NUREG/CR-5429
Vol. 3

UDEC (Universal Distinct Element Code) Version ICG1.5

Verification and Example Problems

Prepared by Mark Board

Itasca Consulting Group, Inc.

Prepared for
U.S. Nuclear Regulatory Commission

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U.S. Nuclear Regulatory Commission
Washington, DC 20555
NRC FIN D1016

ABSTRACT

UDEC (Universal Distinct Element Code) is a two-dimensional distinct element program written for the static and dynamic analysis of the mechanical, thermal and hydrologic behavior of jointed rock masses. This program has been applied to a wide variety of problems in civil construction, mining, nuclear waste disposal, and geologic modeling. This document presents the theoretical basis for the mathematical models, the details of solution procedures, user's manual and presentation of verification and example problems. A description of the program support and documentation methodology which is employed is also given. This document is given in three volumes: Volume 1 — Description of Mathematical Models and Numerical Methods, Volume 2 — User's Manual, and Volume 3 — Verification and Example Problems. These three volumes are intended to satisfy the requirements and guidelines set forth in Final Technical Position and Documentation of Computer Codes For High-Level Waste Management (NUREG-0856).

VOLUME 3

VERIFICATION AND EXAMPLE PROBLEMS

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3.1 INTRODUCTION

This document is the third in a series of three volumes which provide documentation on the UDEC code, Version ICG1.5 as prescribed in NUREG-8056, Final Technical Position on Documentation of Computer Codes For High-Level Waste Management. Volume 3 provides documentation on the assessment and support of the UDEC code for mechanical, thermal and fluid flow analyses of geotechnical materials. Section 3.2 of this volume presents verification problems in which UDEC is compared to analytical solutions for problems in mechanical, thermal and fluid flow analysis. These problems are given to provide assurance of the correct operation of the various component parts of the UDEC program. Section 3.3 presents an example thermomechanical problem analyzed with UDEC. This problem is necessarily more complex than those presented in Section 3.2, and therefore has no analytical solution. The problem chosen exercises features commonly used in geomechanical analyses involving decaying heat sources. Finally, the procedures used by Itasca Consulting Group in development and testing of the UDEC code are presented in Section 3.4.

3.2 UDEC VERIFICATION PROBLEMS

The objective of this section, and code verification in general, is to demonstrate, to a suitable level of tolerance, correspondence between the code's solution to a particular problem and an independent solution to the same problem. In most cases, the independent solution against which UDEC results are compared is an analytic (i.e., closed-form) solution. The problems presented here supplement the verification problems included in the UDEC User's Manual, which is Volume 2 of this document.

It is impractical to present verification problems for all of UDEC's capabilities. However, all classes of problems important to repository design are presented here, including mechanical, thermal, thermo-mechanical, and fluid flow. Table 3.1-1 presents the complete set of UDEC verification problems and points out the location of problems presented in Volume 2, UDEC User's Manual.

Each problem consists of a description of the physical problem, the numerical idealization, and the results. For several problems, results are presented to show the influence of different choices involving discretization, solution technique, conditions during solution, etc., to assist the user in making similar choices for other problems. For all problems, a representative UDEC input data file is included. In some cases, a FORTRAN source code used to compute analytical solutions is also included.

All problems were executed using UDEC Version ICG1.5, operating on either a DSI-780 coprocessor board (manufactured by Definicon Systems, Inc.) or a 80386-based microcomputer. This version complied for operation on an 80386-based microcomputer running DOS3.x uses SVS FORTRAN 386, the PHARLAP linker, and ICG X-AM DOS extender. The screen graphics support for this version is handled through a FORTRAN-linkable library (SCITECH plotting package). The version for the DSI-780 coprocessor is complied using SVS FORTRAN V2.6. The screen graphics support is handled through a FORTRAN-linkable library (SCI-GRAF modules, which are sold by Definicon Systems Inc.)

Table 3.1-1

UDEC VERIFICATION PROBLEMS

Mechanical Problems — Quasi-Static

Cyclic Loading of a Specimen with a Slipping Crack

Sliding Block Between Two Slightly Skewed Rigid Walls

Thick-Walled Cylinder Subject to Internal Pressure

Elasto-Plastic Response of an Unlined Circular Tunnel in a Biaxial Stress Field

Circular Tunnel Problems Involving Use of Boundary Elements

Part A — Tunnel in an Elastic Medium with a Biaxial Stress Field

Part B — Tunnel in an Elastic-Plastic Medium with a Hydro Static Stress Field

Part C — Lined Tunnel in an Elastic Medium with a Biaxial Stress Field

Elastic Behavior of Jointed Medium (see UDEC User's Manual, pp. B-1 to B-10)

Crack Shear by Reduced Friction (see UDEC User's Manual, pp. C-1 to C-6)

Rough Footing on Cohesive Material (see UDEC User's Manual, pp. D-1 to D-5)

Mechanical Problems — Dynamic

Line Source in an Infinite Elastic Medium with a Discontinuity

Slip Induced by Harmonic Shear Wave (see UDEC User's Manual, pp. A-1 to A-9)

Table 3.1-1
(continued)

Thermal Problems

Steady-State Temperature Distribution Along a Tapered Fin (see UDEC User's Manual, pp. 7-37 to 7-39)

One-Dimensional Steady-State Heat Conduction and Convection Through a Composite Wall (see UDEC User's Manual, pp. 7-40 to 7-42)

Thermal Response of a Heat-Generating Slab (see UDEC User's Manual, pp. 7-43 to 7-45)

Transient Temperature Distribution in an Orthotropic Bar (see UDEC User's Manual, pp. 7-51 to 7-53)

Thermo-Mechanical Problems

Thermo-Elastic Response of a Hollow Thick Wall Cylinder

Infinite Slab with Applied Heat Flux (see UDEC User's Manual, pp. 7-46 to 7-50)

Fluid Flow Problems

Steady-State Fluid Flow with Free Surface (see UDEC User's Manual, pp. F-1 to F-7)

3.2.1 Mechanical Problems — Quasi-Static

The following quasi-static mechanical problems are presented in this section or can be found as noted:

Cyclic Loading of a Specimen with a Slipping Crack

Sliding Block Between Two Slightly Skewed Rigid Walls

Thick-Walled Cylinder Subject to Internal Pressure

Elasto-Plastic Response of an Unlined Circular Tunnel in a Biaxial Stress Field

Circular Tunnel Problems Involving Use of Boundary Elements

Part A — Tunnel in an Elastic Medium with a Biaxial Stress Field

Part B — Tunnel in an Elastic-Plastic Medium with a Hydro Static Stress Field

Part C — Lined Tunnel in an Elastic Medium with a Biaxial Stress Field

Elastic Behavior of Jointed Medium (see UDEC User's Manual, pp. B-1 to B-10)

Crack Shear by Reduced Friction (see UDEC User's Manual, pp. C-1 to C-6)

Rough Footing on Cohesive Material (see UDEC User's Manual, pp. D-1 to D-5)

3.2.1.1 Cyclic Loading of a Specimen with a Slipping Crack

Problem Statement

This problem concerns an elastic block with an inclined internal closed crack (Fig. 3.2.1.1-1) subject to a cycle of uniaxial loading.

A constant axial displacement u_a is applied to one end of the block, and the other end is fixed. The resulting load causes inelastic slip on the crack. At some point, the sense of displacement on the end of the block is reversed until the original position is re-established. Olsson (1982) showed that the stress-displacement relation for the loaded specimen is composed of three distinct components (Fig. 3.2.1.1-2):

(1) a loading segment (OA) which involves elastic deformation and slip along the crack;

(2) an initial unloading segment (AB), where the crack does not slip; and

(3) a final unloading segment (BO), again with elastic deformation and slip.

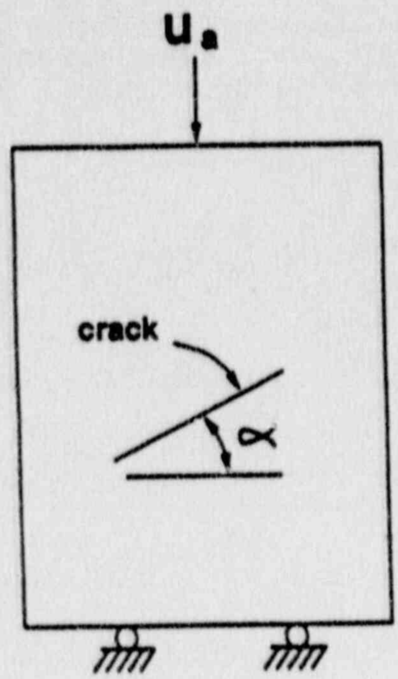


Fig. 3.2.1.1-1 Specimen with Embedded Crack

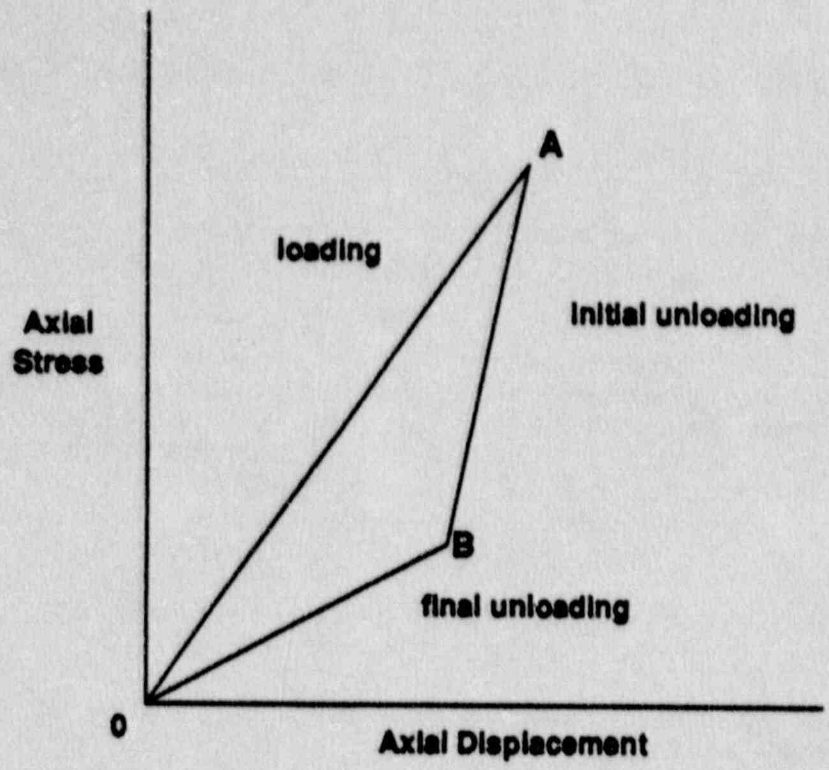


Fig. 3.2.1.1-2 Stress-Displacement Relation for Elastic Specimen with Embedded Crack Subjected to Uniaxial Load Cycle [after Olsson, 1982]

Objective

The objective of this problem is to test joint constitutive relations in UDEC. Other code functions tested by this problem include:

(a) the ability of the code to model solid elastic behavior;

(b) the ability of the code to model quasi-static behavior using adaptive damping; and

(c) the ability of the code to use displacement boundary conditions.

Physical Problem

A single inclined crack is located in an elastic medium. The mechanical properties of the medium are listed below:

elastic modulus (E')	88.9 MPa
Poisson's ratio (ν')	0.26
height (H)	2 m
width (W)	1 m

The properties of the crack are as follows:

joint normal stiffness (K_n)	220 GPa/m
joint shear stiffness (K_s)	220 GPa/m
joint friction angle (ϕ)	16°
joint inclination (α)	45°
slipping portion of crack (l)	0.54 m

Conceptual Model

Several investigators have proposed simple conceptual models of a single, closed crack to explain phenomena associated with the deformational response of jointed rock [e.g., Walsh (1965) and Jaeger and Cook (1976)]. One such model is a single crack embedded in an elastic solid subjected to a cycle of uniaxial compression.

Brady et al. (1985) present relations for the three slopes in Fig. 3.2.1.1-2 in terms of the elastic stiffness of the solid, the elastic and frictional properties of the crack, and the orientation of the crack. The conceptual model is illustrated in Fig. 3.2.1.1-3.

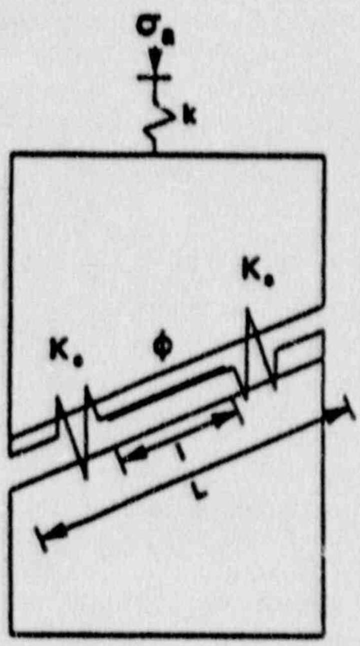


Fig. 3.2.1.1-3 Conceptual Model of Elastic Specimen Containing Embedded Crack

In the conceptual model, k is the equivalent axial elastic stiffness of the specimen, including the through-going discontinuity. The equivalent elastic stiffness is given by

$$\frac{1}{k} = \frac{H}{WE'} + \frac{\cos^2 \alpha}{K_n L} + \frac{\sin^2 \alpha}{K_s L} \qquad (3.2.1.1\text{-}1)$$

where $L = W/\cos\alpha$.

It should be noted that the term (H/WE') in Eq. (1) represents the uniaxial elastic stiffness of the solid in the conceptual model for plane stress conditions. The analysis in UDEC is based on plane strain conditions. However, the formal equivalence between the plane stress and plane strain conditions is represented by the relations between Young's modulus and Poisson's ratio, for plane strain and plane stress:

$$E = \frac{1 + 2\nu'}{(1 + \nu')^2} E' \qquad (3.2.1.1\text{-}2)$$

$$\nu = \frac{\nu'}{1 + \nu'} \qquad (3.2.1.1\text{-}3)$$

where E and ν are the Young's modulus and the Poisson's ratio for plane strain, and

E′ and ν′ are the equivalent plane stress parameters.

The stiffnesses for the three slopes are given, therefore, as

$$\text{slope OA} = \frac{k}{1 + \dfrac{k \sin\alpha \sin(\alpha-\phi)}{K_s\ (L - 1)\ \cos\phi}} \qquad (3.2.1.1\text{-}4)$$

$$\text{slope AB} = k \qquad (3.2.2.1\text{-}5)$$

$$\text{slope BO} = \frac{k}{1 + \dfrac{k \sin\alpha \sin(\alpha+\phi)}{K_s\ (L - 1)\ \cos\phi}} \qquad (3.2.2.1\text{-}6)$$

Assumptions

The material in which the crack is embedded is linearly elastic, homogeneous, and isotropic. The numerical analysis assumes that the specimen is restrained perpendicular to the plane of analysis (i.e., plane strain conditions). It is further assumed that the crack can be represented by a single through-going discontinuity with only the central section of the discontinuity allowed to slip. The ends of the discontinuity are prevented from slipping by setting the frictional resistance to a high value over these regions.

Computer Model

In the UDEC analysis, the elastic blocks are discretized into constant strain finite difference triangles as shown in Fig. 3.2.1.1-4.

The following alternatives for the joint constitutive relation have been studied:

Case A — standard linear deformation, Coulomb friction model

Case B — continuously-yielding model

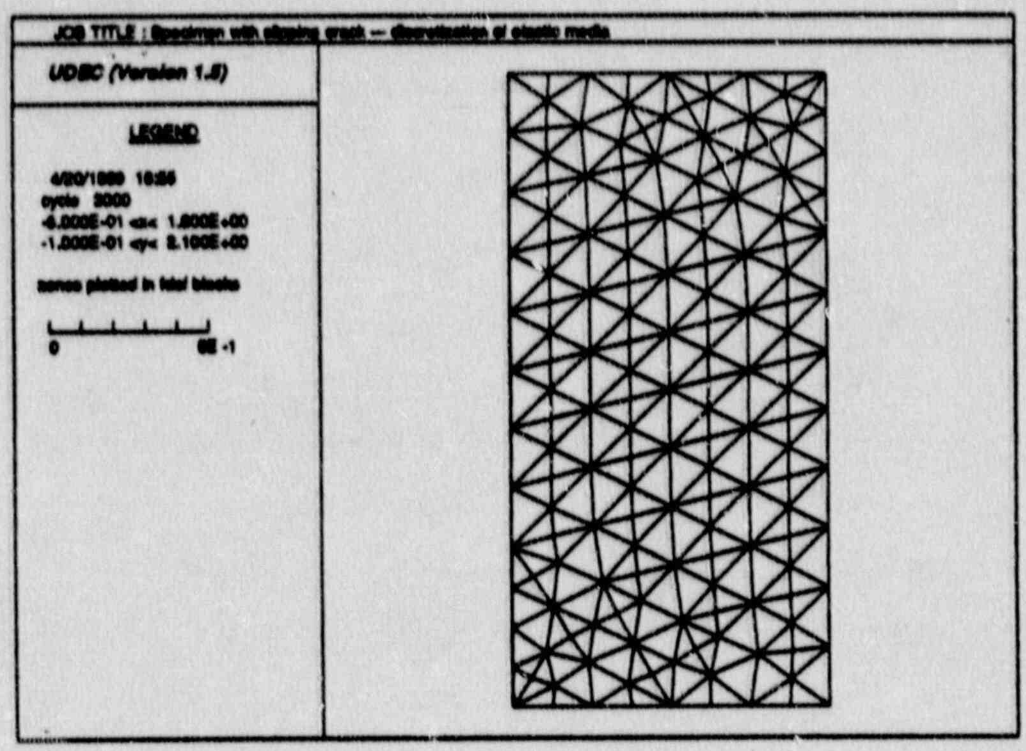


Fig. 3.2.1.1-4 Discretization of Elastic Medium into Constant Strain Finite Difference Triangles

In all problems, the elastic, non-slipping sections of the crack were modeled using the standard Coulomb model (JCONS=2), with the friction parameter set high enough to prevent any slip. In all problems, the center section of the crack was assigned parameters which would permit slip to occur. The specific UDEC parameters used for each joint relation are as presented in Table 3.2.1.1-1.

Table 3.2.1.1-1

JOINT PARAMETERS

Coulomb Friction (JCONS=2)	Continuously-Yielding (JCONS=3)
JKN = 220 GPa/m	JKN = 220 GPa/m
JKS = 220 GPa/m	JKS = 220 GPa/m
JFRIC = 0.287	JFRIC = 0.287
	JEN = 0
	JES = 0
	JIF = 0.279 rad
	JR = 1.0e^{-10} m

The Coulomb model is a linear elastic-perfectly plastic constitutive relation and corresponds with the concepts used in developing the expressions for three stiffnesses [Eqs. (4-6)] in the conceptual model. The other joint constitutive relation is non-linear and, therefore, does not comply with the concepts used to develop the conceptual model. The parameters selected for the continuously- yielding model were found by fitting this model to the results for a Coulomb joint in direct shear under constant normal stress. For the continuously-yielding model, the normal stress-normal displacement relation used in this study is linear, with K_n = 220 GPa/m, but the shear behavior is non-linear. The shear stress-shear displacement response for the continuous-yielding model, based on the parameters defined in Table 3.2.1.1-1 approximates the Coulomb slip, as shown in Fig. 3.2.1.1-5.

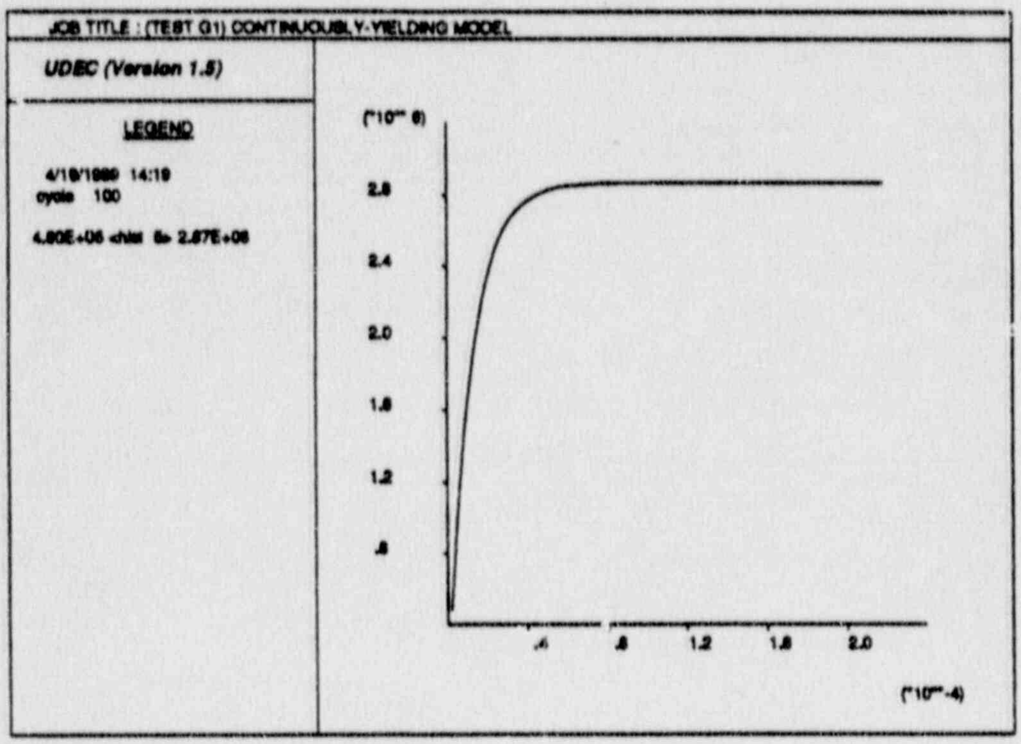


Fig. 3.2.1.1-5 Joint Shear Stress (Pa) versus Shear Displacement (m) for Constant Normal Stress Direct Shear Test Using the Continuously-Yielding Model (normal stress = 10 MPa)

Results

The results for each of the joint constitutive models is compared with the conceptual model in Table 3.2.1.1-2. Global stiffnesses were calculated directly from UDEC results using average vertical stresses and maximum vertical displacements (found using **PRINT MAX**) for each load step. The table shows good agreement for both models. Graphical results for the complete load cycle for the Coulomb model are shown in Fig. 3.2.1.1-6.

TABLE 3.2.1.1-2

COMPARISON OF UDEC RESULTS USING VARIOUS JOINT MODELS WITH CONCEPTUAL MODEL SOLUTION FOR CYCLIC LOADING OF A SPECIMEN WITH A SLIPPING CRACK

	Conceptual Model	Coulomb Model		Continuously-Yielding Model	
Loading Segment	Stiffness (GPa/m)	Stiffness (GPa/m)	Error (%)	Stiffness (GPa/m)	Error (%)
Load (OA)	36.34	36.04	0.82	36.11	0.65
Unload (AB)	38.89	38.91	-0.05	38.77	0.31
Unload (BO)	34.52	34.14	1.1	34.18	0.98

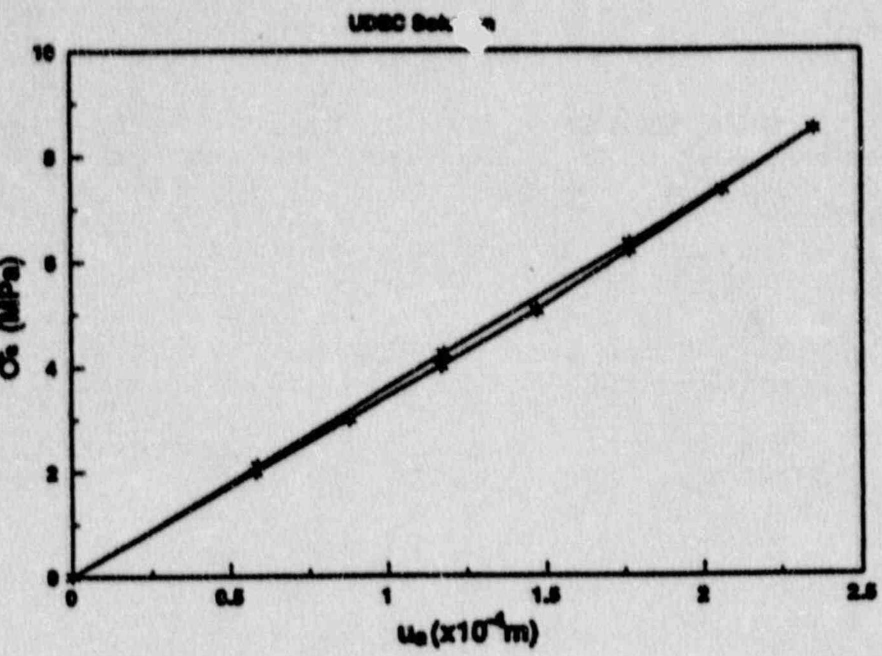


Fig. 3.2.1.1-6 Axial Stress versus Axial Displacement for the Problem Involving Load Cycling for a Specimen with Slipping Crack Modeled with the Coulomb Friction Law

Discussion

There is no analytical solution to the problem of an elastic body with an internal inclined slipping crack, because stress conditions at each end of the crack are very complex. Nevertheless, the simple conceptual model described here captures the essential features of the problem (i.e., three distinctly different global stiffnesses) observed in cyclic loading. The UDEC results agree well with the conceptual model. However, the results agree less closely as the length of the slipping crack increases with respect to the width of the specimen. This observation is expected because the conceptual model assumes uniform distribution of normal stress on the crack and elastic extensions, and stress concentrations (particularly joint normal stress) become more significant as the length of the slipping crack increases.

Parameters for the continuously-yielding model were not optimized to give the "best" results. It is conceivable that other parameters could give even closer agreement with the conceptual model.

References

Brady, B.H.G., M. L. Cramer and R. D. Hart. "Preliminary Analysis of a Loading Test on a Large Basalt Block" (Tech. Note), Int. J. Rock Mech. Min. Sci. & Geomech. Abst., 22, 345-348 (1985).

Jaegar, J. C., and N.G.W. Cook. Fundamentals of Rock Mechanics, 2nd Ed., pp. 329-333. London: Chapman and Hall, 1976.

Olsson, W. A. "Experiments on a Slipping Crack," Geophys. Res. Letters, 9(8), pp. 797-800 (1982).

Walsh, J. B. "The Effect of Cracks on the Compressibility of Rock," J. Geophys. Res., 70, 381-389 (1965).

Input Data Files

Coulomb Model

```
set log on
* verification test a
* load cycling a specimen with a slipping crack
* friction angle = 16 degrees
*
* crack extension - no slip
prop mat=1 d=2850 k=48.25e9 g=35.277e9 jkn=220e9 jks=220e9
jf=100.0
* crack properties,  Coloumb friction model
prop mat=2 d=2850 k=48.25e9 g=35.277e9 jkn=220e9 jks=220e9
jf=0.287
round 0.001
*
block 0,0 0,2 1,2 1,0
split 0 .5 1 1.5
gen 0 1 0 2 auto 0.2
ch jmat=1 jcon=2
change 0.3 0.7 0.74 1.28 jmat=2
damp auto
hist n=15 ydis 0.5 2.0 syy 0.5 2.0 syy 0.2 2.0 syy 0.8 2.0 type 1
*
*  fix the bottom boundary
*
bound -0.1,1.1 -0.1 .1 yvel=0
*
* y-disp. increment (load step 1)
*
*bound -0.1 1.1 1.9 2.1 yvel=-0.1221
bound -0.1 1.1 1.9 2.1 yvel=-0.061
cyc 200
*
*
bound -0.1 1.1 1.9 2.1 yvel=-0.0
cyc 100
pr max
*
*
* y disp. increment (load step 2)
*
*bound -0.1 1.1 1.9 2.1 yvel=-0.1221
bound -0.1 1.1 1.9 2.1 yvel=-0.061
cyc 200
*
```

<u>Coulomb Model</u> (continued)

```
bound -0.1 1.1 1.9 2.1 yvel=-0.0
cyc 100
pr max
*
* y disp. increment (load step 3)
*
*bound -0.1 1.1 1.9 2.1 yvel=-0.1221
bound -0.1 1.1 1.9 2.1 yvel=-0.061
cyc 200
*
bound -0.1 1.1 1.9 2.1 yvel=-0.0
cyc 100
pr max
*
* y disp. increment (load step 4)
*
*bound -0.1 1.1 1.9 2.1 yvel=-0.1221
bound -0.1 1.1 1.9 2.1 yvel=-0.061
cyc 200
*
bound -0.1 1.1 1.9 2.1 yvel=-0.0
cyc 100
pr max
*
* y disp. decrement (unload step 1)
*
*bound -0.1 1.1 1.9 2.1 yvel=0.0611
bound -0.1 1.1 1.9 2.1 yvel=0.0305
cyc 200
*
bound -0.1 1.1 1.9 2.1 yvel=-0.0
cyc 100
pr max
*
* y disp. decrement (unload step 2)
*
*bound -0.1 1.1 1.9 2.1 yvel=0.0611
bound -0.1 1.1 1.9 2.1 yvel=0.0305
cyc 200
*
bound -0.1 1.1 1.9 2.1 yvel=-0.0
cyc 100
pr max
*
```

Coulomb Model (continued)

```
* y disp. decrement (unload step 3)
*
*bound -0.1 1.1 1.9 2.1 yvel=0.0611
bound -0.1 1.1 1.9 2.1 yvel=0.0305
cyc 200
*
bound -0.1 1.1 1.9 2.1 yvel=-0.0
cyc 100
pr max
*
* y disp. decrement (unload step 4)
*
*bound -0.1 1.1 1.9 2.1 yvel=0.0611
bound -0.1 1.1 1.9 2.1 yvel=0.0311
cyc 200
*
bound -0.1 1.1 1.9 2.1 yvel=-0.0
cyc 100
pr max
*
* y disp. decrement (unload step 5)
*
*bound -0.1 1.1 1.9 2.1 yvel=0.0611
bound -0.1 1.1 1.9 2.1 yvel=0.0305
cyc 200
*
bound -0.1 1.1 1.9 2.1 yvel=-0.0
cyc 100
pr max
*
* y disp. decrement (unload step 6)
*
*bound -0.1 1.1 1.9 2.1 yvel=0.0611
bound -0.1 1.1 1.9 2.1 yvel=0.0305
cyc 200
*
bound -0.1 1.1 1.9 2.1 yvel=-0.0
cyc 100
pr max
*
save prob61x.sav
ret
```

Continuously-Yielding Model

```
set log on
* verification test a
* load cycling a specimen with a slipping crack
* friction angle = 16 degrees
*
* crack extension - no slip
prop mat=1 d=2850 k=48.25e9 g=35.277e9 jkn=220e9 jks=220e9
jf=100.0
* crack properties,  continuously yielding joint model
prop mat=2 d=2850 k=48.25e9 g=35.277e9 jkn=220e9 jks=220e9
jf=0.287
prop m 2 jen 0 jes 0 jif 0.279 jr 1e-10
*
round 0.001
*
block 0,0 0,2 1,2 1,0
split 0 .5 1 1.5
gen 0 1 0 2 auto 0.2
ch jmat=1 jcon=2
change 0.3 0.7 0.74 1.28 jmat=2 jcons=3
*
damp auto
hist n=15 ydis 0.5 2.0 syy 0.5 2.0 syy 0.2 2.0 syy 0.8 2.0 type 1
*
*  fix the bottom boundary
*
bound -0.1,1.1 -0.1 .1 yvel=0
*
* y-disp. increment (load step 1)
*
*bound -0.1 1.1 1.9 2.1 yvel=-0.1221
bound -0.1 1.1 1.9 2.1 yvel=-0.061
cyc 200
*
*
bound -0.1 1.1 1.9 2.1 yvel=-0.0
cyc 100
pr max
*
*
* y disp. increment (load step 2)
*
```

Continuously-Yielding Model (continued)

```
*bound -0.1 1.1 1.9 2.1 yvel=-0.1221
bound -0.1 1.1 1.9 2.1 yvel=-0.061
cyc 200
*
bound -0.1 1.1 1.9 2.1 yvel=-0.0
cyc 100
pr max
*
* y disp. increment (load step 3)
*
*bound -0.1 1.1 1.9 2.1 yvel=-0.1221
bound -0.1 1.1 1.9 2.1 yvel=-0.061
cyc 200
*
bound -0.1 1.1 1.9 2.1 yvel=-0.0
cyc 100
pr max
*
* y disp. increment (load step 4)
*
*bound -0.1 1.1 1.9 2.1 yvel=-0.1221
bound -0.1 1.1 1.9 2.1 yvel=-0.061
cyc 200
*
bound -0.1 1.1 1.9 2.1 yvel=-0.0
cyc 100
pr max
*
* y disp. decrement (unload step 1)
*
*bound -0.1 1.1 1.9 2.1 yvel=0.0611
bound -0.1 1.1 1.9 2.1 yvel=0.0305
cyc 200
*
bound -0.1 1.1 1.9 2.1 yvel=-0.0
cyc 100
pr max
*
* y disp. decrement (unload step 2)
*
*bound -0.1 1.1 1.9 2.1 yvel=0.0611
bound -0.1 1.1 1.9 2.1 yvel=0.0305
cyc 200
*
```

Continuously-Yielding Model (continued)

```
bound -0.1 1.1 1.9 2.1 yvel=-0.0
cyc 100
pr max
*
* y disp. decrement (unload step 3)
*
*bound -0.1 1.1 1.9 2.1 yvel=0.0611
bound -0.1 1.1 1.9 2.1 yvel=0.0305
cyc 200
*
bound -0.1 1.1 1.9 2.1 yvel=-0.0
cyc 100
pr max
*
* y disp. decrement (unload step 4)
*
*bound -0.1 1.1 1.9 2.1 yvel=0.0611
bound -0.1 1.1 1.9 2.1 yvel=0.0311
cyc 200
*
bound -0.1 1.1 1.9 2.1 yvel=-0.0
cyc 100
pr max
*
* y disp. decrement (unload step 5)
*
*bound -0.1 1.1 1.9 2.1 yvel=0.0611
bound -0.1 1.1 1.9 2.1 yvel=0.0305
cyc 200
*
bound -0.1 1.1 1.9 2.1 yvel=-0.0
cyc 100
pr max
*
* y disp. decrement (unload step 6)
*
*bound -0.1 1.1 1.9 2.1 yvel=0.0611
bound -0.1 1.1 1.9 2.1 yvel=0.0305
cyc 200
*
bound -0.1 1.1 1.9 2.1 yvel=-0.0
cyc 100
pr max
*
save prob62x.sav
ret
```

3.2.1.2 Sliding Block Between Two Slightly Skewed Rigid Walls

Problem Statement

This problem is derived from a similar problem in Wart et al. (1984) and concerns an elastic block between two near parallel walls (Fig. 3.2.1.2-1). A pressure is applied to one edge of the block, such that the block moves, the initial gap is closed and the normal stress on the contact faces between the block and the walls increases. The increased normal stress causes an increase in the shear resistance through friction on the surface and the block stops. The problem involves computation of the block displacement parallel to the direction of applied pressure.

