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ANALYTIC REBALANCE TECHNIQUE FOR PRESSURE CALCULATION IN TWO-PHASE FLOW SYSTEMS

by

C. C. Miao, V. L. Shah, J. L. Krazinski, and W. T. Sha

Components Technology Division

ARGONNE NATIONAL LABORATORY

Prepared for the

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Office of Nuclear Regulatory Research
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TABLE OF CONTENTS

LIST OF FIGURES

NOMENCLATURE

D	defined in Eq. [3.6]
FF	defined in Eq. [3.7]
FN	defined in Eq. [3.8]
g_j	gravitational acceleration in j-direction
G_1	defined in Eq. [3.10]
G_2	defined in Eq. [3.9]
GH	defined in Eq. [4.4]
$\hat{GH}$	defined in Eq. [4.8]
$\hat{\hat{GH}}$	defined in Eq. [4.21]
K	interfacial drag coefficient
P	pressure
P'	pressure correction
$S_{m\ell x}$, $S_{m\ell y}$, $S_{m\ell z}$	resistance force of liquid per unit volume in x, y and z directions, respectively
S_{mgx}, S_{mgy}, S_{mgz}	resistance force of gas per unit volume in x, y and z directions, respectively
Γ	mass generation rate per unit volume
$X_{\ell f}$, $Y_{\ell f}$, $Z_{\ell f}$	defined in Appendix 2
$X_{\ell n}$, $Y_{\ell n}$, $Z_{\ell n}$	defined in Appendix 2
X_{gf}, Y_{gf}, Z_{gf}	defined in Appendix 2
X_{gn}, Y_{gn}, Z_{gn}	defined in Appendix 2
u, v, w	velocity components in x, y and z directions respectively
δt	time step
δx, δy, δz	control volume dimensions
μ	viscosity
ρ	density
τ	viscous stress
θ_g, θ_ℓ	void fractions for gas and liquid phases

NOMENCLATURE (Contd.)

β	defined in Eq. [3.2]
γ_x, γ_y, γ_z	surface permeability in x, y, and z-directions
γ_v	volume porosity
γ_z	defined in Eq. [4.9]
	<u>subscript</u>
g	gas
ℓ	liquid
m	mixture
	<u>superscript</u>
t	rebalance

ABSTRACT

The present report describes the Analytic Rebalance Technique for the solution of two-phase fluid flow problems. The procedure is based on direct solution of only a section of the flow field at a time along with rebalancing of the remaining flow field. The derivation of the equations and the step by step solution procedure are presented for developing a computer program for analyzing two-phase flows.

1. INTRODUCTION

Although two-phase flow conditions are encountered in many engineering situations and the analysis of hypothetical loss of coolant or transient overpower accident situations in nuclear reactors are of significant interest, there does not exist a single numerical procedure which is stable, efficient and can analyze three-dimensional transient two-phase flow conditions satisfactorily. Most of the solution procedures are either unstable or have a very low rate of convergence. In the present report we are presenting a solution procedure, named "Analytic Rebalance Technique (ART), which we believe has a potential to be efficient and stable.

In ART, we are using the quasi-continuum two-fluid model to describe the two-phase flow conservation equations with both phases having different densities, velocities and temperatures. The method is an extension of the Separation of Matrix Technique (SMT) used in HEV2D [1,2]. The SMT requires a large computer storage, and is therefore restricted to problems with a small number of computational cells. In addition, the formulations in HEV2D [1] were based on the assumption that liquid phase is incompressible. In ART, we solve only a section of the flow field at a time and use the rebalancing technique for the remaining of the flow field. Consequently, ART can handle three-dimensional large flow field problems. In solving a section of the flow field we still follow the SMT; therefore, we retain the computational speed feature of the SMT.

ART is a coupling of whole pressure field iteration with regional solution procedure. The finite difference Poisson type equation for pressure has been derived by combining the momentum equations and mixture continuity equation as in ICE [3]. In the present formulations, the assumption of liquid being incompressible has been removed. In addition, the formulations are carried out in such a way that we preserve the advantages of both iteration and direct solution procedures; namely, low computer storage and high accuracy. The boundary conditions and whole field resistances are taken into account during every rebalance calculation. Hence, the computed pressure field from each rebalance calculation is expected to be closer to physical reality and thus help achieve quicker convergence. In addition, ART is arranged to take the full advantage of the Separation of Matrix Technique [1,2] and therefore, significant savings in computation time are expected.

The temperatures, densities, and void fractions can be computed explicitly or semi-implicitly by using phase continuity and energy equation and the equation state. Since the solution procedure for the pressure is the major topic of this report, methods to calculate those field variables other than pressure and velocities will be presented separately.

2. GOVERNING EQUATIONS: QUASI-CONTINUUM [4.5.6]

2.1 Continuity Equations

Liquid

$$\gamma_v \frac{\partial}{\partial t} (\theta_\ell \rho_\ell) + \frac{\partial}{\partial x} (\gamma_x \theta_\ell \rho_\ell u) + \frac{\partial}{\partial y} (\gamma_y \theta_\ell \rho_\ell v) + \frac{\partial}{\partial z} (\gamma_z \theta_\ell \rho_\ell w) = \gamma_v \Gamma_\ell \qquad (2.1)$$

Gas

$$\gamma_v \frac{\partial}{\partial t} (\theta_g \rho_g) + \frac{\partial}{\partial x} (\gamma_x \theta_g \rho_g u) + \frac{\partial}{\partial y} (\gamma_y \theta_g \rho_g v) + \frac{\partial}{\partial z} (\gamma_z \theta_g \rho_g w) = \gamma_v \Gamma_g \qquad (2.2)$$

Combined Continuity Equation

$$\gamma_v \frac{\partial(\rho_m)}{\partial t} + \frac{\partial}{\partial x} (\gamma_x \rho_m u) + \frac{\partial}{\partial y} (\gamma_y \rho_m v) + \frac{\partial}{\partial z} (\gamma_z \rho_m w) = 0 \qquad (2.3)$$

where $\rho_m = \theta_\ell \rho_\ell + \theta_g \rho_g$

2.2 Momentum Equations

Liquid (x-direct

$$\gamma_v \frac{\partial}{\partial t} (\rho_\ell \theta_\ell u_\ell) + \frac{\partial}{\partial x} (\gamma_x \rho_\ell \theta_\ell u_\ell^2) + \frac{\partial}{\partial y} (\gamma_y \rho_\ell \theta_\ell u_\ell v_\ell) + \frac{\partial}{\partial z} (\gamma_z \rho_\ell \theta_\ell u_\ell w_\ell)$$

$$= - \gamma_v \theta_\ell \frac{\partial p}{\partial x} + \gamma_v \theta_\ell \rho_\ell g_x + \gamma_v K(u_g - u_\ell) + S_{m\ell}$$

$$+ \frac{\partial}{\partial x} (\gamma_x \tau_{xx} \theta_\ell) + \frac{\partial}{\partial y} (\gamma_z \tau_{xy} \theta_\ell) + \frac{\partial}{\partial z} (\gamma_z \tau_{xz} \theta_\ell) \qquad (2.4)$$

Gas (x-direction)

$$\gamma_v \frac{\partial}{\partial t} (\rho_g \theta_g u_g) + \frac{\partial}{\partial x} (\gamma_x \rho_g \theta_g u_g^2) + \frac{\partial}{\partial y} (\gamma_y \rho_g \theta_g u_g v_g) + \frac{\partial}{\partial z} (\gamma_z \rho_g \theta_g u_g w_g)$$

$$= - \gamma_v \theta_g \frac{\partial p}{\partial x} + \gamma_v \theta_g \rho_g u_{gx} + \gamma_v K(u_\ell - u_g) + S_{mg}$$

$$+ \frac{\partial}{\partial x} (\gamma_x \tau_{xx} \theta_g) + \frac{\partial}{\partial y} (\gamma_y \tau_{xy} \theta_g) + \frac{\partial}{\partial z} (\gamma_z \tau_{xz} \theta_g) \qquad (2.5)$$

Momentum equations for y and z-directions are similar to equations 2.4 and 2.5.

