

# Reduction of Structural Degrees of Freedom

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### Abstract

Guyan reduction is used extensively to reduce the degrees of freedom of a structural model prior to performing dynamic analysis. A modification to Guyan reduction is developed which results in the same eigenvalue problem as Guyan reduction but gives an improved transformation for expanding the reduced mode shapes. The SAPV computer program is modified to include both reduction methods.

Four sample problems are then solved using both of the reduction methods and the results are compared with an exact solution. For each of the problems the number of degrees of freedom retained for the dynamic analysis is varied. The errors in frequencies are found to be small while the errors in member forces can be quite large. The errors resulting from the modified reduction method are much smaller and more uniform throughout the structure than those resulting when Guyan reduction is used. Member force errors are much larger in systems where there are large differences in element stiffnesses (e.g., piping systems) than in systems where the stiffnesses is about the same (e.g., frames).

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## 1.0 INTRODUCTION

The development of discrete models for structural frameworks requires that compromises be made when deciding on the total number of degrees of freedom to be retained. A precise description of the structure may require many more degrees of freedom than are acceptable from a computational viewpoint. Limitations of the degrees of freedom to be retained arise from both restrictions embedded in the available software and economic considerations of the cost of generating the computer solutions.

Such limitations are usually more serious for dynamic problems than for static problems. Most of the large structural software packages therefore provide the user with the option of describing the structure with more degrees of freedom than are retained for dynamic response computations. Degrees of freedom to be retained in the dynamic analysis are specified by the user and a transformation relating the remaining degrees of freedom to these retained is used to reduce the order of the system (Guyan Reduction, Ref. 1, is used most often).

This dynamic reduction is often (Refs. 1,2,3) performed based upon a transformation which neglects inertial effects for the degrees of freedom to be eliminated. The transformation is then used to generate mass and stiffness matrices for the reduced system that approximate the kinetic and potential energies of the complete system. There are few data available to assess the extent to which this dynamic reduction introduces errors into various aspects of the problem.

The errors introduced in three small problems (cantilever beam, gabled frame, and small piping loop) are discussed in Ref. 4. It is shown that Guyan Reduction may result in very significant errors in the member loads

(20-40 percent for models which appear reasonable) while the modal frequencies appear quite good. A relatively simple modification of Guyan Reduction results in significantly smaller errors.

The purpose of the work reported here is to extend the studies reported in Ref. 4 to include larger structural systems and in particular structural systems found in nuclear power plants. The specific objectives are twofold. First, an assessment is made of the errors in large structural systems when both Guyan and Modified Guyan Reduction are used. Second, the economies that may be achieved by using these reduction methods as compared with obtaining complete solutions (i.e., no reduction) are investigated.

To achieve these objectives both Guyan Reduction and Modified Guyan Reduction are added to the SAPV (Ref. 5) computer program. The current version of SAPV has no reduction capability. SAPV was selected because it has the capabilities (size and element library) to analyze the structures of interest and the program is relatively easy to modify (as compared with other general purpose structural programs).

The analysis upon which the reduction methods are based is described in Section 2.0. The implementation of these methods into SAPV are discussed in Section 3.0 (Appendix A contains a listing of the modification to SAPV and Appendix B contains a users guide indicating how this version of SAPV may be accessed at Brookhaven National Laboratory). The application to sample problems is described in Section 4.0 and the work is summarized in Section 5.0.

## 2.0 ANALYSIS

The normal flow of computations in the SAPV program involves:

(i) formation of the mass and stiffness matrixies; (ii) calculating the required eigenvalues and eigenvectors; (iii) performing the response spectrum analysis; and (iv) evaluating modal displacements and member loads. When the reduction methods are added, this flow is interrupted after (i). At this stage: reduced mass and stiffness matrixies are developed; the eigenvalues and eigenvectors for the reduced system are determined; and the eigenvectors are expanded from the reduced degrees of freedom to all degrees of freedom. The normal flow of SAPV computation at (iii) is then reentered and the solution concluded as a normal SAPV run. The analysis upon which each of these three new computations are based is outlined.

### 2.1 REDUCTION METHODS

The problems of solving for the eigenvectors and forced response of a linear structural system described in terms of its mass and stiffness matrixies are considered. In particular it is of interest to consider those problems wherein some reduction in the total number of degrees of freedom is attempted before any solutions are found.

Consider the eigenvalue problem for a discrete structural system where it is desired to eliminate secondary degrees of freedom ( $X_s$ ) and retain only primary degrees of freedom ( $X_p$ ) thereby reducing the order of the equations to be solved. The partitioned equations of motion describing free vibration of the system may be written as:

$$\begin{bmatrix} M_{pp} & M_{ps} \\ - & + - - \\ M_{sp} & M_{ss} \end{bmatrix} \begin{Bmatrix} \ddot{x}_p \\ - \\ \ddot{x}_s \end{Bmatrix} + \begin{bmatrix} K_{pp} & K_{ps} \\ - & - - \\ K_{sp} & K_{ss} \end{bmatrix} \begin{Bmatrix} x_p \\ - \\ x_s \end{Bmatrix} = 0 \quad (1)$$

where  $M$  = mass matrix

$K$  = stiffness matrix

$\ddot{x}$  = acceleration

For each mode of the eigenvalue problem

$$\ddot{x} = \begin{Bmatrix} \ddot{x}_p \\ - \\ \ddot{x}_s \end{Bmatrix} = -\lambda^2 \begin{Bmatrix} x_p \\ - \\ x_s \end{Bmatrix} \quad (2)$$

where  $\lambda$  = eigenvalue

Substituting Eq. (2) into (1) gives,

$$-\lambda^2 \begin{bmatrix} M_{pp} & M_{ps} \\ - & + - - \\ M_{sp} & M_{ss} \end{bmatrix} \begin{Bmatrix} x_p \\ - \\ x_s \end{Bmatrix} + \begin{bmatrix} K_{pp} & K_{ps} \\ - & - - \\ K_{sp} & K_{ss} \end{bmatrix} \begin{Bmatrix} x_p \\ - \\ x_s \end{Bmatrix} = 0 \quad (3)$$

Elimination of the secondary degrees of freedom may be accomplished from the second of the above equations to give,

$$x_s = -T x_p$$

$$\text{where, } T = (\lambda^2 M_{ss} - K_{ss})^{-1} (\lambda^2 M_{sp} - K_{sp}) \quad (4)$$

Substituting Eq. (4) into the first of Eq. (3) yields the eigenvalue problem in the reduced degrees of freedom ( $x_p$ ),

$$\lambda^2 (M_{pp} - M_{ps} T) x_p = (K_{pp} - K_{ps} T) x_p \quad (5)$$

Unfortunately the transformation matrix (T) involves the eigenvalue which is not known.

Standard methods of solving the eigenvalue problem may be used to find the solution of Eq. 5 provided a new transformation matrix ( $T$ ) is computed each time a new estimate of the eigenvalue ( $\lambda$ ) is found. Thus one would begin with the ( $\lambda=0$ ) solution, determine the eigenvalue, calculate a new transformation based upon the updated eigenvalue and repeat the process until the eigenvalue no longer changes. Several methods of carrying out this iterative process have been considered. For some problems an almost linear relationship has been found to exist between the eigenvalue assumed in calculating the transformation matrix and the eigenvalue determined as the solution of Eq. (5). This would happen if  $(\lambda^2 M_{ss})$  and  $(\lambda^2 M_{sp})$  are small as compared with  $(K_{ss})$  and  $(K_{sp})$  respectively (see Eq. 4).

This suggests an interesting possibility. If  $(\lambda^2 M_{ss} - K_{ss})^{-1}$  is expanded about  $(K_{ss})^{-1}$  and terms associated with  $(\lambda^4)$  are neglected as compared with terms associated with  $(\lambda^2)$ .

$$T = K_{ss}^{-1} K_{sp} + \lambda^2 (-K_{ss}^{-1} M_{sp} + K_{ss}^{-1} M_{ss} K_{ss}^{-1} K_{sp}) \quad (6)$$

This definition of ( $T$ ) has an advantage over (4) in that an inverse need not be recalculated at each step in the iteration on ( $\lambda$ ).

This requirement for an iterative solution is eliminated if Eq. (6) is substituted into the first of (3) and again the  $(\lambda^4)$  terms neglected as compared to the  $(\lambda^2)$  terms to give,

$$\begin{aligned} \lambda^2 (M_{pp} - M_{ps} K_{ss}^{-1} K_{sp} - K_{ss}^{-1} M_{sp} + K_{ps} K_{ss}^{-1} M_{ss} K_{ss}^{-1} K_{sp}) X_p = \\ (K_{pp} - K_{ps} K_{ss}^{-1} K_{sp}) X_p \end{aligned} \quad (7)$$

Eigenvalues can be determined directly from Eq. (7) without a requirement for iteration. This is the eigenvalue problem solved using both Guyan and Modified Guyan Reduction. Since the full mass matrix in SAPV is diagonal the reduced mass matrix is,

$$M^* = M_{pp} + K_{ps} K_{ss}^{-1} M_{ss} K_{ss}^{-1} K_{sp} \quad (8)$$

The reduced stiffness matrix is,

$$K^* = K_{pp} - K_{ps} K_{ss}^{-1} K_{sp} \quad (9)$$

## 2.2 EIGENVALUE SOLUTION

The SAPV computer program uses a determinant search algorithm to solve for the eigenvalues of small problems and a iteration algorithm for large problems (Ref. 6). An attempt was made to use the subspace iteration algorithm for the reduced system but difficulties were encountered unless considerable care was taken in selecting the trial vectors. This difficulty occurred because the reduced mass matrix is full while the full mass matrix (used in SAPV) is diagonal.

A standard inverse iteration with Gram-Schmidt orthogonalization procedure (Ref. 6) is used. The inverse iteration method may be described as follows. The eigenvalue problem is,

$$\lambda^2 M^* X_p = K^* X_p \quad (10)$$

A trial vector ( $X_1$ ) is selected and the following iteration performed so that ( $X_k$ ) goes to a mode shape as the iteration index (k) increases,

$$\begin{aligned}
 K^* \bar{X}_{K+1} &= M^* X_K \\
 \lambda_{K+1}^2 &= \bar{X}_{K+1}^T M^* \bar{X}_{K+1} / c & (11) \\
 X_{K+1} &= \bar{X}_{K+1}^T / c \\
 c &= \bar{X}_{K+1}^T M^* \bar{X}_{K+1}
 \end{aligned}$$

This iteration converges to the mode associated with the smallest eigenvalue. Modes higher than the first are determined by requiring all the vectors ( $X_k$ ) be orthogonal to the lower modes already evaluated, thereby suppressing those modes in the iteration. Gram-Schmidt orthogonalization is used for this purpose and is done as follows when the  $i^{th}$  mode is being calculated

$$X = \bar{X} - \sum_{k=1}^{i-1} Z_k^T M^* \bar{X} \quad (12)$$

where,  $\bar{X}$  = trial vector

$X$  = trial vector mode orthogonal to all  
of the lower modes  $Z_k$

### 2.3 EXPANSION OF REDUCED EIGENVECTORS

The eigenvectors found in (2.2) are of a length equal to the number of reduced degrees of freedom. If member loads are to be calculated these eigenvectors must be expanded to include all degrees of freedom.

Guyan Reduction calculates the secondary (eliminated) degrees of freedom from,

$$X_s = -K_{ss}^{-1} K_{sp} X_p \quad (13)$$

The Modified Guyan Reduction calculates the secondary degrees of freedom from Eq. (6) or if the original mass matrix is diagonal,

$$X_s = - \left[ K_{ss}^{-1} K_{sp} + \lambda^2 (K_{ss}^{-1} M_{ss} K_{ss}^{-1} K_{sp}) \right] X_p \quad (14)$$

### 3.0 SAPV MODIFICATIONS

The changes discussed in Section 2.0 have been incorporated into the SAPV program at Brookhaven National Laboratory (BNL). The program is operational on the BNL CDC-7600 computer system. The main program in SAPV is modified and thirteen new subroutines are added. All of the SAPV subroutines are unaltered. A listing of the modified main program and the thirteen new subroutines is contained in Appendix A. Detailed instructions for using the BNL SAPV program incorporating these changes is given in Appendix B. In this section of the report the general form of the modification, input data requirements, and a description of each of the programs are given.

#### 3.1 GENERAL FORM OF MODIFICATION

The SAPV Computer Program is modified so that the total static degrees of freedom may be reduced prior to performing a dynamic analysis. In the previous section of the report two reduction methods are developed:

- (i) Guyan Reduction which is the standard reduction used in most structural software packages. (Eq. 7) is the eigenvalue problem and (Eq. 13) is used to expand the modes after solution of the eigenvalue problem.

(ii) Modified Guyan Reduction which uses the same (Eq. 7) for solution of the eigenvalue problem. (Eq. 14) is used to expand the modes rather than (Eq. 13) used in the Guyan Reduction.

These two reduction methods are incorporated into SAPV as indicated on Figure 1. As may be seen the modification is made by interrupting the normal flow of computations in SAPV at two locations:

(i) SAP5.88\*- The type of reduction (or no reduction) is read in together with the primary degrees of freedom to be retained.

These data are then used to rearrange the equation numbers (defined in the ID matrix) so that the primary degrees of freedom occur first.

(ii) SAP5.296\*- The reduction is performed and the eigenvalue solution obtained for the reduced system. The modes are expanded to the total degrees of freedom and stored in the appropriate locations so that the normal SAP response analysis may be performed to complete the solution.

Each of the subroutines listed in Figure 1 are discussed in Section 3.3.

### 3.2 INPUT DATA REQUIREMENTS

Only one addition is required to be made to the normal SAPV data input requirements as given in Ref. 5. After the "III Nodal Point Data" and before the "IV Element Data", the following card(s) are inserted:

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\*Refers to line numbers in the BNL SAPV Program

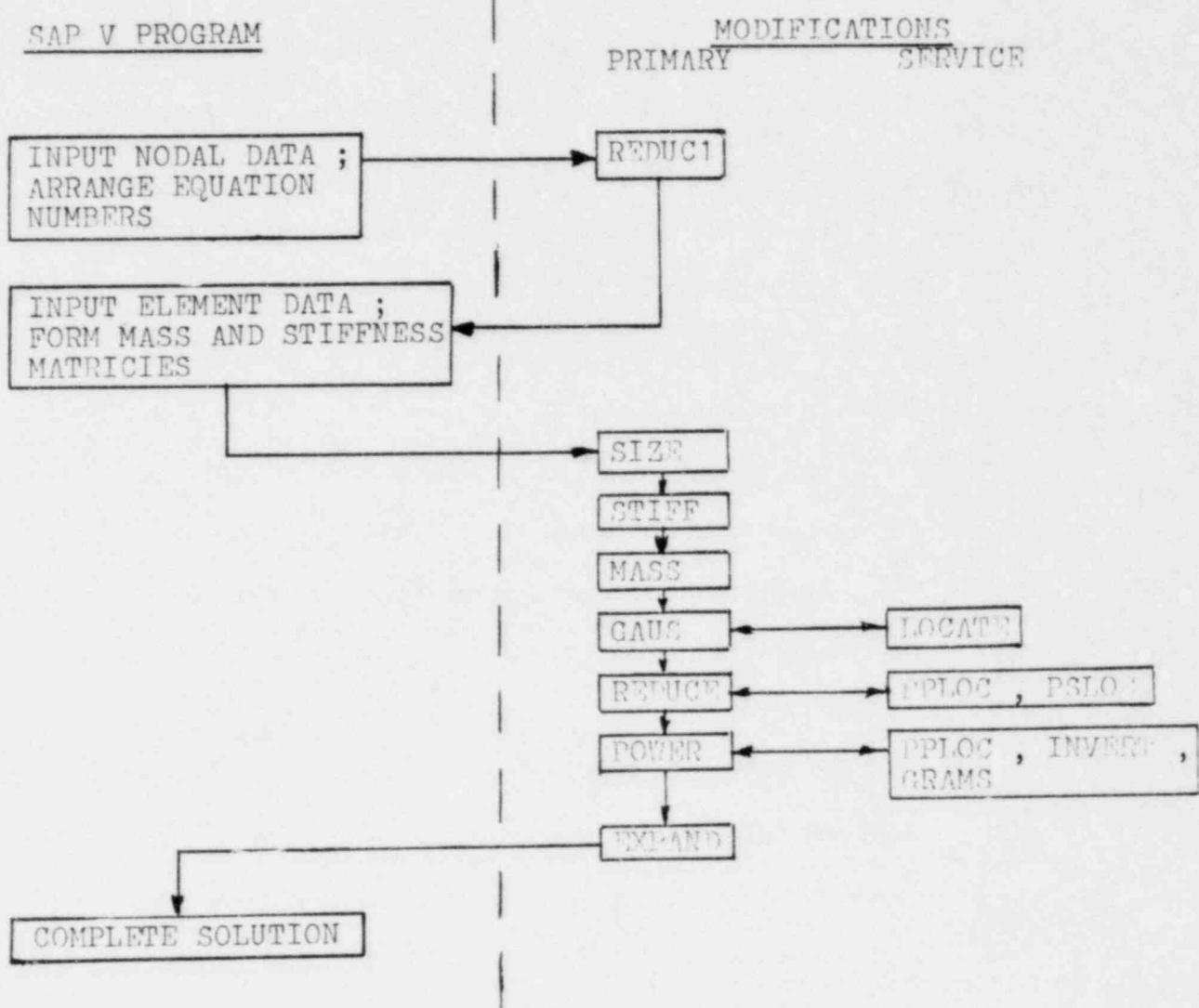


Fig. 1 GENERAL FORM OF SAP V MODIFICATION

<u>No of Cards</u>	<u>Format</u>	<u>Variables</u>	<u>Definition</u>
1	15	NTYP	NTYP = 0; No Reduction (Standard SAP run) = 2; Guyan Reduction = 3; Modified Guyan Reduction
<u><math>2 \times NRDOF + 1</math></u> 14	1415	NRDOF, (MASN(I), MADF(I), I = 1, NRDOF)	NRDOF= Number of Primary Degrees of Freedom MASN = Node Number of Primary Degree of Freedom I MADF = Direction (1-6) of Primary Degree of Freedom I

Also the Card VIII "Mode Shapes and Frequencies" is omitted if NTYP is not equal to 0.

### 3.3 DESCRIPTION OF SUBROUTINES

Before describing the subroutines common variables are defined and storage allocations are discussed.

#### 3.3.1 STORAGE ALLOCATION

Most of the data required in Core is stored in vector (A) in the SAP main program and only put into matrix from within the subroutines so that the size of the matricies may be adjusted to the required sizes.

All data stored in (A) at BNL is put into Large Core Memory (Level 2). The data in (A) at the call to each of the primary subroutines is listed in Table 1. Control parameters are defined in Table 2. Note that the storage in (A) is rearranged between the subroutines REDUCE and POWER. Tape storaged is discussed where applicable in the subroutines.

### 3.3.2 REDUC1

The additional input data described in Section 3.2 is read into the program through this subroutine. The SAP generated matrix of equation numbers (ID) is read from Tape 8 and reordered so that the primary degrees of freedom are the first numbered equations. The primary and secondary degree of freedom equation numbers are kept in the order that they appeared in the original ID matrix. The new ID matrix is written on Tape 8.

The SAP band width minimizer occurs after this location in the program. It should not be used or it will upset the equation number reordering that is performed here. (Therefore MINBND should be read in as 0 on the Master Control Card).

### 3.3.3 SIZE

The size parameters required for the solution are generated and printed in this routine. The following parameters, which are defined in Table 2, are calculated here: NST, NRDOF, NS, NS1, NS2, NS3, NOT, and MBS. In addition the required size of the vector (A) used in the various phases of the program is determined and printed.