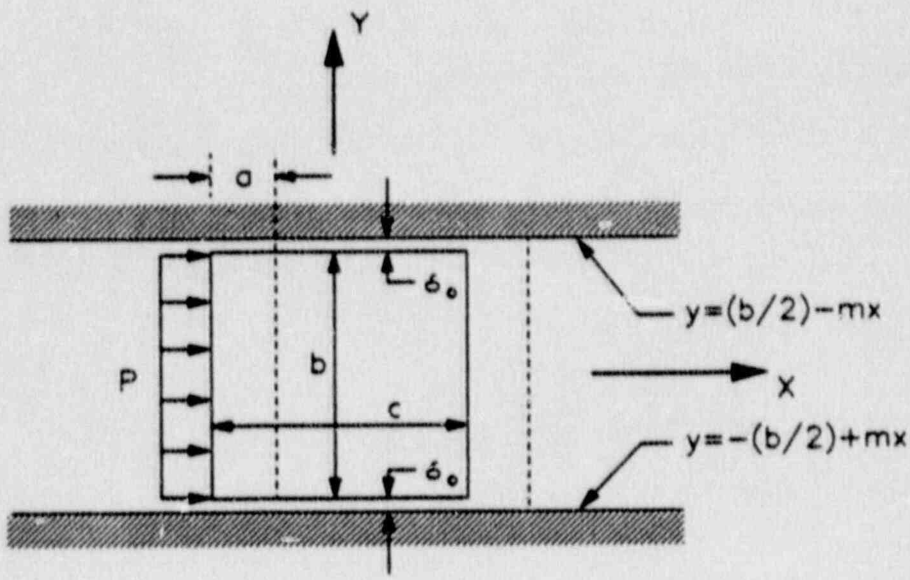


Fig. 3.2.1.2-1 Sliding Block Between Two Slightly Skewed Rigid Walls

Objective

The objective of this problem is to demonstrate:

(a) correct joint constitutive relation implementation;

(b) computation of correct stresses and displacements for a problem involving nonlinear geometry and constitutive relations; and

(c) ability of the code to handle relatively large displacements.

Physical Problem

Input Specifications - The problem is solved for the following input values:

Geometry (see Fig. 3.2.1.2-1)

block height	$b = 1.0m$
block length	$c = 1.0m$
skew slope	$m = 1.0 \times 10^{-3}$
initial gap for both surfaces	$\delta_o = 1.0 \times 10^{-5}m$

Material Properties

modulus of elasticity of the block	E = 20,000 MPa
Poisson's ratio of the block	$\upsilon = 0.25$
friction angle of the sliding surfaces	$\phi = 30°$
joint normal stiffness	JKN = 80,000 MPa

Loads

pressure	P = 0.5, 1.0, and 2.0 MPa

Assumptions

The assumptions related to the analytical solution of this problem include:

(1) the block has a linear stress-strain relation;

(2) the skew angle is small and therefore all of the resistance to sliding is by friction and the pressure on the block is uniform over the top and bottom surfaces; and

(3) the walls are incompressible relative to the block.

Analytical Solution

The analytical solution is based on simple geometrical and stress-strain relations. As the block slides due to imposed pressure, P, the gap will close and the normal stress across the joint will increase due to increased confinement given by the skewed walls. The sliding distance can be divided into two parts as follows:

$$a = a_\delta + a_\sigma \qquad (3.2.1.2\text{-}1)$$

where a = distance of sliding to reach equilibrium,

a_δ = distance of sliding until the initial gap is closed, and

a_σ = distance of sliding as the normal stress increases.

The distance of sliding before gap closure is:

$$a_\delta = \frac{\delta_o}{m} \qquad (3.2.1.2\text{-}2)$$

where δ_o = the initial gap at both the top and bottom of the block, and

m = slope of the skewed walls (see Fig. 3.2.1.2-1).

The block will stop sliding when the frictional resistance equals the applied load. The shear stress due to friction is given by

$$|\tau_f| = -\mu\sigma = \sigma \tan\phi \qquad (3.2.1.2\text{-}3)$$

where τ_f = shear stress due to friction,

σ = normal stress across the joint,

μ = coefficient of friction of the joint, and

ϕ = friction angle of the joint.

Using the stress-strain and the geometric relations between the sliding distance and strain, the normal stresses across each joint is given by:

$$\sigma = \frac{2m\ a_\sigma}{b} E^* \qquad (3.2.1.2\text{-}4)$$

where b = block height, and

E^* = equivalent elasticity of the block joint system

The equivalent elasticity of the block joint system is given by

$$\frac{1}{E^*} = \frac{1}{E} + \frac{2}{JKN \bullet b} \qquad (3.2.1.2\text{-}5)$$

where E = modulus of elasticity for the block, and

JKN = joint normal stiffness.

The friction forces on each sliding surface can then be found by substituting Eq. (4) into Eq. (3). Summing the forces in the x-direction and rearranging terms gives

$$a_\sigma = \frac{Pb^2}{4cmE^* \tan \phi} \qquad (3.2.1.2\text{-}6)$$

where c = length of each sliding surface.

The above solution is for plane stress conditions. The solution for plane strain conditions can be found by substituting $E/(1-\nu^2)$ for E in Eq. (5).

<u>Computer Model</u>

Because of the symmetry about the y=0 line, only the upper half of the problem is studied. The elastic block is discretized into constant strain finite difference triangles (Fig. 3.2.1.2-2). The problem is run using maximum zone edge length of 0.2, 0.1 and 0.05 meters. Symmetry conditions are specified by assigning a zero vertical velocity to the lower horizontal boundary. The initial gap is obtained by assigning an appropriate vertical velocity to the upper rigid block and allowing it to move upward the specified distance. Once the upper block reaches the correct position it is immobilized.

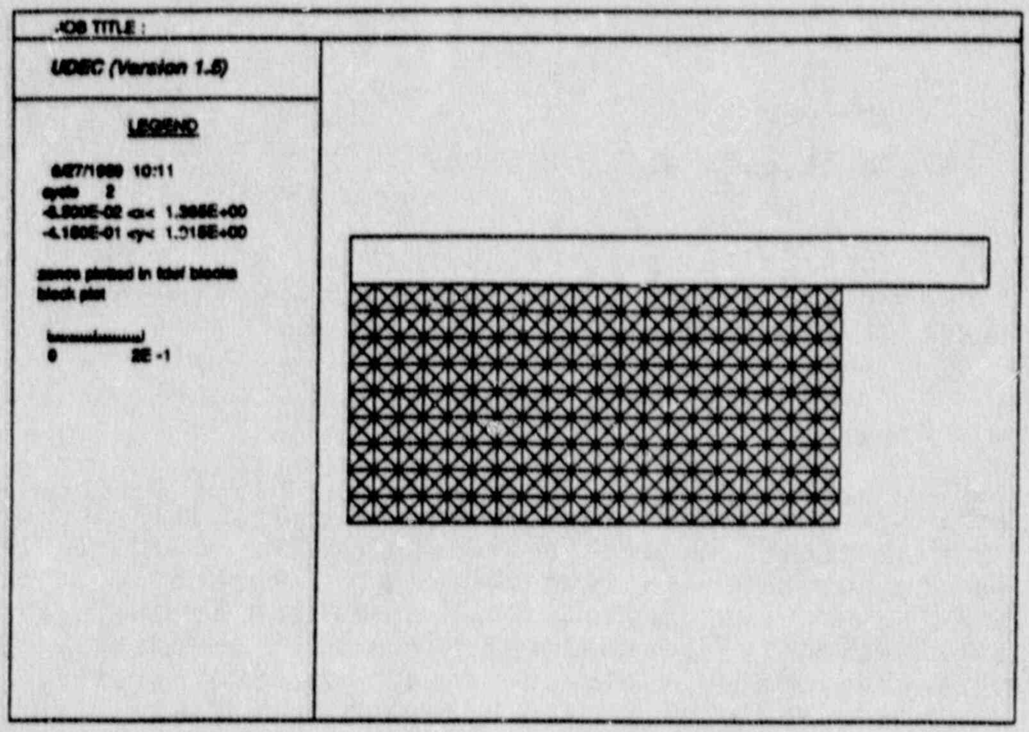


Fig. 3.2.1.2-2 Position of Upper Rigid Block and Discretization of Elastic Block into Constant Strain Finite Difference Triangles

Results

The results for various zone size assumptions are shown in Table 3.2.1.2-1.

Table 3.2.1.2-1

COMPARISON OF UDEC RESULTS USING VARIOUS ZONE SIZES WITH ANALYTICAL SOLUTION FOR SLIDING BLOCK BETWEEN TWO SLIGHTLY SKEWED RIGID BLOCKS
(Results shown are total sliding distance, m.)

Pressure (MPa)	Analytic Solution	Zone* Size	UDEC Results	Error (%)
0.5	0.0256	0.055	0.0262	2.3
		0.125	0.0272	6.3
		0.25	0.0295	15.2
1.0	0.0412	0.055	0.0423	2.7
		0.125	0.0432	4.9
		0.205	0.0505	22.6
2.0	0.0724	0.055	0.0748	3.3
		0.125	0.0758	4.7
		0.25	0.0866	19.6

*actual maximum zone edge length

Discussion

The results shown in Table 3.2.1.2-1 indicate that the result is somewhat dependent on the discretization of the elastic block. This dependence results from the non-uniform vertical stress which is present in the elastic block at equilibrium. The analytic solution assumes that the pressure on the block is uniform over the top and bottom surfaces as given by Eq. (4). For P = 0.5 MPa, Eq. (4) indicates that the normal stress on the top and bottom surface would be about 0.8 MPa. Figure 3.2.1.2-3 shows the equilibrium distribution of normal stress on the top surface of the block, and indicates that the distribution of normal stress on joint is not uniform.

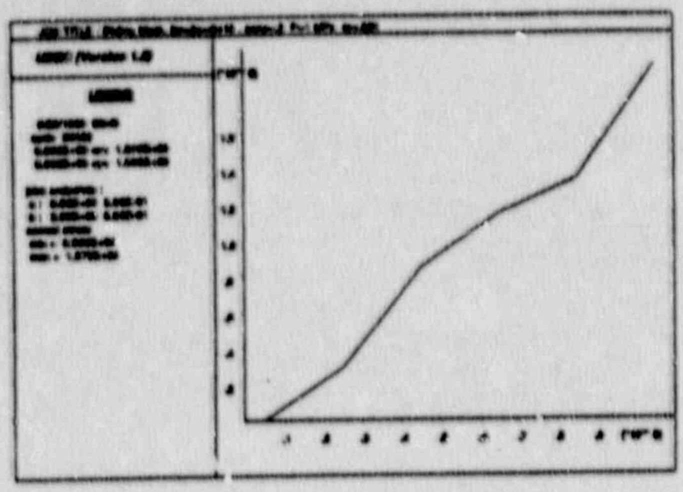
The analytic solution does not include discussion of the joint shear stiffness parameter. The joint shear stiffness, JKS, is related to the joint shear stress, σ, by the amount of shear displacement when the surfaces are in contact, i.e.

$$\tau = JKS\ a_{\sigma} \qquad (3.2.1.2\text{-}7)$$

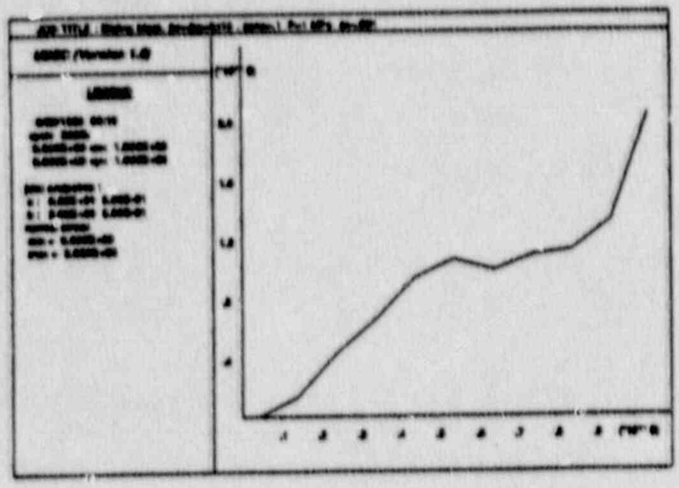
subject to the limitations of Eq. (3). In this problem, the analytic solution is therefore valid for joint shear stiffness greater than the minimum values necessary to achieve equilibrium shear stress within the calculated sliding distance. This value is obtained by dividing the limiting shear stress, σ, at a specified pressure, P, by the amount of sliding distance as the normal stress increases, a_{σ}. For P = 1 MPa, τ = 0.5 MPa and a_{σ} = 0.0312 m; JKS must therefore be 16 MPa/m or greater.

Reference

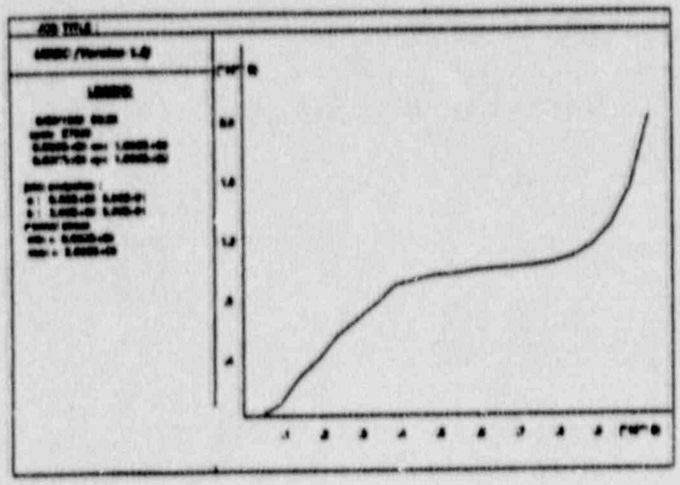
Wart, R. J., E. L. Skiba and R. H. Curtis. Benchmark Problems for Repository Design Models. NUREG/CR-3636. February 1984.



(a)



(b)



(c)

Fig. 3.2.1.2-3 Distribution of Normal Stress on Top Surface of Sliding Elastic Block for Three Different Zone Discretizations

Data Input File

```
* Sliding block between two slightly skewed rigid walls
* Load P=1 MPa
* Joint stiffnesses :jkn=jks=8e10

round 0.001

* set geometry using symmetry:
block 0 0 0 .6 1.3 .6 1.3 0
cr 0 .5 1.3 .4987
cr 1 0 1 .5
del 1 1.5 0 .5

* sliding block is fully deformable:
gen 0 .5 0 1 quad .05

* histories:
hist ncyc 100
hist unbal
hist xdis .5 0
hist ndis 384
hist sdis 384
hist sstr 384
hist xhist

* material properties
prop mat=1 de=2000 g=8e9 k=1.333e10 jkn=8e10 jks=8e10 jfric=.577

* initialization  by loading slightly, and unloading:
bo 0 1.3 -.01 .01 yvel=0
rset -1 86 17
cy 1
rset 0 86 17

* open the gap between the block and the wall:
rset 2.6813246 86 10
cy 1

damp auto

* fix the wall:
rset 0 86 10
fix 0 1.1 0.5 .6

* set linear loading
bo -.1 .2 -.1 .5 xhist lin st -1e6 0 0
cy 1000

cy 27000
save sbljll.sav
quit
```

3.2.1.3 Thick-Walled Cylinder Subject to Internal Pressure

Problem Statement

This problem is adapted from Wart et al. (1984) and concerns plane strain elastic-plastic analysis of a thick-walled cylinder subjected to an internal pressure (Fig. 3.2.1.3-1).

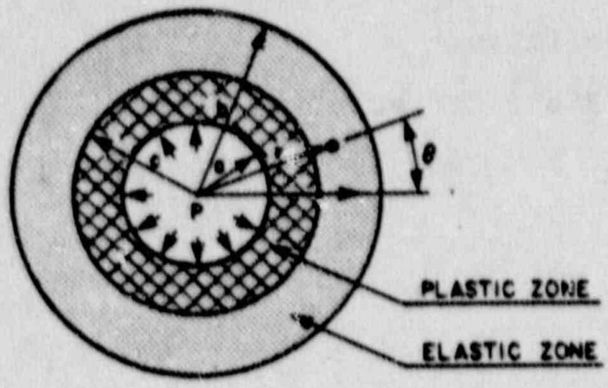


Fig. 3.2.1.3-1 Elastic-Plastic Analysis of a Thick-Walled Cylinder

Objective

The objective of this problem is to test the solution process for elastic-plastic material properties against an analytic solution. This problem has finite boundaries and thus, the accuracy of the analysis should depend only on the fineness of the user defined mesh. Specific aspects of the code tested by this problem include:

(1) application of pressure boundary conditions; and

(2) computation of plastic stresses and deformations.

Problem Specifications

The following problem specifications apply (see Fig. 3.2.1.3-1):

Physical Dimensions	
inside radius	4 m
outside radius	6 m
Elastic Constants	
modulus of elasticity	50 GPa
Poisson's ratio	0.20
Mohr-Coulomb Yield Criteria	
cohesion	170 MPa
friction	0°
Internal Pressure	100, 115, 130 MPa

Assumptions

Assumptions which are implicit in the analytic solution include:

(1) the material is homogeneous and isotropic;

(2) plane strain conditions;

(3) strains are small; and

(4) the Mohr-Coulomb Yield Criterion applies.

Analytical Solution

The analytical solution for part (a) of this problem is given by Ford and Alexander (1977). Prior to initial yield, the stresses and displacements are:

$$\sigma_r = - \frac{P\,[(b/r)^2 - 1]}{(b/a)^2 - 1} \qquad (3.2.1.3\text{-}1)$$

$$\sigma_\theta = -\frac{P\,[(b/r)^2 + 1]}{(b/a)^2 - 1} \qquad (3.2.1.3\text{-}2)$$

$$u = \frac{P\,(1 + \nu)}{E\,[(b/a)^2 - 1]}\left[(1 - 2\nu)\,r + \frac{b^2}{r}\right] \qquad (3.2.1.3\text{-}3)$$

where r = radial coordinate,

σ_r, σ_θ = stresses,

u = radial displacement,

P = internal pressure,

a,b = inside and outside radii, respectively,

ν = Poisson's ratio, and

E = modulus of elasticity.

Yielding is based on the Mohr-Coulomb failure criterion which states that the yield occurs at a constant maximum shear stress, k_T. The internal pressure at initial yield, P_y, by the Mohr-Coulomb criterion is:

$$P_y = k_T\,[1 - (a/b)^2] \qquad (3.2.1.3\text{-}4)$$

After initial yielding, a plastic zone will be created which will interface with an outer elastic zone at radius, c. The stresses in the elastic and plastic zones are:

Plastic Zone $r<c$	Elastic Zone $r>c$
$\sigma_r = 2\ k_T \left[-\ln \frac{c}{r} - \frac{1}{2}\left[1 - \frac{c^2}{b^2} \right] \right]$	$\sigma_r = -k_T \frac{c^2}{b^2} \left[\frac{b^2}{r^2} - 1 \right]$

(3.2.1.3-5)

$$\sigma_\theta = 2\ K_T \left[-\ln \frac{c}{r} + \frac{1}{2}\left[1 + \frac{c^2}{b^2} \right] \right] \qquad \sigma_\theta = k_T \frac{c^2}{b^2} \left[\frac{b^2}{r^2} + 1 \right]$$

(3.2.1.3-6)

Within the elastic zone, the displacements are given by:

$$u = \frac{k_T\ (1 + \upsilon)\ c^2}{E\ b^2} \left[(1 - 2\upsilon)\ r + \frac{b^2}{r} \right] \qquad (3.2.1.3\text{-}7)$$

The value of c is determined from the boundary condition that $\sigma_r = -P$ at $r = a$, which leads to:

$$\frac{P}{2k_T} = \ln \frac{c}{a} + \frac{1}{2}\left[1 - \frac{c^2}{b^2} \right] \qquad (3.2.1.3\text{-}8)$$

A copy of the FORTRAN source code used to compute the analytic solution for this problem is provided in Appendix 3-2.1.3-A.

Computer Model

The computer model consists of one-quarter of the cylinder, with symmetry conditions imposed on the horizontal and vertical surfaces. Because UDEC requires at least two blocks to be modeled, the quarter-cylinder was divided along the line $\theta = 45°$ by a "glued" discontinuity. The two blocks were discretized into zones with maximum edges lengths of 0.3, 0.5, or 1m. Figure 3.2.1.3-2 shows the resulting discretization for maximum edge length of 0.3 m.

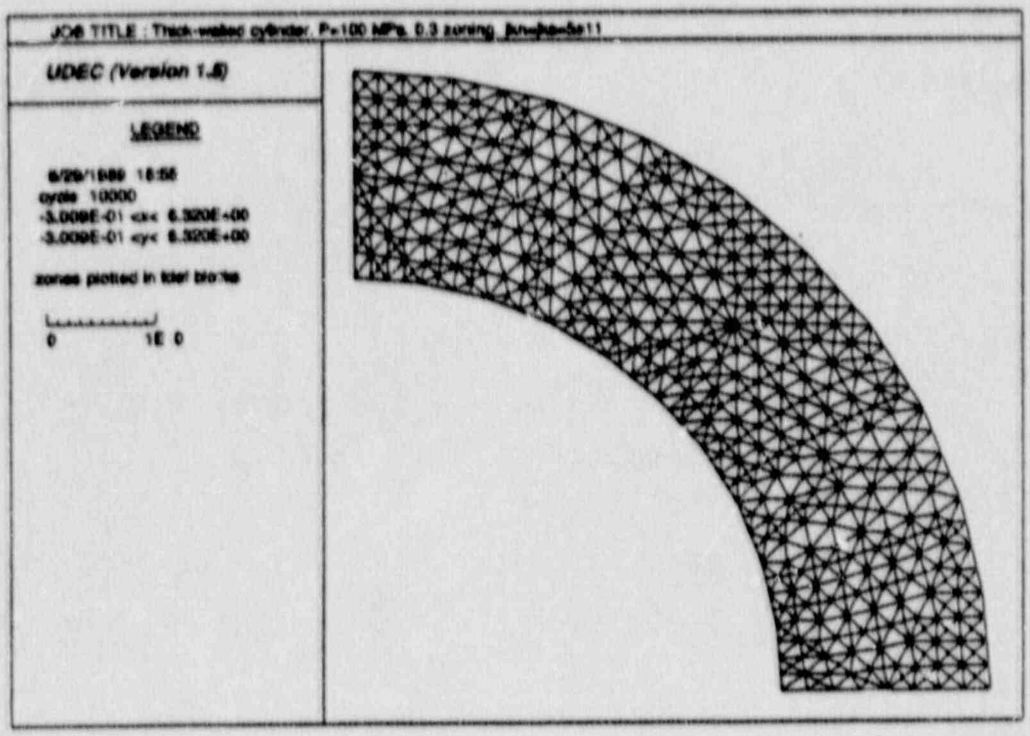


Fig. 3.2.1.3-2 Discretization of Thick-Walled Cylinder into Constant Strain Finite Difference Triangles (maximum zone edge length = 0.3 m)

Results

For each internal pressure, and each of the three discretizations, the UDEC results are compared to the analytic results in terms of plastic radius, c, tangential stresses at both the inner and outer wall, and outer wall displacements, as shown in Table 3.2.1.3-1.

Table 3.2.1.3-1

COMPARISON OF UDEC AND ANALYTIC RESULTS FOR THICK-WALLED CYLINDER PROBLEMS

ANALYTIC SOLUTION					UDEC RESULTS			
c (m)	$\sigma_{\theta r=a}$ (MPa)	$\sigma_{\theta r=b}$ (MPa)	u (mm)	Zone size* (m)	c (m)	$\sigma_{\theta r=a}$ (MPa)	$\sigma_{\theta r=b}$ (MPa)	u (mm)
Internal Pressure = 100MPa								
4.12	240	160	18	0.3	4.23	235	155	20
				0.5	4.16	250	160	19
				1.0	4.10	260	160	19
Internal Pressure = 115MPa								
4.52	224	193	22	0.3	4.65	225	190	23
				0.5	4.66	230	195	23
				1.0	4.49	230	185	22
Internal Pressure = 130MPa								
5.11	247	210	28	0.3	5.18	240	210	29
				0.5	5.19	250	210	29
				1.0	5.33	235	210	29

*input maximum zone edge length

The plastic radius, c, shown in Table 3.2.1.3-1 was calculated for UDEC results as follows:

N_e = number of elastic zones

N_p = number of plastic zones

N_t = number of total zones

$$\frac{N_e}{N_t} = \frac{\text{Plastic Area}}{\text{Total Area}} = \frac{N_t - N_e}{N_t} = \frac{\pi\ (c^2 - 4^2)}{\pi\ (b^2 - 4^2)}$$

$$\frac{N_t - N_e}{N_t} = \frac{c^2 - 16}{20}$$

$$c = \left[20\ \frac{N_t - N_e}{N_t} + 16 \right]^{1/2}$$

The values of tangential stress for UDEC were obtained from contour plots of maximum principal stresses. Outer wall displacements were determined from displacement plots or displacement histories.

Discussion

The results shown in Table 3.2.1.3-1 indicate reasonable agreement between the analytic solution and UDEC results for maximum zone edge lengths of 0.3 to 1.0 meters. All results shown in Table 3.2.1.3-1 were obtained using joint normal stiffness equal to the joint shear stiffness equal to 500GPa/m.

Reference

Ford, Hugh, and J. M. Alexander. Advanced Mechanics of Materials. New York: Halsted Press, 1967.

Wart, R. J., E. L. Skiba and R. H. Curtis. Benchmark Problems for Repository Design Models. NUREG/CR-3636. February 1984.

Data Input File

```
* Thick-walled cylinder subjected to internal pressure

he
Thick-walled cylinder. P=130 MPa. jkn=jks=4.5e11

round=.01
bl 0 0 0 6 6 6 6 0
arc 0 0 4 0 90 10
arc 0 0 6 0 90 10
del 0 3 0 3
cr 0 0 5 5
del 4.5 6 4.5 6

prop m=1 de=2000 k=2.78e10 g=2.08e10 jkn=4.5e11 jks=4.5e11
prop m=1 coh=170e6 fric=0 ten=1e20
prop m=1 jcoh=170e6 jfric=0 jten=1e20

gen edge 1
ch cons=3

damp auto

hi ncyc 100
hi unbal
hi ydis 0 4
hi ydis 0 6

bo 0 4.2 0 4.2 st -130e6 0 -130e6
bo 3.9 7 -.1 .1 yvel 0
bo -.1 .1 3.9 7 xvel 0

cy 1500
save tcsave.035

quit
```

Appendix 3.2.1.3-A

```
c     Analytical solution for Thick-Walled Cylinder

      dimension c(200),d(200)
      real kt,nu
      character ch

      open (unit=11,file='tcres')

c     set parameters
      a=4.
      b=6.
      kt=170.e6
      nu=.2
      e=50000.e6
5     write (*,10)
10    format (' internal pressure  (MPa, compression>0) ? ')
      read (*,*) p
      p=p*10.**6

      write (11,11)
11    format ('Analytical solution for Thick-Walled Cylinder',//)

c     compute plastic zone radius
      dd=9999.
      do 100 i=1,200
      c(i)=4.+(b-a)/200.*(i-1)
      d(i)=p/2./kt-alog(c(i)/a)-.5*(1.-c(i)**2/b**2)
      if (abs(d(i)).lt.dd) then
      dd=abs(d(i))
      cc=c(i)
      endif
100   continue

      write (*,110) cc
      write (11,105) p
      write (11,110) cc
105   format ('internal pressure:',f14.1,' Pa')
110   format (' plastic zone radius:',f10.7)

c     compute stresses in plastic zone, for inner wall (r=a)
      sigr=2.*kt*(-alog(cc/a)-.5*(1-cc**2/b**2))
      sigt=2.*kt*(-alog(cc/a)+.5*(1+cc**2/b**2))
      write (*,120) sigr,sigt
      write (11,120) sigr,sigt
```

```
c     compute stresses and displacement for outer wall (r=b)
      sigr=0.
      sigt=2.*kt*cc**2/b**2
      u=kt*(1+nu)*cc**2/e/b**2*((1.-2.*nu)*b+b)
      write (*,130) sigr,sigt
      write (11,130) sigr,sigt
      write (*,140) u
      write (11,140) u

120   format (' stresses for inner wall: sigr=',f14.1,'
     *sigt=',f14.1)
130   format (' stresses for outer wall: sigr=',f14.1,'
     *sigt=',f14.1)
140   format (' outer wall displacement: u=',f6.3,/)

      write (*,150)
150   format (' another load (y/n) ?')
      read (*,160) ch
160   format (a1)
      if (ch.eq.'y'.or.ch.eq.'Y') goto 5

      close (11)

      stop
      end
```

3.2.1.4 Elasto-Plastic Response of an Unlined Circular Tunnel in a Biaxial Stress Field

Problem Statement

Crushing failure is identified as an important mechanism by which unlined tunnels may fail. Crushing is treated as a static phenomenon and involves massive failure around the excavation due to large-scale plastic flow. The purpose of this verification example is to demonstrate the ability of UDEC to model large-scale plastic flow. The verification was accomplished by comparing UDEC results to those from a closed-form solution which includes plastic flow behavior.

The problem involves a circular tunnel subjected to a non-hydrostatic static load. The medium surrounding the tunnel is treated as an elasto-plastic material with failure defined by a Mohr-Coulomb yield function. The dilatancy of the material at failure is defined by the plasticity flow rule, which is characterized by the dilatancy angle. Both fully-dilatant and non-dilatant material behaviors are verified.

Objective

The objective of this problem is to test the elasto-plastic material model used to describe the non-linear deformational behavior of fully-deformable blocks in UDEC. This test specifically addresses the ability of the code to simulate plastic flow accurately.

Physical Problem

A tunnel is excavated in a rock mass which is isotropic and elasto-plastic. The following parameters and values are used to describe the elastic and plastic behavior:

Young's modulus (E)	1.7×10^6 psi	(11.72 GPa)
Poisson's ratio (ν)	0.25	
cohesion (C)	1443 psi	(9.9 MPa)
angle of internal friction (ϕ)	30°	

The strength parameters, C and ϕ, correspond to an unconfined compressive strength, q, of 5000 psi (34.5 MPa). The tests are performed to verify the representation of dilatancy in UDEC. In the first test, no dilatancy is permitted (i.e., the dilatancy angle is set equal to zero). In the second test, fully-dilatant behavior is allowed, with

$$\psi = \phi = 30°$$

where ψ is the dilation angle.

A two inch (51mm) diameter circular tunnel is used for this test. A non-hydrostatic loading path is applied as an external load starting with no initial stress. The major principal stress, σ_1, applied in the vertical direction, and the minor principal stress, σ_3, applied in the horizontal direction, are both increased simultaneously to peak values of σ_1 = 12500 psi (86 MPa) and σ_3 = 7500 psi (52 MPa). The loading is in steps keeping the ratio of σ_1/σ_3 constant at 2. The load steps, (expressed in normalized form) shown in Table 1 were used. Tunnel closures (expressed as a percentage of the tunnel radius, a) are monitored at the springline and the crown.

Table 3.2.1.4-1

LOADING STEPS USED IN ANALYSIS OF CIRCULAR TUNNEL IN A NON-HYDROSTATIC STRESS FIELD

Step	$\frac{\sigma_1 + \sigma_3}{q}$	$\frac{\sigma_1 - \sigma_3}{q}$
	1.0	0.25
1	1.5	0.375
2	2.0	0.5
3	2.5	0.625
4	3.0	0.75
5	3.5	0.875
6	4.0	1.0

Analytical Solution

Two conventional closed-form techniques used for preliminary analyses of circular tunnels subjected to far-field mechanical loading are the solutions presented by Newmark et al. (1970) and Hendron and Aiyer (1971). These solutions idealize the problem as a static, two-dimensional analysis of a circular tunnel in a hydrostatic stress field. The surrounding medium is treated as an elasto-plastic material with failure defined by a Mohr-Coulomb yield function. The dilatancy of the material at failure is defined by the plasticity flow rule, which is characterized by the dilatancy angle. The Newmark solution assumes a fully non-associated flow rule (i.e., no dilatancy occurs at failure). The Hendron and Aiyer solution assumes a fully associated flow rule (i.e., the dilatancy angle equals the friction angle).

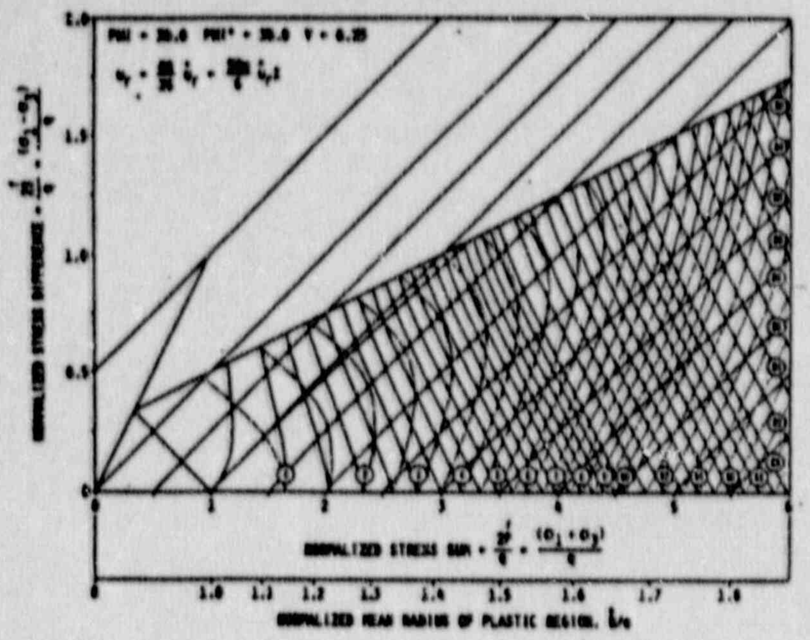
Detournay (1983) provided an extension to the solution for non-hydrostatic loading by the development of a semi-analytical technique. This approach applies for arbitrary dilatancy of the material and, therefore, makes the solutions of Newmark and Hendron and Aiyer special cases of the Detournay solution. For this reason, the Detournay solution was selected as a more rigorous verification test of UDEC.

