NUREG/CR-1560
UCRL-15218
RD, RM

Structural Uncertainty in Seismic Risk Analysis

Seismic Safety Margins Research Program

Manuscript Completed: March 1980
Date Published: October 1980

Prepared by
T. K. Hasselman and S. S. Simonian
J. H. Wiggins Company

Lawrence Livermore Laboratory
7000 East Avenue
Livermore, CA 94550

Prepared for
Division of Reactor Safety Research
Office of Nuclear Regulatory Research
U.S. Nuclear Regulatory Commission
Washington, D.C. 20555
NRC FIN No. A0130

8011030659

ABSTRACT

This report documents the formulation of a methodology for modeling and evaluating the effects of structural uncertainty on predicted modal characteristics of the major structures and substructures of commercial nuclear power plants. The uncertainties are cast in the form of normalized random variables which represent the demonstrated ability to predict modal frequencies, damping and modal response amplitudes for broad generic types of structures (steel frame, reinforced concrete and prestressed concrete). Data based on observed differences between predicted and measured structural performance at the member, substructure, and/or major structural system levels are used to quantify uncertainties and thus form the data base for statistical analysis. Proper normalization enables data from non-nuclear structures, e.g., office buildings, to be included in the data base. Numerous alternative methods are defined within the general framework of this methodology.

The report also documents the results of a data survey to identify, classify and evaluate available data for the required data base. A bibliography of 95 references is included. Deficiencies in the currently identified data base are exposed, and remedial measures suggested. Recommendations are made for implementation of the methodology.

TABLE OF CONTENTS

TABLE OF CONTENTS
(continued)

LIST OF FIGURES

LIST OF FIGURES
(continued)

LIST OF FIGURES
(continued)

LIST OF TABLES

SUMMARY

This report presents a methodology for modeling structural uncertainty in the major structures and substructures of nuclear power reactor facilities. The methodology is aimed at representing this uncertainty in terms of three normalized random variables which relate to modal frequency, modal amplitude and modal damping. It is shown how the distributions of these random variables can be derived from available analysis and test data. Once defined in this manner, the random variables can be sampled for numerical simulation of structural response to seismic excitation. With the use of these three random variables and a structural model, the natural frequencies, mode shapes and modal damping for an arbitrarily specified structure and an arbitrary number of modes can be varied randomly and independently of each other.

A new approach has been taken in developing the present methodology. Whereas the conventional approach relies on the a priori definition of random structural properties and their distributions (or statistical moments) which must be established largely on the basis of conjecture, this approach does not. The problem is viewed here in much the same way as that of modeling structural damping. In general it cannot be done by inductive means. Structural uncertainty can only be deduced by comparison of analytical predictions with experimental measurements for generically similar structures whose dynamic response is computed and measured relative to the way the structure will behave in an actual earthquake, i.e., in its lower natural modes of vibration.

The methodology accommodates the use of either generic structure and/or substructure modal data (e.g., theoretically predicted and experimentally measured modal data for steel frame buildings, reinforced concrete buildings, etc.), or structure-specific data obtained from seismic tests or forced vibration tests on the particular structure being modeled.

The use of structure-specific data is of course expected to reduce modeling uncertainty appreciably, but will not eliminate it entirely. This methodology will enable the reduction of uncertainty to be quantified, thus demonstrating the benefit of seismic testing.

Finally, this report presents and summarizes the results of a literature survey conducted to identify the various types, quantities and sources of data which can be used with the proposed methodology. Much more data are believed to be available in foreign technical journals as well as in unpublished reports within the United States and abroad. The potential benefit of acquiring such data is considered to be great.

1. INTRODUCTION

1.1 Background and Problem Statement

Seismic risk for critical structures and facilities has been defined in many ways, ranging from the probability of experiencing a certain ground motion intensity, to the probability of experiencing a certain level of structural response, to the probability of experiencing failure in a critical failure mode, to the probability of system failure which is the ultimate consequence of failure in a number of interdependent subsystems comprising a functional system. The probability that radioactive gas will be released from a nuclear power reactor during a LOCA triggered by an earthquake is the measure of seismic risk chosen for the SSMRP program. Structural uncertainty is only one of many contributors to this overall measure of seismic risk.

The task of analyzing this risk is enormously complex. A numerical simulation approach has been selected as a means of coping with the time-dependent, nonlinear, nondeterministic nature of the problem. Not only must the functional performance of all aspects of the nuclear power plant be understood in detail, but they must be understood well enough to make valid engineering approximations to the degree that the total problem remains tractable while (a) all significant functional features of the plant are represented, (b) all significant sources of uncertainty are represented, and (c) models are sufficiently realistic and flexible to accommodate the learning process which the analysts must undergo in the process of developing and learning to use them.

Structural modeling is recognized as being one of the significant sources of uncertainty in the network of systems and subsystems. There are countless sources of uncertainty which in turn contribute to that which we categorically call "structural uncertainty". These include uncertainties in material properties, construction, modeling, analysis and even experimental observation to the extent that physical reality may never be perfectly known. We shall therefore define structural uncertainty (generally speaking) to be the difference between that which we can predict based on analytical modeling and analysis, and that which we might physically observe.

There are two distinct kinds of uncertainty which must be recognized in this general problem area. One relates to a population of structures, and the other to a particular structure. The SSMRP program is directed toward the evaluation of seismic risk for particular power plants, i.e., particular structures. To some extent, however, we are forced to use population statistics in modeling the uncertainties of substructures and elements of major structural systems. We shall adopt the concept of generic classification as a means of compartmentalizing the various uncertainties which contribute to overall structural uncertainty, thereby creating a general framework of analysis which permits us to use available data and potentially future data in an optimum manner. We shall endeavor to minimize structural uncertainty by being as specific as our knowledge will permit.

We encounter many difficulties when contemplating an effort such as this. Some of them are listed below:

- There are probably more sources of uncertainty than we can name.

- Even for those which we can name, there are relatively few sources of data upon which to base any quantitative estimates, let alone statistical estimates with high confidence.

- And, even if we could identify all sources of uncertainty and appropriately quantify them, we would face the immense task of combining them computationally.

- Finally, if we could do all of that, we would still have to reduce the resulting uncertainty to a few significant parameters which capture both the qualitative and quantitative essense of the problem.

The constraints imposed by having to model structural uncertainty for numerical simulation in the overall risk analyses (SEISIM) computer program are severe. Nevertheless, the challenge of the task is to do so in a way that is reasonable, easy to understand, and which can be traced through a rigorous mathematical formulation to concrete data, rather than conjecture.

Structural response will be provided to the SEISIM program in a tabular format representing maximum (peak) response statistics at some 200 - 400 locations for a range of earthquakes. These statistics will be generated by the SMACS program via time-history response analysis of multi-degree-of-freedom SSI models. Structural modeling and analysis will take place independently of the SSI response analysis to produce a reduced modal representation of the structure portion of the SSI system. Structural uncertainty will be represented in the modal characteristics of the structure.

In light of the foregoing background discussion, the problem which this study addresses may be stated as follows: Given the likely existence of limited, incomplete, and possibly incompatible data relating to structural uncertainties for nuclear power plants, as well as other types of structures, formulate an analytical methodology for maximizing the usefulness of these data to produce realistic data-based statistics of modal characteristics for major structures of nuclear power plants, for purposes of numerical simulation in SSMRP.

1.2 Objectives

Specific objectives have been defined in structuring a technical approach to this problem. They reflect parallel efforts aimed at identifying sources and types of data upon which to base the uncertainty modeling, and an evaluation of modeling alternatives aimed at synthesizing computational techniques and procedures which are compatible with LLL's structural analysis plan, and which will at the same time accommodate available forms of data. The data oriented objectives are stated as follows:

(1) Identify data and sources of data for potential use in modeling structural uncertainty for seismic risk analysis.

(2) Evaluate the format and adequacy of these data relative to alternative modeling and analysis procedures.

(3) Classify and categorize the data for subsequent use in analysis.

Objectives defined to guide the formulation of modeling and analysis procedures are:

(4) Ensure that modeling techniques and analysis methods are compatible with available (and potentially available) data, and that they make maximum use of the data in the following ways:

- be capable of utilizing element, substructure, and/or major nuclear power plant structure data,

- be capable of using generic as well as structure-specific seismic/vibration test data at the substructure and major structure levels,

- be capable of using generic data from other than nuclear power structures.

(5) With respect to vibration data at the substructure and major structure levels, utilize mode shape as well as frequency and damping information to evaluate uncertainties in modal response time-histories and/or amplitude.

(6) Account for correlation among the various modal properties of a structure or substructure.

(7) Identify combinations of modal parameters which may be used for expressing structural uncertainty more concisely.

(8) Consider the use of empirical analysis for reducing the statistical degrees of freedom required to represent the modal characteristics of major structures.

The end product of this combined data evaluation and mathematical formulation effort should be a unified methodology which meets the following objectives:

(9) Provides a means of assessing the adequacy of available data relative to the needs of SSMRP, and subsequent identification

of additional specific data requirements.

(10) Provides sufficient flexibility for user experimentation and learning in its adaptation to SSMRP.

(11) Will produce realistic results which are in basic agreement with the preponderance of theoretical and experimental evidence available to date.

The present study has endeavored to satisfy these objectives insofar as possible.

1.3 Scope

The scope of the present study includes a rather extensive literature search. A bibliography has been organized to present data sources in three distinct categories:

- Member (or element) data
- Substructure (including nuclear power plant as well as non-nuclear power plant) data
- Major structure (exclusively nuclear power plant) data

Data from these sources have been classified into three generic groups

- Structural steel
- Reinforced concrete
- Prestressed or post-tensioned concrete

corresponding to the major generic types of substructures which comprise the following major structures of existing nuclear power plants:

- Reactor building
- Auxiliary-fuel-turbine building complex

A general formulation of structural modeling and uncertainty analysis is presented and discussed. Simplified examples are included for purposes of illustration.

The scope of the present study does not include the specification of computational procedures, nor does it include any numerical demonstration problems of a realistic nature. The simplified examples offered herein are purely hypothetical.

2. ORGANIZATION OF AVAILABLE DATA

2.1 Sources of Uncertainty

For the purposes of collecting and analyzing available data, it is useful to classify sources of uncertainty into a number of major classes. In this report five major classes of uncertainty are identified and listed in Table 2-1.

I. Material Properties

II. Construction

III. Effects of Nonseismic Loading

IV. Modeling Techniques

V. Analysis Methods

Although the boundaries between the above mentioned classes often intersect and may in fact be inseparable, this classification is intuitively appealing and each class is sufficiently distinct to justify such separation. The decisions made and actions taken during design and analysis of engineering structures involve all of the five classes mentioned above and adversely affect our predictive ability of structural behavior. Errors made in predicting the response of the structure can further be classified under two categories: (a) random, and (b) systematic errors. Each of the five sources of uncertainty mentioned above may contribute to both types of error.

Below, we present a sample listing of the sources of uncertainty identified under each of the five classes. Although the list is fairly detailed,

Table 2-1. Sources of Structural Uncertainty

I. MATERIAL PROPERTIES

(a) CONCRETE

(i) COMPRESSIVE STRENGTH
(ii) TENSILE STRENGTH
(iii) SECANT MODULUS
(iv) TANGENT MODULUS
(v) POISSON'S RATIO
(vi) DENSITY
(vii) DAMPING
(viii) NONLINEAR STRESS-STRAIN PROPERTIES
(ix) CREEP AND SHRINKAGE

(b) STRUCTURAL STEEL [HOT ROLLED]

(i) YOUNG'S MODULUS, COMPRESSIVE
(ii) YOUNG'S MODULUS, TENSILE
(iii) SHEAR MODULUS
(iv) POISSON'S RATIO
(v) VARIABILITY OF GEOMETRIC SHAPE PROPERTIES
- MOMENTS OF INERTIA: I_{xx}, I_{yy}, I_{xy}
- CROSS-SECTIONAL AREAS: A

(vi) YIELD STRESS
(vii) DENSITY
(viii) DAMPING
(ix) NONLINEAR STRESS-STRAIN PROPERTIES
(x) CREEP

(c) REINFORCING STEEL [(i) - (x) as in (b) above]

II. CONSTRUCTION

(a) GEOMETRIC VARIATIONS OF MANUFACTURED COMPONENTS FROM DESIGN SPECIFICATIONS

(i) CONCRETE BEAMS: OVERALL WIDTH AND DEPTH
(ii) CONCRETE COLUMNS: CROSS SECTIONAL DIMENSIONS
(iii) CONCRETE SLABS: OVERALL DEPTH
(iv) LOCATION OF TOP AND BOTTOM REBARS OF BEAMS AND SLABS

Table 2-1. Sources of Structural Uncertainties (cont'd)

(v) SPACING OF STEEL IN COLUMNS

(vi) STIRRUP SPACING IN BEAMS

(vii) STIRRUP SPACING IN COLUMNS

(viii) LENGTH OF COLUMNS

(ix) LENGTH OF SPANS

(x) OUT-OF-PLANE IRREGULARITY

(b) PRESTRESSING OR POST TENSIONING

(i) GEOMETRIC VARIABILITY OF DIMENSIONS

(ii) VARIABILITY IN APPLIED PRESTRESSING FORCES

(c) JOINTS

(i) VARIABILITY OF JOINT DETAILS FROM SPECIFICATIONS

(ii) INTRODUCTION OF ADDITIONAL JOINTS DUE TO CONTINUITY REQUIREMENTS

(d) FASTENERS

(i) VARIABILITY OF APPLIED TORQUES TO PRESTRESS BOLTS

III. EFFECTS OF NONSEISMIC LOADING

(a) CHANGES TO STRUCTURAL PROPERTIES

(i) REDUCTION OF STIFFNESS DUE TO STATIC LOADING (E.G. LIVE AND DEAD LOADS).

(ii) THERMAL LOAD EFFECTS

- CHANGE IN MEMBER STIFFNESSES
- GEOMETRICAL DISTORTION LEADING TO ADDITIONAL CHANGES IN MEMBER STIFFNESS.
- EFFECT OF TEMPERATURE ON MATERIAL PROPERTIES

(iii) CHANGES IN STIFFNESS DUE TO INTERNAL STRESSES CAUSED BY FOUNDATION MOVEMENT

(b) COMBINED LOADS EFFECTS

(i) VARIABILITY OF LINEAR RANGE FOR SEISMIC LOADING

(ii) VARIABILITY IN RESISTANCE CAPACITY FOR SEISMIC LOADING

(iii) UNCERTAINTY IN NONSEISMIC LOADS

Table 2-1. Sources of Structural Uncertainties (cont'd)

IV. MODELING TECHNIQUES

(a) DISCRETE PARAMETER IDEALIZATION OF COMPLICATED DISTRIBUTED PARAMETER SYSTEMS

- (i) SELECTION OF MESH SIZE AND NODAL GEOMETRY
- (ii) ASSUMED DISPLACEMENT FIELDS
- (iii) ASSUMED DISTRIBUTION OF MASS
- (iv) COORDINATE REDUCTION
- (v) SELECTION OF DYNAMIC DEGREES OF FREEDOM
- (vi) MODAL TRUNCATION
- (vii) SELECTION OF RESPONSE COORDINATES

(b) BOUNDARY CONDITIONS

- (i) ELEMENT-TO-ELEMENT
- (ii) SUBSTRUCTURE-TO-SUBSTRUCTURE
- (iii) EXTERNAL TO MAJOR STRUCTURAL SYSTEM

(c) FLUID-STRUCTURE INTERACTION

(d) DAMPING

- (i) ASSUMED EQUIVALENT VISCOUS DAMPING
- (ii) ASSUMED COMPLEX MODULOUS DAMPING
- (iii) ASSUMED UNCOUPLED MODAL DAMPING
- (iv) VARIABILITY IN MEASURED MODAL DAMPING RATIOS

(e) MODELING OF COMPOSITE MATERIAL ACTION

(f) EFFECTS OF NON-STRUCTURAL ELEMENTS

(g) NONLINEARITIES

- (i) MATERIAL
  - NONLINEAR STRESS-STRAIN
  - STIFFNESS DEGRADATION DUE TO CRACKING AND CREEP OF CONCRETE AND YIELDING OF STEEL
- (ii) GEOMETRIC
  - GAPS
  - IMPACT

Table 2-1. Sources of Structural Uncertainties (cont'd)

- JOINT SLIPPAGE - COULOMB FRICTION
- LARGE DISPLACEMENTS AND ROTATIONS
- BUCKLING

(iii) AMPLITUDE AND CYCLE/HISTORY DEPENDENCE

- STIFFNESS
- DAMPING

V. ANALYSIS METHODS

(a) MODAL ANALYSIS

(i) CONVERGENCE OF NUMERICAL ITERATION
(ii) ORTHOGONALITY OF EIGENVECTORS
(iii) SKIPPED MODES
(iv) NUMERICAL INSTABILITY
(v) ROUND-OFF ERROR

(b) RESPONSE SPECTRUM ANALYSIS

(i) ASSUMPTION OF RANDOM PHASING
(ii) APPROXIMATIONS OF MODAL CORRELATION AND COMBINATION
(iii) FORCING FUNCTION UNCERTAINTY

(c) TIME-HISTORY ANALYSIS

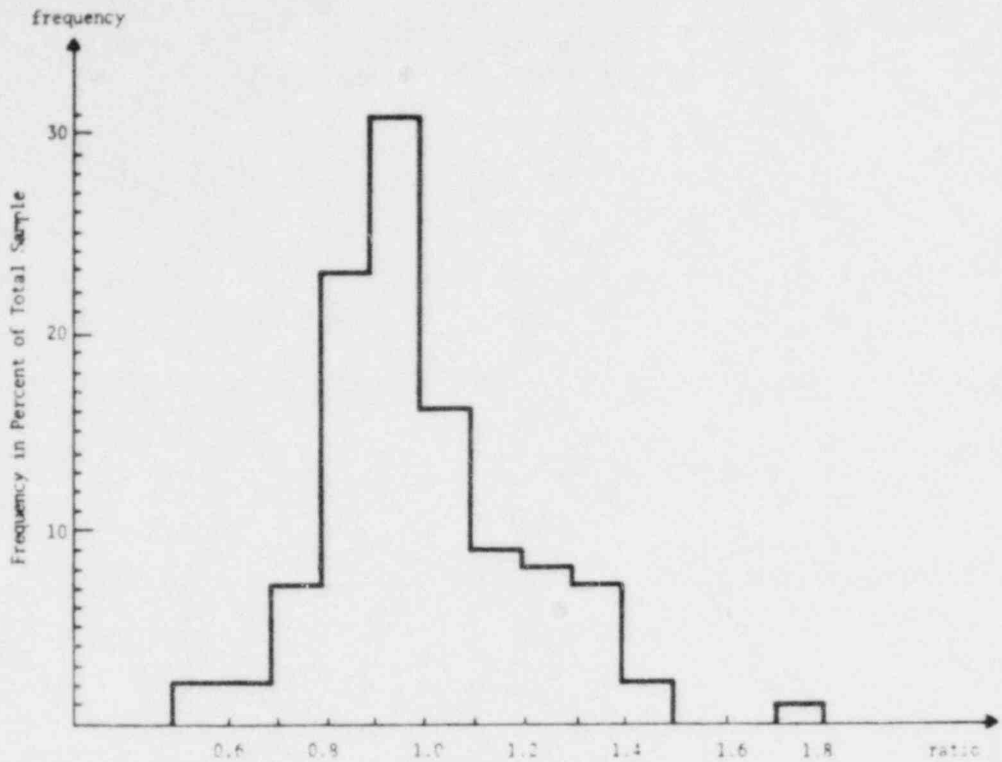
(i) NUMERICAL CONVERGENCE
(ii) NUMERICAL RESOLUTION
(iii) FORCING FUNCTION UNCERTAINTY

it must nevertheless be considered incomplete. It is included for two reasons: (1) as a reminder of the almost endless potential sources of uncertainty compared with the limited amount of available data, and (2) to be suggestive of how uncertainties may be grouped for engineering analysis.

2.2 Sources and Types of Data

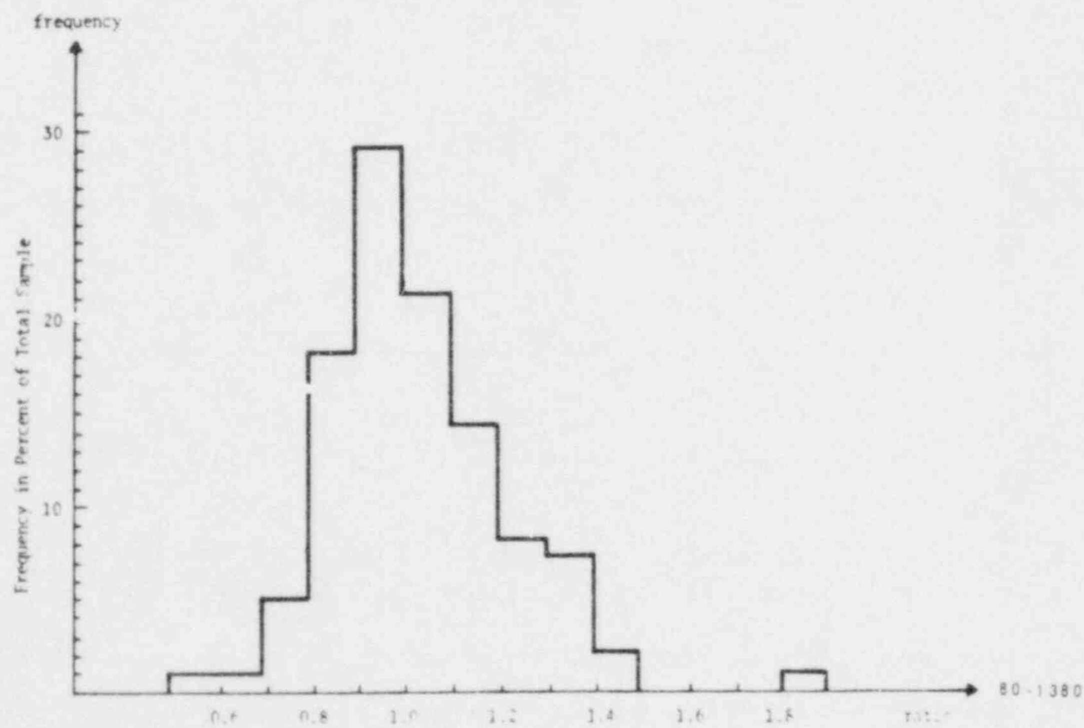
A wide variety of test data on structural components as well as on complete structures are scattered in the literature. Recently a comprehensive summary on the constitutive properties of some construction material (e.g., concrete, reinforcing steel and structural steel) has appeared in the literature [17, 18 and 8]. A summary of data on the geometrical variability of reinforced concrete member dimensions is reported in [19]. The data reported in the above references are in the form of the first two statistical moments, namely, the means and standard deviations and a recommended probability distribution function. The types of data reported in the above references clearly belong to categories I and II defined in Section 2.1. Further, the variability of the ratio of measured to computed deflections of simply supported reinforced concrete beams is reported in [1 and 23]. In [1] normal and lognormal probability density functions are fitted to the data (see Figures 2-1 to 2-6). In references [10 to 12], the results reported in [1] are utilized to construct a probability distribution function for the ratio of measured to predicted effective EI values (see Figure 2-7, and 2-8).

A comprehensive summary of data on measured versus calculated natural periods of a wide range of structures is reported in [31]. In Figures 2-9 to 2-11 we reproduce the results in [31] for both small and large amplitude vibrations; we note that these figures include the parameters of both gamma and lognormal distributions fitted to the data.



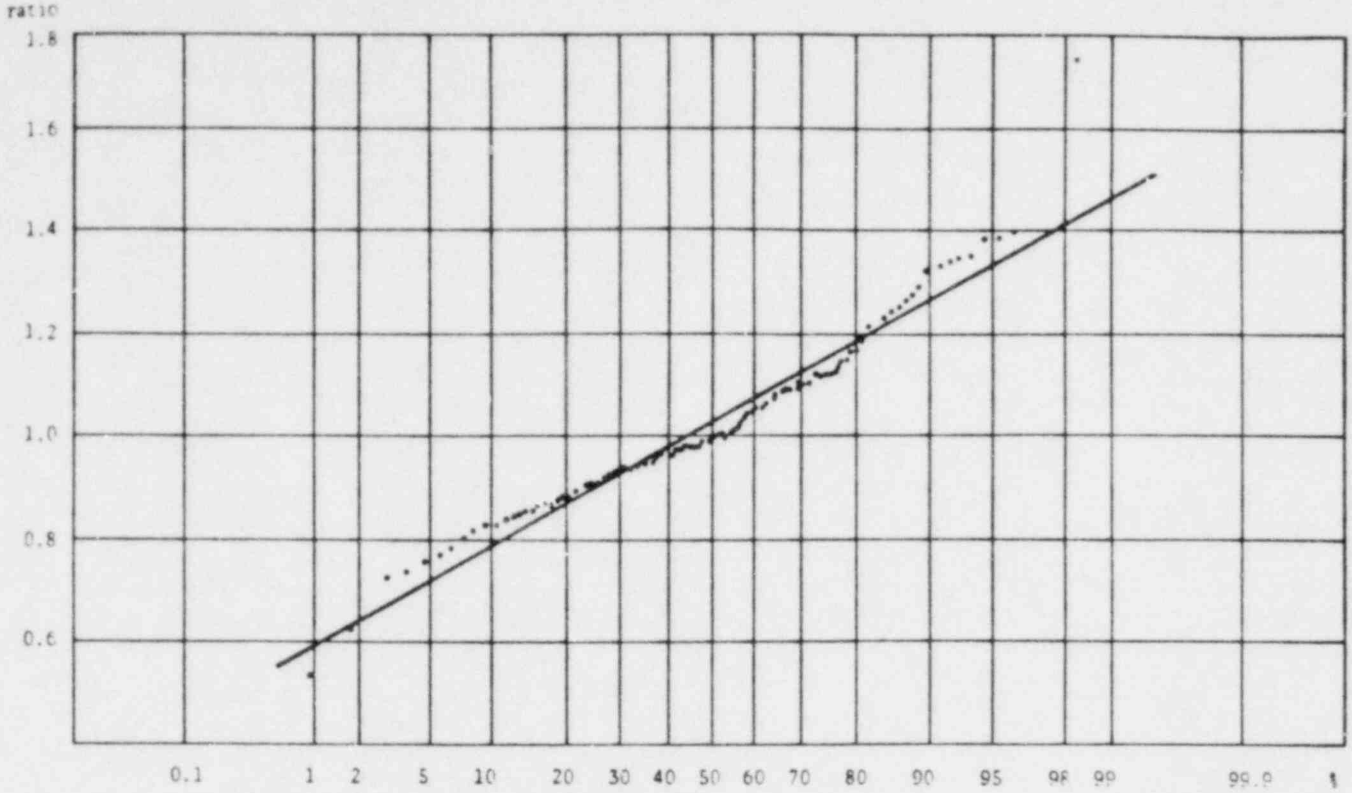
Reference: ACI Committee 435, 1972 [1]

Figure 2-1. Histogram of Ratio of Actual Deflection-to-Deflection Calculated by ACI 318-63 METHOD



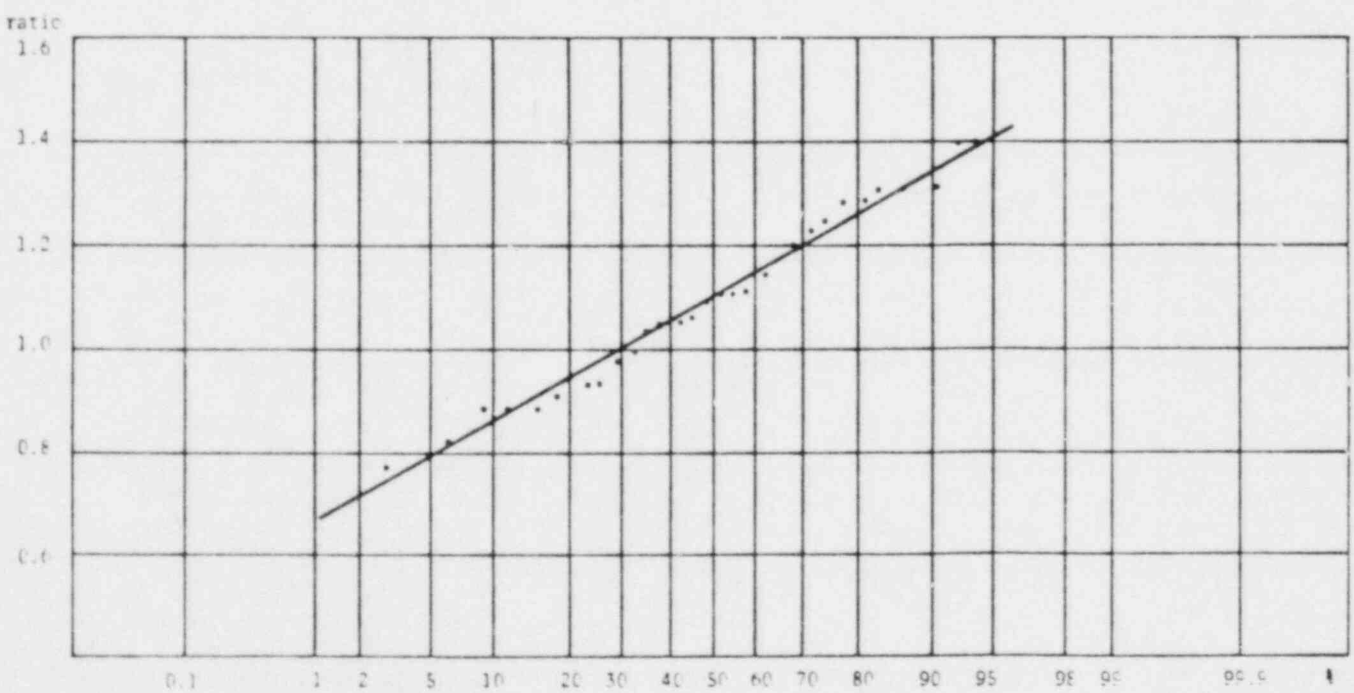
Reference: ACI Committee 435, 1972 [1]

Figure 2-2. Histogram of Ratio of Actual Deflection-to-Deflection Calculated by ACI 318-71 METHOD



Reference: ACI Committee 435, 1972 [1]

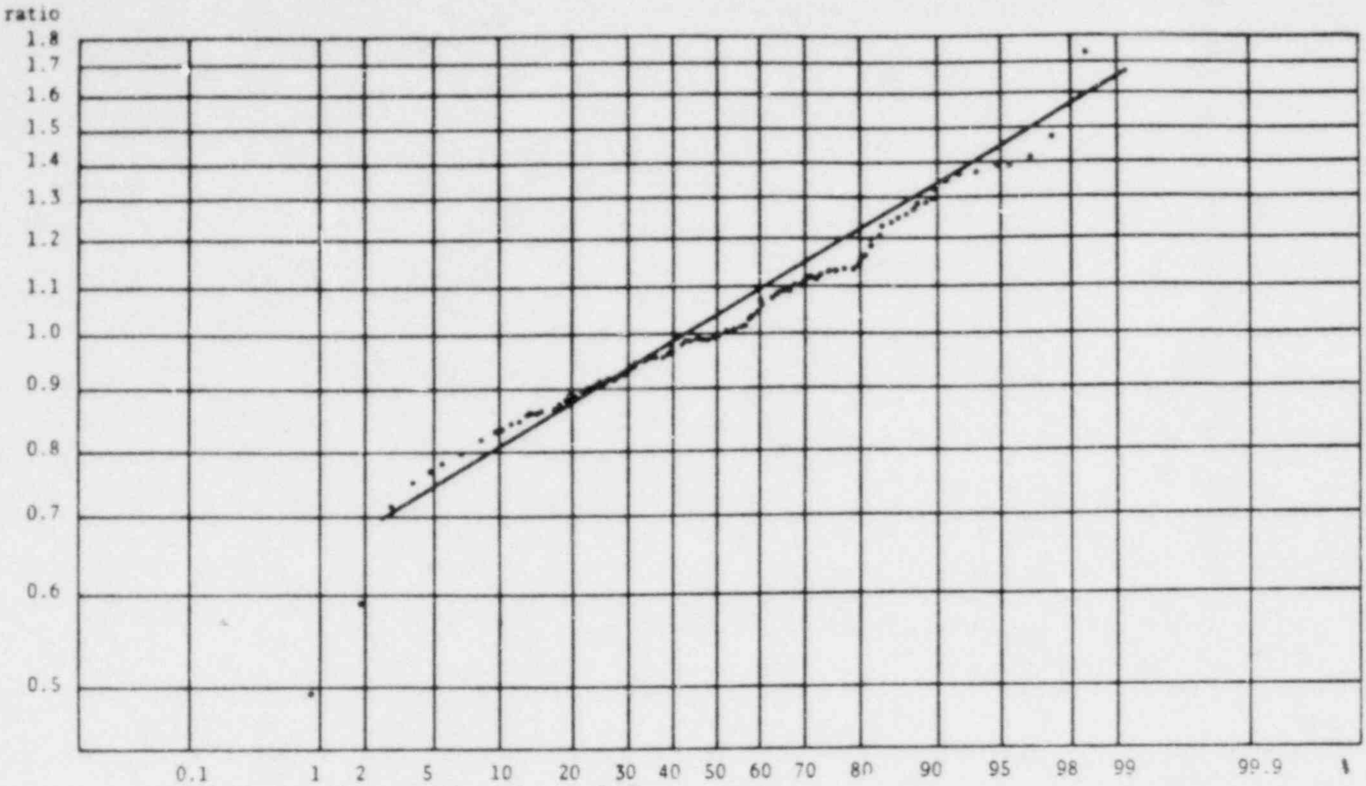
Figure 2-3. Plot of Ratio of Actual Short-Term Deflection-to-Deflection Calculated by ACI 318-71 Code Method (normal model)



Reference: ACI Committee 435, 1972 [1]