Table 1  
DATA STORAGE IN VECTOR A

Subroutine	Variable	Size	Location in A
REDUC1	ID = Matrix giving equation number	6*NUMNP	1
	IDN = Revised ID	6*NUMNP	1+6*NUMNP
SIZE	X = One band of stiffness matrix	NRQB*MBAND	1
	XL = One band of load matrix	NEQB	1+NRQB*MBAND
STIFF	K <sub>pp</sub> see Sect. 2.0	NS1	1
	K <sub>ps</sub> see Sect. 2.0	NS2	NR2=1+NS1
	K <sub>ss</sub> see Sect. 2.0	NSEQ*MBS	NR3=NR2+NS2
	X = Dummy K <sub>ss</sub>	NSEQ*MBS	NR4=NR3+NSEQ*MBS
	XL = Dummy Load	NSEQ	NR5=NR4+NSEQ*MBS
MASS	XM = Diagonal Mass	NST	NR3
GAUS	A1 = One block K <sub>ss</sub>	NSEQ*MBS	NR4=NR3+NST
	A2 = Second block	NSEQ*MBS	NR5=NR4+NSEQ*MBS
	B = Dummy load	NST	NR6=NR5+NSEQ*MBS
	G = Dummy load	NST	NR7=NR6+NSEQ*MBS
REDUCE	K <sub>pp</sub> and K*	NS1	1
	K <sub>ps</sub>	NS2	NR2
	XM	NST	NR3
	M*	NS1	NR4
	XSS = Dummy	NS	NR5=NR4+NS1
	T1 (one row)	NRDOF	NR6=NR5+NRDOF
	TO (one block)	NTO*NRDOF	NR7=NR6+NRDOF

-continued-

Table 1 cont'd

POWER	$K^*$	NS1	1
	$M^*$	NS1	$NR2=1+NS1$
	EV = Eigenvectors	NRDOF*MODE	$NR3=NR2+NS1$
	XD = Factored K	NRDOF*NRDOF	$NR4=NR3+NRDOF*MODE$
	B = Dummy	NRDOF	$NR5=NR4+NRDOF*NRDOF$
	BB = Dummy	NRDOF	$NR6=NR5+NRDOF$
	EDUM = Eigenvalues	MODE	$NR7=NR6+NRDOF$
EXPAND	EV	NRDOF*MODE	$NR3$
	TO	NT0*NRDOF	$NR4=NR3+NRDOF*MODE$
	T1	NRDOF	$NR5=NR4+NT0*NRDOF$
	ESAP = Block of expanded nodes	NTQB*MODE	$NR6=NR5+NRDOF$

Table 2  
DEFINITION OF CONTROL PARAMETERS

Variable	Definition
NUMNP	Number of nodes
NEQB	Number of equations in one block of stiffness matrix
MBAND	Bandwidth of stiffness matrix
NS1	Number of elements in upper half of $K_{pp}$
NS2	Number of elements in $K_{ps}$
NSEQ	Number of equations in block of $K_{ss}$
MBS	Bandwidth of $K_{ss}$
NST	Number of unrestrained degrees of freedom
NS	Number of secondary degrees of freedom
NRDOF	Number of primary degrees of freedom
NTO	Number of equations in one block of $T_0$
MODE	Number of modes

### 3.3.4 STIFF

The stiffness matrix is generated in the normal SAP routines and stored banded and blocked on Tape 4. This tape is read and  $(K_{pp})$  and  $(K_{ps})$  are formed and kept in Large Core Memory (see Table 1). The secondary degree of freedom stiffness  $(K_{ss})$  is also determined and stored banded and blocked on a new Tape 16.

### 3.3.5 MASS

The diagonal mass matrix as stored on Tape 9 is read and used to form the vector of all masses. This vector remains in Large Core Memory.

### 3.3.6 GAUS

This subroutine forms  $(K_{ss}^{-1})$  using elimination as described in Ref. 6. It operates essentially the same as SESOL in the standard SAPV except that it generates "load vectors" required to compute one column of  $(K_{ss}^{-1})$  at a time. The full column of  $(K_{ss}^{-1})$  is stored in Tape 17. The elimination operation is set to operate on a banded symmetric matrix stored in block form. In this instance the  $K_{ss}$  matrix is read in from Tape 16.

The transfer list to this subroutine contains the parameters NT1, NT2, NT3, and INDEX. If INDEX = 1 the inverse of the matrix stored on Tape NT1 is calculated and stored one column at a time on tape NT3. If INDEX = 2 the matrix stored on Tape NT1 is factored and the result is stored on Tape NT2. If INDEX = 3 the equation Ax = B is solved. (A) is stored on NT1 and B is transferred through the list. The solution (x) is stored in (B). The parameters have the following values in this application.

NT1=16  
NT2=18  
NT3=17  
INDEX=1

### 3.3.7 REDUCE

The transformation matrices and the reduced stiffness and mass matrices are determined. The following sequence of calculations are made:

(i)  $T_o = K_{ss}^{-1} * K_{sp} \dots$  (first term in Eq. 6)

The transformation matrix  $T_o$  is calculated one block at a time (NTO rows) and the result stored on Tape 18.  $K_{ss}^{-1}$  is read from Tape 17.

(ii)  $T_1 = K_{ss}^{-1} * M_{ss} * T_o \dots$  (second term in Eq. 6)

The transformation is calculated one now at a time and stored on Tape 16. It is written over and destroys  $K_{ss}$ .

(iii)  $K^* = K_{pp} - K_{ps} * T_o$

The reduced stiffness matrix ( $K^*$ ) is calculated and written over  $K_{pp}$  which is no longer needed.

(v)  $M^* = M_{pp} + K_{ps} * T$

The reduced mass matrix ( $M^*$ ) is calculated and the upper half stored in a new location.

### 3.3.8 POWER

The power method using inverse iteration is used to calculate the lowest (MODE) modes for the reduced mass and stiffness matrices. The iterations are continued until the change in eigenvalue ( $\lambda^2$ ) is less than 0.01%. The error at each iteration is printed. After all eigenvalues have been generated, they are written onto Tape 7 as required for the remainder of the SAPV program.

### 3.3.9 EXPAND

The eigenvectors are expanded to include all unrestrained degrees of freedom according to either Eq. (13) or (14) depending on whether the reduction is Guyan or the Modified Guyan. The full vectors are then stored on Tape 7 after the eigenvalues.

### 3.3.10 LOCATE

This subroutine locates the (i,j) element in  $K_{ss}$  when  $K_{ss}$  is stored in block form.

### 3.3.11 PPLOC

This subroutine locates the (i,j) element in  $K_{pp}$ .

### 3.3.12 PSLOC

This subroutine locates the (i,j) element in  $K_{ps}$ .

### 3.3.13 INVERT

This subroutine factors the reduced stiffness matrix ( $K^*$ ) as required in the POWER subroutine. It operates the same as the GAUS routine except for the format of the matrix being reduced.

### 3.3.14 GRAMS

This subroutine performs Gram-Schmidt orthogonalization as described in Section 2.2. At each iteration in POWER the current eigenvector is made orthogonal to all eigenvectors previously calculated.

#### 4.0 SAMPLE PROBLEMS

The modified SAPV computer program is used to study four sample problems: a ten story building; the ten story building with a relatively light structure attached to the top story; a nuclear power plant piping system; and a nuclear power plant containment building. For each of the problems solutions are obtained; with no reduction; with Guyan Reduction; and with the Modified Guyan Reduction.

#### 4.1 TEN STORY BUILDING

The first problem considered is the ten story by three bay frame as shown on Figure 2. The node numbers used for the SAP data are shown by each of the nodes and the element numbers are shown underlined. All of the columns (elements number 1 through 40) have a cross-sectional area of 60 square inches and a moment of inertia of 1500 inches.<sup>4</sup> The beams (elements 41 through 70) have a cross-sectional area of 100 square inches and a moment of inertia of 1000 inches.<sup>4</sup> The material of all members has a modulus of elasticity of 30,000 ksi and a density of 490 pounds per cubic foot.

The columns are fixed at the base (nodes 1 through 4) and the response spectrum shown in Figure 3 (solid) is input in the horizontal direction. The frame is solved with no reduction and then with both Guyan and Modified Guyan Reduction. For each reduction method four solutions are obtained retaining 3, 4, 6 and 10 degrees of freedom for the dynamic analysis. In all cases the degrees of freedom retained are in the horizontal direction; with the specific degrees of freedom given as:

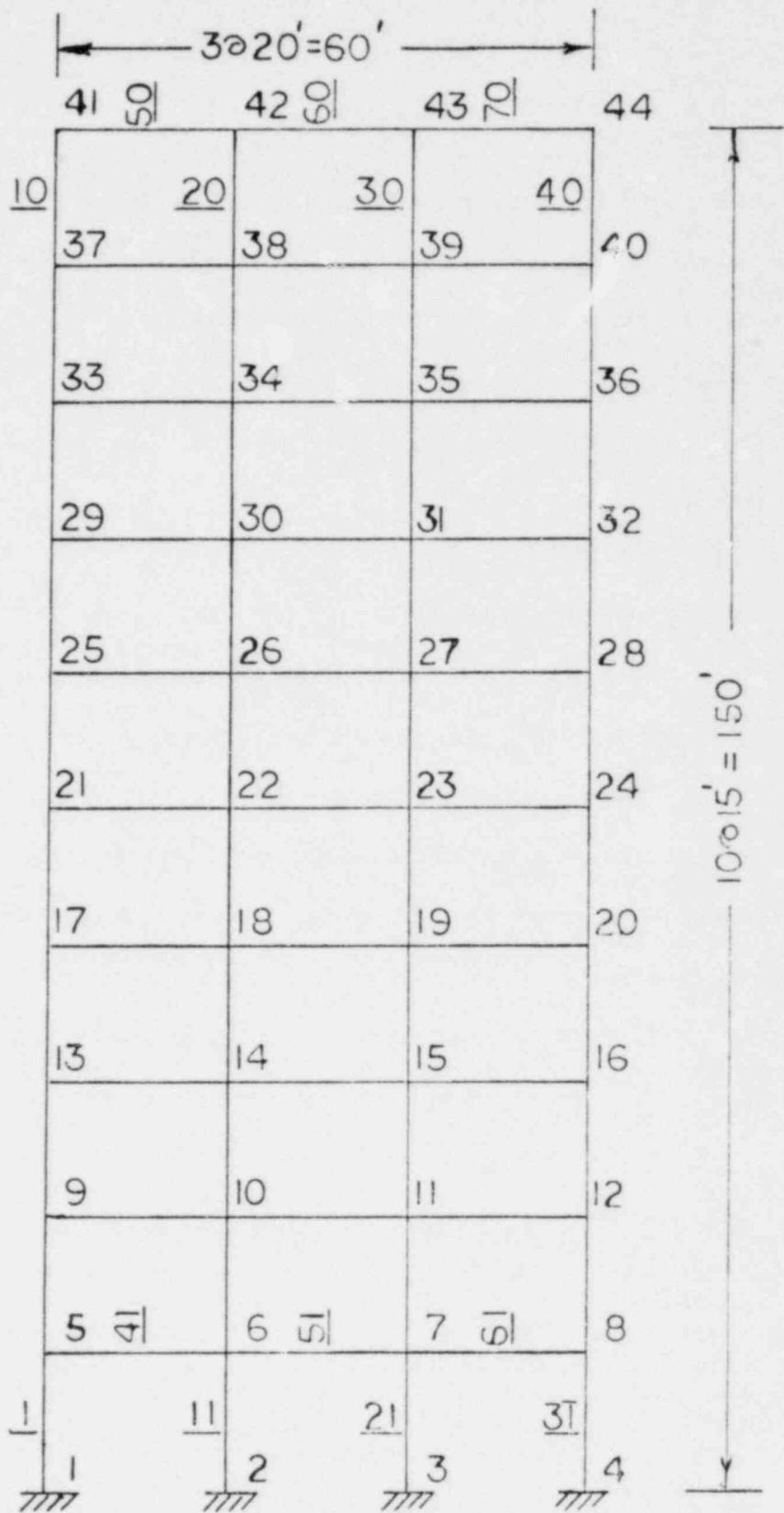


Fig. 2 TEN STORY FRAME

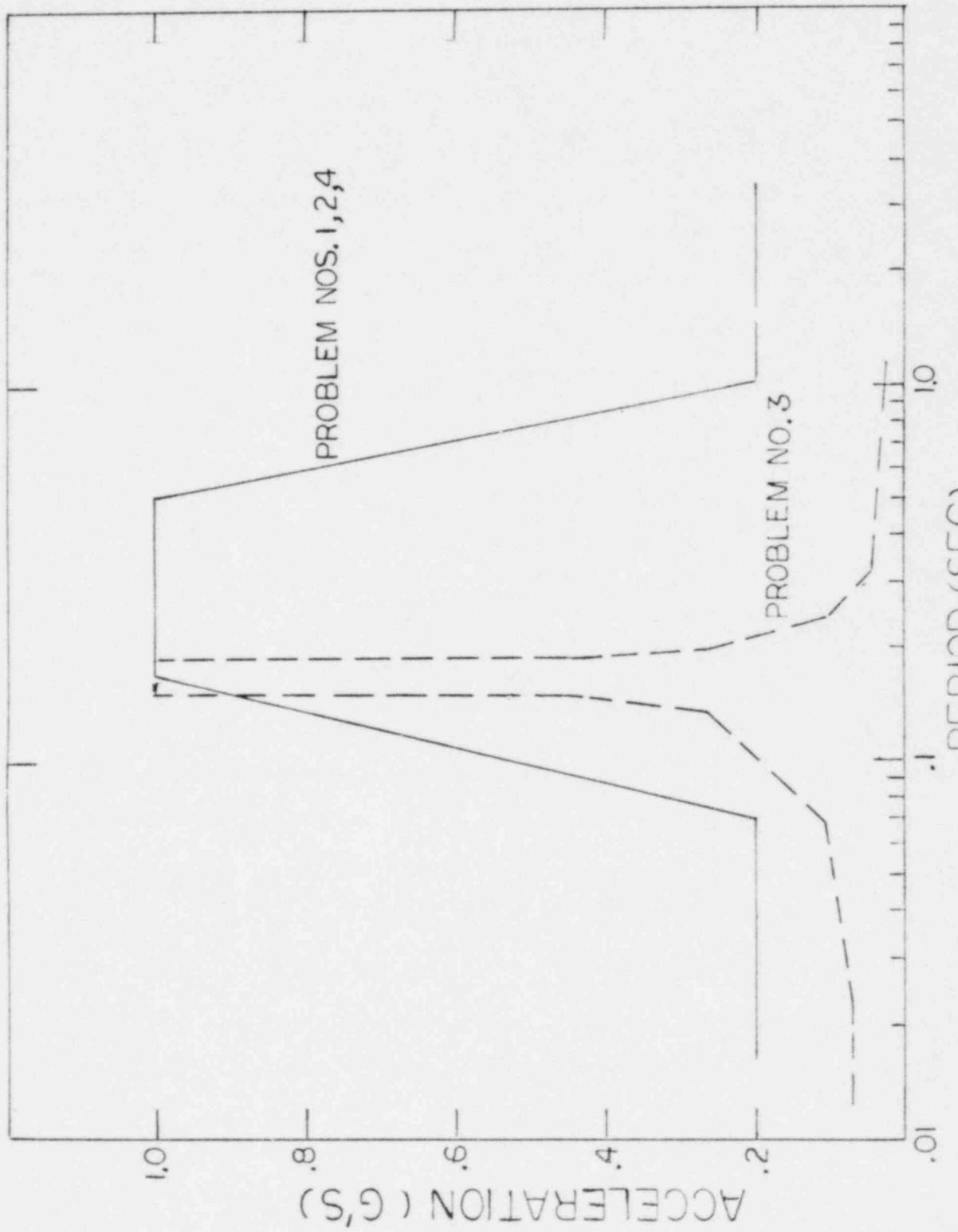


Fig. 3 BASE RESPONSE SPECTRA

Number of Dynamic Degrees of Freedom	Degrees of Freedom - Horizontal Displacement at Nodes
3	11, 26, 41
4	12, 23, 34, 41
6	10, 17, 28, 35, 38, 41
10	6, 9, 16, 19, 22, 25, 32, 35, 38, 41

Three modes are included in the solution and the results for frequencies, lower story columns (1 and 11), upper story columns (10 and 20), lower story beams (41 and 51) and upper story beams (50 and 60) are given in Table 3. As may be seen the results for frequencies are quite good regardless of the type of reduction or number of dynamic degrees of freedom retained in the model. Of course this would not hold true if any particular mode could not be reasonably represented in terms of the degrees of freedom retained. This is the reason for the departure in the mode 3 frequency as calculated with Guyan Reduction from the exact solution.

So that the errors in member bending moments may be evaluated, the percentage errors in four of the columns and four of beams are computed from the data in Table 3 and given in Table 4. The Modified Guyan Reduction gives excellent results (less than 2% error) for all reductions, even when only 3 degrees of freedom are retained. Guyan Reduction gives equally good results for the lower story members (11 and 41) but very poor results for the upper story members unless 6 or more degrees of freedom are retained. This variation in accuracy with location in the structure would make it difficult to review data for a Guyan Reduction run to determine the reliability of the results. It also appears that the Modified Guyan Reduction gives errors of one sign (i.e., a lower bound) while Guyan Reduction gives errors which can be positive or negative.

Table 3

## Ten Story Frame Frequencies and Member Bending Moments

Type of Reduction	No. of Dynamic Degrees of Freedom	FREQUENCIES (CPS)			COLUMN MOMENTS				BEAM MOMENTS			
		MODE 1	MODE 2	MODE 3	1	10	11	20	41	51	50	60
Exact		0.86	2.64	4.61	1520.	150.	1738.	315.	1390.	1196.	150.1	193.1
G	3	0.87	2.85	4.85	1535.	344.	1756.	630.1	1402.	1206.	334.	340.7
U	4	0.86	2.71	5.07	1521.	198.7	1740.	387.4	1393.	1198.	198.7	225.3
Y	6	0.86	2.66	4.72	1514.	151.2	1732.	314.1	1391.	1197.	151.2	192.7
A												
N	10	0.86	2.64	4.61	1520.	149.9	1738.	312.5	1390.	1196.	149.9	192.1
M.												
G	3	0.87	2.85	4.85	1496.	148.7	1710.	311.0	1367.	1177.	148.7	190.6
I	4	0.86	2.71	5.07	1513.	149.6	1730.	313.6	1383.	1190.	149.6	192.6
Y	6	0.86	2.66	4.72	1519.	149.7	1736.	314.0	1388.	1195.	149.7	192.9
A												
N	10	0.86	2.64	4.61	1521.	149.8	1738.	314.2	1390.	1196.	149.8	193

Table 4  
%Errors in Member Moments

No. of Deg. of Freedom	Guyan Reduction Member No.				Modified Guyan Reduction Member No.			
	11	41	20	60	11	41	20	60
3	1.0	0.9	100.0	76.4	-1.6	-1.6	-1.3	-1.3
4	0.1	0.2	23.0	16.7	-0.5	-0.5	-0.4	-0.3
6	-0.3	0.1	-0.3	-0.2	-0.1	-0.1	-0.3	-0.1
10	0	0	-0.8	-0.5	0	0	-0.2	0

The running time for both Guyan and Modified Guyan Reduction solution are about the same (Modified Guyan is about 0.5% slower). The solution times on the BNL CDC 7600 computer are 6.95, 7.28, 7.30 and 7.90 seconds respectively for the 3, 4, 6, and 10 degree of freedom problem. The computer implications of the reduction schemes are discussed in Section 4.5.

#### 4.2 SMALL STRUCTURE ADDED TO TEN STORY BUILDING

The small structure shown in Figure 4 is added to the central bay of the top floor of the ten story building. The dashed lines represent the main structural frame as described on Figure 2 while the solid lines are the added structure. The new structure is attached to nodes 38 and 39 of the main frame. Member numbers in the new frame are underlined and node numbers shown next to each node.

All members of the added frame have a cross-sectional area of 60 square inches and a moment of inertia of 3 inches.<sup>4</sup> The relatively large cross-sectional area is used to simulate added mass to the small structure. Material properties of the small structure are the same as the main frame. In a separate computer run the small structure, is found to have frequencies of 1.78 cps and 5.86 cps when isolated from the main frame. Therefore its modes of vibration would be expected to interact with the modes of the ten story building.

The response spectrum of Figure 3 is again used as input at the base of the ten story frame and solutions obtained with no reduction and with both Guyan and Modified Guyan Reduction.

