

Example:

$$\lambda\bar{\bar{M}} = \begin{bmatrix} \lambda m_{11} & \lambda m_{12} & \lambda m_{13} \\ \lambda m_{21} & \lambda m_{22} & \lambda m_{23} \\ \lambda m_{31} & \lambda m_{32} & \lambda m_{33} \end{bmatrix}$$

Multiplication of matrices

To obtain element ij of the product matrix, we form the inner product (sum of the individual products) of row i of the first matrix with column j of the second matrix.

Thus:

$$\bar{\bar{M}}\bar{\bar{K}} = \begin{bmatrix} m_{11} & m_{12} & m_{13} \\ m_{21} & m_{22} & m_{23} \\ m_{31} & m_{32} & m_{33} \end{bmatrix} \times \begin{bmatrix} k_{11} & k_{12} & k_{13} \\ k_{21} & k_{22} & k_{23} \\ k_{31} & k_{32} & k_{33} \end{bmatrix} = \bar{\bar{P}}$$

$$\bar{\bar{P}} = \begin{bmatrix} m_{11}k_{11}+m_{12}k_{21}+m_{13}k_{31} & m_{11}k_{12}+m_{12}k_{22}+m_{13}k_{32} & m_{11}k_{13}+m_{12}k_{23}+m_{13}k_{33} \\ m_{21}k_{11}+m_{22}k_{21}+m_{23}k_{31} & m_{21}k_{12}+m_{22}k_{22}+m_{23}k_{32} & m_{21}k_{13}+m_{22}k_{23}+m_{23}k_{33} \\ m_{31}k_{11}+m_{32}k_{21}+m_{33}k_{31} & m_{31}k_{12}+m_{32}k_{22}+m_{33}k_{32} & m_{31}k_{13}+m_{32}k_{23}+m_{33}k_{33} \end{bmatrix}$$

Therefore $\quad p_{ij} = \sum_{n=1}^{3} m_{in} \; k_{nj}$

Matrix multiplication is not commutative in most instances

$$\bar{\bar{A}}\bar{\bar{B}} \quad \bar{\bar{B}}\bar{\bar{A}}$$

Also it is clear that the number of columns in the first matrix must be the same as the number of rows in the second matrix for the product to exist.

Associative and distributive laws hold:

$$(\bar{\bar{A}}\bar{\bar{B}})\bar{\bar{C}} = \bar{\bar{A}}(\bar{\bar{B}}\bar{\bar{C}})$$
$$\bar{\bar{A}}(\bar{\bar{B}} + \bar{\bar{C}}) = \bar{\bar{A}}\bar{\bar{B}} + \bar{\bar{A}}\bar{\bar{C}}$$
$$(\bar{\bar{A}} + \bar{\bar{B}})\bar{\bar{C}} = \bar{\bar{A}}\bar{\bar{C}} + \bar{\bar{B}}\bar{\bar{C}}$$

However, if $\bar{\bar{A}}\bar{\bar{B}} = \bar{\bar{A}}\bar{\bar{C}}$, this does not necessarily mean that $\bar{\bar{B}} = \bar{\bar{C}}$.

$$\bar{\bar{A}} = \begin{bmatrix} 1 & 0 \\ 2 & 0 \end{bmatrix} \quad \bar{\bar{B}} = \begin{bmatrix} 2 & 3 \\ 4 & 1 \end{bmatrix} \quad \bar{\bar{C}} = \begin{bmatrix} 2 & 3 \\ 1 & 5 \end{bmatrix} \quad \bar{\bar{A}}\bar{\bar{B}} = \begin{bmatrix} 2 & 3 \\ 4 & 6 \end{bmatrix} \quad \bar{\bar{A}}\bar{\bar{C}} = \begin{bmatrix} 2 & 3 \\ 4 & 6 \end{bmatrix}$$

Special matrices

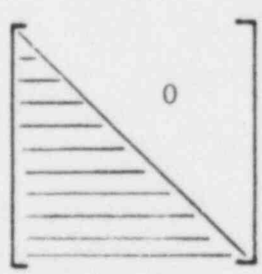
Diεgonal matrix

$$\begin{bmatrix} d_{11} & & 0 \\ & \ddots & \\ 0 & & d_{nn} \end{bmatrix}$$

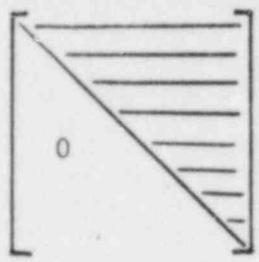
Unit or identity matrix

$$\begin{bmatrix} 1 & & 0 \\ & \ddots & \\ 0 & & 1 \end{bmatrix}$$

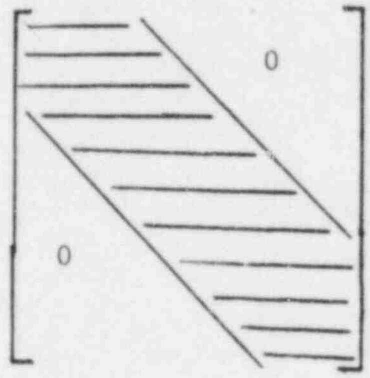
Lower triangular matrix, non-zero elements on the main diagonal and below



Upper triangular matrix, non-zero elements on the main diagonal and above



Banded matrix - zero elements everywhere except along a band or strip running diagonally through the matrix usually centered on the main diagonal



If there are 3 diagonals, centered on the main diagonal, the matrix is called tridiagonal.

APPENDIX D

APPENDIX D

COURSE PROBLEMS

Problem 1: 2-D Areal

The purpose of this problem is to become familiar with data input and the effects of truncation error. The effects of a single well pumping from an "infinite", homogeneous, isotropic aquifer will be simulated and compared with the analytical solution developed by Theis.* Use the following aquifer characteristics:

$$T = 0.01\ ft^2/s$$
$$S = 0.0001$$
$$Q_w = -1.0\ ft^3/s,\ r_w = 0.5\ ft$$

Use a grid block of 1,000 ft on a side for the node containing the well. Expand the grid by 1.5 times for adjacent blocks from the well to the grid perimeter. Use a 21 by 21 grid (well will be at node 11, 11). Simulate drawdown for 10 days with an initial condition of $s(x,y,0) = 0$ for all x and y. Try 1, 6, and 15 time steps with a time step expansion factor of 1.5.

Compare the drawdown in the well ($r = r_w$) after 10 days of pumping with the analytical value as given by the Theis equation. Plot values of drawdown versus r^2 at $t = 10$ days and compare with the analytical values. Plot drawdown versus time for the 15 time step simulation and compare with the analytical values. Plots should be made on log-log paper.

Additional problem:
Simulate a "leaky" aquifer and compare with the Hantush-Jacob solution. Assume that the confining bed has the following properties:

$$K_z = 2 \times 10^{-10}\ ft/s$$
$$S_s = 0.001\ ft^{-1}$$
$$\text{Thickness} = 100\ ft$$

* Complete references for the analytical solutions used in this appendix are given on page D33.