It is important to note that all three solutions are based on infinitesimal (small) strain theory, which assumes that the initial geometry of a deforming body is not appreciably altered during the deformation process. The consequence of this assumption is discussed later.

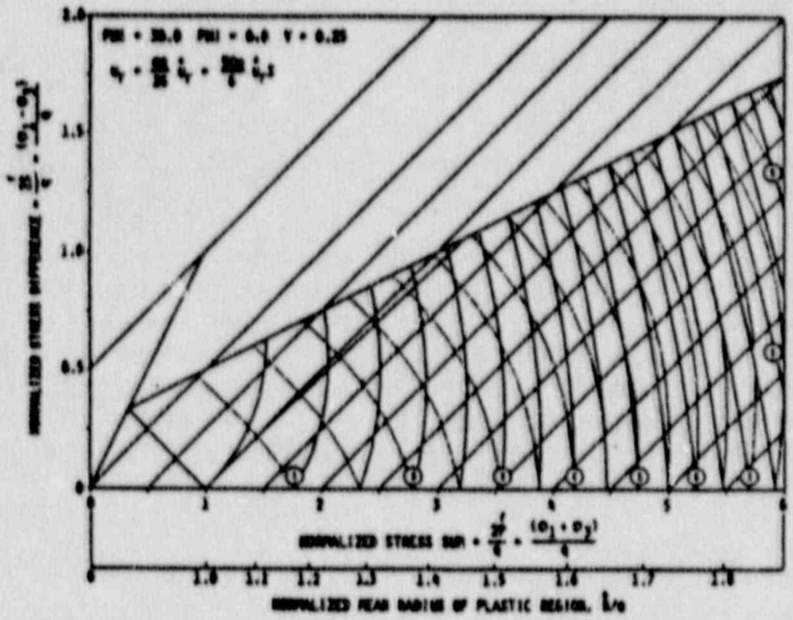
Detournay developed a set of design charts which consist of contours of springline and crown displacement. Figure 3.2.1.4-1 presents two charts, one for dilation angle equal to 30°, and one for dilation angle equal to zero. The normalized displacements for any particular free field stresses can be read from the charts by interpolating between the plotted contours. Actual radial displacements (U_r) can be calculated from the normalized displacements (U^*_r) from :

$$U_r = \frac{a\,q}{2G}\; U_r^*$$

where G is the elastic shear modulus of the material.



(a) fully-dilatant ($\phi = \psi = 30°$)



(b) no dilatancy ($\phi = 30°$; $\psi = 0$)

Fig. 3.2.1.4-1 Normalized Radial Displacements (U_r^*) of the Springline (Solid) and Crown (Dashed) Results of Closed Form Solution [after St. John et al., 1984]

Alternatively, the percentage closure (u_r) can be expressed:

$$u_r = \frac{50q}{G} \quad U_r^* \quad (\%)$$

These displacements apply for the case of a tunnel excavation in a rock mass previously stressed to the far-field stress state. The charts therefore calculate displacements due to the initial state of stress. The displacements induced by additional external loading differ from those calculated by the charts by an amount equal to the elastic displacements that would occur in the absence of the tunnel. The corrections for added external loading are:

at the crown:

$$(\Delta U_r^*)_c = (1-2\nu) \quad \frac{\sigma_1 + \sigma_3}{2q} + \frac{\sigma_1 - \sigma_3}{2q}$$

at the springline:

$$(\Delta U_r^*)_s = (1-2\nu) \quad \frac{\sigma_1 + \sigma_3}{2q} - \frac{\sigma_1 - \sigma_3}{2q}$$

The percentage closure for added external loading is then

$$u_r = \frac{50q}{G} (U_r^* + \Delta U_r^*)$$

The calculated closures for the physical problem described above are summarized in Table 3.2.1.4-2.

These results demonstrate the significant influence of dilatancy on the deformation of the tunnel at the springline. For these problem conditions, the closure at the springline is nearly three times greater for the dilatant material versus non-dilatant material, while the closure at the crown is virtually not affected.

Table 3.2.1.4-2

CALCULATED CLOSURE FROM DETOURNAY SOLUTION

	$\frac{\sigma_1 - \sigma_3}{q}$	$\frac{\sigma_1 + \sigma_3}{q}$	Springline			Crown		
			U_r^*	ΔU_r^*	u_r%	U_r^*	ΔU^*_r	u_r%
ψ=0	0.375	1.5	0.55	0.19	0.27	1.25	0.56	0.67
	0.50	2.0	0.9	0.25	0.42	1.75	0.75	0.92
	0.625	2.5	1.5	0.31	0.66	2.4	0.94	1.23
	0.75	3.0	2.25	0.38	0.97	3.15	1.12	1.57
	0.875	3.5	3.1	0.44	1.30	3.8	1.31	1.88
	1.0	4.0	4.0	0.5	1.65	4.7	1.5	2.28
ψ=30°	0.375	1.5	0.95	0.19	0.42	1.25	0.56	0.66
	0.50	2.0	2.0	0.25	0.83	1.75	0.75	0.92
	0.625	2.5	3.75	0.31	1.49	2.35	0.94	1.21
	0.75	3.0	6.0	0.38	2.35	3.05	1.12	1.53
	0.875	3.5	9.0	0.44	3.47	3.8	1.31	1.88
	1.0	4.0	12.8	0.50	4.89	4.75	1.5	2.30

Assumptions

The material deformation model used in UDEC is based upon finite strain theory. Comparisons between small and large strain calculations made by others (e.g., Carter et al., 1977) demonstrate that at a given strain level, compressive stresses will be higher for a large strain calculation than for a small strain calculation. This difference is attributed to the change in stress rate vector as well as the change in strain rate vector, which is accounted for in the large strain formulation and leads to increased stress concentration with increased deformation. The small strain formulation used in the closed-form solutions thus will give a more conservative (higher) calculation for tunnel closure than that calculated with the large strain formulation.

The large closure produced for the given problem conditions, particularly at the associated flow state, poses a rigorous test for the failure model used in UDEC. Problems which involve large strain and collapse require a numerical scheme which allows locally incompressible plastic flow.

Constant-strain triangular elements such as those used in UDEC tend to inhibit incompressible plastic flow and may produce an excessively stiff and incorrect calculation for plastic flow. Nagtegaal et al. (1974) discuss procedures to improve the representation of plastic flow for triangular elements. One technique is the mixed-discretization procedure (Marti and Cundall, 1982), which reduces the constraints on plastic flow by using different numerical discretization for the isotropic and deviatoric parts of the strain tensor. This scheme works well for uniform grids composed of equal pairs of triangular elements.

Mixed-discretization is not used in UDEC because the creation of arbitrarily-shaped blocks makes the discretization of uniform grids of paired triangular elements difficult. An alternative approach used in UDEC for this test problem is to first divide the model such that a grid of diagonally-opposed triangles can be generated immediately adjacent to the excavation. Nagtegaal et al, (1974) showed that meshes composed of diagonally-opposed triangles also will produce a good representation for plastic flow.

Computer Model

The UDEC model used for the given test problem is illustrated in Fig. 3.2.1.4-2. The model is one quadrant of the tunnel and surrounding rock. The bottom and left boundaries shown in the figure are lines of symmetry. The model is divided into a series of concentric "rings" with increasing spacing between "ring" cuts. In this way, the block zoning can be increased away from the hole. In the first few "rings" adjacent to the hole, it is possible to create a mesh of diagonally-opposed triangular zones. The zoning is shown in Fig. 3.2.1.4-3.

The tunnel closure results are very sensitive to the location of the model boundaries. Goodman (1980, p. 236) notes that plastic behavior of the region in the vicinity of a tunnel has the effect of extending the influence of the tunnel a considerable distance into the surrounding rock. For elasto-plastic behavior, a distance 10 tunnel radii from the tunnel is required to bring the stress perturbation to within 10% of the initial stress state. For this problem the model outer boundaries are located 20 radii from the tunnel.

The model consists of 11 ring blocks divided into 2600 zones. The joints between the blocks are "glued" by setting the cohesion and tensile strength of the contacts to values much higher than the applied loads. The normal and shear stiffnesses of the joints are set equal to 100,000 GPa/m (286,000 ksi/in), which produces an equivalent elastic modulus for the model within 1.5% of the given Young's modulus.

The model loading is in accordance with the load steps defined in Table 3.2.1.4-1 and closures are calculated at the crown and springline of the tunnel.

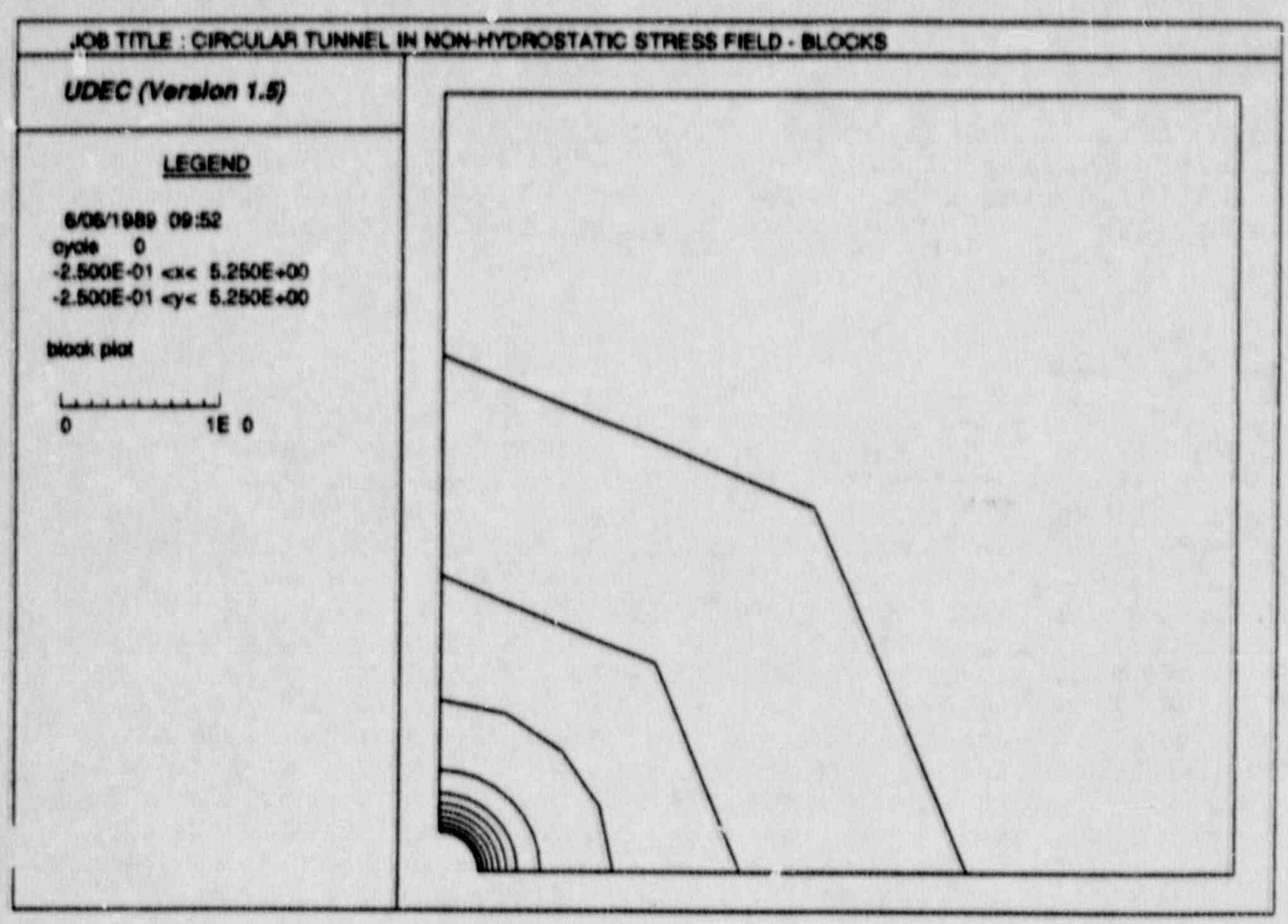
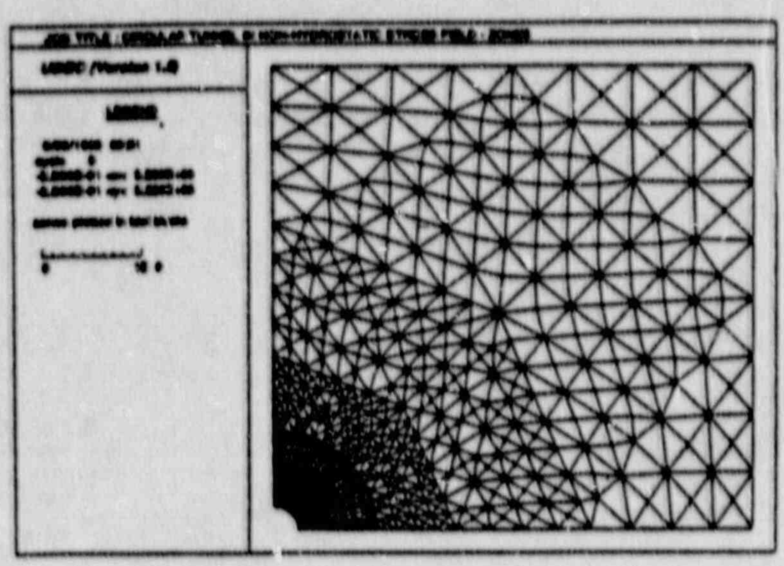
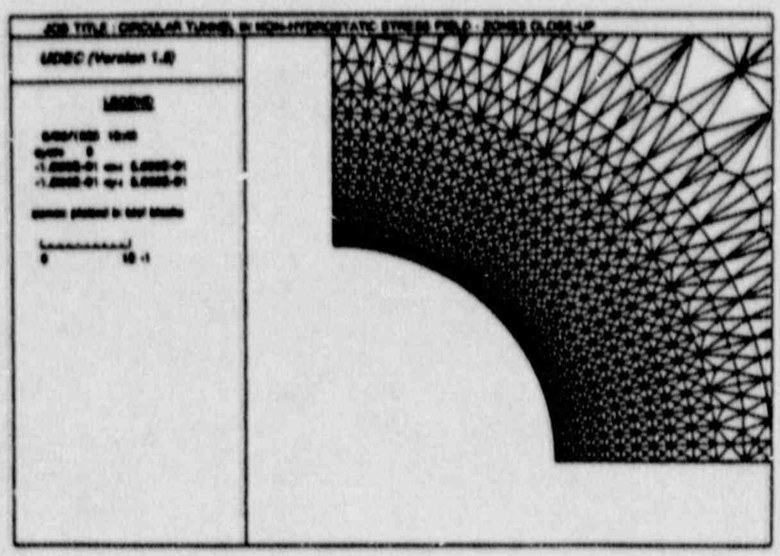


Fig. 3.2.1.4-2 Plot of "Glued" Joints in UDEC Model Used to Improve Zonal Discretization in Model



(a) problem discretization



(b) problem discretization near tunnel periphery

Fig. 3.2.1.4-3 UDEC Model

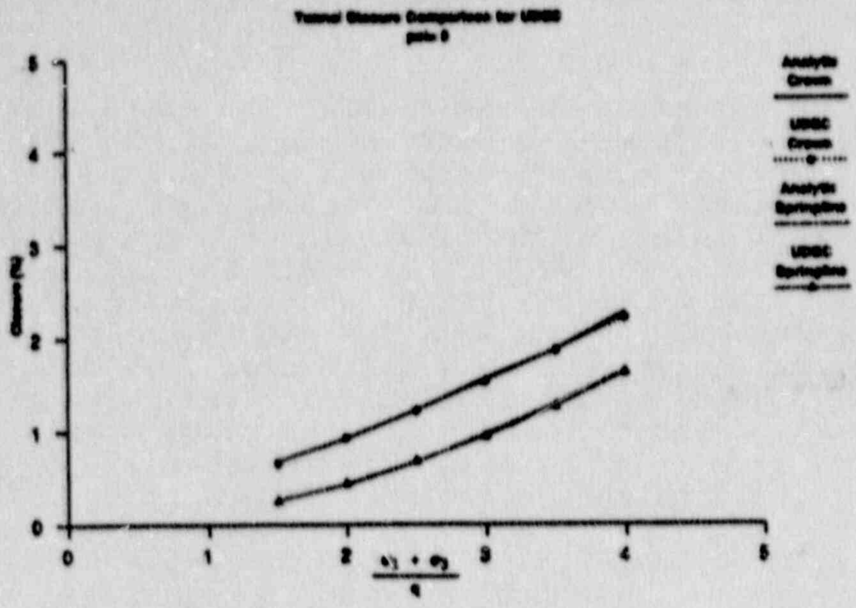
Results

The comparison of the UDEC results, for non-dilatant and fully-dilatant material behavior, to the Detournay solution is given in Table 3.2.1.4-3 and graphically in Fig. 3.2.1.4-4.

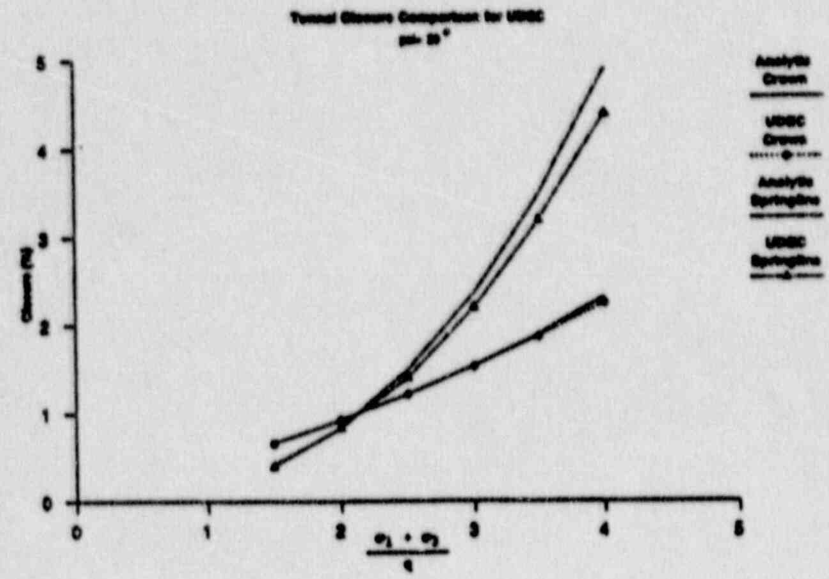
Table 3.2.1.4-3

COMPARISON OF UDEC RESULTS TO DETOURNAY SOLUTION

		Crown Closure			Springline Closure		
		analytic solution (%)	UDEC %	error %	analytic solution (%)	UDEC %	error %
Elastic		0.620	0.620	0	0.207	0.204	-1.4
Elasto-plastic ($\psi = 0°$)							
Step	1	0.67	0.655	-2.2	0.27	0.256	-5.2
	2	0.92	0.927	+0.8	0.42	0.435	3.6
	3	1.23	1.224	-0.5	0.66	0.677	2.6
	4	1.57	1.538	-2.0	0.97	0.942	-2.9
	5	1.88	1.871	-0.9	1.30	1.268	-2.5
	6	2.28	2.221	-2.6	1.65	1.629	-1.3
Elasto-plastic ($\psi = 30°$)							
Step	1	0.66	0.654	-0.9	0.42	0.394	-6.2
	2	0.92	0.922	0.2	0.83	0.831	0.1
	3	1.21	1.209	-0.1	1.49	1.397	-6.2
	4	1.53	1.523	-0.5	2.35	2.187	-6.9
	5	1.88	1.859	-1.1	3.47	3.178	-8.4
	6	2.30	2.232	-3.0	4.89	4.378	-10.5



(a) elasto-plastic (ψ = 0°)



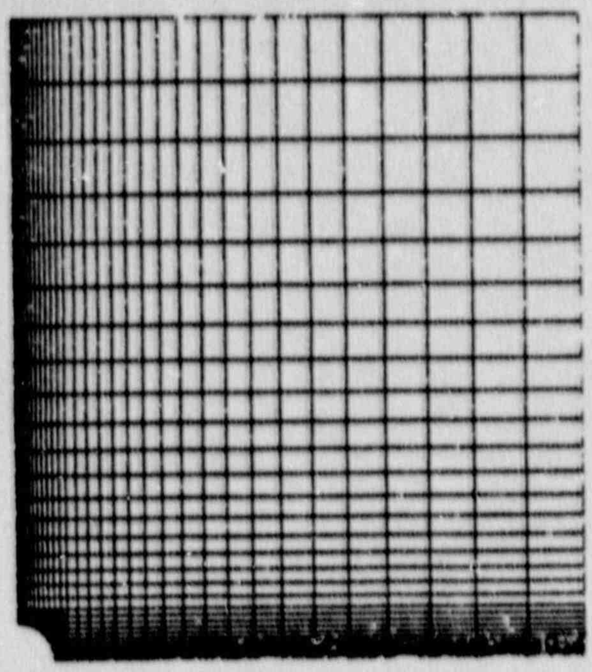
(b) elasto-plastic (ψ = 30°)

Fig. 3.2.1.4-4 Comparison of Crown and Springline Closures for UDEC and Analytic Solutions

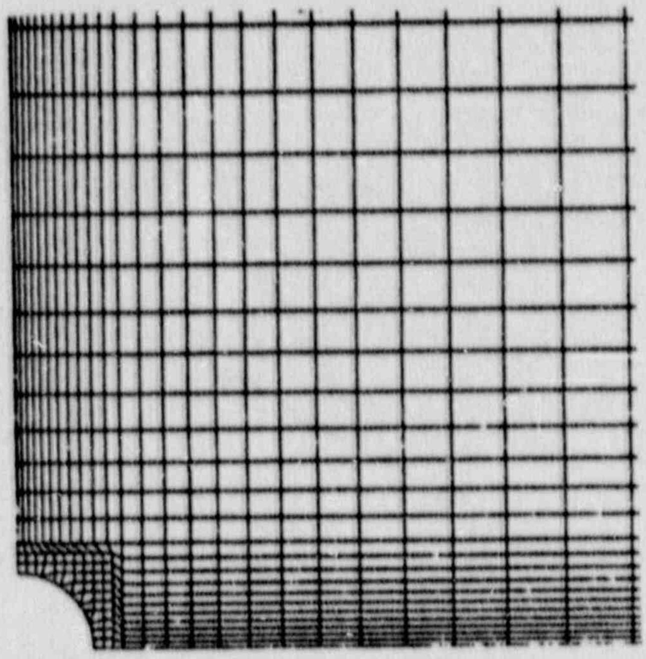
Discussion

In general, the agreement is reasonable; the average error can be attributed to the differences between the small and large strain formulation. The identical problem was also solved with the continuum finite difference code FLAC (Itasca, 1988), which can perform calculations in either small strain or large stain mode. The FLAC grid is shown in Fig. 3.2.1.4-5. The results for the two modes are compared to the Detournay solution in Fig. 3.2.1.4-6. The agreement between the solution and the results given by FLAC for the small strain mode is quite close, as shown by the figure. The closure results from FLAC for the large strain mode are consistently lower than those for the small strain solution, as much as 10% lower for the springline closure at the peak load. The results agree with those from the UDEC solution.

The plasticity model appears to perform correctly in UDEC. A fine mesh and model boundaries at least 10 tunnel radii from the tunnel are required, though, to produce accurate displacement calculations for plasticity analysis with the code.

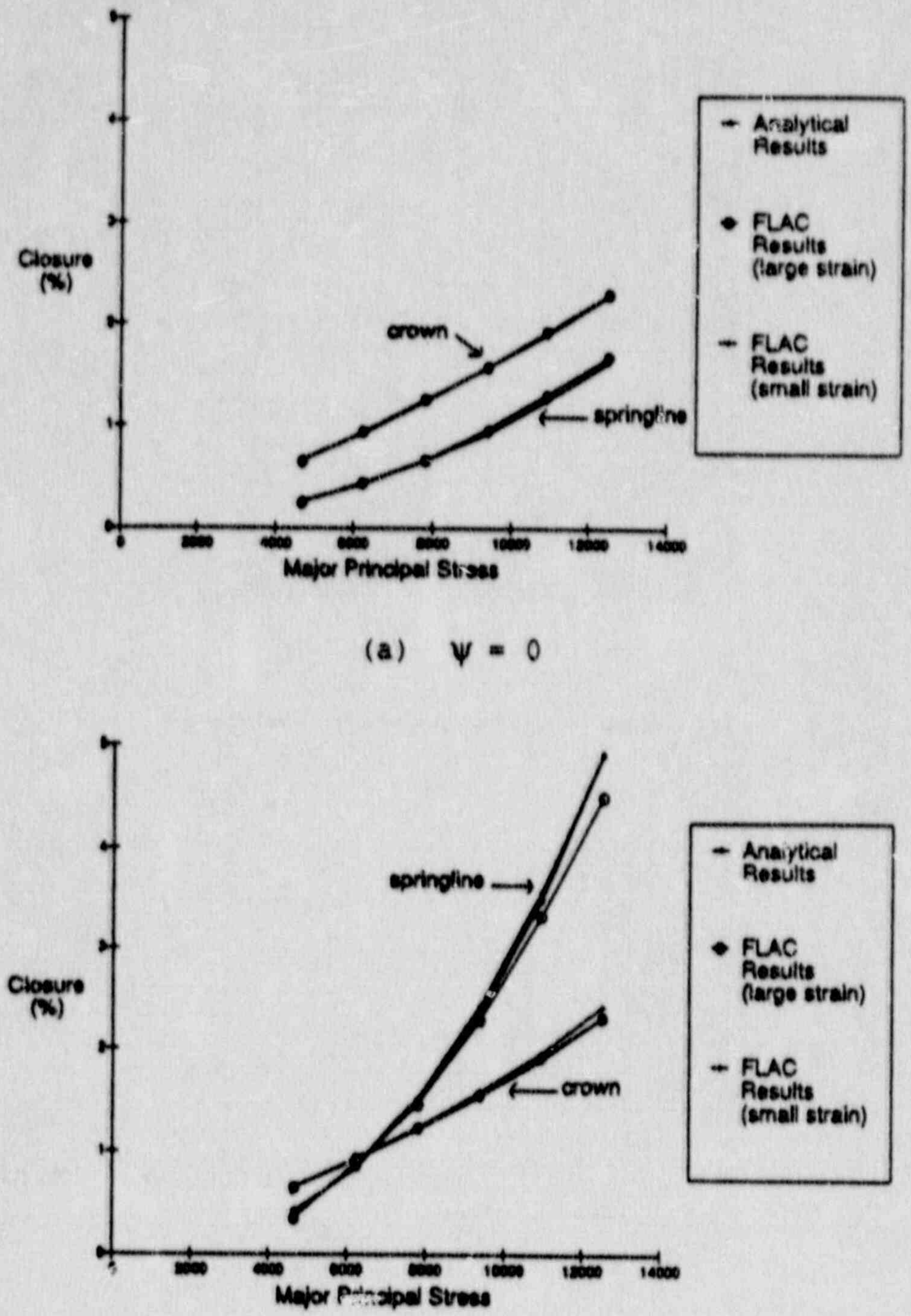


(a) whole mesh



(b) close-up view around tunnel

Fig. 3.2.1.4-5 FLAC Model



(a) ψ = 0

(b) ψ = 30°

Fig. 3.2.1.4-6 Tunnel Closure Comparison for FLAC

References

Carter, J. P., J. R. Booker and E. H. Davis. "Finite Deformation of an Elasto-Plastic Soil," Int. J. Num. & Analy. Meth. Geomech., 1, 25-43 (1977).

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Marti, J. and P. A. Cundall. "Mixed Discretization Procedure for Accurate Solution of Plasticity Problems," Int. J. Num. Meth. Eng., 6, 129-139 (1982).

Nagtegaal, J. C., D. M. Parks and J. R. Rice. "On Numerically Accurate Finite Element Solutions in the Fully Plastic Range," Comp. Meth. Appl. Mech., 4, 153-177 (1974).

Newmark, N. M. et al. "Ground Motion Technology Review," SAMSO, TR-70-114, Nathan M. Newmark Consulting Engineering Services (Urbana, Illinois), April 1970.

St. John, C. M., E. Detournay and C. Fairhurst. "Design Charts for a Deep Circular Tunnel Under Non-Hydrostatic Loading," in Rock Mechanics in Productivity and Protection (Proceedings of 25th Symposium on Rock Mechanics, Northwestern University, June, 1984), pp. 849-856. New York: American Institute of Mining, Metallurgical and Petroleum Engineers, 1984.

Data Input Files

```
set log on
*
* verification test
* circular tunnel in a non-hydrostatic stress field
* Mohr-Coulomb material
* associated flow:
* friction angle = 30 degrees , dilation angle = 30 degrees
*
round = 0.002
block 0 0 0 5 5 5 5 0
*
tunnel 0 0 3.32   8
tunnel 0 0 1.9    8
tunnel 0 0 1.1    16
tunnel 0 0 0.65   32
tunnel 0 0 0.5    128
tunnel 0 0 0.435  64
tunnel 0 0 0.385  64
tunnel 0 0 0.345  64
tunnel 0 0 0.309  64
tunnel 0 0 0.280  64
tunnel 0 0 0.254  64
*
delete 0 0.15 0 0.15
*
gen 0 0.175 0 0.175  edge 0.014
gen 0 0.19  0 0.19   edge 0.016
gen 0 0.21  0 0.21   edge 0.019
gen 0 0.24  0 0.24   edge 0.023
gen 0 0.27  0 0.27   edge 0.026
gen 0 0.31  0 0.31   edge 0.040
gen 0 0.40  0 0.40   edge 0.09
gen 0.5 0.6 0.5 0.6  edge 0.12
gen 0.6 1.1 0.6 1.1  edge 0.21
gen 1.4 1.7 1.4 1.7  edge 0.36
gen                  edge 0.63
*
save epd30b.sav
*
* define material properties
prop mat=1 den=1850 k=7.814e9 g=4.69e9
prop mat=1 coh=9.95e20 fri=.5774 dil=0.5774
* glue joints
prop mat=1 jkn=1e14 jks=1e14 jcoh=1e20 jten=1e20
*
damp auto 0.5 0.99 1.02
```

```
mscale on
insitu stress -1 0 -1
hist ncyc 50 ty 1 xdis (0.254,0) ydis (0,0.254) damp unbal
*
* initial load - elastic
bound -0.1,5.1   4.9,5.1    stress (0,0,-32.32e6)
bound  4.9,5.1 -0.1,5.1     stress (-19.39e6,0,0)
bound -.01,.01 -0.1,5.1     xvel 0.0
bound -0.1,5.1 -0.01,0.01   yvel 0.0
*
set dscan 10000
*
cycle 6000
*
save epd30e.sav
prop mat=1 coh=9.95e20 fri=.5774 dil=0.5774
*
reset damp
*
* allow plastic failure
change cons 3
prop mat 1 coh 9.95e6
*
cycle 6000
*
save epd30p.sav
*
reset damp
*
* step 1
bound -0.1,5.1   4.9,5.1    stress (0,0,-10.77e6)
bound  4.9,5.1 -0.1,5.1     stress (-6.47e6,0,0)
bound -.01,.01 -0.1,5.1     xvel 0.0
bound -0.1,5.1 -0.01,0.01   yvel 0.0
*
cycle 6000
*
save epd30p2.sav
*
reset damp
*
* step 2
bound -0.1,5.1   4.9,5.1    stress (0,0,-10.77e6)
bound  4.9,5.1 -0.1,5.1     stress (-6.47e6,0,0)
bound -.01,.01 -0.1,5.1     xvel 0.0
bound -0.1,5.1 -0.01,0.01   yvel 0.0
*
cycle 6000
```

```
*
save epd30p2.sav
*
reset damp
*
* step 3
bound -0.1,5.1   4.9,5.1     stress (0,0,-10.77e6)
bound  4.9,5.1 -0.1,5.1      stress (-6.47e6,0,0)
bound -.01,.01 -0.1,5.1      xvel 0.0
bound -0.1,5.1 -0.01,0.01    yvel 0.0
*
cycle 6000
*
save epd30p2.sav
*
reset damp
*
* step 4
bound -0.1,5.1   4.9,5.1     stress (0,0,-10.77e6)
bound  4.9,5.1 -0.1,5.1      stress (-6.47e6,0,0)
bound -.01,.01 -0.1,5.1      xvel 0.0
bound -0.1,5.1 -0.01,0.01    yvel 0.0
*
cycle 6000
*
save epd30p2.sav
*
reset damp
*
* step 5
bound -0.1,5.1   4.9,5.1     stress (0,0,-10.77e6)
bound  4.9,5.1 -0.1,5.1      stress (-6.47e6,0,0)
bound -.01,.01 -0.1,5.1      xvel 0.0
bound -0.1,5.1 -0.01,0.01    yvel 0.0
*
cycle 6000
*
save epd30p2.sav
*
return
```

```
set log on
*
* verification test
* circular tunnel in a non-hydrostatic stress field
* Mohr-Coulomb material
* non-associated flow:
* friction angle = 30 degrees , dilation angle = 0 degrees
*
*
round = 0.002
block 0 0 0 5 5 5 5 0
*
tunnel 0 0 3.32   8
tunnel 0 0 1.9    8
tunnel 0 0 1.1    16
tunnel 0 0 0.65   32
tunnel 0 0 0.5    128
tunnel 0 0 0.435  64
tunnel 0 0 0.385  64
tunnel 0 0 0.345  64
tunnel 0 0 0.309  64
tunnel 0 0 0.280  64
tunnel 0 0 0.254  64
*
delete 0 0.15 0 0.15
*
gen 0 0.175 0 0.175  edge 0.014
gen 0 0.19  0 0.19   edge 0.016
gen 0 0.21  0 0.21   edge 0.019
gen 0 0.24  0 0.24   edge 0.023
gen 0 0.27  0 0.27   edge 0.026
gen 0 0.31  0 0.31   edge 0.040
gen 0 0.40  0 0.40   edge 0.09
gen 0.5 0.6 0.5 0.6  edge 0.12
gen 0.6 1.1 0.6 1.1  edge 0.21
gen 1.4 1.7 1.4 1.7  edge 0.36
gen                  edge 0.63
*
save epd0b.sav
*
* define material properties
prop mat=1 den=1850 k=7.814e9 g=4.69e9
prop mat=1 coh=9.95e20 fri=.5774 dil=0.0
* glue joints
prop mat=1 jkn=1e14 jks=1e14 jcoh=1e20 jten=1e20
*
damp auto 0.5 0.99 1.02
mscale on
```

```
insitu stress -1 0 -1
hist ncyc 50 ty 1 xdis (0.254,0) ydis (0,0.254) damp unbal
*
* initial load - elastic
bound -0.1,5.1   4.9,5.1     stress (0,0,-32.32e6)
bound  4.9,5.1 -0.1,5.1      stress (-19.39e6,0,0)
bound -.01,.01 -0.1,5.1      xvel 0.0
bound -0.1,5.1 -0.01,0.01    yvel 0.0
*
set dscan 10000
*
cycle 6000
*
save epd0e.sav
*
reset damp
*
* allow plastic failure
change cons 3
prop mat 1 coh 9.95e6
*
cycle 6000
*
save epd0p.sav
*
reset damp
*
* step 1
bound -0.1,5.1   4.9,5.1     stress (0,0,-10.77e6)
bound  4.9,5.1 -0.1,5.1      stress (-6.47e6,0,0)
bound -.01,.01 -0.1,5.1      xvel 0.0
bound -0.1,5.1 -0.01,0.01    yvel 0.0
*
cycle 6000
*
save epd0p2.sav
*
reset damp
*
* step 2
bound -0.1,5.1   4.9,5.1     stress (0,0,-10.77e6)
bound  4.9,5.1 -0.1,5.1      stress (-6.47e6,0,0)
bound -.01,.01 -0.1,5.1      xvel 0.0
bound -0.1,5.1 -0.01,0.01    yvel 0.0
*
cycle 6000
*
save epd0p2.sav
```

```
*
reset damp
*
* step 3
bound -0.1,5.1   4.9,5.1    stress (0,0,-10.77e6)
bound  4.9,5.1 -0.1,5.1     stress (-6.47e6,0,0)
bound -.01,.01 -0.1,5.1     xvel 0.0
bound -0.1,5.1 -0.01,0.01   yvel 0.0
*
cycle 6000
*
save epd0p2.sav
*
reset damp
*
* step 4
bound -0.1,5.1   4.9,5.1    stress (0,0,-10.77e6)
bound  4.9,5.1 -0.1,5.1     stress (-6.47e6,0,0)
bound -.01,.01 -0.1,5.1     xvel 0.0
bound -0.1,5.1 -0.01,0.01   yvel 0.0
*
cycle 6000
*
save epd0p2.sav
*
reset damp
*
* step 5
bound -0.1,5.1   4.9,5.1    stress (0,0,-10.77e6)
bound  4.9,5.1 -0.1,5.1     stress (-6.47e6,0,0)
bound -.01,.01 -0.1,5.1     xvel 0.0
bound -0.1,5.1 -0.01,0.01   yvel 0.0
*
cycle 6000
*
save epd0p2.sav
*
return
```

3.2.1.5 Circular Tunnel Problems Involving Use of Boundary Elements

Problem Statement

This problem concerns stress analysis of a long circular opening in an infinite medium under various boundary conditions and material properties (see Fig. 3.2.1.5-1). Three variations to this problem will be considered:

(1) Part A: tunnel in an elastic medium with a biaxial stress field;

(2) Part B: tunnel in an elastic-plastic medium with a hydrostatic stress field; and

(3) Part C: lined tunnel in an elastic medium with a biaxial stress field.