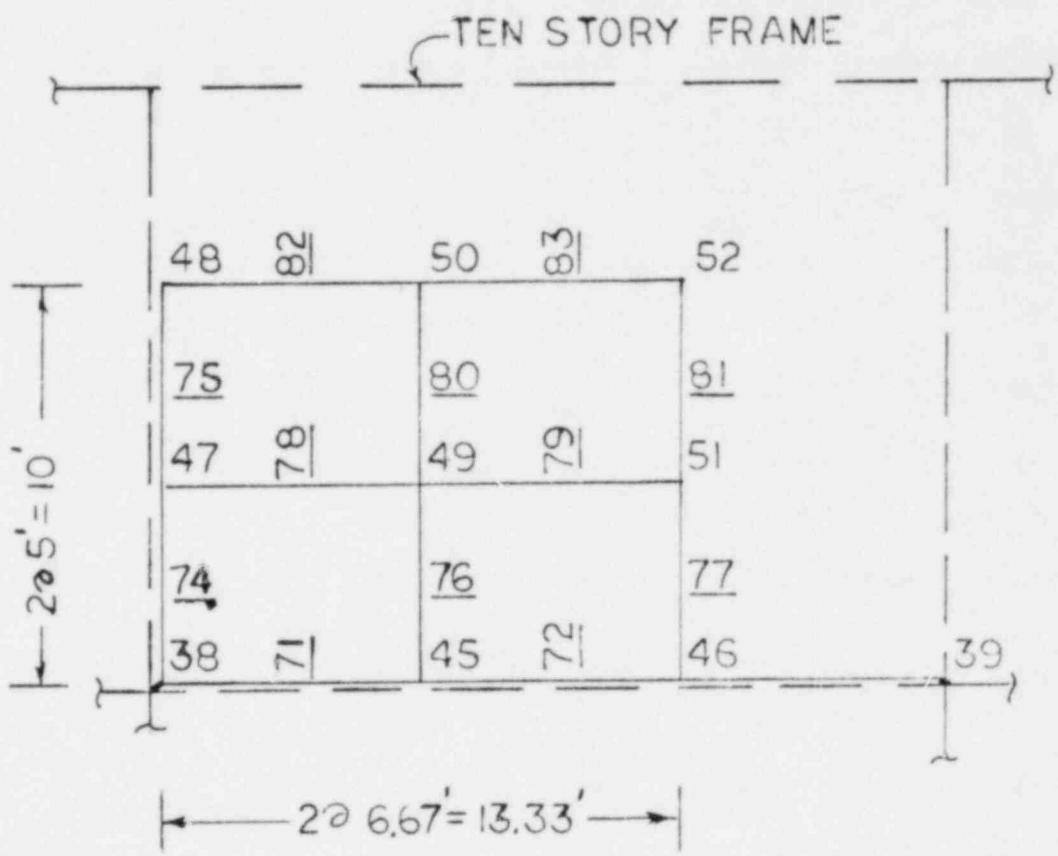


Fig. 4. SMALL STRUCTURE

Six dynamic degrees of freedom in the main structure are retained for all reduction runs since these are shown (Table 4) to give negligible errors in the main frame. Four different reductions are used for the degrees of freedom in the small structure as defined in Table 5.

Table 5  
Reduction in Small Structure

Case	Degrees of Freedom Retained*
1	50 (horizontal)
2	50 (horizontal), 49 (horizontal)
3	48(horizontal and vertical), 52 (horizontal and vertical)
4	All horizontal and vertical

\*The horizontal degrees of freedom at the following nodes in the main structure are retained: 10, 17, 28, 35, 38, 41

The results of the computer solution are shown on Tables 6 and 7. Five modes are considered and the calculated frequencies are shown in Table 6. The results for the first four modes are good for all of the cases while there is considerable variability in the fifth mode.

The member moments are shown on Table 7 and percentage errors for each of the reduction runs is computed and tabulated in Table 8. As may be seen the Modified Guyan Reduction gives significantly better results than the Guyan Reduction. It may again be noted that the errors for Guyan Reduction vary significantly from member to member for the same reduction scheme while the errors are much closer to the same when the Modified Reduction is used.

Table 6

## Frequencies in Frame with Small Structure

Case (Table 5)	Frequencies (CPS)				
	Mode 1	Mode 2	Mode 3	Mode 4	Mode 5
Exact	.81	1.98	2.67	4.58	4.88
1	.82	2.02	2.69	4.71	7.14
2	.82	1.90	2.69	4.71	6.58
3	.82	1.82	2.69	4.71	4.99
4	.82	1.98	2.68	4.70	4.88

Table 7  
Bending Moments in Small Structure

Reduction	Case 5 (Tables)	Moments in Member No.												
		71	72	73	74	75	76	77	78	79	80	81	82	83
Exact		23.2	9.1	20.7	75.0	21.1	32.2	28.2	52.7	74.4	30.7	11.7	22.8	11.7
G. U. A. N.	1	19.2	6.5	16.9	60.1	28.1	25.7	22.1	51.1	27.8	42.2	19.9	28.1	19.9
	2	21.8	10.8	19.4	74.8	20.3	32.7	28.8	51.7	26.1	31.1	12.9	21.6	12.9
	3	20.3	5.3	18.0	60.1	28.7	25.2	21.5	51.8	26.3	41.8	18.8	28.7	18.8
	4	23.1	9.0	20.6	74.9	21.1	32.1	28.2	52.6	24.4	30.7	11.6	22.8	11.5
M. G. U. Y. A. N.	1	22.4	8.7	20.0	72.4	20.6	31.1	27.3	51.1	23.9	30.1	11.6	22.4	11.6
	2	23.1	9.2	20.6	75.1	21.1	32.3	28.3	52.7	24.6	30.8	11.8	22.8	11.8
	3	22.5	8.8	20.1	72.4	20.7	31.1	27.3	51.2	23.8	30.1	11.5	22.4	11.5
	4	23.2	9.0	20.7	75.1	21.1	32.2	28.3	52.8	24.5	30.8	11.7	22.9	11.7

Table 8  
%Errors in Small Structure Moments

Case (Tables)	Guyan Reduction Member No.					Modified Guyan Reduction Member No.				
	72	76	79	80	83	72	76	79	80	83
1	-28.6	-20.2	+13.9	+37.5	+70.1	-4.4	-3.4	-2.0	-2.0	-0.9
2	+18.7	+1.6	+7.0	+1.3	+10.3	+1.1	+0.3	+0.8	+0.3	+0.9
3	-41.8	-21.7	+7.8	-14.3	+60.7	-3.3	-3.4	-2.5	-2.0	-1.7
4	-1.1	-0.3	0	0	-0.9	-1.1	0	+0.4	+0.3	0

#### 4.3 PIPING SYSTEM

The piping system shown in Figure 5 is considered. The details of this system are taken from Reference 7 (Piping Benchmark Problem No. 5). Elements 1 through 19 are 14 inch diameter pipes with a wall thickness of 0.438 inches. Element 20 through 32 are 12 inch diameter pipes with a wall thickness of 1.312 inches. Weights of 3864 pounds are placed at nodes 19, 22, and 27 to simulate valve weights and to test the effect of lumped masses. The pipe supports are modeled with boundary elements having stiffnesses of  $10^7$ ,  $10^7$ ,  $10^7$ , 450, 800 and 600 pounds per inch for elements between nodes 7-37, 18-38, 18-39, 13-34, 25-35, and 31-36 respectively.

The piping system is subjected to the response spectrum shown on Figure 3 (dashed). The response spectrum is input in the two horizontal directions (X,Z) and 0.67 of the spectrum is input in the vertical direction (Y). An exact solution is obtained and solutions are found using 7 sets of reduced degrees of freedom with both Guyan and the Modified Guyan Reductions. The primary degrees of freedom for each of the reduction schemes is shown in Table 9. Note that there are 186 unrestrained degrees of freedom but only the 93 translational degrees of freedom have mass attached.

Five modes are included in the calculation with the frequencies shown in Table 10. The same frequencies are found with Guyan and Modified Guyan Reduction.

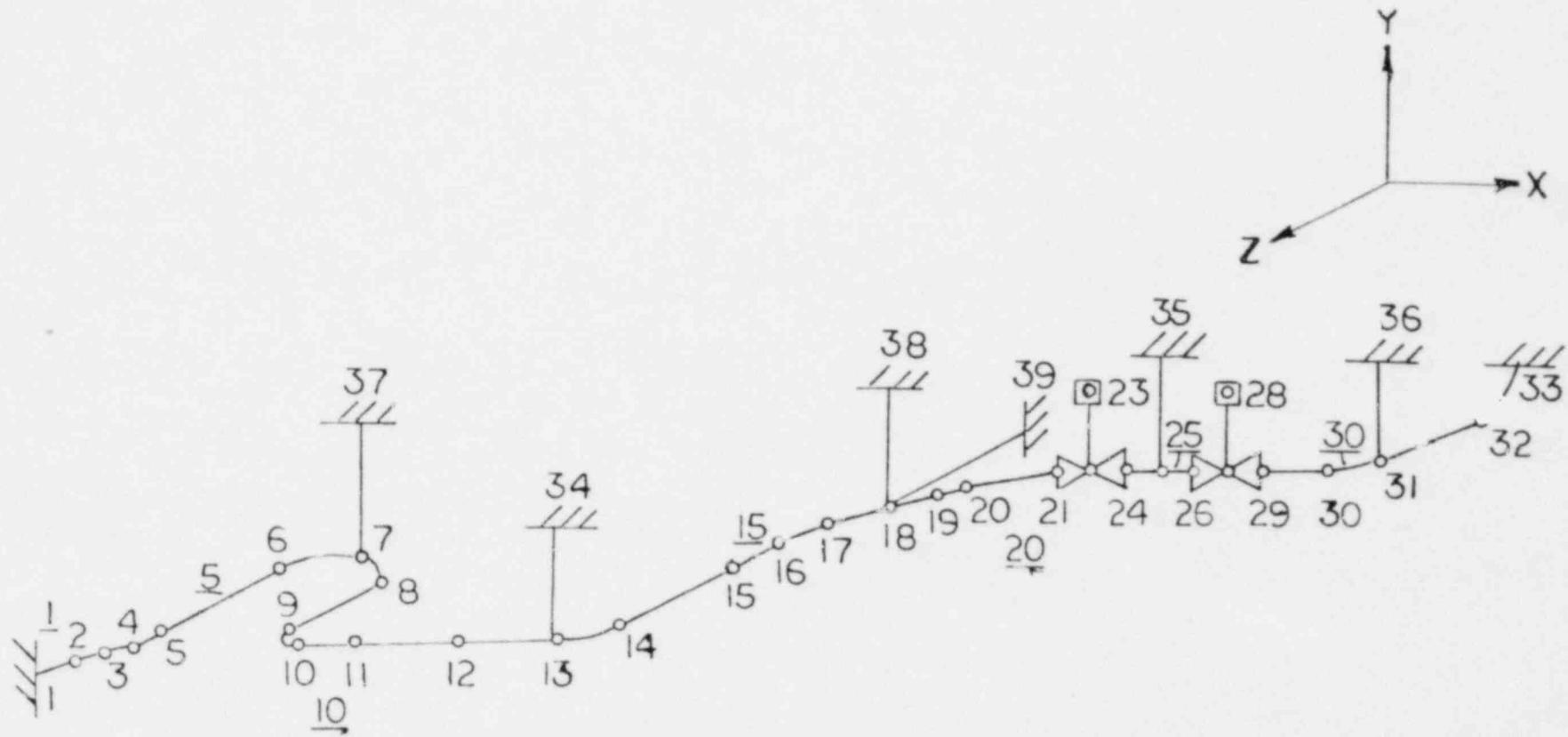


Fig. 5 PIPING SYSTEM

Table 9  
Reduction Schemes for Piping System

Case	No. of Primary Degrees of Freedom	Degrees of Freedom Retained
A	3	9-X;9-Y;9-Z
B	3	9-X;15-Y;22-Z
C	5	9-X;15-Y;18-X;22-X;27-Z
D	10	5-Y;9-X;9-Z;13-Z;15-Y;18-X;22-X;22-Y;27-Z;31-Z
E	15	5-X;5-Y;7-Z;9-X;9-Z;13-Z;15-X;15-Y;18-X;22-X;22-Y;27-X;27-Z; 31-X;31-Z
F	21	5-X;5-Y;7-X;7-Z;9-X;9-Y;9-Z;13-X;13-Z;15-X;15-Y;15-Z;18-X; 22-X;22-Y;22-Z;27-X;27-Y;27-Z;31-X;31-Z
G	26	5-X;5-Y;5-Z;7-X;7-Z;9-X;9-Y;9-Z;11-X;11-Y;11-Z;13-X;13-Z; 15-X;15-Y;15-Z;18-X;22-X;22-Y;22-Z;27-X;27-Y;27-Z;31-X; 31-Z;31-Y

Table 10  
Piping System Frequencies

Reduction Type (Table 9)	Mode 1	Mode 2	Frequencies Mode 3	(CPS)	Mode 4	Mode 5
EXACT	5.71	6.14	12.85	13.32	14.90	
A	6.21	7.11	14.70	-	-	
B	6.30	6.32	16.40	-	-	
C	6.28	6.32	14.30	20.2	42.1	
D	6.18	6.22	13.90	14.20	15.70	
E	6.15	6.20	13.70	14.00	15.60	
F	5.86	6.14	13.00	13.30	15.00	
G	5.75	6.14	12.90	13.30	14.90	

The frequency computations for the piping system may be seen to behave quite differently than for the frame structures. Ten percent errors occur in the first mode frequency until 21 dynamic degrees of freedom are retained.

The forces in all of the piping members for the cases run are listed in Table 11. The P column is the maximum thrust force in the pipe element while the M column is the maximum bending moment in the pipe. There is a great deal of variability in the results but it may be noted that large errors result when Guyan Reduction is used while Modified Guyan Reduction significantly reduces the errors. To illustrate some of these effects percentage errors for "typical" elements are shown in Table 12.

Table 11

## PIPING FORCES

Model		MEMBER NUMBER											
		1		2		3		4		5		6	
		P	M	P	M	P	M	P	M	P	M	P	M
Exact		1558	171500	1560	143600	1847	137800	1847	134300	1016	73300	965	52350
G U Y A N	MODEL A	1277	193200	1278	156600	1676	148200	1676	143500	1577	75400	1577	496690
	MODEL B	1505	127800	1506	110800	1667	107400	1667	105900	520	59990	520	58020
	MODEL C	1494	126900	1494	110000	1655	106700	1655	105100	516	59550	516	57600
	MODEL D	1069	158400	1070	127900	1381	117000	1381	117000	1346	60940	1346	42610
	MODEL E	1966	158300	1967	129900	2251	124000	2251	192560	1005	53260	1005	39060
	MODEL F	1775	171300	1776	141400	2046	138800	2046	136000	960	73040	960	53170
	MODEL G	1612	174300	1614	145900	1905	139700	1905	136500	857	74070	857	53690
M O D. G U Y A N	MODEL A	1551	171800	1522	143700	1843	137600	1843	134400	1020	73300	970	51940
	MODEL B	1487	158000	1488	132900	1747	124700	1746	124700	894	68380	853	50580
	MODEL C	1455	154900	1456	130300	1710	124900	1710	122200	877	67050	837	49470
	MODEL D	1296	140700	1297	117700	1531	112700	1531	110100	836	60910	793	60910
	MODEL E	1296	141500	1297	118500	1534	113500	1532	110800	839	60560	795	60560
	MODEL F	1574	173500	1575	145200	1867	139100	1866	135800	1031	74180	980	52820
	MODEL G	1577	174100	1579	145700	1871	139500	1871	136300	1034	74410	983	52930

Table 11 (cont'd)

## PIPING FORCES

Model		MEMBER NUMBER											
		7		8		9		10		11		12	
Exact		P	M	P	M	P	M	P	M	P	M	P	M
		1302	80730	729	66880	585	17660	501	29440	192	45870	166	52230
GUYAN	MODEL A	1676	96080	1577	80910	324	11760	1071	31220	1071	50290	1071	62820
	MODEL B	1667	75960	520	51830	520	38700	1005	12580	1005	43520	1005	63740
	MODEL C	1655	75420	516	51480	516	37890	962	11850	962	42580	962	62680
	MODEL D	1381	79850	1346	67720	340	16480	527	23030	527	40660	527	53190
	MODEL E	1327	77930	1383	66810	309	14460	471	22760	471	39090	471	55290
	MODEL F	1095	77740	681	69340	364	18750	449	25060	449	42420	449	55440
	MODEL G	1076	81030	721	69780	536	18480	671	32590	101	43990	101	53640
MOD. GUYAN	MODEL A	1294	80800	733	67040	586	14530	475	28280	155	46220	176	52440
	MODEL B	1277	76120	681	62000	576	11710	498	22310	199	43850	188	52470
	MODEL C	1252	74520	669	60680	566	13340	488	23990	197	43110	183	51570
	MODEL D	1090	67280	589	56610	481	15740	439	33520	177	49780	143	49780
	MODEL E	1097	67730	588	56280	483	15210	436	33330	171	49520	143	49520
	MODEL F	1316	81500	737	67570	591	19160	510	30480	202	46450	176	52890
	MODEL G	1319	81730	741	67760	593	19040	510	30590	201	46650	174	53070

Table 11 (cont'd)

## PIPING FORCES

Model		MEMBER NUMBER											
		13		14		15		16		17		18	
		P	M	P	M	P	M	P	M	P	M	P	M
Exact		395	52230	161	45820	182	19800	204	19690	516	66870	1049	70290
G U Y A N  1371	MODEL A	1071	61020	324	47110	324	19090	324	26360	973	71620	1383	72970
	MODEL B	1005	65290	519	53680	519	26260	519	18010	1144	67610	1110	67680
	MODEL C	962	64470	516	53530	516	30850	516	16510	1137	81590	2608	81670
	MODEL D	527	55730	426	38710	426	38710	426	19110	397	116100	1586	116200
	MODEL E	471	55290	528	38600	528	38630	528	18590	978	115200	1477	115300
	MODEL F	597	55440	309	44780	578	21970	578	10270	819	67530	1295	71470
	MODEL G	584	53640	268	45190	392	20640	392	15080	532	67680	1288	71660
M O D. G U Y A N	MODEL A	416	52440	77	45180	108	14830	134	4638	504	67370	860	70580
	MODEL B	408	52470	56	46890	86	22060	120	22650	542	65390	965	68900
	MODEL C	396	51570	65	46170	101	25520	135	26250	538	64760	1037	68050
	MODEL D	334	47450	140	37400	148	34140	160	34090	415	71900	862	74540
	MODEL E	338	47270	43	37430	154	34170	168	34070	419	71890	869	74240
	MODEL F	404	52890	170	46540	192	19100	213	19070	510	67940	1077	71300
	MODEL G	403	53070	172	46660	192	20440	213	20430	518	68210	1079	71590

Table 11 (Cont'd)

## PIPING FORCES

Model		MEMBER NUMBER											
		19		20		21		22		23		24	
		P	M	P	M	P	M	P	M	P	M	P	M
Exact		1317	78150	1321	81610	1254	81390	72	4047	1430	78040	1443	64550
GUYANA	MODEL A	1331	75220	1381	75520	1071	70980	0	0	1071	53590	1071	36210
	MODEL B	1106	68140	1108	72920	1005	102400	0	0	1005	102400	1005	77020
	MODEL C	2607	78100	2603	76970	3150	78110	0	0	2584	67600	2584	61190
	MODEL D	1588	109400	1586	94710	1671	71140	0	0	2004	61000	2004	53600
	MODEL E	1480	108400	1479	75280	1424	71030	0	0	1496	60830	1496	53780
	MODEL F	1294	79140	1294	82440	1212	82460	0	0	1447	80970	1447	66320
	MODEL G	1288	79400	1288	82550	1204	82590	0	0	1445	80760	1445	66080
MODGU YAN	MODEL A	1066	77460	1069	80100	1147	78910	8	2292	1093	70150	1090	52290
	MODEL B	1095	77150	1098	77380	1130	78630	44	3677	1074	77120	1072	62930
	MODEL C	1344	76230	1348	75780	1230	77630	51	3664	1528	77630	1549	67360
	MODEL D	1081	78370	1084	75080	1057	67860	88	5035	1187	65090	1198	53780
	MODEL E	1118	77940	1122	74490	1102	68110	93	5002	1288	64790	1302	53400
	MODEL F	1336	79170	1341	82310	1250	82260	66	3524	1422	79450	1436	66150
	MODEL G	1339	79490	1344	82480	1295	82440	67	3504	1424	79460	1437	66060

Table 11 (Cont'd)

