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THE STABILITY OF TWO-PHASE FLOW LOOPS AND RESPONSE TO SHIP'S MOTION

by

E. S. Beckjord

September 26, 1960

Prepared under
AEC CONTRACT AT(04-3)189, PA #5

GENERAL ELECTRIC

ATOMIC POWER EQUIPMENT DEPARTMENT

SAN JOSE, CALIFORNIA

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Approved by: D H Imhoff
D. H. Imhoff, Manager
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SUMMARY

We investigate the dynamics of stationary two-phase flow loops and of those accelerated by ship's motion, and determine the controlling parameters. The results apply to boiling water reactors, and enable prediction and design of stable two-phase flow in reactors. An analogue computor circuit is developed for calculating loop transients. Analytical predictions of stability and transient response are compared with A.P.E.D. Heat Transfer Loop test results:

1. Analysis correctly predicted the trend of four tests progressing from very stable flow to a case on the verge of instability.

2. The analogue predicts the temporary decrease in inlet velocity which occurs when the loop heating power is increased. The quantitative comparison for a heating rate increase of 7% is:

	Observed	Predicted
inlet velocity transient	-32%	-30.5%
oscillation period	3.0 sec.	2.4 sec.
damping factor	0.066	0.1

Analogue computation of the effect on the T-7 core of a 20% of normal gravitational acceleration increase gives these results:

pressure drop	20%
chimney water velocity	+ 2.7%
inlet water velocity	+ 8.5%
average steam void	-11.7%

Analogue computation of the effect of a 20% heating increase in channel 1 of 2 coupled parallel channels similar to T-7 core channels gives these results:

Channel 1	
inlet water velocity	+ 5%
average steam void	+17%
Channel 2	
inlet water velocity	- 0.4%
average steam void	+ 0.9%

The following conclusions are drawn on the factors which determine loop dynamics:

1. The natural period of the loop is governed by the transit time of fluid across the two-phase vertical section. If oscillatory, the period will be between 4/3 T and 2T, where T is the transit time.

2. The primary cause of loop instability is subcooling with high steam voids, because subcooling makes the natural circulation driving head decrease when the inlet water velocity increases, and vice-versa. A design line of subcooling limit versus operating pressure is given.

3. Unstable flow loops can be stabilized by velocity head losses, such as at an orifice in the downcomer, and by the use long downcomer pipes and consequent high single phase fluid inertia.

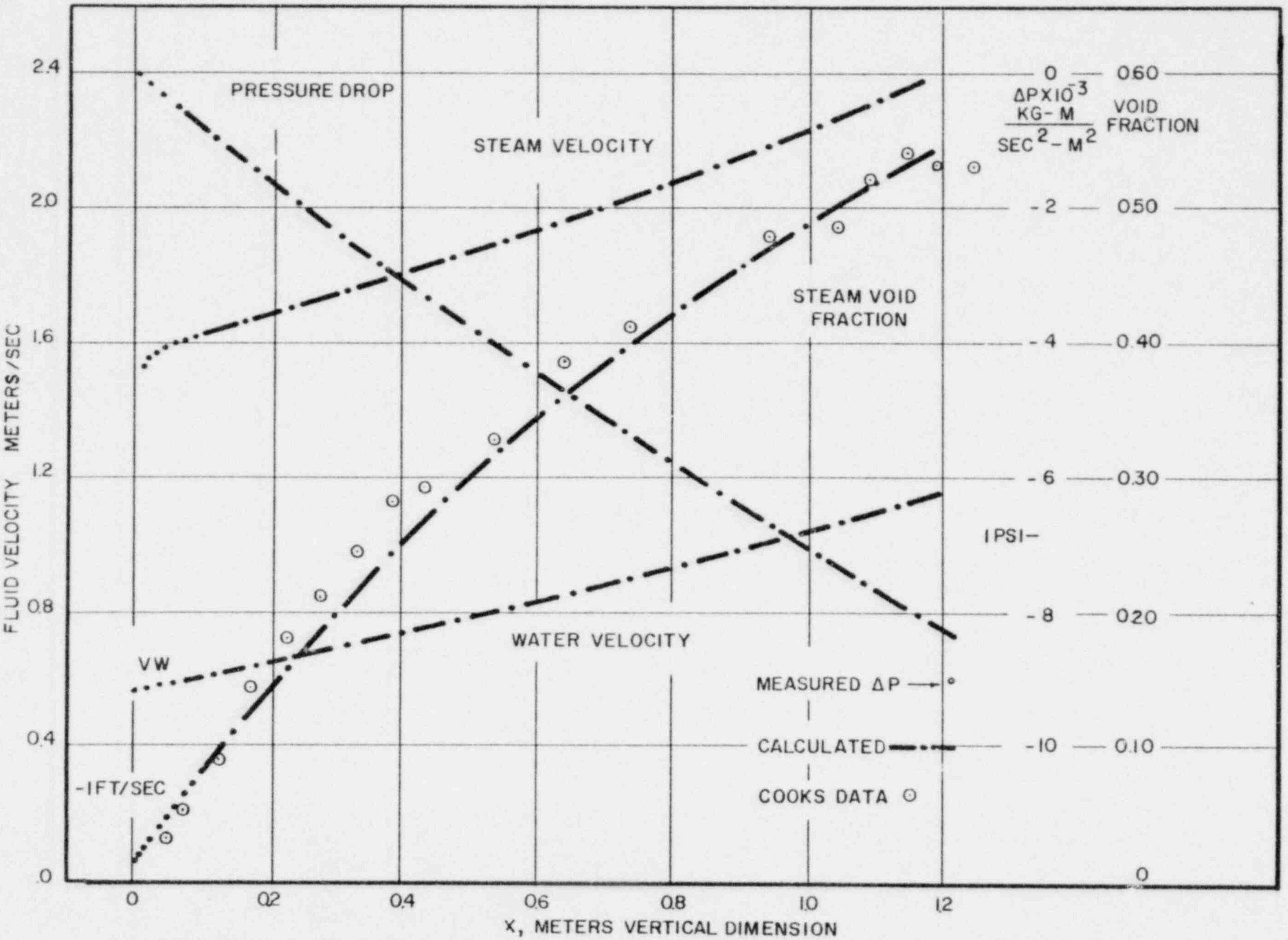
4. Friction pressure drops and velocity head losses in the riser can help to stabilize, provided inlet subcooling is not excessive. If it is excessive, the riser friction pressure drop and head losses actually decrease when inlet water velocity increases. The result is a negative incremental pressure drop which destabilizes. This point is important to the design of reactors with internal steam separation.

The work reported here was performed under contract AT(04-3)-189 PA #5 with the San Francisco Operations Office of the U. S. Atomic Energy Commission.

INTRODUCTION

The purpose of this work is to explain the dynamics of two-phase saturated steam and water flow loops in general, and particular the effects of ship's motion accelerations on such loops. The dynamics of both stationary loops and accelerated loops have obvious bearing on the design of stable boiling water reactors.

The basic questions of two-phase flow in pipes or channels of boiling water reactors are (a) what is the pressure drop? (b) what is the phase volume fraction or phase relative velocity?, and (c) is the flow steady or pulsating? In the past empirical correlations [1], [2] of pressure drop and of volume fractions with quality have been used to deal with the first two questions. Recently a method was introduced[3], which relates pressure drop and volume fraction by the force between the phases. This method is based upon the flow system geometry, the pressure, the heat input, and the single-phase friction factor of pipes. It originates with work by Levy [4]. Calculations of Cook's [5] and Marchaterre's [6] experimental data consisting of void fraction and pressure drop were made by this method, and the calculated values were within the band of experimental error of the measured values, for test runs between 275 and 614 psia. One of the calculated cases from reference (3) is shown in Figure 1. The calculation is made from operating pressure, inlet velocity, heating rate, and channel hydraulic diameter, but without the use of any correlation of void fraction with quality. Because the equations of this method are derived from hydrodynamic forces, they can be applied to the third question of flow



COMPARISON OF MEASURED AND EXPERIMENTAL VOID FRACTION OF COOK'S RUN 445

FIGURE 1

stability and transients. The correlation methods, on the other hand, are not based on dynamical flow relations of force and inertia, and hence are not descriptive of transient problems, without some patching up to account for the forces.

In this report, the equations and assumptions of the dynamical method are described and applied to a general two-phase, natural circulation flow loop. The conditions of stability for the linearized equations are derived, and the predictions are compared with the results of flow stability experiments conducted on the A.P.E.D. Heat Transfer Loop. An analogue computor circuit for solution of the equations is developed, and is applied to the ship's pitching motion problem, and to a parallel flow channel problem as well. Finally the main cause of two-phase flow pulsations, which is the dependence of the natural circulation driving head on inlet water velocity and subcooling, is discussed. The methods of stabilizing loops, as indicated by the physical parameters in the equations, are pointed out.

ANALYSIS OF TWO-PHASE FLOW LOOPS

The analysis of two-phase flow loops is divided into three topics: the mechanics of two-phase flow, the application of the mechanics to a flow loop, and the solution for small disturbances.

1. THE MECHANICS OF TWO-PHASE FLOW

The essence of the two-phase mechanics described in reference (3) is the statement of Newton's third law for each phase:

> The net force acting on the phase, plus the inertia of the phase, plus the phase to pipe shear force, plus the phase to phase shearing force, plus the phase weight equal zero.

The assumptions to give mathematical expression to the problem are as follows:

1. Steam and water flowing concurrently are in saturated equilibrium.
2. The flow process is adiabatic.

3. The transverse pressure gradient is negligible compared to the axial pressure gradient.
4. The shear stress between the phases and between each phase and the channel wall are analytic functions of the phase velocities, the phase volume fractions, and flow rate ratios.
5. The flow system pressure drop is small enough compared to the absolute pressure that the saturation enthalpies and densities are essentially constant.
6. The fluid mechanical energy is negligible compared to the thermal energy.

The two-phase flow equations follow from application of Gauss' law to momentum flow, and mass and energy conservation. Let $\underline{P}$ be pressure, $\underline{u}$ steam void fraction, $\underline{W}$ and $\underline{S}$ water and steam velocities, ρ density, Γ shear stress per unit length, $\underline{g}$ gravity, $\underline{q}$ heat rate per unit volume, and $\underline{h}$ enthalpy per unit mass. The equations take the form:

for water pressure gradient,

$$(1-u)\frac{\partial P}{\partial x} + \frac{\partial}{\partial x}(1-u)\rho_w W^2 + \frac{\partial}{\partial t}(1-u)\rho_w W + \Gamma_w - \Gamma_{ws} + (1-u)\rho_w g = 0 \qquad (1)$$

for steam pressure gradient,

$$u\frac{\partial P}{\partial x} + \frac{\partial}{\partial x} u\rho_s S^2 + - u\rho_s S_s + \Gamma_s + \Gamma_{ws} + u\rho_s g = 0 \qquad (2)$$

for conservation of mass

$$\frac{\partial}{\partial x}(1-u)\rho_w W + \frac{\partial}{\partial t}(1-u)\rho_w + \frac{\partial}{\partial x} u\rho_s S + \frac{\partial}{\partial t} u\rho_s = 0 \qquad (3)$$

and for conservation of energy

$$\frac{\partial}{\partial x}(1-u)\rho_w W h_w + \frac{\partial}{\partial t}(1-u)\rho_w h_w + \frac{\partial}{\partial x} u\rho_s S h_s + \frac{\partial}{\partial t} u\rho_s h_s = q(x,t) \qquad (4)$$

Normally the fluid pressure drop around a loop is small compared to the operating pressure in cases of interest, and the interchange between the phases that results from flashing or condensation is negligible outside of the heated section of the loop, from assumption (5). Thus, above the heated section, the water terms and the steam terms of equation (3) can be set separately equal to zero.

It is necessary to define the shear stress per unit length terms, Γ_W, Γ_S, and Γ_{WS}. For water to pipe stress per unit length, the following relation is assumed.

$$\Gamma w = f(Re)\,\frac{pw}{4A}\,\left(\tfrac{1}{2}\,\rho_w W^2\right).$$

f = D'Arcy Wiesbach friction factor (5)

pw = perimeter common to water and pipe

A = pipe section area

The friction factor depends on Reynolds number, which is taken to be that of water flow only. The common water perimeter is in general difficult to determine without measurement. It is equal to or less than the pipe perimeter. A way of calculating it suggested by S. Levy is described in Appendix A. Analogously for steam, the relation is

$$\Gamma s = f(Re)\,\frac{ps}{4A}\,\left(\tfrac{1}{2}\rho s S^2\right) \qquad (6)$$

where Reynolds number is calculated for steam only.