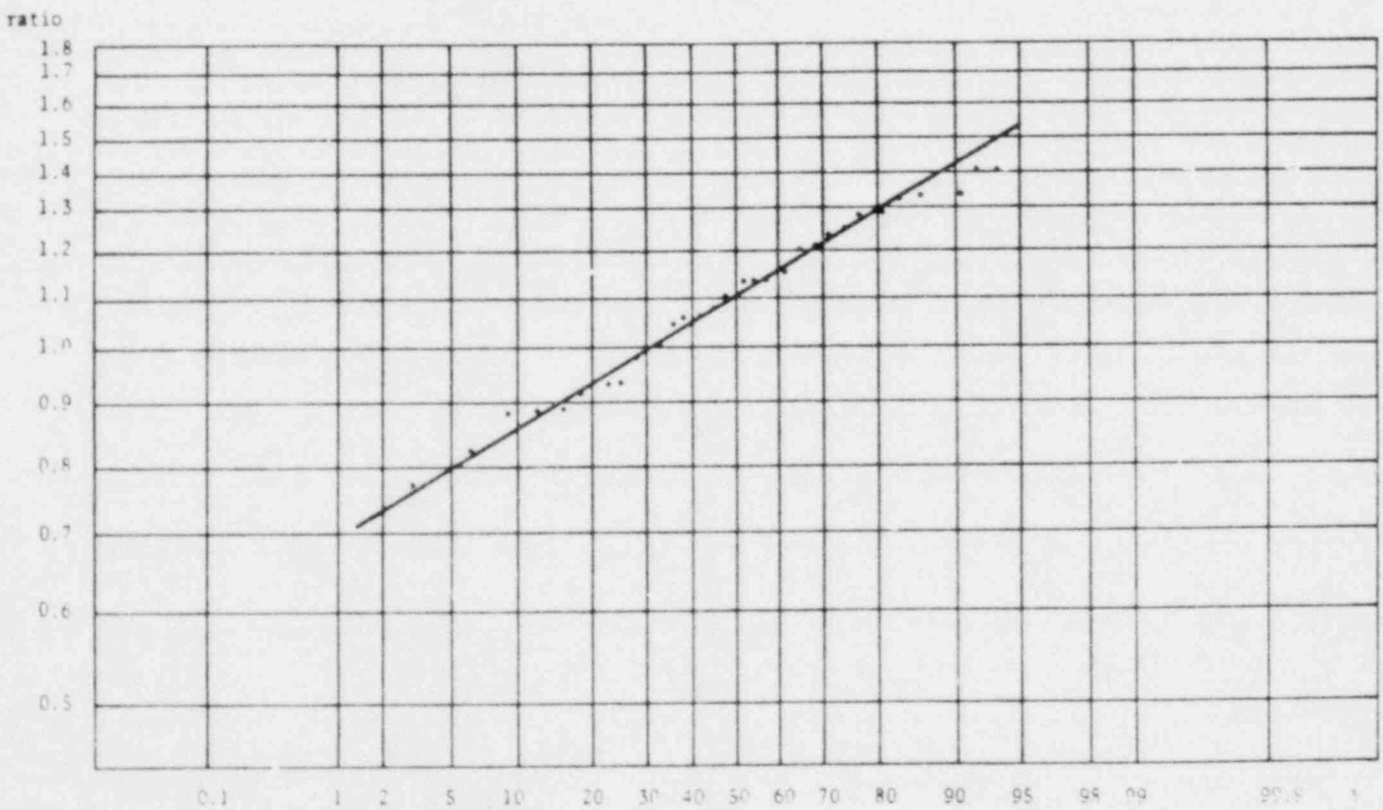
80-1380

Figure 2-4. Plot of Ratio of Actual Long-Term Deflection-to-Deflection Calculated by ACI 318-71 Method (normal model)



Reference: ACI Committee 435, 1972 [1]

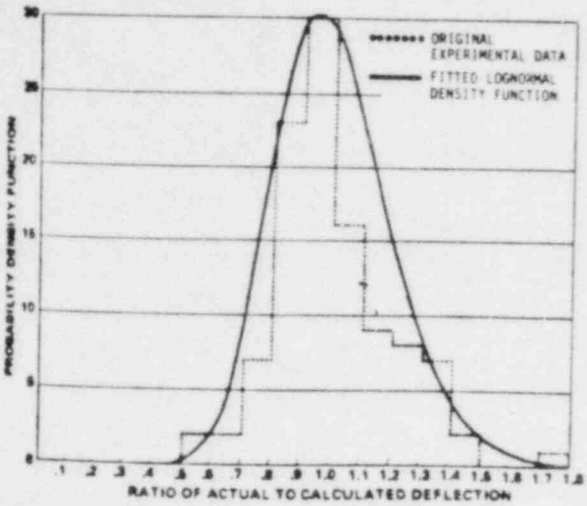
Figure 2-5. Plot of Ratio of Actual Short-Term Deflection-to-Deflection Calculated by ACI 318-71 Method (log-normal model)



Reference: ACI Committee 435, 1972 [1]

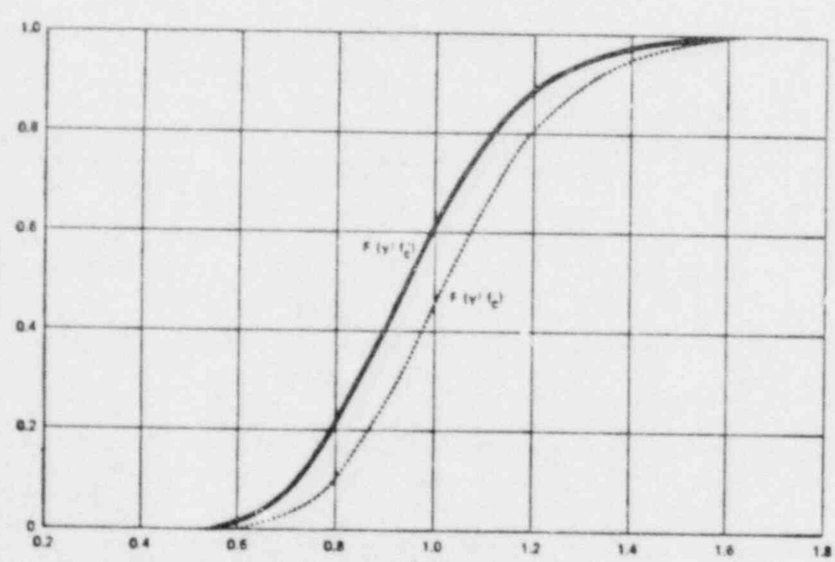
80-1380

Figure 2-6. Plot of Ratio of Actual Long-Term Deflection-to-Deflection Calculated by ACI 318-71 (log-normal model)



Reference: Hamilton, C.W., and Hadjian, A.H., 1976 [12]

Figure 2-7. Probability Density Function Fitted to Experimental Data



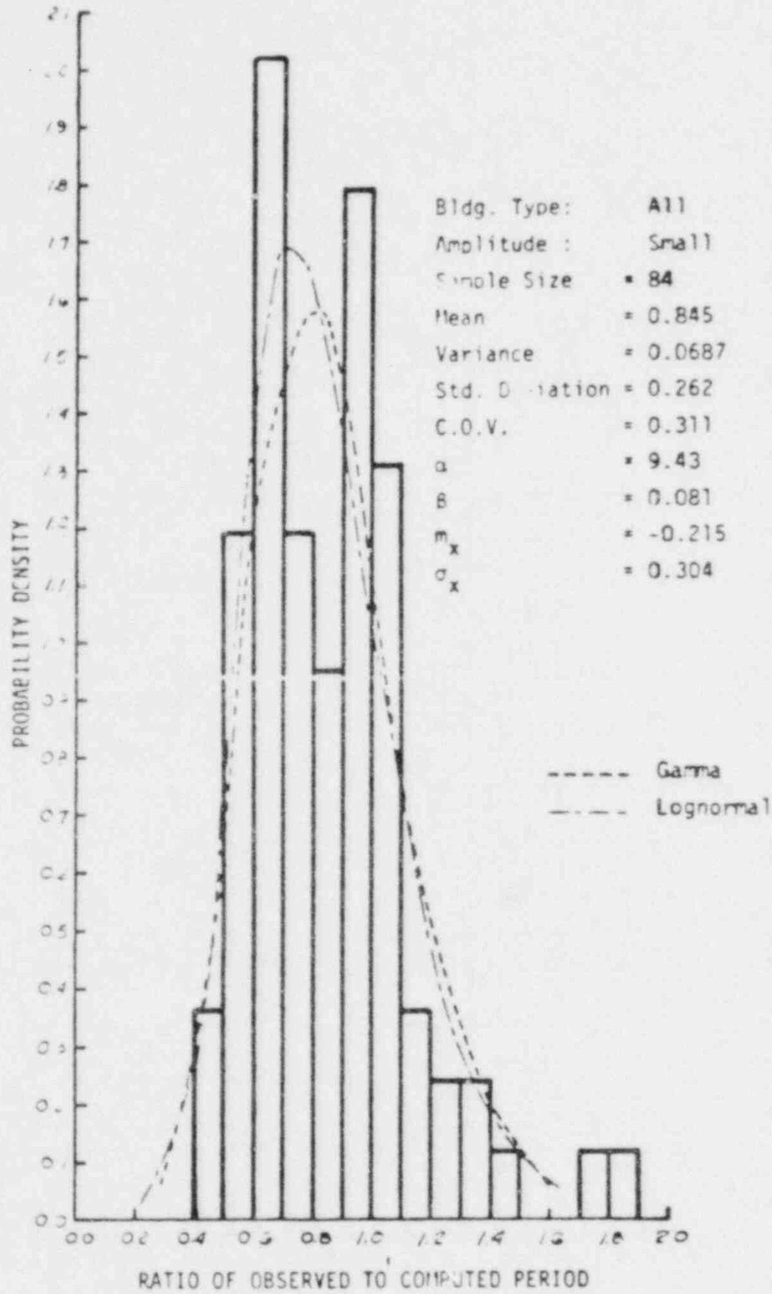
Conditional distributions of stiffness ratios for actual (f_c) and specified (f'_c) compressive strengths.

p	$y_p \mid f_c$	$y_p \mid f'_c$
0.01	0.6459	0.5854
0.05	0.7384	0.6726
0.10	0.7931	0.7243
0.15	0.8324	0.7615
0.25	0.8938	0.8200
0.35	0.9461	0.8700
0.45	0.9955	0.9176
0.50	1.0205	0.9416
0.55	1.0461	0.9662
0.65	1.1007	1.0190
0.75	1.1651	1.0813
0.85	1.2510	1.1647
0.90	1.3130	1.2247
0.95	1.4102	1.3195
0.99	1.6125	1.5176

80-1380

Reference: Hamilton, C.W., and Hadjian, A.H., 1976 [12]

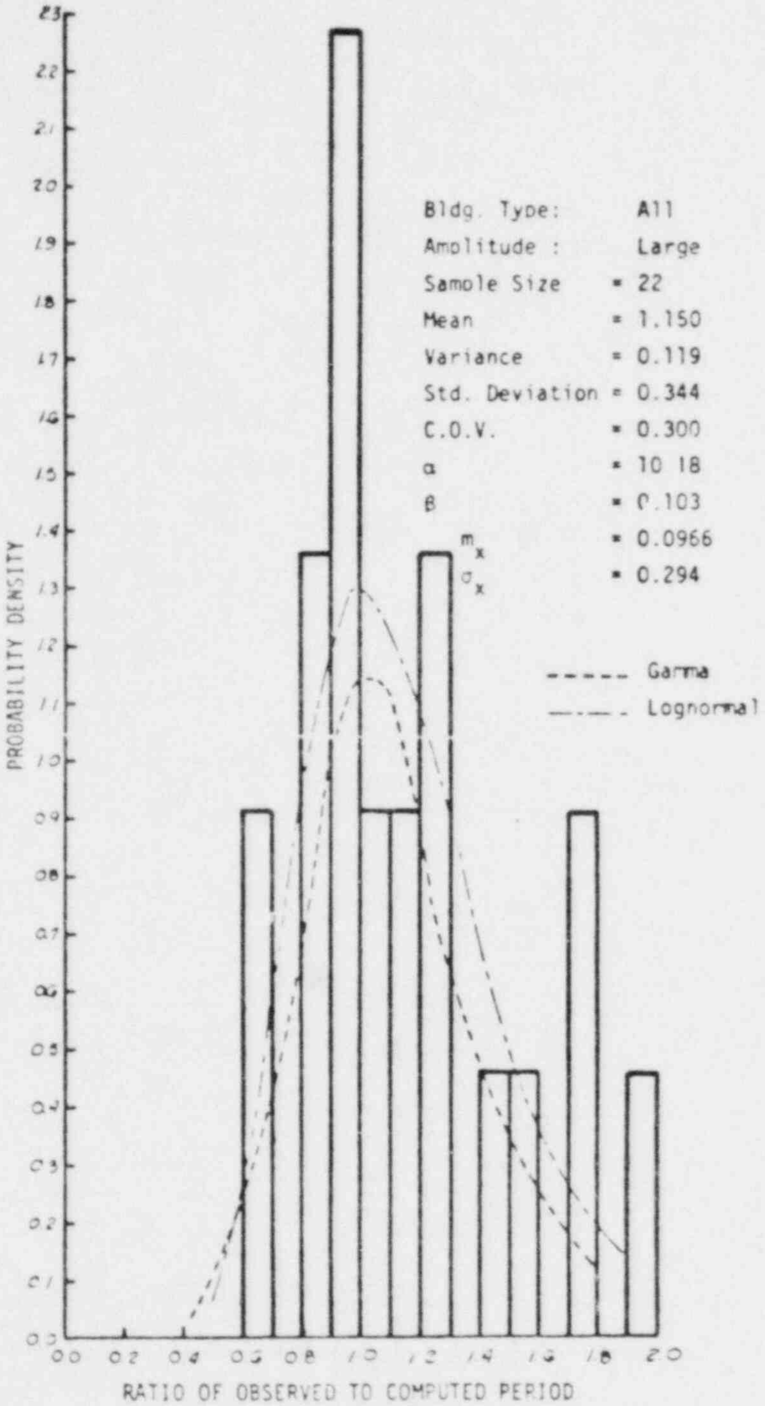
Figure 2-8. Cumulative Distributions of Ratios of Actual-to-Calculated EI-products



Reference: Haviland, R., 1976 [31]

80-1380

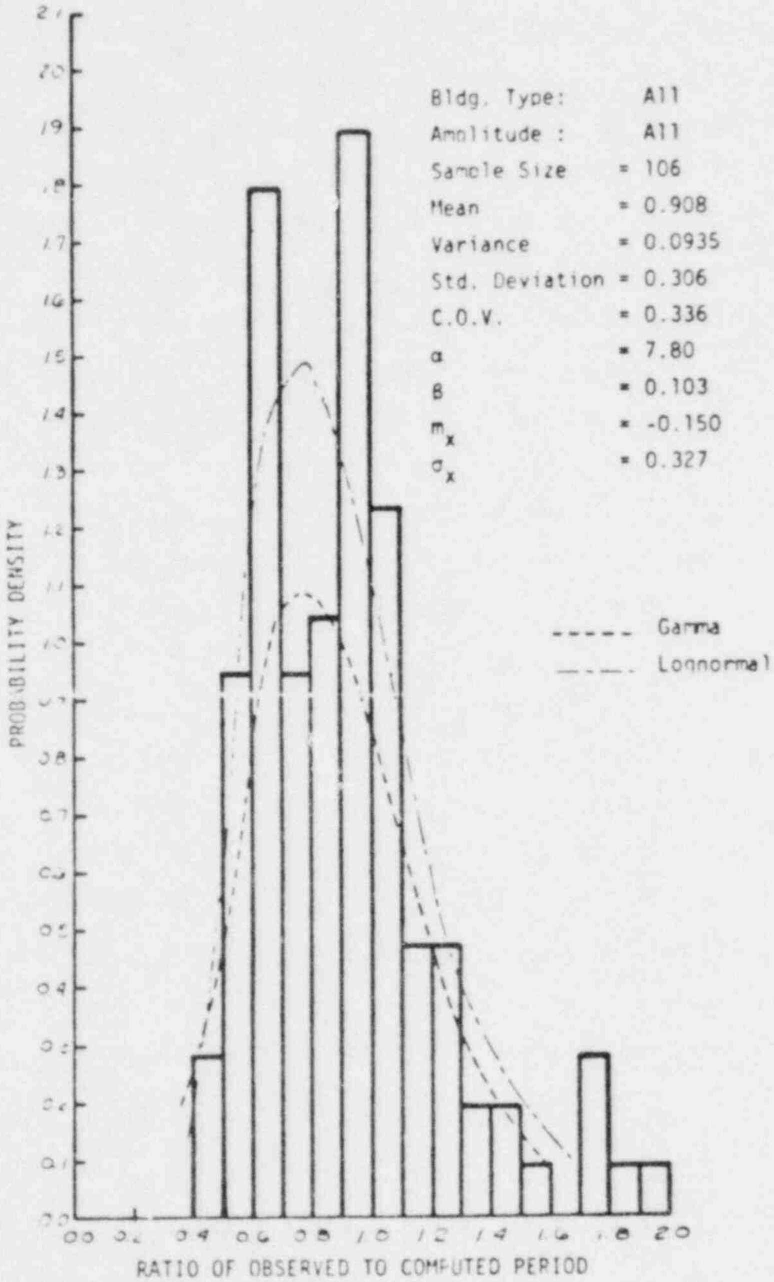
Figure 2-9. Histogram of Ratios of Observed to Computed Period Determinations for Small Amplitude Vibrations of All Building Types



Reference: Haviland, R., 1976 [31]

80-1380

Figure 2-10. Histogram of Ratios of Observed to Computed Period Determinations for Large Amplitude Vibrations of All Building Types



Reference: Haviland, R., 1976 [31]

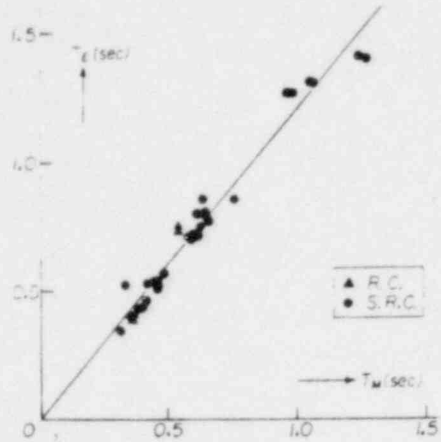
80-1380

Figure 2-11. Histogram of Ratios of Observed to Computed Period Determinations for Small and Large Amplitude Vibrations of All Building Types

Evidence of lengthening of fundamental periods of building structural systems at higher amplitudes of vibration is well recognized in the literature. In [31, 49, 54] a summary of this amplitude dependence of natural periods is reported for data taken in Japan and the U.S.A. Data reported in [31, 49, 54] are reproduced in Figures 2-12 to 2-16 for steel, reinforced concrete and lumped steel-reinforced concrete structures. Further evidence of amplitude-history dependence of natural periods of vibration is summarized in the above mentioned references [31] and [54], the results of which are reproduced in Figures 2-17 to 2-20 for steel and reinforced concrete structures and mixed steel-reinforced concrete structures.

So far, the data reported above are for individual components and building structures. Realistic data on eigenproperties of major substructures, which are typically a coupled combination of a number of substructures and their foundation,are rare in the literature. Obviously, most nuclear power plant facilities are examples of major structures; however, due to their inherently massive and complicated nature very few tests are reported in the free literature. However, a literature search revealed some data on system natural periods, and to a lesser extent, mode shape data. An ambitious program of small and large amplitude vibration tests with state-of-the-art mathematical modeling techniques was recently undertaken on a decommissioned nuclear power plant in the Republic of West Germany [58]; tests of this nature are expected to yield valuable insight on the dynamic characteristics of complex structures.

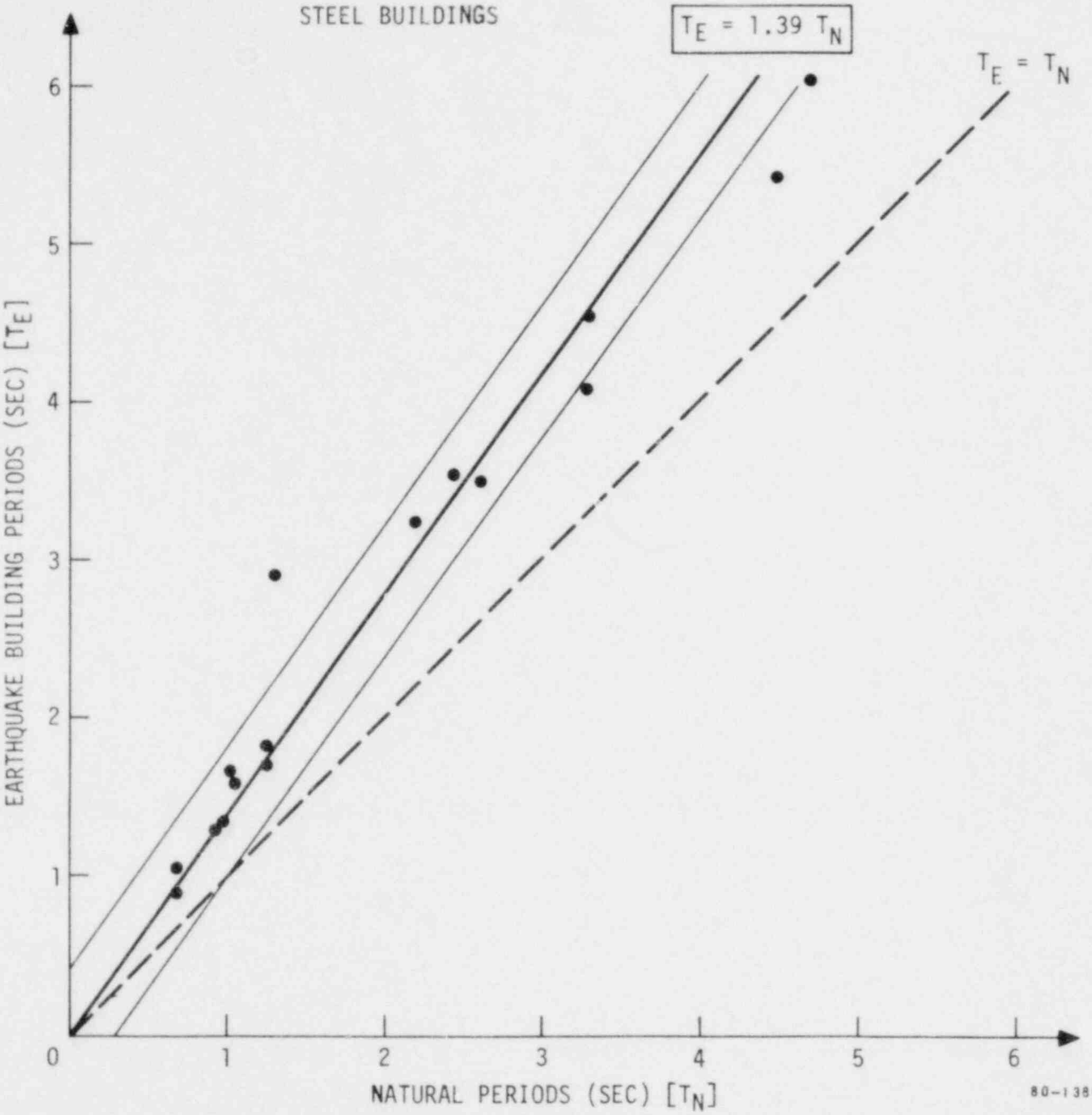
Energy dissipation of structural systems in the elastic range is normally computed using the concept of equivalent viscous damping [31]. In many cases, this method of determining damping values of actual buildings from vibration tests is sufficient; however, precise and quantitative descriptions of damping mechanisms in structures are not yet available.



80-1380

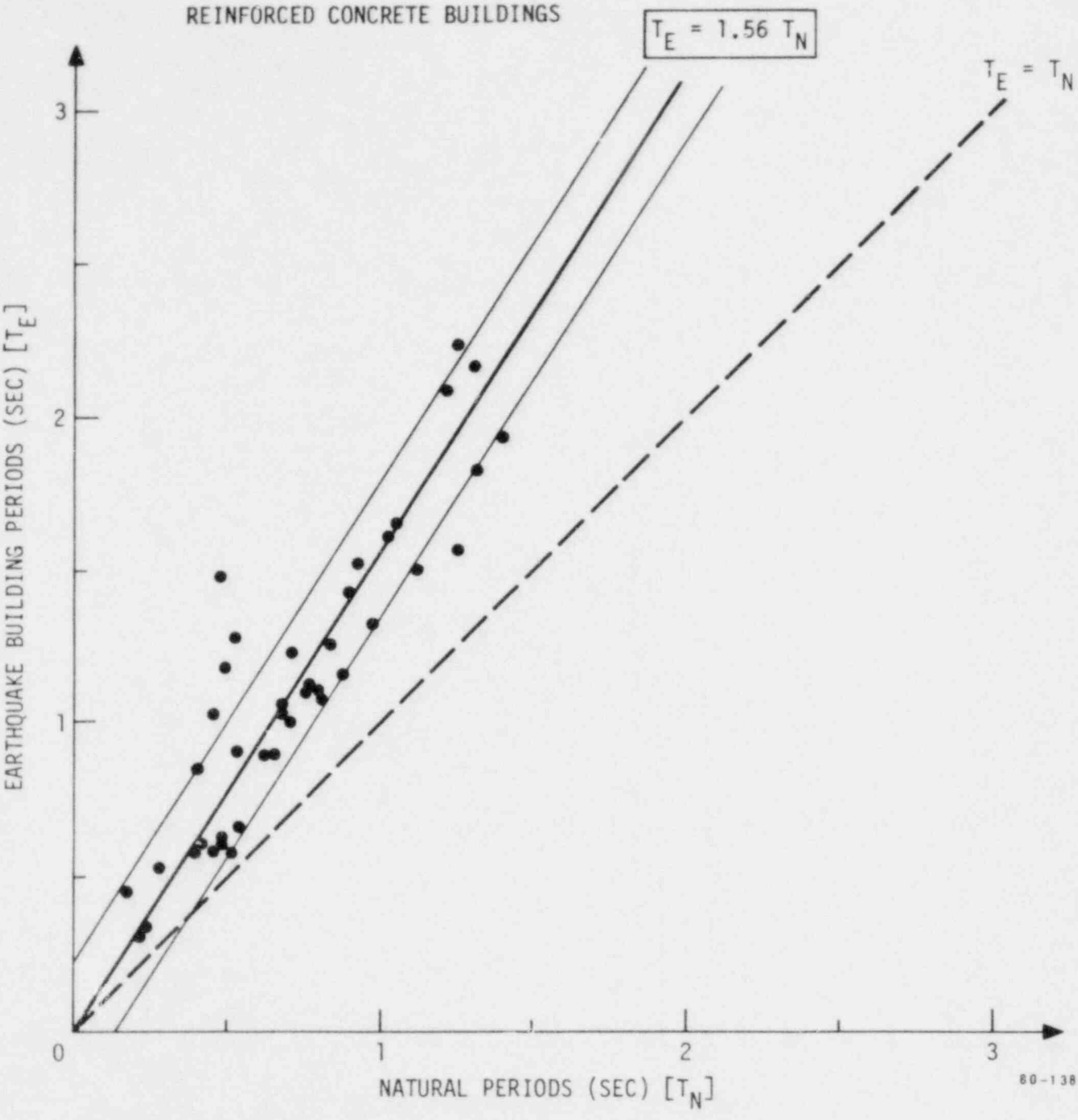
Reference: Tanaka, T. et. al., 1969 [49]

Figure 2-12. Relation Between the Fundamental Periods of the Buildings Estimated from the Strong-Motion Seismograph Records (T_E) and Those from the Records of Microtremors in the Buildings (T_M)



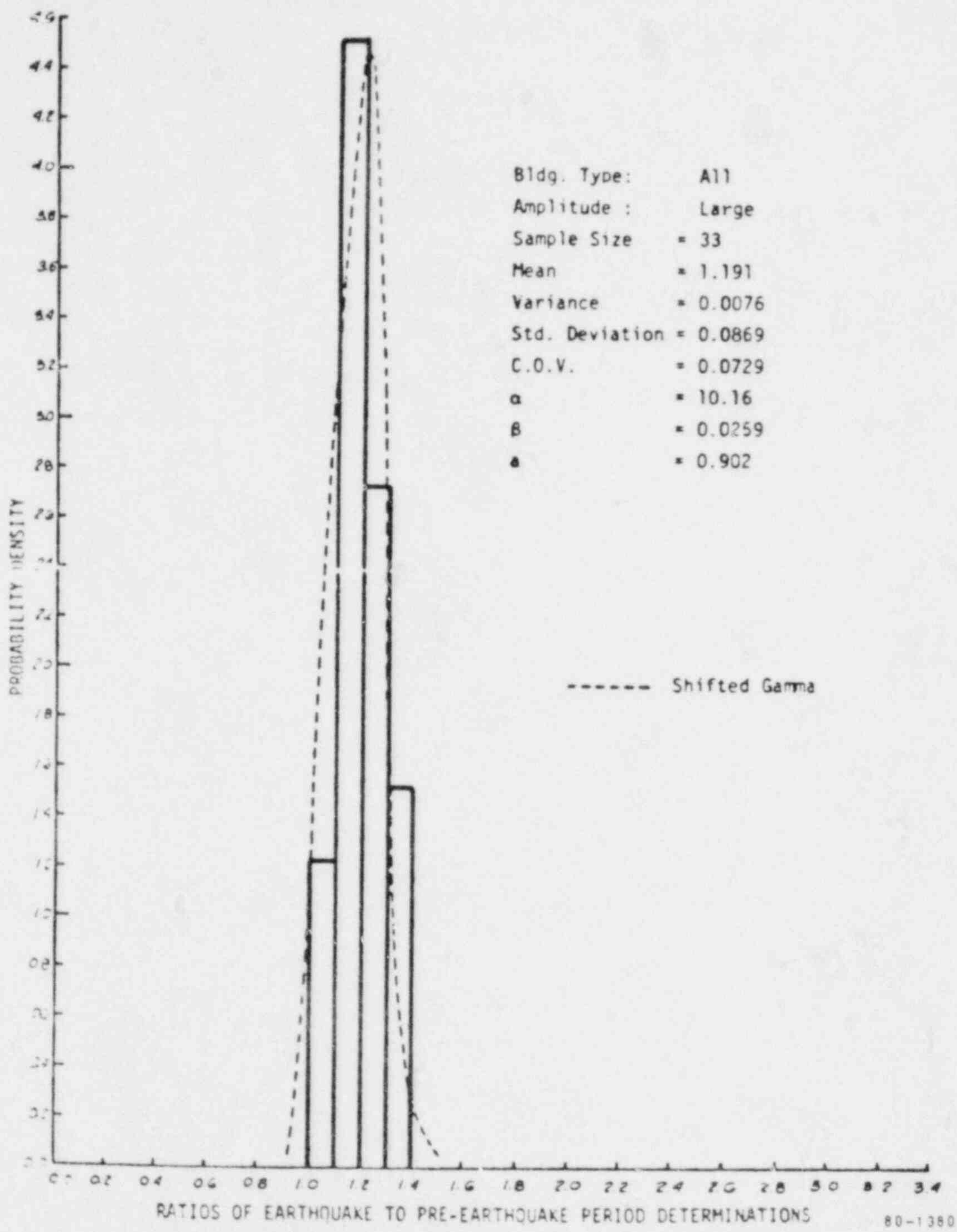
Reference: 1971 San Fernando Earthquake Data [54]

Figure 2-13. Relation Between Natural Periods and Earthquake Building Periods. Steel Buildings.



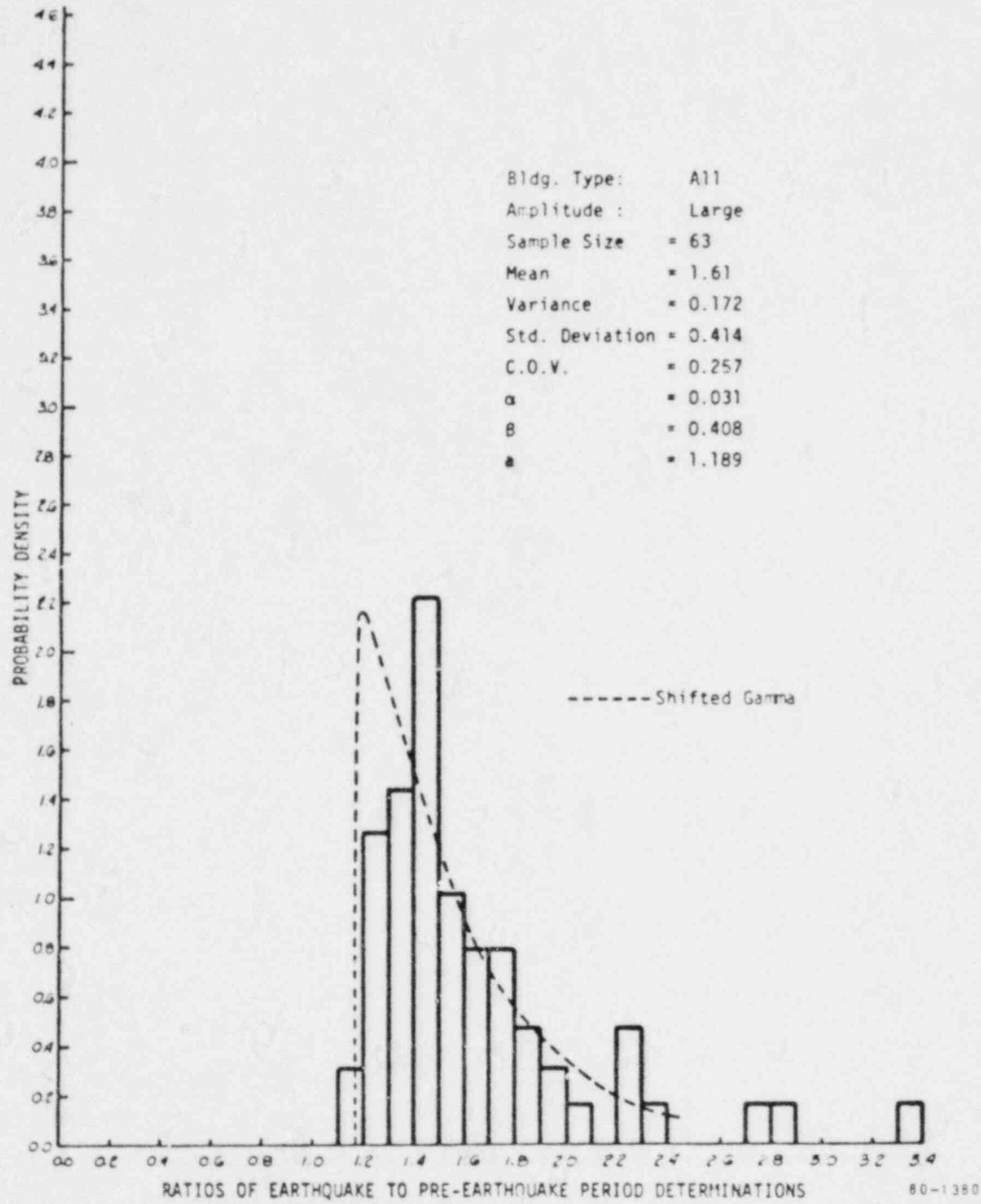
Reference: 1971 San Fernando Earthquake Data [54]

Figure 2-14. Relation Between Natural Periods and Earthquake Building Periods. Reinforced Concrete Buildings.



