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MARK I CONTAINMENT PROGRAM
SCALING ANALYSIS FOR MODELING INITIAL AIR CLEARING CAUSED BY REACTOR SAFETY/RELIEF VALVE DISCHARGE
TASK NUMBER 6.2.1

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ABSTRACT

A generalized method of similitude is introduced and applied to develop scaling relationships for a General Electric Mark I suppression pool. A scale model is proposed to model suppression pool wall loads due to air flow through a T-quencher discharge device. The scaling relationships developed provide the means for relating scale model parameters (i.e., pressure, velocity,) to full scale.

1. INTRODUCTION

1.1 GENERAL

During normal (and sometimes abnormal) operation of a nuclear reactor, high pressure steam must be eliminated from the main steam lines. This steam is bled off through the Safety/Relief Valve (S/RV) system to be condensed in the reactor suppression pool. The typical S/RV system consists of quick opening valves at the main steam lines, each of which is followed by a length of large diameter piping and a discharge device. The discharge device is located at the bottom of a toroidally shaped containment (the "torus") that is partially filled with water. The torus encircles the base of the reactor pressure vessel (Figure 1-1).

When the S/RV is activated, first the water is discharged in the piping, then the air followed by the steam. The torus wall loads caused by water and steam expulsion are small compared with those caused by the air discharge. To reduce torus wall loads, it is suggested that quencher discharge devices be installed to replace the existing ramshead discharge devices (Figure 1-2). To test these devices, a small scale model and test facility will be built.

1.2 OBJECTIVES

This report defines the scaling parameters to be used for modeling initial air clearing caused by reactor safety/relief valve discharge.

The report is divided into seven sections and contains three appendices.