The water to steam shear stress must be zero when the fluid is all water or all steam, and it surely depends on the number and size of the bubbles, or characteristic radii of curvature of the interface between the phases when flow is not in bubble form. For water to steam shear stress per unit length, the relation is taken to be

$$\Gamma_{WS} = f(Re)\,\frac{4\Pi r_B{}^2}{4\cdot\frac{4}{3}\Pi r_B{}^2}\,\tfrac{1}{2}\rho(S-W)^2 u(1-u) \qquad (7)$$

r_B bubble radius.

The Reynolds number used is based on bubble flow, and the density is that of the predominant phase. The hydraulic diameter of bubble flow is based on the volumetric definition of Gunter and Shaw (7). For bubble radius the geometric mean of values suggested by Zuber and Tribus, Ref. (8), is used:

$$r_B = \sqrt{3}\ \sqrt{\tau/(\rho w - \rho s)\ g} \qquad (8)$$

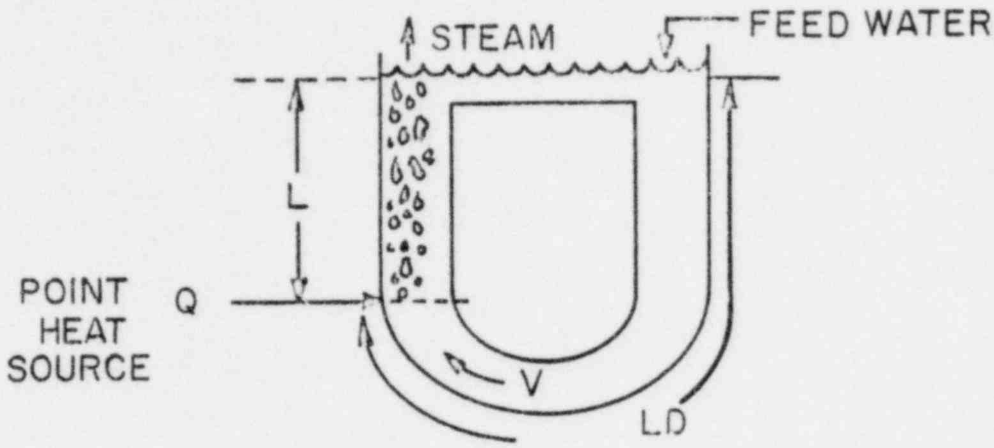
where τ = surface tension of water

Substitution of the auxiliary equations (5) through (8) in the principle equations (1) through (4) gives a system of four equations and four unknowns, p, u, w, and s. If the boundary conditions are known, the solution is determined.

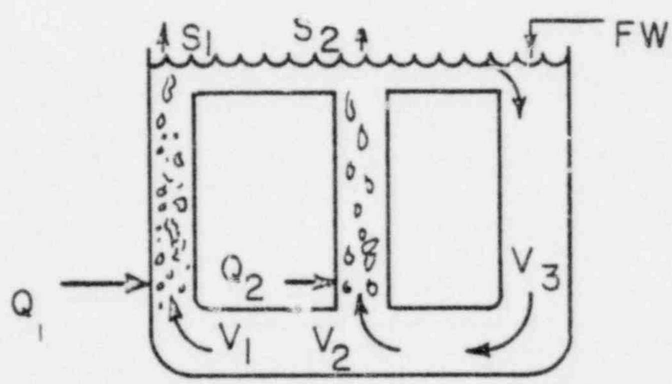
2. Application to the Flow Loop

The simple, single channel natural circulation loop of Figure 2 is chosen for the sake of clarity in presentation of the essentials in loop dynamics. Two assumptions help to simplify algebra considerably. The first is that feedwater flow is adjusted to keep inlet water subcooling constant: this assumption eliminates downcomer transit time effects. The general case is discussed in Appendix B. The second assumption, which eliminates energy storage effects in the heated section, is that heat is injected at a point rather than along a finite section. The equations can, of course, be generalized to account for a finite heated length.

The equations of the preceding section can be applied directly to the two-phase portion of the loop. One additional equation is necessary which relates pressure drop in the downcomer to single phase water velocity. With five equations and the five unknowns, p, u, w, s, and v, the problem solution is determined, but is not very tractable. One way of simplifying the solution is to work with average values of the variables in the two-phase section. The price of this simplification is loss of accuracy through ignoring velocity and void fraction gradients. With this assumption, we will not describe



SINGLE CHANNEL



COUPLED PARALLEL CHANNELS

NATURAL CIRCULATION LOOPS

FIGURE 2

slugging flow where, for example, water is rushing in at the bottom and falling back from the top of the section.

The average pressure gradient across the two-phase portion of the loop is a useful quantity to work with, because it is necessary to calculate the average steam void in the channel in order to ascertain the driving force. We will integrate and average equations (1) and (2) over the vertical length, L. The average void fraction is defined to be R,

$$R = \frac{1}{L}\int_{o}^{L} u \, dx \tag{9}$$

Integration of the acceleration terms is done in the following way. We make use of the identity

$$\frac{\partial(1-u)W^2}{\partial x} + \frac{\partial(1-u)W}{\partial t} = W\left[\frac{\partial(1-u)W}{\partial x} + \frac{\partial(1-u)}{\partial t}\right] + (1-u)W\frac{\partial W}{\partial x} + (1-u)\frac{\partial W}{\partial t} \tag{10}$$

and the fact that the water and steam portions of equation (3) are separately equal to zero away from the heating point, and finally that at the heating point, equations (3) and (4) may be combined, with the result:

$$\frac{\partial(1-u)W}{\partial x} + \frac{\partial(1-u)}{\partial t} = \begin{cases} o & x > o \\ -\dfrac{q(x,t)}{\rho_w(hs - hw)} & x = 0 \end{cases} \tag{11}$$

Use of the mean value theorem gives for equation (1)

$$(1-R)\frac{\Delta P}{\Delta x} + \rho w\left[(1-R)\frac{W^2}{2L} - \frac{v^2}{2L} + (1-R)\frac{dW}{dt} - \frac{W}{L}\left(\frac{Qs}{A\rho w\,(hs - hw)}\right) + (1-R)g\right] + \frac{1}{L}\int_{o}^{L}(\Gamma_w - \Gamma_{ws})\,dx = 0$$

and for equation (2)

$$R\,\frac{\Delta P}{\Delta x} + \rho_s \left[\frac{RS^2}{2L} + R\,\frac{dS}{dt} + \frac{S}{L}\left(\frac{Q_s}{A\rho_s\,(h_s - h_w)}\right) + Rg\right]$$

$$+ \frac{1}{L}\int_0^L (\Gamma s + \Gamma ws)\,dx = 0 \qquad (13)$$

where Qs is the steaming heat rate. The steaming rate itself is derived from conservation of energy:

$$Q_{total} = Q_{steaming} - Q_{subcooling} \qquad (14)$$

$$Q_{subcooling} = \rho_w A v\,(h_{sc} - h_w) \qquad (15)$$

where h_{sc} = inlet subcooled water enthalpy

To continue with reduction of the pressure drop equations, we add equations (12) and (13), divide by water density, and eliminate the steaming rate variable with equations (14) and (1[illegible] e result is

$$\frac{1}{\rho_w}\,\frac{\Delta P}{\Delta x} + (1 - R)\,\frac{[illegible]}{2L} - \frac{v^2}{2L} + (1 - R)\,\frac{dW}{dt} + \frac{R}{\beta}\,\frac{S\Delta S}{L} + \frac{R}{\beta}\,\frac{dS}{dt} + \frac{(S - W)}{L}\,\frac{s}{A_{\rho w}(h_s - h_w)}$$

$$+ (1 - R)_g + \frac{R}{\beta}\,g + G_w\,\frac{W^2}{2} + \frac{G_s}{\beta}\,\frac{S^2}{2} = 0. \qquad (16)$$

where $\beta = \frac{\rho w}{\rho s}$

At qualities less than 10%, and with water to steam density ratios greater than 10, the steam pressure drop terms in equation (16) may be neglected without serious error. The reactor conditions of current interest are within these limits. Outside of the limits, account should be taken of the steam acceleration and head terms. Accordingly we will neglect the term with $S\Delta S$, and with $(S-W)Q_s$, which is the acceleration of water that has become steam at the heated point. The time derivative of steam velocity will also be dropped, because the steam reaches its equilibrium velocity relative to water in times much shorter than the fluid transit time of risers in cases of current interest.

Equation (16) thus becomes

$$\frac{1}{\rho_W}\ \frac{\Delta P}{\Delta x} + (1 - R)\frac{W^2}{2L} - \frac{v^2}{2L} + (1 - R)\frac{dW}{dt}\ G_W\frac{W^2}{2} + (1 - R)g = 0. \tag{17}$$

In similar fashion equation (13) becomes

$$\frac{1}{\rho_W}\ \frac{\Delta P}{\Delta x} + G_{ws}\,(1 - R)\,\frac{(S - W)^2}{2} \tag{18}$$

where $G_{ws} = f\,(R_e)\,\dfrac{4\ \pi\ r_B^2}{4\cdot\frac{4}{3}\ \pi\ r_B^3}$

for the case of water fraction predominent.

The sum of pressure drops around the closed loop is zero. Therefore the pressure drop across the two phase section is the static head of the section height less the velocity head losses in the downcomer. Thus

$$\frac{1}{\rho_W}\frac{\Delta P}{\Delta x} + g - \frac{K}{L}\ \frac{v^2}{2} - \frac{L_D}{L}\ \frac{dv}{dt} = 0. \tag{19}$$

where K = single phase head loss coefficient

and L_D = total single phase length.

In the riser above the point of heat injection, the mass and energy conservation equations reduce to conservation of water and of steam separately. As a result, the volume flow through all coss sections of the riser is the same at any instant of time. Thus

$$(1 - R)\,W + RS = \Big[1 - u(x,0)\Big]\,W(x = 0) + u(x = 0)\,S(x = 0) \tag{20}$$

$$= \left(\frac{\rho_W}{\rho_s} - 1\right)\frac{Q}{A\rho_W(h_s - h_W)} - v\left(1 + \frac{h_{sc} - h_W}{h_s - h_W} - \frac{\rho_W}{\rho_s}\ \frac{h_{sc} - h_W}{h_s - h_W}\right)$$

A boundary condition at the two-phase section is required. This is obtained from equations (14) and (15), and from mass conservation at the heating point.

The equation is

$$(1 - u)\, W(x = 0) = v - \frac{Q - A\rho_w\, v(h_w - h_{sc})}{A\rho_w(h_s - h_w)} \tag{21}$$

where h_{sc} = inlet subcooled water enthalpy.

Finally we need the average steam void as a function of time:

$$R = \frac{1}{L}\int_0^L u(x, t)\, dx \cong \frac{1}{L}\int_{t=\frac{L}{w}}^{t} u(o, \tau)\, w d\tau. \tag{22}$$

Equation (21) is an approximation which assumes that a change in void fraction is transported across the section at the velocity of water.

Equations (17) through (22) are a set of non-linear differential and algebraic equations in the six variables, P, R, u(x=o), S, W, and v which approximate the simple loop dynamical system. The set has no solution in closed form for even simple initial disturbances, and is most accurately solved numerically.

3. Solution for Small Disturbances

For small disturbances the loop dynamics can be investigated by linearization of the equations: that is to say, by taking small variations of the variables about a steady-state solution, and ignoring second and higher order terms and cross products in the equations which relate the variables. The expanded form of the variables is

$$\frac{\Delta P}{\Delta x} = \left(\frac{\Delta P}{\Delta x}\right)_o + \Delta\left(\frac{\Delta P}{\Delta x}\right)$$

$$S - W = (s_o - w_o) + \Delta(S - W)$$

$$W = W_o + \Delta W$$

$$u = u_o + \Delta u$$

$$R = u_o + \Delta R$$

$$v = v_o + \Delta v$$

The set of variables with the subscript zero comprise the steady state solution, which is found by setting the incremental variables equal to zero, and by solving equations (17) through (21) simultaneously. Equation (22) is not required for the steady-state solution, because the average value of void fraction is u_o. With linear differential equations it is also convenient to apply the Laplace Transformation, and write the equations in matrix-operator form. The transformation is straightforward except in the case of the average void, R, which is

$$\mathcal{L}\text{xform}\,(R) = h\left\{\frac{1}{L}\int_{t-\frac{L}{W}}^{t} u(t)W\,dt\right\} = \frac{W}{L\sigma}\left(1 - \epsilon^{-\frac{L\sigma}{W}}\right)\mathcal{L}(u) \qquad (23)$$

where σ = complex frequency RAD/SEC.

