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by T. SARPKAYA

Evolution of the Wake in Unsteady Flow and Transverse Forces

The present state of knowledge of time-dependent flows in general and of the transverse forces in particular is very unsatisfactory. An improved understanding of some basic unsteady flows and the application of this knowledge to new design techniques will prove useful and should provide substantial improvements in performance, reliability, and costs of many fluid devices.

Since almost any aspect of fluid motion includes unsteady effects in some situations, it is logical to inquire what generally distinguishes unsteady and quasi-steady behavior, and why are they different? In the broad class of problems that can be treated by linear theory, unsteady effects are important when some time scale of the physical motion is comparable to the basic fluid dynamic time scale: that is, when L/Ut is of order 1 or greater. Here L, U, and t are the characteristic length, velocity and time, respectively. Truly unsteady flow problems are nonlinear where either the equations of motion or the boundary conditions, or both, contribute strong nonlinearities. The aforementioned time scale is still relevant, but the nonlinear effects may preclude the simple linear addition of unsteady phenomena to steady solutions, regardless of the frequency. The nonlinear unsteady effects are, of course, more difficult to analyze and model.

Impulsively-started and uniformly-accelerating flows are common examples of non-steady boundary layers and have some practical importance particularly when an accident sets the fluid impulsively in motion about the bodies immersed in it. Furthermore, impulsively started flow is one of those unsteady flow situations for which analytical and numerical solutions exist at least for small times and relatively low Reynolds numbers.

At the very early stages of motion, the vorticity does not have enough time to diffuse. Thus, the boundary layers are very thin and the flow is essentially irrotational. The fluid force acting on the body is primarily inertial and the inertia coefficient is $C_m = 1 + C_a$, C_a being the added mass coefficient obtained from the potential theory. For bodies without sharp corners (e.g., circular cylinder), the separation does not occur immediately. Furthermore, it does not necessarily initiate at the downstream stagnation point (as in the case of an elliptic cylinder). For two-dimensional cylinders, it can be shown (Schlicting 1968), that the separation begins after a time t_s at a place where the absolute value of dU/dx is largest. For a circular cylinder started impulsively from rest to a constant velocity, the distance covered until separation begins is s = 0.351R, r being the radius of the cylinder. The separation begins at the rear stagnation point.

For a uniformly accelerating cylinder the same distance is s = 0.52R and obviously greater than that for the case of impulsive motion. For a sphere impulsively set in motion s = 0.392R. The distance covered by the sphere until separation begins is larger, as in the case of the circular sylinder, when the sphere is accelerated uniformly from rest. Evidently,

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the rate of acceleration as well as the history of motion is important in the calculation of the relative distance covered prior to the occurrence of separation. For bodies with sharp corners separation starts immediately and the added mass coefficient is not necessarily equal to that given by the potential theory for the unseparated flow.

Theoretical investigations of the impulsively started motion of the circular cylinder in a fluid otherwise at rest are confined mostly to early times and very low Reynolds numbers (about 100). Such a motion was first considered by Blasius (1908) and his work was later extended by Goldstein and Rosenhead (1936), Görtler (1944), Schuh (1953), Watson (1955), and Wundt (1955). It was found, as noted earlier, that the boundary layer separates from the surface of the cylinder after a certain time lapse, the time and location of the separation depending on the Reynolds number and the bluffness of the body. The separation point then moves rapidly around the cylinder until at large times they coincide with the points of laminar separation for steady flow.

Finite-difference techniques have been employed by several investigators (Payne 1958; Hirota & Miyakoda 1965; Kawaguti & Jain 1966; Ingham 1968; Jain & Rao 1969; Son & Hanratty 1969; Thoman & Szewczyk 1969; Collins & Dennis 1973; Wu & Thompson 1973; Wang 1967; Panikker & Lavan 1975; Coutanceau & Bouard 1977).

There are very few experimental data for either impulsively-started or uniformly accelerated flow at sufficiently high Reynolds numbers, i.e., for Reynolds numbers in the order of 10,000. There are no experimental data for Reynolds numbers in the supercritical and transcritical regimes. This is partly because of experimental difficulties encountered in establishing impulsively-started or uniformly-accelerated flows and partly because of the instrumentation required to measure the transient quantities involved. In fact, the force that acts on the cylinder in impulsively started flow has been measured directly only by Sarpkaya (1966).

Schwabe (1935) used a circular cylinder with a radius of R = 1.7 inch. Experiments were conducted in an open channel with water. The velocity of the cylinder was U = 0.328 inch/sec. The Reynolds number was about 600. A careful examination of Schwabe's work tends to indicate that Schwabe's drag coefficient is about twice the steady-state value, and still increasing with time when the cylinder has moved about 9 body radii. It appears that considerable amount of experimental error may have been involved in the evaluation of the pressures and hence in the resulting drag coefficient.

Bingham, Weimer, and Griffith (1952) carried out a number of experiments in a shock tube to observe the influence of Reynolds and Mach number on the impulsive loading of a 0.5 inch cylinder. The pressures were determined from the density fields and the drag coefficients were calculated by an integration of the pressures. NONE of the foregoing investigations dealt with the transverse force resulting from the vortex shedding.

The impulsive flow has long been regarded as analogous to the evolution of separated flow about slender bodies moving at high angles of attack in the subsonic to moderately supersonic-velocity range. The approximate flow similarity between the development of the cross-flow with distance along the inclined body of uniform diameter and the development of flow with time on a cylinder in impulsive flow is known as the "cross--flow" analogy. This analogy was first suggested by Allen & Perkins (1951) and subsequently used by many other researchers to calculated in-plane normal force and out-of-plane force (side force normal to the plane of flight). A detailed discussion of the analogy, extensive measurements for various nose-shapes and body combinations, and the most pertinent references may be found in (Thomson & Morrison 1971; Bostock 1972; and Lamont & Hunt 1973).

According to the analogy the progressive development of the wake along the body when viewed in cross-flow planes is similar to the growth with time of the flow behind a two-dimensional cylinder started impulsively from rest. Close to the body nose no wake exists whereas further downstream two symmetrically disposed vortices form on the lee side. These vortices are fed by vortex sheets containing boundary-layer fluid which has separated from the body. Further along the body first one and then the other of these vortices detaches and moves downstream at an angle to the free stream. Other vortices form on the lee side of the body at increasing distance and behave in a similar manner. This process continues along the body length and a flow cross-section taken at right angles to the axis far from the nose has the appearance of a vortex street. The flow pattern is depicted in Figs. 1 and 2 and resembles a space-time plot of the flow past a two-dimensional cylinder started impulsively from rest. The evolution of the vortices from a symmetric to an asymmetric configuration is clearly visible in Fig. 2.

