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Snubber Sensitivity Study

Prepared by A. T. Onesto

Energy Technology Engineering Center

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U.S. Nuclear Regulatory Commission

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AUTHOR

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Snubber Sensitivity Study

STATEMENT OF PROBLEM

Develop information which will provide the basis for structural analysis and design rules for systems and components which utilize snubbers as supports. Results will be used to assure that dynamic response characteristics of snubber supported systems and components will be bounded within acceptable limits.

ABSTRACT:

Snubbers are used widely throughout the nuclear industry as seismic restraints. The validity of the analysis of snubber-supported systems depends on their realistic characterization. The purpose of this work was to: 1) identify those parameters which characterize hydraulic and mechanical snubbers which significantly affect snubber dynamic response; 2) determine the response sensitivity to variations of these parameters. Based upon the results of the foregoing, simplified design and analysis procedures are proposed, to maintain system response within acceptable limits.

Germane results of a test program to evaluate the effects of snubber mismatch in multiple support applications are included in this report.

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1.0 INTRODUCTION

The object of this effort under NRC contract FIN No. B3076-8 is to develop simplified design and analysis rules for snubber supported systems which will bound snubber response within acceptable limits. Simplified rules or guidelines are formulated from the results of numerous analytical studies performed under this contract and presented in this report. In addition to these analytical studies, results of a test program performed under NRC contract FIN No. D30558 are included.

The guidelines for snubber usage presented have been formulated solely from the limited analytical and test results presented herein and must therefore be considered preliminary in nature only. Modification or changes should be expected in time as additional knowledge is gained through further analysis and testing.

Only seismic applications of snubbers are considered in this study. Furthermore, the analysis is limited to two specific snubber designs which are presently most widely used for seismic application: the nonlocking mechanical snubber with an acceleration sensing activation mechanism and the hydraulic snubber with a velocity dependent activation level and load dependent release rate.

The analytical work presented represents the results of a two phase, two year study. The first phase (FY 78) consisted of an analytical evaluation and parametric study of various snubber characteristics with the following objectives: 1) Identify snubber structural and performance parameters which significantly affect snubber dynamic response characteristics, and 2) Determine the sensitivity of the snubber response to variations in each parameter identified in 1. The second phase (FY 79) represents an extension of analytical studies initiated in FY 78 and the formulation of simplified design and analysis guidelines from both analytical studies.

The test data included in this report are taken from the results of a program designed to: 1) Evaluate changes in response that occur when a single large snubber is replaced by two smaller snubbers with approximately the same total rated capacity and 2) Evaluate the effects of snubber mismatch for multiple snubber applications. These results are included as part of this report to broaden the scope of the simplified rules.

The discussion section of this report contains the justification and discussion regarding the guidelines presented in Appendix B. The discussion of the guidelines references various sections of the Appendices of this report. These references are of the form Y.X.X.X, where Y and X.X.X. identifies the referenced Appendix and the referenced figure, paragraph or section number in Appendix Y, respectively. Material referenced in the DISCUSSION from other locations in the DISCUSSION will have the form X.X.X. The Appendices of this report are as follows:

Appendix A - Fiscal Year 1979 Analytical Study
Appendix B - Guidelines for Simplified Design and Analysis Procedures for Snubbers
Appendix C - Fiscal Year 1978 Analytical Study
Appendix D - Test Data from Single Versus Multiple Snubber Test Program

2.0 SUMMARY

This report summarizes the work performed by ETEC for NRC under Contract FIN #B3076-8.

2.1 Program Objectives

The program objectives were to:

1) Identify structural and performance parameters which significantly affect snubber dynamic response characteristics.

2) Determine the sensitiv..y of the snubber response and the corresponding effects on the snubber supported system to variations of each parameter identified above.

3) Develop simplified analyses techniques and design rules which bound the response of the system within acceptable limits.

Tasks (1) and (2) represent analytical studies of the externally evident parameters associated with 1) acceleration activated mechanical snubbers and 2) velocity activated poppet-type hydraulic snubbers.

These studies were based primarily on single degree of freedom lumped mass systems and simple piping systems subjected to harmonic and seismic loadings. The guidelines in (3) were based on the results of the first two tasks. Inasmuch as they were based on a limited number of studies, modifications may be required as future efforts continue.

2.2 Summary of Results

2.2.1 Performance Parameters

Analysis has indicated the externally evident parameters nich most significantly affect snubber dynamic response are:

i) Clearance (lost motion)

ii) Activation level

iii) Release rate

iv) Stiffness

v) Friction (drag)

2.2.2 Response Sensitivity

Although it is recognized that there is some interaction between the various snubber parameters and system response, these effects were not considered in this study. Brief summaries of results for each of the performance parameters identified in 2.2.1 are provided below.

2.2.2.1 Clearance/Lost Motion - This is the response parameter that has the greatest effect on system response. For clearances in excess of .05 inch the response changes cannot be predicted without detailed non-linear analysis. Clearance is responsible for impact loads, decreased effective support stiffness and a reduced response sensitivity to other snubber parameters. The ideal situation is one of minimum clearance or lost motion.

2.2.2.2 Activation Level - The activation level is that velocity or acceleration at which the snubber restrains dynamic motion. The velocity activated hydraulic snubber is effective in reducing system response provided that the activation level does not exceed 50 inch/minute. The activation level of the acceleration activated mechanical snubber should not exceed .02g when used as a seismic restraint. The study indicated that the effective stiffness of the acceleration activated mechanical snubber is frequency dependent and the possibility exists that dynamic interaction between the snubber and supported component may result in response amplification rather than response reduction.

2.2.2.3 Release Rate - An expression was derived from the maximum allowable release rat. . a hydraulic snubber in terms of the maximum snubber load and the mass of the component being supported. The snubber release rate should not exceed X_B, where

$$X_B = .50 \times \left(\frac{R_L}{W}\right) \text{ inches/minute}$$

where

R_L = Rated Load (lbs.)

W = Component Weight (lbs.)

The release rate of the acceleration activated mechanical snubber is the same as its activation level therefore, the activation level results control selection of this parameter.

2.2.2.4 Stiffness - Since the "effective" stiffness of a snubber is generally greater than that for the snubber support assembly (clamp, transition tube extension, back-up support structure, etc.) the snubber response characteristics, e.g., damping, activation level, release rate, etc., may be "washed out" by the added flexibility. The results of the work presented in this study indicate that the combined "effective" stiffness of the snubber support assembly must be at least twenty times greater than the piping or component stiffness to be totally effective in reducing response.

Snubber stiffness should be evaluated independently of clearance/lost motion, activation level or release rate. The stiffness should be based on the structural compliance only. The stiffness of a hydraulic snubber can often be represented in terms of fluid compressibility

$$k_{snubber} = \frac{EA^2}{V} \text{ lb/in}$$

where

E = Bulk Modulus at operating temperature (lb/in^2)
V = Cylinder Volume (in^3)
A = Piston Area (in^2)

The stiffness of the mechanical snubber is equal to the stiffness in tension or compression of a locked snubber, whichever is less.

2.2.2.5 Friction (Drag) - The study of friction (Coulomb) indicates that in general, small friction loads have negligible effects on dynamic response. However, the response may be affected when the friction loads exceed 40% of the applied loading. Drag or friction loads may however affect normal operating stresses (thermal expansion) at much lower values.

2.2.3 Simplified Design and Analysis Guidelines

A complete set of design and analysis guidelines are presented in Appendix B. The guidelines are intended to apply to the utilization of specific designs of hydraulic and mechanical snubbers as seismic restraints for component or piping systems. Guidelines relating to multiple snubber support applications based on an independently NRC sponsored test program are also included. A brief summary of selected topics follows.

2.2.3.1 Allowable Parameter Ranges - Ranges were based on the results of 2.2.2.

2.2.3.2 Snubber Selection - Surveys of past snubber failures indicate that consideration should be given to the most common failure mode of the type of snubber to be selected. Hydraulic snubbers usually "fail" in a fr condition, whereas mechanical snubbers generally fail in a locked c n. Based on the foregoing only, it appears that long straight pipe runs w few snubbers should be supported with mechanical snubbers whereas pip g systems with many bends and short runs should be supported with hydraulic snubbers.

2.2.3.3 Linear Representation of a Snubber - There does not appear to be a satisfactory linear representation (spring or rigid support) that will permit system response and snubber reaction loads to be predicted with an accuracy sufficient to justify their use for seismic loading when clearance is present. The best simple representation of a snubber is a nonlinear representation consisting of a linear spring with a gap set equal to the total clearance of the component. This representation enables both response and reaction loads to be predicted with sufficient accuracy in most cases, provided all response parameters are bounded within the limits described in 2.2.2. However, a linear analysis may be made provided the total clearance is less than .05 inch, and the load and stresses are multiplied by the appropriate load factors. Snubber reaction loads and stresses shall be increased by 100% for clearances greater than .0 but less than .02 inch. Snubber reaction loads and stresses shall be increased by a factor of 4 for clearances greater or equal to .02 inch but less than .05 inches. Detailed nonlinear analysis is required for systems with .05 inch or greater clearance.

2.2.3.4 Multiple Snubber Usage - The guidelines for multiple snubber usage are based on a single test program described in Reference 1.

2.2.3.4.1 Snubber Mismatch - Mismatch of snubber end fitting clearance in multiple snubber supports has a greater effect on load sharing of parallel mounted snubbers than mismatch of activation level or release rate. Uniform load sharing of multiple snubber supports (within 10% of the total load) can be expected for hydraulic snubbers when end fitting clearance differentials are less than .01 inches and the activation level and release rate are between 8 and 25 inches/minute and 4 and 14 inches/minute, respectively.

Equal load sharing of multiple snubber supports should not be expected if the end fitting clearance mismatch exceeds .01 inches. If the mismatch clearance differential exceeds .01 inches but is less than .04 inches, peak loads shall be assumed twice the uniform load sharing value. Mismatch of end fitting clearance shall not exceed .04 inches.

2.2.3.4.2 Design Considerations for Hydraulic and Mechanical Snubber Pairs - The load sharing of a hydraulic snubber pair is more sensitive to mismatch of end fitting clearance than the load sharing of a mechanical snubber pair for harmonic input.

The load sharing of a mechanical snubber pair is more sensitive to mismatch of end fitting clearance than the load sharing of a hydraulic snubber pair for seismic input.

3.0 DISCUSSION

Justification for the Guidelines for Simplified Design and Analysis Procedures in Appendix B, herein after referred to as "Guide" is presented in this section. References to material contained elsewhere in this report will be described in the Introduction section. However, references to the Guide will be contained in the [] brackets.

3.1 Snubber Selection [B.2.1.2]

The selection of a snubber for a specific application should be based on its response characteristics, most probable failure mode and operating characteristics. The most probable failure mode of a snubber is related to its design features.

Mechanical snubber failures in the "locked condition" may be due to ball screw mechanism failures, contamination of the ball screw mechanism, or by wearing of internal parts. "Free condition" failures may be associated with failure of the capstan spring tangs or excessive torque drum wear. Mechanical snubbers are susceptable to fatigue failures resulting from steady-state low level vibrations.

Hydraulic snubbers fail most often in a free state. Loss of hydraulic fluid resulting from seal deterioration, reservoir leakage or piston rod scoring is the principle cause of failure. Valve failures can produce a locked failure condition, however these failures are rare.

3.2 Stiffness Requirements [B.2.1.2]

The minimum support stiffness requirements are intended to assure that response at the support can be bounded within acceptable limits. The study presented in C.6.1 addresses this problem. The approach was based on establishing an upper bound for the stiffness of a support above which further increases in stiffness causes insignificant changes in response. This approach enables lower limits of the combined stiffness of all support hardware between the ground and the center of the component to be specified. Support hardware includes all items between ground and component such as backup support structure, clevis, pins, extension tubes, snubbers and all clamping hardware. Stevenson (2) presents a study of specific pipe sizes and support spacing which supports the results of the study presented in C.6.1.

The evaluation of the hydraulic snubber stiffness based on fluid bulk modulus (E_o), cylinder volume (V) and piston area (A) is described in Section A.3. The Lissajous figures (load-deflection curves) shown in A.2.1 through A.2.5 indicate that the stiffness $\frac{dF}{dX}$, is insensitive to the release (bleed) rate, clearance and forcing frequency for loads less than 10 percent of the rated load, R_L. This initial slope or stiffness can be estimated from the fluid compressibility $k_F = E_oA^2/V$. Since the release rate and clearance have little effect on dF/dX for loads less than 80 percent of R_L, the stiffness should be expressed as $dF/dX \approx k_F = EA^2/V$. The response studies of A.3 also indicate that k_F is the best linear representation of the hydraulic snubber stiffness.

The same reasoning is believed to hold true for mechanical snubbers, i.e., the effective stiffness is represented best in terms of its structural compliance.

3.3 Allowable Parameter Ranges [B.2.1.3]

The proposed criteria "allowable" parameter ranges are applicable both to piping systems and component supports. The criteria do not assure that structural adequecy or functional integrity are maintained, but attempt to assure that the results of linear analytical studies bound the true response with reasonable assurance. The three specific criteria used to establish allowable parameter ranges are:

a) The response sensitivity of the characterization parameter is in the stable range and the response is therefore predictable;

b) The response at the snubber location resulting from seismic excitation will be less than .10 inch;

c) Displacement response and impact loads can be bounded with reasonable assurance that the predicted values will not be exceeded.

Relative to the above:

a) The sensitivity of system response to the characterization parameter is inconclusive because of the complex nature of the forcing function or impact characteristics of the snubber (See Figures A.5.2, A.8.3, A.8.4, C.7.1.7 etc). Since reasonable assurance of limiting system response is sought, the parameter was limited to that range which will assure predictable response characteristics.

b) The allowable response of .10 inch resulting from a seismic disturbance is based on the following reasoning. Assuming snubber spacing suggested by paragraph 121.1.4 of Reference (3) for pipe hangers, deflections of .10 inch at the seismic supports will produce stresses which will not exceed 1500 psi. Since design seismic loadings are generally less than 1.0g, the resulting static stresses should not exceed 1500 psi. The dynamically induced stresses due to impact effects will also be in a predictable range if the overall response does not exceed .10 inch. This is indicated by the results of the study presented in Section A.8. It is assumed that stresses can be related directly to impact loads, therefore these terms will be interchanged throughout this discussion.

3.3.1 Activation Level/Release Rate [B.2.1.3.1]

Although the activation level (lock velocity) of a hydraulic snubber has, in most cases a negligible effect on the Lissajous curve (See Figure A.2.5), this parameter does have a pronounced effect on system response (See Figures C.7.2.2 through C.7.2.6 and Figures C.7.2.8 and C.7.2.9).

The $(\frac{C}{M})$ parameter utilized for the study of the activation level of hydraulic snubbers can be related to the release rate of the snubber. The release rate, $\dot{X}_B$, is related to the (C/M) parameter as follows:

Assume

$$(\frac{C}{M}) = f \text{ sec}^{-1}$$

Then

$$\frac{(R_L/\dot{X}_B)}{M} = f \text{ sec}^{-1}$$

$$\dot{X}_B = \frac{R_L}{Mf} \text{ sec}$$

$$\dot{X}_B = (\frac{R_L}{W}) \frac{386.4}{f} \text{ sec}$$

$$\dot{X}_B = \frac{386.4}{f} (\frac{R_L}{W}) \text{ in/sec}$$

$$\dot{X}_B = \frac{386.4(60)}{f} (\frac{R_L}{W}) \text{ in/min} \qquad \text{(A)}$$

$$\dot{X}_B = 23(10^4)(\frac{R_L}{W})/f \text{ in/min}$$

where

$\dot{X}_B$ = bleed rate (in/min)

R_L = snubber rated load

W = component weight

M = component mass

f = value of (C/M) parameter

For a properly designed system, the rated load will usually be 2 to 6 times greater than the maximum anticipated load. Hence, using the results of A.8.3 which indicate that the maximum anticipated impact load is 4 to 5 times the nonimpact value, a range of 10-30 appears reasonable for the (R_L/W) ratio. Assuming $(R_L/W) = 20$, the (C/M) values are related directly to release rate by

$$X_B = \frac{4.6X10^6}{f}$$

Thus,

$$\text{If } \left(\frac{C}{M}\right) = \left\{\begin{matrix} 10^7 \text{ sec}^{-1} \\ 10^6 \text{ sec}^{-1} \\ 10^5 \text{ sec}^{-1} \end{matrix}\right\} , \quad \dot{X}_B \simeq \left\{\begin{matrix} .5 \text{ in/min} \\ 5. \text{ in/min} \\ 50. \text{ in/min} \end{matrix}\right\}$$

The data (Figures C.7.2.5, C.7.2.6, C.7.2.8 and C.7.2.9) indicate that response is insensitive to $(\frac{C}{M}) > 10^5$, that is, bleed rates less than 50 inches/minute. The same trend is indicated (Figures A.7.1 and A.7.2) from the results of the refined hydraulic snubber model.

Equation (A) can be used to establish the upper limit of the allowable release rate assuming that response is insensitive to $(\frac{C}{M}) > 5X10^4$. In particular, In particular,

$$X_B < .50 \left(\frac{R_L}{W}\right) \text{ in/min}$$

The system response is more sensitive to activation level than to release rate (See Figures C.7.2.5 through C.7.2.9). In all cases, the maximum response is limited to less than .1 inch if the activation level is less than 50 inch/minute and the bleed rate limits are satisfied.

The response curves presented in Figures C.7.1.4 through C.7.1.6 indicate that a "resonance" or peak response condition may be encountered when the mechanical snubber is subjected to harmonic oscillations. The data also suggests that this resonant condition occurs at low frequencies (< 3 Hz). Although Figure C.7.1.5 indicates that maximum response will be less than .10 inch at forcing frequencies greater than 3 Hz and Acceleration Threshold Parameter (ATP) values less than .02g (10 inch/sec^2), Figure C.7.1.7 indicates that response may exceed .10 inch if a seismic loading is applied rather than a harmonic loading as in C.7.1.5. Based on the results of these two studies, the upper limit for the allowable ATP limit is .02g (10 inch/sec^2). There is considerable uncertainty associated with system response when the ATP is greater than 10 inch/sec^2. Figures C.7.1.5, C.7.1.6 and C.7.1.7 indicate that response amplification may occur at larger ATP values, therefore the .02g limit is established.

3.3.2 Friction [B.2.1.3.4]

Figures C.7.4.1, C.7.4.3 and C.7.4.4 indicate that friction has little effect on system dynamic response unless the friction load exceeds 40 percent of the applied load.

3.3.3 Clearance [B.2.1.3.3]

The results of Figures A.8.3 through A.8.5 show that consistent trends in system response can be observed only when clearance is less than .05 inch. Furthermore, Figures C.7.3.1 and C.7.3.2 indicate that support flexibility reduces the effects of impact loading on stresses, i.e. the added flexibility of the support reduces the deterimental effects of clearance.

The ability to bound system response and impact loads for clearance values less than .05 inch is the primary reason for establishing this limit for clearance.

3.4 Desirable Parameter Limits [B.2.1.4]

3.4.1 Activation Level [B.2.1.4.1]

The results of studies regarding the activation level (Figures C.7.1.5 through C.7.1.8, C.7.2.2 through C.7.2.9, C.7.2.12, C.7.3.5, C.7.3.6, and Figures A.7.1 and A.7.2) indicate that response will be reduced whenever the activation level is reduced providing the forcing frequency is greater than 3 Hz. The 3 Hz frequency stipulation is noted since, for low frequency applications of mechanical snubbers, (Figure C.7.1.5) reductions in the activation level may increase the system response.

3.4.2 Release Rate [B.2.1.4.2]

Analytical studies (Figure C.7.2.7 and Figures A.7.1 and A.7.2) indicate that the minimum response does not occur at the minimum bleed rate. Based on the results of the indicated studies, the system response will be minimized when the bleed rate is between 5 and 30 inch/minute.

3.4.3 Clearance [B.2.1.4.3]

In general, impact loads and displacements are minimized when clearance is minimized. Clearance has an effect of softening the support. Consequently, cases may exist where the natural frequency of the system will shift away from the driving frequency as the gap increases thereby reducing the system response and impact loads (Figure C.7.3.4). When the input waveform is complex such as an earthquake, the response trends are often unpredictable. In some cases (as shown in A.8.3) the impact loads increase with increasing clearance while in other cases, they decrease slightly before increasing. Considering both displacement response and impact loads, minimum clearance is desirable.

3.4.4 Friction [B.2.1.4.4]

Friction loads have negligible effect on system dynamic response unless they exceed 40 percent of the applied load (Figure C.7.4.1). Friction loads of this magnitude are generally undesirable from the standpoing of thermal expansion. Therefore, it is desirable to have low friction loads.

3.4.5 Stiffness [B.2.1.4.5]

The displacement response generally can be reduced by increasing the effective stiffness of the snubber (Figure A.6.2 and Figures C.7.5.4 and C.7.5.5). The impact loading may either increase or decrease depending on the magnitude of the clearance (Figure A.8.3) or the relationship between the natural frequency and the forcing frequency. The difference between the maximum impact load and the zero clearance snubber load is generally much greater than the difference between the zero clearance and the minimum impact load. Therefore, it is reasonable to presume that whenever clearance is present, impact loads will be greater than or equal to the zero clearance loads. The zero clearance state is therefore preferred . Figures C.7.3.1 and C.7.3.2 indicate that added snubber flexibility is desired when clearance is present.

3.5 Linear Representation of a Snubber [B.2.2.2]

A linear analysis of a system may be performed provided that the clearance at the snubber is less than or equal to 0.05 inch. The 0.05 inch value represent the upper limit of clearance which permits reasonable assurance that response will be maintained within acceptable limits. If the gap is greater than 0.05 inch, the impact loads become dependent on the initial condition or distribution of gap at the onset of loading (Figure A.8.7). Figure A.8.5 shows that the dynamic impact load can be as large as 4 times the zero clearance value.

The 0.02 inch gap is that value which will assure that displacement response and impact loads will be predictable and also the impact load factor will not exceed twice the zero clearance value. Considering the results of Figure A.8.3, the impact load factor will be less than 2 if (Δ/X_B) is less than .003.

Assuming X_B = 6.0 in. (San Fernando 1971, Figure C.5.7),

$$\Delta < .020 \text{ inch}$$

Therefore, 0.02 inch represents an upper limit for clearance where displacements and impact loads are predictable and impact loads are less than twice the zero clearance values.

3.6 Multiple Snubber Supports [B.2.3]

Justification for guidelines for multiple snubber supports is provided from results of a test program from which Appendix D was extracted (see Reference 1).

Results of the test program are summarized as follows:

1) End fitting clearance has a greater effect on load sharing of dual snubber supports than mismatch of activation level or release rate. For zero end fitting clearance and any combination of activation level and release rate between 8 to 25 in/min and 4 to 14 in/min, respectively, equal load sharing (50%/50% to within 3%) was observed. However, for end fitting clearance differentials of 0.05 in., 30%/70% load sharing distributions were obtained.
2) The effects of end fitting clearance on support reactions were extremely variable. Different trends were obtained

for the various support types (rigid strut, hydraulic snubber or mechanical snubber), support configuration (single or dual) and inputs (seismic or sine). Table 2.3.1 in Appendix D summarizes the trends observed for the single and matched pair tests.

3) For each type (rigid strut, mechanical snubber or hydraulic snubber) of matched pair of snubbers and given type of loading (seismic or sine):

    i) For zero clearance, the total reaction force for the pair was less than the reaction force for a single snubber of the same type of loading. Table 2.3.2 in Appendix D lists the results.

    ii) For nonzero clearances, the single snubber force may be greater or less than the total load for the pair.

These data have been used to formulate the guidelines presented in B.2.3.1 and B.2.3.2. Other data obtained from the test program and summarized in Appendix D (specifically Figures 2.3.1 and 2.3.2) have been utilized to present the general design considerations presented in B.2.3.3.

The zero end fitting clearance requirement for equal load sharing has been modified to reflect lost motion differentials that existed in the test hardware (snubbers). The zero end fitting clearance requirement shall be modified to include both lost motion and end fitting clearance, and shall not exceed .01 inch.

REFERENCES

1. A. T. Onesto, "Single Versus Dual Snubber Installations", ETEC-TDR-80-12, (July 31, 1980).

2. J. D. Stevenson, L. A. Bergman, "The Effect of Support Stiffness on Seismic Response of Piping Systems", Piping Restraint Effects on Piping Integrity, ASME Technical Session Publication PVP-37, (June 1979).

3. ANSI B31.1, American National Standard Code for Pressure Piping, 1977 Edition, (May 1, 1977).

APPENDIX A

FISCAL YEAR 1979

ANALYTICAL STUDY

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TABLES

FIGURES

FIGURES

A.1 Mathematical Model of Hydraulic Snubber

Details of a mathematical model of hydraulic snubbers are presented in this section. Analytical expression for the rate of change of snubber load is derived in terms of various kinematic, dynamic and characterization parameters and the integration of this expression to obtain Lissajous curves is described.

First we consider the case when the snubber is activated (See Fig. A1.1,. Kinematically, the change in volume of the fluid in the cylinder, dV, during an increment of time, dt, is given by

$$dV = \dot{X}Adt \tag{1}$$

where

$\dot{X}$ = Piston velocity
A = Piston area

Since the change in volume is due to the effect of fluid compressibility and bleeding, we also have

$$dV = dF\frac{V}{EA} + CAFdt \tag{2}$$

where

F = Force in piston
dF = Increment of piston force due to compressibility
V = Volume of fluid
A = Area of piston
E = Bulk modulus of fluid
$C = \frac{\dot{X}_B}{R_L} = \frac{\text{Bleed Rate}}{\text{Rated Load}}$ (see Figure A.1.2)

Equations (1) and (2) give

$$\frac{dF}{dt} = \left(\frac{EA^2}{V}\right)[\dot{X} - CF], \text{ when the snubber is activated} \quad (3)$$

Clearly $\frac{dF}{dt} = 0$ when the snubber is not activated (4)

Hence, by Equations (3) and (4),

$$\frac{dF}{dt} = \begin{cases} \left(\frac{EA^2}{V}\right)[\dot{X} - CF], & \text{when the snubber is activated} \\ 0 & \text{, otherwise} \end{cases} \quad (5)$$

During snubber characterization test, snubber Lissajous curves for loads up to the rated loads are generated over a range of frequencies (usually 3Hz to 33Hz). These curves are obtained by subjecting the snubber to sinusoidal displacements $X = A \sin \Omega$ where the displacement amplitude, A, is initially small, increasing A until the rated load of the snubber is reached and then recording the resulting load-deflection plot.

Similar curves can be generated analytically by integration of Equation (5) with $\dot{X} = A\Omega \cos \Omega t$. Integration over several input cycles are necessary to obtain stable curves.

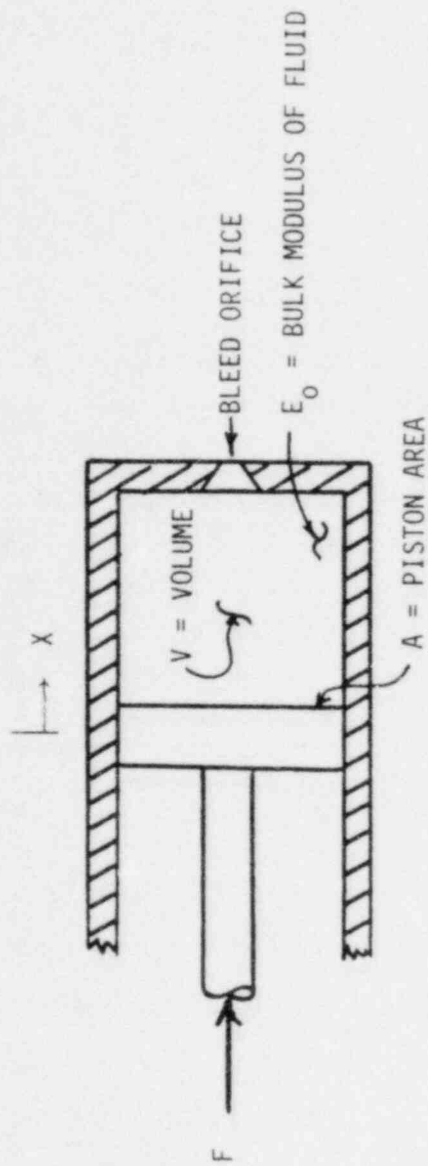


FIGURE A.1.1 HYDRAULIC SNUBBER MODEL

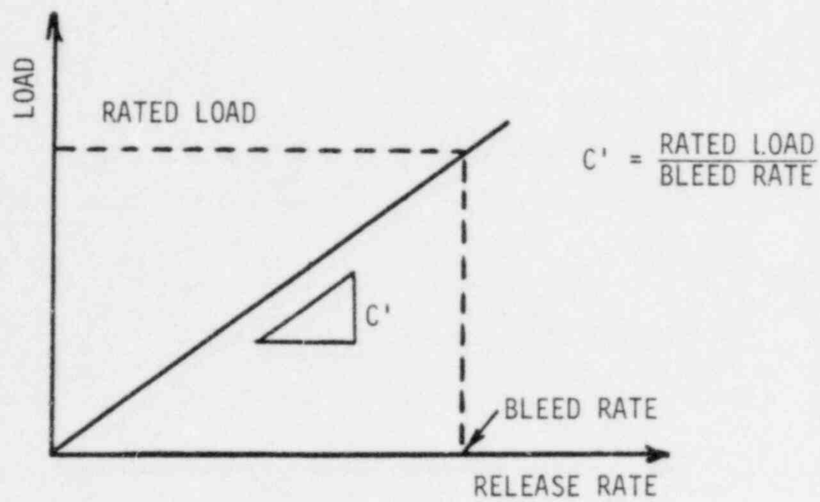


FIGURE A.1.2 EVALUATION OF DAMPING COEFFICIENT

A.2 Load-Deflection Characteristics of Hydraulic Snubbers

The results of a study of the effects of clearance, snubber geometry, fluid bulk modulus, activation level and release rate on the shape of Lissajous curves are presented in this section. The curves shown in this section were generated in accordance with the procedure described in Section A.1.

Figure A.2.1 shows the 3Hz curve for the "base" snubber that was selected for this study. The stiffness characteristics were based on the following data which are comparable to those for a typical 15K lb. snubber:

Piston Area	-	5 in^2
Cylinder Volume	-	30 in^3
Fluid Bulk Modulus	-	200,000 lb/in^2
Release Rate	-	50 in/min
Activation Level	-	10 in/min
Rated Load Capacity	-	12,500 lb
Clearance	-	.02 in

The effects of changes in frequency, gap, release rate and activation level on the base curve of Figure A.2.1 are shown in Figures A.2.2, A.2.3, A.2.4 and A.2.5, respectively.

Although Figure A.2.5 shows that the shape of the Lissajous curve is insensitive to changes in activation level, the studies of this Appendix and Appendix C indicate that significant changes in system response occur when the activation level varies between 0 to 80 in./min. Hence, system response cannot be related to dynamic stiffness per se.

The insensitivity of the shape of the Lissajous curve to changes in activation level is investigated in Figure A.2.6. The upper right quadrant shows the Lissajous curves for three clearance configurations. The lower right quadrant shows the velocity deflection characteristics for the snubber input. Points A', B", and C" represent the zero load points at which the hydraulic snubber disengages during the loading cycle for the 0, .01, and .02 inch clearance curves respectively. Points A', B', and C' indicate the points where the snubber activates, and points A, B, and C represent the velocities at which activation occurs. It can be seen that if the activation level(lock velocity)is less than 67 in/min that variations in lock velocity will have no effect on the Lissajous curve for the zero clearance curve. Similarly, for the 0.01 inch clearance call if the activation level does not exceed 72 in/min, there will be no effect on the Lissajous curve. The data indicate that for the given response parameters (bleed rate, forcing frequency, amplitude, fluid properties, etc...) that changes in load-deflection characteristics will occur only if the activation level(lock velocity)exceeds 90% of the peak input velocity (79.18 in/min). Figure A.2.7 indicates the load-deflection characteristics change for this system when the activation level (lock velocity)varies between 90 and 100% of the peak input velocity.

Figure A.2.8 shows the load-deflection characteristics for the same system at a higher forcing frequency (8Hz). This data indicates that much higher lock velocities are required to change the load-deflection characteristics. The activation level (lock velocity) must be increased beyond 200 in/min to change these properties.

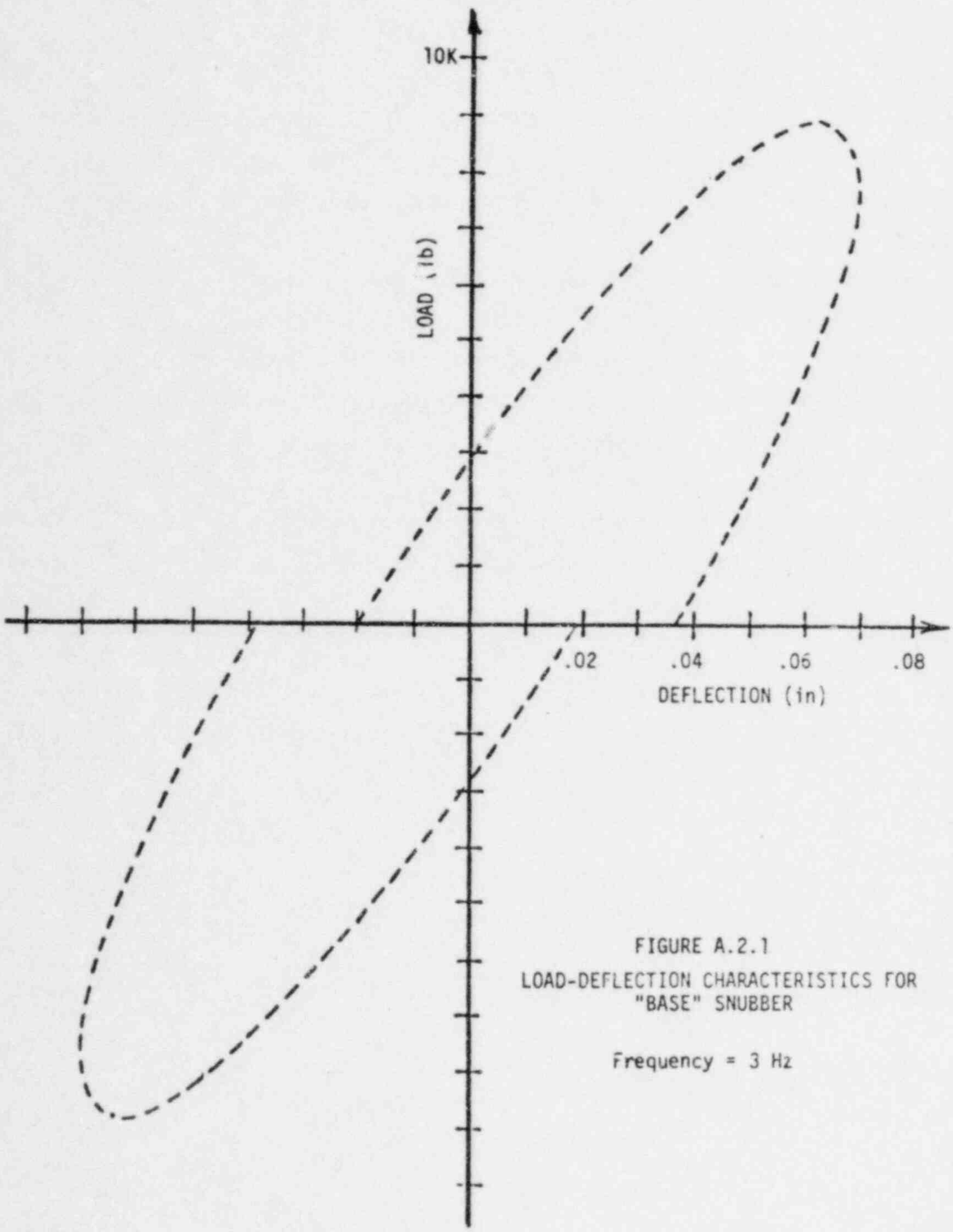


FIGURE A.2.1

LOAD-DEFLECTION CHARACTERISTICS FOR "BASE" SNUBBER

Frequency = 3 Hz

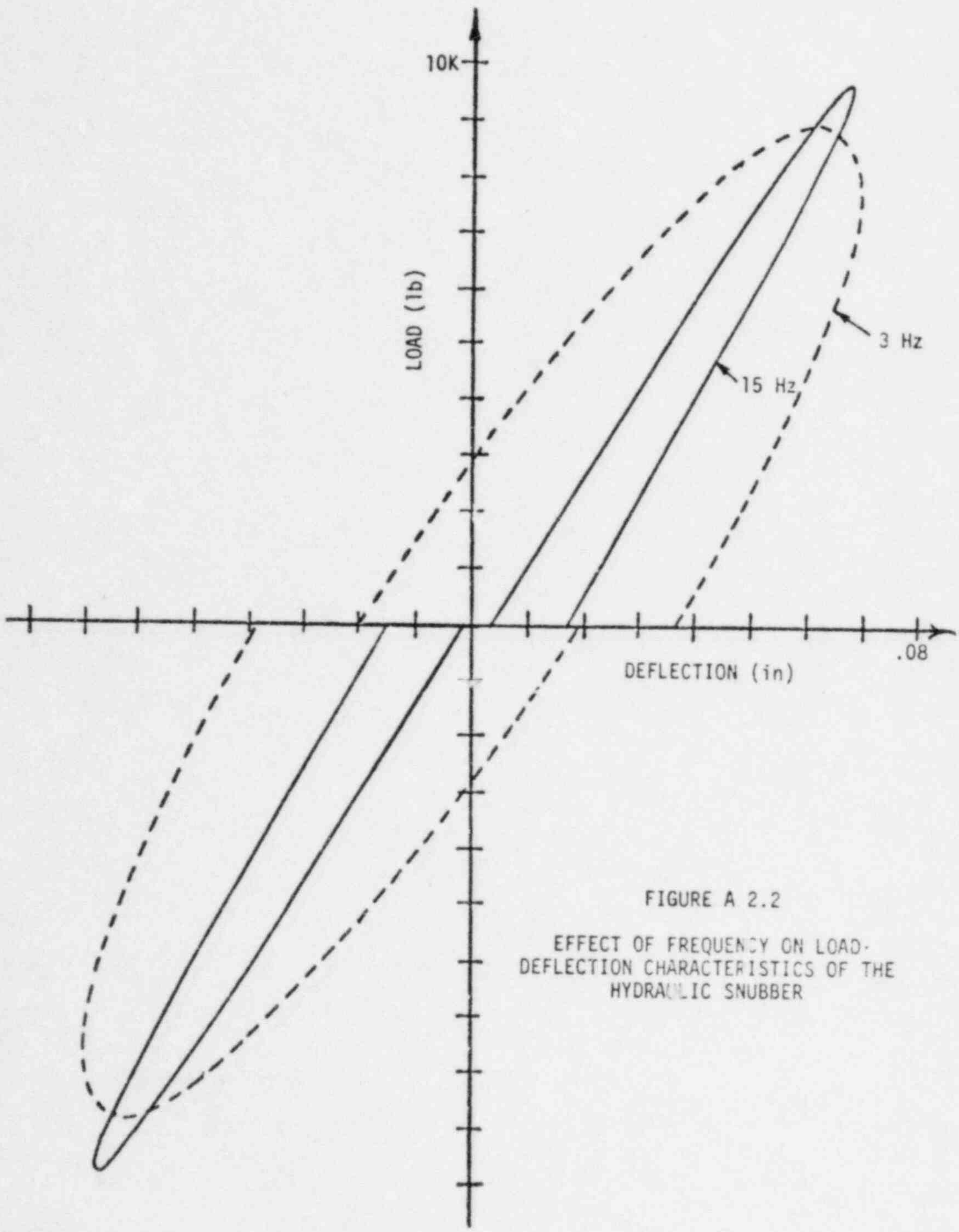


FIGURE A 2.2

EFFECT OF FREQUENCY ON LOAD-DEFLECTION CHARACTERISTICS OF THE HYDRAULIC SNUBBER

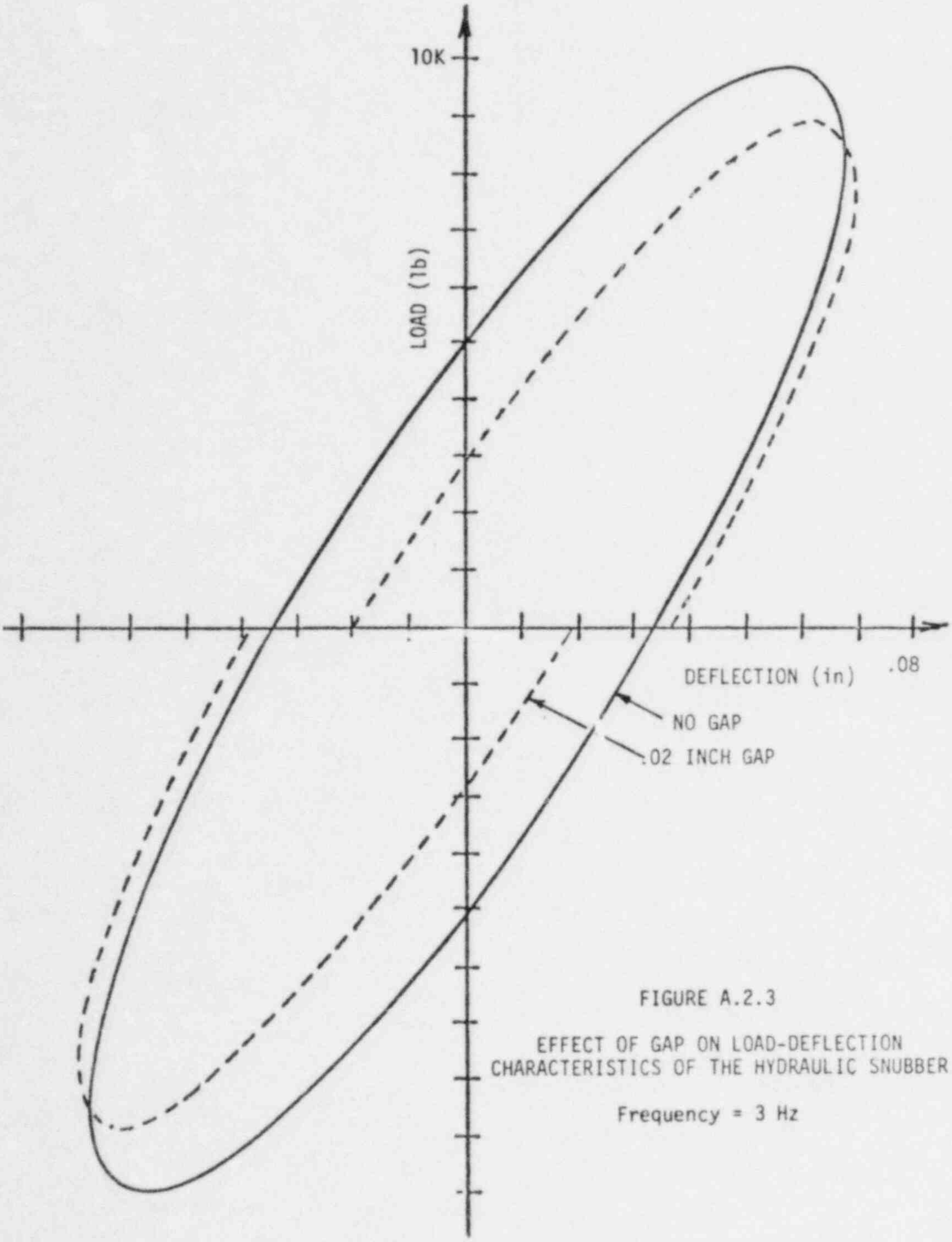


FIGURE A.2.3

EFFECT OF GAP ON LOAD-DEFLECTION CHARACTERISTICS OF THE HYDRAULIC SNUBBER

Frequency = 3 Hz

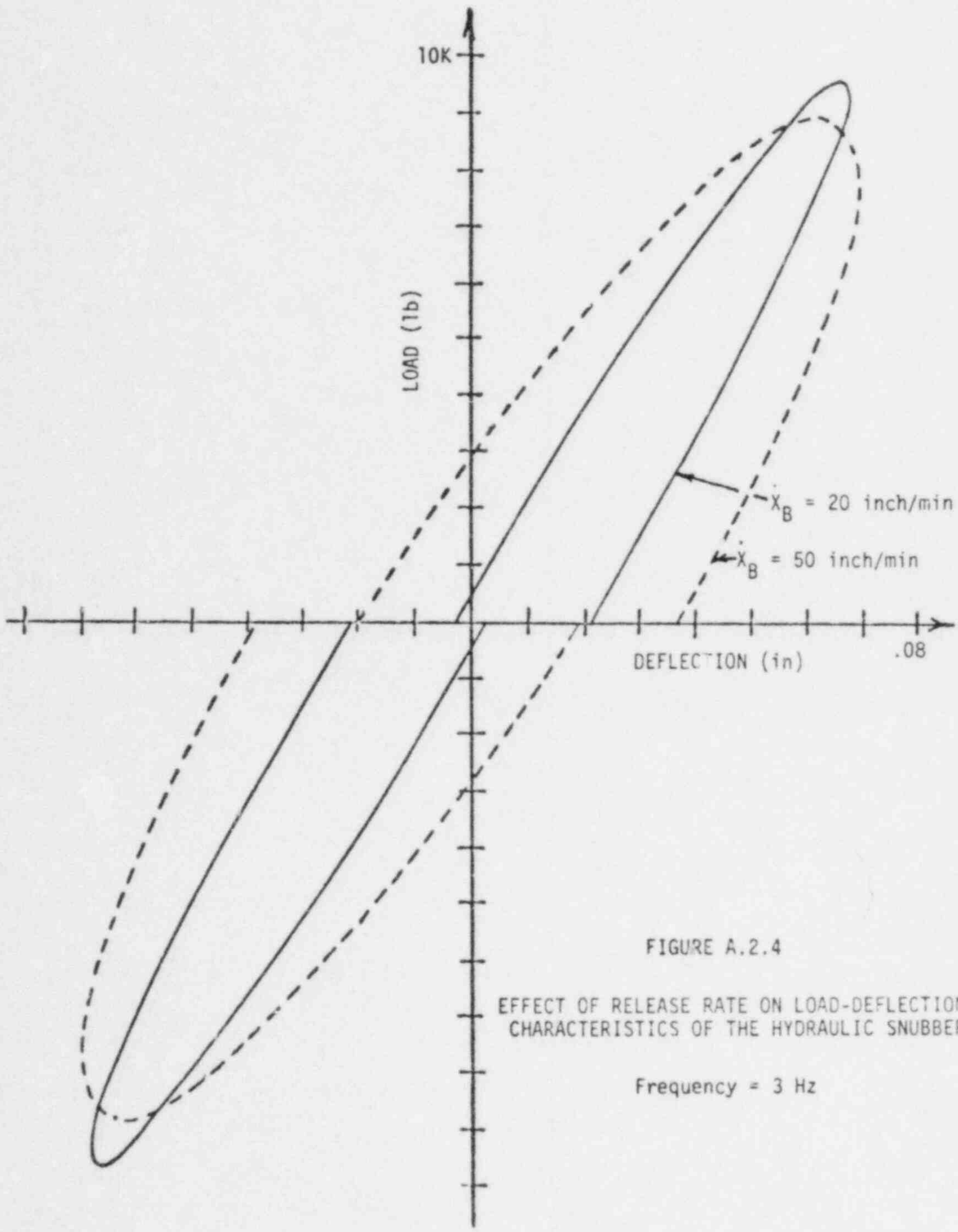


FIGURE A.2.4

EFFECT OF RELEASE RATE ON LOAD-DEFLECTION CHARACTERISTICS OF THE HYDRAULIC SNUBBER

Frequency = 3 Hz

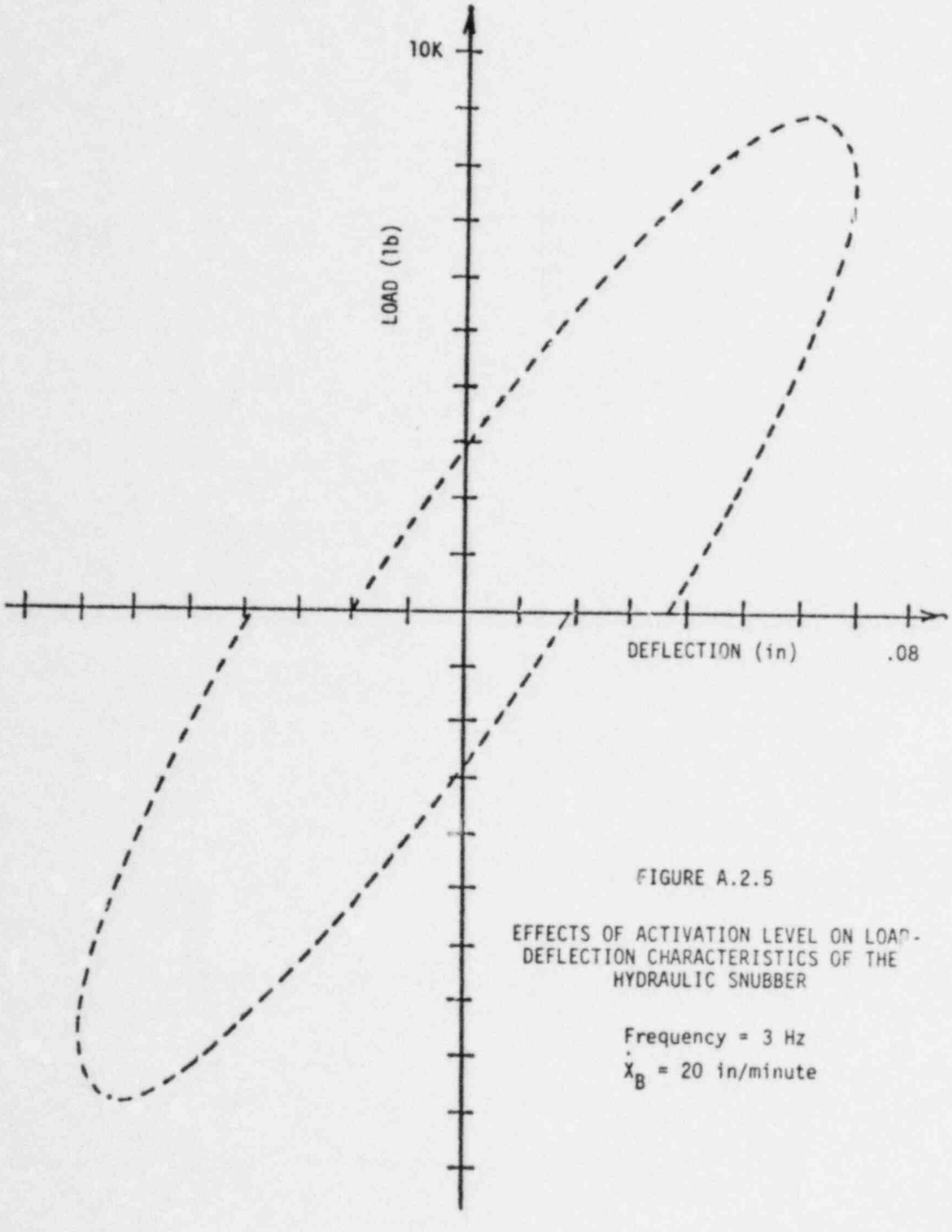


FIGURE A.2.5

EFFECTS OF ACTIVATION LEVEL ON LOAD-DEFLECTION CHARACTERISTICS OF THE HYDRAULIC SNUBBER

Frequency = 3 Hz
$\dot{X}_B$ = 20 in/minute

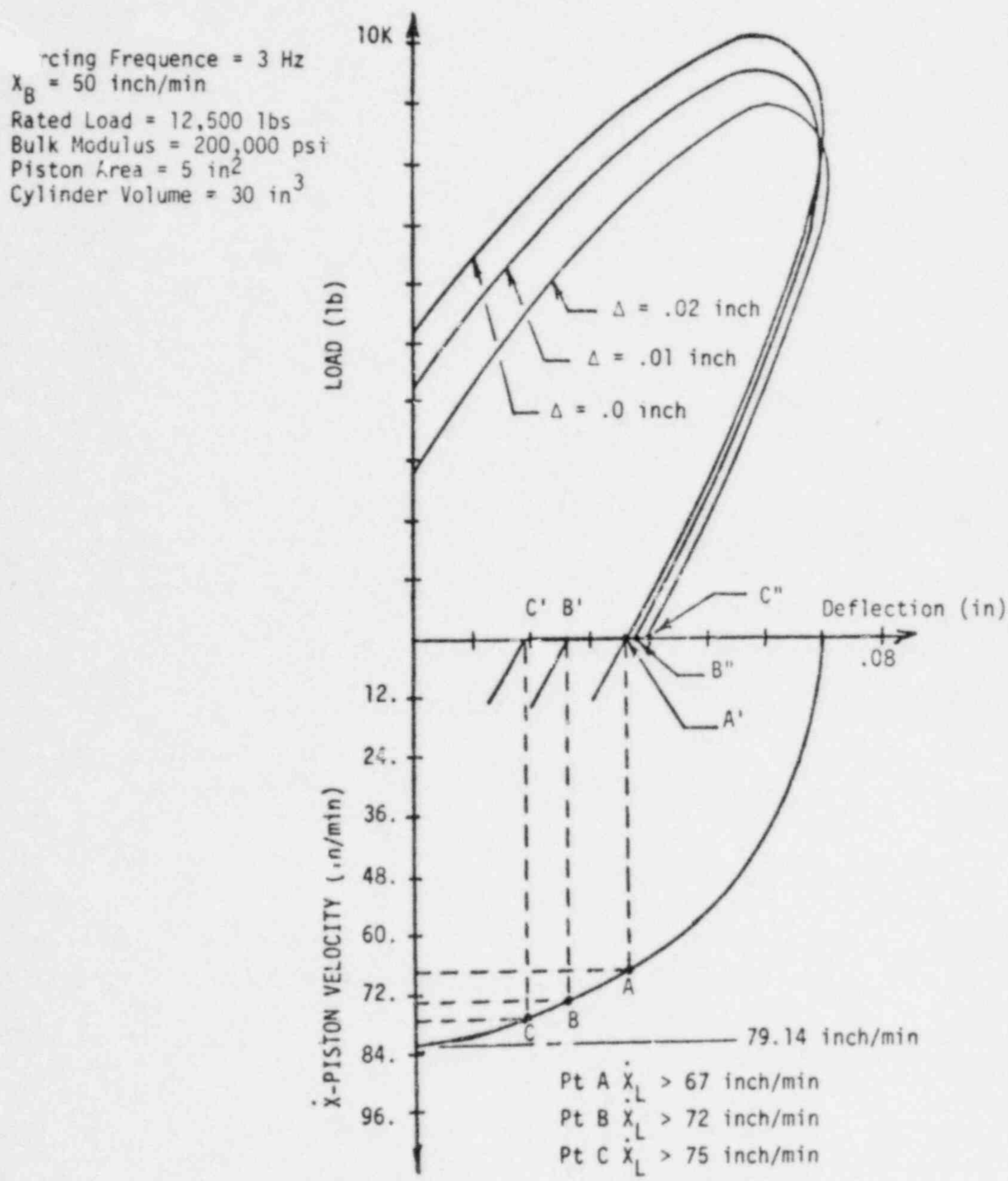
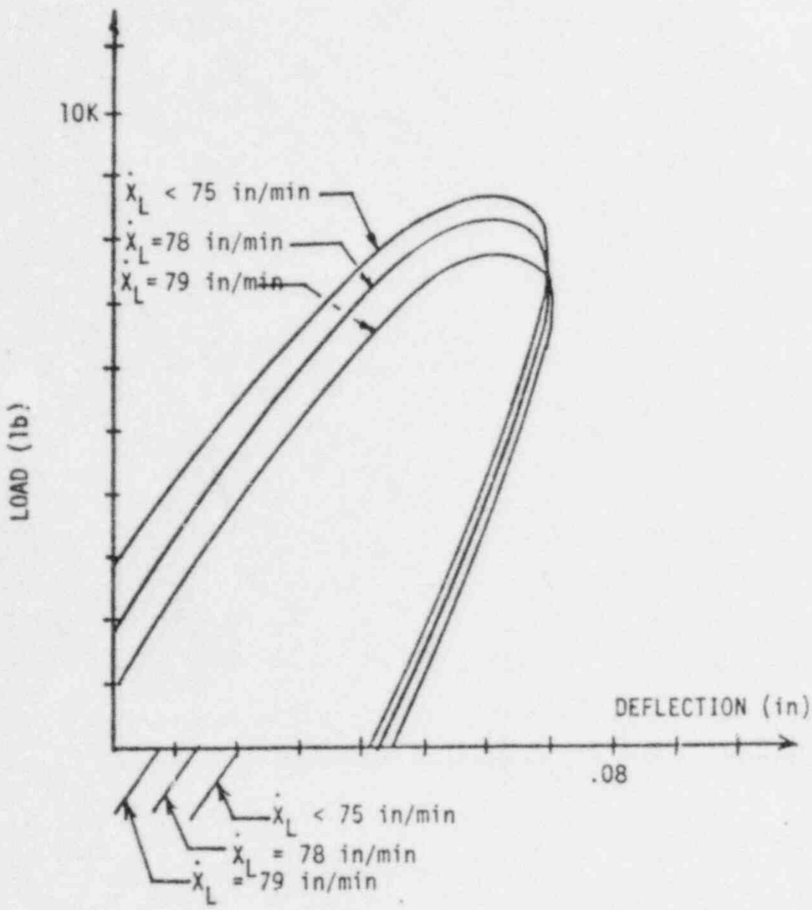


FIGURE A.2.6 ACTIVATION LEVEL REQUIREMENTS FOR HYDRAULIC SNUBBER AT LOW FREQUENCY EXCITATION



NOTE: If $\dot{X}_L$ > 80 in/min the snubber is not activated (Load = 0 for entire cycle)

Frequency = 3 Hz

Clearance = .02 inch

FIGURE A.2.7 ACTIVATION LEVEL EFFECTS ON LOAD-DEFLECTION CHARACTERISTICS

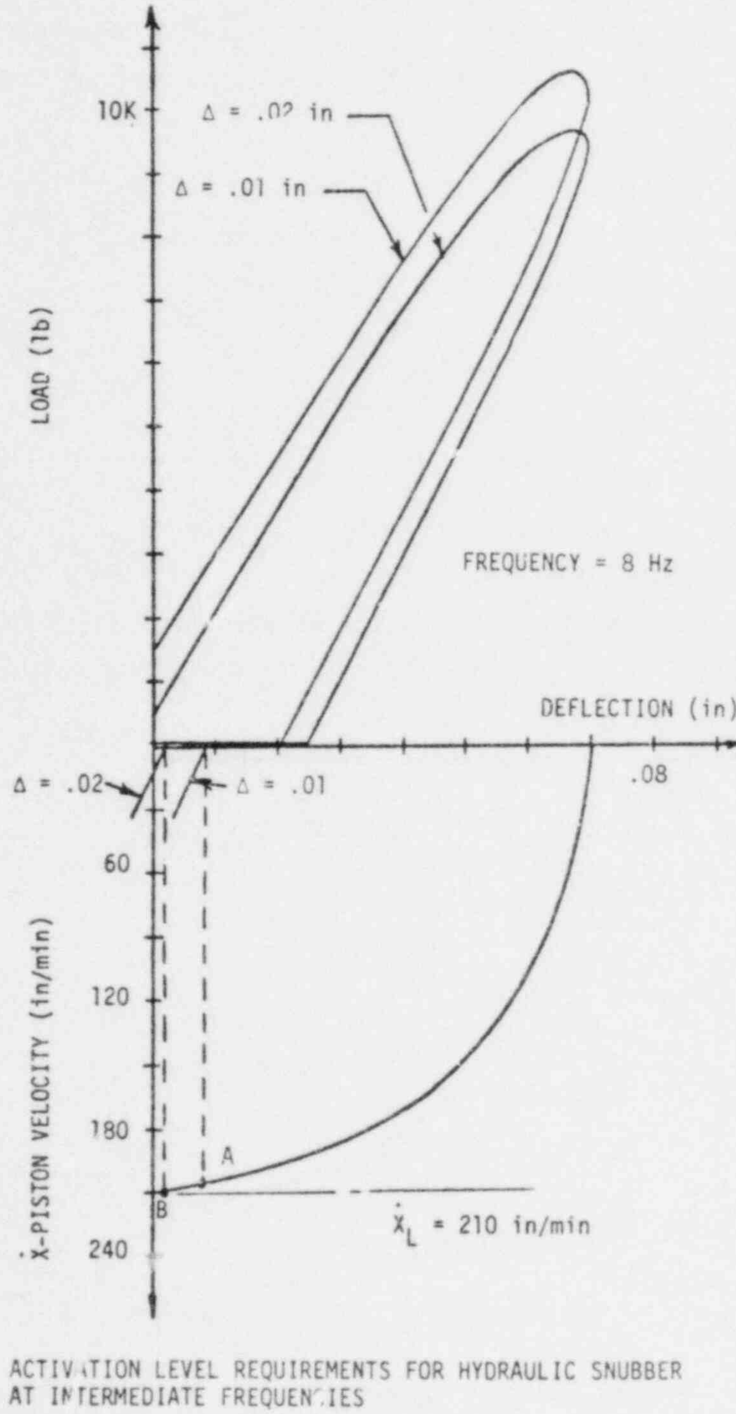


FIGURE A.2.8 ACTIVATION LEVEL REQUIREMENTS FOR HYDRAULIC SNUBBER AT INTERMEDIATE FREQUENCIES

A.3 Effective Stiffness of Hydraulic Snubbers

The results of an examination of and relationships between various definitions of snubber stiffness are presented in this section. These results will be utilized in the next section which addresses the problem of "best snubber stiffness representation."

First we show that in the case of zero clearance, the dynamic stiffness K, as defined in Figure A.3.1, can be approximated by its static stiffness, K_{st}, due to fluid compressibility, where

$$K_{st} = \frac{EA^2}{V}$$

where E, A and V are as defined in Section A.1.

To this end, we examine the Lissajous curves of Figure A.3.1. These curves were based on the same data assumed in Section A.2 except that the gap, release rate and frequencies are as indicated in the figure. From the referenced data, K_{st} = 166,660 lb/in, which is approximately equal to the K, values shown in Figure A.3.1.