## PIPING FORCES

Model		MEMBER NUMBER											
		25		26		27		28		29		30	
		P	M	P	M	P	M	P	M	P	M	P	M
Exact		1454	57430	1468	51970	68	3718	1781	47670	1800	41430	1815	42340
GUYAN	MODEL A	1071	25480	1071	14930	0	0	1071	19270	1071	29400	1071	33580
	MODEL B	1005	61450	1005	46270	0	0	1005	22980	1005	26050	1005	30700
	MODEL C	2584	61010	2584	67090	0	0	2584	67090	2584	50790	2584	49820
	MODEL D	2004	50950	2004	52850	0	0	2004	52850	2004	39330	2004	40060
	MODEL E	1496	51610	1496	55380			1963	55380	1963	42850	1963	43070
	MODEL F	1447	58970	1447	53760	0	0	1827	50850	1827	42180	1827	42980
	MODEL G	1445	58690	1445	53420	0	0	1820	50390	1820	42000	1820	42890
MODUGUYAN	MODEL A	1088	41010	1085	29760	5	1079	1030	11740	1027	25390	1024	31660
	MODEL B	1071	54400	1070	46570	13	2552	1082	36280	1084	27530	1086	31200
	MODEL C	1565	62440	1587	59200	21	3615	2042	57700	2069	47240	2090	46530
	MODEL D	1207	47790	1218	43160	56	3776	1454	39410	1470	34370	1482	35340
	MODEL E	1313	47670	1327	43410	64	3856	1614	40630	1632	36940	1646	37610
	MODEL F	1446	59160	1461	53840	64	3021	1780	49630	1800	42100	1815	42790
	MODEL G	1448	59000	1462	53610	63	2983	1780	49330	1799	42030	1814	42800

Table 11 (Cont'd)

## PIPING FORCES

Model		MEMBER NUMBER											
		31		32		33		34		35		36	
Exact		P	M	P	M	P	M	P	M	P	M	P	M
		1737	44290	1739	50700	121.3		905.3		849.5		81.6	
G U Y A N	MODEL A	1329	34060	1329	27650	138.8		53.8		509.4		16.6	
	MODEL B	1520	37990	1520	62180	100.6		1539.		1768.		99.7	
	MODEL C	7428	46310	2428	80590	116.8		1804.		642.2		116.9	
	MODEL D	2203	40660	2103	77260	381.8		3097.		1069.		156.1	
	MODEL E	2515	40890	2516	77920	377.4		3209.		1024.		155.4	
	MODEL F	1743	44850	1743	54060	232.0		1291.		1022.		77.4	
	MODEL G	1745	44760	1745	55220	102.1		976.7		961.4		84.5	
M O D. G U Y A N	MODEL A	1503	35240	1504	35740	68.1		66.9		505.3		17.3	
	MODEL B	1470	40270	1471	64850	114.5		257.2		813.8		109.6	
	MODEL C	1792	45980	1796	77740	130.7		349.3		1001		126.0	
	MODEL D	1457	44870	1459	71640	210.2		931.8		768.9		165.7	
	MODEL E	1543	44710	1546	71260	210.0		1038.0		733.4		164.6	
	MODEL F	1732	45030	1734	53510	124.1		831.9		927.5		81.9	
	MODEL G	1737	44980	1739	55020	125.1		834.3		914.7		86.9	

Table 11(Cont'd)

## PIPING FORCES

Model	MEMBER NUMBER					
	37		38			
	P	M	P	M	P	M
Exact	17.4	3.4				
G	MODEL A	13.8	1.6			
U	MODEL B	17.7	3.2			
Y	MODEL C	18.1	4.3			
A	MODEL D	14.9	3.3			
N	MODEL E	14.9	3.3			
	MODEL F	17.8	3.6			
	MODEL G	17.9	3.6			
M	MODEL A	16.1	2.3			
O	MODEL B	17.8	3.5			
D.	MODEL C	18.1	4.2			
G	MODEL D	13.8	3.0			
U	MODEL E	13.7	2.9			
Y	MODEL F	17.8	3.5			
A	MODEL G	17.9	3.6			

TABLE 12  
PERCENTAGE ERRORS IN PIPING ELEMENTS

Case (Table 9)	GUYAN REDUCTION										
	PIPE ELEMENTS								PIPE SUPPORTS		
	1		5		12		18		7-37	18-38	13-34
	P	M	P	M	P	M	P	M			
A	-18.	12.7	+55.2	+2.9	545.2	+20.3	+31.8	+3.8	+14.4	-94.1	-79.7
B	-3.4	-25.5	-48.8	-18.2	505.4	+22.0	+5.8	-3.7	-17.1	+70.0	+22.2
C	-4.1	-26.0	-49.2	-18.8	479.5	+120.0	148.6	+16.2	-3.7	+99.3	+43.3
D	-31.4	-7.6	+32.5	-16.9	217.5	+1.8	+51.2	+65.3	214.8	242.1	+91.3
E	+26.2	-7.7	-1.1	-27.3	183.7	+5.5	+40.8	+64.0	211.1	254.5	+90.4
F	+13.9	-0.1	-5.5	-0.4	170.5	+6.1	+123.5	+1.7	91.3	+42.6	-5.1
G	+3.5	+1.6	-15.6	+1.1	-39.2	+2.7	+22.8	+1.9	-15.8	7.9	+3.6

TABLE 12 (Cont'd)

## PERCENTAGE ERRORS IN PIPING ELEMENTS

Case (Table 9)	MODIFIED GUYAN										
	PIPE ELEMENTS				PIPE SUPPORTS						
	1		5		12		18		7-37	18-38	13-34
	P	M	P	M	P	M	P	M			
A	-.4	+0.2	+.4	0	+6.0	+.4	-18.0	+0.4	-43.9	-92.6	-78.8
B	-4.6	-7.9	-12.0	-6.7	+13.3	+.5	-8.0	-2.0	-5.6	-71.6	+34.3
C	-6.6	-9.7	-13.7	-8.5	+10.2	-1.3	-1.1	-3.2	+7.7	-61.4	+54.4
D	-16.8	-18.0	-17.7	-16.9	-13.9	-4.5	-17.8	+6.0	73.3	+2.9	103.1
E	-16.8	-17.5	-17.4	-17.4	-13.9	-5.2	-17.2	+5.6	73.1	+14.7	101.7
F	+1.0	+1.2	+1.5	+1.2	+6.0	+1.3	+2.7	+1.4	2.3	-8.1	+0.4
G	+1.2	+1.5	+1.8	+1.5	+4.8	+1.6	+2.9	+1.8	3.1	-7.8	+6.5

The following conclusion may be drawn from the data on Table 12:

- (i) When reduction schemes are used the bending moment data appears to be most reliable with the thrust and pipe support data the poorest.
- (ii) The Modified Guyan Reduction gives significantly better results than the Guyan Reduction. The maximum error for pipe thrust and moment for the latter case is 18% no matter which reduction scheme is used. The maximum error for pipe thrust and moment is 545% when Guyan Reduction is used. The comparable maximum errors for pipe support loads are 103% and 254% respectively for the both reduction methods.
- (iii) Reduction schemes F and G yield good results (maximum error 8.1%) when Modified Guyan Reduction is used while scheme G still yields rather poor (39% error in pipe 12 thrust) results for Guyan Reduction.
- iv) The good results are obtained for scheme G because degrees of freedom normal to the restraints provided by the pipe supports are included as dynamic degrees of freedom. The degree of freedom active in the direction of the pipe support should probably be included to achieve good results for the pipe support loads with Guyan Reduction.

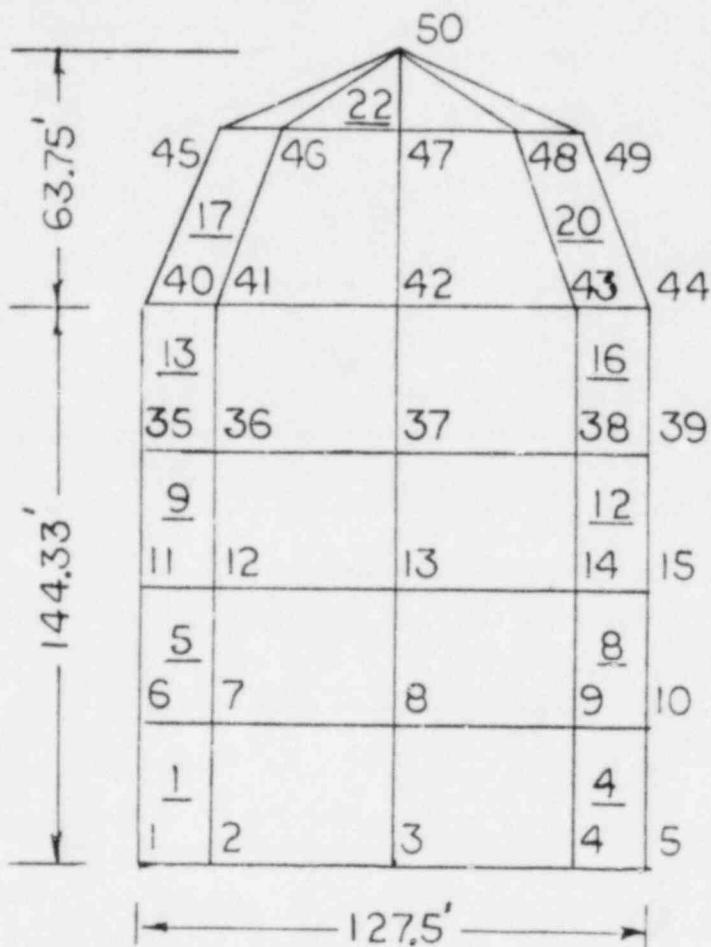
#### 4.4 REACTOR CONTAINMENT BUILDING

The reactor containment building shown in Figure 6 is subjected to the response spectrum of Figure 3 in the X direction. Plate elements 1 through 24 are used to model the exterior containment. The reactor floor is modeled with plate elements 25 through 32. A wall supporting the plate is modeled with plate element 33 through 40. All plate elements are taken as concrete with a thickness of 36 inches. Beam elements 41 through 43 are used to model the reactor. Symmetric boundary conditions are specified at all nodes at ( $Y = 0$ ). These boundary conditions restrain the following degrees of freedom: displacement in the Y direction and rotations about the X and Z axes.

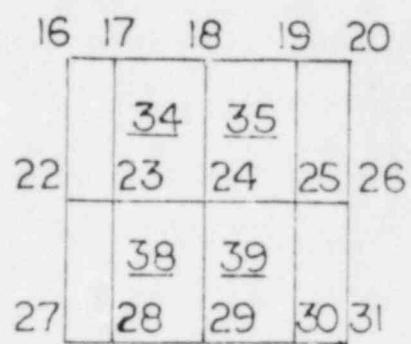
An exact solution is obtained and solutions using both Guyan and Modified Guyan Reduction for the reduced models shown in Table 13. There are 102 unrestrained translational degrees of freedom.

Table 13  
Reduction Schemes for Containment Building

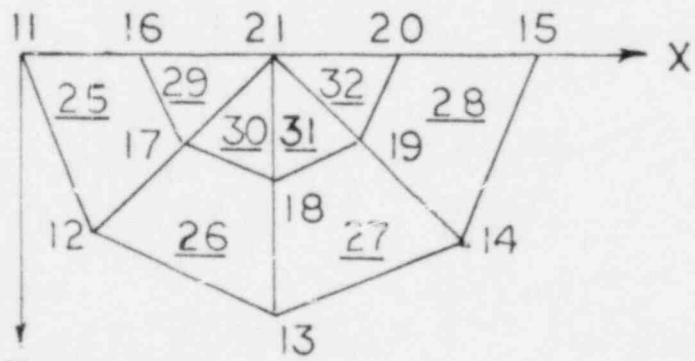
Case	No. of Primary Degrees of Freedom	Degrees of Freedom Retained
A	3	21-X;34-X;50-X
B	6	13-Y;21-X;34-X;40-Z;44-Z;50-X
C	13	11-X;11-Z;15-X;15-Z;21-X;21-Z;34-X; 40-X;40-Z;42-Y;44-X;44-Z;50-X
D	23	11-X;11-Z;13-X;13-Y;15-X;15-Z;16-X; 16-Z;18-X;18-Y;20-X;20-Z;21-X;21-Z; 33-X;34-X;40-X;40-Z;42-X;42-Y;44-X; 44-Z;50-X
E	46	11-X;11-Z;13-X;13-Y;15-Y;15-Z;;6-X; 16-Z;16-YY;17-X;17-Y;17-Z;17-YY; 17-ZZ;18-X;18-Y;18-Z;18-XX;18-YY; 18-ZZ;19-X;19-Y;19-Z;19-XX;19-YY; 19-ZZ;20-X;20-Z;20-YY;21-X;21-Z;21-YY; 33-X;33-Z;33-YY;34-X;34-Z;34-YY;40-X; 40-Z;42-X;42-Y;44-X;44-Z;50-X



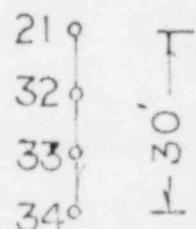
ELEVATION



INTERNAL WALL



REACTOR FLOOR



REACTOR

Fig. 6 CONTAINMENT BUILDING

Ten modes are included in the solution with the resulting frequencies shown in Table 14.

Table 14  
Frequencies for Containment Building

Case	Frequency (CPS) for Mode									
	1	2	3	4	5	6	7	8	9	10
EXACT	3.84	9.53	9.89	10.2	10.3	10.7	10.9	14.5	14.8	15.8
A	4.09	13.3	113.	-	-	-	-	-	-	-
B	4.08	12.6	13.6	20.6	22.2	113.	-	-	-	-
C	3.98	9.55	11.2	12.7	13.0	13.6	14.4	23.7	24.5	29.5
D	3.89	9.55	10.4	12.5	13.2	13.4	13.6	20.1	22.5	25.4
E	3.89	9.54	10.4	12.5	13.1	13.3	13.5	20.0	22.3	23.3

As with the piping system it appears that several of the modes from the exact model are missing from the reduced model.

Stresses in typical plate element and the peak bending moment in the reactor beam model are shown in Table 15.

Table 15  
Stresses and Moments in Containment Building

Case	Peak Stress in Plate Element						Peak Moment in Beam
	1	5	13	25	29	33	
EXACT	649	481	186	72	83	138	47,740
G A	545	419	225	65	175	109	2,527,000
U Y B	548	426	244	63	180	109	2,738,000
A N C	600	465	222	87	149	122	2,148,000
D	635	475	192	46	104	140	350,900
E	635	475	192	56	89	141	45,640
M. A G	569	420	166	60	24	219	103,300
U Y B	570	422	167	60	29	117	56,450
A C N	601	444	173	68	82	127	45,170
D	632	466	178	71	93	135	46,410

The plate stresses for all of the elements appear to be quite good. Modified Guyan Reduction appears to be a little better than Guyan Reduction but not to the same degree as for the first three problems considered. The very significant difference shows up with the peak moment in the beam model of the reactor. Guyan Reduction yields totally unreliable results while Modified Guyan gives good results except for Reduction scheme A. As with the stiff pipe supports the floor provides a very stiff support for the reactor and undoubtedly leads to these very poor results. Guyan Reduction gives a reasonable value for this moment only when the rotational degree of freedom connecting the beam to the plate is included as a primary degree of freedom. Note that there is no mass attached to this degree of freedom.

#### 4.5 COMPUTER REQUIREMENTS

For the four sample problems considered the Modified Guyan and Guyan Reduction solutions required almost exactly the same computer time. Both of the reduction methods required solution times about 3 times longer than would be required using the complete SAPV solution. Undoubtedly some of this difference resulted from "patching in" the reduction modification. Changes were made in a manner to expedite the code modification rather than to develop an efficient code. If this facet of the problem were to be investigated further it would be recommended that the current code be used to pinpoint the calculations that require the most time. This could then be used to evaluate the likelihood that the SAPV with reduction would be expected to run faster than SAPV without reductions.

#### 5.0 CONCLUSIONS AND RECOMMENDATIONS

Based upon the four sample problems the following conclusions are made:

1. The errors resulting from Guyan Reduction are significantly larger than those which result from Modified Guyan Reduction. For example the errors in member bending moments in the ten story structure are always less than 2% when Modified Guyan Reduction is used even when only 3 degrees of freedom are retained. The same errors for Guyan Reduction are 100% when 3 degrees of freedom are retained and only become acceptable when 6 or more degrees of freedom are retained.
2. For a given reduction scheme the errors resulting from Modified Guyan Reduction are much more uniform for all members in the problem than when Guyan Reduction is used.

Again using the ten story frames as an example for the same reduction scheme (3 degrees of freedom retained) the error in the bending moments of members 20 and 41 are 100% and 1% when Guyan Reduction is used. The corresponding errors with Modified Guyan Reduction are 1.3% and 1.6%.

This variation in error which Guyan Reduction exhibits would make it very difficult to validate a given set of results. If we looked at the forces in member 41, the results for the 3 degrees of freedom model would appear to be quite good.

3. Frequencies calculated with Guyan Reduction appear to be quite good as long as the degrees of freedom retained can adequately represent the mode shapes of interest.
4. The bending moments in piping systems predicted based upon Guyan Reduction are more reliable than are the axial pipe loads. For example element 12 has errors in the axial pipe load in the range of 200% to 300% (for different degrees of freedom retained) while the same errors in moments are in the range of 5% to 20%. The same errors when Modified Guyan Reduction is used are 6 to 14% for axial load and 1% to 5% for bending moments. This again illustrates the better and more uniform results obtained using Modified Guyan Reduction.
5. Much larger errors result with the reduction schemes when there are both "stiff" and "soft" elements in the system. This occurs in the piping problem (supports relative to pipe) and in the containment building problem (floor in plane stiffness to reactor beam model). In either case Guyan Reduction gives totally unreliable results unless

care is taken to isolate the stiff element by containing it within degrees of freedom retained. The same effect is also observed with Modified Guyan Reduction but again the errors are much smaller.

The following recommendations are made:

1. Computer codes which contain Guyan Reduction should be modified to incorporate Modified Guyan Reduction.
2. If computer solutions are to be made using Guyan Reduction care should be taken to identify the stiff elements in the model and to retain all degrees of freedom around these elements as primary degrees of freedom.
3. Additional parametric studies should be made with the following model parameters varied: structural geometry and type; relative stiffness of members; and relative mass. Based in these studies criteria should be developed relating errors to the reduction scheme used. This should be done for both Guyan and Modified Guyan Reduction. The results of such a study would serve two purposes. First, they would provide some insight to the errors which may exist in Guyan Reduction runs made. Second, such criteria could provide the basis for selecting the "optimum" set of primary degrees of freedom to be retained.

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4. Miller, C. A., "Dynamic Reduction of Structural Models", Proceedings American Society of Civil Engineers, Structural Division, Vol. 106, No. ST10, October 1980.
5. Bathe, R. J., Wilson, F. L., Peterson, F. E., "A Structural Analysis Program for Static and Dynamic Response of Linear Systems", Earthquake Engineering Research Center, University of California, June 1973.
6. Bathe, K.J., Wilson, E. L., "Numerical Methods in Finite Element Analysis", Prentice-Hall, 1976.
7. Bezler, P., Hartzman, M. Reich, M., "Piping Benchmark Problems", NUREG/CR-1677, Vol. 1, Brookhaven National Laboratory, August 1980.\*

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\*Available for purchase from the NRC/GPO Sales Program, U.S. Nuclear Regulatory Commission, Washington, DC 20555, and the National Technical Information Service, Springfield, VA 22161.

