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Regional Relationships Among Earthquake Magnitude Scales

Seismic Safety Margins Research Program

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FOREWORD

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GLOSSARY OF SYMBOLS

A	Ground motion amplitude	
A_0, A_0'	Trace amplitudes for earthquake with $M_L = 0$	
F F_1', F_2'	Compressional-wave amplitude factors	
h		Focal depth
L	Fault rupture length	
M	Estimate of the Richter magnitude, based on signal duration	
$M(\tau)$ $M'(t)$ $M''(F - P)$	Magnitudes based on signal duration	
m_b		Body-wave magnitude
$m_b(L_g)$		Body-wave magnitude based on L_g -wave amplitude
M_L	Local (Richter) magnitude	
M_Q	Corrected body-wave magnitude (Marshall et al., 1979)	
M_S, M_S'	Surface-wave magnitudes	
Q_α	Seismic quality factor	
$S(\Delta, h)$	Empirical calibration function in the expression for m_b	
t	Duration of surface waves	
T	Ground motion period	
T_0	Wave period for magnitude determination (20 s for M_S scale; 1 s for m_b scale)	
V	Rupture velocity	
α	Compressional-wave velocity	
Δ	Distance between earthquake source and receiver	
τ	Signal duration (Eqs. 6 and 7); rise time	

ABSTRACT

The seismic body-wave magnitude m_b of an earthquake is strongly affected by regional variations in the Q structure, composition, and physical state within the earth. Therefore, because of differences in attenuation of P-waves between the western and eastern United States, a problem arises when comparing m_b 's for the two regions. A regional m_b magnitude bias exists which, depending on where the earthquake occurs and where the P-waves are recorded, can lead to magnitude errors as large as one-third unit. There is also a significant difference between m_b and M_L values for earthquakes in the western United States. An empirical link between the m_b of an eastern U.S. earthquake and the M_L of an equivalent western earthquake is given by $M_L = 0.57 + 0.92(m_b)_{\text{East}}$. This result is important when comparing ground motion between the two regions and for choosing a set of real western U.S. earthquake records to represent eastern earthquakes.

EXECUTIVE SUMMARY

Earthquake magnitude is a useful source parameter, which enters into various correlations between the ground motion of an earthquake and its strength. Since C. F. Richter first introduced the concept of magnitude in 1935, a number of different magnitude scales have been developed, each to overcome some deficiencies of earlier scales. As each new scale has been developed, an attempt has been made to correlate it with other scales, but no general correlations are available. Attempts at correlation can be only partially successful, as the various magnitude scales are empirical and are based on rather narrow samplings of the total radiated seismic energy, often at different ends of the frequency spectrum. The magnitude of an earthquake is complexly related to the parameters which control the level and duration of strong ground motion. Most correlations between local ground motion and the distance from the epicenter are based on magnitude. Considering the central role that magnitude plays in observational seismology and the fact that a number of different magnitude scales exist, it is of practical interest to know how the different scales are interrelated and how each varies from one region of the United States to another.

In this report, we examine the various magnitude scales commonly used and the interrelationships among them, primarily as a first step in the development of an earthquake ground motion model for the eastern United States. It is shown that problems exist with all of the magnitude scales being used in the United States. When using regional catalogs, for example, it is often necessary to determine how the reported magnitudes were determined. Often such information is not available, although the potential errors are quite large.

Both the M_S and m_b scales were designed to be universal scales. However, both M_S and m_b magnitudes are often determined beyond the applicable range of the equations used to define the two scales. M_S has an advantage over m_b in this regard, because the M_S scale saturates at a

higher value ($M_S \approx 8.6$). However, the M_S magnitudes are not generally available for moderate to small earthquakes in most earthquake catalogs. The m_b magnitudes are more generally available; however, the m_b scale appears to saturate at $m_b \approx 7.3$. There is also much greater variation in the way m_b is determined. In particular, a significant change in the m_b scale occurred in the early 1960s when the WWSSN was established. This change in instrumentation used to determine m_b values had a significant effect on estimated magnitudes (post-1960 values are lower) and the saturation level of the m_b scale. The older, longer-period instruments recorded larger m_b magnitudes than can be recorded with the WWSSN instruments. In addition, great care must be taken when selecting the m_b magnitudes of western U.S. earthquakes, because the values are often in considerable error due to the fact that they were determined at distances less than 25° and were not properly corrected.

We discuss the need for a regional correction to the m_b magnitudes for shallow earthquakes located in low-Q regions such as the western United States. We find that the m_b 's of western U.S. earthquakes are about 0.3 units smaller than those for similar earthquakes in higher-Q regions such as the eastern United States. We base our conclusion on the following evidence:

- Theoretical considerations of the effect of the difference in Q between the western and eastern United States lead to a correction factor of $\Delta m_b \approx 0.3$.
- The data of Guyton (1964), shown in Fig. 2, provide evidence of lower estimated m_b 's in low-Q areas compared to high-Q areas, for earthquakes occurring elsewhere.
- A comparison of the m_b and M_S values for eastern U.S. earthquakes and western U.S. earthquakes shows that the m_b 's of the latter region are lower by about $\Delta m_b \approx 0.3$ units (Fig. 5).
- The considerable evidence from explosions shows that the western U.S. data give low m_b 's compared to explosions in higher-Q areas.
- Western U.S. earthquakes appear to have smaller m_b 's for the same M_S than do earthquakes elsewhere in the world.

The M_L and the $m_b(L_g)$ scales are regional scales and should not be used (without careful calibration) in other regions. The M_L scale is the most generally available magnitude for western U.S. earthquakes, and $m_b(L_g)$ is the most readily available for eastern U.S. earthquakes. The $m_b(L_g)$ and m_b scales are closely related; however, since m_b is determined from P-waves and $m_b(L_g)$ from surface waves, we can expect some variation between the two due to source mechanism.

We conclude that one might expect to be able to relate m_b to M_L . The relation given by Eq. 15 seems to bear this out, suggesting $M_L \sim 0.92m_b$. We conclude that the relations between m_b and M_S , and between M_S and M_L are more complex, as M_S scales differently from either M_L or m_b . Thus, we find $m_b \sim 0.6M_S$, and Nuttli (1979) found $M_L \sim 0.63M_S$, indicating substantial scaling differences.

The most useful result of this study is the relation given by $M_L = 0.57 + 0.92(m_b)_{\text{East}}$, which provides a link between the m_b of an eastern U.S. earthquake and the corresponding M_L of an equivalent western earthquake. This result is of practical importance when comparing ground motion between the two regions and for choosing a set of real western U.S. records to represent eastern U.S. earthquakes.

INTRODUCTION

Magnitude is still the most directly measurable and useful source parameter of an earthquake. Its scientific and practical uses are numerous: tectonophysicists, for example, use magnitudes to study the occurrence patterns of earthquakes, and structural engineers use them as measures of the strengths of seismic sources. More broadly, magnitude is the source parameter most often used to specify the strength of an earthquake, based on records of local ground motion. This reflects the original concept of the magnitude scale--that earthquakes releasing the same amount of seismic energy should be assigned the same magnitude.

Since Richter (1935) first introduced the concept of earthquake magnitude, a number of different magnitude scales have been developed--generally to overcome deficiencies of earlier scales. As each new scale has been developed, an attempt has been made to correlate it with other scales. However, these attempts can be only partially successful, as the various magnitude scales are empirical and are based on rather narrow samplings of the total radiated seismic energy, often at different ends of the frequency spectrum. In addition, the most common magnitude scales were defined before source models of earthquakes were developed, making it difficult to directly relate the physical parameters of earthquake source models to earthquake magnitude without a number of simplifying assumptions. (It is now generally recognized that the magnitude of an earthquake is complexly related to the parameters that control the level and duration of strong ground motion.) Nevertheless, most correlations between local ground motion and the distance from the epicenter are based on magnitude. Other parameters that might be used to improve this correlation are poorly (if at all) understood; hence, it is simply not yet possible to develop a more refined model.

Considering the central role that magnitude plays in seismology and the fact that a number of different magnitude scales exist, it is useful to know how the scales are interrelated and to understand regional differences that might affect the use of the same scale in different parts of the world. The interrelations of the various magnitude scales have received some study, for

example, by Gutenberg and Richter (1956), Duda and Nuttli (1974), Brazee (1976), and Nuttli (1979). In particular, the relation between the body-wave magnitude m_b and the surface-wave magnitude M_S has been extensively studied because of the usefulness of the relationship in discriminating between explosions and earthquakes (see, for example, Basham, 1969; Marshall and Basham, 1972; and Marshall et al., 1979).