The equations are normalized by the substitution of a dimensionless frequency variable $\mu = \frac{L\sigma}{W}$, which converts the time scale from seconds to transit time units. The set of equations in matrix form is given in equation (24). The disturbances we are interested in are normally heat input, and gravity changes caused by ship's motion. These disturbances are, therefore, placed on the right-hand side of equation (24).

$$
\begin{bmatrix}
1 & 0 & (1-u_o)\frac{W_o}{L}\mu + (1-u_o)\frac{W_o}{L} + G_w W_o & 0 & -g - \frac{W_o^{\,2}}{2L} & -\frac{v_o}{L} \\
1 & G_{ws}(S_o-W_o)(1-u_o) & 0 & 0 & -G_{ws}\frac{\left(S_o-W_o\right)^2}{2} & 0 \\
1 & 0 & 0 & 0 & 0 & -\frac{L_D}{L}\frac{W_c}{L}\mu - \frac{K_{vo}}{L} \\
0 & u_o & 1 & 0 & (S_o - W_o) & -(1 + \alpha - \alpha\beta) \\
0 & 0 & 1 - u_o & -W_o & 0 & -(1+\alpha) \\
0 & 0 & 0 & -\frac{1-\epsilon^{-\mu}}{\mu} & 1 & 0
\end{bmatrix}
\cdot
\begin{bmatrix}
\frac{1}{\rho_w}\Delta\frac{\Delta P}{\Delta x} \\
\Delta(s-w) \\
\Delta W \\
\Delta u \\
\Delta R \\
\Delta v
\end{bmatrix}
=
\begin{bmatrix}
u_o \Delta g \\
0 \\
\Delta g \\
(\beta - 1)\Delta\gamma \\
\Delta\gamma \\
0
\end{bmatrix}
\qquad (24)
$$

where $\alpha = \frac{h_w - h_{sc}}{h_s - h_w}$

$\beta = \frac{\rho_w}{\rho_s}$

$\gamma = \frac{Q}{A\rho_w\,(h_s - h_w)}$

We are interested in both the transient response of the flow loop and in its stability. First let us consider stability. The system is stable if the transient response to a disturbance decreases exponentially. The theory of ordinary linear differential equations with constant coefficients shows this to be the case if the real parts of the roots of the characteristic equation are negative. In the problem at hand the characteristic equation is simply the determinant of the matrix operator of equation (24) set equal to zero. Reduction of this determinant gives

$$B_1 \mu^2 + \left[B_2 + B_3 \frac{1 - \epsilon^{-\mu}}{\mu}\right]\mu + B_4 + B_5 \frac{1 - \epsilon^{-\mu}}{\mu} = 0 \tag{25}$$

where $B_1 = \dfrac{L_D/L}{1 + \alpha} \cdot \dfrac{W_o u_o}{LG_{ws}(S_o - W_o)(1 - u_o)}$

$$B_2 = 1 - \frac{\alpha\beta}{1 + \alpha} + \frac{\mathbf{v}_o}{L} \cdot \frac{u_o}{G_{ws}(S_o - W_o)(1 - u_o)} + \frac{Kv_o u_o}{(1+\alpha)G_{ws}(S_o - W_o)1 - u_o}$$

$$+ \frac{L_D/L}{1 - \alpha}\left[\frac{u_o}{1 - u_o} \cdot \frac{G_w W_o}{G_{ws}(S_o - W_o)(1 - u_o)} + \frac{1}{1 - u_o}\right]$$

$$B_3 = \left(1 + \frac{L_D/L}{1 - \alpha}\right)\left(\frac{S_o - W_o}{W_o}\right)\left(1 + \frac{1}{2(1 - u_o)}\right) - \frac{L_D/L}{1 + \alpha} \cdot \frac{u_o\left(g + \frac{W_o^2}{2L}\right)}{W_o G_{ws}(S_o - W_o)1 - u_o}$$

$$B_4 = \frac{LG_w}{1 - u_o} \cdot \left[1 - \frac{\alpha\beta}{1 + \alpha} + \frac{\mathbf{v}_o/L}{1 + \alpha} \cdot \frac{u_o}{G_{ws}(S_o - W_o)(1 - u_o)}\right] + \frac{K}{1 + \alpha} \cdot$$

$$\frac{\mathbf{v}_o}{w_o}\left[\frac{1}{1 - u_o} + \frac{u_o}{1 - u_o} \cdot \frac{G_w W_o + W_o/L}{G_{ws}(S_o - W_o)(1 - u_o)}\right]$$

$$B_5 = \left[\frac{K}{1 + \alpha} \cdot \frac{\mathbf{v}_o}{w_o} + \frac{LG_w + 1}{1 - u_o}\right]\left(\frac{S_o - W_o}{W_o}\right)\left(1 + \frac{1}{2(1 - u_o)}\right) +$$

$$\left(\frac{L_g}{W_o^2} + \frac{1}{2}\right)\left[\frac{u_o}{1 - u_o} + \frac{\alpha\beta}{1 + \alpha} - \frac{u_o K \mathbf{v}_o}{LG_{ws}(S_o - W_o)(1 - u_o)}\right]$$

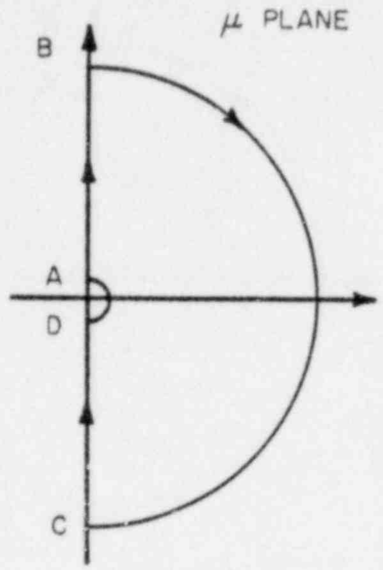
To find the roots of equation (25) is a time consuming task. The question of stability, however, can be answered by an indirect method. Let us call the left hand side of equation (25) a function F of the complex variable μ. To find out if F has zeroes in the right half of the μ plane, we draw a contour enclosing the right half plane, as shown in Fig. 3(a) and map the contour on the F plane. By inspection of equation (25), we see that F has no poles in the right half plane excluding the origin. If F has zeroes in the right half μ plane, then from Cauchy's Index Theorem the contour on the F - plane must encircle the origin once for each zero in the clockwise direction. In Fig. 3(b) a representative F contour is shown for the case of no such zeroes. Fig. 3(c) shows a contour indicating two zeroes in the right half plane.

In order to determine whether or not a two phase loop is stable, we can calculate the coefficients of equation (25) from the steady-state solution, and plot the F contour for the positive imaginary values of frequency beginning with 0, until the contour crosses the imaginary axis. As a practical matter, the system is stable if the crossing is above the origin, and unstable if below.

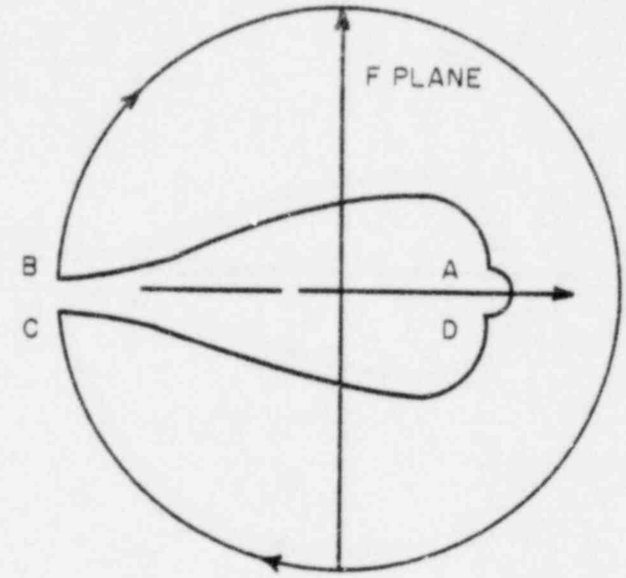
The transient solution of equation (24) for arbitrary disturbances is readily done by means of an analogue computor. We shall return to this topic after discussion of some experimental results.

COMPARISON OF PREDICTION AND EXPERIMENT

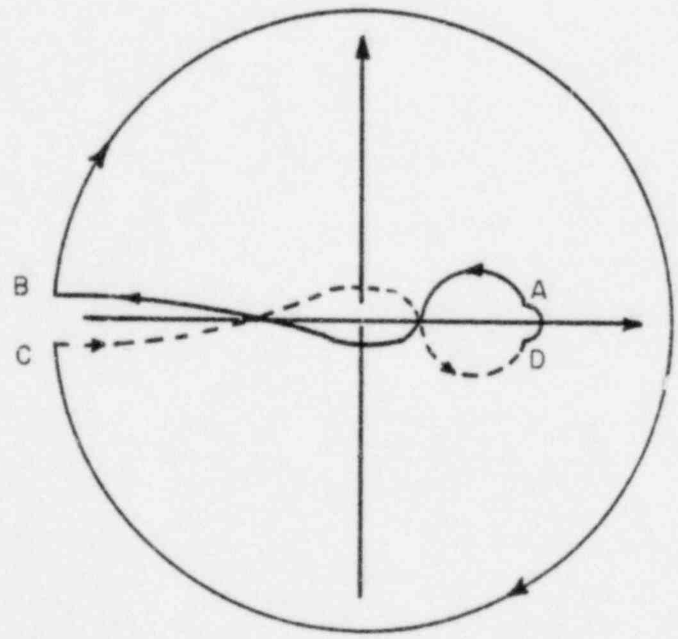
Hydraulic instability tests on the A.P.E.D. large heat transfer loop were performed and reported in Reference (9). At the time of test, the analysis had not been finished. When the analysis was finished, it became apparent that the test data was incomplete and could not be adequately checked with prediction. Several of the tests were therefore repeated, and the missing information, which was pressure drop across the two phase portion of the loop, was obtained. The experimental



(A) CLOSED CONTOUR ENCIRCLING RIGHT-HALF COMPLEX FREQUENCY PLANE



(B) F CONTOUR - NO ENCIRCLEMENT - NO ROOTS IN RIGHT-HALF PLANE.



(C) F CONTOUR - TWO ENCIRCLEMENT - TWO ROOTS IN RIGHT-HAND PLANE

INSPECTION OF CHARACTERISTIC EQUATION TO LEARN WHEREABOUTS OF ROOTS

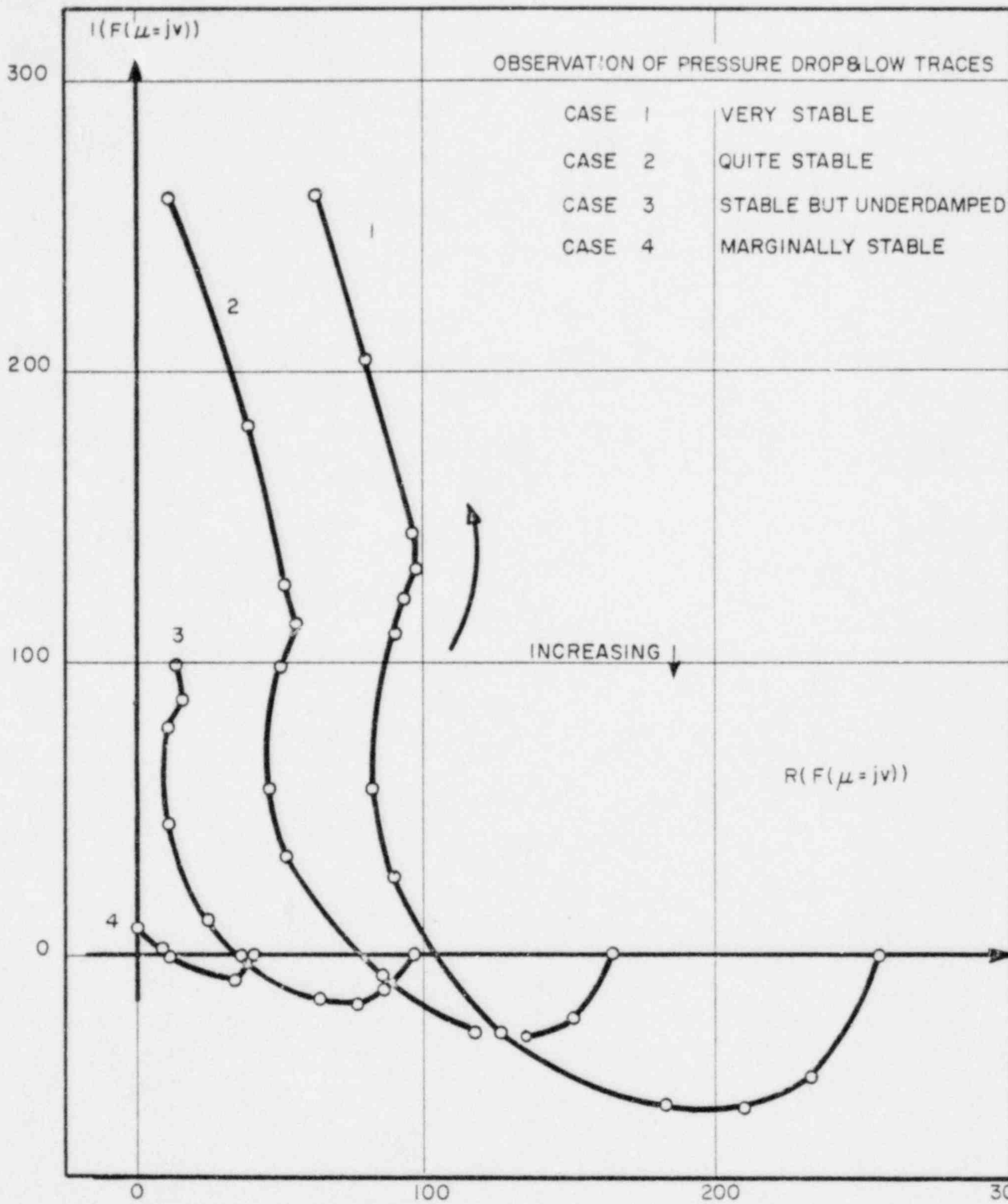
FIGURE 3

values of heat input, subcooling, inlet velocity and two-phase total pressure drop were used to solve the analytical equations for steam void fraction (which could not at the time be measured). This value for void fraction was trustworthy to require accuracy because it checked closely with Larson's void fraction versus quality data (Ref. 2).