Also, from Figure A.3.2, it is clear that the following relationship holds between the alternate stiffness parameters K_1 and K_2

$$K_1 > K_2$$

where

$$K_1 = \frac{P_M}{X_M - G/2}$$

$$K_2 = \frac{P_M}{X_M}$$

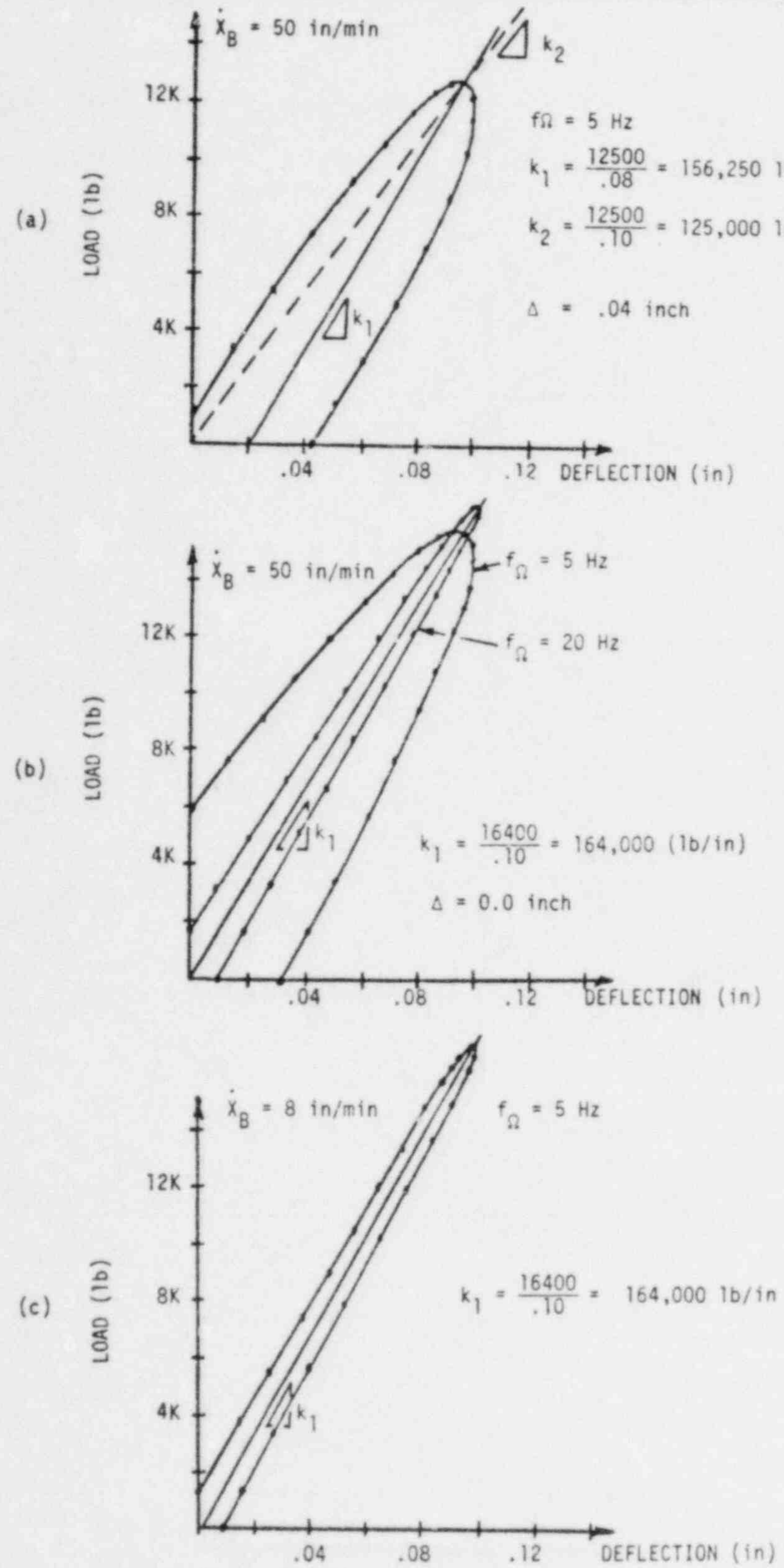


FIGURE A.3.1 SNUBBER STIFFNESS VALUES

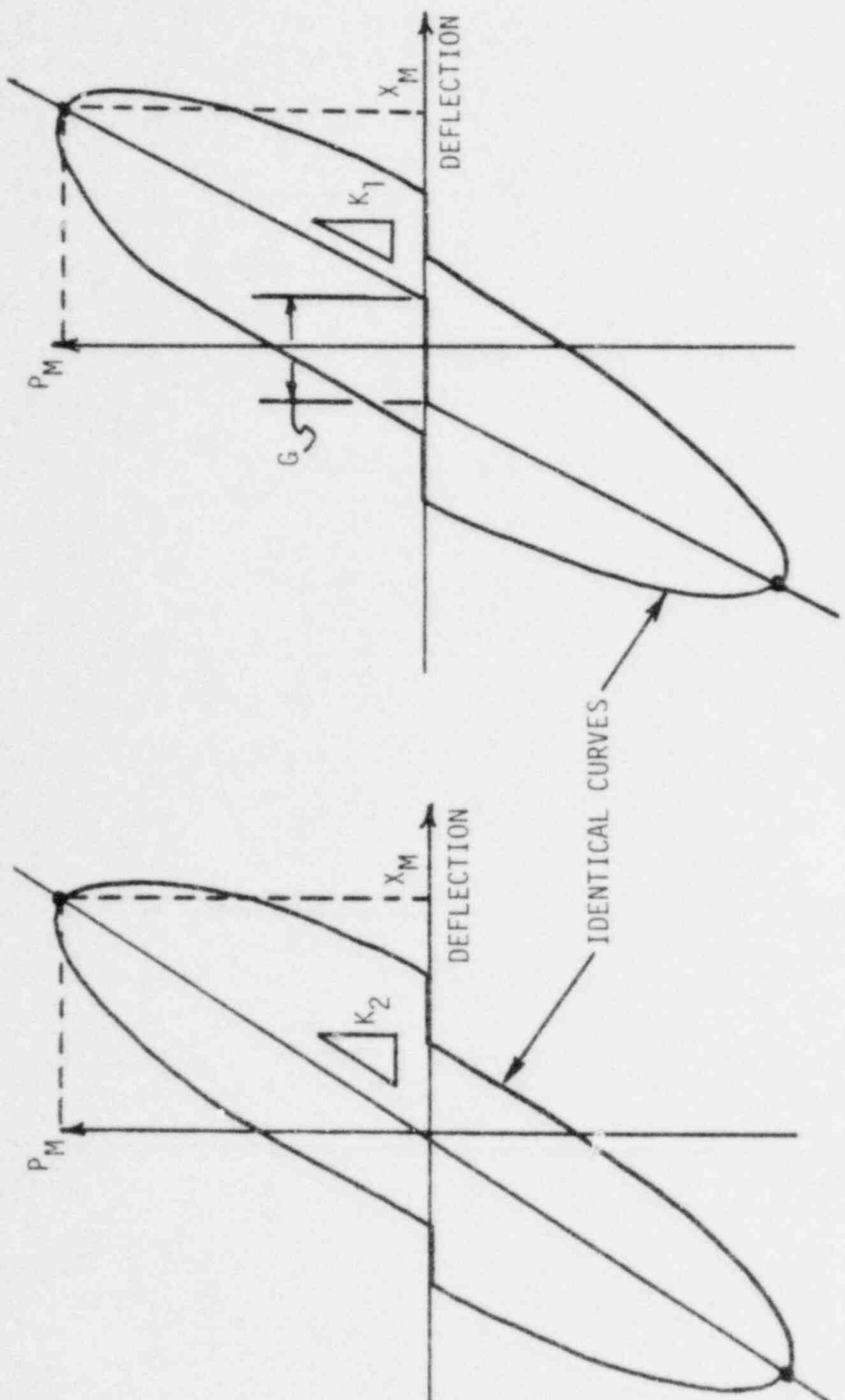


FIGURE A.3.2 ALTERNATE STIFFNESS METHODS

A.4 Effect of Snubber Stiffness Representation on System Response

The problem of the best snubber stiffness representation for the accurate prediction of system response is addressed in this section.

The problem was investigated with the aid of the SDOF lumped mass system shown in Figure A.4.1. The responses of this system to the El Centro seismic event were obtained for various snubber stiffness representations and are compared below. The detailed stiffness representations in the comparisons were based on the data of Section A.2 with various lock velocities and bleed rates as defined below and the K_1 and K_2 representations as defined in Figure A.3.2. Thus, for the first of the preceding, Figure A.3.1 (b) and (c) are typical Lissajous curves.

We first consider the case of zero clearance. Table A.4.1 summarizes the results of several response studies. The data indicate that for this case system response can be predicted with reasonable accuracy when the lock velocity is less than 40 in/min. The data also indicate that maximum snubber reaction loads will be overpredicted with the spring representation for activation levels below 40 in/min and underpredicted if exceeded.

The results of studies for an actual nonzero clearance case are summarized in Table A.4.2. They indicate that an "effective" stiffness representation must include the clearance as part of its characterization.

If the static stiffness representation with zero gap is utilized, the maximum system displacement can be approximated by increasing the resulting displacement by one-half the clearance, e.g.:

$$\begin{aligned} \delta &= 0.02154 + 0.5 \times 0.4 \\ &= 0.4154 \text{ in (cf } 0.4304 \text{ in)} \end{aligned}$$

The impact loads are however underestimated.

TABLE A.4.1

COMPARISON OF SYSTEM RESPONSE FOR VARIOUS SNUBBER STIFFNESS REPRESENTATIONS FOR ZERO CLEARANCE CASES

SNUBBER STIFFNESS REPRESENTATIONS[1]							SYSTEM RESPONSES	
DETAILED MODEL			SPRING REPRESENTATIONS				MAXIMUM DISPLACEMENT (In)	MAXIMUM SNUBBER FORCE (lbs)
GAP (In)	LOCK VELOCITY (In/Min)	BLEED RATE (In/Min)	GAP (In)	K (lb/in)	K_1 (lb/in)	K_2 (lb/in)		
0	10	8					.01078	1674
0	10	50					.01565	1523
			0			164,000	.01275	2090
0	20	50					.02237	1917
0	35	50					.02929	1862
0	50	50					.06926	2848

[1] See Second Paragraph of Section A.4 for Definition

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TABLE A.4.2

COMPARISON OF SYSTEM RESPONSE FOR VARIOUS SNUBBER STIFFNESS REPRESENTATIONS FOR NONZERO CLEARANCE CASES

SNUBBER STIFFNESS REPRESENTATIONS[1]							SYSTEM RESPONSES	
DETAILED MODEL			SPRING REPRESENTATIONS				MAXIMUM DISPLACEMENT (In)	MAXIMUM SNUBBER FORCE (lbs)
GAP (In)	LOCK VELOCITY (In/Min)	BLEED RATE (In/Min)	GAP (In)	K (lb/in)	K_1 (lb/in)	K_2 (lb/in)		
.04	10	50					.04524	4298
			.04		156,250		.04304	3600
			0			125,000	.03047	3808
			0		156,250		.02154	3366

[1] See Second Paragraph of Section A.4 for Definition

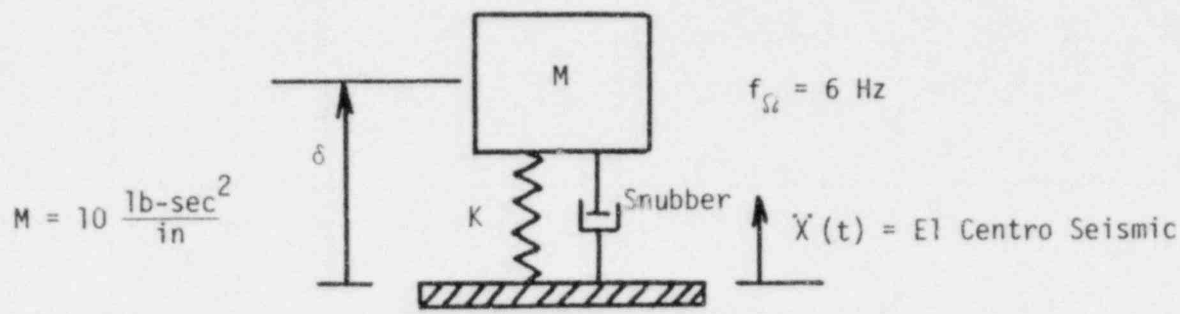


FIGURE A.4.1 SDOF LUMPED MASS MODEL

A.5 Effect of Other Snubber Representation on System Response

The results of comparisons of system responses for two different snubber representations are contained in this section. The basic system considered for the comparisons is of the type shown in Figure A.4.1, i.e., a SDOF lumped mass system. The two snubber representations considered were the usual linear viscous model and the detailed model of Section A.1. The models were subjected to both harmonic base excitations and the El Centro seismic event input. The system natural frequency for harmonic inputs was 8Hz but varied between 1Hz and 13Hz for the seismic input.

A comparison of the maximum displacement between the two representations when subjected to harmonic inputs is shown in Figure A.5.1. The figure indicates that larger displacements are predicted by the detailed snubber model of Section A.1 and the difference in maximum displacement between the two models increases as the input frequency increases. The results also showed that in the case of the detailed snubber model, the rate of load change in the snubber increases as the frequency increases. Since the fluid exits the orifice at a faster rate, the cylinder pressure decreases at a faster rate thereby reducing the restraining load on the mass and increasing the displacements.

Observations similar to the above were also obtained for the seismic input case (See Figure A.5.2). Local resonant conditions may however occur as indicated by the 3Hz peak response.

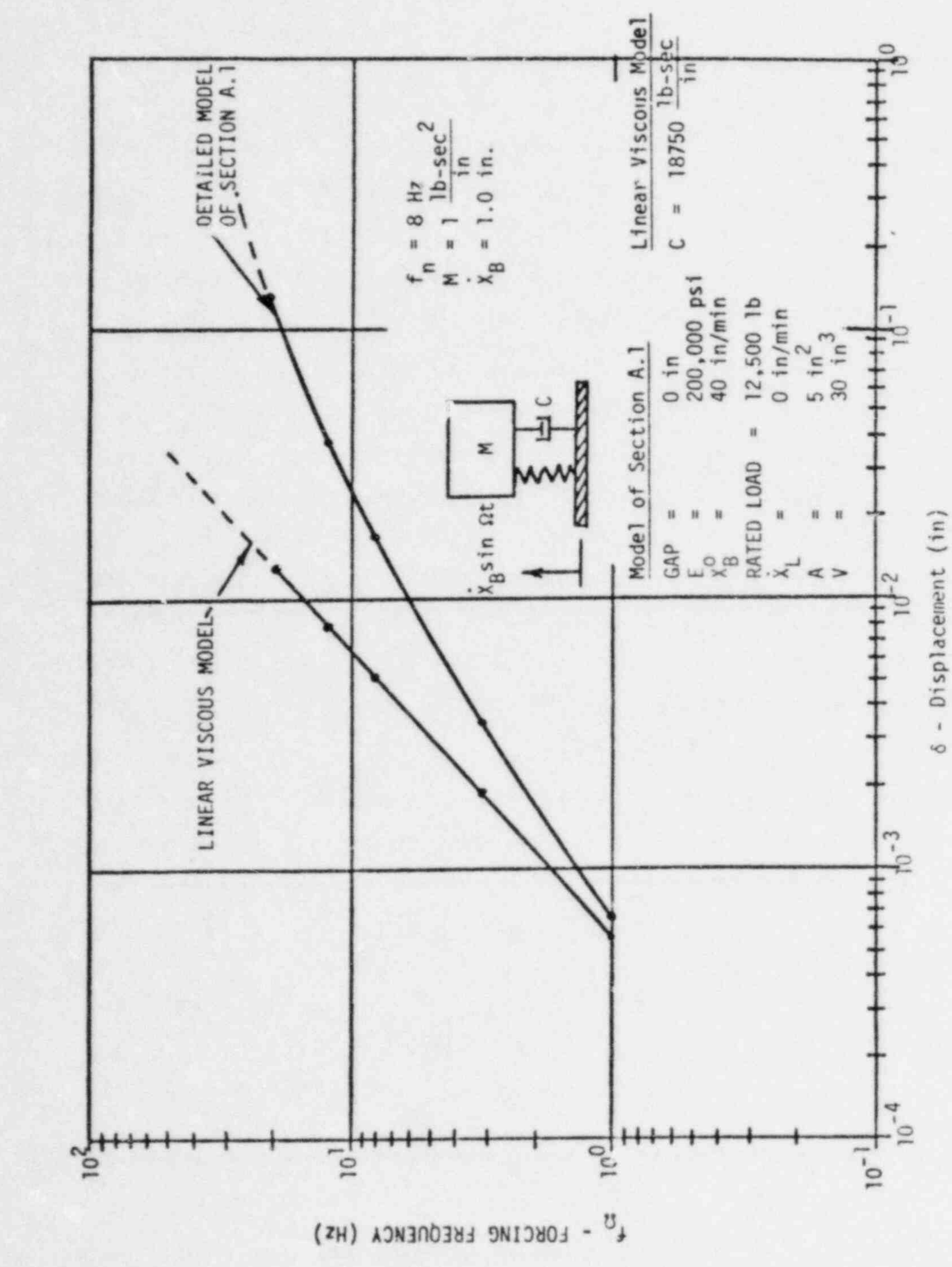


FIGURE A.5.1 · COMPARISON OF RESPONSE CHARACTERISTICS

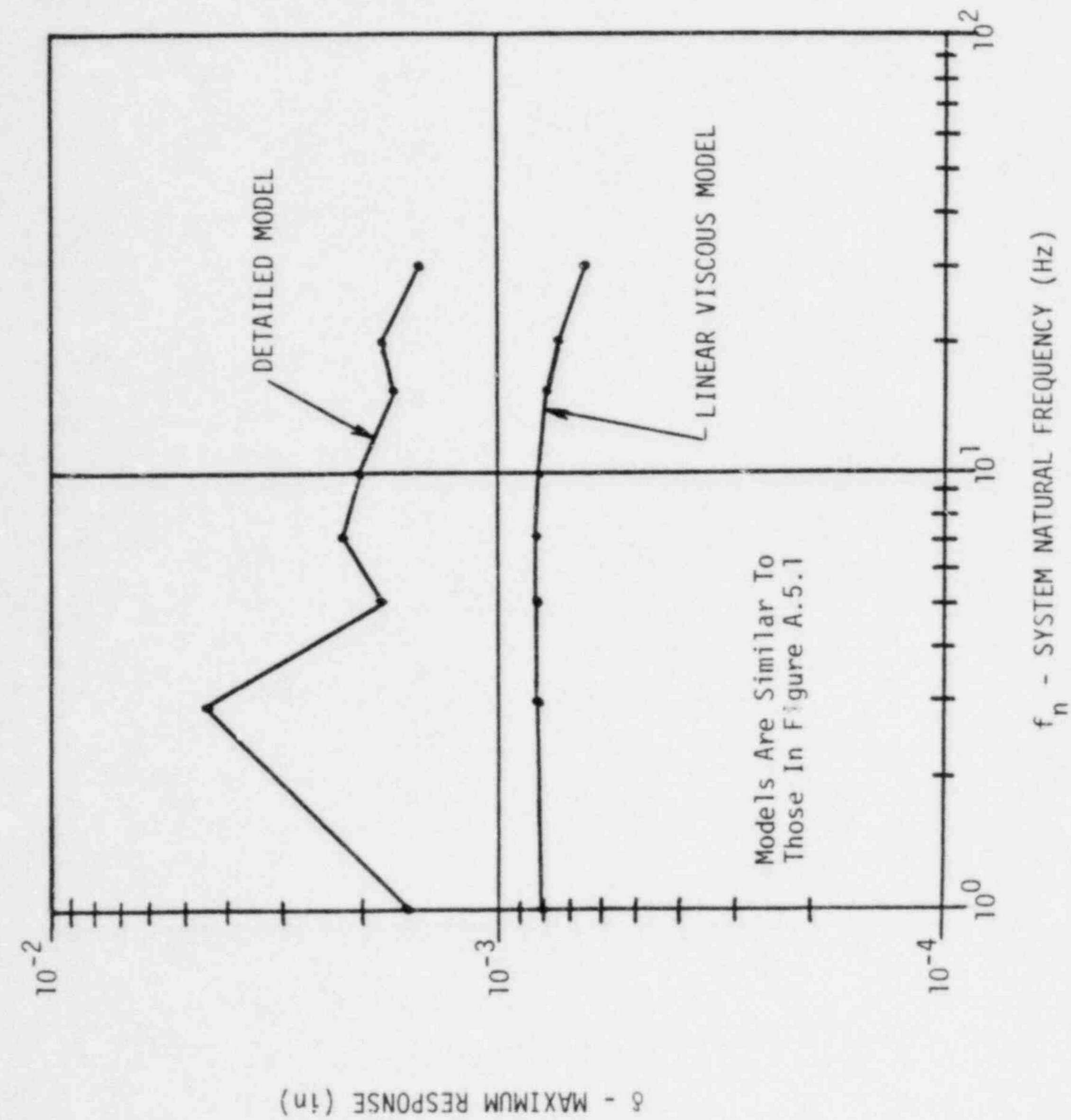


FIGURE A.5.2 COMPARISON OF RESPONSE CHARACTERISTICS FOR EL CENTRO SEISMIC INPUT

A.6 Analytic Solution for SDOF System Under Harmonic Input With Detailed Snubber Representation

An analytical solution for the SDOF system shown in Figure A.6.1 when subjected to harmonic inputs $X = X_B \sin \Omega t$ is reviewed in this section. The detailed snubber representation was used with zero clearance. Equation (5) of Section A.1 therefore becomes

$$\frac{dF}{dt} = k(\dot{\delta} - CF) \tag{1}$$

where

$$k = \frac{EA^2}{V}$$

$$\dot{\delta} = \text{Piston velocity}$$

In terms of dimensionless parameters the response of this system can be expressed as,

$$\left(\frac{\delta}{X_B}\right) = \frac{\beta^2(\beta^2 + Q^2)}{\sqrt{[(1 - \beta^2)(\beta^2 + Q^2) + \gamma\beta^2]^2 + [Q\beta\gamma]^2}} \tag{2}$$

where

$$Q = \left(\frac{ck}{\omega}\right) \tag{3a}$$

$$\gamma = \left(\frac{k}{K}\right) \tag{3b}$$

$$\beta = \left(\frac{\Omega}{\omega}\right) \tag{3c}$$

Figure A.6.2 shows a plot of the response function, Eq. (2) for a family of "Q" curves for a system where $\gamma = (k/K) = 100$. The curve for $Q = 0$ is identical to the classical response curve of a SDOF having a natural frequency of $\sqrt{\frac{K}{m}(1 + \gamma)}$ and no damping. Unlike the classical frequency response curves for which increased damping always produces decreases in response, the response is increased by increasing the "damping", Q, when $\beta < \sqrt{\frac{2 + \gamma}{2}}$. The range of interest of Q is from .1 to 3.0 and the range of γ is from 10 to 100.

Figure A.6.2 indicates the maximum response occurs when the forcing frequency (Ω) is equal to the natural frequency of the system ($\sqrt{\frac{K}{m}(1 + \gamma)}$). This figure indicates that the maximum expected response will be minimized when ($1 < \gamma < 10$).

Figure A.6.3 indicates the maximum response as a function of δ for various Q values.

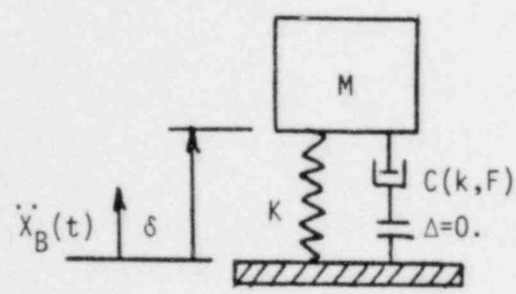


FIGURE A.6.1 S.D.O.F. STRUCTURAL MODEL

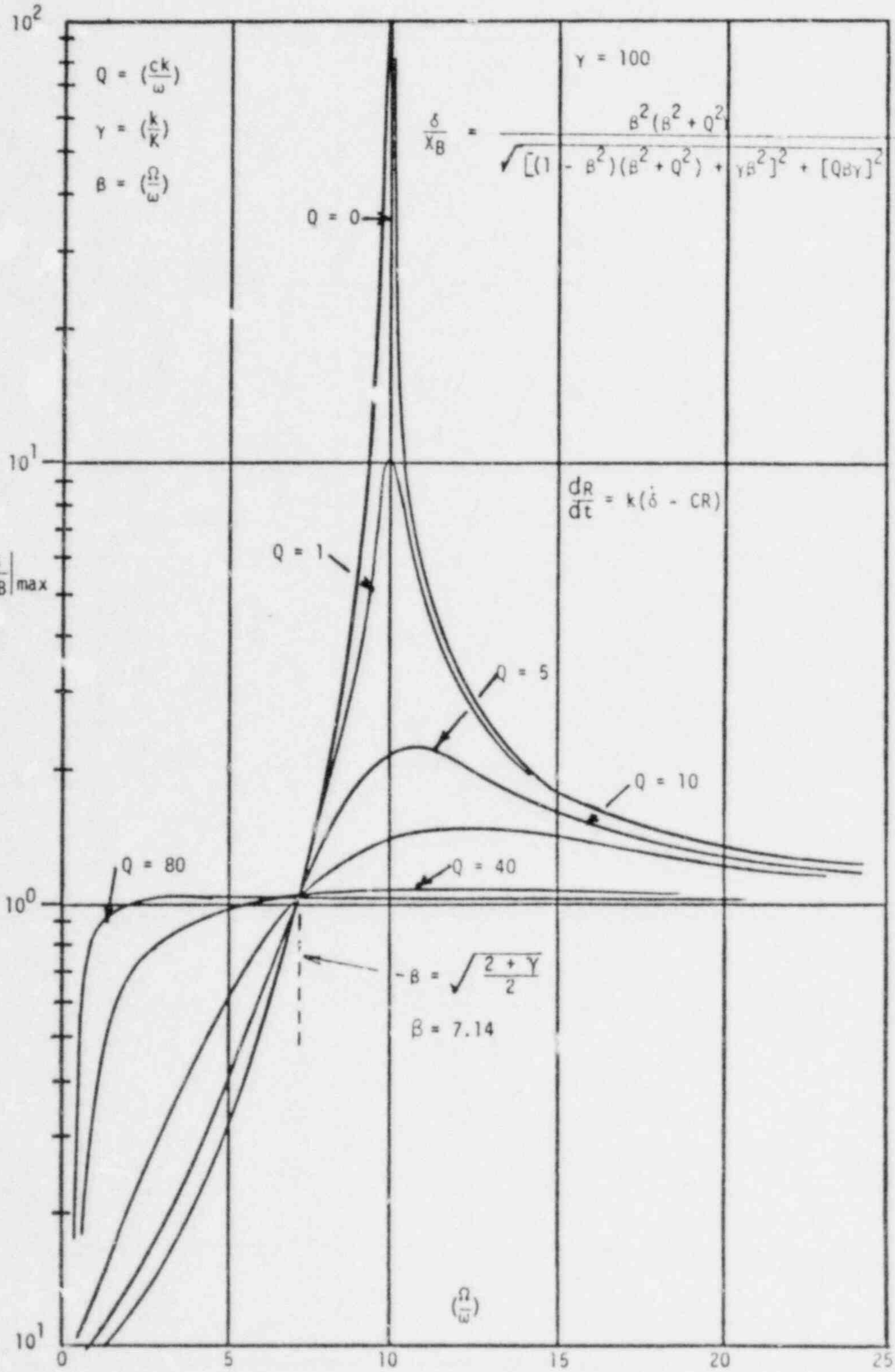


FIGURE A.6.2 SYSTEM RESPONSE FUNCTION VS β

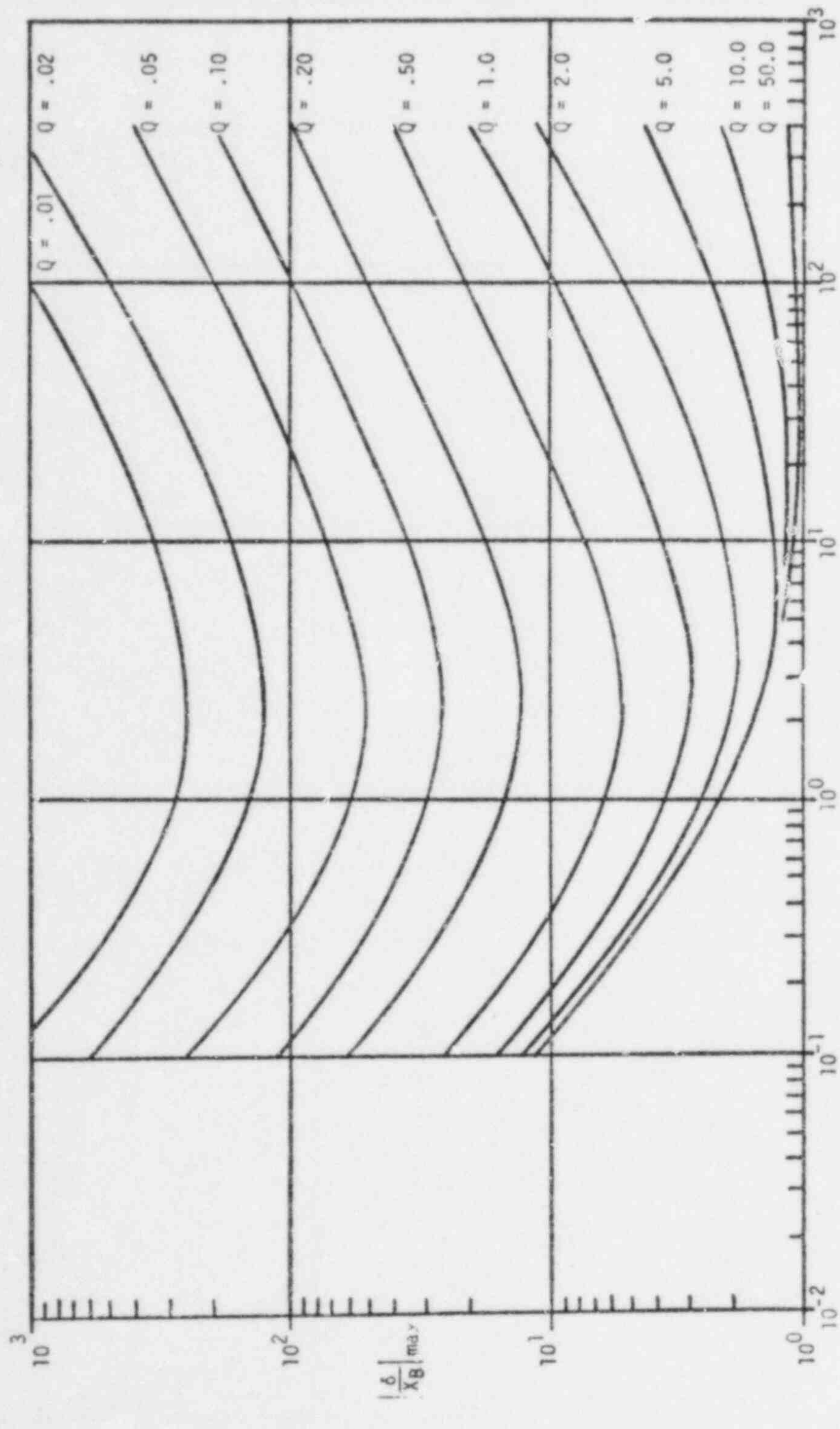
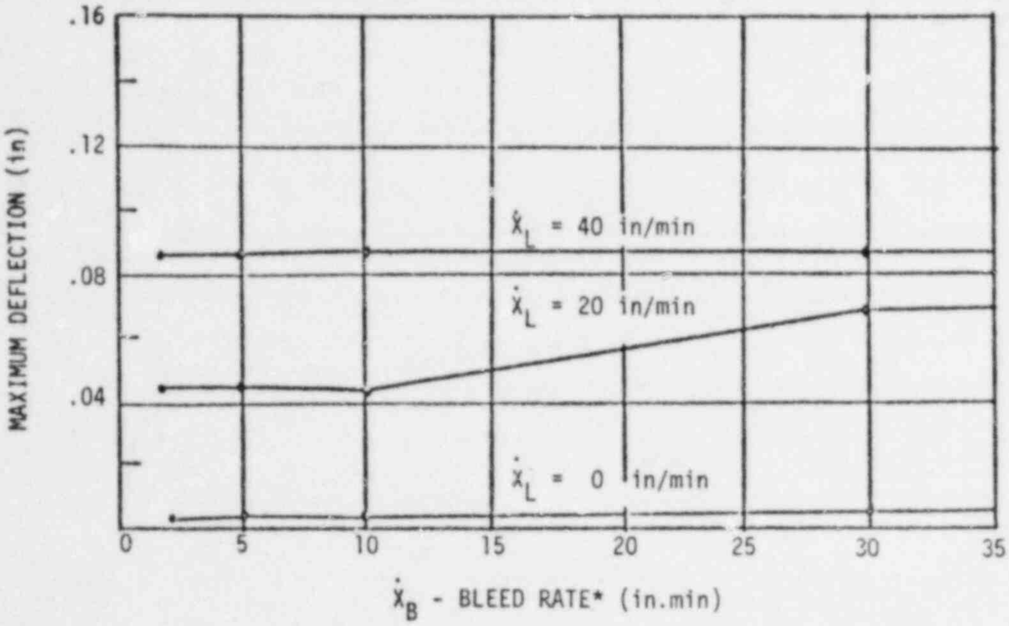


FIGURE A.6.3 SYSTEM RESPONSE FUNCTION VS Y

A.7 Effect of Bleed Rate and Lock Velocity for SDOF System Under Seismic Input With Detailed Snubber Representation

The SDOF utilized in this study is similar to that shown in Figure A.6.1. The natural frequency of the unrestrained, i.e. unsnubbed, system was 2Hz and the mass was 10 lb. sec^2/in. Bleed rates ($\dot{X}_B$) between 2 and 35 in/min. and lock velocities ($\dot{X}_L$) between 0 and 60 in/min. were considered.

The maximum snubber loads and maximum displacements are shown in Figures A.7.1 and A.7.2 for the first 2 and 5 seconds, respectively. The figures show that, in general, the response is insensitive to the bleed rate but maximum displacement and loads increase with increasing lock velocities.



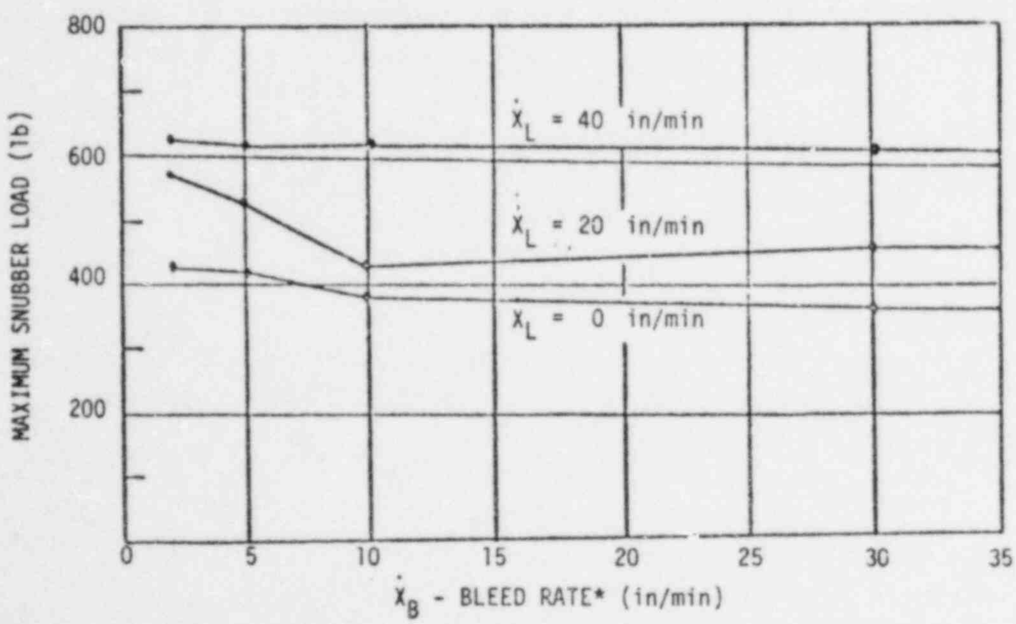


FIGURE A.7.1 RESPONSE AS A FUNCTION OF BLEED RATE FOR SEISMIC INPUT (0 TO 2 SECONDS)

*RATED LOAD = 12,500 lb

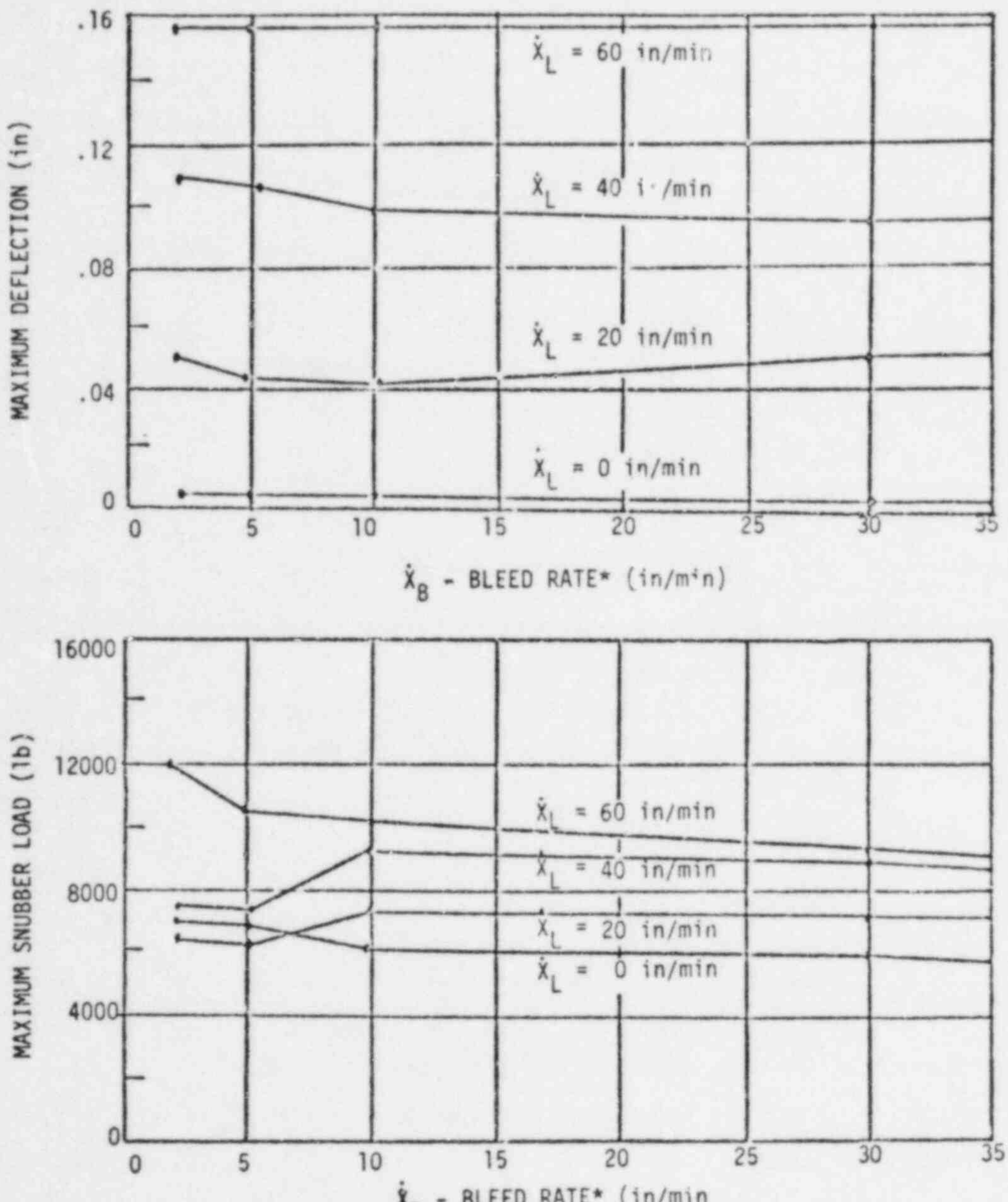


FIGURE A.7.2 RESPONSE AS A FUNCTION OF BLEED RATE FOR SEISMIC INPUT (0 TO 5 SECONDS)

*RATED LOAD = 12500 lb

A.8 Effect of Gap on System Response

Previous studies have indicat d that gaps or clearances have marked effects on system responses in that small changes in clearance may have significant effects on response. The results of further studies to evaluate the effects of clearance on system response for both h monic and seismic loadings are presented in this section. These studies were based on the SDOF system of Figures A.8.1 and A.8.2.

The series of sketches shown in Figure A.8.3 indicates the mathematical representation of snubber clearance - the method used in this study. The center of the unloaded snubber (center of gap) changes with time as shown. This is different than a spring with gap since the unloaded position for the spring does not change with time.

The response of the system is expressed in terms of the following dimensionless parameters;

$$(\frac{F_d}{F_s}),\ (\frac{\Delta}{X_B}),\ (\frac{k}{K}),\ (\frac{\Omega}{\omega}),\ (\zeta)$$

The parameter (F_d/F_s) is the ratio of the "dynamic impact load" when clearance is considered and the "static" or zero clearance load.

The results for harmonic loadings are shown in Figures A.8.4 and A.8.5. The results are based on the assumption $\zeta = 0.10$. Since the damping is not small transient response was attenuated very rapidly and system response reached steady state conditions after a few cycles.

Figure A.8.4 shows the steady state impact load characteristics. These characteristics are based on the maximum values occurring during the 6 - 8th cycles and, in view of the foregoing, can be considered to be the maximum steady state values. Data for several (Ω/ω) values where (Ω/ω) varies from 0.3 to 3.0 are shown. The values of β selected were based on the results of C.6.1.1 where it was shown that the (k/K) ratio should be greater than 20 if the support stiffness (k) is to effect the dynamic response of the system. The data indicates that impact load ratios do not exceed 1.50 for the cases studied.

Figure A.8.5 shows the peak value of the transient response for the same three systems, i.e. values of β considered in Figure A.8.4. The data indicates that the dynamic impact load can be as large as 4 or 5 times the zero clearance value.

Figures A.8.4 and A.8.5 show that the impact loads decrease to zero with increasing (Δ/X_B). This is to be expected since system response does not exceed the snubber clearance (Δ).

The results for seismic loads are shown in Figures A.8.6 and A.8.7. Figure A.8.6 shows the maximum relative displacement response and maximum snubber load as a function of snubber gap. The range of gaps investigated is much greater than realistic values. The study indicates that displacement response trends are much more predictable and stable than snubber reaction load trends. However for snubber clearance less than .10 inch, both displacement response and snubber loads are stable.

In Figure A.8.6 a dotted line is shown besides the displacement response curve. This dotted line indicates that protion of response that is due to clearance. The data indicates nearly a linear relationship between clearance and displacement response. As expected and shown in the figure, impact loads decrease to zero when the gap (Δ) exceeds the system response.

The initial clearance configuration prior to the application of loading affects the maximum snubber load and system response. Figure A.8.7 shows the maximum snubber load as a function of initial gap configuration. The curve shows that the maximum snubber load is dependent on the clearance configuration when the total gap exceeds .10 inches.

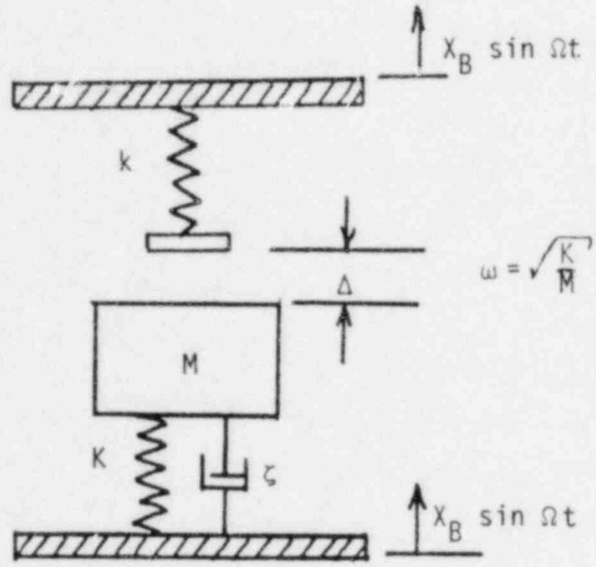


FIGURE A.8.1 HARMONIC INPUT MODEL

$E = 200{,}000$ psi
$\dot{X}_B = 4.$ in/min
$\dot{X}_L = 10.$ in/min
$A = 5\ in^2$
$V = 30\ in^3$

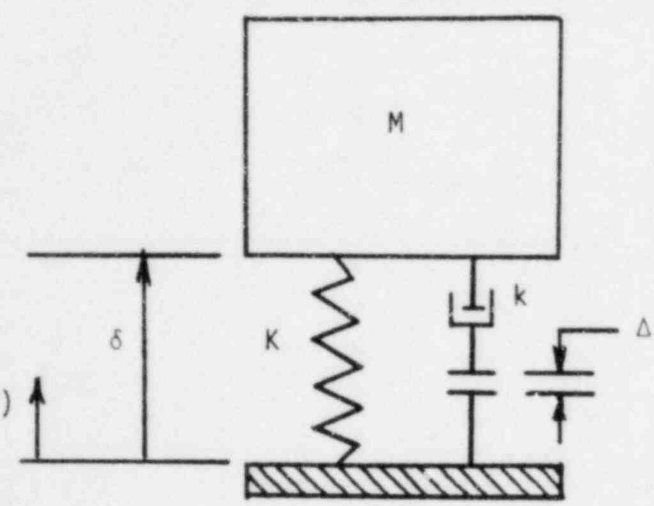


FIGURE A.8.2 SEISMIC INPUT MODEL

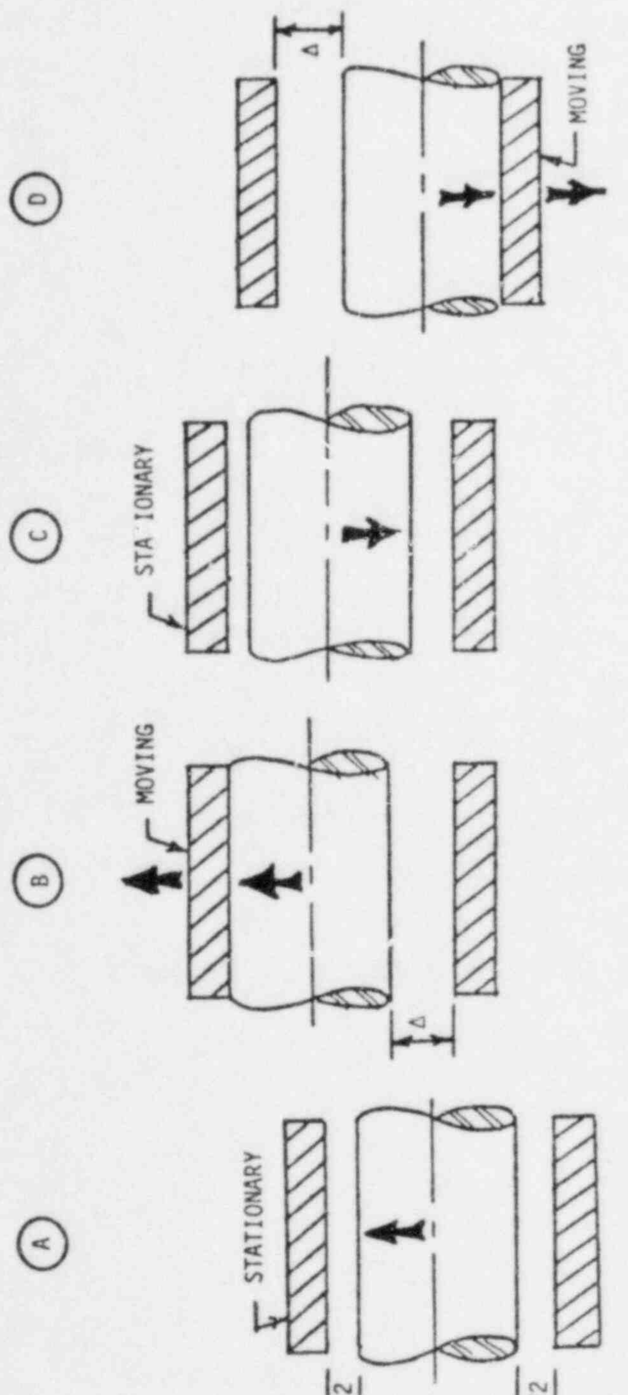


FIGURE A.8.3 STRUCTURAL MODEL FOR SNUBBER GAP

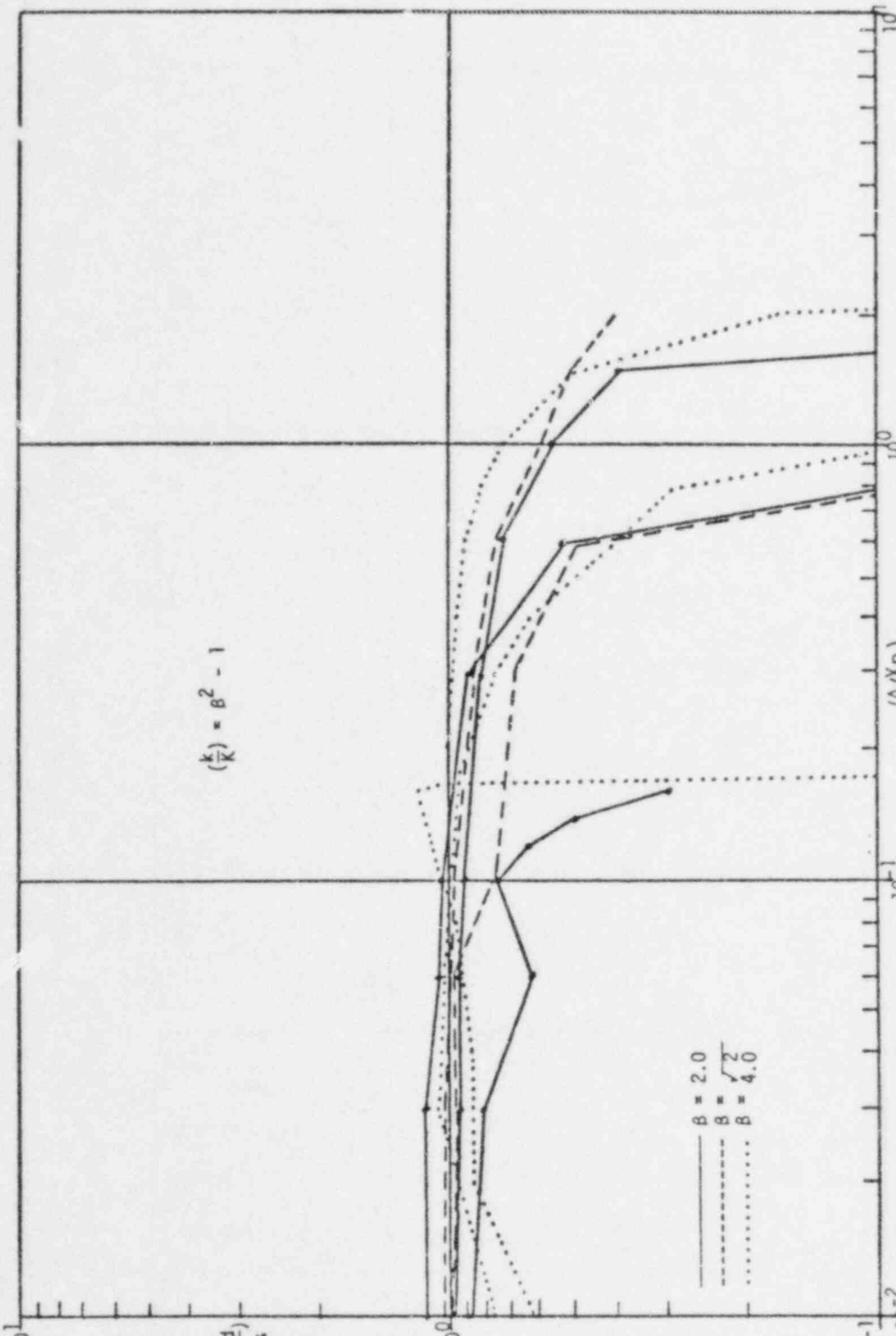


FIGURE A.8.4 MAXIMUM IMPACT LOAD FACTORS (STEADY STATE) - HARMONIC INPUT

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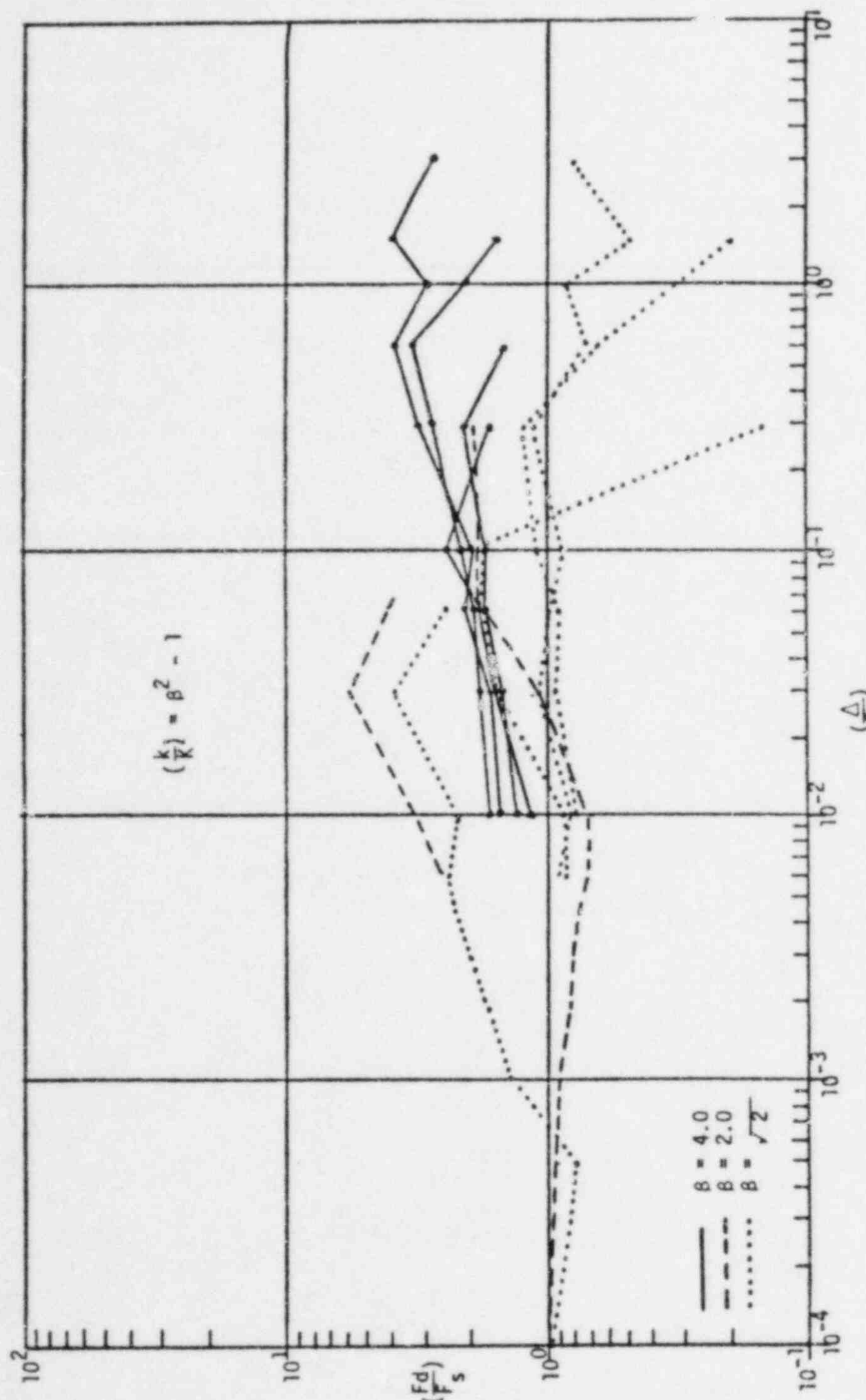

FIGURE A.8.5 MAXIMUM IMPACT LOAD FACTORS (TRANSIENT)- HARMONIC INPUT

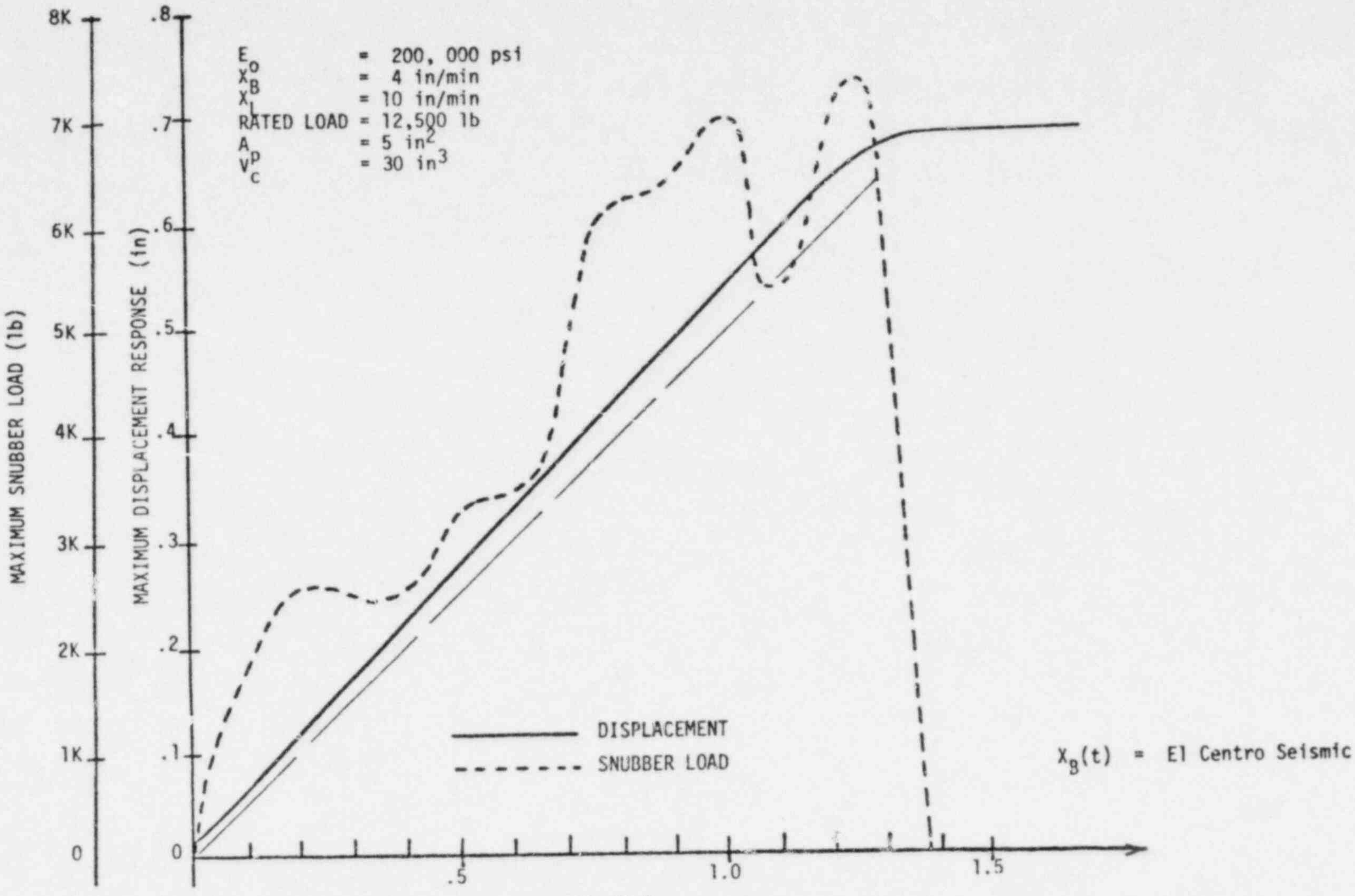

FIGURE A.8.6 RESPONSE OF SDOF SYSTEM AS A FUNCTION OF A LARGE SNUBBER GAP

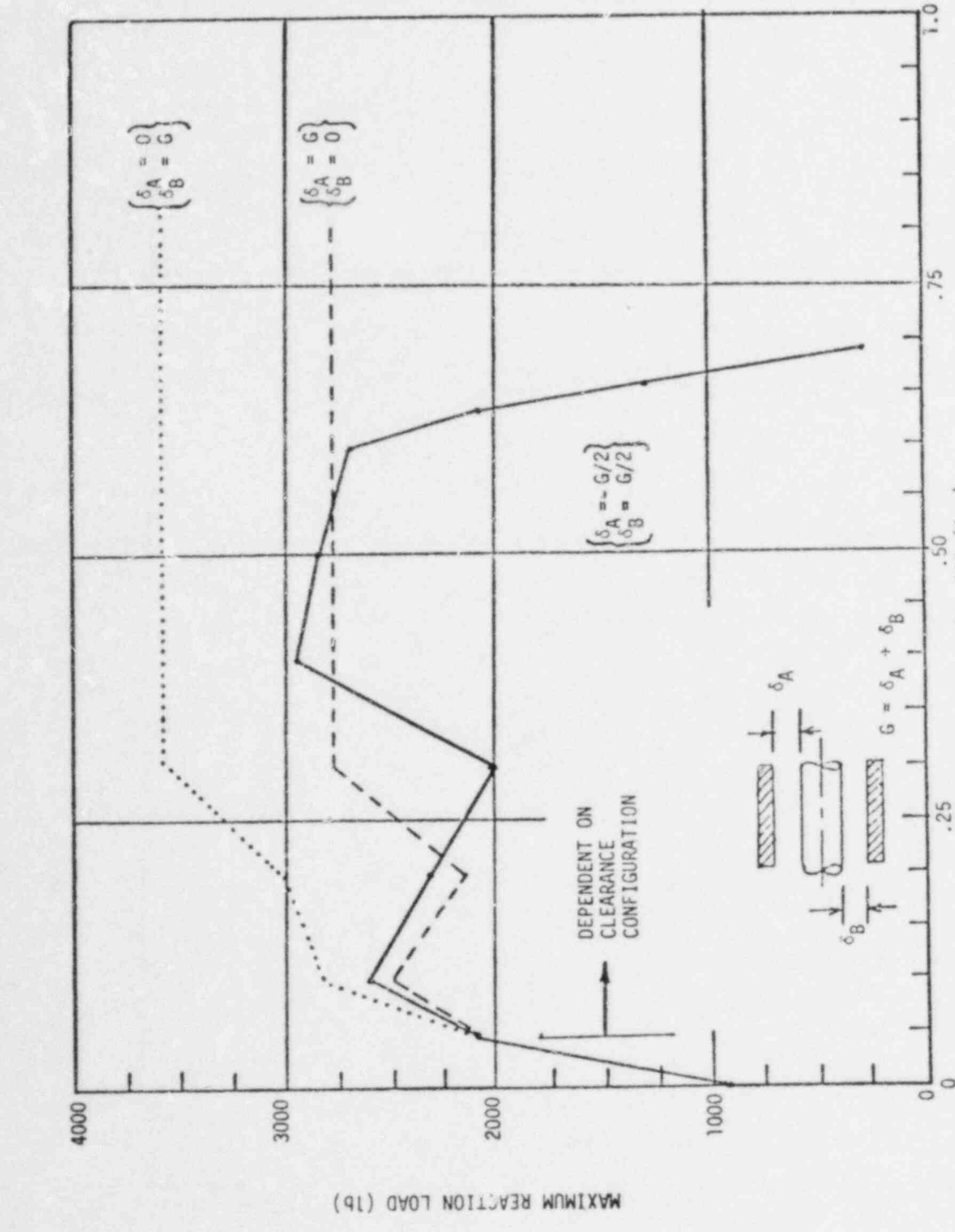

FIGURE A.8.7 CLEARANCE CONFIGURATION EFFECT ON SNUBBER LOADS

A.9 Effects of Mass Magnitude on System Response

A brief study to investigate the effects of component mass on response was made. The study indicates that there exists an optimum mass size for a given snubber at which response may be minimized, or conversely an optimum snubber for a given component size. Figure A.9.1 summarizes the results of this study.