In this paper, we examine the various magnitude scales commonly used and the interrelations among them, primarily as a first step toward developing an earthquake ground motion model for the eastern United States. (The conterminous United States east of the Rocky Mountains is referred to here as the eastern United States; the conterminous United States west of the Rockies is referred to as the western United States.) We are interested in such a model as a supplement to the sparse data available for this region. Only a few recordings of strong ground motion (with peak accelerations over 0.01 g) exist for the eastern United States, and most of these are from a single earthquake ($m_b = 5$), recorded at distances greater than 90 km. Because of this shortage of records, it is not possible for the structural engineer to develop relations between ground motion, earthquake magnitude, and distance, using eastern records alone. Instead, to develop estimates of ground motion in the eastern United States, he must turn to data recorded elsewhere. In doing so, he encounters potential problems; for example, the attenuation of seismic energy is much lower in the eastern United States than in the western United States. Furthermore, earthquakes in the East are not observed to cause surface rupture; hence, the source model might be systematically different in the East than in the West (for example, it might involve a higher stress drop and a smaller area). These are all matters of some concern. A ground motion model will address these matters, but as a first step, we must ensure that earthquake magnitudes are correct, that equal magnitudes represent equal potential for generating strong ground motion.

Earthquake magnitudes are routinely measured throughout the world, but it is a difficult task to know just what a given recorded magnitude represents in terms of the size of the earthquake or the seismic energy it generated. There are a number of reasons for this. First, regional differences in attenuation can--and do--lead to differences in estimates of the magnitude. The effect of these regional attenuation differences, in turn, depends upon the magnitude

scale used, where the earthquake occurred, and when the measurement was taken. Second, the different magnitude scales are not equivalent. They are not related in the same way to the various source parameters of the earthquake (for example, stress drop and fault dimension), nor do equivalent numerical values imply equivalent potential to generate damaging ground motion. Third, the various magnitude scales are often used incorrectly. For example, the M_S scale is based on waves of 20-s period; however, waves of much higher frequency are often used. Other examples include m_b values obtained from data recorded less than about 25° from the epicenter and surface-wave magnitudes obtained at stations less than 20° from the epicenter. Finally, different instruments can lead to different estimates of magnitude unless careful instrument corrections are made.

REGIONAL CORRECTIONS TO MAGNITUDE SCALES

The magnitude of an earthquake determined at any station using any of the various magnitude scales must be corrected before it can be compared to other estimates of the magnitude of the same earthquake. The need for these corrections arises from several factors, including:

- Regional variations in anelastic attenuation of seismic waves.
- Regional variations in the manner the magnitude is determined.
- Influence of local station geology on the incoming seismic waves.

All of these factors are difficult to account for, yet each introduces a potential for significant variations in estimates of the magnitude of a given earthquake. If data are available from enough stations for a given earthquake, the local station factors might well average out. However, the first two factors can introduce important systematic biases. In this study, we are concerned primarily with regional biases, not with local site corrections, despite their significance.

REGIONAL CORRECTIONS TO THE m_b SCALE

Historically, Gutenberg (1945) collected a set of data in the form of amplitude to period ratios (A/T) for P, PP, and SH waves, recorded at teleseismic distances. He found that the ratio remained approximately constant over the range of period T that was measured; thus, he defined the body-wave magnitude m_b as

$$m_b = \log_{10}(A/T) + S(\Delta, h) \quad , \quad (1)$$

where $S(\Delta, h)$ is an empirically determined term that accounts for the source-receiver distance Δ and the focal depth h . Thus, the term $S(\Delta, h)$ can be considered as the calibration function. The ratio A/T is the ground motion in nanometers per second. Contour curves of $S(\Delta, h)$ for P, PP, and SH waves

are given by Richter (1958, pp. 688-689). Evernden (1967) found it necessary to adjust the distance-depth correction of Gutenberg and Richter at distances of less than 20° .

Since the World-Wide Standard Seismograph Network (WWSSN) was installed in the early 1960s, the body-wave magnitude has been determined almost exclusively from the vertical component of the P-wave ground motion at a period of approximately one second. Before 1960 longer-period instruments were used to determine m_b . Experience has shown that estimates of body-wave magnitude based on records from long-period instruments are about 0.3 to 0.6 units higher than estimates based on the short-period instrument used in the WWSSN (see, for example, Romney, 1964; and Geller and Kanamori, 1977). In addition, m_b values for a single earthquake, determined from body waves at different seismograph stations, commonly vary by 0.5 or more (see Guyton, 1964), despite corrections for differences in epicentral distance among the stations. This variation, which corresponds to differences in amplitude of a factor of three or more, is generally attributed to azimuthal, instrumental, and geological differences among the stations. The corrections for the azimuthal and instrumental variations are routinely made. Corrections for the geological and other site-specific effects are more difficult.

Among the geological factors that affect estimates of m_b are regional variations in the Q structure, composition, and physical state within the earth. Studies of the structure and geophysical properties of the crust and upper mantle since the mid-1960s indicate distinct differences in the upper mantle between tectonic regions. For example, we know that within either the eastern or western United States the regional differences are not very pronounced, but that between these two parts of the country the nature and vertical extent of the upper mantle low-velocity zones clearly differ. These differences bear directly on earthquake magnitude determinations. We can, for example, demonstrate a reduction in the m_b values produced by the passage of a compressional wave through a zone of low Q , such as that in the western United States.

Suppose a layer in the earth has constant values for both velocity and Q . The compressional-wave amplitude for a vertically incident wave traversing that layer is reduced by the amplitude factor $F = \exp(-\pi fx/Q_\alpha)$, where f is the frequency, x is the thickness of the layer, α is the compressional-wave velocity, and Q_α is the seismic quality factor. If x is 150 km, f is 1 Hz,

and α is 7.6 to 8.2 km/s, then the factor F can be calculated for any value of Q_α . The results of a few sample calculations are presented in Table 1. It is seen that a 150-km-thick layer having a Q_α value of about 500 or more will cause only a very small reduction in amplitude. On the other hand, a Q_α value of 20 would have a drastic effect on compressional-wave amplitudes.

TABLE 1. Amplitude factors for a compressional wave traversing a 150-km-thick layer of constant velocity and constant Q , for different values of Q .

Q	Amplitude factor (F)	
	for $\alpha = 7.6$ km/s	for $\alpha = 8.2$ km/s
20	0.05	0.06
50	0.29	0.32
100	0.54	0.57
200	0.73	0.75
300	0.81	0.83
500	0.88	0.89
1000	0.94	0.94

To see the effect such amplitude variations might have on estimates of M_b , let F_1' and F_2' be the observed compressional-wave amplitudes leading to magnitude determinations $(m_b)_1$ and $(m_b)_2$, respectively, in accordance with Eq. 1. The difference between these determinations can be expressed as

$$\Delta m_b = |(m_b)_2 - (m_b)_1|, \quad (2)$$

or

$$\Delta m_b = \log_{10} (F_2' / F_1'), \quad (3)$$

where the subscript 2 has now been chosen to represent the high- Q zone.

If the amplitude differences are produced by different Q_α values in a 150-km-thick layer, then we can take the values of F_2' and F_1' from Table 1, corresponding, say, to the eastern United States (shield and platform regions) and the western United States (tectonically active regions), respectively.

From Eq. 3, we calculate expected differences in m_b values produced by compressional waves passing through the two regions. Figure 1 illustrates our results for various combinations of Q_α values which might represent the eastern and western United States. The ratio of the amplitude factors, F_1'/F_2' , is plotted in Fig. 1a, and the magnitude bias Δm_b , as a function of various Q_α values in the 150-km layer, is given in Fig. 1b. The range of Q_α values for the "low-Q zone" in Fig. 1 might be taken as representative for the upper mantle beneath western North America (particularly the Basin and Range Province). The Q_α values for the "high-Q zone" are likewise representative of the East. Assuming a 150-km-thick path segment, therefore, we would expect m_b values for events in the western United States to be a few tenths of a magnitude unit lower than equivalent events in the eastern United States. If two "identical" earthquakes occurred--one located in the eastern United States and one located in the western United States--the m_b of the earthquake in the East would appear to be larger than the m_b of the western U.S. earthquake by Δm_b . Thus, in choosing earthquakes in the West to compare to the earthquakes in the East, we should select those earthquakes with

$$(m_b)_{\text{West}} \approx (m_b)_{\text{East}} - \Delta m_b \quad . \quad (4)$$

In general, for equivalent seismic sources, it appears that the m_b of an eastern U.S. earthquake differs from that of a western earthquake by $\Delta m_b \approx 1/3$. Attenuation is higher in the western United States than in the older, stable regions of the East, and this magnitude bias appears to be well correlated with tectonic structure and lateral variations in attenuation characteristics. Typically, the upper mantle Q_α in the East is about 1000 (Solomon and Toksoz, 1970) and that in the West is about 100 (Solomon, 1972). From Fig. 1b, the magnitude bias expected from this attenuation difference is about 1/3.