3. DERIVATION OF POISSON TYPE DIFFERENCE EQUATION

To simplify our illustration of ART, the mesh spacing δx, δy, and δz in x, y and z direction, respectively, are assumed to be uniform, however, this assumption can be readily removed. Furthermore, the indices may be simplified as:

$$(\text{Variable}) = (\text{Variable})_{i,j,k}$$

and

$$(\text{Variable})_{i+1/2} = (\text{Variable})_{i+1/2,j,k}$$

The staggered mesh system is shown in Fig. 1. The pressure, densities, enthalpies and void fractions are defined at the center of the control volume at (i,j,k) and velocities are defined at the surfaces of the control volume such as (i+1/2,j,k), (i,j+1/2,k)...etc. With this convention, we now write the finite difference equation of liquid momentum, Eq. (2.4) in x-direction,

$$\frac{(\gamma_V\theta_\ell\rho_\ell u_\ell)^{n+1}_{i+1/2} - (\gamma_V\theta_\ell\rho_\ell u_\ell)^{n}_{i+1/2}}{\partial t}$$

$$= (X_{\ell f})_{i+1/2} + (X_{\ell n})_{i+1/2} + \gamma_{V_{i+1/2}}$$

$$\frac{1}{\partial x}\left[(\theta_\ell p)^{n+1}_{i} - (\theta_\ell p)^{n+1}_{i+1}\right] \qquad (3.1)$$

where $(X_{\ell f})$ contains momentum flux term, $(X_{\ell n})$ contains the remaining terms (body force, viscous force, momentum source due to mass transfer, interfacial coupling) of momentum equation. $(X_{\ell f})_{i+1/2}$ is evaluated explicitly during the pressure iteration in a given time step. On the other hand, $(X_{\ell n})$ is to be updated in each iteration (see Appendix 2). In order to express pressure related to surface permeability instead of volume porosity, we define

$$\beta_x = \frac{\gamma_x}{\gamma_V} \qquad (3.2)$$

Equation (3.1) can be rewritten as:

$$(\gamma_x\theta_\ell\rho_\ell u_\ell)^{n+1}_{i+1/2} = (\gamma_x\theta_\ell\rho_\ell u_\ell)^{n}_{i+1/2} + \delta t\left[\beta_x(X_{\ell f})\right]_{i+1/2}$$

$$+ \delta t\left[\beta_x(X_{\ell n})\right]_{i+1/2} + (\gamma_x)_{i+1/2}$$

$$\frac{\delta t}{\delta x}\left[(\theta_\ell p)^{n+1}_{i} - (\theta_\ell p)^{n+1}_{i+1}\right] \qquad (3.3)$$

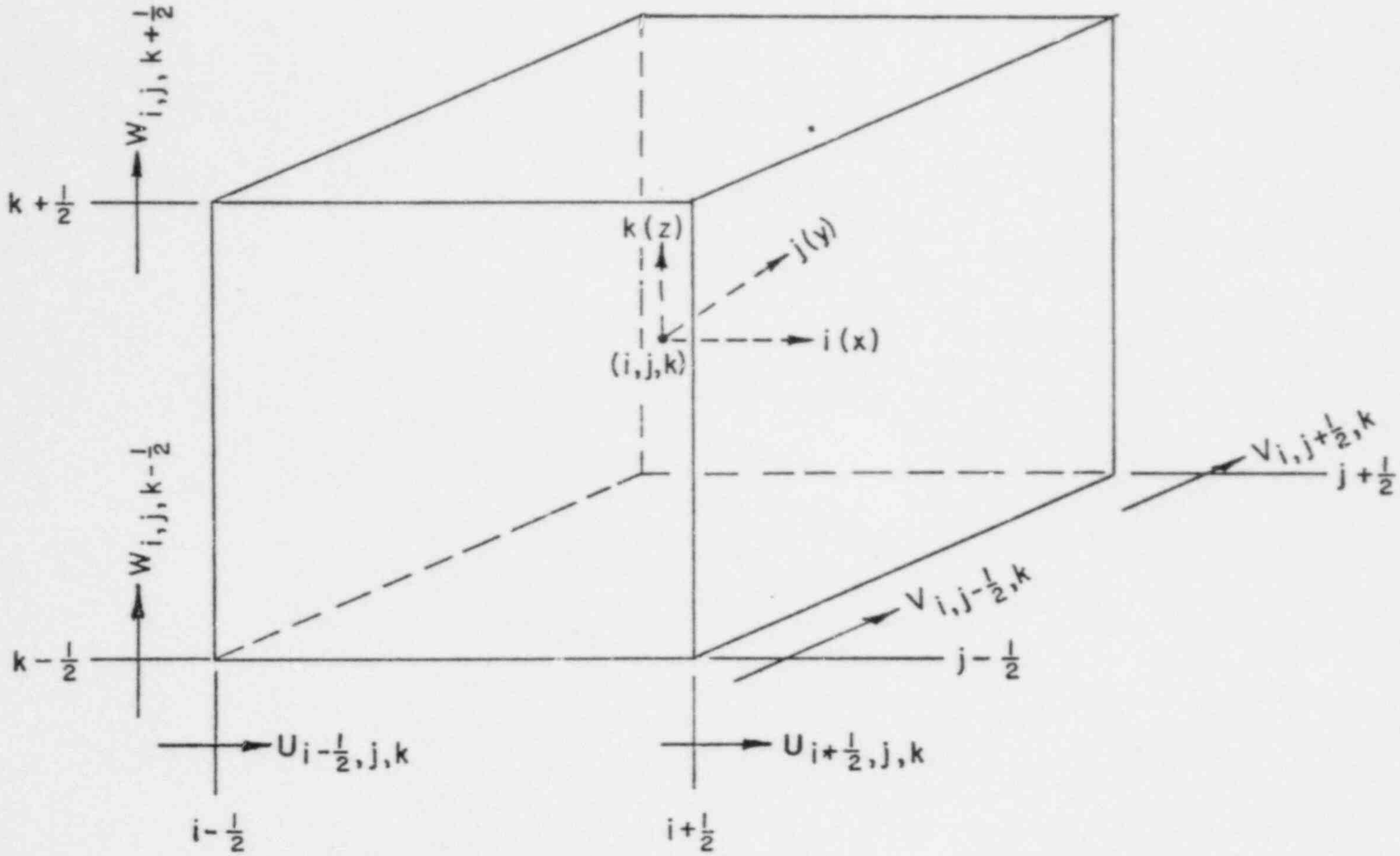

Fig. 1. Staggered Mesh System

Similarly, finite difference momentum equations for gas phase and for other directions (y and z) are derived and presented in Appendix 3.