The data indicate that the response of a specific system excited by a seismic disturbance is minimized when the system mass is between 100-200 pounds. This conclusion is independent of the natural frequency but slightly sensitive to fluid bulk modulus. When very large masses are supported ($m > 10^3 \frac{lb\text{-}sec^2}{in}$) the ability of the snubber to minimize system or component response is greatly reduced. When the mass is small ($< 10^{-3} \frac{lb\text{-}sec^2}{in}$) the response is dependent on input magnitude and frequency characteristics.

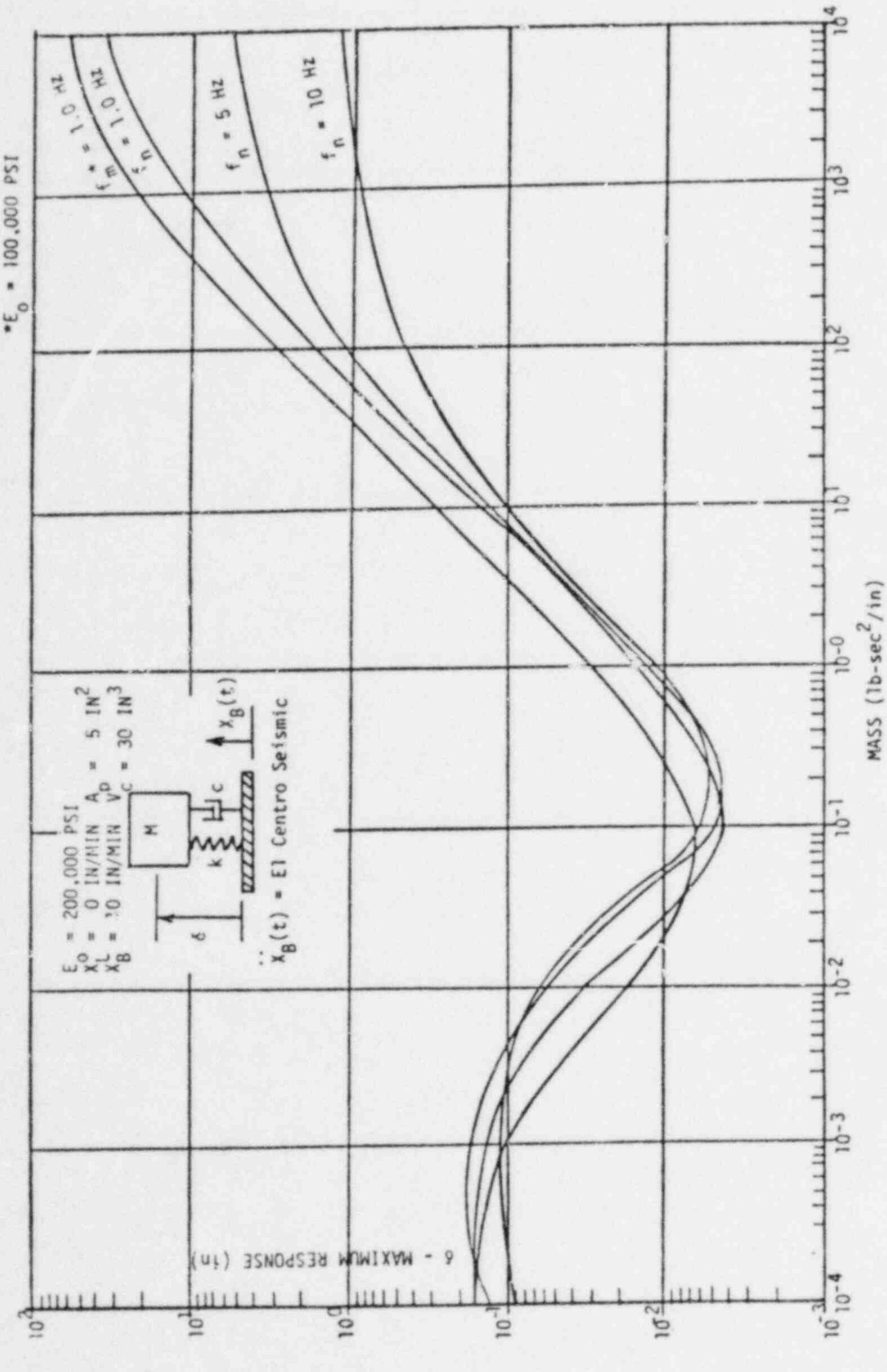

FIGURE A.9.1 EFFECT OF MASS ON SYSTEM RESPONSE

APPENDIX B

GUIDELINES FOR SIMPLIFIED DESIGN AND ANALYSIS PROCEDURES FOR SNUBBERS

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TABLES

- A B S T R A C T -

There are concerns about the reliability of snubbers when used as seismic supports. The Nuclear Regulatory Commission (NRC) developed a Task Action Plan to establish consistent analysis and design rules, qualification testing procedures, and preservice and inservice testing requirements to ensure snubber operability. This document which was developed under the NRC Task Action Plan, provides an analysis and design methodology which will reasonably assure that snubber system response will be bounded within acceptable limits.

B.1 INTRODUCTION

B.1.1 Background

The material presented herein is the result of an effort identified in the Nuclear Regulatory Commission (NRC) Task Action Plan (Reference B1) which was developed to resolve outstanding issues pertaining of snubber operability assurance. An overview of this Task Action Plan is contained in Reference B2.

This guide is a direct result of the analytical and test efforts described in References B3, B4 and B5 and were performed under contract to the NRC. The results of these programs were formulated into the Snubber Guide (Guide) presented herein.

B.1.2 Scope

This guide addresses design and analysis guidelines which relate to the operability of a specific hydraulic snubber design and the specific mechanical snubber design described in B.1.3. These guidelines provide a uniform approach to verifying snubber performance requirements by addressing parameters that affect snubber response such as activation level, release rate, clearance, friction and stiffness. Guidelines relating to multiple snubber support applications are also presented.

The guidelines are based on numerous studies of simple piping and lumped mass structural models subjected to harmonic and seismic loadings. However, since this material is based on a limited number of studies, modifications to the guidelines may be required as future efforts continue.

B.1.3 Applicability

This document is intended to apply to the utilization of mechanical and hydraulic snubbers as seismic restraints for component or piping systems. The use of snubbers for other applications is not implied. The guidelines are applicable for systems supported by: 1) Acceleration activiated mechanical snubbers; and/or 2) Velocity activated hydraulic snubbers with load dependent release rate. A snubber support may consist of a single snubber or two snubbers acting in parallel with either radial or tangential attachments. Multiple snubbers acting in series are not considered in these guidelines.

B.2.0 SIMPLIFIED DESIGN AND ANALYSIS GUIDELINES

The guidelines address the need to establish a consistent design and analysis methodology for snubber supported systems. In addition to specific requirements which must be met, information is provided which might be useful in establishing good design practices.

B.2.1 Design Guidelines

The design guidelines are intended to apply to snubber selection, snubber assembly design (clamps, extension tubes, fittings, etc.), support requirements and snubber characterization parameter limits.

Specific items that are addressed include backup structural stiffness requirements, allowable and desirable ranges for snubber characterization parameters (actuation level, release rate, clearance, friction and flexibility), sizing requirements, allowable mismatch for multiple snubber applications and cautionary statements regarding specific design details.

Related items that are not covered by this document include snubber placement, system design requirements (piping layout, etc.) and environmental effects such as exposure to radiation.

Both the allowable and desirable ranges of characterization parameters are intended to provide reasonable assurance that the snubber and hence system response will be bounded. The former range will limit the response to acceptable values, and the latter range will further reduce the response. These ranges are summarized in Table 1.

B.2.1.1 Snubber Selection

General criteria to be considered in the selection of size and type of snubber to be used for a particular application are addressed in this section.

Surveys of past snubber failures (Reference 2) indicate that consideration should be given to the most common failure mode of the type of snubber to be selected. Hydraulic snubbers usually "fail" in a free condition due to fluid loss or seal deterioration. This failure mode does not affect thermal expansion stresses in the piping system but renders the snubber ineffective as a seismic restraint. Mechanical snubbers, on the other hand, generally fail in a locked condition. Although this failure does not affect the ability to function as a seismic restraint, excessive thermal stresses may be imposed on the piping system as a result of this failure mode. Based on the foregoing only, it appears that long straight pipe runs with few snubbers should be supported with mechanical snubbers whereas piping systems with many bends and short runs should be supported with hydraulic snubbers.

Snubber selection should initially be based on the maximum expected loads. The snubbers selected should be the smallest size available whose rated capacity will not be exceeded by the expected loads. This will minimize the possibility of overstressing of the system due to the restraint of thermal growth by snubber frictional or drag forces. These forces are usually expressed as a percentage of the snubber rated capacity.

Environmental effects should also be considered in snubber selection. Thus, if low frequency loads with long durations are anticipated, a locking type snubber should be considered provided that thermal expansion effects after lockup are not critical. Furthermore, since the physical properties of the fluid strongly affect the lock velocity and bleed rates of hydraulic snubbers and are temperature sensitive, consideration should be given to operating temperature ranges to assure that the activation level and release rate of hydraulic snubbers will remain within acceptable limits. The ability of a snubber to function after long term sustained vibrations such as created by a pump or fluid flow should also be considered.

B.2.1.2 Stiffness Requirements

Minimum support stiffness requirements are recommended to maintain system response within acceptable limits. The support stiffness is the effective stiffness of all the hardware between the geometric center of the component/pipe being supported and the location of the input seismic disturbance exclusive of clearance. Support stiffness shall be based on, but not limited to, flexibility of structural members, snubber and snubber extensions, clamp/fittings and component distortion.

The snubber support stiffness should not be less than 20 times greater than the stiffness of the pipe/component at the point of application. Pipe/component stiffness is expressed as:

$$k_{PIPE} = \frac{1}{\text{Pipe Flexibility}}$$

The pipe flexibility represents the deflection produced by a unit load at the geometric center of the component at the plane of attachment, with other snubber locations having rigid supports. Therefore:

$$\frac{1}{k_{SNUBBER}} + \frac{1}{k_{CLAMP}} + \frac{1}{k_{SUPPORT}} + \text{- - -} \leq \frac{1}{20\ k_{PIPE}}$$

The individual stiffness calculations required for the above expression is made without consideration to clearance. Clearances associated with the clamp or other supporting hardware are added to the existing and fitting clearance. The snubber stiffness is based on its structural compliance which is independent of frequency.

The stiffness of a hydraulic snubber can be expressed in terms of fluid compressibility

$$k_{SNUBBER} \approx \frac{EA^2}{V}$$

E = Bulk modulus (@ operating temperature)

V = Cylinder volume

A = Piston area

The stiffness of a mechanical snubber is equal to the stiffness in compression or tension of a locked snubber, whichever is less.

B.2.1.3 Allowable Parameter Ranges

Allowable parameter ranges are established with the aim of providing reasonable assurance that: 1) The response will be maintained within acceptable limits; and 2) The response can be reasonably predicted with simple analytical techniques thereby precluding the use of sophisticated nonlinear analytical procedures.

The allowable parameter ranges established for the hydraulic and mechanical snubber characterization parameters are based on the following criteria:

a) The response sensitivity to the characterization parameter is in a stable range and the response is therefore predictable;
b) The response at the snubber resulting from seismic excitation will be less than .10 inch;
c) Displacement response and impact loads can be bounded with reasonable assurance that predicted values will not be exceeded.

The parameter limits established herein may be exceeded if it can be demonstrated analytically or by test that system response can be maintained within acceptable limits. Allowable parameter ranges are summarized in Table 1.

B.2.1.3.1 Activation Level

The activation level is defined as that velocity or acceleration at which free motion of the snubber actuator or piston ceases and restricted motion begins.

Although the activation level of a hydraulic snubber is independent of its bleed rate, the response is a function of both quantities. The activation level of a hydraulic snubber shall not exceed 40 inches/minute when the bleed rate is greater than 5 inches/minute. The activation level may exceed 40 inches/minute but not exceed 50 inches/minute if the bleed rate is less than or equal to 5 inches/minute.

The activation level of a mechanical snubber which is equal to its release rate and defined in terms of its acceleration shall not exceed .02g. Application of the mechanical snubber shall be limited to environments where low frequency loadings (<3 Hz) are not anticipated.

B.2.1.3.2 Release Rate

The release rate is defined as the rate of snubber axial movement under load after the snubber is activated. The release rate of the mechanical snubber is the same value as its activation level and independent of load. The release rate of the hydraulic snubber is independent of its activation level and is proportional to the applied load.

The release rate of a hydraulic snubber is commonly defined in terms of its bleed rate and rated load capacity. The bleed rate is defined as the release rate at the snubber rated load. The bleed rate of the hydraulic snubber used for component and piping systems shall not exceed $\dot{X}_B$, where,

$$\dot{X}_B = .50 \times \left(\frac{\text{RATED LOAD}}{\text{COMPONENT WT}}\right) \text{ inch/minute}$$

If the snubber is used to restrain piping, the component weight represents the equivalent piping weight. The equivalent weight is the weight loading at the snubber assuming all snubbers are locked with the gravity loadings acting in the direction of the snubber.

B.2.1.3.3 Clearance

The response of a piping system or component supported with snubbers is highly dependent on the clearances located at the supports. This is especially true of impact loads. Evaluation of clearance at a specific support location shall be based on snubber free play, end fitting clearances, pipe clamp tolerances, and other clearances not indicated. The support clearance is the summation of individual gaps existing between the snubber backup support structure and the center of gravity (or geometry) of the component being supported. The total gap shall not exceed .05 inch.

B.2.1.3.4 Friction

Friction or resistance to free movement of the snubber does not have an adverse effect on its ability to act as a seismic restraint. The loads in general restrain thermal growth and hence increase thermal expansion stresses. Since the snubber drag loads are expressed as a percentage of the snubber rated load capacity, snubbers having a rated capacity much larger than required should not be used. The system response due to seismic excitation will decrease as the snubber friction increases. The response of the system becomes very sensitive to further increases in friction when the friction load approaches about 40% of the applied load. Further increases in friction load will often preclude system response.

B.2.1.4 Desirable Parameter Ranges

The desirable response parameter ranges in this section are intended to further reduce system response and maximize confidence in maintaining response parameters which this study identifies as having the greatest effect on system response.

B.2.1.4.1 Activation Level

The displacement response at the snubber location will be minimized if the activation level is minimized. The activation level should be only as great as that necessary to prevent lockup during thermal expansion or other non-seismic loading events. Activation level has little influence on snubber reaction loads and piping stresses within its allowable range. The activation level of the snubber also has little influence on the effective stiffness within this range.

B.2.1.4.2 Release Rate

The displacement response of a hydraulic snubber will be minimized when the release rate exceeds 5 inch/minute, where the exact value depends on system resonant frequency, activation level, clearance, and effective stiffness properties of the snubber. Generally the displacement response is minimized when the release rate is greater than 5 inch/minute but less than

30 inch/minute. When the release rate has a low value (< 5 inch/minute) the snubber shows very little capacity to dampen response with a velocity dependent reaction component but acts instead as a linear spring. The snubber reaction load will generally decrease as the bleed rate increases with the greatest decrease occurring as the bleed rate increases from 0 to 30 inches/minute.

The release rate of the mechanical snubber is not an independent parameter but is equal to its activation level. Therefore B.2.1.4.1 is applicable to the mechanical snubber release rate.

B.2.1.4.3 Clearance

Displacement response and snubber reaction loads and stresses are minimized when clearance is minimized. The ideal condition is one of zero clearance. As the clearance decreases, response becomes more predictable (stable) and impact effects are minimized. In specific rare cases, response may decrease as clearance increases due to a shift in resonance away from the driving frequency. Impact loads at a support will decrease as clearance increases from a zero value. After a minimum impact load condition is reached, usually when the clearance is about 1% of the input ground displacement, impact loads increase with increasing clearance until the clearance approaches the free system response value at which time the impact loads decrease quickly to zero.

B.2.1.4.4 Friction

From the standpoint of reducing thermal expansion stresses, the friction loads associated with a snubber should be as small as possible. Frictional forces will have negligible effects on the dynamic response of the system provided they are less than 40% of the snubber reaction forces.

B.2.1.4.5 Stiffness

The response of a system is generally sensitive to the "effective" stiffness of a snubber. The snubber stiffness is affected by the structural compliance of the device, the activation level, the release rate, and clearance. An increase in clearance reduces the "effective" stiffness of the snubber, the activation level has insignificant effects on stiffness, and an increase in release rate produces a slight decrease in "effective stiffness.

The ideal snubber configuration is one where stiffness is very high and clearance is negligible. When significant clearance (> .02 inch) is present, large stiffness values are not always desirable. In this situation, reduction of the stiffness will reduce the impact reaction loads of the snubber. In the presence of significant clearance values, the snubber stiffness should be reduced until deflections become critical, and as a result the impact loads and stresses may be reduced significantly.

B.2.2 Analysis Guidelines

The analysis procedures/guidelines are intended to give assistance to the analyst by providing means of reasonably assuring that system response will be maintained within acceptable limits. Guidance is given to the analyst through simplified analysis procedures and recommendations for mathematical modeling of structural elements. Specific attention is given to linear representation of snubbing devices, characterization parameter limits, effective snubber stiffness and analysis procedures (time history, response spectrum, etc.).

A rigorous analysis of a snubber-supported system requires sophisticated analysis techniques due to the highly nonlinear nature of the snubber. The analyst can in certain situations predict with reasonable accuracy the response of the nonlinear component or piping system by using linear analysis procedures. Guidelines are presented which will assist the analyst in achieving this objective.

Mathematical modeling of snubbers, comparison of analysis procedures, linear analysis of nonlinear systems, and simplified analyses procedures are considered.

B.2.2.1 Analysis Procedure

The following analysis procedures are available for analyzing systems or components utilizing snubbers.

a) Time History Analysis (Nonlinear)
b) Time History Analysis (Linear)
c) Response Spectrum Analysis (Linear)
d) Static Inertia Analysis (Nonlinear/linear)

A brief discussion of each procedure is given.

B.2.2.1.1 Time History Analysis (Nonlinear)

This method is a rigorous method for analyzing snubber support systems. The effects of the characterization parameters and the seismic input can be treated rigorously without introducing uncertainties resulting from simplifying assumptions. The method, however, is costly to implement and susceptible to instabilities associated with various integration schemes available to the analyst.

B.2.2.1.2 Time History Analysis (Linear)

This method is less costly to implement than nonlinear analysis procedures and is less susceptible to numerical instabilities. The use of this linear analysis procedure requires the representation of the nonlinear snubber as a linear spring (see B.2.2.2).

B.2.2.1.3 Response Spectrum Analysis

This analysis method is strictly limited to linear systems. This method can be used provided the snubbers can be represented as linear springs. When the range of characterization parameters does not permit representation of the snubber as a linear support, a modified response spectrum (R.S.) procedure may be adopted. When performing a modified R.S. analysis, the results are modified to reflect the use of equivalent linear snubbers in place of the actual nonlinear snubbers.

B.2.2.1.4 Static Inertia Analyses

This method is commonly used to satisfy basic building code requirements and is permitted by Appendix A of the "Code of Federal Regulations," 10 CRF 100 only when it can be demonstrated that this method provides adequate conservatism for the analysis of Category I and Category II components. The method is based on the application of a lateral and sometimes vertical static gravitational loading. Inasmuch as this method does not inherently take into account the dynamic response characteristics of the system, the conservatism or unconservatisms provided by this method are highly dependent on the geometric configuration of the system. Since this document does not address overall system design considerations, the static inertia analysis is precluded from further discussion.

B.2.2.2 Linear Representation of Snubber

Snubbers have usually been represented as fixed anchors or linear springs for the purpose of linear dynamic analysis. Neither is a rigorous representation of the snubber. A linear representation is permitted when both the system response and snubber impact loads (as predicted by rigorous time-history analyses) do not differ from those predicted by linear analyses by more than 20%. This may be verified by testing or comparisons of analytical studies.

There does not appear to be a satisfactory linear representation (spring or rigid support) that will permit system response and snubber reaction loads to be predicted with an accuracy sufficient to justify their use for seismic loading when clearance is present. The best simple representation of a snubber is a nonlinear representation consisting of a linear spring with a gap set equal to the total clearance of the component. This representation enables both response and reaction loads to be predicted with sufficient accuracy in most cases, provided all response parameters are bounded within the limits described in B.2.1.3. However, a linear analysis may be used, provided the total clearance is less than .05 inch, and the load and stresses are multiplied by the appropriate load factors. Snubber reaction loads and stresses shall be increased by 100% for clearances greater than .0 but less than .02 inch. Snubber reaction loads and stresses shall be increased by a factor of 4 for clearances greater or equal to .02 inch but less than .05 inches. Detailed nonlinear analysis is required for systems with .05 inch or greater clearance.

Clearance and structural compliance have the greatest effect on system response when the characterization barameters are within the allowable ranges indicated in B.2.1.3. Displacement response can be determined with greater confidence than snubber reaction loads, particularly when support clearance in excess of .02 inch is present. This situation exists for two reasons: first, the complexity of the seismic input wave form combined with clearance creates a situation of unpredictability; and second, the high stiffness of the snubber creates a situation where, mathematically, the impact load is sensitive to the integration time increment and solution technique. Hence the need for load factors with the linear analysis.

B.2.3 Multiple Snubber Usage

The guidelines addressed in this section are presented in response to concerns regarding the practice of using several small snubbers in place of a single large snubber. These guidelines are based on the results of a test program discussed in Reference B5.

B.2.3.1 Snubber Mismatch

Mismatch of snubber end fitting clearance in multiple snubber supports has a greater effect on load sharing of parallel mounted snubbers than mismatch of activation level or release rate. Uniform load sharing of multiple snubber supports (±10%) can be expected for hydraulic snubbers when end fitting clearance differentials plus lost motion differentials are less than .01 inch and the activation level and release rate are between 8 and 25 inches/minute and 4 and 14 inches/minute, respectively.

Equal load sharing of multiple snubber supports should not be expected if the end fitting clearance mismatch exceeds .01 inches. If the mismatch clearance differential exceeds .01 inches but is less than .04 inches, peak loads shall be assumed twice the uniform load sharing value. Mismatch of end fitting clearance shall not exceed .04 inches.

B.2.3.2 Design Considerations for Hydraulic and Mechanical Snubber Pairs

The load sharing of a hydraulic snubber pair is more sensitive to mismatch of end fitting clearance than the load sharing of a mechanical snubber pair for harmonic input.

The load sharing of a mechanical snubber pair is more sensitive to mismatch of end fitting clearance than the load sharing of a hydraulic snubber pair for seismic input.

B.3 REFERENCES

1) J. P. Knight, J. M. Kovacs: NUREG-0371, "Snubber Operability Assurance," Task Action Plan A-13, October 1977; USNRC, Washington, D. C. 20555

2) J. M. Kovacs: "An Overview of Efforts to Resolve Safety Issues Associated with Snubbers Utilized in Nuclear Power Generating Facilities," ASME PVP-3[illegible], "Piping Restraint Effects on Piping Integrity," 1979, Pp. 5-24

3) A. T. Onesto: "Snubber Sensitivity Study Final Report, FY 1978," ETEC-TDR-78-17, September 29, 1978

4) A. T. Onesto: "A Snubber Response Sensitivity Study," ASME PVP-37, "Piping Restraint Effects on Piping Integrity," 1979, Pp. 41-61

5) A. T. Onesto: "Single Versus Dual Snubber Installations," ETEC-TDR-80-12, July 31, 1980

TABLE B.1 ALLOWABLE AND DESIRABLE SNUBBER RESPONSE PARAMETER RANGES

PARAMETER	TYPE	ALLOWABLE RANGE	DESIRABLE RANGE
ACTIVATION LEVEL	H	< 40 in/min if $\dot{X}_B$ > 5 in/min < 50 in/min if $\dot{X}_B$ < 5 in/min	Low as possible, but great enough to preclude lockup during thermal expansion.
	M	< .02g	
RELEASE RATE ($\dot{X}_B$)	H	$< .50 \times \left(\frac{\text{Rated Load}}{\text{Component Wt}}\right)$ in/min	5 → 30 in/min
	M	See Activation Level	See Activation Level
CLEARANCE	B	< .05 in.	0
FRICTION	B	< 40% Rated Load	As large as possible but small enough to maintain thermal stresses within acceptable limits.
STIFFNESS	B	$> \frac{20}{\alpha - 20\alpha_H}$	$>> \frac{20}{\alpha - 20\alpha_H}$

NOTES:

H = HYDRAULIC (Velocity Activated, Poppet Type)
M = MECHANICAL (Acceleration Activated)
B = BOTH

α = Flexibility of pipe @ snubber location (in/lb)

α_H = Combined flexibility of all hardware between the center of the component and "ground" (in/lb)

APPENDIX C

FISCAL YEAR 197S
ANALYTICAL STUDY

COVER SHEET

ETEC-TDR-78-17
Document No.
(This document contains 160 pages.)

ETEC-TDR-78-17
Document No.

Title: Snubber Sensitivity Study
Final Report, FY 78

Author(s): A. T. Onesto

TECHNICAL DATA RECORD

AUTHOR
A. T. Onesto *A. T. Onesto*
DEPT. & GROUP NO.
720-211

TITLE
Snubber Sensitivity Study Final Report, FY 78

PROGRAM
NRC Contract
FIN #B3076-8

S/A NO

STATEMENT OF PROBLEM

Develop information which will provide the basis for structural analysis and design rules for systems and components which utilize snubbers as supports. Results will be used to assure that dynamic response characteristics of snubber supported systems and components will be bounded within acceptable limits.

ABSTRACT:

The sensitivity of mechanical and hydraulic snubber parameters to system displacements, stresses and forces are analyzed. Acceleration threshold, clearance and friction are evaluated for mechanical snubbers while hydraulic snubber investigations include lock velocity, bleed rate, unlock loading, clearance and friction. The back-up structure is influential for both types of snubbers and although not a snubber parameter, per se, is treated like a parameter. Forcing functions are utilized, and include both harmonic and time history seismic inputs to the mathematical models. Mathematical models are used to simulate snubber characteristics. Special mathematical techniques are developed for economical use in piping programs. Acceptable parameter ranges are established, based on criteria for the various mechanical and hydraulic snubber characteristics.

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FIGURES

1. INTRODUCTION

The objectives of this program under NRC contract FIN #B3076-8 are to perform analytical evaluations and parametric studies to:

(1) Identify structural and performance parameters which significantly affect snubber dynamic response characteristics;

(2) Determine the sensitivity of the snubber response and the corresponding effects on the snubber supported system to variations in each parameter identified above; and

(3) Develop simplified analyses techniques and design rules which will bound the response of the system within acceptable limits.

The areas of work covered in (1) and (2) were completed this fiscal year, FY 78, and work under (3) is planned for completion in FY 79.

Simple analytical models with simple loadings were used initially to establish dimensionless response parameters permitting insights into the basic problem. The use of steady-state harmonic response calculations was invaluable as a tool. After the simple models and loadings were examined the more sophisticated models and loadings were studied. Figure 1.1 illustrates both the simple and more sophisticated models.

Snubber parameters, e.g., acceleration threshold parameter and bleed rate, were considered invariant quantities, i.e., unaffected by feedback with the restrained system. Although it is recognized that there is some interaction between the various snubber parameters and system response, these effects were not considered in this study. The work does not consider variables that effect parameter magnitudes such as hydraulic fluid properties, entrapped air, and inertia effects.

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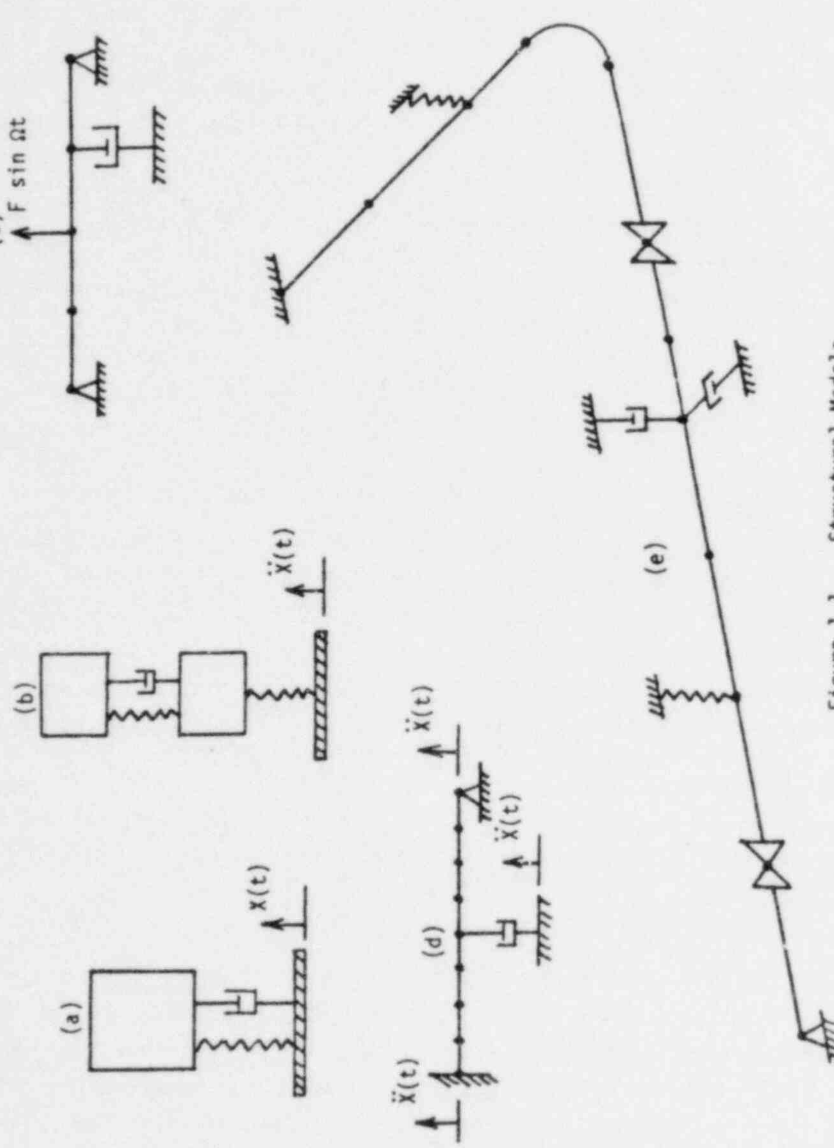

Figure 1.1 Structural Models

The first two objectives of the program are related to response sensitivity, i.e., the amount of change in response due to a corresponding change in a parameter. The load or response magnitude is considered when evaluating or determining the acceptable parameter ranges that maintain acceptable response.

The evaluation of system response considers not only displacement response but also stress and snubber reaction load response characteristics. In general, displacement response is important when considering the response of components such as pumps, steam generators, and valves. Stress and reaction load responses, on the other hand, are often related better to piping systems.

The following five tasks are used as a basis for meeting the program objectives.

TASK 1 - IDENTIFY PARAMETERS AND DETERMINE PROBABLE RANGES OF PARAMETER VALUES

This task involved determining which snubber parameters affect dynamic response. Significant parameters associated with mechanical snubbers include the acceleration threshold parameter, clearance, and friction. Parameters of significance with hydraulic snubbers include lock velocity, bleed rate, friction, and clearance. The identification of parameters and probable ranges were made based on Nuclear Regulatory Commission (NRC) and vendor suggestions.

Consideration was also given to stiffness and dynamic effects of the snubber back-up support structure. These effects are influential in attenuating or modifying parameter response sensitivity.

TASK 2 - DEVELOP SNUBBER ANALYTICAL MODELS

Under this task mathematical models were established for the hydraulic and mechanical snubbers. Using the mathematical model for a snubber, a solution technique based on a modified modal analysis procedure was developed. This technique is useful in incorporating snubber parameters since it is not susceptable to convergence problems, and requires minimal computer time. Special purpose computer codes were used for limited studies while a general purpose piping code SUPERPIPE, Reference 1, was used for more complex analyses.

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TASK 3 - SELECT APPROPRIATE TIME HISTORY FORCING FUNCTIONS

The selection of forcing functions is important if meaningful results are to be obtained. Harmonic loadings provide insights into physically understanding the problem. Seismic loadings are then used in evaluating or relating response sensitivity to actual conditions. The four most used, strong motion seismic events for the design of facilities have been employed as "benchmarks" for this study. They include: El Centro (California-1940); Taft (California-1952); San Fernando (California-1971); and Washington (Washington-1949). These seismic events represent varying earthquake magnitudes and frequencies.

TASK 4 - EVALUATE RESPONSE SENSITIVITY FOR PARAMETERS ESTABLISHED IN TASK 1

This part of the study consisted of forming response curves for the various structural models and for the various loadings with the aim of graphically presenting the response sensitivity data. Consideration is given to displacement, stress and loads.

TASK 5 - ESTABLISH AN ACCEPTABLE RANGE OF PARAMETERS THAT WILL BOUND SYSTEM RESPONSE WITHIN ACCEPTABLE LIMITS

Establishing the acceptable parameter ranges requires the consideration of all the findings of the previous tasks. This is one area where the actual magnitude was considered in the time history forcing functions for the four seismic events utilized. The acceptable ranges were based on the dynamic response characteristics of displacement, stress and snubber loadings.

The results or conclusions obtained from this study are based on a limited observation contained herein. Experimental and analytical work by other authors was not considered. The effort scheduled for FY 79 will supplement this work and as a result, future refinement of the acceptable ranges for the various parameters is expected.

2. SUMMARY

This study interrogated the externally evident parameters associated with mechanical and hydraulic snubbers. The basic approach taken was to first use simple and fundamental models. As more sophisticated systems were developed analytical comparisons were made with the fundamental investigations. Summaries of each parameter are presented below to highlight results of the snubber sensitivity study for FY '78.

ACCELERATION THRESHOLD PARAMETER (A.T.P.) — Analytical studies of simple systems reveal that the acceleration threshold parameter has the potential of increasing system response rather than reducing the response as might be expected when snubbers are used. Analytical studies indicate that an amplification may exist when compared to the base motion when the forcing frequency is less than the natural frequency of the snubbed system or when low forcing frequencies (< 3 Hz) are applied to the system. For the more complex seismic inputs, amplification also occurs in some cases. Acceptable ranges for this parameter depend on the frequency response characteristics of the system being restrained.

VISCOUS PARAMETERS (LOCK VELOCITY, BLEED RATE, RELEASE CRITERIA) — Response sensitivity of the viscous parameters is not greatly dependent on the forcing frequency or on the magnitude of the applied load. The release criteria (condition when velocity changes from bleed velocity to free velocity) has the least effect on response of the viscous parameters. Lock velocity has the greatest effect on system response. The response of highly loaded systems (based on benchmark seismic loadings) are in general reduced to 5 percent of the unrestrained response for realistic bleed rates when the lock velocity is less than 1 in/sec. A realistic bleed rate (< 1 in/sec) reduced response to acceptable limits since the snubber represents a highly "damped" or critically damped restraint.

FRICTION LOADS — The study of friction (coulomb) indicates that small friction loads have negligible effect on system response. For the case of harmonically excited simple system, the response is affected significantly only after the friction load exceeds 40 percent of the applied load. The same trends were observed for nonharmonic (seismic) loadings.

CLEARANCE — Clearance reduces the effectiveness of other snubber parameters in reducing the response of the system (stress and displacement). Snubber reaction loads, however, may be increased or decreased by clearance depending on the magnitude of the load and the amount of clearance, compared to the free displacement (no support) condition.

BACK-UP STIFFNESS — Since the "effective" stiffness of a snubber is generally greater than its back-up support structure, the snubber response characteristics, e.g., damping properties, acceleration limits, etc., may be "washed out" by flexible supporting structures. The results of the work presented in this study indicate that the combined "effective" stiffness of the back-up structure and snubber must be at least twenty times greater than the piping or component stiffness to be totally effective in reducing response. In terms of hydraulic snubber viscous parameters the support stiffness should be greater than 1.2 ($C\Omega$) and 1000 C/M where C is the snubber rated load (lb) divided by the bleed rate (in/sec), Ω is the predominant forcing frequency in rad/sec, and M is the support structure effective mass.

SUPPORT DYNAMIC INTERACTION — Since the "effective" snubber stiffness is in general quite large, the snubber acts as a "rigid" connection between the supporting structure and supported system. A classical resonant condition can occur if the forcing frequency coincides with the combined system-support natural frequency. Based on this situation it is advisable to have a supporting structure with a higher natural frequency than the supported system. Ideally it is desirable to have the supporting structure with a natural frequency that is twice that of the supported structure. In addition to the specific parameters mentioned above the study indicated other results some of which are discussed below.

The response analysis of a specific single degree of freedom system with a snubber having negligible clearance indicated that the response resulting from each of four different seismic inputs was nearly the same. Therefore, it can be concluded that the response of a component is not greatly dependent on the seismic input, whereas the response of a piping system may or may not be. This depends on the dynamic amplification resulting from the forcing frequency and its coincidence with the natural frequency.

Although snubber response appears to be insensitive to changes in component load and forcing frequency, this may not be the case for the piping systems.

Table 2.1 shows the range of snubber parameters that ensure a properly designed system will operate within acceptable limits. These ranges are based solely on the work presented to date in this report and may be revised pending results of the tasks scheduled for FY '79.

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TABLE 2.1

ACCEPTABLE LIMITS FOR SNUBBER PARAMETERS

Acceleration Threshold Parameter	<0.[illegible] g
Lock Velocity	<60 in/min
Bleed Rate ($C = \frac{\text{rated load}}{\text{bleed rate}}$)	$>(10)^4$ lb-sec/in
($C/M = \frac{C}{\text{mass}}$)	$>(10)^3$ sec^{-1}
Friction Load	40% rated load
Back-Up Structural Stiffness	
Viscous Parameters$^{(1)}$	$k > 1.2(C\Omega)$
Effective Snubber Stiffness$^{(2)}$	$K/K^* > 20$
Clearance	
No Other Parameters$^{(3)}$	$(\delta_G/\delta_F) < 0.15$
With Other Parameters$^{(3)}$	$(\delta_G/\delta_F) < 0.10$

NOTES:

(1) C = Rated Load/Bleed Rate (lb-sec/in)
Ω = Predominant Forcing Freq. (rad/sec)

(2) k = Support Stiffness (lb/in)
$K^* = [\text{Pipe Flexibility}]^{-1}$ (lb/in)

(3) δ_G = Clearance Gap (in)
δ_F = Unrestrained Response (in)

3. SNUBBER PARAMETERS

The scheme adopted for identifying snubber parameters was to isolate parameters applicable to mechanical snubbers, hydraulic snubbers and those common to both. Following analyses of the individual parameters, combinations are addressed.

3.1 The Mechanical Snubber

The mechanical snubber is a device that operates on the principle of limiting the acceleration of any pipe movement, relative to the snubber base, to a threshold value of a specified "g" level, Figure 3.1. Should a disturbance accelerate the pipe in either direction, a braking force is applied within the arrestor to restrict the acceleration to specified limits. Thermal expansion which is a gradual movement is not restricted. A particular feature of the snubber is that its performance is independent of the force being applied. The design of the unit is completely symmetrical. Braking action in both tension and compression loading is identical.

3.1.1 Acceleration Threshold - This snubber parameter is unique to the mechanical arrestor. It represents the limit acceleration of pipe movement relative to the base of the snubber. For the "ideal" mechanical arrestor, unrestrained pipe movement is permitted when pipe accelerations are less than a specified "g" level. At acceleration rates equal to or greater than the threshold parameter, the snubber reacts with a load that does not permit pipe movement to accelerate at a rate greater than the threshold value. The acceleration threshold is a "built-in" feature of the snubber. It cannot be adjusted or modified. The acceleration threshold level is normally set from 0.02 g to 0.08 g. Snubbers can be designed for other values that might be desired. Consider the Pacific Scientific Snubber, one that is in extensive use. Here the threshold values are basically a function of the capstan spring rate and the inertia mass, Figure 3.1. Other type mechanical snubbers are assumed to have the same acceleration threshold characteristics.

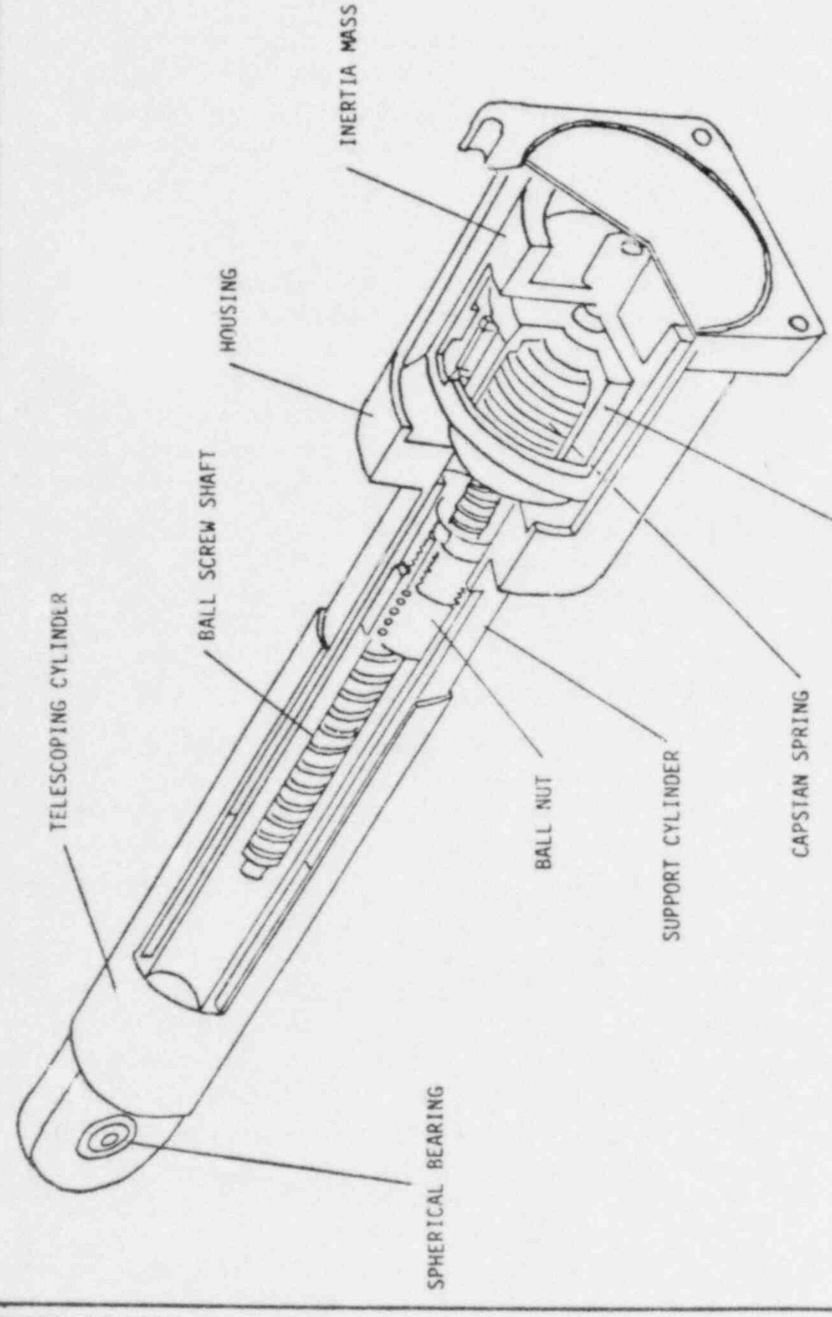

Figure 3.1 Cutaway of Typical Pacific Scientific Arrestor

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At impact loads greater than the rated capacity, the capstan spring may slip as it engages the torque transfer drum. This "safety" device prevents failure of the device and permits accelerations greater than the acceleration threshold limit. Since snubbers should not be used above their rated capacity, this situation is not being considered.

3.1.2 Clearance - The primary source of clearance is established by setting the ball nut mechanism. This is where the linear translational motion caused by a seismic disturbance is converted to rotational motion of the ball screw and drum assembly.

3.1.3 Friction - Breakaway or friction loads are created when parts rub or roll upon each other. Mechanical snubbers have fairly low breakaway loads, usually about 1 percent of the rated capacity. These loads are generated in the ball nut mechanism, bearing sleeves, and in the telescoping cylinder. The friction loads are sensitive to corrosion and certain types of contamination. Situations may arise where friction or breakaway forces exceed this 1 percent value.

3.2 The Hydraulic Snubber

The hydraulic arrestor is a velocity sensitive device used for restraining pipe motion during dynamic loading while permitting free thermal motion. A control valve permits unrestricted motion at low velocities, whereas resistance to motion is encountered at high velocities. The operational characteristics of the hydraulic snubber are related in general to two parameters - the locking velocity, and the bleed velocity. The locking velocity is the velocity at which free flow of the hydraulic fluid through the valve is stopped, and resistance to movement develops. At this velocity, a poppet valve closes and the fluid flow is restricted through a bleed orifice. When the hydraulic fluid is channeled through the "bleed" system, restraining forces are created which are proportional to the bleed velocity, Figure 3.2. The bleed and locking velocities can be set independently of each other and can be changed from the nominal values that have been set at the factory.

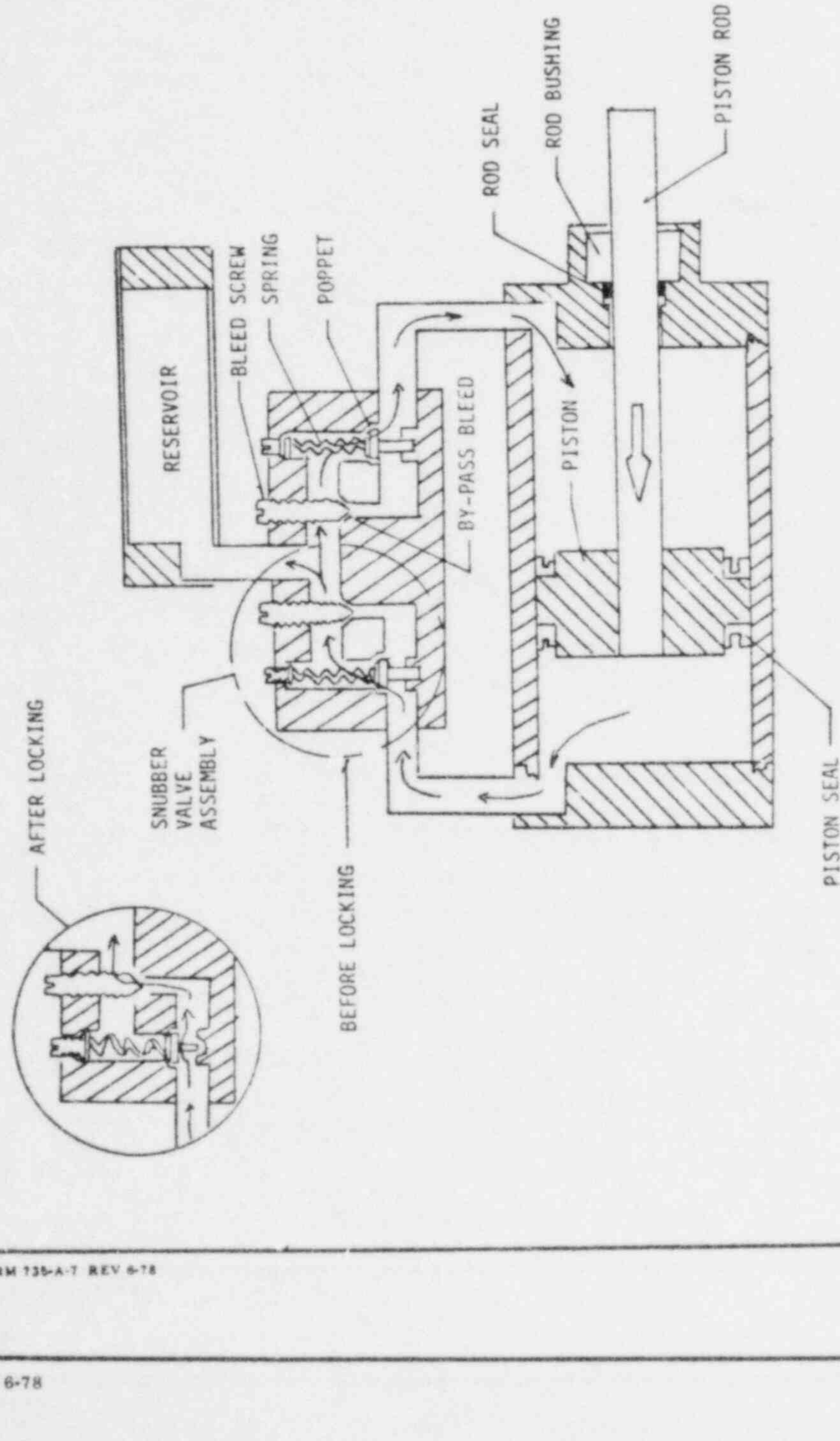

Figure 3.2 Schematic of the Snubber Cylinder and Control Valve

3.2.1 Lock Velocity - The lock velocity is that actuator or piston velocity at which free motion of the actuator stops and restricted motion begins. Prior to a seismic event, all motion occurring with a velocity less than this value will be unrestrained. The actuator will also lock when acceleration exceeds a given level.

3.2.2 Bleed Rate - The bleed rate is the restrained motion that occurs after snubber lock up occurs (see Locking Velocity). The piston velocity is proportional to the actuator load, that is, a velocity dependent force is produced as a result of this action.

The physical properties of the fluid have significant effects on the operation of hydraulic snubbers. The stiffness properties of snubbers are particularly sensitive to the effective bulk modulus and, of course, the bulk modulus is quite sensitive to air entrapment and temperature. If air pockets exist in the system, dead bands may also exist.

3.2.3 Clearance - For hydraulic snubbers this is unrestrained motion, at piston velocities greater than the locking velocity. It occurs because of mechanical tolerances at end fittings and entrapped air in the hydraulic fluid. The unrestrained motion acts as a clearance or dead band.

3.2.4 Friction - The friction or breakaway loads result from the actuator rod bushing and seal, and the piston seal. Corrosion effects tend to increase these loads.

3.2.5 Unlock Loading - This is the actuator load due to hydraulic pressure which causes the snubber to become unlocked; that is, free flow of hydraulic fluid takes place in the snubber as the poppet valve opens.

3.3 Parameter Combinations

The snubber parameters that affect response can be divided into two categories. The first category represent those parameters which affect the relative motion between the pipe (component) attachment and the arrestor support. And

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the second category represent those parameters which effect the snubber support motion. Although the second category of parameters may not be related directly to the snubber component, it is important that these characteristics be recognized.

Some parameters that may be considered in the first category include: acceleration threshold parameter, dead bands (gaps due to tolerances), linear stiffness characteristics due to the mechanical structure, nonlinear stiffness characteristics due to fluid compression or air entrapped in the hydraulic system, and viscous parameters such as lock velocities and bleed rates. Parameters that may be considered as belonging to the second category include: back-up or structural response characteristics, nonlinearities due to snubber orientation (single snubber usage), and clearances or local support flexibility associ[illegible] with installation such as loose connections.

Figures 3.3.1 and 3.3.2 show analytical schematics of the mechanical and hydraulic snubbers. Included in these schematics are the snubber support mass and stiffness properties, and the snubber attachment flexibility characteristics. The figures describe "generalized" snubbers with various possible parameters.

3.3.1 Mechanical Snubber Model - For the mechanical snubber the schematic shows two stiffness terms, one a displacement dependent characteristic $k(X)$, the other a velocity dependent stiffness characteristic $k(\dot{X})$, a load dependent parameter C_f, which is a velocity dependent reaction, and X_L which represents a load due to motion constraints. It should be noted that the spring constants are not necessarily linear. They could be either hardening, softening or a combination of both. The respective clearances for each of these parameters are also shown. The displacement dependent stiffness characteristic represents the flexibility of the internals of the snubber, i.e., the telescoping cylinder, the ball screw shaft, and threads. The velocity dependent stiffness $k(\dot{X})$ is a nebulous entity but is probably best visualized as resistance to changes in motion of the rotating inertia mass. The C_f, or velocity dependent load may actually represent the frictional resistance of moving parts such as screw shaft - ball nut friction, or sliding interfaces like the telescoping cylinder. Some of the

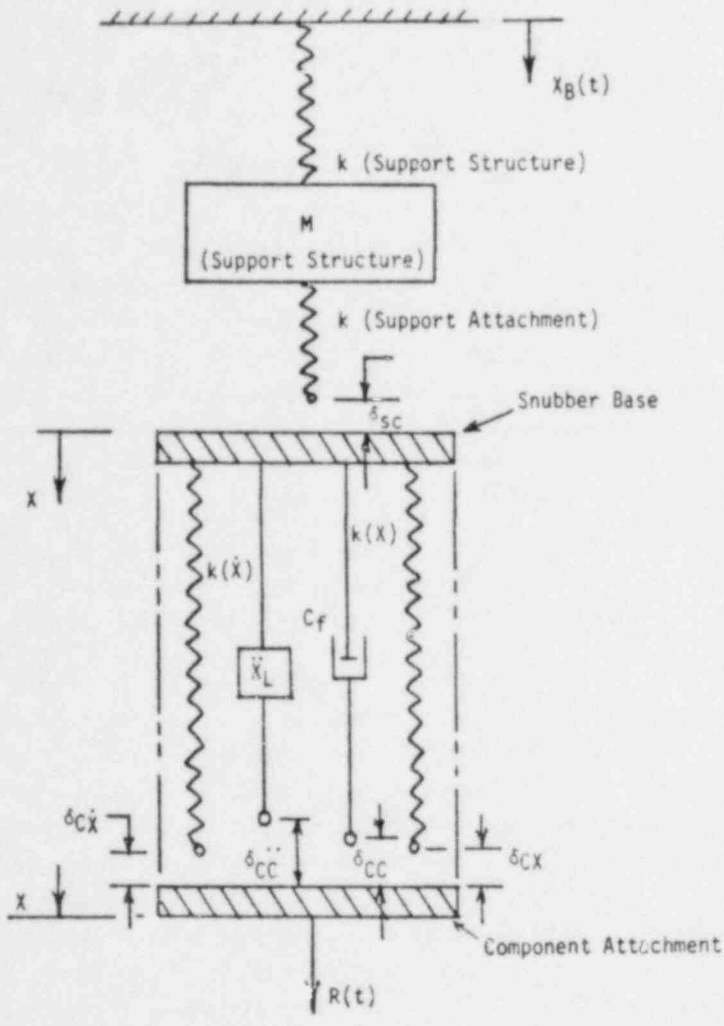

Figure 3.3.1 Mechanical Snubber Model

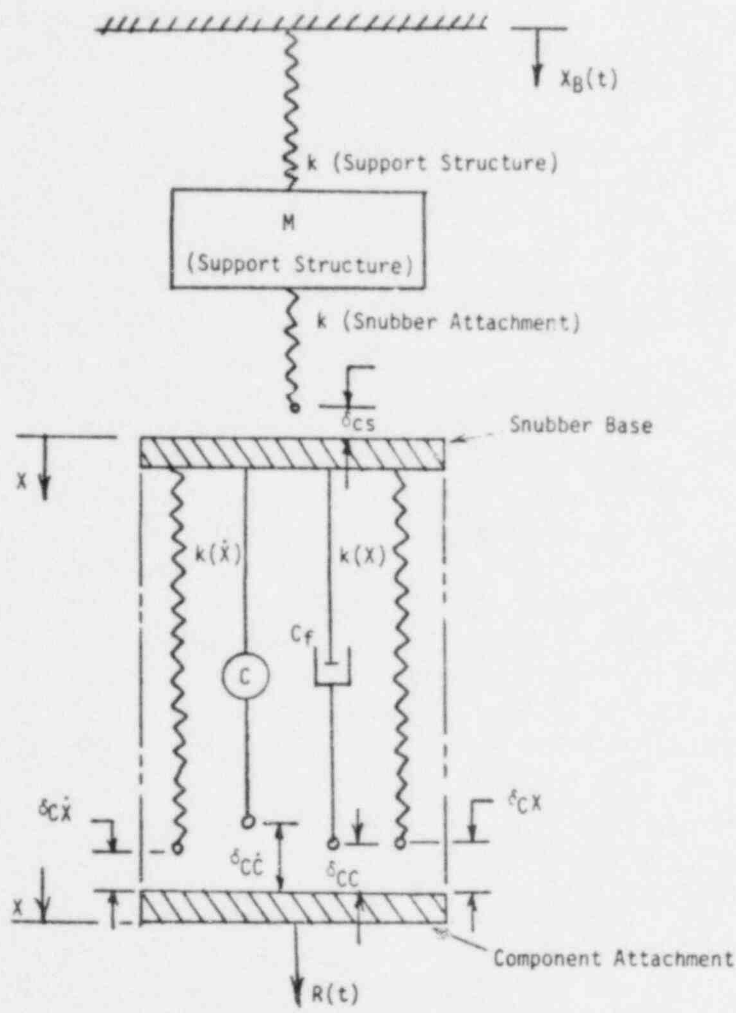

Figure 3.3.2 Hydraulic Snubber Model

frictional characteristics may be coulomb type; however, they result in an "effective" velocity dependent loading. The acceleration threshold reaction is an acceleration dependent parameter which results from the interaction between the capstan spring and the inertial mass.

The clearance δ_{CX} and $\delta_{C\dot{X}}$ associated with structutal stiffness of the overall component may be related to tolerances between internal parts and gaps in end attachments. The clearance δ_{CC} which is primarily due to friction is usually small since friction exists even without relative motion. The acceleration constraint clearance $\delta_{C\ddot{X}}$ is related to slop or tolerances between the ball screw shaft, and the ball nut. It is difficult to categorize these clearances according to origin, consequently clearances are lumped together as a single item.

3.3.2 Hydraulic Snubber Model - For the hydraulic snubber, the load parameter C represents a viscous load proportionality factor relating load to velocity. This is an important parameter that results from the bleed velocity setting for the snubber, and to a lesser extent the lock velocity and "free flow" pressures exerted on the hydraulic fluid. It is the classical viscous damping coefficient with one exception; it may be a function of velocity and displacement. Frictional forces may also be considered in this category. The flexibility parameters $k(X)$ and $k(\dot{X})$ are similar to those of the mechanical snubber, that is, a combination of "external" structural flexibilities and "internal" flexibilities. The internal flexibilities, however, are dependent on the properties of the hydraulic fluid such as air entrapment and entrainment, bulk modulus, and viscosity. The spring constants are not necessarily linear, and could be either, hardening, softening, or a combination of both.

Clearances from external sources such as installation tolerances are treated the same for both hydraulic and mechanical snubbers. Internal sources differ; the greatest source of clearance is from entrapped or entrained air in the hydraulic snubber. Since clearances are related to the hydraulic fluid they will be considered as part of the nonlinear restorative characteristic of the fluid.

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4. STRUCTURAL/ANALYTICAL MODELS

The selection of a mathematical model involves consideration of system response characteristics, load types, restraint characteristics, and the interaction with other dynamic systems, e.g., the snubber and supporting stru... re. Utilization of simple structural models formed the basis for meeting the goals of the study. The single degree of freedom lumped mass oscillator was used intensively in the study. The simple oscillator model enables one to readily isolate parameters and establish dimensionless combinations including both system response characteristics and snubber parameters. The simple oscillator also minimizes the number of variables that affect system response.

The response of the lumped mass oscillator when related to a more complex system can be viewed as an anchor motion or base excitation input to the system which is restrained by the snubber.

When a simple system is used to develop response sensitivity characteristics, one must realize that input and restraint effects become more pronounced than they would be for a complex system. For example, consider the single degree of freedom lumped mass sytems shown in Figure 4.1, here each system has the same characteristic equation,

$$m\ddot{X} + C\dot{X} + kX = f(t).$$

The response magnitude is a function of f(t) and is different for each model shown. The load expressions for each of the six cases are shown in Table 4.1.

The response sensitivity to the viscous parameter, C, can be studied using at least five different structural models. Although it is possible to logically single out one model as more realistically representing a given situation, the reference sensitivity of snubber parameters suggests that any one or all of the models can be used to meet the goals of this program.

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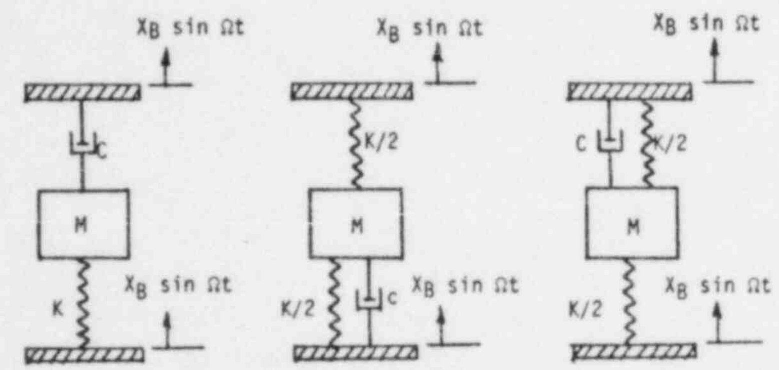

Figure 4.1 1-D.O.F. Boundary Condition Variations

TABLE 4.1

FORCING FUNCTION PERMUTATIONS

Case	f(t)
1	$kX_B\,[\sin \Omega t]$
2	$kX_B\,[\frac{1}{2} \sin \Omega t + (C\Omega/k) \cos \Omega t]$
3	$kX_B\,[\frac{1}{2} \sin \Omega t]$
4	$kX_B\,[\sin (\Omega t - \phi) + (C\Omega/k) \cos \Omega t]$
5	$kX_B\,[\frac{1}{2} \sin \Omega t + \frac{1}{2} \sin (\Omega t - \phi) + (C\Omega/k) \cos \Omega t]$
6	$kX_B\,[\frac{1}{2} \sin \Omega t + \frac{1}{2} \sin (\Omega t - \phi) + (C\Omega/k) \cos \Omega t]$

Depending on the component or system configuration, the snubber may affect the system response by restraining motion, by acting as a load or input attenuator, or a combination of the two. Component sensitivity studies are based on various types of loadings and supports. This allows an overall response sensitivity to be established. Studies consider base excitation, force inputs, and displacement acceleration inputs.

In addition to the simple oscillator, more complicated structures have been studied, such as, a multi-degree of freedom lumped mass model, simple beam models and piping loops. These more sophisticated models are used to verify the results of the study of simple systems. Consideration of the more sophisticated system permits insights into multi-mode effects and reduced response sensitivity.

The lumped mass models represent a series of models with different natural frequencies and forcing functions. Figures 1.1(c) and 1.1(d) represent two of several beam models used in this study. Figure 1.1(e) represents the "typical" piping loop with components that are found in reactor piping such as valves, springs, straight pipe runs and elbows. The valve weights, spring rates, and lengths with dynamic characteristics - in particular, the natural frequency and displacement and stress responses - approximate those found in typical reactor installations. Without the snubber installation the first three natural frequencies of this model are 2.311 Hz, 4.191 Hz and 7.251 Hz. With the snubber installed the first three frequencies are 5.854 Hz, 9.881 Hz and 11.292 Hz.

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It is recognized that the static and dynamic properties of the back-up structure contribute to or modify the response characteristics of the snubber. Compliance with the dynamic coupling of the systems are investigated as a separate task in this study.

5. FORCING FUNCTIONS

Use of harmonic forcing functions in simple oscillators allows the development of dimensionless expressions thus reducing the number of independent variables. With the response characteristics dependent on the forcing frequency, the use of actual seismic inputs is essential to the development of a comprehensive understanding of snubber parameter sensitivity.

Seismic loadings selected for this study represent the four most widely used seismic inputs for the design of nuclear facilities (Table 5.1).