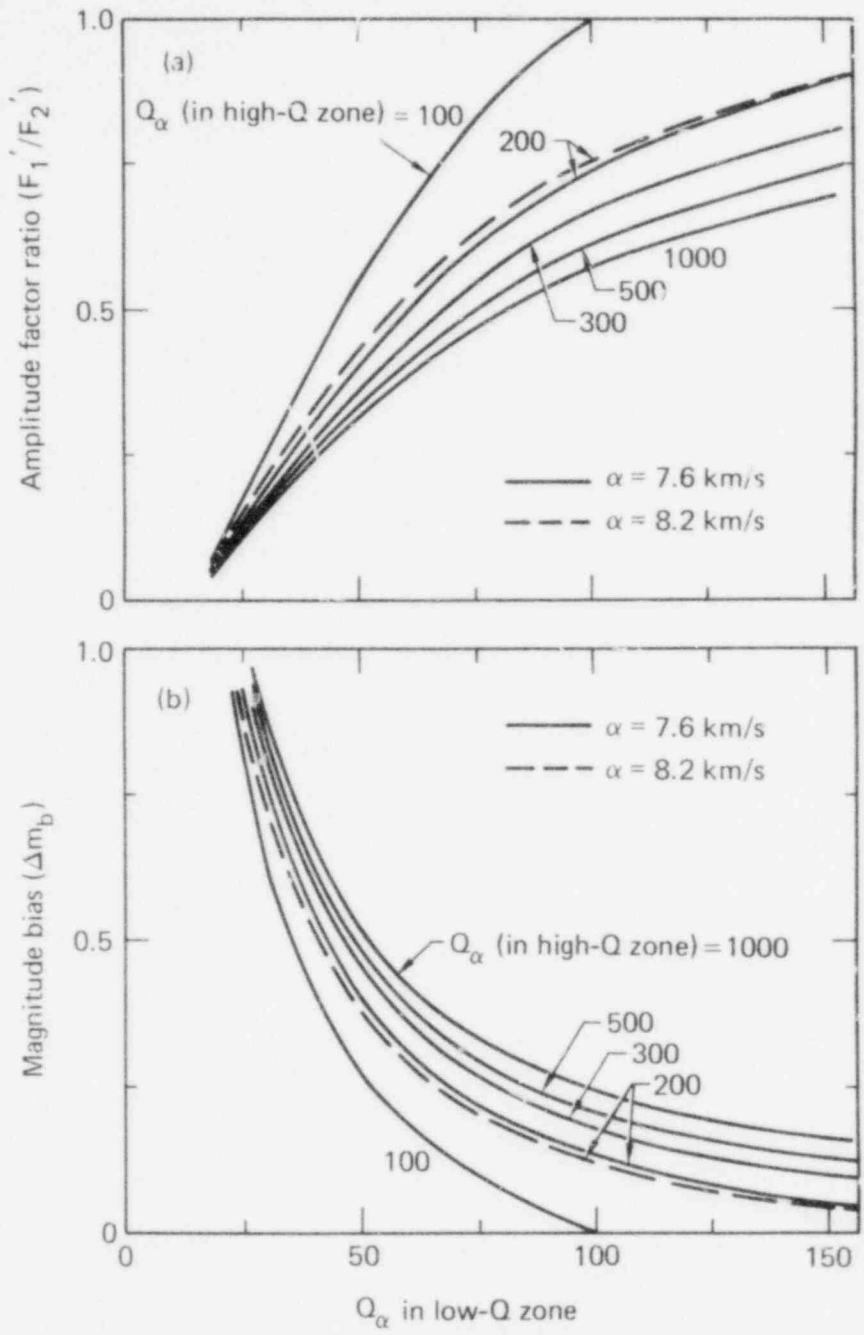


FIG. 1. Amplitude factor ratio (a) and the magnitude bias (b) as functions of various Q_α values in a 150-km-thick layer. The effect of different α values (7.6 vs 8.2 km/s) is shown for the case of Q_α (in the high-Q zone) = 200.

Many studies of the amplitudes and frequencies of body waves have detected anomalously high seismic-wave absorption in the upper mantle in the western United States (see, for example, Guyton, 1964; Molnar and Oliver, 1969; Solomon and Toksoz, 1970; Evernden and Clark, 1970; and Ward and Toksoz, 1971). These studies consistently report higher attenuation in the West than in the East, sometimes by a factor of three. There also appears to be a correlation between the unusual attenuation and observed patterns of P- and S-wave slowing (late arrival times), upper-mantle electrical conductivity, and heat flow, as noted by Der et al. (1975).

Marshall et al. (1979) formulated a new definition of body-wave magnitude, designated m_Q to distinguish it from the standard body-wave magnitude m_b :

$$m_Q = m_b + RC + SC + DC \quad ,$$

where RC is the correction for attenuation in the upper mantle at the receiver end of the wave path, SC is the similar correction for attenuation near the source of the disturbance, and DC is the correction for the depth of the source. Separating the correction into parts emphasizes that the values of the correction terms can be different in different parts of the world and for different depths of the seismic event source. The attenuation is assumed to be greatest in the upper mantle and negligible over most of the deep portion of the wave path--hence the separate corrections at the source and receiver ends of the wave path. The aim of this m_Q scale is to remove any regional bias in standard magnitude measurements of distant seismic events, including underground explosions.

Of particular interest to our work is the work of Guyton (1964). He has performed an important analysis of the maximum peak-to-peak amplitude in the first four cycles of the P-waves, and of the period of the P-waves. Using the U.S. Coast and Geodetic Survey (USCGS) epicentral data, m_b values were calculated in the usual way. For earthquakes recorded at 26 different stations within the conterminous United States, calculated magnitudes ranged from 4.0 to 6.8, with 60% of these earthquakes between 5.0 and 6.0. Only

well-recorded earthquakes within the distance range 20° to 100° were selected to avoid complexity from crustal and core phases. Events not well-recorded at seven or more stations were discarded in Guyton's study. Guyton adopted the Wichita Mountains, Oklahoma, station (WMO) as a stable standard for comparison. Table 2, after Guyton (1964), gives for each station the number of observations, the average amplitude ratio to the WMO station, the standard error of the mean (σ/\sqrt{n}) in the determination of the ratio, and the equivalent deviation in magnitude units.

Figure 2, again after Guyton (1964), shows the geographic distribution of the recording stations, with their average magnitude deviations partially contoured to distinguish among the high positive values, low positive values, and negative values of Δm_b . All stations with negative deviations are located along the Pacific border and in the southwestern United States. Stations with low positive deviations are located in the Pacific Northwest, the Rocky Mountains, and the Wichita-Quachita Mountains of the south-central United States. High positive deviations occur at stations on the Colorado Plateau and east of the Rocky Mountains. What is observed in Fig. 2 is that Δm_b is not related to either distance or azimuth from the epicentral areas where most of the earthquakes originated. The recorded amplitudes of earthquakes are related instead to the regional geological setting of the recording station.

REGIONAL CORRECTIONS TO THE M_L SCALE

As originally defined by Richter (1935), the local magnitudes M_L were calculated from amplitudes recorded on Wood-Anderson torsion instruments and were calculated only for southern California earthquakes. To assign absolute values of magnitudes, Richter arbitrarily defined the zero-magnitude earthquake to be one for which the maximum trace amplitude at a distance of 100 km is 1 μm . If $A_0(\Delta)$ expresses the dependence of the maximum trace amplitude A_0 of the zero-magnitude earthquake on epicentral distance Δ , then M_L is given by

$$M_L = \log_{10} A(\Delta) - \log_{10} A_0(\Delta) \quad , \quad (5)$$

TABLE 2. Average amplitude ratio to WMO and equivalent deviation in magnitude, for various recording stations (after Guyton, 1964).

Station	Number of earthquakes	Average amplitude ratio	Standard error of the mean	Average deviation in magnitude determination
AR WS	53	1.56	0.38	0.19
BL WV	22	3.61	1.37	0.56
CP CL	112	0.74	0.07	-0.13
DH NY	90	2.30	0.56	0.36
DR CO	41	1.21	0.28	0.08
FM UT	112	1.14	0.14	0.06
FS AZ	28	1.32	0.16	0.12
GV TX	21	2.24	0.37	0.35
HB OK	29	2.02	0.32	0.30
HL ID	40	1.12	0.31	0.05
HN ME	18	1.55	0.28	0.19
KN UT	41	1.57	0.27	0.20
LC NM	117	0.81	0.08	-0.09
MM TN	28	1.60	0.24	0.20
MN NV	107	0.90	0.12	-0.05
MP AR	29	1.06	0.25	0.02
MV CL	113	0.69	0.08	-0.16
NG WS	33	1.50	0.22	0.18
PM WY	30	1.00	0.13	0
PT OR	33	1.23	0.31	0.09
SE MN	30	2.62	0.51	0.42
SJ TX	94	2.07	0.25	0.32
SS TX	35	1.32	0.23	0.12
TP CL	105	0.96	0.12	-0.02
WI NV	119	0.96	0.09	-0.02
WN SD	29	3.61	0.64	0.56