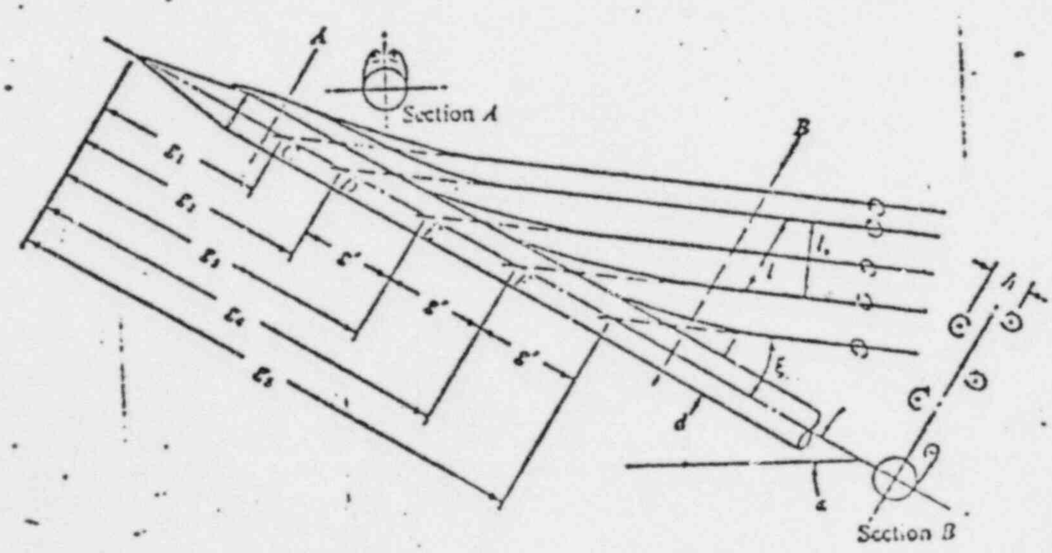

Fig. 1 Sketch of wake from slender cone-cylinder at large incidence and the impulsive-flow analogy.

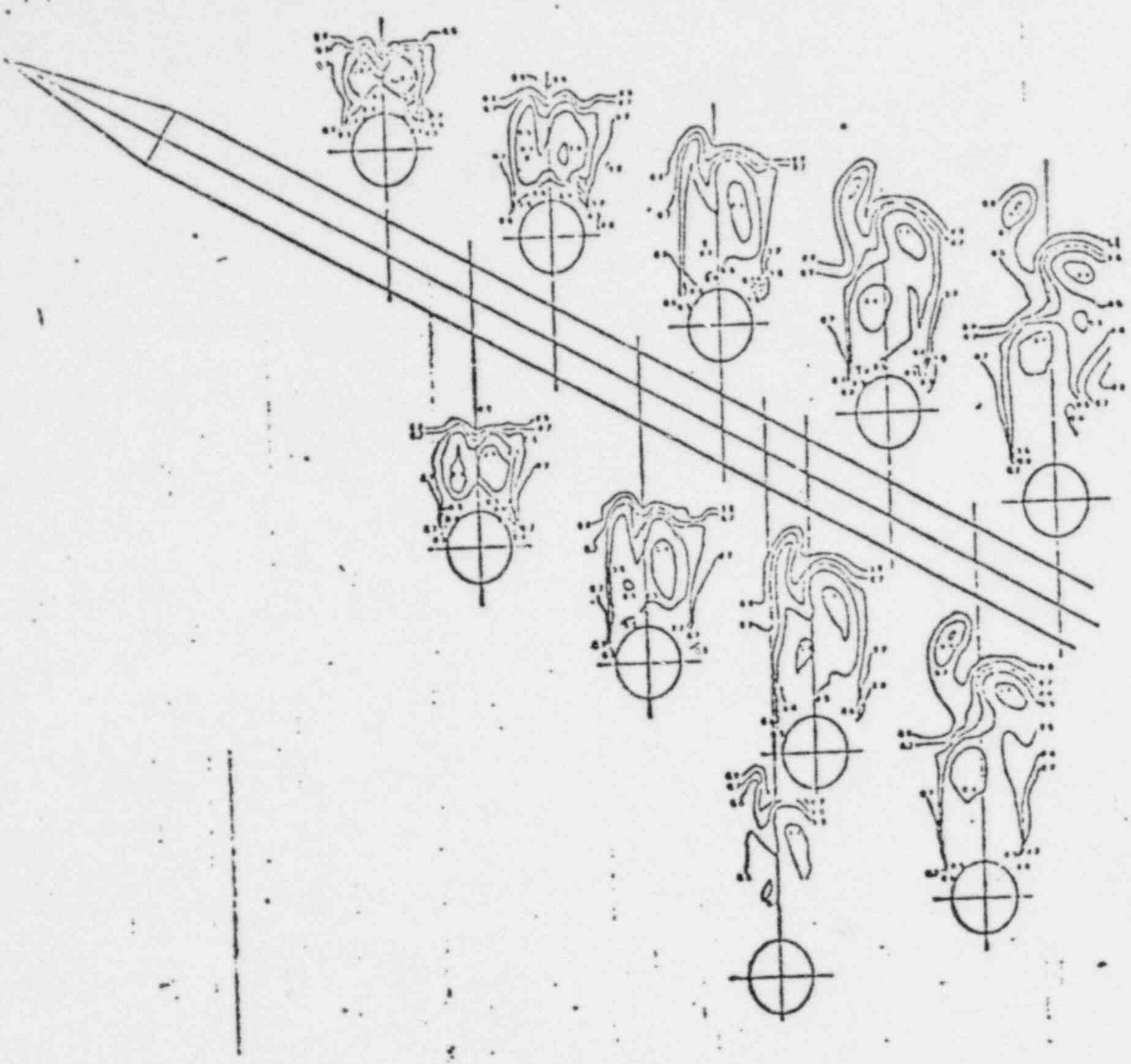

Fig. 2 Evolution of the vortices behind the body at 30 degree incidence.

The evolution of the drag and transverse-force coefficients for various bodies at various angles of attack has been measured by Lamont & Hunt (1976). Figures 3 through 5 show the most representative results. Evidently, the in-line force reaches a maximum at a time of about 5, i.e., at Ut/R = 5, and then gradually decreases to its steady-state value of 1.2. The reason for the initial rise in the drag coefficient is the rapid accumulation of vorticity in the two symmetric vortices. Evidently, the Reynolds number is in the subcritical range. A similar rise in the drag coefficient is also expected for flows in the supercritical Reynolds number range although there is, at present, no experimental data to support this conjecture (See Sarpkaya 1966).

Figures 3 and 4 show that the lift coefficient (non-dimensionalized by means of the cross-flow dynamic head and the cylinder diameter) has a typical value of about 0.6 in the early stages of motion (Ut/R = 12). Subsequently, the lift coefficient decreases to a value of about 0.3. The reasons for the foregoing observations are as follows. The weakest vortex is always the first vortex shed in any unsteady flow started from rest. The strongest vortex is always the second vortex. In fact the first maximum in the lift curve occurs at the time of the shedding of the second vortex. Vortices which are subsequently shed have strengths intermediate to the first two. Partly because of this reason and partly because of the fact that the vortices are situated relatively further downstream, the lift coefficient shows some decrease following the shedding of the first few vortices. The additional decay in C_L for very large values of Ut/R is applicable only to the bodies at incidence and represents the transition between the developing flow over the front part of the body and the effect of the blunt end. Thus, one should not assume, on the basis of the analogy, that the lift coefficient in an accelerating flow will continue to decrease with time. The fact that this is not so will be demonstrated shortly.