FORTRAN CODING FORM

~~PROGRAM~~ Data: Problem 1

PROGRAMMER

DATE

PAGE 1 OF 1

```
PROBLEN -- 2-D AREAL / THEIS SIMULATION
NRC CLASS / PROB. 1
                    SIP  CHEC                   NUME
       21        21        1        25
Blank
        1         1      .01        0.        0.        0.        0.        10
       1.        1.       1.
Blank
Blank
Blank
       0.                                             STRT
    .0001                                             S
      .01                                             T
    1000.         1                                   DELX
       1.    38.443    25.629    17.086    11.391     7.594     5.063     3.375
     2.25       1.5        1.       1.5      2.25     3.375     5.063     7.594
   11.391    17.086    25.629    38.443        1.
    1000.         1                                   DELY
       1.    38.443    25.629    17.086    11.391     7.594     5.063     3.375
     2.25       1.5        1.       1.5      2.25     3.375     5.063     7.594
   11.391    17.086    25.629    38.443        1.
        1         0         1       10.        15       1.5       .19
       11        11      -1.0       0.5
```

U. S. G. S.

FINITE-DIFFERENCE MODEL
FOR
SIMULATION OF GROUND-WATER FLOW

JANUARY, 1975

****

PROBLEM -- 2-D AREAL / THEIS SIMULATION   NRC CLASS / PROB. 1

****

SIMULATION OPTIONS: SIP CHEC NUMF

NUMBER OF ROWS = 21
NUMBER OF COLUMNS = 21
NUMBER OF WELLS FOR WHICH DRAWDOWN IS COMPUTED AT A SPECIFIED RADIUS = 1
MAXIMUM PERMITTED NUMBER OF ITERATIONS = 25

WORDS OF Y VECTOR USED = 7724 ← VALUE MUST BE LESS THAN THE DIMENSION OF VARIABLE Y IN SOURCE PROGRAM

NUMBER OF PUMPING PERIODS = 1
TIME STEPS BETWEEN PRINTOUTS = 1

ERROR CRITERION FOR CLOSURE = .1000000E-01
STEADY STATE ERROR CRITERION = 0.

SPECIFIC STORAGE OF CONFINING BED = 0.
EVAPOTRANSPIRATION RATE = 0.
EFFECTIVE DEPTH OF ET = 1.000000

MULTIPLICATION FACTOR FOR TRANSMISSIVITY IN X DIRECTION = 1.000000
IN Y DIRECTION = 1.000000

STARTING HEAD = 0.

STORAGE COEFFICIENT = .1000000E-03

TRANSMISSIVITY = .1000000E-01

GRID SPACING IN PROTOTYPE IN X DIRECTION

```
1000.   38443.   25629.   17086.   11391.   7594.   5063.   3375.   2250.   1500.   1000.   1500.
2250.   3375.   5063.   7594.   11391.   17086.   25629.   38443.   1000.
```

GRID SPACING IN PROTOTYPE IN ' DIRECTION

```
1000.   38443.   25629.   17086.   11391.   7594.   5063.   3375.   2250.   1500.   1000.   1500.
2250.   3375.   5063.   7594.   11391.   17086.   25629.   38443.   1000.
```

SOLUTION BY THE STRONGLY IMPLICIT PROCEDURE

[VALUES ARE CALCULATED BY THE PROGRAM BASED ON PROBLEM CHARACTERISTICS]

BETA= 1.00

```
10 ITERATION PARAMETERS:  0.        .7883319E+00   .9551966E+00   .9905165E+00   .9979927E+00  0.
                   .7883319E+00   .9551966E+00   .9905165E+00   .9979927E+00
```

PUMPING PERIOD NO. 1:   10.00 DAYS

NUMBER OF TIME STEPS=   15

DELT IN HOURS =   .183

MULTIPLIER FOR DELT =   1.500

1 WELLS

I	J	PUMPING RATE	WELL RADIUS
11	11	-1.00	.50

[OUTPUT FOR TIME STEPS 1 THRU 14 HAS BEEN OMITTED]

```
1                    TIME STEP NUMBER =        15                    1
-----------------------------------------------------------------------

          SIZE OF TIME STEP IN SECONDS=       288659.20

               TOTAL SIMULATION TIME IN SECONDS=        864000.00
                                        MINUTES=         14400.00
                                          HOURS=           240.00
                                           DAYS=            10.00
                                          YEARS=              .03

      DURATION OF CURRENT PUMPING PERIOD IN DAYS=           10.00
                                          YEARS=              .03


CUMULATIVE MASS BALANCE:              L**3                 RATES FOR THIS TIME STEP:                 L**3/T
------------------------                                   -------------------------

      SOURCES:                                                                 STORAGE =             1.0001
      --------                                                                RECHARGE =             0.0000
               STORAGE =           864050.75                             CONSTANT FLUX =             0.0000
              RECHARGE =                0.00                                   PUMPING =            -1.0000
         CONSTANT FLUX =                0.00                        EVAPOTRANSPIRATION =             0.0000
         CONSTANT HEAD =                0.00                            CONSTANT HEAD:
               LEAKAGE =                0.00                                        IN =             0.0000
         TOTAL SOURCES =           864050.75                                       OUT =             0.0000
                                                                              LEAKAGE:
      DISCHARGES:                                         FROM PREVIOUS PUMPING PERIOD =             0.0000
      -----------                                                                TOTAL =             0.0000
    EVAPOTRANSPIRATION =                0.00
         CONSTANT HEAD =                0.00                              SUM OF RATES =              .0001
      QUANTITY PUMPED =            864000.00
              LEAKAGE =                 0.00
      TOTAL DISCHARGE =            864000.00

   DISCHARGE-SOURCES =               -50.75
  PER CENT DIFFERENCE =                -.01  <----  ABSOLUTE VALUE SHOULD BE
                                                    LESS THAN 0.5 PERCENT

MAXIMUM HEAD CHANGE FOR EACH ITERATION:                      VALUES MAY OSCILLATE BUT SHOULD
---------------------------------------                      HAVE A DECREASING TREND

    1.3027    1.3991    1.1898     .2677     .0571     .0102     .0027

MAXIMUM CHANGE IN HEAD FOR THIS TIME STEP =      3.000
------------------------------------------------------
                                                     SUMMARY OF ITERATIONS REQUIRED FOR
TIME STEP :  1  2  3  4  5  6  7  8  9 10 11 12 13 14 15
-----------                                          SOLUTION AT EACH TIME STEP.  IF VALUES
                                                     INCREASE SIGNIFICANTLY, THE INCREASE
ITERATIONS:  2  2  2  2  2  2  3  3  3  3  4  5  5  5  6  <----  IN TIME STEP SIZE MAY BE TOO LARGE.
-----------
```

DRAWDOWN

1	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
2	0.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	0.0
3	0.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	0.0
4	0.0	.0	.0	.0	.0	.0	.0	.1	.1	.1	.1	.1	.1	.1	.0	.0	.0	.0	.0	.0	0.0
5	0.0	.0	.0	.0	.1	.2	.4	.5	.5	.5	.5	.5	.5	.5	.4	.2	.1	.0	.0	.0	0.0
6	0.0	.0	.0	.0	.2	.8	1.6	2.1	2.4	2.5	2.5	2.5	2.4	2.1	1.6	.8	.2	.0	.0	.0	0.0
7	0.0	.0	.0	.0	.4	1.6	3.6	5.4	6.5	7.0	7.0	7.0	6.5	5.4	3.6	1.6	.4	.0	.0	.0	0.0
8	0.0	.0	.0	.1	.5	2.1	5.4	9.4	12.4	13.9	14.2	13.9	12.4	9.4	5.4	2.1	.5	.1	.0	.0	0.0
9	0.0	.0	.0	.1	.5	2.4	6.5	12.4	18.6	22.9	24.2	22.9	18.6	12.4	6.5	2.4	.5	.1	.0	.0	0.0
10	0.0	.0	.0	.1	.5	2.5	7.0	13.9	22.9	33.0	39.9	33.0	22.9	13.9	7.0	2.5	.5	.1	.0	.0	0.0
11	0.0	.0	.0	.1	.5	2.5	7.0	14.2	24.2	39.9	71.1	39.9	24.2	14.2	7.0	2.5	.5	.1	.0	.0	0.0
12	0.0	.0	.0	.1	.5	2.5	7.0	13.9	22.9	33.0	39.9	33.0	22.9	13.9	7.0	2.5	.5	.1	.0	.0	0.0
13	0.0	.0	.0	.1	.5	2.4	6.5	12.4	18.6	22.9	24.2	22.9	18.6	12.4	6.5	2.4	.5	.1	.0	.0	0.0
14	0.0	.0	.0	.1	.5	2.1	5.4	9.4	12.4	13.9	14.2	13.9	12.4	9.4	5.4	2.1	.5	.1	.0	.0	0.0
15	0.0	.0	.0	.0	.4	1.6	3.6	5.4	6.5	7.0	7.0	7.0	6.5	5.4	3.6	1.6	.4	.0	.0	.0	0.0
16	0.0	.0	.0	.0	.2	.8	1.6	.1	2.4	2.5	2.5	2.5	2.4	2.1	1.6	.8	.2	.0	.0	.0	0.0
17	0.0	.0	.0	.0	.1	.2	.4	.5	.5	.5	.5	.5	.5	.5	.4	.2	.1	.0	.0	.0	0.0
18	0.0	.0	.0	.0	.0	.0	.0	.1	.1	.1	.1	.1	.1	.1	.0	.0	.0	.0	.0	.0	0.0
19	0.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	0.0
20	0.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	0.0
21	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	