Difference equation of combined continuity equation can be written as:

$$\gamma_V \frac{(\rho_m)^{n+1} - (\rho_m)^n}{\partial t} = \frac{1}{\delta x}\left[(\gamma_x\theta_g\rho_g u_g)^{n+1}_{i-1/2} - (\gamma_x\theta_g\rho_g u_g)^{n+1}_{i+1/2}\right]$$
$$+ \frac{1}{\delta y}\left[(\gamma_y\theta_g\rho_g v_g)^{n+1}_{j-1/2} - (\gamma_y\theta_g\rho_g v_g)^{n+1}_{j+1/2}\right]$$
$$+ \frac{1}{\delta z}\left[(\gamma_z\theta_g\rho_g w_g)^{n+1}_{k-1/2} - (\gamma_z\theta_g\rho_g w_g)^{n+1}_{k+1/2}\right]$$
$$+ \frac{1}{\delta x}\left[(\gamma_x\theta_\ell\rho_\ell u_\ell)^{n+1}_{i-1/2} - (\gamma_x\theta_\ell\rho_\ell u_\ell)^{n+1}_{i+1/2}\right]$$
$$+ \frac{1}{\delta y}\left[(\gamma_y\theta_\ell\rho_\ell v_\ell)^{n+1}_{j-1/2} - (\gamma_y\theta_\ell\rho_\ell v_\ell)^{n+1}_{j+1/2}\right]$$
$$+ \frac{1}{\delta z}\left[(\gamma_z\theta_\ell\rho_\ell w_\ell)^{n+1}_{k-1/2} - (\gamma_z\theta_\ell\rho_\ell w_\ell)^{n+1}_{k+1/2}\right] \qquad (3.4)$$

Noting that both n+1-step densities and voiding fraction are to be estimated by n-step known quantities, we may substitute $(\gamma_x\theta_\ell\rho_\ell u_\ell)^{n+1}_{i+1/2}$ from Eq. (3.3)

and similar expressions for $(\ _y\theta_\ell\rho_\ell v_\ell)^{n+1}_{j+1/2}$, $(\gamma_z\theta_\ell\rho_\ell w_\ell)^{n+1}_{k+1/2}$,

$(\gamma_x\theta_g\rho_g u_g)^{n+1}_{i+1/2}$, $(\gamma_y\theta_g\rho_g v_g)^{n+1}_{i+1/2}$, and $(\gamma_z\theta_g\rho_g w_g)^{n+1}_{k+1/2}$ (see Appendix 3) into the combined continuity Eq. (3.4), and we get

$$\gamma_V(\rho_m^{n+1} - \rho_m^n)$$
$$= \delta t\left[\frac{(\gamma_x\theta_g\rho_g u_g)^n_{i-1/2} - (\gamma_x\theta_g\rho_g u_g)^n_{i+1/2}}{\delta x} + \frac{(\gamma_y\theta_g\rho_g v_g)^n_{j-1/2} - (\gamma_y\theta_g\rho_g v_g)^n_{j+1/2}}{\delta y}\right.$$
$$\left. + \frac{(\gamma_z\theta_g\rho_g w_g)^n_{k-1/2} - (\gamma_z\theta_g\rho_g w_g)^n_{k+1/2}}{\delta z}\right] + \delta t^2\left\{\frac{\left[\beta(X_{gf})\right]_{i-1/2} - \left[\beta(X_{gf})\right]_{i+1/2}}{\delta x}\right.$$
$$\left. + \frac{\left[\beta(Y_{gf})\right]_{j-1/2} - \left[\beta(Y_{gf})\right]_{j+1/2}}{\delta y} + \frac{\left[\beta(Z_{gf})\right]_{k-1/2} - \left[\beta(Z_{gf})\right]_{k+1/2}}{\delta z}\right\}$$

$$+ \delta t^2 \left\{ \frac{\left[\beta(X_{gn})\right]_{i-1/2} - \left[\beta(X_{gn})\right]_{i+1/2}}{\delta x} + \frac{\left[\beta(Y_{gn})\right]_{j-1/2} - \left[\beta(Y_{gn})\right]_{j+1/2}}{\delta y} \right.$$

$$\left. + \frac{\left[\beta(Z_{gn})\right]_{k-1/2} - \left[\beta(Z_{gn})\right]_{k+1/2}}{\delta z} \right\} + \delta t \left[\frac{(\gamma_x \theta_\ell \rho_\ell u_\ell)^n_{i-1/2} - (\gamma_x \theta_\ell \rho_\ell u_\ell)^n_{i+1/2}}{\delta x} \right.$$

$$\left. + \frac{(\gamma_y \theta_\ell \rho_\ell v_\ell)^n_{j-1/2} - (\gamma_y \theta_\ell \rho_\ell v_\ell)^n_{j+1/2}}{\delta y} + \frac{(\gamma_z \theta_\ell \rho_\ell w_\ell)^n_{k-1/2} - (\gamma_z \theta_\ell \rho_\ell w_\ell)^n_{k+1/2}}{\delta z} \right]$$

$$+ \delta t^2 \left\{ \frac{\left[\beta(X_{\ell f})\right]_{i-1/2} - \left[\beta(X_{\ell f})\right]_{i+1/2}}{\delta x} + \frac{\left[\beta(Y_{\ell f})\right]_{j-1/2} - \left[\beta(Y_{\ell f})\right]_{j+1/2}}{\delta y} \right.$$

$$\left. + \frac{\left[\beta(Z_{\ell f})\right]_{k-1/2} - \left[\beta(Z_{\ell f})\right]_{k+1/2}}{\delta z} \right\} + \delta t^2 \left\{ \frac{\left[\beta(X_{\ell n})\right]_{i-1/2} - \left[\beta(X_{\ell n})\right]_{i+1/2}}{\delta x} \right.$$

$$\left. + \frac{\left[\beta(Y_{\ell n})\right]_{j-1/2} - \left[\beta(Y_{\ell n})\right]_{j+1/2}}{\delta y} + \frac{\left[\beta(Z_{\ell n})\right]_{k-1/2} - \left[\beta(Z_{\ell n})\right]_{k+1/2}}{\delta z} \right\}$$

$$+ \delta t^2 \left\{ \frac{1}{\delta x^2} \left[(\gamma_x)_{i-1/2} P^{n+1}_{i-1} + (\gamma_x)_{i+1/2} P^{n+1}_{i+1} - \left((\gamma_x)_{i-1/2} + (\gamma_x)_{i+1/2}\right) P^{n+1} \right] \right.$$

$$+ \frac{1}{\delta y^2} \left[(\gamma_y)_{j-1/2} P^{n+1}_{j-1} + (\gamma_y)_{j+1/2} P^{n+1}_{j+1} - \left((\gamma_y)_{j-1/2} + (\gamma_y)_{j+1/2}\right) P^{n+1} \right]$$

$$+ \frac{1}{\delta z^2} \left[(\gamma_z)_{k-1/2} P^{n+1}_{k+1} + (\gamma_z)_{k+1/2} P^{n+1}_{k+1} \right.$$

$$\left.\left. - \left((\gamma_z)_{k-1/2} + (\gamma_z)_{k+1/2}\right) P^{n+1} \right] \right\} \tag{3.5}$$

To simplify the expression of Eq. (3.5), we define:

$$D_{i,j,k} = \delta t \left[\frac{(\gamma_x \theta_g \rho_g u_g)^n_{i-1/2} - (\gamma_x \theta_g \rho_g u_g)^n_{i+1/2}}{\delta x} \right.$$

$$\left. + \frac{(\gamma_y \theta_g \rho_g v_g)^n_{j-1/2} - (\gamma_y \theta_g \rho_g v_g)^n_{j+1/2}}{\delta y} + \frac{(\gamma_z \theta_g \rho_g w_g)^n_{k-1/2} - (\gamma_z \theta_g \rho_g w_g)^n_{k+1/2}}{\delta z} \right]$$

$$+ \delta t \left[\frac{(\gamma_x \theta_\ell \rho_\ell u_\ell)^n_{i-1/2} - (\gamma_x \theta_\ell \rho_\ell u_\ell)^n_{i+1/2}}{\delta x} + \frac{(\gamma_y \theta_\ell \rho_\ell v_\ell)^n_{j-1/2} - (\gamma_y \theta_\ell \rho_\ell v_\ell)^n_{j+1/2}}{\delta y} \right.$$

$$\left. + \frac{(\gamma_z \theta_\ell \rho_\ell w_\ell)^n_{k-1/2} - (\gamma_z \theta_\ell \rho_\ell w_\ell)^n_{k+1/2}}{\delta z} \right] \tag{3.6}$$