The coefficients of equation (25) were calculated from the data, and the partial $\underline{F}$ contours for four cases are shown in Fig. 4. The four cases proceed from Case 1 - very stable, to Case 4 - marginally stable. From the discussion in the preceeding paragraph, it is clear that the predictions of the four cases would be qualitatively the same, because the contour of the first case crosses far above the origin, and the fourth almost at the origin. Let us now look at the inlet flow and power traces for the first three cases, in Fig. 5, 6, and 7. The disturbances were imposed on the loop by small changes of the loop heating rate. The first case is strongly damped, the second slightly less damped, and the third is noisy and overshoots. The fourth case - the trace has been misplaced - was observed to show lightly damped oscillations after the power change. The degree of damping of response in the different cases was controlled by a valve setting in the downcomer line.

THE TRANSIENT RESPONSE OF TWO-PHASE FLOW LOOPS

Transient traces of flow loop operation are important to developing quantitative knowledge of two-phase flow mechanics. For example, in developing a model for BWR dynamics, it was hypothesized that a sudden increase of heat input in the reactor would cause the inlet water velocity to decrease temporarily as a result of the impulse of steam formation increase. Recently analogue computer work on the two-phase flow equations showed the same velocity effect, and it



STABILITY PLOT - F CONTOUR FROM EXPERIMENTAL DATA

FIGURE 4

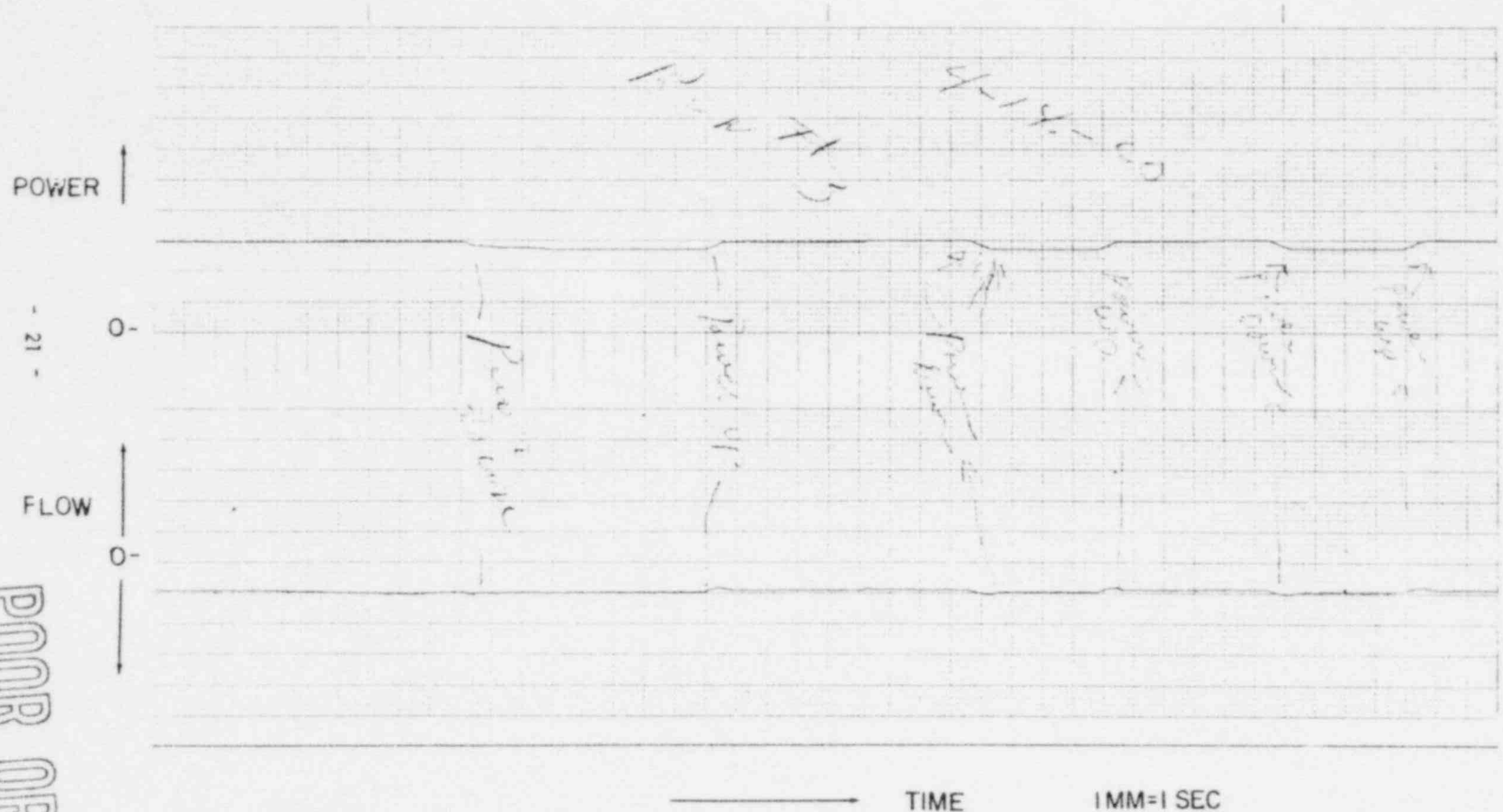


FIGURE 5 HEAT TRANSFER LOOP

FLOW TRANSIENT CASE 1

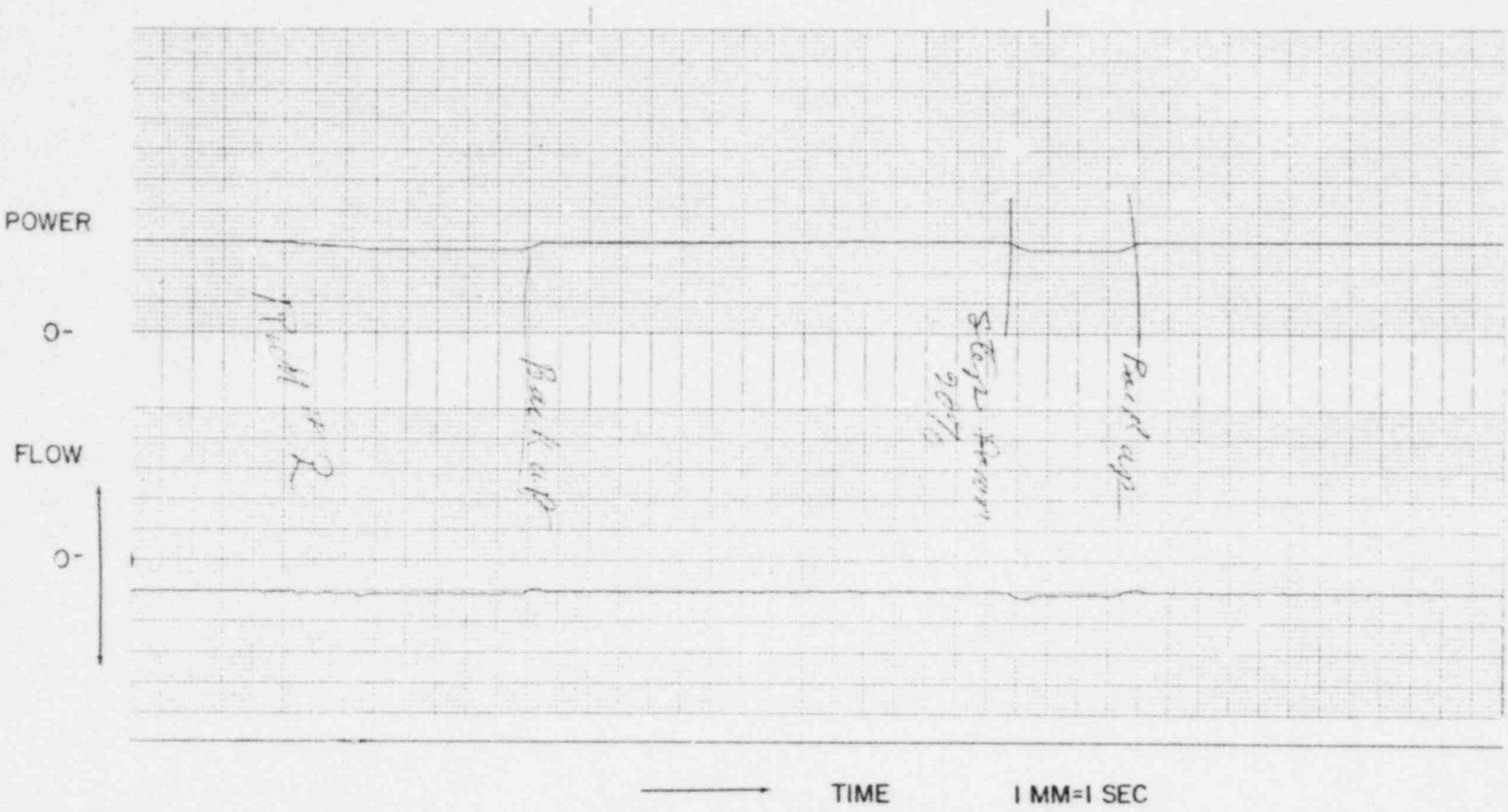


FIGURE 6 HEAT TRANSFER LOOP

FLOW TRANSIENT CASE 2

FIGURE 7 HEAT TRANSFER LOOP

FLOW TRANSIENT CASE 3

was obvious that a simple experiment could be performed to measure it. The heat increase and decrease experiment was performed first at 30 psia on the visual burnout loop designed by E. Janssen. Two typical charts of power input to the loop and inlet water velocity are shown in Figures 8 and 9. The experiment was repeated twelve times and in each case a flow decrease pip appeared after a power increase and vice versa.

A similar experiment was carried out on the heat transfer loop at 1000 psia under more carefully controlled conditions than are feasible on the atmospheric loop. Typical test results are shown in the chart of Figure 10. The same effect is also evident in Figures 5, 6, and 7. In every test run, the inlet water velocity initially decreased after the heat rate increased and settled to a new equilibrium value which was higher than the initial value if the loop was operating on the rising portion of the natural circulation flow curve, or lower if on the descending part of the curve. In Figure 10 the power was increased twice and decaying oscillations ensued.