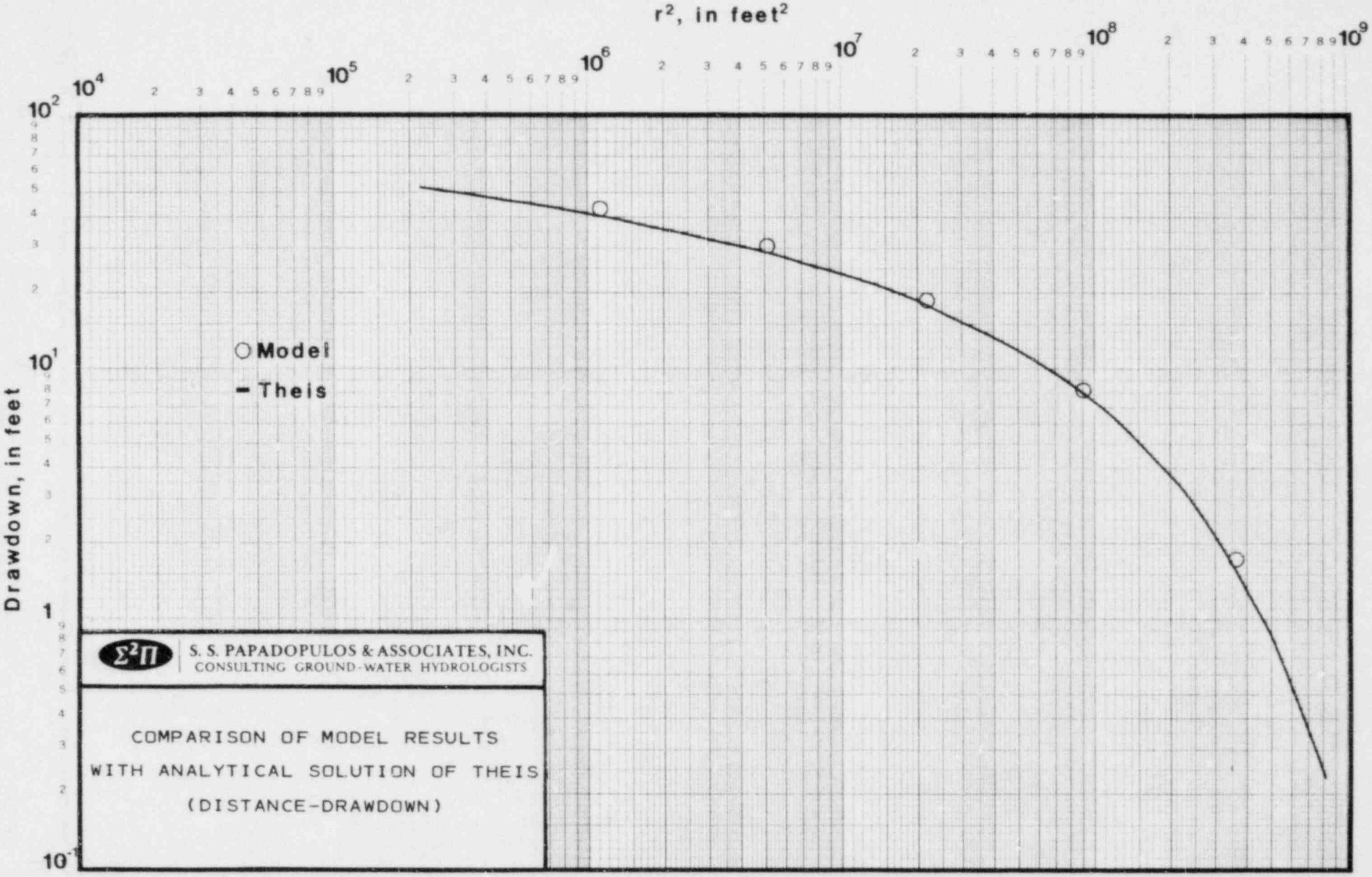
D6

0.0

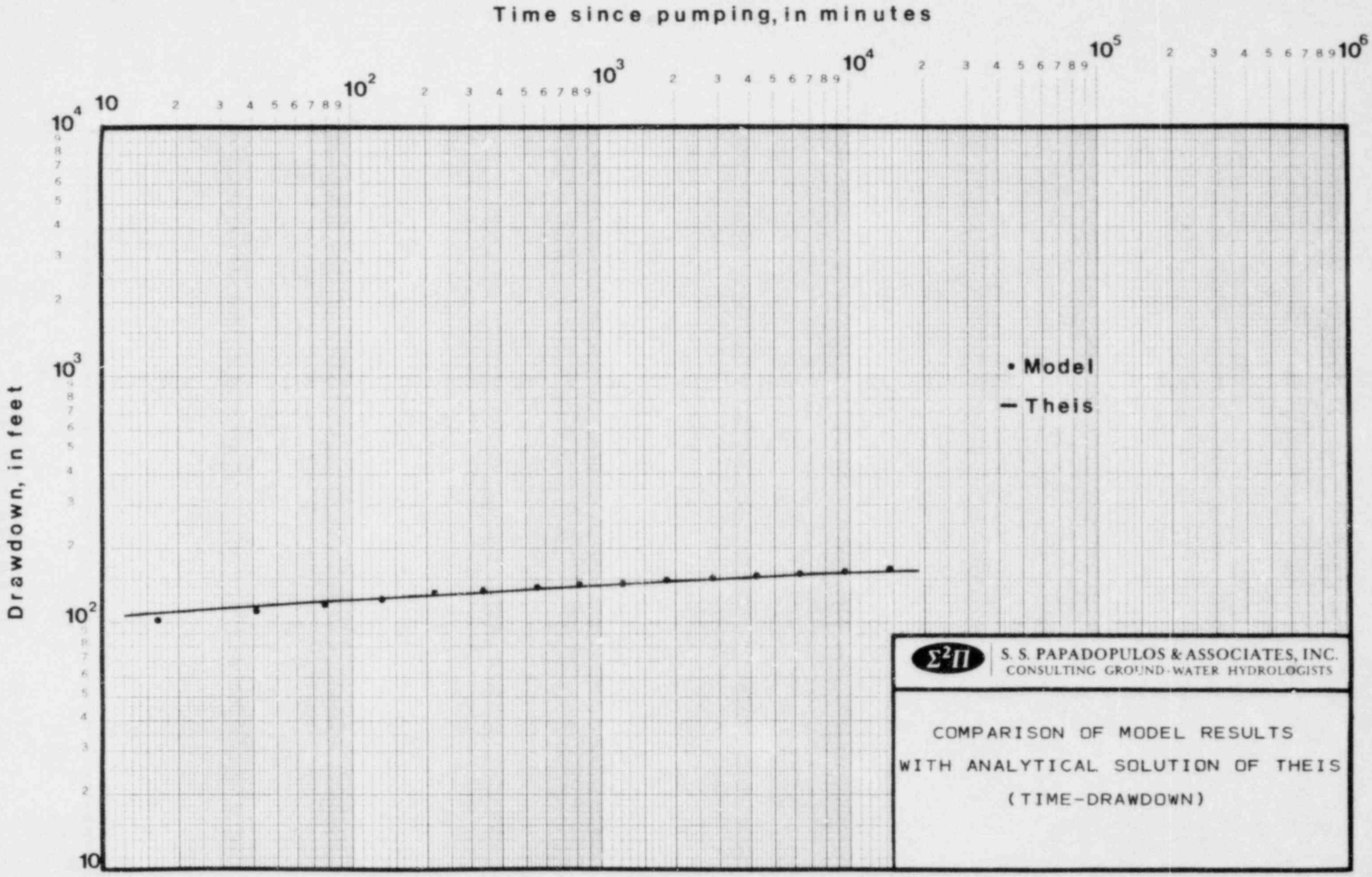
HEAD AND DRAWDOWN IN PUMPING WELLS

I	J	WELL RADIUS	HEAD	DRAWDOWN
11	11	.50	-167.06	167.06

RESULTS OF EXTRAPOLATION TO REAL WELL RADIUS



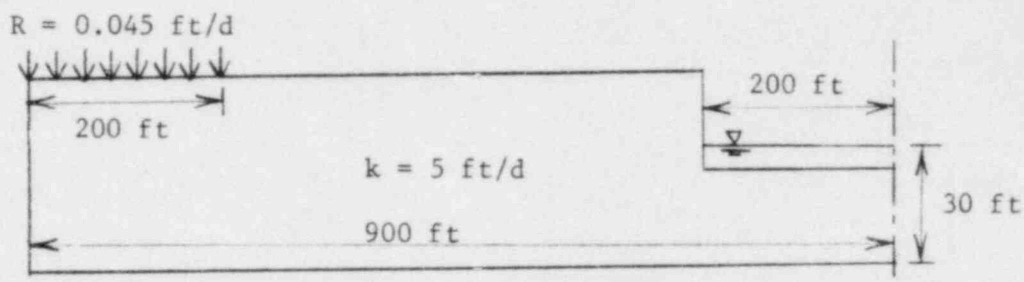
COMPARISON OF MODEL RESULTS WITH ANALYTICAL SOLUTION OF THEIS (DISTANCE-DRAWDOWN)



COMPARISON OF MODEL RESULTS WITH ANALYTICAL SOLUTION OF THEIS (TIME-DRAWDOWN)

Problem 2: 2-D Cross-section

A long waste pit, 200 ft. wide, is leaking at a rate of 0.045 ft/d into an underlying aquifer. The aquifer is composed of sand with a permeability of 5 ft/d and extends 700 ft on both sides of a river. The waste pit is parallel to the river at the edge of the aquifer. The aquifer is 50 ft thick and the river level is 30 ft above the aquifer base. The river is 5 ft deep and 400 ft wide.



1) Compute the steady-state head distribution created by the leakage for anisotropy ratios of 1:1, 10:1, and 100:1.
2) What adjustments should be made after the initial solution?
3) How would one calculate the time required for leachate to reach the river?

FORTRAN CODING FORM

~~PROGRAM~~ Data: Problem 2

PROGRAMMER

DATE

PAGE 1 OF 2

STATEMENT LABEL | FORTRAN STATEMENT | STATEMENT SEQUENCE

```
PROBLEM -- 2-D CROSS-SECTION
NRC CLASS / PROB. 2
                           IP  CHEC                     HEAD
         7         20         0       150
CONT              4.        1.        25        0.        1.FEET
         1         1     .001         0.        0.        1.        0.        10
        1.        1.        1.
Blank
Blank
Blank
       30.                                         STRT
        1.         1                               S
   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0
   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0
   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0
   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0  -1  -1  -1  -1   0
   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0
   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0
   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0
        5.         1                               K
   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0
   0   1   1   1   1   1   1   1   1   1   1   1   1   1   1   0   0   0   0   0
   0   1   1   1   1   1   1   1   1   1   1   1   1   1   1   0   0   0   0   0
   0   1   1   1   1   1   1   1   1   1   1   1   1   1   1   1   1   1   1   0
   0   1   1   1   1   1   1   1   1   1   1   1   1   1   1   1   1   1   1   0
   0   1   1   1   1   1   1   1   1   1   1   1   1   1   1   1   1   1   1   0
   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0
       50.                                         DELX
       10.                                         DELY
         1         0         4        1.         1        1.       24.
         2         2      2.25
         2         3      2.25
         2         4      2.25
```

PRODUCT NO. 5905

FORTRAN CODING FORM