Figure 5 shows the variation of the maximum lift coefficient with two representative Reynolds numbers. Evidently, in the subcritical range (i.e., Re in the order of 50,000), the lift coefficient is about 1.3 and decreases rapidly with increasing Reynolds number. It should be observed that the Reynolds number at 110,000 corresponds to the critical range in which the lift coefficient reaches its minimum value.

As noted earlier, the foregoing results have been obtained by Lamont & Hunt (1976) through the use of a slender body at an angle of attack. Furthermore, they have been presented herein as an indirect evaluation of the lift coefficient through the use of the impulsive flow analogy. One must, therefore, keep in mind that these coefficients are only for an impulsively started flow about a cylinder and hold true only in the regions of flow where the impulsive-flow analogy is not expected to hold true. In summary then one finds that the lift coefficient in an impulsively started flow about a cylinder may reach a typical value of about 0.6 in the early stages of motion for Reynolds numbers in the order of 100,000. For smaller Reynolds numbers, the maximum lift coefficient may be as large as 1.2. Lamont & Hunt did not obtain any data for higher Reynolds numbers.

A very extensive literature search has revealed that no investigator has ever measured the transverse force on a body immersed in a time-dependent flow. The only exception being the work carried out by the present writer with uniformly accelerating flows. Sarpkaya (1977-b) conducted a series of experiments in a vertical water tunnel of 2 ft by 2 ft cross-section with circular cylinders ranging from 3 inch to 5 inch diameter. The flow was subjected to uniform acceleration of desired constant magnitude (dU/dt ranged from 0.2 ft/s^2 to 20 ft/s^2).

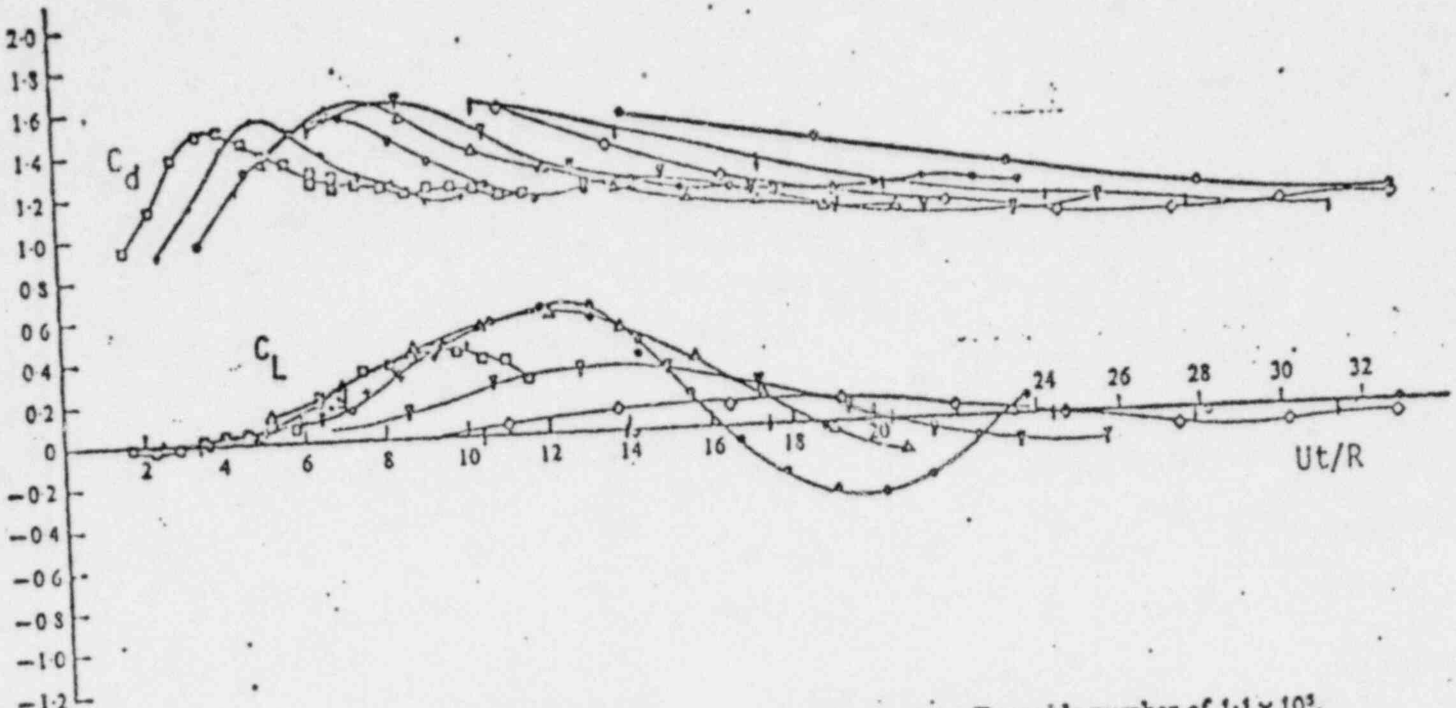

Fig. 3 Experimental force coefficients for the 2R ogive nose at a Reynolds number of $1{\cdot}1 \times 10^5$.