TABLE 5.1
SEISMIC EVENTS INPUTS

Event	Location	Richter Magnitude	Max G
Imperial Valley Earthquake	El Centro, California	6.7	.348
Kern County Earthquake	Taft, California	7.7	.196
San Fernando Earthquake	San Fernando, California	6.4	.253
Western Washington Earthquake	Olympia, Washington	7.1	.280

Information was obtained from the CAL TECH Earthquake Engineering Research Laboratory, Reference 2, with test instrument corrected and fully documented digitized acceleration, velocity and displacement data. This data was processed through the Rockwell computers and the CRT plots are shown for the various seismic disturbances in Figures 5.1 through 5.9. Acceleration traces are shown in Figure 5.1 through 5.4 and displacement traces are shown in Figure 5.5 through 5.8. The response spectral data for these siesmic events are shown in Figure 5.9.

The records used in the investigation were measured only in the United States and predominantly in California. The conclusions are applicable to the west coast of the United States. Slower ground motion attenuation rates in the east and mid-west suggest that ground motions in these regions could have different frequency contents. Nevertheless, the seismic design of major structures in the east and mid-west have of necessity, been based on measured records from the west This practice will no doubt continue until a sufficiently large ensemble of strong motion records can be obtained from the east and mid-west.

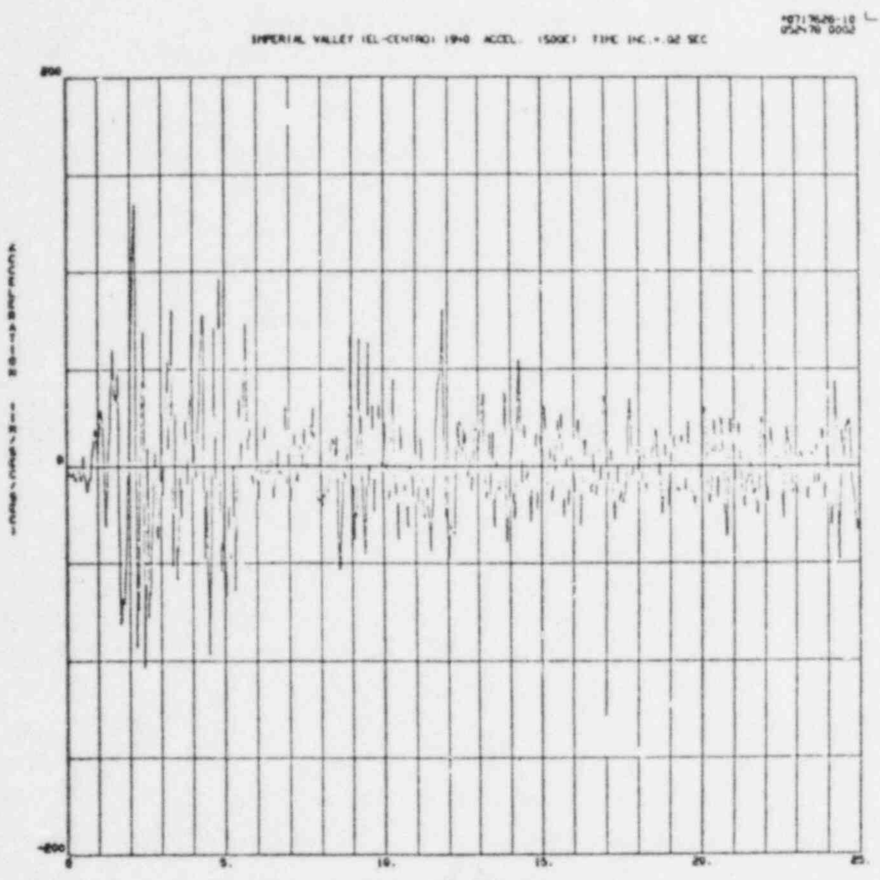

Figure 5.1 El Centro Acceleration Seismic Input

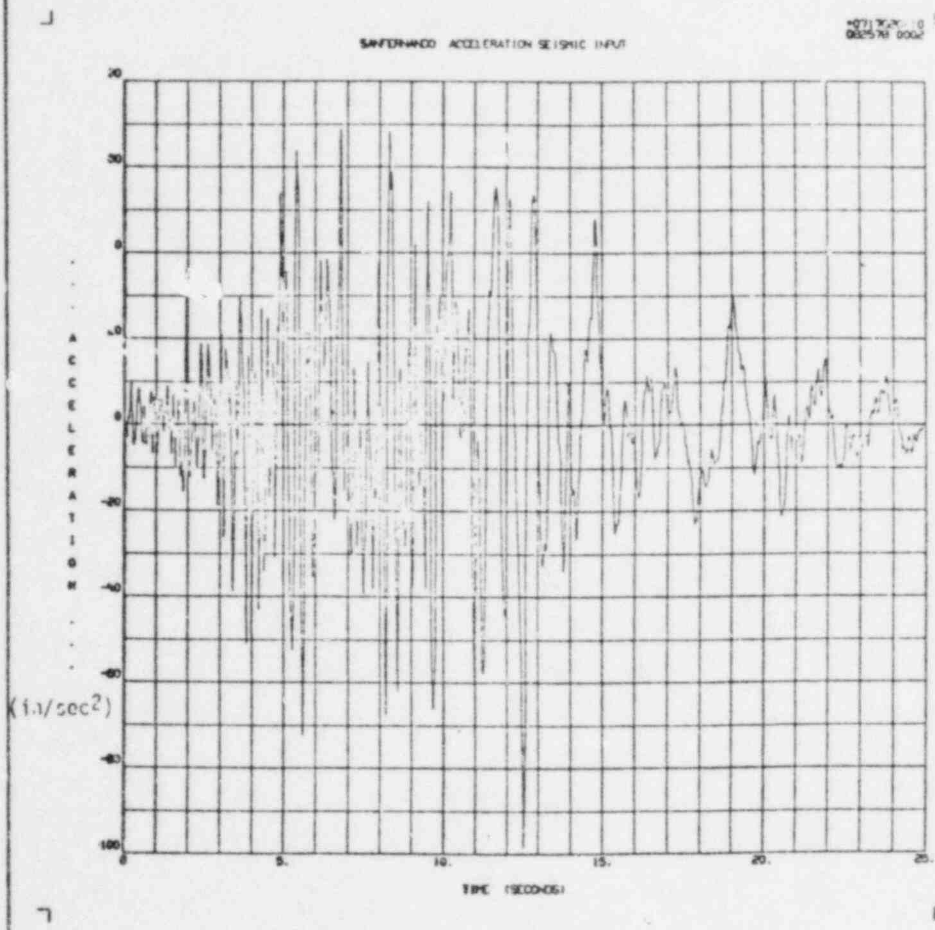

Figure 5.2 San Fernando Acceleration Seismic Input

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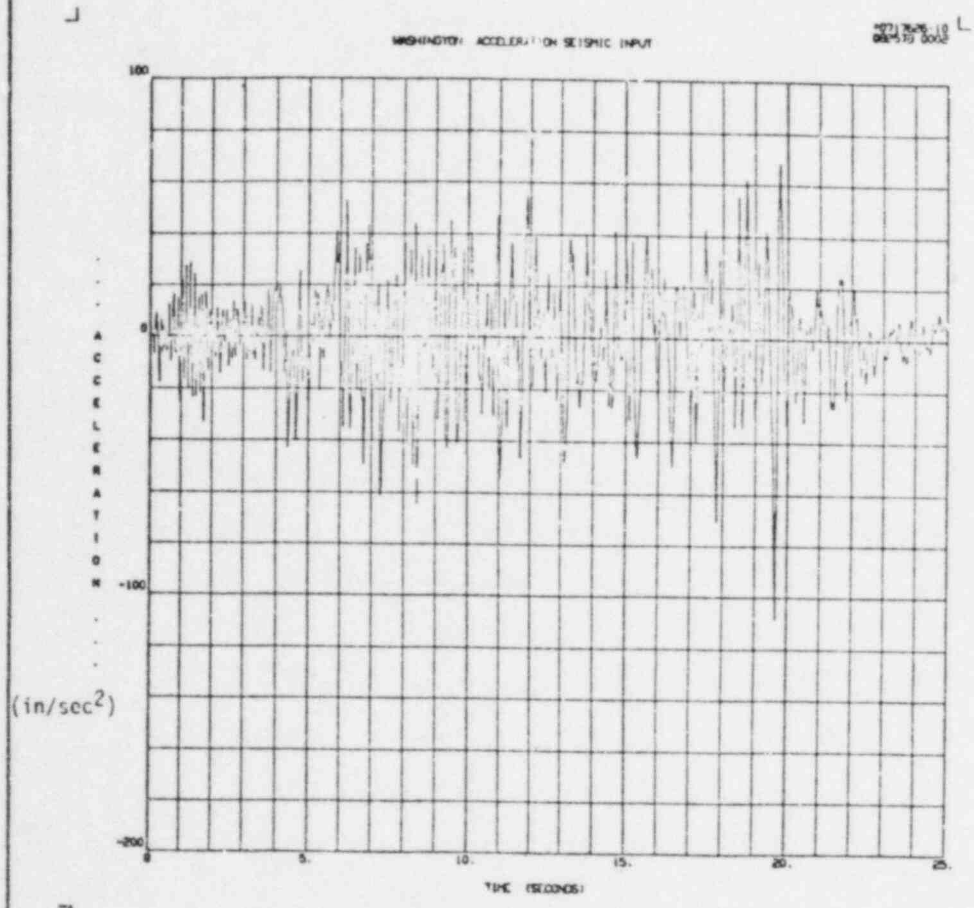

Figure 5.3 Washington Acceleration Seismic Input

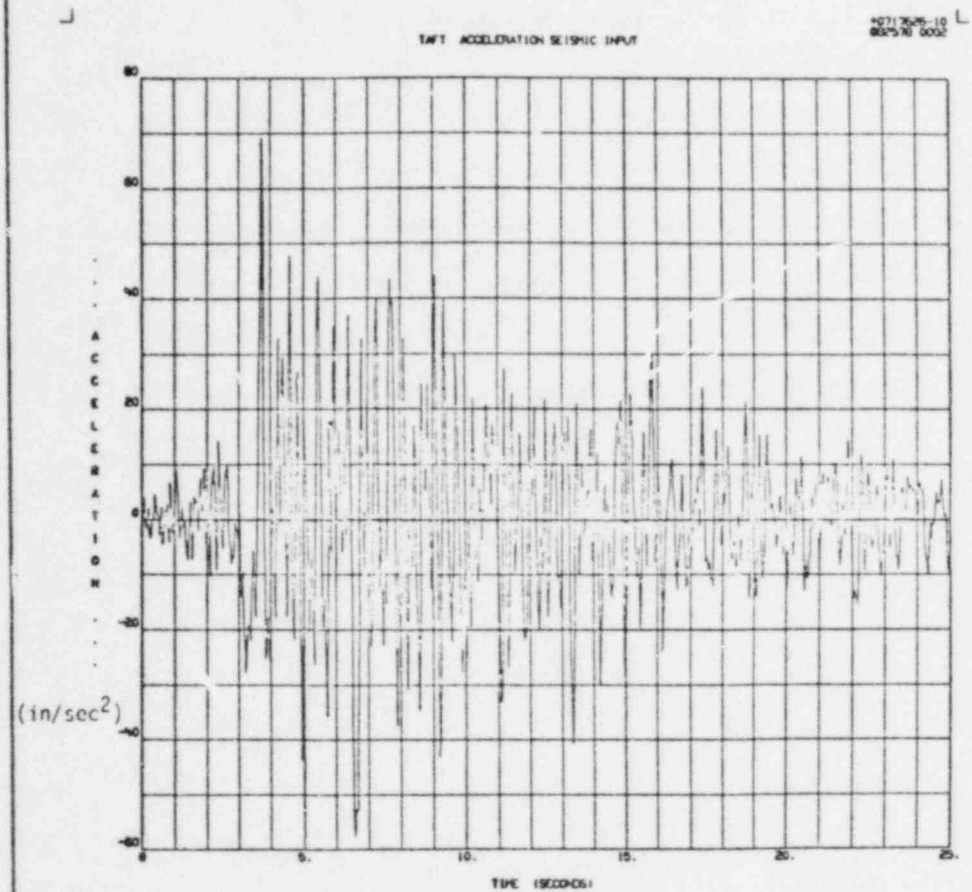

Figure 5.4 Taft Acceleration Seismic Input

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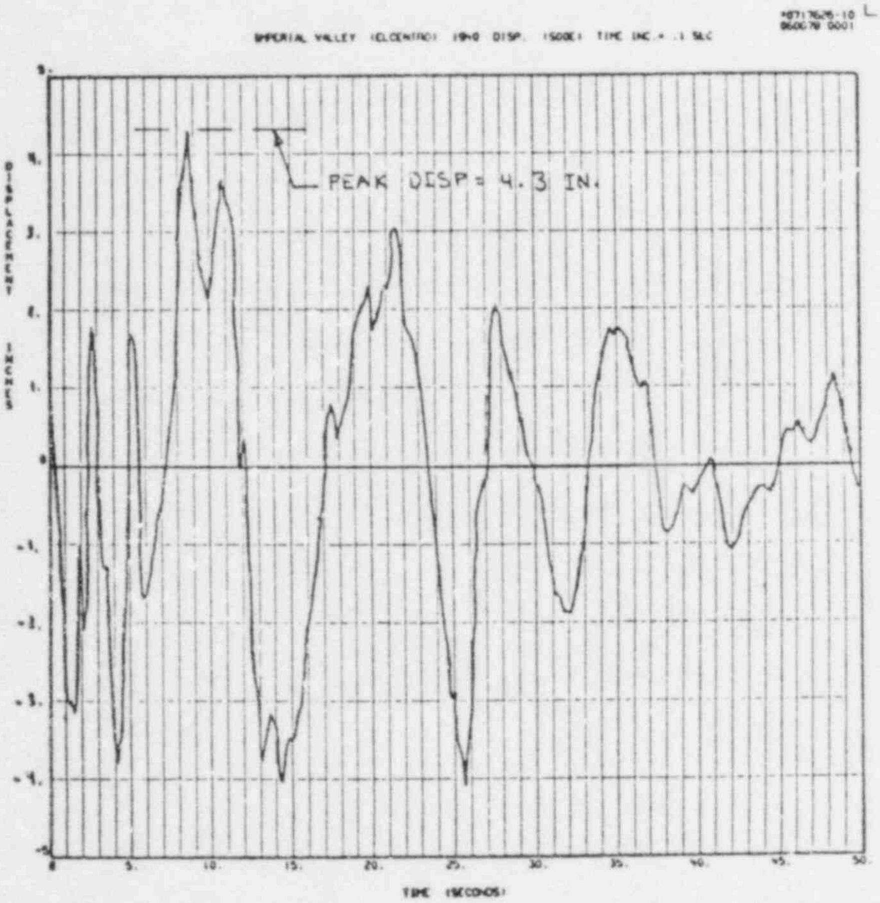

Figure 5.5 El Centro Displacement Input

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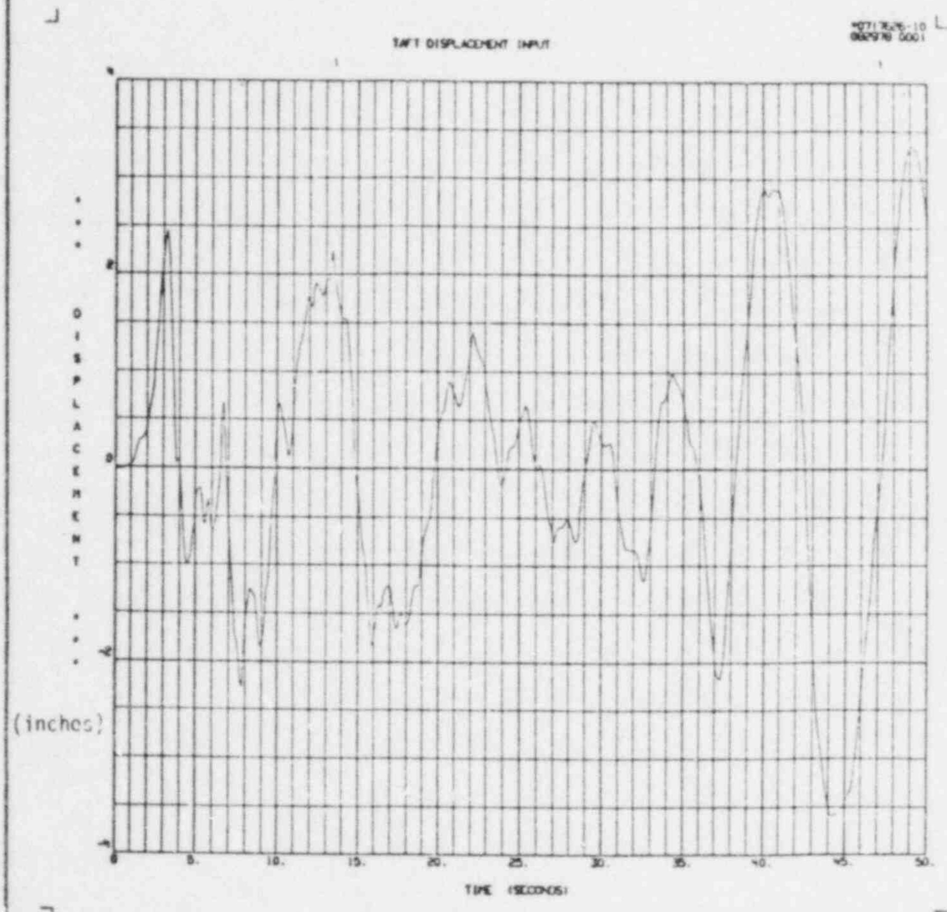

Figure 5.6 Taft Displacement Input

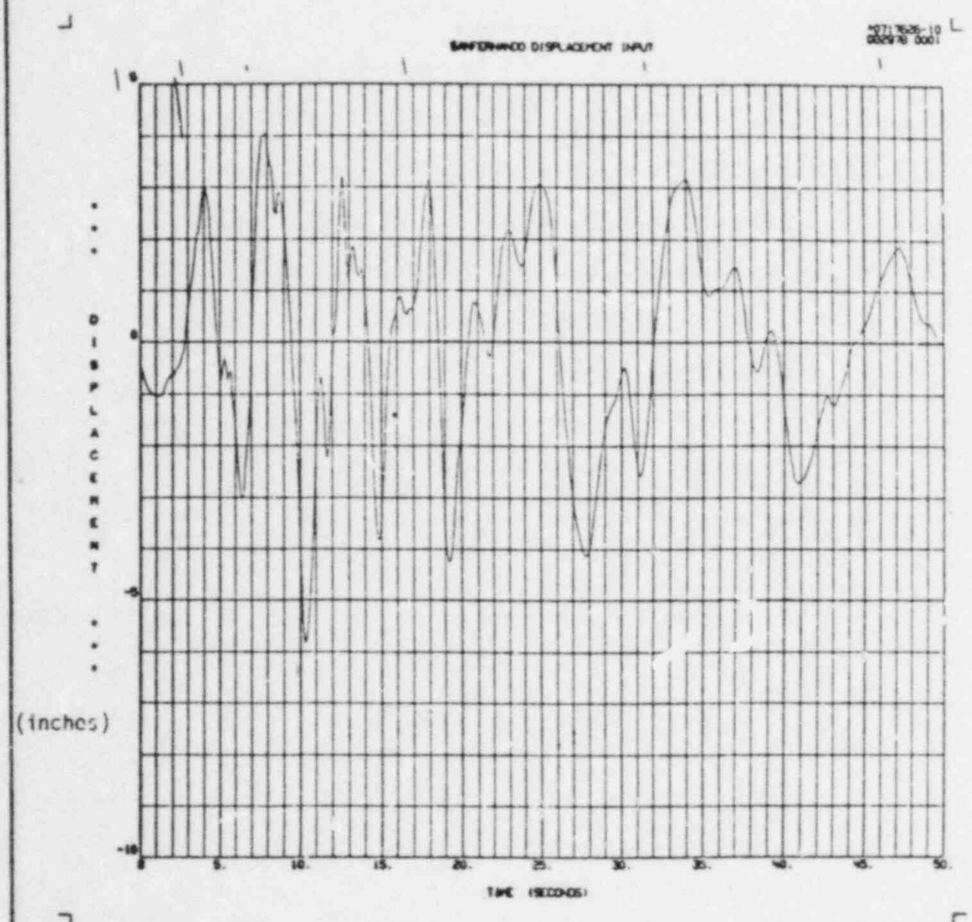

Figure 5.7 San Fernando Displacement Input

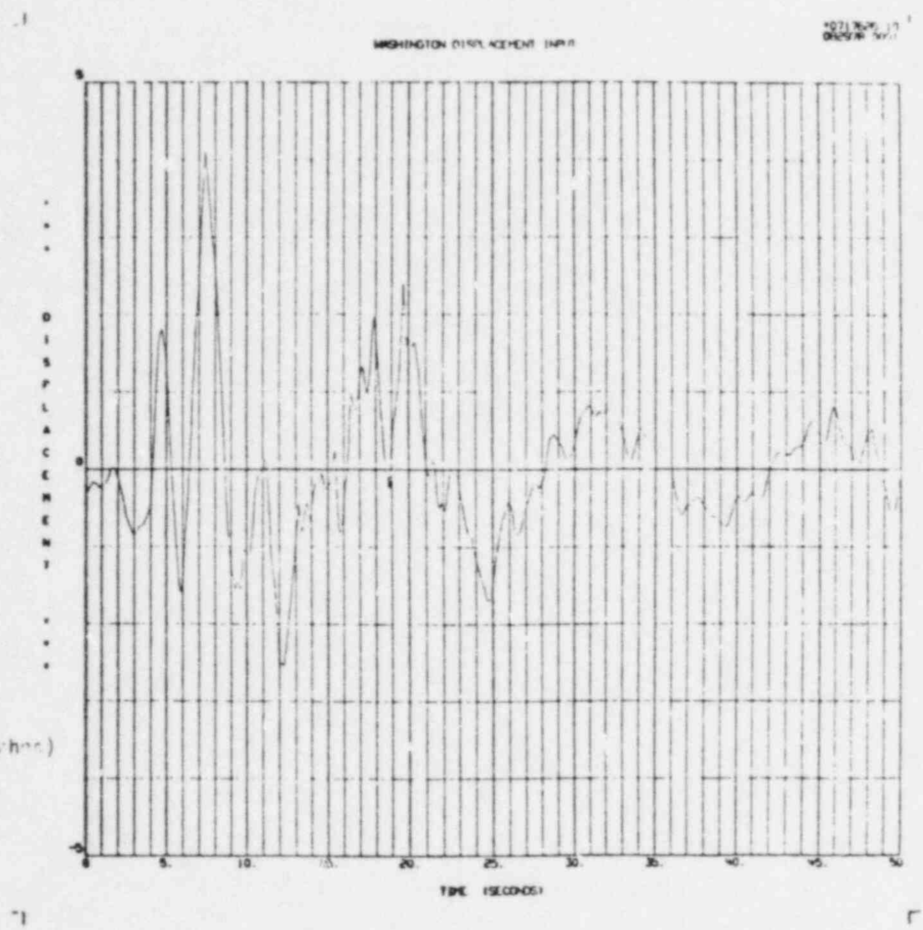

Figure 5.8 Washington Displacement Input

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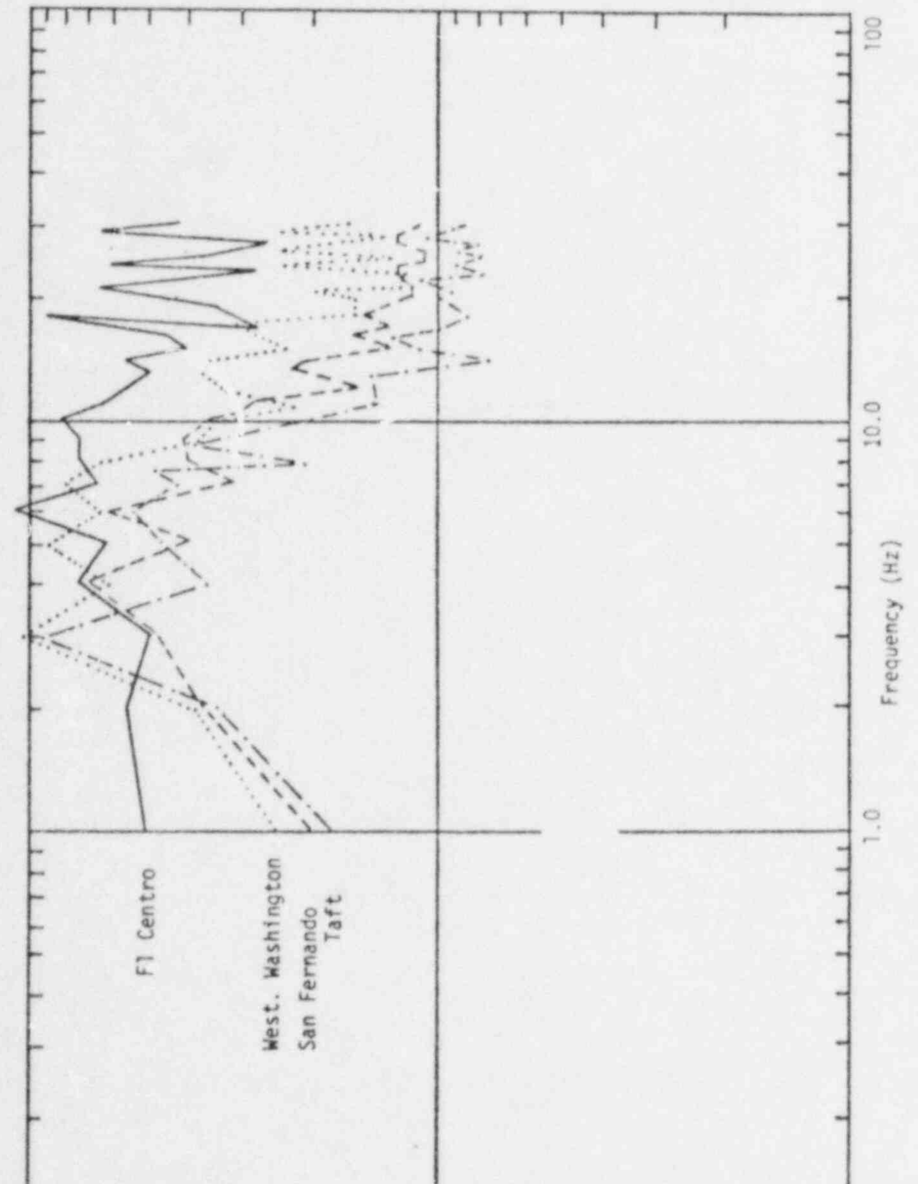

Figure 5.9 Acceleration Spectrum (Large Motion Earthquakes)

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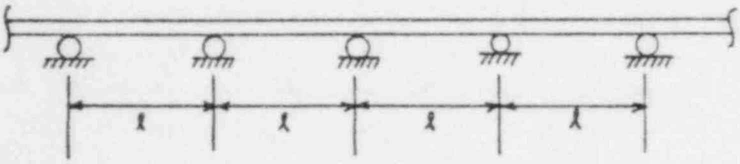
6. SUPPORT CHARACTERISTICS

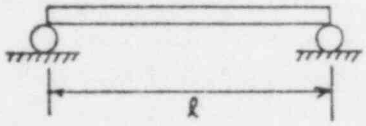
6.1 Back-up Structural Stiffness

Typical dynamic analyses of components and piping systems consider the snubber as a rigid restraint. This is considered particularly valid when the motion during a dynamic disturbance is very small and applies when clearances are nonexistent, or loads are low compared to the rated capacity of the snubbers. The response of the system, however, can be influenced by the flexibility of the supporting structure.

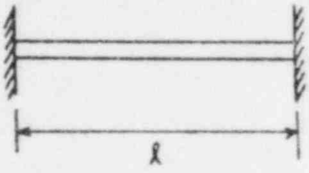
Consider an infinitely long pipe supported at uniform distances.



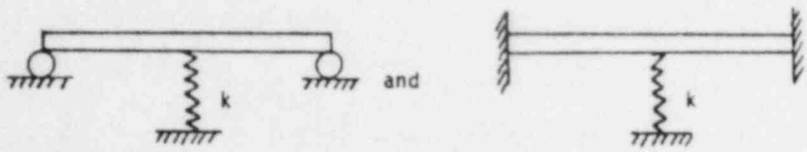
The fundamental mode of this system can be found using a single span.



The second mode can be found by changing the boundary conditions from pin-pin to fixed-fixed.



Supports on the infinitely long pipe can be related to anchors in a general piping system. The resonant frequency of the system can best be changed by placing a snubber at the center of the span.



Using the two systems in the figure, the effects of snubber stiffness on system frequencies can be evaluated by plotting the dimensionless parameters of restrained natural frequency/unrestrained natural frequency, (f_n/f_1), and the spring stiffness/pipe stiffness, $(k/K°)$ as shown in Figure 6.1.1. The flexibility at the point of snubber attachment is α. The pipe stiffness, K°, is evaluated on the basis of pipe flexibility at the snubber location. K° is, therefore, a function of not only EI (modulus of elasticity and bending moment of inertia) and ℓ for a general system but also a function of hanger and anchor locations and geometry.

For the snubber to be active in changing the system frequencies, its "effective" stiffness needs to be at least twenty times greater than the pipe stiffness (K°). That is, the snubber is most effective in altering the response of the system when $(k/K°) \geq 20$. This is the case when response can be directly related to system natural frequencies. This can be verified by noting system response, rather than system natural frequencies, as a function of the stiffness parameter, $(k/K°)$. The results for a simple beam model are shown in Figure 6.1.2 where peak transient response for an undamped system is shown as a function of "effective" snubber stiffness. Figure 6.1.3 presents the results for the El Centro seismic loading.

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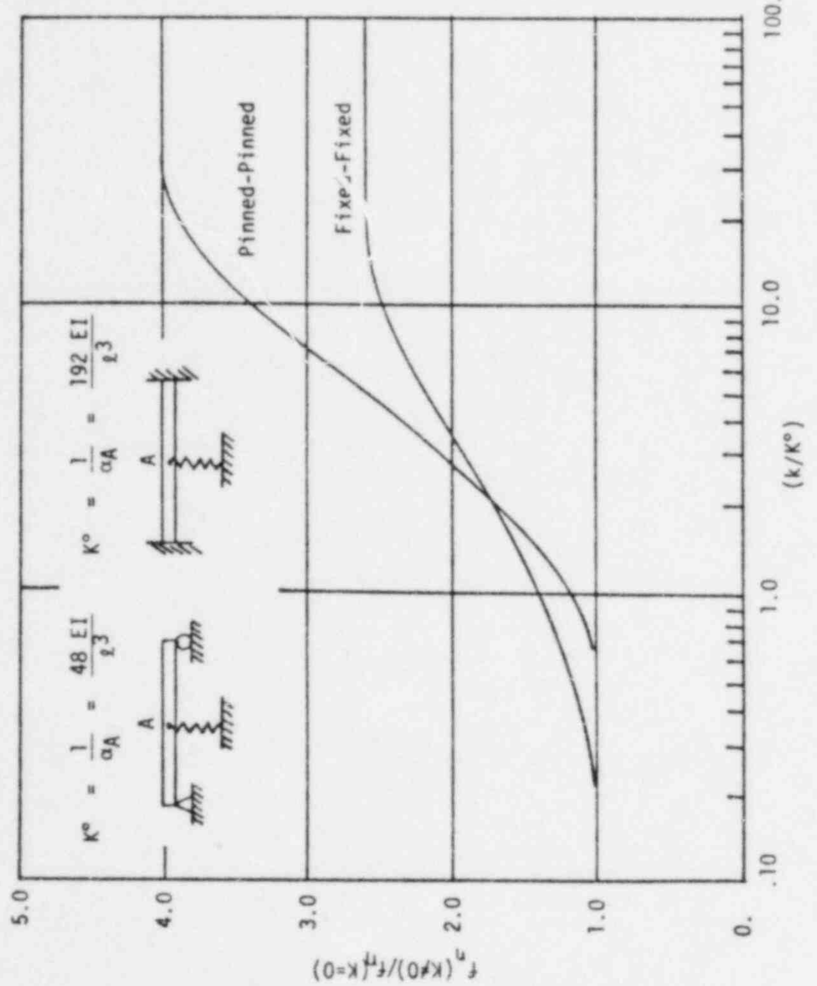

Figure 6.1.1 Snubber Stiffness vs Natural Frequency

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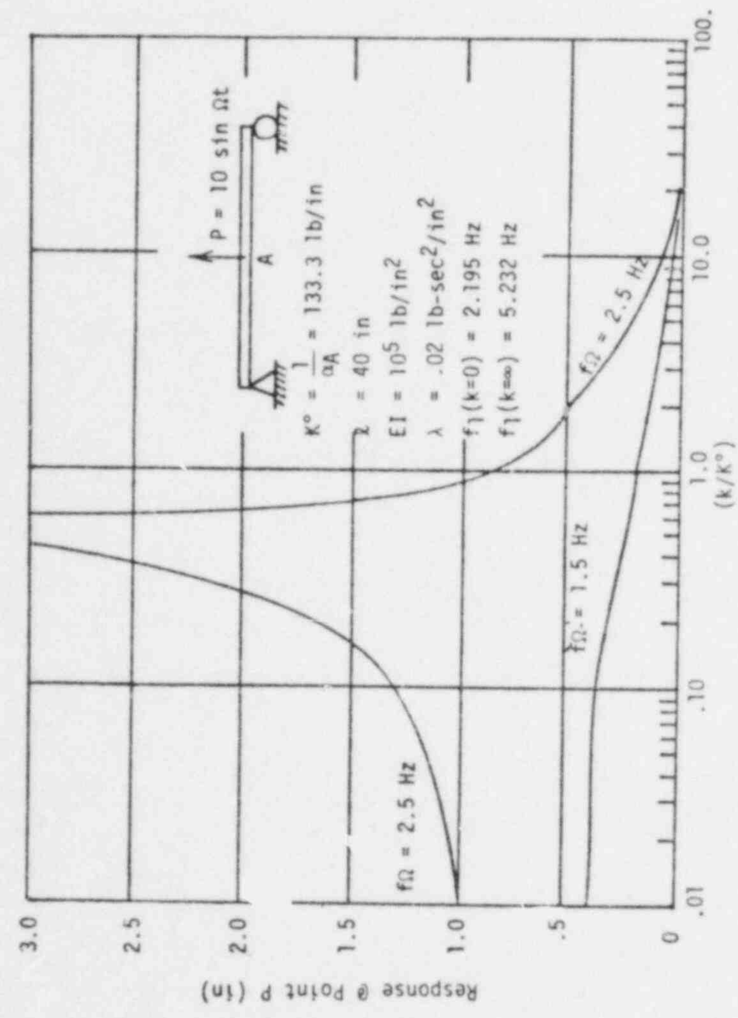


Figure 6.1.2 Response Spectra for Simply Supported Beam

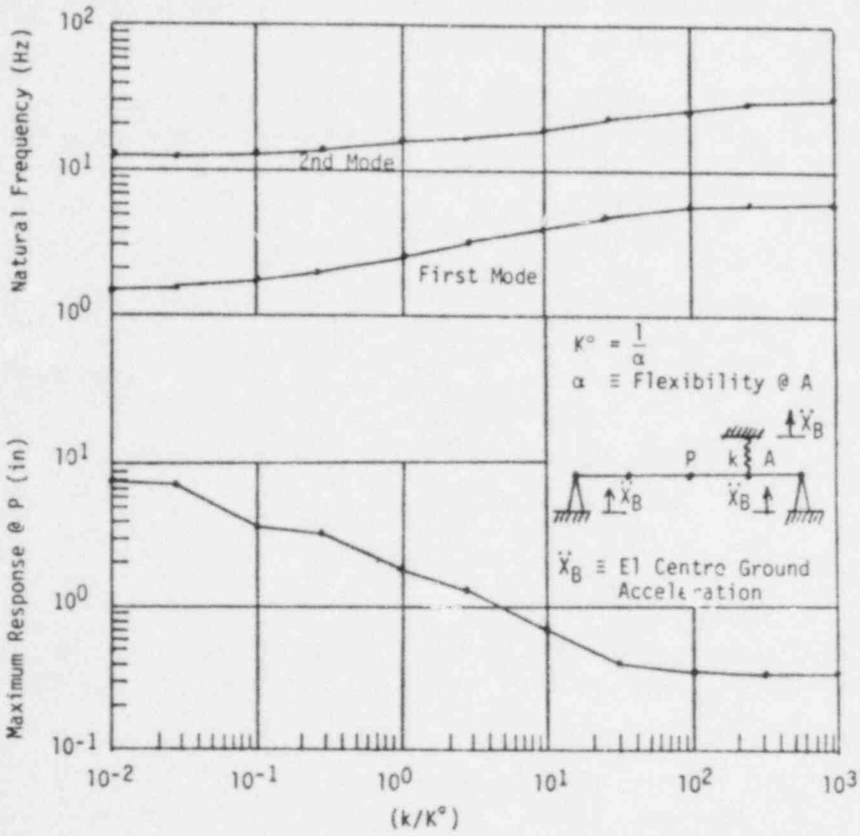
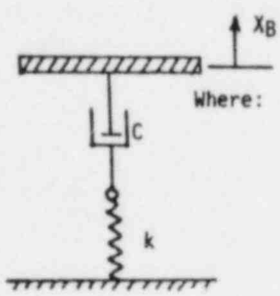


Figure 6.1.3 Beam Response to El Centro Seismic Input

Two different harmonic forcing functions and one seismic loading were examined to evaluate the effects of resonance on the forcing frequency. In all cases, the response was found to be insensitive to k for $(k/K°) > 10$. This same trend is expected to exist regardless of the complexity of the structure, provided reasonable engineering design practices are used. For example, if the snubber were placed very close to either support, the response sensitivity to $(k/K°)$ would become less pronounced. This would represent poor engineering choice because the effect on the response would be minimal.

The hydraulic snubber efficiencies require additional consideration of the backup structure. This can best be demonstrated by the dashpot-spring arrangement subjected to a harmonic excitation,



Where: $C \equiv \frac{\text{RATED LOAD}}{\text{BLEED RATE}}$

$\Omega \equiv$ Predominant Forcing Frequency (rad/sec)

This system crudely represents a hydraulic snubber "C" with a back-up support stiffness "k" associated with it. The steady state force in the snubber is,

$$F = CX_B\Omega \left[\left(\frac{a^2}{1 + a^2}\right) \cos \Omega t + \left(\frac{a}{1 + a^2}\right) \sin \Omega t\right],$$

where,

$$a = \left(\frac{k}{c\Omega}\right)$$

The development of the above relationship is given in Section 9.

If efficiency of the support is .ned as the ratio of the maximum load developed to the maxim.m possible load, the following is obtained,

$$\eta = \sqrt{\frac{a^2}{1 + a^2}} = \sqrt{\frac{\left(\frac{k}{C\Omega}\right)^2}{1 + \left(\frac{k}{C\Omega}\right)^2}}$$

This expression gives a method for determining support stiffness for a given forcing function, and type of hydraulic snubber. Figure 6.1.4 shows a plot of the efficiency (η) versus ($k/C\Omega$). For low efficiencies a minimal effect is experienced for reducing response. On the other hand effectiveness is enhanced in reducing response for high efficiencies. If a minimum 0.75 efficiency is desired from Figure 6.1.4, it can be seen that k must be greater than 1.2 ($C\Omega$).

Knowing k and ($C\Omega$), Figure 6.1.5 can be used to evaluate the effectiveness of the snubber.

6.2 Support Dynamics

In the discussion of back-up structural stiffness, it was presumed that the dynamic characteristics of the supporting structure did not attenuate the input loading or couple dynamically with components and piping systems. Dynamic coupling, however, can occur and the response of the system can be greatly influenced by its dynamic characteristics.

The response characteristics of structures should be considered when designing components and piping systems. This practice is generally used in the design of nuclear facilities. Even when detailed dynamic analyses are not required the spacing of piping supports and snubbing devices are often selected so that the frequency of the piping will be significantly different from that of main structures. Building structures typically have a resonant frequency range of 2 → 4 Hz. For this reason, component and piping systems are designed to have a natural

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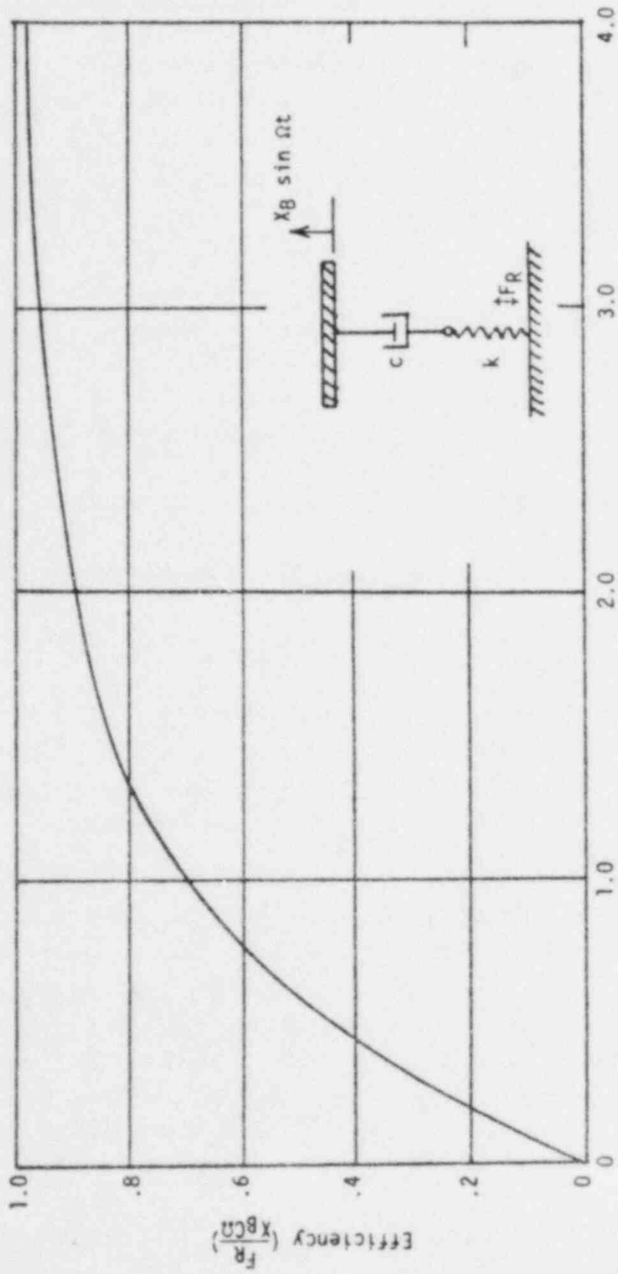

Figure 6.1.4 Hydraulic Snubber Efficiency

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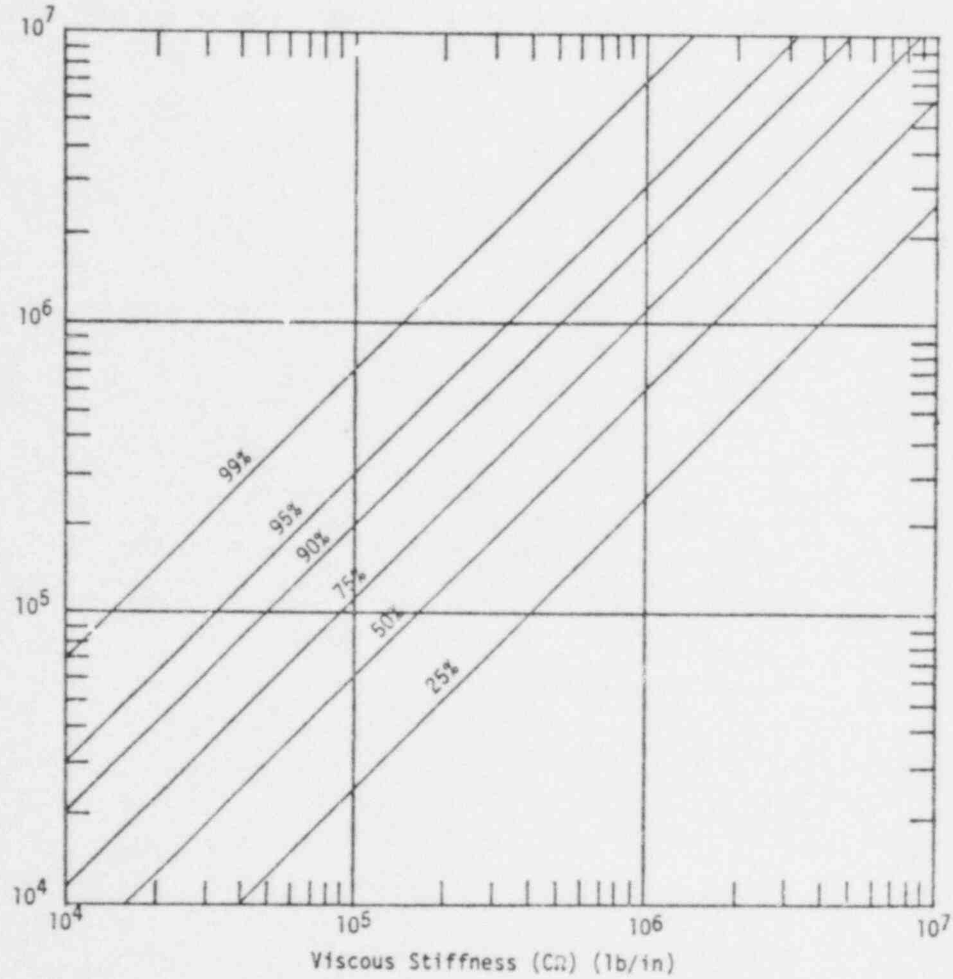

Figure 6.1.5 Support Efficiency

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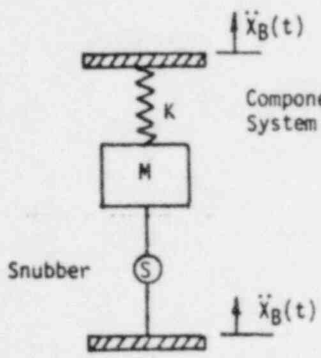
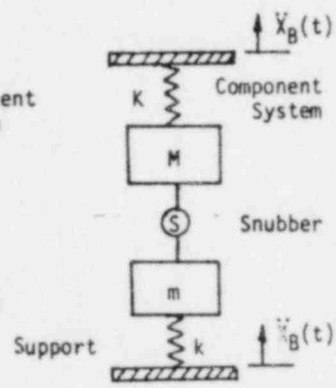
frequency of at least 8 Hz. Although a higher frequency would be desirable, 8 Hz is considered to be a realistic and acceptable value.

The basic structural feature selected to represent dynamic characteristics of structures supporting snubbed components and systems is one where the ground motion input is applied at all anchors and all snubber locations. The model is an expansion of the generalized model as illustrated below.



Generalized Model



Model for Studying Dynamic Characteristics

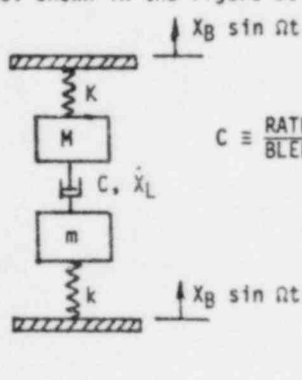
The analytical solution for the response of the system can be found in Section 9, Analytical Methods.

Consider the structural model shown in the figure below:

$\frac{C}{M} = 10{,}000 \text{ sec}^{-1}$

$\frac{m}{M} = 1.0$

$\frac{1}{2\pi}\sqrt{\frac{K}{m}} = 8 \text{ Hz}$

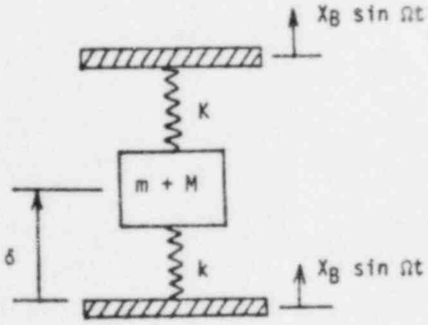


$C \equiv \frac{\text{RATED LOAD}}{\text{BLEED RATE}}$

Viscous parameters are considered in this structural mode. The parameters K (stiffness) and M (mass) represent the equivalent properties of the system for which the response is studied and k and m those for the supporting structure. The system and supporting structure natural frequency are $\frac{1}{2\pi}\sqrt{\frac{K}{M}}$ and $\frac{1}{2\pi}\sqrt{\frac{k}{m}}$, respectively. Coupling is accomplished using the snubbing device. In this example, the "uncoupled" supporting structure has a natural frequency of 8 Hz and is excited by a 6 Hz harmonic base excitation.

The maximum response can be evaluated for various values of supporting structure natural frequencies (k/m). Thus the effects of support frequency in combination with the viscous parameters can be observed with the effects of viscous properties. The results of this study are shown graphically in Figure 6.2.1.

The response of the masses, representing the two ends of the snubber, are nearly equal. A response peak is noted similar to a classical response-frequency curve. The small difference in relative motion between both ends of the snubber relative to the overall motion of the masses suggests that the effective stiffness of the snubber is much greater than the stiffness of the component system or the supporting structure. Since the effective stiffness is much greater than the apparent dynamic stiffness of the supporting structure the overall response of the component system (K, M) can be approximated from the model shown in the figure below.



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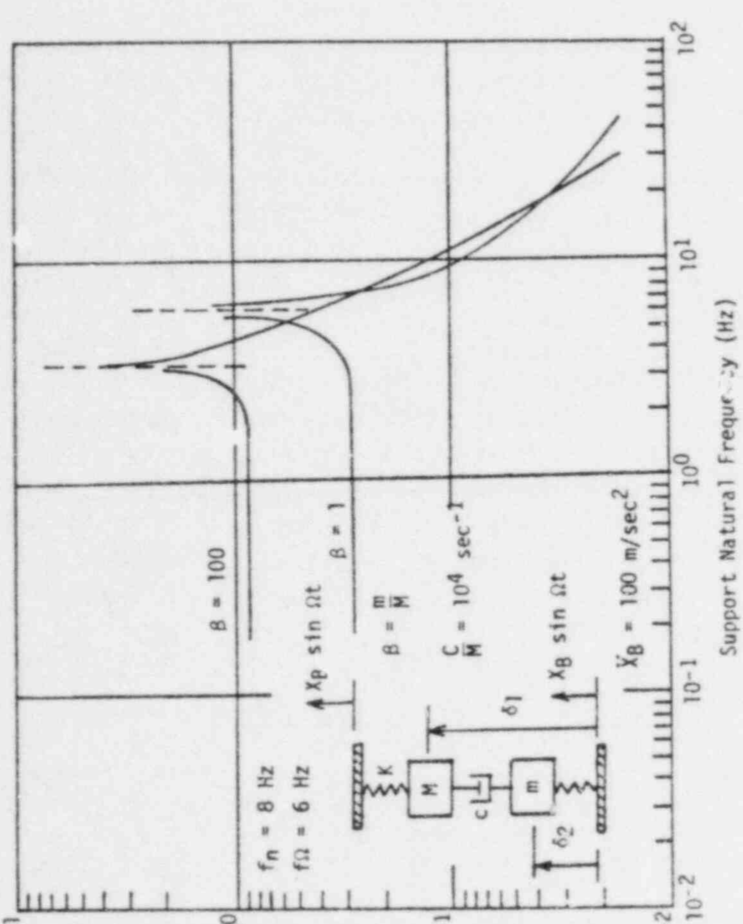

Figure 6.2.1 Support Dynamic Characteristics

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The response of the system can be approximated by

$$\delta = \frac{\ddot{X}_B}{\frac{K}{M}\left(\frac{1}{1+\beta}\right) + \frac{k}{m}\left(\frac{1}{1+\beta}\right) - \Omega^2} ,$$

where,

$$\beta = \left(\frac{m}{M}\right) .$$

The natural frequency of the coupled system is

$$f_{ns} = \frac{1}{2\pi} \sqrt{\frac{K}{M}\left(\frac{1}{1+\beta}\right) + \frac{k}{m}\left(\frac{\beta}{1+\beta}\right)} .$$

When the forcing frequency approaches f_{ns} the response will increase as a resonant condition is created.

7. DETAIL PARAMETER STUDIES

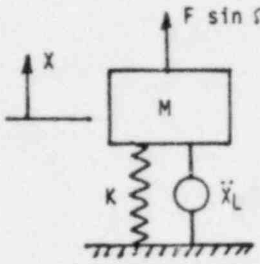
7.1 Acceleration Threshold Parameter

The acceleration threshold parameter (A.T.P.) is a characteristic that is unique to the mechanical snubber. It limits acceleration of the pipe movement relative to the base of the snubber.

The effects of the A.T.P. on system response were studied using the single degree of freedom lumped mass oscillators and simple beam models shown in Figure 7.1.1 and the more complex piping system shown in Figure 1.1(e). The investigation considered steady-state and transient response characteristics resulting from harmonic loadings, and transient response characteristics resulting from seismic inputs.

Consider the simple system shown below excited by a harmonic loading. The response of the system was evaluated by numerical methods (Section 9). The transient response for the first few cycles of loading is shown in Figure 7.1.2.

$f_n = \frac{1}{2\pi}\sqrt{\frac{k}{m}} = 1$ Hz



$f_\Omega = 8$ Hz

$\frac{F}{m} = 10$ in/sec^2

The response of the lumped mass system does not respond at either the forcing frequency or the natural frequency, but responds with a complex response which appears to be the summation of two harmonics, one the forcing frequency and the

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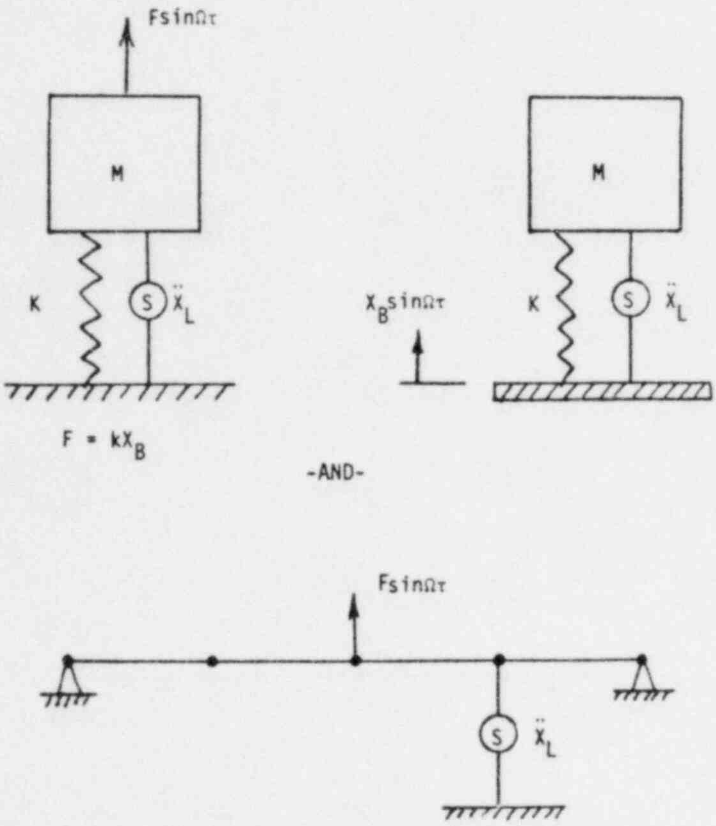

Figure 7.1.1 A.T.P. Structural Models

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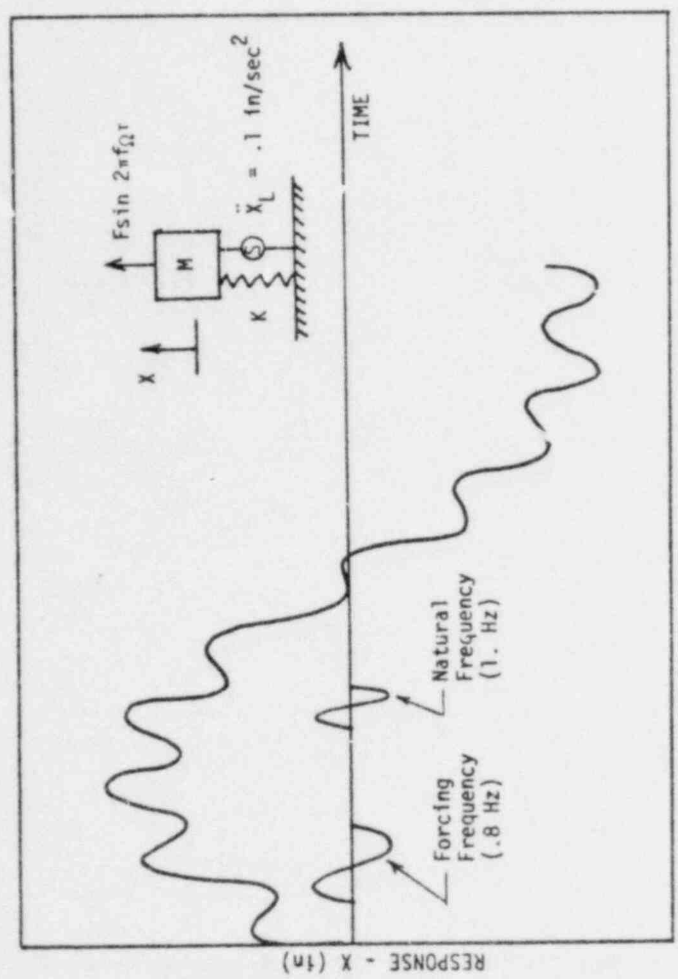

Figure 7.1.2 Oscillator Response Due to A.T.P.

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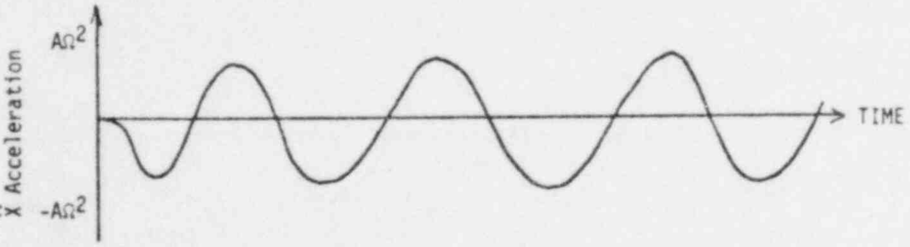
other a much lower frequency component. This lower frequency component is referred to as the shadow frequency.

To describe this shadow frequency, consider the steady-state response of a single degree of freedom, lumped mass as a result of an harmonic excitation, without the influence of the acceleration limit - acceleration threshold parameter. The steady-state response, velocity and acceleration can be expressed by

$$X = A_o \sin(\Omega t - \emptyset),$$
$$\dot{X} = A_o \Omega \cos(\Omega t - \emptyset),$$
$$\ddot{X} = A_o \Omega^2 \sin(\Omega t - \emptyset)$$

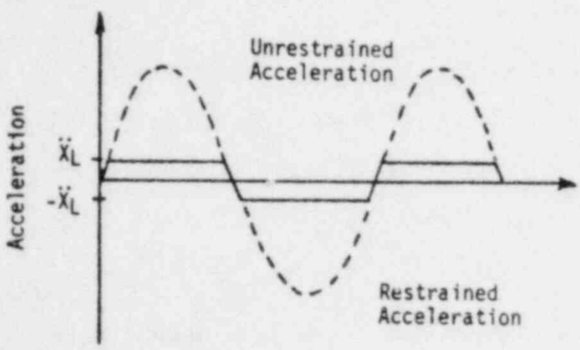
where, $\emptyset$ is a phase angle between the applied force and the response.

The system acceleration is an harmonic function which is 180° out of phase with the displacement. If the acceleration were plotted as a function of time it would be a continous sine wave shown below.

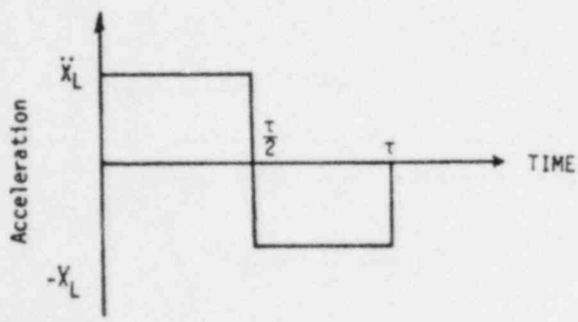


This assumes an Acceleration Threshold Parameter where, $\ddot{X}_L << A_o\Omega^2$.

Applying the acceleration limits to the system acceleration response curve, the system acceleration response becomes that shown by the solid line in the figure below.



If $\ddot{X}_L << A_o\Omega^2$, the acceleration characteristics become, for the first cycle



Where τ is the period of the forcing function. If the acceleration is integrated from $0 \rightarrow \tau$ one finds that there is a net movement or shift of the mass from the neutral position as shown in Figure 7.1.3.

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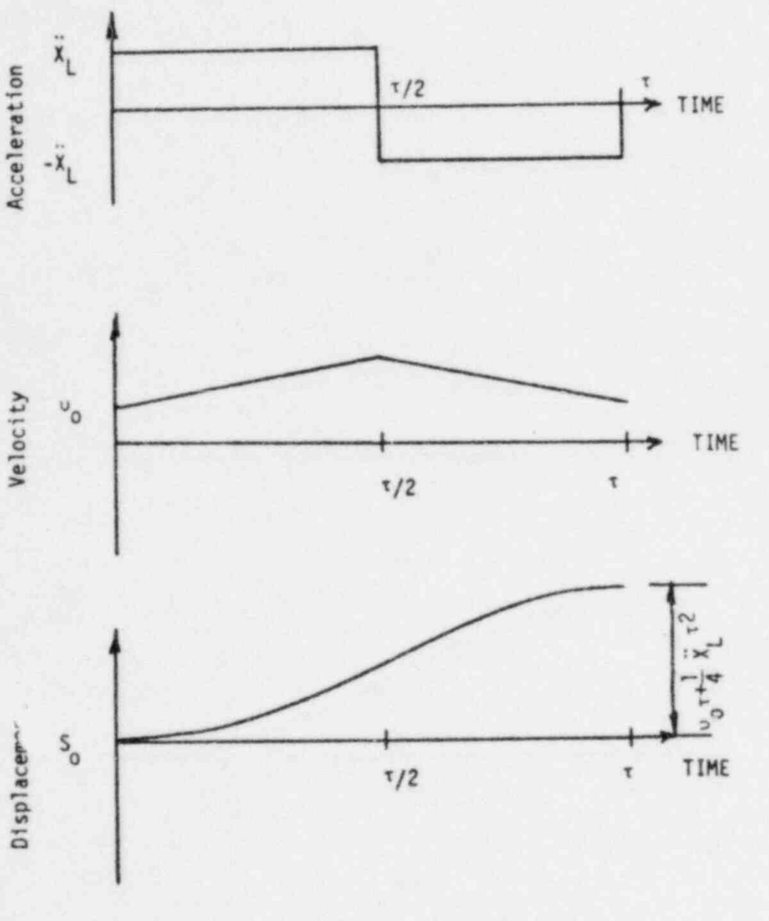

Figure 7.1.3 A.T.P. Residual Amplitude

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The mass undergoes an incremental displacement $\Delta X = 1/4\ \ddot{X}_L \tau^2$. If this situation were to continue to exist, the mass would continue to move away from the neutral position at a rate of $1/4\ \ddot{X}_L \tau^2$/cycle. As the mass moves away from its neutral position an additional force equal to kX is applied to the mass causing a shift in amount of time that $+\ddot{X}_L$ applies and $-\ddot{X}_L$ applies. For the first cycle,

$$\ddot{X}_L \text{ exists when } 0 \leq t \leq \tau/2$$

and,

$$\ddot{X}_L \text{ exists when } \tau/2 \leq t \leq \tau$$

For the second cycle, in which the applied load (resulting acceleration) is changed because of the added force (kX) the following situation exists,

$$\ddot{X}_L \text{ exists when } 0 \leq t \leq (\tau/2 - \Delta)$$

and,

$$\ddot{X}_L \text{ exists when } (\tau/2 - \Delta) \leq t \leq \tau$$

The Δ v. .ue for each cyle of loading varies because the total force (F sin Ω - kX) varies. The value of Δ increases during each cycle of loading until the maximum displacement (zero velocity) of the shadow response occurs, at which time, Δ reaches a maximum Δ_{max}. The cycle then continues with Δ decreasing until the time shift reaches its negative maximum $-\Delta_{max}$ at which time the shadow response is at its minimum position. The shadow response represents the time integral of the acceleration, as shown below.