~~PROGRAM~~ Data: Problem 2

DATE

PROGRAMMER

PAGE 2 OF 2

STATEMENT LABEL	CONT	FORTRAN STATEMENT	STATEMENT SEQUENCE
		2 5 2.25	

PRODUCT NO. 5905

```
                                   U. S. G. S.
                             FINITE-DIFFERENCE MODEL
                                       FOR
                         SIMULATION OF GROUND-WATER FLOW

                                 JANUARY, 1975

**********************************************************************************************************
ROBLEM -- 2-D CROSS-SECTION                                          NRC CLASS, PROB. 2
**********************************************************************************************************

IMULATION OPTIONS:                          SIP      CHEC                                      HEAD

                                                   NUMBER OF ROWS =     7
                                                NUMBER OF COLUMNS =    20
   NUMBER OF WELLS FOR WHICH DRAWDOWN IS COMPUTED AT A SPECIFIED RADIUS =     0
                            MAXIMUM PERMITTED NUMBER OF ITERATIONS =   150

                                            WORDS OF Y VECTOR USED =   2697

                    ON ALPHAMERIC MAP:
                           MULTIPLICATION FACTOR FOR X DIMENSION =   4.000000
                           MULTIPLICATION FACTOR FOR Y DIMENSION =   1.000000
                                       MAP SCALE IN UNITS OF    FEET
                                    NUMBER OF FEET      PER INCH =   25.00000
                               MULTIPLICATION FACTOR FOR DRAWDOWN =  0.
                                   MULTIPLICATION FACTOR FOR HEAD =   1.000000

                                      NUMBER OF PUMPING PERIODS =     1
                                   TIME STEPS BETWEEN PRINTOUTS =     1

                                      ERROR CRITERION FOR CLOSURE =    .1000000E-02
                                    STEADY STATE ERROR CRITERION =  0.

                                SPECIFIC STORAGE OF CONFINING BED =  0.
                                        EVAPOTRANSPIRATION RATE =  0.
                                         EFFECTIVE DEPTH OF ET =   1.000000

              MULTIPLICATION FACTOR FOR TRANSMISSIVITY IN X DIRECTION =   1.000000
                                                     IN Y DIRECTION =   1.000000

                                                    STARTING HEAD =  30.00000
```

STORAGE COEFFICIENT MATRIX

1	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000								
2	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000								
3	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000								
4	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
	0.00000	-1.00000	-1.00000	-1.00000	-1.00000	0.00000								
5	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000								
6	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000								
7	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	000	0.00000
	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000								