	⊡	+	o	△	▽	◇	↑	⊙
α (deg)	30	40	50	60	65	70	74	75

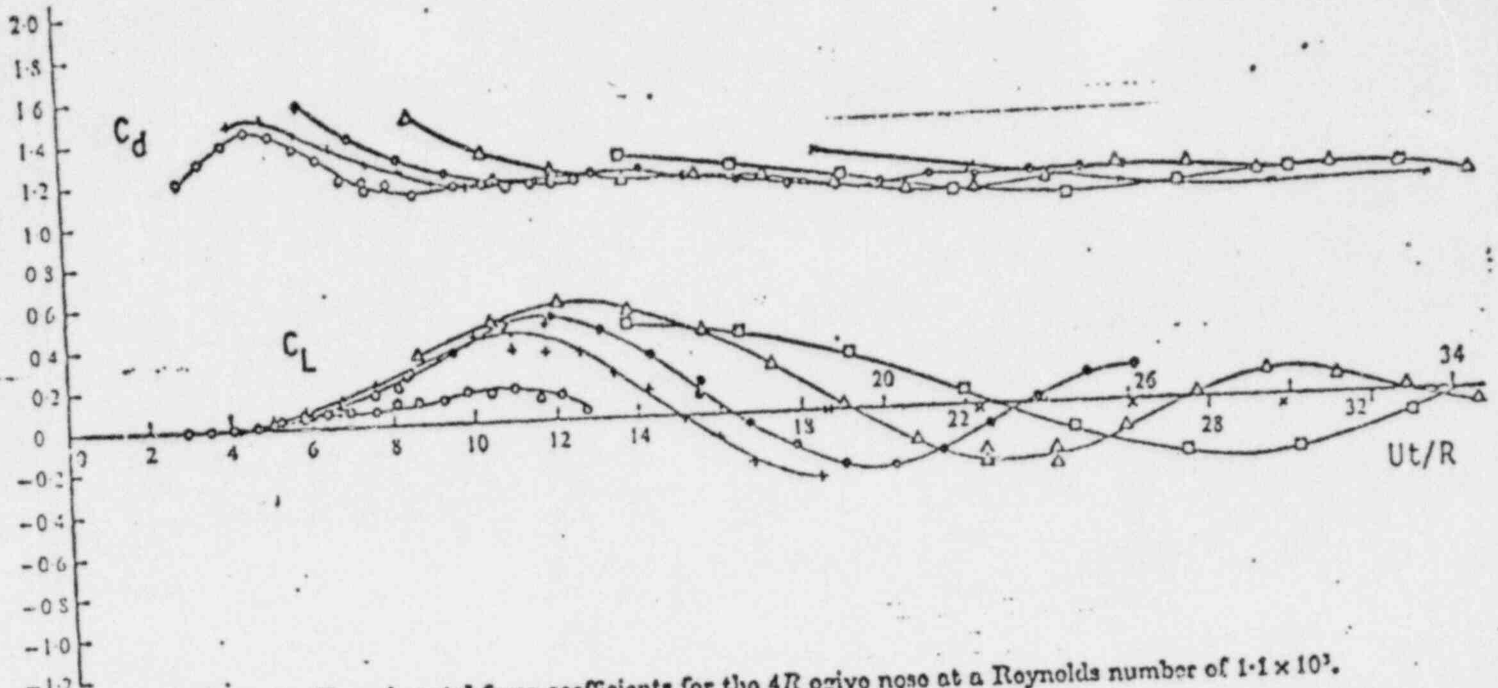

Fig. 4 Experimental force coefficients for the 4R ogive nose at a Reynolds number of $1{\cdot}1 \times 10^5$.

	○	+	⊙	△	□	×
α (deg)	30	40	50	60	70	75

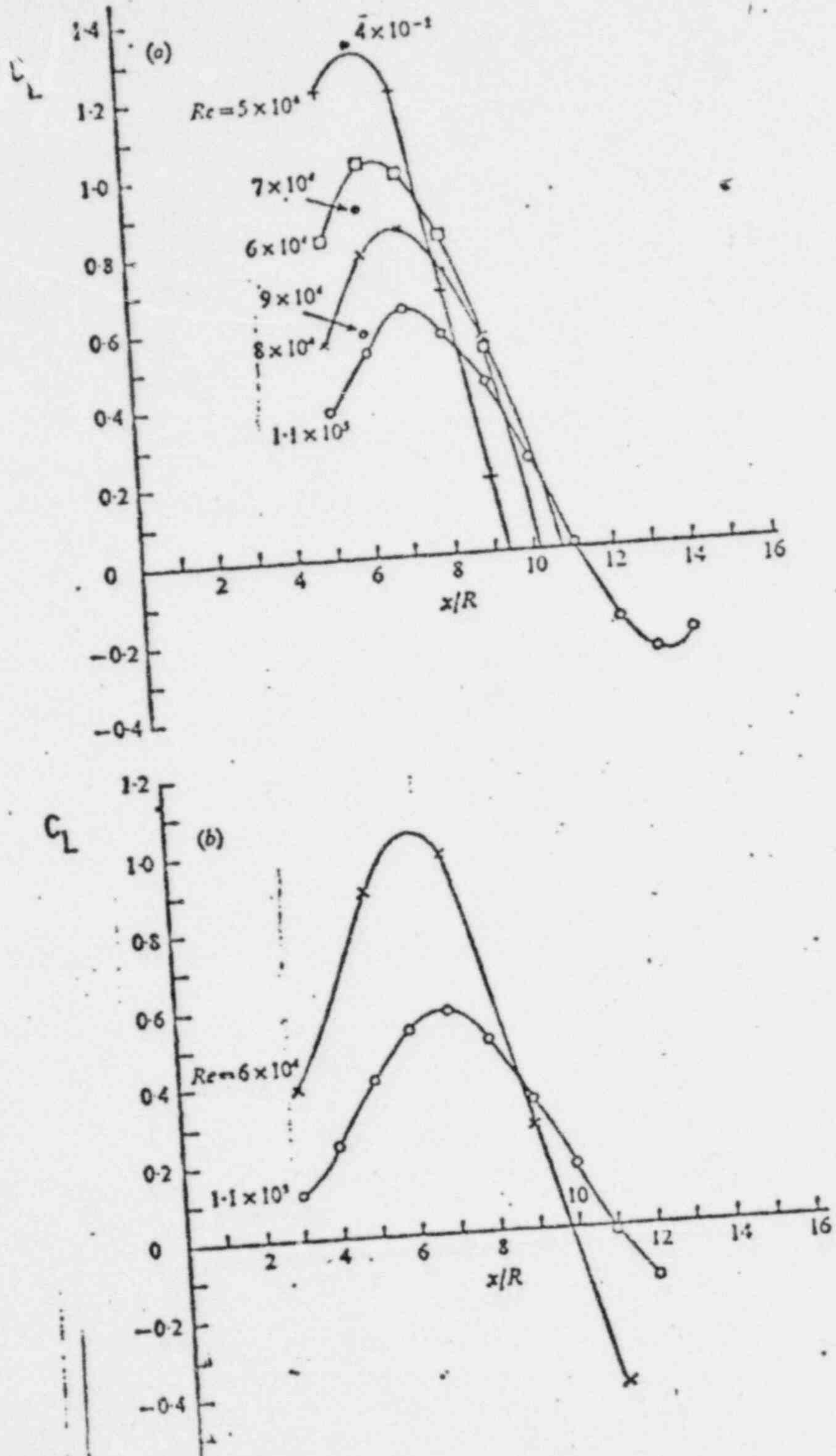

Fig. 5 Effect of Reynolds number on out-of-plane force for (a) the 4R and (b) the 2R ogive nose; $\alpha = 60°$.

The purpose of the investigation was partly to measure the transverse force and partly to re-examine the drag force obtained approximately 15 years ago (Sarpkaya et al. 1963). The drag force measurements have yielded nearly identical results with a scatter less than 5%.

The transverse force was non-dimensionalized with the dynamic head of the instantanious velocity and the diameter of the cylinder, i.e., C_l = (Instantanious value of the transverse force)/($0.5\rho LDU^2$) where U is the corresponding instantanious velocity. The lift coefficients obtained in this manner are then plotted as a function of $Ut/R = 2s/R$ where s is the displacement of the fluid (i.e., $s = 0.5Ut = 0.5.dU/dt.t^2$).

Figures 6 through 9 show the results for various cylinders and accelerations. Each figure contains the results of two or more repeat runs with the same cylinder at the same acceleration. Also shown in each figure are the corresponding instantaneous Reynolds numbers.

Figure 6 for which Re = 60,000 at Ut/R = 50, shows that the lift coefficient remains fairly constant at a value of about 0.5. Figure 7, for which Re = 120,000 at Ut/R = 50, shows that the lift coefficient begins with a maximum value of about 0.4, then decreases to about 0.3 as the Reynolds number goes through values in the order of 60,000, and then increases slightly to about 0.4. Figure 8, for which Re = 400,000 at Ut/R = 50, shows a similar variation. Finally, Figure 9 for which Re = 600,000 at Ut/R = 50, exhibits a nearly constant lift coefficient of about 0.3.