The loop conditions which produced the flow trace in Fig. 10 were used to obtain coefficients for an analogue computor solution of equation (24), which is shown in Fig. 11. The actual flow trace and the computor trace are quite similar, and compare quantitatively as follows for a heating rate increase of 7%:

	Observed	Computed
Inlet velocity transient	- 32%	- 30.5%
Oscillation period	3.0 sec.	2.4 sec.
Damping factor	0.066	0.1

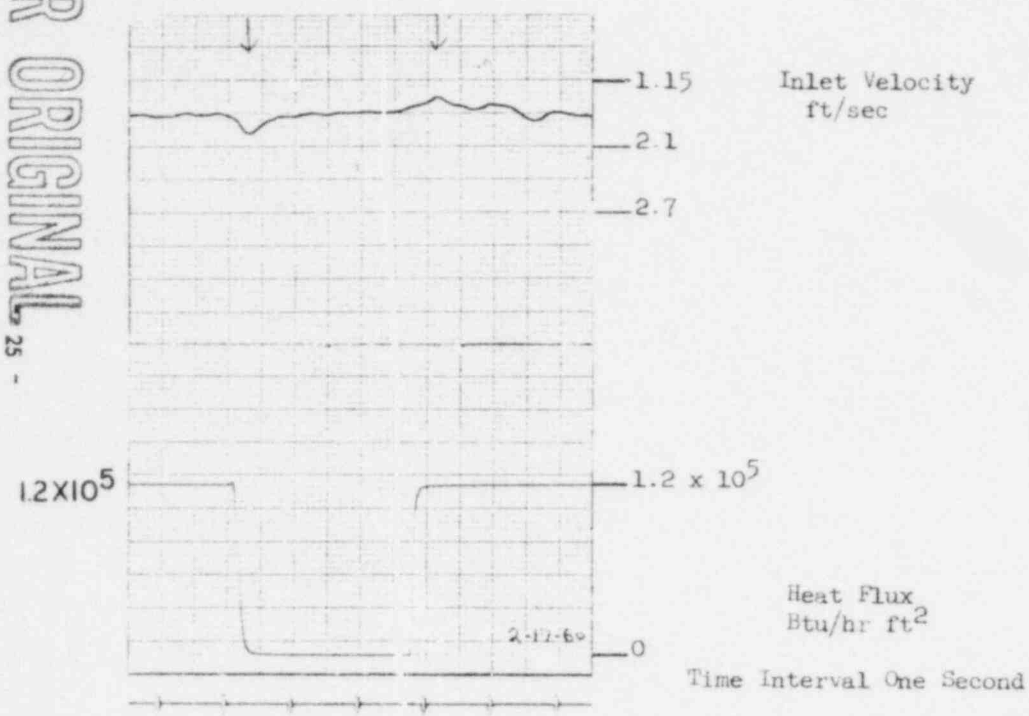


Fig 8

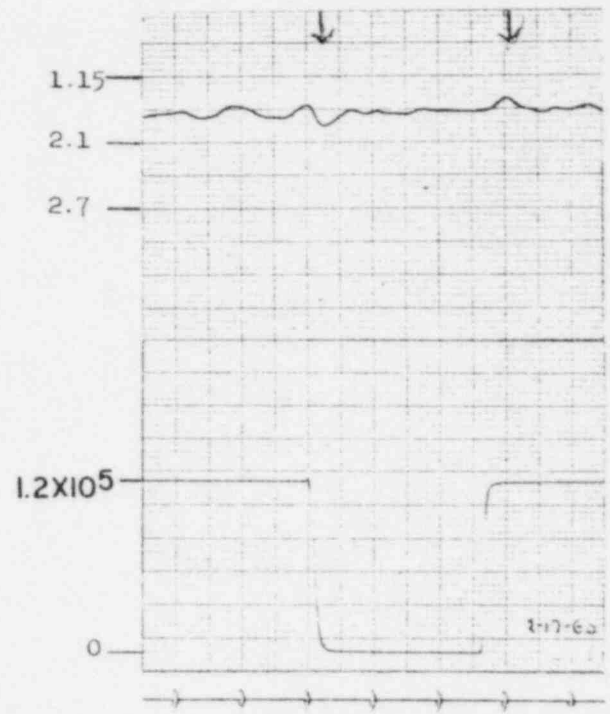


Fig. 9

FLOW VARIATION WITH PULSED INPUT POWER
PERFORMED AT 30 PSIA ON
JANSEN VISUAL BURNOUT LOOP

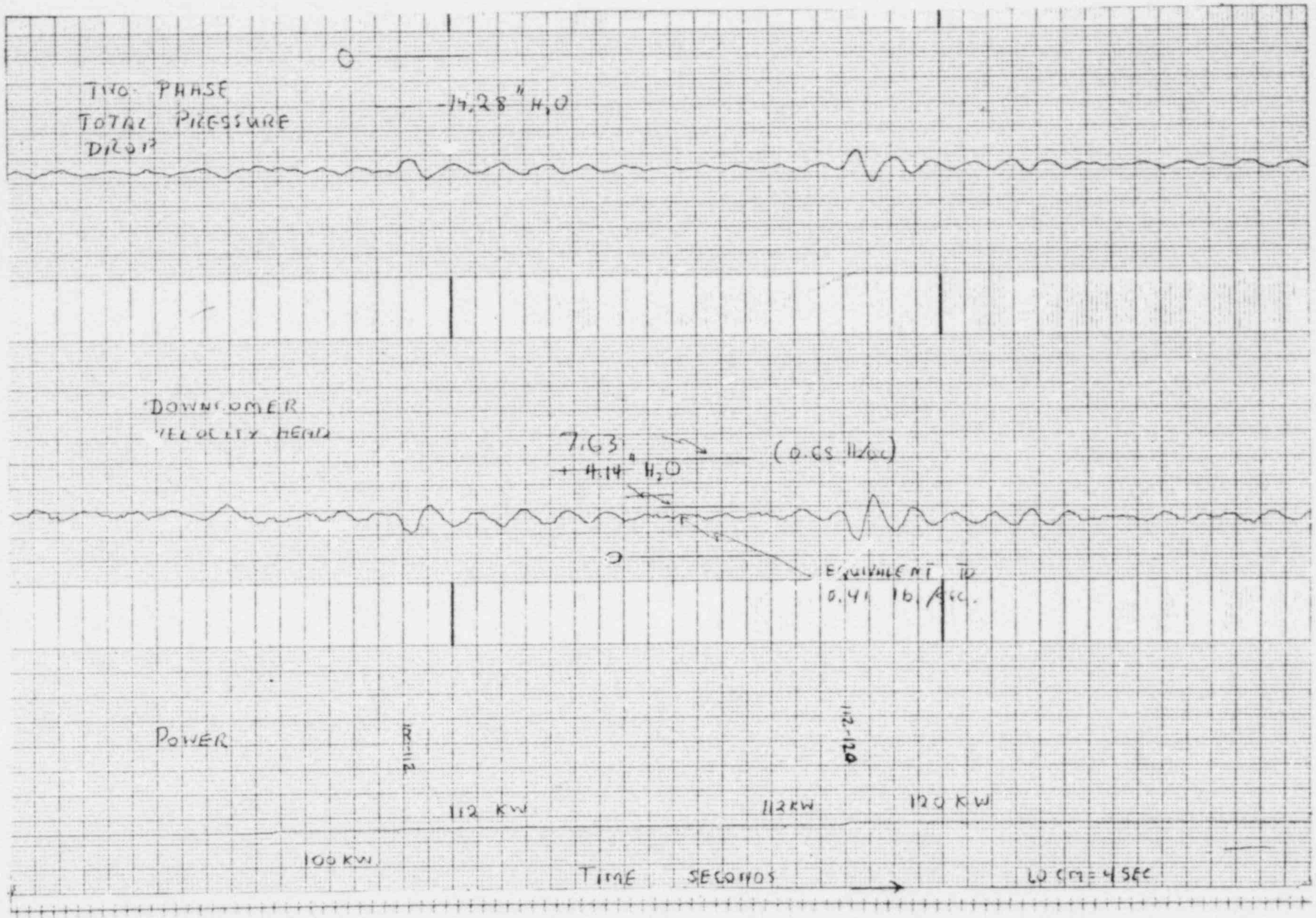


FIGURE 10 HEAT TRANSFER LOOP FLOW TRANSIENT TEST

POOR ORIGINAL

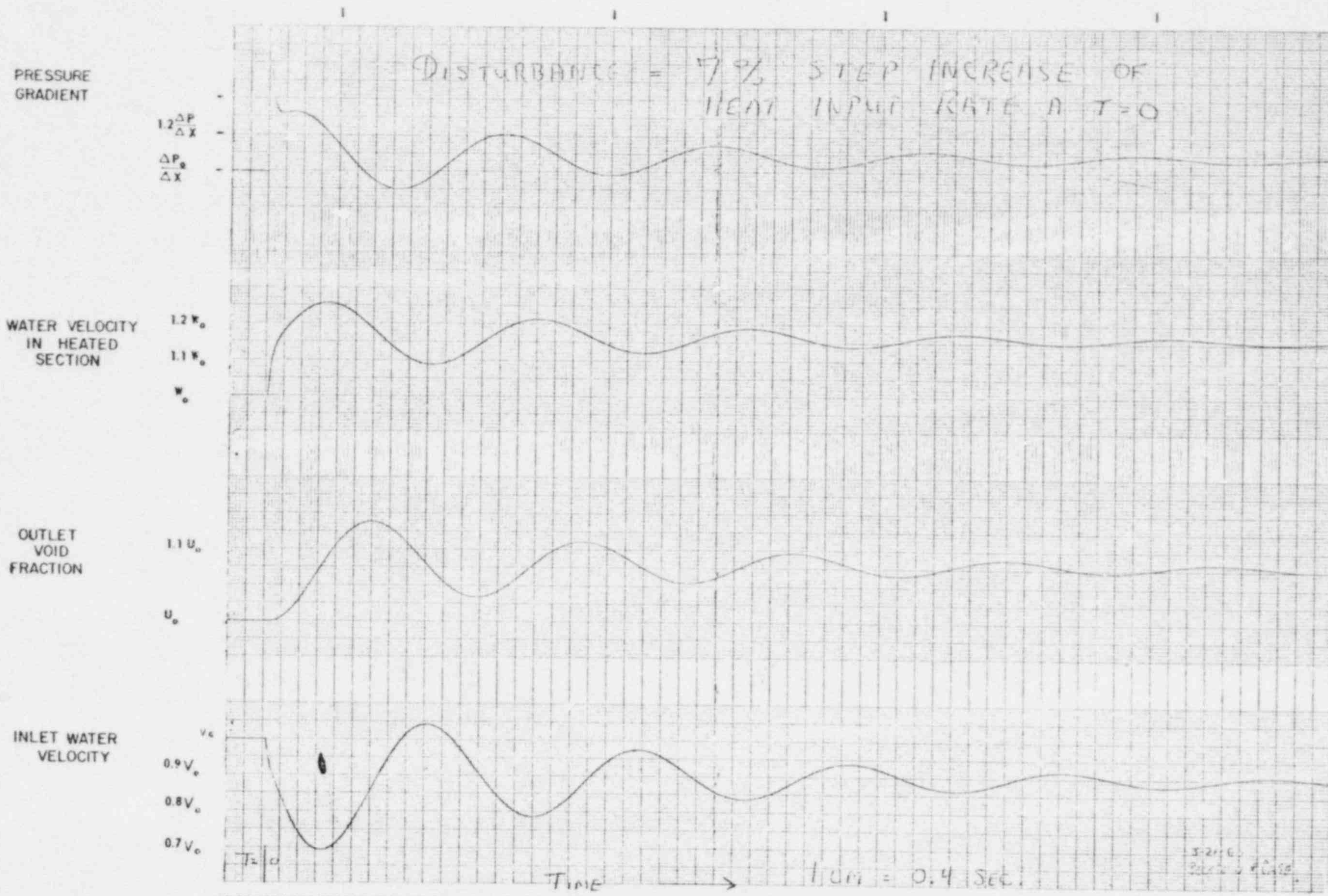


FIGURE II ANALOGUE COMPUTOR TRACE OF FLOW TRANSIENT

The error in period and damping factor is attributable at least in part to the approximation of riser transit time used in the analogue simulation. The approximation errs at high frequencies.