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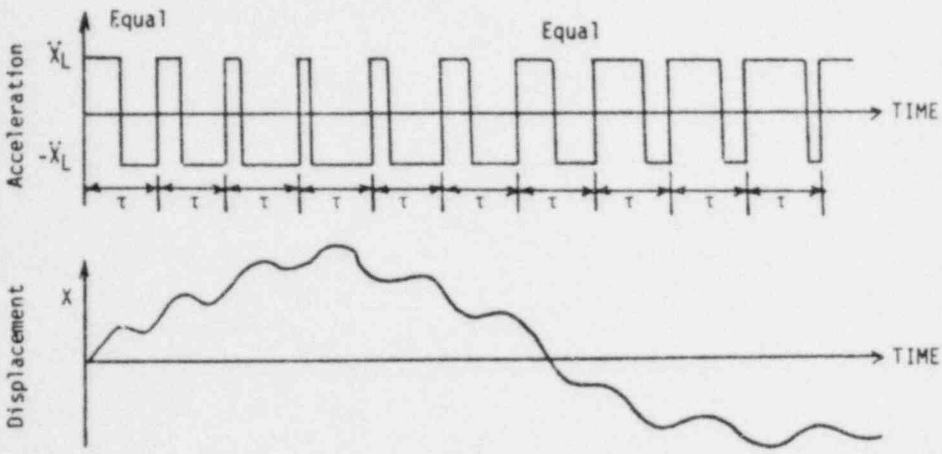

This shows how the shadow response is created. The higher frequency perturbations seen in Figure 7.1.2 have a period equal to the period of the forcing function.

Considering the same lumped mass oscillator used in the previous example the response characteristics have been determined for various frequency ratios β, where

$$\beta = \frac{\text{Forcing Frequency}}{\text{Natural Frequency}}$$

This is shown in Figure 7.1.4 as a function of the A.T.P. The curve indicates the response sensitivity to the A.T.P. One can see that the response sensitivity is a function of forcing frequency and A.T.P. value.

Utilizing numerical analysis procedures the response spectrum for a single degree of freedom lumped mass is shown in Figure 7.1.5. The response of the system as a function of the forcing frequency is shown for several acceleration threshold parameter valu(The dashed line shown on this figure represents the

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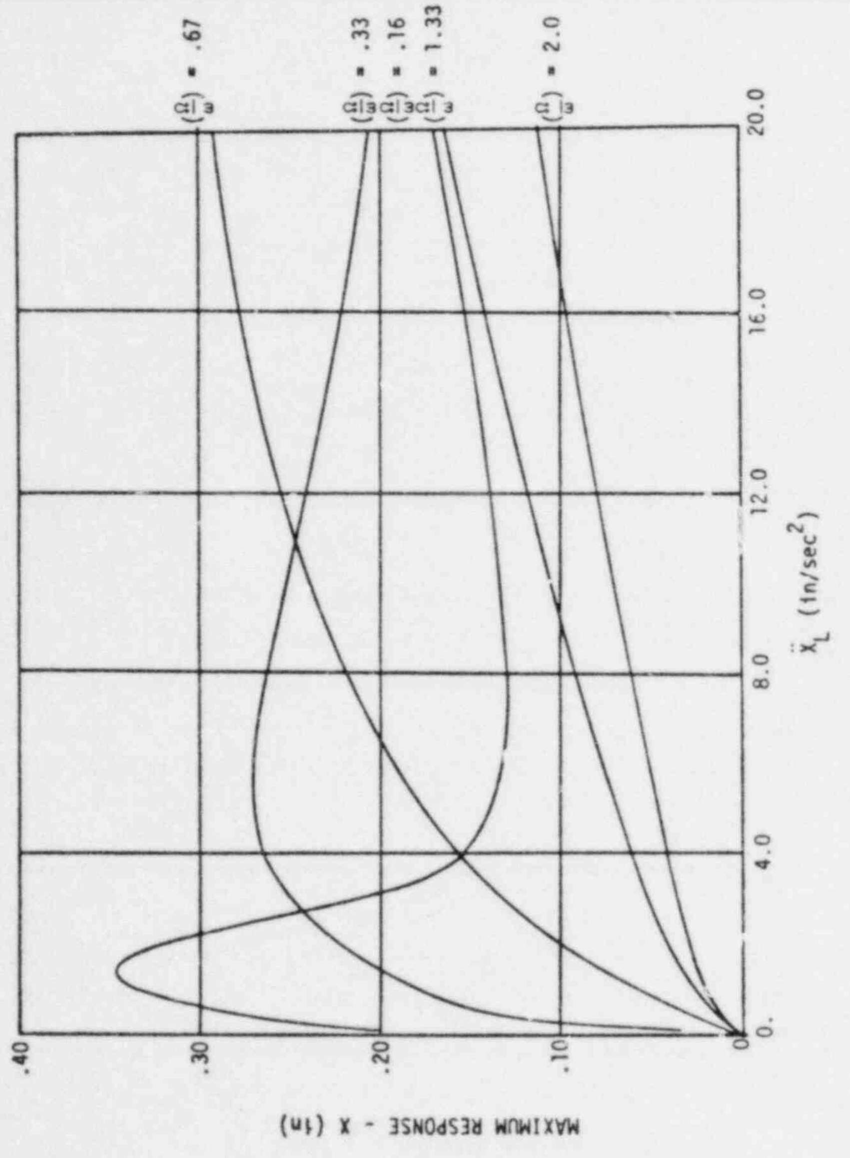

Figure 7.1.4 Response Characteristics Due to Acceleration Threshold Parameter

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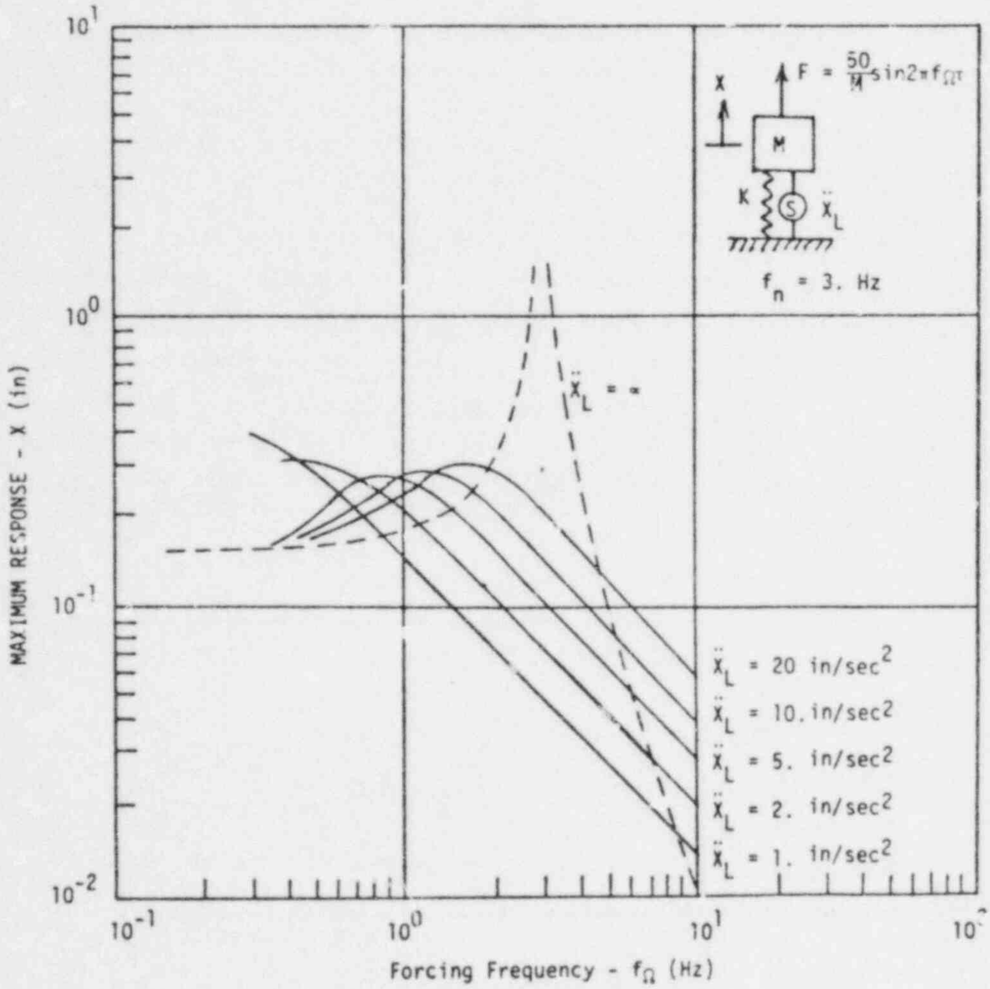

Figure 7.1.5 Response Spectrum for Lumped Mass Oscillator

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response if no snubber were present, ($\ddot{X}_L = \infty$). The results presented for this one case indicate that it is possible to obtain a greater response, with an ideal snubber possessing an acceleration threshold characteristic, than one would get if no snubber were present.

The results f 's particular case indicate that for low frequency harmonic excitations (< 2 h .ne "restrained" response of the simple oscillator will exceed the unrestrained response for A.T.P. values that were considered. This situation also exists at high frequencies, however, with a diminished response amplitude. Therefore, the low frequency response characteristics are more important.

To develop greater insight into the response sensitivity of simple systems towards the A.T.P. the following evaluation is made. In the subsequent analyses dimensionless ratios are utilized.

The lumped mass systems are subjected to a harmonic base excitation, where $\ddot{X}_L$ represents the A.T.P. If $\ddot{X}_L \ll |\ddot{X}|$, where $|\ddot{X}|$ is the amplitude of the acceleration of the system without the A.T.P., the response of the system can be evaluated in terms of dimensionless parameters.

Beginning with the differential equations of motion for each system,

$$[\text{Sys (a)}] \quad \ddot{\delta} + \omega^2 \quad = X_B\Omega^2 \sin \Omega t, \qquad \ldots (7.1.1a)$$

and,

$$[\text{Sys (b)}] \quad \ddot{X} + \omega^2 X = X_B\omega^2 \sin \Omega t, \qquad \ldots (7.1.b)$$

the following differential equation of motion representing the response for the system can be developed. (See Section 9, Analytical Methods.)

$$\ddot{Q} = \mathbf{C} \text{ sgn } |\sin \Omega t - Q| \qquad \ldots (7.1.2)$$

For system (a)

$$Q = \left(\frac{\omega}{\Omega}\right)^2 \frac{\delta}{X_B}, \quad \ell = \left(\omega^2 \frac{\ddot{X}_L}{|\ddot{X}_B|}\right) , \qquad ...\ (7.1.3a)$$

and, for system (b),

$$Q = \left(\frac{X}{X_B}\right), \quad \ell = \left(\Omega^2 \frac{\ddot{X}_L}{|\ddot{X}_B|}\right) \qquad ...\ (7.1.3b)$$

Equation (7.1.2) is solved using computerized numerical techniques. The results obtained from the solution of Equation (7.1.2) are shown in Figure 7.1.6. The response curve shows the peak values of response (X/X_b) as a function of ($\ddot{X}_L/|\ddot{X}_B|$). Although the pseudo response $\ddot{Q}$ (see Equation 7.1.2) appears to be a function of both ℓ and Ω, the Ω can be eliminated since ℓ also contains the frequency parameter. Therefore, the remaining parameters are (X/X_B) and ($\ddot{X}_L/|\ddot{X}_B|$). The relative displacement δ for system (a) and the absolute displacement X for system (b) are interchangeable in Figure 7.1.6.

The insights that can be derived from Figure 7.1.6 is limited, yet important. The observation most pertinent to this study concerns the response parameter (X/X_B). Results of the study indicate that (X/X_B) > 1 for certain values of ($\ddot{X}_L/|\ddot{X}_B|$). The implications of this are not obvious at first. It appears, however, that there can be response amplification in the presence of A.T.P., ($\ddot{X}_L$), consequently it is possible that response for a given system can be increased due to the A.T.P.

Next, consideration was given to non-harmonic input, specifically seismic ground motion. For the case of the lumped mass model, the sensitivity of the dynamic response to A.T.P. has been evaluated and presented in Figure 7.1.7. The El Centro ground acceleration was applied to a 6 Hz and 12 Hz lumped mass oscillator. The results indicate that as the $\ddot{X}_L$ value increases, the response increases. However, the response of the 12 Hz oscillator indicates, shaded area of Figure 7.1.7, that for certain values of the acceleration threshold parameter,

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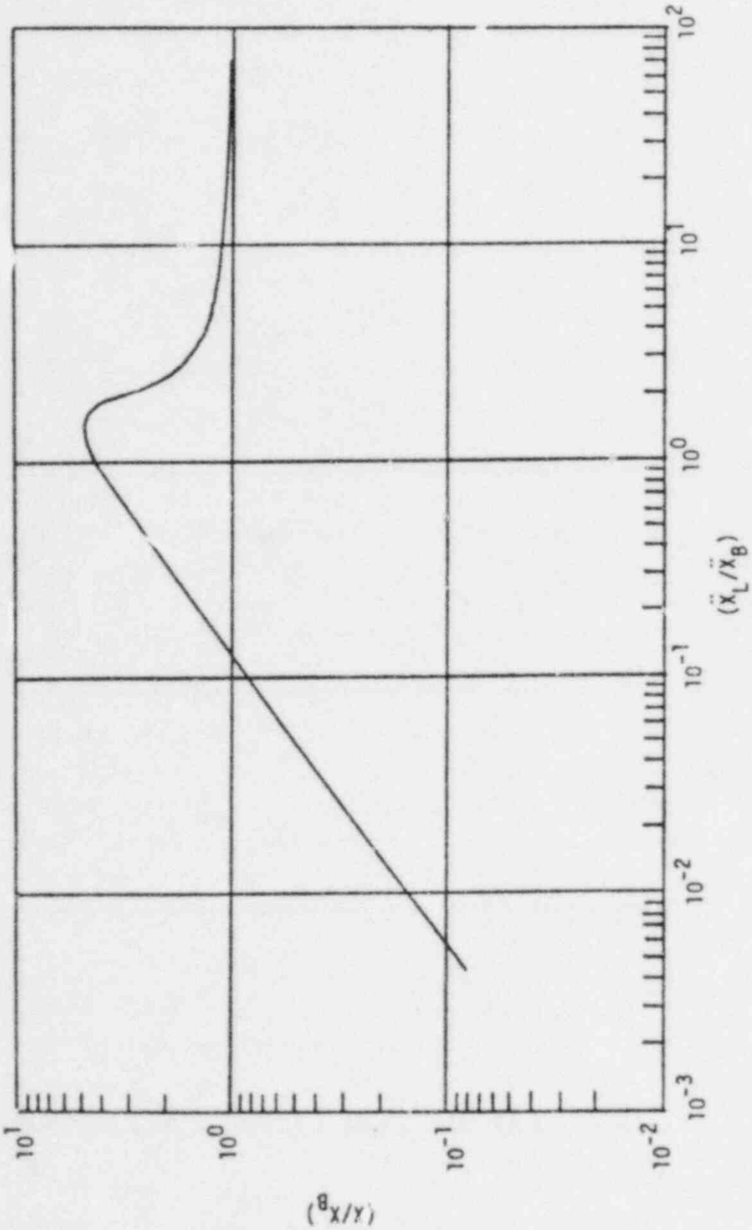

Figure 7.1.6 Acceleration Threshold Parameter Harmonic Input

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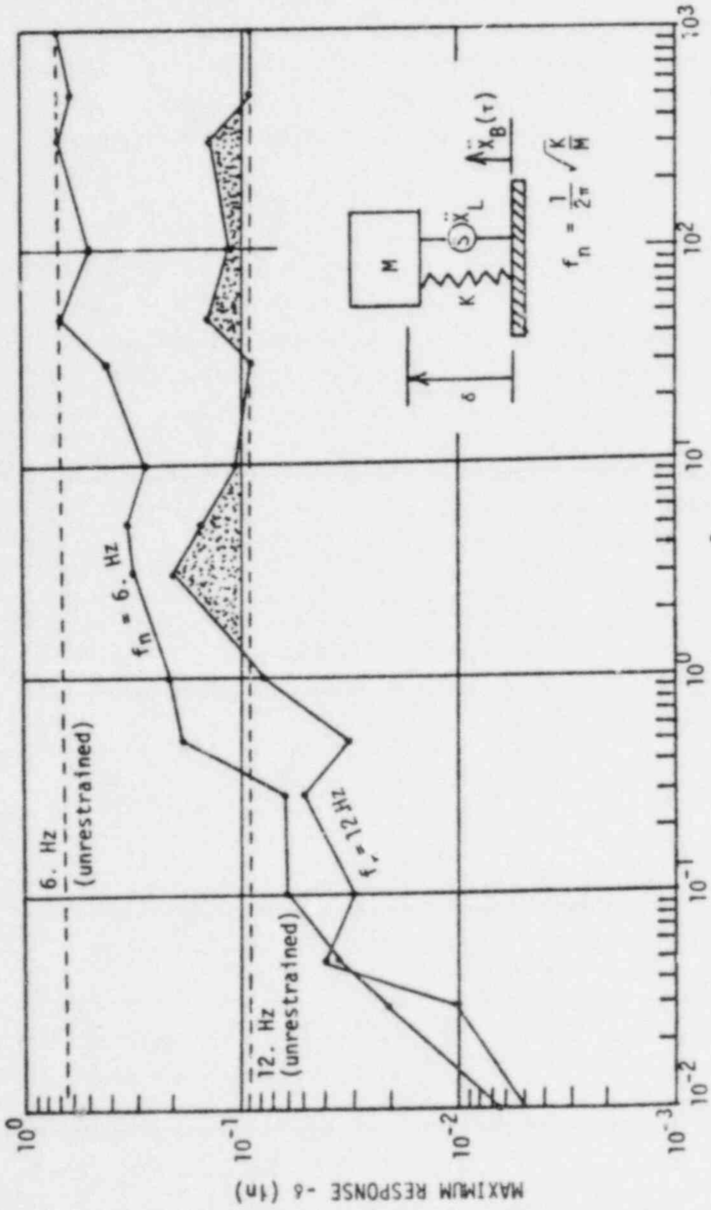

Figure 7.1.7 A.T.P. Response Due to El Centro Input

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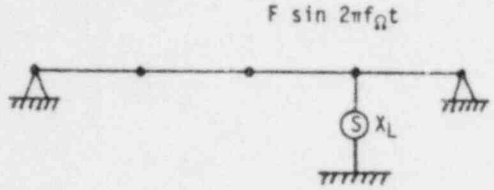
($\ddot{X}_L$), the response was greater than if the A.T.P. were not considered. Also shown in Figure 7.1.7 is the response of both systems in the absence of $\ddot{X}_L$. Although the response is greater for specific values of $\ddot{X}_L$ for the 12 Hz system, the total response is an order of magnitude smaller than the response for the 6 Hz system.

The next step in this study was to develop a structural model that would permit the response characteristics of simple beam models to be studied as a function of the effects of the acceleration threshold parameter. An analyses procedure that could solve this problem would be general in nature and be applicable to piping analyses. The analyses procedure that was developed is presented in Section 9.

The basic model incorporated lumped mass finite element techniques, utilizing modified modal analyses procedures. The result for the model are shown below are presented.

f_Ω = 2.5 Hz

F = 10 lbs



f_{n1} = 2.19 Hz

f_{n2} = 8.71 Hz

At this point in the study, it is appropriate to look at the results of one specific system. The system described above was subjected to an harmonic loading while attention was given to the transient displacement response. Although, only a few cycles of response were studied, trends could be easily detected.

The model shows a simple beam that was used for purposes of studying variations of the snubber parameter, $\ddot{X}_L$, acceleration threshold parameter. Figure 7.1.8 presents the results of this preliminary study. Shown on this figure are the response traces for the initial transient phases of loading for various $\ddot{X}_L$ values.

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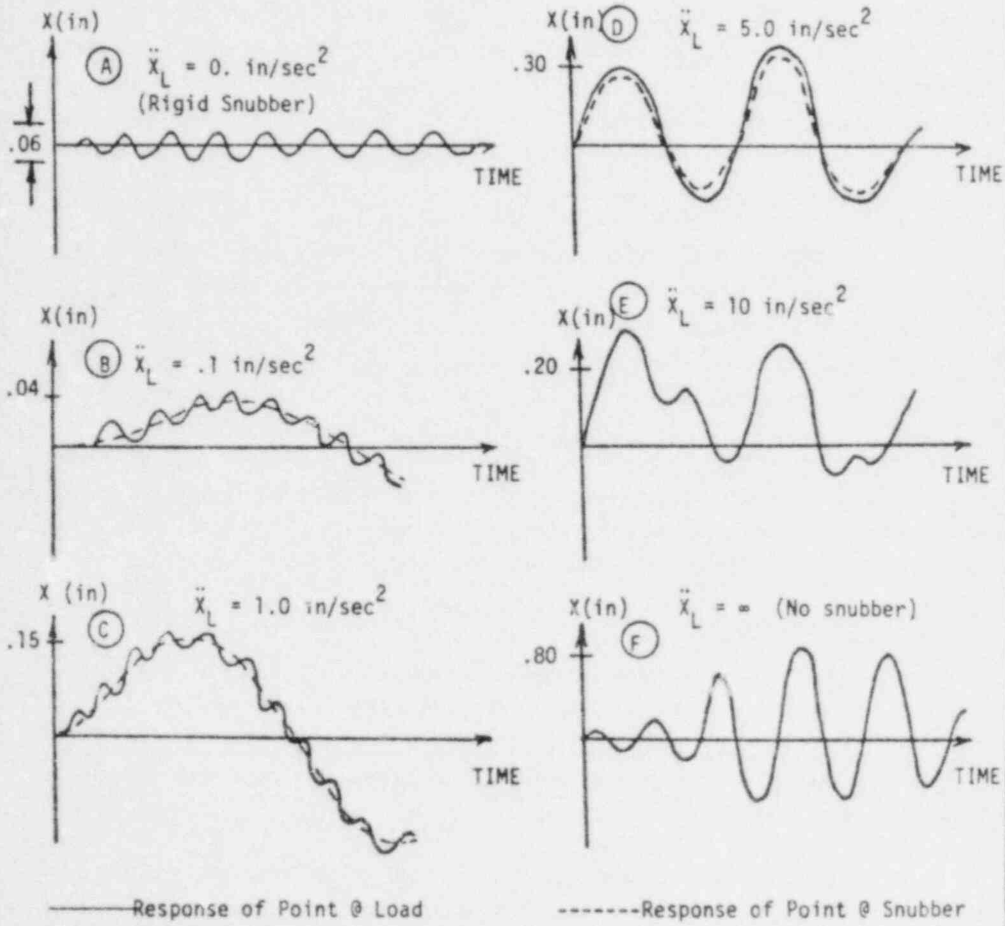

Figure 7.1.8 Beam Response Due to Acceleration Threshold Parameter

Figure A shows the response when $\ddot{X}_L = 0$, or when the snubber acts as a rigid support. Figure B shows the response when $\ddot{X}_L = 0.1$ in/sec^2. The superimposed dashed line in this figure is the response of the beam at the point where the snubber is attached. From this figure, one can see how the low cycle shadow response starts to appear as a superimposed response. Figure C indicates an even more pronounced effect which is seen in the increased response and Figure D shows the total response when the snubber response is at its apparent maximum.

When $\ddot{X}_L = 5$ in/sec^2, the steady-state response, that is, the component responding at the forcing frequency, is completely dominated by the low cycle shadow response. The response of the point being forced follows very closely the response of the snubber. As $\ddot{X}_L$ is increased, the snubber becomes "inactive" during certain portions of the applied loading. At this time, the response begins to take on a complex form as shown in Figure E. And finally when the snubber is removed ($\ddot{X}_L = \infty$), the response becomes predictable.

It is apparent from this example that if the standard engineering practice of representing a snubber as a rigid restraint were used ($\ddot{X}_L = 0$) - A, the response could be underpredicted by a factor as high as 10. ($\ddot{X}_L$ - 5 in/sec^2) - D.

7.2 Viscous Parameters

Viscous properties represent a series of three parameters that are related to hydraulic snubbers and are "lumped" together. These parameters are: 1) lock velocity; 2) bleed rate; and 3) unlock criteria. Their functions are to initiate snubber operation, control snubber reaction, and disengage the snubber during dynamic loadings.

The operation of the snubber can be described briefly as follows: Pipe movement is permitted until the velocity reaches the locking velocity, at which time the snubber engages. This locking velocity is dependent on the viscosity of the hydraulic fluid and the preset compression of the poppet spring. After lockup, additional displacement of the snubber piston is through the bleed mechanism. The bleed velocity is a function of the load on the piston. The flow rate through the bleed orifice varies approximately linearly with the load on the piston rod. After lockup occurs, the piston velocity at the rated load of the snubber is defined as the "bleed rate" of the snubber. When the piston load drops below a certain value the poppet spring opens and free flow occurs with the snubber disengaged.

The viscous parameters are highly dependent on the physical properties of the hydraulic fluid, i.e., operation of the snubber is sensitive to both the viscosity and bulk modulus of the fluid. Since the scope of the study involves only response sensitivity to the snubber parameters, this study will not include hydraulic fluid physical variations or any other considerations directly involved with operation of the snubbing device that control the magnitudes of the viscous parameters. It is recognized that temperature and fluid volume have an effect on hydraulic snubber performance, however, this study only considers the parameter and not the cause.

The bleed rate is a load dependent property which is defined in terms of the rated load of the snubber. Since bleed rate and rated load are connected with each other a new parameter referred to as the damping parameter "C" is introduced. This parameter has units $[\frac{\text{FORCE-TIME}}{\text{LENGTH}}]$ which are the same as the classical viscous damping coefficient and represents the ratio of the snubber rated load to bleed rate at the rated load,

$$C = \frac{\text{RATED LOAD}}{\text{BLEED RATE}}$$

Therefore, throughout the following analysis this term will be used in place of bleed rate.

Considering the range of viscous parameter values of hydraulic snubbers, i.e., the rated load capacities and nominal bleed rate values, the snubbers have viscous damping parameters C that are much greater than the critical damping value for that of the system. Consideration of resonant influences is not required in the investigation of viscous parameters. The effect of resonance on system response may have a slight effect for large $\dot{X}_L$, lock velocity values.

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This effect is not consi ered significant in the practical range of lock ve oci- ties ($\dot{X}_L$ < 1.0 in/sec).

For commercial snubbers the range of interest for the viscous damping parameter is $10^4 < C < 10^6$ lb-sec/in. The practical range of interest for the lock velocity is $0.01 < \dot{X}_L < 1.0$ in/sec.

Consider the single degree of freedom lumped mass oscillator, excited by a harmonic base motion. The initial transient response for this system is shown in Figure 7.2.1. Since the system is critically damped the transient response does not contain free vibration components. The transient response consists of the steady-state component plus an exponential decay component. Due to the initial shift developed during the first cycle of loading the maximum response occurs during the first loading cycle for harmonic excitations. The actual peak-to-peak response maximum occurs later in time. Consequently a reasonable estimate for the maximum displacement would be twice the steady-state amplitude. Any trends that are established are based on steady-state harmonic analysis and reflect response trends for the transient response regime.

With the large number of permutations of the various system parameters, the present study was limited to a few selected cases. Since the dynamic response appears to be more sensitive to lock velocity than to the other viscous parameters, this parameter is discussed first.

Since applied force, forcing frequency, and natural frequency are not viscous parameters the first task was to determine the effects on system response. The response data presented in Figure 7.2.2 is used to evaluate forcing frequency effects on the lock velocity response sensitivity. In this study various harmonic excitations were considered in connection with a given dynamic system. The response at small lock velocities ($\dot{X}_L$ < 0.03 in/sec) for the frequencies considered is practically independent of lock velocity. This is reasonable since the snubber is active during almost the entire excursion. In this case the response can be approximated by

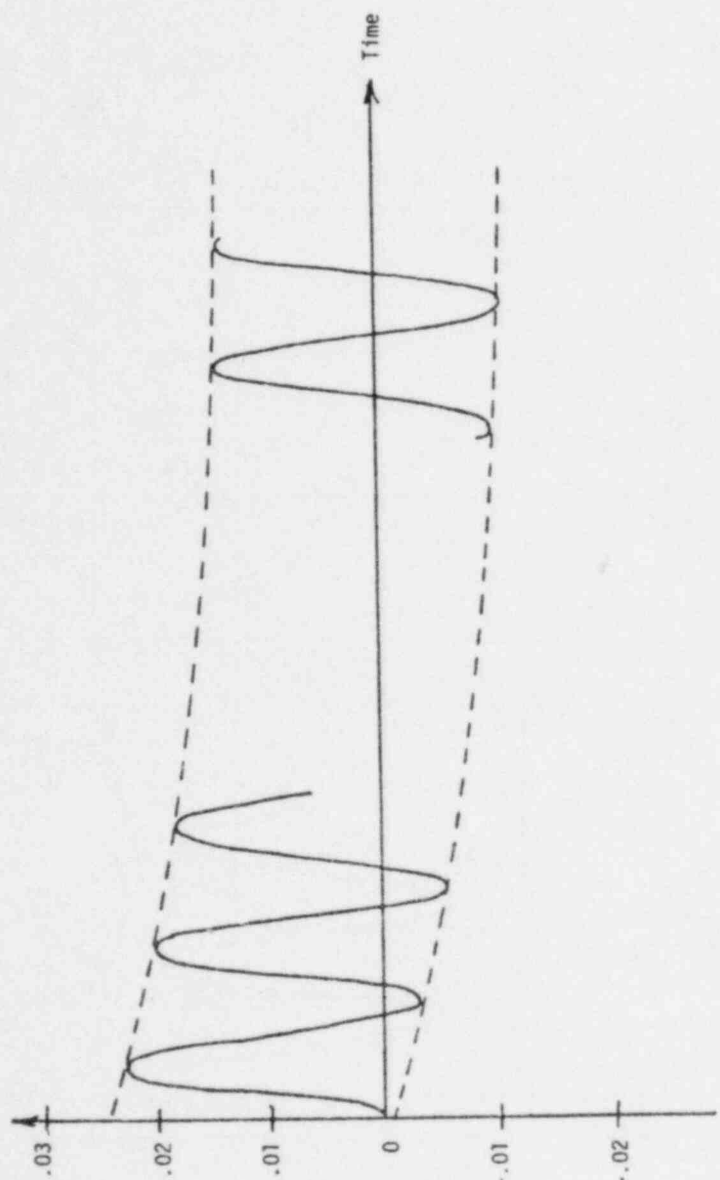

Figure 7.2.1 Transient Response for Harmonic Input
(Critically Damped System)

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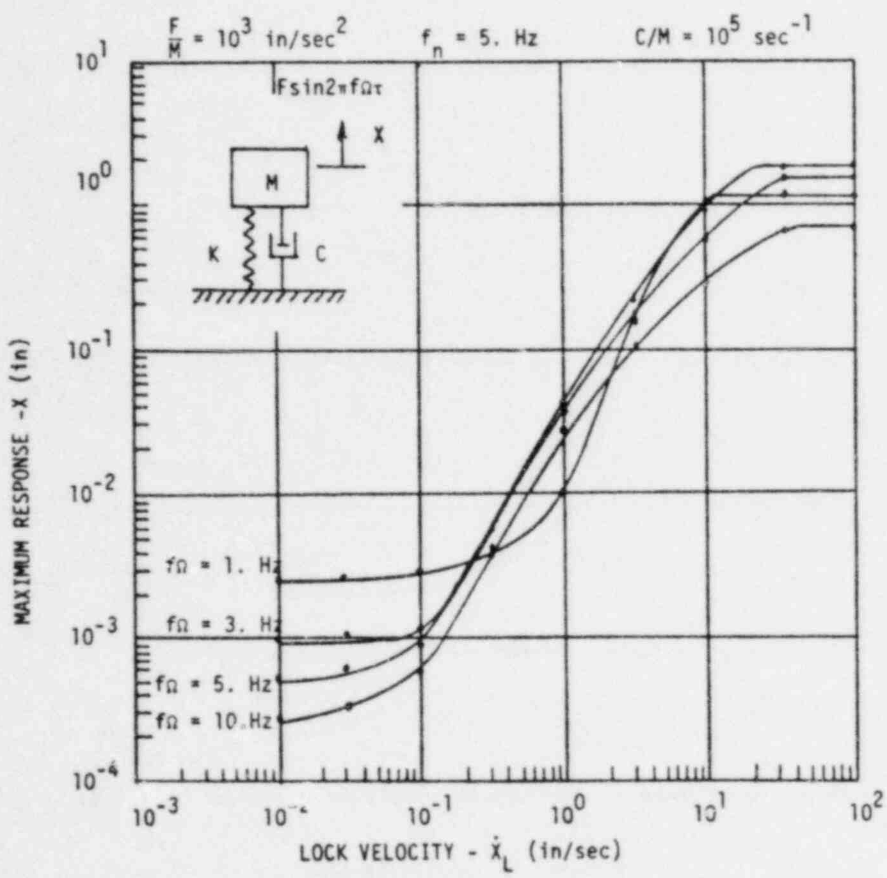

Figure 7.2.2 Forcing Frequency Effects on Viscous Response

$$\frac{X}{X_{stat}} = \frac{1}{2\xi \left(\frac{\Omega}{\omega}\right)}$$

where

ζ = Critical damping ratio >> 1

Ω = Forcing frequency

ω = Natural frequency

The response for large lock velocities ($\dot{X}_L$ > 10 in/sec) is also independent of lock velocity. In this case the response velocity does not exceed the lock velocity and snubber restraint is not experienced. For this region the response is

$$\frac{X}{X_{stat}} = \frac{1}{\sqrt{[1 - (\frac{\Omega}{\omega})^2 + (2\xi \frac{\Omega}{\omega})^2}}$$

Between the two extremes of lock velocity, 0.03 < $\dot{X}_L$ < 10 in/sec, the response sensitivity to the lock velocity appears to be similar for each of the different harmonic forcing frequencies studied. The free response amplitude is reduced by 95 percent for lock velocities less than 1 in/sec (Figure 7.2.2).

An important question in seismic design is "What range of locking velocity will limit the response of the system to acceptable limits?" For practical purposes it is reasonable to presume that any snubbing device that reduces motion to five percent of its unrestrained value is acceptable. Based on these limited observations locking velocities less than [illegible] will perform this task satisfactorily.

Since the previous results were obtained for a specific load value, (F/m), the next step was to determine what effects the applied load has on system response. The results shown in Figure 7.2.3 are used for making this determination. As in the previous example a harmonic loading was employed. However,

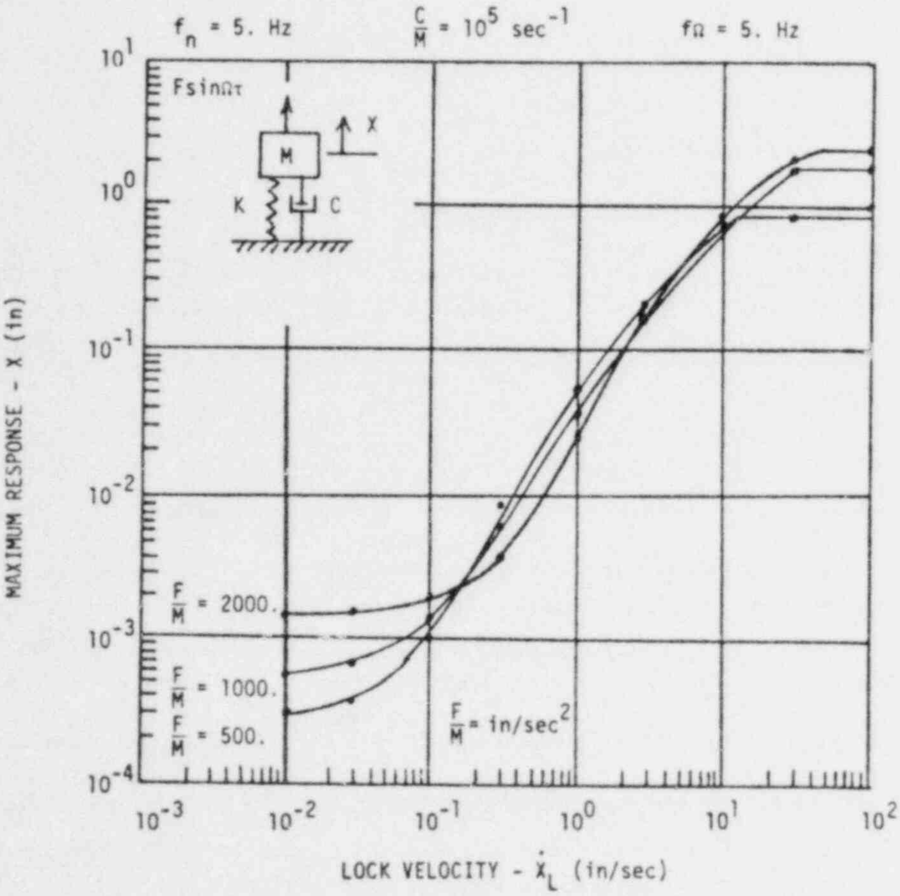

Figure 7.2.3 Load Effects on Viscous Response

in this example the applied frequency is fixed and is equal to the undamped natural frequency of the oscillator.

At low lock velocities, $\dot{X}_L < 0.03$ in/sec, the response is proportional to applied load, (F/m). The response is similar to that of a "classical" highly damped lumped mass oscillator. For very high lock velocities, the response is also proportional to the applied load, with the same response characteristics as a single degree of freedom undamped lumped mass oscillator. For the intermediate values of lock velocity the applied load, (F/m), appears to have a less significant effect on response than at the lock velocity extremes. It appears from these limited observations that the unrestrained free response amplitude will be reduced by 95 percent for lock velocities less than 1 in/sec regardless of the applied load, (F/m).

Since large motion seismic events may produce ground accelerations which approach 1 g a practical value of (F/m) can be deduced from this. A typical loading is

$$\frac{F}{m} = \frac{W}{W/g} \text{ (1.g loading) } = g = 386.4 \ \frac{\text{in}}{\text{sec}^2}$$

and

$$\left(\frac{F}{m}\right) \approx 350 \rightarrow 400 \ \left(\frac{\text{in}}{\text{sec}^2}\right)$$

Figure 7.2.4 indicates the response sensitivity for a different viscous damping parameter, $C/m = 10^3$ (sec^{-1}). This value is considerably lower than those encountered in typical systems, however, it indicates similar sensitivity trends to those observed for $C/m = 10^5$ (sec^{-1}).

The next step was to evaluate response sensitivity for various C (bleed rate) values as a function of lock velocity. Two sets of data were generated for this purpose and both studies utilize harmonic excitations. The input for these

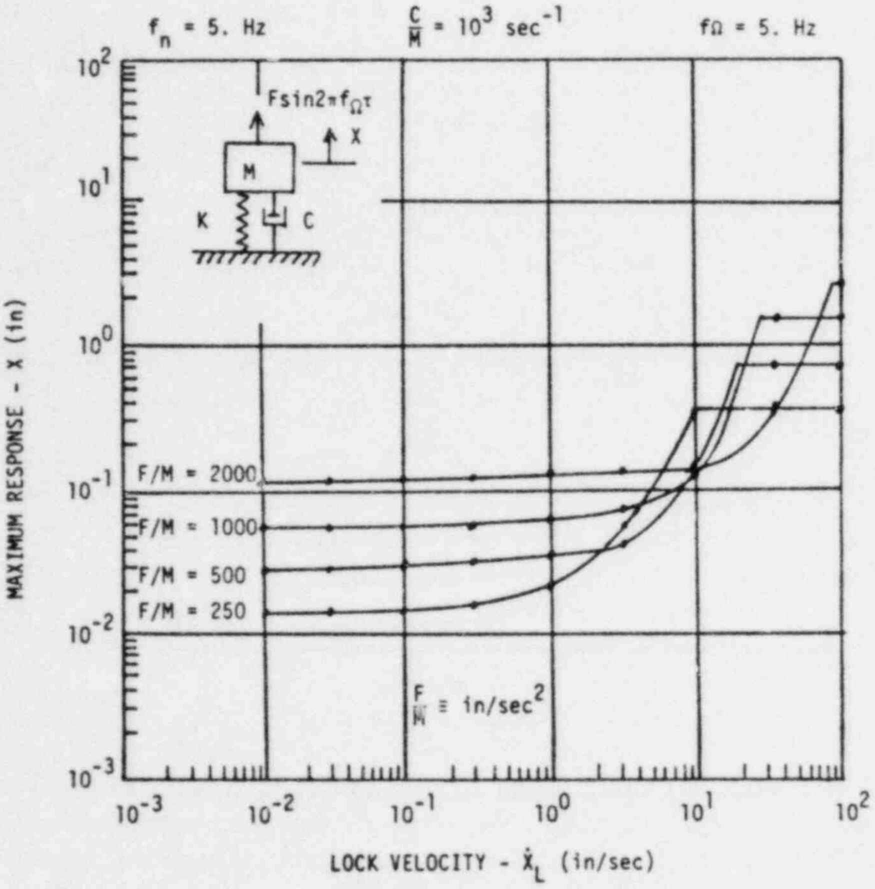

Figure 7.2.4 Load Effects on Viscous Response

studies, the results of which are shown in Figures 7.2.5 and 7.2.6 was a harmonic base acceleration with a peak acceleration of 0.348g and a frequency of 6 Hz. This acceleration level was selected because it has the same maximum input level as the El Centro earthquake. The present study was extended to consider this earthquake for additional comparisons. Figure 7.2.5 shows the response results for a 6 Hz natural frequency system while Figure 7.2.6 shows the results for a 12 Hz resonant system.

Inspection of Figures 7.2.5 and 7.2.6 indicates that the response characteristics are nearly identical for lock velocities less than 1 in/sec independently of the damping value. For heavily damped systems the response is independent of the system natural frequency. It can be shown that the steady state relative displacement can be approximated by

$$\delta = \left(\frac{m}{C\Omega}\right)|\ddot{X}_B| = \left(\frac{\Omega m}{C}\right)X_B \quad ,$$

where

δ = Relative displacement (in)

$|\ddot{X}_B|$ = Base acceleration (in/sec^2) $(\Omega^2 X_B)$

C = Damping (lb-sec/in)

Ω = Harmonic forcing frequency (rad/sec), and

m = System mass (lb-sec^2/in)

However, as the lock velocity increases the response becomes increasingly sensitive to the natural frequency of the system and when the lock velocity is sufficiently large the response l[illegible]es that of an undamped oscillator whose steady state response can be expressed as

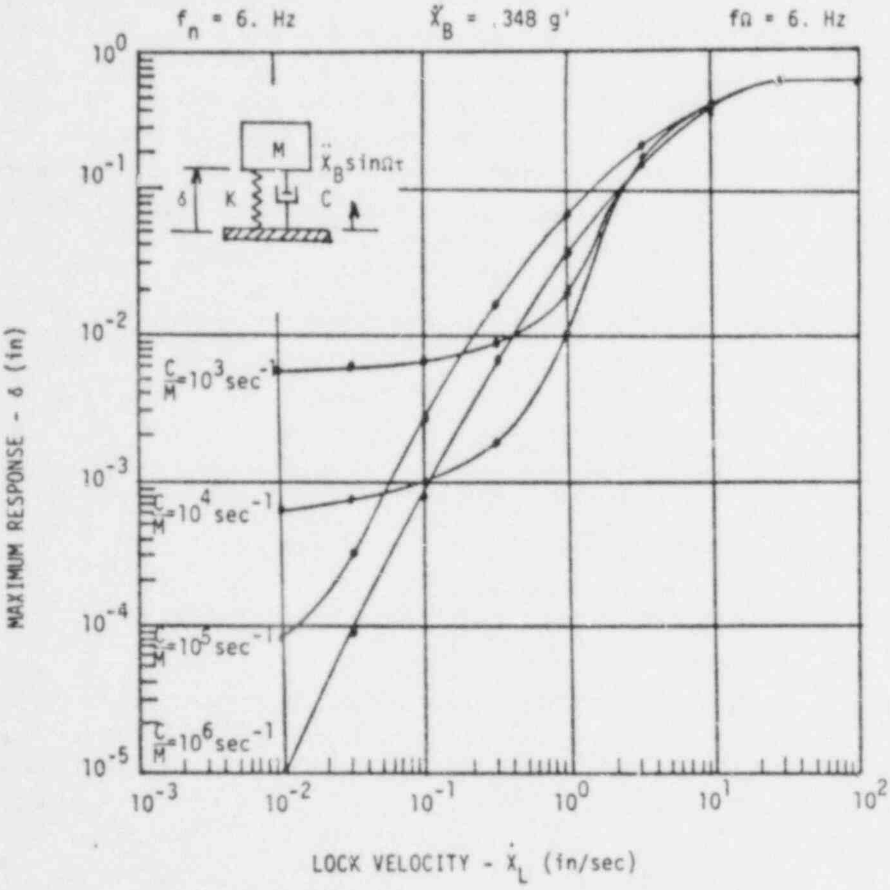

Figure 7.2.5 Bleed Rate Effects on Viscous Response

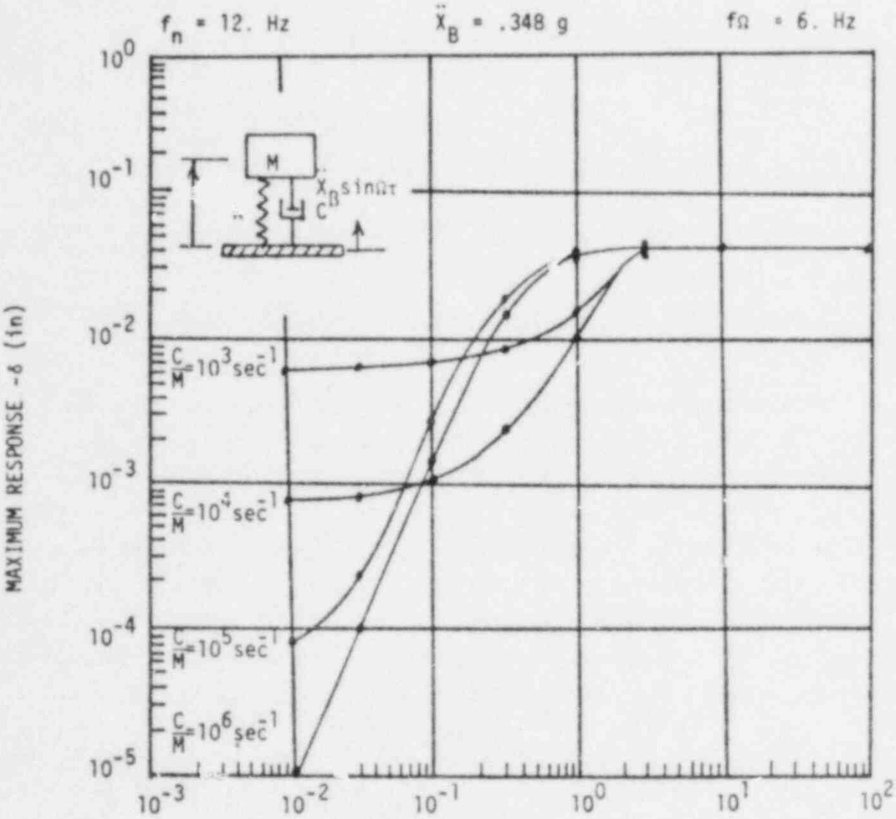

Figure 7.2.6 Bleed Rate Effects on Viscous Response

$$\delta = \frac{X_B \beta^2}{(1 - \beta^2)} = \frac{|\ddot{X}_B| \beta^2}{\Omega^2 (1 - \beta^2)}$$

where

Ω = Forcing frequency (rad/sec)

ω = Natural frequency (rad/sec)

$\beta = \Omega/\omega$

X_B, $|\ddot{X}_B|$ = Base displacement and accelerations, respectively - (in), (in/sec^2)

It appears that the response characteristics are almost identical for both systems for lock velocities less than 1 in/sec, however, for higher lock velocities the response becomes sensitive to the resonant characteristics of the system.

Since the response amplification was small for the 12 Hz system, relatively low lock velocities are required to reduce the response to 95 percent of the unrestrained value - $\dot{X}_L < 0.2$ in/sec. It is anticipated that later studies will show that since the actual response is low, a detrimental effect will not be experienced for much larger lock velocities. Further insights can be obtained if response is plotted as a function of the viscous parameter $C \equiv (C/m)$ for various lock velocities. Figure 7.2.7 indicates that for a given lock velocity response may increase as the bleed rate is reduced (C is increased). This situation appears to exist when the lock velocity exceeds 0.3 in/sec.

The results presented for the limiting cases $\dot{X}_L = 30$ in/sec and $\dot{X}_L = 0$ in/sec can be explained, however, a physical interpretation of this phenomenon for intermediate values of $\dot{X}_L$ is not available at this time.

Figures 7.2.8 and 7.2.9 indicate response characteristics for the systems investigated in Figures 7.2.5 and 7.2.6, however, the input is the El Centro earthquake ground motion rather than harmonic input. The system response

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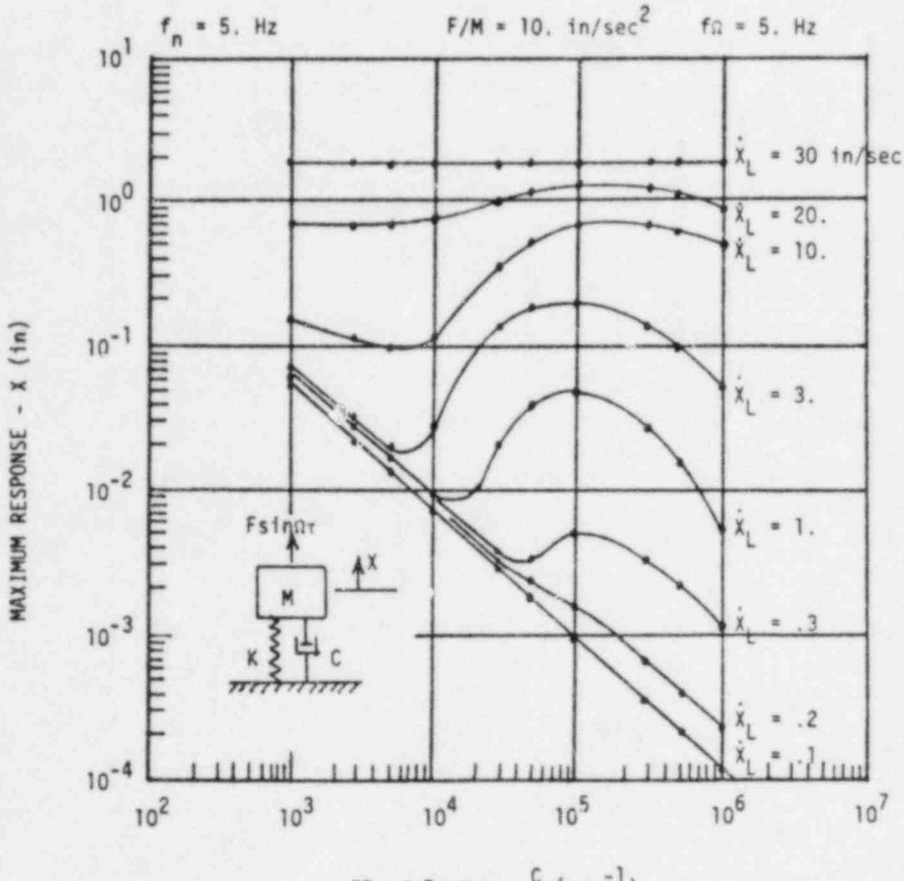


Figure 7.2.7 Lock Velocity Effects on Viscous Response

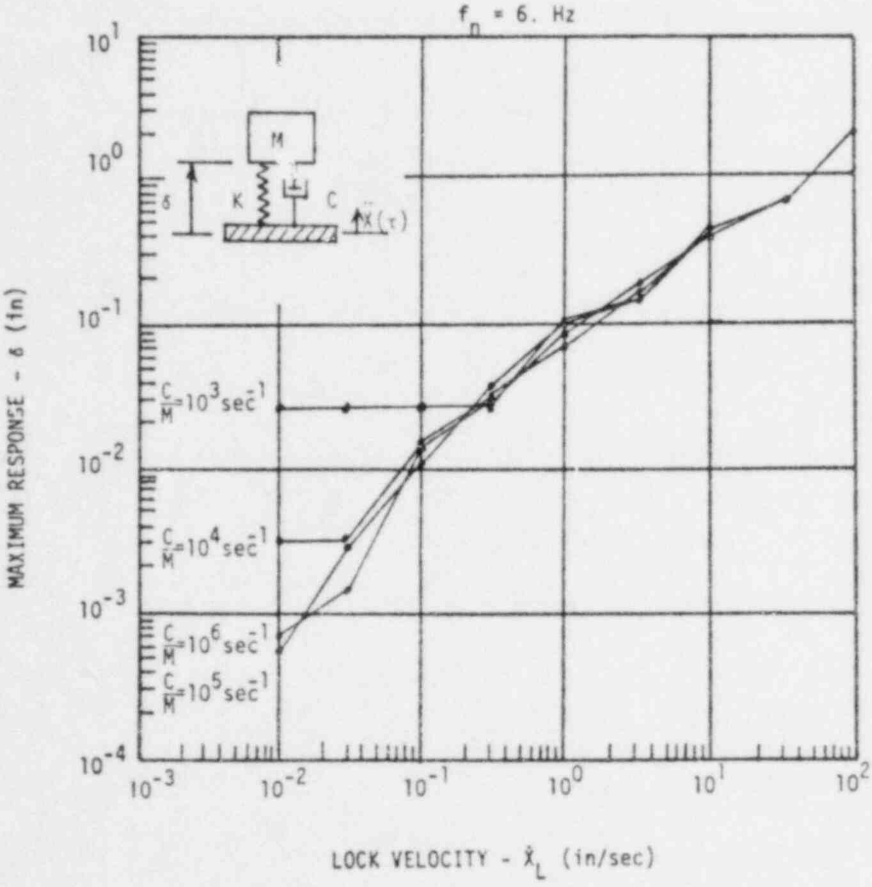
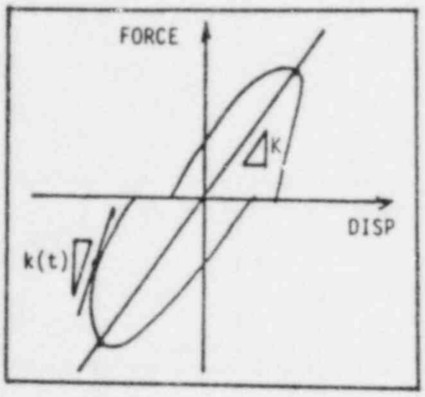


Figure 7.2.8 Bleed Rate Effects on Viscous Response
(Seismic Input)



The steady state load deflection relationship is shown above for a hydraulic snubber subjected to a harmonic excitation (a fixed frequency). One way of determining the effective stiffness is to draw a straight line between the positive and negative peak responses as shown.

This represents the effective or average stiffness. The instantaneous stiffness is the instantaneous slope of the response curve, i.e., k = k(t). The effective stiffness range for a 2½-inch bore hydraulic snubber is approximately 100,000 to 300,000 lb/in.

The stiffness characteristic of snubbers is the one most important parameter affecting dynamic response of piping systems. This applies to the "effective" stiffness of the snubbing devices, Figure 7.5.1, or to "normal" linear stiffness characteristics, i.e., simple linear springs. The "effective" stiffness of a snubber is a function of the following parameters: fluid bulk modulus, clearance, lock and bleed velocities, acceleration and velocity threshold parameters, and snubber support structure characteristics.

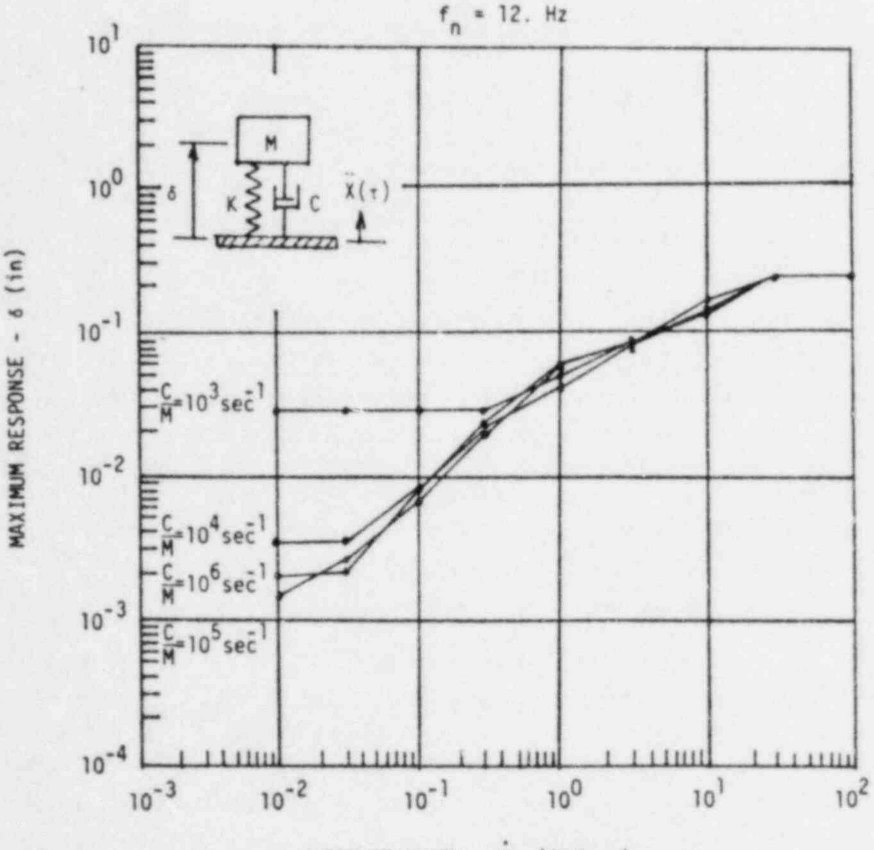


Figure 7.2.9 Bleed Rate Effects on Viscous Response

characteristics are very similar for El Centro motion and for harmonic inputs (both 0.348 g maximum acceleration) as can be seen from Figures 7.2.5, 7.2.6, 7.2.8 and 7.2.9. For realistic viscous damping parameters ($10^4 < C < 10^6 \frac{\text{lb-sec}}{\text{in}}$) the response appears to be insensitive to the damping (bleed rate) for lock velocities greater than 0.03 in/sec (0.18 in/min). The response can be represented by a straight line on log-log paper as indicated in Figures 7.2.8 and 7.2.9. Although the maximum input g levels are the same for both models, the El Centro response is slightly greater than the harmonic input because the harmonic response represents the 1st cycle maximum response whereas the El Centro maximum response represents the maximum during the entire loading. Plotting the first cycle maximum response, the actual maximum response will occur for critically damped systems for all cases where $\dot{X}_L$ is sufficiently low enough so that the snubber can apply a restraining load. For the cases studied, $\dot{X}_L < 10$ in/sec or 600 in/min. Figures 7.2.10 and 7.2.11 show the transient response during the first 20 seconds of the seismic disturbance.

Taking a specific example where the system frequency is 6 Hz and C/m = 10,000 sec^{-1}, the response due to four different seismic distrubances is calculated and plotted in Figure 7.2.12. Response sensitivity and magnitude are similar for lock velocities that are not representative of extreme values. The largest differences in response occur at the extremes, i.e., when the response is independent of the lock velocity (fully engaged or fully disengaged). This characteristic property is also exhibited in Figure 7.2.3, where the response due to harmonic excitations is studied, and in Figures 7.2.8 and 7.2.9 where seismic input is studied. The results of this particular investigation suggest that the variations in frequency content and magnitude of the four seismic disturbances studied have very little effect on response sensitivity; the most significant factor is the amount of time that the velocity of the component causes the snubber to be engaged. For example, if the velocity of the component never exceeds the snubber lock velocity, the snubber will have no effect on system response, or if the unrestrained response velocity is large compared to the snubber lock velocity, the response will be insensitive to changes in lock velocity.