APPENDIX A  
LISTING OF SAPV  
MODIFICATIONS

# POOR ORIGINAL

78 / 78 OPT = 1

FTN 4, 5+6/94

11/19/20 17:24:07

# POOR ORIGINAL

PROGRAM SAPS 76/76 OPT=1

FTN 4.5+414

11/19/80 17.24.07

```

      C      S(1)=S(1)
      READ (3) LL,NF,KDYN,NEQB,NEQ,NBLOCK
      CALL SAPLOT(LL,NF,KDYN,NEQB,NEQ,NBLOCK)
      CALL SECOND(T(8))
      DO 8 I=9,12
      C      T(1)=T(8)
      C      S(1)=S(8)
      GO TO 90
      C
      C      RE-START MODE ACTIVATED IF NODYN.EQ.-2 OR NODYN.EQ.-3
      C
      14 IF(NODYN.LT.0) GO TO 20
      C      INPUT JOINT DATA
      C
      70      N2=N1+6*NUMNP
      N3=N2+NUMNP
      N4=N3+NUMNP
      N5=N4+NUMNP
      N6=N5+NUMNP
      CALL ERROR(N6)
      C
      80      CALL INPUTJ(A(N1),A(N2),A(N3),A(N4),A(N5),NUMNP,NEQ)
      C      STORE THE NODAL COORDINATES FOR LATER USE
      C
      85      WRITE ((3) (A(I),I=N2,N5)
      C      FORM ELEMENT STIFFNESSES
      C
      90      INPUT PRIMARY DOF AND REORDER ALL DOF SO THAT PRIMARY ARE FIRST
      C
      CALL REDUCI(A(N1),A(N6),NTYP,NUMNP,NRDOF)
      CALL SECOND(T(2))
      C
      MBAND=0
      NUMEL=0
      95      REWIND 1
      REWIND 2
      C
      DO 900 M=1,NELTYP
      READ (5,100) NPAR
      C+++
      DATA Porthole SAVE
      IF(MODEX.EQ.1) WRITE (NTB) NPAR
      NTI=1
      IF (MINBND .GE. 1 .AND. MODEX .EQ. 0) NTI=4
      IF (MINBND .GE. 1 .AND. MODEX .EQ. 1) NTI=7
      WRITE (NTI) NPAR
      NUMEL=NUMEL+NPAR(2)
      MTYPE=NPAR(1)
      C
      CALL ELTYPE(MTYPE)
      C
      900 CONTINUE
      IF (KOUNT.GE.1) STOP
      C      ADD AN END-OF-FILE FLAG ON FILE 13
      SAPS 58
      SAPS 59
      SAPS 60
      SAPS 61
      SAPS 62
      SAPS 63
      SAPS 64
      SAPS 65
      SAPS 66
      SAPS 67
      SAPS 68
      SAPS 69
      SAPS 70
      SAPS 71
      SAPS 72
      SAPS 73
      SAPS 74
      SAPS 75
      SAPS 76
      SAPS 77
      SAPS 78
      SAPS 79
      CORR2 4
      SAPS 80
      SAPS 81
      SAPS 82
      SAPS 83
      SAPS 84
      SAPS 85
      SAPS 86
      SAPS 87
      SAPS 88
      CAM 5
      CAM 6
      CAM 7
      CAM 8
      SAPS 89
      SAPS 90
      SAPS 91
      SAPS 92
      SAPS 93
      SAPS 94
      SAPS 95
      SAPS 96
      SAPS 97
      SAPS 98
      SAPS 99
      SAPS 100
      SAPS 101
      SAPS 102
      SAPS 103
      SAPS 104
      SAPS 105
      SAPS 106
      SAPS 107
      SAPS 108
      SAPS 109
      CORR3 10
      SAPS 110
      SAPS 111

```

# POOR ORIGINAL

PROGRAM SAPS

78/78 OPT=1

FTN 4.5+414

11/19/80 17.24.0

```

115      C
          M=-1
          IF ((PLT .GT. 0)
          XWRITE (13) M,M
          C
          MINIMIZE THE BANDWIDTH
          C
          IF (MBAND .GT. 0) CALL SAPMIN
          C
          DETERMINE BLOCKSIZE
          C
          ADDSTF
          C
          NEQB=(MTOT - 4*LL)/(MBAND + LL + 1)/2
          C
          OVER-RIDE THE SYSTEM MATRIX BLOCKSIZE WITH THE INPUT (NON-ZERO)
          C VALUE, KEQB.
          C THIS OVER-RIDE ENTRY IS TO ALLOW PROGRAM CHECKING OF MULTI-
          C BLOCK ALGORITHMS WITH WHAT WOULD NORMALLY BE ONE BLOCK DATA.
          C
          IF (KEQB.LT.NEQB) NEQB = KEQB
          C
          GO TO (690,700,700,700,730,700), KDYN
          C
          STATIC SOLUTION
          C
          690 CONTINUE
          NEQBI=(MTOT - MBAND)/(2*(MBAND+LL) + 1)
          NEQB2=(MTOT - MBAND - LL*(MBAND-1))/(3*LL + MBAND + 1)
          IF (NEQBI.LT.NEQB) NEQB=NEQBI
          IF (NEQB2.LT.NEQB) NEQB=NEQB2
          NBLOCK = (NEQ-1)/NEQB + 1
          IF (NEQB.GT.NEQ) NEQB=NEQ
          IF ((PLT.EQ.0)) GO TO 695
          NTOT=20*NUMNP+NEQB*NBLOCK*LL
          IF (MTOT.GT.NTOT) GO TO 790
          WRITE(6,2020)NTOT
          MODEX=1
          695 NTOT=8*NUMNP+NEQB*NBLOCK*LL+1
          IF (MTOT.GT.NTOT) GO TO 790
          WRITE(6,2030)NTOT
          MODEX=1
          GO TO 790
          C
          EIGENSOLUTION
          C
          700 IF (NEQB.LT.NEQ) GO TO 710
          NIM=3
          NC=NF + NIM
          NVM=6
          NCA=NEQ*MAX0(MBAND,NC)
          NTOT=NCA + 4*NEQ + 2*NVM*NEQ + 5*NC
          NE10=0
          IF (INTOT.LE.MTOT) GO TO 720
          C
          SAP5   112
          SAP5   113
          SAP5   114
          SAP5   115
          SAP5   116
          SAP5   117
          SAP5   118
          SAP5   119
          SAP5   120
          SAP5   121
          SAP5   122
          SAP5   123
          SAP5   124
          SAP5   125
          SAP5   126
          SAP5   127
          SAP5   128
          SAP5   129
          SAP5   130
          SAP5   131
          SAP5   132
          SAP5   133
          SAP5   134
          SAP5   135
          SAP5   136
          SAP5   137
          SAP5   138
          SAP5   139
          SAP5   140
          SAP5   141
          SAP5   142
          SAP5   143
          SAP5   144
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          SAP5   146
          SAP5   147
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          SAP5   164
          SAP5   165
          SAP5   166
          SAP5   167
          SAP5   168

```

# POOR ORIGINAL

PROGRAM SAPS

78/78 OPT=1

FTN 4.5+414

11/19/80 17.24.0

	C	2. SUBSPACE ITERATION ALGORITHM	SAPS	169
	C		SAPS	170
175		710 NV=MINO(2*NF,NF+8)	SAPS	171
		IF (NAD.NE.0) NV=NAD	SAPS	172
		NEQB1=(MTOT-MBAND)/(2*MBAND+1)	SAPS	173
		NEQB2=(MTOT-MBAND-2*NV-NV*(MBAND-2))/(3*NV+MBAND+1)	SAPS	174
		NEQB3=(MTOT-3*NV-NV-3*NV)/(2*NV+MBAND+1)	SAPS	175
		NEQB4=(MTOT-6*NV)/(1+MBAND)	SAPS	176
180		IF (NEQB1.LT.NEQB) NEQB=NEQB1	SAPS	177
		IF (NEQB2.LT.NEQB) NEQB=NEQB2	SAPS	178
		IF (NEQB3.LT.NEQB) NEQB=NEQB3	SAPS	179
		IF (NEQB3.LT.NEQB) NEQB=NEQB3	SAPS	180
		IF (NEQB4.LT.NEQB) NEQB=NEQB4	SAPS	181
185		NEIG=1	SAPS	182
	C	720 CONTINUE	SAPS	183
		NBLOCK=(NEQ-1)/NEQB+1	SAPS	184
		IF (NEQB.GE.NEQ) NEQB=NEQ	SAPS	185
190	C	HISTORY OR SPECTRUM ANALYSIS	SAPS	186
	C		SAPS	187
	C	KREM = 1000	SAPS	188
195		NTOT = NBLOCK*NEQB*NF + KREM	SAPS	189
		IF (MTOT.LT.NTOT)	SAPS	190
		XWRITE(6,320) NTOT	SAPS	191
		IF (IPLT.EQ.0) GO TO 725	SAPS	192
		NTOT=20*NUMNP+NEQB*NBLOCK*NF+2*NF	SAPS	193
		IF (MTOT.GE.NTOT) GO TO 790	SAPS	194
200		WRITE(6,2020) NTOT	SAPS	195
		MODEX=1	SAPS	196
	725	NTOT=6*NUMNP+NEQB*NBLOCK*NF+NF	SAPS	197
		IF (MTOT.GE.NTOT) GO TO 790	SAPS	198
		WRITE(6,2030) NTOT	SAPS	199
205		MODEX=1	SAPS	200
		GO TO 790	SAPS	201
	C	STEP-BY-STEP DIRECT INTEGRATION	SAPS	202
	C		SAPS	203
210	730	CONTINUE	SAPS	204
	C	DISPLACEMENT COMPONENTS FOR DIRECT OUTPUT (*NSD*)	SAPS	205
	C	NN2 = NEQ	SAPS	206
	C	DISPLACEMENT COMPONENTS REQUIRED FOR RECOVERY OF ALL OF THE	SAPS	207
	C	REQUESTED ELEMENT STRESS COMPONENTS (*NSS*)	SAPS	208
215	C	NN3 = NEQ	SAPS	209
	C	1. DECOMPOSITION	SAPS	210
	C		SAPS	211
	C	MEQB1 = (MTOT-NN2-NN3-NEQ-MBAND)/(2*MBAND+1)	SAPS	212
220	C	2. TIME INTEGRATION PHASE	SAPS	213
	C		SAPS	214
	C	MEQB2 = (MTOT-MBAND-2*(NN2+NN3)-5*NEQ)/(MBAND+1)	SAPS	215
225	C	IF (MEQB1.LT.NEQB) NEQB = MEQB1	SAPS	216
		IF (MEQB2.LT.NEQB) NEQB = MEQB2	SAPS	217
		IF (NEQB.GT.NEQ) NEQB = NEQ	SAPS	218
			SAPS	219
			SAPS	220
			SAPS	221
			SAPS	222
			SAPS	223
			SAPS	224
			SAPS	225