$$
\begin{aligned}
(FF)_{i,j,k} = \delta t^2 \Bigg\{ & \frac{\left[\beta(X_{gf})\right]_{i-1/2} - \left[\beta(X_{gf})\right]_{i+1/2}}{\delta x} \\
& + \frac{\left[\beta(Y_{gf})\right]_{j-1/2} - \left[\beta(Y_{gf})\right]_{j+1/2}}{\delta y} + \frac{\left[\beta(Z_{gf})\right]_{k-1/2} - \left[\beta(Z_{gf})\right]_{k+1/2}}{\delta z} \Bigg\} \\
+ \delta t^2 \Bigg\{ & \frac{\left[\beta(X_{\ell f})\right]_{i-1/2} - \left[\beta(X_{\ell f})\right]_{i+1/2}}{\delta x} + \frac{\left[\beta(Y_{\ell f})\right]_{i-1/2} - \left[\beta(Y_{\ell f})\right]_{j+1/2}}{\delta y} \\
& + \frac{\left[\beta(Z_{\ell f})\right]_{k-1/2} - \left[\beta(Z_{\ell \delta})\right]_{k+1/2}}{\delta z} \Bigg\}
\end{aligned}
\tag{3.7}
$$

$$
\begin{aligned}
(FN)_{i,j,k} = \delta t^2 \Bigg\{ & \frac{\left[\beta(X_{gn})\right]_{i-1/2} - \left[\beta(X_{gn})\right]_{i+1/2}}{\delta x} \\
& + \frac{\left[\beta(Y_{gn})\right]_{j-1/2} - \left[\beta(Z_{gn}\right]_{j+1/2}}{\delta y} + \frac{\left[\beta(Z_{gn})\right]_{k-1/2} - \left[\beta(Z_{gn})\right]_{k+1/2}}{\delta z} \Bigg\} \\
+ \delta t^2 \Bigg\{ & \frac{\left[\beta(X_{\ell n})\right]_{i-1/2} - \left[\beta(X_{\ell n})\right]_{i+1/2}}{\delta x} + \frac{\left[\beta(Y_{\ell n})\right]_{j-1/2} - \left[\beta(Y_{\ell n})\right]_{j+1/2}}{\delta y} \\
& + \frac{\left[\beta(Z_{\ell n})\right]_{k-1/2} - \left[\beta(Z_{\ell n})\right]_{k+1/2}}{\delta z} \Bigg\}
\end{aligned}
\tag{3.8}
$$

$$
(G_2)_{i,j,k} = \gamma_v(\rho_m^{n+1} - \rho_m^n)_{i,j,k} \tag{3.9}
$$

and

$$
(G_1)_{i,j,k} = (D)_{i,j,k} + (FF)_{i,j,k}. \tag{3.10}
$$

Hence, we can rewrite equation (3.5) as

$$\delta t^2 \left\{ \frac{1}{\delta x^2} \left[(\gamma_x)_{i-1/2} + (\gamma_x)_{i+1/2} \right] + \frac{1}{\delta y^2} \left[(\gamma_y)_{j-1/2} + (\gamma_y)_{j+1/2} \right] \right.$$

$$\left. + \frac{1}{\delta z^2} \left[(\gamma_z)_{k-1/2} + (\gamma_z)_{k+1/2} \right] \right\} P^{n+1}$$

$$= G_1 + G_2 + (FN) + \delta t^2 \left\{ \frac{1}{\delta x^2} \left[(\gamma_x)_{i-1/2} \; P^{n+1}_{i-1} + (\gamma_x)_{i+1/2} \; P^{n+1}_{i+1} \right] \right.$$

$$+ \frac{1}{\delta y^2} \left[(\gamma_y)_{j-1/2} \; P^{n+1}_{j-1} + (\gamma_y)_{j+1/2} \; P^{n+1}_{j+1} \right]$$

$$\left. + \frac{1}{\delta z^2} \left[(\gamma_z)_{k-1/2} \; P^{n+1}_{k-1} + (\gamma_z)_{k+1/2} \; P^{n+1}_{k+1} \right] \right\} \tag{3.11}$$

The above equation is the general form of Poisson type difference equation.

4. ANALYTIC REBALANCE TECHNIQUE

4.1 Poisson Type Equation for Pressure Correction

Now we can proceed to iterate equation (3.9) from n-step towards n+1 step. We define t correction step of pressure as:

$$P^t = P^{t-1} + P' \tag{4.1}$$

Where t = 1, 2, 3..... → n+1 and P' is the pressure correction. Obviously, for t = 0, we can write

$$P^{t=0} = P^n. \tag{4.2}$$

Substituting equation (4.1) into equation (3.11), a Poisson type equation of pressure correction can be obtained as:

$$\delta t^2 \left\{ \frac{1}{\delta x^2} \left[(\gamma_x)_{i-1/2} + (\gamma_x)_{i+1/2} \right] + \frac{1}{\delta y^2} \left[(\gamma_y)_{j-1/2} + (\gamma_y)_{j+1/2} \right] + \frac{1}{\delta z^2} \left[(\gamma_z)_{k-1/2} + (\gamma_z)_{k+1/2} \right] \right\} P'$$

$$= (GH)^t + \delta t^2 \left\{ \frac{1}{\delta x^2} \left[(\gamma_x)_{i-1/2} P'_{i-1} + (\gamma_x)_{i+1/2} P'_{i+1} \right] + \frac{1}{\delta y^2} \left[(\gamma_y)_{j-1/2} P'_{j-1} + (\gamma_y)_{j+1/2} P'_{j+1} \right] + \frac{1}{\delta z^2} \left[(\gamma_z)_{k-1/2} P'_{k-1} + (\gamma_z)_{k+1/2} P'_{k+1} \right] \right\} \tag{4.3}$$

where

$$(GH)^t = G_1 + G_2 + (FN)$$

$$+ \delta t^2 \left\{ \frac{1}{\delta x^2}\left[(\gamma_x)_{i-1/2} + (\gamma_x)_{i+1/2}\right] + \frac{1}{\delta y^2}\left[(\gamma_y)_{j-1/2} + (\gamma_y)_{j+1/2}\right] \right.$$

$$\left. + \frac{1}{\delta z^2}\left[(\gamma_z)_{k-1/2} + (\gamma_z)_{k+1/2}\right] \right\} P^t$$

$$- \delta t^2 \left\{ \frac{1}{\delta x^2}\left[(\gamma_x)_{i-1/2} P^{t-1}_{i-1} + (\gamma_x)_{i+1/2} P^{t-1}_{i+1}\right] + \frac{1}{\delta y^2}\left[(\gamma_y)_{j-1/2} P^{t-1}_{j-1} \right.\right.$$

$$\left.\left. + (\gamma_y)_{j+1/2} P^{t-1}_{j+1}\right] + \frac{1}{\delta z^2}\left[(\gamma_z)_{k-1/2} P^{t-1}_{k-1} + (\gamma_z)_{k+1/2} P^{t-1}_{k+1}\right] \right\} \quad (4.4)$$

4.2 Some Definitions

Few terms are defined for the purpose of simplicity (see Fig. 2).

(i) Offense Zone: Detailed cell pressure is to be computed in this zone. In this zone, we solve a Poisson type equation.

(ii) Defense Zone: In this zone, averaged plane pressure increments are to be calculated. Two-phase cells are assumed to be in Numerical Quasi-Incompressible State (NQIS).

(iii) Floating Plane Pressure (FPP): In Defense Zone, the known plane pressure profiles from previous rebalance calculations are used. As the averaged plane increments are formulated as unknown, the plane pressure profiles are floating in Defense Zone.