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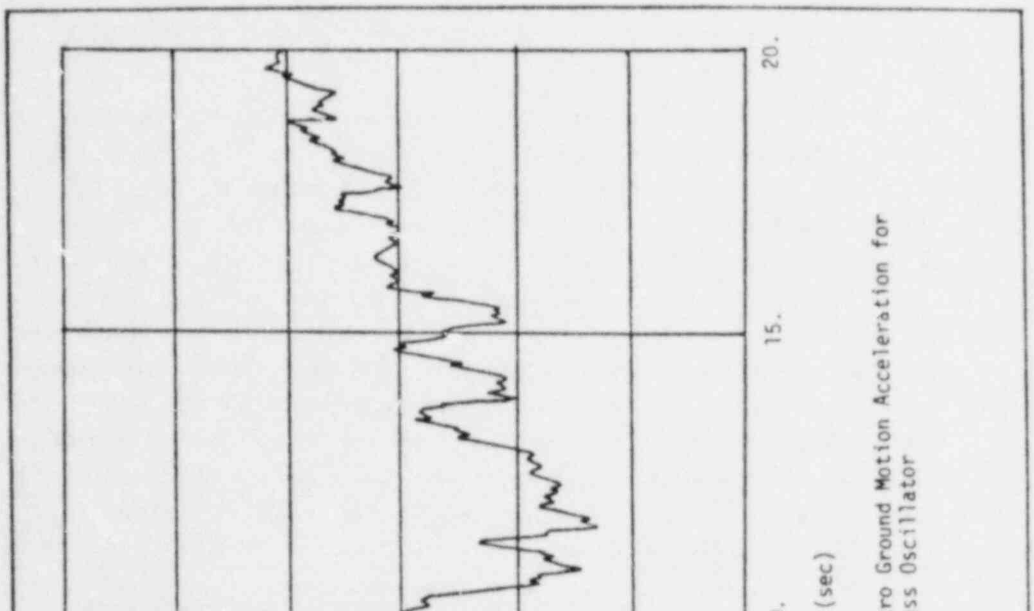

Figure 7.2.10 Response to El Centro Ground Motion Acceleration for 12. Hz Lumped Mass Oscillator

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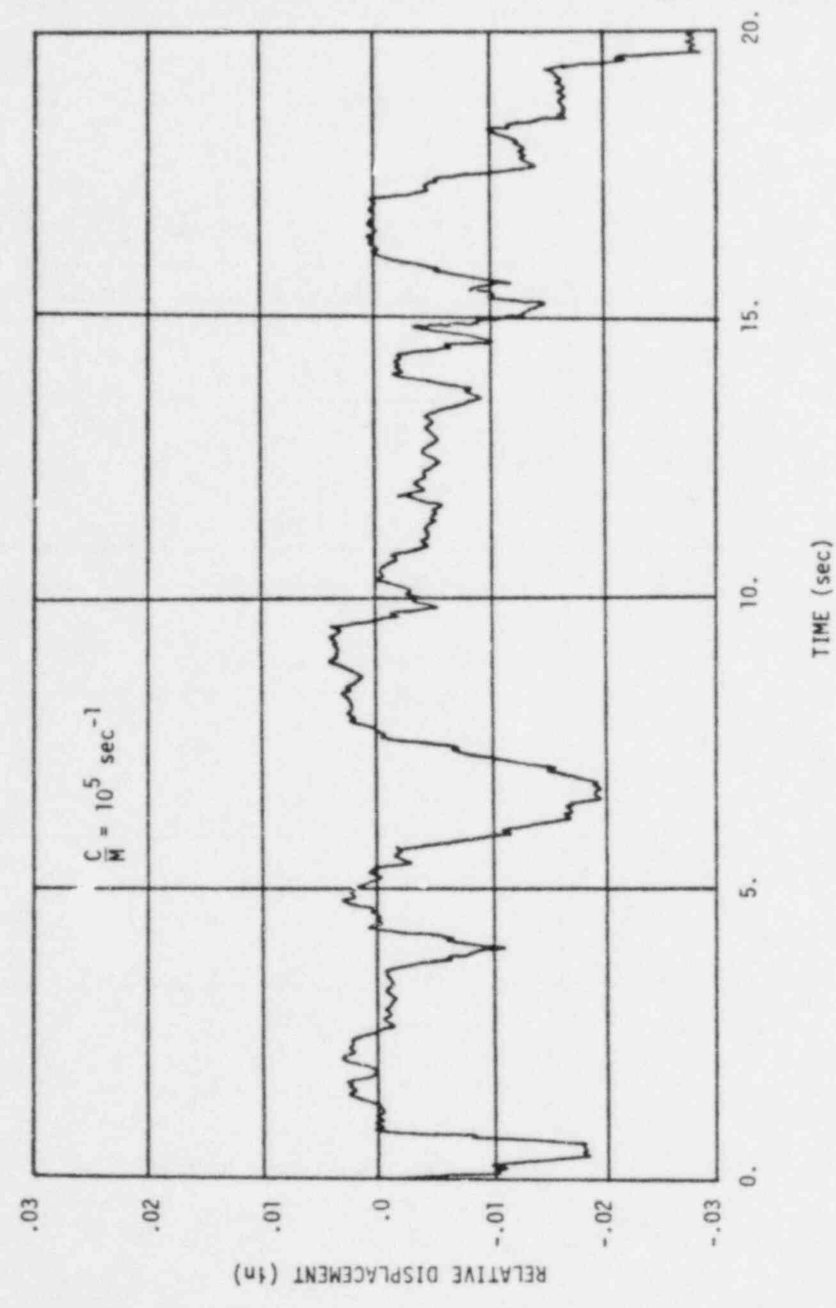

Figure 7.2.11 Response to El Centro Ground Motion Acceleration for 6. Hz Lumped Mass Oscillator

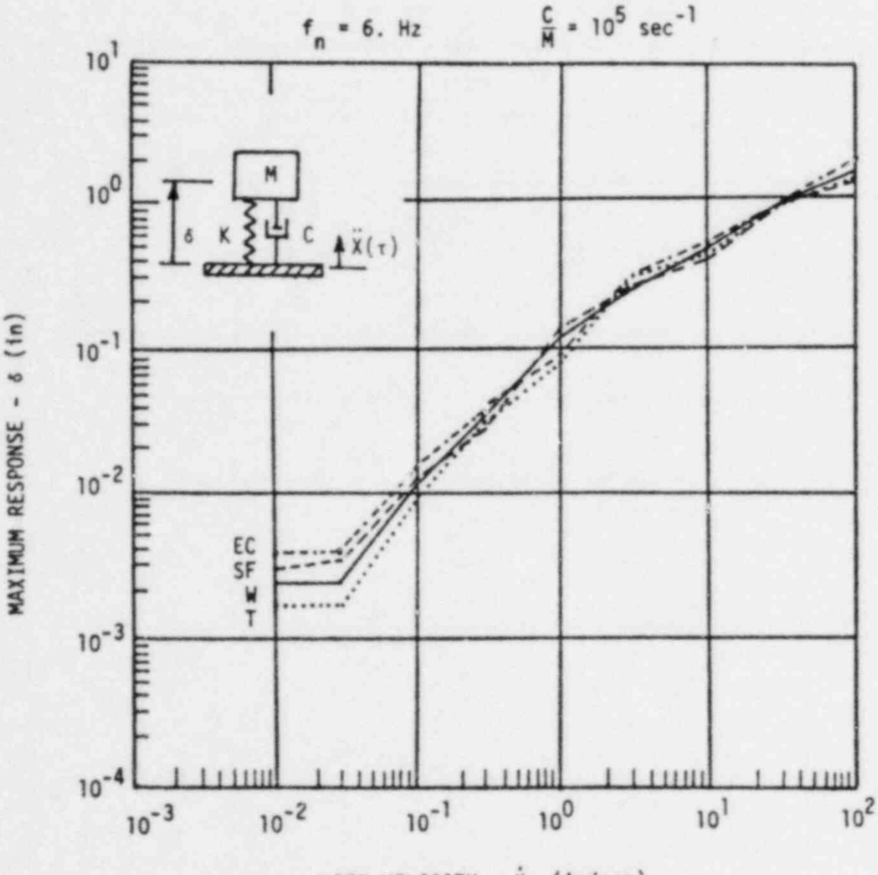

Figure 7.2.12 Seismic Effects on Viscous Response

Most of the studies concerning the viscous parameters have been made using the single degree of freedom oscillator. A logical extension of these studies would be to consider the snubber parameters in relation to a "typical" piping loop. The response analyses of a system enables one to evaluate sensitivity changes resulting for increased structural complexity. The system that will be used is shown in Figure 7.2.13. Although this model does not represent an actual system, it is typical of piping loops in that it has many features found in typical loops. It contains spring hangers, valves, rigid anchors, and a piping routing (although very simple).

Assume a typical hydraulic snubber is utilized in the model, i.e., a 50,000 pound rated load, a 10 in/minute bleed rate and a 10 in/minute lock velocity. The response of the system due to El Centro seismic excitation is studied. Three conditions are examined. Case 1) investigates the response without a snubber; Case 2) investigates the response with a typical (realistic) hydraulic snubber; and Case 3) investigates the response with a rigid snubber. The results of the study are summarized in the Table below. Deflections and stresses are presented for keypoints.

	W/O SNUBBER	WITH SNUBBER	RIGID SUPPOPT
Stress @ Fixed Anchor	12846 PSI	4691 PSI	31[illegible]7 PSI
Stress @ Snubber	4910 PSI	301 PSI	2336 PSI
Disp. @ Snubber	1.595 in.	.306 in.	0 in.

Consider comparing the results of the complex structural model with response characteristics developed from a simple 1-degree of freedom oscillator shown in Figure 7.2.9. Using the entire mass of the piping loop in evaluating c/m its value is 1,100 sec^{-1}. Based on the data presented in Figure 7.2.9, the ratio of the restrained response to free response is,

$$\frac{\delta_R}{\delta_F} = \frac{.35}{2.00} = .175$$

Based on the response, of the piping loop the same ratio is

$$\frac{\delta_R}{\delta_F} = \frac{.306}{1.595} = .191$$

The agreement appears good for this particular case. Consequently, the results for the simple single degree of freedom oscillators are relevant with regard to complex systems.

7.3 Clearance

Clearance is associated with nearly all snubber designs and with snubber installation hardware. The actual magnitude varies from negligibly small values to accumulative values ranging from 0.125 to 0.250 inch (extreme cases). The clearance effects become particularly important when load reversals take place. For this case, the actual clearance becomes equal to the full "plus to minus" range of the dead band, hence large clearance values may need to be considered.

When determining the effects of clearance on the snubbed system, the clearance parameter is nearly always considered with another snubber parameter. The consideration of the clearance parameter in the absence of other snubber parameters does not represent a realistic situation since snubbers in general are not rigid nor is the back-up support structure completely stiff. Certain insights can be gained, however, by investigating the effects of clearance with a rigid support.

Figure 7.3.1 shows the structural model that was used to determine the sensitivity of load, stress, and displacement to snubber gap. Stresses were evaluated for a series of gaps ranging from 0 (fixed support) to infinity (no support). The dimensionless ratio (δ_G/δ_F) is plotted on the abscissa. For values of (δ_G/δ_F) > 1, the response is unaffected by the snubber gap because it is greater than free response.

MODE	NATURAL FREQUENCY - Hz	
	WITH SNUBBERS	WITHOUT SNUBBERS
1	5.954	2.311
2	9.781	4.191
3	11.292	7.254
4	15.161	10.005
5	21.161	15.527

24" SCH 160 PIPING

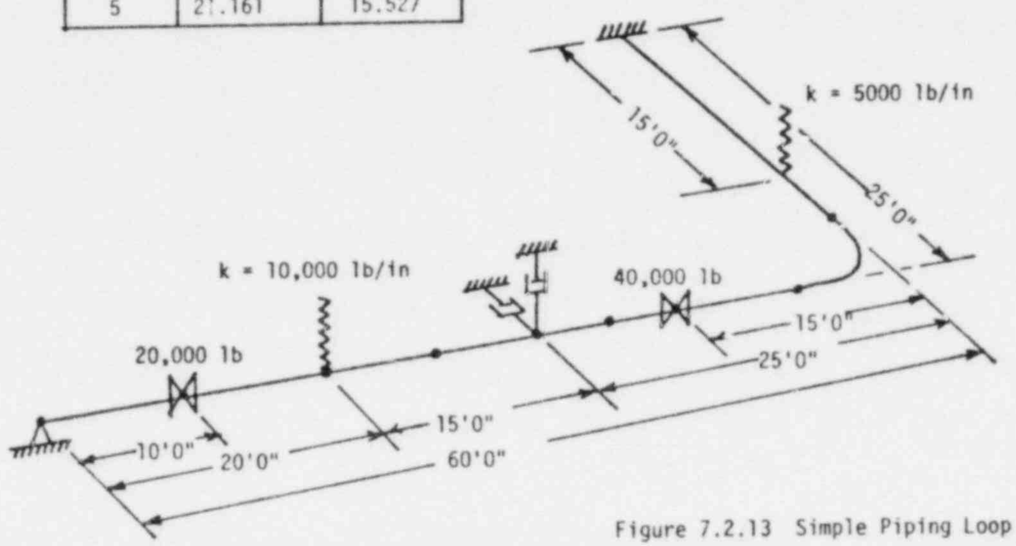

Figure 7.2.13 Simple Piping Loop

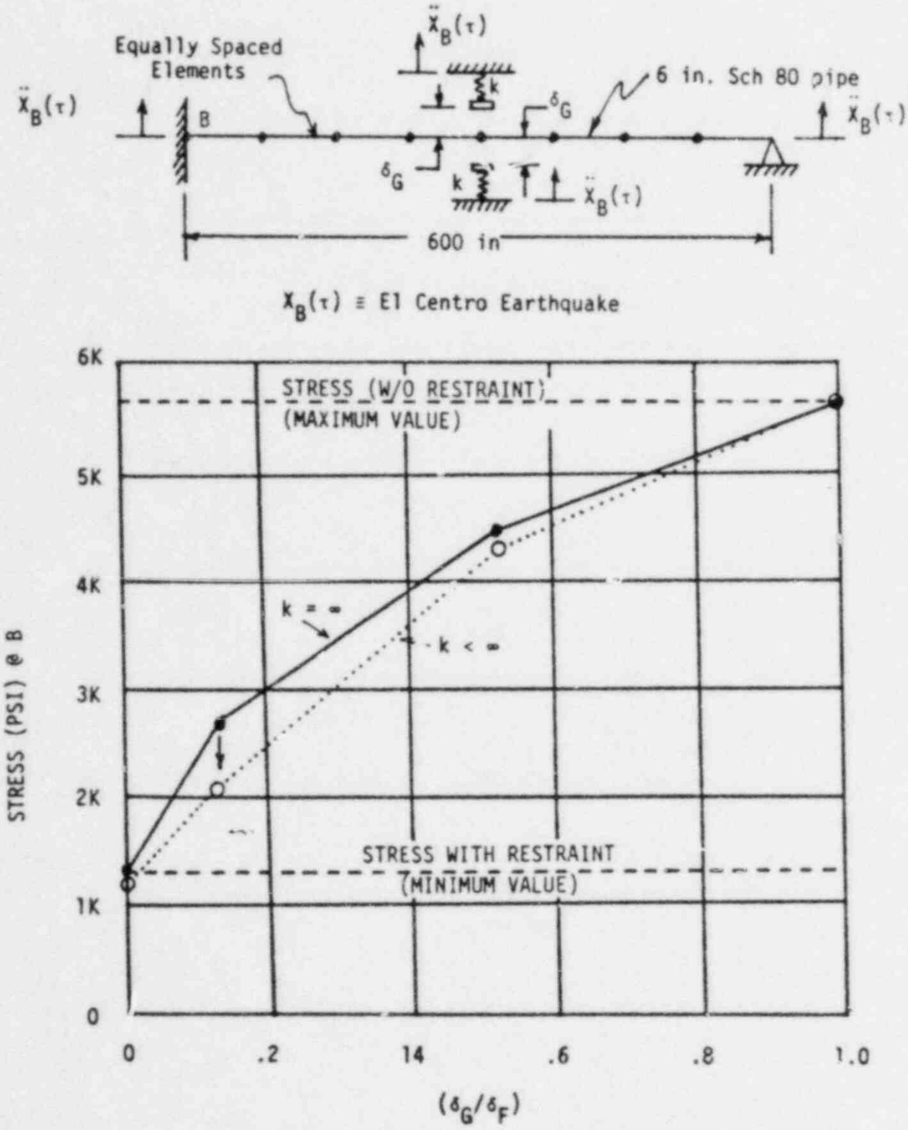

Figure 7.3.1 Clearance Effects On Stress

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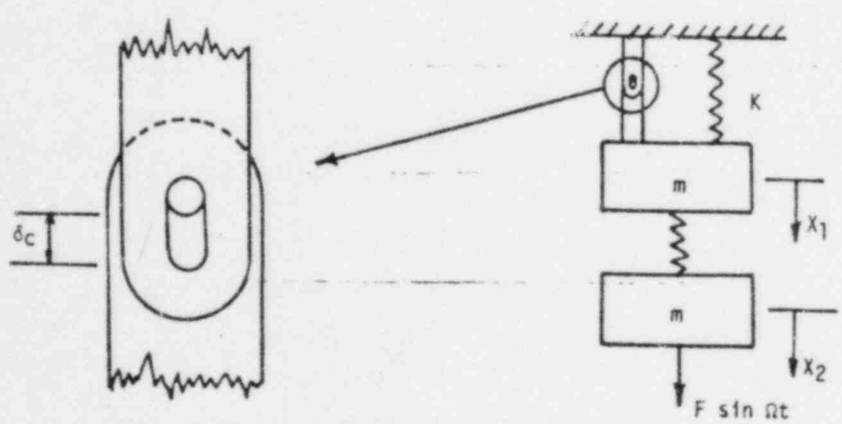
If the restraint were not truly rigid as is the case with a snubber, the stress would be reduced for a given (δ_G/δ_F) value as shown in Figure 7.3.1. The reduction in snubber reactor load (Figure 7.3.2) reduces the stress at point B. Increased support (snubber) flexibility does not, in general, reduce stresses at all points in a system. Figure 7.3.3 shows the effect of clearance on displacement response. The response at the snubber increases, with structural support flexibility. If the flexibility of the snubber is very large (soft spring), the natural frequency of the system may be affected and the response characteristics of the system will change.

Displacement response is reduced when the snubber is added to the system. This may not be the case for stresses, in particular, at the snubber location. The principle on which the snubbers are designed is that if the motion of a point in a system is reduced the overall response is reduced. Since the clearance parameter permits added motion, the overall reduction of system response is diminished with clearance. Situations can exist where response is either increased or decreased by the introduction of clearance.

Consider the lumped mass system shown below:



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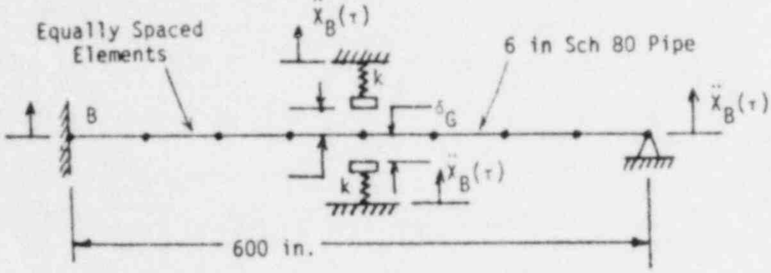


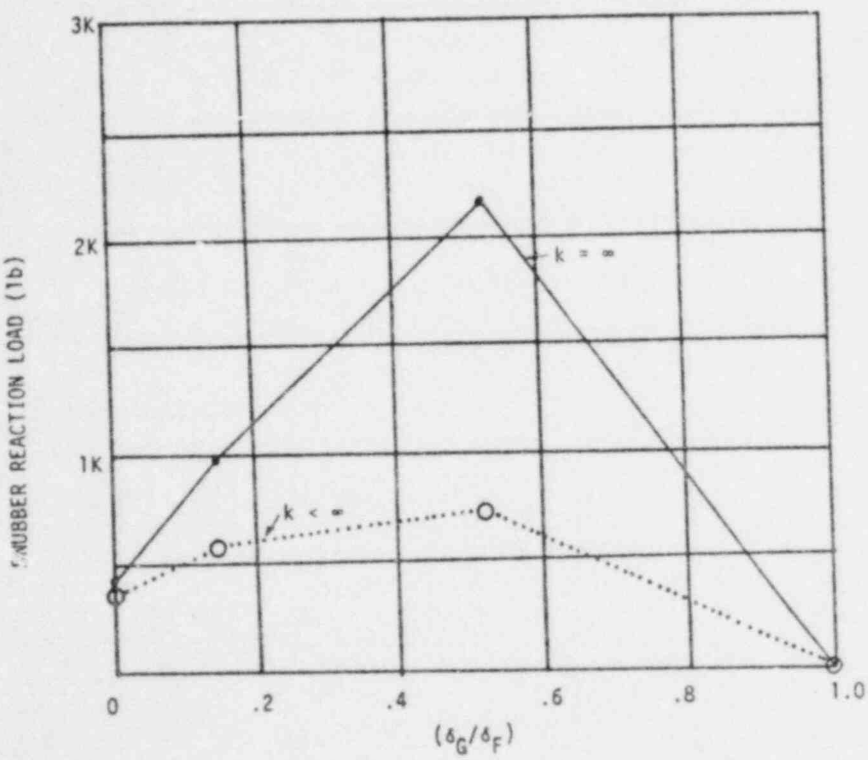
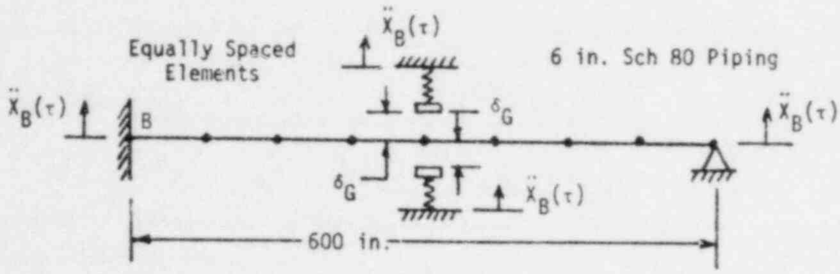


Figure 7.3.2 Clearance Effects on Snubber Reaction



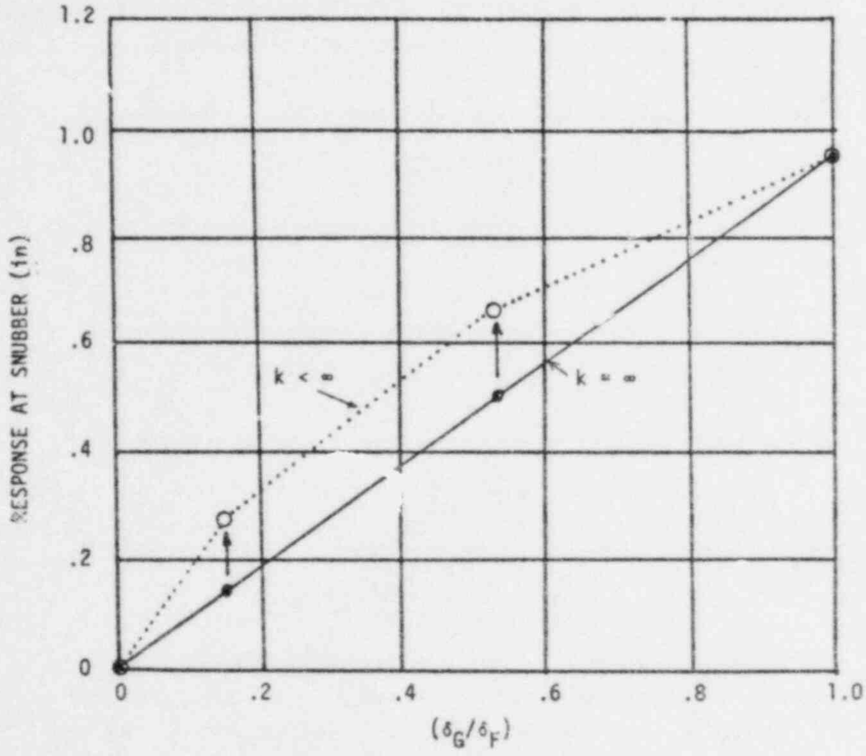


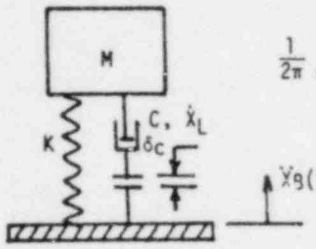
Figure 7.3.3 Clearance Effects on Displacement Response

In this illustration, it is possible to create a situation where response X_2 increases as δ_c increases, and another situation where X_2 decreases as δ_c increases. This is accomplished by changing the forcing frequency f_Ω. Figure 7.3.4 shows the results.

Clearance affects response by attenuating the snubber parameter. Situations can occur, however, where the response is affected as a result of changes in the resonant frequency of the system. This is shown in the previous example.

The following study considers the effect of clearance and viscous parameters on the response of a simple system excited by El Centro seismic event. The investigation considers two values of the viscous parameter C (or C/M).

Consider the system shown:



$$\frac{1}{2\pi}\sqrt{\frac{k}{m}} = 8 \text{ Hz}$$

The clearance is in series with the viscous parameters. The response is calculated as a function of lock velocity. This model was selected because the response is more sensitive to this parameter than other viscous parameters. Figures 7.3.5 and 7.3.6 show the results of the study.

Based on these observations, the component response is restricted to the clearance band for lock velocities less than 1 in/sec. The response increases as the lock velocity increases until the undamped response is reached. As the clearance increases, a decreased sensitivity of response to the viscous parameters is experienced. It appears from these observations that clearances reduce the response sensitivity to a given parameter, in this case the viscous parameter.

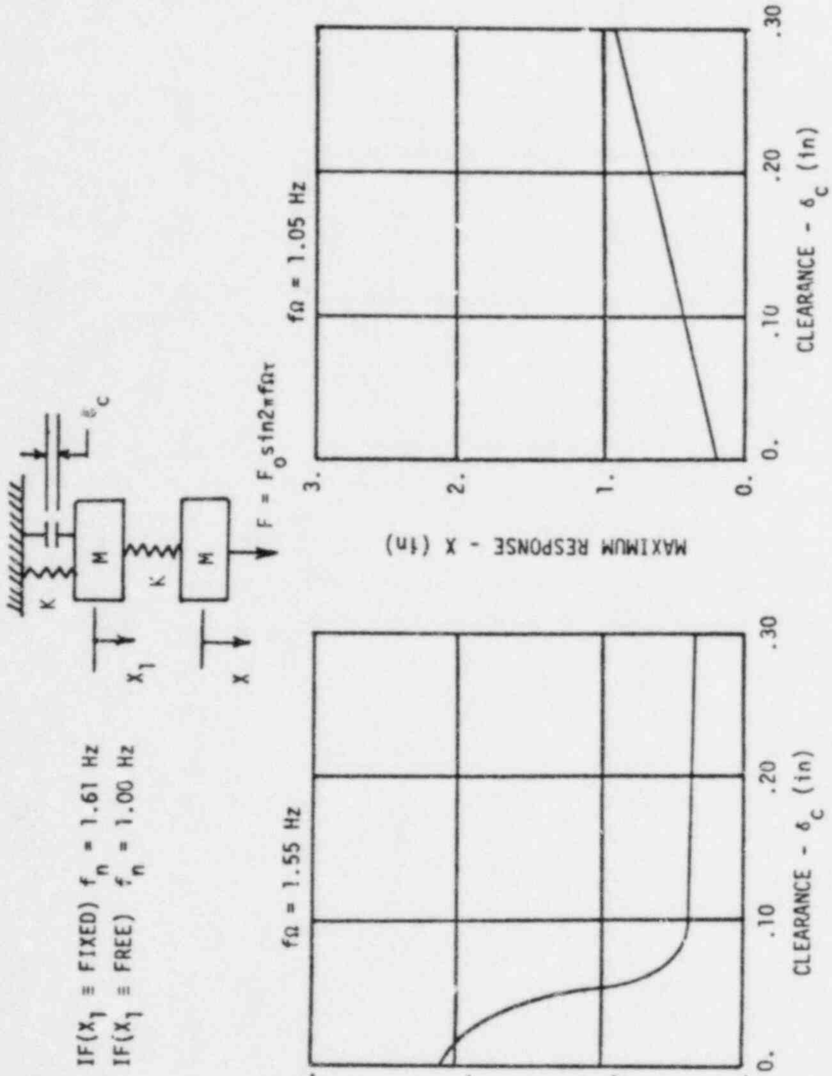


Figure 7.3.4 Clearance Effects on System Response

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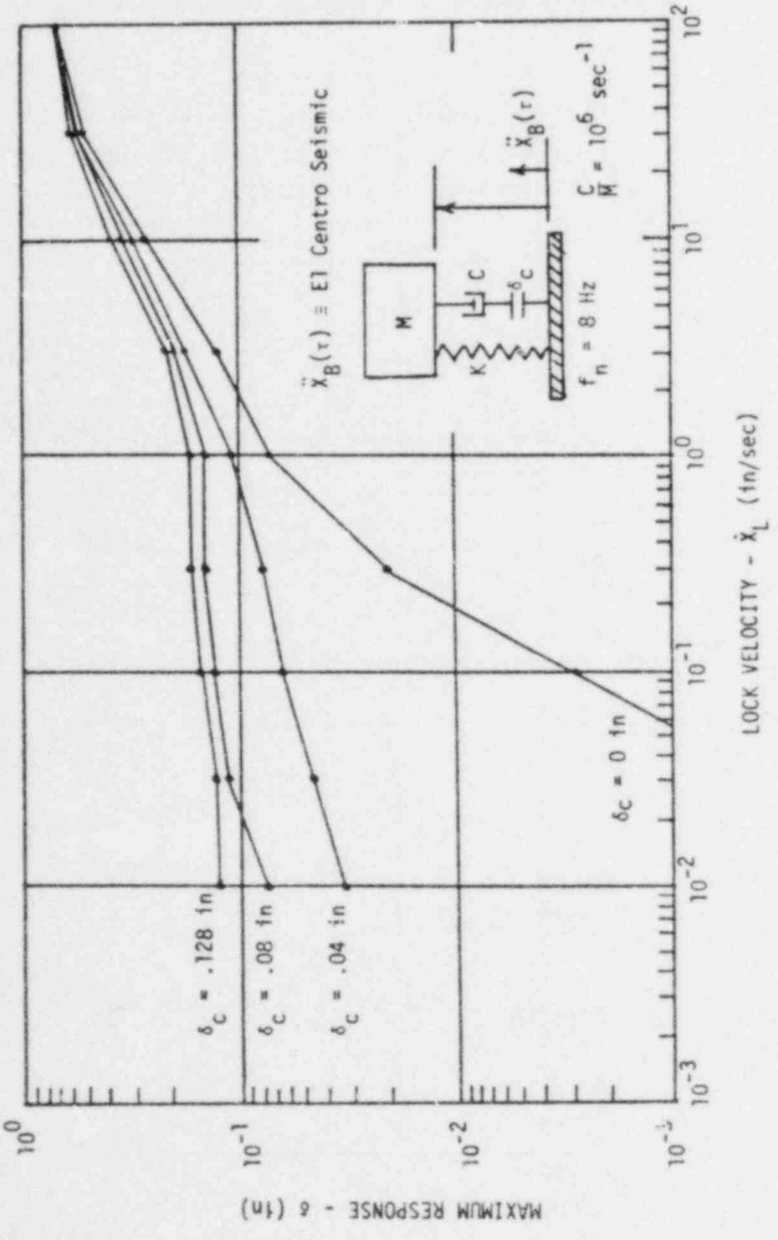

Figure 7.3.5 Effects of Clearance on System Response, C/M = 10^6 sec^{-1}

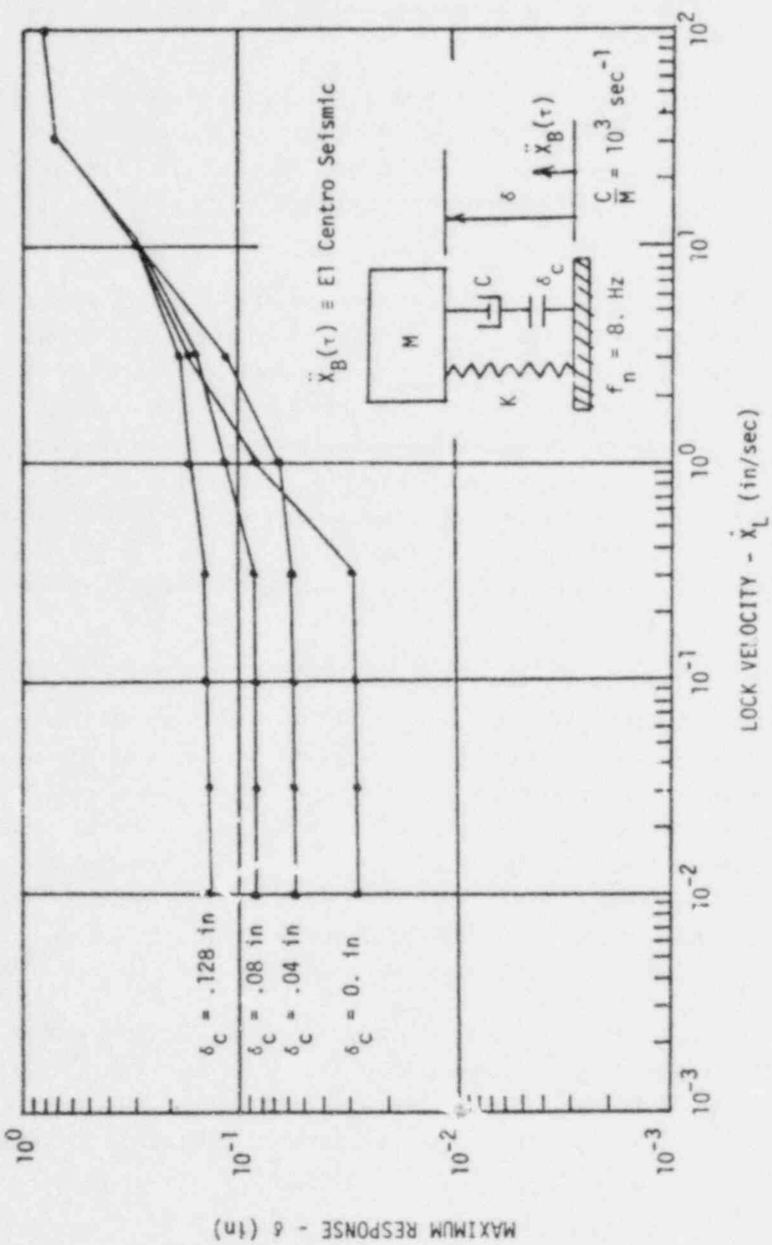

Figure 7.3.6 Effects of Clearance on System Response, C/M = 10^3 sec-1

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7.4 Breakaway Characteristics

Breakaway loads are those loads that restrain movement until the reaction load reaches the "breakaway" or release value, at which time, motion can take place. The motion is opposed by a force equal to the breakaway value. Movement ceases when the reaction load becomes less than the breakaway value.

There are two frictional characteristics to consider. First, there is the initial breakaway load or static friction, and second, the dynamic friction which is the resisting load during movement. The static friction is, in general, greater than the dynamic friction. The response of the system is greatly dependent on the dynamic breakaway loads (friction) and less dependent on static friction. The maximum loading for seismic disturbances usually occurs several seconds after the earthquake begins. It is probable that the static restraint force will be exceeded during the early stages of the seismic disturbance and the dynamic frictional characteristic will apply when the greatest loading occurs.

Den Hartog, Reference 3, studied the breakaway frictional characteristics for harmonic steady-state loadings for simple lumped mass oscillators. His work considered one variance in the frictional laod which in some cases caused the motion to stop in each cycle. As a result of this work, response curves were generated showing the response sensitivity with frictional loads. Figure 7.4.1 shows the response characteristics for a simple lumped mass system in terms of dimensionless parameters. This response curve indicates that the resistive force F_R has little effect on steady-state response when its value is less than 40 percent of the applied load, P.

A detailed look at the transient phase of the harmonic loading (Figure 7.4.2) indicates that the maximum total response probably occurs during the first cycle of loading and may approach twice the steady-state value. The cyclic range may be greater during the steady-state phase of the response.

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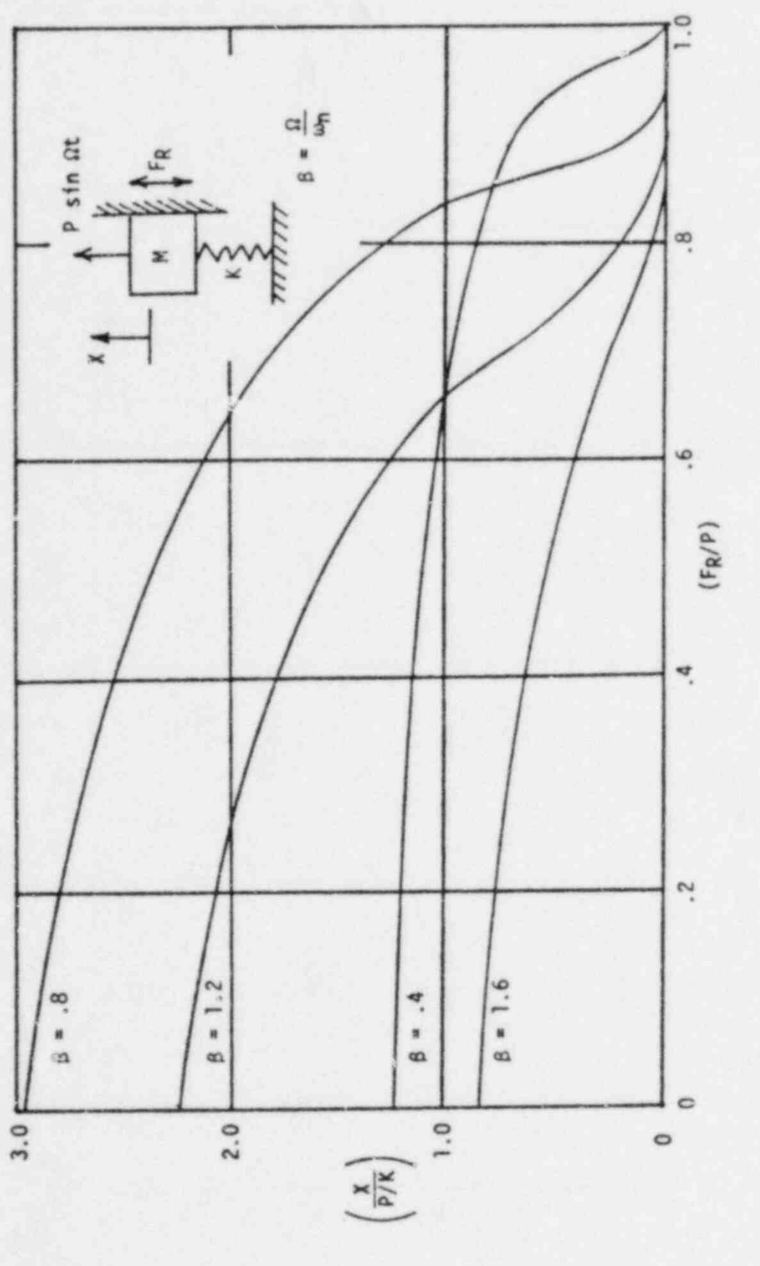

Figure 7.4.1 Friction Effects on Steady State Harmonic Response

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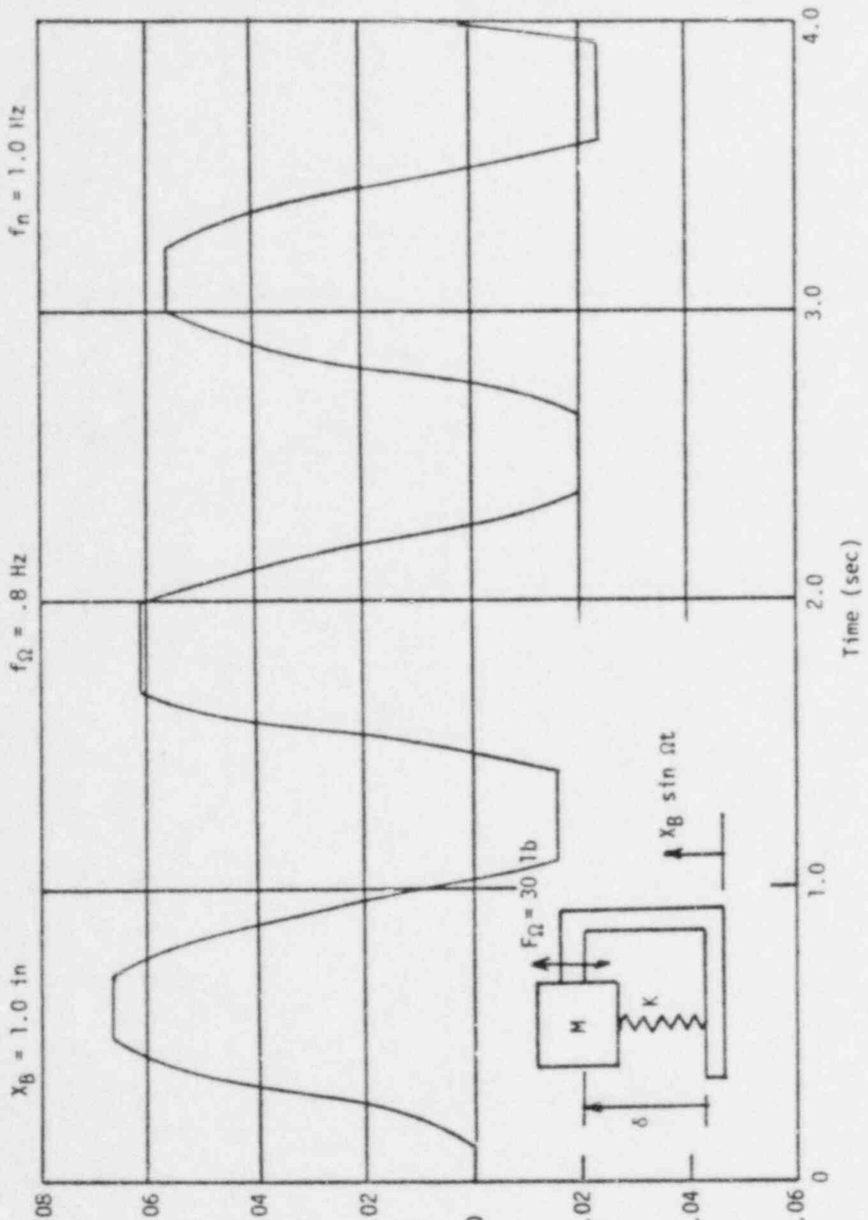
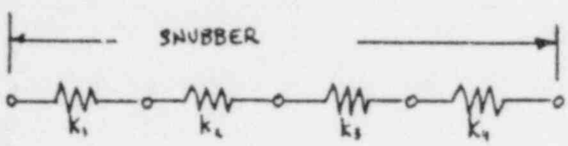


Figure 7.4.2 Transient Response Considering Friction Loads

The response sensitivity considering breakaway frictional characteristics and seismic base acceleration is shown in Figures 7.4.3 and 7.4.4. The response is shown for two different dynamic systems, one having a natural frequency of 6 Hz and the other of 12 Hz. The results show similar trends or characteristics as those established for the harmonic loadings.

7.5 Stiffness Parameter

The load-deflection characteristic of a snubbing device is referred to as the snubber stiffness. The stiffness parameter may be built into the snubber by virtue of subcomponent flexibilities or it may be associated with mounting hardware or structural support flexibilities.



In the above figure the overall stiffness is expressed as:

$$k_{eq} = \frac{1}{\frac{1}{k_1} + \frac{1}{k_2} + \frac{1}{k_3} + \frac{1}{k_4}}$$

The individual stiffness or flexibilities could represent, for example, k_1 (support flexibility), k_2 (free air compressibility), k_3 (attaching hardware flexibility), and k_4 (piston flexibility). If k_1 is small, the overall stiffness of the snubber will be small, and the remaining large k's will have little effect on the overall stiffness of the snubber.

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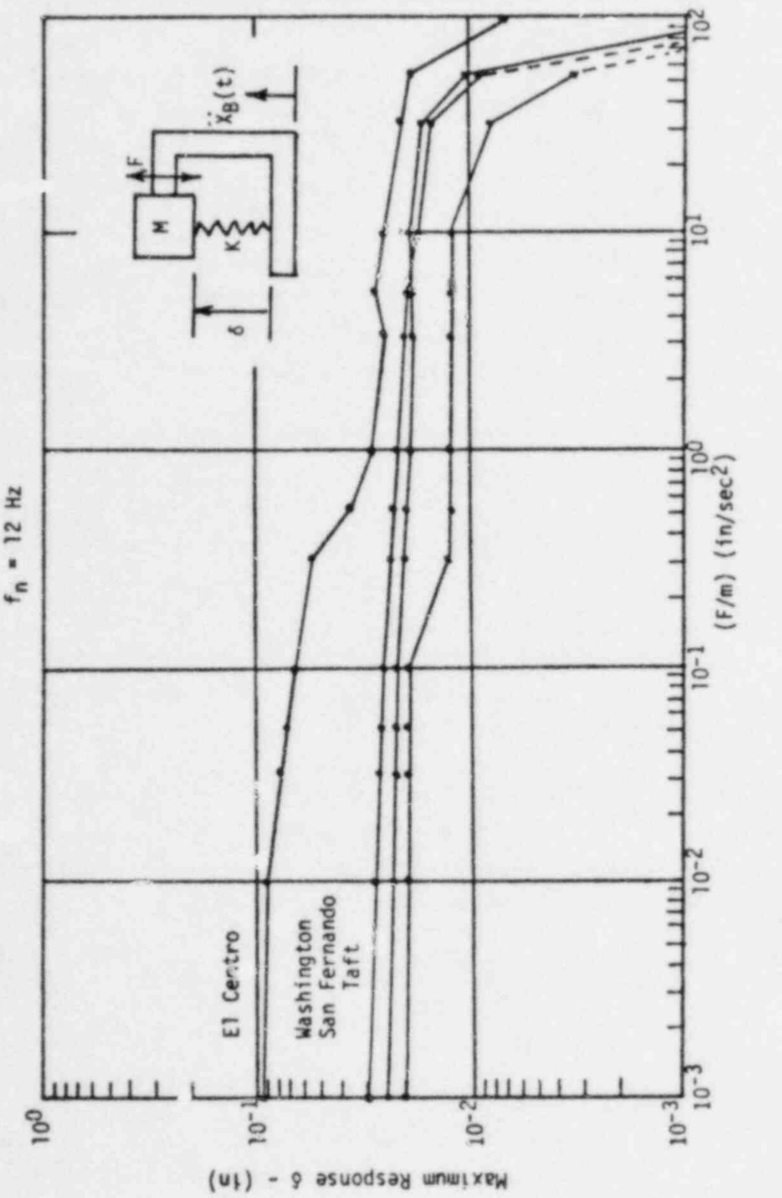

Figure 7.4.3 Response Sensitivity Due to Friction for Seismic Loadings (12 Hz System)

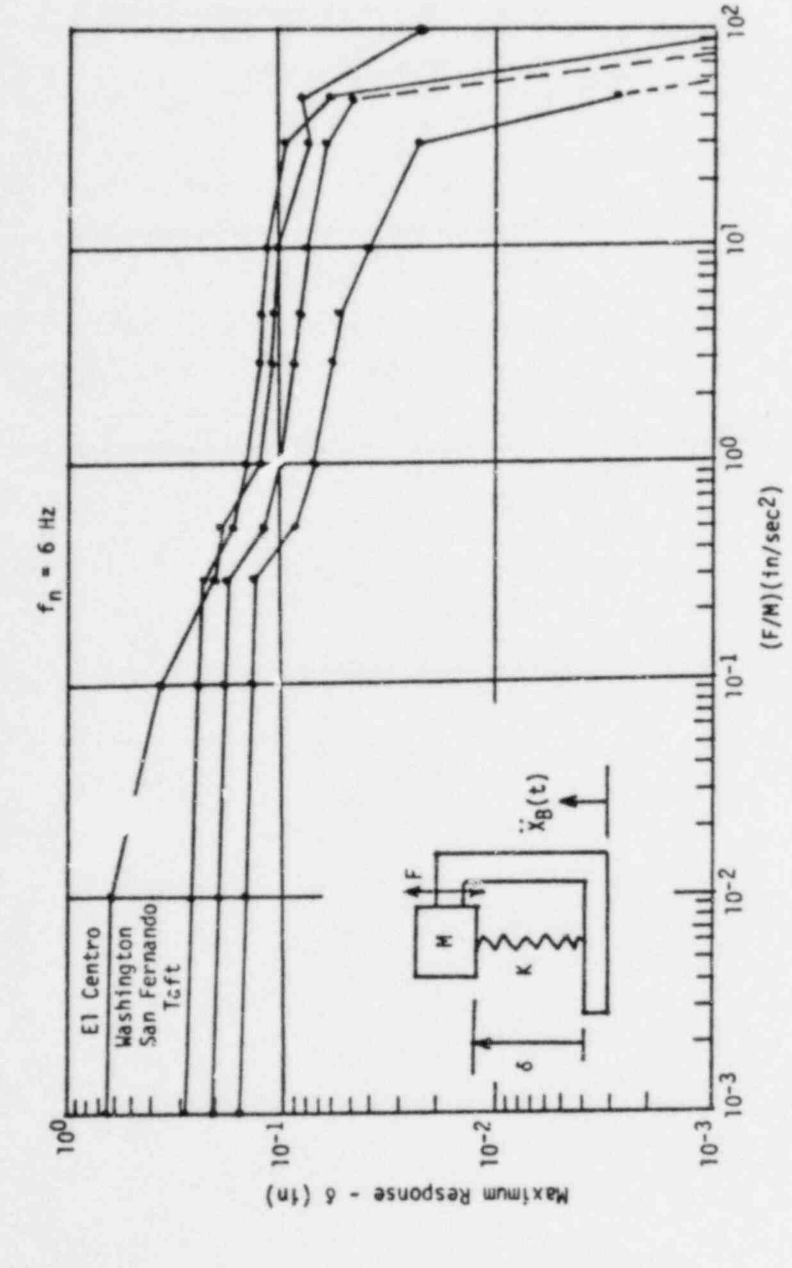

Figure 7.4.4 Response Sensitivity Due to Friction for Seismic Loadings (6 Hz System)

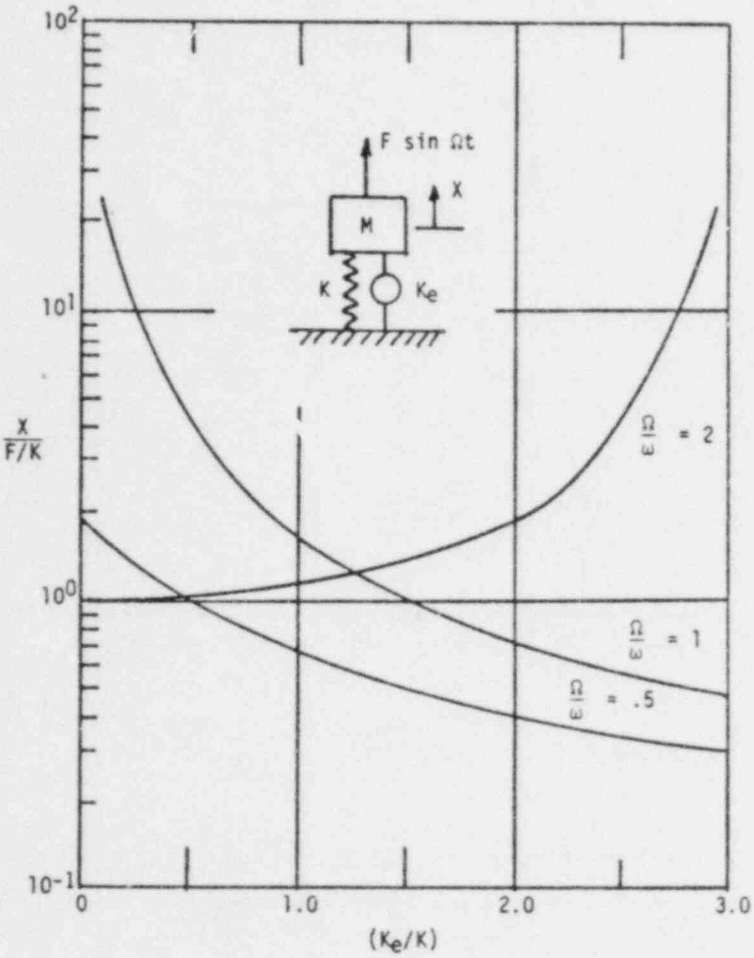

Figure 7.5.1 Response Envelope For 1-D.O.F. Oscillator

Although hydraulic and mechanical snubbers dissipate energy, this is not the primary mechanism for limiting the overall response of a system. The snubbing device actually works by limiting the response of the system at the point of attachment or in a less direct way by modifying the natural frequency characteristics of the system. Since the frequency or response characteristics of a system are sensitive to support stiffness, the "effective" stiffness of snubbers become important.

Consider the simple single mass, undamped, oscillator system shown in Figure 7.5.1. The response envelope or spectra for this system is altered by the effective snubber stiffness where the frequency response characteristics increase as a result of the additional stiffness introduced by the snubber. This can be seen in Figure 7.5.1 from the curve labeled $(\Omega/\omega) = 2$. In this situation, the snubber stiffness tends to drive the resonant or natural frequency of the structure towards the forcing frequency, increasing system response. The same situation would exist if a base excitation was employed rather than an applied oscillating force.

Ideally, to dampen the oscillations of the single degree of freedom system, one would choose an effective snubber stiffness of $K_e >> k$. The resonant frequency of the system would be much greater than the forcing frequency and the response would deteriorate to that of a static loading.

The "effective" stiffness properties are, in general, nonlinear for snubbers. This nonlinearity results from asymmetric response characteristics, e.g., in hydraulic snubbers the actuator rod is on one side of the piston. Nonlinear load deflection characteristics can have one of the following type forms.

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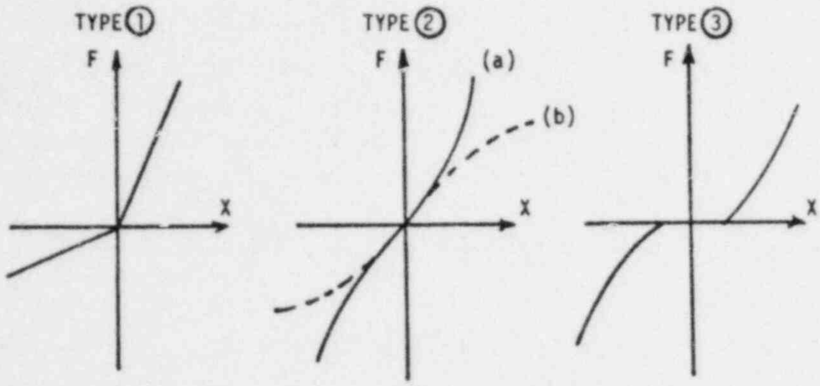
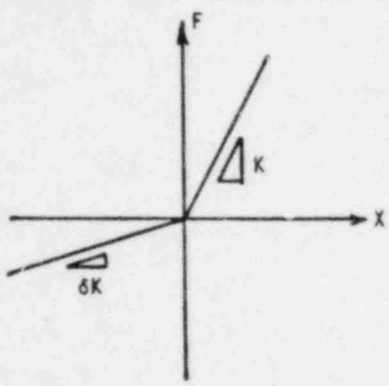


The load-deflection characteristic of TYPE 3 involves the clearance parameter, and is addressed in the section on Clearances. Nonlinear stiffness characteristics similar to those indicated by TYPES 1 and 2 are the starting points for determining the sensitivity of response to the parameter γ, which will be referred to as the bilinear stiffness ratio.



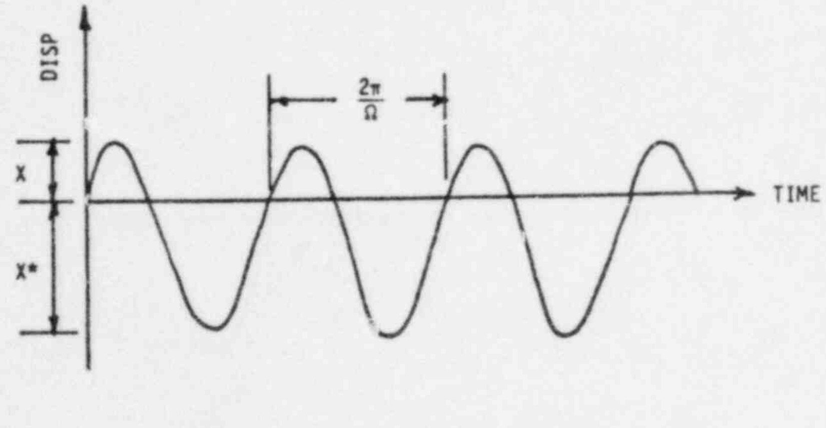
The figure above for TYPE 1 shows the load deflection characteristics for the support system. Inasmuch as K and γK can be interchanged without changing the conclusions we will investigate the range where $0 \leq \gamma \leq 1$.

Figure 7.5.2 shows how the natural frequency of a single lumped mass varies as a function of the bilinear ratio γ. Considering the typical linear system, where $\gamma = 1$, the results indicate that the rate of change of the natural frequency as a function of γ is greatest when $\gamma = 1$, that is, $(\partial fn/\partial\gamma)_{max}$ for a linear spring. Consequently, a change in natural frequency can be expected as bilinear effects are introduced.

For steady-state harmonic loading, Figure 7.5.3 shows how the system steady-state response varies with the bilinear ratio parameter γ. The response is similar to a linear system and the maximum response occurs when the forcing frequency is equal to, or approaches, the natural frequency of the system.

The primary effects are, (1) sensitivity of the natural frequency to the bilinear stiffness ratio γ, and (2) the nearness of the forcing frequency to the "modified" natural frequency.

The response peak-to-peak displacement is twice the maximum response X* when $\gamma = 1$, and less when $\gamma \neq 1$. This should be considered when evaluating the total response range.



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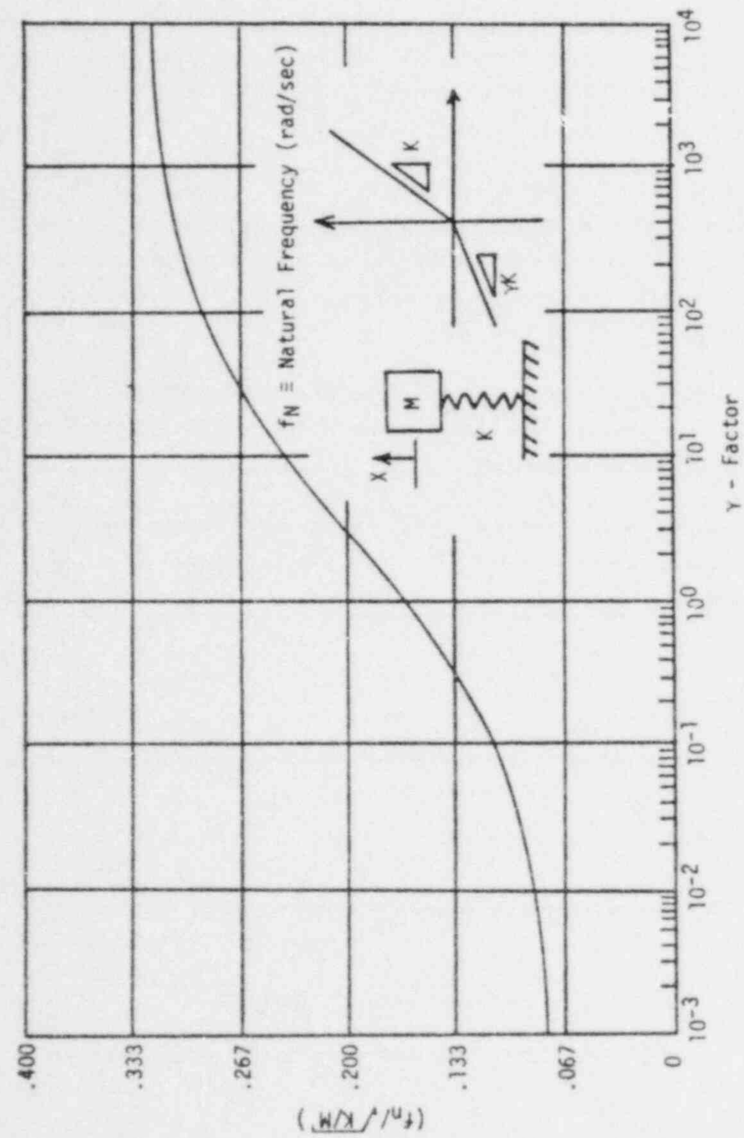

Figure 7.5.2 Resonant Frequency Versus Bilinear Stiffness Ratio

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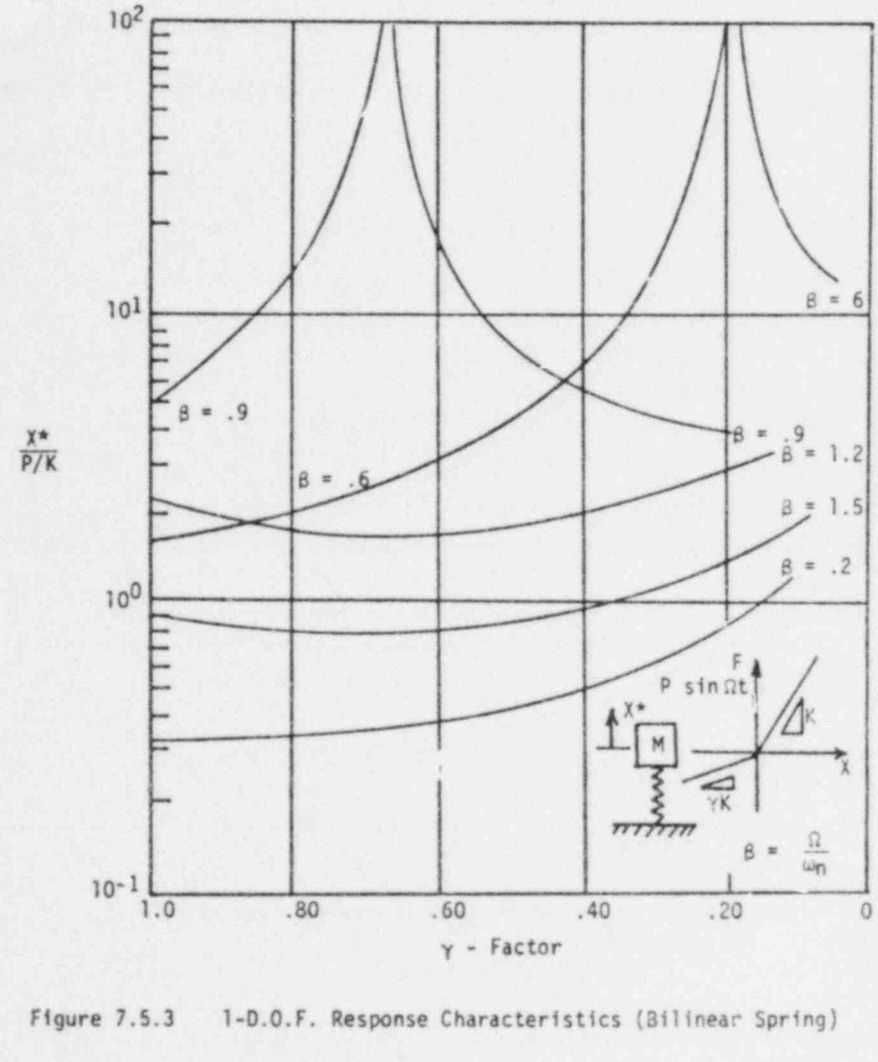
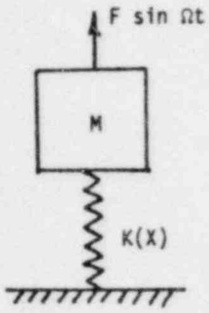
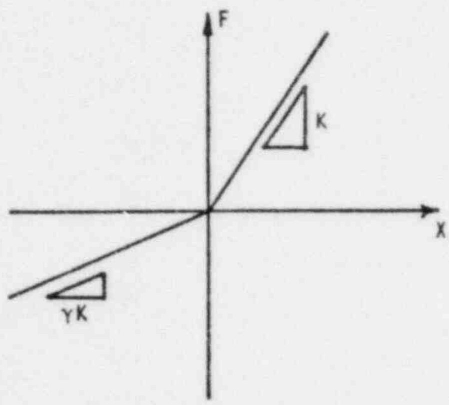


Figure 7.5.3 1-D.O.F. Response Characteristics (Bilinear Spring)





To determine the steady state forced response of the dynamic system shown above, consider the equation

$$M\ddot{X} + K^*X = F \sin \Omega t \qquad \ldots (7.5.1)$$

K^* is an equivalent linear spring. It replaces the bilinear spring while approximately producing the same response - the so called average energy approach.