Several conclusions may be drawn from the foregoing. Firstly, the lift coefficient is not uniquely determined by the Reynolds number (e.g., note that the lift coefficient at Re = 60,000 in Figs. 6 and 7 are not the same), The history of the flow appears to play some role in the evolution of the transverse force. Secondly, there does not appear to be any dramatic changes in the lift-force coefficient as the Reynolds number goes through values in the order of 100,000. This may either mean that the accelerating flow has not yet gone through the critical transition even though the Reynolds number exceeded values of about 300,000 or that the passage through the transition region in a uniformly accelerating flow is not as dramatic as it is in steady flows. Finaly, it is evident from a comparison of Figs. 3 and 4 with Figs. 6 through 9 that the lift coefficients predicted through the use of the impulsive flow analogy are fairly close to those obtained with uniformly accelerating flows. Suffice it to note that the impulsively started flows are more likely to yield an initially larger lift coefficient because of the more rapid accumulation of vorticity in the symmetrically growing vortices.

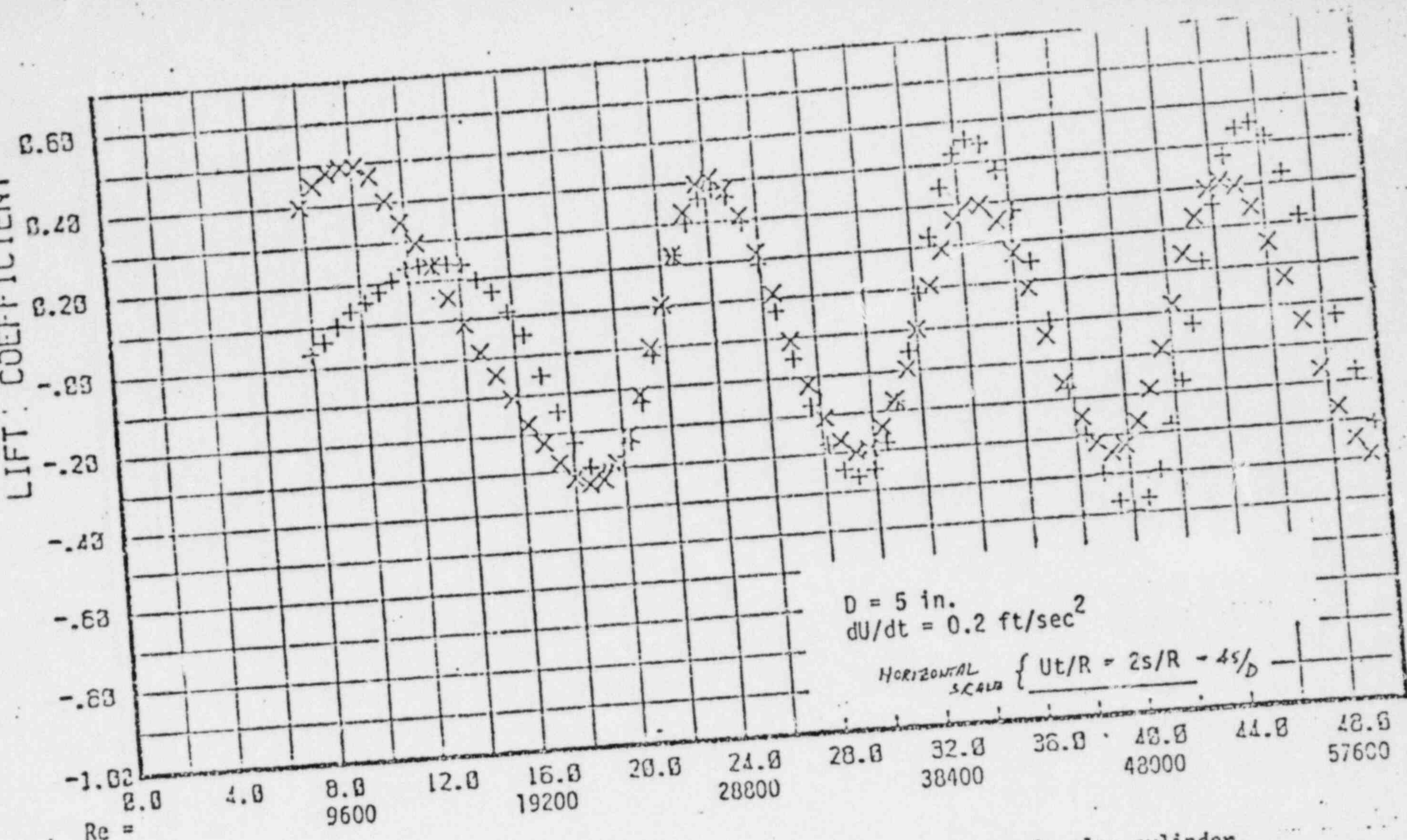

Fig. 6 Lift coefficient for a uniformly accelerated flow about a circular cylinder.