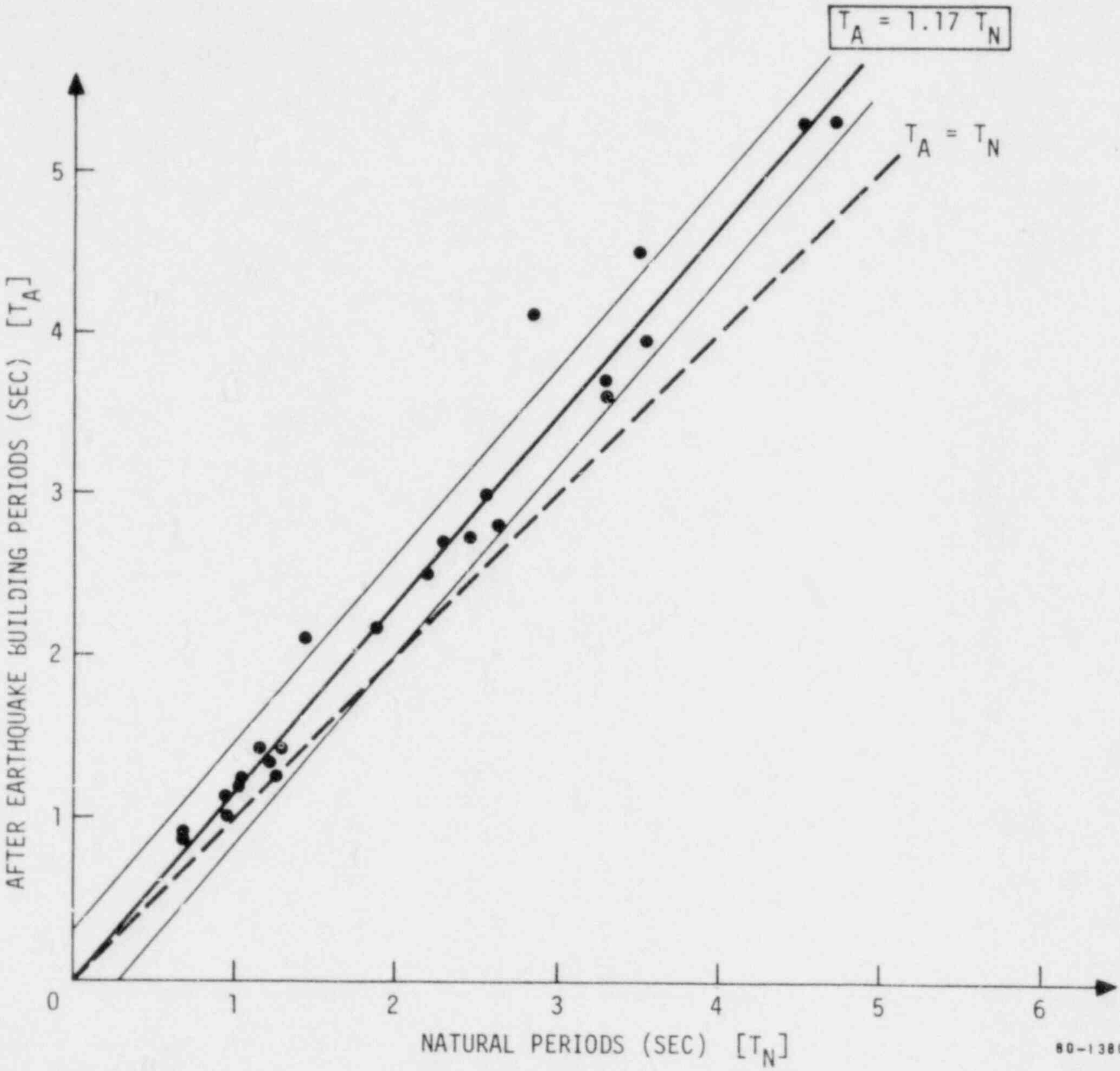
Reference: Haviland, R., 1976 [31]

Figure 2-15. Histogram of Ratios of Earthquake to Pre-Earthquake Period Determinations for Buildings Subjected to an Earthquake Near Tokyo



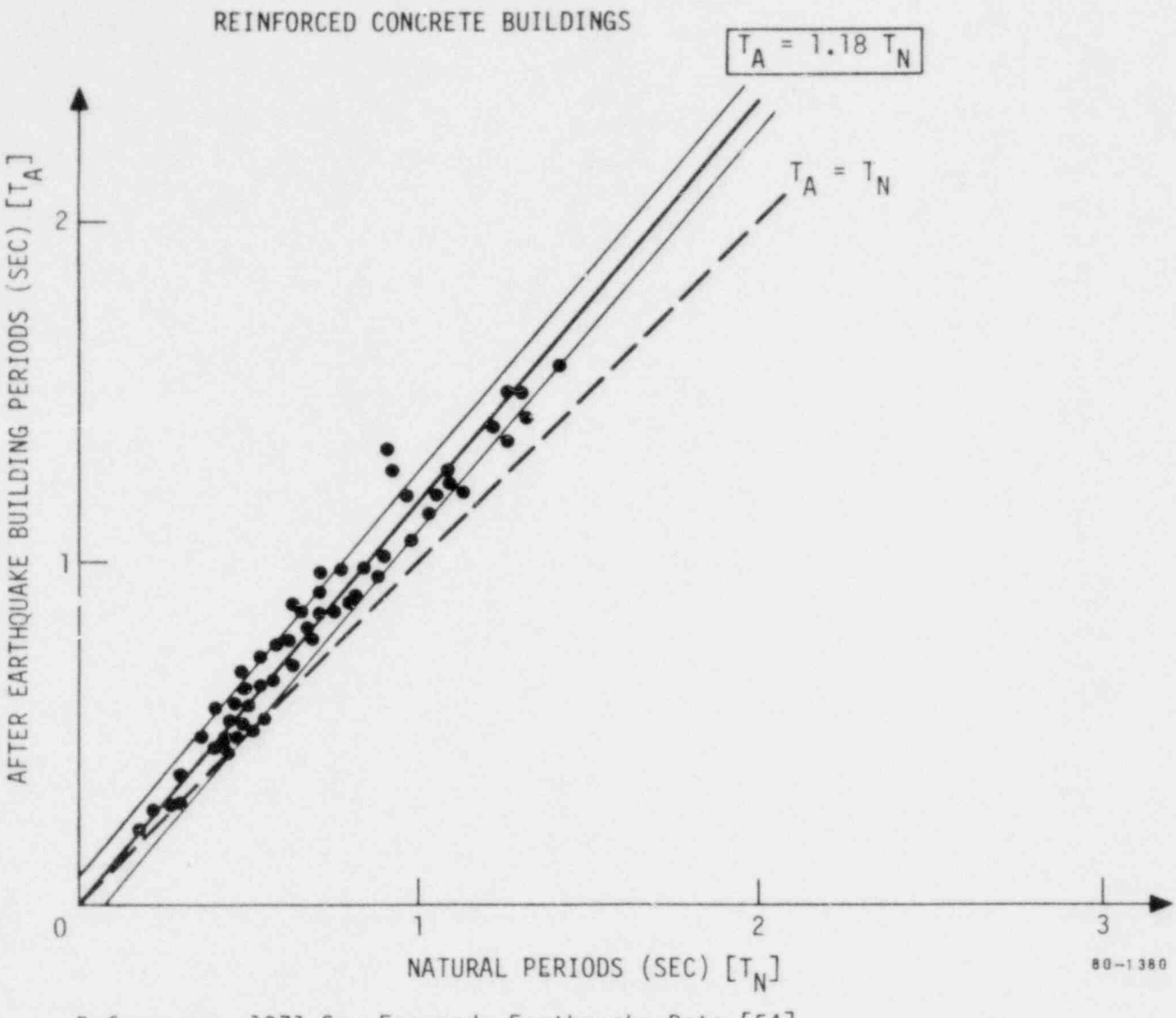
Reference: Haviland, R., 1976 [31]

Figure 2-16. Histogram of Ratios of Earthquake to Pre-Earthquake Period Determinations for Buildings Subjected to the San Fernando Earthquake



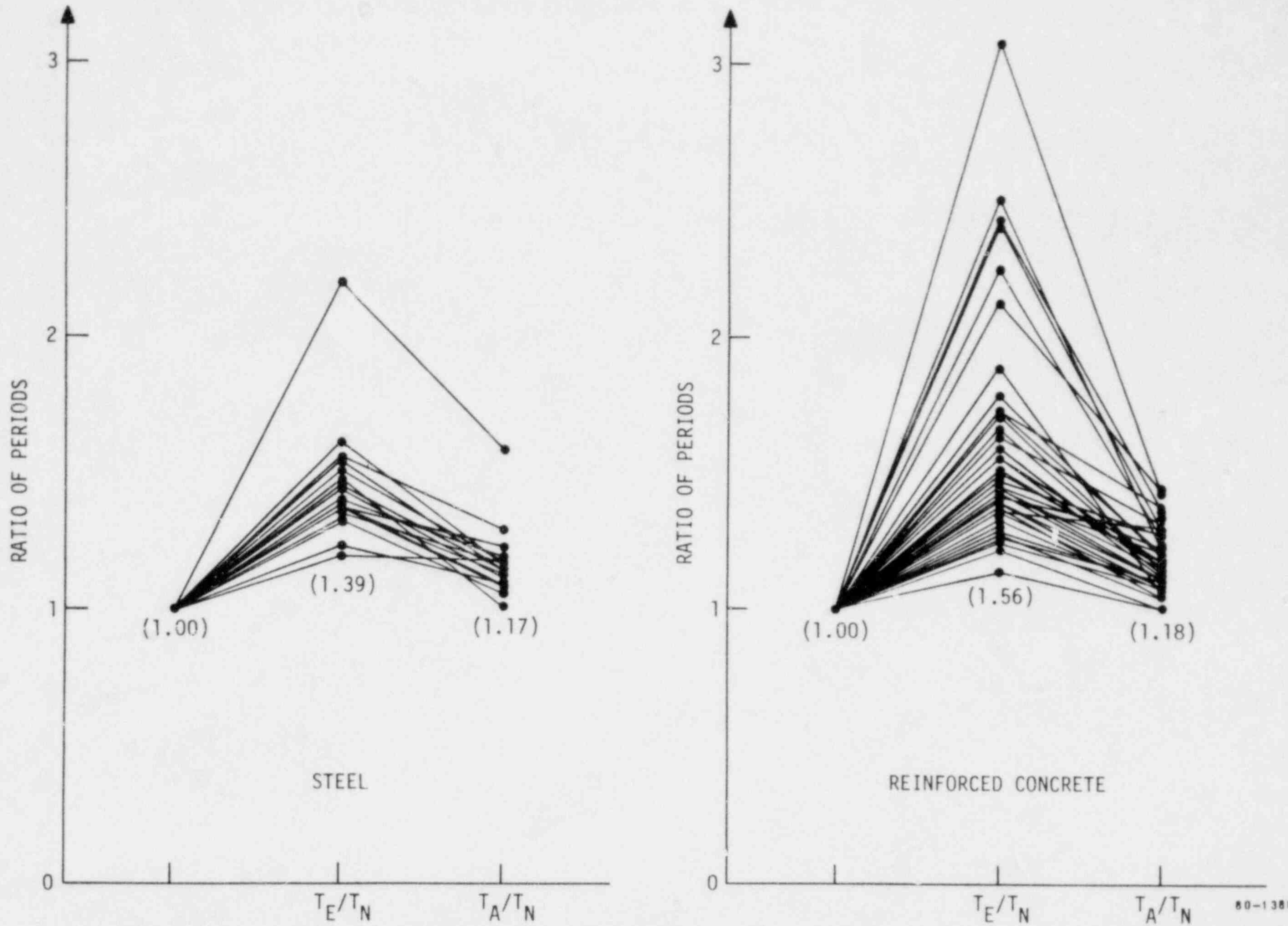
Reference: 1971 San Fernando Earthquake Data [54]

Figure 2-17. Relation Between Natural Periods and After Earthquake Building Periods. Steel Buildings.



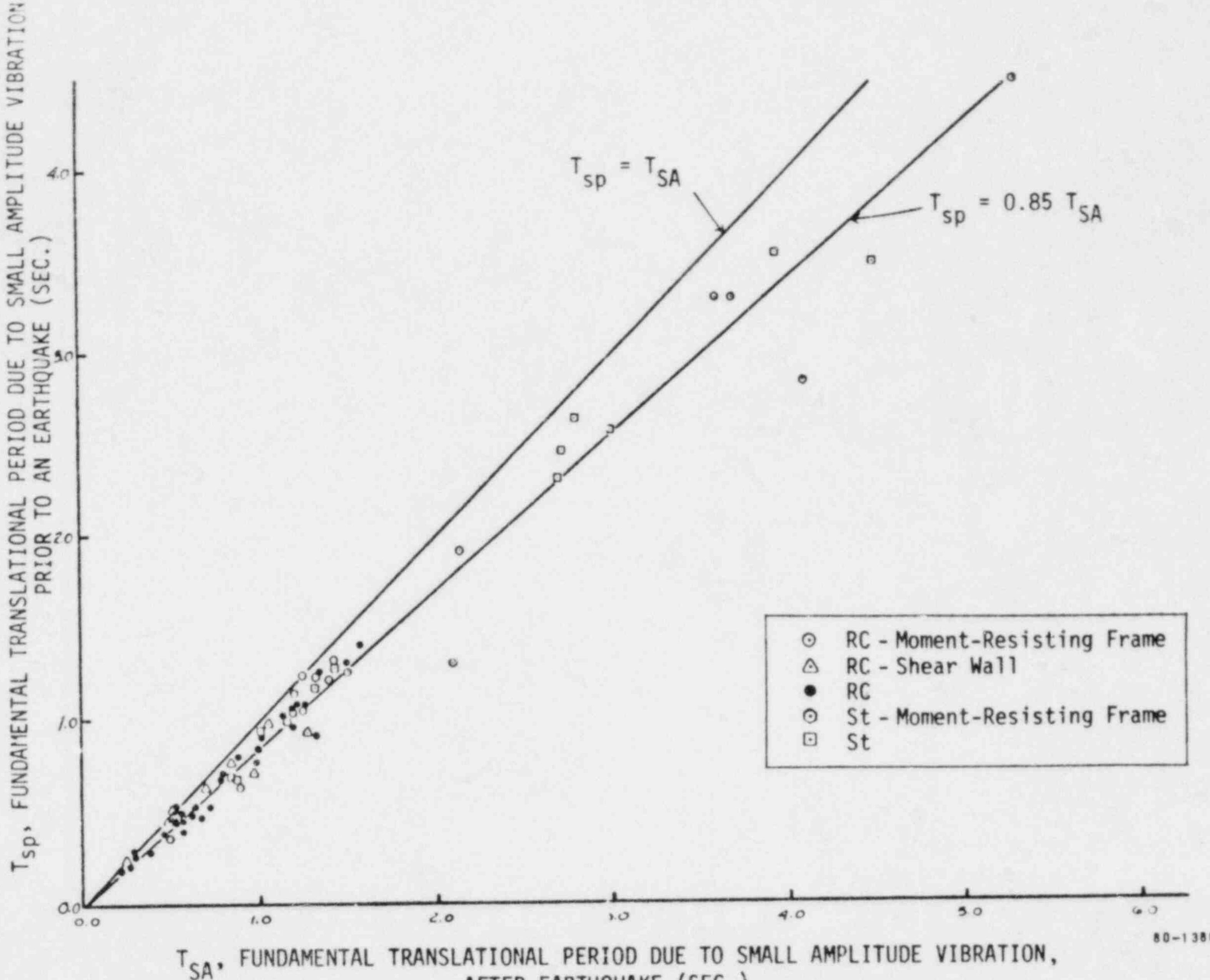
Reference: 1971 San Fernando Earthquake Data [54]

Figure 2-18. Relation Between Natural Periods and After Earthquake Building Periods. Reinforced Concrete Buildings.



Reference: 1971 San Fernando Earthquake Data [54]

Figure 2-19. Relations Between Pre-Earthquake Natural Periods, Earthquake Building Periods and After Earthquake Building Periods



Reference: Haviland, R., 1976 [31]

Figure 2-20. Pre- VS. Post-Earthquake Period Determinations for Buildings Subjected to San Fernando Earthquake

Although no quantitative measure exists, there is qualitative evidence from measurements that damping increases with vibration amplitude [31]. A summary of data on fundamental mode damping is given in [31]. In Figures 2-21 to 2-23 we reproduce the results of [31] for damping values for small amplitude vibrations for steel, reinforced concrete and composite buildings, respectively. Additional data on damping values for nuclear power plants are reported in [39 and 59].

2.3 Classification of Data

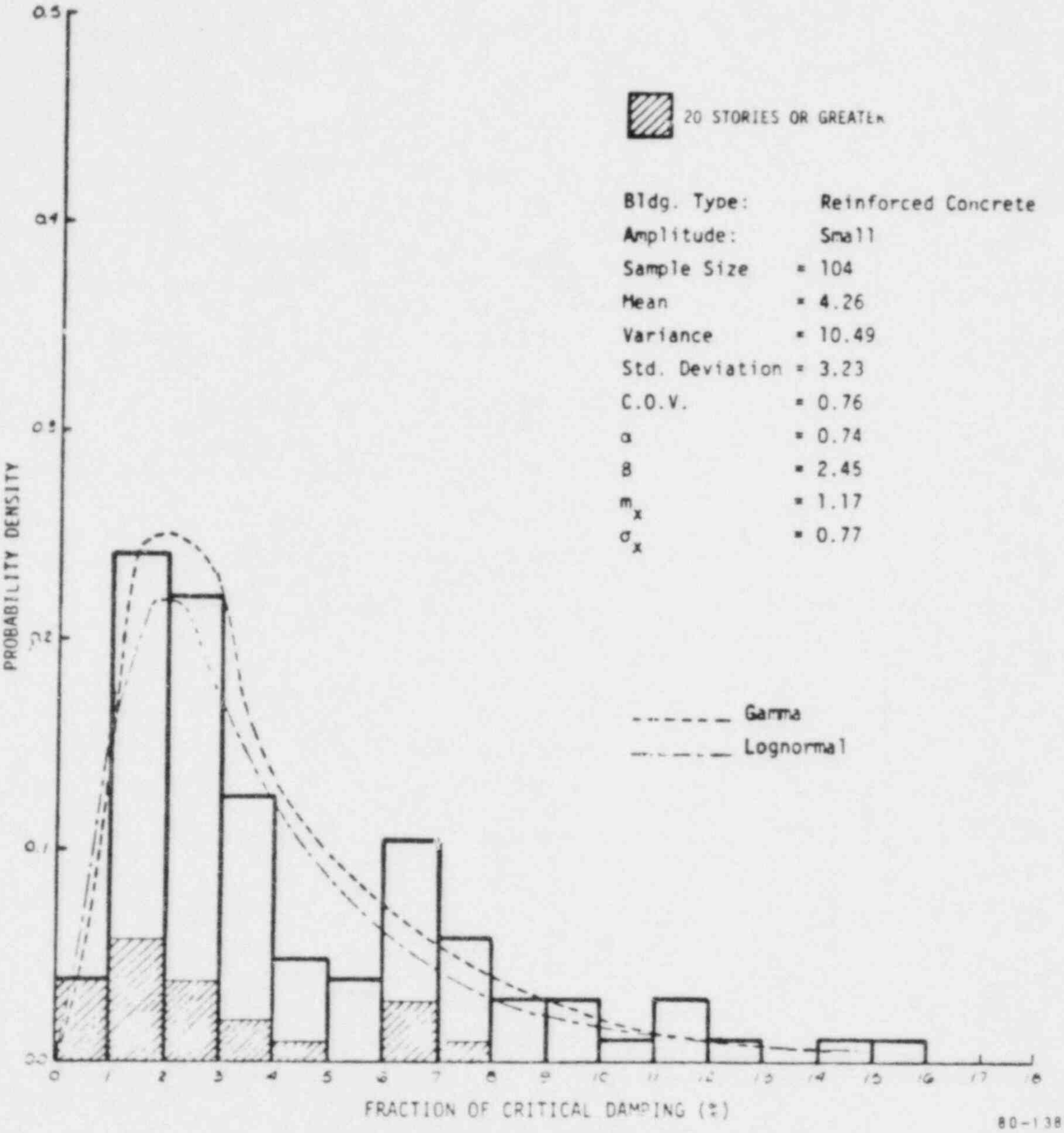
Consistent with the methodology developed elsewhere in this report it is instructive to classify the data into three categories, namely: (a) Major Structures, (b) Substructures, and (c) Structural Elements (members). We discuss below each of the three categories of data.

Major Structures

In this category we include large structural systems which may be a number of structures that are coupled to each other either through sharing a common foundation, being physically attached or a combination of the two (clearly, nuclear power plants belong to this category). Single structures which are a collection of a set of distinct substructures connected together, may also be classified to belong to this category (e.g., modular structures and precast or prestressed structures). Data in this category are expected to bear information on coupling effects of the various interconnected structures.

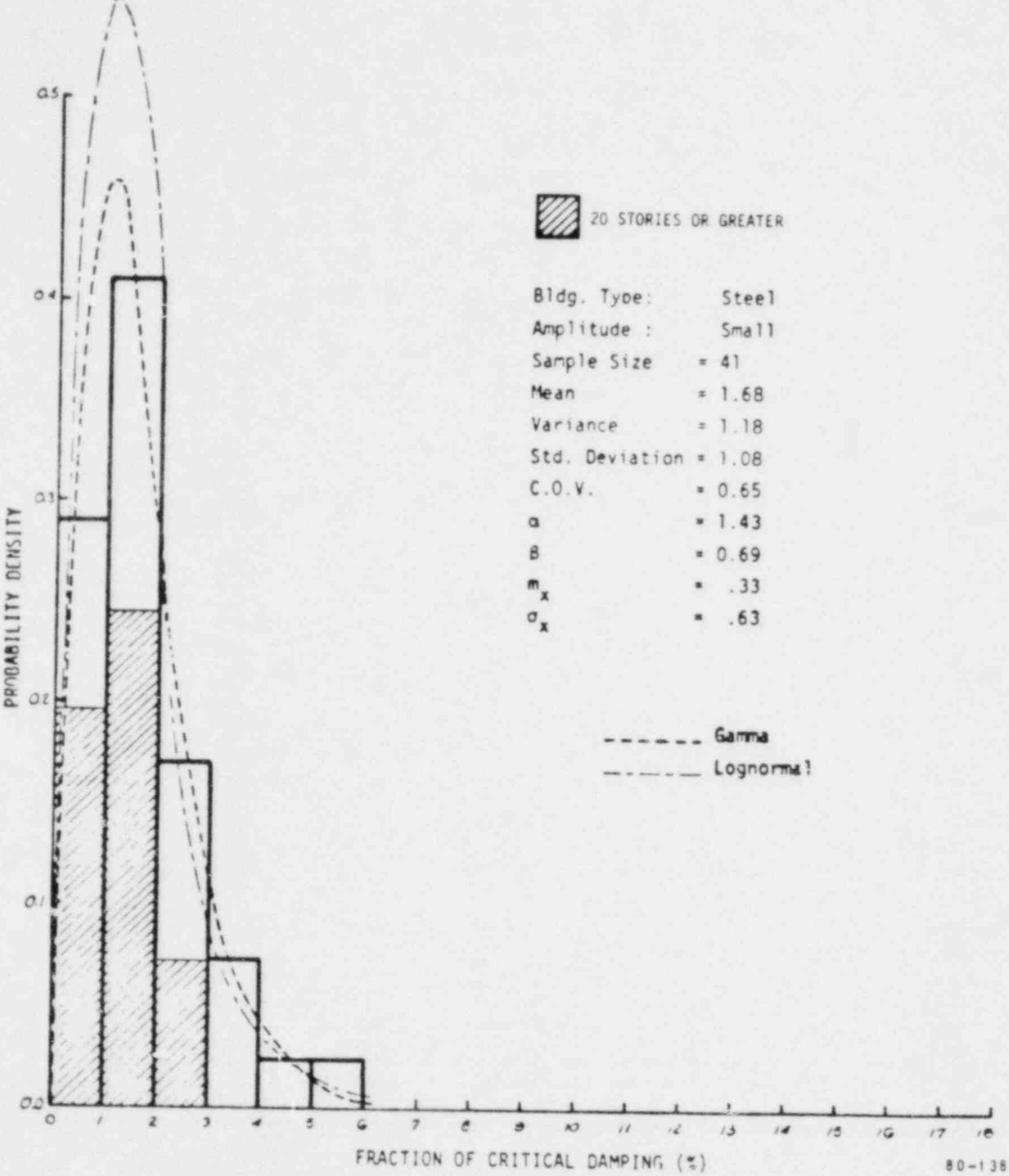
Substructures

To this category belong any collection of components which as a unit is designed to perform a specific function. From this definition, it is



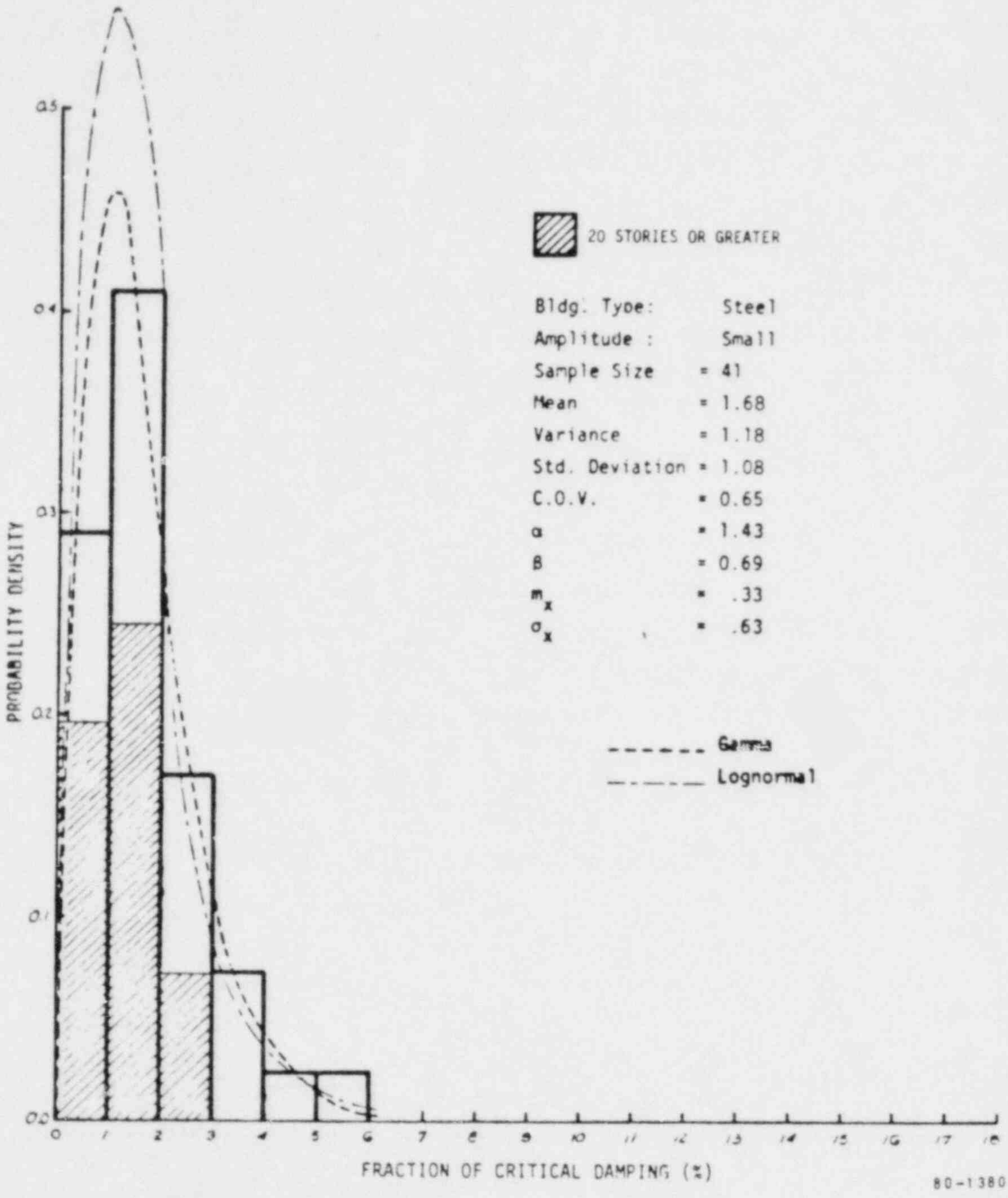
Reference: Haviland, R., 1976 [31]

Figure 2-21. Histogram of Damping Determinations for Small Amplitude Vibrations of Reinforced Concrete Buildings



Reference: Haviland, R., 1976 [31]

Figure 2-22. Histogram of Damping Determinations for Small Amplitude Vibrations of Steel Buildings



Reference: Haviland, R., 1976 [31]

Figure 2-23. Histogram of Damping Determinations for Small Amplitude Vibrations of Composite Buildings clear that a number of substructures may emerge for the same collection of components, when viewed by different individuals. This is perfectly acceptable and is precisely the intent. The only requirement is the availability of test data (such as modal data) for the particular substructure chosen. It is also clear that a full-size building may be regarded as a substructure when viewed as a simple member of a major structural system as defined in (a). The inclusion of this last category as a substructure is significant, since building data can profitably be utilized to furnish us with additional information in the analysis of uncertainty of nuclear power plant facilities.

Structural Elements (members)

This category includes the individual elements of structures which when interconnected, form the complete structure or substructure. Clearly, connectors and fasteners belong to this family; unfortunately, their stiffness properties are the least understood. Existing methods of uncertainty analysis make exclusive use of only this type of data [3 to 7], [7 to 9], [20 to 21].

3. ANALYTICAL FORMULATION

The mathematical formulation of methods for evaluating structural uncertainty will have a substructures orientation. This means that the basic building blocks of major structural systems are considered to be substructures, as opposed to simple structural elements such as beams, columns, shell elements, etc. This is not to say that standard finite element modeling codes cannot be used. On the contrary, it is expected that standard codes will be used to generate substructure mass and stiffness matrices, as well as the sensitivity coefficients (first order partial derivatives) required to perform a linearized statistical analysis of modal parameters. The following subsections discuss the coordinate systems and their corresponding equations of motion, the selection of parameters to represent structural uncertainty, and a general formulation of the structural uncertainty analysis recommended for SMACS. Before engaging the detailed development of methodology, however, a brief overview and summary is offered to help the reader establish some perspective.

Section 3.1 lays the foundation for the methodology by describing the coordinate systems and corresponding equations of motion for any given major structural system, and its substructures. These equations should be sufficiently general to relate directly to the structural models used in SMACS, as well as to those which represent the various structures from which uncertainty data must be extracted. Modal coordinates have been chosen for the latter. However, mode shapes define the modal coordinates; because the assumption of structural uncertainty implies that analytically predicted mode shapes will differ from the (hypothetical) "true" modes, it is convenient to define two modal coordinate systems. Thus, three coordinate systems in all (one involving <u>nodal</u> displacements and two involving <u>modal</u> displacements) are required. Parallel sets of coordinate systems at the substructure and major structural system levels therefore result in a total of 3(N+1) coordinate systems being defined, where N is the number of substructures comprising the major structural system, and the "1" added to N accounts for the additional set required for the major structural system itself.

Section 3.2 proceeds to derive the modal parameters recommended to embody structural uncertainty at the substructure level. The intent is to choose these parameters in such a way as to (1) account for all significant contributions to structural uncertainty (i.e., including all of the potential sources listed in Table 2-1 as well as those which may have been overlooked), and (2) provide a common basis for relating the observed uncertainties (differences between predicted and measured structural characteristics) among various structures or substructures within a given generic family.

Section 3.3 suggests an alternative set of parameters for embodying structural uncertainty. Whereas those discussed in Section 3.2 relate to the modal properties of structures, those of Section 3.3 are identified with the physical partitioning of a substructure in the sense that uncertainties can be localized and segregated according to physical characteristics such as bending stiffness, shear stiffnesses, structural mass, equipment mass, etc. The mapping of one type of uncertainty (e.g., modal) into the other (localized physical) is not a one-for-one mapping. In fact the two representations of uncertainty result from two different ways of "slicing the cake" so to speak. Each presents a different view of the total picture. Transformations between one view and the other should enhance the understanding of both. One of the advantages of representing uncertainty in terms of localized physical characteristics is that contributions from such things as operating loads, deterioration, maintenance, and the extrapolation of small to large amplitude behavior are more easily accounted for.

Section 3.4 presents a general methodology for evaluating structural uncertainty. This methodology is considered to be a broad general framework within which various methods may be defined. The concept is illustrated by a flow diagram through which many paths can be charted. A total of twelve optional paths (methods) is presented, each beginning with different kinds of structural uncertainty data, but all leading to a covariance matrix involving three random variables. The three random variables represent uncertainty in modal frequency, amplitude and damping and are cast in dimensionless form so that their distributions can be sampled, and the sample variates scaled according to the particular characteristics of the structure being modeled. This enables a represenation of the variation in the modal parameters of the structure being analyzed.

The reader may benefit by jumping ahead a ways at this point, to catch a better glimpse of where the methodology ends up. The methodology is presented in Section 3.4 by "walking through" each of the twelve options, one step at a time. As each new situation is encountered, it is explained. As each successive option is described, reference is made to earlier described options for points of similarity and/or difference. Option 1 is particularly important in this regard, because it explains each step of a complete method, from start to finish. The other eleven options all refer back to Option 1, either directly or indirectly.