TRANSMISSIVITY
MATRIX

```
--------------------------------------

1  0.00000  0.00000  0.00000  0.00000  0.00000  0.00000  0.00000  0.00000  0.00000  0.00000  0.00000  0.00000  0.00000  0.00000
   0.00000  0.00000  0.00000  0.00000  0.00000  0.00000

2  0.00000  5.00000  5.00000  5.00000  5.00000  5.00000  5.00000  5.00000  5.00000  5.00000  5.00000  5.00000  5.00000  5.00000
   5.00000  0.00000  0.00000  0.00000  0.00000  0.00000

3  0.00000  5.00000  5.00000  5.00000  5.00000  5.00000  5.00000  5.00000  5.00000  5.00000  5.00000  5.00000  5.00000  5.00000
   5.00000  0.00000  0.00000  0.00000  0.00000  0.00000

4  0.00000  5.00000  5.00000  5.00000  5.00000  5.00000  5.00000  5.00000  5.00000  5.00000  5.00000  5.00000  5.00000  5.00000
   5.00000  5.00000  5.00000  5.00000  5.00000  0.00000

5  0.00000  5.00000  5.00000  5.00000  5.00000  5.00000  5.00000  5.00000  5.00000  5.00000  5.00000  5.00000  5.00000  5.00000
   5.00000  5.00000  5.00000  5.00000  5.00000  0.00000

6  0.00000  5.00000  5.00000  5.00000  5.00000  5.00000  5.00000  5.00000  5.00000  5.00000  5.00000  5.00000  5.00000  5.00000
   5.00000  5.00000  5.00000  5.00000  5.00000  0.00000

7  0.00000  0.00000  0.00000  0.00000  0.00000  0.00000  0.00[illegible]  0.00000  0.00000  0.00000  0.00000  0.00000  0.00000  0.00000
   0.00000  0.00000  0.00000  0.00000  0.00000  0.00000

                                    DELX =   50.00000

                                    DELY =   10.00000

                         SOLUTION BY THE STRONGLY IMPLICIT PROCEDURE
                         *******************************************

                                    BETA= 1.00

10 ITERATION PARAMETERS:  0.            .7248616E+00   .9242989E+00   .9791717E+00   .9942693E+00  0.

                          .7248616E+00  .9242989E+00   .9791717E+00   .9942693E+00

                            PUMPING PERIOD NO.   1:       1.00 DAYS
                            --------------------------------------

                            NUMBER OF TIME STEPS=        1

                                   DELT IN HOURS =     24.000

                            MULTIPLIER FOR DELT =      1.000

                                       4 WELLS
                                       -------

                        I        J    PUMPING RATE   WELL RADIUS

                        2        2        2.25
                        2        3        2.25
                        2        4        2.25
                        2        5        2.25
```

1 TIME STEP NUMBER = 1 1

SIZE OF TIME STEP IN SECONDS= 86400.00

TOTAL SIMULATION TIME IN SECONDS=	86400.0[illegible]
MINUTES=	1440.[illegible]
HOURS=	24.[illegible]0
DAYS=	1.[illegible]0
YEARS=	.[illegible]0

DURATION OF CURRENT PUMPING PERIOD IN DAYS=	1.00
YEARS=	.00

CUMULATIVE MASS BALANCE: L**3

SOURCES:

STORAGE =	0.00
RECHARGE =	0.00
CONSTANT FLUX =	777600.00
CONSTANT HEAD =	0.00
LEAKAGE =	0.00
TOTAL SOURCES =	7[illegible]7600.00

DISCHARGES:

EVAPOTRANSPIRATION =	0.00
CONSTANT HEAD =	773160.26
QUANTITY PUMPED =	0.00
LEAKAGE =	0.00
TOTAL DISCHARGE =	773160.26

DISCHARGE-SOURCES =	-4439.74
PER CENT DIFFERENCE =	-.57

RATES FOR THIS TIME STEP: L**3/T

STORAGE =	0.0000
RECHARGE =	0.0000
CONSTANT FLUX =	9.0000
PUMPING =	0.0000
EVAPOTRANSPIRATION =	0.0000
CONSTANT HEAD:	
IN =	0.0000
OUT =	-8.9486
LEAKAGE:	
FROM PREVIOUS PUMPING PERIOD =	0.0000
TOTAL =	0.0000
[illegible]UM OF RATES =	.0514

D16

MAXIMUM HEAD CHANGE FOR EACH ITERATION:

.6660	.6682	1.1902	1.9209	3.0386	.2142	.2695	.4570	.9791	2.0303
.1470	.1932	.3455	.7255	1.5769	.11[illegible]2	.1535	.2761	.5756	1.2436
.0906	.1203	.2169	.4530	.9922	.07[illegible]6	.0969	.1745	.3634	.7859
.0573	.0760	.1371	.2864	.6274	.0459	.0613	.1104	.2298	.4970
.0362	.046[illegible]	.0867	.1811	.3968	.0290	.0388	.0698	.1453	.3144
.0229	.0304	.0549	.1146	.2510	.0184	.0245	.0441	.0919	.1988
.0145	.0192	.0347	.0725	.1587	.0116	.0155	.0279	.0581	.1257
.0092	.0122	.0219	.0458	.1004	.0073	.0098	.0177	.0368	.0795
.0058	.0077	.0139	.0290	.0635	.0046	.0062	.0112	.0233	.0503

.0037	.0049	.0088	.0183	.0402	.0029	.0039	.0071	.0147	.0318
.0023	.0031	.0056	.0116	.0254	.0019	.0025	.0045	.0093	.0201
.0015	.00[illegible]9	.0035	.0073	.0161	.0012	.0016	.0028	.0059	.0127
.0009									

[illegible] CHANGE IN HEAD FOR THIS TIME STEP = 23.000

ME STEP : 1

ERATIONS:120

PLOT OF HYDRAULIC HEAD

```
+---------+---------+   900.00
1                   1
1        R  30  30  1
1                   1
1                   1
1        R  30  30  1
+                   +   800.00
1                   1
1        P  30  30  1
1                   1
1                   1
1        R  30  30  1
+                   +   700.00
1                   1
133  33  33  33  33 1
1                   1
1                   1
135  35  35  35  35 1
+                   +   600.00
1                   1
136  36  36  36  36 1
1                   1
1                   1
13.  38  38  38  38 1
+                   +   500.00
1                   1  X DIS-
140  40  40  40  40 1 TANCE IN
1                   1
1                   1
142  42  42  42  42 1 FEET
+                   +   400.00
1                   1
143  43  43  43  43 1
1                   1
1                   1
145  45  45  45  45 1
+                   +   300.00
1                   1
147  47  47  47  47 1
1                   1
1                   1
149  49  49  49  49 1
+                   +   200.00
1                   1
151  51  51  51  51 1
1                   1
1                   1
152  52  52  52  52 1
+                   +   100.00
1                   1
153  53  53  53  53 1
1                   1
1                   1
153  5   53  53  53 1
+---------+---------+     0.00

0.0      25.00     50.00
```

DISTANCE FROM ORIGIN IN Y DIRECTION, IN FEET

XPLANATION

= CONSTANT HEAD BOUNDARY
$$ = VALUE EXCEEDED 3 FIGURES
ULTIPLICATION FACTOR = 1.000)

HEAD MATRIX

1	30.0	30.0	30.0	30.0	30.0	30.0	30.0	30.0	30.0	30.0	30.0	30.0	30.0	30.0	30.0	30.0	30.0	30.0	30.0	30.0
2	30.0	54.0	53.5	52.6	51.3	49.4	47.6	45.8	44.0	42.2	40.4	38.6	36.8	35.1	33.4	30.0	30.0	30.0	30.0	30.0
3	30.0	53.9	53.5	52.6	51.2	49.4	47.5	45.8	44.0	42.2	40.4	38.6	36.8	35.0	33.4	30.0	30.0	30.0	30.0	30.0
4	30.0	53.9	53.4	52.5	51.2	49.4	47.6	45.8	44.0	42.2	40.4	38.6	36.8	35.0	33.2	30.0	30.0	30.0	30.0	30.0
5	30.0	53.8	53.4	52.5	51.1	49.4	47.6	45.8	44.0	42.2	40.4	38.6	36.8	35.0	33.1	30.2	30.0	30.0	30.0	30.0
6	30.0	53.8	53.4	52.5	51.1	49.4	47.6	45.8	44.0	42.2	40.4	38.6	36.8	35.0	33.1	30.3	30.0	30.0	30.0	30.0
7	30.0	30.0	30.0	30.0	30.0	30.0	30.0	30.0	30.0	30.0	30.0	30.0	30.0	30.0	30.0	30.0	30.0	30.0	30.0	30.0