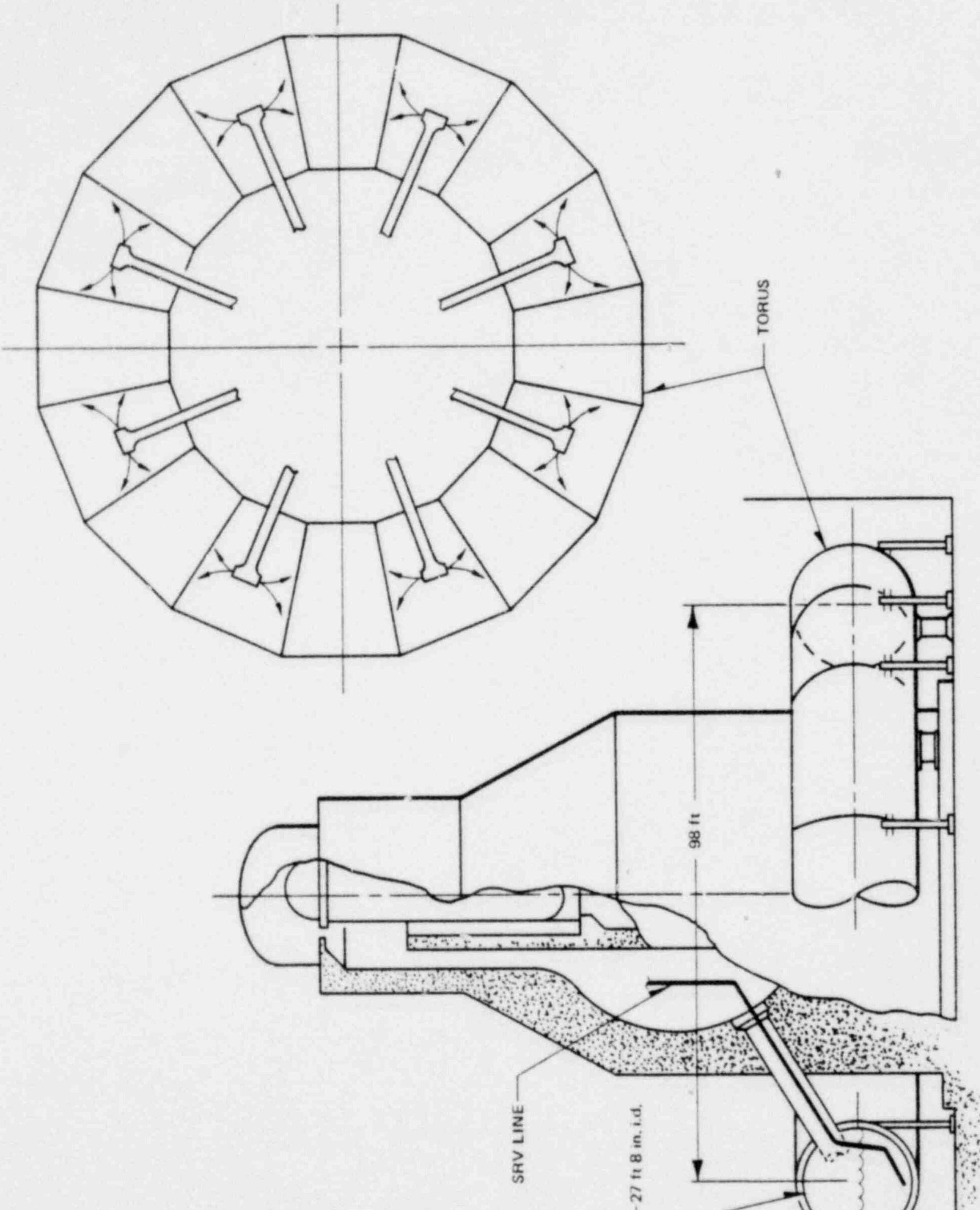

Figure 1-1. Safety/Relief Valve Torus Arrangement

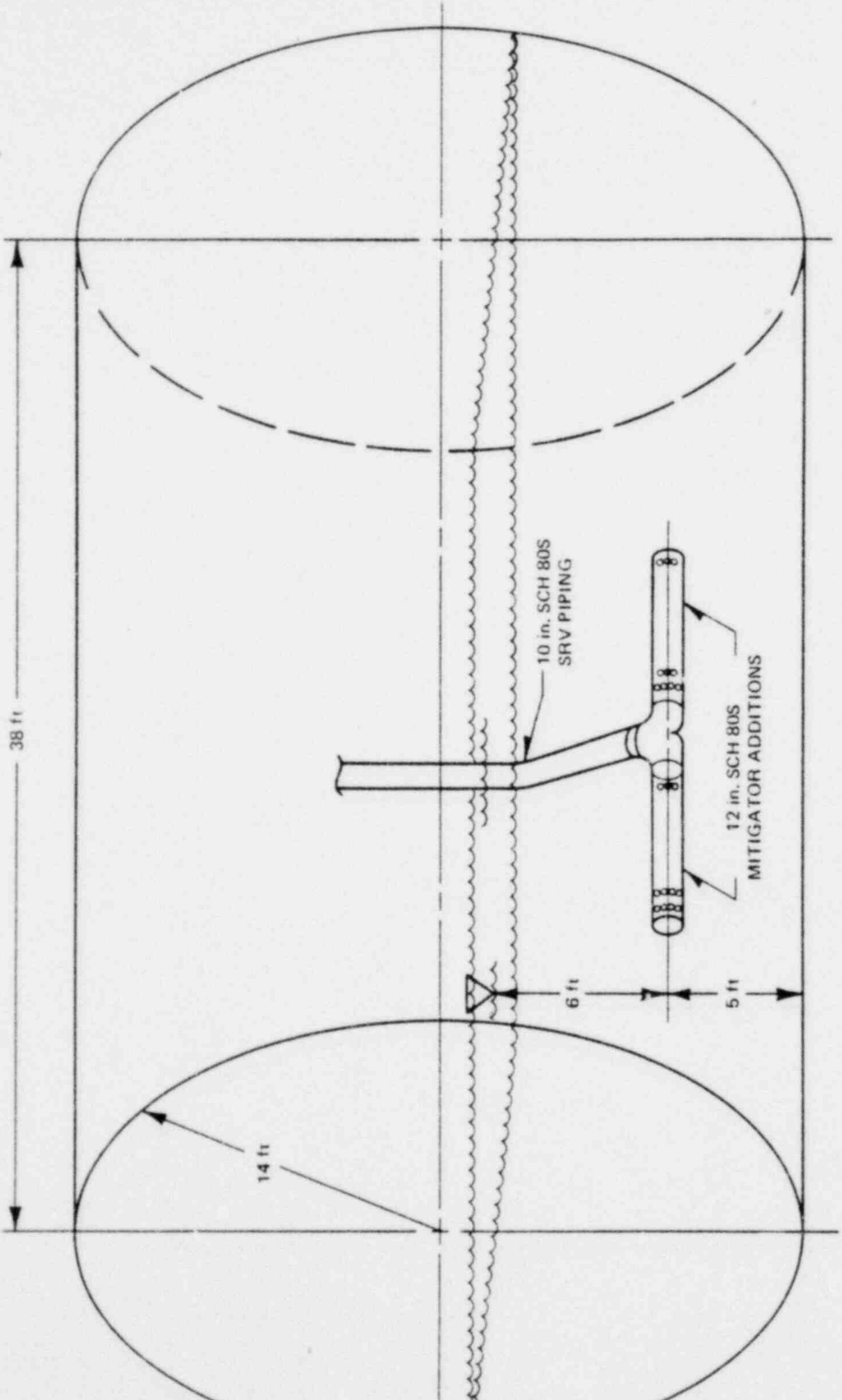

Figure 1-2. Flow Mitigating Addition to Ramshead Device - Monticello Plant

2. MODELING

2.1 GENERAL

Scale modeling is a powerful tool for solving technical problems when the governing equations are not readily solved and when physical behavior can be readily observed in another scale. Application of scaling procedures to unsteady fluid mechanics has not been extensive, however.
extensive, however.