## POOR ORIGINAL

```

PROGRAM SAPS      76/76   OPT=1          FTN 4.5+414        11/19/80  17.24.01
                                                              
100      NBLOCK = (NEQ-1)/NEQB +1          SAPS      226
101      C                                     SAPS      227
102      C      3. INPUT PHASE             SAPS      228
103      C
104      C      NUMBER OF TIME FUNCTIONS ( NFN )
105      C      NUMBER OF TIME FUNCTIONS (*NFN*)
106      C      NN2 = 10                   SAPS      229
107      C      MAXIMUM NUMBER OF FUNCTION DEFINITION POINTS (*MXLP*)
108      C      NN3 = 40                   SAPS      230
109      C
110      C      NN4 = 6*NUMNP + 2*NN2*NEQ    SAPS      231
111      IF(NN4.GT.MTOT)                  SAPS      232
112      XWRITE (6,320) NN4               SAPS      233
113      NN4 = NEQ*2*(NN2+1) + NN2*(1+2*NN3)  SAPS      234
114      IF(NN4.GT.MTOT)                  SAPS      235
115      XWRITE (6,320) NN4               SAPS      236
116      C
117      C      790 IF (NEQB .GE. 2) GO TO 792  SAPS      237
118      IF (NEQ .LT. 3) GO TO 792       SAPS      238
119      WRITE (6,2010) NEQB            SAPS      239
120      NEQB=2                      SAPS      240
121      NBLOCK=(NEQ-1)/NEQB + 1        SAPS      241
122      MODEX=1                     SAPS      242
123      CONTINUE
124      C
125      C      INPUT NODAL LOADS        SAPS      243
126      C
127      NSB=NEQB*NBLOCK            SAPS      244
128      N2=N1+6*NUMNP            SAPS      245
129      N3=N2+LL*NSB            SAPS      246
130      NN=NN3+NSB              SAPS      247
131      CALL ERRKR(NN)
132      WRITE (6,2011) NEQ,MBAND,NEQB,NBLOCK
133      C
134      CALL SECOND(T(3))
135      C
136      CALL INL(A(N1),A(N2),A(N3),NUMNP,NEQB,NSB,LL,NBLOCK)
137      C
138      CALL SECOND(T(4))
139      C
140      CHECK FOR UNRESTRAINED DEGREES OF FREEDOM
141      C
142      N3=N2+NEQ
143      CALL CHICK(A(N1),A(N2),NEQ,NUMEL,NELTYP,NUMNP)
144      C
145      FORM TOTAL STIFFNESS
146      C
147      NE2B=2*NEQB
148      N2=N1+NEQB*MBAND
149      N3=N2+NEQB*LL
150      NN=NN3+4*LL
151      NN2=N1+NE2B*MBAND
152      NN3=NN2+NE2B*LL
153      NN4=NN3+4*LL
154      C
155      CALL ADDSTF (A(N1),A(NN2),A(NN3),A(NN4),NUMEL,NBLOCK,NE2B,LL,MBAN
156      XD,ANORM,NNV)          SAPS      248
157

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POOR ORIGINAL

PROGRAM SAPS 78/78 OPT=1

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	C	SAP5	283
	C C	SAP5	284
	HT AND CG CALCULATIONS	SAP5	285
	C	SAP5	286
290	IF (IHTCG .EQ. 0) GO TO 18	SAP5	287
	N2=N1+NUMNP*6	SAP5	288
	N3=N2+NUMNP	SAP5	289
	NA=N3+NUMNP	SAP5	290
	NS5=NA+NUMNP	SAP5	291
	NS6=NS5+NSB	SAP5	292
295	CALL ERROR(NS)	SAP5	293
	CALL HTCG(A(N1)),A(N2),A(N3),A(N4),A(N5),NUMNP,NEQB,NEQ,NBLOCK,NSB	SAP5	294
	X)	SAP5	295
	18 CONTINUE	SAP5	296
	CALL SECOND(T(5))	CAM	9
300	C	CAM	10
	IF NTYP.NE.0 GO TO ROUTINES TO FORM EIGENVECTORS	CAM	11
	C	CAM	12
	IF (INTYP.EQ.0) GO TO 100	CAM	13
305	C DETERMINE SIZE REQUIREMENTS	CAM	14
	NST=NEQ	CAM	15
	MBW=MBAND	CAM	16
	NS=NST-NRDOF	CAM	17
	NS1=(NRDOF*NROOF+NROOF)/2	CAM	18
	IF (NRDOF.GT.MBW) GO TO 101	CAM	19
310	NS2=(MBW-NRDOF)*NRDOF+((NRDOF-1)*(NRDOF-1))/2	CAM	20
	GO TO 102	CAM	21
	101 NS2=(MBW*MBW+MBW)/2	CAM	22
	102 NOT=NS-NRDOF	CAM	23
	NADD=0	CAM	24
315	MODE=NF	CAM	25
	MD=MODE+NADD	CAM	26
	IF (MD.GT.NRDOF) MD=NRDOF	CAM	27
	IF (MODE.GT.NRDOF) MODE=NRDOF	CAM	28
	IF (NF.GT.NRDOF) NF=NRDOF	CAM	29
320	NR2=1+NEQB*MBAND	CAM	30
	CALL SIZE(A(1)),A(NR2),MTOT,NEQB,NBLOCK,MBAND)	CAM	31
	NRI=1	CAM	32
	NR2=NR1+NS1	CAM	33
	NR3=NR2+NS2	CAM	34
325	NR4=NR3+NSEQ*MB5	CAM	35
	NR5=NR4+NSEQ*MBAND	CAM	36
	CALL STIFF(A(NR1)),A(NR2),A(NR3),A(NR4),A(NR5),NBLOCK,NEQB,NRDOF,	CAM	37
	I NTYP,NST,MBW,NS,NS1,NS2,NS3,NOT,MBS,MODE,MD,MBLRED,NSEQ,NT01	CAM	38
	NSZ=NST	CAM	39
330	C CALL MASS(A(NR3)),NEQB,NBLOCK,NSZ)	CAM	40
	INVERT KSS AND STORE ON TAPE 17	CAM	41
	NR4=NR3+NST	CAM	42
	NR5=NR4+NSEQ*MB5	CAM	43
335	NR6=NR5+NSEQ*MB5	CAM	44
	NR7=NR6+NST	CAM	45
	CALL DAUS(A(NR4)),A(NR5),A(NR6),A(NR7),NSEU,NS,MBS,MBLRED,	CAM	46
	I 18,19,17,1)	CAM	47
	C FORM K <sup>T</sup> AND M <sup>T</sup>	CAM	48
	NR5=NR4+NS1	CAM	49
	NR6=NR5+NS	CAM	50
340	NR7=NR6+NRDOF	CAM	51
	CALL REDUCE (A(NR1)),A(NR2),A(NR3),A(NR4),A(NR5),A(NR6),A(NR7),	CAM	

# POOR ORIGINAL

PROGRAM SAPS 76 / 78 OPT = 1

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1      NRDOF,NITYP,NSH,NS,NS1,NS2,NS3,NOT,NEQS,MODE,      CAM
2  MO,MILRED,NSEQ,NTD)      CAM
3  C      SAVE MASS ON 15 FOR NORMALIZATION OF EIGENVECTORS    CAM
4  REWIND 15      CAM
5  DO 105 I=1,NST      CAM
6  105 WRITE(15)(A(NR3+I-1))      CAM
7  REWIND 15      CAM
8  DO 104 I=1,NS1      CAM
9  104 A(NR2+I-1)=A(NR4+I-1)      CAM
10 NR1=1      CAM
11 NR2=NR1+NS1      CAM
12 NR3=NR2+NS1      CAM
13 NR4=NR3+NRDOF+MD      CAM
14 NR5=NR4+NRDOF+NRDOF      CAM
15 NR6=NR5+NRDOF      CAM
16 NR7=NR6+NRDOF      CAM
17 CALL POWER(A(NR1),A(NR2),A(NR3),A(NR4),A(NR5),A(NR6),A(NR7),      CAM
18 NS1,NRDOF,MODE)      CAM
19 NR4=NR3+NRDOF+MD      CAM
20 NR5=NR4+NTD*NRDOF      CAM
21 NR6=NR5+NRDOF      CAM
22 NR7=NR6+NEQB*MODE      CAM
23 CALL EXPAND(A(NR3),A(NR4),A(NR5),A(NR6),NRDOF,MD,MODE,NTD,NEQB,      CAM
24 1,NEBLOCK,NTYP,NS,A(NR7),NST)      CAM
25 100 CONTINUE      CAM
26
27 C      S O L U T I O N   P H A S E      SAPS
28 C      IF (GEOST) GO TO 30      SAPS
29 20 GO TO (30,40,50,60,70,80), KDYN      SAPS
30
31 C      S T A T I C   S O L U T I O N      SAPS
32 C      30 IF(MODEX.EQ.0) GO TO 32      SAPS
33 T(6)=T(5)      SAPS
34 S(6)=S(5)      SAPS
35 T(7)=T(5)      SAPS
36 S(7)=S(5)      SAPS
37 IF (IPLT .GT. 0 .AND..NOT.GEOST) CALL SAPLOT(LL,NF,KDYN,NEQB,NEQ      SAPS
38 X,NEBLOCK)      SAPS
39 CALL SECOND(T(8))      SAPS
40 DO 31 I=9,12      SAPS
41 31 T(I)=T(8)      SAPS
42 C      S(I)=S(8)      SAPS
43 00 TO 90      SAPS
44
45 C      32 REWIND 12      SAPS
46 CALL SOLEQ      SAPS
47 CALL SECOND(T(6))      SAPS
48 T(7)=T(8)      SAPS
49 S(7)=S(8)      SAPS
50 IF (IPLT .GT. 0) CALL SAPLOTLL,NF,KDYN,NEQB,NEQ,NEBLOCK)      SAPS
51 CALL SECOND(T(8))      SAPS
52 DO 33 I=9,12      SAPS
53 33 T(I)=T(8)      SAPS
54 C      S(I)=S(8)      SAPS
55 IF (.NOT. GEOST) GO TO 90      SAPS

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# POOR ORIGINAL

PROGRAM SAPS	76/76 OPT=1	FTN 4.5+414	11/19/80 17.24.07
400	1WRITE (6,2000) NUMEL,NELGEO	SAP5	329
	CALL ADDGE0 (A(N1),A(NN2),NE2B,MBAND,NUMEL,NBLOCK,LL,ANORM)	SAP5	330
	GO TO 20	SAP5	331
C		SAP5	332
C	EIGENVALUE EXTRACTION	SAP5	333
405	C 40 T(6) = T(5)	SAP5	334
C	S(6)=S(5)	SAP5	335
	CALL SOLEIG	SAP5	336
	CALL SECOND(T(7))	SAP5	337
410	IF (IPLT .GT. 0) CALL SAPLOT(LL,NF,KDYN,NEQB,NEQ,NBLOCK)	SAP5	338
	CALL SECOND(T(8))	SAP5	339
	T(9) = T(8)	SAP5	340
	T(10)= T(8)	SAP5	341
	T(11)=T(8)	SAP5	342
415	T(12)=T(8)	SAP5	343
C	S(9) = S(8)	SAP5	344
C	S(10)= S(8)	SAP5	345
C	S(11)= S(8)	SAP5	346
C	S(12)= S(8)	SAP5	347
420	GO TO 90	SAP5	348
C		SAP5	349
C	FORCED DYNAMIC RESPONSE ANALYSIS	SAP5	350
C		SAP5	351
425	50 T(6) = T(5)	SAP5	352
C	S(6)=S(5)	SAP5	353
	IF (NDYN.LT.0) GO TO 52	SAP5	354
	CALL SOLEIG	SAP5	355
	CALL SECOND(T(7))	SAP5	356
	CALL SECOND(T(7))	SAP5	357
430	IF (IPLT .GT. 0) CALL SAPLOT(LL,NF,KDYN,NEQB,NEQ,NBLOCK)	SAP5	358
	CALL SECOND(T(8))	SAP5	359
	GO TO 54	SAP5	360
52	DO 53 I=1,7	SAP5	361
53	T([+1])=T([1])	SAP5	362
435	C S([+1])=S([1])	SAP5	363
	REWIND 2	SAP5	364
	READ (2) NEQ,NBLOCK,NEQB,MBAND,N1,NF,(QQQ(I),I=1,NF)	SAP5	365
	REWIND 7	SAP5	366
	I MAX=NEQB+NF	SAP5	367
440	READ (7) (A(I)),I=1,NF	SAP5	368
	DO 56 L=1,NBLOCK	SAP5	369
56	READ (7) (A(I)),I=1,IMAX)	SAP5	370
54	CALL HISTRY	SAP5	371
	CALL SECOND(T(9))	SAP5	372
445	T(10)= T(9)	SAP5	373
	T([1])= T(9)	SAP5	374
	T([2])= T(9)	SAP5	375
C	S([10])= S(9)	SAP5	376
C	S([11])= S(9)	SAP5	377
C	S([12])= S(9)	SAP5	378
450	GO TO 90	SAP5	379
C		SAP5	380
C	RESPONSE SPECTRUM ANALYSIS	SAP5	381
C		SAP5	382
455	60 T(6) = T(5)	SAP5	383
C	S(6) = S(5)	SAP5	384
		SAP5	385

# POOR ORIGINAL

PROGRAM SAPS 76/76 OPT=1

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        IF(NTYP.NE.0) GO TO 105
        IF(NDYN.LT.0) GO TO 62
        CALL SOLEIG
105 CONTINUE
        CALL SECOND(T(7))
        IF (IPLT.GT.0) CALL SAPLOTILL,NF,KDYN,NEQB,NEQ,NBLOCK)
        CALL SECOND(T(8))
        T(9)=T(8)
485      C   S(9)=S(8)
        GO TO 64
62 DO 63 I=1,8
63 T(I+1)=T(I)
        C   S(I+1)=S(I)
470      REWIND 2
        READ (2) NEQ,NBLOCK,NEQB,MBAND,N1,NF
        REWIND 7
        IMAX=NEQB+NF
        READ (7) (A(I),I=1,IMAX)
        DO 66 L=1,NBLOCK
66 READ (7) (A(I),I=1,IMAX)
64 IF(NRSC.LE.0)NRSC=1
        DO 69 KAPG=1,NRSC
69 CALL RESPEC
        CALL SECOND(T(10))
        T(11)=T(10)
        T(12)=T(10)
        C   S(11)=S(10)
        C   S(12)=S(10)
485      GO TO 90
        C   STEP-BY-STEP (DIRECT INTEGRATION) ANALYSIS
        C
        70 DO 71 I=6,10
71 T(I)=T(5)
        C   S(I)=S(5)
        CALL STEP
        CALL SECOND(T(11))
        T(12)=T(10)
        C   S(12)=S(10)
        GO TO 90
        C   FREQUENCY RESPONSE ANALYSIS
        C
500      80 T(8)=T(5)
        C   S(8)=S(5)
        IF (NDYN.LT.0) GO TO 82
        CALL SOLEIG
        CALL SECOND(T(7))
        IF (IPLT.GT.0) CALL SAPLOTILL,NF,KDYN,NEQB,NEQ,NBLOCK)
        CALL SECOND(T(8))
        T(9)=T(8)
        T(10)=T(8)
        T(11)=T(8)
        C   S(9)=S(8)
        C   S(10)=S(8)
        C   S(11)=S(8)
        GO TO 88

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# POOR ORIGINAL

PROGRAM SAPS 78/76 OPT=1

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      82 DO B3 I=1,10          SAPS 442
  515   83 T(I+1)=T(I)          SAPS 443
      C S(I+1)=S(I)          SAPS 444
      REWIND 2          SAPS 445
      READ (2) NEQ,NBLOCK,NEQB,MBAND,N1,NF          SAPS 446
      REWIND 7          SAPS 447
  520   IMAX=NEQB*NF          SAPS 448
      READ (7) (A(I),I=1,NF)          SAPS 449
      DO B6 L=1,NBLOCK          SAPS 450
      86 READ (7) (A(I),I=1,IMAX)          SAPS 451
      88 CALL FREQ          SAPS 452
  525   CALL SECOND(T(12))          SAPS 453
      C
      C COMPUTE AND PRINT OVERALL TIME LOG          SAPS 454
      C
  530   90 TT = 0.0          SAPS 455
      C ST = 0.000          SAPS 456
      GO TO 5          CORR2 6
      DO 95 I=1,11          SAPS 458
      T(I) = T(I+1)-T(I)          SAPS 459
      TT = TT + T(I)          SAPS 460
  535   95 CONTINUE          SAPS 461
      C S(I) = S(I+1)-S(I)          SAPS 462
      C ST = ST + S(I)          SAPS 463
      C
      C WRITE (6,203) (T(K),S(K),K=1,11),TT,ST          SAPS 464
  540   WRITE (6,203) (T(K),K=1,11),TT          SAPS 465
      C
      C GO TO 5          SAPS 466
      C
  545   201 FORMAT (3BHIEQUATION PARAMETERS, //          SAPS 467
      X 34H TOTAL NUMBER OF EQUATIONS    =,15.          SAPS 468
      X /34H BANDWIDTH                =,15.          SAPS 469
      X /34H NUMBER OF EQUATIONS IN A BLOCK =,15.          SAPS 470
      X /34H NUMBER OF BLOCKS        =,15)          SAPS 471
      203 FORMAT (1HI,31HOVERALL TIME LOG, //          SAPS 472
  550   X 5X,30HNODEAL POINT INPUT     =, FB.2 /          SAPS 473
      X 5X,30HELEMENT STIFFNESS FORMATION =, FB.2 /          SAPS 474
      X 5X,30HLOAD INPUT             =, FB.2 /          SAPS 475
      X 5X,30HTOTAL STIFFNESS FORMATION =, FB.2 /          SAPS 476
      X 5X,30HSTATIC ANALYSIS       =, FB.2 /          SAPS 477
      X 5X,30HEIGENVALUE EXTRACTION =, FB.2 /          SAPS 478
      X 5X,30HSTRUCTURE PLOTTING   =, FB.2 /          SAPS 479
      X 5X,30HFORCED RESPONSE ANALYSIS =, FB.2 /          SAPS 480
      X 5X,30HRESPONSE SPECTRUM ANALYSIS =, FB.2 /          SAPS 481
      X 5X,30HSTEP-BY-STEP INTEGRATION =, FB.2 /          SAPS 482
      X 5X,30HRESPONSE ANALYSIS      =, FB.2 //          SAPS 483
      X 5X,30HTOTAL SOLUTION TIME   =, FB.2 /1          SAPS 484
      C
      320 FORMAT (// 47H ** WARNING. ESTIMATE OF STORAGE FOR A DYNAMIC,          SAPS 485
      X 32H ANALYSIS EXCEEDS AVAILABLE CORE,          SAPS 486
  565   X //,* REQUIRED MTOT=*,110)          SAPS 487
      C
      C
      1001 FORMAT (1H15)
  570   2000 FORMAT (1H0TOTAL NUMBER OF ELEMENTS           **,15..          SAPS 488
      X           * NUMBER OF ELEMENTS WITH GEOMETRIC STIFFNESS**,15)          SAPS 489
      2010 FORMAT (*0*** WARNING *** THE AVAILABLE BLANK COMMON (MTOT) IS *,          SAPS 490
      X* TOO SMALL*,/,,* COMPUTED NO. OF EQUATIONS PER BLOCK=*,110,/*          SAPS 491
      X* PROCEED IN THE DATA CHECK MODE WITH NEQB=2*)          SAPS 492
      2020 FORMAT(*0*** FATAL ERROR ** AVAILABLE CORE IS TOO SMALL*          SAPS 493
      1,* FOR PLOTTING, REQUIRED MTOT=*,110/
      2* CONTINUE IN THE DATA CHECK MODE*)          SAPS 494
      2030 FORMAT(*0*** FATAL ERROR ** AVAILABLE CORE IS INADEQUATE*          SAPS 495
      1,* REQUIRED MTOT=*,110/*CONTINUE IN THE DATA CHECK MODE*)          CORR2 7
      END          SAPS 496

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# POOR ORIGINAL

SUBROUTINE REDUC1	78/78 OPT=1	FTN 4.5+IN	11/19/80 17.24.07
1	SUBROUTINE REDUC1 (ID, IDN, NTYP, NUMNP, NRDOF)	REDUC1	2
	DIMENSION MASN(200), MADF(200), ID(NUMNP,6), IDN(NUMNP,6)	CAM	78
	LEVEL 2, ID, IDN	REDUC1	4
5	C INPUT PRIMARY DOF AND REORDER ALL DOF SO THAT PRIMARY DOF ARE FIRST	REDUC1	5
	NTYP = 0 NO REDUCTION	REDUC1	6
	1 EXACT REDUCTION	REDUC1	7
	2 GUYAN REDUCTION	REDUC1	8
	3 MODIFIED GUYAN	REDUC1	9
10	C M1(I) = NODE NUMBER OF I PRIMARY DOF , I=1,NRDOF	REDUC1	10
	C MADF(I) = DOF NUMBER (1 TO 6)	REDUC1	11
	C NRDOF = NUMBER OF PRIMARY DOF < 200	REDUC1	12
	READ(5,300) NTYP	CAM	80
	300 FORMAT(14I5)	REDUC1	14
15	IF(NTYP.GT.1) GO TO 301	REDUC1	15
	WRITE(6,302)	REDUC1	16
	302 FORMAT(5X,1BHNO REDUCTION OF DOF)	REDUC1	17
	RETURN	REDUC1	18
20	301 IF(NTYP.EQ.2) WRITE(6,303)	REDUC1	19
	IF(NTYP.EQ.3) WRITE(6,304)	REDUC1	20
	303 FORMAT(5X,1SHGUYAN REDUCTION)	REDUC1	21
	304 FORMAT(5X,24HMODIFIED GUYAN REDUCTION)	REDUC1	22
	READ(5,300) NRDOF, (MASN(I),MADF(I),I=1,NRDOF)	REDUC1	23
	WRITE(6,305)	REDUC1	24
25	305 FORMAT(20X,1IHPRIAMRY DOF/10X,4HNODE,5X,3HDOF)	REDUC1	25
	WRITE(6,306) (MASN(I),MADF(I),I=1,NRDOF)	REDUC1	26
	306 FORMAT(9X,15,4X,15)	REDUC1	27
	C	REDUC1	28
30	C READ ID FROM TAPE 8 AND WRITE NEW ID ON TAPE 8	REDUC1	29
	REWIND 8	REDUC1	30
	READ(8) ID	REDUC1	31
	IP=1	REDUC1	32
	IS=NRDOF+1	REDUC1	33
35	C*****	REDUC1	34
	WRITE(6,307)	REDUC1	35
	307 FORMAT(5X,1BORIGINAL ID =)	REDUC1	36
	DO 310 I=1,NUMNP	REDUC1	37
	WRITE(6,311) (ID(I,J),J=1,6)	REDUC1	38
40	311 FORMAT(5X,6(2X,15))	REDUC1	39
	DO 312 J=1,6	REDUC1	40
	IF(ID(I,J).EQ.0) GO TO 312	CAM	41
	NODE=MASN(IP)	CAM	81
	MDF=MADF(IP)	CAM	82
	IF(I.EQ.NODE.AND.J.EQ.MDF) GO TO 313	CAM	83
45	C SECONDARY DOF	CAM	84
	IDN(I,J)=IS	CAM	85
	IS=IS+1	CAM	86
	GO TO 312	CAM	87
50	C PRIMARY DOF	CAM	88
	313 IDN(I,J)=IP	CAM	89
	IP=IP+1	CAM	90
	312 CONTINUE	CAM	91
	310 CONTINUE	CAM	92
55	C	REDUC1	93
	WRITE ONTO TAPE 8 AND PRINT NEW ID	REDUC1	58
	C	REDUC1	59
	REWIND 8	REDUC1	60
	WRITE(8) IDN	REDUC1	61
60	WRITE(6,320)	REDUC1	62
	320 FORMAT(5X,BHNEW ID =)	REDUC1	63
	DO 321 I=1,NUMNP	REDUC1	64
	WRITE(6,311) (IDN(I,J),J=1,6)	REDUC1	65
	DO 321 J=1,6	REDUC1	66
	ID(N,J)=IDN(I,J)	REDUC1	67
65	321 CONTINUE	REDUC1	68
	REWIND 8	REDUC1	69
	RETURN	REDUC1	70
	END	REDUC1	71
		REDUC1	72
		REDUC1	73

# POOR ORIGINAL

SUBROUTINE SIZE	78/78 OPT=1	FTN 4.5+4.14	11/19/80 17.24.07
1	SUBROUTINE SIZE(X,XL,MTOT,NEQB,NBLOCK,MBAND)	CAM	94
	COMMON/CONTL/NRDOF,NTYP,NST,MBW,NS,NS1,NS2,NS3,NOT,MBS,MODE,	SIZE	95
	I,MD,MLRED,NSEQ,NTD	SIZE	96
	DIMENSION X(NEQB,MBAND),XL(NEQB)	CAM	97
5	LEVEL 2,X,XL	CAM	98
	C FIND BANDWIDTH OF KSS	SIZE	99
	MB5=0	SIZE	100
	REWIND 4	SIZE	101
	NPP=NRDOF/NEQB	SIZE	102
10	IRW=0	SIZE	103
	IF(NPP.EQ.0) GO TO 1	SIZE	104
	DO 2 I=1,NPP	SIZE	105
	IRW=IRW+NEQB	SIZE	106
	READ(4) ((X(IC,JC),JC=1,NEQB),JC=1,MBAND),(XL(IL),IL=1,NEQB)	CAM	107
15	2 CONTINUE	SIZE	108
	I NF=NRDOF-(IRW+1)	SIZE	109
	NSEC=0	SIZE	110
20	3 READ(4) ((X(IC,JC),JC=1,NEQB),JC=1,MBAND),(XL(IL),IL=1,NEQB)	CAM	111
	NSEC=NSEC+NEQB-NF+1	SIZE	112
	NLST=NEQB	CAM	113
	IF(NSEC.GT.NS) NLST=NEQB-(NSEC-NS)	CAM	114
	IF(NSEC.GT.NS) NSEC=NS	SIZE	115
	DO 4 I=NF,NLST	CAM	116
	MTST=0	CAM	117
25	DO 7 J=1,MBAND	CAM	118
	7 IF(X(I,J).NE.0.) MTST=J	CAM	119
	IF(MTST.GT.MBS) MBS=MTST	CAM	120
	4 CONTINUE	CAM	121
	NF=1	SIZE	122
30	IF(NSEC.NE.NS) GO TO 3	SIZE	123
	REWIND 4	SIZE	124
	NS3=NS+MBS	SIZE	125
	WRITE(6,100) NST,NRDOF,NS,NS1,NS2,NS3,NOT,MBS	SIZE	126
35	100 FORMAT(//5X,26HSUMMARY OF SIZE PARAMETERS/)	SIZE	127
	1 5X,34HNUMBER OF UNRESTRAINED DOF (NST) =,15/	SIZE	128
	2 5X,31HNUMBER OF PRIMARY DOF (NRDOF) =,15/	SIZE	129
	3 5X,30HNUMBER OF SECONDARY DOF (NS) =,15/	SIZE	130
	4 5X,19HSIZE OF KPP (NS1) =,15/	SIZE	131
	5 5X,19HSIZE OF KPS (NS2) =,15/	SIZE	132
40	6 5X,19HSIZE OF KSS (NS3) =,15/	SIZE	133
	7 5X,18HSIZE OF TD (NOT) =,15/	SIZE	134
	8 5X,18HBANDWIDTH OF KSS =,15)	SIZE	135
	C DETERMINE BAND SIZE OR KSS	SIZE	136
	C FOR STIFF	SIZE	137
45	NSEQ=(MTOT-NS1-NS2-NEQB-MBW)/MBS	SIZE	138
	C FOR KSS-1	SIZE	139
	NDUM=(MTOT-NS1-NS2-3*NST)/(2*MBS)	SIZE	140
	IF(NDUM.LT.NSEQ) NSEQ=NDUM	SIZE	141
	IF(NSEQ.OE.NS) GO TO 5	SIZE	142
50	MLRED=NS/NSEQ+1	SIZE	143
	GO TO 6	SIZE	144
	5 NSEQ=NS	SIZE	145
	MLRED=1	SIZE	146
55	6 NT0=(MTOT-2*NS1-NS2-NS-NST-NRDOF)/NRDOF	SIZE	147
	IF(NT0.GT.NS) NT0=NS	SIZE	148
	WRITE(6,101) MLRED,NSEQ,NT0	SIZE	149
60	101 FORMAT(//5X,21HNO OF BLOCKS IN KSS =,15/	SIZE	150
	1 5X,24HNO OF EQUATIONS IN KSS =,15/	SIZE	151
	2 5X,23HNO OF EQUATIONS IN TD =,15/	SIZE	152
	NA4=2*NS1+NRDOF*(MD+NRDOF+1+MD)	SIZE	153
	WRITE(6,102) NA4	SIZE	154
65	102 FORMAT(5X,27HSIZE OF A DURING EIGENSOL =,15)	SIZE	155
	RETURN	SIZE	156
	END	SIZE	157

SUBROUTINE STIFF 78/78 OPT=1 FTN 4.5+414 11/19/80 17.24.07

```

1      SUBROUTINE STIFF(XKPP,XKPS,XKSS,X,XL,NBLOCK,NEQB,NRDOF,NTYP,NST,
1      MBLK,NS,NS1,NS2,NS3,NOT,MBS,MODE,MD,MBLRED,NSEQ,NT0)
1      DIMENSION XKPP(NS1),XKPS(NS2),XKSS(NSEQ,MBS),X(NEQB,MBS),XL(NEQB)
1      LEVEL 2,XKPP,XKPS,XKSS,X,XL
5      C      FORM KPP KPS,KSS FROM TOTAL STIFFNESS ON TAPE 4 WRITE KSS ON
5      C      TAPE 16 IN BLOCKS
5      C
10     REWIND 16
10     ILPP=0
10     ILPS=0
10     IRS=0
10     IBS=0
10     IROW=0
15     DO 1 I=1,NBLOCK
15     READ(4) ((X(IC,JC),IC=1,NEQB),JC=1,MBS),(XL(IL),IL=1,NEQB)
15     DO 2 I=1,NEQB
15     IROW=IROW+1
15     IF(IROW.GT.NST) GO TO 2
15     IF(IROW.GT.NRDOF) GO TO 3
20     C      KPP OR KPS
20     DO 4 J=1,MBS
20     IF(J+IROW-1.GT.NRDOF) GO TO 5
25     C      KPP
25     ILPP=ILPP+1
25     XKPP(ILPP)=X(1,J)
25     GO TO 4
30     C      KPS
30     5 ILPS=ILPS+1
30     XKPS(ILPS)=X(1,J)
30     4 CONTINUE
30     GO TO 2
35     C      KSS
35     3 IRS=IRS+1
35     DO 6 J=1,MBS
35     XKSS(IRs,J)=X(1,J)
35     IF(IRs.LT.NSEQ) GO TO 2
35     WRITE(16) XKSS
40     IRS=0
40     2 CONTINUE
40     1 CONTINUE
40     REWIND 16
40     RETURN
45     END

```

## POOR ORIGINAL

SUBROUTINE MASS 78/78 OPT=1 FTN 4.5+414 11/19/80 17.24.07