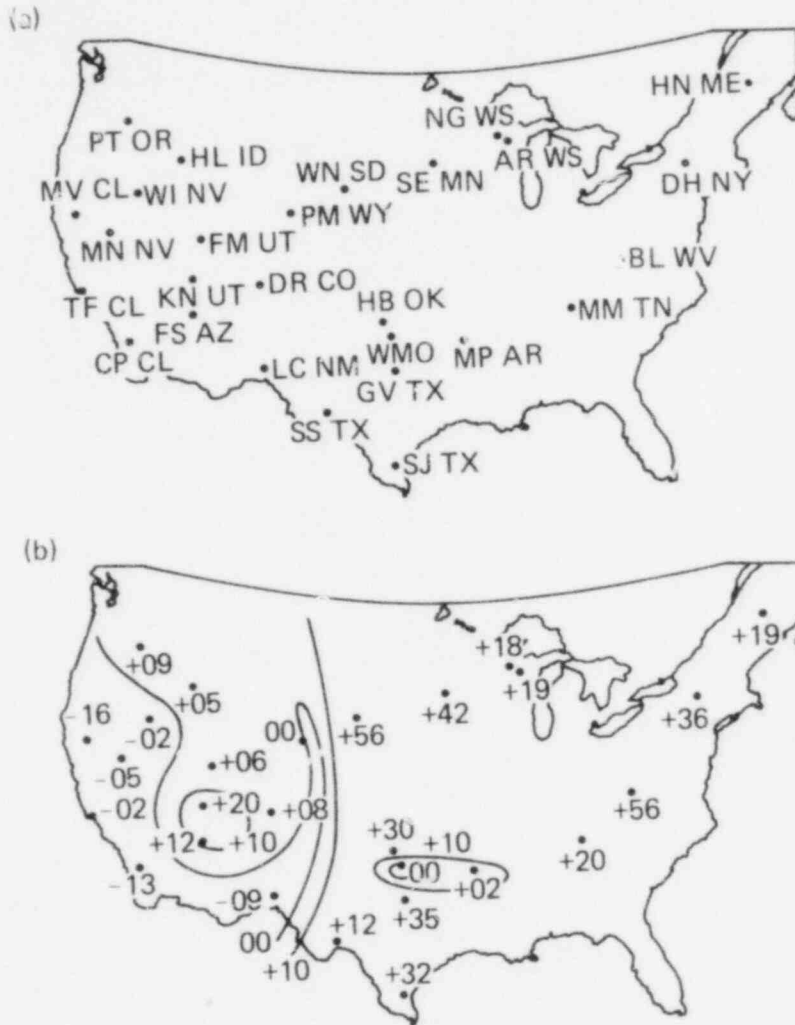


FIG. 2. Geographic distribution of the station sites (a) and their average magnitude deviations (b) from the reference WMO station (in hundredths of the m_b unit). (After Guyton, 1964.)

where A is the maximum amplitude on the Wood-Anderson seismogram for an earthquake of at a distance Δ . The values of A and A_0 are expressed in millimeters. A table of $(-\log_{10} A_0)$ as a function of distance Δ (in kilometers) is conveniently found in the text by Richter (1958, p. 342).

Certain consequences follow from this definition of the local magnitude. By specifying the Wood-Anderson torsion seismograph, with a natural period of 0.8 s and a damping factor of 0.8, Richter effectively fixed the passband of the ground motion to be considered. For example, such a seismograph has a magnification of 2800 for periods of 0 to about 0.5 s, and on a log-log plot of magnification vs period, the curve falls off rapidly with a slope of -2 at periods greater than about 0.5 s. In the passband of 0 to 0.5 s, the kind of wave motion (that is, P, S, and surface waves) that produces the maximum trace amplitude changes with epicentral distance in some unknown manner that depends on focal depth, focal mechanism, and the mantle-crust structure. The falloff of amplitude with epicentral distance as specified originally by the reference or zero-magnitude earthquake applies only to southern California.

Wide variations in the amplitude vs distance relations over the surface of the earth's crust are expected, because the attenuation of seismic waves having periods between 0 and 0.5 s is determined by the absorptive properties of the earth and by the structure and physical properties of the upper layers of the earth's crust. Thus, it is necessary to restrict the use of the M_L scale to California (Richter, 1958). This has not always been done, and the attenuation law developed by Richter has been used in other geological settings. Such use can lead to significant errors; for example, Nuttli (1973) showed how much lower the attenuation of seismic energy is in the eastern United States compared to the western United States. However, regional corrections for extending its use to other geologic settings have not been developed for the M_L scale.

In recent years, many local seismic networks have been set up to study small earthquakes and microearthquakes. Many of these networks use high-gain, short-period vertical instruments; they do not include any Wood-Anderson torsion seismometers. The use of such high-gain instruments allows wave amplitudes to be measured for only the smallest local earthquakes; thus, a different approach must be employed to estimate magnitudes.

Earthquake magnitude can be estimated independently of signal amplitude by using an empirical formula of the general form

$$M(T) = a_1 + a_2 \log_{10} T + a_3 \Delta + a_4 h , \quad (6)$$

where τ is the signal duration in seconds, Δ is the epicentral distance in kilometers, h is the focal depth in kilometers, and a_1 through a_4 are empirical constants. The idea to use the signal duration originates from work of Bisztricsany (1958). He determined the relationship between earthquakes of magnitude 5 to 8 and the duration of the surface waves at epicentral distances between 4° and 160° . He suggested that

$$M'(t) = 2.92 + 2.25 \log_{10} t - 0.001\Delta^\circ ,$$

where M' is the magnitude (between 5 and 8), t is the duration of surface waves, and Δ° is the epicentral distance in degrees.

Solov'ev (1965) applied this technique in the study of the seismicity of Sakhalin Island, USSR, but used the total duration instead of the duration of the surface wave. Tsumura (1967) studied in detail the determination of earthquake magnitude from the total duration of oscillation in seconds ($F - P$), for local earthquakes recorded by the Wakayama, Japan, micro-earthquake network. He found an empirical formula

$$M''(F - P) = -2.53 + 2.85 \log_{10} (F - P) + 0.0014 \Delta ,$$

where M'' is a new magnitude in the range 3 to 5, as determined by the Japan Meteorological Agency, and Δ is the epicentral distance in kilometers. This M'' is the magnitude used today in Japan for local earthquakes.

An empirical formula for estimating the Richter magnitude of local earthquakes, using signal durations, has now been established for the National Center for Earthquake Research by Lee et al. (1972):

$$M = -0.87 + 2.00 \log_{10} \tau + 0.0035\Delta , \quad (7)$$

where M is an estimate of Richter magnitude, τ is the signal duration in seconds, and Δ is the epicentral distance in kilometers. Lee et al. established this M scale by analyzing the relation among Richter magnitude, signal duration, and epicentral distance for 351 earthquakes in central

California. This magnitude scale has been widely used since 1972. Lee et al. found that the M_L of an earthquake can be estimated by Eq. 7 to within about $\pm 1/4$ unit.

It should be noted that the signal duration in Eq. 7 is defined from the P-arrival to the point in the coda where the largest peak-to-peak amplitude on a Geotech Model 6585 film viewer (20x magnification) is less than 1 cm. Thus, it is not possible to compare Eq. 7 to, say, the results of Tsumura (1967), because of instrument differences. The paper by Aki and Chouet (1975) shows that one would expect a strong regional influence on the coda length-- particularly as defined by Lee et al. (1972). Thus, local magnitude scales defined by the use of the coda length must be calibrated for each region. Such work is just getting underway.

REGIONAL CORRECTIONS TO THE M_S SCALE

In 1936 Gutenberg and Richter (1936) extended their method for determining local magnitude M_L to earthquakes occurring anywhere in the world. The distribution of earthquakes and seismograph stations was such that the only practical way of accomplishing this aim was to make use of seismograms recorded at teleseismic distances (greater than about 2000 km). At these distances, the most prominent seismic waves on the broad passband seismograms are the surface waves, especially those with a period of about 20 s. This period corresponds approximately to a group velocity minimum (thus an amplitude maximum) of Love and Rayleigh waves.

For selected earthquakes, Gutenberg and Richter (1936) found the amplitude variation of the maximum of the horizontal component of 20-s-period surface-wave motion, then determined amplitude as a function of epicentral distance. To fix the scale absolutely, they needed to set a reference level for the curve, which they obtained from California earthquakes whose local magnitudes were determined independently. The surface-wave magnitude M_S was then defined as

$$M_S = \log_{10} A(\Delta) - \log_{10} A_0(\Delta) , \quad (8)$$

where A_0 is the amplitude associated with an $M_S = 0$ earthquake. Values of A_0 (in micrometers) were determined empirically by Gutenberg and Richter; values for $(-\log_{10} A_0)$ have been tabulated by Richter (1958, p. 346) as a function of distance Δ (in degrees).

The M_S scale is particularly useful for the study of large earthquakes (say, greater than $m_b = 6$). However, the scale is limited in its application to earthquakes which generate measurable 20-s-period surface waves; this in turn implies shallow earthquakes as a rule. Nonetheless, the use of the M_S scale is advantageous, because there is little lateral variation in the attenuation of 20-s-period surface waves anywhere in the world (Richter, 1958). Equation 8 is therefore universally valid.