The design of a scale model requires that all governing phenomena be transformed in a way that the relative influences are preserved in another scale. When the governing equations cannot be written, the Buckingham Pi theorem may provide model laws. The success of the use of this theorem depends largely on the user's intuition. Use of this theorem does not guarantee that all governing effects will be included or that negligible effects will be excluded. When governing forces in a problem are identified, the method of similitude provides certain model laws in terms of force ratios for a geometrically similar system, but it often fails to provide other necessary model constraints. A generalization of the method of similitude (Kline, 1965) provides all the required model laws for a complete formulation of the governing equations and boundary conditions, and establishes a starting point for scaling procedures in unsteady fluid mechanics.

2.2 METHODOLOGY

A complete formulation, which includes the governing equations and boundary equations, is nondimensionalized so that all dependent variables and their derivatives become of the order of 1.0 in magnitude, designated by O(1). Dimensionless similarity parameters appear as coefficients in the formulation; a comparison of their relative numerical size determines which are sufficiently small to be neglected. In a properly scaled model, the remaining parameters must be numerically equal to full scale values. Finally, model test results are employed with the nondimensional variables to relate behavior in another scale.

Mass, momentum, and energy conservation principles are usually formulated in terms of macroscopic-dependent properties, such as pressure, density, and velocity. Other equilibrium state properties are uniquely determined from appropriate state equations, whether algebraic, tabular, or graphical. If properties are designated by ϕ^K, K equals 1,2, initial properties ϕ^{Ki} are disturbed by prescribed activity at the system boundaries. Examples of boundary disturbances are val. ration, position motion, submerged gas discharge in liquid, local heating, or local pressure variation on a liquid surface. Generally, the disturbance itself provides reference value ϕ^{KR} of one or more properties associated with the problem. Using a superscript "o" to denote nondimensional quantities, dependent variables occurring in derivatives are written in the form

$$\phi^{Ko} \triangleq (\phi^K - \phi^{Ki})/\phi^{KR} = \phi^{Ki}) \quad (1)$$

which includes both the initial and references values, and is 0 (equation 1). Time and space variables are independent with arbitrary zeroes, and therefore are nondimensionalized as

$$t^o \triangleq t/\tau; \; x^o \triangleq x/X \; ; \; y^o \triangleq y/Y \; ; \; z^o \triangleq z/Z \quad (2)$$

References time τ and displacements X, Y, Z in equation (2) are specified to be consistent with the phenomena to be modeled such that the change in t^o and x^o, y^o, z^o are 0 (equation 1); e.g., 0 to 1, or 3 to 4. It follows that the nondimensional derivatives such as $\partial\phi^{Ko}/\partial t^o$, $\partial\phi^{Ko}/\partial x^o$, and $\partial^2\phi^{Ko}/\partial x^{o2}$ are are 0 (equation 1) and their respective coefficients $(\phi^{KR} - \phi^{Ki})/\tau$, $(\phi^{KR} - \phi^{Ki})/X$, and $(\phi^{KR} - \phi^{Ki})/X^2$ later become factors with other constants in the problem to provide similarity parameters. Some system parameters have direction-dependent variables which appear in the derivatives. Pressure changes, for example, may depend on direction as a result of gravitational forces; thus, a nondimensional pressure can be defined for each direction p^{ox}, p^{oy}, p^{oz}.

The form selected for nondimensional derivative variables in equation (1) automatically provides initial values that are zero when ϕ^K is uniform throughout the region of interest. If ϕ^K is not initially uniform in the region, its initial distribution must be considered in the model laws.

Unsteady flow systems frequently encountered in practice involve three-to-one-dimensional flows and time-dependent nodal flows with negligible space-dependence. The simplification from three-to-one dimensional governing equations is acceptable when changes are predominantly in one direction. Nodal equations are acceptable when parameters such as temperature and pressure are equal throughout the system. Generally speaking, more system detail is achieved by using greater dimensions.

Certain parameters are "control" parameters; for example, lengths, areas, initial temperatures, and initial pressures are controllable. Other parameters are "consequences" of the control parameters; for example, behavior of mass flow rate, internal energy, and velocities depend on what control parameters are chosen.

Once a scale model is specified, the governing equations should be reexamined using the small scale reference values to be certain no new parameters have been introduced.