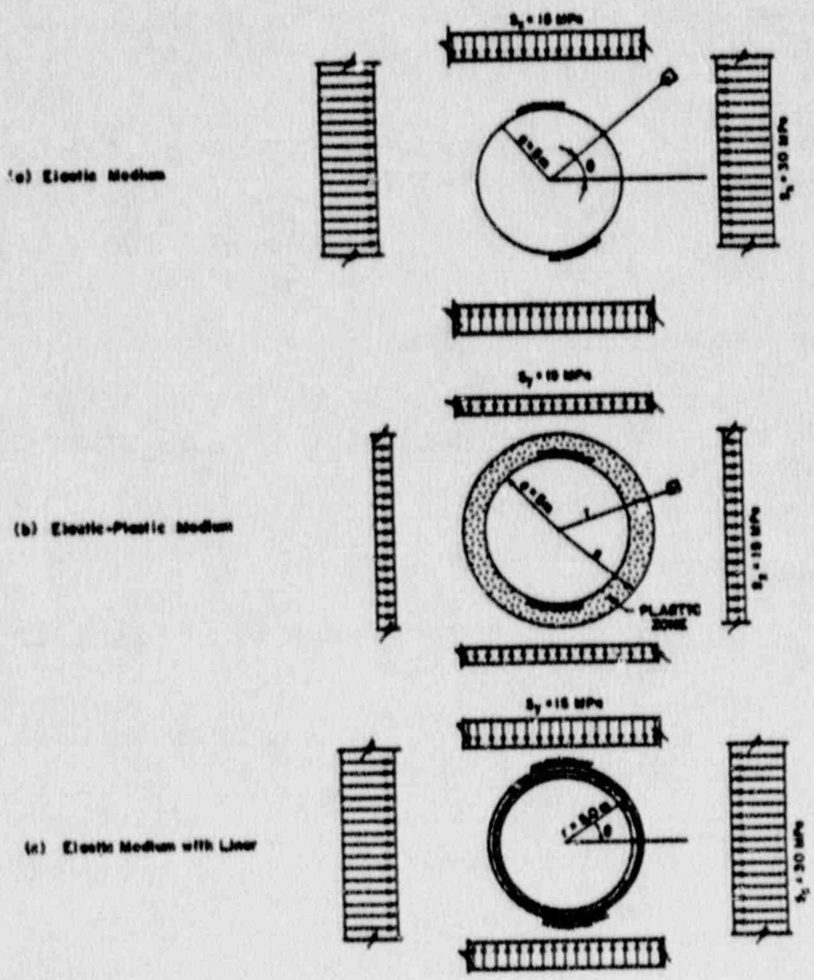

Fig. 3.2.1.5-1 Three Variations to the Circular Tunnel Problem [(after Wart et al., 1984]

Upon excavation of a tunnel, the in-situ stresses within the rock or soil mass are redistributed from a uniform orthogonal stress field to a more complex stress distribution. Stress concentrations around a tunnel cause elastic deformations at the periphery and, if the yield strength of the material is exceeded, result in plastic deformations and redistribution of stresses due to yielding of the material. In the case of plastic yielding, a yield zone will develop around the tunnel beyond which the stresses will be elastic. These processes are modeled by parts A and B of this problem.

Part C of this problem involves the interaction of a structural tunnel lining and an elastic media. Although the actual design of a tunnel lining is more complex, this problem checks the basic interaction between the two types of material for non-axisymmetric loadings.

Objective

This problem has the advantage of being similar to repository problems as well as having a closed-form analytical solution. Several aspects of the computer model will be tested by this problem:

(1) the ability of the code to simulate an infinite medium by boundary elements;

(2) the determination of displacements and stresses in a non-symmetric problem in two dimensions;

(3) the computation of plastic stresses and deformations; and

(4) the interaction between structural lining and rock or soil mass.

Physical Problem

The tunnel is excavated in a rock mass which is isotropic and elastic (Parts A and C) or elasto-plastic (Part B).

The following parameters and values are used to describe the problem.

Geometry

excavated tunnel radius (m)	$a = 5$

Material Properties

modulus of elasticity (GPa)	$E = 6$
Poisson's ratio	$\nu = 0.2$
cohesion (MPa)	$k = 10$
friction	$\phi = 20°$
density (kg/m³)	$\rho = 3000$

In-Situ Stresses

<u>Parts A and C</u>

horizontal stress (MPa)	$S_x = 30$
vertical stress (MPa)	$S_y = 15$

<u>Part B</u>

Horizontal Stress (MPa)	$S_x = 15$
vertical stress (MPa)	$S_y = 15$

Tunnel Lining Properties (Part C)

thickness (m)	$t = 0.5$
modulus of elasticity (GPa)	$E = 20$
Poisson's ratio	$\nu = 0.20$
density (kg/m³)	$\rho = 3000$

Note that the density is not required by the analytical solution, but some value must be provided in UDEC. The solution is independent of the choice of density.

Analytical Solutions

Part A - The analytical solution to Part A is the well-known Kirsch solution as reported by Goodman (1980).

Part B - The analytical solution to Part B is derived from Salencon (1969).

Part C - The analytical solution to Part C is given by Einstein and Schwartz (1979).

A FORTRAN computer code was written to calculate the analytic solution for each of the three cases. The computer code and results are listed in Appendix 3.2.1.5-A.

Assumptions

Assumptions which are implicit in the theoretical solutions include the following:

(1) plane strain conditions apply, with one of the principal stresses aligned with the tunnel axis; and

(2) the material is homogeneous, isotropic and weightless.

The following assumptions apply to individual parts of the problem.

Part A — The medium is linear-elastic.

Part B — The medium behaves as an elastic-perfectly plastic solid obeying a Mohr-Coulomb yield criterion.

Out-of-plane stresses do not affect plastic yielding.

Part C — The liner and medium are both linear-elastic materials.

The lining is installed coincidentally with tunnel excavation.

The lining is bonded to the surrounding material so no slip or separation of material occurs.

Computer Model

For each part, two different discretizations were used as shown in Figs. 3.2.1.5-2 and -3. In both geometries the inner and outer radii were 5.0 m and 30.0 m, respectively. Also, boundary elements were coupled to gridpoints in the outer boundary in both cases. "Glued" joints were used to provide the needed discretization in each case.

In Part C of the problem, interaction of a structural lining with the surrounding material is modelled. For this part, the lining was divided into 24 linear segments. To satisfy the conditions of perfect bonding between the lining and surrounding material, high interface stiffness and strength parameters were specified.

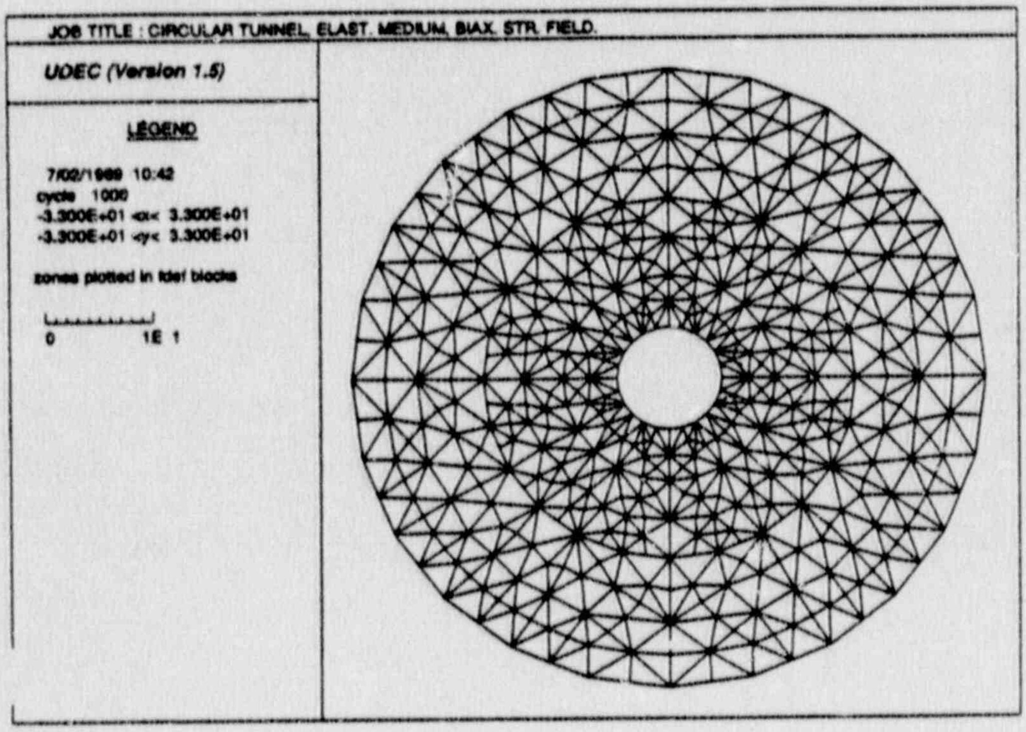

Fig. 3.2.1.5-2 Coarse Zoning Used in Circular Tunnel Problems

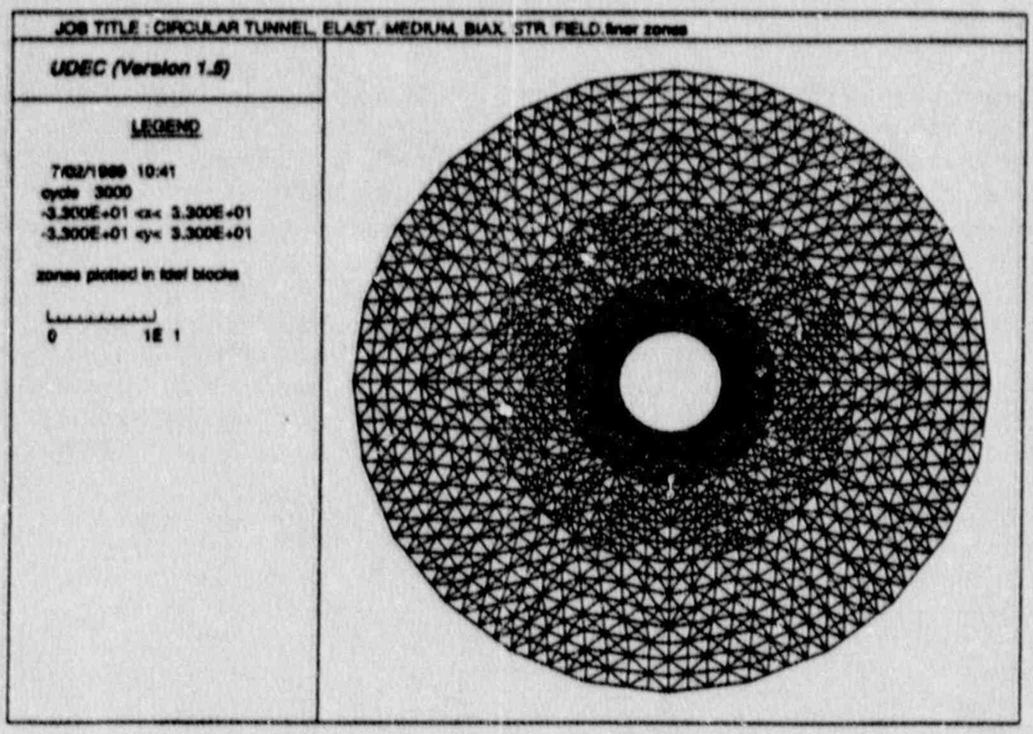

Fig. 3.2.1.5-3 Finer Zoning Used in Circular Tunnel Problems

Results

Part A — The results for Part A are compared graphically with the analytic solution in Figs. 3.2.1.5-4 through - 8. All results shown are for a line inclined 30° counterclockwise from the x-axis (i.e., 30° from the major principal stress direction). In nearly all cases, the finer zoning resulted in improved correspondence with the analytical solution.

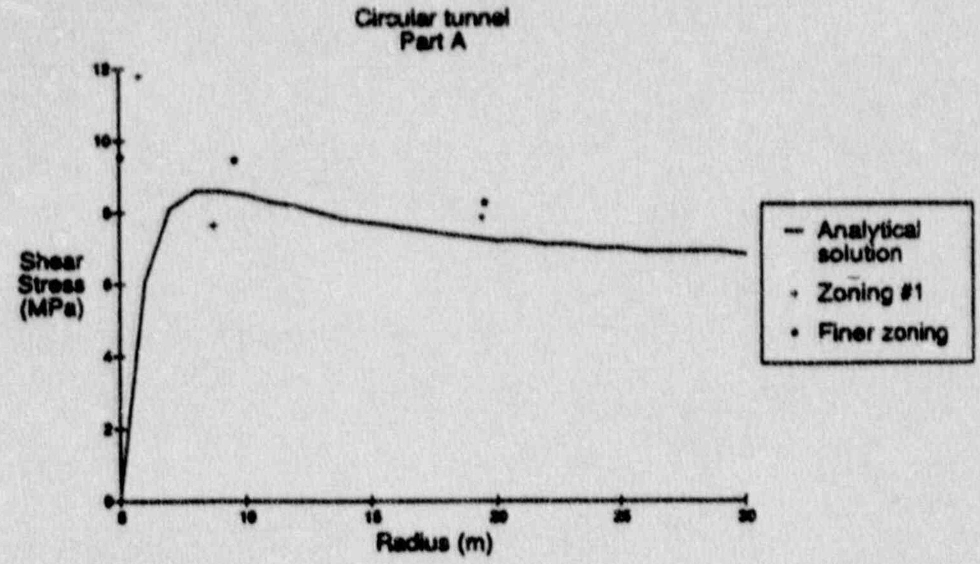

Fig. 3.2.1.5-4 Comparison of UDEC Results of Shear Stress versus Radial Distance Along a Line θ = 30° with Analytical Solution for the Case of a Tunnel in an Elastic Medium with a Biaxial Stress Field

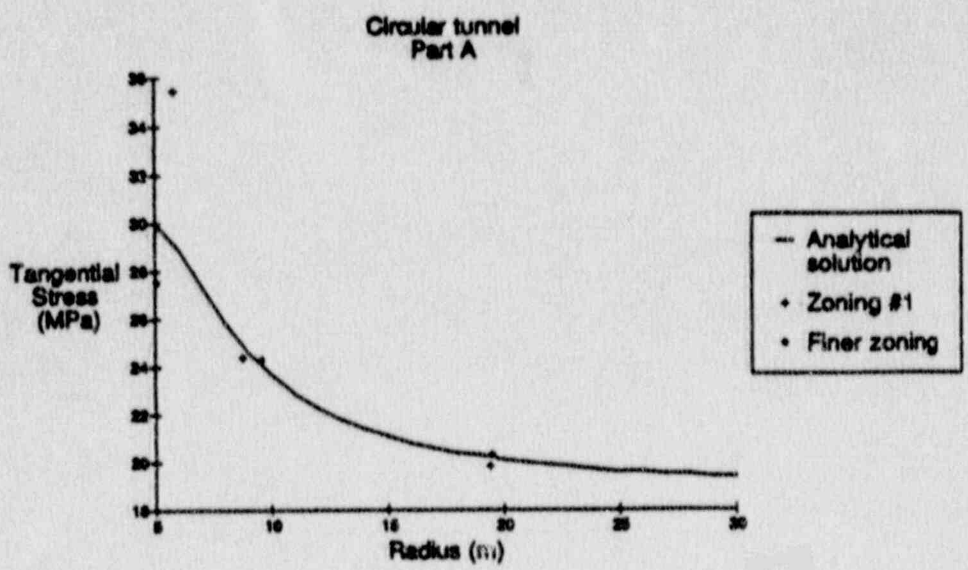

Fig. 3.2.1.5-5 Comparison of UDEC Results of Tangential Stress versus Radial Distance Along a Line θ = 30° with Analytical Solution for the Case of a Tunnel in an Elastic Medium with a Biaxial Stress Field

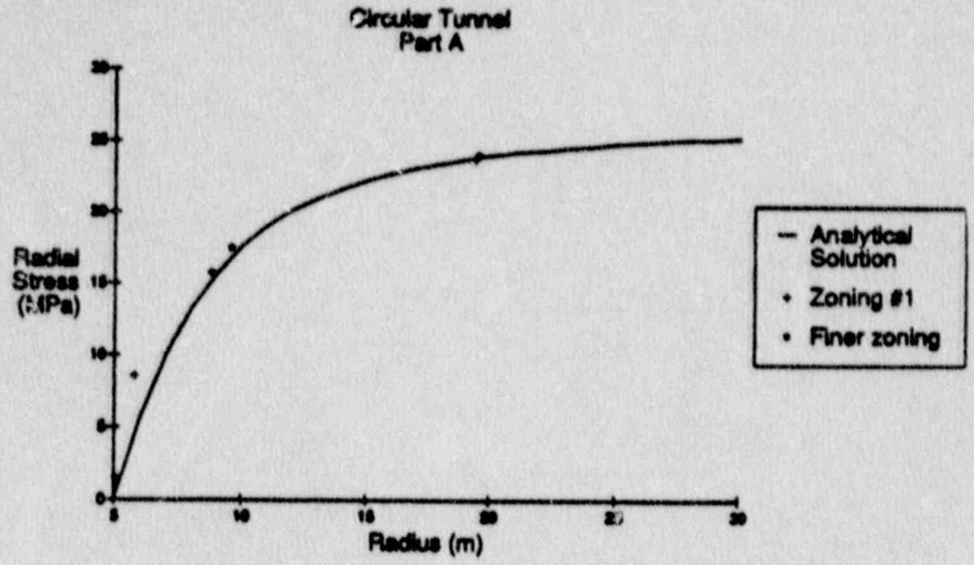

Fig. 3.2.1.5-6 Comparison of UDEC Results of Radial Stress versus Radial Distance Along a Line θ = 30° with Analytical Solution for the Case of a Tunnel in an Elastic Medium with a Biaxial Stress Field

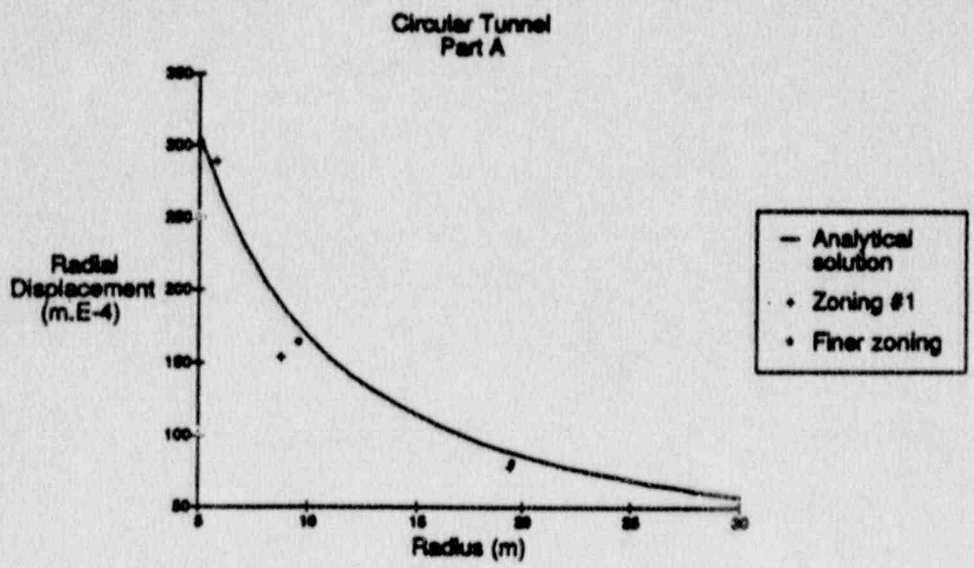

Fig. 3.2.1.5-7 Comparison of UDEC Results of Radial Displacement versus Radial Distance Along a Line θ = 30° with Analytical Solution for the Case of a Tunnel in an Elastic Medium with a Biaxial Stress Field

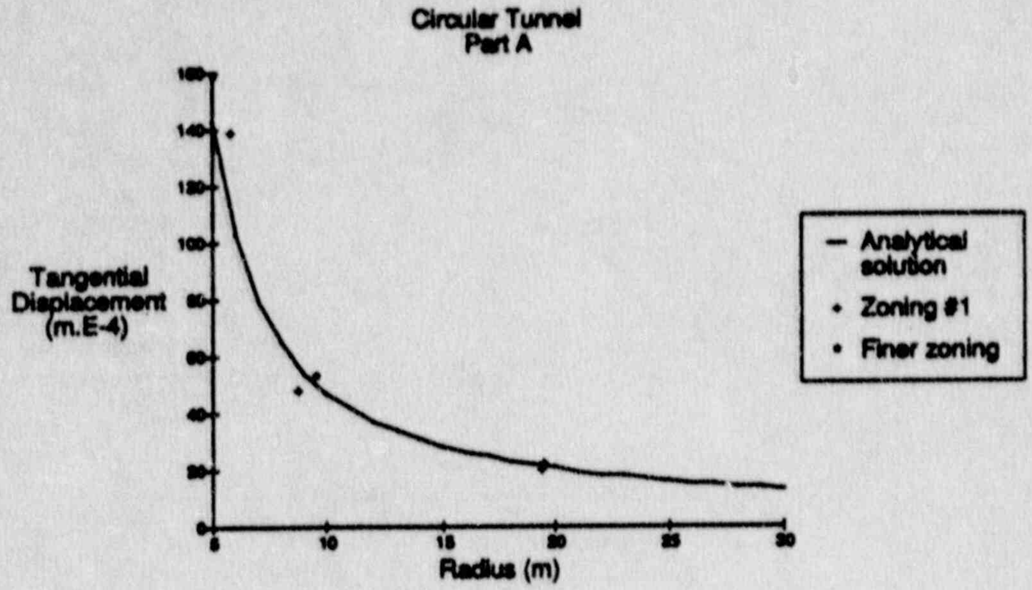

Fig. 3.2.1.5-8 Comparison of UDEC Results of Tangential Displacement versus Radial Distance Along a Line $\theta = 30°$ with Analytical Solution for the Case of a Tunnel in an Elastic Medium with a Biaxial Stress Field

Part B — The results of Part B are compared graphically with the analytic solution in Figs. 3.2.1.5-9 and -10. The calculated radius to the elastic-plastic interface based on the analytic solution is 5.28 m. For UDEC the corresponding radius was determined by calculating the ratio of plastic zones to elastic zones in the central area where the zone size was constant. This procedure is explained in more detail in the thick-walled cylinder problem. The radius to the elastic-plastic interface was found to be 6.0 m for the coarse zoning and 5.33 m for the fine zoning, or errors of 8% and 3%, respectively.

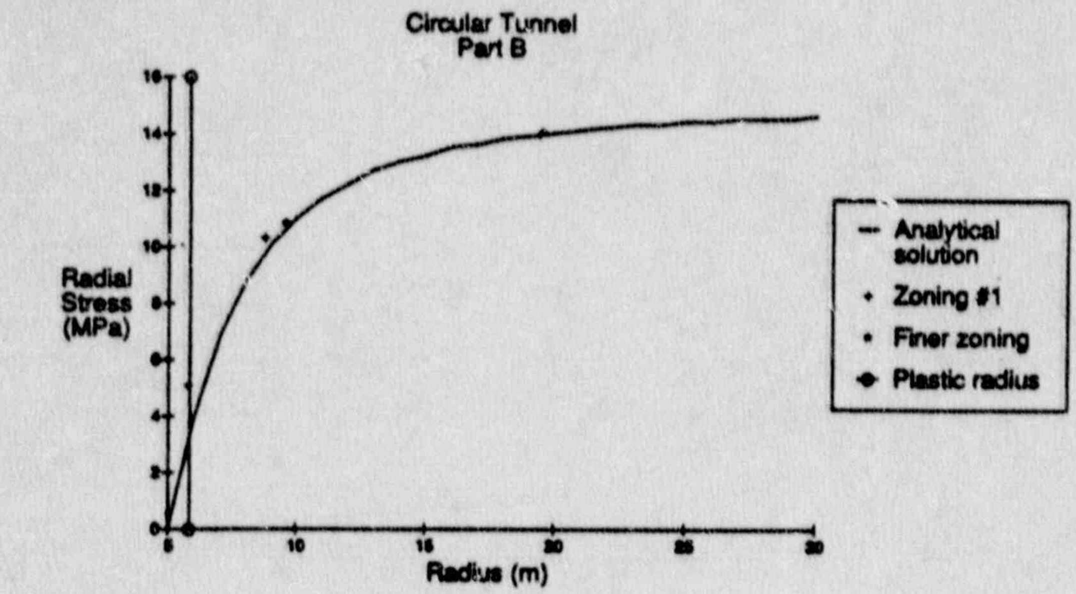

Fig. 3.2.1.5-9 Comparison of UDEC Results for Radial Stress versus Radial Distance with Analytic Solution for the Case of a Tunnel in an Elastic-Plastic Medium with a Hydrostatic Stress Field

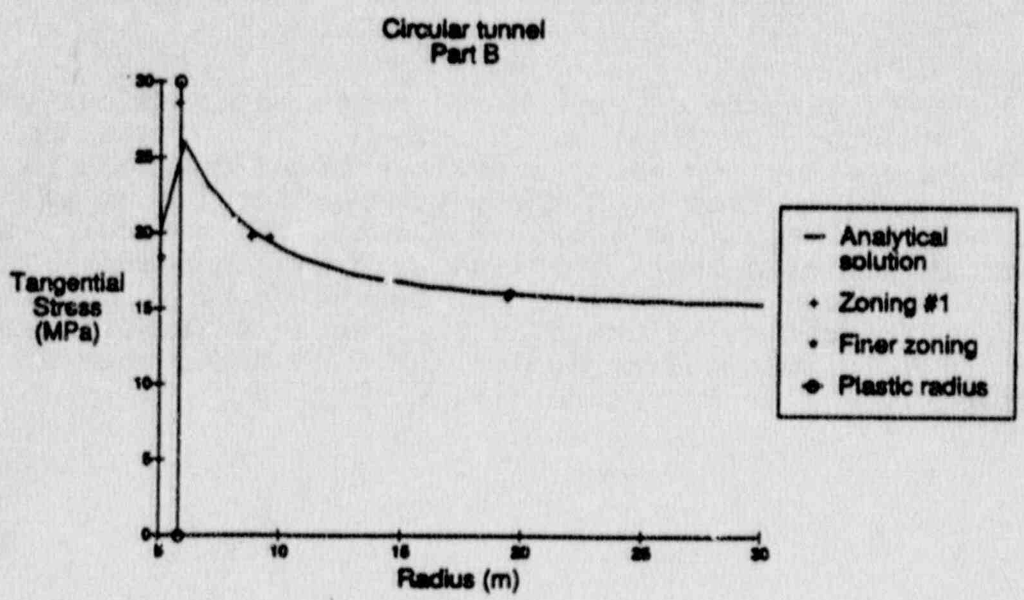

Fig. 3.2.1.5-10 Comparison of UDEC Results for Tangential Stress versus Radial Distance with Analytic Solution for the Case of Tunnel in an Elastic-Plastic Medium with a Hydrostatic Stress Field

Part C — The UDEC results for Part C are presented in terms of lining thrust, moment, and radial displacement in Figs. 3.2.1.5-11 through -13. Results shown are for the first quadrant. Results for the other quadrants are similar. The structural lining logic in UDEC assumes plane stress conditions. In order to simulate plain strain conditions, modified values for E must be used in the UDEC input for tunnel lining properties. Results of these corrections are shown in the figures.

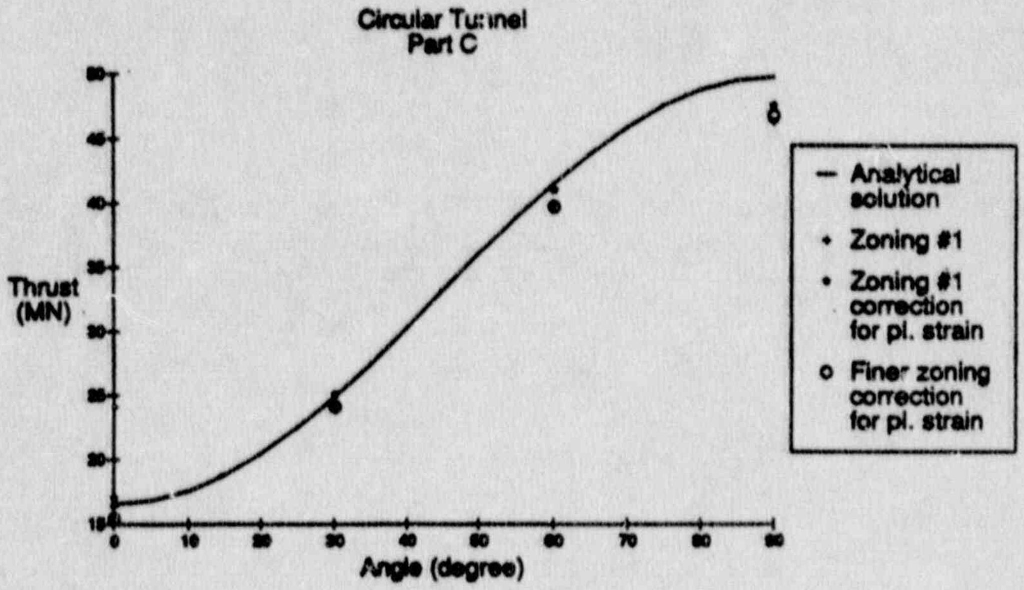

Fig. 3.2.1.5-11 Comparison of UDEC Results for Lining Thrust with Analytical Solution for the Case of a Lined Tunnel in an Elastic Medium with a Biaxial Stress Field

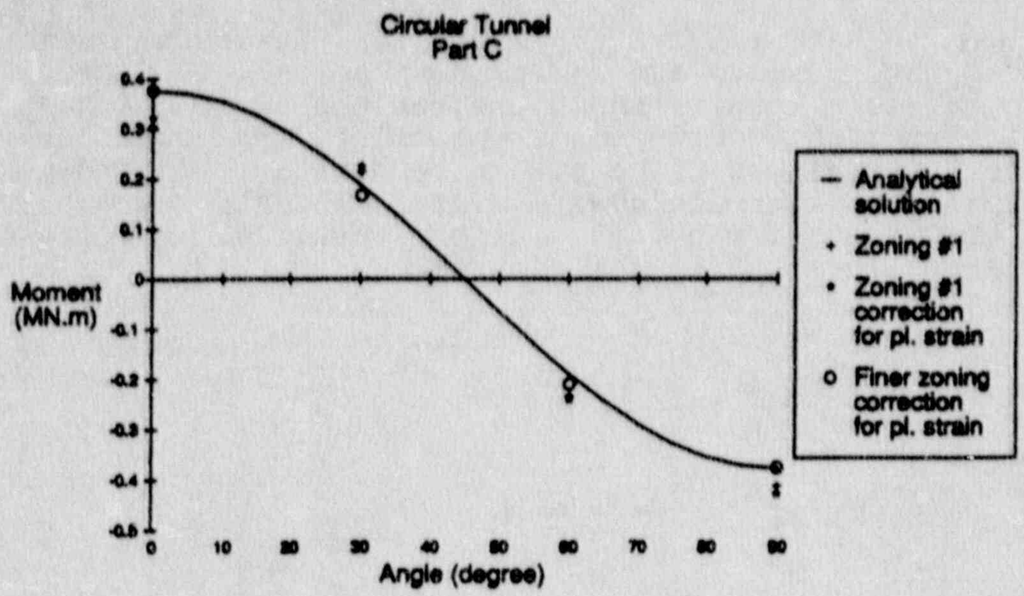

Fig. 3.2.1.5-12 Comparison of UDEC Results for Lining Moment with Analytical Solution for the Case of a Lined Tunnel in an Elastic Medium with a Biaxial Stress Field

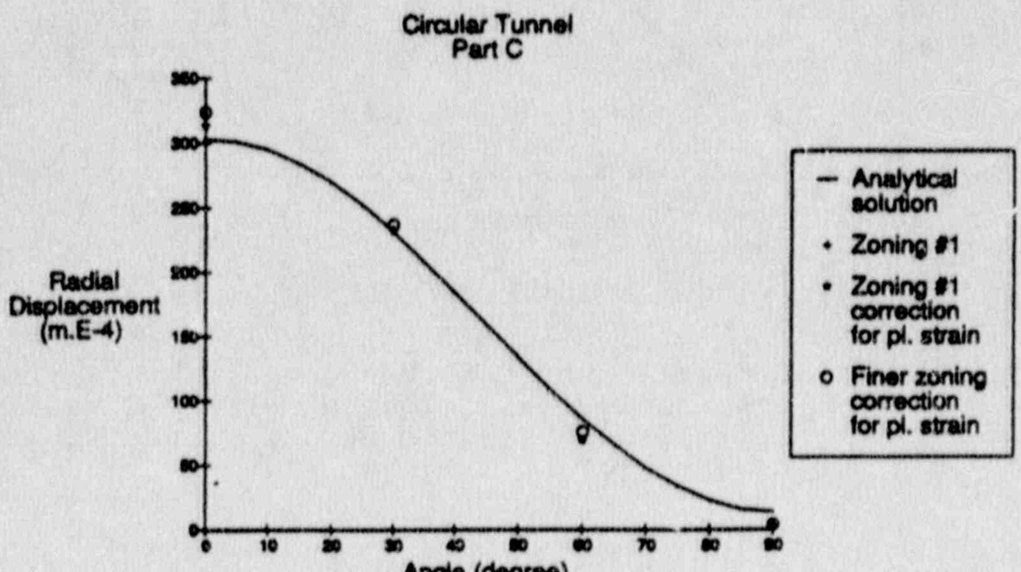

Fig. 3.2.1.5-13 Comparison of UDEC Results for Lining Radial Displacement with Analytical Solution for the Case of a Lined Tunnel in an Elastic Medium with a Biaxial Stress Field

Einstein, Herbert H., and Charles W. Schwartz. "Simplified Analysis for Tunnel Supports," J. of Geotech. Eng. Div., 499-518 (April 1973).