We can therefore define

$$P^t_{i,j,k_o} = P^{t-1}_{i,j,k_o} + \bar{P}'_{k_o}. \quad (4.5)$$

for all cells in plane k_o. Here it represents the t^{th} rebalance, and k_o is a plane in the Defense Zone. $\bar{P}'_{k_o}$ is averaged k-plane pressure correction.

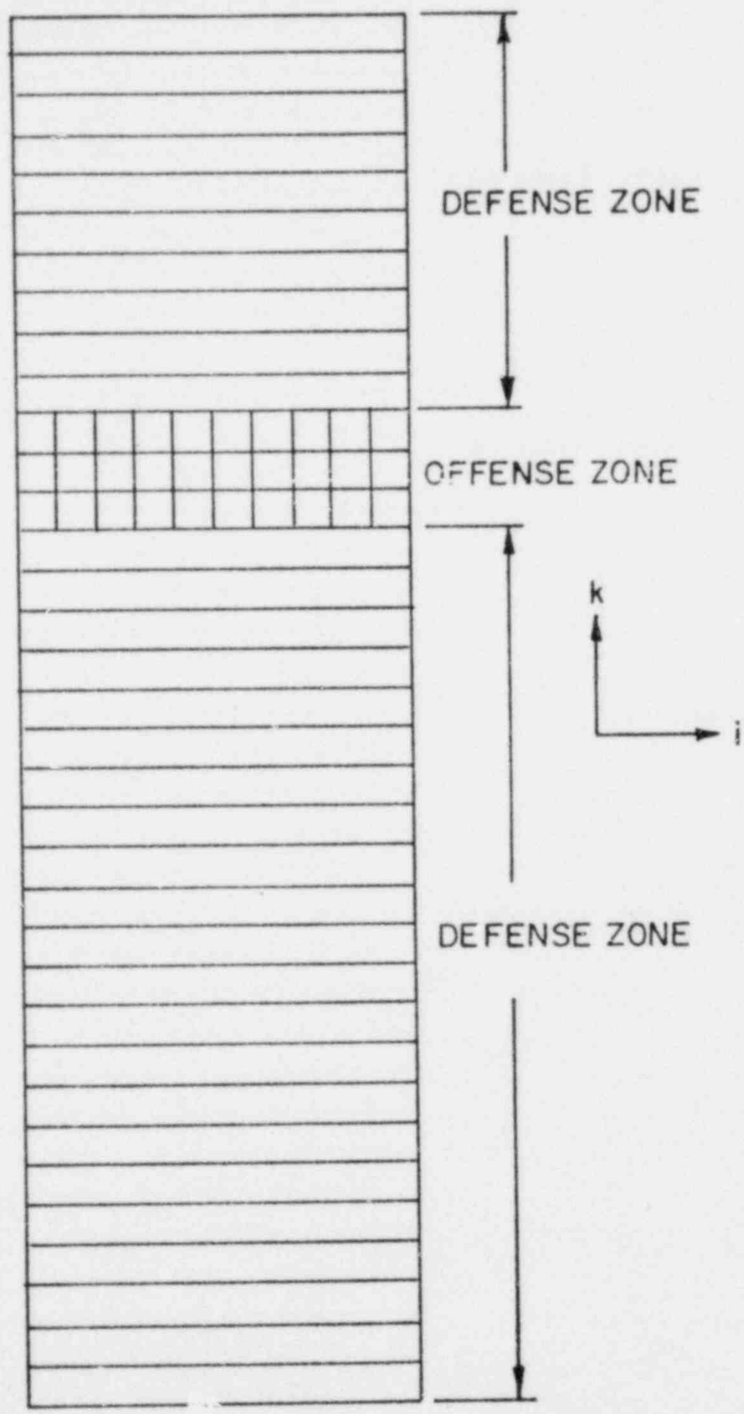

Fig. 2. ART Mesh System

(iv) Numerical Quasi-Incompressible State(NQIS): Two-phase densities are maintained from previous rebalance calculation whenever these cells are in Defense Zone. Their values are computed and updated only when Offense Zone moves to cover these cells. Hence, NQIS does not interfere with the real compressible characteristic of two-phase flow. It is a pure numerical arrangement in solution process. Its role in ART will be discussed further in the convergence section.

(v) Rebalance: During each rebalance, a reduced system of Poisson type equation is formed and solved.

(vi) Iteration: One iteration is considered complete when rebalance calculation sweeps the entire field.

(vii) We define

$$P^{t}_{i,j,k} = P^{t-1}_{i,j,k} + P'^{t}_{i,j,k} \tag{4.6}$$

for all cells in the Offense Zone. During t-rebalance, all pressures in the Offense Zone are considered to be unknown.

4.3. Plane Integration in Defense Zone

To simplify our illustration of ART, we are considering the case of $(\gamma_z)_{i,k-1/2} = (\gamma_z)_{i,k+1/2}$ in the remaining part of this report. The general case without assuming $(\gamma_z)_{k-1/2} = (\gamma_z)_{k+1/2}$ will be presented in a separate report.

Considering equation (4.3), it can be integrated with respect to i and j to obtain an expression;

$$\frac{\delta t^2}{\delta t^2} \sum_{i,j} \left[(\gamma_z)_{k-1/2} + (\gamma_z)_{k+1/2}\right] P'_k = \sum_{i,j} (GH) + \frac{\delta t^2}{\delta z^2} \sum_{i,j} \left[(\gamma_z)_{k-1/2}\, P'_{k-1} + (\gamma_z)_{k+1/2}\, P'_{k+1}\right] \tag{4.7}$$

To simplify equation (4.7) further, we introduce the notations

$$\hat{(GH)} = \sum_{i,j} (GH)_{i,j,k} \tag{4.8}$$

and

$$(\hat{\gamma}_z)_{k+1/2} = \sum_{i,j} (\gamma_z)_{i,j,k+1/2} . \qquad (4.9)$$

Finally, we can examine equation (4.7) in the following cases (Fig. 2).

(i) For each k plane in Defense Zone, but not adjacent to Offense Zone ($k<k_o-2$ or $k>k_o+2$), equation (4.7) can be simplified as:

$$2\hat{\gamma}_z \frac{\delta t^2}{\delta z^2} \bar{P}'_k = (\hat{GH})_k + \hat{\gamma}_z \frac{\delta t^2}{\delta z^2} (\bar{P}'_{k-1} + \bar{P}'_{k+1}). \qquad (4.10)$$

(ii) For the case $k = k_o+2$, we have

$$2\hat{\gamma}_z \frac{\delta t^2}{\delta z^2} \bar{P}'_k = (\hat{GH})_k + \frac{\delta t^2}{\delta z^2} \sum_{i,j} (\gamma_z)_{i,j,k-1/2} \, P'_{i,j,k} + \hat{\gamma}_z \frac{\delta t^2}{\delta z^2} \bar{P}'_{k+1} \qquad (4.11)$$

and

(iii) For the case $k = k_o-2$, the expression will be

$$2\hat{\gamma}_z \frac{\delta t^2}{\delta z^2} \bar{P}'_k = (\hat{GH})_k + \hat{\gamma}_z \frac{\delta t^2}{\delta z^2} \bar{P}'_{k-1} + \frac{\delta t^2}{\delta z^2} \sum_{i,j} (\gamma_z)_{i,j,k} \, P'_{i,j,k+1} \qquad (4.12)$$

We now have a total of $(K_{max}-3) + 3n$ linear equations with $(K_{max}-3) + 3n$ unknowns in this reduced system. All equations related to Defense Zones can be integrated again with respect to z. This will be illustrated in the following section.