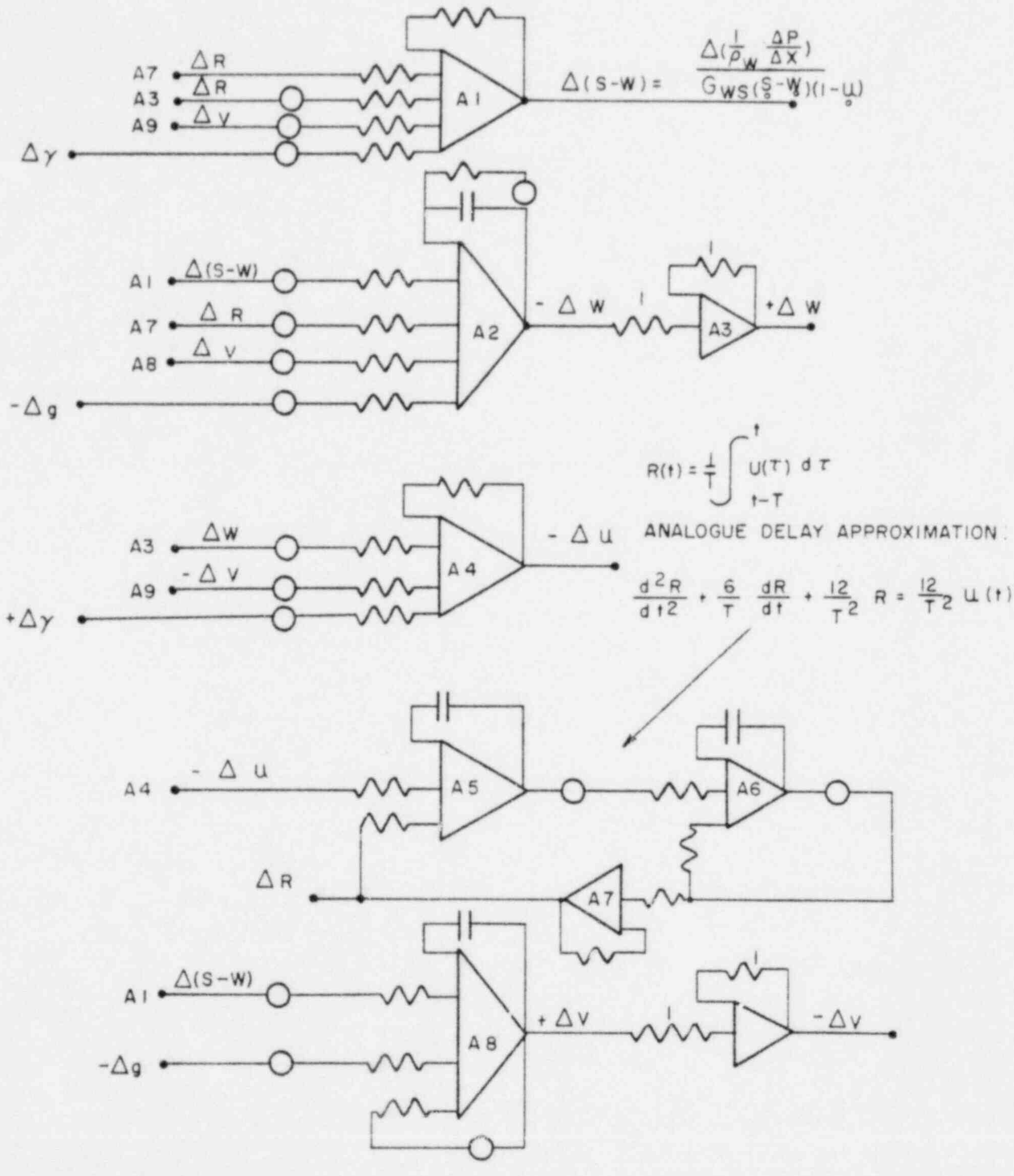
The pressure gradient trace in Figure 11 deserves comment. The input step disturbance causes an impulse to appear initially. This impulse is a mathematical consequence of the assumption that the fluids in the two-phase mixture are incompressible. It is therefore not physical, as is clearly demonstrated in the test results.

ANALOGUE COMPUTATION OF TRANSIENTS

The analogue computor circuit which was used to calculate the traces in Fig. 11 is shown in Fig. 12. The circuit is composed of linear networks and follows directly from equation (24).

The analogue computation of flow transients was developed to determine ship's motion effects on reactor dynamics. The principal effects are (1) the acceleration of pitching and rolling which affects all channels symmetrically and (2) the effects of sloshing which may affect channels across the core assymetrically. To conclude the discussion, therefore, analogue simulation of varying acceleration and of hydraulic coupling of parallel two-phase flow channels, which is necessary for the study of assymetric effects, is presented.

Natural circulation in the two-phase flow loop is caused by the difference in weight between two fluid columns: the mean density of the riser column is less than the density of the downcomer, because part of the riser volume contains steam. Weight or gravity force appears in the loop pressure drop equations with the term containing the factor "g", which is the acceleration of gravity. When the flow loop is accelerated, the weight of the columns



ANALOG COMPUTER CIRCUIT FOR TRANSIENT SOLUTION OF LOOP DYNAMICS

FIGURE 12

changes by the component of acceleration force which parallels the gravitational vector. In ship's motion the accelerations are transverse and rotational, as well as along the riser flow direction. The flow loop is confined by channels and thus the transverse accelerations affect the mass distrubution of the two-phase mixture across the channel. The change in fluid distribution across the channel will in turn affect the pressure drops along the flow direction. In a first approximation, however, the transverse effects are assumed to be negligible. Under this assumption, the transients that follow acceleration changes can be computed by variation of the weight term in equations (17), (18), and (19) by the amount of the axial component of acceleration.

The solution following a step change of 20% of gravitational acceleration along the vertical axis is shown in Fig. 13 for conditions of the T-7 Tanker Reference Core. The traces show the inlet water velocity to reach a higher equilibrium value and the average channel steam void a lower value, as is expected. The small dip in average channel water velocity, W, immediately following the step is intriguing. Since the velocities are measured from the channel frame of reference, the water velocity in the heated section will initially decrease because the channel is accelerated upward. Shortly, however, the heavier downcomer leg predominates and increases both inlet and channel velocities. The equilibrium values after the 20% acceleration increase are:

pressure drop	+ 20 %
chimney water velocity	+ 2.7%
inlet water velocity	+ 8.5%
average void fraction	- 11.7%

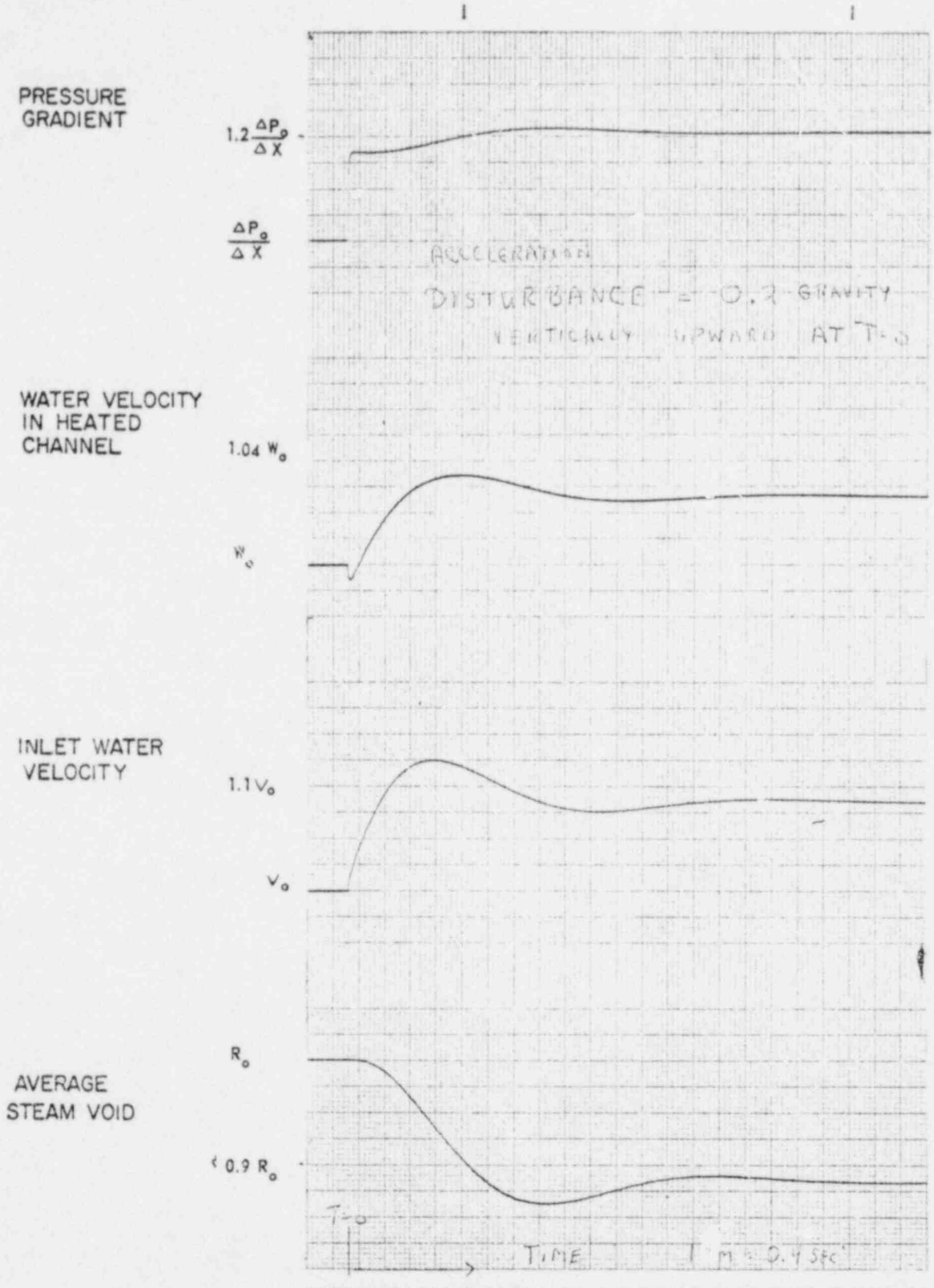


FIGURE 13 FLOW TRANSIENT CAUSED BY ACCELERATION

The damped natural period is 2.6 seconds. The results given in this paragraph are purely theoretical. A comparison of theory and experiments on an accelerated, rocking two-phase flow loop at 1000 psia are reported in Ref. (10).

The simulation of parallel channel flow problems is a simple extension of the mechanics of single channel problems. The parallel channels are coupled by a common downcomer velocity head loss, v_3, as shown in Figure 2(b). One set of equations is written for each heated channel, including the individual head losses. An additional equation relates the common head loss to the driving pressure. The effect of an increase in heat rate in one channel on the other channel of a coupled, initially symmetrical system is shown in Figure 14. The parameters represent two channels similar to the T-7 Reference Core Channels. In this problem the velocity head loss in the common leg was chosen to be equal to the separate leg losses. The result is to decrease the flow in channel whose input of heat is unchanged, by decreasing the driving pressure. Also, it is noted, the swing of variables in one channel is out of phase with the swing of corresponding variables in the other channel. The equilibrium values following a 20% increase of heat rate in channel 1 are:

Channel 1	
inlet water velocity	+ 5%
average steam void fraction	+17%
Channel 2	
inlet water velocity	-0.4%
average steam void fraction	+0.9%

The effects of parallel channel hydraulic coupling are explored more fully in Ref. (11).

A discussion of downcomer transit time effects is given in Appendix B.