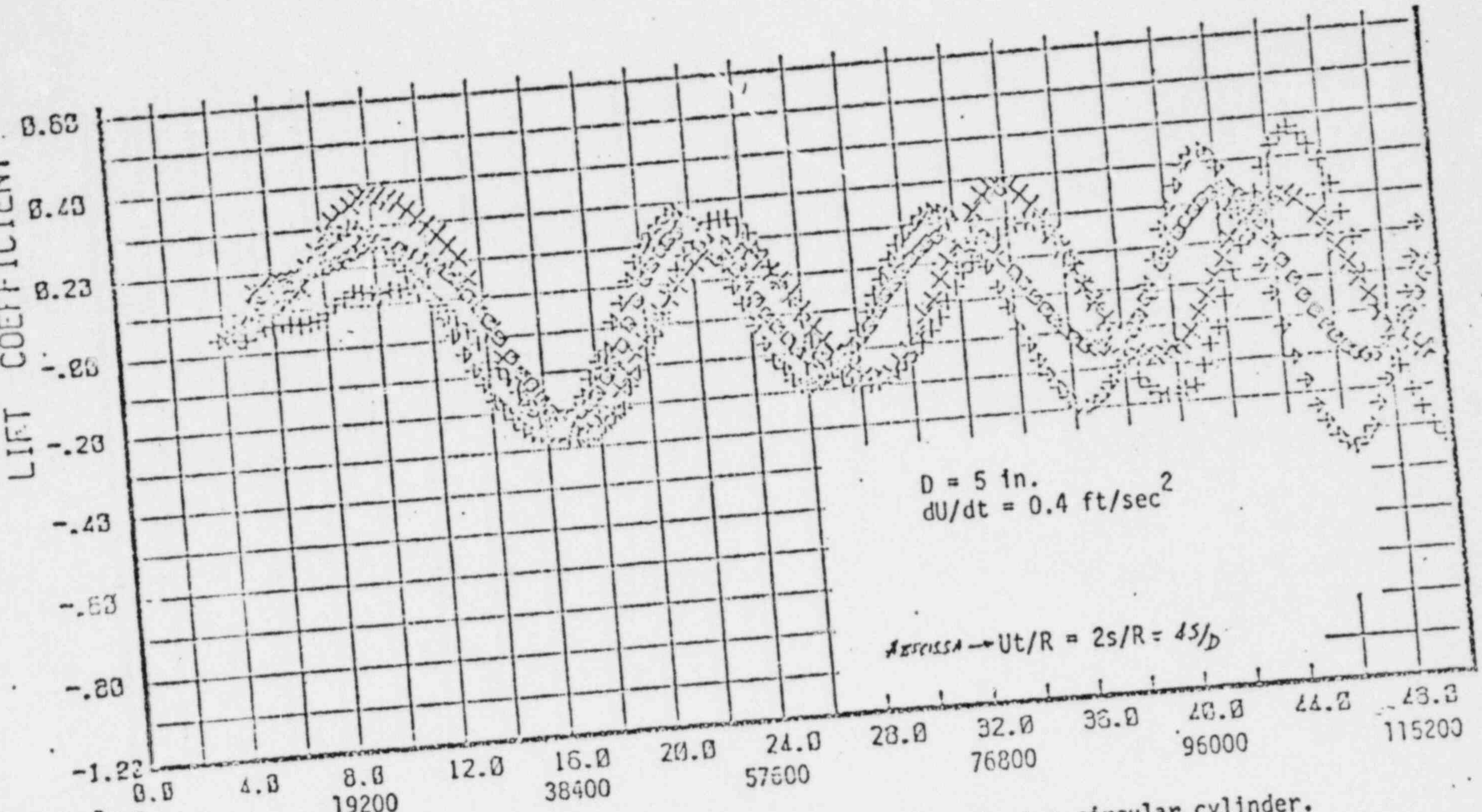

Fig. 7 Lift coefficient for a uniformly accelerated flow about a circular cylinder.

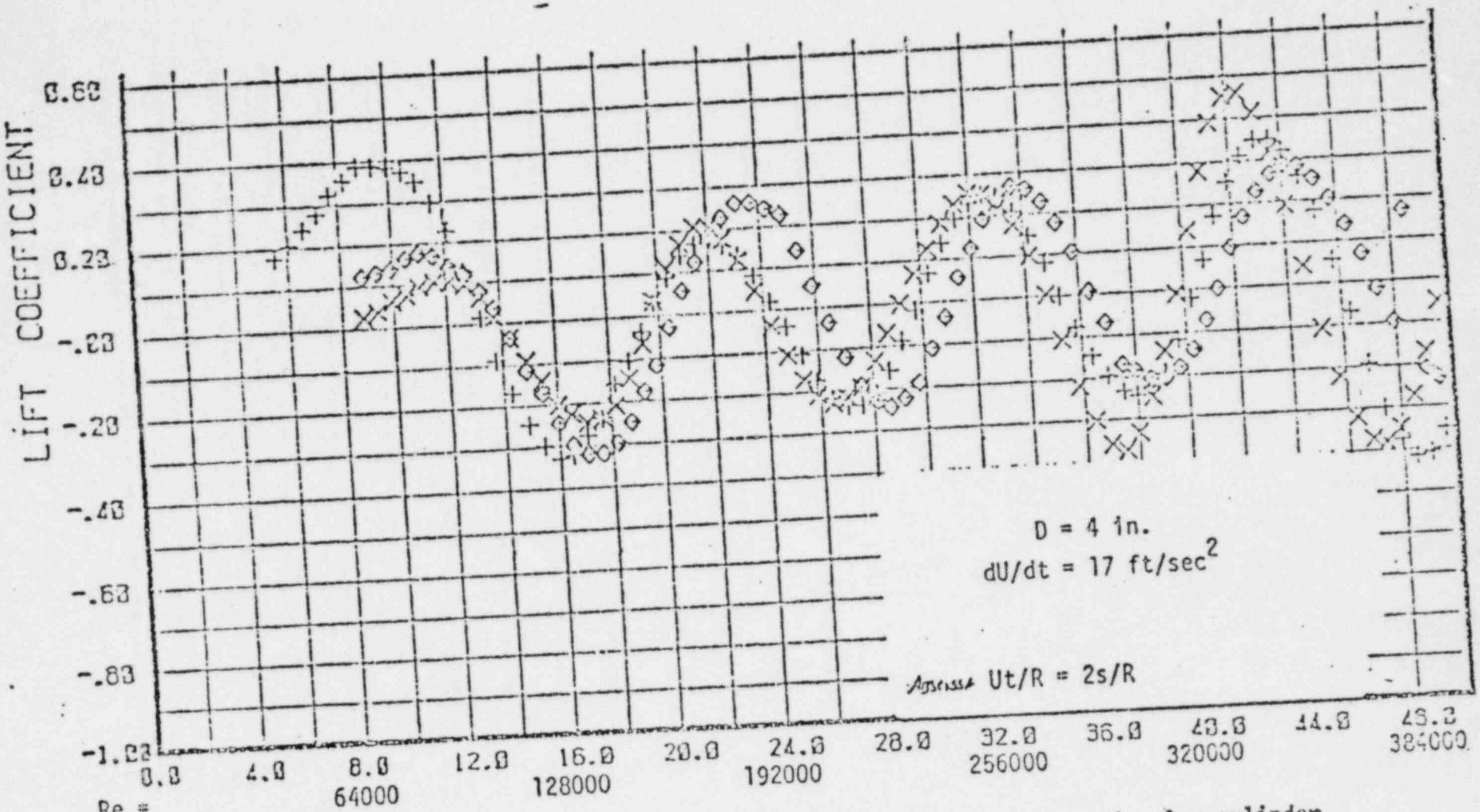

Fig. 8 Lift coefficient for a uniformly accelerated flow about a circular cylinder.