```

1      SUBROUTINE MASS(XM,NEQB,NBLOCK,NST)
1      DIMENSION XM(NST)
1      LEVEL 2,XM
5      C      LOAD MASS VECTOR
5      C
5      REWIND 9
5      IROW=1
5      DO 1 I=1,NBLOCK
5      JLST=IROW+NEQB-1
5      IF(JLST.GT.NST) JLST=NST
5      READ(9) (XM(J),J=IROW,JLST)
5      IROW=JLST+1
10     1 CONTINUE
10     REWIND 9
10     RETURN
10     END

```

# POOR ORIGINAL

SUBROUTINE GAUS	76/78 OPT=1	FTN 4.5+414	11/19/80 17.24.07
1	SUBROUTINE GAUS(A1,A2,B,G,NSB,NST,MBS, NBLK,NT1,NT2,NT3,INDEX)	GAUS	2
	DIMENSION A1(NSB,MBS),A2(NSB,MBS),B(NST),G(NST)	GAUS	3
	LEVEL 2,A1,A2,B,G	GAUS	4
5	C INDEX = 1 FIND INVERSE OF A (ON TAPE NT1) STORE RESULTS ON TAPE NT3	GAUS	5
	C 2 FACTOR A STORE RESULTS ON TAPE NT2	GAUS	6
	C 3 SOLVE A X = B ENTER WITH 2 FIRST TO FACTOR A	GAUS	7
	IF (INDEX.EQ.3) GO TO 1	GAUS	8
10	C FACTOR A	GAUS	9
	C REWIND NT1	GAUS	10
	REWIND NT2	GAUS	11
15	N1=MBS-1	GAUS	12
	N2=NSB-MBS+1	GAUS	13
	IBMX=NBLK-1	GAUS	14
	IF(NBLK.EQ.1) IBMX=1	GAUS	15
	DO 10 IB=1,IBMX	GAUS	16
20	IF(IB.EQ.1) READ(NT1) A1	GAUS	17
	IF(NBLK.NE.1) READ(NT1) A2	GAUS	18
	JSTRT=2	GAUS	19
	IF((IB.GT.1) JSTRT=JLST+1	GAUS	20
	JLST=2,*NSB	GAUS	21
25	IF((IB.GT.1) JLST=JSTRT+N1+N2-1	GAUS	22
	IF(JLST.GT.NST) JLST=NST	GAUS	23
	DO 11 J=JSTRT,JLST	GAUS	24
	MSK=J-MBS	GAUS	25
	IF(MSK.LT.1) MSK=1	GAUS	26
30	CALL LOCATE(MSK,J,A,A1,A2,MBS,NSB,IB)	GAUS	27
	G(MSK)=A	GAUS	28
	IF=MSK+1	GAUS	29
	IL=J-1	GAUS	30
	DO 12 I=IF,IL	GAUS	31
35	CALL LOCATE(I,J,A,A1,A2,MBS,NSB,IB)	GAUS	32
	G(I)=A	GAUS	33
	KF=MSK	GAUS	34
	IF((I-MBS.GT.KF) KF=I-MBS	GAUS	35
	KL=I-1	GAUS	36
40	IF(KF.GT.KL) GO TO 12	GAUS	37
	DO 13 K=KF,KL	GAUS	38
	CALL LOCATE(K,I,A,A1,A2,MBS,NSB,IB)	GAUS	39
45	13 G(I)=G(I)-A*G(K)	GAUS	40
	12 CONTINUE	GAUS	41
	IF=J-MBS	GAUS	42
	IF((IF.LT.1) IF=1	GAUS	43
	IL=J-1	GAUS	44
	DO 14 I=IF,IL	GAUS	45
50	KA=I-(IB-1)*NSB	GAUS	46
	IF(KA.GT.NSB) GO TO 15	GAUS	47
	LA=J-(IB-1)*NSB-KA+1	GAUS	48
	IF(LA.GT.MBS) GO TO 14	GAUS	49
	A1(KA,LA)=G(I)/A1(KA,1)	GAUS	50
	GO TO 14	GAUS	51
55	15 KA=I-(IB-MBS	GAUS	52
	LA=J-(IB-MBS-KA+1)	GAUS	53
	IF(LA.GT.MBS) GO TO 14	GAUS	54
	A2(KA,LA)=G(I)/A2(KA,1)	GAUS	55
		CAM	119
		GAUS	56

# POOR ORIGINAL

SUBROUTINE GAUS	78/78 OPT=1	FTN 4.5+414	11/19/80 17.24.07
	14 CONTINUE	GAUS	57
60	KF=J-MBS	GAUS	58
	IF(KF.LT.1) KF=1	GAUS	59
	KL=J-1	GAUS	60
	KA=J-(IB-1)*NSB	GAUS	61
	DO 18 K=KF, KL	GAUS	62
65	CALL LOCATE(K, J, A, A1, A2, MBS, NSB, IB)	GAUS	63
	IF(KA.GT.NSB) GO TO 17	GAUS	64
	A1(KA,1)=A1(KA,1)-A*G(K)	GAUS	65
	DO 18	GAUS	66
17	KA=J-IB*NSB	GAUS	67
	A2(KA,1)=A2(KA,1)-A*G(K)	GAUS	68
70	16 CONTINUE	GAUS	69
	11 CONTINUE	GAUS	70
	WRITE(NT2) A1	GAUS	71
	IF(NBLK.EQ.1) GO TO 10	GAUS	72
	DO 18 IT=1, NSB	GAUS	73
75	DO 18 JT=1, MBS	GAUS	74
	18 A1(IT, JT)=A2(IT, JT)	GAUS	75
10	10 CONTINUE	GAUS	76
	REWIND NT1	GAUS	77
	REWIND NT2	GAUS	78
80	IF(INDEX.EQ.2) RETURN	GAUS	79
	REWIND NT3	GAUS	80
C	SET UP LOAD FOR INVERT STORE RESULTS ON TAPE NT3	GAUS	81
	DO 20 ICOL=1, NST	GAUS	82
	DO 21 J=1, NST	GAUS	83
85	21 B(J)=0.	GAUS	84
	B(ICOL)=1.	GAUS	85
	DO TO 1	GAUS	86
22	WRITE(NT3) B	GAUS	87
20	CONTINUE	GAUS	88
90	REWIND NT3	GAUS	89
	RETURN	GAUS	90
C	SOLVE A X = B HAVING FACTORED A ON TAPE NT2 RETURN SOLUTION IN B	GAUS	91
C		GAUS	92
95	C REDUCE B	GAUS	93
I	G(1)=B(1)	GAUS	94
	IBMX=NBLK-1	GAUS	95
	IF(NBLK.EQ.1) IBMX=1	GAUS	96
	DO 30 IB=1, IBMX	GAUS	97
100	IF(IB.EQ.1) READ(NT2) A1	GAUS	98
	IF(NBLK.NE.1) READ(NT2) A2	GAUS	99
	JSTRT=2	GAUS	100
	IF(IB.GT.1) JSTRT=JSTRT+1	GAUS	101
	JLST=2.*NSB	GAUS	102
105	IF((IB.GT.1) JLST=JSTRT+N1+N2-1	GAUS	103
	IF((JLST.GT.NST) JLST=NST	GAUS	104
	DO 31 I=JSTRT, JLST	GAUS	105
	G(I)=B(I)	GAUS	106
	KS=1-MBS	GAUS	107
110	IF(KS.LT.1) KS=1	GAUS	108
	KL=I-1	GAUS	109
	DO 32 K=KS, KL	GAUS	110
	CALL LOCATE(K, I, A, A1, A2, MBS, NSB, IB)	GAUS	111
32	G(I)=G(I)-A*G(K)	GAUS	112
		GAUS	113

# POOR ORIGINAL

SUBROUTINE GAUS	75/75 OPT=1	FTN 4.5+414	11/19/80 17.24.07
115	31 CONTINUE	GAUS	114
	DO 33 IT=1,NSB	GAUS	115
	DO 33 JT=1,MBS	GAUS	116
	33 A1(IT,JT)=A2(IT,JT)	GAUS	117
120	30 CONTINUE	GAUS	118
C	BACKSUBSTITUTE	GAUS	119
	REWIND NT2	GAUS	120
	IROW=0	GAUS	121
	DO 40 IB=1,NBLK	GAUS	122
	READ(NT2) A1	GAUS	123
125	IMX=NSB	GAUS	124
	IF((IROW+IMX.GT.NST)) IMX=NST	GAUS	125
	IFST=IROW+1	GAUS	126
	ILST=IROW+NSB	GAUS	127
	DO 41 I=IFST,ILST	GAUS	128
130	41 G(I)=G(I)/A1(I-IROW,1)	GAUS	129
	IROW=IROW+ILST-IFST+1	GAUS	130
	40 CONTINUE	GAUS	131
	REWIND NT2	GAUS	132
	B(NST)=G(NST)	GAUS	133
135	IB=NBLK-1	GAUS	134
	IF((IB.LT.1)) IB=1	GAUS	135
	47 IDUM=IB-1	GAUS	136
	IF((IDUM.LE.0)) GO TO 42	GAUS	137
	DO 43 I=1, IDUM	GAUS	138
140	43 READ(NT2) A1	GAUS	139
	42 READ(NT2) A1	GAUS	140
	IF((NBLK.EQ.1)) GO TO 44	GAUS	141
	READ(NT2) A2	GAUS	142
145	44 ISRT=(IB-1)*NSB+NI+1	GAUS	143
	ILST=ISRT+NI+2*M2	GAUS	144
	IF((ILST.GT.NST)) ILST=NST	GAUS	145
	IF((IB.EQ.1)) ISRT=2	GAUS	146
	NI=ILST-ISRT+1	GAUS	147
	DO 45 I=1,NI	GAUS	148
150	IROW=ILST-1+1	GAUS	149
	KS=IROW-MBS	GAUS	150
	IF((KS.LT.1)) KS=1	GAUS	151
	KL=IROW-1	GAUS	152
	DO 46 K=KS,KL	GAUS	153
155	CALL LOCATE(K,IROW,A,A1,A2,MBS,NSB,IB)	GAUS	154
	46 G(K)=G(K)-A*B(IROW)	GAUS	155
	B(IROW-1)=0(IROW-1)	GAUS	156
	45 CONTINUE	GAUS	157
160	IB=IB-1	GAUS	158
	REWIND NT2	GAUS	159
	IF((IB.GT.0)) GO TO 47	GAUS	160
	IF((INDEX.EQ.1)) GO TO 22	GAUS	161
	RETURN	GAUS	162
	END	GAUS	163

# POOR ORIGINAL

SUBROUTINE LOCATE	78/78 OPT=1	FTN 4.5+414	11/19/80	17.24.07
1	SUBROUTINE LOCATE(I,J,A,A1,A2,MBS,NSB,IB)		LOCATE	2
	DIMENSION A1(NSB,MBS),A2(NSB,MBS)		LOCATE	3
	LEVEL 2,A1,A2		LOCATE	4
5	ILOW=(IB-1)*NSB		LOCATE	5
	IHIGH=IB*NSB		LOCATE	6
	INDEX=3		LOCATE	7
	IF(I.GT.ILOW.AND.I.LE.IHIGH) INDEX=1		LOCATE	8
	ISHG=(IB+1)*NSB		LOCATE	9
10	IF(I.GT.IHIGH.AND.I.LE.ISHG) INDEX=2		LOCATE	10
	GO TO (1,2,3),INDEX		LOCATE	11
	I=I-(IB-1)*NSB		LOCATE	12
	L=J-(IB-1)*NSB-K+1		LOCATE	13
	IF(L.GT.MBS) GO TO 5		CAM	120
15	A=A1(K,L)		LOCATE	14
	RETURN		LOCATE	15
20	2 K=I-IB*NSB		LOCATE	16
	L=J-IB*NSB-K+1		LOCATE	17
	IF(L.GT.MBS) GO TO 5		CAM	121
	A=A2(K,L)		LOCATE	18
	RETURN		CAM	122
25	5 A=0.		LOCATE	19
	RETURN		CAM	123
	3 WRITE(6,4) I,J,IB		LOCATE	20
	4 FORMAT(//5X,39HEMOR IN LOCATE ELLMENT NOT IN A1 OR A2/		LOCATE	21
	I 10X,3H1 = .15/10X,3H1 = .15/10X,4H1B = .15)		STOP	22
	END		LOCATE	23
			LOCATE	24

SUBROUTINE PPLOC	78/78 OPT=1	FTN 4.5+414	11/19/80	17.24.07
1	SUBROUTINE PPLOC(IR,JC,ILOC)		PPLOC	2
	COMMON/CONTL/ NDOF,NTYP,NST,MBW,NS,NS1,NS2,NS3,NOT,MBS,MODE,		PPLOC	3
	1 MD,MBLRED,MSEQ,NT0		PPLOC	4
5	C LOCATES I,J IN VECTOR STORING KPP		PPLOC	5
	IDR=IR		PPLOC	6
	JDC=JC		PPLOC	7
	IF((IR.LT.JC) GO TO 3		PPLOC	8
	IDR=JC		PPLOC	9
	JDC=IR		PPLOC	10
10	3 CONTINUE		PPLOC	11
	NL=NDOF		PPLOC	12
	ILOC=0		PPLOC	13
	IS=IDR-1		PPLOC	14
	IF((IS.EQ.0) GO TO 1		PPLOC	15
15	DO 2 I=1,IS		PPLOC	16
	ILOC=ILOC+NL		PPLOC	17
2	NL=NL-1		PPLOC	18
	I ILOC=ILOC+JDC-IDR+1		PPLOC	19
	RETURN		PPLOC	20
20	END		PPLOC	21

# POOR ORIGINAL

SUBROUTINE REDUCE 78/78 OPT=1

FTN 4.5+414

11/19/80 17.24.07

```

1      SUBROUTINE REDUCE(XKPP,XKPS,XM,XMR,XSS,T1,T0,
1                         NDOF,NTYP,NST,MW,NS,NS1,NS2,NS3,NOT,MBS,MODE).
2      ND,MBL,RED,NSEQ,NT0)
3      DIMENSION XKPP(NS1),XKPS(NS2),XM(NST),XMR(NS1),XSS(NS),T1(NDOF),
4      T0(NT0,NDOF)
5      LEVEL 2,XKPP,XKPS,XM,XMR,XSS,T1,T0
C      FORM T0 = XSS-1 * KSP ONE BLOCK AT A TIME AND STORE ON 1B
C
10     NBL=NS/NT0+1
11     IF(NS.EQ.NT0) NBL=1
12     REWIND 1B
13     REWIND 17
14     DO 1 IB=1,NBL
15     DO 2 I=1,NT0
16     DO 2 J=1,NDOF
2      T0(I,J)=0.
17     DO 3 I=1,NT0
18     IROW=(IB-1)*NT0+1
19     IF(IROW.GT.NS) GO TO 3
20     READ(17) XSS
21     DO 5 J=1,NDOF
22     DO 5 K=1,NS
23     CALL PSLOC(2,K,J,LOC)
24     IF(LOC.EQ.0) GO TO 5
25     T0(I,J)=T0(I,J)+XSS(K)*XKPS(LOC)
5      CONTINUE
3      CONTINUE
1      WRITE(1B) T0
26     REWIND 17
27     REWIND 1B
C      FORM T1 = XSS-1 * MSS + T0 ONE ROW AT A TIME AND SAVE ON 16
C
35     REWIND 1B
36     DO 10 I=1,NS
37     READ(17) XSS
38     DO 11 IM=1,NS
39     XSS(IM)=XSS(IM)*XM(IM+NDOF)
40     DO 12 J=1,NDOF
41     T1(J)=0.
42     DO 13 IB=1,NBL
43     READ(1B) T0
44     DO 14 J=1,NDOF
45     DO 14 K=1,NT0
46     KROW=(IB-1)*NT0+K
47     IF(KROW.GT.NS) GO TO 14
48     T1(J)=T1(J)+XSS(KROW)*T0(K,J)
49     T1(J)=T1(J)+XSS(KROW)*T0(K,J)
50     14 CONTINUE
51     13 CONTINUE
52     WRITE(1B) T1
53     REWIND 1B
54     10 CONTINUE
55     REWIND 1B
56     REWIND 17
C      COMPUTE KPP = KPP - KPS + T0 AND STORE IN KPP
      