Problem 3: 3-D Areal

The purpose of this problem is to become familiar with data input and the effects of truncation error. The effects of a partially penetrating well pumping from an "infinite", homogeneous, isotropic aquifer will be simulated and compared with an analytical solution by Hantush. Assume,

$K = 0.001$ ft/s

$S_s = 10^{-6}$ ft^{-1}

$Q_w = 3$ ft^3/s

Aquifer thickness = 300 ft

Screened interval of well: lower 1/3 of aquifer

Compute the drawdown after 0.5 hours and compare with the analytical data tabulated below. To reduce computing time, only one quadrant of the x-y plane will be solved. The solutions in the remaining quadrants are mirror images. Use a 44 ft, square block at node (2,2) for the well and expand block sizes by a factor of 1.5. Use a 12 by 12 grid. Try 1, 3, and 6 time steps with 3 and 6 layers. If time permits, simulate the same problem in the multi-aquifer mode.

Radius in feet

	31	80	161	284	470	748	1166	1792	2732
Analytical	9.91	6.39	4.40	3.20	2.35	1.63	1.01	0.50	0.16
3L, 1 t s	10.48	6.47	4.19	2.89	2.03	1.37	0.83	0.44	0.18
3L, 3 t s	10.79	6.78	4.49	3.17	2.27	1.55	0.94	0.47	0.17
3L, 6 t s	10.85	6.84	4.55	3.22	2.32	1.60	0.97	0.48	0.17
6L, 1 t s	10.07	6.25	4.11	2.87	2.03	1.37	0.83	0.44	0.18
6L, 3 t s	10.39	6.56	4.41	3.15	2.27	1.55	0.94	0.47	0.17
6L, 6 t s	10.23	6.62	4.47	3.21	2.32	1.60	0.97	0.48	0.17

FORTRAN CODING FORM

~~PROGRAM~~ Data: Problem 3

PROGRAMMER

DATE

PAGE 1 OF 1

STATEMENT LABEL	CONT	FORTRAN STATEMENT	STATEMENT SEQUENCE

```
PROBLEM -- 3-D PARTIALLY PENETRATING WELL
NRC CLASS / PROB.  3
        12        12         3        30         0
DRAW      MASS                                    EQN3
         1         6      .001         5
        1.        1.      500.         1.100000600
Blank
Blank
Blank
        0.
        0.
        0.
      1.-6
      1.-6
      1.-6
      1.-3                            1.         1        1.
      1.-3                            1.        1.        1.
      1.-3                            1.        1.        1.
       44.         1
        1.        1.       1.5    2.25     3.375     5.063     7.594    11.391
    17.086    25.629    38.443        1.
       44.         1
        1.        1.       1.5    2.25     3.375     5.063     7.594    11.391
    17.086    25.629    38.443        1.
      100.
         1         0         1    .02083         3       1.5      .071
         1         2         2    -.0075
```

PRODUCT NO. 5905

```
                              NUMBER OF ROWS =   12
                           NUMBER OF COLUMNS =   12
                            NUMBER OF LAYERS =    3

          MAXIMUM PERMITTED NUMBER OF ITERATIONS =   30

                NUMBER OF CONSTANT HEAD NODES =    0

SIMULATION OPTIONS:  DRAW          MASS                                    EQN3

                        WORDS OF VECTOR Y USED =   6423

                     NUMBER OF PUMPING PERIODS =    1
                  TIME STEPS BETWEEN PRINTOUTS =    6

                    ERROR CRITERIA FOR CLOSURE =   .1000000E-02

     ON ALPHAMERIC MAP:
          MULTIPLICATION FACTOR FOR X DIMENSION =   1.000000
          MULTIPLICATION FACTOR FOR Y DIMENSION =   1.300000
                          MAP SCALE IN UNITS OF
                   NUMBER OF           PER INCH =   500.0000
        MULTIPLICATION FACTOR FOR DRAWDOWN =   1.000000     PRINTED FOR LAYERS 1 0 0 0 0 0 0 0 0
            MULTIPLICATION FACTOR FOR HEAD =  0.             PRINTED FOR LAYERS 0 0 0 0 0 0 0 0 0

                          STARTING HEAD =  0.             FOR LAYER  1

                          STARTING HEAD =  0.             FOR LAYER  2

                          STARTING HEAD =  0.             FOR LAYER  3

                   STORAGE COEFFICIENT =   .1000000E-05 FOR LAYER  1

                   STORAGE COEFFICIENT =   .1000000E-05 FOR LAYER  2

                   STORAGE COEFFICIENT =   .1000000E-05 FOR LAYER  3

                        TRANSMISSIVITY =   .1000000E-02 FOR LAYER  1

      DIRECTIONAL TRANSMISSIVITY MULTIPLICATION FACTORS FOR LAYER  1
                                     X =   1.000000
                                     Y =   1.000000
                                     Z =   1.000000

                        TRANSMISSIVITY =   .1000000E-02 FOR LAYER  2

      DIRECTIONAL TRANSMISSIVITY MULTIPLICATION FACTORS FOR LAYER  2
                                     X =   1.000000
                                     Y =   1.000000
                                     Z =   1.000000

                        TRANSMISSIVITY =   .1000000E-02 FOR LAYER  3

      DIRECTIONAL TRANSMISSIVITY MULTIPLICATION FACTORS FOR LAYER  3
                                     X =   1.000000
                                     Y =   1.000000
                                     Z =   1.000000
```

GRID SPACING IN PROTOTYPE IN X DIRECTION

44.	44.	66.	99.	149.	223.	334.	501.	752.	1128.	1691.	44.

GRID SPACING IN PROTOTYPE IN Y DIRECTION

44.	44.	66.	99.	149.	223.	334.	501.	752.	1128.	1691.	44.

DELZ = 100.0000

SOLUTION BY THE STRONGLY IMPLICIT PROCEDURE
****

5 ITERATION PARAMETERS: 0. .9158952E+00 .9929264E+00 .9994051E+00 .9999500E+00

PUMPING PERIOD NO. 1: .02 DAYS

NUMBER OF TIME STEPS= 3

DELT IN HOURS = .070

MULTIPLIER FOR DELT = 1.500

1 WELLS

K	I	J	PUMPING RATE
1	2	2	-.01

```
:                    TIME STEP NUMBER =          3                    :

                    SIZE OF TIME STEP IN SECONDS=          852.50

                         TOTAL SIMULATION TIME IN SECONDS=          1799.71
                                                  MINUTES=            30.00
                                                    HOURS=              .50
                                                     DAYS=              .02
                                                    YEARS=              .00

              DURATION OF CURRENT PUMPING PERIOD IN DAYS=               .02
                                                    YEARS=              .00


     CUMULATIVE MASS BALANCE:              L**3                 RATES FOR THIS TIME STEP:                L**3/T
     ------------------------                                   -------------------------

         SOURCES:                                                                STORAGE =                 .7500
         --------                                                               RECHARGE =                0.0000
                  STORAGE =          1349.47                               CONSTANT FLUX =                0.0000
                 RECHARGE =             0.00                                     PUMPING =                -.7500
            CONSTANT FLUX =             0.00                       EVAPOTRANSPIRATION =                0.0000
            CONSTANT HEAD =             0.00                              CONSTANT HEAD:
                  LEAKAGE =             0.00                                          IN =                0.0000
            TOTAL SOURCES =          1349.47                                         OUT =                0.0000
                                                                                LEAKAGE:
         DISCHARGES:                                          FROM PREVIOUS PUMPING PERIOD =                0.0000
         -----------                                                               TOTAL =                0.0000
       EVAPOTRANSPIRATION =             0.00
            CONSTANT HEAD =             0.00                                SUM OF RATES =                -.0000
          QUANTITY PUMPED =          1349.78
                  LEAKAGE =             0.00
          TOTAL DISCHARGE =          1349.78

        DISCHARGE-SOURCES =              .31
       PER CENT DIFFERENCE =             .02


FLOW TO TOP LAYER =  -.2498958       FLOW TO BOTTOM LAYER =  -.4999069          POSITIVE UPWARD

MAXIMUM HEAD CHANGE FOR EACH ITERATION:
---------------------------------------

     .0377      .0856      .3387      .1279      .0262      .0025      .0069      .0140      .0141      .0007

TIME STEP :  1  2  3
----------

ITERATIONS: 10   9   9
----------
```