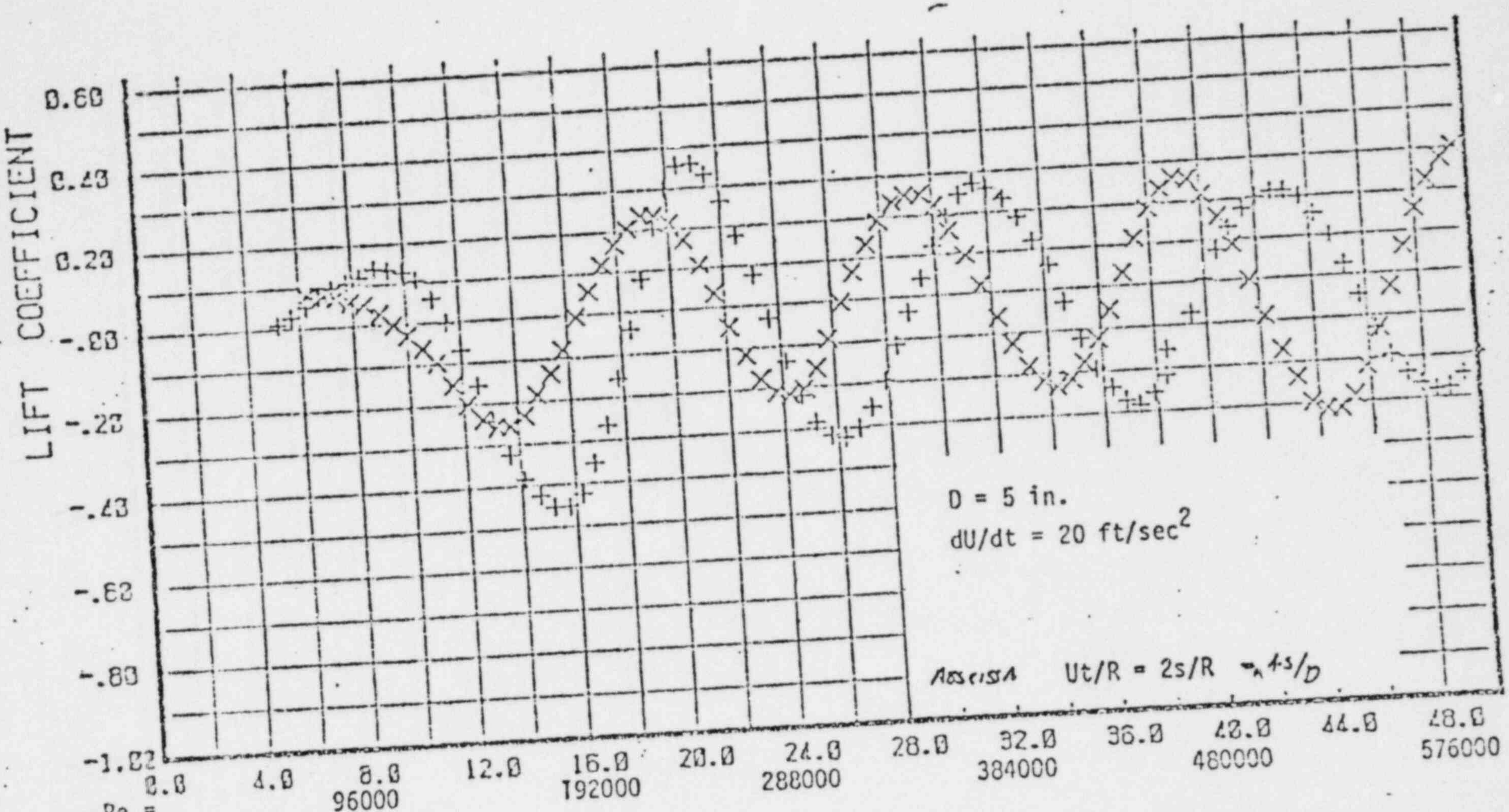

Fig. 9 Lift coefficient for a uniformly accelerated flow about a circular cylinder.

LIST OF REFERENCES CITED

Allen, H. J. and Perkins, E. W. (1951), "A Study of Effects of Viscosity on Flow Over Slender Inclined Bodies of Revolution," NACA Rep. 1048.

Bingham, H. H., Weimer, D. K., and Griffith, W. (1952), "The Cylinder and Semi-cylinder in Subsonic Flow," Princeton Univ. Dept. Phys., Tech. Report 11-13.

Bishop, R. E. D. and Hassan, A. Y. (1964), "The Lift and Drag Forces on a Circular Cylinder in a Flowing Fluid," Proc. Roy. Soc., (London), Vol. 277A, 32-50.

Blasius, H. (1908) "Grenzchichten in Flussigkeiten mit kleiner Reibung," Z. Math. Physik Vol. 56, 1.

Bostock, B. R. (1972) "Slender Bodies of Revolution at Incidence," Ph. D. Thesis, Churchill College, Univ. of Cambridge.

Bursnall, W. J. and Loftin, L. K. (1951), "Experimental Investigation of the Pressure Distribution about a Yawed Circular Cylinder in the Critical Reynolds Number Range," NACA Tech. Note 2463.

Carper, H. J. and Dodge, F. T., (1977), "An Assessment of Existing Literature Relative to Drag on Submerged Structures," SwRI Report on Project No. 02-4899 to G. E. Company.

Collins, W. M. and Dennis, S. C. R. (1973), "Flow Past an Impulsively Started Circular Cylinder," J. Fluid Mech. Vol. 60, 105.

Coutanceau, M. and Bouard, R. (1977), "Experimental Determination of the Main Features of the Viscous Flow in the Wake of a Circular Cylinder in Uniform Flow. Part 2. Unsteady Flow," J. Fluid Mech. Vol. 79, Part 2, 257-272.

Det Norske Veritas, Norway (1974), "Rules for the Design, Construction and Inspection of Fixed Offshore Structures,"

Drescher, H. (1956), "Messung der auf querangestromte Zylinder ausgeubten zeitlich veranderten Drucke," Z. Flugwiss, Vol. 4 (1/2), 17-21.

el Baroudi, M. Y. (1960), "Measurement of Two Point Correlation of Velocity near a Circular Cylinder Shedding a Karman Vortex Street," U.T.I.A., Tech. Note 31.

Etkin, B. and Korbacher, G. K. and Keefe, R. T. (1958)," Acoustic Radiation from a Stationary Cylinder in a Fluid Stream (Aeolian Tones)," U.T.I.A., Rev. No. 13.

Fabula, A. G. (1957), "Ellipse-Fitting Approximation of Two-Dimensional Normal Symmetric Impact of Rigid Bodies on Water," Proceedings of the 5th Midwestern Conference on Fluid Mechanics, (Ann Arbor, Michigan), 299-315.

Fung, Y. C. (1960), "Fluctuating Lift and Drag Acting on a Cylinder in a Flow at Supercritical Reynolds Number," J. Aerospace Sci., Vol. 27, 801-814.

Gerrard, J. H. (1961), "An Experimental Investigation of the Oscillating Lift and Drag of a Circular Cylinder Shedding Turbulent Vortices," J. Fluid Mech. Vol. 11, Part 2.

Gerrard, J. H. (1965), "A Disturbance-Sensitive Reynolds Number Range of the Flow Past a Circular Cylinder," J. Fluid Mech., Vol. 22, Part 1.

Glenny, D. E. (1966), "A Review of Flow Around Circular Cylinders, Stranded Cylinders and Struts Inclined to the Flow Direction," Australian Defence Scientific Service ARL Mech. Eng. Note 284.

Goldstein, S. and Rosenhead, L. (1936), "Boundary-Layer Growth," Proc. Cambridge Phil. Soc. Vol. 32, 392.

Görtler, H. (1944), "Verdrängungswirkung der Laminaren Grenzschicht und Druckwiderstand," Ingr.-Archiv Vol. 14, 286-305.

Hirota, I. and Miyakoda, K. (1965), "Numerical Solution of Karman Vortex Street Behind a Circular Cylinder," J. Met. Soc. Japan, Vol. 43, 30.

Hoerner, S. F. (1958), Fluid-Dynamic Drag, Published by the Author, Midland Park, N. J.

Humphreys, J. S. (1960), "On a Circular Cylinder in a Steady Wind at Transition Reynolds Numbers," J. Fluid Mech., Vol. 9, 603-612.

Ingham, D. B. (1968), "Note on the Numerical Solution for Unsteady Viscous Flow Past a Circular Cylinder," J. Fluid Mech. Vol. 31, 815.

Jain, P. C. and Rao, K. S. (1969), "Numerical Solution of Unsteady Viscous Incompressible Fluid Flow Past a Circular Cylinder," Phys. Fluids Suppl. Vol. 12, II 57.

Kaplan, P. and Silbert, M. N. (1976), "Impact Forces on Platform Horizontal Members in the Splash Zone," Proceedings of the Offshore Technology Conference, Paper No. 2498, Houston, TX.

Kawaguti, M. and Jain, P. C. (1966), "Numerical Study of a Viscous Fluid Past a Circular Cylinder," J. Phys. Soc. Japan, Vol. 21, 10, 2055.

Keefe, R. T. (1961), "An Investigation of the Fluctuating Forces Acting on a Stationary Cylinder in a Subsonic Stream, and of the Associated Sound Field," U.T.I.A., Rept. 76.