Vanek et al. (1962) have proposed a new M_S magnitude scale:

$$M_S' = \log_{10} \left(\frac{A}{T} \right)_{\text{Max}} + 1.66 \log_{10} \Delta^0 + 3.3 \quad (9)$$

Equation 9, the so-called "Prague formula," employs a geographic average of various distance-normalizing terms. It also incorporates the period to account for continental propagation paths for which the maximum trace amplitude, measured by broad-band instruments, might not occur at a period near 20 s. Thus, the Prague formula can be used over any epicentral distance range, and the magnitude determination is not restricted to a fixed period, as it is in the case of Eq. 8.

It was observed by Marshall and Basham (1972) that their data, obtained at distances greater than about 25° , gave values for M_S' close to M_S , generally within 0.2 magnitude units: $M_S' \approx M_S + 0.2$. However, these authors also noted that M_S' values computed from measurements at less than 25° and at periods shorter than 20 s are larger than M_S' values computed from distant observations ($\Delta > 25^\circ$) at periods near 20 s. For example, Basham (1969) determined the magnitude of the Greeley event, a Nevada Test Site (NTS) underground explosion, as $M_S' = 6.1$, using regional data and Eq. 9. This may be compared with $M_S = 5.1$, the value estimated from long-range observations using Eq. 8.

Problems of this nature have led to considerable uncertainty and confusion in observational seismology, particularly in attempts to compare small events observed only at short distances with larger events observed at greater distances. In pursuit of a solution to these problems, Marshall and Basham (1972) proposed that for distances up to 25° the distance correction term, $\log_{10} A_o(\Delta)$ in Eq. 8, be replaced by $0.8 \log_{10} A_o'(\Delta)$, and that at large teleseismic distances it be the same as Gutenberg's original term, which in turn is very close to the correction used in the Prague formula by Vanek et al. (1962). The $A_o'(\Delta)$ baseline level in the distance range between 0° and 25° is adjusted so that magnitude determinations from the new formula will give results essentially the same as those obtained when M_S (due to Eq. 8) or M_S' (due to Eq. 9) is calculated from seismograms recorded at large epicentral distances.

The main point of the above discussion is that the reported M_S values can be in considerable error for smaller earthquakes, unless care is taken to properly correct for the travel path and frequency of the wave used to compute M_S . No further discussion of the complex corrections required for M_S is given in this report, because M_S does not play a significant role in our attempts to build a ground motion model.

REGIONAL CORRECTIONS TO THE $m_b(L_g)$ SCALE

Nuttli (1973) developed an m_b scale based on the measured amplitude of L_g -waves. The L_g -waves are generated in the continental crust at a characteristic velocity of 3.54 km/s, and their arrivals are sharply recorded in seismograms. These L_g -waves travel over long continental paths with relatively little attenuation, but they are abruptly cut off when the path has even a small oceanic segment. The L_g -waves are channel waves in the crust which are superimposed on long-period surface waves, and they are more closely related to m_b than to M_S . Nuttli (1973), using measured L_g -waves in the central United States, computed $m_b(L_g)$ magnitudes from the wave amplitudes as follows:

$$m_b(L_g) = 3.75 + 0.90 \log_{10} \Delta + \log_{10} (A/T), \text{ for } 0.5^\circ \leq \Delta \leq 4^\circ, \quad (10)$$

and

$$m_b(L_g) = 3.30 + 1.65 \log_{10} \Delta + \log_{10} (A/T), \text{ for } 4^\circ \leq \Delta \leq 30^\circ, \quad (11)$$

where A is the maximum ground amplitude (zero to peak, in micrometers) of the 1-s L_g -waves and Δ is the epicentral distance in degrees. It appears from the work of Street et al. (1975) that the $m_b(L_g)$ magnitudes calculated from Eqs. 10 and 11 are consistent with the standard m_b values for earthquakes as small as $m_b = 3.0$.

Street (1976) investigated the applicability of Eq. 11 to the northeastern United States. He concluded that the anelastic attenuation was somewhat higher in the Northeast (about 0.11 deg^{-1} compared to about 0.07 deg^{-1} in the central United States); however, the effect on the $m_b(L_g)$ magnitude, as determined by Eq. 11, was not significant (less than 0.1 units), provided that the use of Eq. 11 was restricted to distances between 4° and 16° .

Bollinger (1979) concluded that Eq. 11 could also be used to estimate earthquake magnitudes in the southeastern United States, provided that epicentral distances are limited to 2000 km.

RELATIONS BETWEEN MAGNITUDE SCALES

The magnitude of earthquakes in the eastern United States are generally measured in terms of m_b and $m_b(L_g)$. For a few large earthquakes, M_S is also available. For the western United States, M_L is generally available; m_b and M_S are sometimes available. In other regions of the world, m_b and M_S are sometimes available, but in some cases, only M_L is available. As discussed above, M_L values are of little use, because they have not been corrected for differences in attenuation between the western United States and other regions.

Chinnery (1979) examined a number of regional catalogs and concluded that only in the last few years have attempts been made to measure M_S on a routine basis. The m_b catalog, on the other hand, was more often complete. Because of the wide availability of m_b values, we focus here primarily on the relation of m_b to the other magnitude scales.

Before obtaining empirical correlations, however, it is of some use to examine how m_b , M_S , and M_L relate to other earthquake source parameters from a theoretical point of view. Unfortunately, as pointed out by Kanamori and Anderson (1975), magnitude is the most difficult earthquake source parameter to relate to other important source characteristics, such as strain energy release, fault offset, stress drop, source dimension, and seismic moment. These other parameters can be related to one another fairly easily with rather simple models; however, their relationships to magnitude require a spectral description of the seismic source. Such a description, in turn, demands a complete time and space history of the faulting or stress-release mechanism. For example, the various magnitudes are calculated from seismic-wave amplitudes (all but M_L at a given period), while the seismic moment is calculated from the limit of the long-period spectral level. M_S , m_b , and $m_b(L_g)$ are somewhat simpler to relate to the other source parameters than is M_L , because they are related to a given wave type at a given period. M_L on the other hand is proportional to the maximum amplitude read on a Wood-Anderson torsion seismometer, irrespective of wave type or its period.

Using a simplified dynamic source model based on the work of Haskell (1964), Kanamori and Anderson (1975) argued that

$$M_S \text{ or } m_b \sim \begin{cases} \log L^2, & \text{if } \tau < \frac{T_0}{\pi} \text{ and } \frac{L}{V} > \frac{T_0}{\pi}, \\ \log L^2, & \text{if } \tau > \frac{T_0}{\pi} \text{ and } \frac{L}{V} < \frac{T_0}{\pi}, \\ \log L, & \text{if } \tau > \frac{T_0}{\pi} \text{ and } \frac{L}{V} > \frac{T_0}{\pi}, \\ \log L^3, & \text{if } \tau < \frac{T_0}{\pi} \text{ and } \frac{L}{V} < \frac{T_0}{\pi}, \end{cases}$$

where

- L = fault rupture length,
- τ = rise time,
- V = rupture velocity (about 3 km/s),
- T_0 = 20 s for M_S and 1 s for m_b .

For the m_b scale, the critical value for the rise time is $\tau = \frac{1}{\pi} \approx 0.3$ s, and the critical value for the fault rupture length is $L \approx VT_0/\pi \approx (3)(1)/\pi \approx 1$ km. We might expect most earthquakes of interest to have $L > 1$ km and $\tau > 0.3$ s; for this class of earthquakes,

$$m_b \sim \log L .$$

For the M_S scale, the critical values of τ and L are, respectively, about 6 s and $VT_0/\pi \approx 60$, $\tau \approx 20$ km. There are not many values of τ available; however, among the data given by Kanamori and Anderson (1975), all but one value of τ fell between 0.7 s and 6 s. Therefore, for earthquakes with $L < 20$ km ($M_S < 6.5$), we see that

$$M_S \sim \log L^3 ;$$

whereas, for events with $L > 20$ km ($M_S > 6.5$),

$$M_S \sim \log L^2 .$$

For very large earthquakes with slow rise times,

$$M_S \sim \log L .$$

(The value of $M_S \approx 6.5$ is highly approximate, as there is a considerable scatter in the relation between L and magnitude.) Therefore, for earthquakes which are similar to explosive sources, that is, which have small rupture areas and very rapid rise times, both M_S and m_b vary as $\log L^3$.

It should also be noted that both m_b and M_S are strongly influenced by the source mechanism of the earthquake (von Seggern, 1970). However, the m_b 's and M_S 's are influenced in different ways; for example, dip slip, relative to strike slip, enhances m_b by as much as 0.8 units while suppressing M_S by as much as 0.3 units.