D25

PLOT OF DRAWDOWN   LAYER   1

```
+----------+----------+----------+----------+----------+----------+----------+----------+----------+   4500.00
:                                                                                                   :
:                                                                                                   :
:                                                                                                   :
0  0  0  0     0       0             0                   0                            0         :
:                                                                                                   :
+                                                                                                   +   4000.00
:                                                                                                   :
:                                                                                                   :
:                                                                                                   :
:                                                                                                   :
+                                                                                                   +   3500.00
:                                                                                                   :
:                                                                                                   :
:                                                                                                   :
:                                                                                                   :
+                                                                                                   +   3000.00
:                                                                                                   :
:                                                                                                   :
0  0  0  0     0       0             0                   0                            0         :
:                                                                                                   :
:                                                                                                   :
+                                                                                                   +   2500.00
:                                                                                                   :  X DIS-
:                                                                                                   : TANCE IN
:                                                                                                   :
:                                                                                                   :
:                                                                                                   :
+                                                                                                   +   2000.00
:                                                                                                   :
:                                                                                                   :
0  0  0  0     0       0             0                   0                            0         :
:                                                                                                   :
:                                                                                                   :
+                                                                                                   +   1500.00
:                                                                                                   :
:                                                                                                   :
:                                                                                                   :
1  1  1  1     1       1             0                   0                            0         :
:                                                                                                   :
+                                                                                                   +   1000.00
:                                                                                                   :
:                                                                                                   :
2  2  1  1     1       1             0                   0                            0         :
:                                                                                                   :
:                                                                                                   :
2  2  2  2     1       1             0                   0                            0         +    500.00
:                                                                                                   :
:                                                                                                   :
3  3  3  2     1       1             0                   0                            0         :
4  4  3  2     2       1             0                   0                            0         :
7  4  3  2     2       1             0                   0                            0         :
1  4- 3- 2+--- 2----+- 1------+---- 0---+---------+--- 0----+---------+---------+- 0------+      0.00

0.0    500.00  1000.00  1500.00  2000.00  2500.00  3000.00  3500.00  4000.00  4500.00

                 DISTANCE FROM ORIGIN IN Y DIRECTION, IN
```

EXPLANATION

R = CONSTANT HEAD BOUNDARY
*** = VALUE EXCEEDED 3 FIGURES
MULTIPLICATION FACTOR = 1.000

DRAWDOWN, LAYER 1

1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
2	0.00	10.79	6.78	4.49	3.17	2.27	1.55	.94	.47	.17	.05	0.00
3	0.00	6.78	5.62	4.23	3.11	2.25	1.55	.94	.47	.17	.05	0.00
4	0.00	4.49	4.23	3.54	2.91	2.19	1.52	.93	.47	.17	.05	0.00
5	0.00	3.17	3.11	2.91	2.54	2.03	1.46	.91	.46	.17	.05	0.00
6	0.00	2.27	2.25	2.19	2.03	1.74	1.32	.86	.44	.17	.04	0.00
7	0.00	1.55	1.55	1.52	1.46	1.32	1.07	.74	.40	.16	.04	0.00
8	0.00	.95	.94	.93	.91	.86	.74	.56	.33	.13	.04	0.00
9	0.00	.47	.47	.47	.46	.44	.40	.33	.21	.10	.03	0.00
10	0.00	.17	.17	.17	.17	.17	.16	.13	.10	.05	.02	0.00
11	0.00	.05	.05	.05	.05	.04	.04	.04	.03	.02	.01	0.00
12	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00

DRAWDOWN LAYER 2

1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
2	0.00	5.23	4.70	3.90	3.05	2.26	1.55	.94	.47	.17	.05	0.00
3	0.00	4.70	4.40	3.77	3.00	2.24	1.54	.94	.47	.17	.05	0.00
4	0.00	3.90	3.77	3.41	2.84	2.18	1.52	.93	.47	.17	.05	0.00
5	0.00	3.05	3.00	2.84	2.51	2.02	1.46	.91	.46	.17	.05	0.00
6	0.00	2.26	2.24	2.18	2.02	1.73	1.32	.86	.44	.17	.04	0.00
7	0.00	1.55	1.55	1.52	1.46	1.32	1.07	.74	.40	.16	.04	0.00
8	0.00	.95	.94	.93	.91	.86	.74	.56	.33	.13	.04	0.00
9	0.00	.47	.47	.47	.45	.44	.40	.33	.21	.10	.03	0.00
10	0.00	.17	.17	.17	.17	.17	.16	.13	.10	.05	.02	0.00
11	0.00	.05	.05	.05	.05	.04	.04	.04	.03	.02	.01	0.00
12	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00

DRAWDOWN, LAYER 3

1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
2	0.00	4.06	3.92	3.55	2.95	2.24	1.55	.94	.47	.17	.05	0.00
3	0.00	3.92	3.81	3.47	2.91	2.22	1.54	.94	.47	.17	.05	0.00
4	0.00	3.55	3.47	3.24	2.78	2.17	1.52	.93	.47	.17	.05	0.00
5	0.00	2.95	2.91	2.78	2.48	2.02	1.46	.91	.46	.17	.05	0.00
6	0.00	2.24	2.23	2.17	2.02	1.73	1.32	.86	.44	.17	.04	0.00
7	0.00	1.55	1.54	1.52	1.46	1.32	1.07	.74	.40	.16	.04	0.00
8	0.00	.95	.94	.93	.91	.86	.74	.56	.33	.13	.04	0.00
9	0.00	.47	.47	.47	.46	.44	.40	.33	.21	.10	.03	0.00
10	0.00	.17	.17	.17	.17	.17	.16	.13	.10	.05	.02	0.00
11	0.00	.05	.05	.05	.05	.04	.04	.04	.03	.02	.01	0.00
12	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00