Goodman, Richard E. Introduction to Rock Mechanics. New York: John Wiley & Sons, 1980.

Salencon, J. "Contraction Quasi-Statique d'une Cavite a Symmetrie Spherique ou Cylindrique dans un Milieu Elastoplastique," Annales des Ponts et Chaussees, 4, 213-216 (1969).

Wart, R. J., E. L. Skiba and R. H. Curtis. Benchmark Problems for Repository Design Models. NUREG/CR-3636. February 1984.

<u>Data Input Files</u>: UDEC Input Files

*Circular Tunnel (Part A and Part C)

```
head
CIRCULAR TUNNEL, ELAST. MEDIUM, BIAX. STR. FIELD.
round 0.05

*set geometry, without excavation.
block circular 0 0 30 24
crack -30 0 30 0
crack 0 -30 0 30
tunnel 0 0 5 24
tunnel 0 0 10 24
tunnel 0 0 17.5 24

*create zoning (3 different sizes)
gen 15 25 15 25 edge 6
gen 7.5 10 7.5 10 edge 4
gen 3 6 3 6 edge 3
gen -30 -15 -30 -15 edge 6
gen -10 -7.5 -10 -7 edge 4
gen -6 -3 -6 -3 edge 3
gen -30 -15 15 30 edge 6
gen -10 -7.5 7.5 10 edge 4
gen -6 -3 3 6 edge 3
gen 15 25 -25 -15 edge 6
gen 7.5 10 -10 -7.5 edge 4
gen 3 6 -6 -3 edge 3
save ctgeo.001

damp auto

*set stresses (oriented at 30 degrees)
bound stress -26.25 7.495 -18.75
insitu   stress -26.25 7.495 -18.75

*material properties
prop m=1 d=.003 k=3.33e3 g=2.5e3 jkn=6e5 jks=6e5
prop m=1 jfric=10.0 jcoh=10e7 jtens=10e7

*histories (displacements and stresses) at radii=5,10,20.
hist n=20 xdis 5 0 xdis 10 0 xdis 20 0
hist n=20 ydis 5 0 ydis 10 0 ydis 20 0
hist n=20 sxx  5 0 sxx  10 0 sxx  20 0
hist n=20 syy  5 0 syy  10 0 syy  20 0
hist n=20 sxy  5 0 sxy  10 0 sxy  20 0

mscale on
```

```
*cycle until equilibrium
cy 400
save ctsave.eq

*excavate
del -3.5 3.5 -3.5 3.5

*set boundary elements
be gen -30 30 -30 30
be mat=1
be fix 0 0 0 0
be stiff

*cycle until new equilibrium
cyc 600
save ctsave.a01


*Part C:restart from first equilibrium (before excavation)
rest ctsave.eq
he
CIRCULAR TUNNEL WITH SUPPORT, ELAST. MEDIUM, BIAX. STR.
FIELD.kn=ks=1e4

*excavate
del -3.5 3.5 -3.5 3.5

*set support and its properties
stru  0 0 7.5 6 24 2 .5
prop m=2 dens=.003 k=1.11e4 g=8.33e3 kn=1e4 ks=1e4
prop m=2 cfric=10  ccoh=10e7  ctens=10e7

*set boundary elements
be gen -30 30 -30 30
be mat=1
be fix 0 0 0 0
be stiff

*cycle until new equilibrium
cyc 1100
save ctsave.c01
quit
```

```
*Circular tunnel. (Part B and Part D)


head
CIRCULAR TUNNEL, ELAST-PLAST. MEDIUM, HYDROST. STR. FIELD.
round 0.05

*set geometry (before excavation)
block circular 0 0 30 24
crack -30 0 30 0
crack 0 -30 0 30
tunnel 0 0 5 24
tunnel 0 0 10 24
tunnel 0 0 17.5 24

*create zoning (3 different sizes)
gen 15 25 15 25 edge 6
gen 7.5 10 7.5 10 edge 4
gen 3 6 3 6 edge 3
gen -30 -15 -30 -15 edge 6
gen -10 -7.5 -10 -7 edge 4
gen -6 -3 -6 -3 edge 3
gen -30 -15 15 30 edge 6
gen -10 -7.5 7.5 10 edge 4
gen -6 -3 3 6 edge 3
gen 15 25 -25 -15 edge 6
gen 7.5 10 -10 -7.5 edge 4
gen 3 6 -6 -3 edge 3

damp auto

*give state of stresses (hydrostat.)
bound stress -15 0 -15
insitu   stress -15 0 -15

*set material properties
prop m=1 d=.003 k=3.33e3 g=2.5e3 jkn=6e5 jks=6e5
prop m=1 jfric=.364 jcoh=7 jtens=10e7

*histories (displacements and stresses) at radii=5,10,20.
hist n=20 xdis 5 0 xdis 10 0 xdis 20 0
hist n=20 ydis 5 0 ydis 10 0 ydis 20 0
hist n=20 sxx  5 0 sxx  10 0 sxx  20 0
hist n=20 syy  5 0 syy  10 0 syy  20 0
hist n=20 sxy  5 0 sxy  10 0 sxy  20 0

mscale on
```

```
*cycle until equilibrium
cy 400
save ctsave.eqb

*excavate
del -3.5 3.5 -3.5 3.5

*set boundary elements
be gen -30 30 -30 30
be mat=1
be fix 0 0 0 0
be stiff

*cycle until new equilibrium
cyc 600
save ctsave.b01


*Part D:restart from first equilibrium (before excavation)
rest ctsave.eqb

he
CIRC.TUNNEL WITH SUPPORT, ELAST.PLAST. MEDIUM, HYDROST. STR.
FIELD.kn=ks=1e4.

*excavate
del -3.5 3.5 -3.5 3.5

*set support and its properties
stru  0 0 7.5 6 24 2 .5
prop m=2 dens=.003 k=1.11e4 g=8.33e3 kn=1e4 ks=1e4
prop m=2 cfric=10  ccoh=10e7  ctens=10e7

*set boundary elements
be gen -30 30 -30 30
be mat=1
be fix 0 0 0 0
be stiff

*cycle until new equilibrium
cyc 1100
save ctsave.d01
quit
```

Appendix 3.2.1.5-A

Computer Program and Results for Analytic Solutions for Circular Tunnel Problems

```
c    analytical solution for circular tunnel

     real nu,k,kp,nul,k0,il,mc,m

     pi=4.*atan(1.)
     e=6000.e6
     nu=.20
     k=10.e6
     q=2.*k
     friction=20.
     phi=friction/180.*pi
     a=5.
     theta=30.

     open (unit=11,file='ctres')
     rewind (11)

     ee=e*1.e-6
     qq=q*1.e-6
     write (11,5) ee,nu
5    format (' Rock Young"s modulus:',f8.1,/,' Poison"s ratio:',f3.1)
     write (11,6) qq,friction
6    format (' UCS:',f6.1,/,' Friction:',f4.1,/)

c    part a---------------------------------

c    Kirsh's solution (Goodman)

     write (11,10)
10   format (' ---------PART a ---------',/)

     sx=-30.e6
     sy=-15.e6
     tet=theta*pi/180.
     s1=sx+sy
     s2=sx-sy

     r=5.
     write (11,48) theta
48   format (' theta=',f4.1,/)
     write (11,49)
```

```
49  format(' r     sigrr     sigtt     sigrt      u        v',/)

    do 100 i=1,26

    ar=a**2/r**2

    sigrr=1.e-6*.5*(s1*(1.-ar)+s2*(1.+3.*ar**2-4.*ar)*cos(2.*tet))
    sigtt=1.e-6*.5*(s1*(1.+ar)-s2*(1.+3.*ar**2)*cos(2.*tet))
    sigrt=-1.e-6*.5*s2*(1.-3.*ar**2+2.*ar)*sin(2.*tet)

    g=e/2./(1.+nu)

    u=s1/4./g*a**2/r+s2/4./g*a**2/r*(4.*(1.-nu)-a**2/r**2)*cos(2.*tet)
    v=-s2/4./g*a**2/r*(2.*(1.-2.*nu)+a**2/r**2)*sin(2.*tet)

    write (11,50) r,sigrr,sigtt,sigrt,u,v
50  format (f4.1,3f10.1,2f12.4)
    if (r.eq.5..or.r.eq.10..or.r.eq.20.) write (11,52)
52  format('-----------------------------------------------------')

    r=r+1.

100 continue

c   part b------------------------------

c   Salencon's solution

    write (11,110)
110 format (/,' ---------PART b----------')

    p=15.e6
    kp=(1.+sin(phi))/(1.-sin(phi))
    r0=a*(2./(kp+1.)*(p+q/(kp-1.))/(q/(kp-1.)))**(1./(kp-1.))
    r=5.

    write (11,111) r0
111 format (' plastic zone radius:',f12.6,/)
    write (11,112)
112 format(' r     sigrr     sigtt  ',/)

    do 200 i=1,26

    if (r.gt.r0) goto 150
```

```
c     plastic zone:
      sigrr=1.e-6*(-q/(kp-1.)+(q/(kp-1.))*(r/a)**(kp-1.))
      sigtt=1.e-6*(-q/(kp-1.)+kp*(q/(kp-1.))*(r/a)**(kp-1.))
      goto 160

c     elastic zone:
150   sigre=1./(kp+1.)*(2.*p-q)
      sigrr=1.e-6*(p-(p-sigre)*(r0/r)**2)
      sigtt=1.e-6*(p+(p-sigre)*(r0/r)**2)

160   write (11,170) r,sigrr,sigtt
170   format (f4.1,2f10.1)
      if (r.eq.5..or.r.eq.10..or.r.eq.20.) write (11,172)
172   format('------------------------')

      r=r+1.

200   continue

c     part c--------------------------------

c     Einstein and Schwartz solution

      write (11,210)
210   format (/,'----------PART c--------------'/)
      write (11,211)
211   format (' theta   thrust   moment  shear   displ.'/)

      r=5.
      t=.5
      el=20000.e6
      nul=.20
      k0=2.
      il=t**3/12.
      theta=0.

      c=e*r*(1.-nul**2)/(1.-nu**2)/el/t
      f=e*r**3*(1.-nul**2)/el/il/(1.-nu**2)

      a0=c*f*(1.-nu)/(c+f+c*f*(1.-nu))
      bet=((6.+f)*c*(1.-nu)+2.*f*nu)/(3.*f+3.*c+2.*c*f*(1.-nu))
      b2=c*(1.-nu)/(2.*(c*(1.-nu)+4.*nu-6.*bet-3.*bet*c*(1.-nu)))
      a2=bet*b2

      do 300 i=1,19

      tet=theta/180.*pi
```

```
      t=sy*r*.5*((1.+k0)*(1.-a0)+(1.-k0)*(1.+2.*a2)*cos(2.*tet))
      m=sy*r**2/4.*(1.-k0)*(1.- 2.*a2+2.*b2)*cos(2.*tet)
      s=-sy*r*.5*(1.-k0)*(1.-2.*a2+2.*b2)*sin(2.*tet)
      u=sy*r*(1.+nu)/e*.5*((1.+k0)*a0+(1.-k0)*(4.*(1.-nu)*b2-2.*a2)*
     * cos(2.*tet))

      t=t*1.e-6
      m=m*1.e-6
      s=s*1.e-6

      write (11,250) theta,t,m,s,u
250   format (f5.1,3f10.4,f10.6)
      if (theta.eq.0..or.theta.eq.30.
     *.or.theta.eq.60..or.theta.eq.90.) write (11,255)
255   format('------------------------------------------')

      theta=theta+5.

300   continue

c     part d--------------------------------

c     no solution

      close(11)
      stop
      end
```

Rock Young's modulus: 6000.0
Poison's ratio:0.2
UCS: 20.0
Friction:20.0

---------PART a ---------

theta=30.0

r	sigrr	sigtt	sigrt	u	v
5.0	0.0	-30.0	0.0	-0.0307	0.0143
6.0	-5.6	-28.9	6.1	-0.0266	0.0103
7.0	-10.0	-27.3	8.1	-0.0233	0.0079
8.0	-13.3	-25.8	8.6	-0.0206	0.0065
9.0	-15.7	-24.6	8.6	-0.0185	0.0054
10.0	-17.6	-23.7	8.5	-0.0168	0.0047
11.0	-19.0	-22.9	8.3	-0.0153	0.0042
12.0	-20.1	-22.3	8.2	-0.0141	0.0037
13.0	-20.9	-21.8	8.0	-0.0131	0.0034
14.0	-21.6	-21.4	7.8	-0.0122	0.0031
15.0	-22.2	-21.1	7.7	-0.0114	0.0028
16.0	-22.7	-20.8	7.6	-0.0107	0.0026
17.0	-23.1	-20.6	7.5	-0.0101	0.0025
18.0	-23.4	-20.4	7.4	-0.0095	0.0023
19.0	-23.7	-20.3	7.3	-0.0090	0.0022
20.0	-24.0	-20.1	7.2	-0.0086	0.0021
21.0	-24.2	-20.0	7.2	-0.0082	0.0019
22.0	-24.3	-19.9	7.1	-0.0078	0.0018
23.0	-24.5	-19.8	7.1	-0.0075	0.0018
24.0	-24.6	-19.7	7.0	-0.0072	0.0017
25.0	-24.8	-19.6	7.0	-0.0069	0.0016
26.0	-24.9	-19.6	6.9	-0.0066	0.0015
27.0	-25.0	-19.5	6.9	-0.0064	0.0015
28.0	-25.1	-19.5	6.9	-0.0061	0.0014
29.0	-25.1	-19.4	6.9	-0.0059	0.0014
30.0	-25.2	-19.4	6.8	-0.0057	0.0013

---------PART b----------
plastic zone radius: 5.819942

r	sigrr	sigtt
5.0	0.0	20.0

6.0	4.0	26.0
7.0	6.9	23.1
8.0	8.8	21.2
9.0	10.1	19.9
10.0	11.0	19.0
11.0	11.7	18.3
12.0	12.2	17.8
13.0	12.7	17.3
14.0	13.0	17.0
15.0	13.2	16.8
16.0	13.5	16.5
17.0	13.6	16.4
18.0	13.8	16.2
19.0	13.9	16.1
20.0	14.0	16.0
21.0	14.1	15.9
22.0	14.2	15.8
23.0	14.3	15.7
24.0	14.3	15.7
25.0	14.4	15.6
26.0	14.4	15.6
27.0	14.5	15.5
28.0	14.5	15.5
29.0	14.5	15.5
30.0	14.6	15.4

----------PART [illegible]--------------

theta	thrust	moment	shear	displ.
0.0	-16.5595	0.3767	0.0000	-0.030344
5.0	-16.8109	0.3710	-0.0262	-0.030125
10.0	-17.5574	0.3540	-0.0515	-0.029472
15.0	-18.7765	0.3263	-0.0753	-0.028406
20.0	-20.4310	0.2886	-0.0969	-0.026960
25.0	-22.4707	0.2422	-0.1154	-0.025177
30.0	-24.8336	0.1884	-0.1305	-0.023111

35.0	-27.4479	0.1289	-0.1416	-0.020826
40.0	-30.2341	0.0654	-0.1484	-0.018390
45.0	-33.1077	0.0000	-0.1507	-0.015878
50.0	-35.9813	-0.0654	-0.1484	-0.013366
55.0	-38.7675	-0.1289	-0.1416	-0.010931
60.0	-41.3818	-0.1884	-0.1305	-0.008645
65.0	-43.7447	-0.2422	-0.1154	-0.006580
70.0	-45.7844	-0.2886	-0.0969	-0.004797
75.0	-47.4389	-0.3263	-0.0753	-0.003351
80.0	-48.6579	-0.3540	-0.0515	-0.002285
85.0	-49.4045	-0.3710	-0.0262	-0.001632
90.0	-49.6559	-0.3767	0.0000	-0.001412

3.2.1.6 Elastic Behavior of Jointed Medium

This problem is given in Volume 2 of this document, UDEC User's Manual, pp. B-1 through B-10.

3.2.1.7 Crack Shear by Reduced Friction

This problem is given in Volume 2 of this document, UDEC User's Manual, pp. C-1 through C-6.

3.2.1.8 Rough Footing on Cohesive Material

This problem is given in Volume 2 of this document, UDEC User's Manual, pp. D-1 through D-5.

3.2.2 Mechanical Problems — Dynamic

The following dynamic mechanical problems are presented in this section or can be found as noted.

Line Source in an Infinite Elastic Medium with a Discontinuity

Slip Induced by Harmonic Shear Wave (see UDEC User's Manual, pp. A-1 to A-9)

3.2.2.1 Line Source in an Infinite Elastic Medium With a Discontinuity

Problem Statement

This problem concerns the dynamic behavior of a single discontinuity under explosive loading. The problem shown in Fig. 3.2.2.1-1 consists of a planar crack of infinite lateral extent in an elastic medium and a dynamic load at some distance, h, from the discontinuity. This problem was modeled using UDEC to determine the dynamic response of the discontinuity. The closed-form solution to this problem was derived by Day (1985) as a special symmetric condition for the general problem of slip of an interface due to a dynamic point source (Salvado and Minster (1980). The results from numerical and analytical solutions are compared and discussed.

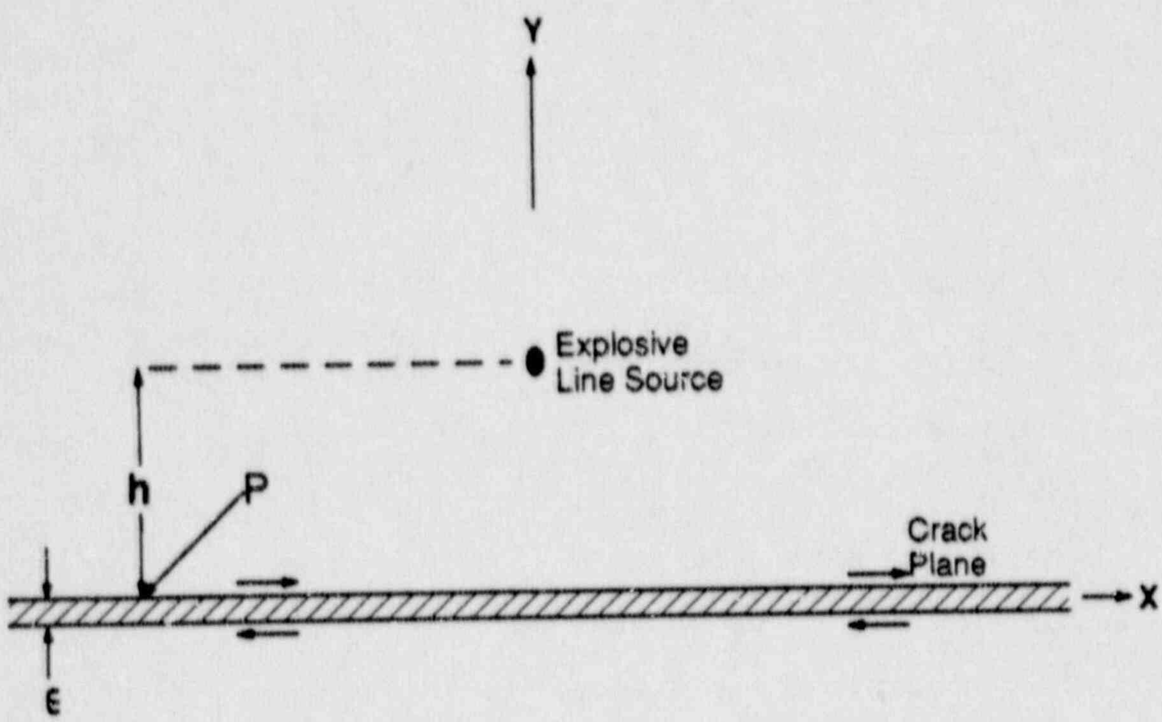

Fig. 3.2.2.1-1 Problem Geometry for an Explosive Source Near a Slip-Prone Discontinuity

Objective

The objective of this problem is to test the following functions of UDEC:

(a) the ability to model dynamic performance of a jointed rock mass;

(b) the ability to simulate a high frequency dynamic wave emanating from a buried explosion; and

(c) the ability to simulate non-reflecting boundary conditions.

Closed-Form Solution

The closed-form solution for crack slip as a function of time was derived by Day (1985) and is given by

$$\delta u(x,t) = \frac{2\, m_o\, \beta^2}{\pi\, \rho\, \alpha^2} \;\mathrm{Re}\left[\frac{p\, \eta_\alpha\, \eta_\beta}{R(p)}\right] \left(\tau + \frac{2r}{\alpha}\right)^{-1/2} \tau^{-1/2}\, H(\tau)$$

(3.2.2.1-1)

where $r = (x^2 + h^2)^{1/2}$, distance from the point source to the point on the crack where the slip is monitored,

$H(\tau)$ = step function,

$\tau = t - (r/\alpha)$

m_o = source strength,

α = velocity of pressure wave,

β = velocity of shear wave,

ρ = density,

$\eta_\alpha = (\alpha^{-2} - p^2)^{1/2}$, $\mathrm{Re}\ \eta_\alpha \geq 0$,

$\eta_\beta = (\beta^{-2} - p^2)^{1/2}$, $\mathrm{Re}\ \eta_\beta \geq 0$,

$$p = \frac{1}{r^2}\left[(\tau + \frac{r}{\alpha})x + i\ (\tau + \frac{2r}{\alpha})^{1/2}\ \tau^{1/2}\ h \right]$$

The slip response of the discontinuity for any source history S(t) can be obtained by convolution of Equation (3.2.2.1-1) and the source function S(t). Figure 3.2.2.1-2 shows the dimensionless analytical results of slip history at a point P for a smooth step function

$$S(t) = \begin{cases} 0.5\ (1 - \cos(\pi t/0.6)) & t < 0.6 \\ 1.0 & t \geq 0.6 \end{cases} \qquad (3.2.2.1\text{-}2)$$

and for the following values of the variables:

$$\alpha^2 = 3\beta^2$$

$$h = x$$

$$\gamma = 0$$

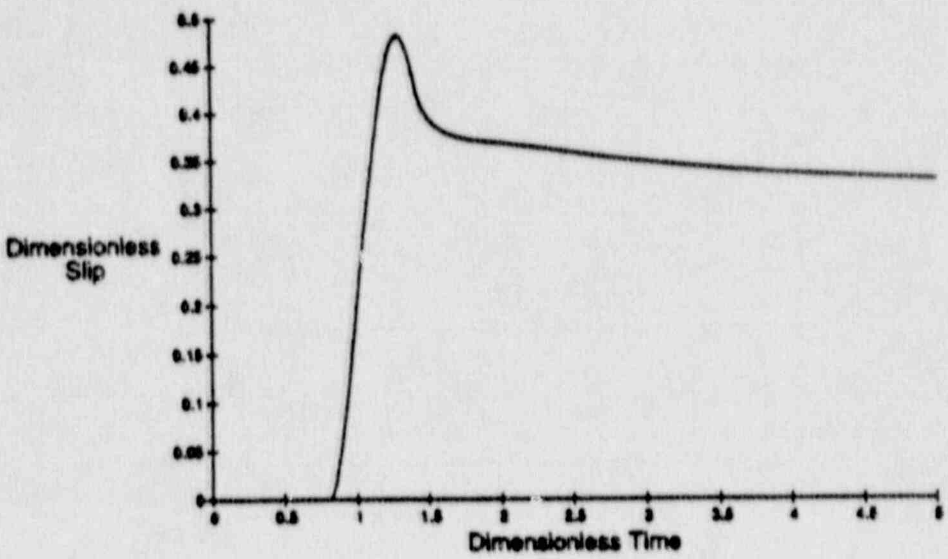

Fig. 3.2.2.1-2 Dimensionless analytical results of slip history at point P (from Day, 1985) [dimensionless slip = $(4h\rho\beta^2/m_o)\delta u$, dimensionless time = $t\beta/h$]

Numerical Model

1. **Model Set-up**

Fig. 3.2.2.1-3 shows the problem geometry modeled by UDEC. The source is located at the origin of the co-ordinate axes and the discontinuity is located at y = -h. The y-axis is a line of symmetry and non-reflecting boundaries were used on the other three sides of the model. The dynamic input was applied at the semi-circular boundary of radius 0.05h. The slip movement is monitored at point P on the discontinuity.

The continuous medium was modeled with elastic, fully deformable blocks, as shown in Fig. 3.2.2.1-4, and each block was further discretized into triangular finite-difference zones. All the joints except for the discontinuity are "glued" with high normal and shear stiffness and cohesion so as to model a continuous elastic medium. The discontinuity was assigned zero shear strength, a high normal stiffness, and high tensile strength in order to meet the assumptions implied in the analytical solution.

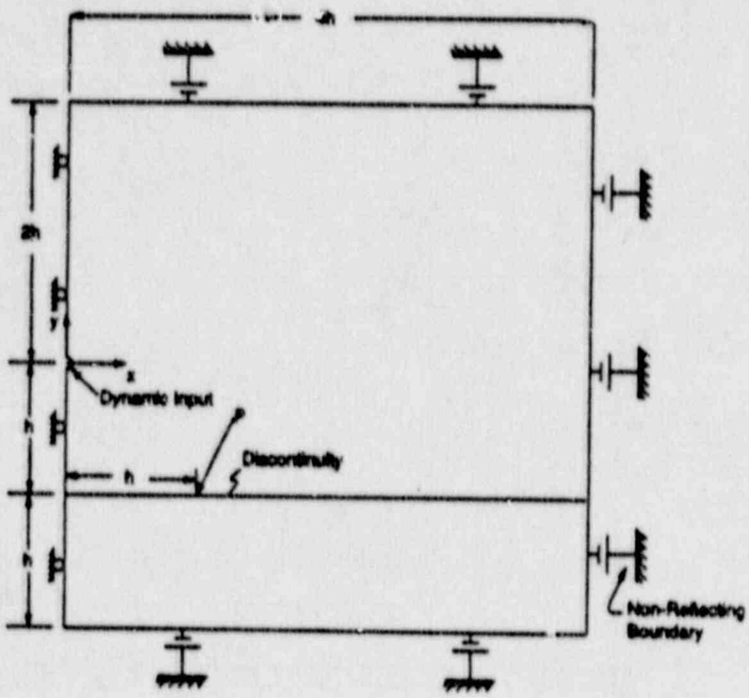

Fig. 3.2.2.1-3 Problem Geometry and Boundary Conditions for Numerical Model

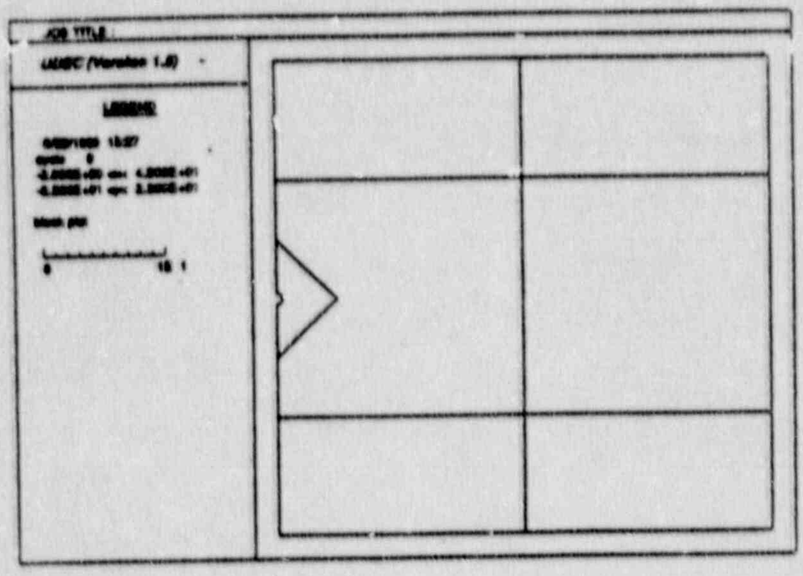

Fig. 3.2.2.1-4 UDEC Model Showing Semi-Circular Source and "Glued" Joints Used to Provide Appropriate Zoned Discretization

2. Properties of Joints and Continuous Medium

A) Material Properties

		Typical Units
Geometric Scale:	h = 10	(m)
Block Properties:		
Mass density (ρ)	= 1	(kg/m^3)
Shear modulus (G)	= 100	(Pa)
Bulk modulus (K)	= 166.67	(Pa)
P-wave velocity (α)	= 17.32	(m/sec)
S-wave velocity (β)	= 10.00	(m/sec)

B) Joint Properties:

The following joint constitutive relations are used:

(i) Mohr Coulomb model; and

(ii) continuously-yielding model.

The specific UDEC parameters used for each joint relation are as follows:

		Typical Units
(i) Mohr Coulomb Model (Jcons=2)		
	JKN = 10,000	(Pa/m)
	JKS = 0.1	(Pa/m)
	JFRIC = 0	
(ii) Continuously Yielding Model (Jcons=3)		
	JKN = 10,000	(Pa/m)
	JKS = 0.1	(Pa/m)
	JFRIC = 0.00001	
	JEN = 0	
	JES = 0	
	JIF = $1.0e^{-10}$	rad
	JR = $1.0e^{-4}$	m

3. **Dynamic Loading**

Two kinds of dynamic input load were applied at the source: (1) pressure input and (2) velocity input. To avoid problems with the singularity at the source, both the inputs were applied over a surface distant 0.05h from the nominal point source.

(a) Pressure Input

The radial pressure applied on the semi-circular boundary was calculated from the static solution in an infinite medium, due to Love (1946). The radial stress at a distance r from a compressive line source is given by

$$\sigma_{rr} = \frac{1}{2\pi} \frac{2G}{\lambda+2G} \frac{1}{r^2} m_o \qquad (3.2.2.1\text{-}3)$$

where $\lambda = \frac{2\nu G}{1-2\nu}$, and

ν = Poisson's ratio

For the properties used in this problem the stress component σ_{rr} at distance r = 0.05h (h=10m) is 0.4244 Pa. The time history of the applied pressure is given by Eq. (3.2.2.1-2) and is shown in Fig. 3.2.2.1-5.

(b) Velocity Input

Radial velocities corresponding to the dynamic solution for a line source in an infinite medium were enforced at the semi-circular boundary. The velocities were calculated in the following manner.

The solution for the displacement due to a center of dilation in an infinite medium, due to Achenbach (1973), is described by the expression

$$u_i = \frac{1}{4\pi C_p^2} \frac{\partial}{\partial x_i} \left[\frac{1}{r} f(t - r/C_p) \right] \qquad (3.2.2.1\text{-}4)$$

where $r^2 = x^2 + y^2 + z^2$,

C_p = P-wave velocity, and

$f(t)$ = source time history.

Integration of Eq. (3.2.2.1-4) along the z-axis leads to the solution for a line source of compression (Lemos, 1987) when f(t) is taken as a step function,

$$f(t) = \begin{cases} 0, & t < 0 \\ 1, & t \geq 0 \end{cases} \qquad (3.2.2.1\text{-}5)$$

The two-dimensional solution for radial displacement becomes

$$u = -\frac{1}{2\pi C_p} \frac{t}{r^2} \left[\frac{t^2 C_p^2}{r^2} - 1 \right]^{-1/2}, \quad t > r/C_p \qquad (3.2.2.1\text{-}6)$$

where $r^2 = x^2 + y^2$.

The corresponding velocity is

$$v = -\frac{1}{2\pi C_p} \frac{1}{r^2} \left[\frac{t^2 C_p^2}{r^2} - 1 \right]^{-3/2}, \quad t > r/C_p \qquad (3.2.2.1\text{-}7)$$

The actual input velocity record at r = 0.05h as shown in Fig. 3.2.2.1-6 was obtained by convoluting Eqs. (3.2.2.1-7) and (3.2.2.1-2).