The last six pages of Option 1, beginning with Equation (3.43) are of central importance. It is here that the equations of structural response are presented showing how the three normalized random variables are defined. Also contained in these few pages is a discussion of how the statistics of system eigenvalues and eigenvectors (λ and ϕ) can be processed to obtain meaninfgul statistics for the three normalized random variables. Certain assumptions are tentatively suggested, subject to proper numerical verification. Among these is the assumption, for example, that eigenvalue and eigenvector statistics tend to be uncorrelated, at least in those cases where generic uncertainty data (as opposed to structure-specific data) control the confidence of the estimates. It is important to recognize, however, that these assumptions are not essential to the methodology; one has the choice of making alternative assumptions, or no assumptions at all in which case the computational effort required during numerical simulation may increase substantially.

Perhaps the most important feature of the proposed methodology is its ability to utilize frequency and mode shape data from non-nuclear structures such as steel frame and reinforced concrete buildings, as a basis for quantifying the uncertainty in structural response predictions made for the major structural systems of nuclear power plants. The benefit offered by this feature is that predicted response uncertainties can be quantified on the basis of actual data which relate directly to observed differences between predicted and measured behavior of similar structures.

3.1 Coordinate Systems and Equations of Motion

Since structural characteristics are the primary concern of this study, only the homogeneous equations of motion will be written here. Equations of motion for the ith substructure are therefore

$$M^i \ddot{x}^i + C^i \dot{x}^i + K^i x^i = 0 \qquad (3.1)$$

where x^i denotes a vector of nodal displacements, and where M^i, C^i and K^i denote respectively the mass, damping and stiffness matrices of the ith substructure corresponding to that coordinate system. For the time being, C^i will remain undefined. The mass and stiffness matrices will be of the form typically generated by standard finite element modeling codes. No restrictions are placed on M^i and K^i except that they be symmetric and positive definite. Thus, for example M^i and K^i could represent the mass and stiffness matrices of a substructure after static reduction has been performed to reduce the general coordinate vector x^i to a smaller set of dynamic degrees of freedom.

It will be assumed that the "structure" of the model represented by these equations of motion is correct (i.e., given the right parameter values, M^i and K^i, these equations of motion would indeed represent the actual substructure) but that the true parameter values are not precisely known. We shall denote our original estimate of M^i and K^i by $°M^i$ and $°K^i$ respectively, and denote the differences between these estimates and the (presently unknown) "true" values by ΔM^i and ΔK^i. Thus

$$M^i = °M^i + \Delta M^i \tag{3.2a}$$

$$K^i = °K^i + \Delta K^i \tag{3.2b}$$

The structural modeling effort will thus lead to the undamped eigenproblem

$$(°K^i - °\lambda_j^i \, °M^i)°\phi_j^i = 0 \tag{3.3}$$

where $°\lambda_j^i$ denotes the j<u>th</u> predicted eigenvalue for the i<u>th</u> substructure (a scalar), and $°\phi_j^i$ denotes its corresponding eigenvector. Consistent with the present notational convention, the symbols λ_j^i and ϕ_j^i will be used later on to denote the "true" j<u>th</u> eigenvalue and eigenvector.

The <u>predicted</u> mode shapes can be used to transform the equations of motion to a set of truncated modal coordinates, p^i.

$$x^i = {}^\circ\phi \; p^i \tag{3.4}$$

where $^\circ\phi$ is in general considered to be a rectangular matrix (more rows than columns). Applications of this transformation to Equation (3.1) (using $^\circ M^i$, $^\circ C^i$ and $^\circ K^i$ in place of M^i, C^i and K^i) results in

$${}^\circ m^i \, \ddot{p}^i + {}^\circ c^i \, \dot{p}^i + {}^\circ k^i \, p^i = 0 \tag{3.5}$$

where

$${}^\circ m^i = ({}^\circ\phi^i)^T \, {}^\circ M^i \, {}^\circ\phi^i \equiv I \tag{3.6a}$$

$${}^\circ c^i = ({}^\circ\phi^i)^T \, {}^\circ C^i \, {}^\circ\phi^i \tag{3.6b}$$

$${}^\circ k^i = ({}^\circ\phi^i)^T \, {}^\circ K^i \, {}^\circ\phi^i \equiv {}^\circ\lambda^i \tag{3.6c}$$

assuming that mode shapes are normalized to give unit modal mass. Notationally, I denotes an identity matrix and $^\circ\lambda^i$ denotes a diagonal matrix of predicted frequencies squared ($^\circ\lambda^i_{jk} \equiv {}^\circ\omega^i_j \, {}^\circ\omega^i_k \, \delta_{jk}$, where δ_{jk} is the kronecher delta).

By analogy with Equations (3.2) we shall assume that the "true" parameter values, m^i and k^i are related to those of our initial estimate by

$$m^i = {}^\circ m^i + \Delta m^i = I + \Delta m^i \tag{3.7a}$$

$$k^i = {}^\circ k^i + \Delta k^i = {}^\circ\lambda^i + \Delta k^i \tag{3.7b}$$

The substructure eigenproblem involving m^i and k^i in the p^i coordinate system may be stated

$$(k^i - \lambda_j^i m^i) \psi_j^i = 0 \tag{3.8}$$

where λ_j^i and ψ_j^i denote the "true" j<u>th</u> eigenvalue and corresponding eigenvector in the p^i coordinate system. The true eigenvector in the x^i coordinate system would then be given by

$$\phi_j^i = {}^\circ\phi^i \psi_j^i \tag{3.9}$$

Modal damping cannot be predicted the way modal mass and stiffness can. It is customary to rely on experimental measurements. Experimental measurements by definition relate to the actual structure, and strictly speaking should be associated therefore with the "true" structural model, in the "true" modal coordinate system. We shall associate this coordinate system with the coordinate vector q^i and define the transformation from p^i to q^i by

$$p^i = \psi^i q^i \tag{3.10}$$

or from x^i to q^i by

$$x^i = {}^\circ\phi^i \psi^i q^i = \phi^i q^i \tag{3.11}$$

Equations of motion are thus written in the q^i coordinate system as

$$I \ddot{q}^i + \xi^i \dot{q}^i + \lambda^i q^i = 0 \tag{3.12}$$

where in general the substructure modal damping matrix ξ^i will not be diagonal. Although the off-diagonal terms of ξ^i have in the past been neglected in substructuring type analysis, it is, strictly speaking, incorrect to do so [89]. We shall examine the implication of this statement later on. Suffice it to say for the time being that the diagonal elements, ξ^i_{jj} of the matrix ξ^i are given by the familiar relationship

$$\xi^i_{jj} = 2\, \zeta^i_j\, \omega^i_j \tag{3.13}$$

where $\omega^i_j \equiv \sqrt{\lambda^i_j}$ and ζ^i_j is the critical damping ratio of the jth mode of the ith substructure.

A completely parallel development can be given for a major structure comprised of a number of substructures. The results are identical to Equations (3.1) through (3.13) with all the "i" superscripts removed. It is therefore unnecessary to repeat the derivation. What is required, however, is to define how one proceeds from the isolated substructure equations to the coupled equations of motion representing the major structural system which is comprised of interactive substructures.

There are basically two ways in which this can be done. Both will be considered here, although the first is likely to be preferred for the sake of convenience. For the moment we shall overlook the fact that our structural modeling gives inaccurate predictions, and proceed as though the true structural parameters are known. Returning to the substructure equations of motion in the x^i coordinate system, we shall formally assemble the equations of motion for the major structure in the following block-diagonal form

$$m^{\dagger}\, \ddot{x}^{\dagger} + c^{\dagger}\, \dot{x}^{\dagger} + k^{\dagger}\, x^{\dagger} = 0 \tag{3.14}$$

where

$$x^{\dagger} = \left\{ \begin{matrix} x^{1} \\ x^{2} \\ \cdot \\ \cdot \\ \cdot \\ x^{i} \\ \cdot \\ \cdot \\ \cdot \\ x^{N} \end{matrix} \right\} \tag{3.15}$$

considering the major structure to be composed of N substructures, and where accordingly

$$m^{\dagger} = \begin{bmatrix} m^{1} & & & & & & & \\ & m^{2} & & & & & & \\ & & \cdot & & & & & \\ & & & \cdot & & & & \\ & & & & \cdot\; m^{i} & & & \\ & & & & & \cdot & & \\ & & & & & & \cdot & \\ & & & & & & & \cdot\; m^{N} \end{bmatrix} \tag{3.16}$$

and $c^{\dagger}$ and $k^{\dagger}$ are defined in a similar fashion. We shall consider the substructure equations of motion to be coupled by application of the constraint equations

$$G\, x^{\dagger} = 0 \tag{3.17}$$

where G is a rectangular matrix consisting of a number of rows equal to the number of constraint equations, and a number of columns equal to the length of the vector $x^{\dagger}$. We shall apply a reordering transformation, E, to $x^{\dagger}$ so as to separate the dependent coordinates, x^{d}, from those considered to be independent, x.

Thus we can write

$$G\ E \begin{Bmatrix} x^d \\ \hdashline x \end{Bmatrix} \equiv \bar{G} \begin{Bmatrix} x^d \\ \hdashline x \end{Bmatrix} = 0$$

By partitioning $\bar{G}$ we may solve for x^d in terms of x.

$$[\bar{G}_1 \ \vdots \ \bar{G}_2] \begin{Bmatrix} x^d \\ \hdashline x \end{Bmatrix} = 0$$

$$x^d = -\bar{G}_1^{-1}\ \bar{G}_2\ x$$

$$\begin{Bmatrix} x^d \\ \hdashline x \end{Bmatrix} = \begin{bmatrix} -\bar{G}_1^{-1}\ \bar{G}_2 \\ \hdashline I \end{bmatrix} x$$

It follows formally that

$$x^{+} = E \begin{bmatrix} -\bar{G}_1^{-1}\ \bar{G}_2 \\ \hdashline I \end{bmatrix} x \equiv \beta x \qquad (3.18)$$

where β is a rectangular matrix commonly referred to as the compatibility matrix. The coupled equations of motion for the major structure can then be written

$$M\ \ddot{x} + C\ \dot{x} + K\ x = 0 \qquad (3.19)$$

where $M = \beta^T\ M^{+}\ \beta$, and where C and K are similarly defined.

The equations of motion written in the various coordinate systems, and the transformations which relate them are summarized diagrammatically in Figure 3-1.

An alternative substructuring procedure for writing the structural system equations of motion in terms of substructure equations of motion is what is usually referred to as "modal synthesis" [90, 81, 88]. For comparison with Figure 3-1 which illustrates a direct substructuring procedure, the modal synthesis approach is diagrammed in Figure 3-2. In this case one must be particularly concerned with the boundary conditions assumed in calculating the substructure modes so that unacceptably large errors do not result from modal truncation at the substructure level. Since the first method (or "direct" method) is considered to be the preferred one here, the modal synthesis approach will not be discussed in further detail. It will be mentioned later on when discussing the treatment of structural uncertainty because it does appear to offer one advantage over the direct method. That advantage will be pointed out in Section 3.3.

3.2 Selection of Parameters to Represent Structural Uncertainty

One of the obvious conclusions to be drawn from Section 2 is that there are many more acknowledged sources of uncertainty than there are data to quantify them. Even if the data were available, a tremendous data processing effort would be required to compute the corresponding statistics of modal properties. Confidence in these computed values might be low, to the extent that significant sources of uncertainty have been overlooked. This situation is quite similar, in fact, to that of trying to model damping in complex structures. There are too many damping mechanisms about which too little is known, and there is no guarantee that all of the significant ones can be identified. In the case of damping, we must resort to direct measurement of composite

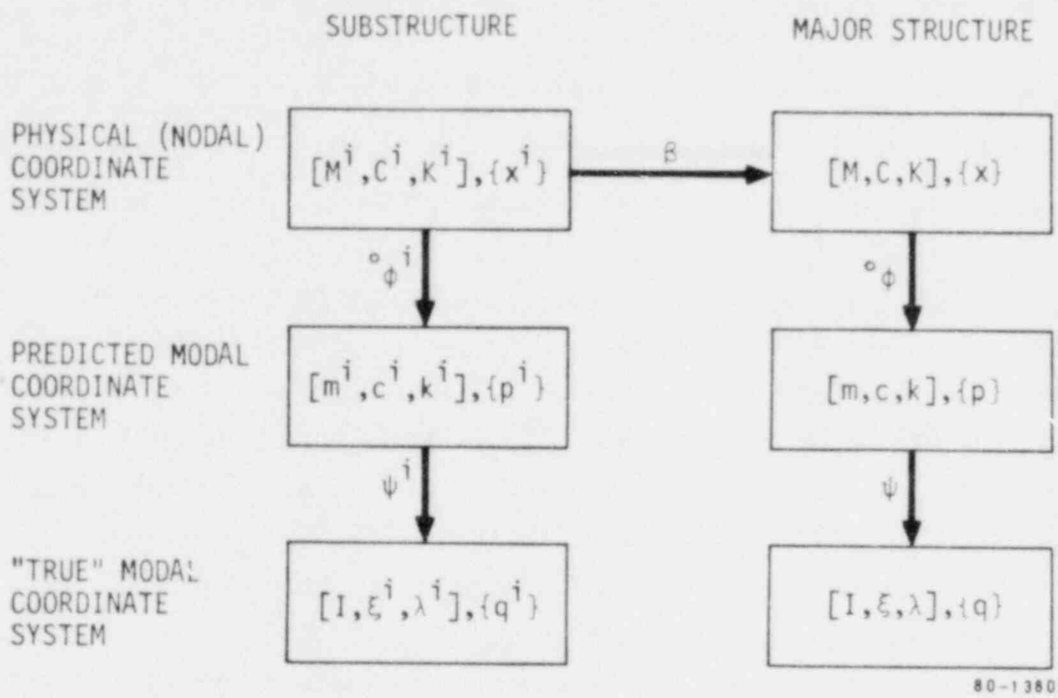


Figure 3-1. Diagram of Coordinate Systems and Related Transformations for Substructuring using Direct Method

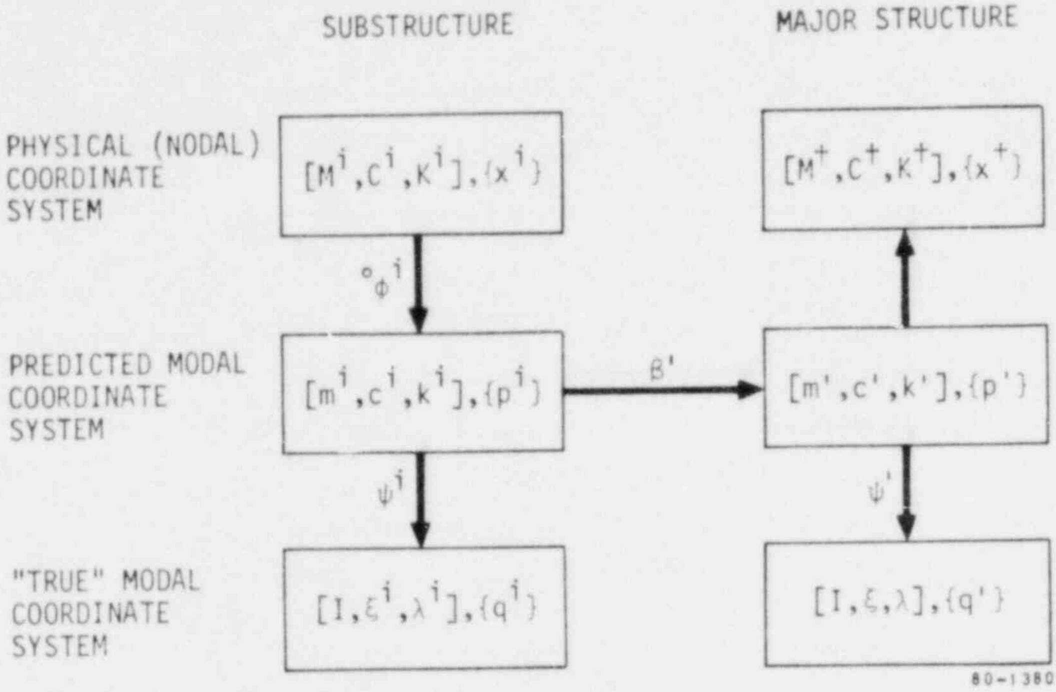


Figure 3-2. Diagram of Coordinate Systems and Related Transformations for Substructuring using Modal Synthesis Method

structural damping. A similar approach is suggested for the estimation of structural uncertainty.

Normally, the measurement of damping requires dynamic testing of the complete structure so that the damping mechanisms which contribute to damping in a particular mode of vibration are exercised in a realistic way. Such a direct approach would also be desirable for measuring structural uncertainty. The analogous situation here would be to have both the predicted and measured modal data for a family of similar major structures, from which statistics of the differences could be computed. Clearly, however, this is not possible.

Conceivably, the next best thing would be to have such data from families of generic substructures of which the major structure is comprised. This is the essence of the approach proposed here. The basic plan is to take uncertainties in the modal properties of generic substructures, and combine them to predict uncertainties in the modal properties of major structural systems.

A crucial question arises here. What parameters can be defined to represent uncertainty in substructure modal properties? Obviously, the difference between predicted and measured resonant frequencies would constitute one type of parameter. But what about mode shape? It is often (but not always) true that frequencies can be predicted more accurately than mode shape. This would imply that mode shape uncertainties ought to be important in representing structural uncertainty. The problem here is to transfer knowledge of mode shape uncertainty from one structure to another within the same generic category. The generic categories defined in Section 1 were structural steel, reinforced concrete, and prestressed (or post-tensioned) concrete. These are very broad categories in which modal characteristics obviously will vary considerably. Some convenient form of normalization is required in order to do a statistical analysis.

An interesting possibility is suggested by the form of Equations (3.5, 3.6 and 3.7). It may be recalled that these equations were written in the substructure <u>predicted</u> modal coordinate space. Theoretically, the mass and stiffness matrices will be diagonal as shown in (3.6a) and (3.6c). However, since the theoretical model at best only approximates the (unknown) "true" modal properties, we expect that the "true" m^i and k^i will have off-diagonal terms which are not precisely zero in the predicted modal coordinate system. In fact we consider the diagonal matrices $°m^i = I$ and $°k^i = °\lambda^i$ to be perturbed by the matrices Δm^i and Δk^i. The elements of these matrices provide a basis for representing uncertainty at the substructure level in terms of modal characteristics. This assertion becomes clearer after a perturbation analysis is carried out to express Δm^i and Δk^i explicitly in terms of measured vs. predicted modal parameters.

The results of the perturbation analysis yield the following relationships:

$$\Delta m^i_{jj} = - 2\Delta\eta^i_{jj} \tag{3.20a}$$

$$\Delta m^i_{jk} = - \left(\Delta\eta^i_{jk} + \Delta\eta^i_{kj}\right) \tag{3.20b}$$

$$\Delta k^i_{jj} = \Delta\lambda^i_j + °\lambda^i_j \, \Delta m^i_{jj} \tag{3.20c}$$

$$\Delta k^i_{jk} = -°\lambda^i_j \, \Delta\eta^i_{jk} - °\lambda^i_k \, \Delta\eta^i_{kj} \, ; \quad j \neq k \tag{3.20d}$$

where $°\lambda^i_j$ are the theoretically predicted eigenvalues and

$$\Delta\lambda_j^i = \hat{\lambda}_j^i - {}^\circ\lambda_j^i \qquad (3.21a)$$

$$\Delta\eta_{jk}^i = \hat{\psi}_{jk}^i - \delta_{jk}^i \qquad (3.21b)$$

$$\hat{\psi}_{jk}^i = \left({}^\circ\phi_j^i\right)^T {}^\circ M^i \, \hat{\phi}_k^i \qquad (3.21c)$$

where $\hat{\lambda}_j^i$ and $\hat{\phi}_j^i$ are the measured (experimentally estimated) "true" eigenvalues and eigenvectors, respectively.

Now Δm^i and Δk^i begin to resemble the kind of terms we might expect to represent modal uncertainty. It may be noted in particular that Δk_{jj}^i contains $\Delta\lambda_j^i$, the difference between predicted and measured eigenvalues, and that

$$\hat{\psi}_{jk}^i = \left({}^\circ\phi_j^i\right)^T {}^\circ M^i \, \hat{\phi}_k^i$$

is a scalar measure of the cross orthogonality between the predicted mode shape ${}^\circ\phi_j^i$ and the measured mode shape $\hat{\phi}_k^i$. Clearly, if the "true" modes could be predicted perfectly <u>and</u> measured perfectly, then $\Delta\lambda^i \equiv 0$, $\psi^i \equiv I$ and $\Delta m^i = \Delta k^i = 0$.

It is significant to note that by virtue of the fact that ${}^\circ\phi_j^i$ and $\hat{\phi}_k^i$ occur in vector product form, many measurements are reduced to a single scalar value. This is certainly appealing from both a computational and a data storage point of view.

One further thing must be done before any statistical analysis can be performed. The parameters Δk_{jk}^i must be first normalized by a scaling transformation to remove the frequency dependence. We shall define this transformation as follows:

$$\Delta\tilde{k}^i = \left({}^\circ\lambda^i\right)^{-1/2} \Delta k^i \left({}^\circ\lambda^i\right)^{-1/2}$$

$$= \left({}^\circ\omega^i\right)^{-1} \Delta k^i \left({}^\circ\omega^i\right)^{-1} \tag{3.22}$$

where ${}^\circ\omega^i_j$ are the predicted undamped natural frequencies of the i<u>th</u> substructure. Since ${}^\circ\omega^i$ is a diagonal matrix, it follows that

$$\Delta\tilde{k}^i_{jj} = \frac{\Delta\lambda^i_j}{{}^\circ\lambda^i_j} - 2\,\Delta\eta_{jj} \tag{3.23a}$$

$$\Delta\tilde{k}^i_{jk} = -\left(\frac{{}^\circ\omega^i_j}{{}^\circ\omega^i_k}\right)\Delta\eta_{jk} - \left(\frac{{}^\circ\omega^i_k}{{}^\circ\omega^i_j}\right)\Delta\eta_{kj} \tag{3.23b}$$

We shall now define a set of generic substructure parameters as m^i_{jk} and $\tilde{k}^i_{jk}$ where

$$m^i_{jk} = \delta_{jk} + \Delta m^i_{jk} \tag{3.24a}$$

$$\tilde{k}^i_{jk} = \delta_{jk} + \Delta\tilde{k}^i_{jk} \tag{3.24b}$$

We shall array these parameters in the parameter vector $\tilde{r}^i$ such that

$$\tilde{r}^i = \left\{ \begin{array}{c} m_{11} \\ m_{12} \\ \cdot \\ \cdot \\ \cdot \\ m_{22} \\ m_{23} \\ \cdot \\ \cdot \\ \cdot \\ m_{nn} \\ \hdashline \tilde{k}_{11} \\ \tilde{k}_{12} \\ \cdot \\ \cdot \\ \tilde{k}_{22} \\ \tilde{k}_{23} \\ \cdot \\ \cdot \\ \tilde{k}_{nn} \end{array} \right\}^i \tag{3.25}$$

The covariance matrix of $\tilde{r}^i$ is then

$$S^i_{\tilde{r}\tilde{r}} = E\left[\Delta\tilde{r}^i \left(\Delta\tilde{r}^i\right)^T\right] \tag{3.26}$$

In particular, a set of scalar equations is obtained,

$$\left(S^i_{\tilde{r}\tilde{r}}\right)_{jk} = \frac{1}{N-1} \sum_{\ell=1}^{N} \left[\left(\Delta\tilde{r}^i_j\right)\left(\Delta\tilde{r}^i_k\right)\right]_\ell \tag{3.27}$$

averaging over N substructures.

It is anticipated that many of the off-diagonal terms of the covariance matrix, $S^i_{\tilde{r}\tilde{r}}$, will tend to be zero. Reasonable assumptions can probably be made in this regard. Conceivably, it may even be valid to assume that $S^i_{\tilde{r}\tilde{r}}$ is diagonal. This would have to be investigated on the basis of actual data as well as engineering judgement.

Once $S^i_{\tilde{r}\tilde{r}}$ is estimated for each generic class of substructures, the covariance matrix S^i_{rr} for a particular substructure may be computed by applying the eigenvalue scaling transformation in reverse, i.e., for

$$\left(S^i_{rr}\right)_{jk} = E\left[\left(\Delta r^i_j\right)\left(\Delta r^i_k\right)\right] , \tag{3.28}$$

$$\left(S^i_{rr}\right)_{jk} = \left(S^i_{\tilde{r}\tilde{r}}\right)_{jk} \tag{3.29a}$$

whenever $\Delta r^i_j = \Delta m^i_{\ell m}$ and $\Delta r^i_k = \Delta m^i_{no}$;

$$\left(S^i_{rr}\right)_{jk} = {}^\circ\omega^i_n \, {}^\circ\omega^i_o \left(S^i_{\tilde{r}\tilde{r}}\right)_{jk} \tag{3.29b}$$

whenever $\Delta r^i_j = \Delta m^i_{\ell m}$ and $\Delta r^i_k = \Delta k^i_{no}$;

$$\left(S^i_{rr}\right)_{jk} = {}^\circ\omega^i_\ell \, {}^\circ\omega^i_m \left(S^i_{\tilde{r}\tilde{r}}\right)_{jk} \tag{3.29c}$$

whenever $\Delta r^i_j = \Delta \tilde{k}^i_{\ell m}$ and $\Delta r^i_k = \Delta m^i_{no}$;

$$\left(S^i_{rr}\right)_{jk} = {}^\circ\omega^i_\ell \, {}^\circ\omega^i_m \, {}^\circ\omega^i_n \, {}^\circ\omega^i_o \left(S^i_{\tilde{r}\tilde{r}}\right)_{jk} \tag{3.29d}$$

whenever $\Delta r^i_j = \Delta \tilde{k}^i_{\ell m}$ and $\Delta r^i_k = \Delta \tilde{k}^i_{no}$.

It is important to recognize here that the frequencies used to normalize the substructure data in Equations (3.22) and (3.23a, b) are those of the substructures comprising the data base, while those used in Equations (3.29a) through (3.29d) are those of the particular substructure of the major structural system being modeled.

The two principal advantages of using substructure modal parameters to represent structural uncertainty are (1) they reflect the combined effects of all sources of uncertainty present in the substructure, even those which might not be specifically identified, and (2) there appear to be at least some data upon which to base quantitative statistical estimates for generic classes of substructures.

There are some disadvantages, however, which should be recognized also. One disadvantage is that the parameters may be (at least initially) difficult to interpret physically. There is of course no problem with the $\Delta\lambda/{}^{\circ}\lambda$ terms. This relationship between predicted and measured eigenvalues is universally appreciated. The cross-orthogonality coefficients, $\hat{\psi}$, on the other hand are more difficult to interpret from the standpoint of structural variability and modeling error, even though they are frequently used to make qualitative comparisons between predicted and measured modes. The importance of cross-correlation among the parameters, $\tilde{r}_j^i$, is even more difficult to envision. The degree to which a particular mathematical formulation lends itself to physical interpretation at intermediate steps is an important quality in modern engineering technology.

Another disadvantage, although perhaps only a temporary one, is that conventional structural analysis computer programs are usually not structured to transform modal representations of substructure mass and stiffness matrices directly to coupled system coordinates (e.g., Figure 3-2).

In view of these two potential obstacles, the substructure uncertainties which are computed for parameters related to the p^i coordinate system, will be transformed back to parameters to be subsequently defined in the x^i coordinate system.

3.3 Alternative Parameters for Representing Structural Uncertainty

Since the x^i coordinate system could conceivably represent nodal displacements of a detailed finite element mesh, it would be theoretically possible to define structural uncertainty in terms of such things as the axial stiffness (EA), shear stiffness (GA_s), flexural stiffness (EI), and joint stiffness (K_θ), etc., of individual structural members. However, as pointed out previously, the necessary data are not available and the computational requirements are too great to make this alternative feasible.

Some investigators (e.g., [9]) have gone to the opposite extreme of assuming that uncertainty can be represented by simple scaling coefficients on the mass and stiffness matrices, i.e., $M = \alpha_1(°M)$ and $K = \alpha_2(°K)$. This approach is unrealistic inasmuch as it allows for no variability in mode shape. A similarly convenient but more satisfying approach is to use multiple scaling coefficients on a linear combination of submatrices which nominally sum to the original theoretical matrices [93].

$$M = °\overline{M} + \sum_{j=1}^{n} \alpha_j \ °M_j \tag{3.30a}$$

$$K = °\overline{K} + \sum_{j=n+1}^{m} \alpha_j \ °K_j \tag{3.30b}$$

where by definition, $°\overline{M}$, $°\overline{K}$, $°M_j$ and $°K_j$ are all square matrices of the same order as M and K such that

$$°M = °\overline{M} + \sum_{j=1}^{n} °M_j \tag{3.31a}$$

$$°K = °\overline{K} + \sum_{j=n+1}^{m} °K_j \tag{3.31b}$$

We can of course generalize this technique for the substructures approach by adding the "i" superscripts.

$$M^i = °\overline{M}^i + \sum_{j=1}^{n^i} \alpha_j^i \; °M_j^i \tag{3.32a}$$

$$K^i = °\overline{K}^i + \sum_{j=n^i+1}^{m^i} \alpha_j^i \; °K_j^i \tag{3.32b}$$

In such a formulation it is assumed that all uncertainty is embodied in the scaling parameters, α_j^i, all of which have mean values of unity. The submatrices, $°M_j^i$ and $°K_j^i$ may be defined in various ways. For example, an intermediate level in the structural hierarchy between member and substructure (e.g., frame, bay, etc.) could be associated with the α's. The stiffness variates for some of these might be further subdivided to distinguish between axial stiffness (braced frames), shear stiffness (shear walls and thick concrete shells) and flexural stiffness (moment resisting frames). All of these submatrices can be generated with standard modeling codes simply by selectively setting the unwanted parameter values to zero and generating a partial matrix.

Before going on to the next section which addresses the complete methodology, it will be worthwhile to point out another aspect of modeling structural uncertainty, another dimension of the problem, so to speak. Ang and Cornell [80] suggest a formulation which considers uncertainty due to "basic variability", separately from that due to modeling inaccuracy. Basic variability is often referred to as random error, while modeling inaccuracy is considered to be systematic error. From the standpoint of analysis (identification), the two are often inseparable and indistinguishable. From the standpoint of synthesis (modeling) the conceptual distinction may be significant in the sense that recognizing two sources of uncertainty instead of one can lead to a higher estimate of overall uncertainty.

The concept may be generalized further if desired to give separate consideration to various types of uncertainty which enter the analysis at different stages. Suppose we consider the following steps of analysis:

(a) Identify materials and construction.

(b) Generate linear model for small amplitude behavior.

(c) Determine effects of static loading (live load and dead load).

(d) Estimate effects of deterioration and maintenance.

(e) Extrapolate to large amplitude behavior.

Under these considerations we might in general replace each of the parameters, α_j^i, in Equation (3.32b) for example, by the product

$$\alpha_j^i = \alpha_j^{i(a)} \alpha_j^{i(b)} \alpha_j^{i(c)} \alpha_j^{i(d)} \alpha_j^{i(e)} \tag{3.33}$$

Some of these α's may have mean values different from unity so that the bias effects of static loading, deterioration, and extrapolation to large amplitude can be accounted for. The main point to recognize here is that the uncertainties are approximately additive, provided that they are relatively small, i.e.,

$$\sigma^2 \approx \sigma_a^2 + \sigma_b^2 + \sigma_c^2 + \sigma_d^2 + \sigma_e^2 \tag{3.34}$$

where σ^2 denotes the variance of a parameter and subscripts correspond to the steps of analysis listed above.

It has been stated a number of times that insufficient data are available to make direct statistical estimates of these parameters. The following section will discuss how these measures of uncertainty can be assigned, and subsequently estimated using a Bayesian procedure. Part of the reason for expanding the α's as shown in Equation (3.33) is to reveal the impact of modeling uncertainty on theoretical predictions. Its contribution is difficult to quantify at this level but should not be overlooked.

3.4 General Methodology for Evaluating Structural Uncertainty

This section will develop and describe a general and comprehensive methodology for evaluating structural uncertainty. It is emphasized from the outset, however, that there are many different paths through the methodology, the selection of which will depend on the availability of data and the amount of effort one may choose to invest. The end result in any case will be a set of three (probably uncorrelated) statistical variates which define the generic uncertainties in computed values of modal frequency, modal amplitude and damping for a given major structural system. A significant feature of this result is that uncertainties in frequency, mode shape and damping are properly represented in seismic response calculations. It is anticipated that such parameters could be used in computing structural response either by the response spectrum method or by the time-history method of analysis.

From the standpoint of helping the reader establish a proper perspective, it may be stated that under the assumption that mode shapes are deterministic (or have negligible uncertainty relative to modal frequency), the statistical variates, modal frequency and damping, are simply

$\Delta\lambda/{}^{\circ}\lambda = \Delta(\omega^2)/{}^{\circ}\omega^2$ = modal frequency variate (similar to Figures 2-9 through 2-11)

ζ = modal damping variate (Figures 2-21 through 2-23)

where the above symbols represent scalar quantities in this case.

The ensuing development will begin with a presentation of the general methodology, followed by a limited discussion of the various paths which may be charted through the methodology. One of these paths will be subsequently illustrated by example (Appendix A).

Figure 3-3 contains a flow chart summarizing the general methodology. The nomenclature used in this flow chart is defined in Table 3-1. As before, the "i" superscript will denote properties of the ith substructure, the "°" left superscript will denote properties of the original theoretical model, the "^" notation over a symbol will denote experimental measurements or test data. The "g" superscript is used to denote generic structure or substructure data, which includes both theoretical predictions and experimental data. The asterisks (*) are used to denote revised (improved) estimates of the parameters.