4.4 Example of z-direction Integration in Upper Defense Zone

Let k_o+1 be the upper plane in Offense Zone, and k_o+7 be the upper boundary as shown in Fig. (3). The equations corresponding to the upper Defense Zone can be written as:

$$2(\hat{\gamma}_z) \frac{\delta t^2}{\delta z^2} \bar{P}'_{k_o+6} = (\hat{GH})_{k_o+6} + (\hat{\gamma}_z) \frac{\delta t^2}{\delta z^2} \bar{P}'_{k_o+5} \tag{4.12}$$

$$2(\hat{\gamma}_z) \frac{\delta t^2}{\delta z^2} \bar{P}'_{k_o+5} = (\hat{GH})_{k_o+5} + (\hat{\gamma}_z) \frac{\delta t^2}{\delta z^2} (\bar{P}'_{k_o+6} + \bar{P}'_{k_o+4}) \tag{4.13}$$

$$2(\hat{\gamma}_z) \frac{\delta t^2}{\delta z^2} \bar{P}'_{k_o+4} = (\hat{GH})_{k_o+4} + (\hat{\gamma}_z) \frac{\delta t^2}{\delta z^2} (\bar{P}'_{k_o+5} + \bar{P}'_{k_o+3}) \tag{4.14}$$

$$2(\hat{\gamma}_z) \frac{\delta t^2}{\delta z^2} \bar{P}'_{k_o+3} = (\hat{GH})_{k_o+3} + (\hat{\gamma}_z) \frac{\delta t^2}{\delta z^2} (\bar{P}'_{k_o+4} + \bar{P}'_{k_o+3}) \tag{4.15}$$

$$2(\hat{\gamma}_z) \frac{\delta t^2}{\delta z^2} \bar{P}'_{k_o+2} = (\hat{GH})_{k_o+2} + (\hat{\gamma}_z) \frac{\delta t^2}{\delta z^2} \bar{P}'_{k_o+3}$$

$$+ \frac{\delta t^2}{\delta z^2} \sum_{i,j} (\hat{\gamma}_z)_{i,j,(k_o+2)-1/2} P'_{i,j,(k_o+1)} \tag{4.16}$$

Substituting equation (4.12) into (4.13), we can obtain

$$3(\hat{\gamma}_z) \frac{\delta t^2}{\delta z^2} \bar{P}'_{k_o+5} = \left[2(\hat{GH})_{k_o+5} + (\hat{GH})_{k_o+6}\right] +2(\hat{\gamma}_z) \frac{\delta t^2}{\delta z^2} \bar{P}'_{k_o+4} \tag{4.17}$$

Then, substituting equation (4.17) into (4.14), a similar equation can be derived as:

$$4(\hat{\gamma}_z) \frac{\delta t^2}{\delta z^2} \bar{P}'_{k_o+4} = \left[3(\hat{GH})_{k_o+4} + 2(\hat{GH})_{k_o+5} + (\hat{GH})_{k_o+6}\right] + 3(\hat{\gamma}_z) \frac{\delta t^2}{\delta z^2} \bar{P}'_{k_o+3} \tag{4.18}$$

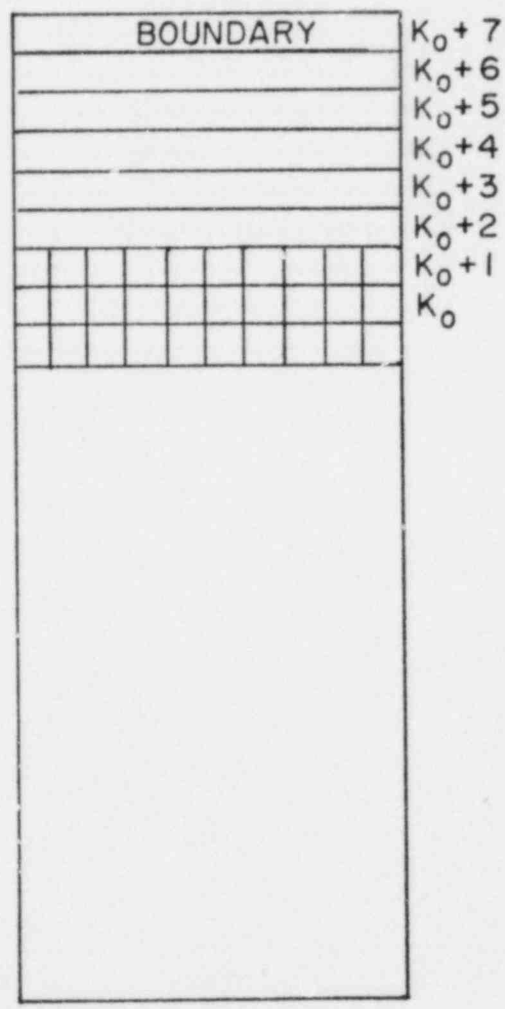

Fig. 3 Integration Scheme for Upper Defense Zone

Then, we substitute equation (4.18) into (4.15), to obtain:

$$5(\hat{\gamma}_z)\,\frac{\delta t^2}{\delta z^2}\,\bar{P}'_{k_o+2} = \left[4(\hat{GH})_{k_o+3} + 3(\hat{GH})_{k_o+4} + 2(\hat{GH})_{k_o+5} + (\hat{GH})_{k_o+6}\right] + 3(\hat{\gamma}_z)\,\frac{\delta t^2}{\delta z^2}\,\bar{P}'_{k_o+3} \tag{4.19}$$

Finally, we substitute equation (4.19) into (4.16). The equation for k_o+2 can be written as:

$$6(\hat{\gamma}_z)\,\frac{\delta t^2}{\delta z^2}\,\bar{P}'_{k_o+2} = \left[5(\hat{GH})_{k_o+2} + 4(\hat{GH})_{k_o+3} + 3(\hat{GH})_{k_o+4} + 2(\hat{GH})_{k_o+5} + (\hat{GH})_{k_o+6}\right] + 5\,\frac{\delta t^2}{\delta z^2}\left(\sum_{i}^{J_{max}}\ \sum_{j}^{\hat{}max}\right)(\hat{\gamma}_z)_{i,j(k_o+2)-1/2}\ P'_{i,j,k_o+1} \tag{4.20}$$

We define

$$(\hat{\hat{GH}})_{k_o+2} = 5(\hat{GH})_{k_o+2} + 4(\hat{GH})_{k_o+3} + 3(\hat{GH})_{k_o+4} + 2\,(\hat{GH})_{k_o+5} + (\hat{GH})_{k_o+6} \tag{4.21}$$

Then, equation (4.20) can be simplified as:

$$\frac{6}{5}\,(\hat{\gamma}_z)\,\frac{\delta t^2}{\delta z^2}\,\bar{P}'_{k_o+2} = \frac{1}{5}\,(\hat{\hat{GH}})_{k_o+2} + \frac{\delta t^2}{\delta z^2}\left(\sum_{i}^{J_{max}}\ \sum_{j}^{I_{max}}\right)(\gamma_z)_{(k_o+2)-1/2}\ P'_{i,j,k_o+1} \tag{4.22}$$

The term, $(\hat{\hat{GH}})_{k_o+2}$ in equation (4.22) includes upper boundary condition and the closely approximated resistance in upper Defense Zone. Upper boundary has been taken into account first in $(\hat{GH})_{k_o+6}$. Then, step by step, it is integrated into $(\hat{\hat{GH}})_{k_o+2}$.

Similarly, the lower Defense Zone can be integrated as outlined above.

4.5 Coupling with Separation of Matrix Technique (SMT)

As shown in preceding examples, a reduced Poisson type system can be reduced again to $(3n + 2) = [3 (I_{max}) \cdot (J_{max}) + 2]$ equation with $(3n + 2)$ unknowns. Hence, a system with $(3n + 2)$ equations can be arranged as:

$$Ax = B \tag{4.23}$$

As in HEV2D [2], A varies only in diagonal for rebalance formulation. Therefore, the inverse of A can be computed first and used in every rebalance to reduce the $(3n + 2)$ system to 2.