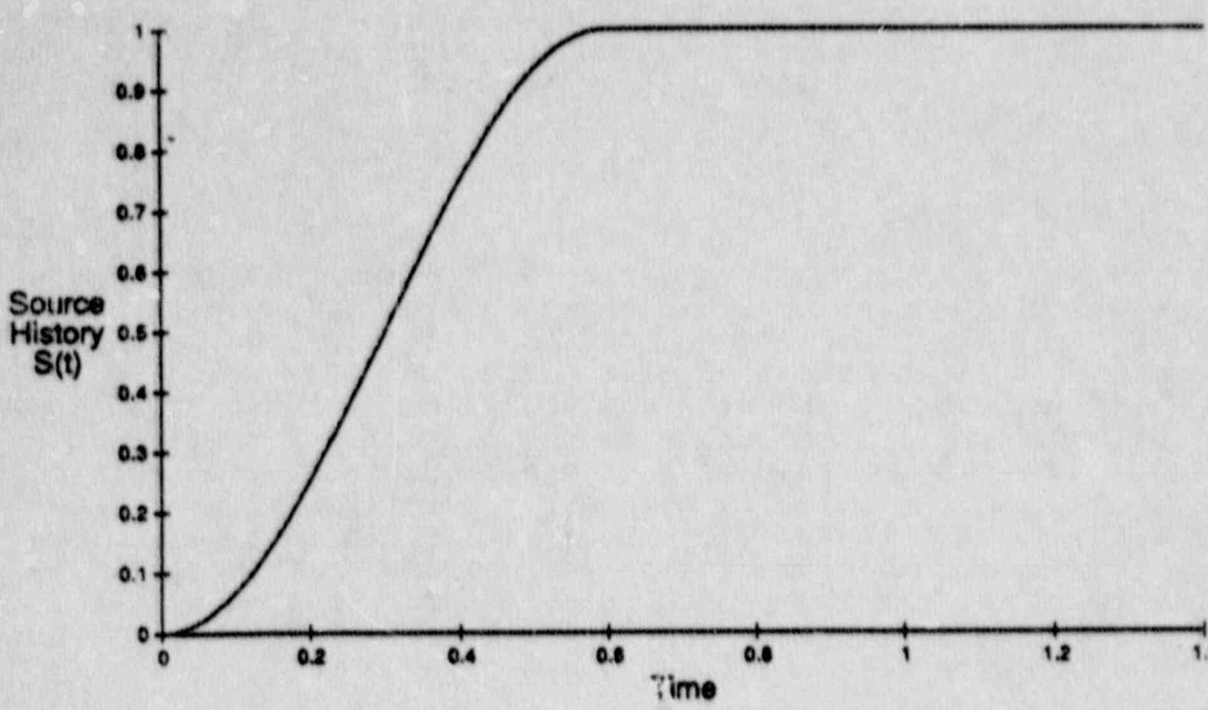

Fig. 3.2.2.1-5 Input Radial Pressure Time History Prescribed at r = 0.05h

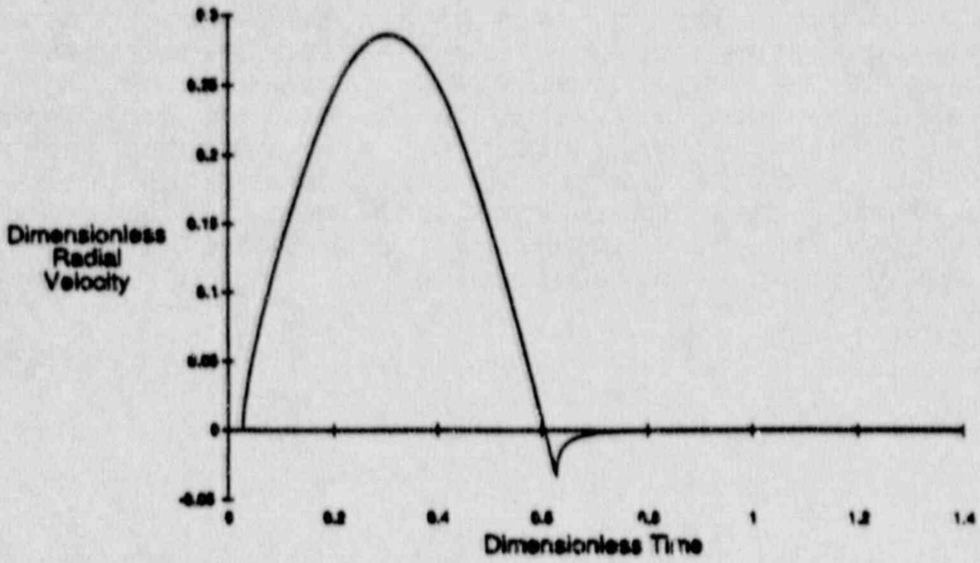

Fig. 3.2.2.1-6 Input Radial Velocity Time History Prescribed at r = 0.05h [dimensionless velocity = $(h^2\rho\beta/m_o)v$, dimensionless time = $t\beta/h$]

Results

The results of the analysis are considered in terms of four criteria: (a) dynamic input, (b) mesh size, (c) joint model, and (d) boundary conditions.

(a) Dynamic Input

The dimensionless slip at point P vs dimensionless time for the Coulomb joint model is shown in Fig. 3.2.2.1-7. This compares the results from UDEC for velocity input and pressure input with the analytical solution. The velocity input gives a better match with the analytical solution than the pressure input. The error at the peak slip for velocity input is 5.21% and that of pressure input is 9.81%. This suggests that the velocity boundary provides an accurate representation of the dynamic stress at r=0.05h compared to the pressure input. The reason for this is that in the pressure input, the source function is simply scaled by static stress magnitude and neglects the inertial effects of dynamic stress at the input boundary.

(b) Mesh Size

The results shown in Fig. 3.2.2.1-7 were obtained with a mesh of maximum zone length of 0.065h. The slip response on the discontinuity involves higher frequency components because of zero friction along the discontinuity and this requires finer mesh for accurate representation. It has been shown by Lemos (1987) that if the maximum zone length is 0.033h then the UDEC solution due to velocity input is within 1% of the analytical solution and the pressure input is within 2.5%. These results suggest a requirement of 35 zones within the distance of the dominant wavelength of the input wave in order to provide good accuracy.

(c) Joint Model

Figure 3.2.2.1-7 shows the results of joint slip based on the Coulomb joint model. The Coulomb joint model is a linear elastic, perfectly-plastic constitutive relation. Figure 3.2.2.1-8 shows the results with the continuously yielding joint model which represents a non-linear joint constitutive relation. For the joint parameters chosen for the continuously-yielding model, the slip response is virtually identical to the Coulomb model for both pressure and velocity input.

It must be noted here that computations were performed successfully for the Coulomb joint model and the continuously-yielding joint model on a 80286-based personal computer with DSI-780 co-processor board. On an 80386-based personal computer the program did not run successfully for the continuously-yielding joint model because of an error in the ATAN2 function in the compiler (SVS FORTRAN 386). This problem can be rectified at some time in the future.

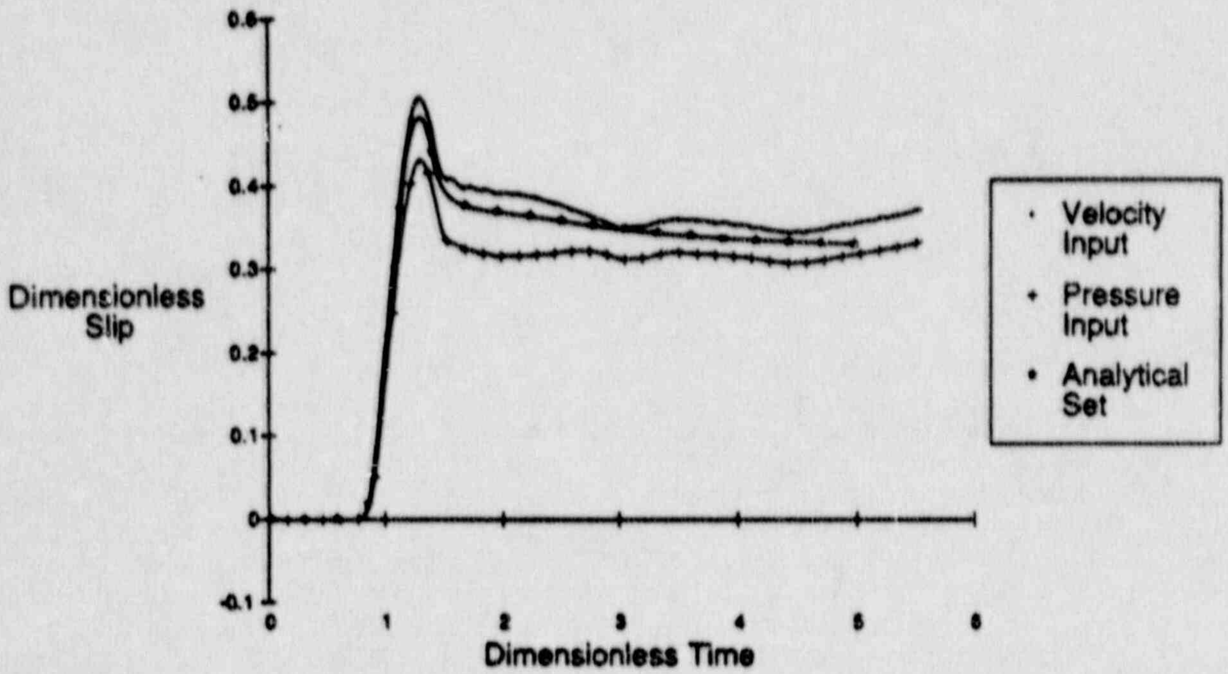

Fig. 3.2.2.1-7 Comparison of Dimensionless Slip at Point P With Coulomb Joint Model [dimensionless slip = $(4h\rho\beta^2/m_o)\delta u$, dimensionless time = $t\beta/h$]

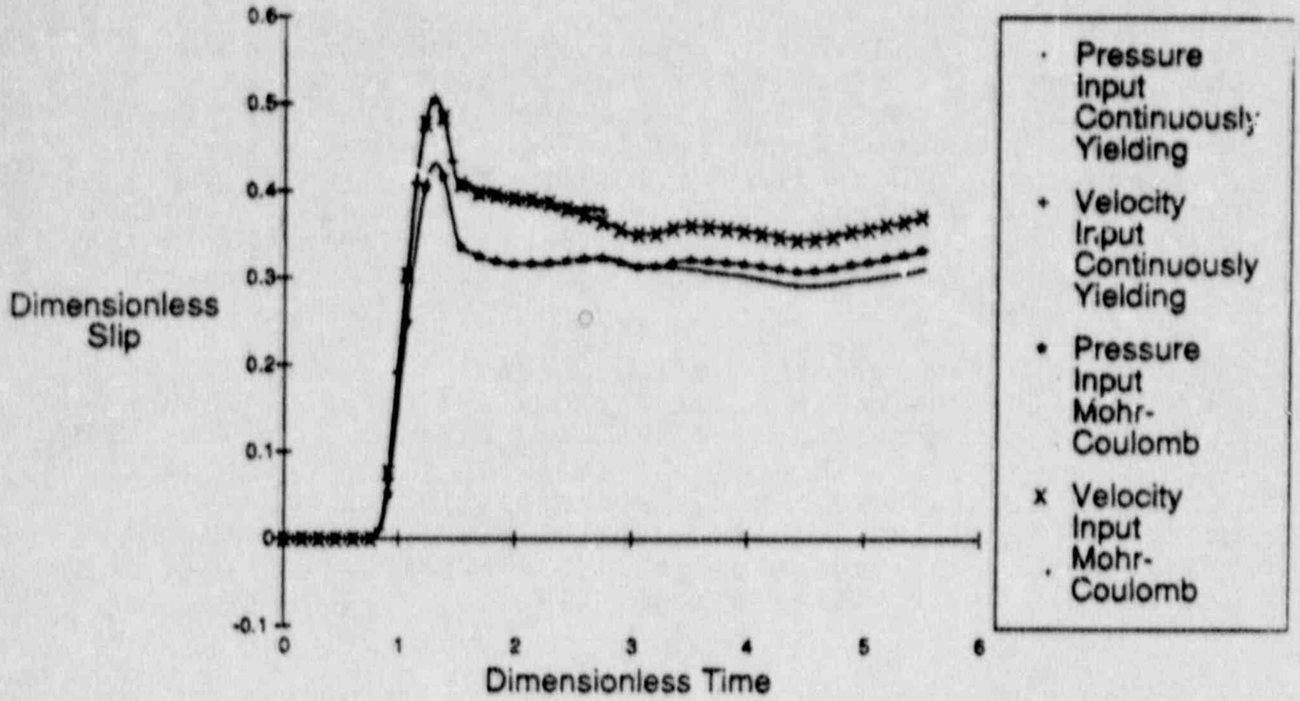

Fig. 3.2.2.1-8 Comparison of Dimensionless Slip for Coulomb and Continuously Yielding Joint Models [dimensionless slip = $(4h\rho\beta^2/m_o)\delta u$, dimensionless time = $t\beta/h$]

(d) Boundary Conditions

As seen in Fig. 3.2.2.1-3, non-reflecting boundaries are used along the top, bottom and right boundaries and line of symmetry boundary condition are used on the left boundary. The viscous boundaries, designed to absorb normally incident P- and S-waves, cannot be fully effective in this dynamic slip problem because the discontinuity crosses the boundary. Viscous boundaries, however, are preferable to roller boundaries. Lemos (1987) studied the effects of boundary reflection on slip response by varying the model size and obtained improved performance with a model size of 4hx4h. As shown in Fig. 3.2.2.1-3, this problem geometry has been employed in this analysis.

References

Achenbach, J. D. Wave Propagation in Elastic Solids. New York: North-holland Publishing Company, 1975.

Day, S. M. "Test Problem for Plane Strain Block Motion Codes," S-Cubed Memorandum, May 1, 1985.

Lemos, J. "A Distinct Element Model for Dynamic Analysis of Jointed Rock with Application to Dam Foundations and Fault Motion," Ph.D. Thesis, University of Minnesota, June 1987.

Love, A. E. H. A Treatise on the Mathematical Theory of Elasticity. New York: Dover Publications, 1946.

Salvado, C., and J. B. Minster "Slipping Interfaces: A Possible Source of S Radiation from Explosive Sources," Bull. Seism. Soc. Amer., 70, 659-670 (1980).

Input Data File

```
**************************************************************************
*
*  Verification problem for dynamic analysis using UDEC1.5
*
*    Joint model:    Coulomb
*    Dynamic Input:  Pressure
*
**************************************************************************
*
*  INITIAL PROBLEM GEOMETRY
*
*   create block geometry
*
round 0.002
bl 0,-20 0,-.5 0.1913,-0.4619 0.3536,-0.3536 0.4619,-0.1913 &
   0.5,0 0.4169,0.1913 0.3536,0.3536 0.1913,0.4619 0,0.5    &
   0,20 40,20 40,-20
*
crack -5,-10 45,-10
crack -5,10 45,10
crack 20,-21 20,21
crack -1,-6 6,1
crack -1,6 6,-1
*
jdel
crack 5.01,0 21,0
jdel
*
* create finite difference zones
*
gen 0,40 -20,20 auto 0.65
*
save verf31bl.sav
*
*-----------------------------------------------------------------------
*  set material and joint properties
*
prop mat=1 d=1.0 k=166.67 g=100.0 &
           jkn=10000.0 jks=0.1 &
           tens=1.0e6 jtens=1.0e6
prop mat=2 jkn=10000.0 jks=10000.0 &
           jtens=1.0e6 coh=1.0e6 jcoh=1.0e6
```

```
*
change -1,41 -10.1,-9.9 ang -1 1 jmat=1 jcons=2
change -1,41 -21,-10.1  jmat=2   jcons=2
change -1,41 -9.9,21    jmat=2   jcons=2
*
*  set boundary material property
bound mat=1
*
*  set viscous boundary conditions along three sides
*
bound -1,41 -20.1,-19.9 xvisc yvisc
bound -1,41 19.9,20.1   xvisc,yvisc
bound 39,41 -21,21      xvisc,yvisc
*bound -0.1,0.6 -0.6,0.6 stress -1,0,-1
*  set stress boundary conditions along the semi-circular notch
bound -0.1,0.6 -0.6,0.6 stress -0.4244,0,-0.4244
*  set symmetry boundary conditions along the remaining side
bound -0.1,0.1 -21,21   xvel=0
*
*  set time function of the applied stress
bound hist sine 30 0.6
*
bound hist=func
*
insitu stress -1.0e-9,0,-1.0e-9
*
*  set histories
* contact address at coordinate 10,-10 is 1445
*
hist n=10 yvel (0,.6) xvel (.6,0) yvel (.6,0) yvel (0,-.6)
hist xvel (1.0,0.) yvel (1.0,0) xvel (10.,0) yvel (10.,0) xvel
(39.5,0)
hist yvel (39.5,0) syy (.6,0) sxx (.6,0) syy (39.5,0) sxx (39.5,0)
hist add=1445,15
*
cyc 4000
save ver31st.sv2
ret
*
```

Input Data File

```
*******************************************************************
*
*  Verification problem for dynamic analysis using UDEC1.5
*
*    Joint model:    Coulomb
*    Dynamic Input:  Velocity
*
*******************************************************************
*
*  INITIAL PROBLEM GEOMETRY
*
*   create block geometry
*
round 0.002
bl 0,-20 0,-.5 0.1913,-0.4619 0.3536,-0.3536 0.4619,-0.1913 &
   0.5,0 0.4169,0.1913 0.3536,0.3536 0.1913,0.4619 0,0.5    &
   0,20 40,20 40,-20
*
crack -5,-10 45,-10
crack -5,10 45,10
crack 20,-21 20,21
crack -1,-6 6,1
crack -1,6 6,-1
*
jdel
crack 5.01,0 21,0
jdel
*
gen 0,40 -20,20 auto 0.65
*
save verf31bl.sav
*------------------------------------------------------------------
*
*  set material and joint properties
*
prop mat=1 d=1.0 k=166.67 g=100.0 &
           jkn=10000.0 jks=0.1 &
           tens=1.0e6 jtens=1.0e6
prop mat=2 jkn=10000.0 jks=10000.0 &
           jtens=1.0e6 coh=1.0e6 jcoh=1.0e6
*
change -1,41 -10.1,-9.9 ang -1 1 jmat=1 jcons=2
change -1,41 -21,-10.1  jmat=2   jcons=2
change -1,41 -9.9,21    jmat=2   jcons=2
*
```

```
*  set boundary material property
bound mat=1
bound -1,41 -20.1,-19.9 xvisc yvisc
bound -1,41 19.9,20.1   xvisc,yvisc
bound 39,41 -21,21      xvisc,yvisc
bound -0.1,0.1 -21,21   xvel=0
*
*  set velocity boundary conditions along the semi-circular boundary
bo -.05,.05 -.55,-.45 xvel=0 yvel=-1.0
bo .17,.21 -.48,-.45 xvel=0.383 yvel=-0.924
bo .33,.37 -.37,-.33 xvel=0.707 yvel=-0.707
bo .43,.47 -.21,-.17 xvel=0.924 yvel=-0.383
*
bo .48,.52 -0.05,0.05 xvel=1.0 yvel=0.0
*
bo .41,.45 .17,.21 xvel=.924 yvel=.383
bo .33,.37 .33,.37 xvel=.707 yvel=.707
bo .17,.21 .43,.47 xvel=0.383 yvel=0.924
bo -0.05,0.05 .45,.55 xvel=0 yvel=1
*
*  read time variation of velocity input from an external data file
*  cilvdx.out is output from program cilvpr.for
*
bound hread=1 cilvdx.out
*
bound hist=1
*
insitu stress -1.0e-9,0,-1.0e-9
*
*  set histories
* contact address at coordinate 10,-10 is 1445
*
hist n=10 yvel (0,.5) xvel (.5,0) xvel (.35,0) yvel (.35,.35)
hist xvel (.19,-.46) yvel (.19,-.46)
hist  add=1445,15
*
cyc 4000
save ver31v1.sv2
stop
*----------------------------------------------------------------
*
```

Input Data File

```
******************************************************************
*
*  Verification problem for dynamic analysis using UDEC1.5
*
*    Joint model:    Continuously-Yielding
*    Dynamic Input:  Pressure
*
******************************************************************
*
*  INITIAL PROBLEM GEOMETRY
*
*   create block geometry
*
round 0.002
bl 0,-20 0,-.5 0.1913,-0.4619 0.3536,-0.3536 0.4619,-0.1913 &
   0.5,0 0.4169,0.1913 0.3536,0.3536 0.1913,0.4619 0,0.5    &
   0,20 40,20 40,-20
*
crack -5,-10 45,-10
crack -5,10 45,10
crack 20,-21 20,21
crack -1,-6 6,1
crack -1,6 6,-1
*
jdel
crack 5.01,0 21,0
jdel
*
gen 0,40 -20,20 auto 0.65
*
save verf31bl.sav
*-----------------------------------------------------------------
*  set material and joint properties
*
*
prop mat=1 d=1.0 k=166.67 g=100.0 &
           jkn=10000.0 jks=0.1  jfric 0.00001 &
           tens=1.0e6 jtens=1.0e6 jen=0 jes=0 jif=1e-10 jr=1.0e-4
prop mat=2 jkn=10000.0 jks=10000.0 &
           jtens=1.0e6 coh=1.0e6 jcoh=1.0e6
*
```

```
*
change -1,41 -10.1,-9.9 ang -1 1 jmat=1 jcons=3
change -1,41 -21,-10.1  jmat=2   jcons=2
change -1,41 -9.9,21    jmat=2   jcons=2
*
*  set boundary material property
bound mat=1
*
*  set viscous boundary conditions along three sides
*
bound -1,41 -20.1,-19.9 xvisc yvisc
bound -1,41 19.9,20.1   xvisc,yvisc
bound 39,41 -21,21      xvisc,yvisc
*
*  set stress boundary conditions along the semi-circular notch
bound -0.1,0.6 -0.6,0.6 stress -0.4244,0,-0.4244
*  set symmetry boundary conditions along the remaining side
bound -0.1,0.1 -21,21   xvel=0
*
*  set time function of the applied stress
bound hist sine 30 0.6
*
bound hist=func
*
insitu stress -1.0e-9,0,-1.0e-9
*
*  set histories
* contact address at coordinate 10,-10 is 1445
*
hist n=10 yvel (0,.6) xvel (.6,0) yvel (.6,0) yvel (0,-.6)
hist xvel (1.0,0.) yvel (1.0,0) xvel (10.,0) yvel (10.,0) xvel
(39.5,0)
hist yvel (39.5,0) syy (.6,0) sxx (.6,0) syy (39.5,0) sxx (39.5,0)
hist add=1445,15
*
cyc 4000
save ver41st.sv2
ret
*
*-----------------------------------------------------------------
*
```

Input Data File

```
******************************************************************
*
*  Verification problem for dynamic analysis using UDEC1.5
*
*    Joint model:     Continuously-Yielding
*    Dynamic Input:   Velocity
*
******************************************************************
*
*  INITIAL PROBLEM GEOMETRY
*
*   create block geometry
*
round 0.002
bl 0,-20 0,-.5 0.1913,-0.4619 0.3536,-0.3536 0.4619,-0.1913 &
   0.5,0 0.4169,0.1913 0.3536,0.3536 0.1913,0.4619 0,0.5    &
   0,20 40,20 40,-20
*
crack -5,-10 45,-10
crack -5,10 45,10
crack 20,-21 20,21
crack -1,-6 6,1
crack -1,6 6,-1
*
jdel
crack 5.01,0 21,0
jdel
*
gen 0,40 -20,20 auto 0.65
*
save verf31bl.sav
*
*-----------------------------------------------------------------
*
*
*  set material and joint properties
*
prop mat=1 d=1.0 k=166.67 g=100.0 &
           jkn=10000.0 jks=0.1  jfric 0.00001 &
           tens=1.0e6 jtens=1.0e6 jen=0 jes=0 jif=1e-10 jr=1.0e-4
prop mat=2 jkn=10000.0 jks=10000.0 &
           jtens=1.0e6 coh=1.0e6 jcoh=1.0e6
*
```

```
change -1,41 -10.1,-9.9 ang -1 1 jmat=1 jcons=3
change -1,41 -21,-10.1  jmat=2   jcons=2
change -1,41 -9.9,21    jmat=2   jcons=2
*
*  set boundary material property
bound mat=1
*  set viscous boundary conditions along three boundaries
bound -1,41 -20.1,-19.9 xvisc yvisc
bound -1,41 19.9,20.1   xvisc,yvisc
bound 39,41 -21,21      xvisc,yvisc
*  set symmetry boundary conditions along the remaining boundary
bound -0.1,0.1 -21,21   xvel=0
*
*  set velocity boundary conditions along the semi-circular boundary
bo -.05,.05 -.55,-.45 xvel=0 yvel=-1.0
bo .17,.21 -.48,-.45 xvel=0.383 yvel=-0.924
bo .33,.37 -.37,-.33 xvel=0.707 yvel=-0.707
bo .43,.47 -.21,-.17 xvel=0.924 yvel=-0.383
*
bo .48,.52 -0.05,0.05 xvel=1.0 yvel=0.0
*
bo .41,.45 .17,.21 xvel=.924 yvel=.383
bo .33,.37 .33,.37 xvel=.707 yvel=.707
bo .17,.21 .43,.47 xvel=0.383 yvel=0.924
bo -0.05,0.05 .45,.55 xvel=0 yvel=1
*
*  read time variation of velocity input from an external data file
*  cilvdx.out is output from program cilvpr.for
*
*
bound hread=1 cilvdx.out
*
bound hist=1
*
frac 0.05 .5
insitu stress -1.0e-9,0,-1.0e-9
*
*  set histories
* contact address at coordinate 10,-10 is 1445
*
hist n=10 yvel (0,.5) xvel (.5,0) xvel (.35,0) yvel (.35,.35)
hist xvel (.19,-.46) yvel (.19,-.46)
hist add=1445,15
*
*
cyc 4000
save ver41vl.sv2
stop
*------------------------------------------------------------------
```

APPENDIX 3.2.2.1-A

COMPUTER PROGRAM

Code Name : CILVPR.FOR

```
c****************   Dynamic verification problem   *************
c*
c*    This program evaluates the radial velocity input profile at
c*     r=0.05h
c*************************************************************
c
      common a(5000,5), ta(5000)
      real v(5000),fp(5000),vh(5000)
      character*80 title
c*
      cl=17.32
      per=1.2
      tt=1.4
      x=.5
      nt=1000
      nx=0
c
c
      write (*,*) ('cl   per   tt   x    nt  ')
      write (*,*) cl,per,tt,x,nt
      read(*,100) char
 100  format(a1)
c
c      if (x.le.0.0) go to 200
      nx=nx+1
c
      pi=4.0*atan(1.0)
      w=2.0*pi/per
      dt=tt/nt
      ca=-1.0/(2.0*pi*cl)
      cb=ca/(x*x)
      cc=cb*dt
c
      do 20 i=1,nt
      t=(i-1)*dt
      if (t.lt.0.5*per) then
        fp(i)=0.5*w*sin(w*t)
```

```
cxxxx           fp(i)=0.5*w*w*cos(w*t)
            nfp=i
         else
            fp(i)=0.0
         endif
   20    continue
c
         t0=x/cl
         j0=t0/dt
         j0=j0+1
c
         do 30 j=1,nt
            if(j.lt.j0) then
                vh(j)=0.0
            else
                t=t0+0.5*dt+(j-j0)*dt
                cf=t*cl/x
                cf2=cf*cf
                cs=sqrt(cf2-1.0)
c
c   velocity
           cg=(cf2-1.0)**1.5
            vh(j)=cc/cg
c   displacement
cxxxx                cg=cs/t
cxxxx                vh(j)=cc/cg
         endif
   30    continue
c
         v(1)=0.0
         do 60 i=2,nt
c        t=(i-1)*dt
         v(i)=0.0
         j1=min(nfp,i-1)
cccc      if (j1.lt.j0) goto 60
         do 40 j=1,j1
c        v(i)=v(i)+fp(j)*vh(i-j+1)
         v(i)=v(i)+fp(j)*vh(i-j)
   40    continue
   60    continue
         vmax=0.0
         do 80 i=1,nt
         ta(i)=(i-1)*dt
         a(i,nx)=v(i)
         vi=abs(v(i))
         vmax=amax1(vmax,vi)
   80    continue
   90    format ('  x= ',f6.3,'  nt= ',i5,5x,'  max= ',e12.4)
```

```
c
c       if (x.le.0.0) then
         open (3,file='cilvdx.out')
         write (3,*) title
         write (3,101) nt,dt
         write (3,102) (ta(j),a(j,nx),j=1,nt)
         close (3)
c       endif
  101 format (2x,i5,2x,f10.4)
  102 format (2(2x,e10.4))
c
      stop
      end
```

APPENDIX 3.2.2.1-B

COMPUTER PROGRAM

Code Name : LSJEM.FOR

```
c*************    Dynamic verification problem *****************
c
c*    This program evaluates the dynamic response of the slip of
c*    a single discontinuity of infinite extent caused by an
c*    explosive loading. Analytical solution of a line source in
c*    an elastic medium with a  discontinuity is given by
c*    S.M. Day ( see equation (4) and (6) in S-CUBED memorenda
c     from S.M.Day to R. Hart, May 1, 1985).
c*****************************************************************
          dimension duf(2000),fil(2000)
          common /gplot/ nt,tt(2000),du(2000)
          complex cp,cetap,cetas,cr
c
      open(2,file='line.out')
c
c  input data nt=1000, dt=0.005, x=1 h=1 gamma=0 per=0.6 rho=1.0
c
 999  write (*,888)
 888  format (' nt dt x h gamma per rho',/)
      read(*,*) nt
      if(nt.eq.0) goto 1000
      read(*,*) dt,x,h,gamma,per,rho
      pi=3.14159
      vp=sqrt(3.)
      vs=1.
      xmin=0
      ymin=0
      r=sqrt(x*x+h*h)
      do 1 i=1,nt
      t=float(i)*dt
      tt(i)=t
      tau=t-r/vp
      if(tau.gt.0.) then
       t2r2=sqrt(t**2-(r/vp)**2)
        cp=cmplx(t*x/r**2,t2r2*h/r**2)
       cetap=csqrt(1./vp**2-cp**2)
       cetas=csqrt(1./vs**2-cp**2)
       cr=(1.-2.*vs**2*cp**2)**2+4.*vs**4*cetap*cetas*cp**2
       cr=cr+2.*vs*cetas*gamma
       dut=2.*vs**2/(pi*rho*vp**2)
```

```
       dut=dut*real(cp*cetap*cetas/cr)/t2r2
       du(i)=dut
      else
            du(i)=0.
      end if
    1 continue
      nf=int(per/dt+0.0001)
      if(nf.gt.1000) goto 1200
      do 2 j=1,nf
      ph=float(j)*dt/per
      if(ph.lt.1.) then
        fil(j)=sin(pi*ph)
      else
        fil(j)=0.
      end if
    2 continue
      sum=0.
      do 5 j=1,nf
    5 sum=sum+fil(j)
      do 4 i=1,nt
      duf(i)=0
      n=min(nf,i)
      do 3 j=1,n
    3 duf(i)=duf(i)+du(i-j+1)*fil(j)
    4 duf(i)=duf(i)/sum
      dmx=0.
      write(2,400)
  400 format (/,'   time      norm. slip',/)
c
      do 6 i=1,nt
      if(mod(i,10).eq.0) then
      time=float(i)*dt
      ftduf=4.*duf(i)
      write(2,500) time,ftduf
  500   format(1p,e12.4,5x,1p,e12.4)
      endif
      if(duf(i).gt.dmx) then
      dmx=duf(i)
      tmx=float(i)*dt
      endif
    6 continue
      ftdmx=4.*dmx
      print *,'max value of du = ',ftdmx
      print *,'time at max du = ',tmx
      go to 999
```

```
c
1200  write(*,898)
 898  format(' nf exceeds fil dimension')
1000  stop
      end
```

3.2.2.2 Slip Induced by Harmonic Shear Wave

This problem is given in Volume 2 of this document, UDEC User's Manual, pp. A-1 to A-9.

3.2.3 Thermal Problems

The following thermal problems can be found as noted.

Steady-State Temperature Distribution Along a Tapered Fin (see UDEC User's Manual, pp. 7-37 to 7-39)

One-Dimensional Steady-State Heat Conduction and Convection Through a Composite Wall (see UDEC User's Manual, pp. 7-40 to 7-42)

Thermal Response of a Heat-Generating Slab (see UDEC User's Manual, pp. 7-43 to 7-45)

Transient Temperature Distribution in an Orthotropic Bar (see UDEC User's Manual, pp. 7-51 to 7-53)

3.2.3.1-1 Steady-State Temperature Distribution Along a Tapered Fin

This problem is given in Volume of this document, UDEC User's Manual, pp. 7-37 through 7-39.

3.2.3.2-1 One-Dimensional Steady-State Heat Conduction and Convection Through a Composite Wall

This problem is given in Volume 2 of this document, UDEC User's Manual, pp. 7-40 through 7-42.

3.2.3.3 Thermal Response of a Heat-Generating Slab

This problem is given in Volume 2 of this document, UDEC User's Manual, pp. 7-43 through 7-45.

3.2.3.4 Transient Temperature Distribution in an Orthotropic Bar

This problem is given in Volume 2 of this document, UDEC User's Manual, pp. 7-51 through 7-53.

3.2.4 Thermo-Mechanical Problems

The following thermal problems are presented in this section or can be found as noted.

Thermo-Elastic Response of a Hollow Thick Wall Cylinder

Infinite Slab with Applied Heat Flux (see UDEC User's Manual, pp. 7-46 to 7-50)

3.2.4.1 Thermo-Elastic Response of a Hollow Thick Wall Cylinder

Problem Statement

This problem concerns determination of thermal stresses in a long cylinder subjected to steady-state heat flow from the interior to the exterior. The problem definition requires only an internal temperature, external temperature, and elastic constants for the analytical calculation of both the temperature and stress distribution.

Objective

The objective of this problem is to test the coupled thermo-mechanical capability of UDEC. The problem checks the following specific aspects of the code:

(1) heat conduction algorithms for both implicit and explicit calculations; and

(2) determination of induced stresses from temperature changes through the coefficient of linear thermal expansion.