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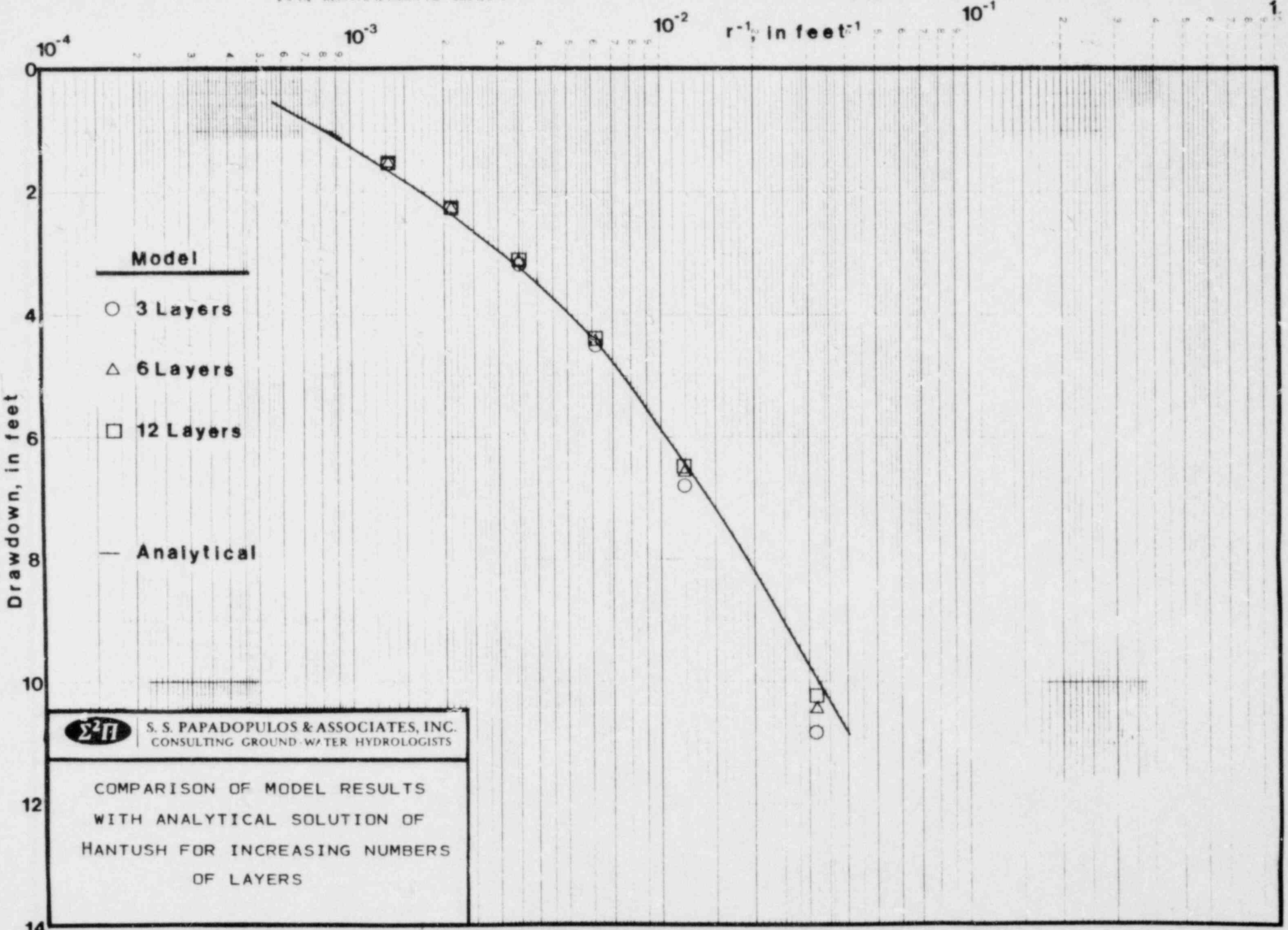

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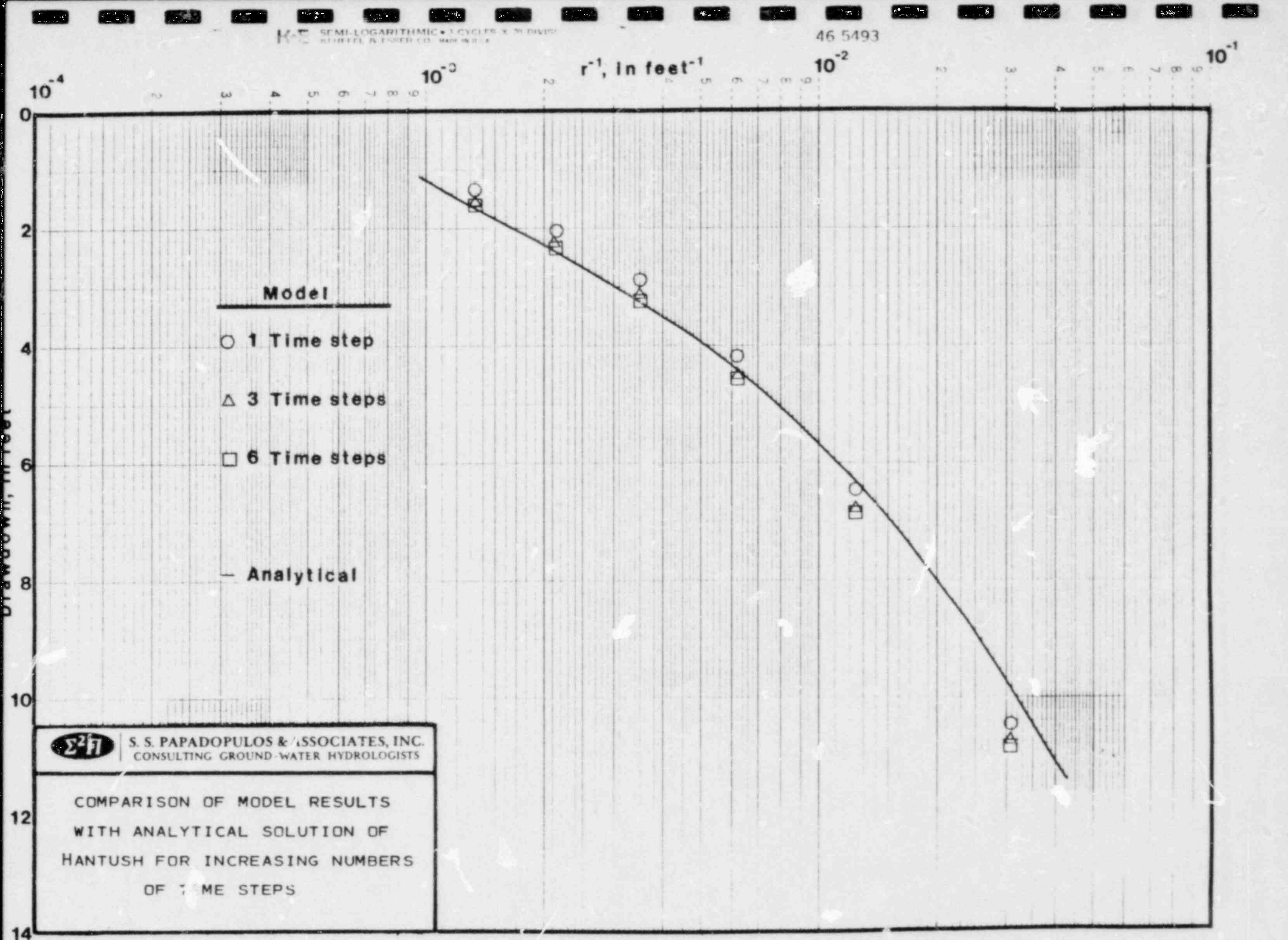
COMPARISON OF MODEL RESULTS WITH ANALYTICAL SOLUTION OF HANTUSH FOR INCREASING NUMBERS OF LAYERS



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References for analytical solutions:

Hantush, M.S., 1961, Drawdown around a partially penetrating well: Proc. Am. Soc. Civil Eng., v. 87, No. HY4.

Hantush, M.S., and Jacob, C.E., 1955, Nonsteady radial flow in an infinite leaky aquifer: Am. Geophys. Union Trans., v. 36, No. 1, pp. 95-100.

Theis, C.V., 1935, The relation between the lowering of the piezometric surface and the rate and duration of discharge of a well using ground-water storage: Am. Geophys. Union Trans., v. 16, pp. 519-524.

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2.		
3.		
4.		
5.		

	Action		File		Note and Return
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	Comment		Investigate		Signature
	Coordination		Justify		

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