Lamont, P. J. and Hunt, B. L. (1976), "Pressure and Force Distributions on a Sharp-Nosed Circular Cylinder at Large Angles of Inclination to a Uniform Subsonic Stream," J. Fluid Mech. Vol. 76, Part 3, 519-559.

Macovsky, M. (1958), U.S. Navy Dept. D. Taylor Model Basin Rept. 1190.

Mair, W. A. and Maull, D. J. (1971), "Aerodynamic Behaviour of Bodies in the Wakes of Other Bodies," Phil. Trans. Roy. Soc. London, Vol. 269A, 425-437.

McGregor, D. M. (1957), "An Experimental Investigation of the Oscillating Pressures on a Circular Cylinder in a Fluid Stream," U.T.I.A., Tech. Note 14.

Miller, B. L. (1977), "Wave Slamming Loads on Horizontal Circular Elements of Offshore Structures," The Royal Institution of Naval Architects Meeting Paper No. 5, April 6, 1977, London.

Milwitzky, M. (1952), "Generalised Theory for Seaplane Impact," NACA Report 1103.

Miyazaki, H. and Sparrow, E. M. (1976), "Potential Flow Solution for Cross-Flow Impingement of a Slot Jet on a Circular Cylinder," ASME Paper No. 76-FE-F.

Moody, F. J. (1977-a), "Analytical Model for Estimating Drag Forces on Rigid Submerged Structures Caused by LOCA and Safety Relief Valve Ramshead Air Discharges," G. E. Company Report NEDE-21471, (Proprietary Information).

------ (1977-b), "Analytical Model for Liquid Jet Properties for Predicting Forces on Rigid Submerged Structures," G. E. Company Report NEDE-21472, (Proprietary Information).

------ (1977-c), "Forces on Submerged Structures in Unsteady Flow," Paper presented at the Thermal Reactor Safety Meeting, Sun Valley, Idaho, July 31-Aug. 5, 1977.

Morison, J. R. et al., (1950), "The Force Exerted by Surface Waves on Piles," Petroleum Transactions, Vol. 189, 149-157.

Ojha, S. K. and Gollakota, S. (1977), "Jet Impingement on a Curved Surface," AIAA Journal, Vol. 15, No. 4, 453.

Panikker, P. K. and Lavan, Z. (1975), "Flow Past Impulsively Started Bodies Using Green's Functions," J. Comp. Phys. Vol. 18, 4[illegible].

Parkinson, G. V. et al. (1968), "Mechanisms of Vortex-Excited Oscillation of Bluff Cylinders," Symp. Wind Effects on Buildings and Structures, Loughborough.

Payne, R. B. (1958), "Calculations of Unsteady Viscous Flow Past a Circular Cylinder," J. Fluid Mech. Vol. 4, 81.

Phillips, O. M. (1956), "The Intensity of Aeolian Tones," J. Fluid Mech. Vol. 1 ,607-624.

Prendegast, V. (1958) ,"Measurement of Two-Point Correlations of the Surface Pressure on a Circular Cylinder," U.T.I.A. Tech. Note 23 ,

Roshko, A. (1961), "Experiments on the Flow Past a Circular Cylinder at very High Reynolds Number," J. Fluid Mech. Vol. 10, 345-356.

Sarpkaya, T. et al. (1963), "Vortex Formation and Resistance in Unsteady Flow," Trans. ASME, Jour. Appl. Mechs. Vol. 30, 16-24.

Sarpkaya, T. (1966), "Separated Flow about Lifting Bodies and Impulsive Flow about Cylinders," AIAA Jour. Vol. 4, No. 3 ,414-420.

Sarpkaya, T. (1976), "Vortex Formation and Resistance in Harmonic Flow about Smooth and Rough Circular Cylinders at High Reynolds Numbers," Naval Postgraduate School Technical Report No: NPS-59SL-76021.

Sarpkaya, T. (1977-a), "Impact of Harmonically Oscillating Water Surface on Circular Cylinders," (to appear).

Sarpkaya, T. (1977-b), "Uniformly-Accelerating, Impulsively-Started, and other Types of time-Dependent Flow about Cylinders," (to appear in the Engineering Mechanics Journal of ASCE).

Schlichting, H. (1968), Boundary-Layer Theory, McGraw-Hill, New York, (6th ed.), (See p. 400-420).

Schmidt, L. V. (1965), "Measurement of Fluctuating Air Loads on a Circular Cylinder," J. Aircraft, Vol. 2, No. 1, 49-55.

Schuh, H. (1953), "Calculation of Unsteady Boundary Layers in Two-Dimensional Laminar Flow," ZFW Vol. 1, 122-131.

Schuh, H. and Persson, B. (1964), "Heat Transfer on Circular Cylinders Exposed to Free-Jet Flow," International Journal of Heat and Mass Transfer, Vol. 7, 1257-1271.

Schwabe, M. (1935), "Uber Druckermittlung in der nichtsationaren ebenen Stromung," Ingenieur-Archiv, Vol. 6, 34-50.

Smith, R. A., Moon, W. T. and Kao, T. W. (1972), "Experiments on Flow About a Yawed Circular Cylinder," ASME Paper No. 72-FE-2.

Son, J. S. and Hanratty, T. J. (1969), "Numerical Solution for the Flow Around a Cylinder at Reynolds Numbers of 40, 200, and 500," J. Fluid Mech. Vol. 35, 369.

Thoman, D. C. and Szewczyk, A. A. (1969), "Time-Dependent Viscous Flow Over a Circular Cylinder," Phys. Fluids Suppl. Vol. 12, II.

Thomson, K. D. and Morrison, D. F. (1971), "The Spacing, Position and Strength of Vortices in the Wake of Slender Cylindrical Bodies at Large Incidence," J. Fluid Mech. Vol. 50, Pt. 4 751-783.

Trimble, T. H. (1977), "Normal and Tangential Force on Cylinders at Small Inclinations to the Flow," Unpublished ARL work, (See Glenny, D. E. 1966)

Tsui, Y. T. (1977), "On Wake-Induced Flutter of a Circular Cylinder in the Wake of Another," ASME Paper No. 77-APM-28.

von Karman and Wattendorf, F. (1929), "The Impact on Sea-Plane Floats During Landing," NACA TN 321.

Vickery, B. J. (1966), "Fluctuating Lift and Drag on a Long Cylinder of Square Cross-Section in a Smooth and in a Turbulent Stream," J. Fluid Mech. Vol. 25, 481-494.

Wang, C. Y. (1967), "The Flow Past a Circular Cylinder which is Started Impulsively from Rest," J. Math. Phys. Vol. 46, 195.

Watson, E. J. (1955), "Boundary-Layer Growth," Proc. Roy. Soc. Vol. 231A, 104-116.

Wu, J. C. and Thompson, J. F. (1973), "Numerical Solution of Time-Dependent Incompressible Navier-Stokes Equations using an Integro-Differential Formulation," Computers & Fluids, Vol. 1, 197.

Wundt, H. (1955), "Wachstum der Laminaren Grenzschicht an schrag angestromten Zylindern bei Anfahrt aus der Ruhe," Ingr.-Archiv. Vol. 23, 212-230.