Physical Problem

The following values and parameters are used to describe the problem.

Geometry

- inside radius (m) $a = 0.5$
- outside radius (m) $b = 20$

Material Properties

- modulus of elasticity (GPa)	$E = 6$
- Poisson's ratio	$\nu = 0.25$
- coefficient of linear thermal expansion (1/°C)	$\alpha = 5$
- conductivity (W/m °C)	$K = 5$
- specific heat (J/kg °C)	$C = 900$
- density (kg/m^3)	$\rho = 2000$

Constant Temperatures

- internal surface (°C)	$T_i = 200$
- external surface (°C)	$T_e = 50$

Note that the input parameters for conductivity, specific heat, and density are not required by the analytical solution, but are required by most codes, including UDEC, for thermal analysis.

Analytical Solution

The analytical solution to this problem is given by Timoshenko and Goodier (1970). From the heat flow equation, the temperature distribution for the geometry and boundary conditions of this problem can be defined as:

$$T = \frac{T_i - T_e}{\ln(b/a)} \ln \frac{b}{r} + T_e \qquad (3.2.4.1\text{-}1)$$

where r = radial coordinate,

T = temperature at radius r,

T_i, T_e = internal, external temperatures,

b = external radius, and

a = internal radius.

The stress equilibrium equation in polar coordinates is:

$$\frac{\partial \sigma_r}{\partial r} + \frac{\sigma_r - \sigma_\theta}{r} = 0 \qquad (3.2.4.1\text{-}2)$$

where σ_r, σ_θ = radial, tangential stresses.

Using this equation, the elastic stress-strain relation and the definition of strain in terms of displacements, appropriate integrations can be carried out to give stress in terms of the temperature distribution. These are given by:

$$\sigma_r = \frac{\alpha E}{1 - \nu} \frac{1}{r^2} \left[\frac{r^2 - a^2}{b^2 - a^2} \int_a^b Tr\,dr - \int_a^r Tr\,dr \right] \qquad (3.2.4.1\text{-}3)$$

$$\sigma_\theta = \frac{\alpha E}{1 - \nu} \frac{1}{r^2} \left[\frac{r^2 + a^2}{b^2 - a^2} \int_a^b Tr\,dr + \int_a^r Tr\,dr - Tr^2 \right] \qquad (3.2.4.1\text{-}4)$$

By substituting Eq. (3.2.4.1-1) in the above, the stresses can be found from the interior and exterior surface temperatures as follows:

$$\sigma_r = \frac{\alpha E(T_i - T_e)}{2(1 - \nu)\ \ln\ (b/a)} \left[-\ \ln \frac{b}{r} - \frac{a^2}{(b^2 - a^2)} \left[1 - \frac{b^2}{r^2} \right] \ln \frac{b}{a} \right]$$

(3.2.4.1-5)

$$\sigma_\theta = \frac{\alpha E(T_i - T_e)}{2(1 - \nu)\ \ln(b/a)} \left[1 - \ln \frac{b}{r} - \frac{a^2}{(b^2 - a^2)} \left[1 + \frac{b^2}{r^2} \right] \ln \frac{b}{a} \right]$$

(3.2.4.1-6)

The analytical solution expressed above has been written as a Fortran program to permit comparison with the numerical solution. The program and results are appended.

Assumptions

The following assumptions are implicit in the analytical solution:

(1) the medium is linear elastic, homogeneous and isotropic;

(2) elastic properties and conductivities are not temperature dependent;

(3) heat flux is constant and steady-state conditions have been achieved; and

(4) plane strain conditions apply.

Computer Model

The UDEC model used for the given test problem consists of the first quadrant of the cylinder. The bottom (x-axis) and left (y-axis) boundaries are lines of symmetry. The model is divided into a series of concentric arcs with increasing spacing between the arc cuts. In this way, the block zoning can be increased away from the hole. The ring cuts or joints are "glued" by setting the strength and stiffness parameters high. Models were run with 7, 13 and 17 arcs. The zoning for the 7 arc model and the 17 arc model are shown in Figs. 3.2.4.1-1 and 3.2.4.1-2, respectively.

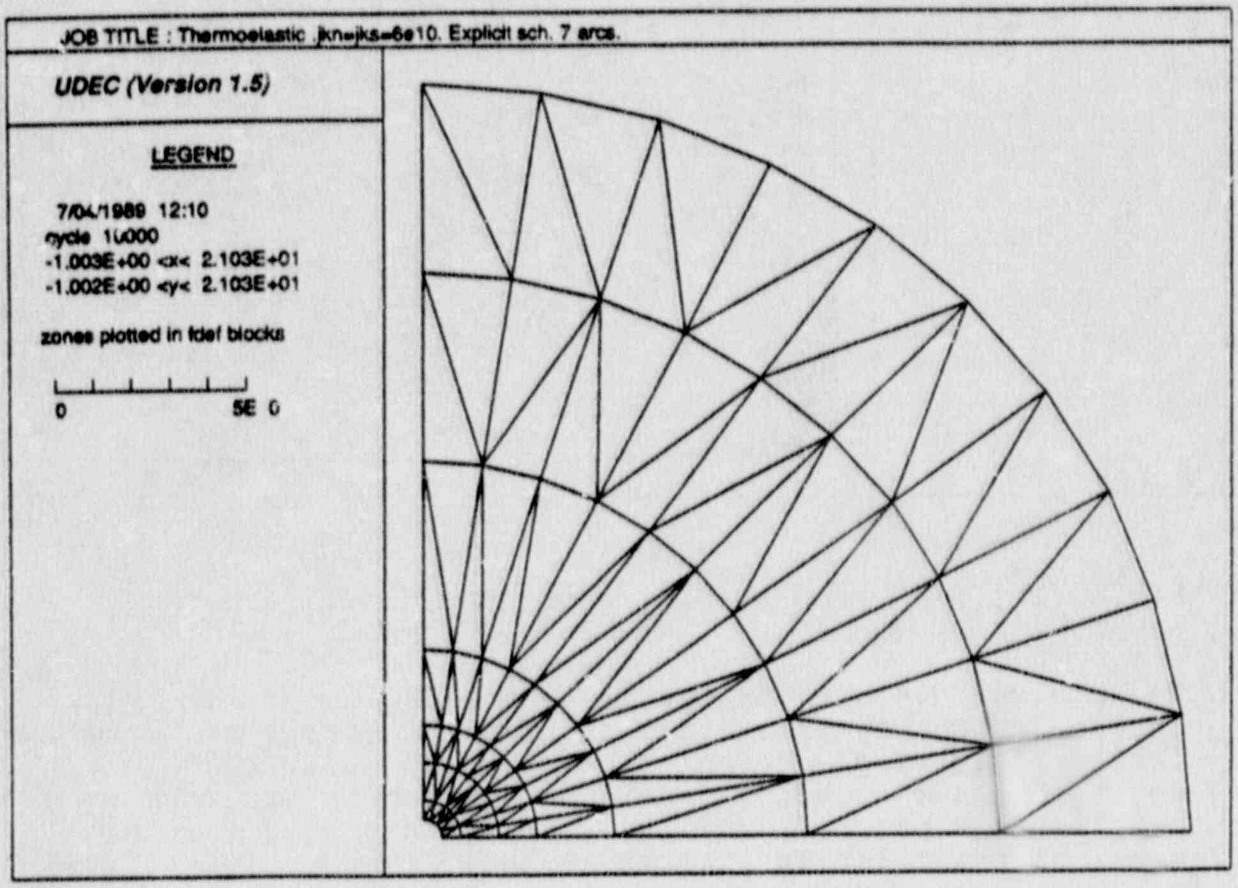

Fig. 3.2.4.1-1 Problem Discretization for Seven Arc Model

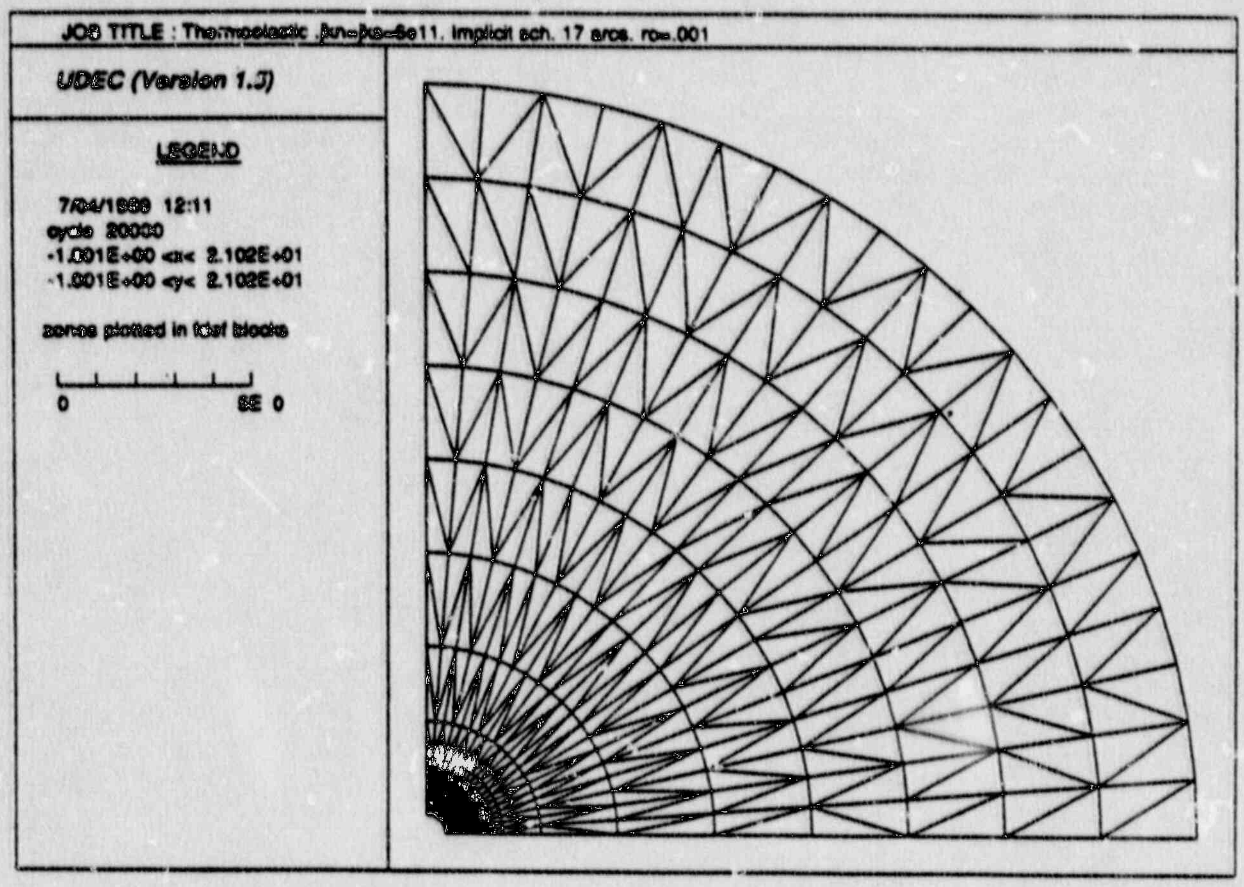

Fig. 3.2.4.1-2 Problem Discretization for 17 Arc Model

In UDEC, the rounding length specified for a problem determines the distance from the corner of a block to the point where forces and displacements with adjacent blocks are actually calculated. In general, smaller rounding lengths lead to more accurate results. In this problem, various combinations of joint normal stiffness, rounding length, thermal solution method (i.e., implicit or explicit), and number of arcs were used as shown in Table 3.2.4.1-1. In all runs, the joint shear stiffness was set equal to the joint normal stiffness.

Table 3.2.4.1-1

SOLUTION PARAMETERS USED IN THERMO-ELASTIC ANALYSIS OF A HOLLOW THICK WALL CYLINDER

Run	Arcs	Thermal Solution Scheme	Rounding Length (mm)	Joint Stiffness (GPa/m)
1	7	Expl.	10	60
2	7	Impl.	10	60
3	13	Expl.	10	60
4	13	Impl.	10	60
5	13	Impl.	10	600
6	13	Impl.	10	6000
7	17	Impl.	1	600

Results

The results for radial and tangential stress versus radial distance are shown in Figs. 3.2.4.1-3 and 3.2.4.1-4, respectively. In general, the UDEC results agree fairly well with the analytical solution. Theoretically, there should be no difference between the results obtained using the implicit or explicit thermal solution schemes. However, differences can result if either scheme is not run to the same "equilibrium".

Reference

Timoshenko, S. P. and J. N. Goodier. Theory of Elasticity, 3rd Ed., New York: McGraw-Hill, 1970.

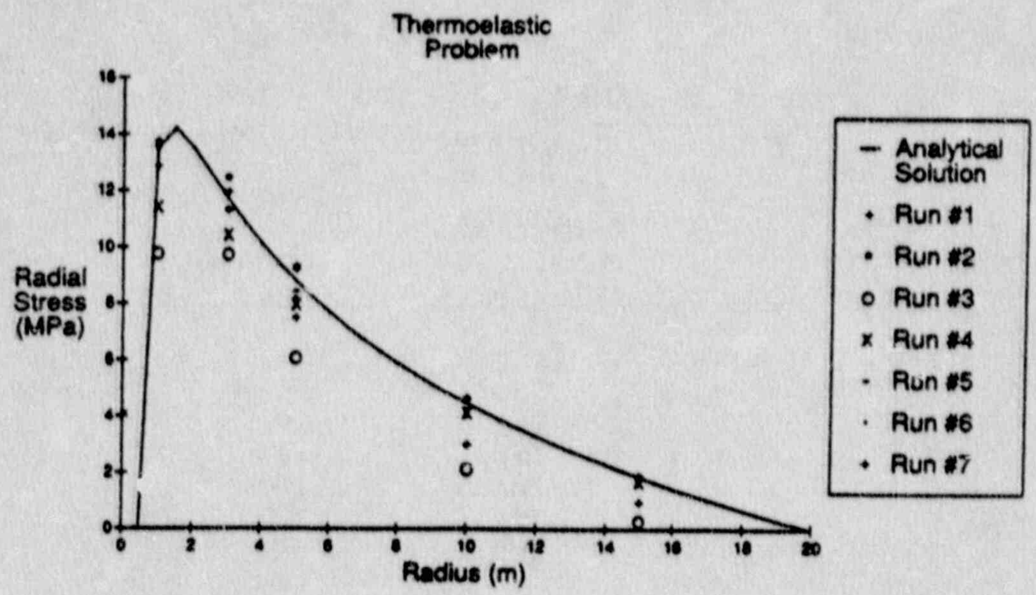

Fig. 3.2.4.1-3 Comparison of UDEC Results of Radial Stress With Analytical Solution for the Case of a Hollow Thick-Walled Cylinder Subjected to Thermal Loading

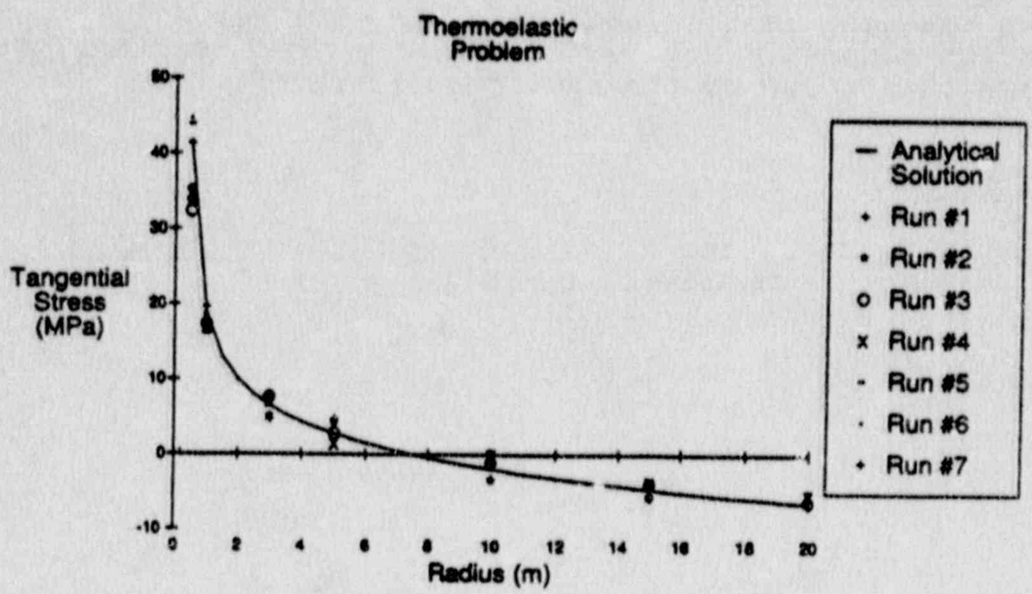

Fig. 3.2.4.1-4 Comparison of UDEC Results of Tangential Stress with Analytical Solution for the Case of a Hollow Thick-Walled Cylinder Subjected to Thermal Loading

Data Input File

```
************************************************************************
* Verification test - Thermoelastic Response of a Hollow,
* Thick Wall Cylinder.  This data file is for implicit
* Thermal solution with fine discretization.
************************************************************************

* configure thermal problem
thermal

head
Thermoelastic jkn=jks=6e11. Implicit sch. 17 arcs. ro=.001

round=.001

*set geometry.Many blocks to get better zoning
bl 0 0 0 20 20 20 20 0
arc 0 0 .5 0 90 20
arc 0 0 .55 0 90 20
arc 0 0 .6 0 90 20
arc 0 0 .7 0 90 20
arc 0 0 .825 0 90 20
arc 0 0 1  0 90 20
arc 0 0 1.25 0 90 20
arc 0 0 1.5 0 90 20
arc 0 0 2  0 90 20
arc 0 0 3  0 90 20
arc 0 0 5  0 90 20
arc 0 0 7.5 0 90 20
arc 0 0 10 0 90 20
arc 0 0 12.5 0 90 20
arc 0 0 15 0 90 20
arc 0 0 17.5 0 90 20
arc 0 0 20 0 90 20

*excavate
del 0 .3 0 .3

*set joint at 45 degr.
cr 0 0 20 20

*delete outside
del 10 20 10 20
```

```
*set material properties
prop m=1 de=2000 k=4e9 g=2.4e9 jkn=6e11 jks=6e11
prop m=1 jcoh=1e20 jfric=0 jten=1e20
prop m=1 thexp=40e-6 cond=5 spec=900

*create zoning
gen edge 20

damp auto

*initial temperature=323 K
initem 323 0 20 0 20

*temperature=323.00 fixed at radius=   20.000000
tfix   323.000000   19.932182   20.006165   -0.156918    1.726100
tfix   323.000000   19.735310   19.956804    1.413231    3.284640
tfix   323.000000   19.416763   19.784405    2.974668    4.822929
tfix   323.000000   18.978502   19.490026    4.517765    6.331483
tfix   323.000000   18.423237   19.075483    6.033008    7.801002
tfix   323.000000   17.754385   18.543337    7.511055    9.222425
tfix   323.000000   16.976070   17.896864    8.942794   10.586987
tfix   323.000000   16.093094   17.140049   10.319398   11.886278
tfix   323.000000   15.110896   16.277561   11.632380   13.112288
tfix   323.000000   14.035536   15.314716   12.873645   14.257454
tfix   323.000000   12.873642   14.257452   14.035538   15.314719
tfix   323.000000   11.632378   13.112285   15.110898   16.277563
tfix   323.000000   10.319395   11.886276   16.093096   17.140051
tfix   323.000000    8.942790   10.586985   16.976072   17.896866
tfix   323.000000    7.511050    9.222421   17.754387   18.543339
tfix   323.000000    6.033001    7.800997   18.423239   19.075485
tfix   323.000000    4.517758    6.331478   18.978504   19.490026
tfix   323.000000    2.974661    4.822923   19.416761   19.784405
tfix   323.000000    1.413224    3.284633   19.735310   19.956804
tfix   323.000000   -0.156926    1.726093   19.932182   20.006165

*temperature=473.00 fixed at radius=    0.500000
tfix   473.000000    0.426314    0.506699   -0.025000    0.275000
tfix   473.000000    0.231699    0.451314    0.231699    0.451314
tfix   473.000000   -0.025000    0.275000    0.426314    0.506699

*thermal histories
thist ntcycl=500
thist temp .5 0
thist temp 15 0
thist temp 19 0
```

```
*mechanical history
h ncyc=500
h nstr 927

*boundary conditions
bo .4 20.1 -.1 .1 yvel 0
bo -.1 .1 .4 20.1 xvel 0

*run thermal problem (implicit). Tolerance:1/2*minimum zone size
run temp=500 delt=5000 age=5e8 step=2000000 implicit tol=.025

*then run mechanical problem
cy 20000
save thme9.sav

quit
```

```
*******************************************************************
* Verification test - Thermoelastic Response of a Hollow,
* Thick Wall Cylinder.  This data file is for explicit
* Thermal solution with coarse discretization.
*******************************************************************

* configure thermal problem
thermal

head
Thermoelastic jkn=jks=6e10. Explicit sch. 7 arcs.

round=.01

*set geometry (one quarter of cylinder).Many blocks to get good zoning
bl 0 0 0 20 20 20 20 0
arc 0 0 .5 0 90 4
arc 0 0 1  0 90 5
arc 0 0 2  0 90 10
arc 0 0 3  0 90 10
arc 0 0 5  0 90 10
arc 0 0 10 0 90 10
arc 0 0 15 0 90 10
arc 0 0 20 0 90 10

*excavate
del 0 .3 0 .3

*set joint at 45 degr. for tangential stress recording
cr 0 0 20 20

*delete outside of cylinder
del 10 20 10 20

*set material properties
prop m=1 de=2000 k=4e9 g=2.4e9 jkn=6e10 jks=6e10
prop m=1 jcoh=1e20 jfric=0 jten=1e20
prop m=1 thexp=40e-6 cond=6 spec=900

*create zoning
gen edge 20

damp auto

*initial temperature : 323 K (everywhere)
initem 323 0 20 0 20
```

```
*temperature=323.00 fixed at radius=   20.000000
tfix  323.000000  19.932182  20.006165  -0.156918   1.726100
tfix  323.000000  19.735310  19.956804   1.413231   3.284640
tfix  323.000000  19.416763  19.784405   2.974668   4.822929
tfix  323.000000  18.978502  19.490026   4.517765   6.331483
tfix  323.000000  18.423237  19.075483   6.033008   7.801002
tfix  323.000000  17.754385  18.543337   7.511055   9.222425
tfix  323.000000  16.976070  17.896864   8.942794  10.586987
tfix  323.000000  16.093094  17.140049  10.319398  11.886278
tfix  323.000000  15.110896  16.277561  11.632380  13.112288
tfix  323.000000  14.035536  15.314716  12.873645  14.257454
tfix  323.000000  12.873642  14.257452  14.035538  15.314719
tfix  323.000000  11.632378  13.112285  15.110898  16.277563
tfix  323.000000  10.319395  11.886276  16.093096  17.140051
tfix  323.000000   8.942790  10.586985  16.976072  17.896866
tfix  323.000000   7.511050   9.222421  17.754387  18.543339
tfix  323.000000   6.033001   7.800997  18.423239  19.075485
tfix  323.000000   4.517758   6.331478  18.978504  19.490026
tfix  323.000000   2.974661   4.822923  19.416761  19.784405
tfix  323.000000   1.413224   3.284633  19.735310  19.956804
tfix  323.000000  -0.156926   1.726093  19.932182  20.006165

*temperature=473.00 fixed at radius=   0.500000
tfix  473.000000   0.426314   0.506699  -0.025000   0.275000
tfix  473.000000   0.231699   0.451314   0.231699   0.451314
tfix  473.000000  -0.025000   0.275000   0.426314   0.506699

*thermal histories (to check thermal equilibrium)
thist ntcycl=500
thist temp .5 0
thist temp 15 0
thist temp 19 0

*mechanical histories at different joints (to check equil.)
h ncyc=500
h nstr 9078
h nstr 2240
h nstr 4664
h nstr 4370
h nstr 7088
h nstr 8264
h nstr 6794
h sstr 9078
h sstr 2240
h sstr 4664
h sstr 4370
h sstr 7088
h sstr 8264
h sstr 6794
```

```
*set boundary conditions
bo .4 20.1 -.1 .1 yvel 0
bo -.1 .1 .4 20.1 xvel 0

*run thermal problem until equilibrium (explicit procedure)
run temp=500 step=2000000

*then run mechanical problem
cy 10000
save thme1.sav

quit
```

Fortran Code For Analytical Solution For Thermo-Elastic Response of a Thick Walled Cylinder

```
c     analytical solution for thermoelastic response of cylinder

      dimension c(200),d(200)
      real nu

      open (unit=11,file='thres')

c     set parameters
      a=.5
      b=20
      nu=.25
      e=6000.e6
      alf=40.e-6

c     thermal loading
      ti=200.
      te=50.

      write (11,100)
      write (11,105) e,nu
      write (11,110) alf
      write (11,115) ti,te
      write (11,120)
100   format ('THERMOELASTIC PROBLEM',/)
105   format ('Young''s modulus:',f14.1,/,'Poisson''s ratio:',f5.3)
110   format ('Coefficient of thermal expansion:',f14.6)
115   format ('Thermal loading:',/,'    ti:',f5.1,/,'    te:',f5.1,//)
120   format ('  radius    sigr(MPa)    sigt(MPa)',/)

c     compute stresses
      a1=alf*e*(ti-te)/(2.*(1.-nu)*alog(b/a))
      r=.5
      do 200 i=1,40
      sigr=a1*(-alog(b/r)-a**2/(b**2-a**2)*(1.-b**2/r**2)*alog(b/a))
      sigt=a1*(1.-alog(b/r)-a**2/(b**2-a**2)*(1.+b**2/r**2)*alog(b/a))
      sigr=sigr*1.e-6
      sigt=sigt*1.e-6
      write (11,130) r,sigr,sigt
130   format (f5.2,2f14.3)
135   format ('-----------------------------------')
      if (r.eq..5.or.r.eq.1..or.r.eq.3.) write (11,135)
      if (r.eq.5..or.r.eq.10..or.r.eq.15..or.r.eq.20.) write (11,135)
      r=r+.5
```

```
200   continue
      close (11)
      stop
      end
```

THERMOELASTIC PROBLEM

Young's modulus: 6000000000.0
Poisson's ratio:0.250
Coefficient of thermal expansion: 0.000040
Thermal loading:
ti:200.0
te: 50.0

radius	sigr (MPa)	sigt (MPa)
0.50	0.000	-41.524
1.00	-13.502	-19.003
1.50	-14.199	-13.030
2.00	-13.495	-9.991
2.50	-12.583	-7.998
3.00	-11.691	-6.519
3.50	-10.865	-5.339
4.00	-10.111	-4.355
4.50	-9.423	-3.510
5.00	-8.794	-2.768
5.50	-8.216	-2.107
6.00	-7.681	-1.509
6.50	-7.185	-0.963
7.00	-6.723	-0.462
7.50	-6.290	0.003
8.00	-5.883	0.436
8.50	-5.499	0.841
9.00	-5.136	1.222
9.50	-4.792	1.581
10.00	-4.465	1.921
10.50	-4.153	2.244
11.00	-3.855	2.552
11.50	-3.570	2.845
12.00	-3.297	3.126
12.50	-3.034	3.395
13.00	-2.782	3.653
13.50	-2.539	3.901
14.00	-2.305	4.140
14.50	-2.079	4.370
15.00	-1.860	4.593

15.50	-1.648	4.808
16.00	-1.443	5.016
16.50	-1.245	5.217
17.00	-1.052	5.413
17.50	-0.864	5.603
18.00	-0.662	5.787
18.50	-0.505	5.966
19.00	-0.332	6.141
19.50	-0.164	6.311
20.00	0.000	6.476

3.2.4.2 Infinite Slab with Applied Heat Flux

This problem can be found in the UDEC User's Manual, pp. 7-46 to 7-50.

3.2.5 Fluid Flow Problems

The following fluid flow problem can be found as noted.

Steady-State Fluid Flow with Free Surface (see UDEC User's Manual, pp. F-1 to F-7)

3.2.5.1 Steady-State Fluid Flow with Free Surface

This problem is given in Volume 2 of this document, UDEC User's Manual, pp. F-1 through F-7.

3.3 EXAMPLE THERMOMECHANICAL ANALYSES OF A WASTE EMPLACEMENT DRIFT

This problem consists of transient thermal mechanical simulation of the behavior of a drift in which heat producing waste is placed vertically beneath the floor. The specific problem presented here is adapted from Christianson (1989).

Assumptions and Idealizations

The emplacement drift being modeled is in the center of an emplacement panel. This assumption allows symmetry to be imposed reducing the computation time. The emplacement of waste in the panel is assumed to be instantaneous.

The analyses ignore any effects of the joint on the thermal conductivity of the rock mass. Based on the results of field tests involving thermal conductivity of rock masses, this assumption appears reasonable. The analyses also ignore the effects of fluid (i.e., air and water) convection in the rock mass and emplacement room. The analyses ignore effects of boiling of pore water which could affect heat transfer rates. The thermal properties used assume fully saturated conditions.

A linear stiffness Mohr-Coulomb joint model is used in this analysis. While more complex models exist, such as the continuously yielding model (Cundall, 1988) and the Barton-Bandis model (Barton, 1982), these models vary in detail of the behavior, but the fundamental effects are similar.

In UDEC, each joint is explicitly modeled with variable spacing and persistence. The matrix in UDEC is assumed to behave elastically. This means that inelastic behavior is allowed to occur only in the joints. Figure 3.3-1 illustrates the pattern of joints used in the UDEC modeling.

Conceptual Considerations

Vertical emplacement of waste is being considered in this analysis. It is assumed that the general conclusions will also apply to the horizontal emplacement alternative. Figure 3.3-2 illustrates the vertical emplacement concept. In this example, the spent fuel (SF) and defense high-level waste (DHLW) are assumed to each have a 7.5 ft. pitch.

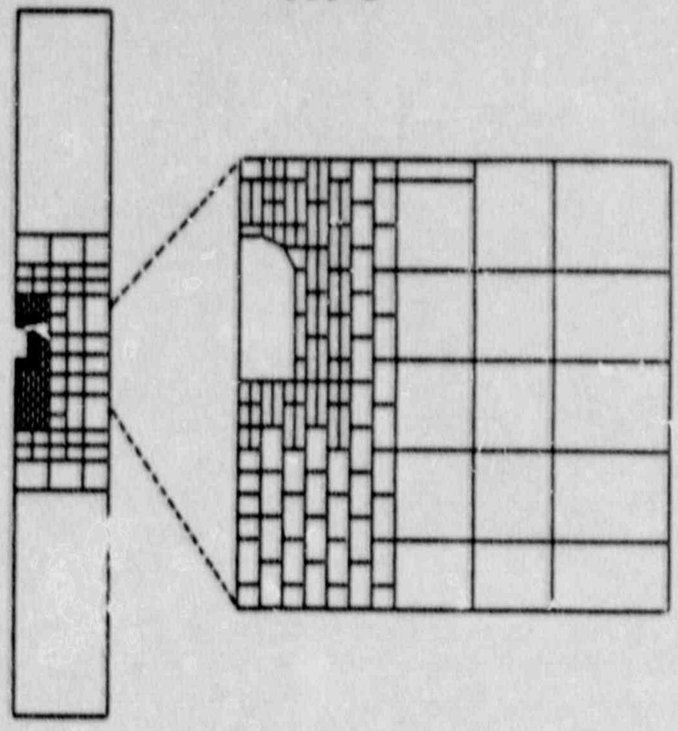

Fig. 3.3-1 UDEC Geometry Used for Example Thermomechanical Analysis of a Waste Emplacement Drift (with blowup of drift area)

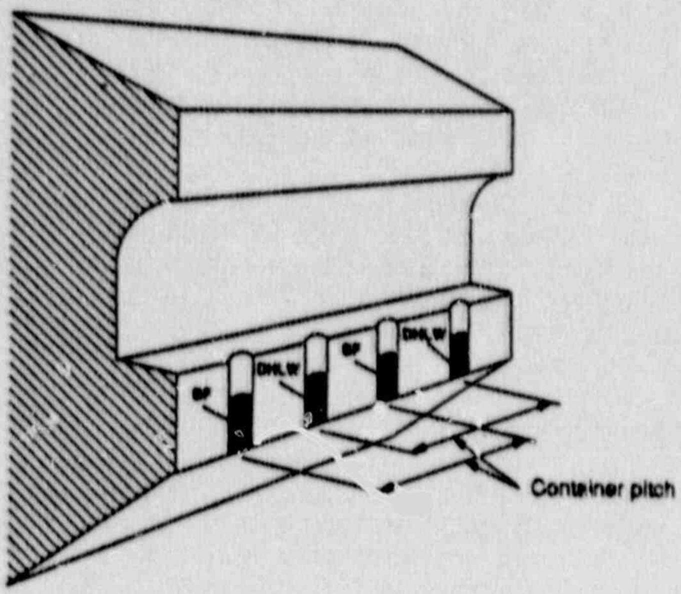

Fig. 3.3-2 Vertical Emplacement Concept

Using two-dimensional models requires that the discrete location of the waste containers be distributed uniformly along the disposal room. In the case of vertical emplacement, this means the location of a vertical heat-generating trench at the center of the floor along the axis of the room. Because of the transient nature of the problem as well as the geometric layout of the waste, the "trench" concept is expected to be an adequate idealization of the emplacement.

Figure 3.3-3 illustrates the conceptual model of the vertical and waste emplacement. Because of symmetry, only one half of the disposal room and pillar needs to be included. The thermal boundary conditions are adiabatic. The two horizontal boundaries have been removed sufficiently far from the heat generating waste to remain at the initial temperature of 26 °C for the time period simulated.