Because M_L is not restricted to a particular wave type or period, it is somewhat more difficult to develop relations between M_L and the other earthquake source parameters, or between M_L and M_S or m_b . Gutenberg and Richter (1956) provided estimates of the periods of the waves whose peak amplitudes have been used to determine M_L values. The estimates range from 0.1 to 1.7 s and average about 0.3 s, equal to about 3 Hz. Thus, for M_L the critical values of rise time and rupture length are approximately 0.1 s and 0.3 km, respectively. Thus, we might expect $M_L \sim \log L$, even for small earthquakes where $m_b \sim \log L^3$. From these results, we might expect to see a reasonable relationship between M_L and m_b , except for relatively small earthquakes. We might also expect some scatter in the relation (even in the range where both magnitudes scale as $\log L$), because m_b might be somewhat more influenced by mechanism than M_L . Because M_L is not restricted to a particular wave type at a particular period, it is very difficult to determine what influence source mechanism might have on it. In addition, m_b might be

influenced somewhat by rise time, which is related to the effective stress. Events with very high stress drops and short rise times could influence the relation between M_L and m_b . (Recall that the critical τ for m_b is 0.3 s.)

From this brief review, we see that a reasonable basis exists for relating m_b to M_L . The relation between m_b and M_S or between M_L and M_S should, however, be much more complex, because M_S scales as $\log L^3$ over much of the range of interest. Only for very small earthquakes will M_L and m_b scale as $\log L^3$. It would appear that the $m_b - M_S$ and $M_L - M_S$ differences are source related. Therefore, attempting to use an empirical relation between m_b and M_S to relate earthquakes in different regions of the world (or for that matter in the same region) would not be a very good approach. Nor would it appear valid to go from M_S in one region to M_S in the West, and finally to M_L . On the other hand, it appears reasonably valid to go from m_b in one region to m_b in the western United States, then to M_L .

For the above reasons, we have not investigated the relation between M_L and M_S . (Nuttli [1979], however, provides an equation relating M_L and M_S). We do examine in some detail the relation between m_b and M_S , because regional differences between M_S and m_b for explosions provide evidence for the necessary regional correction to the m_b scale.

RELATION BETWEEN m_b AND M_L

In this section, we want to relate m_b to M_L . In particular, we want to obtain a relation which provides an appropriate M_L value for a western U.S. earthquake, given an m_b value for an eastern U.S. earthquake. (The correlation of m_b values for earthquakes occurring in two different regions by accounting for differences in regional attenuation has already been discussed.) The problem of interrelating the two magnitude scales is difficult, because, as discussed above, the two scales were empirically and independently derived, using different wave types. In addition to these theoretical problems, consideration must be given to the fact that the m_b

scale effectively changed in the early 1960s. This change, as noted earlier, occurred because of the change in the instruments used to determine m_b . These changes resulted in substantially lower m_b values than those reported by Gutenberg and Richter (1956) for the same M_S or M_L . Thus, it is necessary either to restrict the m_b data to those gathered since 1963 or to recompute the m_b 's calculated prior to 1963, applying appropriate instrument corrections.

Finally, it should be noted that determinations of m_b 's for California earthquakes of $m_b \leq 5.5$ have special problems. Ideally, m_b should only be determined from P-waves at distances Δ of greater than 25° (2800 km). At shorter distances, regional variations in upper-mantle structure affect the body-wave magnitude calibration function. The Gutenberg-Richter (1956) m_b -calibration curve, which is used by the National Earthquake Information Services (NEIS), is particularly poor at these distances (Veith and Clawson, 1972; Nuttli, 1980). For California earthquakes of $m_b \leq 5.5$, most of the P-wave amplitude data used by NEIS come from U.S. stations at distances less than 25° , and thus their m_b values for these earthquakes are suspect. (This point has been made in personal communications from O. W. Nuttli.) This was the reason Nuttli et al. (1979) recomputed m_b values for all the earthquakes they used in their study, by going back to film copies of seismograms. Earthquakes of $m_b \geq 5.5$ are less of a problem, as their P-waves may be recorded throughout the world by a large number of stations at distances greater than 25° ; however, depending upon the stations used, the NEIS tape can be significantly in error.

To get a feel for the difference between the NEIS tape and the corrected m_b values, we have plotted in Fig. 3 m_b vs M_L for western U.S. earthquakes, showing both the NEIS m_b values and the values recomputed by Nuttli et al. (1979). For the NEIS data, we selected only the earthquakes that occurred in California (so that we have meaningful M_L 's) between 1963 and 1977 and that have $M_L \geq 4.5$. This lower M_L limit was chosen to insure that the M_L 's were recorded at a number of stations (to help reduce station bias) and to insure that both M_L and m_b scale as $\log L$. As our data base, we used the NEIS m_b values and the Berkeley and Pasadena M_L values. The M_L values are, therefore, self-consistent, and none were calculated from

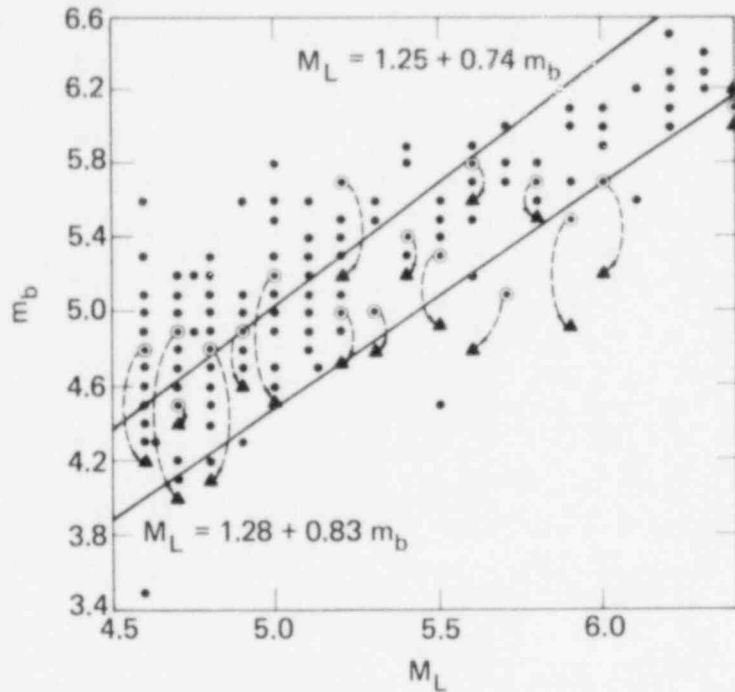


FIG. 3. Relationship between m_b and M_L for western U.S. earthquakes between 1963 and 1977 ($M_L \geq 4.5$). The solid circles are the NEIS m_b values. The triangles are due to Nuttli et al. (1979), who recomputed values from the NEIS data tapes. The straight lines are discussed in the text.

signal durations alone. Nuttli et al. (1979) recomputed the m_b 's for only a few of the earthquakes on the NEIS data tape. The changes in the m_b 's are shown in Fig. 3. Basham (1969) found similar differences when he compared Canadian m_b 's with USCGS values for m_b . He also noted a large scatter in the USCGS values.

A least-squares line, assuming that the relationship between m_b and M_L is linear, was fit to the NEIS data. We obtained the following relations:

$$M_L = 1.25 + 0.74m_b, \quad \text{where } m_b \text{ is error-free,} \quad (12a)$$

and

$$m_b = 0.30 + 0.95M_L, \quad \text{where } M_L \text{ is error-free.} \quad (12b)$$

It is more likely that neither m_b nor M_L is error-free; for such a case, the middle line drawn between the two curves given by Eqs. 12a and 12b may best represent the true relationship. The equation of this curve is

$$M_L = 0.54 + 0.88m_b \quad . \quad (13)$$

Using the data of Nuttli et al. only, we found

$$M_L = 1.28 + 0.83m_b, \quad \text{where } m_b \text{ is error-free} \quad , \quad (14a)$$

and

$$m_b = 0.99M_L - 0.39, \quad \text{where } M_L \text{ is error-free} \quad . \quad (14b)$$

Assuming approximately equal errors in m_b and M_L values, then neither Eq. 14a nor Eq. 14b is correct. The actual relation lies somewhere within this scatter of data. We approximate the relation between M_L and m_b by

$$M_L = 0.85 + 0.92m_b \quad . \quad (15)$$

Equations 12a and 14a are shown in Fig. 3; Eqs. 12a through 15 are compared in Fig. 4.

A perusal of these limited data and the regression equations (Eqs. 13 and 15) suggests that, given an m_b , the scatter in M_L is about 0.5 units and that, at least for these data and for the range of magnitudes plotted, m_b is less than M_L by about a half unit for a given earthquake.