For the single degree of freedom oscillatory system shown above, the natural frequency can be determined from

$$\omega_n^2 = \frac{K^*}{m} \qquad \ldots (7.5.2)$$

However, the natural period of vibration is

$$f_n = \frac{1}{\pi}\sqrt{\frac{K}{m}} \Big/ \left(1 + \sqrt{\frac{1}{\gamma}}\right)$$

or,

$$\omega_n = \frac{2\pi}{\pi}\sqrt{\frac{K}{m}} \Big/ \left(1 + \sqrt{\frac{1}{\gamma}}\right) \qquad \ldots (7.5.3)$$

Squaring (7.5.3) gives

$$\omega_n^2 = 4\,(K/m)\Big/\left(1 + \sqrt{\frac{1}{\gamma}}\right)^2 \qquad \text{... (7.5.4)}$$

Equating the frequency equations (7.5.2) and 7.5.4) yields

$$\frac{K^*}{m} = 4\,\left(\frac{K}{m}\right)\Big/\left(1 + \sqrt{\frac{1}{\gamma}}\right)^2 \qquad \text{... (7.5.5)}$$

and solving for K*,

$$K^* = \frac{4K}{\left(1 + \sqrt{\frac{1}{\gamma}}\right)^2} \qquad \text{... (7.5.6)}$$

Substituting equation (7.5.6) into equation (7.5.1) gives

$$MX + \left[\frac{2}{\left(1 + \sqrt{\frac{1}{\gamma}}\right)}\right]^2 KX = F \sin \Omega t \qquad \text{... (7.5.7)}$$

Solving for the steady-state response

$$X = \frac{F}{K\left[\frac{2}{\left(1 + \sqrt{\frac{1}{\gamma}}\right)}\right]^2 - M\Omega^2} \qquad \text{... (7.5.8)}$$

Or in terms of the natural frequency,

$$X = \frac{(F/K)}{\left[\frac{2}{\left(1 + \sqrt{\frac{1}{\gamma}}\right)}\right]^2 - \left(\frac{\Omega}{\omega_{no}}\right)^2} \qquad \text{... (7.5.9)}$$

where

$$\omega^2_{no} = \left(\frac{K}{m}\right)$$

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Equations (7.5.8) and (7.5.9) represent the steady-state response in the +X direction. To find the response (steady-state) in the -X direction, equate the potential energys for $X < 0$ with that when $X > 0$,

$$\frac{1}{2} KX^2 = \frac{1}{2} \gamma K(X^*)^2 \qquad \ldots (7.5.10)$$

X^* represents the motion in the -X direction. Therefore from equation (7.5.10),

$$\left(\frac{X^*}{X}\right)^2 = \frac{1}{\gamma}$$

and solving for X^*,

$$X^* = \sqrt{\frac{1}{\gamma}}\, X \qquad \ldots (7.5.11)$$

If $0 < \gamma < 1$, the maximum response is

$$X^* = \sqrt{\frac{1}{\gamma}}\, X \qquad \ldots (7.5.12)$$

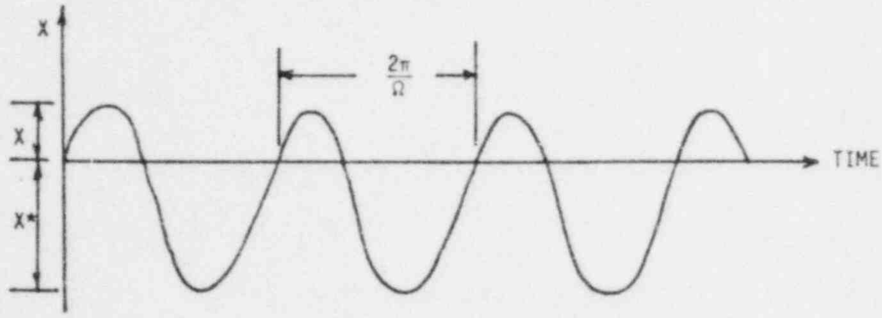
Considering equation (7.5.9) and

$$\frac{X^*}{F/k} = \frac{X^*}{X_{static}}$$

for the static case

$$\frac{X^*}{X_{static}} = \frac{\sqrt{\frac{1}{\gamma}}}{\left[\frac{2}{\left(1 + \sqrt{\frac{1}{\gamma}}\right)}\right]^2 - \left(\frac{\Omega}{\omega_{no}}\right)^2} \qquad \ldots (7.5.13)$$

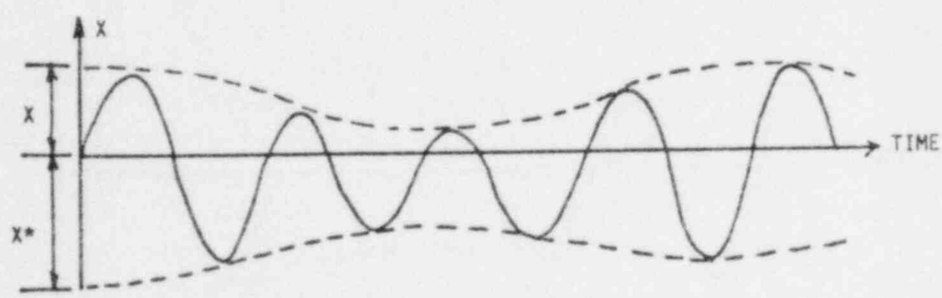
The steady-state response can graphically be represented by:



Equation (7.5.13) does not consider damping. This can be incorporated as follows

$$\frac{X^*}{X_{st}} = \sqrt{\frac{1}{\gamma}}\ \frac{1}{\sqrt{\left\{\left[\frac{2}{1+\sqrt{\frac{1}{\gamma}}}\right]^2 - \left(\frac{\Omega}{\omega_{no}}\right)^2\right\}^2 + \left[2\zeta\left(\frac{\Omega}{\omega_{no}}\right)\right]^2}} \quad \ldots (7.5.14)$$

Equations (7.5.9), (7.5.13), and (7.5.14) represent envelope values for the response. These results are for an average energy approach. In reality there is beating phenomenon that occurs as shown below. This results from $K \neq \gamma K$.



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This beating period is a function of the forcing frequency and the natural frequency.

$$\tau_B = \frac{1}{f_\Omega - f_n}$$

$$\tau_B = \frac{1}{f_n - \dfrac{\sqrt{K/M}}{\pi\left(1 + \sqrt{\dfrac{1}{\gamma}}\right)}} \qquad \ldots (7.5.15)$$

The figure above shows the steady-state response of the system. The total displacement range is

$$X_R = (X + X^*) = \frac{\sqrt{\dfrac{1}{\gamma}}\,(F/K)(1 + \sqrt{\gamma}\,)}{\left[\dfrac{2}{1 + \sqrt{\dfrac{1}{\gamma}}}\right]^2 - \left(\dfrac{\Omega}{\omega_{no}}\right)^2} \qquad \ldots (7.5.16)$$

The results indicate a lumped mass, supported by a bilinear spring system, has response characteristics similar to that of a linear system. The bilinearity stiffness ratio effects the system response by changing the natural frequency. The response characteristics of a system with a specific γ ratio are similar to a linear system with forcing frequencies. The principle difference between a linear and bilinear supported system is the asymmetric response that occurs about the neutral position.

The results obtained from the simple, single degree of freedom analysis indicate the same trends should exist for more complicated lumped mass systems or continuous beam models. Since the bilinear system response can be evaluated in terms of an effective spring rate K^*, the response trends can be expected to follow those of linear supported systems. A more detailed discussion is presented later in this section.

The stiffness or load-deflection characteristics described in TYPE 2a is an effect that is commonly referred to as a "hardening" spring, that is, as the deflection increases so does the stiffness. The "hardening" spring characteristic is probably the most common naturally occurring effect, and is most often found in hydraulic and soil systems. There is a' › another stiffness property referred to as the softening spring. For this, the stiffness decreases with increasing deflection. This effect is described as TYPE 2b. This situation is probably less common in nature than the hardening effect. The most common examples can be attributed to nonlinear geometric effects and inelastic behavior. As far as snubber systems are concerned, the hardening characteristic is the most relevant.

The response of a simple single degree of freedom oscillation having "hardening" or "softening" stiffness characteristics has been studied in great detail by Duffing, Reference 4.

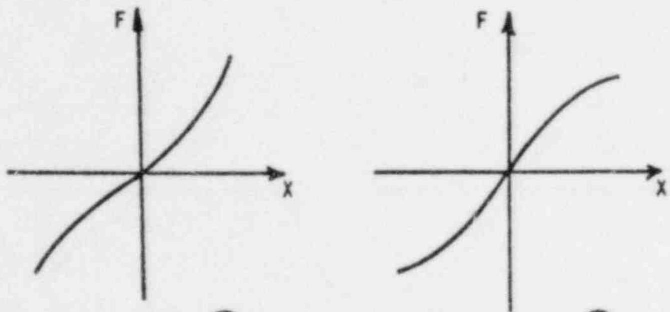
Duffing's equation for a system with a nonlinear restoring force is,

$$m\ddot{X} + k(X \pm \kappa^2 X^3) = F \sin \Omega t \qquad \ldots (7.5.17)$$

Where the reaction load is,

$$R = k(X \pm \kappa^2 X^3) \qquad \ldots (7.5.18)$$

When the upper sign of κ is used, the system is said to have a "hardening" spring, while the lower sign means the system is restored by a "softening" spring.



HARDENING (TYPE 2a) SOFTENING (TYPE 2b)

The steady state response frequency equation for the single degree of freedom system is

$$1 \pm \frac{3\kappa^2 X^2}{4} - \frac{\Omega^2}{\omega_n^2} = (\pm)\frac{F}{kX} \qquad \ldots (7.5.19)$$

The parenthetical plus-or-minus (±) sign refers to X being in phase (+) or π radians out of phase (-) with the exciting force F.

By letting the excitation displacement approach 0 in equation (7.5.19), the relation between the natural frequency and the amplitude of the non-linear system can be found.

$$\omega_n = \sqrt{\frac{k}{m}}\sqrt{1 \pm \frac{3\kappa^2 X^2}{4}} \qquad \ldots (7.5.20)$$

Referring to equation (7.5.19) the response frequency relationship for a hardening spring is

$$\left(1 - \frac{\Omega^2}{\omega^2}\right) + \frac{3\kappa^2 X^2}{4} - \frac{F}{kX} = 0$$

or,

$$\left(\frac{3\kappa^2}{4}\right)X^3 + \left[1 - \left(\frac{\Omega}{\omega}\right)^2\right]X - \frac{F}{k} = 0 \qquad \ldots (7.5.21)$$

and for a softening spring is

$$\left(\frac{3\kappa^2}{4}\right)X^3 + \left[1 - \left(\frac{\Omega}{\omega}\right)^2\right]X - \left(\frac{F}{k}\right) = 0 \qquad \ldots (7.5.22)$$

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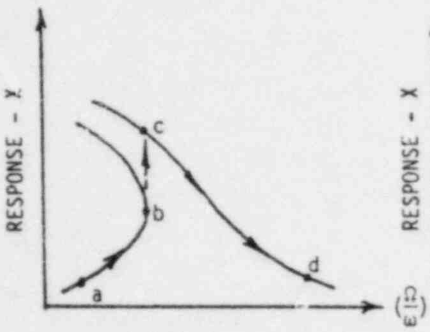
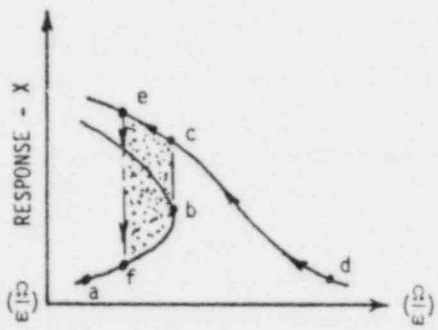
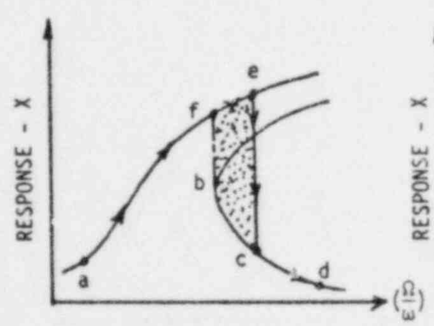
Phenomena that occur as a result of the "hardening" or "softening" support characteristics are the jump phenomena. Jump phenomena for "softening" springs are illustrated as follows

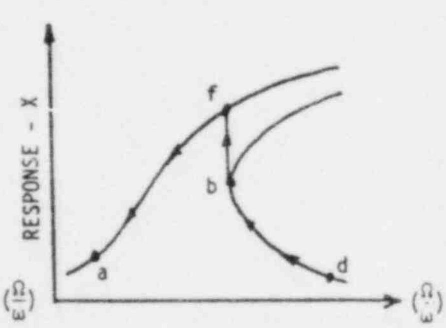




With increasing frequency of excitation, the amplitude gradually increases from "a" until "b" is reached. It then jumps to a larger value at "c" and diminishes as the forcing frequency is increased. In decreasing the frequency from some point "d", the response increases to some point beyond "c", to point "e" where the response will drop to point "f", and continue to decrease from that point. Stability analysis shows the middle branch is unstable (shaded area).

For a "hardening" spring, the jump phenomena are illustrated as follows





With increasing excitation frequency, the amplitude increases beyond "f" to some point "e" where the response drops to "c" and continues to decrease. If the excitation frequency decreases from point "d", the response will increase to point "b" where it will jump to point "f" and then continue to decrease. Just as with the "softening" spring, the middle branch is unstable.

When the drop-jump phenomenon occurs, it is usually preceeded by an accidental unsteadiness or extraneous disturbance. When the instability occurs there is a phase change between the response and the forcing function which gives rise to a transient superimposed motion at the time of the drop-pump.

The effects of the "hardening" (softening) parameter κ^2 on dynamic response can be evaluated using the frequency response equation

$$\pm \left(\frac{3\kappa^2}{4}\right)X^3 + \left[1 - \left(\frac{\Omega}{\omega}\right)^2\right] X - \left(\frac{F}{K}\right) = 0$$

Figures 7.5.4 and 7.5.5 show the variation of response amplitude X as a function of κ^2, parametric with forcing frequency and magnitude, (F/k) and (Ω/ω), constant. These plots apply to a single degree of freedom system.

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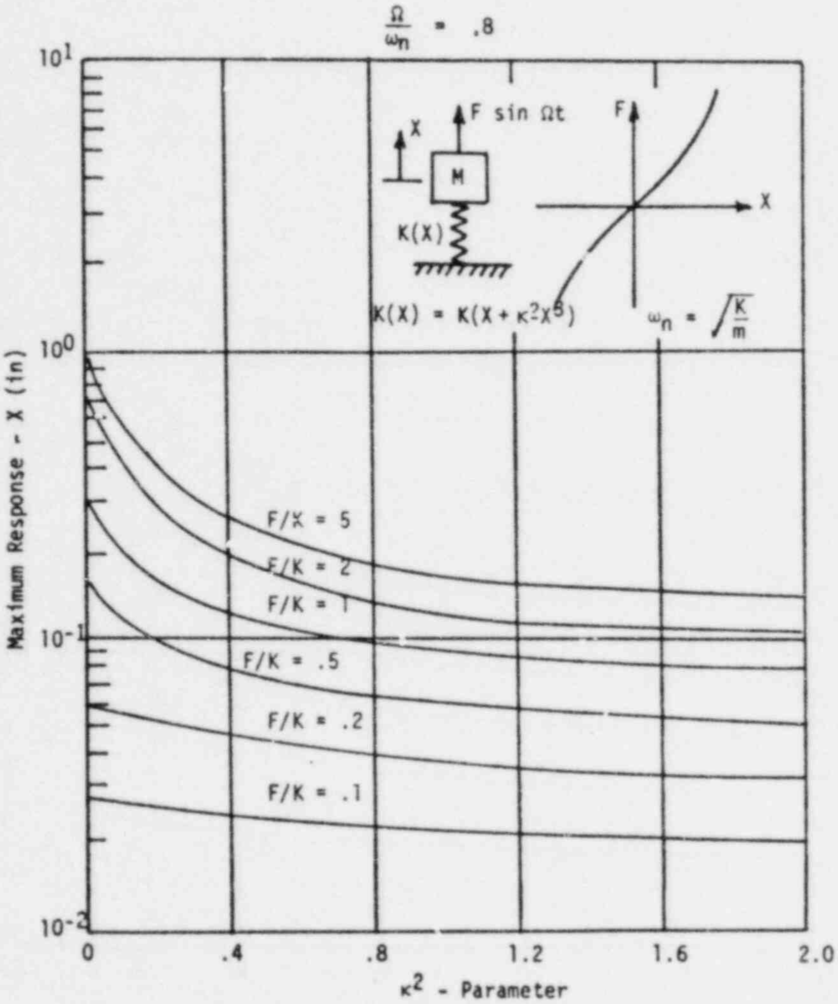

Figure 7.5.4 Hardening Parameter Effects on System Response

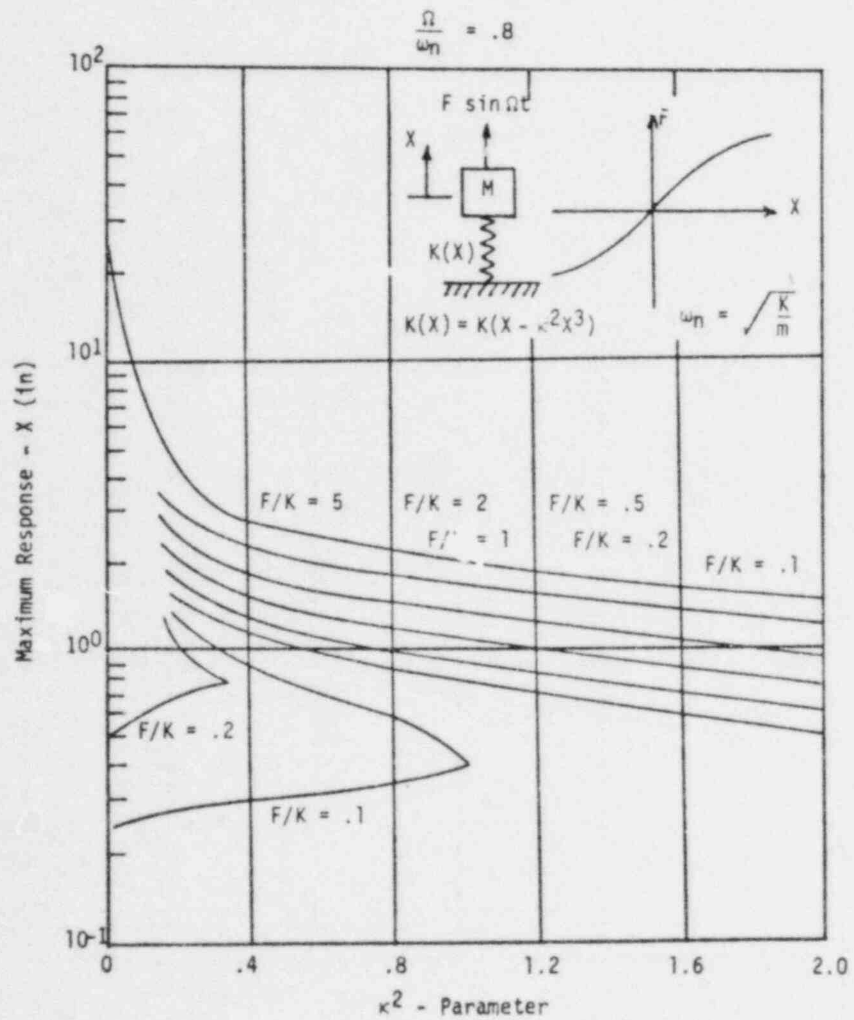

Figure 7.5.5 Softening Parameter Effects on System Response

8. PARAMETER RANGES

Once the parameters affecting system response are determined, the next step is to establish parameter ranges that will assure system responses are bounded within acceptable limits.

Parameter ranges may be controlled directly by the snubber manufacturer, or by installation practices and procedures. Examples of parameters controlled directly by the customer or manufacturer are the acceleration threshold, lock velocity, and snubber stiffness characteristics. Parameters which can best be related to installation practices would include, end fitting clearances, back-up support stiffnesses, positioning (orientation), and environmental considerations such as radiation and temperature. The clearance parameter is unique in that is can be influenced by both the manufacturer and by installation procedures. Other parameters may be influenced more by probabilistic occurrences, rather than by manufacturer or installers (designers) control. This would include air entrainment or entrapment, and increased friction caused by corrosion, and contamination. It is the intent of this work to consider ranges of parameters that exceed the limits of current design practice.

Probable parameter ranges were established by using vendor literature and information obtained by NRC. These ranges were extended to consider values that would be pertinent to this study. The minimum range of parameters addressed by this study are presented in Table 8.1.

The establishment of acceptable parameter ranges is not an easy task in light of all the variables involved. Consideration must be given to back-up structure effects, load magnitudes, displacement-stress-load responses, parameter combinations, and response characteristics of the supported component or system. The purpose of this section is to discuss which results were used in establishing acceptable parameter ranges. The conclusions are based solely on the data generated in this study.

TABLE 8.1

SNUBBER PARAMETER RANGES

Parameter	Range to be Investigated
Dead Band	.0 to .125 in
End Fitting Tolerancd	.0 to .125 in
Breakaway Loads	1 to 3%
Frequency Range	3 to 33 Hz
Acceleration Threshold	0 to .2 g
Lock Velocity	0 to 1 in/sec
Bleed Rate (@ Rated Load)	.001 to 1 in/sec
Snubber Support Stiffness	10^2 to 10^6 lb/in

The back-up structural support dynamic and stiffness characteristics influence snubber performance used as a basis for establishing parameter ranges. The following describes the basis for establishing acceptable parameter ranges.

8.1 Viscous Parameters

The viscous parameters that effect response are lock velocity and bleed rate. The results of the study indicate that the forcing frequency does not have a significant effect on response, Figure 7.2.2, nor does the applied load, Figure 7.2.3, for realistic values of bleed rate. Response curves such as those indicated in Figures 7.2.5 and 7.2.6 are representative of system response. Component responses are maintained within acceptable limits provided the lock velocity is less than 1 in/sec. Figure 7.2.6 does not substantiate this value based on percent reduction of response, however, it does support this conclusion based on actual maximum response. Therefore, the established limit appears reasonable. The bleed rate is related to the viscous constant C as follows.

$$C = \frac{\text{RATED LOAD}}{\text{BLEED RATE}}$$

In general, C values are large, e.g., a 2,000 lb snubber with a bleed rate of 6 in/min has a viscous constant value of 2 (10^4)(lb-sec/in). Sample studies done to date indicate that if reasonable lock velocities are maintained (< 1 in/sec) that a bleed rate less than the lock velocity will assure a bounded response. Without putting excessive constraints on the viscous parameter, the following limits will be established,

$$(C/m) > 10^3 \text{ sec}^{-1}$$

or

$$C > 10^4 \frac{\text{lb-sec}}{\text{in}}$$

8.2 Friction Loads

The effects of friction on system response are negligible, i.e., the friction loads must be at least 40 percent of the applied load before dynamic response is significantly affected. Based on the results developed for harmonic, Figure 7.4.1, and seismic, Figure 7.4.3, the frictional characteristics have insignificant effects on dynamic response when less than 40 percent of the rated load of the snubber.

8.3 Acceleration Threshold Parameter

The establishment of an acceptable A.T.P. range is a complex issue. There apparently is considerable interaction between system response characteristics and the A.T.P. as indicated in Figure 7.1.4. The possibility of increased rather than reduced response, Figures 7.1.4, 7.1.6, and 7.1.7, complicates the issue. The results shown graphically in Figure 7.1.6 indicate that the response is nonlinear with load magnitude ($\ddot{X}_B$). Based on the results presented in Figure 7.1.6, an A.T.P. ($\ddot{X}_L$) equal to .001 g would assure a 90-95 percent reduction in free response. This appears restrictive, however, if wise design practices are used in realistic limit of .04 g should bound the response within acceptable limits. This limit may be modified when further studies are completed.

8.4 Support Stiffness Requirements

Current design practice does not consider support stiffness requirements. This study indicates that with relatively high "effective" stiffness of snubbers, the back-up structural support stiffness properties may effect snubber response characteristics. Back-up structural stiffness requirements have been established in this study so that the snubber can be "effective." For example, it would be unreasonable to attach a snubber having an effective stiffness of $0.3(10^6)$ lb/in to a supporting structure having an effective stiffness of 5,000 lb/in for a massive component. If the back-up support stiffness is very low there could be a dynamic interaction that would affect response. Considering the combined "effective" stiffness of both the back-up structure and the snubber, the "effective" stiffness should be at least twenty times greater than the stiffness (1/flexibility) of the pipe at the snubber location. The factor of twenty can be verified from Figures 6.1.1, 6.1.2, and 6.1.3. In terms of the viscous constant, where C is the ratio of the rated load to bleed rate, the back-up structure stiffness should be 1.2 ($C\Omega$) where Ω is the predominant forcing frequency in rad/sec. Figure 6.1.4 shows the response efficiency of the support. The efficiency represents the actual load transferred into the base compared to the maximum load that can be transferred.

8.5 Clearance

Clearance is always associated with another snubber parameter. Limited observations indicate that clearance attentuates the ability of the snubber parameter to reduce the displacement response amplitude, Figures 7.3.3, 7.3.5, and 7.3.6. Sufficient information has not been generated to date to establish absolute clearance limits for all the snubber parameters. However, based on observations, clearances are estimated to be 15 percent of the unrestrained response for rigid supports and 10 percent of the unrestrained response for snubbers.

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9. ANALYTICAL METHODS

The method used for solving the transient response problem must be capable of considering parameters such as nonlinear restorative forces $F_R = F(X)$, viscous forces $F_V = F(\dot{X})$, clearances, and motion constraints. Analytical techniques must have the capabilities of considering these parameters singly or in any combination.

Although the list of snubber parameters is considerably longer than those parameters listed above, the snubber parameters do fall into one of the above categories. For example, the nonlinear stiffness characteristic associated with entrained air in hydraulic fliuids is categorized as a nonlinear restorative force while the velocity dependent load associated with this fluid is categorized as a viscous force.

The analytical method that is chosen must be general in nature, so that large component or piping system with complex geometries may be analyzed. It must be flexible enough to allow for modification so additional parameters may be added. It must be efficient so that computer costs are to be kept at a minimum. The method that can best be used to satisfy the previously mentioned requirements is a modified modal analysis procedure utilizing a load correcting algorithm.

Prior to describing the analysis method that will be used to solve the general problem, some of the analysis procedures that were abandoned in favor of the modified modal analysis procedure will be discussed.

Equations of motion for nonlinear systems can usually be solved approximately by using step-by-step integration procedures. Many well known numerical methods involve extrapolation or interpolation formulas for the solution which are applied in a series of small but finite time intervals. Some of the more popular methods are based on assumed acceleration functions between integration time steps. Of particular importance are the constant, average, and linear acceleration methods - the later two involving interaction procedures. The acceleration analysis method works quite well for simple 1 or 2 degree of freedom systems but encounters stability problems when the complexity of the structural model increases or the natural frequencies become high. Many elaborate analysis procedures have been developed to overcome most of these problems; however, their complexity and long running computer times do not make these methods suitable for the solution of the snubber problem. Another major drawback of the acceleration analysis method is the selection of an integration time interval that will produce a stable or at least conditionally stable solution. In many cases very small integration time steps are required for a stable solution, and even smaller time steps may be required for an accurate solution. Figure 9.1 shows how the accuracy of a simple single degree of freedom solution varies as a function of the integration time interval for the constant acceleration method.

The modal analysis procedure has an advantage over general step-by-step integration procedures in that many of the stability problems can be eliminated quite easily for large systems of equations. Although the modal analysis procedure requires the evaluation of system natural frequencies and modeshapes, standard analysis methods can easily be adopted for this purpose. When analyzing complex systems it is often necessary to calculate only a few modes to get reasonably accurate results; consequently, the solution time is reduced considerably.

Due to the nature of the nonlinear parameters used to describe the snubbers, standard modal analysis procedures cannot be used to solve the problems. However, modifications can be made to this standard procedure to permit the adoption of this procedure.

The modal analysis technique has become a popular analysis tool since the advent of finite element analysis. The snubber analyses procedure developed here will be incorporated in an existing finite element statics and dynamics piping program SUPERPIPE. The development of the procedure beings with the differential equations of motion in terms of the mass and stiffness matrices as shown in Equation (9.1).

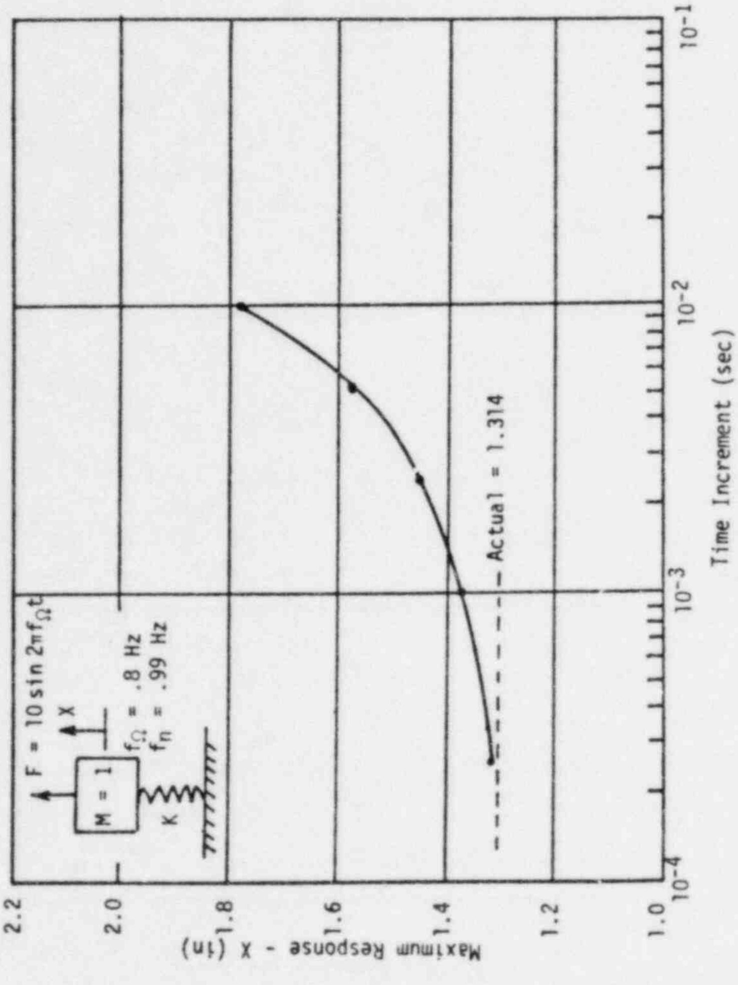

Figure 9.1 Response Versus Time Increment Sensitivity

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$$\begin{bmatrix} M & 0 \\ 0 & M^* \end{bmatrix} \begin{Bmatrix} \ddot{X} \\ \ddot{X}_S \end{Bmatrix} + \begin{bmatrix} K_{11} & K_{1S} \\ K_{S1} & K_{SS} \end{bmatrix} \begin{Bmatrix} X \\ X_S \end{Bmatrix} = \begin{Bmatrix} F(t) \\ R^* \end{Bmatrix} \qquad \ldots (9.1)$$

X_S, $\ddot{X}_S$ = Known displacement and acceleration (excluding snubber locations)
X, $\ddot{X}$ = Unknown displacement and acceleration
$\{F(t)\}$ = Specified forces
$\{R^*\}$ = Unknown reaction forces

Separating (9.1) into parts we obtain two matrix equations,

$$[M]\{\ddot{X}\} + [K_{11}]\{X\} = \{F(t)\} - [K_{1S}]\{X_S\} \qquad \ldots (9.2a)$$

and

$$-\{R^*\} + [K_{S1}]\{X\} = -[M^*]\{\ddot{X}_S\} - [K_{SS}]\{X_S\} \qquad \ldots (9.2b)$$

where the known quantities are on the right side of the equations.

By solving (9.2a) for $\{X\}$ all the displacements can be determined.

Examination of (9.2a) indicates that the homogenous portions involve only those degrees of freedom that are unknown. Therefore, if modal analysis procedures are to be used the frequencies and mode shapes are determined for the system with all known movements removed or fixed.

$$[M]\{\ddot{X}\} + [K]\{X\} = \{0\} \qquad \ldots (9.3)$$

The modeshapes and frequencies are determined for the homogenous differential equation shown in (9.3).

To solve (9.2a) assuming the modal coordinate transformation,

$$\{X\} = [A]\{Z\}, \text{ and } \{\ddot{X}\} = [A]\{\ddot{Z}\} \qquad \ldots (9.4)$$

Where $[A]$ = Matrix of eigenvectors normalized such that $[A]^T[M][A] = [I]$,

$$[M][A]\{\ddot{Z}\} + [K][A]\{Z\} = \{F(t) - [K_{1S}]\{X_S\}\} \qquad \ldots (9.5)$$

By multiplying both sides of Equation (9.5) by $[A]^T$ we obtain

$$[A]^T[M][A]\{\ddot{Z}\} + [A]^T[K][A]\{Z\} = [A]^T(f(t) - [K_{1S}]\{X_S\}) \quad \ldots(9.6)$$

From the principle of orthogonality,

$$[A]^T[K][A] = [\omega_n^2]$$
$$[A]^T[M][A] = [1]$$

Therefore, Equation (9.6) becomes

$$[1]\{\ddot{Z}\} + [\omega_n^2]\{Z\} = \{P(t)\} \quad \ldots(9.7)$$

This is a series of uncoupled equations, each representing the response of a single degree of freedom oscillator.

$$\ddot{Z}_i + \omega_{ni}^2 Z_i = P_i(t) \quad \ldots(9.3)$$

Knowing, Z_i, $\dot{Z}_i$, and $\ddot{Z}_i$ we can proceed to solve for X_i, $\dot{X}_i$, and $\ddot{X}_i$, where

$$\{X\} = [A]\{Z\}, \quad \{\dot{X}\} = [A]\{\dot{Z}\}, \quad \{\ddot{X}\} = [A]\{\ddot{Z}\} \quad \ldots(9.9)$$

The modal analysis procedure is limited to those problems where boundary restraints do not vary during the course of the analysis.

For the nonlinear snubber problem, boundary restraints do vary with time. A modification to this procedure is employed where [A] is also a function of time.

A procedure which permits the use of modal analysis techniques has been adopted, the procedure utilizes a technique whereby reaction loads are determined so that specified displacements or accelerations can be made without modifying the modeshape matrix [A] or system frequencies.

The procedure involves the solution for a given time increment, for example, $X(t)$, $\ddot{X}(t)$ are known and it is desired to calculate $X(t + \Delta t)$ and $\ddot{X}(t + \Delta t)$.

The solution is obtained using the normal modal analysis techniques described. The results at $(t + \Delta t)$ are compared with whatever boundary restraints are specified. If, for example, an acceleration limit is set at $\ddot{X}_L$, all snubber locations are checked to see if

$$|\ddot{X}|_i \geq \ddot{X}_L \qquad \text{...(9.10)}$$

If this situation exists one realizes that X_i at $(t + \Delta t)$ has to be

$$X(t + \Delta t)_i = X(t) + \underbrace{\dot{X}(t)\Delta t + \tfrac{1}{2}\ddot{X}_L \Delta t^2}_{\Delta X_i} \qquad \text{...(9.11)}$$

Basically X_i is unknown if $|\ddot{X}|_i < \ddot{X}_L$, and X_i is known if $|\ddot{X}|_i \geq \ddot{X}_L$.

Having determined X_i for all "m" snubber locations where $|\ddot{X}|_i \geq \ddot{X}_L$ one obtains the snubber specified displacement vector,

$$\{\Delta X\} \qquad \text{...(9.12)}$$

Returning to Equation (9.3), the homogeneous part of the differential equation is formed which include· ·tive degrees of freedom at the snubbers,

$$[M]\{\ddot{X}\} + [K]\{X\} = \{R\} \qquad \text{...(9.13)}$$

where

$R_i = 0$ - At all non-snubber locations,

$R_i = 0$ - At all snubber locations where $|\ddot{X}|_L < \ddot{X}_L$

and $R_i \neq 0$ - At m locations where $|\ddot{X}|_i \geq \ddot{X}_L$

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Knowing $\{\ddot{X}(t)\}$, $\{\dot{X}(t)\}$, and $\{X(t)\}$, we solve equation (9.13) by modal analysis techniques for changes in X_i at the m snubber locations, by incrementing on a one by one basis the non-zero R loads. Thus we can find the "dynamic" influence coefficients at the m active snubber supports.

$$\frac{\partial X_1}{\partial R_1} = \frac{\Delta X_1}{\Delta R_1} \qquad \ldots(9.14)$$

$$\frac{\partial X_2}{\partial R_1} = \frac{\Delta X_2}{\Delta R_1}$$

$$\vdots \;\; \text{(etc.)} \;\; \vdots$$

The total deflections can be written as

$$\Delta X_1 = \frac{\partial X_1}{\partial R_1}\Delta R_1 + \frac{\partial X_1}{\partial R_2}\Delta R_2 + - - - - - + \frac{\partial X_1}{\partial R_n}\Delta R_n$$

$$\Delta X_2 = \frac{\partial X_2}{\partial R_1}\Delta R_1 + \frac{\partial X_2}{\partial R_2}\Delta R_2 + - - - - - + \frac{\partial X_2}{\partial R_n}\Delta R_n$$

$$\vdots \qquad \vdots \qquad \vdots \qquad \vdots$$

$$n = \frac{\partial X_n}{\partial R_1}\Delta R_1 + \frac{\partial X_n}{\partial R_2}\Delta R_2 + - - - - - + \frac{\partial X_n}{\partial R_n}\Delta R_n$$

or in matrix form,

$$\{\Delta X\} = \left[\frac{\partial X}{\partial R}\right]\{\Delta R\} \qquad \ldots(9.15)$$

Since we know $\{\Delta X\}$ from equation (9.12)

$$\{\Delta R\} = \left[\frac{\partial X}{\partial R}\right]^{-1}\{\Delta X\} \qquad \ldots(9.16)$$

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One can now proceed to determine the response at $(t + \Delta t)$ and satisfy the requirements of equation (9.10), or others that may be imposed on the system, by solving equation (9.2a) with the additional load term $\{\Delta R\}$.

Hence knowing $\{X(t)\}$, $\{\dot{X}(t)\}$, $\{F(t + \Delta t\}$, and $\{\Delta R\}$,

$$[M]\{\ddot{X}\} + [K]\{X\} = (\{F(t)\} - [K_{1S}]\{X_S\}) + \{\Delta R\} \qquad \ldots(9.17)$$

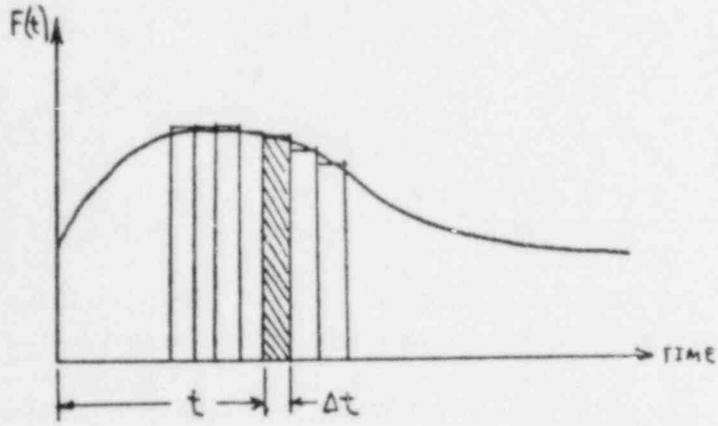
can be solved for $\{X(t + \Delta t)\}$, $\{\dot{X}(t + \Delta t)\}$, and $\{\ddot{X}(t + \Delta t)\}$. Here,

$\Delta R_i = 0$ - All non-snubber locations
$\Delta R_i = 0$ - All snubber locations $|\ddot{X}|_i < \ddot{X}_L$
$\Delta R_i \neq 0$ - All snubber locations where $|\ddot{X}|_i > \ddot{X}_L$

The solution for the next time increment can be solved by the same method.

Returning to Equation (9.8), the solution to the uncoupled equations can be found exactly if P(t) is known as a function of time. P(t) is not necessarily limited to harmonic functions, but exact solutions can be obtained for other functions such as ramps, steps, impulses, etc. However, seismic or dynamic disturbances cannot be expressed mathematically as a function of time, but rather as a series of pairs of points defining the excitation vs time. An exact solution will not be obtained but instead an approximate solution will be obtained, one that will be a function of the integration time step that has been chosen.

The solution of Equation (9.8) for an arbitrary excitation can be solved by using techniques presented by Duhamel, commonly referred to as a Duhamels Integral, the Convolution Integral, or Superpositon integral. Briefly, the forcing function is broken up into a series of constant load steps as shown below.



At time (t) a step excitation F(t) acting from (t) to (t + Δt) exists. The response of the single degree of freedom oscillator described by Equation (9.8) can be expressed as

$$X(t) = A_o e^{-\zeta\omega_n t}\cos(\omega_d t - \gamma) - \frac{F^*}{\omega_n\omega_d} e^{-\zeta\omega_n t}\cos(\omega_d t - \Psi) + \frac{F^*}{\omega_n^2} \quad (9.18)$$

where

$$A_o = \sqrt{\dot{x}_o^2 + 2\zeta\omega_n x_o\dot{x}_o + \omega_n^2 x_o^2} \,/\, \omega_d$$

$$\tan\gamma = \frac{\dot{x}_o + \zeta\omega_n x_o}{\omega_d x_o}$$

$$\tan\Psi = \frac{\zeta}{\sqrt{1 - \zeta^2}}$$

$$\omega_d = \frac{\omega_n}{\sqrt{1 - \zeta^2}}$$

X_o, $\dot{X}_o$ = Initial con[illegible]ions, i.e., the displacement and velocity respectively at t

ζ = Critical damping ratio

F^* = Constant force furing time $t \rightarrow (t + \Delta t)$

If the force F^* does not vary during Δt the solution can be calculated exactly. Therefore to solve the snubber problem, Δt is chosen so that $F(t)$ does not vary significantly during Δt. As a matter of fact, if $F(t)$ does not vary with time, Δt can be increased without significantly affecting the results. Consequently, the computation may be reduced drastically. Knowing the condition at t, equation (9.18) can be used for determining the response at $(t + \Delta t)$. The displacement, velocity, and acceleration can be expressed as follows:

$$X(t + \Delta t) = A_o e^{-\zeta\omega_n\Delta t}\cos(\omega_d\Delta t - \gamma) - \frac{F^*}{\omega_n\omega_d} e^{-\zeta\omega_n\Delta t}\cos(\omega_d\Delta t - \Psi) + \frac{F^*}{\omega_n^2} \quad \ldots(9.19a)$$

$$\dot{X}(t + \Delta t) = e^{-\zeta\omega_n\Delta t}\left[\frac{-\zeta}{\sqrt{1 - \zeta^2}}\,\omega_d A_o\cos(\omega_d\Delta t - \gamma) - A_o\omega_d\sin(\omega_d\Delta t - \gamma) \quad \ldots(9.19b)\right.$$

$$\left. + \frac{F^*}{\omega_n}\left(\frac{\zeta}{\sqrt{1 - \zeta^2}}\right)\cos(\omega\ \Delta t - \Psi) + \frac{F^*}{\omega_n}\sin(\omega\ \Delta t - \Psi)\right]$$

$$\ddot{X}(t + \Delta t) = -A_o\left(\frac{\zeta}{\sqrt{1 - \zeta^2}}\right)\omega_d e^{-\zeta\omega_n\Delta t}\left(-\omega_d\sin(\omega [illegible] \quad \zeta\omega_n\cos(\omega_d\Delta t - \gamma)\right) \ldots(9.19c)$$

$$-A_o\omega_d e^{-\zeta\omega_n\Delta t}\left(\omega_d\cos(\omega_d\Delta t - \gamma) - \zeta\omega_n\sin(\omega_d\Delta t - \gamma)\right)$$

$$+ \frac{F^*}{\omega_n}\left(\frac{\zeta}{\sqrt{1 - \zeta^2}}\right)e^{-\zeta\omega_n\Delta t}\left(-\omega_d\sin(\omega_d\Delta t - \Psi) - \zeta\omega_n\cos(\omega_d\Delta t - \Psi)\right)$$

$$+ \frac{F^*}{\omega_n}\ e^{-\zeta\omega_n\Delta t}\left(\omega_d\cos(\omega_d\Delta t - \Psi) - \zeta\omega_n\sin(\omega_d\Delta t - \Psi)\right)$$

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The analysis approach has been discussed in general terms in the preceeding section. The following section will be concerned with the computational procedures. Analysis procedures for handling nonlinear restorative forces, viscous forces, clearances, and motion constraints will be discussed. The various procedures will be combined to form the total snubber analytical model (computer code).

This particular form of loading can be separated into two categories: (1) nodal movements that are constrained throughout the loading cycle; and (2) nodal movements that are constrained during portions of the loading cycle. The first class or category of constraints are those that can be handled by standard modal analysis techniques. These movements are in general the specified loading such as pipe anchor movements or support motions. The solution for this type of input is presented in the previous section and since this technique is not unique, a detailed description of the procedure is omitted. However, the second category is unique to the snubber analysis and will be presented in detail.

The detail of the analysis procedure is in the form of a flow chart. The snubber parameter that the analysis procedure applies to is the acceleration threshold parameter. Basically, the nodal accelerations are limited to a specified limit $\ddot{X}_L$. The flow chart in Figure 9.2 indicates the general approach for solving the problem when considering acceleration limits.

Viscous loads are those loads or reactions that are a function of velocity. The force may be proportional to the velocity as it is in classical viscous damping or it may be any generalized function $F_v = F_v(\dot{X})$. The calculation and implementation of viscous snubber reactions is quite simple inasmuch as the boundary conditions for the piping or component system do not vary with time. Viscous loadings are created by hydraulic snubbers, and it should be noted that there is no connection between these viscous effects and the modal damping referred to in Equation (9.8). This damping (percent critical damping) refers to the piping or component system damping and not the discrete damping of the snubbers. The flow chart in Figure 9.3 indicates the general approach for solving the problem of viscous damping forces.

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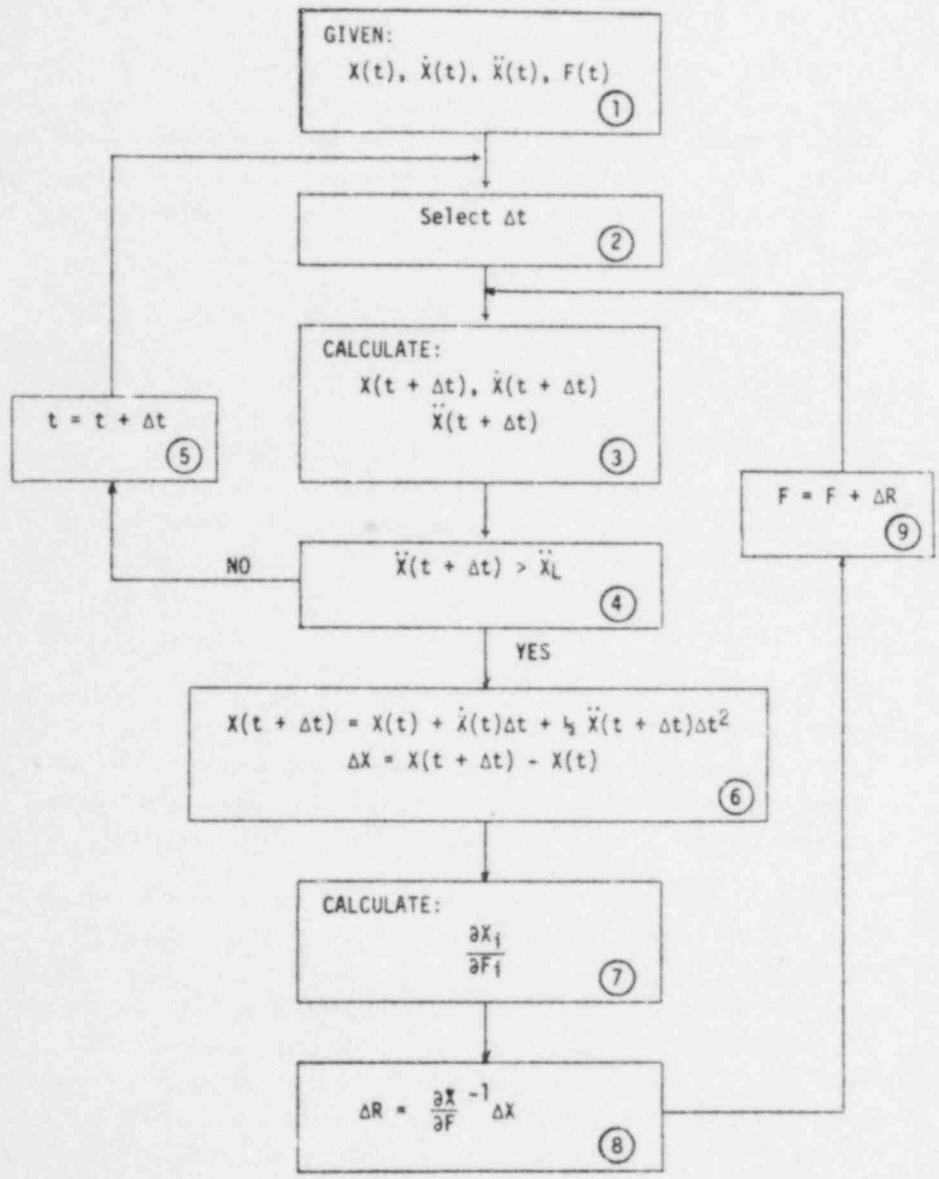

Figure 9.2 Response Due to Acceleration Threshold Parameter

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FIGURE 9.2 (Cont.)

STEP 1: All anchor motions or specified displacements, loads and system parameters are known at t = 0.

STEP 2: The selection of a time increment is made based on either input data frequency characteristics, system natural frequency characteristics, or both. In general, the time increment is constant throughout the analysis, but this is not a restriction. The time increment (Δt) may be varied as the solution progresses to minimize computer time.

STEP 3: Knowing the conditions at t and the loads at $(t + \Delta t)$ the response at $(t + \Delta t)$ can be calculated. [See Equations (9.10), (9.8), and 9.4).]

STEP 4: After calculating the response at $(t + \Delta t)$ each snubber location motion is checked to see if the acceleration is greater than the acceleration threshold parameter (limit acceleration). If all accelerations are less than or equal to $\ddot{X}_L$, the solution calculated at $(t + \Delta t)$ is correct. If not, go to STEP 6.

STEP 5: Increment the time and proceed to calculate the response for the next integration time interval.

STEP 6: Since certain accelerations exceed the limit values, the predicted displacements from STEP 3 will not be correct. The response at these locations can be calculated since $\ddot{X}_L$ is known. Knowing what was calculated from STEP 3 and what should be, from STEP 6, the displacement increment is known:

$$\Delta X = X(t + \Delta t) - X(t)$$

Therefore at each snubber location where the acceleration exceeds $\ddot{X}_L$, the displacement increment ΔX is known for the time step going from $t \rightarrow (t + \Delta t)$.

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FIGURE 9.2 (Cont.)

STEP (7): In order to calculate $(\partial X_i/\partial F_j)_i$ terr , a unit change impulse load is applied at snubber location j (whe $\ddot{X} > \ddot{X}_L$) and the change of displacement ΔX is noted at each snul er location where $\ddot{X}(t + \Delta t) > \ddot{X}_L$. This solution is represented by Equation (9.13). Therefore in "m" locations have snubber responses where $\ddot{X}(t + \Delta t) > \ddot{X}_L$ then a (mxm) matrix of $(\partial X_i/\partial F_j)$ terms will be formed.

STEP (8): Having obtained the matrix of "flexibility" terms $(\partial X_i/\partial F_j)$, the snubber reaction loads ΔR can be calculated since ΔX_i are known from STEP (6).

STEP (9): The snubber reaction loads calculated in STEP (8) are added to the applied load vector $F(t + \Delta t)_i$ and the response calculation of STEP (3) is repeated. The desired response determined in STEP (6) will not be obtained.

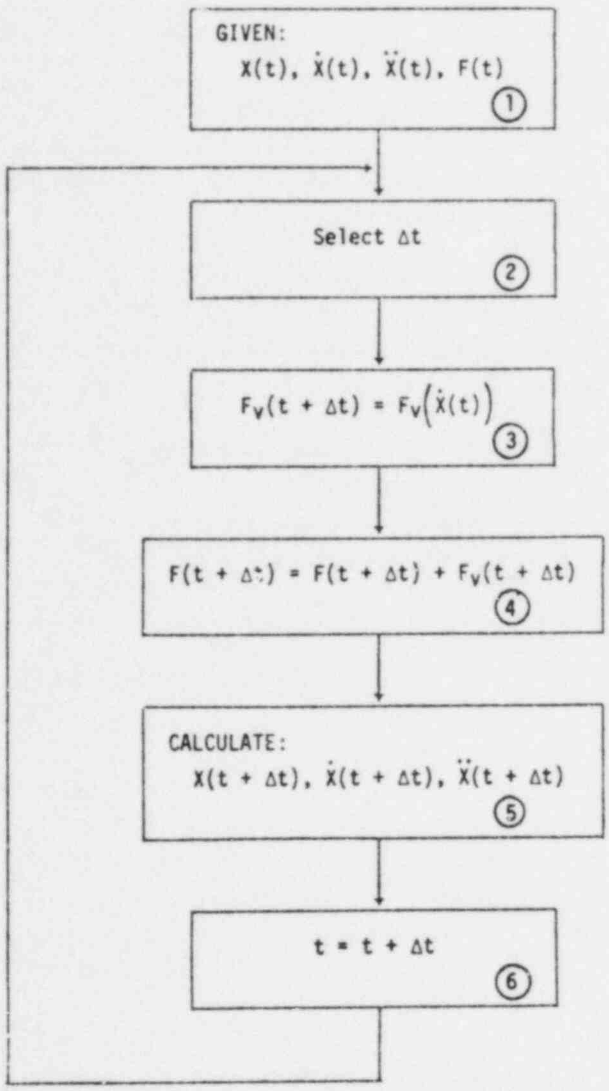

Figure 9.3 Response Due to Viscous Parameter
(Without Consideration of Unlocking)

FIGURE 9.3 (Cont.)

STEP 1: All anchor motions or specified displacements, loads, and system parameters are known at t = 0.

STEP 2: The selection of time increment is made. (See STEP 2 - Motion Constraints.)

STEP 3: The calculation of the viscous force at $(t + \Delta t)$ is based on the velocity at t. This approach adds an insignificant error provided Δt is not large - which removing the need for an iterative solution scheme.

STEP 4: The viscous force is added to the applied force to get the total force acting on the system at $(t + \Delta t)$.

STEP 5: The response at $(t + \Delta t)$ is calculated based on the applied force obtained in STEP 4 and the response at t, i.e., $X(t)$, $\dot{X}(t)$, and $\ddot{X}(t)$.

STEP 6: The integration time interval is incremented and the next integration step is initiated.

A problem that needs to be addressed for viscous hydraulic snubber reaction loads is implementing the criteria when the snubber "unlocks". First, there is the locking velocity parameter that dictates when the snubber becomes active. Next, there is the bleed rate which effects the response when the snubber is active. Finally there is the criteria which indicates the snubber becomes inactive. Several criteria can be established for causing the snubber to become inactive, some of these include:

a) Reaction load < R_{CRIT}
b) $X_i < \dot{X}_{CRIT}$
c) $X < X_{CRIT}$

The problem is one of establishing which parts of the response cycle have an "active" snubber. This complicates the flow diagram analysis procedure shown in Figure 9.3. The procedure used for this analysis is shown in Figure 9.4.

Nonlinear restorative loads are those that are other than linear functions of displacement response $F_R = F_R(X)$. The linear restorative forces are in general treated as linear spring rates, while the higher order terms are treated as nonlinear restorative forces. The consideration of these forces in the dynamic analysis is identical to the procedure used for solving the viscous problem. The procedure differs only in STEP 3 (Figure 9.3) where the nonlinear translational load is calculated rather than the velocity dependent load. The snubber characteristic that is more exemplified by this nonlinearity, is caused by hydraulic fluid with entrained air.

Clearances are usually associated with other response parameters that act in conjunction with them. For example a clearance may exist with viscous damping, or nonlinear restoring forces; or, a clearance may exist between the system and a rigid anchor. Clearances may be symmetric or asymmetric about an equilibrium point. Basically a clearance represents a dead band in the response-reaction characteristics of a snubber. The analysis procedure used when clearances are considered is basically similiar to that when no clearance

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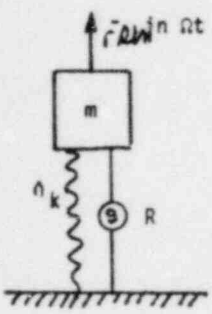
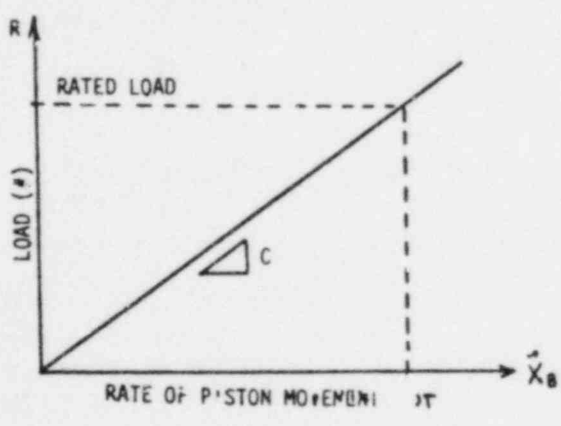
exists. However, there exists a check in which the response is compared to clearance values at a specific time, and if the response is in the dead-band region, corrective steps or load modifications are not made. For example consider the viscous damping parameter procedure. If clearance is included the flow chart is modified from that shown in Figure 9.3 to that shown in Figure 9.5.

Although the previously discussed analysis technique works well when damping effects are small, numerical problems are encountered when the discrete damping parameter C = RATED LOAD/BLEED RATE is large. For a single degree of freedom lumped mass model, a large value appears to be $C > 20\ M\omega_n$. The numerical instabilities can be minimized by using very small integrating time steps; however, for most practical problems the running time would be prohibitively long. Therefore, to solve problems when viscous damping parameters are large, an alternate analysis procedure has been adopted. This procedure, which requires a load correction step for each integration step, is stable for large discrete viscous damping parameters, $C >> 20\ M\omega_n$.





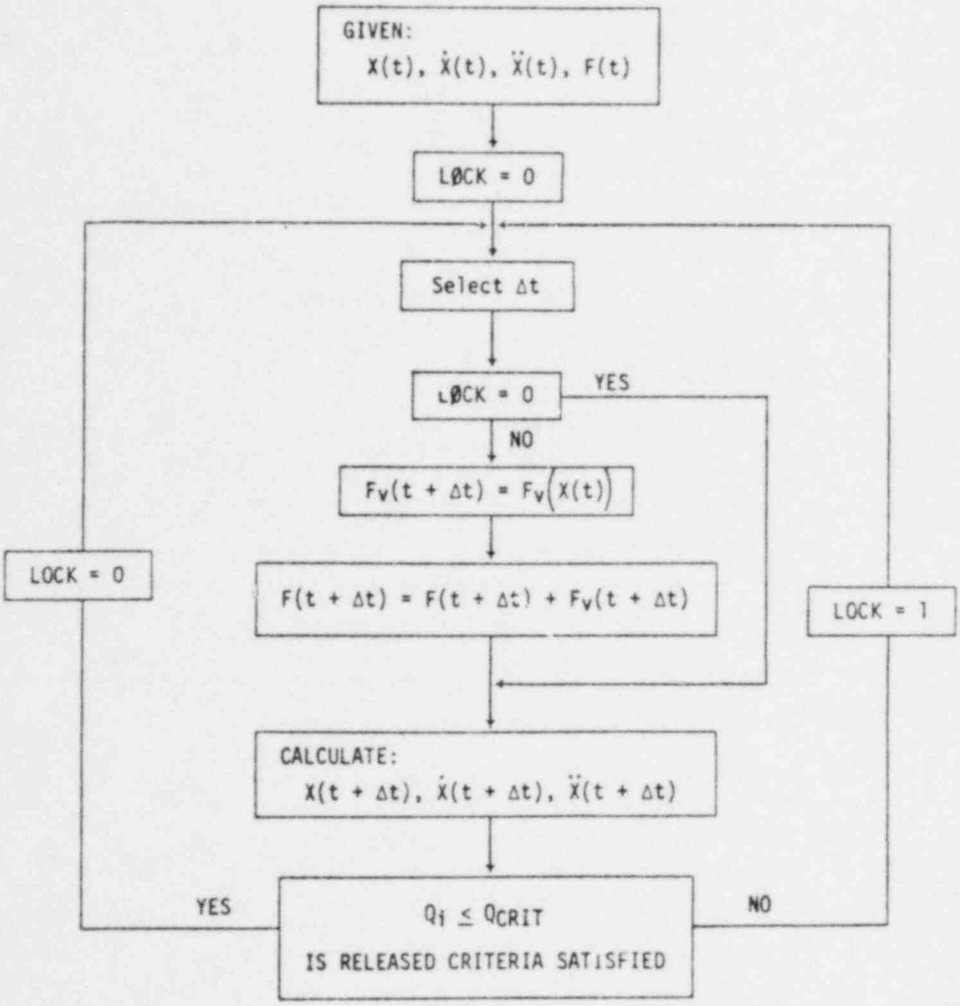


Figure 9.4 Response Due to Viscous Paramaters (Includes Unlocking)

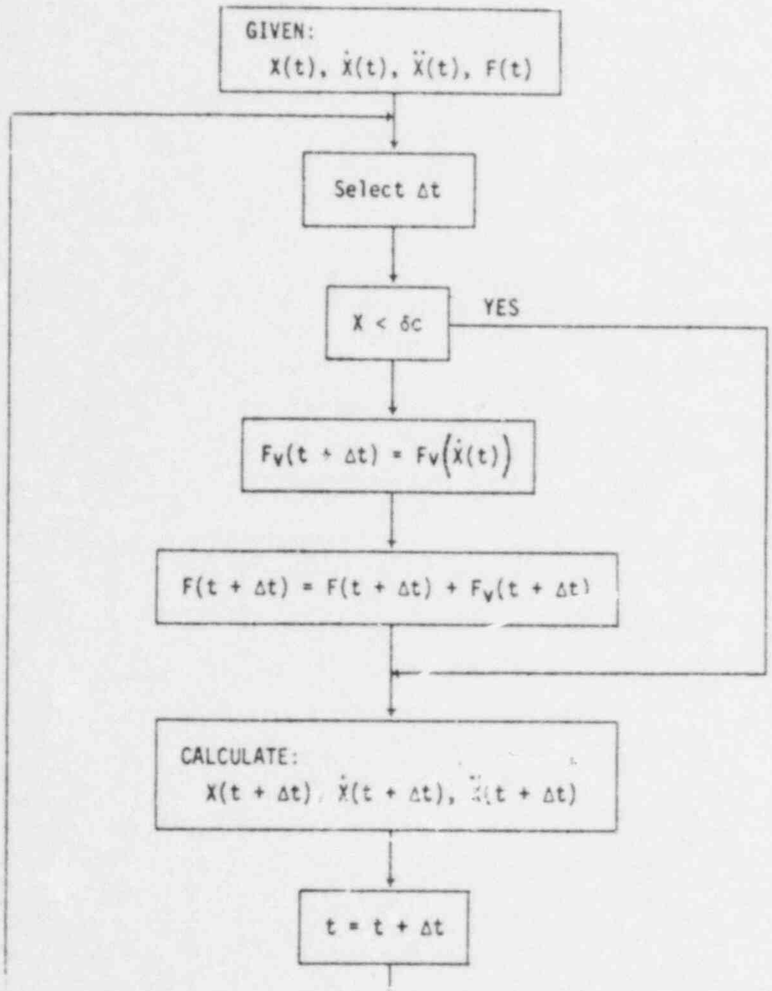


Figure 9.5 Response Due to Viscous and Clearance Parameters

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Looking at a single degree of freedom lumped mass, we can proceed to develop an analysis procedure that will not be sensitive to the magnitude of C. The analysis technique is based on constant velocity between integration time steps.

The response at $(t + \Delta t)$ can be expressed by

$$X_i(t + \Delta t) = X_i(t) - \dot{X}_{B_i}(R)\Delta t \qquad \ldots(9.20)$$

The same response can be expressed in terms of the reaction loads,

$$X_i(t + \Delta t) = X_i(F, t, R) = X_i(R) \qquad \ldots(9.21)$$

since F, t are constant during Δt.