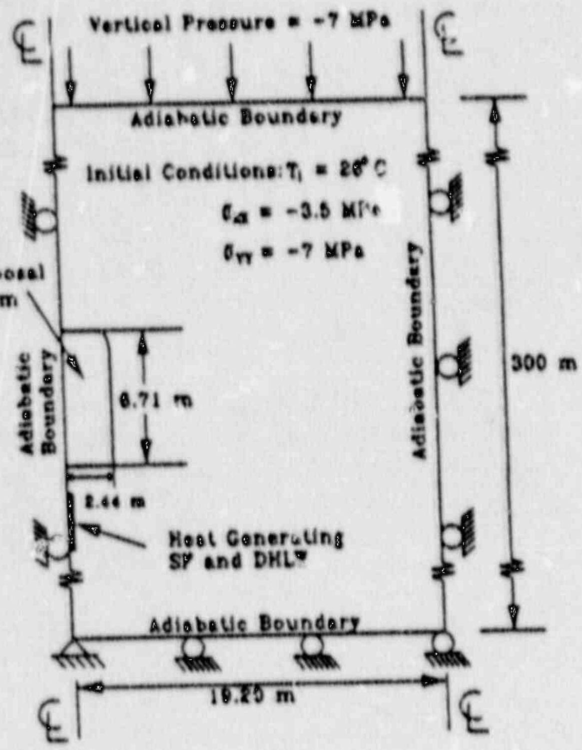

Fig. 3.3-3 Conceptual Model of Vertical Emplacement Concept (compressive stresses assumed negative)

The kinematic boundary conditions are also shown in Fig. 3.3-3, and are such that the two vertical boundaries are restricted from moving in the horizontal direction, while free to move in the vertical direction. The lower horizontal boundary is restricted from moving in the vertical direction, while free to move in the horizontal direction. The upper horizontal boundary is a free-to-move pressure boundary. The initial vertical and horizontal stresses applied to the models are -7 MPa and -3.5 MPa. Note, that compressive stresses are negative.

Waste Form Characteristics

The initial power of a SF container at the time of emplacement is set conservatively to 3.2 kW. The initial power of the DHLW container is chosen as 0.42 kW. Also in this example, the power output of the two waste types is combined and treated as spent fuel.

The thermal decay characteristics of SF given by Peters (1983) for ten-year old waste:

Spent Fuel $$P(t) = 0.54 \exp(-\ln(0.5)t/89.3) + 0.44 \exp(-\ln(0.5)t/12.8)$$

where P(t) = normalized power, and

t = time in years.

The normalized power as a function of time, as described from the above equations as well as that given by Mansure (1985) for SF are shown in Fig. 3.3-4. As seen, the two approximations for SF are very similar.

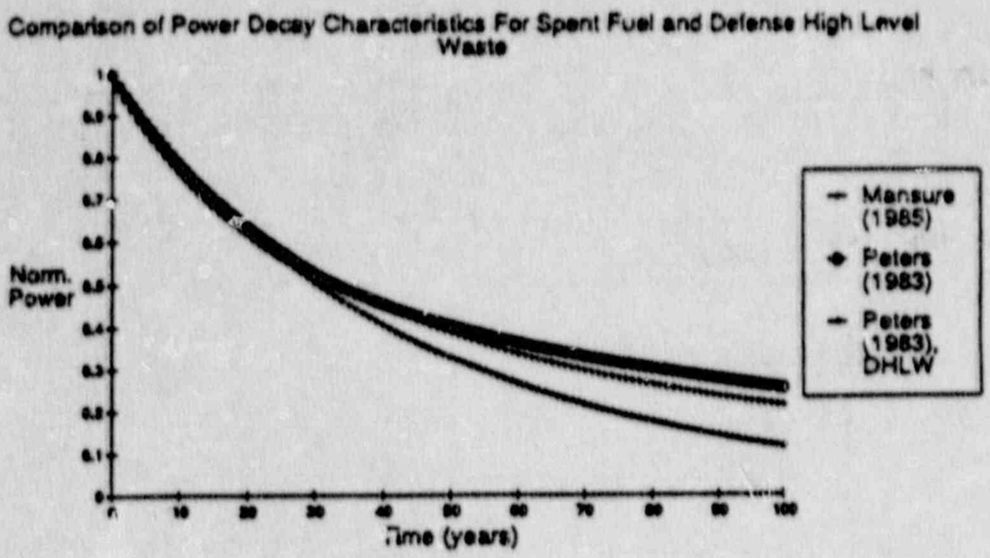

Fig. 3.3-4 Normalized Power as Function of Time

Material Properties

The base thermal and mechanical properties used are shown in Table 3.3-1.

Table 3.3-1

THERMAL AND MECHANICAL PROPERTIES USED IN EXAMPLE THERMOMECHANICAL ANALYSIS OF A WASTE EMPLACEMENT DRIFT

Property		Units
Rock Mass Property		
Bulk Density	2.34	g/cc
E	15.1	GPa
Poisson's ratio	0.20	
k (sat)	2.07	W/m K
Cp (sat)	2.25	j/cm^3 K
Therm Exp.	10.7E-06	1/K
Joint property		
Kn	1E+05	MPa/m
Ks	1E+05	MPa/m
Cohesion	1.0	MPa
Friction	0.8	Coef
Dilation	0.0	Degrees

Modeling Sequence

The input instructions used to generate the UDEC results are appended. The modelling sequence used was:

- EXCAVATION OF THE DRIFT AT TIME = 0

  (Deformations and stresses are determined throughout the rock.)

- INITIAL WASTE EMPLACEMENT AT TIME = 0

  (Heat transfer calculations start. The drift is not ventilated during this period. Adiabatic boundaries are assumed for the emplacement drift.)

- THERMOMECHANICAL RESPONSE AT 50 YEARS

  (The thermal/mechanical response of the rock is predicted at 50 years.

Results

The results of the analyses are shown in Figs. 3.3-5 to 3.3-9. Figures 3.3-5 and 3.3-6 show the stress and displacement distributions which result from drift excavation. The temperature distribution at 50 years is shown in Fig. 3.3-7. Figure 3.3-8 shows the stress distribution at 50 years. The extent of joint shear displacement is shown in Fig. 3.3-9. In UDEC, shear displacement magnitudes are expressed by plotting multiple parallel joints along a joint. The thicker lines have experienced more shear displacement than thinner lines.

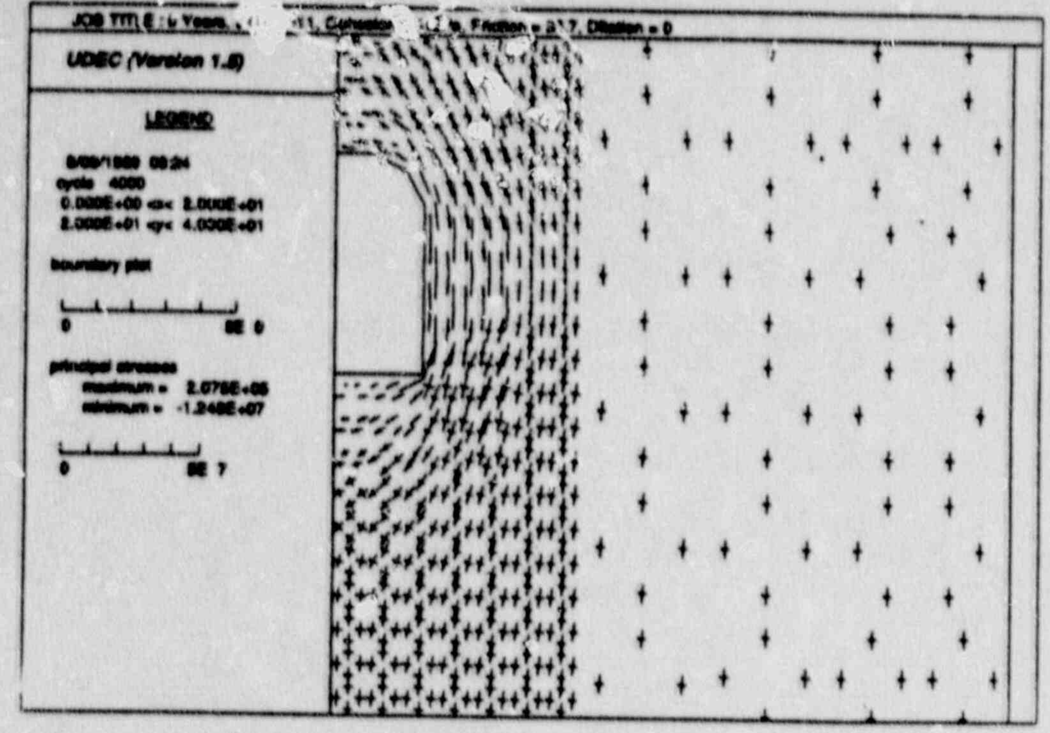

Fig. 3.3-5 Principal Stress Distribution in the Rock at Time = 0 Years

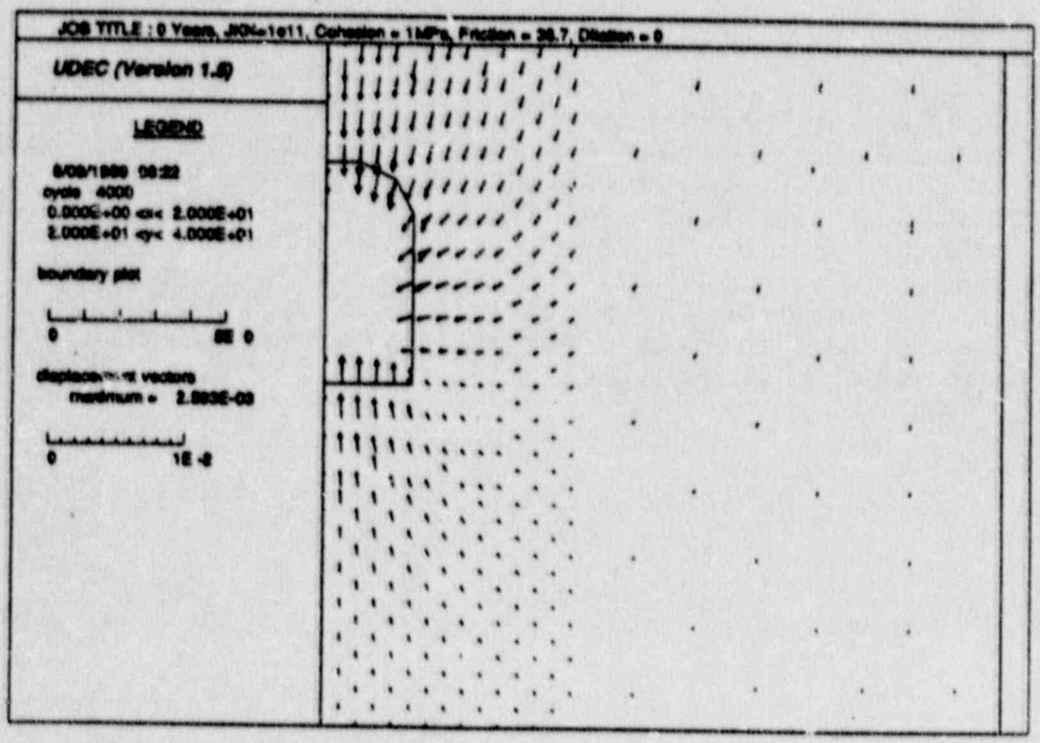

Fig. 3.3-6 Displacement Vector of the Rock at Time = 0 Years

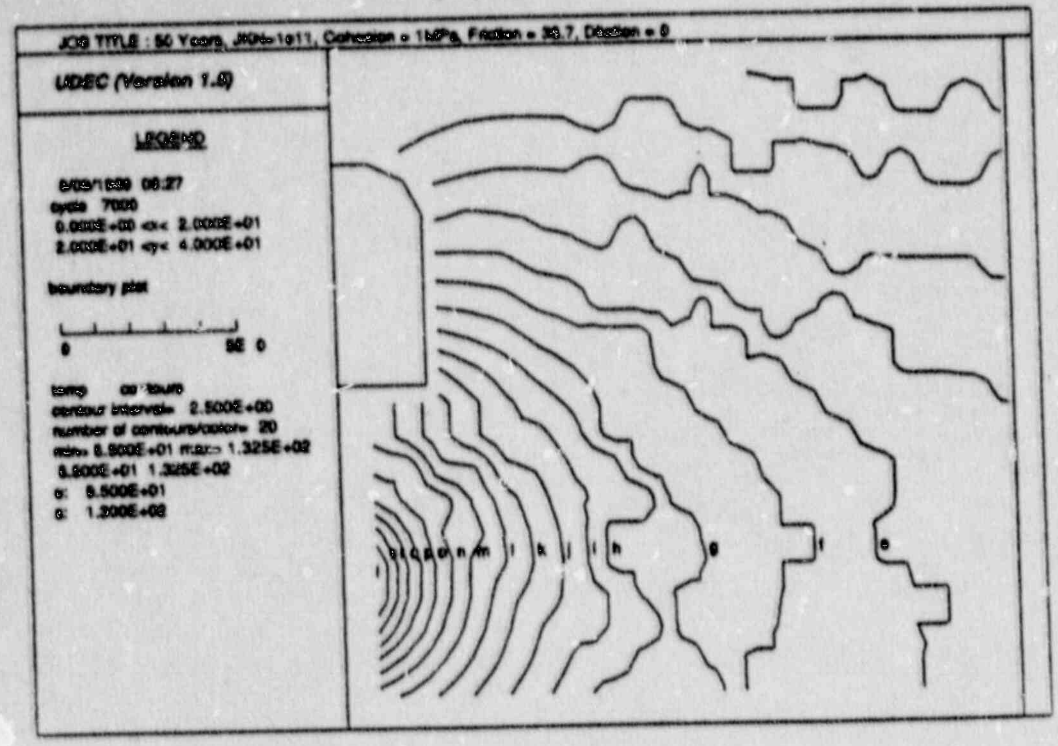

Fig. 3.3-7 Temperature Distribution in the Rock at Time = 50 Years

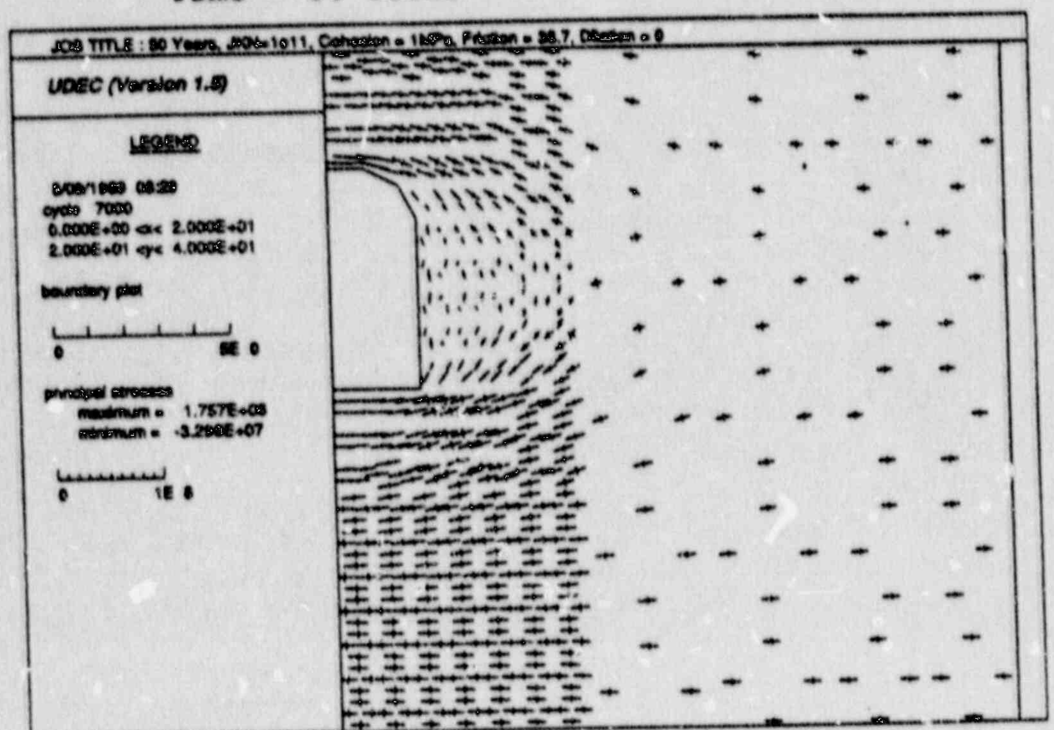

Fig. 3.3-8 Principal Stress Distribution in the Rock at Time = 50 Years

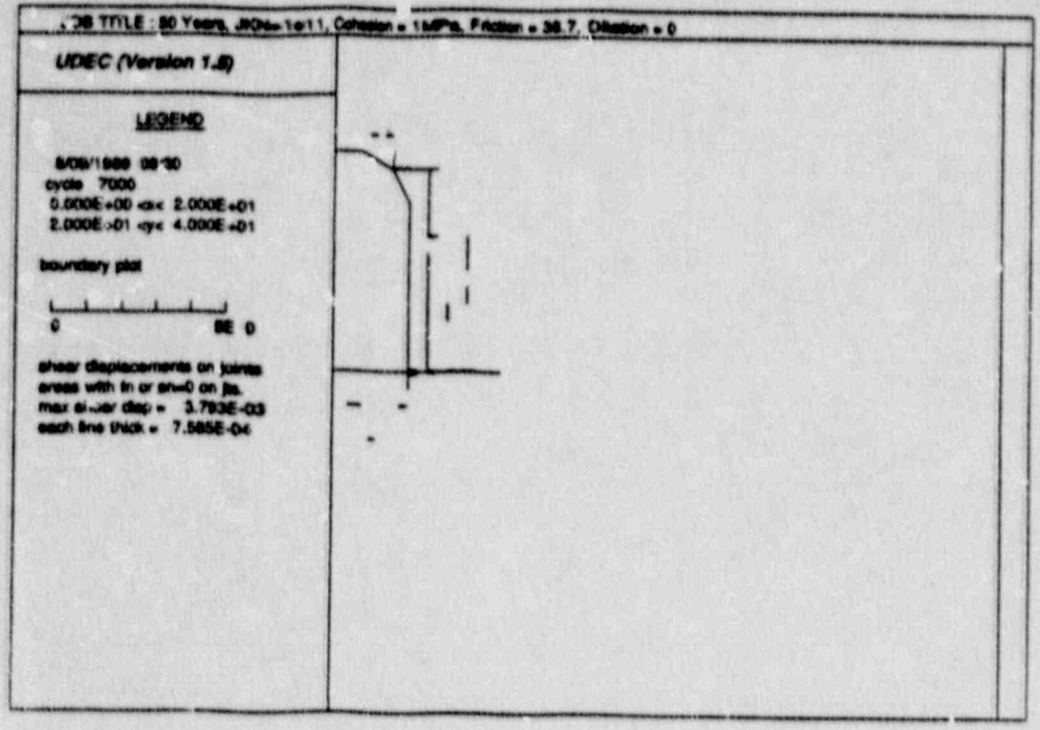

Fig. 3.3-9 Shear Displacement Along the Joints at Time = 50 Years

References

Barton, Nick. "Importance of Joint Parameters on Deformations Observed in Dynamically Loaded Models of Large Excavations," Proceedings of the Workshop on Seismic Performance of Underground Facilities (February 11-13, 1981), pp. 243-247. Oak Ridge: U.S. DOE, 1982.

Christianson, Mark. "Sensitivity of the Stability of a Waste Emplacement Drift to Variation in Assumed Rock Joint Parameters in Welded Tuff," U.S. Nuclear Regulatory Commission, NUREG/CR-5336, April 1989.

Cundall, P. A. "Formulation of a Three-Dimensional Distinct Element Model — Part I: A Scheme to Detect and Represent Contacts in a System Composed of Many Polyhedral Blocks," Int. J. Rock Mech., Min. Sci. & Geomech. Abstr., 25, 107-116 (1988).

Mansure, A. J. "Expected Temperatures for Spent Fuel Borehole Walls and Drifts," Memo to R. J. Flores, Sandia National Laboratories, Sandia Keystone Memo 6310-85-8, April 15, 1985.

Peters, Ralph R. "Thermal Response to Emplacement of Nuclear Waste in Long, Horizontal Boreholes," Sandia National Laboratories, SAND82-2497, April 1983.

Data Input File

```
**********************************************************************
*                                                                    *
*                                                                    *
*    T H E R M A L / M E C H A N I C A L   A N A L Y S I S           *
*                                                                    *
*  Input file to UDEC 1.5 for determining emplacement room behavior. *
*  Vertical emplacement scheme ...                                   *
*                                                                    *
**********************************************************************
*
thermal
set enh
head
TUFF 90 degree dip - 140m model
round=.005
block 0,-40 0,100 19.2,100 19.2,-40
**********************
* large block cracks
**********************
split 0,43 19.2,43
split 0,16 19.2,16
*
*************************
* emplacement room cracks
*************************
crack 0.0,36.5  1.0,36.5
crack 1.0,36.5  2.0,36.0
crack 2.0,36.0  2.5 35.0
crack 2.5,27.0  2.5,40.0
crack 0.0,30.0  6.0,30.0
*
*************************
* heavily jointed region
*************************
jreg  0,16 0,43 7,43 7,16
jset  0,0  1,0 1,0 2,0 1,0
jset  0.0  1,0 1,0 2,0 0,1
jset 90,0 30,0 0,0 1,0
split 7,16 7,43
*
**************************
* make split for heaters
**************************
split 0,28 1,28
split 0,26 1,26
split 0,24 1,24
*
```

```
*
***************************
* additional fine cracks
***************************
split  0.5,27 0.5,30
split  0.5,37 0.5,40
split  1.5,27 1.5,30
split  1.5,36.1 1.5,40
split  2.5,27 2.5,30
split  2,27   3,27
split  3.5,27 3.5,40
split  4.5,27 4.5,40
split  1,37 2,37
split  1,39 2,39
split  2,36 3,36
split  2.5,35.9 2.5,40
*
***************************
* bottom region
***************************
split 0,4 19.2,4
split 0,10 19.2,10
split 0,13 19.2,13
split 3.5,10 3.5 16
split 7,4 7,16
split 10.5,10 10.5,16
split 14,4 14,16
*
***************************
* top region
***************************
split 0,46 19.2,46
split 0,49 19.2,49
split 0,55 19.2,55
split 3.5,49 3.5 43
split 7,55 7,43
split 10.5,49 10.5,43
split 14,55 14,43
*
*
***************************
* right side
```

```
**************************
split 10.5,16 10.5,43
split 14,16 14,43
split 7,19 10.5 19
split 7,23 19.2 23
split 7,27 19.2 27
split 7,31 19.2 31
split 7,35 19.2 35
split 7,39 10.5 39
*
jdel
*
* generate zones
*
gen 0,7  16,43  auto 1.4
gen 7,20  16,43  auto 4.2
gen 0,20  4,16  auto 4.2
gen 0,20 -40,4  auto 14
gen 0,20  43,55  auto 4.2
gen 0,20  55,100 auto 14
*
* define material properties and initial conditions
*
change jcons=5 mat=1
change cons=1 mat=1
*
*--- ASSIGN MATERIAL PROPERTIES (REF: SCP-CDR CHAP. 2, SEC. 2.3.1)
*--- USING THE JOINT PROPERTIES AND "ROCK MASS" PROPERTIES.
*--- USING THE 'DESIGN' VALUES FROM
*--- TABLES 2-4, 2-6, AND 2-7.
*--- THE ROCK IS CHARACTERIZED AS AN ELASTIC/PLASTIC MATERIAL
*--- WITH VERTICAL AND HORIZONTAL.  A MOHR-COULOMB FAILURE
CRITERION
*--- IS USED FOR THE JOINTS ...
*
*
*--- Rock Mass:
prop mat=1 k  = 8.39e9  g   = 6.29e9  dens = 2340
*
*--- Rock Joints:
prop mat=1 jkn = 1.0e11  jks  = 1.0e11  jcoh = 1.0e6 &
        jdil=  .000  jfric =  0.800  jtens=  0  &
        kn  =  1.0e3  ks   =  1.0e3
*
*--- THERMAL PROPERTIES OF THE ROCK ...
*   (Ref: SCP-CDR Chap. 2, Sec. 2.3.1.9, Table 2-9)
prop mat=1 con =  2.07  thexp =  1.07e-5 spec =  961
*
```

```
*--- DEFINE THE INITIAL STRESS FIELD (MPa)...
*--- REFERENCE: SCP-CDR CHAP. 2, SEC. 2.3.1.9
*   (The initial vertical stress is about -7 MPa at
*    the disposal room horizon. The horizontal stress
*    is determined as 0.5 x SYY.)
*
insitu -.1 19.2 -40.1 100.1 stress -3.5e6 0 -7.0e6  ygrad 11700 0 23400
*
*--- SET THE INITIAL TEMPERATURE TO 26 DEG. CELSIUS ...
initem 26 -1,19.2 -41,101
*
grav 0,-9.8
*
*--- SET KINEMATIC BOUNDARY CONDITIONS ...
*   (The two vertical boundaries are symmetry planes, thus,
*    they are restricted from moving in the horizontal (x)
*    direction. The bottom horizontal boundary is restricted
*    from moving in the vertical (y) direction. The top
*    horizontal boundary is a free-to-move pressure boundary.
*    The pressure is acting downward, and is equal to the
*    initial vertical stress.)
*
bound -.1 19.3  99.9 100.1 str -3.5e6 0 -7e6 ygrad 11700 0 23400
bound -.1 .1 -40.1 100.1 xvel 0
bound 17.9 19.3 -40.1 100.1 xvel 0
bound -.1 19.3 -40.1 -39.9 yvel 0
*
* run time parameters
*
damp auto
mscale on
*
* cycle to equilibrium
cy 1000
reset disp
reset jdisp
*
*
*
*--- EXCAVATE THE DISPOSAL ROOM ...
*
delete 0,2.5 30,35
delete 2,2.3 35,35.5
delete 0,1.55 35,36.2
*
* set history points
*
*
```

```
*--- DEFINE POINTS FOR WHICH TEMP. HISTORIES ARE RECORDED ...
*
reset hist
this ntc=500 type 1
this tem 0.0 30   * floor center
this tem 0.0 36.7 * crown center
this tem 2.5 30   * floor rib intersection
* histories along a line out from heater center
this tem 1 25 tem 2,25 tem 3,25 tem 5,25 tem 9,25 tem 18,25
*
*--- DEFINE POINTS FOR WHICH MECH HISTORIES ARE RECORDED ...
*
hist nc=100
hist ydis 0.0, 36.5
hist ydis 0.0, 30.0
hist xdis 2.5, 33.0
hist ydis 1.5, 36.2
hist sxx  0.0, 36.5
hist sxx  0.0, 30.0
hist syy  2.5, 33.0
hist type 1
*  perform mechanical calculations for t=0 years
cycle 3000
head
0 Years, JKN=1e11, Cohesion = 1MPa, Friction = 38.7, Dilation = 0
save m0.sav
*
*--- ASSIGN THE DECAYING HEAT SOURCE WHICH SIMULATES THE
*--- COMMINGLED SF AND DHLW ...
*   (The thermal decay characteristics are from Peters, 1983,
*    SAND-2497. The initial heat generating power per meter
*    of room length is 713.5 W. Because of symmetry only half
*    of this power is applied. Note that the decay coefficients
*    have dimension 1/sec and not 1/year, which is commonly
*    used in the literature ...
*    decay constants for SF are also used for the DHLW.
*
thapp -.1,.1 23,27 flux 48.17 -2.46079e-10
thapp -.1,.1 23,27 flux 41.03 -1.716788e-9
*
*--- START THE HEAT TRANSFER SOLUTION USING THE EXPLICIT
SCHEME ...
*
run delt 8e4 t=200 s=100000 age=1.58e9 impl
head
50 Years thermal results only
sav t50.sav
*
```

```
* perform mechanical calculations for t=50 years
cycle 3000
head
50 Years, JKN=1e11, Cohesion = 1MPa, Friction = 38.7, Dilation = 0
save m50.sav
ret
```

3.4 CODE MAINTENANCE AND SUPPORT

Itasca Consulting Group maintains a formal system for performing maintenance and updating of the UDEC code, as well as procedural controls for ensuring Quality Assurance. The Quality Assurance program* provides a formalized system for:

(1) testing of the code against relevant analytic solutions or test problems;

(2) reporting and documentation of errors in code logic, and fixing of these errors;

(3) Maintaining a system of unique identifiers for the code to identify all changes;

(4) Documentation of all code modification; and

(5) Peer code reviews.

Figure 3.4-1 gives an organization chart showing the management of the UDEC code with the Itasca Consulting Group.

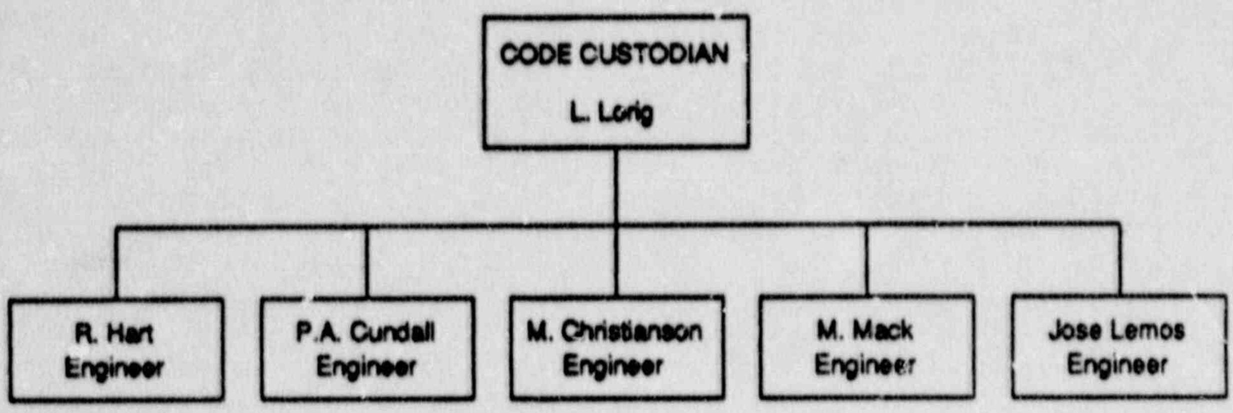

Fig. 3.4-1 Organization Chart Illustrating Management of UDEC Code Maintenance and Support

* Quality Assurance plan has not been submitted for formal NRC approval.

The code custodian is Dr. Loren Lorig who maintains code documentation, defines the required work effort, and arranges internal and peer review as necessary. Since the UDEC code is marketed to the general engineering community, modifications and additions are made on a continuing basis. Depending upon the modification, individuals with varying expertise may be required for the developments. It is the reponsibility of the code custodian to assign the work effort to development engineers listed under Fig. 3.4-1.

Code Modifications

Code modifications may result from two sources: (1) errors discovered in the code logic; or (2) planned additions to the code logic. A development plan for code updates is defined by Itasca, with updates issued on a roughly yearly basis. These updates may include items such as the addition of heat transfer logic, fluid flow, etc., and thus may be considered major additions. Code modifications resulting from errors or minor code changes or additions are handled by the code custodian on an as-needed basis, or any Itasca engineer at his discretion.

Two records are kept of code modifications: (1) paper hard copy; and (2) a disk "mod" file. The code custodian (with or without consultation with other staff) defines the necessary scope of work to be performed and completes a code modification form (Fig. 3.4-2). This form is kept in the permanent UDEC files. An additional form, called the UDEC.MOD file is kept on floppy disk in the disk archives at Itasca. As the code developer performs modification to the code, he updates this file with a complete description of the changes made. The UDEC.MOD file therefore provides a history of code modifications. A hard copy is kept in the permanent UDEC files.

CODE MODIFICATION

CODE ____________ VERSION ____________

PROPOSED MODIFICATION

PURPOSE

NEW COMMAND/KEYWORD ____________

MODIFICATION PLAN (INCLUDE LIST OF ROUTINES TO BE CHANGED, ADDED OR DELETED)

PERSON RESPONSIBLE ____________

EST. TIME REQUIRED ____________

REVIEWED BY ____________

FILL-IN AFTER MODIFICATION COMPLETED

COMPLETION DATE ____________

COMMENTS

REVIEWER ____________ DATE ____________

ATTACH LISTING OF MODIFIED CODE

Fig. 3.4-2 Code Modification Form

A simple numbering system is followed for setting a unique code identification number. Any code modification which does not result in a need to change the save files created by the code is considered a <u>minor modification</u>. This is because major modifications require additional offsets to be added to the linked listed arrays in the program. Therefore, previous versions of the code will not be compatible with the new array structure, and it is impossible to restart the old save files. The basic version numbering scheme is

Version ICGA.XY

where A is an integer starting with 1, X is an integer which is incremented whenever a major modification is made, and Y is a one or two-digit number which is incremented whenever minor modifications are made. The Y value is reset to zero each time a major modification is made; the previous minor modifications are considered to be part of the ensuing major modification. The example below illustrates a series of minor and major modifications.

```
Version ICG1.400
        1.410┐
             │
             │-   6 minor modifications
             │
        1.46 ┘
        1.50      Save file change constituting a major
                  modification, version 1.50 includes all
                  previous modifications.
```

The history of each minor and major modification can be traced via the UDEC.MOD file.

Code Verification

Code verification involves performing a set series of problems with analytical solutions which exercize all critical functions of the code operation. The problem set used for verification is given in chapters 7 and 8 of Vol. II, User's Manual. New code logic may require additional problems to be added to this set.

Technical Review

Technical peer review of the UDEC code is conducted at the discretion of the code custodian. Dr. Peter Cundall, author of the original UDEC code is used in this capacity. It is noted that UDEC is actively used for research and design by approximately 50 organizations worldwide. Approximately 30 of these have access to the UDEC source code. In effect, this group constitutes a peer review in that they are submitting the code to a wide variety of problems on a continuous basis. The comments and error reports of this users group form a portion of the basis for code modifications and additions.

Restrictions

Itasca will distribute the UDEC source code to researchers, although generally, only an executable version of the code is distributed. Official copies of the source code are kept in Itasca offices and may be obtained through special arrangement with the UDEC Code Custodian.

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Verification and Example Problems

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10. SUPPLEMENTARY NOTES

11. ABSTRACT (200 words or less)

UDEC is a two-dimensional distinct element code written for analysis of static or dynamic, mechanical, thermomechanical and fracture fluid flow problems in jointed rock or soil. The body to be analyzed is subdivided into a series of blocks which are separated from their neighbors by interface planes which have friction, cohesion and dilation. The blocks themselves may behave as non-linear materials also. The code uses an explicit solution procedure for solving the dynamic equations of motion for the blocks. The large deformation formulation allows interaction between adjacent blocks including slip or separation. General heat transfer logic, fluid flow along the fractures and structural element support are optional features.

12. KEY WORDS/DESCRIPTORS (List words or phrases that will assist researchers in locating the report.)

UDEC, distinct element method, explicit procedure, dynamic, heat transfer, fluid flow, interface planes, large deformation structural elements

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