To establish the M_L value of a western U.S. earthquake that releases the same energy as an eastern earthquake of a given m_b , we proceed in two steps. First, we use Eq. 4 to derive the m_b for an equivalent western earthquake, then we use Eq. 15 to calculate the correct M_L . Or, combining Eqs. 4 and 15,

$$M_L \approx 0.85 + 0.92 \left[(m_b)_{\text{East}} - \Delta m_b \right] \quad ,$$

where $\Delta m_b \approx 0.3$. Hence,

$$M_L = 0.57 + 0.92(m_b)_{\text{East}} \quad . \quad (16)$$

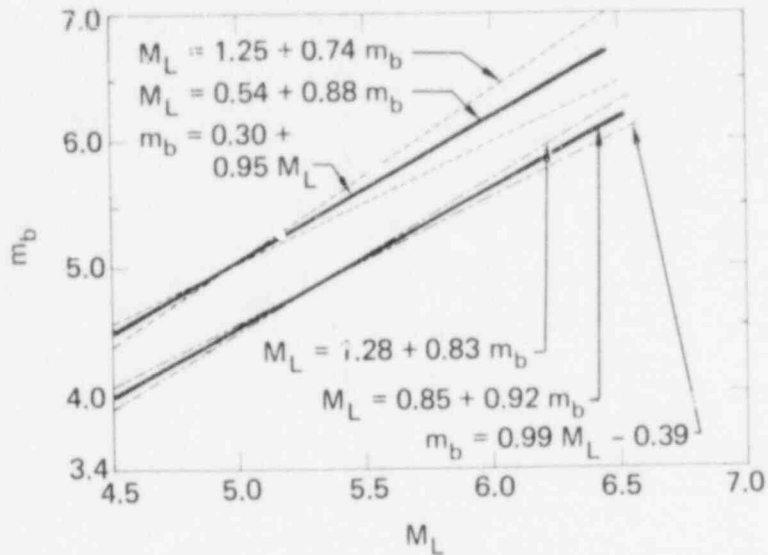


FIG. 4. Comparison of the m_b vs M_L relationships for the western United States. The upper three curves were derived on the basis of the Berkeley and Pasadena M_L values and the NEIS m_b values. The lower curves were derived on the basis of the Berkeley and Pasadena M_L values and the corrected m_b values listed by Nuttli et al. (1979).

Equation 16 is an important result of the present study. This relationship provides a link between the m_b of an eastern U.S. earthquake and the appropriate M_L of a western U.S. earthquake releasing the same amount of energy.

Nuttli (1980) pointed out that the corrections to m_b computed by Nuttli et al. (1979) have a possible bias, because, for the smaller earthquakes, the only P-wave data for distances greater than 25° came from a few eastern U.S., eastern Canadian, and Arctic Canadian stations. For some earthquakes, only two or three stations of this group recorded identifiable P-waves. More often, there were data from approximately 10 stations, which is more satisfactory, but all were within a 90° azimuth spread, which may have led to radiation pattern effects that influenced the m_b values obtained. Nonetheless, considering the worldwide variation in the manner m_b is determined, we feel that, for the purpose of relating the m_b of an eastern earthquake to the M_L of a western earthquake, it is more important to have the same people compute the m_b 's of both the eastern and western U.S.

earthquakes than to be concerned with azimuth effects. Most of the magnitudes of the eastern U.S. earthquakes were computed in a manner consistent with the approach used by Nuttli et al. (1979); thus, the use of Eq. 15 is appropriate for determining the values of the corresponding M_L 's.

Care must be taken not to extrapolate Eqs. 5 through 16 beyond the points where the various scales saturate. Several authors, for example, Chinnery and North (1975), Kanamori and Anderson (1975), and Hanks and Kanamori (1979), have discussed the saturation of magnitude scales. Chinnery (1979) examined the saturation of the m_b scale in some detail. He found that considerable differences exist between various seismic arrays. Thus, depending upon the data set, the m_b scale appears to saturate in the range $6.5 < m_b < 7.3$. Chinnery concluded that the U.S. VELA arrays saturate at higher m_b values than other arrays. The M_L scale also appears to saturate around $M_L \approx 7$, principally because both m_b and M_L are obtained from amplitudes at about 1-s periods. There are, however, no direct data to show that the M_L scale saturates. The M_S scale saturates at much larger magnitude values (about 8.6) than either m_b or M_L (Chinnery, 1979); hence, the saturation of the M_S scale is not a problem for building a ground motion model for the eastern United States.

Other m_b - M_L relations appear in the literature. Our results are naturally very similar to those supported by Nuttli (1979). The difference between our results and his is that we used only the instrumental data from Nuttli et al. (1979), whereas Nuttli (1979) used some instrumentally determined m_b 's and some determined by the intensity-falloff method discussed by Nuttli et al. (1979).

Richter (1958) stated that Gutenberg used every available means to relate m_b to M_L . Gutenberg's preferred result is

$$m_b = 1.7 + 0.8M_L - 0.01M_L^2 .$$

One reason for the difference between our results and Gutenberg's relation is that the m_b 's used by Gutenberg were based on long-period instruments and a rather meager data set.

RELATION BETWEEN m_b AND M_S

As pointed out in the introduction, the relation between the body-wave and surface-wave magnitude scales has received extensive study. We do not propose to review the literature here in great detail. There are two reasons why the m_b - M_S relation is of interest to this study. First, the m_b - M_S studies provide added evidence for a regional bias to the m_b scale and for the adequacy of the correction given by Eq. 4. Second, differences in the m_b and M_S values for given earthquakes may give added insight into differences in earthquake source parameters that could have important influences on the ground motion.

The relation between the body-wave and surface-wave scales is primarily empirical in nature, as the scales themselves are empirical and based on totally different wave types at opposite ends of the frequency spectrum. Because of the regional variations in the m_b scale discussed earlier, it is sometimes difficult to sort out the true relationship between m_b and M_S . (As discussed earlier, it is generally assumed that regional differences in crustal structure and attenuation do not have a significant effect on the M_S values, provided that the M_S 's are determined at distances greater than 20° .) In addition to the direct evidence provided by Guyton (1964), the regional variation in the m_b scale is indicated by the systematic bias in the relationship between m_b and M_S , measured for earthquakes and explosions located in different regions. An example is the difference between the m_b - M_S relations for the eastern and western United States. Figure 5 shows plots of m_b vs M_S for a number of eastern earthquakes, taken from Nuttli et al. (1979) and Nuttli and Zollweg (1974), and for a number of western U.S. earthquakes, taken from Nuttli et al. (1979). (The m_b 's are the values recomputed by Nuttli et al.) Also shown is the least-squares regression line for the western U.S. data. Because there are so few data for eastern U.S. earthquakes, we did not feel that a least-squares fit would be meaningful. We do show in Fig. 5 the line obtained from Eq. 4, with $\Delta m_b = 0.35$. It appears to fit the eastern U.S. data well. There are a number of different interpretations that could be placed on Fig. 5. One is that the m_b 's in the western United States are systematically low by about 0.3 units, as suggested by Eq. 4. On the other hand, considering the scatter in the m_b - M_S relation

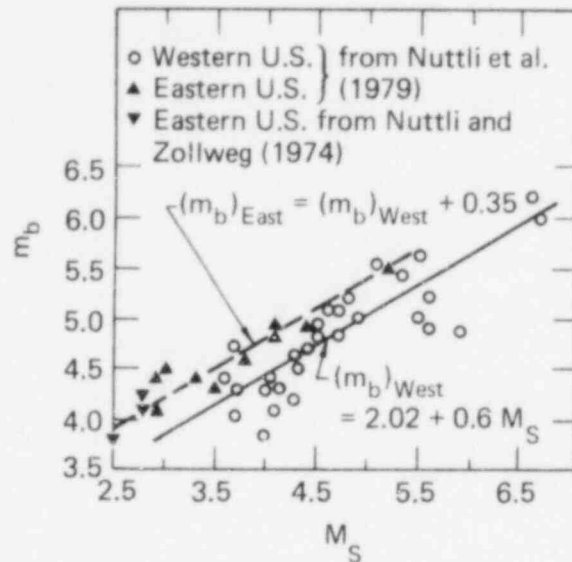


FIG. 5. Plot of m_b vs M_S for eastern U.S. earthquakes listed by Nuttli et al. (1979) and by Nuttli and Zollweg (1974), and for western U.S. earthquakes listed by Nuttli et al. (1979). Also shown is the least-squares fit to the western U.S. data and the correction given by Eq. 4.

for the western U.S. earthquakes, it is clearly not possible to conclude that systematic source differences between earthquakes occurring in the eastern and western United States are not the cause of the regional difference.

The data for surface-wave magnitude (M_S) vs body-wave magnitude (m_b) from a large number of mixed seismic events tend to separate into two distinct populations: one composed of earthquakes and the other of explosions. If the m_b values for an explosion and for an earthquake are equal, the earthquake M_S is generally greater than that for the explosion. This is the basis of the M_S vs m_b seismic discrimination technique.

Liebermann and Pomeroy (1969) and Basham (1969) first noted that the M_S vs m_b data for explosions and earthquakes in the western United States are anomalous with respect to such data for the Aleutians, the Sahara, and the USSR. Marshall and Basham (1972) confirmed that the M_S vs m_b data for explosions in the western United States differ from the M_S vs m_b relations for explosions in the Aleutians, the USSR, and China, but they did not detect any regional differences in the M_S vs m_b data for earthquakes. Separate M_S vs m_b populations were noted for explosions in the western United States

and central Asia by Filson and Bungum (1972). Evernden and Filson (1971) studied explosions in similar testing media at NTS and at Amchitka. The M_S vs yield relation was the same for the two locations, but the m_b vs yield relations were different.