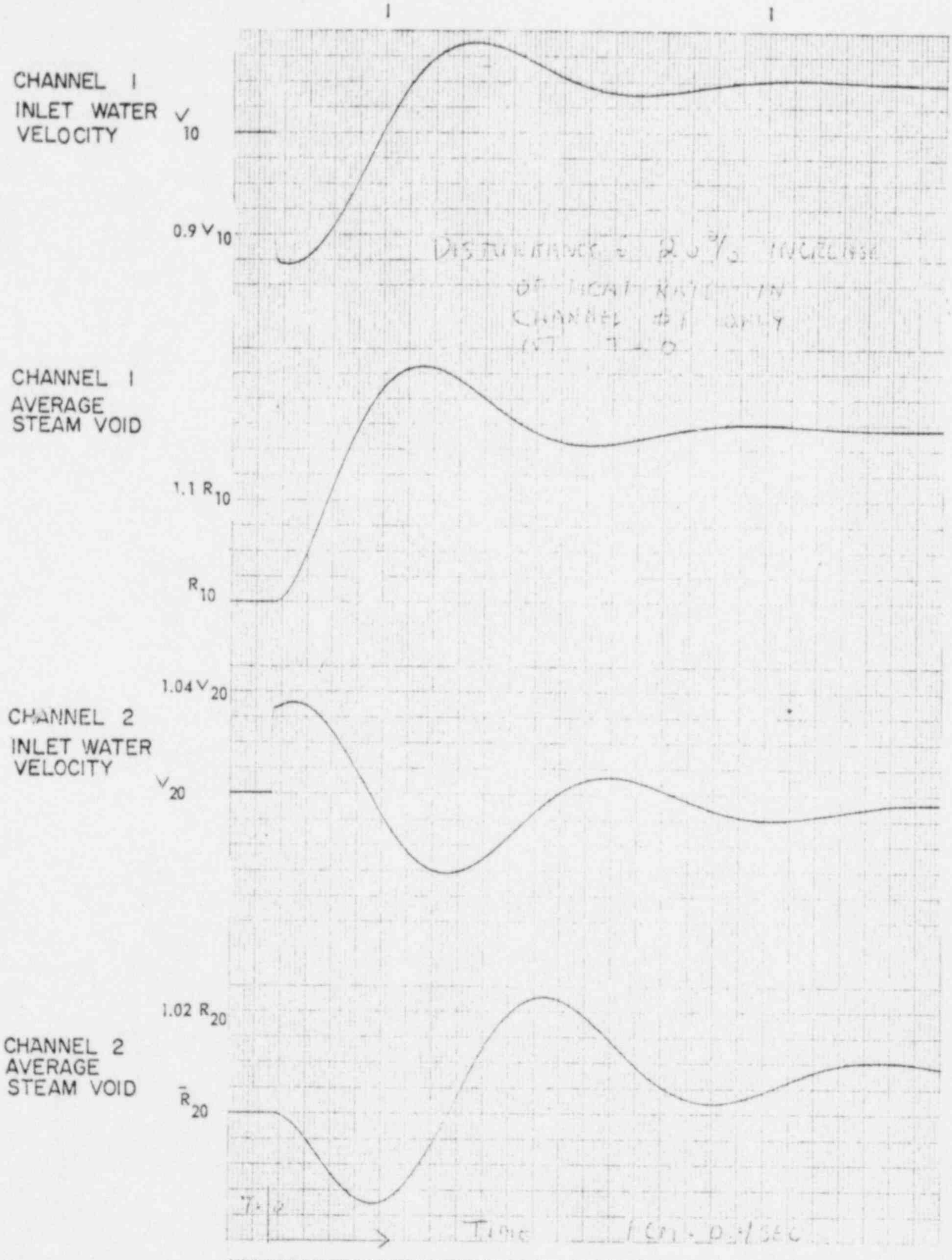


FIGURE 14 PARALLEL CHANNEL TRANSIENT

CONCLUSIONS

The two-phase flow mechanics presented in this report closely check the steady state void measurements of W. H. Cook. The mechanics check his results a priori, using only the input conditions of operating pressure, section geometry, inlet velocity, and heating rate. The loop analysis and the two-phase mechanics together successfully explain the pubations and transients observed on the A.P.E.D. 1000 psia Heat Transfer Loop.

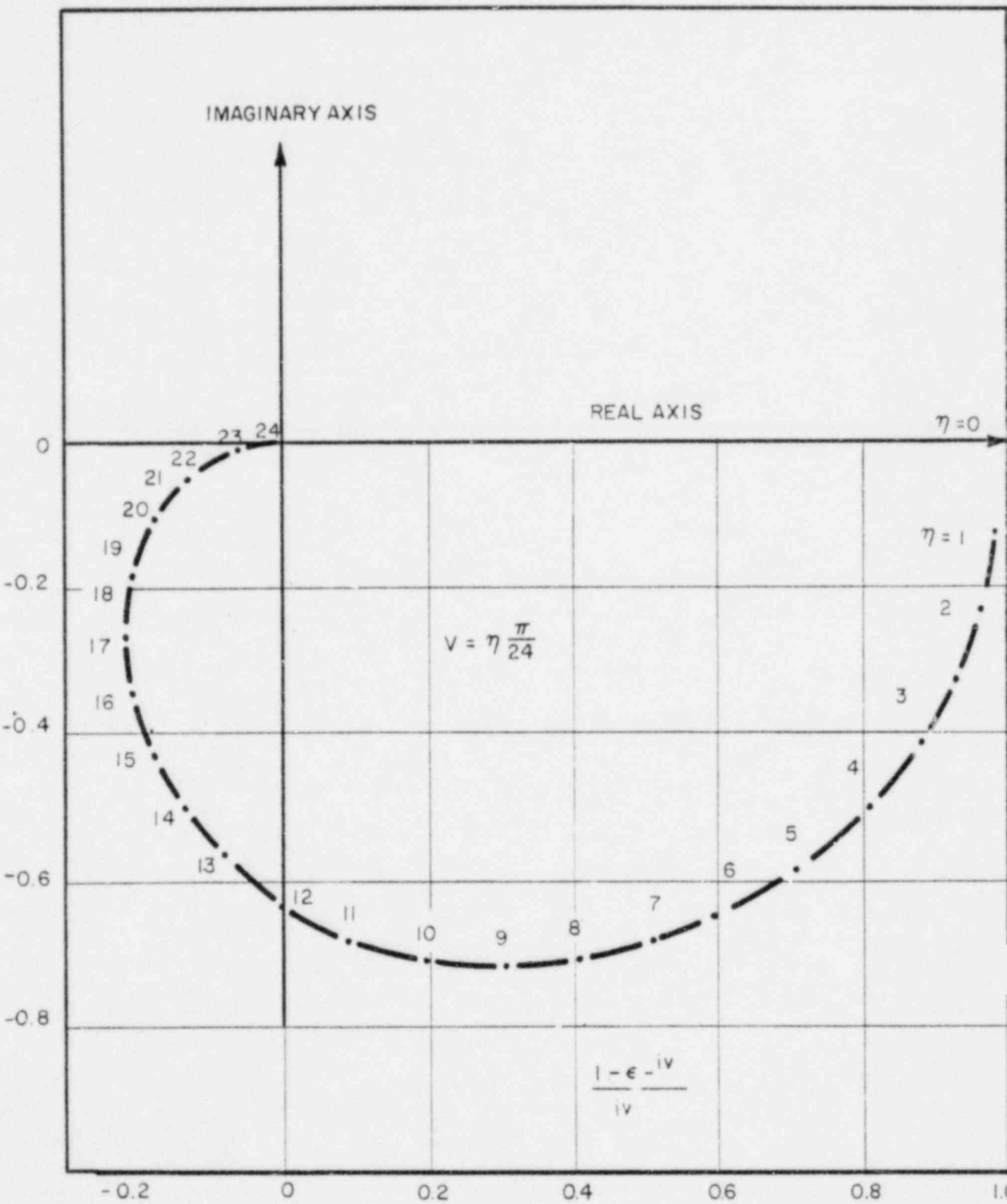
The parameters governing two-phase flow loop dynamics are revealed by studying the loop equations. First we consider fluid transit time of the two-phase section. Because the loop driving force is weight difference of the columns, and because changes in the average void fraction are transported across the riser at the average two-phase fluid velocity, it is clear that the loop dynamics in the time domain will be dominated by the transit time. Another way of stating this fact is that a change in steaming conditions at the inlet builds up an effect on the driving head during one complete transit time of the riser. The mathematical consequences are evident in the time delay term in equation (22). Experimentally it has been noted that loop oscillations when they occur, do so at cyclic periods a bit longer than the transit time. Analysis predicts the period of an oscillatory system 4/3 to 2 times the two-phase section transit time.

The second point is the dependence of steaming rate on inlet water velocity and subcooling. This dependence is the primary cause of flow pulsations in the A.P.E.D. loop, and it is of concern in the range of interest to boiling water reactor design. Paradoxically this cause is thermodynamic and not hydrodynamic. Consider the energy equation:

steaming heat rate = total heat rate -
(subcooling per unit mass) (mass flow rate) (area).

If we suppose inlet velocity increases in a loop that has been in a steady state of operation, the equation states that steaming rate will decrease. The riser steam fraction will then decrease, and with it the driving head during the passage of a transit time. The loop velocity will decelerate as a consequence, and as velocity decreases, the steaming rate will rise. If certain conditions are met, the process will be oscillatory and undamped. The main conditions are that steaming be strongly dependent on inlet velocity, and that the natural circulation driving head be substantial. The conditions can be met with high subcooling, and high heating rate. They can also be met at low heating rates with low inlet velocities, but the latter conditions are of less practical interest. The degree of subcooling is related to the operating pressure and the specific volume ratio of saturated water and steam, as will be discussed.

It is useful to examine the structure of the characteristic equation (25) to find out what the stabilizing and de-stabilizing influences are. We deduced from the complex variable plots in Fig. 3 that the flow loop is stable if the plot of $F(\mu) = F(iv)$ passes from the first to the second quadrant without crossing under the origin as v is increased from zero to high positive frequency, and vice-versa. If the coefficients of the delay function in equation (25), which are B_3 and B_5, are identically zero, the function is a simple quadratic. Furthermore, if the remaining coefficients B_1, B_2, and B_3 are positive, it is obvious that F passes from quadrant 1 to quadrant 2 above the origin. We will now see how destabilization is caused by the delay function $\frac{1 - \epsilon^{-\mu}}{\mu}$ which is plotted in Fig. 15. The numerator is sinusoidal at periods of 2Π, and the function proceeds in a spiral-like form in a clockwise direction under the abscissa for positive and increasing values of v, and its magnitude approaches zero. If the coefficients B_3 and B_5 are now allowed positive values, it is apparent that they displace the F contour at each point by a vector



PLOT OF DELAY FUNCTION

FIGURE 15

ıving the direction of the delay function for each frequency. In the case of B_5, the direction, using compass terms, repeats itself pointing first South-East and then South-West. It is the South-West direction, with $\Pi > v > 2\Pi$ that destabilizes, for a sufficiently large coefficient B_5 will cause the F Contour to pass under the origin. As a practical matter, the frequency of oscillation will be $\Pi < v < \frac{3\Pi}{2}$ hich corresponds to period e between $4/3T$ and 2 T.

A condition of instability is that B_5 must exceed B_4 by about a factor of 4, because the delay function magnitude is attenuated by roughly that factor in its destabilizing direction. The system can nevertheless be stable if the coefficient B_2 is large enough and B_3 is sufficiently smaller than B_2.

The parameters which comprise the coefficients show how to determine system characteristics, and also give considerable physical insight into the problem. The destabilizing coefficient, B_5, is effectively the velocity dependent portion of the static head pressure drop in the riser. It is increased by long risers and by subcooling. It is decreased by high fluid velocity. The stabilizing coefficients are B_2 and B_4. B_4 represents the velocity head losses in both the single phase and two-phase sections, and large head losses will always stabilize the system. B_2 is a combination of head loss terms and fluid inertia terms by virtue of the downcomer to riser length ratio L_d/L, and will also tend to stabilize. The coefficient B_1 is essentially an inertia quantity. As B_1 which is proportional to L_d/L, is increased from zero (by supplying water from an infinite reservoir at the section inlet) its effect on stability proceeds from no effect to possibly adverse, and fi: .lly to an absolutely stabilizing effect, when the downcomer inertia is so large that the velocity cannot change appreciably during one riser transit time.

The effect of subcooling on the velocity dependent static head has been discussed, and is evident in the term $\frac{\alpha\beta}{1+\alpha}$ in coefficient B_5. Subcooling has another important and surprising effect. It reduces the stabilizing influence of riser velocity head loss by means of the term $-\frac{\alpha\beta}{1+\alpha}$ in B_2 and B_4. To understand why, we will examine equation (20). At sufficiently high subcooling, $\frac{\alpha\,\beta}{1+\alpha} > 1$, the riser steam [illegible]or volume flow rate decreases for an increase of inlet velocity. [illegible] is caused by subcooling. Normally the steam slip velocity i[illegible] sensitive to inlet velocity changes, and therefore the void fraction decreases with the steam volume rate. Thus the water acquires a larger flow area. Although water volume rate increases, its velocity may decrease. If the water velocity decreases, so will the velocity head loss. Friction forces that increase with velocity tend to stabilize the loop, whether they are in the single phase or two-phase part of the loop. High subcooling causes riser friction to decrease with inlet velocity increases, and therefore makes the loop less stable. A useful design curve of subcooling limit versus operating pressure is given in Appendix C. The effect of riser velocity head loss is important in the design of reactors with internal steam separation, and the stability of such designs should be checked.

This report extends prior work on two-phase flow mechnnics in Ref. (3), and a simple flow loop model in Ref. (9). The conditions of stability of flow are derived under simplifying assumptions. The work does not deal with transient velocity gradients, such as in slug flow. Two other topics, as well as slug flow, need further exploration. The first is the transient behavior of two-phase flow at an orifice. The second is the steam-water shear stress. The possibility is suggested in the equations of this report, of a decrease of the shearing stress between water and steam at high void fractions which would lead to a higher slip velocity and a lower void fraction. Such behavior could result in an oscillation similar to the one caused by subcooling.