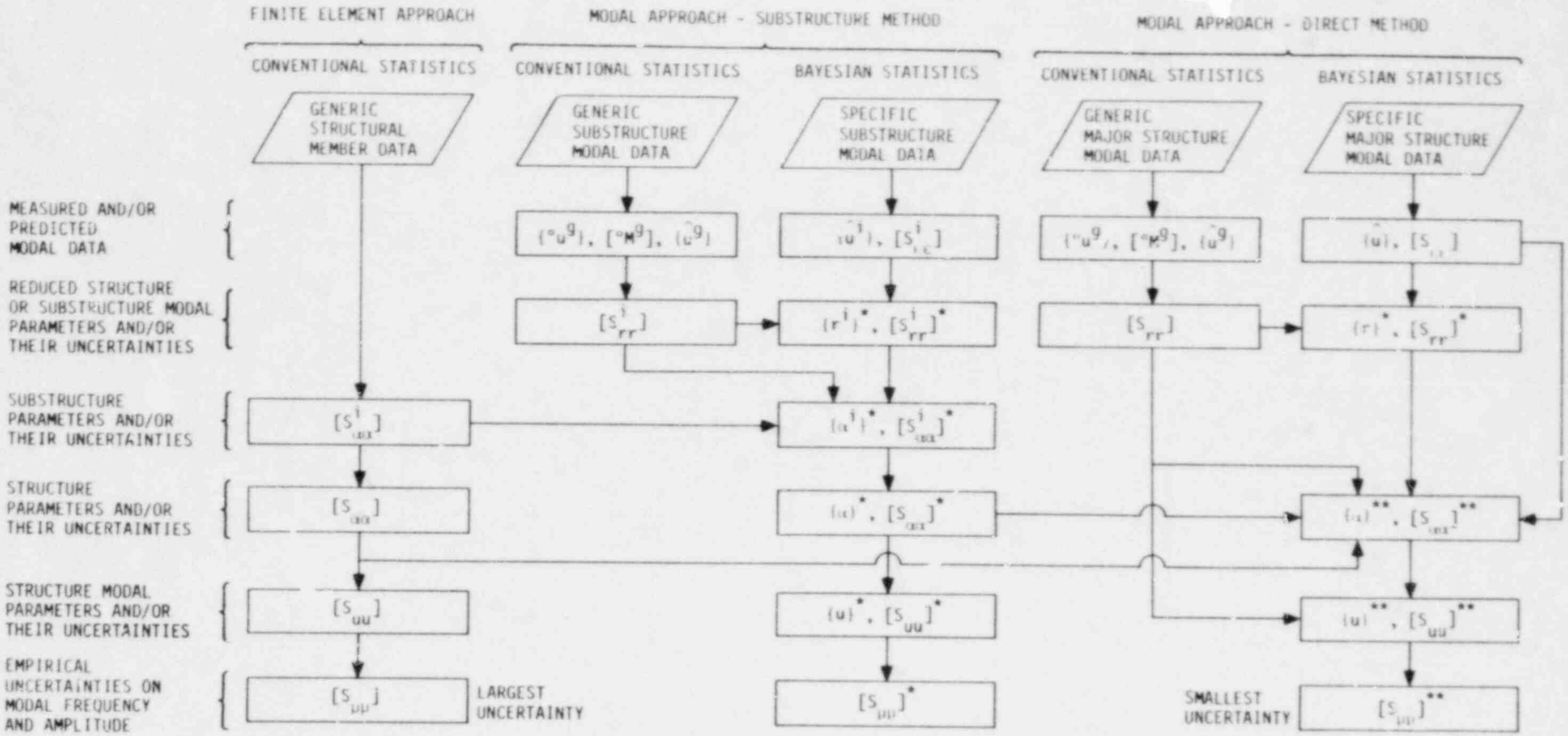


Figure 3-3. Flow Diagram of General Methodology

Table 3-1. Nomenclature for Figure 3-3

$\{\alpha^i\}$	Vector of substructure scaling parameters (treated as random variables) for mass and stiffness submatrices.
$\{\alpha\}$	Vector of scaling parameters for the major structure (contains all $\{\alpha^i\}$'s).
$\{r^i\}$	Vector of reduced substructure modal parameters.
$\{r\}$	Vector of reduced major structure modal parameters.
$\{u^i\}$	Vector of substructure modal parameters (λ_j^i and ϕ_{kj}^i).
$\{u\}$	Vector of major structure modal parameters (λ_j and ϕ_{kj}).
$\{{}^\circ u^g\}$	Vector of analytically predicted modal parameters for a structure or substructure within a generic category.
$\{\hat{u}^g\}$	Vector of experimentally measured modal parameters for a structure or substructure within a generic category.
$\{\mu\}$	A three-element empirical modal parameter vector where μ_1 = a normalized modal frequency squared and μ_2 = a normalized modal amplitude, and μ_3 = modal damping
$\{\varepsilon^i\}$	Vector of experimental errors associated with $\{\hat{u}^i\}$.

Table. 3-1. Nomenclature for Figure 3-3 (cont'd)

$\{\varepsilon\}$	Vector of experimental errors associated with $\{u\}$.
$[^{\circ}M^{g}]$	Original analytical mass matrix used in computing $\{^{\circ}u^{g}\}$ for a structure or substructure within a generic category.
$[S^{i}_{\alpha\alpha}]$	Covariance matrix of the parameter vector $\{\alpha^{i}\}$.
$[S_{\alpha\alpha}]$	Covariance matrix of the parameter vector $\{r^{i}\}$.
$[S^{i}_{rr}]$	Covariance matrix of the parameter vector $\{r^{i}\}$.
$[S_{rr}]$	Covariance matrix of the parameter vector $\{r\}$.
$[S_{uu}]$	Covariance matrix of the parameter vector $\{u\}$.
$[S_{\mu\mu}]$	Covariance matrix of the parameter vector $\{\mu\}$.
$[S^{i}_{\varepsilon\varepsilon}]$	Covariance matrix (usually assumed to be diagonal) of the experimental error vector $\{\varepsilon^{i}\}$.
$[S_{\varepsilon\varepsilon}]$	Covariance matrix (usually assumed to be diagonal) of the experimental error vector $\{\varepsilon\}$.

The eleven figures following Figure 3-3 illustrate distinct options which may be exercised within the general methodology. Additional options are possible. Those which are shown have been selected to demonstrate the methodology. Some are more practical than others. One is recommended for initial implementation. It is illustrated by example in Appendix A.

Option 1: Figure 3-4.

Option 1 is considered to be the conventional approach in the sense that statistics associated directly with the mass and stiffness matrices are propagated through the structural equations to determine corresponding statistics for the modal properties. Under this option, generic uncertainty data related to structural member properties must be utilized along with engineering judgement to establish the covariance matrices $[S^i_{\alpha\alpha}]$. The primary reason for dividing a major structural system into distinct substructures is to try to separate for purposes of analysis those portions of a major structure which may be expected to exhibit different levels of uncertainty and/or damping. The α - parameters defined here are taken to be submatrix scaling coefficients with mean values of unity as defined in Equations (3.32a, b). It is likely that these parameters will be uncorrelated so that $[S^i_{\alpha\alpha}]$ (or $S^i_{\alpha\alpha}$) is diagonal. By definition

$$S^i_{\alpha\alpha} = E[\Delta\alpha^i \ (\Delta\alpha^i)^T] \tag{3.35}$$

where $\Delta\alpha^i_j = (\alpha^i_j - 1)$. In general, the diagonal elements of $S^i_{\alpha\alpha}$ may be of the form suggested by Equation (3.34), where the variance of α reflects the combined effect of several distinct sources of uncertainty. Judgement will be a major factor in establishing these uncertainties. To a degree, the ease or difficulty of making this judgement will depend on how the α's are defined.

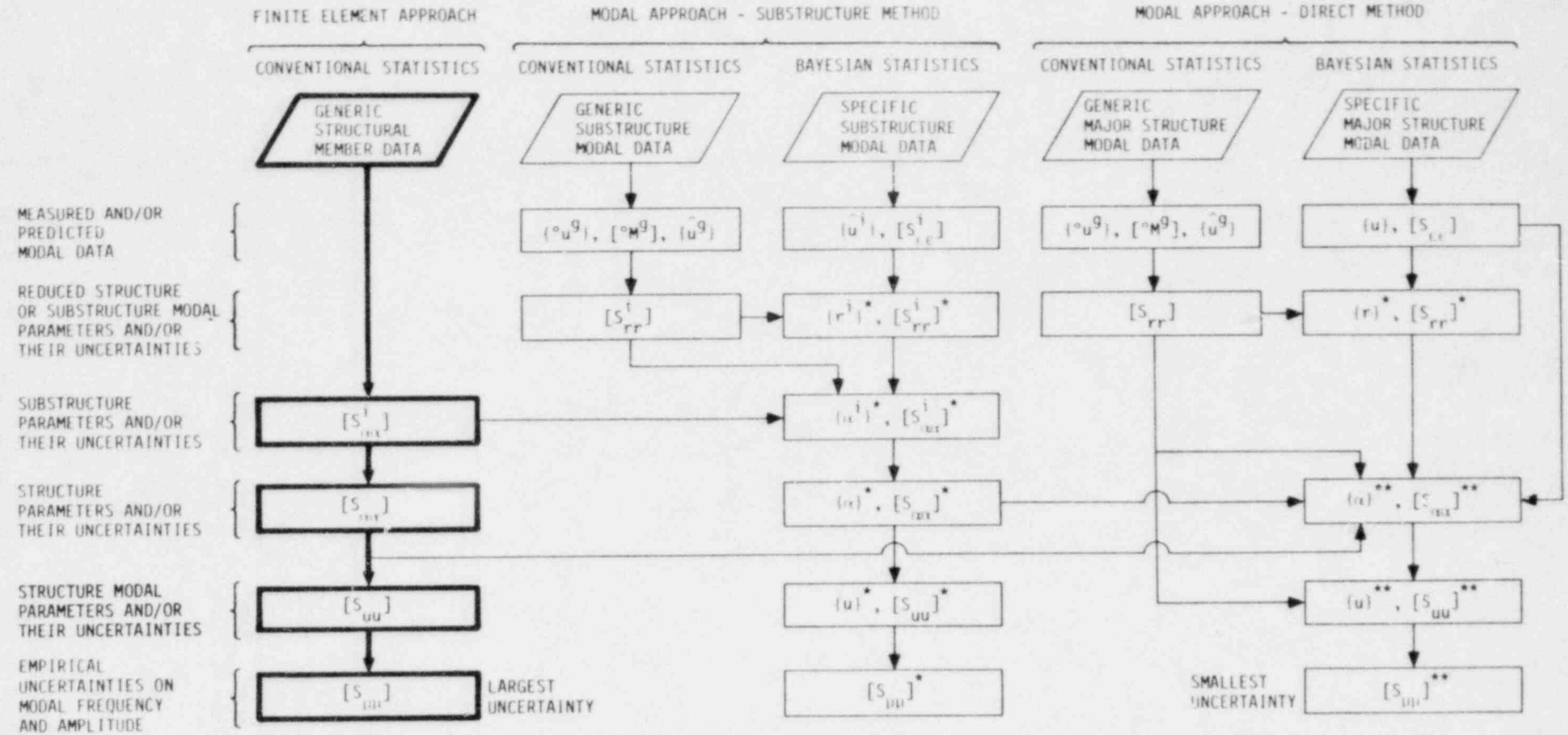


Figure 3-4. Flow Diagram for Option 1

The transformation of $S_{\alpha\alpha}^{i}$ to $S_{\alpha\alpha}$ is direct. We first recognize that

$$\alpha \equiv \{\alpha\} = \begin{Bmatrix} \alpha^{1} \\ \alpha^{2} \\ \cdot \\ \cdot \\ \cdot \\ \alpha^{i} \\ \cdot \\ \cdot \\ \cdot \\ \alpha^{N} \end{Bmatrix} \tag{3.36}$$

It follows then that

$$S_{\alpha\alpha} \equiv [S_{\alpha\alpha}] = \begin{bmatrix} S_{\alpha\alpha}^{1} & & & & & & \\ & S_{\alpha\alpha}^{2} & & & & & \\ & & \cdot & & & & \\ & & & \cdot & & & \\ & & & & \cdot & & \\ & & & & & S_{\alpha\alpha}^{i} & \\ & & & & & & S_{\alpha\alpha}^{N} \end{bmatrix} \tag{3.37}$$

assuming that α^{i} and α^{j} are uncorrelated. Thus, whenever all $S_{\alpha\alpha}^{i}$ are diagonal, $S_{\alpha\alpha}$ will be also.

The next step in the sequence of Option 1 is to transform $S_{\alpha\alpha}$ to S_{uu}, where the vector u is defined as follows:

$$u \equiv \{u\} = \left\{ \begin{array}{c} \lambda_1 \\ \lambda_2 \\ \cdot \\ \cdot \\ \cdot \\ \lambda_n \\ \hline \phi_{11} \\ \phi_{21} \\ \cdot \\ \cdot \\ \cdot \\ \phi_{12} \\ \phi_{22} \\ \cdot \\ \cdot \\ \cdot \\ \phi_{nn} \end{array} \right\} \tag{3.38}$$

As before, the eigenvalues and eigenvectors of a major structural system are denoted by λ_j and ϕ_j respectively. The transformation of $S_{\alpha\alpha}$ to S_{uu} is obtained as follows:

$$S_{uu} = E\left[(u - \overline{u})\ (u - \overline{u})^T\right]$$

$$= E\left[T_{u\alpha}\ \Delta\alpha\ \Delta\alpha^T\ T_{u\alpha}{}^T\right] = T_{u\alpha}\ S_{\alpha\alpha}\ T_{u\alpha}{}^T \tag{3.39}$$

The transformation matrix $T_{u\alpha}$ is called a sensitivity matrix and is the matrix of first partial derivatives,

$$\left(T_{u\alpha}\right)_{\ell m} = \partial u_\ell / \partial \alpha_m \tag{3.40}$$

Whenever $u_\ell = \lambda_j$,

$$\left(T_{u\alpha}\right)_{\ell m} = \frac{\partial \lambda_j}{\partial \alpha_m} = \phi_j^T \left(\frac{\partial K}{\partial \alpha_m} - \lambda_j \frac{\partial M}{\partial \alpha_m} \right) \phi_j \tag{3.41}$$

Whenever $u_\ell = \phi_{kj}$,

$$\begin{aligned} \left(T_{u\alpha}\right)_{\ell m} &= \frac{\partial \phi_{kj}}{\partial \alpha_m} \\ &= \sum_h \phi_{kh} \, \phi_h^T \left[\frac{1-\delta_{hj}}{\lambda_j - \lambda_h} \left(\frac{\partial K}{\partial \alpha_m} - \lambda_j \frac{\partial M}{\partial \alpha_m} \right) - \frac{\delta_{hj}}{2} \frac{\partial M}{\partial \alpha_m} \right] \phi_j \end{aligned} \tag{3.42}$$

The final step in the sequence of Option 1 is to tranform S_{uu} to $S_{\mu\mu}$.

We shall assume that the dynamic equations of motion for the major structural system are transformed from the physical x coordinate system to the modal p coordinate system and solved in the time domain. Whether the equations are solved by time-series integration or by the response spectrum method of analysis is immaterial.

The equations of motion for the theoretical model are

$$^{\circ}M\ \ddot{x} + {}^{\circ}C\ \dot{x} + {}^{\circ}K\ x = f(\varepsilon) \tag{3.43}$$

Forced response is of interest here. The force vector is denoted by $f(t)$; it will be assumed variable separable in space and time such that

$$f(t) = P_x\ g(t) \tag{3.44}$$

where P_x is the spacial distribution vector and $g(t)$ is a scalar function of time. In general, $g(t)$ may represent a single component of ground motion such as ground acceleration in a given horizontal or the vertical direction. The following derivation can be generalized to include all three components of ground motion if desired.

Transformation of Equation (3.43) to the p coordinate system results in

$$I\ \ddot{p} + {}^{\circ}\xi\ p + {}^{\circ}\lambda\ p = {}^{\circ}\Gamma\ g(t) \tag{3.45}$$

where Γ is a vector of modal participation factors

$$^{\circ}\Gamma_j = {}^{\circ}\phi_j^T\ P_x = \sum_{\ell} {}^{\circ}\phi_{\ell j}\ P_{x_\ell} \tag{3.46}$$

The off-diagonal terms of $^{\circ}\xi$ may be neglected provided that the modal frequencies are sufficiently well separated [89], leading to a set of independent equations

$$\ddot{p}_j + 2{}^{\circ}\zeta_j\ {}^{\circ}\omega_j\ \dot{p} + {}^{\circ}\omega_j^2\ p = {}^{\circ}\Gamma_j\ g(t) \tag{3.47}$$

The solution of the above equation may be expressed in integral form as

$$p_j = {}^\circ\Gamma_j \int_0^t h_j(t-\tau)g(\tau)\, d\tau \tag{3.48}$$

where $h_j(t)$, the impulse response function for mode "j", is

$$h_j(t) = \frac{{}^\circ\omega_j}{\sqrt{1-{}^\circ\zeta_j^2}}\, e^{-{}^\circ\zeta_j\, {}^\circ\omega_j t} \sin\left({}^\circ\omega_j \sqrt{1-{}^\circ\zeta_j^2}\right) t \tag{3.49}$$

Finally, response at point "k" on the major structural system is obtained as a sum of the modal contributions

$$x_{kj} = {}^\circ\phi_{kj}\, p_j$$

$$= {}^\circ\phi_{kj}\, {}^\circ\Gamma_j \int_0^t h_j(t-\tau)g(\tau)\, d\tau \tag{3.50}$$

We shall further simplify the form of Equation (3.50) by defining a modal response amplitude parameter, A_{kj}.

$${}^\circ A_{kj} = {}^\circ\phi_{kj}\, {}^\circ\Gamma_j \tag{3.51}$$

At this point we must address the problem of numerical simulation. If the number of independent random variables which may be used to represent structural uncertainty is limited to a few (2, 3 or 4), it would appear that three independent random variables might be defined, each relating to one of the modal parameters $°\lambda_j$, $°A_{kj}$ or $°\zeta_j$. The following parameters are tentatively suggested, pending the analysis of actual data upon which the final selection must be made.

Two parameters are suggested for modeling the randomness of natural frequencies and modal displacements respectively. We can express λ_j and A_{kj} as follows:

$$\lambda_j = \left(\frac{\lambda_j}{°\lambda_j}\right) °\lambda_j \tag{3.52}$$

$$\begin{aligned} A_{kj} &= \left(°\phi_{kj} + \Delta\phi_{kj}\right)\left(°\Gamma_j + \Delta\Gamma_j\right) \\ &\cong °A_{kj} + °\phi_{kj}\,\Delta\Gamma_j + °\Gamma_j\,\Delta\phi_{kj} \\ &= °A_{kj} + \left[°\mathcal{m}_j\left(°\phi_{kj}\,\Delta\Gamma_j + °\Gamma_j\,\Delta\phi_{kj}\right)\right]\left(\frac{1}{°\mathcal{m}_j}\right) \\ &= °A_{kj} + \left(°\mathcal{m}_j\,\Delta A_{kj}\right)\frac{1}{°\mathcal{m}_j} \end{aligned} \tag{3.53}$$

where $°\mathcal{m}_j$ is the computed modal mass of the j<u>th</u> mode. We are now in a position to define two new random variables, μ_1, and μ_2, and will subsequently quantify them so as to justify the following approximations:

$$\mu_1 \cong \lambda_j/°\lambda_j \tag{3.54a}$$

$$\mu_2 \cong °\mathcal{m}_j\,\Delta A_{kj} \tag{3.54b}$$

We shall define the mean values of μ_1 and μ_2 such that $E(\lambda_j) = {}^\circ\lambda_j$ and $E(A_{kj}) = {}^\circ A_{kj}$. Thus

$$E(\mu_1) \equiv \bar{\mu}_1 = 1 \tag{3.55a}$$

$$E(\mu_2) \equiv \bar{\mu}_2 = 0 \tag{3.55b}$$

Information on the variances of μ_1 and μ_2 is contained in the covariance matrix S_{uu}. With the vector u defined by Equation (3.38), we can partition S_{uu} as follows:

$$S_{uu} = \left[\begin{array}{c|c} S_{\lambda\lambda} & S_{\lambda\phi} \\ \hline S_{\lambda\phi} & S_{\phi\phi} \end{array}\right] \tag{3.56}$$

Independence of λ and ϕ would imply $S_{\lambda\phi} = 0$. We shall make this assumption tentatively, subject to verification by the data. It will also be of interest to investigate the possiblity that λ_j and λ_k are uncorrelated for $j \neq k$. In any case, there is more than one way to establish a reasonable probability law for μ_1. We might assume a lognormal probability law and select a variance based on the average of the diagonal terms of $S_{\lambda\lambda}$. A more conservative approach would be to select a variance for μ_1 based on the largest of the diagonal elements of $S_{\lambda\lambda}$. This approach is tentatively recommended, pending experience with actual data. Thus we shall choose

$$E\left[\left(\mu_1 - \bar{\mu}_1\right)^2\right] = \sigma^2_{\mu_1} = \left[(S_{\lambda\lambda})_{jj}/{}^\circ\lambda_j^2\right]_{\max(j)} \tag{3.57}$$

In order to establish a value for $\sigma^2_{\mu_2}$ based on $S_{\lambda\lambda}$, we must first evaluate the variance of Γ_j as defined by

$$\Gamma_j = \phi_j^T P_x = P_x^T \phi_j \tag{3.58}$$

If P_x is deterministic, then

$$\sigma^2_{\Gamma_j} = E\left[(\Gamma_j - {^\circ}\Gamma_j)^2\right] = P_x^T E\left[(\phi_j - {^\circ}\phi_j)(\phi_j - {^\circ}\phi_j)^T\right] P_x$$

$$= P_x^T S_{\phi_j\phi_j} P_x \tag{3.59}$$

The cross correlation between ϕ_{kj} and $\phi_{\ell j}$ is expected to be important in computing the variance of Γ_j. It is anticipated, however, that the correlation between ϕ_{kj} and Γ_j will be relatively weak, and therefore may be neglected. We thus obtain

$$E\left[({^\circ}\!m_j\, \Delta A_{kj})^2\right] = {^\circ}\!m_j^2\, E\left[(\Delta A_{kj})^2\right]$$

$$= {^\circ}\!m_j^2 \left[{^\circ}\phi_{kj}^2\, \sigma^2_{\Gamma_j} + {^\circ}\Gamma_j^2\, \sigma^2_{\phi_{kj}}\right] \tag{3.60}$$

where $\sigma^2_{\phi_{kj}}$ is given by $S_{\phi\phi}$. It also may be noted that ${^\circ}\!m_j$ is given by

$$1/{^\circ}\!m_j = \left[{^\circ}\phi_{kj}^2\right]_{\max(k)} \tag{3.61}$$

under the assumption that $°\phi^T \, °M \, °\phi \equiv I$. It remains only to define $\sigma^2_{\mu_2}$, given $E\left[(°m_j \, \Delta A_{kj})^2\right]$ for all j. It would again be conservative to define μ_2 based on the upper bound

$$\sigma^2_{\mu_2} = E\left[(°m_j \, \Delta A_{kj})^2\right]_{\max(k,j)} \qquad (3.62)$$

Such a recommendation is again made tentatively, pending the analysis of actual data. The normal probability law is suggested for μ_2.

Finally, we must define the uncertainty in modal damping ζ_j. The best data for this purpose are those data which constitute direct measurements of modal damping for the type of major structure being considered, i.e., reactor building or auxiliary-fuel-turbine building complex. Such data are identified in Table 4-3. The data may be combined as shown in Figures 2-21 through 2-23 to establish probability distributions directly for a single "equivalent" random variable, μ_3, such that

$$\zeta_j \equiv \mu_3 \qquad (3.63)$$

Having previously established

$$\lambda_j = \mu_1 \, °\lambda_j \qquad (3.64)$$

$$A_{kj} = °A_{kj} + \mu_2/°m_j \qquad (3.65)$$

along with probability laws for μ_1 and μ_2, we have reached the end of the sequence in Option 1.

Option 2: Figure 3-5.

It may be noted that Option 2 directly parallels Option 1, and in fact is identical to it except for the manner in which $S^i_{\alpha\alpha}$ is established. The procedure for deriving S^i_{rr} was derived in Section 3.2. It remains here only to define the transformation from S^i_{rr} to $[S^i_{\alpha\alpha}]^*$.

The transformation is written [91]

$$[S^i_{\alpha\alpha}]^* = \left[(T^i_{r\alpha})^T(S^i_{rr})^{-1}\ T_{r\alpha}\right]^{-1} \tag{3.66}$$

where elements of the sensitivity matrix $T^i_{r\alpha}$ are given by

$$\left(T^i_{r\alpha}\right)_{\ell h} = \partial r^i_\ell / \partial \alpha^i_h \tag{3.67}$$

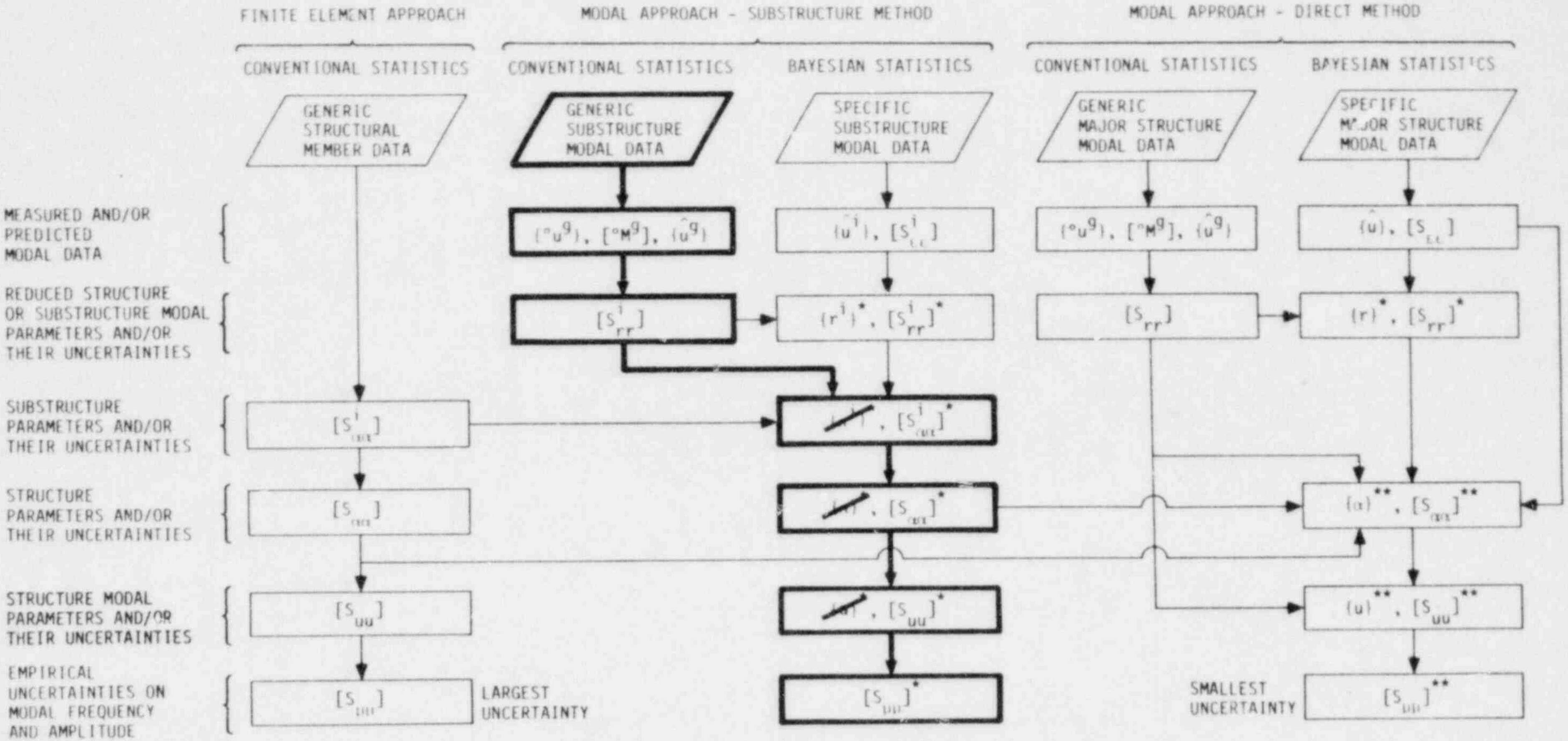
Whenever $1 \leq h \leq n^i$ in Equation (3.32a),

$$\partial r^i_\ell / \partial \alpha^i_h = \partial m^i_{jk} / \partial \alpha^i_h = {}^\circ\phi^i_j\ {}^\circ M^i_h\ {}^\circ\phi^i_k \tag{3.68a}$$

Whenever $n^i + 1 \leq h \leq m^i$ in Equation (3.32b)

$$\partial r^i_\ell / \partial \alpha^i_h = \partial k^i_{jk} / \partial \alpha^i_h = {}^\circ\phi^i_j\ {}^\circ K^i_h\ {}^\circ\phi^i_k \tag{3.68b}$$

It is important to note here that in order to solve for $S^i_{\alpha\alpha}$ as shown, both of the matrices S^i_{rr} and $[(T^i_{u\alpha})^T\ (S^i_{rr})^{-1}\ T_{u\alpha}]$ must be nonsingular. In particular, this means that there must be at least as many r's as α's.



NOTE: The vectors $\{\alpha^i\}^*$, $\{\alpha\}^*$ and $\{u\}^*$ have been crossed out to indicate that revised parameter estimates are not obtained without substructure-specific test data.

Figure 3-5. Flow Diagram for Option 2

Option 3: Figure 3-6.

Option 3 is similar to Option 2 except that generic major structure modal data are used to generate S_{rr}, instead of generic substructure modal data being used to generate S^i_{rr}. S_{rr} is then transformed directly to $S_{\alpha\alpha}$ in a manner similar to that described in Equations (3.66) through (3.68).

Option 4: Figure 3-7.

Option 4 is similar to Option 2, except that $[S^i_{\alpha\alpha}]^*$ is now obtained by the transformation

$$[S^i_{\alpha\alpha}]^* = \left[(S^i_{\alpha\alpha})^{-1} + (T^i_{r\alpha})^T (S^i_{rr})^{-1} T^1_{r\alpha}\right]^{-1} \tag{3.69}$$

instead of Equation (3.66). The covariance matrix $S^i_{\alpha\alpha}$ is obtained as in Option 1. This option was selected for illustration in Appendix A.

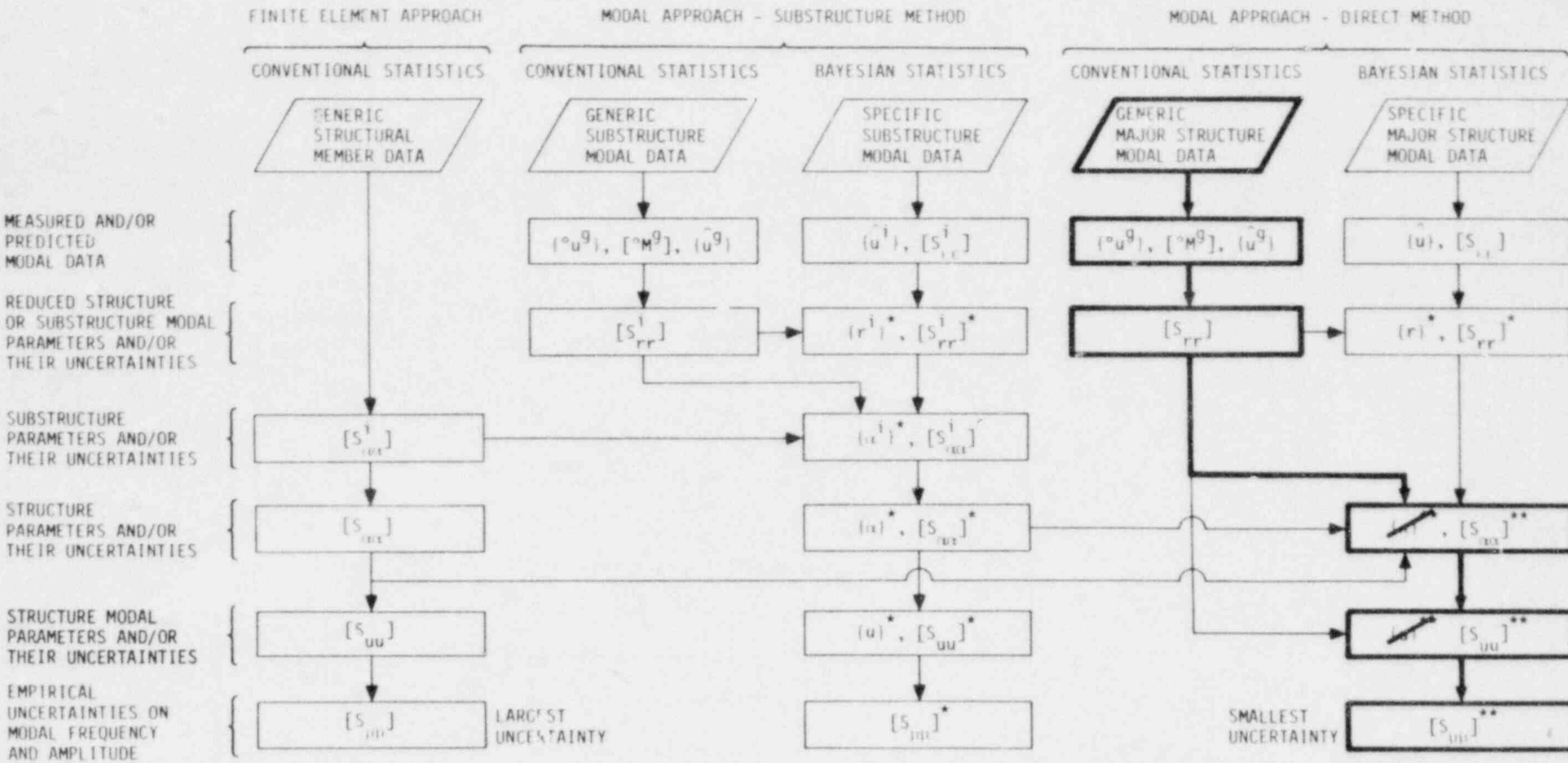
Option 5: Figure 3-8.