These and other studies suggest that, for explosions (and perhaps for earthquakes) in the western United States, either (1) the M_S values are anomalously high, or (2) the m_b values are anomalously low, or (3) there is a combination of the two effects. The discrepancy between the M_S vs m_b data for explosions inside and outside the western United States is at least 0.3 to 0.5 magnitude units.

Two alternative explanations have been proposed for this M_S vs m_b anomaly: (1) tectonic strain release accompanying the explosions at NTS results in increased Rayleigh-wave energy and enhanced M_S values, and (2) m_b values are depressed by higher-than-average body-wave attenuation in the upper mantle beneath the western United States. These explanations are discussed in this section.

Love waves as well as Rayleigh waves have been observed from many underground nuclear explosions at NTS. Analysis of seismic data has indicated that the observed radiation patterns of these waves approximates the theoretical radiation patterns from an orthogonal double-couple (tectonic strain release) superimposed on an isotropic dilatational source (the explosion). The ratio of the tectonic (double-couple) to the explosive (dilatational) component of the surface-wave energies for NTS explosions exceeded 10 for Hardhat and Pile Driver and 3 for Greeley. However, this ratio is about 1 or less for the other 13 observed explosions at NTS (Toksoz and Kehrler, 1972). Ward and Toksoz (1971) concluded that a surface-wave energy ratio of unity is not sufficient to account for the M_S vs m_b anomaly at NTS. Toksoz and Kehrler (1972) found that the additional surface-wave energy in the Hardhat, Pile Driver, and Greeley signals did not significantly affect the M_S vs m_b relation for those explosions. They suggested that this null result may be a consequence of constructive and destructive interference in alternating quadrants between the explosive and tectonic components of the Rayleigh waves. Masse (1973) compared Rayleigh-wave radiation patterns for nine explosions, including some studied by Toksoz and Kehrler (1972), and four cavity collapses at NTS. He concluded that the effect

of any explosion-produced release of tectonic strain on the Rayleigh-wave radiation pattern was small for most of the explosions studied. Since the studies were limited by the available seismic data, these conclusions must be qualified. It is clear, however, that seismic observations do not give strong support to the proposal that tectonic strain release is responsible for the M_S vs m_b anomaly at NTS.

Evernden and Filson (1971) cited evidence of significantly smaller P-wave amplitudes in the western United States than in the eastern United States. They proposed that regional variations in body-wave attenuation are responsible for the M_S vs m_b anomaly at NTS. Ward and Toksoz (1971), after considering and rejecting M_S enhancement by tectonic strain release, concluded that regional variations in attenuation in the upper mantle play an important role in regional differences in the M_S vs m_b relation. Filson and Bungum (1972) suggested that the difference between the M_S vs m_b relations for NTS and central Asia results from higher attenuation beneath the western United States. Solomon (1972) concluded that short-period ($T = 1$ s) body waves and long-period ($T > 20$ s) surface waves are attenuated more in the western than in the eastern United States and that the 10- to 20-s surface waves used to determine M_S are attenuated at about the same rate throughout the United States. He also showed that Basham's (1969) M_S vs m_b data for 28 earthquakes in southwestern North America show an apparent m_b depression of 0.3 to 0.4 units for earthquakes under the Basin and Range Province and the Gulf of California, compared to earthquakes in adjacent areas. Solomon concluded that the anomalous M_S vs m_b pattern in the western United States is at least in part due to the greater-than-average attenuation of P-waves in the upper mantle beneath that area. It is clear that m_b depression by higher-than-average attenuation in the upper mantle is responsible for at least part of the M_S vs m_b anomaly at NTS. As discussed earlier, Marshall et al. (1979) also concluded that considerable evidence exists for high body-wave attenuation in the upper mantle beneath the western United States and for this attenuation having a significant impact on the m_b 's of earthquakes and explosions.

Evernden and Clark (1970) showed that teleseismic P-waves in the western United States are lower in amplitude than those in the East by a factor of three (0.5 magnitude units). They suggested that a region of abnormally low Q

exists at comparatively small depths in the western United States and that this low-Q region is probably related to the abnormally high heat flow to the surface. Booth et al. (1974) determined station corrections for 37 Long Range Seismic Measurement stations in the continental United States, Canada, and the Aleutians. They found a similar pattern of short-period P-wave attenuation in the United States and concluded that signals leaving NTS will be reduced by about 0.25 magnitude units. They attributed the P-wave attenuation pattern to lateral variations of Q in the upper mantle.

RELATION BETWEEN m_b AND $m_b(L)_g$

Both m_b and $m_b(L)_g$ are determined using 1-Hz waves; however, m_b is determined from P-waves and $m_b(L)_g$ from the L phase. Both magnitude scales should scale the same. When Nuttli (1973) developed the $m_b(L)_g$ scale, he adjusted the constants to agree with the recorded m_b 's of the earthquakes he used to develop the scale. However, some difference between the two scales might be expected to develop because of the different wave types involved, depending upon the mechanism of the earthquakes. This does not appear to have been studied in any detail. Available published data indicate good agreement between the two scales.

CONCLUSIONS

From the above discussion, it is clear that there are problems with all of the magnitude scales. When using regional catalogs, it is necessary to determine how the reported magnitudes were determined. Often such information is not available; this presents problems, because the potential errors are quite large.

Both the M_S and m_b scales were designed to be universal scales, valid for comparing earthquakes in various regions. However, we have seen that this is only partly true, because both M_S and m_b magnitudes are often determined beyond the applicable range for the equations used to define the two scales. In this regard, M_S has an advantage over m_b : the M_S scale does not saturate until $M_S \approx 8.6$ (Chinnery, 1979). However, the M_S magnitudes are not generally available in most catalogs for moderate to small earthquakes. The m_b magnitudes are more generally available; however, the m_b scale appears to saturate around $m_b \approx 7.3$. There is also much greater variation in the way m_b is determined. In particular, there was a significant change in the m_b scale in the early 1960s when the WWSSN was established. This change in instrumentation significantly affected estimates of magnitude (post-1960 values are lower) and the saturation level of the m_b scale. The older, longer-period instruments recorded larger m_b magnitudes than can be recorded with the WWSSN instruments. In addition, great care must be taken when selecting the m_b magnitudes of western U.S. earthquakes, because the values are often in considerable error due to the fact that they were determined at distances less than 25° and were not properly corrected.

Because m_b magnitudes are generally more readily available than M_S magnitudes, the m_b scale must be used to relate earthquakes between various regions; however, consideration should be given to earthquake mechanisms and their influence on m_b . The saturation of the m_b scale is also an important consideration, because, as discussed by Chinnery (1979), there is significant variation in the m_b magnitude at which saturation occurs for different stations and arrays.

We discussed in some detail the need for a regional correction to the m_b magnitudes for shallow earthquakes located in low-Q regions such as the western United States. In particular, we determined that the m_b 's of western U.S. earthquakes are about 0.3 units smaller than similar earthquakes in higher-Q regions such as the eastern United States. We based this conclusion on several lines of evidence:

- Theoretical considerations of the effect of the difference in Q between the western and eastern United States lead to a correction factor of $\Delta m_b \approx 0.3$.
- The data of Guyton (1964), shown in Fig. 2, provide evidence of lower estimated m_b 's in low-Q area, compared to high-Q areas, for earthquakes occurring elsewhere.
- A comparison of the $m_b - M_S$ values for eastern U.S. earthquakes and western U.S. earthquakes shows that the m_b 's of the latter are lower by about $\Delta m_b \approx 0.3$ units (Fig. 5).
- The considerable evidence from explosions shows that the western U.S. data give low m_b 's compared to explosions in higher-Q areas.
- Western U.S. earthquakes appear to have smaller m_b 's for the same M_S than do earthquakes elsewhere in the world.

The M_L and the $m_b(L_g)$ scales are regional scales and should not be used (without careful calibration) in other regions. The M_L scale is the most generally available magnitude for western U.S. earthquakes, and $m_b(L_g)$ is the most readily available for eastern U.S. earthquakes. The $m_b(L_g)$ and m_b scales are closely related; however, since m_b is determined from P-waves and $m_b(L_g)$ from surface waves, we can expect some variation between the two due to source mechanism.

We concluded that one could reasonably expect to relate m_b to M_L . The relation given by Eq. 15 seems to bear this out, suggesting $M_L \sim 0.92m_b$. We concluded that the relations between m_b and M_S , and between M_S and M_L are more complex, as M_S scales differently from either M_L or m_b . Thus, we found $m_b \sim 0.6M_S$ and Nuttli (1979) found $M_L \sim 0.63M_S$, indicating substantial scaling differences.

The most useful result of this study is Eq. 16, which provides a link between the m_b of an eastern U.S. earthquake and the corresponding M_L of an equivalent western earthquake. The source mechanisms of the earthquakes used in developing Eq. 16 were not considered, because, for the most part, they were not available. No doubt the scatter seen in Figs. 3 and 5 could be reduced if corrections were made for earthquake source mechanism.

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