```

# POOR ORIGINAL

SUBROUTINE	REDUCE	76/78	OPT=1	FTN 4.5+414	11/19/80	17.24.07
	C					
60	DO 20 I=1,NBL READ(18) T0 DO 21 K=1,NRDOF DO 21 J=K,NRDOF CALL PPLLOC(K,J,ILOC) DO 22 L=1,NTO	55 56 57 58 59 60 61 62 63 64 65 66 67 68 69 70 71 72 73 74 75 76 77 78 79 80 81 82 83 84 85 86 87 88 89 90 91 92 93 94 95 96 97 98 99 100 101 102 103 104 105 106 107 108 109 110 111 112 113 114 115 116 117 118 119 120 121 122 123 124 125 126 127 128 129 130 131 132 133 134 135				
65	LROW=(I8-1)*NTO+L IF(LROW.GT.NS) GO TO 22 CALL PSLOC(1,K,LROW,INDEX) IF(INDEX.EQ.0) GO TO 22 XCPP(ILOC)=XCPP(ILOC)-XKPS(INDEX)*T0(L,J)					
70	22 CONTINUE 21 CONTINUE 20 CONTINUE REWIND 18					
75	C COMPUTE M* = MPP + KPS * T1 AND STORE IN XMR C					
80	DO 30 I=1,NS1 30 XMR(I)=0. DO 31 I=1,NRDOF CALL PPLLOC(1,I,ILOC) 31 XMR(ILOC)=XMR(I)					
85	DO 32 L=1,NS READ(18) T1 DO 33 K=1,NRDOF DO 33 J=K,NRDOF CALL PSLOC(1,K,L,INDEX) IF(INDEX.EQ.0) GO TO 33 CALL PPLLOC(K,J,ILOC) XMR(ILOC)=XMR(ILOC)+XKPS(INDEX)*T1(J)					
90	33 CONTINUE 32 CONTINUE REWIND 18 RETURN END					

	SUBROUTINE PSLOC	78/78 OPT=1	FTN 4.5+414	11/19/80 17.24.07
1	SUBROUTINE PSLOC(IT,I,J,ILOC)		PSLOC	2
	COMMON/CONT1/ NDOF,NTYP,NST,MBW,NS,NS1,NS2,NS3,NCT,MB5,MODE,		PSLOC	3
	I,MD,MBLRED,NSEQ,NTD		PSLOC	4
5	C LOCATE I,J COMPONENT OF KPS IN A IF IT=1 OR J,I COMPONENT		PSLOC	5
	C OF KSP IF IT=2 ...ILOC IS LOCATION OF COMPONENT IN A..IF		PSLOC	6
	C ILOC=0 COMPONENT IS 0		PSLOC	7
	C		PSLOC	8
	ILOC=0		PSLOC	9
10	IF(IT.EQ.2) GO TO 1		PSLOC	10
	IR=I		PSLOC	11
	JC=J		PSLOC	12
	GO TO 2		PSLOC	13
15	I IR=J		PSLOC	14
	JC=I		PSLOC	15
20	2 IF(NDOF.GT.MBW) GO TO 10		PSLOC	16
	IF(JC.GT.MBW-NDOF+IR-1) RETURN		PSLOC	17
	IL=MBW-NDOF+1		PSLOC	18
	IF(IR.EQ.1) GO TO 4		PSLOC	19
25	IN=IR-1		PSLOC	20
	DO 3 IS=1,IN		PSLOC	21
	IL=IL+1		PSLOC	22
30	3 ILOC=ILOC+IL		PSLOC	23
	4 ILOC=ILOC+JC		PSLOC	24
	RETURN		PSLOC	25
35	C		PSLOC	26
	10 IF((IR.LT.NDOF-MBW+1)) RETURN		PSLOC	27
	IF(JC.GT.(IR-NDOF+MBW)) RETURN		PSLOC	28
	KMIN=NDOF-MBW+1		PSLOC	29
40	IF(IR.LE.KMIN) GO TO 11		PSLOC	30
	IN=IR-1		PSLOC	31
	IL=0		PSLOC	32
	DO 12 IS=KMIN,IN		PSLOC	33
	IL=IL+1		PSLOC	34
45	12 ILOC=ILOC+IL		PSLOC	35
	11 ILOC=ILOC+JC		PSLOC	36
	RETURN		PSLOC	37
	END		PSLOC	38
			PSLOC	39

## POOR ORIGINAL

	SUBROUTINE GRAMS	78/78 OPT=1	FTN 4.5+414	11/19/80 17.24.07
1	SUBROUTINE GRAMS(XM,EV,B,DUM,IM,NS1,NDOF,MODE)		CAM	309
	DIMENSION EV(NDOF,MODE),B(NDOF),DUM(NDOF),ALPH(20),XM(NS1)		CAM	310
	LEVEL 2,XM,EV,B,DUM		CAM	311
5	C COMPUTE NEW B ORTHOGONAL TO ALL LOWER MODES		CAM	312
	C BMCH=BOLD - SUM ( (EV(I)*M * BOLD) * EV )		CAM	313
	C		CAM	314
	M=IM-1		CAM	315
10	DO 1 I=1,M		CAM	316
	DO 2 J=1,NDOF		CAM	317
2	DUM(J)=0.		CAM	318
	DO 3 J=1,NDOF		CAM	319
	DO 3 K=1,NDOF		CAM	320
	CALL PPLOC(J,K,ILOC)		CAM	321
15	3 DUM(J)=DUM(J)+XM(ILOC)*B(K)		CAM	322
	ALPH(I)=0.		CAM	323
	DO 4 J=1,NDOF		CAM	324
4	4 ALPH(I)=ALPH(I)+EV(J,I)*DUM(J)		CAM	325
	1 CONTINUE		CAM	326
20	DO 5 J=1,NDOF		CAM	327
	DO 5 I=1,M		CAM	328
5	5 B(J)=B(J)-ALPH(I)*EV(J,I)		CAM	329
	RETURN		CAM	330
	END		CAM	331
			CAM	332

# POOR ORIGINAL

SUBROUTINE	INVERT	78/78	OPT=1	FTN 4.5+414	11/19/80	17.24.07
1						
	SUBROUTINE INVERT(N,MB,A,B,JS,INDEX)			INVERT	2	
	DIMENSION A(N,MB),MS(1000),Q(1000),B(N)			INVERT	3	
	LEVEL 2,A,B			INVERT	4	
5	C			INVERT	5	
	PROGRAM TO OBTAIN A-1 USING GAUSS ELIMINATION AS OUTLINED			INVERT	6	
	C			INVERT	7	
	IN BATHE-WILSON PAGES 246-250....A STORED IN BANDED FORM			INVERT	8	
	C			INVERT	9	
	N EQUATIONS MB BANDWIDTH...INVERSE STORED BANDED IN B			INVERT	10	
10	C			INVERT	11	
	INDEX= 1 FIND INVERSE OF A ONE COL (JS) AT A TIME			INVERT	12	
	C			INVERT	13	
	= 2 FACTOR A			INVERT	14	
	C			INVERT	15	
	= 3 FIND SOL A X = B...ENTER WITH 2 FIRST TO FACTOR A			INVERT	16	
	IF(INDEX.EQ.3) GO TO 21			INVERT	17	
	IF(JS.GT.1.AND.INDEX.EQ.1) GO TO 15			INVERT	18	
	IF(N.LE.1000) GO TO 16			INVERT	19	
15	15 WRITE(B,100) N			INVERT	20	
	100 FORMAT(//5X,27HNO OF EQUATIONS IN INVERT =,15,2X			INVERT	21	
	1 ,12HEXCEEDS 1000)			INVERT	22	
	STOP			INVERT	23	
20	16 CONTINUE			INVERT	24	
	C			INVERT	25	
	CALC SKYLINE FOR ALL COLUMNS			INVERT	26	
	DO 1 J=1,MB			INVERT	27	
	1 MS(J)=1			INVERT	28	
	JC=KB+1			INVERT	29	
	DO 2 J=JC,N			INVERT	30	
25	2 MS(J)=MS(J-1)+1			INVERT	31	
	C			INVERT	32	
	REDUCE A			INVERT	33	
	DO 5 J=2,N			INVERT	34	
	JLOC=J-MS(J)+1			INVERT	35	
	G(MS(J))=A(MS(J),JLOC)			INVERT	36	
30	IS=MS(J)+1			INVERT	37	
	IL=J-1			INVERT	38	
	IF(IS.GT.IL) GO TO 10			INVERT	39	
	DO 6 I=IS,IL			INVERT	40	
	ILOC=J-1+1			INVERT	41	
35	G(I)=A(I,ILOC)			INVERT	42	
	MM=MS(J)			INVERT	43	
	IF(MS(J).GT.MM) MM=MS(J)			INVERT	44	
	KL=I-1			INVERT	45	
	IF(MM.GT.KL) GO TO 6			INVERT	46	
40	DO 7 KR=MM,KL			INVERT	47	
	7 G(I)=G(I)-A(KR,I-KR+1)*G(KR)			INVERT	48	
	8 CONTINUE			INVERT	49	
	10 CONTINUE			INVERT	50	
45	C			INVERT	51	
	IS=MS(J)			INVERT	52	
	IF((IS.GT.IL) GO TO 11			INVERT	53	
	DO 8 I=IS,IL			INVERT	54	
	ILOC=J-1+1			INVERT	55	
50	8 A(I,ILOC)=G(I)/A(I,I)			INVERT	56	
	C			INVERT	57	
	11 CONTINUE			INVERT	58	
	IF((IS.GT.IL) GO TO 5					
	DO 9 KR=IS,IL					
55	9 A(I,J)=A(I,J)-A(KR,J-KR+1)*G(KR)					
	C					
	5 CONTINUE					
	IF((INDEX.EQ.2) RETURN					

# POOR ORIGINAL

SUBROUTINE	INVERT	76/78	OPT=1	FTN 4.5+MIN	11/19/80	17.24.07
	21 IF((INDEX.EQ.3) GO TO 27			INVERT	59	
60	C FORM INVERSE			INVERT	60	
	15 CONTINUE			INVERT	61	
	J=JS			INVERT	62	
	C REDUCE RMS			INVERT	63	
	DO 23 I=1,N			INVERT	64	
65	23 B(I)=0.			INVERT	65	
	B(J)=1.			INVERT	66	
	22 G(I)=B(I)			INVERT	67	
	DO 41 I=2,N			INVERT	68	
	G(I)=B(I)			INVERT	69	
	KS=MS(1)			INVERT	70	
70	KL=I-1			INVERT	71	
	IF(KS.GT.KL) GO TO 41			INVERT	72	
	DO 42 K=KS,KL			INVERT	73	
	ILOC=I-K+1			INVERT	74	
	42 G(I)=G(I)-A(K,ILOC)*G(K)			INVERT	75	
75	41 CONTINUE			INVERT	76	
	C BACKSUBSTITUTE TO GENERATE J COL OF A-1 FROM N TO 1			INVERT	77	
	DO 49 I=1,N			INVERT	78	
	49 G(I)=G(I)/A(1,1)			INVERT	79	
	B(N)=G(N)			INVERT	80	
80	IT=N-1			INVERT	81	
	IF(IT.EQ.0) GO TO 40			INVERT	82	
	DO 45 IN=1,IT			INVERT	83	
	I=N+1-IN			INVERT	84	
	KS=MS(1)			INVERT	85	
85	KL=I-1			INVERT	86	
	DO 46 K=KS,KL			INVERT	87	
	46 G(K)=G(K)-A(K,I-K+1)*B(I)			INVERT	88	
	B(I-1)=G(I-1)			INVERT	89	
	45 CONTINUE			INVERT	90	
90	40 CONTINUE			INVERT	91	
	RETURN			INVERT	92	
	END			INVERT	93	

# POOR ORIGINAL

SUBROUTINE EXPAND	78/78 OPT=1	FTN 4.5+414	11/19/80	17.24.07
1	SUBROUTINE EXPAND(EV,T0,T1,ESAP,NRDOF,MD,MODE,NT0,NEQB,NBLOCK,	CAM	131	
	I NTYP,NS,XM,NST)	CAM	132	
	COMMON/EVAL/EF(20)	CAM	133	
	DIMENSION EV(NRDOF,MD),T0(NT0,NRDOF),T1(NRDOF),ESAP(NEQB,MODE)	CAM	134	
5	I ,XM(NST),XMTOT(20)	CAM	135	
	LEVEL 2,EV,T0,T1,ESAP,XM	CAM	136	
	C EXPAND MODES AND WRITE ONTO 7 IN NBLOCKS	CAM	137	
	C	CAM	138	
10	C STORE TO ONE ROW AT A TIME ON 17	CAM	139	
	REWIND 18	CAM	140	
	REWIND 17	CAM	141	
	NBL=NS/NT0+1	CAM	142	
15	IF(NS.EQ.NT0) NBL=1	CAM	143	
	NROW=0	CAM	144	
	DO 1 IB=1,NBL	CAM	145	
	READ(1B) T0	CAM	146	
20	DO 2 I=1,NT0	CAM	147	
	IF(NROW+I.GT.NS) GO TO 2	CAM	148	
	WRITE(17) (T0(I,J),J=1,NRDOF)	CAM	149	
	2 CONTINUE	CAM	150	
	I NROW=NROW+NT0	CAM	151	
25	C EXPAND MODES	CAM	152	
	C	CAM	153	
	REWIND 16	CAM	154	
	REWIND 17	CAM	155	
	REWIND 18	CAM	156	
30	NROW=0	CAM	157	
	DO 10 IB=1,NBLOCK	CAM	158	
	DO 12 IRW=1,NEQB	CAM	159	
	DO 15 IFR=1,MODE	CAM	160	
35	16 ESAP(IRW,IFR)=0.	CAM	161	
	IF(NROW+IRW.GT.NS+NRDOF) GO TO 12	CAM	162	
	IF(NROW+IRW.GT.NRDOF) GO TO 13	CAM	163	
	DO 20 IFR=1,MODE	CAM	164	
40	20 ESAP(IRW,IFR)=EV(IRW+NROW,IFR)	CAM	165	
	GO TO 12	CAM	166	
	13 READ(17) (T0(I,J),J=1,NRDOF)	CAM	167	
	DO 14 L=1,NRDOF	CAM	168	
	DO 14 IFR=1,MODE	CAM	169	
45	14 ESAP(IRW,IFR)=ESAP(IRW,IFR)-T0(I,L)*EV(L,IFR)	CAM	170	
	IF(NTYP.EQ.2) GO TO 12	CAM	171	
	READ(1B) T1	CAM	172	
	DO 15 L=1,NRDOF	CAM	173	
	DO 15 IFR=1,MODE	CAM	174	
	EIV=(EF(IFR)*5.28)**2.	CAM	175	
50	15 ESAP(IRW,IFR)=ESAP(IRW,IFR)-EIV*T1(L)*EV(L,IFR)	CAM	176	
	12 CONTINUE	CAM	177	
	10 NROW=NROW+NEQB	CAM	178	
	C NORMALIZE VECTORS AND WRITE ON 7	CAM	179	
	DO 30 I=1,NST	CAM	180	
55	30 READ(15) XM(I)	CAM	181	
	REWIND 15	CAM	182	
	REWIND 1B	CAM	183	

POOR ORIGINAL

SUBROUTINE EXPAND	76/76	OPT=1	FTN 4.5+414	11/19/80	17.24.07
	DO 31 I=1,MODE			CAM	188
60	31 XMTOT(1)=0.			CAM	189
	IROM=0			CAM	190
	DO 32 IB=1,NBLOCK			CAM	191
	READ(18) ESAP			CAM	192
	DO 33 JM=1,MODE			CAM	193
	DO 33 I=1,NEQB			CAM	194
65	NROM=IROM+1			CAM	195
	IF(NROM.GT.NST) GO TO 33			CAM	196
	XMTOT(JM)=XMTOT(JM)+ESAP(1,JM)*ESAP(1,JM)*XM(NROM)			CAM	197
	33 CONTINUE			CAM	198
	IROM=IROM+NEQB			CAM	199
70	32 CONTINUE			CAM	200
	REWIND 18			CAM	201
	IROM=0			CAM	202
	DO 34 IB=1,NBLOCK			CAM	203
	READ(18) ESAP			CAM	204
75	DO 35 JM=1,MODE			CAM	205
	DO 35 I=1,NEQB			CAM	206
	IF(IROM+1.GT.NST) GO TO 35			CAM	207
	ESAP(1,JM)=ESAP(1,JM)/SQRT(XMTOT(JM))			CAM	208
80	35 CONTINUE			CAM	209
	IROM=IROM+NEQB			CAM	210
	WRITE(7) ESAP			CAM	211
	34 CONTINUE			CAM	212
	RETURN			CAM	213
	END			CAM	214

POOR ORIGINAL

SUBROUTINE	POWER	78/78	OPT=1	FTN 4.5+414	11/19/80	17.24.07
1						
	SUBROUTINE	POWER	(XX,XM,EV,XD,B,BB,EDUM,NS1,NDOF,MODE)	CAM	215	
	COMMON/EVAL/EF(20)			CAM	216	
	DIMENSION	XX(NS1),XM(NS1),EV(NDOF,MODE),B(NDOF),BB(NDOF)		CAM	217	
5	I,XD(NDOF,NDOF),EDUM(MODE)			CAM	218	
	LEVEL 2,XX,XM,EV,XD,B,BB,EDUM			CAM	219	
	C			CAM	220	
	C INVERSE ITERATION TO COMPUTE MODES LOWEST MODES....			CAM	221	
	C EACH ITERATION STARTS WITH UNIT VECTOR FOLLOWED WITH			CAM	222	
	C GRAM SCHMIDT ORTHOGONALIZATION			CAM	223	
10	C			CAM	224	
	WRITE(6,100) MODE			CAM	225	
	100 FORMAT(//5X,22HINVERSE ITERATION FOR ,15.5H MODES//			CAM	226	
	1 2X,4HMODE,2X,9HITERATION,2X,10HE1GENVALUE,2X,5HERROR)			CAM	227	
	TOL=.0001			CAM	228	
15	I,MAX=30			CAM	229	
	C FACTOR K			CAM	230	
	DO 1 I=1,NDOF			CAM	231	
	DO 1 J=1,NDOF			CAM	232	
	1 XD(I,J)=0.			CAM	233	
20	1 XC(1,I)=1.			CAM	234	
	XC(2,I)=1,NDOF			CAM	235	
	DO 2 K=1,NDOF			CAM	236	
	CALL PPLOC(1,K,1LOC)			CAM	237	
	KCOL=K-1+1			CAM	238	
25	2 XD(I,KCOL)=XX(1,LOC)			CAM	239	
	CALL INVERT(NDOF,NDOF,XD,B,JOUR,2)			CAM	240	
	DO 20 IM=1,MODE			CAM	241	
	C TRIAL VECTOR			CAM	242	
	DO 3 I=1,NDOF			CAM	243	
30	3 B(I)=1.			CAM	244	
	IF((IM.GT.1)) CALL GRAMS(XM,EV,B,BB,IM,NS1,NDOF,MODE)			CAM	245	
	ITER=1			CAM	246	
	ERR=0.			CAM	247	
	C M = X			CAM	248	
35	12 DO 4 I=1,NDOF			CAM	249	
	BB(I)=0.			CAM	250	
	DO 4 J=1,NDOF			CAM	251	
	CALL PPLOC(1,J,1LOC)			CAM	252	
	4 BB(I)=BB(I)+XM(1,LOC)*B(J)			CAM	253	
40	CALL INVERT(NDOF,NDOF,XD,BB,JOUR,3)			CAM	254	
	X1 = M * X			CAM	255	
	DO 5 I=1,NDOF			CAM	256	
	B(I)=0.			CAM	257	
	DO 5 J=1,NDOF			CAM	258	
	CALL PPLOC(1,J,1LOC)			CAM	259	
45	5 B(I)=B(I)+XM(1,LOC)*BB(J)			CAM	260	
	DUM=0.			CAM	261	
	DO 6 I=1,NDOF			CAM	262	
	6 DUM=DUM+B(I)*BB(I)			CAM	263	
50	C X1=M = X1 + K * X / DUM			CAM	264	
	DO 7 I=1,NDOF			CAM	265	
	B(I)=0.			CAM	266	
	DO 8 J=1,NDOF			CAM	267	
	CALL PPLOC(1,J,1LOC)			CAM	268	
55	8 B(I)=B(I)+XX(1,LOC)*BB(J)			CAM	269	
	7 CONTINUE			CAM	270	
	DUMK=0.			CAM	271	
	DO 9 I=1,NDOF					

POOR ORIGINAL

SUBROUTINE POWER	78/78 OPT=1	FTN 4.5+414	11/19/80 17.24.07
	9 DUMK=DUMK+B(1)*BB(1)	CAM	272
80	XLAM=XLAM/DUM	CAM	273
	DUM=SQRT(DUM)	CAM	274
	DO 10 I=1,NDOF	CAM	275
10	B(1)=BB(1)/DUM	CAM	276
	WRITE(6,101) IM,ITER,XLAM,ERR	CAM	277
101	FORMAT(IX,15,3X,15,2X,E13.3,2X,E13.3)	CAM	278
	IF((IM.GT.1) CALL GRAMS(XM,EV,B,BB,IM,NS1,NDOF,MODE)	CAM	279
	ITER=ITER+1	CAM	280
	IF((ITER.GT.2) GO TO 11	CAM	281
	XLAM1=XLAM	CAM	282
	GO TO 12	CAM	283
70	11 IF((ITER.GT.1)MAX) GO TO 15	CAM	284
	ERR=ABS(XLAM-XLAM1)/XLAM	CAM	285
	IF((ERR.LE.TOL) GO TO 13	CAM	286
	XLAM1=XLAM	CAM	287
	GO TO 12	CAM	288
15	15 WRITE(6,102)	CAM	289
	104 FORMAT(5X,25HNO OF ITERATIONS EXCEEDED)	CAM	290
	13 DO 14 I=1,NDOF	CAM	291
14	EV(I,IM)=B(I)	CAM	292
	EF(IM)=SORT(XLAM)/6.28	CAM	293
80	20 CONTINUE	CAM	294
	WRITE(6,110)	CAM	295
	110 FORMAT(//5X,3I)ITERATION CONVERGED MODAL DATA/)	CAM	296
	DO 21 IM=1,MODE	CAM	297
	111 WRITE(6,111) IM,EF(IM)	CAM	298
85	111 FORMAT(5X,5HMODE ,15,12HE1GENVALUE =,E13.3,4H CPS/	CAM	299
	10X,12HMODE SHAPE =)	CAM	300
	112 WRITE(6,112)(EV(I,IM),I=1,NDOF)	CAM	301
	112 FORMAT(5X,6(E13.3,2X))	CAM	302
	EDUM(IM)=EF(IM)*6.28	CAM	303
90	21 CONTINUE	CAM	304
	REWIND 7	CAM	305
	WRITE(7) EDUM	CAM	306
	RETURN	CAM	307
	END	CAM	308

APPENDIX B  
USE OF MODIFIED SAPV AT  
BROOKHAVEN NATIONAL LABORATORY

The following card deck should be used to run the modified SAPV program on Brookhaven National Laboratories' CDC 7600 computer:

MSAP, STMFZ, T(t), P2.

t = running time

ACCOUNT (Name, Number)

Name, Number = Valid Account

ATTACH (OLDPL, SAP5, ID=ZZGAJPH)

ATTACH (CORR, GUYAN, ID=ZZGCAM)

ATTACH (LIB1, FR80LIB)

ATTACH (LIB2, CALCOMPLIB)

LIBRARY (LIB1, LIB2)

UPDATE (F, M=CORR, L=A12)

FTN (I=COMPILE, L=0)

MAP(OFF)

SEGLOAD.

LD SET (PRESET = ZERO)

LGO.

EOR

\* ID CAM

\* D PLOTTT.1

\* I CORR2.2

Card CAM.1 (All CAM - cards are listed in Appendix A).

\* D LEVEL2.1

\* I COMSIZE.3

Cards CAM.2 through CAM.4

\* D LEVEL 2.3

\* I LEVEL 2.2

Card CAM.5

\* I SAP 5.88

Card CAM.6 through CAM.8

\* I SAP 5.296

Card CAM.9 through CAM.76

\* I SAP.5 385

Card CAM.77

\* I SAP.5 387

Card CAM.78

EOR

Segment cards as for normal SAPV run

EOR

Data as specified in SAPV manual Ref. 5 and modified in Section 3.2

EOF

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