Option 5 is a combination of Options 2 and 3 where $[S_{\alpha\alpha}]^{**}$ is obtained by the transformation

$$[S_{\alpha\alpha}]^{**} = \left[(S^*_{\alpha\alpha})^{-1} + (T_{r\alpha})^T (S_{rr})^{-1} T_{r\alpha}\right]^{-1} \tag{3.70}$$

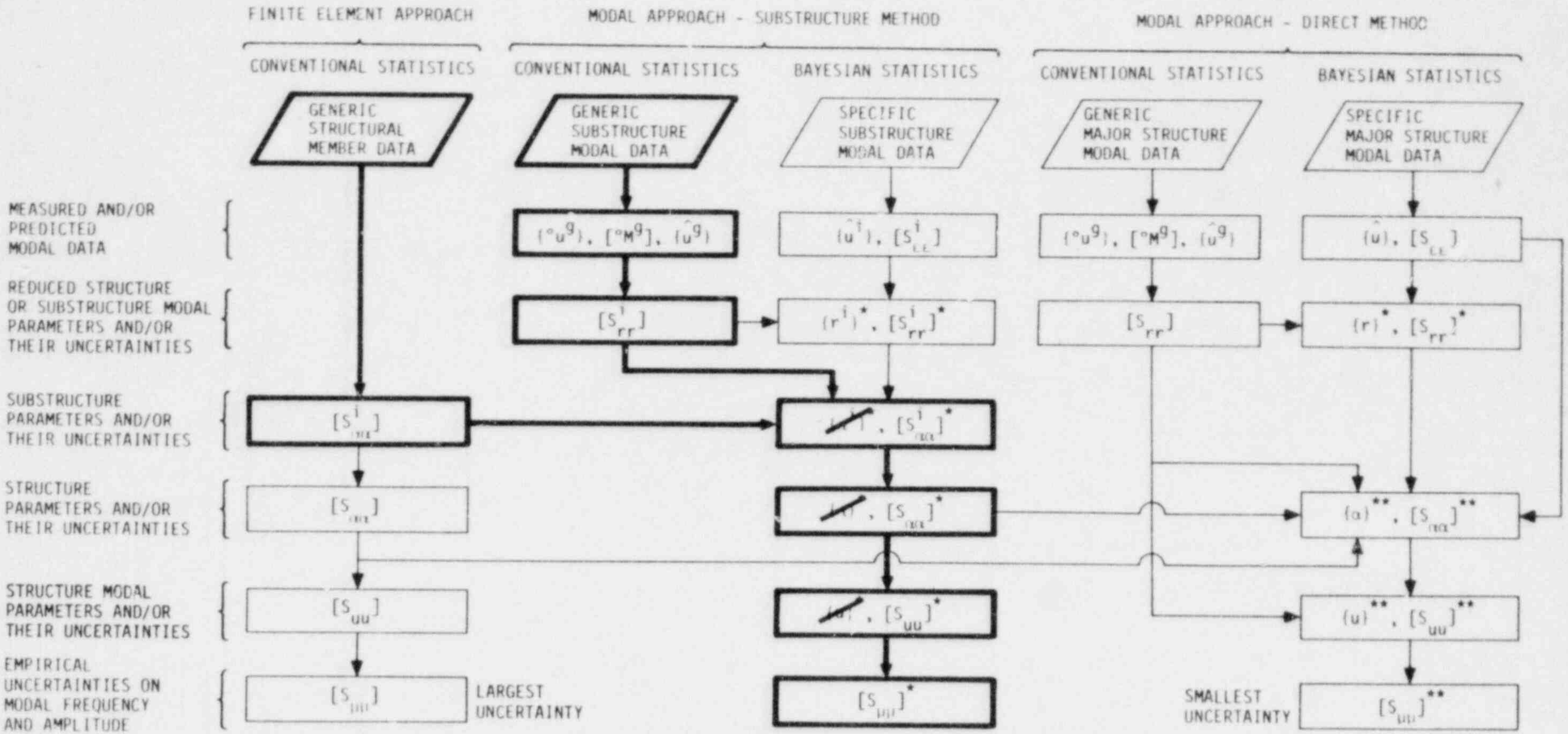
Option 6: Figure 3-9.

Option 6 is a combination of Option 1 and 3 where $[S_{\alpha\alpha}]^{**}$ is obtained by the transformation



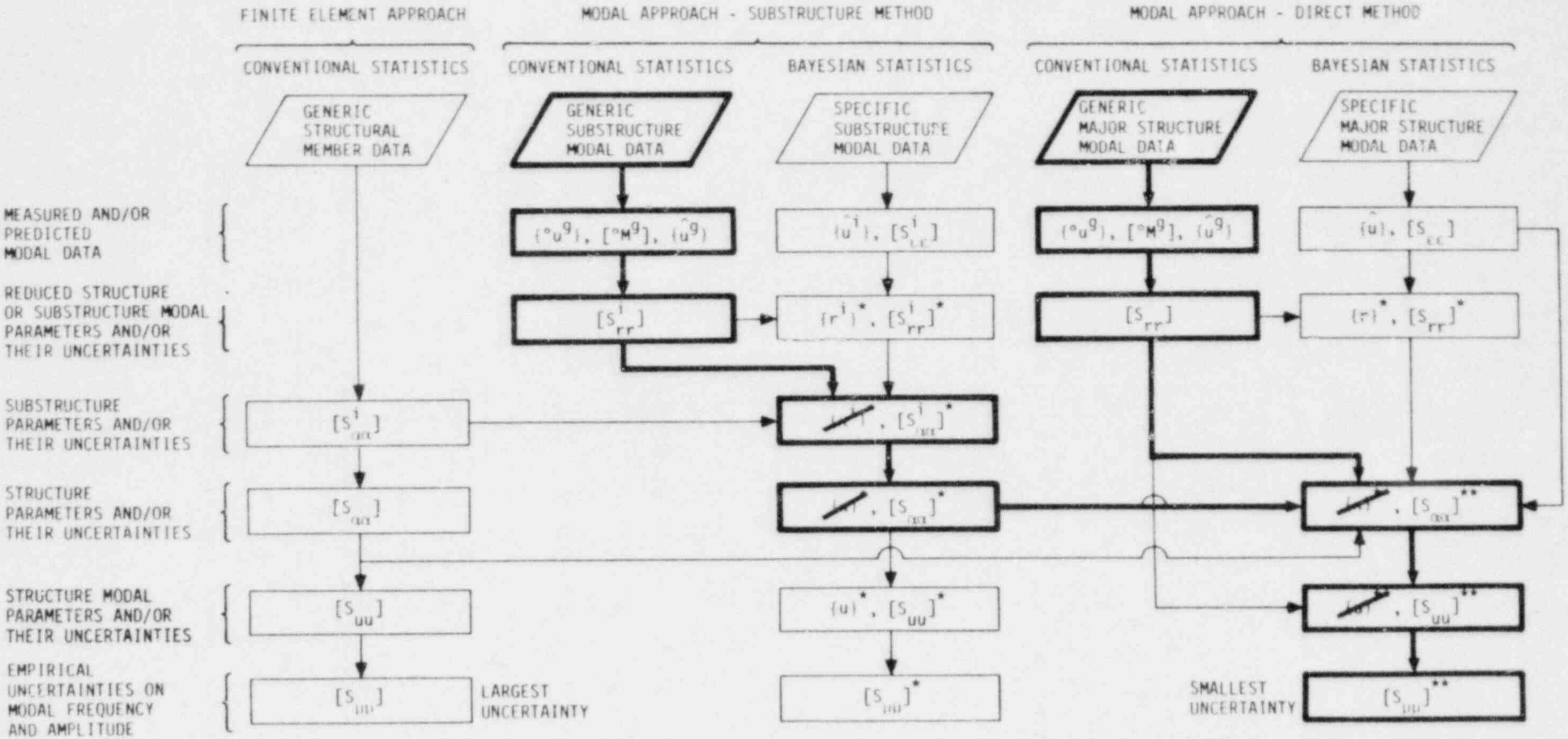
NOTE: The vectors {α}** and {u}** have been crossed out to indicate that revised parameter estimates are not obtainable without major structure-specific test data.

Figure 3-6. Flow Diagram for Option 3



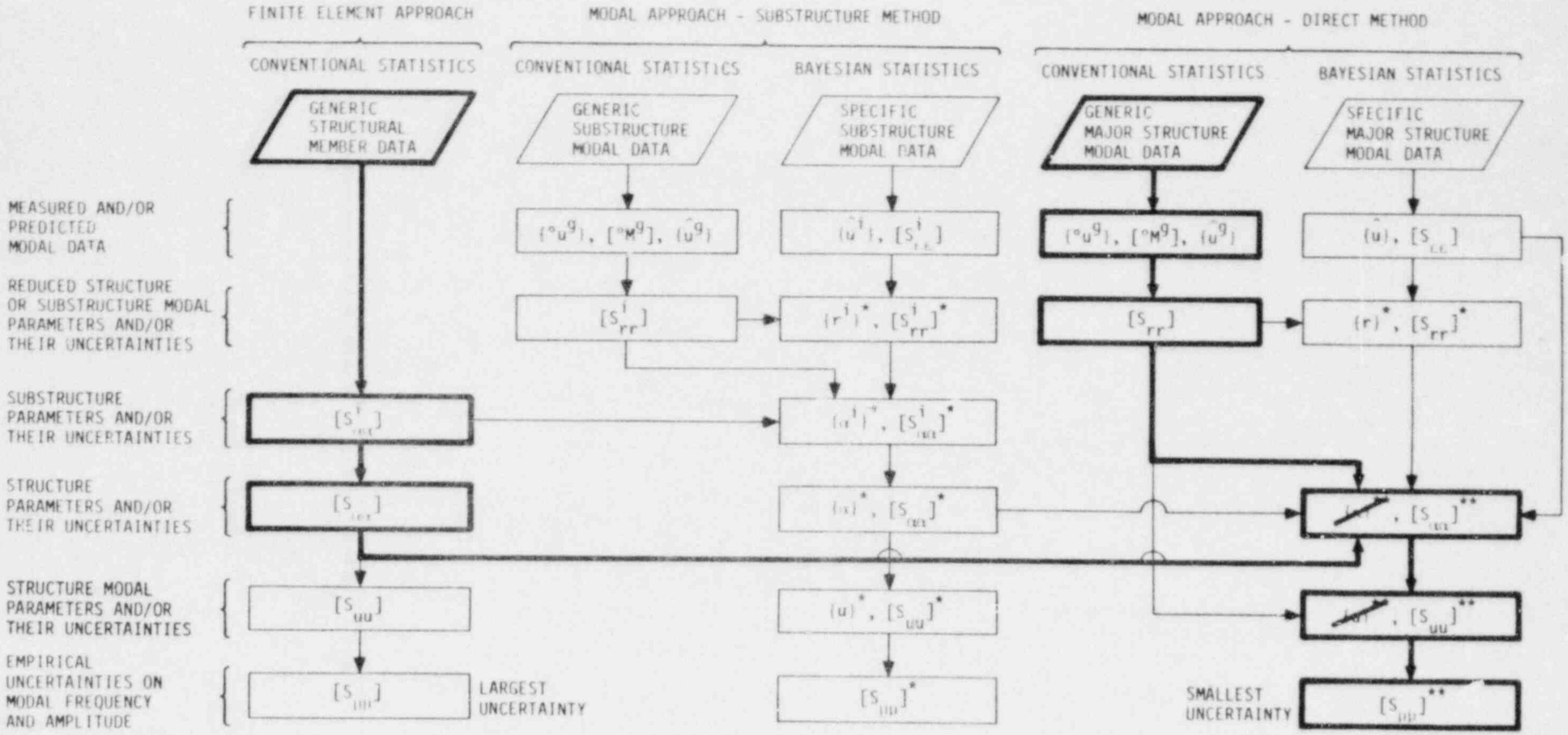
NOTE: The vectors $\{\alpha^i\}^*$, $\{\alpha\}^*$, and $\{u\}^*$ have been crossed out to indicate that revised parameter estimates are not obtainable without substructure-specific test data.

Figure 3-7. Flow Diagram for Option 4



NOTE: The vectors $\{\alpha^i\}^*$ and $\{\alpha\}^*$, and $\{\alpha\}^{**}$ and $\{u\}^{**}$, have been crossed out to indicate that revised parameter estimates are not obtainable without substructure-specific and major structure-specific test data respectively.

Figure 3-8. Flow Diagram for Option 5



NOTE: The vectors $\{\alpha\}^{**}$ and $\{u\}^{**}$ have been crossed out to indicate that revis.d parameter estimates are not obtainable without major structure-specific test data.

Figure 3-9. Flow Diagram for Option 6

$$[S_{\alpha\alpha}]^{**} = \left[(S_{\alpha\alpha})^{-1} + (T_{r\alpha})^T (S_{rr})^{-1} T_{r\alpha}\right]^{-1} \tag{3.71}$$

Option 7: Figure 3-10.

Option 7 is a combination of Options 1, 2 and 3 where in this case, Equations (3.69) and (3.71) both apply.

Option 8: Figure 3-11.

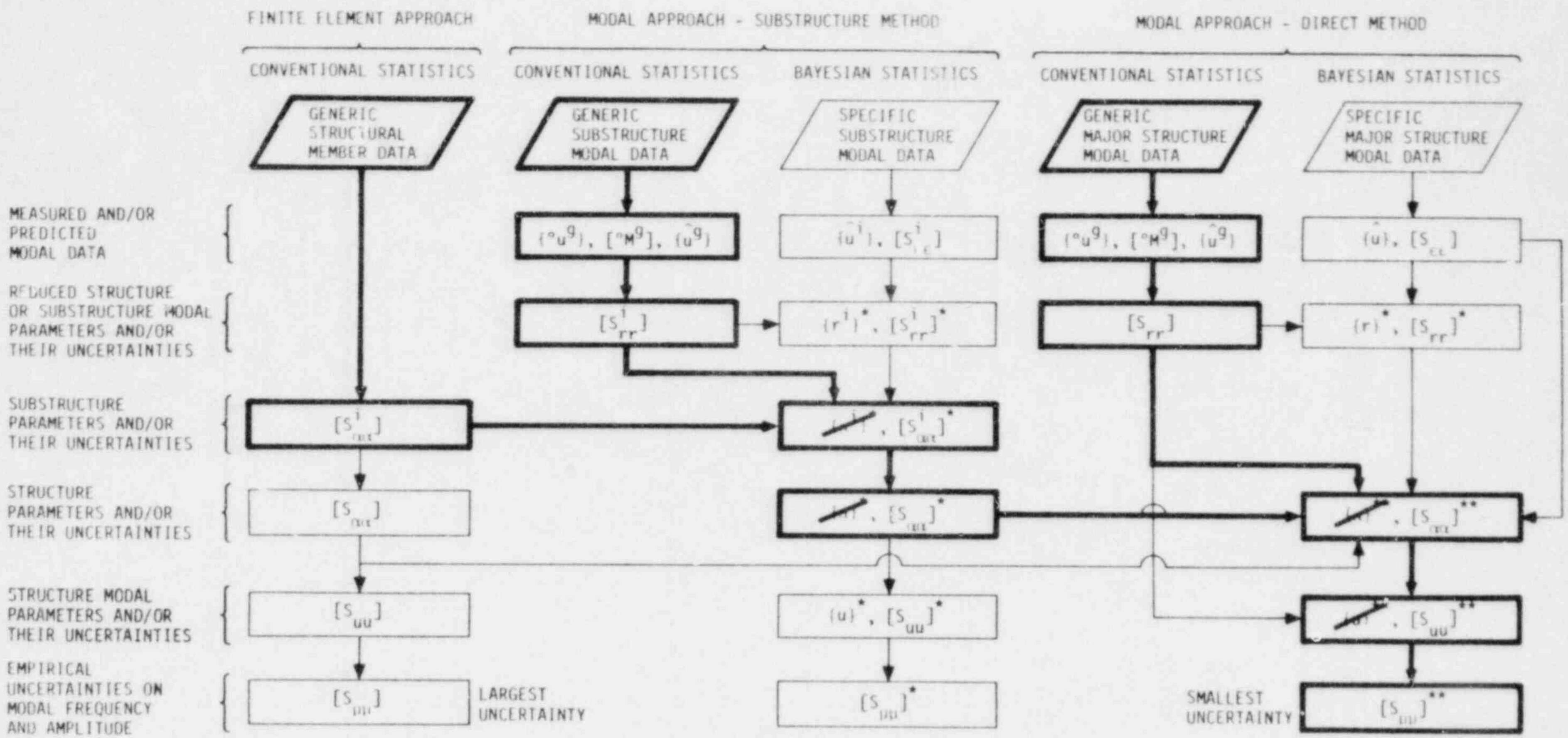
Option 8 introduces the concept of Bayesian parameter estimation based on substructure-specific test data. There is an important philosophical point to be made here. If modal test data are available for the specific substructures which comprise a major structural system (or any one of them for that matter), revised (improved) parameter estimates can be obtained by Bayesian statistical estimation [91]. At the same time, we should expect the corresponding uncertainties to be reduced significantly. Under this Option, $\{r^i\}^*$ is obtained by applying the recursive equation

$$r^i = r_e^i + \left[(S_{rr}^i)^{-1} + (T_{ur}^i)^T (S_{\varepsilon\varepsilon}^i)^{-1} T_{ur}^i\right]^{-1}$$

$$\times \left[(S_{rr}^i)^{-1} (r_o^i - r_e^i) + (T_{ur}^i)^T (S_{\varepsilon\varepsilon})^{-1} (u_o^i - u_e^i)\right] \tag{3.72}$$

until $\{r^i\}$ converges to $\{r^i\}^*$ at which time $[S_{rr}^i]^*$ is computed from the equation

$$[S_{rr}^i]^* = \left[(S_{rr}^i)^{-1} + (T_{ur}^i)^T (S_{\varepsilon\varepsilon}^i)^{-1} T_{ur}^i\right]^{-1} \tag{3.73}$$



NOTE: The vectors $\{\alpha^i\}^*$ and $\{\alpha\}^*$, and $\{\alpha\}^{**}$ and $\{u\}^{**}$, have been crossed out to indicate that revised parameter estimates are not obtainable without substructure-specific and major structure-specific test data respectively.

Figure 3-10. Flow Diagram for Option 7

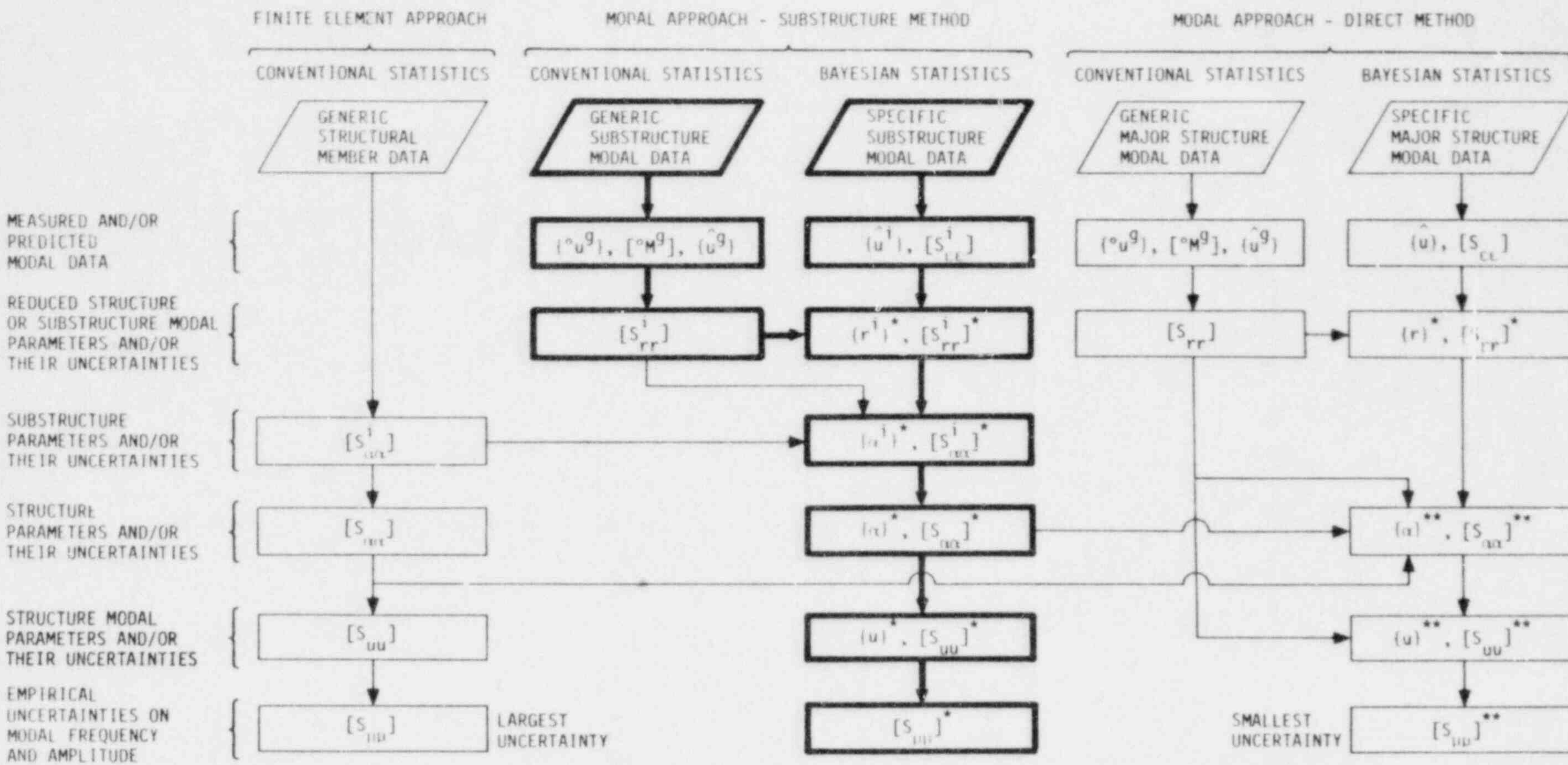


Figure 3-11. Flow Diagram for Option 8

In the above equations, r_o^i denotes the original (a priori) estimates of the parameter vector r^i, r_e^i denotes the most recent estimate from the iterative operation, u_e^i is the theoretically-predicted modal parameter vector based on r_e^i, and u_o^i is the observed (measured) modal parameter vector. The covariance matrix $S_{\varepsilon\varepsilon}^i$ denotes the estimated statistical variances of measurement error (usually assumed to be diagonal) while T_{ur}^i is the sensitivity matrix relating r^i to u^i.

$$\left(T_{ur}^i\right)_{\ell m} = \partial u_\ell^i / \partial r_m^i \tag{3.74}$$

<u>Option 9: Figure 3-12.</u>

Option 9 is similar to Option 8 except that both generic and structure-specific modal data are assumed available for the major structure instead of the substructures.

<u>Option 10: Figure 3-13.</u>

This option represents what is considered to be the most advanced of the more practical options. (The more practical options are considered to include Options1, 2, 4 and 10. The others are considered to be less practical, either because they require generic major structure modal data, specific substructure modal data, or both. These data are believed to be relatively unavailable.) The implication is that structural uncertainty, and thus seismic risk, may be reduced by physically testing the particular structure of interest. It is of interest to note here that the step of computing r^* and S_{rr}^* has been bypassed in this option.

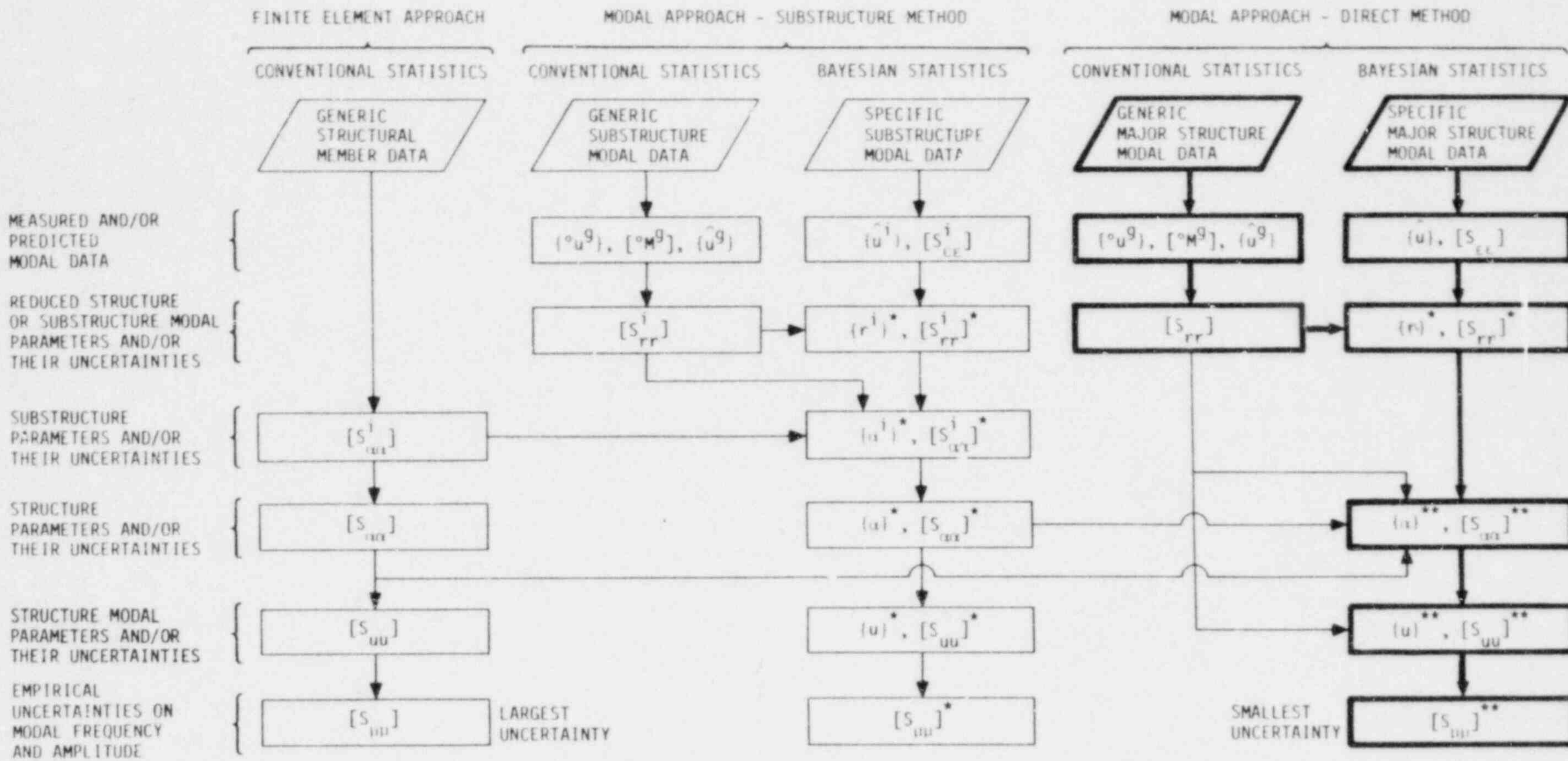
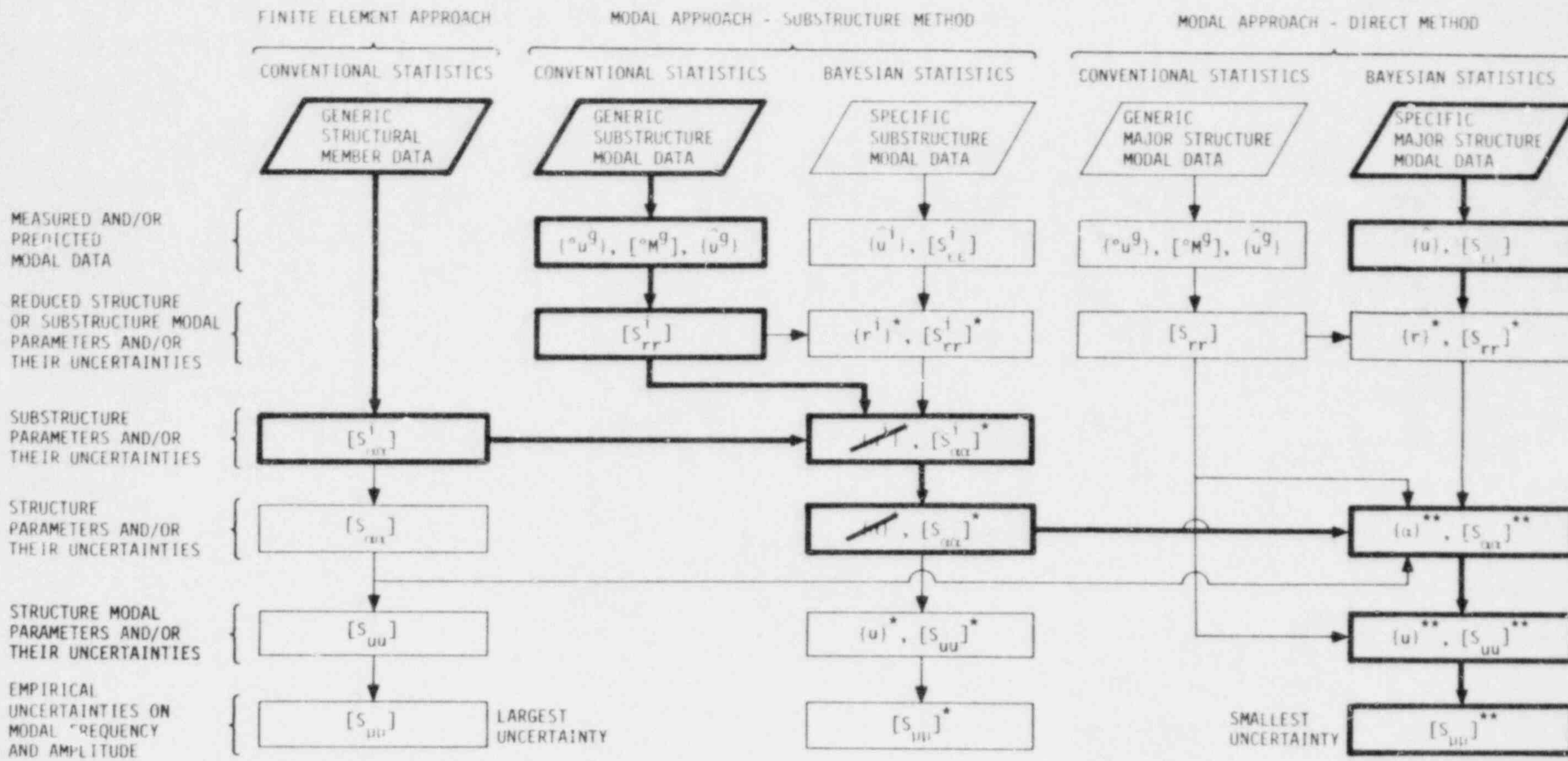


Figure 3-12. Flow Diagram for Option 9



NOTE: The vectors $\{\alpha^i\}^*$ and $\{\alpha\}^*$ have been crossed out to indicate that revised parameter estimates are no: obtainable without substructure-specific test data.

Figure 3-13. Flow Diagram for Option 10

Option 11: Figure 3-14.

This option is shown only to illustrate the most complete use of the present methodology. It is considered to be one of the less practical options because of the data requirements discussed in the previous paragraph.

A few general comments on the methodology are in order at this time. As can be noted in Figure 3-4 and succeeding figures, estimates of the r - parameters are always transformed to corresponding estimates of the α - parameters before transforming to the u - parameters which symbolize frequency and mode shape. As mentioned in Section 3.1, this was done primarily for the sake of convenience, anticipating that most structural analysis codes do not have a modal synthesis capability which is required to make the compatibility transformation from p^i to p' as shown in Figure 3-2. A secondary reason for choosing this approach, however, was to transform the uncertainty parameters into a form where they are more readily understood. As explained in the latter part of Section 3.3, the modal parameters (i.e., the r - parameters) may be difficult to relate to practical experience, at least for a while.

There are two points to make here before concluding the present section. The first is that care must be taken in defining the α - parameters so that they can "accept" the information contained in the r - parameters. If the two are incompatible, significant information may be lost in the transformation. In general, it is thought to be preferrable to define too many α's rather than too few in order to give the r - parameter information "freedom to go where it wants to go," without undue constraint. In order to do this, of course, one must choose an option which utilizes a priori estimates of $S^i_{\alpha\alpha}$ or $S_{\alpha\alpha}$; otherwise the matrix inversion cannot be made, i.e.,

IMAGE EVALUATION
TEST TARGET (MT-3)

6"

MICROCOPY RESOLUTION TEST CHART

IMAGE EVALUATION
TEST TARGET (MT-3)

6″

MICROCOPY RESOLUTION TEST CHART

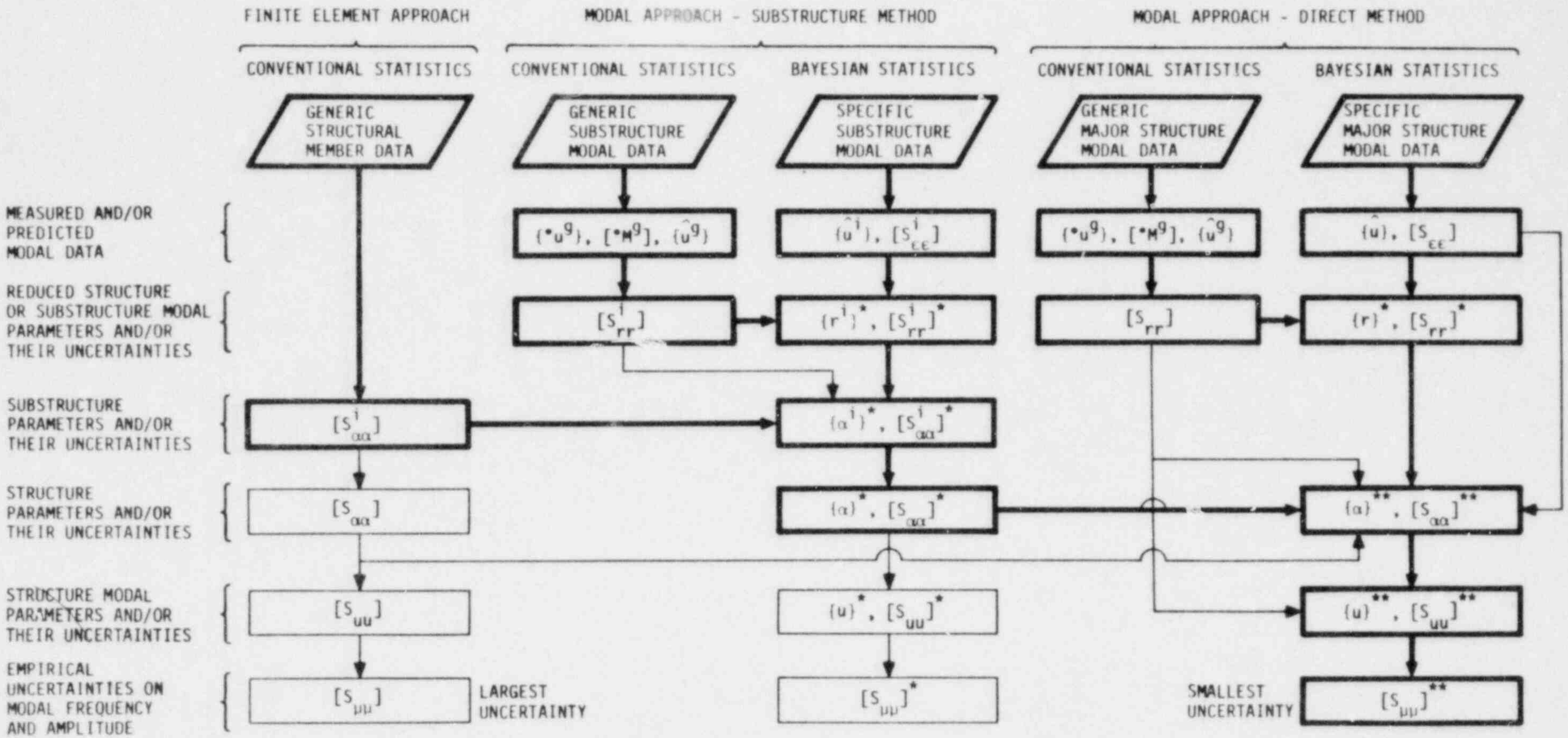


Figure 3-14. Flow Diagram for Option 11

$$S_{\alpha\alpha} = \left[(T_{r\alpha})^T \, (S_{rr})^{-1} \, T_{r\alpha} \right]^{-1}$$

is rank deficient when the number of α's exceed the number of r's. In a sense, the prior estimate of $S_{\alpha\alpha}$ is used to condition the transformation,

$$[S_{\alpha\alpha}]^* = \left[(S_{\alpha\alpha})^{-1} + (T_{r\alpha})^T \, (S_{rr})^{-1} \, T_{r\alpha} \right]^{-1}$$

If $S_{\alpha\alpha}$ is too large, the matrix on the right hand side of the equation may be ill-conditioned. If it is too small, it will dominate S_{rr}. The objective is to make $S_{\alpha\alpha}$ small enough to facilitate matrix inversion, but still large enough (within reason) to allow S_{rr} to control the outcome of $[S_{\alpha\alpha}]^*$.