ACKNOWLEDGEMENTS

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APPENDIX A - CALCULATION OF FRICTION PRESSURE DROP FACTOR

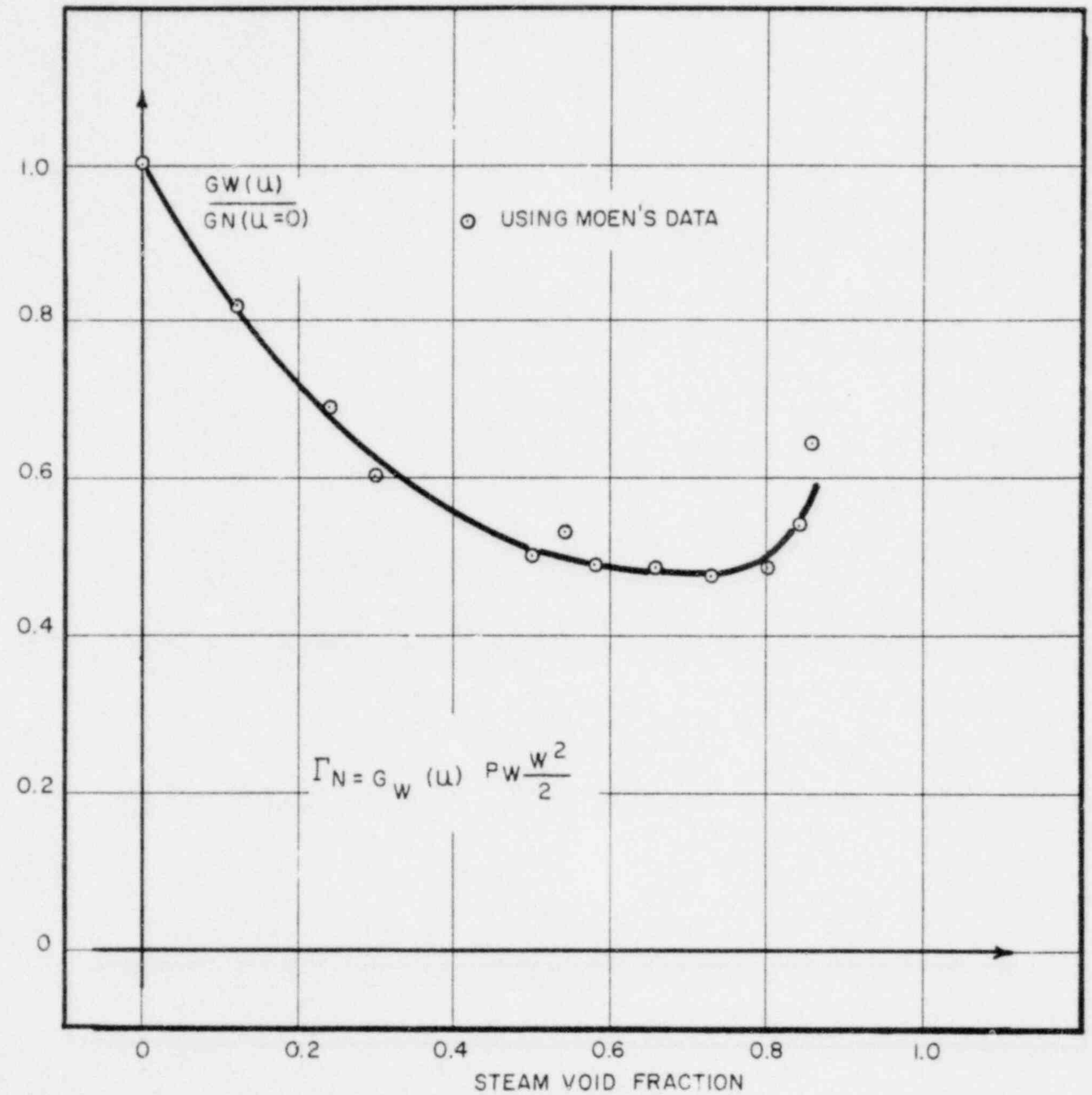
S. Levy suggested a way of calculating the effective water to wall perimeter to obtain two-phase friction pressure drop from established methods. From equation (5) we assume $p_W = p\,F(u)$, where p is the geometrical perimeter, and $F(u)$ is to be determined. We have now

$$\Gamma_W = f\,(Re)\,\frac{P}{4A}\left(\tfrac{1}{2}\rho_W W^2\right) F\,(u),$$

and from the literature

$$\Gamma_W = \frac{\Delta P_{TP}}{\Delta x} = \frac{\Delta P_{TP}}{\Delta P_o} \cdot \frac{\Delta P_o}{\Delta x} = \frac{\Delta P_o}{\Delta x} \quad (1 - x)\,\phi_1^{\ 1.75}$$

using Moen's (Ref. 12) parameter ϕ_1 and is shown in Fig. A. In this fashion Γ_W can be determined from the literature or from experimental data.



DEPENDENCE OF WATER-PIPE FRICTION PRESSURE DROP ON VOID FRACTION

FIG A

APPENDIX B - DOWNCOMER TRANSIT TIME EFFECTS

In the body of the report, the heated section inlet subcooling was assumed to be fixed. In practice, feedwater is controlled to maintain water level, and subcooling varies with feedwater flow and with recirculation flow. The effect of subcooling changes is delayed by the downcomer transit time. Usually the downcomer transit time is many times the riser transit time. For purposes of loop stability, therefore, the inlet subcooling is effectively constant over the period of several riser transit times. For purposes of calculating transit response, subcooling variations and downcomer transit time should, on the contrary, be included.

Two of the loop equations are affected by subcooling changes: the volume flow, equation (20), and the inlet water flow, equation (21). Instead of constant subcooling, let us assume that feedwater flow, C, is constant and equal to the average steaming rate. The average water enthalpy at the feedwater mixing point is therefore

$$h_{sc} = h_w - \frac{C(hw-hc)}{v}$$

The average water enthalpy at the inlet is $h_{sc}(t-T_D)$, where T_D is the downcomer transit time. Incorporation of these relations results in the following changes to equations (20) and (21).

$$RS + (1-R)W = v(t)-(B-1)\frac{h_w-h_c}{h_s-h_w} C \frac{v(t)}{v(t-T_D)} + (B-1)\ \gamma$$

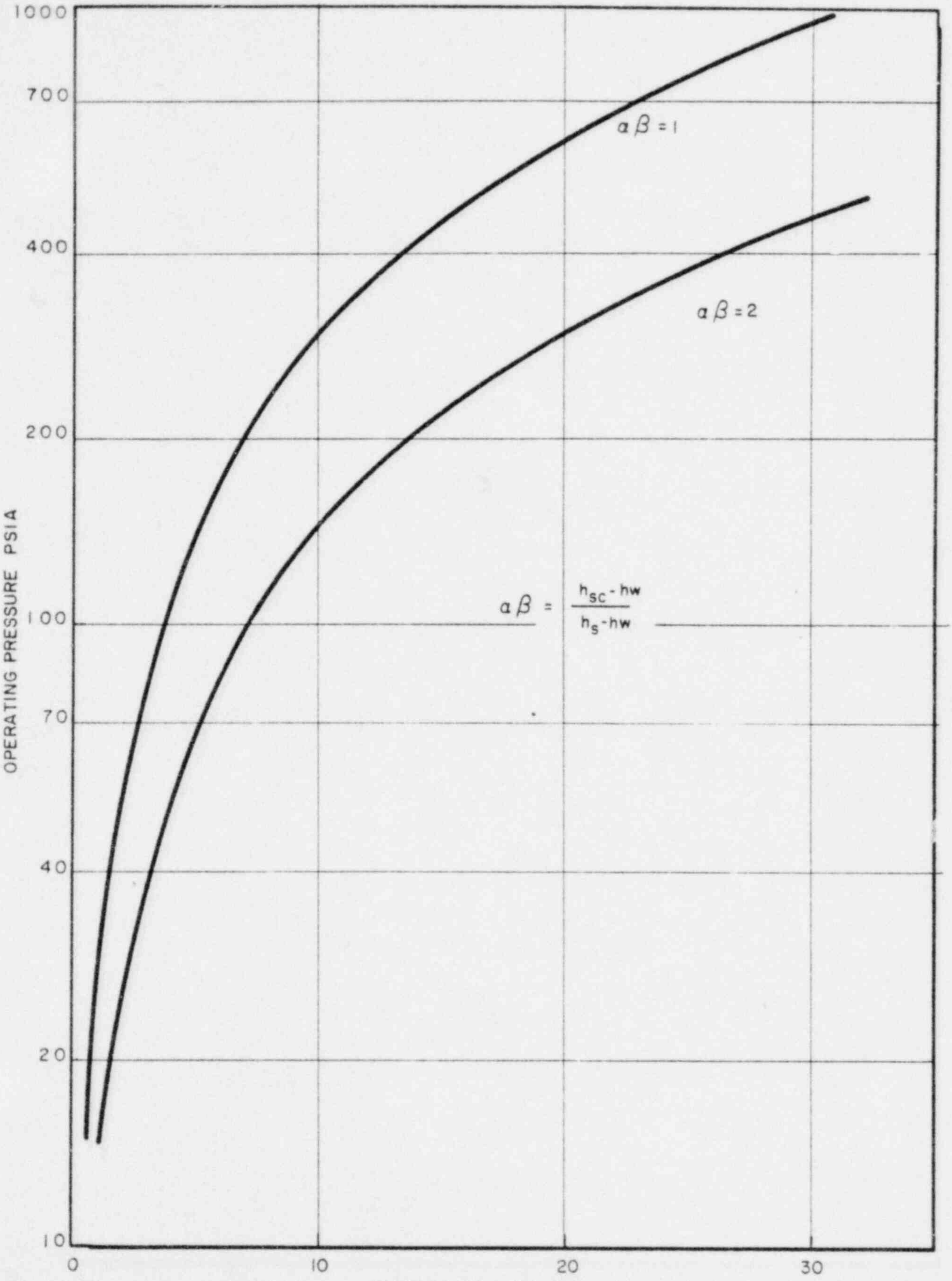
$$(1-u)W = v(t) + \frac{hw-hc}{h_s-h_w} C \frac{v(t)}{v(t-T_D)} - \gamma\ (t)$$

For changes that occur in periods less than the downcomer transit time, the two formulations are equivalent.

For transients of longer duration, the ratio $v(t)/v(t-T_D)$ tends to mitigate the destabilizing tendency of subcooling. Increasing loop velocity tends to increase the proportion of recirculating water to feedwater, and thereby to decrease subcooling. Pulsating flow traces of two-phase flow loops often show a basic frequency of oscillation determined by riser transit time, and a long envelope modulation of the basic frequency of about the downcomer transit time. The pulsation is continuous, but its magnitude increases and decreases cyclically.

APPENDIX C

As yet a convenient formula for predicting two-phase flow loop stability has not been devised, and it is necessary to do the detailed calculations. Because of the prime importance of subcooling to stability, a graph of operating pressure versus subcooling is useful. Two curves of the parameter $\alpha \beta = \frac{p_w}{p_s} \frac{h_{sc} - h_w}{h_s - h_w}$ are plotted in Fig. C. A loop is probably stable if it operates above the curve $\alpha \beta = 1$. If the riser void fraction is high, the region below $\alpha \beta = 1$ will be unstable. At low void fractions the region between $\alpha \beta = 1$ and $\alpha \beta = 2$ will be marginal.



OPERATING PRESSURE VS. SUBCOOLING

FIGURE C

NOMENCLATURE

A	-	Cross sectional area
B	-	Constants of characteristic equation
C	-	Condenser feedwater flow
f	-	Pipe friction factor
F	-	Characteristic function
g	-	Acceleration of gravity
h_w	-	Water saturation enthalpy
h_s	-	Steam saturation enthalpy
h_c	-	Feedwater enthalpy
h_{sc}	-	Subcooled water enthalpy
K	-	Velocity head loss coefficient
L	-	Length of two-phase vertical portion of loop
L_D	-	Length of downcomer
P	-	Pressure
p	-	Perimeter
Q	-	Heat input rate
q	-	Heat input rate per unit coolant volume
R	-	Average steam void
S	-	Steam velocity
τ	-	Surface tension
T	-	Transit time
u	-	Steam void
v	-	Inlet water velocity
w	-	Two-phase fluid water velocity
x	-	Channel vertical dimension

α - $h_{..} - h_{sc}/h_s - h_w$

β - $\rho_w \rho_s$

γ - $Q/A\rho_w(h_s - h_w)$

Γ_w - Water-wall shear stress/unit length

Γ_s - Steam-wall shear stress/unit length

Γ_{ws} - Steam-water shear stress/unit length

μ - $L/w \cdot \sigma$ dimensionless complex frequency

σ - Complex frequency (sec^{-1})

ρ_s - Steam density

ρ_w - Water density