Equating these expressions,

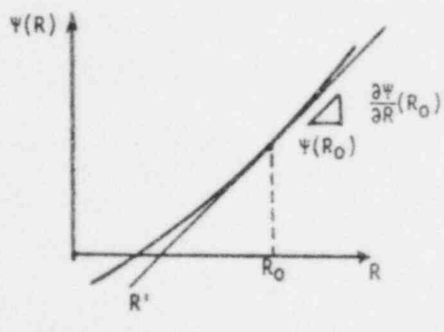
$$X_i(t) - \dot{X}_{B_i}(R)\Delta t = X_i(R) \qquad \ldots(9.22)$$

For the general system consisting of many modes, there is one equation for each snubber support. Bascially we have "m" nonlinear equations in which R must be solved.

In order to solve these equations, we take and rearrange the equation as

$$X_i(t) - \dot{X}_{B_i}(R)\Delta t - X_i(R) = \Psi_i \qquad \ldots(9.23)$$

where $\Psi_i = 0$ when R is correct.



The figure on the previous page shows a graphical representation of the function $\Psi(R)$. A first approximation is

$$\frac{\Psi(R_0)}{R_0 - R} = \frac{\partial \Psi}{\partial R}(R_0) \qquad \ldots(9.24)$$

where

$$\left(\frac{\partial \Psi}{\partial R}\right)_i = \frac{\partial}{\partial R}\left[X(t) - \dot{X}_B(R)\Delta t - X_i(R)\right] = \frac{\partial \dot{X}_B}{\partial R}\Delta t - \frac{\partial X}{\partial R}(R_0)$$

and

$$\left(\frac{\partial \Psi}{\partial R}\right)_i = -\frac{1}{C}\Delta t - \frac{\partial X}{\partial R}(R_0) \qquad \ldots(9.25)$$

Since we are concerned with the general solution, Equation (9.6) becomes

$$\left(\frac{\partial \Psi}{\partial R}\right)_{ij} = -\frac{1}{C}\Delta t - \Sigma \frac{\partial X_i}{\partial R_j}(R_{0j}) \qquad \ldots(9.26a)$$

therefore,

$$\left[\frac{\partial \Psi}{\partial R}\right] = \left[\diagdown\frac{\Delta t}{C}\diagdown\right] + \left[\frac{\partial X}{\partial R}\right] \qquad \ldots(9.26b)$$

Writing Equation (9.24) in matrix form,

$$\frac{\Psi(R_0)}{R_0 - R} = \frac{\partial \Psi}{\partial R}(R_0)$$

$$\Psi(R_0) = \frac{\partial \Psi}{\partial R}(R_0)(R_0 - R)$$

and,

$$\{\Psi(R_0)\} = \left[\frac{\partial \Psi}{\partial R}\right](\{R_0\} - \{R\}) \qquad \ldots(9.27)$$

Solving for {R},

$$\{R\} = \{R_O\} + \left[\frac{\partial\Psi}{\partial R}\right]^{-1} \{\Psi(R_O)\} \qquad \text{... (9.28)}$$

Substituting (9.26b) into (9.28):

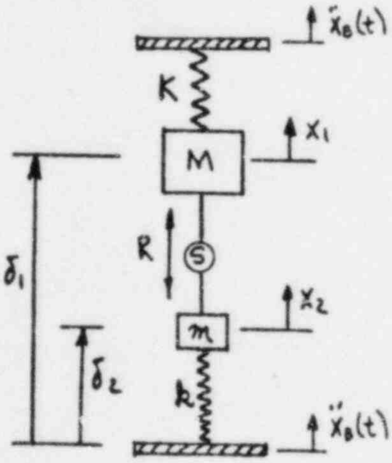
$$\{R\} = \{R_O\} + \left[\left[\frac{\Delta t}{C}\right] + \left[\frac{\partial X}{\partial R}\right]\right]^{-1} \{\Psi(R_O)\} \qquad \text{...(9.29)}$$

Equation (9.29) is a recursion formula that can be used to solve for the snubber reaction loads {R}.

Further examination of Equation (9.29) reveals that since $[\partial X/\partial R]$ is a linear matrix, the equation is linear. Therefore {R} can be solved after one iterative load correction.

One can see, that as "C" becomes large, the $[\Delta t/C]$ matrix becomes small, and since $[\partial X/\partial R]$ is stable, the solution will remain stable.

9.1 Analysis Procedure for Considering Support Dynamic Characteristics



Assume for purposes of analysis that the snubber reaction load is represented simply as R, where R is a function of variables such as accelerations, velocities, or displacement with a nonlinear relationship to load. The two differential equations of motion are:

$$M\ddot{X}_1 + (X_1 - X_B)K = -R \qquad \ldots(9.1.1a)$$

$$m\ddot{X}_2 + (X_2 - X_B)K = R \qquad \ldots(9.1.1b)$$

Expressing the response in terms of the relative displacements δ_1 and δ_2, the differential equations beome

$$M(\ddot{\delta}_1 + \ddot{X}_B) + K\delta_1 = -R \qquad \ldots(9.1.2a)$$

$$m(\ddot{\delta}_2 + \ddot{X}_B) + K\delta_2 = R \qquad \ldots(9.1.2b)$$

where

$$X_1 = \delta_1 + X_B \qquad X_2 = \delta_2 + X_B \qquad \ldots(9.1.3a)$$

$$\ddot{X}_1 = \ddot{\delta}_1 + \ddot{X}_B \qquad \ddot{X}_2 = \ddot{\delta}_2 + \ddot{X}_B \qquad \ldots(9.1.3b)$$

Assume the reaction load R is a function of acceleration as it pertains to the acceleration threshold parameter. That is

$$R = R(\ddot{\delta}_1, \ddot{\delta}_2) \qquad \ldots(9.1.4)$$

The acceleration of each mass assuming R = 0 is

$$\ddot{\delta}_1 = -\ddot{X}_B - (\tfrac{K}{M})\delta_1 \qquad \ldots(9.1.5a)$$

$$\ddot{\delta}_2 = -\ddot{X}_B - (\tfrac{k}{M})\delta_2 \qquad \ldots(9\ 1.5b)$$

Imposed on the system is the following condition,

$$|\ddot{\delta}_1 - \ddot{\delta}_2| \leq \ddot{X}_L \qquad \ldots(9.1.6)$$

If $|\ddot{\delta}_1 - \ddot{\delta}_2| \geq \ddot{X}_L$ then $R \neq 0$, and

$$\ddot{\delta}_1 - \ddot{\delta}_2 = -R(\frac{1}{M} + \frac{1}{m}) - (\frac{K}{M})\delta_1 + (\frac{k}{m})\delta_2 \qquad \ldots(9.1.7)$$

Also,

$$\delta_1 = \delta_2 + X_L \text{ sgn} \left[-(\frac{K}{M})\delta_1 + (\frac{k}{m})\delta_2\right] \qquad \ldots(9.1.8)$$

letting

$$\emptyset = -(\frac{K}{M})\delta_1 + (\frac{k}{m})\delta_2 \qquad \ldots(9.1.9)$$

Then,

$$\ddot{\delta}_1 - \ddot{\delta}_2 = \ddot{X}_L \text{ sgn}(\emptyset) = -R(\frac{1}{M} + \frac{1}{m}) - (\frac{K}{M})\delta_1 + (\frac{k}{m})\delta_2 \qquad \ldots(9.1.10)$$

Solving for R,

$$R = (\frac{mM}{m + M})\left[(\frac{k}{m})\delta_2 - (\frac{K}{M})\delta_1 - \ddot{X}_L \text{ sgn}(\emptyset)\right] \qquad \ldots(9.1.11)$$

Solving for δ_1 from equation (9.1.2a) and equation (9.1.11),

$$\delta_1 = (\frac{1}{M + m})\left[(M + m)\ddot{X}_B - K\delta_1 - K\delta_2 + m\ddot{X}_L \text{ sgn}(\emptyset)\right] \qquad \ldots(9.1.12)$$

And from Equation (9.1.8),

$$\ddot{\delta}_2 = \ddot{\delta}_1 - \ddot{X}_L \text{ sgn}(\emptyset) \qquad \ldots(9.1.13)$$

If we let

$$\beta = (\frac{m}{M}) \qquad \ldots(9.1.14)$$

The following expressions can be used to calculate the system response and the snubber reaction load.

$$\ddot{\delta}_1 = \ddot{X}_B - \left(\frac{K}{M}\right)\left(\frac{1}{1+\beta}\right)\delta_1 - \left(\frac{k}{m}\right)\left(\frac{\beta}{1+\beta}\right)\delta_2 + \left(\frac{\beta}{1+\beta}\right)\ddot{X}_L \operatorname{sgn}(\emptyset) \quad ..(9.1.15)$$

and

$$\left(\frac{R}{M}\right) = \left(\frac{\beta}{1+\beta}\right)\left[\left(\frac{k}{m}\right)\delta_2 - \left(\frac{K}{M}\right)\delta_1 - X_L \operatorname{sgn}(\emptyset)\right] \quad ...(9.1.16)$$

And finally,

$$\ddot{\delta}_2 = \ddot{\delta}_1 - \ddot{X}_L \operatorname{sgn}(\emptyset) \quad ...(9.1.13)$$

where

$$\emptyset = -\left(\frac{K}{M}\right)\delta_1 + \left(\frac{k}{m}\right)\delta_2 \quad ...(9.1.9)$$

If the snubber reaction load is velocity dependent, as for a hydraulic snubber, the following analysis is applicable.

The differential equations of motion are the same as previously developed.

$$\ddot{\delta}_1 + \left(\frac{K}{M}\right)\delta_1 = -\ddot{X}_B - \frac{R}{M} \quad ...(9.1.2a)$$

$$\delta_2 + \left(\frac{k}{m}\right)\delta_2 = -X_B + \frac{R}{\beta M} \quad ...(9.1.2b)$$

where

$$\beta = \frac{m}{M} \quad ...(9.1.14)$$

Since the load is velocity dependent,

$$R = C(\dot{\delta}_1 - \dot{\delta}_2) \quad ...(9.1.17)$$

or,

$$\frac{R}{M} = \frac{C}{M}(\dot{\delta}_1 - \dot{\delta}_2) \qquad \text{...(9.1.18)}$$

where

$$C = \text{Viscous Damping Coefficient} = \frac{\text{RATED LOAD}}{\text{BLEED RATE}}$$

During the time increment Δt, the response changes from $\dot{\delta}_1' \rightarrow \dot{\delta}_{1act}'$ and $\dot{\delta}_2 \rightarrow \dot{\delta}_{2act}'$. Also during the time increment the snubber reaction also changes. The velocity at t and (t + Δt) can be determined from the following expressions:

$$\dot{\delta}_{1act}' = \dot{\delta}_1' - (\frac{\partial \dot{\delta}_1}{\partial R}) \frac{R}{M} \qquad \text{...(9.1.19a)}$$

$$\dot{\delta}_{2act}' = \dot{\delta}_2' + (\frac{\partial \dot{\delta}_2}{\partial R}) \frac{R}{M} \qquad \text{...(9.1.19b)}$$

where

$\dot{\delta}_{1act}'$, $\dot{\delta}_{2act}'$ = Correct velocities at (t + Δt)

$\dot{\delta}_1$, $\dot{\delta}_2$ = Velocity at (t + Δt) when R = 0

$(\frac{\partial \dot{\delta}_1}{\partial R})$, $(\frac{\partial \dot{\delta}_2}{\partial R})$ = Partial derrivatives

Substituting (9.1.19a) and (9.1.19b) into (9.1.18) the following expression is obtained:

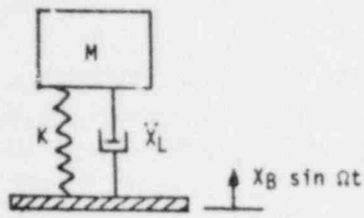
$$(\frac{R}{M}) = (\frac{C}{M})\left[(\dot{\delta}_1' - \dot{\delta}_2') - (\frac{R}{M})(\frac{\partial \dot{\delta}_1}{\partial R} + \frac{\partial \dot{\delta}_2}{\partial R})\right] \qquad \text{...(9.1.20)}$$

or, rearranging and solving for $(\frac{R}{M})$,

$$(\frac{R}{M}) = (\frac{C}{M}) \frac{(\dot{\delta}_1' - \dot{\delta}_2')}{1 + (\frac{C}{M})(\frac{\partial \dot{\delta}_1}{\partial R} + \frac{\partial \dot{\delta}_2}{\partial R})} \qquad \text{...(9.1.21)}$$

The above expression for the reaction load can then be substituted into Equations (9.1.2a) and (9.1.2b) and the solution can be obtained by conventional analysis procedures.

9.2 Analysis of Acceleration Threshold Parameter (System A)



Given the single degree of freedom system shown above, in which $\ddot{X}_L << |\ddot{X}|$, where $|\ddot{X}|$ is the amplitude of the acceleration for the unsnubbed system, the differential equation of motion is

$$m\ddot{X} + kX = kX_B \sin \Omega t \qquad ...(9.2.1)$$

Expressing the response in terms of relative components, where

$$\delta = X - X_B \qquad ...(9.2.2a)$$

$$\dot{\delta} = \dot{X} - \dot{X}_B \qquad ...(9.2.2b)$$

$$\ddot{\delta} = \ddot{X} - \ddot{X}_B \qquad ...(9.2.2c)$$

then Equation (9.2.1) becomes

$$m\ddot{\delta} + k\delta = -m\ddot{X}_B$$

or

$$\ddot{\delta} + \omega^2\delta = -\ddot{X}_B = X_B\Omega^2 \sin \Omega t \qquad ...(9.2.3)$$

Solving Equation (9.2.3) for the relative acceleration,

$$\ddot{\delta} = \Omega^2 X_B \sin \Omega t - \omega^2 \delta \qquad \text{...(9.2.4)}$$

However, since the acceleration is limited to $\ddot{X}_L$, and since $\ddot{X}_L << |\ddot{X}|$, the acceleration of the mass can be expressed as

$$\ddot{\delta} = \ddot{X}_L \text{ sgn}|\Omega^2 X_B \sin \Omega t - \omega^2 \delta| \qquad \text{...(9.2.5)}$$

The following equation is equivalent to Equation (9.2.5)

$$\ddot{\delta} = \ddot{X}_L \text{ sgn}|\sin \Omega t - \frac{\omega \delta}{\Omega^2 X_B}| \qquad \text{...(9.2.6)}$$

Letting

$$Q = \frac{\omega^2 \delta}{\Omega^2 X_B} \qquad \text{...(9.2.7)}$$

then

$$\ddot{Q} = \frac{\omega^2 \delta}{\Omega^2 X_B} \qquad \text{...(9.2.8)}$$

Substituting (9.2.7) and (9.2.8) into (9.2.6) the following expression is obtained:

$$\ddot{Q} = (\frac{\omega^2 X_L}{\Omega^2 X_B}) \text{ sgn}|\sin \Omega t - \frac{\omega^2 \delta}{\Omega^2 X_B}| \qquad \text{...(9.2.9)}$$

or

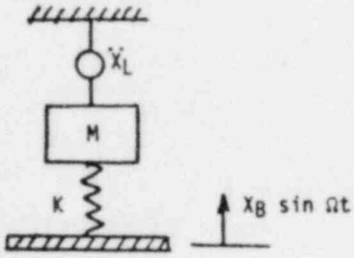
$$\ddot{Q} = \not{C} \text{ sgn}|\sin \Omega t - Q| \qquad \text{...(9.2.10)}$$

where

$$Q = (\frac{\omega}{\Omega})^2 (\frac{\delta}{X_B}) \qquad \text{...(7.2.11a)}$$

$$C = \omega^2 (\frac{\ddot{X}_L}{|\ddot{X}_B|}) \qquad \text{...(9.2.11b)}$$

The same analysis procedure can be followed for the single degree of freedom oscillator shown below.



The results are as follows:

$$\ddot{Q} = \ell \, \mathrm{sgn}|\sin \Omega t - Q| \qquad \ldots(9.2.10)$$

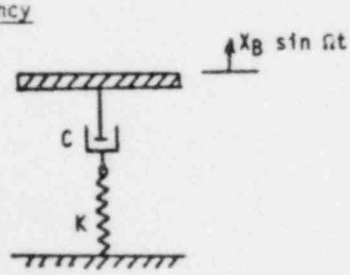
where

$$Q = (X/X_B) \qquad \ldots(9.2.12a)$$

$$\ell = (\Omega^2 \frac{\ddot{X}_L}{|\ddot{X}_B|}) \qquad \ldots(9.2.12b)$$

The solution to Equation (9.2.10) can be obtained by numerical techniques on a computer. It should be pointed out that due to the nature of this equation very small integration time steps are required.

9.3 Viscous Snubber Efficiency



Given the system shown in the above figure. The load transferred into the base is the same as the load in the dashpot which is the same as the load in the spring.

$$F_B(t) = kX_o(t) = C(\dot{X}_B - \dot{X}_o) \qquad \ldots(9.3.1)$$

The snubber efficiency η is defined as the ratio of the snubber load F_d to the maximum snubber load, where the snubber load is

$$F_d = C(\dot{X}_B - \dot{X}_o) \qquad \ldots(9.3.2)$$

therefore,

$$\eta = \frac{|F_d|}{|CX_B\Omega|} \qquad \ldots(9.3.3)$$

The force in the dashpot is

$$F_d = C(\dot{X}_B - \dot{X}_o) \qquad \ldots(9.3.4)$$

and the force in the spring is

$$F_S = kX_o \qquad \ldots(9.3.5)$$

At any instant the forces are equal, therefore

$$C(\dot{X}_B - \dot{X}_o) = kX_o \qquad \ldots(9.3.6)$$

or

$$\dot{X}_o + \left(\frac{k}{C}\right)X_o = \dot{X}_B$$

or

$$\dot{X}_o + aX_o = X_B\Omega \cos \Omega t \qquad \ldots(9.3.7)$$

where

$$a = \left(\frac{k}{C}\right) \qquad \ldots(9.3.8)$$

The solution of (9.3.7) can be found by Laplace Transform methods or other techniques. Solving for $X_o(t)$,

$$X_o(t) = \frac{X_B}{1 + (\frac{k}{C\Omega})^2} \left[(\frac{k}{C\Omega})\cos \Omega t + \sin \Omega t - (\frac{k}{C\Omega})e^{-(\frac{k}{c})t} \right] \quad \ldots(9.3.9)$$

or disregarding transient terms,

$$X_o(t) = \frac{X_B}{1 + (\frac{k}{C\Omega})^2} \left[(\frac{k}{C\Omega})\cos \Omega t + \sin \Omega t \right] \quad \ldots(9.3.10)$$

Therefore,

$$\dot{X}_o(t) = \frac{X_B\Omega}{1 + (\frac{k}{C\Omega})^2} \left[\cos \Omega t - (\frac{k}{C\Omega}) \sin \Omega t \right] \quad \ldots(9.3.11)$$

Substituting (9.3.11) into (9.3.4) the following expression is obtained,

$$F_d = CX_B\Omega \left[(\frac{a^2}{1 + a^2})\cos \Omega t + (\frac{a}{1 + a^2})\sin \Omega t \right] \quad \ldots(9.3.12)$$

From (9.3.3) and (9.3.12),

$$\eta = \left| \frac{F_d}{CX_B\Omega} \right| = \sqrt{(\frac{a^2}{1 + a^2})^2 + (\frac{a}{1 + a^2})^2} \quad \ldots(9.3.13)$$

$$\eta = \sqrt{\frac{a^2}{1 + a^2}} \quad \text{WHERE} \quad a = (\frac{k}{C\Omega})$$

Therefore, the support efficiency is

$$\eta = \sqrt{\frac{(\frac{k}{C\Omega})^2}{1 + (\frac{k}{C\Omega})^2}} \quad \ldots(9.3.14)$$

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10. CONCLUSIONS AND RECOMMENDATIONS

Basic information on snubber response sensitivity was obtained, for mechanical and hydraulic snubbers, using simple analytical models. Sophisticated analytical models, utilizing hydraulic snubber parameters, were also investigated. This data can now be used for the development of simplified analyses and design rules. The results obtained from the analyses of the simple models appeared to be consistent with the results of the more complex models. In future work, it is recommended that ana' 2s of these more complex models be extended to include the following parameters:

1. Mechanical Snubber Acceleration Threshold Parameters (ATP);
2. Mechaical Snubber ATP Combined with a Clearance;
3. Hydraulic Snubber Parameters Combined with a Cleara: :e.

This should verify that the parameter sensitivity developed for the simple models is valid for the more sophisticated models and verify, or refine, the acceptable parameter ranges in Table 2.1. In FY 79 detailed time history studies, using the snubber mathematical models developed herein, will be compared with various simplified analytical procedures, to determine their applicability. Comparisons will be made based on stress response, snubber loadings, and displacement. The simplified analyses procedures that will be developed should assure that system response will be bound within acceptable limits.

NOMENCLATURE

M, m = Mass (lb-sec^2/in)

K, k = Spring rate (lb/in)

K^*, ke = Effective spring rate (lb/in)

$K°$ = Pipe stiffness (lb/in)

Ω = Forcing frequency (rad/sec)

f_Ω = Forcing frequency (Hz)

ω_n = Natural frequency (rad/sec)

f_n = Natural frequency (Hz)

ζ = Critical damping ratio

γ = Bilinearity stiffness ratio

$X, \dot{X}, \ddot{X}$ = Translational displacement, velocity, and acceleration respectively (in., in/sec, in/sec^2)

$\tau, t, \Delta t$ = Time and time increment respectively (sec)

ℓ = length (in)

τ_B = Beating period (sec)

κ^2 = Hardening (softening) coefficient

$\ddot{X}_L$ = Acceleration threshold parameter (in/sec^2)

F, P = Force (lb)

C = Viscous damping coefficient (lb-sec/in)

$\dot{X}_B$ = Bleed velocity (in/sec)

X_B = Base excitation (in)

$\dot{X}_L$ = Lock velocity (in/sec)

α = Flexibility (in/lb)

R = Peaction load (lb)

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δ_C = Clearance (in)

F_d, C_F, F_v = Damping forces (lb)

∞ = Infinity

δ, $\dot{\delta}$, $\ddot{\delta}$ = Relative displacement, velocity and acceleration respectively (in., in/sec, in/sec^2)

F_r = Friction load

η = Support efficiency

a = $(k/C\Omega)$

β = Forcing frequency/natural frequency

REFERENCES

1. A. T. Onesto, "Finite Element Flexibility Analysis of Piping Systems," TI-10-LME-071 (to be published)

2. California Institute of Technology, "Strong Motion Earthquake Accelograms-Index Volume," Report No. EERL 76-02 (August 1976)

3. J. P. Den Hartog, "Forced Vibration with Combined Coulomb and Viscous Friction," ASME Transaction, Vol. 53 (1931)

4. Jacobsen and Ayre, Engineering Vibrations, (McGraw-Hill Book Co. 1958)

APPENDIX D

TEST DATA FROM SINGLE VERSUS MULTIPLE SNUBBER TEST PROGRAM

The data contained in this appendix are extracted from the results of a test program performed by ETEC for the U.S. NRC under a separate contract with the Office of Standards Development. The test objectives were: 1) To evaluate changes in system response that occur when a single large snubber is replaced by two smaller snubbers and 2) To evaluate the influence of mismatch of end fitting clearance, activation level and/or release rate on load sharing capacity of the snubber pair.

The material herein was abstracted from the final report for the test program. Only data pertaining to multiple snubber usage have been included.

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COVER SHEET

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Title: Single Vs Dual Snubber Installations

Author(s): A. T. Onesto

Prepared for the Office of Standards Development

NRC FIN No. 30558

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2. SUMMARY AND RECOMMENDATIONS

The work described herein was initiated in response to NRC concern regarding the practice of using several small snubbers in place of a single large snubber.

2.1 Test Objective

The test objectives were 1) to evaluate changes in system response that occur when a single large snubber is replaced by two smaller snubbers, and 2) to evaluate the influence of mismatch of end fitting clearance, activation level and/or release rate on the load sharing capacity of the snubber pair.

2.2 Test Setup

The test setup is shown in Figures 3.1 through 3.3.

The setup utilizes a horizontal test beam which was cantilevered from a hydraulically activated shaker table. The test beam consisted of a 10 foot long, 6.5 inch outside diameter, 0.5 inch wall carbon steel tube and supported a 1000 pound lead mass at the free end. Snubbers were located between the test beam and a strongback structure which was attached to a seismic mass. The attachment to the test beam consisted of a trapeze type structure which was identified by the NRC as being prototypic for dual snubber installations. The snubber attachment locations between the trapeze structure and the strongback are shown in Figure 3.3. Snubber characterization tests were performed in the location shown in Figure 3.2. During these tests snubbers were located between the shaker table and the strongback in the fixtures shown.

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2.3 Test Results

The following results were obtained from the test data:

1) End fitting clearance has a greater effect on load sharing of dual snubber supports than mismatch of activation level or release rate. For zero end fitting clearance and any combination of activation level and release rate between 8 to 25 in/min and 4 to 14 in/min, respectively, equal load sharing (50%/50% to within 3%) was observed. However, for end fitting clearance differentials of 0.05 in., 30%/70% load sharing distributions were obtained.

2) The effects of end fitting clearance on support reactions were extremely variable. Different trends were obtained for the various support types (rigid strut, hydraulic snubber or mechanical snubber), support configuration (single or dual) and inputs (seismic or sine). Table 2.3.1 summarizes the trends observed for the single and matched pair tests (Tests 5, 8, 9, 17, 20, 21).

3) For each type (rigid strut, mechanical snubber or hydraulic snubber) of matched pair of snubbers and given type of loading (seismic or sine):

   i) For zero clearance, the total reaction force for the pair was less than the reaction force for a single snubber of the same type subjected to the same type of loading. Table 2.3.2 lists the results of Tes. 2 and 17.

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ii) For non zero clearances, the single snubber force may be greater or less than the total load for the pair.

2.4 Recommendations

The results of the test program confirm the concern expressed in the Introduction and, not unexpectedly, also raise additional ones.

Since clearances in snubber assemblies are expected to be much greater than the 0.05 inch investigated in this program and ovalization of holes in clevises had significant effects, it is recommended that:

1) Further testing and analytical studies be implemented to investigate the effects of clearance mismatch over a wider range of parameters investigated.
2) Inspection be initiated to determine the extent of clearance mismatch in existing plants.
3) Design changes be implemented to minimize the effects of wear on end fitting clearance mismatch.

Relative to the first of the above, it is recommended that the effects of varying a single parameter be investigated during testing. This could require the leasing of equipment to control accurately the activation levels and release rates of hydraulic snubbers.

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TABLE 2.3.1

EFFECTS OF CLEARANCE ON SNUBBER LOADS

SUPPORT TYPE	SNUBBER TYPE	INPUT TYPE	EFFECT OF INCREASING CLEARANCE ON SNUBBER LOADS
Single	Rigid	Sine	Increases
Single	Mechanical	Sine	Increases
Single	Hydraulic	Sine	Decreases
Single	Rigid	Seismic	Decreases
Single	Mechanical	Seismic	Decreases
Single	Hydraulic	Seismic	Negligible Change
Matched Pair	Rigid	Sine	Negligible Change
Matched Pair	Mechanical	Sine	Negligible Change
Matched Pair	Hydraulic	Sine	Increase
Matched Pair	Rigid	Seismic	Inconclusive*
Matched Pair	Mechanical	Seismic	Increases
Matched Pair	Hydraulic	Seismic	Decreases

* Bad Data

TABLE 2.3.2

SUPPORT LOADS - SINGLE VS MATCHED PAIR OF SNUBBERS
(ZERO CLEARANCE TESTS)

SUPPORT TYPE	INPUT TYPE	TOTAL LOAD (LBS) SINGLE	MATCHED PAIR
Rigid	Sine	3680	1100
Hydraulic	Sine	4150	2800
Rigid	Seismic	2260	1000*
Hydraulic	Seismic	1820	800

* Questionable Data

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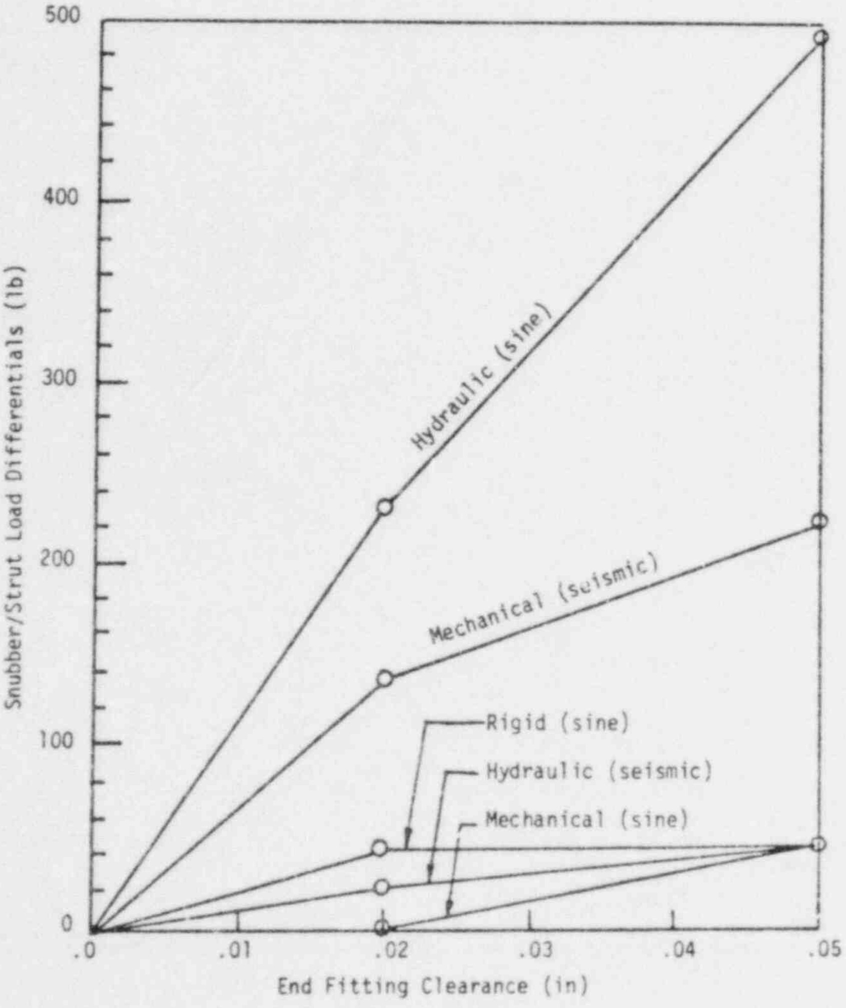

Figure 2.3.1 Load Differentials Resulting from Matched End Fitting Clearance

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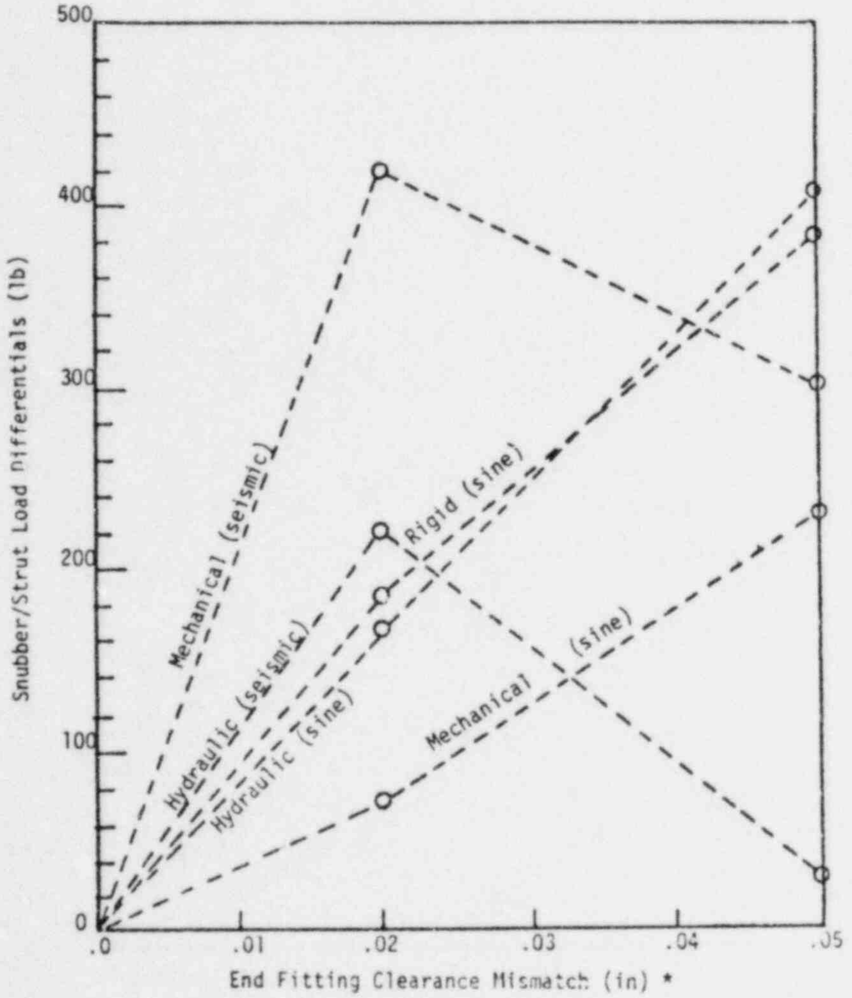

*Clearance varied on one snubber only

Figure 2.3.2 Load Differentials Resulting from Mismatched End Fitting Clearance

3. TEST SYSTEM

The test setup is shown in Figures 3.1 through 3.3. Figure 3.1 shows a plan view of the shaker table and support setup. Section Views A-A and B-B indicate snubbers in their test and characterization positions, respectively. Snubbers were located in either of these positions but not in both simultaneously. Figures 3.2 and 3.3 show the test and characterization positions, respectively, without the snubbers.

The test system consisted of a 10 foot long carbon steel tube (6 1/2 inch outside diameter, 0.5 inch wall) which was cantilevered from a hydraulically activated shaker table. The beam was oriented parallel to the ground and supported a 1000 pound lead mass at the free end and a trapeze structure 4 1/2 feet from the base. Snubbers were attached between the trapeze structure and the strongback structure which was attached to a 50,000 pound seismic mass. The geometry of the trapeze attachment was identified by the NRC as being prototypic and was used upon their recommendations. Appendix C contains the drawings of the support configuration supplied by the NRC.

The design of the test hardware was performed by ETEC and the test was performed at the Rockwell International Autonetics Strategic Systems Division (ASSD). Design drawings are presented in Appendix D. Design modifications to the ASSD facility were required to provide torsional rigidity to the shaker table as a result of large moment loadings produced by the test article and to maintain the table support bearing side loads within facility limitations. The drawings for the hardware for these modifications are contained in Appendix D.

Initial concepts for implementing the program had envisioned using existing piping systems or prototypic piping loops. These concepts were

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abandoned for the following reasons: 1) facilities were not available which could be used for generating large snubber loads without fear of overstressing the existing piping; 2) the fabrication of a prototypic piping loop capable of producing snubber loads in excess of 10,000 pounds was outside the budgetary constraints on the program; 3) A prototypic configuration would not permit control of all parameters affecting system response.

The selected test configuration had the following features:

1) The simple geometric configuration permitted control and measurement of parameters affecting system response such as clearance, support flexibility and load orientation.

2) Large snubber loads and system deflections and stresses could be developed (Snubber loads in excess of 30,000 pounds, piping stresses of approximately 50,000 psi and displacement response of 0.75 inches at the snubber attachment could be developed).

3) The dynamic characteristics of the test article were similar to many short to intermediate length of piping subsystems found in nuclear plants (4.5 Hz unrestrained natural frequency and 7.5 Hz with snubber supports).

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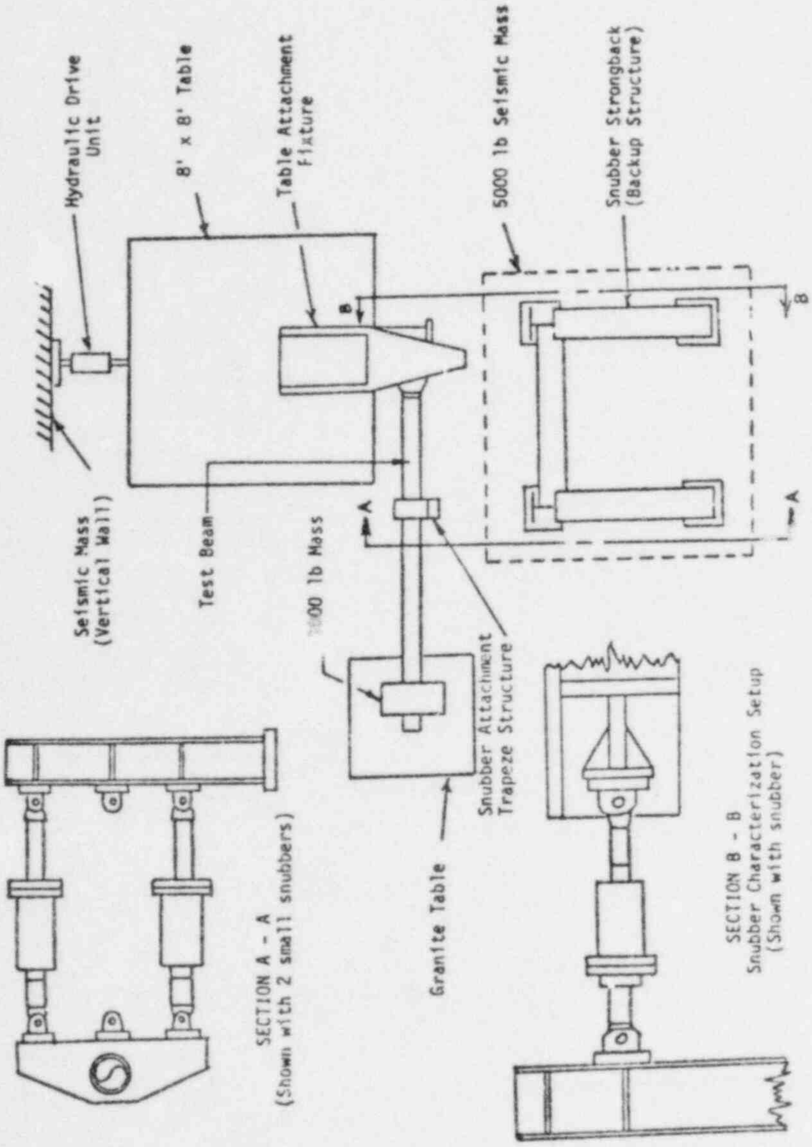

Figure 3.1 Plan View of Test Hardware

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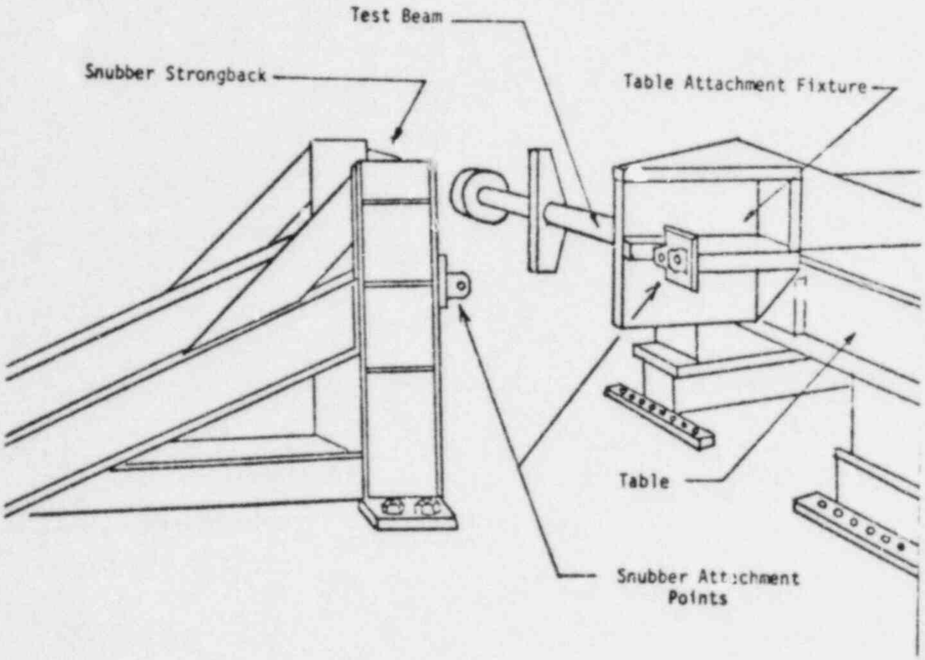

Figure 3.2 Snubber Characterization Fixture

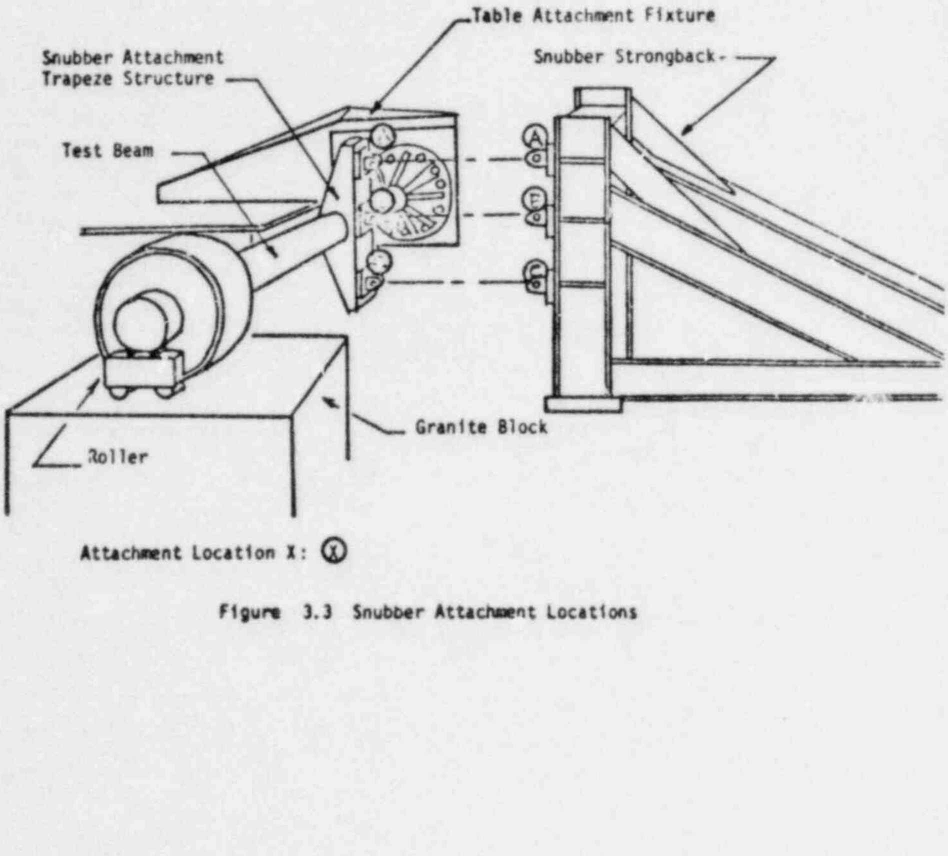

Figure 3.3 Snubber Attachment Locations

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7.5 Mismatch Test

A combined discussion of the sine and seismic mismatch tests is contained in this section. Test 17 is discussed in conjunction with the mismatch tests - Test 23, 25, 27 and 29.

Table 7.5.1 identifies the snubbers and their respective activation levels and release rates used for the various tests. Table 7.5.2 indicates the maximum snubber loads resulting from the sine and seismic inputs.

The release rate used for snubber S/N 2407-4 during Test 29 was so low that the snubber responded in compression with essentially no load release. The load drop off characteristics are shown in Figure E3.3 of Appendix E. The release rates and activation levels used for the other mismatch tests were however closer to nominal settings.

The actual snubber loads that occurred during all mismatch tests except Test 29 had nearly identical time history traces for each loading. There was no noticeable phase shifts resulting from torsional oscillations or differences in lost motion. Figure 7.5.1 through Figure 7.5.4 show snubber pair loads resulting from seismic and sine inputs for tests 23, 25, 27 and 29 respectively.

Table 7.5.3 indicates the percentage of load carried by each snubber. Test 17 and 29 indicate significant variations from equal load sharing The results of Test 17 can be discounted for reasons discussed in Section 7.3. Based on evidence from the zero clearance tests for mechanical snubbers and rigid struts, equal load sharing can be expected for a truly balanced system.

Therefore, Test 29 is the only test which would suggest concern regarding equal load sharing for a mismatched system. Based on the data summarized in Table 7.5.1, nearly equal load sharing can be anticipated for any combination of activation levels and release rates between 8.$\pm$2. to 25. in/min and 4.$\pm$1. to 15. in/min, respectively. It can be presumed that greater variations will produce cause negligible changes in load sharing and only when the snubber load will not release due to low release rates will significant variations in load sharing occur.

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TABLE 7.5.1

MISMATCH SNUBBER PROPERTIES* (IN/MIN)

	TOP SNUBBER**					BOTTOM SNUBBER**				
TEST	SNUBBER	ACTIVATION LEVEL		RELEASE RATE		SNUBBER	ACTIVATION LEVEL		RELEASE RATE	
NO.	S/N	Tension	Compression	Tension	Compression	S/N	Tension	Compression	Tension	Compression
17	2407-4	$8\pm2^{(1)}$	$8\pm2^{(1)}$	$4\pm1^{(1)}$	$4\pm1^{(1)}$	2407-3	$8\pm2^{(1)}$	$8\pm2^{(1)}$	$4\pm1^{(1)}$	$4\pm1^{(1)}$
23	11497	$25.^{(2)}$	$12.^{(2)}$	$7.0^{(2)}$	$4.0^{(2)}$	2407-4	$8\pm2^{(1)}$	$8\pm2^{(1)}$	$4\pm1^{(1)}$	$4\pm1^{(1)}$
25	11497	$8.3^{(2)}$	$7.5^{(2)}$	$8.8^{(2)}$	$14.^{(2)}$	2407-4	$8\pm2^{(1)}$	$8\pm2^{(1)}$	$4\pm1^{(1)}$	$4\pm1^{(1)}$
27	11497	8.3	7.5	8.8	14.	$2407\text{-}4^{(2)}$	$15.^{(2)}$	$18.0^{(2)}$	$14.^{(2)}$	$15.^{(2)}$
29	11497	8.3	7.5	8.8	14.	$2407\text{-}4^{(2)}$	$17.^{(2)}$	$16.0^{(2)}$	$5.0^{(2)}$	$1.2^{(2)}$

* Release rates based on bulk modulus of $1(10^5)$ psi.

** See Figure 3.3 for installation location.

(1) Factory set values.

(2) Setting selected by adjustment, for tests.

TABLE 7.5.2

MISMATCH TESTS SNUBBER LOADS (lb)

	SINE INPUT		SEISMIC INPUT	
TEST NO.	TOP SNUBBER*	BOTTOM SNUBBER*	TOP SNUBBER*	BOTTOM SNUBBER*
17	800.	1200.	480.	720.
23	500.	480.	700.	680.
25	480.	540.	760.	760.
27	480.	460.	680.	720.
29	300.	520.	240.	720.

* See Figure 3.3 for installation location.

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TABLE 7.5.3

MISMATCH TEST LOAD SHARING (PERCENTAGE)

	SINE INPUT		SEISMIC INPUT	
TEST NO.	TOP * SNUBBER	BOTTOM * SNUBBER	TOP * SNUBBER	BOTTOM * SNUBBER
17	40	60	40	60
23	51	49	51	49
25	47	53	50	50
27	51	49	48	52
29	37	63	25	75

* See Figure 3.3 for installation location.

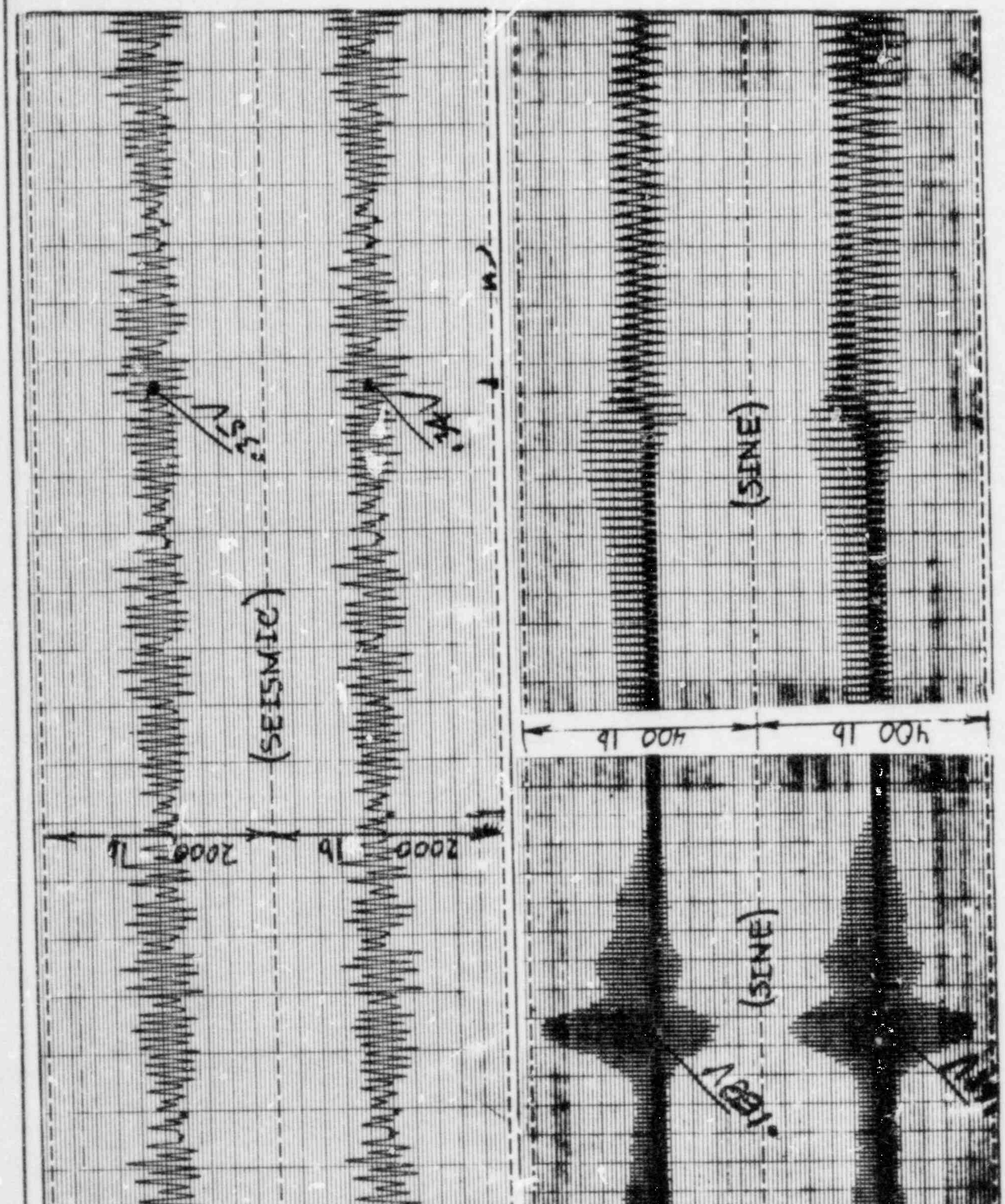

Figure 7.5.1 Mismatch Test 23 Load Sharing (Sine/Seismic, Input)

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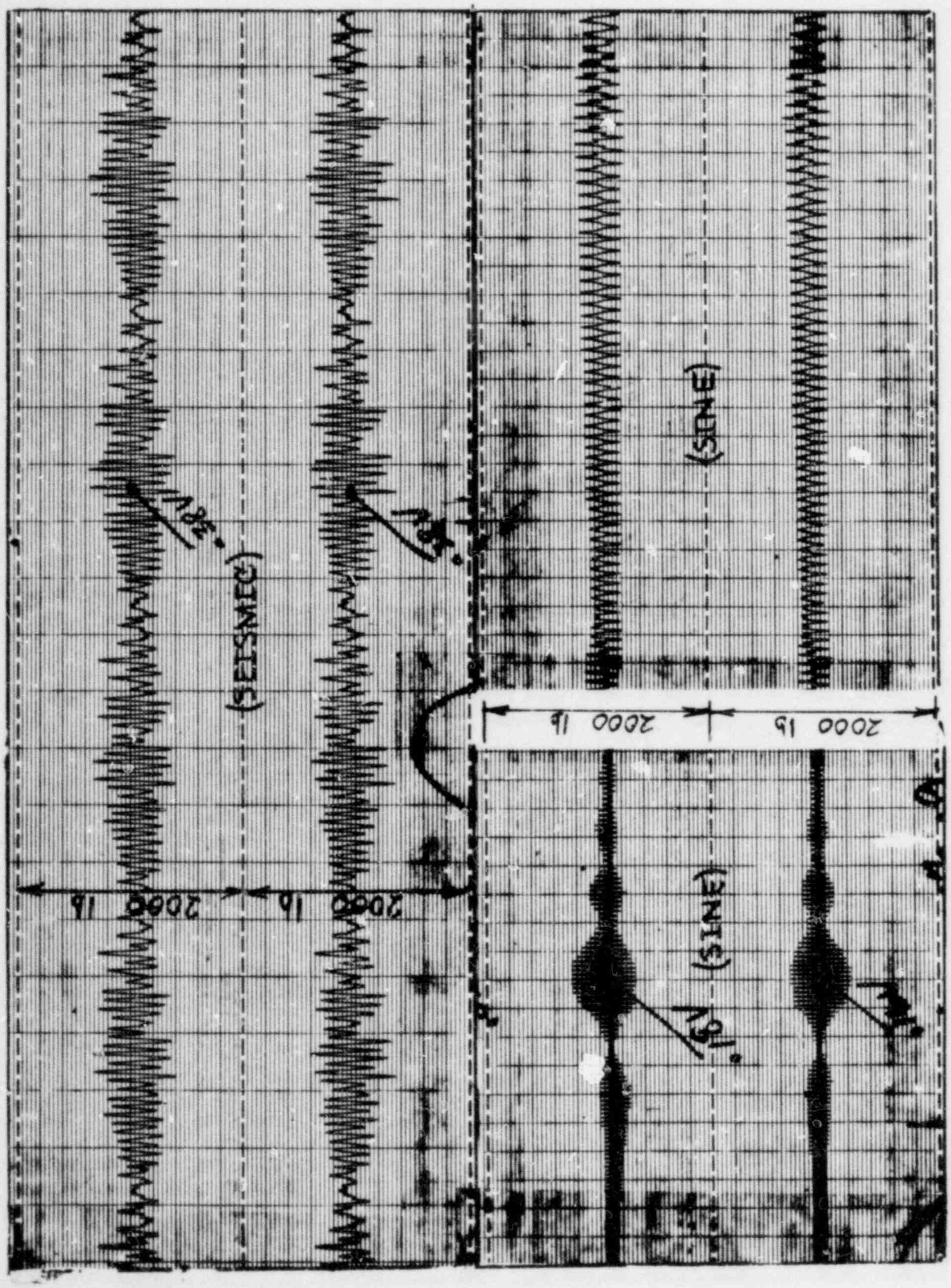

Figure 7.5.2 Mismatch Test 25 Load Sharing (Sine/Seismic Input)

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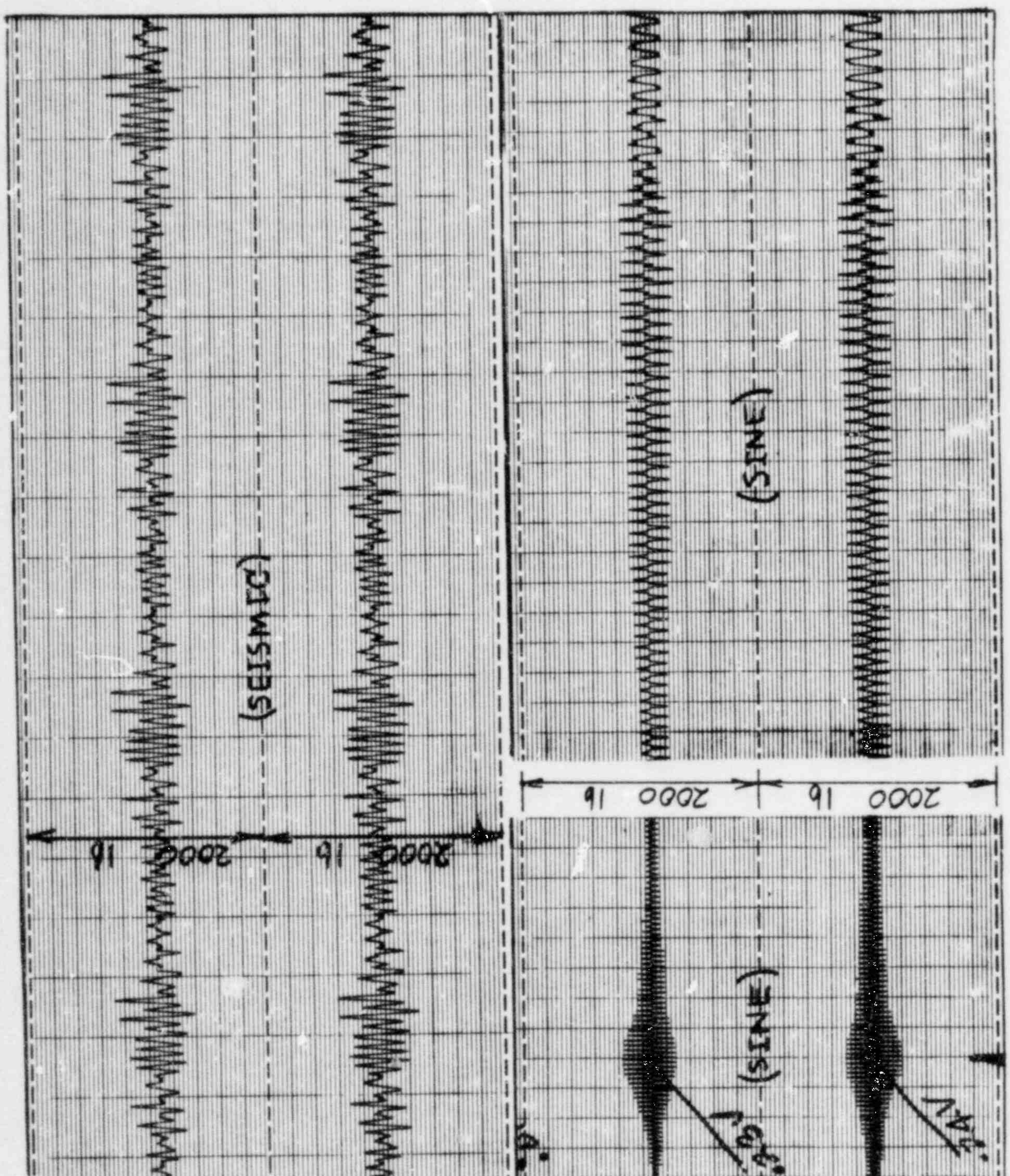

Figure 7.5.3 Mismatch Test 27 Load Sharing (Sine/Seismic Input)

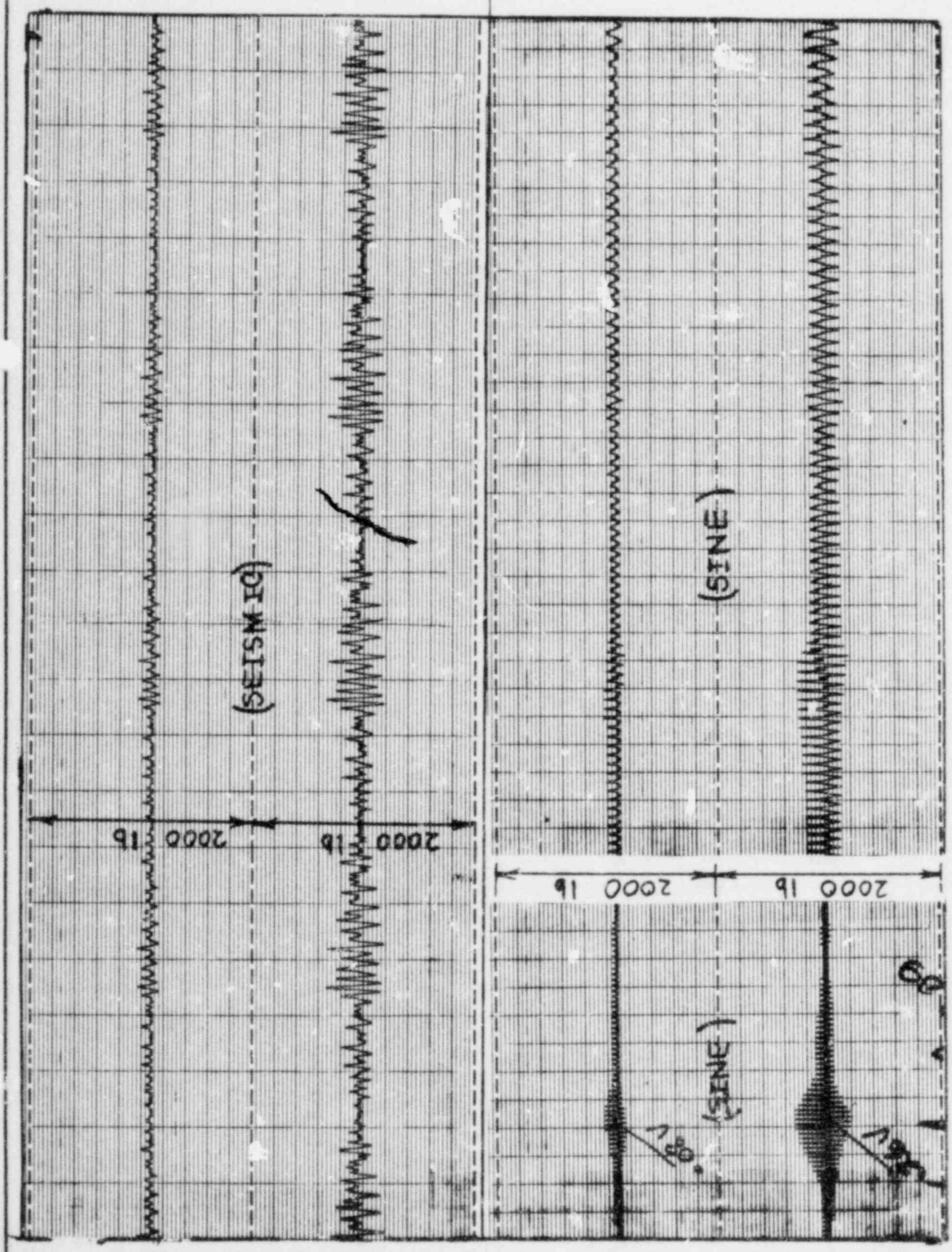

Figure 7.5.4 Mismatch Test 29 Load Sharing (Sine/Seismic Input)

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16. ABSTRACT (200 words or less)

Snubbers are used widely throughout the nuclear industry as seismic restraints. The validity of the analysis of snubber-supported systems depends on their realistic characterization. The purpose of this work was to: 1) identify those parameters which characterize hydraulic and mechanical snubbers which significantly affect snubber dynamic response, 2) determine the response sensitivity to variations of these parameters. Based upon the results of the foregoing, simlified design and analysis procedures are proposed, to maintain system response within acceptable limits.

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