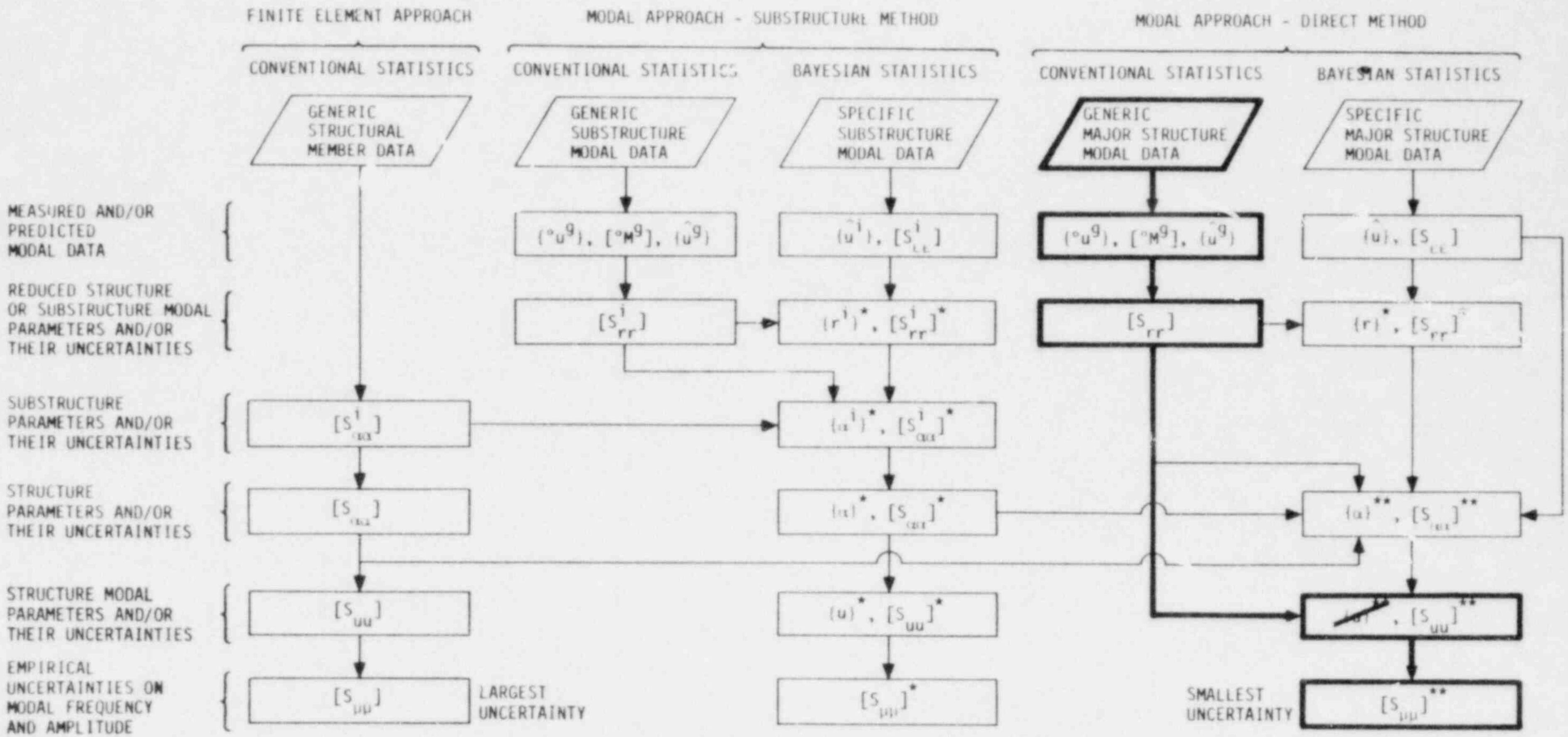
The second point to be made will help to bring this section to a conclusion on "more familiar ground." Although none of the options identified so far take this path, Figure 3-15 does show the possibility of transforming directly from $[S_{rr}]$ to $[S_{uu}]^{**}$. Although this option (designated <u>Option 12</u>) is not considered to be very practical because of the requirement for generic major structure modal data, it definitely provides insight because of the similarity between some of the r and u - parameters. In particular, some of the r - parameters are such that

$$\Delta r_{\ell} = \Delta k_{jj} = \Delta\lambda_j + {}^{\circ}\lambda_j \, \Delta m_{jj}$$

while corresponding u - parameters are such that

$$\Delta u_{\ell} = \Delta\lambda_j$$

If mode-shape uncertainty were negligible, then $\Delta m_{jj} = 0$ and



NOTE: The vector {u}** has been crossed out to indicate that revised parameter estimates are not obtainable without major structure-specific test data.

Figure 3-15. Flow Diagram Option 12

$$\Delta r_{\ell} = \Delta k_{jj} = \Delta\lambda_j \equiv \Delta u_{\ell}$$

This comparison clearly reveals the implication of using only frequency data and neglecting mode shape uncertainty; i.e., the implication is that mode shapes can be predicted accurately!

To demonstrate that this is a poor assumption, we need only to compute values of $\Delta\lambda_j$ and Δm_{jj} from some actual data. References [87 and 92] present respectively the predicted and measured frequencies and mode shapes for a 22 - story steel frame building. The predicted frequency for the second N-S bending mode was 1.077 Hz, whereas the corresponding measured frequency was 1.10 Hz. Based on these frequencies and the corresponding predicted and measured mode shapes, the following values were calculated

$$\Delta k_{22} = \Delta\lambda_2 - {}^{\circ}\lambda_2 \left[2(\hat{\psi}_{22} - 1) \right]$$

$$= 1.97 - 45.97\ [2(.922 - 1)]$$

$$= 1.97 + 7.14 = 9.11$$

This contribution to Δk_{22} from mode shape differential is seen to be 7.14, while that from frequency differential is only 1.97.

It may be noted in general that $\hat{\psi}_{jj}$ is always less than unity, so that $\Delta m_{jj} = -2(\hat{\psi}_{jj} - 1)$ will always be a positive quantity. On the other hand, $\Delta\lambda_j$ may be either positive or negative.

This example illustrates the relative sensitivity of the r - parameters to uncertainty in frequency and mode shape prediction. In general, we expect mode-shape uncertainty to dominate. Cross-orthogonality coefficients on the order of 20% (i.e., $|\hat{\psi}_{jj} - 1| = 0.20$) are not uncommon [93]. It should be understood that the corresponding eigenvalue differential would be 40% (i.e., $2\Delta\eta_{jj}$), reflecting a frequency differential on the order of 20%, which is, relatively speaking, unusual. Mode shape uncertainty therefore <u>should not</u> be neglected, at least until further investigation is undertaken to apply the proposed analysis to actual data, thereby demonstrating the relative importance of frequency and mode shape uncertainty.

4. IMPLEMENTATION

4.1 Assessment of Available Data

4.1.1 Member Level Data (Table 4-1)

After the α_i parameters are selected by the analyst to represent uncertainties in the structure, then the uncertainly analysis of structural response is completely defined provided we have the joint probability distribution functions for the α_i's, or at least their first two statistical moments. Typically α_i may include parameters related to flexural stiffness of members EI, shear stiffness parameters GA_s, joint stiffness parameters K_θ, etc. Hence the most direct data required are the mean values, variances and covariances of conveniently normalized versions of these parameters; unfortunately, data of this type are extremely rare. The closest thing to statistics of EI obtained directly for reinforced concrete (RC) beams is that reported in [12], see also Figure 2-8. In [12], statistical data for the ratio of measured to computed mid-span deflections of simply supported RC beams as reported in [1] are used to construct a probablity distribution function for the ratio of effective measured to computed EI. Even for this case, the statistics can only be regarded as a priori estimates since as-tested component loading and boundary conditions may not be realized for similar components in real structures. The situation becomes aggravated when one attempts to estimate EI indirectly, using models for EI. Indirect estimation of EI for RC using models for EI tends to result in lower confidence because of our inability to correctly model E and I for partially cracked RC members; moreover, only empirical models for E and I exist which are subject to large uncertainty. It is precisely for these reasons that the statistical data reported in [8][17]-[19], for material and geometric properties of materials, are not particularly relevant for the purposes of this study. Hence, the best alternative under the given

Table 4-1. Member Data

TYPE	DESCRIPTION OF DATA	REFERENCES
1	RATIO OF MEASURED TO COMPUTED DEFLECTIONS OF SIMPLY SUPPORTED RC BEAMS	1, 23
2	EFFECTIVE YOUNG'S MODULUS OF RC FROM SHEAR TEST DATA; ALSO SHEAR STRENGTH DATA	2, 16, 24
3	STATISTICAL PROPERTIES FOR RATIO OF EFFECTIVE MEASURED TO PREDICTED EI VALUES FOR SIMPLY-SUPPORTED RC	10, 11, 12, 9
4	STATISTICAL STRENGTH PROPERTIES OF STEEL	8, 18, 16
5	GEOMETRIC VARIABILITY OF RC BEAM, COLUMN AND SLAB DIMENSIONS	19, 16
6	STATISTICAL DATA ON COMPRESSIVE STRENGTH OF CONCRETE	17
7	DAMPING RATIOS FOR CONCRETE, STEEL AND SOIL	16, 39
8	VARIABILITY OF MASSES IN DYNAMIC ANALYSIS	16

circumstances, may be to use subjective engineering judgement to specify the prior estimates of the statistical parameters of the α_i's.

To overcome the unavailability of member-level information, we will seek elsewhere to learn about their properties; we recommend the use of major structure and/or substructure test data to extract the missing information. The basic advantage to be gained in using these data (usually expressed in terms of modal parameters) is that, unlike member-level data, they furnish information on the α_i's taking into account the effect of the particular boundary conditions and strain distributions as they exist in real interconnected structural systems, provided of course that the data contain sufficient information on the particular α_i's of interest. In other words, the α_i's must be identifiable or observable from the data.

4.1.2 Substructure and Major Structure Data (Tables 4-2 and 4-3)

Above we have emphasized the importance of structure and substructure-level data to gain information on the α's. In section 2.2, building data (treated as substructure data) are reported. Upon study of the available data the following comments are in order:

(1) The reported data do not explicitly account for measurement error effects.

(2) The data may include structure-to-structure and/or soil-structure interaction effects.

(3) Most of the reported data do not contain any information on mass distribution and mode shapes.

Table 4-2. Substructure Data

STRUCTURE	$^{\circ}\omega$	$\hat{\omega}$	$^{\circ}\phi$	$\hat{\phi}$	$^{\circ}M$	MEASURED DAMPING	REFERENCES
S$^{(1)}$	X	X	X	X			26
S	X	X	X	X		X	30
S$^{(2)}$	X	X				X	31
S	X	X					32
S$^{(3)}$	X$^{(3)}$	X	X$^{(3)}$	X	X$^{(3)}$	X	33, 51
S$^{(10)}$		X		X			34
S							39
S	X	X	X	X	X$^{(4)}$	X	40
S	X$^{(5)}$	X	X$^{(5)}$	X	X	X	41
S	X	X		X		X	43
S	X$^{(6)}$	X	X	X	X$^{(6)}$	X	44
S	X	X	X	X	X	X	46
S$^{(15)}$		X				X	49
S		X	X	X	X$^{(7)}$	X	52
S	X	X		X		X	53
S$^{(8)}$		X					54
RC$^{(9)}$		X		X			25
RC		X	X	X		X	27
RC	X	X	X	X	X	X	28, 36, 51
RC		X		X	X	X	29

Table 4-2. Substructure Data (Cont.)

STRUCTURE	$°\omega$	$\hat{\omega}$	$°\phi$	$\hat{\phi}$	°M	MEASURED DAMPING	REFERENCES
RC[2]	X	X				X	31
RC[10]		X		X			34
RC[11]		X					35
RC		X		X		X	37
RC						X	39
RC	X[5]	X	X[5]	X	X	X	41
RC		X		X		X	42
RC	X[12]	X	X[12]	X		X	45
RC[13]		X		X			47
RC[14]		X		X		X	48
RC[15]		X				X	49
RC[16]	X	X			X		50
RC	X	X		X		X	53
RC[8]		X					54
PRESTRESSED CONCRETE[17]		X		X			38

NOTES

(1) This is a two story steel frame building with two reinforced concrete-block shear walls along two adjacent sides.
(2) This is a comprehensive summary of statistical data for fundamental frequencies and damping.
(3) Analytical models for this building are reported in: Gobler, H., "Three-Dimensional Modeling and Dynamic Analysis of the San Diego Gas and Electric Company Building," MS Thesis, UCLA, 1969.
(4) May be estimated from given data.
(5) Reported results utilize data from identified stiffness matrices; they are not a priori data.
(6) These values are reported in Reference 4 of our Reference 44.
(7) Scale model structures, mass can be estimated from given data.
(8) Contains data of pre, during and post earthquake fundamental frequency data.
(9) For additional information for this structure, see Reference 71 of our Reference 25.
(10) Several measured mode shapes for various structures are presented in the paper.
(11) Interesting comparison of measured frequencies for identically designed buildings
(12) Are the results of a posteriori model
(13) Reinforced masonary and RC buildings.
(14) Reinforced masonary building.
(15) Based on Japanese data.
(16) These are RC shear wall structures.
(17) Data for a containment structure.

Table 4-3. Major Structure Data

STRUCTURE	$^{\circ}\omega$	$\hat{\omega}$	$^{\circ}\phi$	$\hat{\phi}$	$^{\circ}M$	MEASURED DAMPING	REFERENCE
RC	x	x	x			x	61
RC(1)	x	x		x(2)			65
RC(3)							56
RC & S	x	x	x	x	x	x	58
RC(4)	x	x			x	x	71,72,77
RC & S	x	x	x	x		x	73
RC & S(5)	x	x				x	62
RC						x	59,60

(1) Report data for 250'RC stack, control room floor frequency and equipment frequency.
(2) Only control room floor vertical mode shape published.
(3) This paper is only a summary of an extensive experimental program carried out on a power plant in Italy. Extensive data in all categories are gathered, but not published.
(4) Is a combined reactor building and turbine building. System identification is also used in this paper to improve a prior model.
(5) Includes data on three different nuclear power plants.

(4) The data do not differentiate between specific structure types such as braced frame, moment resisting frame, etc.

The first comment is in order because the determination of natural periods from measured response data is subject to inherent experimental errors due to measurement, processing and interpretation of the derived results. For instance, the achievable accuracy of modal parameters, for a given structure, from random vibration data is reported in [82-86, 94,96]. Hence, based on the above remarks we may at best interpret the data to reflect the combined effects of modeling uncertainty and measurement error. Secondly, the data may contain the effects of structure-to-structure and/or soil-structure interaction effects. These effects may especially be relevant to data obtained from blast and earthquake sources; however, it may be reasonable to expect results derived from very low level ambient vibration tests to be only minimally affected by the above mentioned effects. A serious deficiency of the reported data, at least for the purposes of this study, is the lack of information on the mass distributions of the tested structure. It is felt that this deficiency may be partially remedied by going through the original sources of data; however, this may prove to be a considerable undertaking. The lack of mode shape data is also clearly apparent; these data bear important information on the stiffness and mass distributions of structures.

In general, it is believed that considerably more data exist in unpublished reports than have been identified here. A determined effort to identify specific data requirements followed by data acquisition via appropriate channels should produce sufficient data to implement the procedures defined herein.

4.2 Potential Benefits from Additional Data

A review of the available data which appears in Tables 4-1 to 4-3, reveals that a priori mode shapes and a priori mass distribution information are rather rare, compared to fundamental frequency data. Clearly, there are two basic reasons for this deficiency, (1) economic, and (2) interest. It is obvious that a large number of sensors are required to measure mode shapes; the number of instruments required increases roughly in proportion to the number of modes to be measured. The operating bandwidth of shaking machines and their frequency control resolution further limit the measurable frequencies to the lower modes. Also, the number of recording instruments and personnel required to carry out experiments to determine higher modes have been excessive in the past. Most of these practical constraints are being overcome by technological advancement in the design of sensors, recording equipment, mechanical shakers and analysis units. The availability of minicomputers and analysis units have significantly reduced the effort necessary to extract higher mode data from ambient vibration survey records or random vibration data. In the last decade or so, there has been a growing interest by experimentalists to fit analytical models to their measurement data, and interest by theoreticians to utilize more abundant (in quality and quantity) measured data to construct more realistic mathematical models of physical phenomena. This surge in interest of experimentalists and theoreticians towards the work of each other, is to a significant extent accountable to the emergence of the methodology of system identification in the last decade as well as the widespread use of the digital computer and greatly improved experimental techniques.

Potential benefits to be gained from having additional generic substructure and major structure modal data are considered great. There appears to be no alternative for making a realistic assessment of structural uncertainty, with the exception of dynamic tests performed on the particular substructures of major structures of interest.

Substructure or major structure - specific testing is of course recognized as being the very best way to reduce structural uncertainties in seismic risk analysis. The distinct advantages of this approach should be fully appreciated, so that whenever possible, such information can be used.

5. CONCLUSIONS AND RECOMMENDATIONS

The methodology presented in this report is considered to be new. While its basic elements have been used successfully in other applications, it should be understood that at the present time the proposed methodology is untried and unproven. As such it may be expected to evolve as experience is gained. The following statement of conclusions and recommendations conveys the authors' assessment of significant accomplishments to date, and their view of directions for future development.

5.1 Conclusions

With reference to the problem as stated in Section 1.1 of the Introduction, it is felt that the purpose of this study has been accomplished. An analytical methodology for maximizing the use of presently limited, incomplete and previously incompatible data has been formulated. The methodology should produce results which are based on quantitative data rather than conjecture, and which are in a form suitable for the numerical simulation of seismic response of major structures of nuclear power reactor facilities.

The following conclusions are drawn with respect to specific objectives set forth in Section 1.2.

(1) A literature search to identify data and sources of data for use in modeling structural uncertainty was completed. A bibliography containing 95 references is included in this report.

(2) The format and adequacy of the above data have been evaluated relative to the proposed methodology. Although

the data and sources identified in this report are probably not sufficient to meet anticipated needs, it is believed that once the specific needs are firmly established, the data can be found in unpublished reports within the United States. Until now there has been no particular need for complete sets of predicted and measured modal data, accompanied by the mass matrices which were used to obtain analytically predicted modes. Therefore, the data reported in the literature tend to be incomplete in one way or another. Much of these data should be available if properly pursued. Tables 4-1 through 4-3 show the deficiencies in data compiled to date.

(3) Available data have been classified into three basic types: data related to member uncertainties, data related to substructure uncertainties, and data related to the major structures of nuclear power plants. The bibliography and the summary tables in Section 4 are segregated accordingly. In addition, the summary tables differentiate among structures in three generic categories - structural steel, reinforced concrete, and prestressed (or post - tensioned) concrete.

(4) The proposed methodology is compatible with currently and potentially available data. Structural property data at the member level can be used as well as modal data at the substructure or major structure levels. Engineering judgement may or <u>may not</u> be used at all three levels as desired. Data from structures and substructures other than nuclear power plant structures can be utilized. Specific seismic or forced vibration test data for the

particular structure being modeled can be used in a Bayesian procedure for statistical parameter estimation.

(5) The proposed methodology utilizes mode shape data at the substructure and major structure levels in addition to frequency and damping data. Sample calculations have been made which demonstrate (using actual data) that mode shape uncertainties can be significant and may conceivably dominate frequency uncertainties with respect to characterizing overall structural uncertainty for response prediction.

(6) The proposed methodology retains the correlation information relating the statistics of structural properties and modal parameters until the final step of analysis where frequency and modal amplitude are assumed to be independent. This assumption, however, is made only for purposes of convenience in the numerical simulation and is not an essential part of the methodology.

(7) Dimensionless combinations of modal parameters have been identified for concise representation of structural uncertainty. In addition to the conventional dimensionless parameters such as the ratio of actual to predicted eigenvalue, and actual to critical modal damping, a dimensionless modal amplitude parameter involving modal displacement, modal participation factor and modal mass has been identified.

(8) Empirical analyses of the three dimensionless parameters named above are suggested as a basis for quantifying them for specific applications.

(9) The framework of analysis outlined in this report should be helpful in establishing data requirements for SSMRP

as well as other design, analysis and test programs involving nuclear power plants. The methodology can be used to evaluate the potential benefits of various kinds of data in terms of modeling confidence. For example, calculations can be made which treat steel and reinforced concrete structures either separately, or in combination to evaluate the trade-offs between aggregating available data into broader categories, as opposed to segregating them into narrower categories.

(10) The proposed methodology offers many different options to choose from, all leading to the same end. Twelve different options are presented. Of these, four (Options 1, 2, 4 and 10) are identified as being the more practical ones. Option 4 was selected as the most desirable for initial implementation.

(11) Because the approach taken in formulating the proposed methodology is based on utilizing those data which are most directly representative of prediction uncertainties in seismic response analyses performed to date, the results of analysis produced thereby should be in agreement with the state-of-the-art experience.

In retrospect, the one aspect of the methodology which may have been underemphasized, is that of modeling damping and damping uncertainty. The recommended approach of utilizing direct measurements from similar major structures is considered to be preferable to other alternatives as a general rule. Since damping has been a focal point of attention for some time, there are considerable data available. However, anticipating the inevitable concern over the possible uniqueness of special cases, we might consider the possiblity of synthesizing measurements (or estimates) of modal damping at the substructure level to predict modal damping (and related uncertainties) at the major structure level. Such an undertaking must be approached with great caution, however. Generally speaking, this type of analysis is beyond the present state-of-the-art and is fraught with many subtle pitfalls. It should only be considered in special circumstances where the direct approach cannot be used because of insufficient data, and where suitable data are available at the substructure level. Conditions for "suitability" must be carefully defined in this case.

5.2 Recommendations

Recommendations for the implementation and possible future development of the proposed methodology are summarized here. They represent an extension of the authors' present thoughts in line with the objectives of SSMRP as presently understood.

(1) Specific modeling requirements should be identified relative to the Zion Nuclear Facility which has been selected as the pilot application for SSMRP. Efforts should then be made to fill out the required data base. It is of critical importance here to determine the number of generic substructures within a given category required to define $[S^i_{rr}]$ so that the matrix can be inverted, or conversely, to determine the maximum number of off-diagonal terms of this matrix which can be identified using available data. For the most part, this matrix will have to be assumed diagonal. Justification should be sought for this assumption.

(2) In general, several cross-orthogonality coefficients, $\hat{\psi}_{kj}$,

may be computed for each measured mode shape. A numerical investigation should be made to determine how many are required to capture the essential information contained in the difference between predicted and measured mode shapes.

(3) Numerical examples should be investigated on the basis of available data before making a final selection of the computational option. Option 4 is tentatively recommended, but the final selection should be made only after alternative options have been thoroughly considered with respect to available data.

(4) The quantification of the dimensionless random variables - μ_1, μ_2 and μ_3 - should be investigated thoroughly on the basis of numerical examples using available data. Only tentative recommendations have been made in this regard.

(5) Early consideration should be given to specific interface requirements with other parts of the SMACS code. It may be possible to limit the direct interface to considerations involving only the three random variables named herein. This would require, however, that compatible structural modeling subroutines be used to generate the submatrices, transformations, etc. required to implement the proposed methodology. In other words, this methodology is based on using the same analytical models as will be used in SMACS. The validity of the methodology depends on this.

BIBLIOGRAPHY

Part I: References related to structural member uncertainties.

1. ACI Committe 435, "Variability of Deflections of Simply Supported Reinforced Concrete Beams," ACI Journal, January 1972.

2. Benjamin, J., "Variability Analysis of Shear Wall Structures," 4th World Conference on Earthquake Engineering, Santiago, Chile, 1969, pp. 45-51.

3. Chelapati, C.V. and Wall, I.B., "Probabilistic Assessment of Seismic Risk for Nuclear Power Plants," Paper K1/3, SMIRT, Berlin, 1973.

4. Chen, P.C., "Floor Response Spectra of Buildings with Uncertain Structural Properties," Proc. of the U.S. National Conference on Earthquake Engineering, Ann Arbor, Michigan, June 1975, pp. 519-528.

5. Chen, P.C. and Soroka, W.W., "Multi-Degree Dynamic Response of a System with Statistical Properties," Journal of Sound and Vibration, Vol. 37, No. 4, 1974, pp. 547-556.

6. Collins, J.D. and Thomson, W.T., "The Eigenvalue Problem for Structural Systems with Statistical Properties," AIAA Journal, Vol. 7, No. 4, April 1969.

7. Ellyin, F. and Chandrasekhar, P., "Probabilistic Dynamic Response of Beams and Frames," Journal of Engineering Mechanics Division, ASCE, EM3, June 1977.

8. Galambos, T. and Ravindra, M., "Properties of Steel for Use in LRFD," Journal of Structural Division, ASCE, Vol. NO. ST9, September 1978, pp. 1459-1468.

9. Gallo, M.P. and Ang, A.H-S, "Evaluation of Safety of Reinforced Concrete Buildings to Earthquakes," Technical Report UILU-ENG-76-2018, Dept. of Civil Engr., University of Illinois at Urbana, Champaign, Oct. 1976.

10. Hadjian, A.H. and Hamilton, C.W., "Impact of Soil-Structure Interaction on the Probabilistic Frequency Variation of Concrete Structures," Paper K3/8, SMIRT, London, 1975.

11. Hadjian, A.H. and Hamilton, C.W., "Probabilistic Frequency Variations of Concrete Structures," Paper K3/5, SMIRT, Berlin, 1973.

12. Hamilton, C.W. and Hadjian, A.H., "Probabilistic Frequency Variations of Structure-Soil Systems," Nuclear Engineering and Design, Vol. 38, 1976, pp. 303-322.

13. Hart, G.C., "Eigenvalue Uncertainty in Stressed Structures," Journal of Engineering Mechanics Divsion, ASCE, No. EM3, June 1973, pp. 481-494.

14. Hasselman, T.K. and Hart, G.C., "Modal Analysis of Random Structural Systems," Journal of Engineering Mechanics Divsion, ASCE, No. EM3, June 1972, pp. 561-572.

15. Hoshiya, M. and Shah, H.C., "Free Vibration of Stochastic Beam-Columns," Journal of the Engineering Mechanics Division, ASCE, Vol. 97, No. EM4, Aug. 1971, pp. 1239-1255.

16. Liu, L.D., et al., "Effects of Parameter Variations on Floor Response Spectra," Speciality Conference on Structural Design of Nuclear Plant Facilities, ASCE, Chicago, Illinois, December 1973.

17. Mirza, S.A., et al., "Statistical Description of Strength of Concrete," Journal of Structural Division, ASCE, Vol. 105, No. ST6, June 1979, pp. 1021-1037.

18. Mirza, S.A. and MacGregor, J.G., "Variability of Mechanical Properties of Reinforcing Bars," Journal of Structural Division, ASCE, Vol. 105, No. ST5, May 1979, pp. 921-927.

19. Mirza, S.A. and MacGregor, J.G., "Variations in Dimensions of Reinforced Concrete Members," Journal of Structural Division, ASCE, Vol. 105, No. ST4, April 1979, pp. 751-766.

20. Schiff, A. and Bogdanoff, J., "An Estimation of the Standard Deviation of Natural Frequencies," Journal of Applied Mechanics, Parts 1 and 2, 39, Series E, June 1972.

21. Shinozuka, M. and Astrill, C.J., "Random Eigenvalue Problems in Structural Analysis," AIAA Journal, Vol. 10, No. 4, April 1972.

22. Tagart, S.W. and Torres, M.R., "Seismic Analysis of Nuclear Components Considering Modeling Uncertainties," Preprints 5th World Conference on Earthquake Engineering, International Association for Earthquake Engineering, Rome, June 1973, pp. 2462-2464.

23. Winter, G. and Yu, W.W., "Instantaneous and Long-Time Deflection of Reinforced Concrete Beams Under Working Loads," ACI Journal, Vol. 57, No. 1, July 1960, pp. 29-50.

24. Zsutty, T., "Beam Shear Strength Prediction by Analysis of Existing Data," ACI Journal, Proc. Vol. 55, November 1968.

PART II: References related to substructure modal uncertainties.

25. Becker, J. and Llorente, C., "Seismic Design of Precast Concrete Panel Buildings," Proceedings of a Workshop on Earthquake-Resistant Reinforced Concrete Building Construction, University of California, Berkeley, July 1977.

26. Bouwkamp, J.G. and Blohm, J.K., "Dynamic Response of a Two-Story Steel Frame Structure," Bulletin of the Seismological Society of America, Vol. 56, No. 6, December, 1966, pp. 1289-1303.

27. Englekirk, R.E. and Matthiesen, R.B., "Forced Vibration of an Eight-Story Reinforced Concrete Building," Bulletin of the Seismological Society of America, Vol. 57, No. 3, June, 1967, pp. 421-436.

28. Foutch, D., "A Study of the Vibrational Characteristics of Two Multi-story Buildings," Technical Report EERL 76-03, California Institute of Technology, September 1976.

29. Freeman, S., et al. "Dynamic Response Characteristics of Reinforced Concrete Structures," ASCE/EMD Special Conference on Dynamic Response of Structures, UCLA, Los Angeles, California, March 1976.

30. Funahashi, I., Kinoshita, K., and Aoyama, H., "Vibration Tests and Test to Failure of a 7 Stories Building Survived a Severe Earthquake," Proceedings of the Fourth World Conference of Earthquake Engineering, Santiago, Chile, 1969.

31. Haviland, R., "A Study of the Uncertainties in the Fundamental Translational Periods and Damping Values for Real Buildings," Technical Report, Publication No. R76-12, Department of Civil Engineering, MIT, February 1976.

32. Ibanez, P., "Experimental and Theoretical Analysis of Buildings," ASCE/EMD Special Conference on Dynamic Response of Structures, UCLA, Los Angeles, California, March 1976.

33. Jennings, P.C., Matthiesen, R.B., and Hoerner, J.B., "Forced Vibration of a Tall Steel-Frame Building," International Journal of Earthquake Engineering and Structural Dynamics, Vol. 1, No. 2, Oct. - Dec., 1972.

34. Jennings, D., "Spectrum Techniques for Tall Buildings," 4th World Conference on Earthquake Engineering, Santiago, Chile, 1969.

35. Kircher, C. and Shark, H., "Ambient Vibration Study of Six Similar High-Rise Apartment Buildings," Technical Report No. 14, Department of Civil Engineering, Stanford University, January 1975.

36. Kurciwa, J., "Vibration Test of a Multistory Building," Technical Report, Earthquake Engineering Research Laboratory, California Institute of Technology, Pasadena, California, June 1967.

37. Mayes, R. and Galambos, T., "Large-Scale Dynamic Shaking of 11-Story Reinforced Concrete Building," Proceedings of a Workshop on Earthquake-Resistant Reinforced Concrete Building Construction, University of California, Berkeley, July 1977.

38. Milella, P., et al.,"Ambient Vibration Survey on the Contaminant Structure of PEC Reactor," Paper K5/4, SMIRT, Berlin, 1973.

39. Morrone, A., "Damping Values of Nuclear Power Plant Components," Nuclear Engineering and Design, Vol. 26, 1974. [Condensed from Westinghouse Report WCAP-7921, Nov. 1972]

40. Nielsen, H., "Dynamic Response of a 90-Ft Steel Tower," 4th World Conference on Earthquake Engineering, Santiago, Chile, 1969.

41. Nielsen, H., "Dynamic Response of Multistory Buildings," Technical Report, Earthquake Engineering Research Laboratory, California Institute of Technology, Pasadena, California, June 1964.

42. Osawa, Y., et al.,"Earthquake Measurements in and Around a Reinforced Concrete Building," 4th World Conference on Earthquake Engineering, Santiago, Chile, 1969.

43. Otsuki, Y., et al., "Summarized Report on Dynamic Tests of High-rised Buildings and Cooperative Plan for Large-scale Vibration Tests in Japan," Proceedings of the Fourth World Conference on Earthquake Engineering, Santiago, Chile, 1969.

45. Petrovski, J., et al., "Dynamic Properties of Fourteen Story R.C. Frame Building from Full Scale Forced Vibration Study and Formulation of Mathematical Model," 5th World Conference on Earthquake Engineering, Rome, 1973.

46. Rea, D., Shah, A.A., and Bouwkamp, J.G., "Dynamic Behavior of a High-Rise Diagonally Braced Steel Building," Earthquake Engineering Research Center Report No. EERC 71-5, University of California, Berkeley, California, August, 1971.

47. Shepherd, R. and Jennings, P., "Experimental Investigations - Correlation with Analysis," Proceedings of a Workshop on Earthquake-Resistant Reinforced Concrete Building Construction, University of California, Berkeley, July 1977.