For the case when the Offense Zone is touching the boundary, a system with $(3n + 1)$ equations will be formed, and $A^{-1}(3n+1)$ is needed. Without increasing computer storage, a simple method to find $A_o^{-1}(3n + 1)$ from $A_o^{-1}(3n + 2)$ has been developed and presented in Appendix 1.

5. CONVERGENCE

(a) Offense Zone: Convergence is achieved since conservation of mass is observed by using SMT.

(b) Defense Zone: By the assumption of FPP, the sum of the mass residue of every plane goes to zero. Furthermore, within every plane in the Defense Zone, the mass residue profile does not change and cannot translate from one plane to another. However, when the Offense Zone sweeps through, the mass residue of every cell vanishes; thus speeding up the convergence rate. (see Fig. 4).

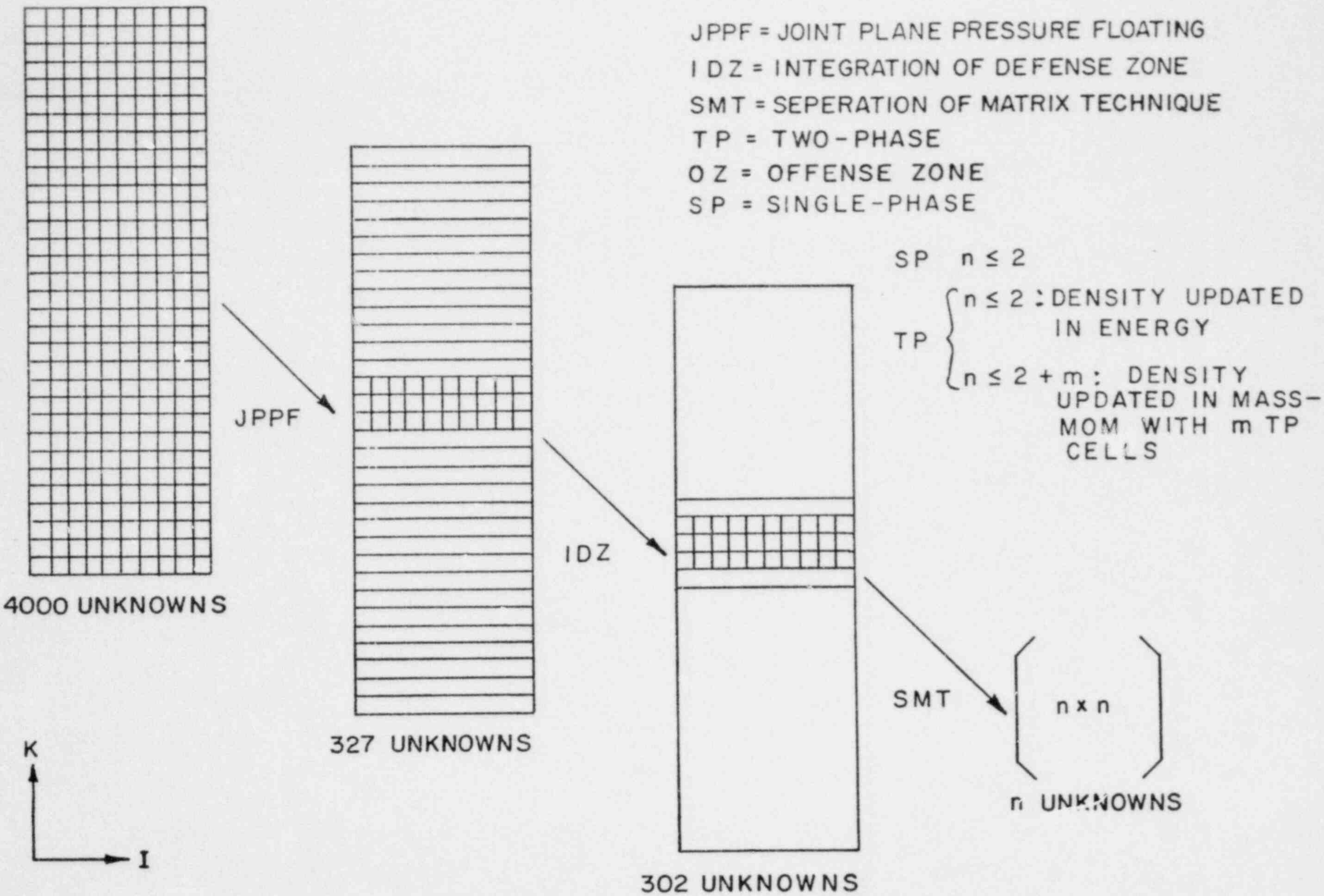

Fig. 4. REDUCTION PROCEDURE IN ART
(X, Y, Z) = (10, 10, 40)

6. CONCLUSION

The unique feature of the ART over other solution techniques for the Navier-Stokes equations is that the effect of entire boundary conditions of a physical system under consideration is accounted for during every sweep of an iteration. Furthermore, it retains advantages of low storage of iterative solution procedures and high accuracy of direct inversion solution methods. Finally, the ART solution procedure can readily alleviate the assumption of a constant correction of pressure drop across a plane during an iteration in the Defense Zone. This can be accomplished by subdividing the plane into more than one region (i.e., contraction or converging and expansion or diverging flow region).

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APPENDIX 1

Derivation of $A_o^{-1}(3n+1)$ from $A_o^{-1}(3n+2)$

Let us examine the case when the Offense Zone is touching the lower boundary. It is obviously, $A_o(3n+1)$ is the major position of $A_o(3n+2)$. Partition $A_o(3n+2)$ as:

$$A_o(3n+2) = \left[\begin{array}{c|c} \begin{matrix} A_{11} \\ (3n+1;3n+1) \end{matrix} & \begin{matrix} A_{11} \\ (3n+1,1) \end{matrix} \\ \hline \begin{matrix} A_{21} \\ (1,3n+1) \end{matrix} & \begin{matrix} A_{22} \\ (1,1) \end{matrix} \end{array}\right] \qquad [A.1.1]$$

where $A_{11}=A_o(3n+1)$. Let

$$A_o^{-1}(3n+2) = \left[\begin{array}{c|c} \begin{matrix} B_{11} \\ (3n+1,3n+1) \end{matrix} & \begin{matrix} B_{12} \\ (3n+1,1) \end{matrix} \\ \hline \begin{matrix} B_{21} \\ (1,3n+1) \end{matrix} & \begin{matrix} B_{22} \\ (1.1) \end{matrix} \end{array}\right] \qquad [A.1.2]$$

Since $A_o^{-1}(3n+2)$ is known, B_{11}, B_{12}, B_{21}, B_{22} are also known.

Let $\zeta = B_{22}^{-1}$, then we have

$$B_{11} = A_{11}^{-1} + (A_{11}^{-1}\ A_{12})\zeta^{-1}\ (A_{21}\ A_{11}^{-1}) \qquad [A.1.3]$$

$$B_{12} = -(A_{11}^{-1}\ A_{12})\zeta^{-1} \qquad [A.1.4]$$

and

$$B_{21} = -\zeta^{-1}(A_{21}\ A_{11}^{-1}) \qquad [A.1.5]$$

Then A.1.3 can be simplified as:

$$B_{11} = A_{11}^{-1} + (-B_{12}\zeta)\zeta^{-1}(-\zeta\ B_{21}) \qquad [A.1.6]$$

or $$A_{11}^{-1} = A_o^{-1}(3n+1) = B_{11} - B_{12}\ B_{21}\zeta \qquad [A.1.7]$$