48. Stephen, R. and Bouwkamp, J., "Dynamic Behavior of an Eleven-Story Masonry Building," Proceedings of a Workshop on Earthquake-Resistant Reinforced Concrete Building Construction, University of California, Berkeley, July 1977.

49. Tanaka, T., et al., "Period and Damping of Vibration in Actual Buildings During Earthquakes," Bulletin of the Earthquake Research Institute, University of Tokyo, Vol. 47, November 1969.

50. Taoka, G., et al., "Dynamic Properties of Tall Shear-Wall Buildings," Journal of Structural Division, ASCE, No. ST2, February 1974.

51. Trifunac, M.D., "Comparisons Between Ambient and Forced Vibration Experiments," International Journal of Earthquake Engineering and Structural Dynamics, Vol. 1, No. 2, Oct. - Dec., 1972.

52. Ueda, C., "Study of the Large Scale Displacement Vibration Test for the 1/25 Scale Model of the 17-Storied Building," 4th World Conference on Earthquake Engineering, Santiago, Chile, 1969.

53. Ward, H.S. and Crawford, R., "Wind-Induced Vibrations and Building Modes," Bulletin of the Seismological Society of America, Vol. 56, No. 4, August, 1966.

54. San Fernando, California Earthquake of February 9, 1971, Vol. I, Part B, U.S. Department of Commerce, National Oceanic and Atmospheric Administration, Washington, D.C., 1973.

BIBLIOGRAPHY (continued)

PART III: References related to major structure modal uncertainties.

55. Bekowich, R., "Analysis and Testing of Prestressed Concrete Nuclear Containment Structures," Paper J4/4, SMIRT, Berlin, 1971.

56. Castoldi, A., Casirati, M., Scotto, F.L., "In-Situ Dynamic Tests and Seismic Records on the RHR System Building ENEL IV Nuclear Power Plant, Caruso (Italy)," Paper K8/4, SMIRT, San Francisco, 1977.

57. Chrostowski, J., et al., "Simulating Strong Motion Earthquake Effects on Nuclear Power Plants Using Explosive Blast," Technical Report UCLA-ENG-7119, University of California, Los Angeles, February 1972.

58. Gundy, W.E., et al., "A Comparison of Vibration Tests and Analysis on Nuclear Power Plant Structures and Piping," Paper K8/6, SMIRT, San Francisco, 1977.

59. Hart, G.C., "Damping in Nuclear Reactor Facilities," Technical Report UCLA-ENG-7336, UCLA, Los Angeles, California, May 1973.

60. Hart, G.C. and Ibanez, P., "Experimental Determination of Damping in Nuclear Power Plant Structures and Equipment," Nuclear Engineering and Design, Vol. 25, No. 1, 1973.

61. Hirasawa, M., et al., "Dynamic Behavior of a Nuclear Reactor Building," Paper K8/2, SMIRT, San Francisco, 1977.

62. Hornbuckle, J., et al., "Forced Vibration Tests and Analysis of Nuclear Power Plants," Nuclear Engineering and Design, Vol. 26, No. 1, 1973.

63. Howard, G., et al., "Seismic Design at Nuclear Power Plants - An Assessment," Final Report EPRI 273, June 1975.

64. Ibanez, P., "Review of Analytical and Experimental Techniques for Improved Structural Dynamic Models," Welding Research Council Bulletin 249, June 1979.

65. Ibanez, P., et al., "In-Situ Testing for Seismic Evaluation of Humboldt Bay Nuclear Power Plant for Pacific Gas and Electric Company," Paper K8/3, SMIRT, San Francisco, 1977.

66. Ibanez, P., et al., "San Onofre Nuclear Generating Station Vibration Tests," Technical Report UCLA-ENG-7037, UCLA, Los Angeles, California, August 1970.

67. Jeary, A.P., "Vibrations of a Nuclear Power Station Charge Hall," Earthquake Engineering and Structural Dynamics, Vol. 4, No. 3, Jan. - Mar. 1976.

68. Jennings, P.C. and Kuroiwa, "Vibration and Soil-Structure Interaction Tests of a Nine-Story Reinforced Concrete Building," Bulletin of the Seismological Society of America, Vol, 58, No. 3, June, 1968, pp. 891-916.

69. Matthiesen, R., et al., "San Onofre Nuclear Generating Station Supplementary Vibration Tests," Technical Report UCLA-ENG-7095, UCLA, Los Angeles, California, December 1970.

70. Matthiesen, R., et al., "BLAST," An Interim Report UCLA-ENG-7081, UCLA, Los Angeles, California, October 1970.

71. Mizuno, N., et al., "Seismic Response Analysis of Nuclear Reactor Buildings Under Consideration of Soil-Structure Interaction with Torsional Behavior," Paper K2/16, SMIRT, San Francisco, 1977.

72. Mizuno, N. and Tsushima, Y., "Experimental and Analytical Studies for BWR Nuclear Reactor Building Evaluation of Soil-Structure Interaction Behavior," Paper K3/2, SMIRT, London, 1975.

73. Muto, K., et al., "Comparative Forced Vibration Test of Two BWR-Type Reactor Buildings," Paper K5/3, SMIRT, Berlin, 1973.

74. Muto, K. and Omatsuzawa, K., "The Earthquake Response Analysis for a BWR Nuclear Power Plant Using Recorded Data," Paper K2/1, SMIRT, Berlin, 1971. See also Nuclear Engineering and Design, Vol. 20, 1972.

75. Rea, D. and Bouwkamp, J.G., "A Source of Damping Produced by the Interaction of Close-Standing Buildings," Bulletin of the Seismological Society of America, Vol. 58, No. 3, June, 1968, pp. 917-1135.

76. Smith, G.B., Ibanez, P., Wong, G., Matthiesen, R.B., "San Onofre Nuclear Generating Station Vibration Tests," Technical Report UCLA-ENG-7037, UCLA, Los Angeles, California, August 1970.

77. Tsushima, Y., "Experimental and Analytical Studies Soil-Structure Interaction Behavior of Nuclear Reactor Building," Takanaka Technical Research Report No. 21, Japan, April 1979.

78. Vasudevan, R., et al., "Modeling of the San Onofre Nuclear Generating Station for Seismic Response," Technical Report UCLA-ENG-7151, UCLA, Los Angeles, California, July 1972.

79. "Vibration Testing and Seismic Analysis of Nuclear Power Plants," a special issue of Nuclear Engineering and Design, Vol. 25, No. 1, 1973, pp. 1-164.

Part IV: General References.

80. Ang, A. H-S. and Cornell, C.A., "Reliability Bases of Structural Safety and Design," Journal of the Structural Division, ASCE, Vol. 100, No. ST9, September 1974, pp. 1755-1769.

81. Craig, R.R. and Bampton, M.C.C., "Coupling of Substructures for Dynamic Analysis," AIAA Journal, Vol. 6, No. 7, July 1968, pp. 1313-1319.

82. Gersch, W., Nielsen, N. and Akaike, H., "Maximum Likelihood Estimation of Structural Parameters from Random Vibration Data," Journal of Sound and Vibration, Vol. 31, No. 3, 1973.

83. Gersch, W., "On the Achievable Accuracy of Structural System Parameter Estimates," Journal of Sound and Vibration, Vol. 34, No. 1, 1974.

84. Gersch, W. and Foutch, D., "Least Squares Estimates of Structural System Parameters Using Covariance Function Data," IEEE Trans. Automat. Control, Vol. AC-19, No. 6, December 1974.

85. Gersch, W., Taoka, G.T. and Liu, R., "Structural System Parameter Estimation by a Two-Stage, Least-Square Method," Journal of Engineering Mechanics Division, ASCE, Vol. 102, No. EM5, October 1976.

86. Gersch, W., et al., "A Two-Stage Least Squares Method for the Identification of Input-Output Vibration Data Systems," Proceedings of the Symposium on Applications of Computer Methods in Engineering, Vol. I, Los Angeles, California, August 1977, pp. 583-592.

87. Gobler, H., "Three-Dimensional Modeling and Dynamic Analysis of the San Diego Gas and Electric Company Building," MS Thesis, University of California, Los Angeles, California, 1969.

88. Hasselman, T.K. and Hart, G.C., "A Minimization Method for Treating Convergence in Modal Synthesis," AIAA Journal, Vol. 12, No. 3, March 1974, pp. 316-323.

89. Hasselman, T.K., "Modal Coupling in Lightly Damped Structures," Technical Note, AIAA Journal, Vol. 14, No. 11, November 1976.

90. Hurty, W.C., "Dynamic Analysis of Structural Systems Using Component Modes," AIAA Journal, Vol. 3, No. 4, April 1965, pp. 678-685.

91. Isenberg, J., "Progressing from Least Squares to Bayesian Estimation," ASME, Paper 79-WA/DSC-16, Presented at the Winter Annual Meeting, New York, N.Y., December 1979.

92. Jennings, P.C., et al., "Forced Vibration of a 22-Story Steel Frame Building," Technical Report EERL 71-01, California Institute of Technology, Pasadena, California, February 1971.

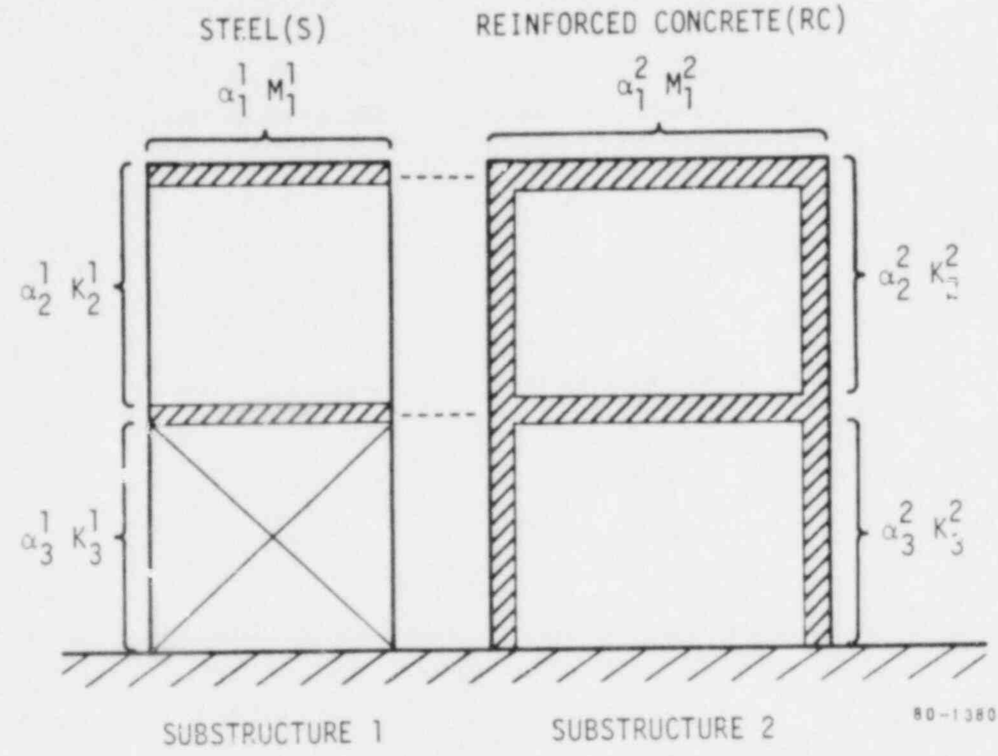
93. Lee, L.T. and Hasselman, T.K., "Dynamic Model Verification of Large Structural Systems," Society of Automotive Engineers, Paper 781047, Presented at the Aerospace Meeting, San Diego, California, November 1978.

94. Taoka, G.T., "Digital Filtering of Ambient Response Data," Proceedings of the ASCE/EMD Specialty Conference on Dynamic Response of Structures, University of California, Los Angeles, March 1976, pp. 253-262.

95. Taoka, G.T., "A Comparison of Structural Identification Methods," Proceedings of the Symposium on Applications of Computer Methods in Engineering, Vol. I, Los Angeles, California, August 1977, pp. 283-291.

APPENDIX A

Illustration of Option 4



We assume that these substructures are connected together by rigid links at the second floor and roof levels.

A priori estimates of statistics of α^i

Substructure 1: $\alpha^1 = \begin{Bmatrix} \alpha_2^1 \\ \alpha_2^1 \\ \alpha_3^1 \end{Bmatrix}$
(S)

Substructure 2: $\alpha^2 = \begin{Bmatrix} \alpha_1^2 \\ \alpha_2^2 \\ \alpha_3^2 \end{Bmatrix}$
(RC)

Assume that we have a priori estimates of $E(\alpha_j^i)$, Cov.(α_j^i, α_k^i): $i = 1,2$; $j,k = 1,2,3$.

$$
\left.
\begin{aligned}
&\text{Substructure 1:}\quad E\,(\alpha^1) = \begin{Bmatrix} {}^\circ\alpha_1^1 \\ {}^\circ\alpha_2^1 \\ {}^\circ\alpha_3^1 \end{Bmatrix} \\[1em]
&\text{Cov. matrix:}\quad S_{\alpha\alpha}^1 = \begin{bmatrix} S_{11}^1 & S_{12}^1 & S_{13}^1 \\ & S_{22}^1 & S_{23}^1 \\ \text{SYM} & & S_{23}^1 \end{bmatrix} \\[1em]
&\text{Substructure 2:}\quad E\,(\alpha^2) = \begin{Bmatrix} {}^\circ\alpha_1^2 \\ {}^\circ\alpha_1^2 \\ {}^\circ\alpha_3^2 \end{Bmatrix} \\[1em]
&\text{Cov. matrix:}\quad S_{\alpha\alpha}^2 = \begin{bmatrix} S_{11}^2 & S_{12}^2 & S_{13}^2 \\ & S_{22}^2 & S_{23}^2 \\ \text{SYM} & & S_{33}^2 \end{bmatrix}
\end{aligned}
\right\}\ \text{Only a priori estimates}
$$

Available generic substructure modal data

Substructure 1: Structural Steel

(i) $\hat{\omega}_1^1$, $\hat{\phi}_1^1$, $°\omega_1^1$, $°\phi_1^1$, $°M^1$, structure "a"

(ii) $\hat{\omega}_1^1$, $\hat{\phi}_1^1$, $°\omega_1^1$, $°\phi_1^1$, $°M^1$, structure "b"

Substructure 2: Reinforced Concrete

(i) $\hat{\omega}_1^2$, $\hat{\phi}_1^2$, $°\omega_1^2$, $°\phi_1^2$, $°M^2$, structure "c"

(ii) $\hat{\omega}_1^2$, $\hat{\phi}_1^2$, $°\omega_1^2$, $°\phi_1^2$, $°M^2$, structure "d"

Note: Each $\hat{\phi}_1^i$, $°\phi_1^i$ is normalized with respect to $°M^i$.

Calculation of $S^i_{\tilde{r}\tilde{r}}$, i=1,2,

$$S^i_{\tilde{r}\tilde{r}} = E\left(\Delta\tilde{r}^i \; \Delta\tilde{r}^{i\,T}\right) = \frac{1}{N-1} \sum_{\ell=1}^{N} \left[\left(\Delta\tilde{r}^i\right)\left(\Delta\tilde{r}^i\right)^T\right]_\ell$$

Substructure 1: (i=1, N=2)

$$\tilde{r}^1 = \begin{Bmatrix} 1 + \Delta m_{11}^1 \\ \\ 1 + \Delta\tilde{k}_{11}^1 \end{Bmatrix}_{\ell=1} , \tilde{r}^1 = \begin{Bmatrix} 1 + \Delta m_{11}^1 \\ \\ 1 + \Delta\tilde{k}_{11}^1 \end{Bmatrix}_{\ell=2}$$

$$\Delta m_{11}^{1} = -2\Delta\eta_{11}^{1} = -2\left(\hat{\psi}_{11}^{1} - 1\right)$$

$$\hat{\psi}_{11}^{1} = {}^{\circ}\phi_{1}^{1^T}\ {}^{\circ}M^{1}\ \hat{\phi}_{1}^{1}, \text{ computed from available data}$$

$$\Delta\tilde{k}_{11}^{1} = \frac{\Delta k_{11}^{1}}{{}^{\circ}\lambda_{1}^{1}} = \frac{\Delta\lambda_{1}^{1}}{{}^{\circ}\lambda_{1}^{1}} + \Delta m_{11}^{1}, \ \lambda=\omega^{2}$$

$$\Delta\lambda_{1}^{1} = \hat{\lambda}_{1}^{1} - {}^{\circ}\lambda_{1}^{1} \text{ ; computed from available data}$$

$$S_{\tilde{r}\tilde{r}}^{1} = \left(\begin{bmatrix} \left(\Delta m_{11}^{1}\right)^{2} & \left(\Delta m_{11}^{1}\right)\left(\Delta\tilde{k}_{11}^{1}\right) \\ \left(\Delta m_{11}^{1}\right)\left(\Delta\tilde{k}_{11}^{1}\right) & \left(\Delta\tilde{k}_{11}^{1}\right)^{2} \end{bmatrix}_{1} + \begin{bmatrix} \left(\Delta m_{11}^{1}\right)^{2} & \left(\Delta m_{11}^{1}\right)\left(\Delta\tilde{k}_{11}^{1}\right) \\ \left(\Delta m_{11}^{1}\right)\left(\Delta\tilde{k}_{11}^{1}\right) & \left(\Delta\tilde{k}_{11}^{1}\right)^{2} \end{bmatrix}_{2}\right)$$

<u>Substructure 2</u>: (i=2, N=2)

$$\tilde{r}^{2} = \begin{Bmatrix} 1 + \Delta m_{11}^{2} \\ 1 + \Delta\tilde{k}_{11}^{2} \end{Bmatrix}_{\ell=1}, \quad \tilde{r}^{2} = \begin{Bmatrix} 1 + \Delta m_{11}^{2} \\ 1 + \Delta\tilde{k}_{11}^{2} \end{Bmatrix}_{\ell=2}$$

$$\Delta m_{11}^{2} = -\ 2\Delta\eta_{11}^{2} = -\ 2\left(\hat{\psi}_{11}^{2} - 1\right)$$

$$\hat{\psi}_{11}^{2} = {}^{\circ}\phi_{1}^{2^T}\ {}^{\circ}M^{2}\ \hat{\phi}_{1}^{2}, \text{ computed from available data}$$

$$\Delta\tilde{k}^2_{11} = \frac{\Delta\lambda^2_1}{{}^\circ\lambda^2_1} + \Delta m^2_{11}$$

$\Delta\lambda^2_1 = \hat{\lambda}^2_1 - {}^\circ\lambda^2_1$, computed from available data

$$S^2_{rr} = \begin{bmatrix} \left(\Delta m^2_{11}\right)^2 & \left(\Delta m^2_{11}\right)\left(\Delta\tilde{k}^2_{11}\right) \\ \left(\Delta m^2_{11}\right)\left(\Delta\tilde{k}^2_{11}\right) & \left(\Delta\tilde{k}^2_{11}\right)^2 \end{bmatrix}_1 + \begin{bmatrix} \left(\Delta m^2_{11}\right)^2 & \left(\Delta m^2_{11}\right)\left(\Delta\tilde{k}^2_{11}\right) \\ \left(\Delta m^2_{11}\right)\left(\Delta\tilde{k}^2_{11}\right) & \left(\Delta\tilde{k}^2_{11}\right)^2 \end{bmatrix}_2$$

Transformations

$$M^i = {}^\circ\overline{M}^i + \alpha^i_1\, {}^\circ M^i_1 \ , \ K^i = {}^\circ\overline{K}^i + \alpha^i_2\, {}^\circ K^i_2 + \alpha^i_3\, {}^\circ K^i_3$$

$$m^i = {}^\circ\phi^{i^T} M^i\, {}^\circ\phi^i \ , \ k^i = {}^\circ\phi^{i^T} K^i\, {}^\circ\phi^i$$

Substructure 1

$$\mathrm{Var}(\tilde{r}^1) = T^1_{r\alpha}\, S^1_{\alpha\alpha}\, T^{1^T}_{r\alpha}$$

$$T^1 = \begin{bmatrix} \dfrac{\partial\tilde{r}^1_1}{\partial\alpha^1_1} & \dfrac{\partial\tilde{r}^1_1}{\partial\alpha^1_2} & \dfrac{\partial\tilde{r}^1_1}{\partial\alpha^1_3} \\ \dfrac{\partial\tilde{r}^1_2}{\partial\alpha^1_1} & \dfrac{\partial\tilde{r}^1_2}{\partial\alpha^1_2} & \dfrac{\partial\tilde{r}^1_2}{\partial\alpha^1_3} \end{bmatrix}$$

$$\frac{\partial \tilde{r}_1^1}{\partial \alpha_1^1} = \frac{\partial}{\partial \alpha_1^1}\left(1 + \Delta m_{11}^1\right) = {}^\circ\phi^{1^T}\ {}^\circ M_1^1\ {}^\circ\phi_1^1,\ \frac{\partial \tilde{r}_1^1}{\partial \alpha_2^1} = \frac{\partial \tilde{r}_1^1}{\partial \alpha_3^1} = 0$$

$$\frac{\partial \tilde{r}_2^1}{\partial \alpha_1^1} = 0,\ \frac{\partial \tilde{r}_2^1}{\partial \alpha_2^1} = \frac{\partial}{\partial \alpha_2^1}\left(1 + \frac{\Delta k_{11}^1}{{}^\circ\lambda_1^1}\right) = \frac{1}{{}^\circ\lambda_1^1}\left({}^\circ\phi_1^{1^T}\ {}^\circ K_2^1\ {}^\circ\phi_1^1\right)$$

$$\frac{\partial \tilde{r}_2^1}{\partial \alpha_3^1} = \frac{\partial}{\partial \alpha_3^1}\left(1 + \frac{\Delta k_{11}^1}{{}^\circ\lambda_1^1}\right) = \frac{1}{{}^\circ\lambda_1^1}\left({}^\circ\phi_1^{1^T}\ {}^\circ K_3^1\ {}^\circ\phi_1^1\right)$$

Substructure 2

$$Var(\tilde{r}^2) = T_{r\alpha}^2\ S_{\alpha\alpha}^2\ T_{r\alpha}^{2^T}$$

$$T^2 = \begin{bmatrix} \frac{\partial \tilde{r}_1^2}{\partial \alpha_1^2} & \frac{\partial \tilde{r}_1^2}{\partial \alpha_2^2} & \frac{\partial \tilde{r}_1^2}{\partial \alpha_3^2} \\ \\ \frac{\partial \tilde{r}_2^2}{\partial \alpha_1^2} & \frac{\partial \tilde{r}_2^2}{\partial \alpha_2^2} & \frac{\partial \tilde{r}_2^2}{\partial \alpha_2^2} \end{bmatrix}$$

$$\frac{\partial \tilde{r}_1^2}{\partial \alpha_1^2} = \frac{\partial}{\partial \alpha_1^2}\left(1 + \Delta m_{11}^2\right) = {}^\circ\phi^{2^T}\ {}^\circ M_1^1\ {}^\circ\phi_1^2\ ,\ \frac{\partial \tilde{r}_1^2}{\partial \alpha_2^2} = \frac{\partial \tilde{r}_1^2}{\partial \alpha_3^2} = 0$$

$$\frac{\partial \tilde{r}_2^2}{\partial \alpha_1^2} = 0, \quad \frac{\partial \tilde{r}_2^2}{\partial \alpha_2^2} = \frac{\partial}{\partial \alpha_2^2}\left(1 + \frac{\Delta k_{11}^2}{{}^\circ\lambda_1^2}\right) = \frac{1}{{}^\circ\lambda_1^2}\left({}^\circ\phi_1^{2^T}\, {}^\circ K_2^2 \; {}^\circ\phi_1^2\right)$$

$$\frac{\partial \tilde{r}_2^2}{\partial \alpha_3^2} = \frac{\partial}{\partial \alpha_3^2}\left(1 + \frac{\Delta k_{11}^2}{{}^\circ\lambda_1^2}\right) = \frac{1}{{}^\circ\lambda_1^2}\left({}^\circ\phi_1^{2^T}\, {}^\circ K_3^2 \; {}^\circ\phi_1^2\right)$$

Updated Estimates of $S_{\alpha\alpha}^i$

Substructure 1: $S_{\alpha\alpha}^{1\,*} = \left[T^{1^T} \, S_{\tilde{r}\tilde{r}}^{i^{-1}} \, T^1 + {}^\circ S_{\alpha\alpha}^{1^{-1}}\right]^{-1}$

Substructure 2: $S_{\alpha\alpha}^{2\,*} = \left[T^{2^T} \, S_{\tilde{r}\tilde{r}}^{2^{-1}} \, T^2 + {}^\circ S_{\alpha\alpha}^{2^{-1}}\right]^{-1}$

Compatibility Transformation

$$x^\dagger = \beta x = \begin{bmatrix} [\beta^1] \\ [\beta^2] \\ \cdot \\ \cdot \\ \cdot \\ [\beta^N] \end{bmatrix} x$$

$$m^\dagger \ddot{x}^\dagger + k^\dagger x^\dagger = 0$$

$$\underbrace{\beta^T m^\dagger \beta}_{M} \ddot{x} + \underbrace{\beta^T k^\dagger \beta}_{K} x = 0$$

$$M = \sum_{i=1}^{N} \beta^{i^T} M^i \beta^i = \sum_{i=1}^{N} \beta^{i^T}\left(\circ\overline{M}^i + \alpha_1^i \circ M_1^i\right)\beta^i$$

$$= \left(\beta^{1^T} \circ\overline{M}^1 \beta^1 + \beta^{2^T} \circ\overline{M}^2 \beta^2\right) + \alpha_1^1\left(\beta^{1^T} \circ M_1^1 \beta^1\right) + \alpha_1^2\left(\beta^{2^T} \circ M_1^2 \beta^2\right)$$

$$M = \circ\overline{M} + \alpha_1^1 \circ M_1 + \alpha_1^2 \circ M_2$$

$$= \circ\overline{M} + \alpha_1 \circ M_1 + \alpha_2 \circ M_2$$

Similarly,

$$K = \sum_{i=1}^{N} \beta^{i^T} K^i \beta^i = \sum_{i=1}^{N} \beta^{i^T}\left(\circ\overline{K}^i + \alpha_2^i \circ K_2^i + \alpha_3^i \circ K_3^i\right) \beta^i$$

$$= \left(\beta^{1^T} \circ\overline{K}^1 \beta^1 + \beta^{2^T} \circ\overline{K}^2 \beta^2\right) + \alpha_2^1\left(\beta^{1^T} \circ K_2^1 \beta^1\right) + \alpha_2^2\left(\beta^{2^T} \circ K_2^2 \beta^2\right)$$

$$+ \alpha_3^1\left(\beta^{1^T} \circ K_3^1 \beta^1\right) + \alpha_3^2\left(\beta^{2^T} \circ K_3^2 \beta^2\right)$$

$$K = \circ\overline{K} + \alpha_2^1 \circ K_3 + \alpha_2^2 \circ K_4 + \alpha_3^1 \circ K_5 + \alpha_3^2 \circ K_6$$

$$= \circ\overline{K} + \alpha_3 \circ K_3 + \alpha_4 \circ K_4 + \alpha_5 \circ K_5 + \alpha_6 \circ K_6$$

Covariance Matrix of α, $S^{*}_{\alpha\alpha}$

$$\alpha = \begin{Bmatrix} \{\alpha^1\} \\ \{\alpha^2\} \end{Bmatrix}, \quad S^{*}_{\alpha\alpha} = \left[\begin{array}{c|c} S^{1*}_{\alpha\alpha} & \underline{0} \\ \hline \underline{0} & S^{2*}_{\alpha\alpha} \end{array}\right], \text{ block diagonal matrix}$$

Calculation of Variances of System Eigenvalues and Eigenvectors: S^{*}_{uu}

$$\text{Let } \{u\} = \begin{Bmatrix} \lambda_1 \\ \lambda_2 \\ \{\phi_1\} \\ \{\phi_2\} \end{Bmatrix}, \text{ note that: } \begin{aligned} u_j &= u_j(\alpha_1, \alpha_2, \alpha_3, \alpha_4, \alpha_5, \alpha_6) \\ \phi_j &= \phi_j(\alpha_1, \alpha_2, \alpha_3, \alpha_4, \alpha_5, \alpha_6) \end{aligned}$$

$$\mathrm{Var}(\lambda_1) = E\left[(\lambda_1 - \bar{\lambda}_1)^2\right]$$

$$= \sum_{\gamma=1}^{6} \sum_{\delta=1}^{6} \frac{\partial \lambda_1}{\partial \alpha_\gamma} \frac{\partial \lambda_1}{\partial \alpha_\delta} \underbrace{\mathrm{Cov.}\,(\alpha_\gamma\ \alpha_\delta)}_{\substack{\text{already computed} \\ \text{elements of } S^{*}_{\alpha\alpha}}}$$

Similarly

$$\mathrm{Var}(\lambda_2) = E\left[(\lambda_2 - \bar{\lambda}_2)^2\right]$$

$$= \sum_{\gamma=1}^{6} \sum_{\delta=1}^{6} \frac{\partial \lambda_2}{\partial \alpha_\gamma} \frac{\partial \lambda_2}{\partial \alpha_\delta} \mathrm{Cov.}(\alpha_\gamma\ \alpha_\delta)$$

where

$$\frac{\partial \lambda_i}{\partial \alpha_\gamma} = \phi_i^T \left(\frac{\partial K}{\partial \alpha_\gamma} - \lambda_i \frac{\partial M}{\partial \alpha_\gamma} \right) \phi_i$$

$$\frac{\partial K}{\partial \alpha_\gamma} = 0, \ \gamma=1,$$

$$\frac{\partial K}{\partial \alpha_\gamma} = {}^\circ K_\gamma, \ \gamma=3,4,5,6$$

$$\frac{\partial M}{\partial \alpha_\gamma} = {}^\circ M_\gamma, \ \gamma=1,2$$

$$\frac{\partial M}{\partial \alpha_\gamma} = 0, \ \gamma=3,4,5,6$$

Eigenvector Variances: $\mathrm{Var}(\phi_i) = E\left[(\phi_i - \bar{\phi}_i)(\phi_i - \bar{\phi}_i)^T\right]$

$$= \sum_{\gamma=1}^{6} \sum_{\delta=1}^{6} \frac{\partial \phi_i}{\partial \alpha_\gamma} \frac{\partial \phi_i}{\partial \alpha_\delta} \ \mathrm{Cov.}(\alpha_\gamma \ \alpha_\delta)$$

where,

$$\frac{\partial \phi_i}{\partial \alpha_\gamma} = {}^\circ\phi \frac{\partial \theta_i}{\partial \alpha_\gamma}$$

where the k<u>th</u> element of vector $\frac{\partial \theta_i}{\partial \alpha_\gamma}$ is given by

$$\left(\frac{\partial \theta_i}{\partial \alpha_\gamma}\right)_k = \phi_k^T \left[\frac{1-\delta_{ki}}{\lambda_i - \lambda_k} \left(\frac{\partial K}{\partial \alpha_\gamma} - \lambda_i \frac{\partial M}{\partial \alpha_\gamma} \right) - \frac{\delta_{ki}}{2} \frac{\partial M}{\partial \alpha_\gamma} \right] \phi_i$$

where δ_{ki} is the Kronecker delta and $\frac{1-\delta_{ki}}{\lambda_i - \lambda_k} \equiv 0$ for $i=k$. The expressions for $\frac{\partial K}{\partial \alpha_\gamma}$, $\frac{\partial M}{\partial \alpha_\gamma}$ have already been computed.

NRC FORM 335 (7-77)

U.S. NUCLEAR REGULATORY COMMISSION

BIBLIOGRAPHIC DATA SHEET

1. REPORT NUMBER *(Assigned by DDC)*
NUREG/CR-1560
UCRL-15218

4. TITLE AND SUBTITLE *(Add Volume No., if appropriate)*
Structural Uncertainty in Seismic Risk Analysis·
Seismic Safety Margins Research Program

2. *(Leave blank)*

3. RECIPIENT'S ACCESSION NO.

7. AUTHOR(S)
T. K. Hasselman & S. S. Simonian,
J. H. Wiggins Co. for Lawrence Livermore National Laboratory

5. DATE REPORT COMPLETED

MONTH	YEAR
March	1980

DATE REPORT ISSUED

MONTH	YEAR
October	1980

9. PERFORMING ORGANIZATION NAME AND MAILING ADDRESS *(Include Zip Code)*
Lawrence Livermore National Laboratory
P.O. Box 808
Livermore, California 94550

6. *(Leave blank)*

8. *(Leave blank)*

12. SPONSORING ORGANIZATION NAME AND MAILING ADDRESS *(Include Zip Code)*
Mechanical Engineering Research Branch
Division of Reactor Safety Research
U.S. Nuclear Regula   Commission
Washington, D. C. 20555

10. PROJECT/TASK/WORK UNIT NO.

11. CONTRACT NO.
FIN A0130

13. TYPE OF REPORT
Technical Report

PERIOD COVERED *(Inclusive dates)*

15. SUPPLEMENTARY NOTES

14. *(Leave blank)*

16. ABSTRACT *(200 words or less)*

This report documents the formulation of a methodology for modeling and evaluating the effects of structural uncertainty on predicted modal characteristics of the major structures and substructures of commercial nuclear power plants. The uncertainties are case in the form of normalized random variables which represent the demonstrated ability to predict modal frequencies, damping and modal response amplitudes for broad generic types of structures (steel frame, reinforced concrete and prestressed concrete). Data based on observed differences between predicted and measured structural performance at the member, substructure, and/or major structural system levels are used to quantify uncertainties and thus form the data base for statistical analysis. Proper normalization enables data from non-nuclear structures, e.g., office buildings, to be included in the data base. Numerous alternative methods are defined within the general framework of this methodology.

The report also documents the results of a data survey to identify, classify and evaluate available data for the required data base. A bibliograph of 95 references is included. Deficiencies in the currently identified data base are exposed, and remedial measures suggested. Recommendations are made for implementation of the methodology.

17. KEY WORDS AND DOCUMENT ANALYSIS

17a. DESCRIPTORS

17b. IDENTIFIERS/OPEN-ENDED TERMS

18. AVAILABILITY STATEMENT
Unlimited

19. SECURITY CLASS *(This report)*
UNCLASSIFIED

20. SECURITY CLASS *(This page)*
UNCLASSIFIED

21. NO. OF PAGES

22. PRICE
$

NRC FORM 335 (7-77)