APPENDIX 2

<u>Liquid</u>

$$X_{\ell f} = - [\frac{\partial}{\partial x}(\rho_{\ell}\theta_{\ell}u_{\ell}^{2}) + \frac{\partial}{\partial y}(\rho_{\ell}\theta_{\ell}u_{\ell}v_{\ell}) + \frac{\partial}{\partial z}(\rho_{\ell}\theta_{\ell}u_{\ell}w_{\ell})] \qquad [A.2.1]$$

$$X_{\ell n} = \{\theta_{\ell}\rho_{\ell}g_{x} + K\,(u_{g}-u_{\ell}) + S_{m\ell x}\}$$

$$+ \{\frac{\partial}{\partial x}[\tau_{xx}\theta_{\ell}] + \frac{\partial}{\partial y}(\tau_{xy}\theta_{\ell}) + \frac{\partial}{\partial z}(\tau_{xz}\theta_{\ell})\} \qquad [A.2.2]$$

$$Y_{\ell f} = - [\frac{\partial}{\partial x}(\rho_{\ell}\theta_{\ell}u_{\ell}v_{\ell}) + \frac{\partial}{\partial y}(\rho_{\ell}\theta_{\ell}v_{\ell}^{2}) + \frac{\partial}{\partial z}(\rho_{\ell}\theta_{\ell}v_{\ell}\,w_{\ell})] \qquad [A.2.3]$$

$$Y_{\ell n} = \{\theta_{\ell}\rho_{\ell}g_{y} + K\,(v_{g}-v_{\ell}) + S_{m\ell y}\}$$

$$+ \{\frac{\partial}{\partial x}(\tau_{xy}\theta_{\ell}) + \frac{\partial}{\partial y}(\tau_{yy}\theta_{\ell}) + \frac{\partial}{\partial z}(\tau_{yz}\theta_{\ell})\} \qquad [A.2.4]$$

$$Z_{\ell f} = - [\frac{\partial}{\partial x}(\rho_{\ell}\theta_{\ell}w_{\ell}u_{\ell}) + \frac{\partial}{\partial y}(\rho_{\ell}\theta_{\ell}v_{\ell}w_{\ell}) + \frac{\partial}{\partial z}(\rho_{\ell}\theta_{\ell}w_{\ell}^{2})] \qquad [A.2.5]$$

$$Z_{\ell n} = \{\theta_{\ell}\rho_{\ell}g_{z} + K(w_{g}-w_{\ell}) + S_{m\ell z}\}$$

$$+ \{\frac{\partial}{\partial x}(\tau_{xz}\theta_{\ell}) + \frac{\partial}{\partial y}(\tau_{yz}\theta_{\ell}) + \frac{\partial}{\partial z}(\tau_{zz}\theta_{\ell})\} \qquad [A.2.6]$$

APPENDIX 2 (continued)

$$X_{gf} = - [\frac{\partial}{\partial x}(\rho_g \theta_g u_g^2) + \frac{\partial}{\partial y} (\theta_g \rho_g u_g v_g) + \frac{\partial}{\partial z} (\Theta_g \ell_g u_g w_g)]$$

$$X_{gn} = \theta_g \rho_g g_x + K(u_\ell - u_g) + S_{mgx}$$

$$+ \frac{\partial}{\partial x} (\zeta_{xx} \theta_g) + \frac{\partial}{\partial y} (\zeta_{xy} \theta_g) + \frac{\partial}{\partial y} (\zeta_{xz} \theta_g)$$

$$Y_{gf} = - [\frac{\partial}{\partial x} (\theta_g \rho_g u_g v_g) + \frac{\partial}{\partial y} (\theta_g \rho_g u_g^2) + \frac{\partial}{\partial z} (\theta_g \rho_g v_g w_g)]$$

$$Y_{gn} = \theta_g \rho_g g_y + K(u_\ell - v_g) + S_{mgy}$$

$$+ \frac{\partial}{\partial x} (\zeta_{xy} \theta_g) + \frac{\partial}{\partial y} (\zeta_{yy} \theta_g) + \frac{\partial}{\partial z} (\zeta_{yz} \theta_g)$$

$$Z_{gf} = - [\frac{\partial}{\partial x} (\theta_g \rho_g w_g u_g) + \frac{\partial}{\partial y} (\theta_g \rho_g v_g w_g) + \frac{\partial}{\partial z} (\theta_g \rho_g w_g^2)$$

$$Z_{gn} = \theta_g \rho_g g_z + K(w_\ell - w_g) + S_{m\ell z}$$

$$+ \frac{\partial}{\partial x} (\zeta_{xz} \theta_g) + \frac{\partial}{\partial y} (\zeta_{yz} \theta_g) + \frac{\partial}{\partial z} (\zeta_{zz} \theta_g)$$

APPENDIX 3

Liquid

$$\frac{(\gamma_v\theta_\ell\rho_\ell u_\ell)^{n+1}_{i+1/2} - (\gamma_v\theta_\ell\rho_\ell u_\ell)^{n}_{i+1/2}}{\delta t}$$

$$= (X_{\ell f})_{i+1/2} + (X_{\ell n})_{i+1/2} + \frac{(\gamma_v)_{i+1/2}}{\delta x_{i+1/2}}\left[(\theta_\ell P)^{n+1}_{i} - (\theta_\ell P)^{n+1}_{i+1}\right] \qquad [A.3.1]$$

$$\frac{(\gamma_v\theta_\ell\rho_\ell v_\ell)^{n+1}_{j+1/2} - (\gamma_v\theta_\ell\rho_\ell v_\ell)^{n}_{j+1/2}}{\delta t}$$

$$= (Y_{\ell f})_{j+1/2} + (Y_{\ell n})_{j+1/2} + \frac{(\gamma_v)_{j+1/2}}{\delta y_{j+1/2}}\left[(\theta_\ell P)^{n+1}_{j} - (\theta_\ell P)^{n+1}_{j+1}\right] \qquad [A.3.2]$$

$$\frac{(\gamma_v\theta_\ell\rho_\ell w_\ell)^{n+1}_{k+1/2} - (\gamma_v\theta_\ell\rho_\ell w_\ell)^{n}_{k+1/2}}{\delta t}$$

$$= (Z_{\ell f})_{k+1/2} + (Z_{\ell n})_{k+1/2} + \frac{(\gamma_v)_{k+1/2}}{\delta z_{k+1/2}}\left[(\theta_\ell P)^{n+1}_{k} - (\theta_\ell P)^{n+1}_{k+1}\right] \qquad [A.3.3]$$

Gas

$$\frac{(\gamma_v\theta_g\rho_g u_g)^{n+1}_{i+1/2} - (\gamma_v\theta_g\rho_g u_g)^{n}_{i+1/2}}{\delta t}$$

$$= (X_{gf})_{i+1/2} + (X_{gn})_{i+1/2} + \frac{(\gamma_v)_{i+1/2}}{\delta x_{i+1/2}}\left[(\theta_g P)^{n+1}_{i} - (\theta_g P)^{n+1}_{i+1}\right] \qquad [A.3.4]$$

$$\frac{(\gamma_v\theta_g\rho_g v_g)^{n+1}_{j+1/2} - (\gamma_v\theta_g\rho_g v_g)^{n}_{j+1/2}}{\delta t}$$

$$= (Y_{gf})_{j+1/2} + (Y_{gn})_{j+1/2} + \frac{(\gamma_v)_{j+1/2}}{\delta y_{j+1/2}}\left[(\theta_g P)^{n+1}_{j} - (\theta_g P)^{n+1}_{j+1}\right] \qquad [A.3.5]$$

$$\frac{(\gamma_v\theta_g\rho_g w_g)^{n+1}_{k+1/2} - (\gamma_v\theta_g\rho_g w_g)^{n}}{\delta t}$$

$$= (Y_{gf})_{k+1/2} + (Y_{gn})_{k+1/2} + \frac{(\gamma_v)_{k+1/2}}{\delta y_{k+1/2}}\left[(\theta_g P)^{n+1}_{k} - (\theta_g P)^{n+1}_{1+1}\right] \qquad [A.3.6]$$

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