3. APPLICATION TO THE MARK I S/RV SYSTEM

The quencher device to be modeled is depicted in Figure 3-1. Using the Monticello plant as a reference, there are eight symmetrically spaced quenchers in the torus; only one need be modeled. When the S/RV valve opens, a complex series of coupled events occur in the pipe and in the pool. These events are listed below.

Pipe Events

A. A shock wave travels down the pipe impacting the water leg surface.

B. Initial air compresses while the initial water slug concurrently accelerates down the pipe and exits into the pool.

C. The air slug exits the pipe in a distributed fashion through the quencher holes.

D. Steam fills the pipe and consenses on the colder walls. The steam charging the S/RV line may condense somewhat or become somewhat superheated depending on the valve discharge rate, pipe size, and air properties.

Pool Events

A. Water accelerates into the pool following shock wave impact.

B. Air enters the pool following the water.

C. Pool water level rises, compressing the torus top air.

The transient extends until steam enters the pool.

Typical plant parameters will be used to establish the necessary reference values. The primary concern in modeling the discharge device is to model

Company Proprietary

Figure 3-1. Typical T-Quencher Detail (GE Company Proprietary) torus wall loads (or pressures). Consequently, other system parameters may be adjusted to give the correct pool pressure responses.

The pool water is generally modeled as a three-dimensional, constant density, incompressible fluid. The governing differential equations in cartesian coordinates are:

Mass

$$\nabla \cdot \vec{V} = 0 \tag{3}$$

$$\rho \frac{DV}{Dt} + \nabla P = \vec{F} + \mu \nabla^2 \vec{V} \tag{4}$$

Energy

$$\rho \frac{De}{Dt} = K \nabla^2 T + \Phi \tag{5}$$

where

$$\frac{D}{Dt} \triangleq \frac{\partial}{\partial t} + \mu \frac{\partial}{\partial x} + v \frac{\partial}{\partial y} + w \frac{\partial}{\partial z} \quad \text{(Material derivative)}$$

$$\vec{F} \triangleq f^x \hat{i} + f^y \hat{j} + f^z \hat{b} \quad \text{(Body force vector)}$$

$$\vec{V} \triangleq u \hat{i} + v \hat{i} + w \hat{k} \quad \text{(Velocity vector)}$$

$$\nabla \triangleq \frac{\partial}{\partial x} \hat{i} + \frac{\partial}{\partial y} \hat{j} + \frac{\partial}{\partial z} \hat{k} \quad \text{(Gradient)}$$

$$\nabla^2 \triangleq \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} \quad \text{(Laplacian)} \tag{6}$$

$$\Phi \triangleq 2\mu \left[\left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial v}{\partial y}\right)^2 + \left(\frac{\partial w}{\partial z}\right)^2 + \frac{1}{2}\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right)^2 + \frac{1}{2}\left(\frac{\partial v}{\partial z} + \frac{\partial w}{\partial y}\right)^2 + \frac{1}{2}\left(\frac{\partial w}{\partial x} + \frac{\partial u}{\partial z}\right)^2 \right]$$

$$+ \lambda \left[\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} \right]^2 \quad \text{(Dissipation function)}$$

Introducing the nondimensional parameters:

$$u^o = \frac{u - u^i}{\Delta u}$$

$$v^o = \frac{v - v^i}{\Delta v}$$

$$w^o = \frac{w - w^i}{\Delta w}$$

$$P^o = \frac{P - P^i}{\Delta P}$$

$$e^o = \frac{e - e^i}{\Delta e} \qquad (7)$$

$$T^o = \frac{T - T^i}{\Delta T}$$

$$\phi^o = \frac{\phi - \phi^i}{\Delta \phi}$$

$$x^o = \frac{x}{X}$$

$$y^o = \frac{y}{Y}$$

$$z^o = \frac{z}{Z}$$

$$t^o = \frac{t}{\tau}$$

where

$$\Delta(\ ) \triangleq (\ )^R - (\ )^i$$

Inserting equation (7) into equations (3), (4), and (5) results in a normalized form of the governing equations:

Mass

$$\left(\frac{\Delta u}{X}\right) u_x^o + \left(\frac{\Delta v}{Y}\right) v_y^o + \left(\frac{\Delta w}{Z}\right) w_z^o = 0 \tag{8}$$

Momentum

$$\begin{aligned}
\rho \Bigg\{ & \left(\frac{\Delta u}{\tau}\right) u_t^o \, \hat{i} + \left(\frac{\Delta v}{\tau}\right) v_t^o \, \hat{j} + \left(\frac{\Delta w}{\tau}\right) w_t^o \, \hat{k} \\
& + (u^o \Delta u + u^i) \left[\left(\frac{\Delta u}{X}\right) u_x^o \, \hat{i} + \left(\frac{\Delta v}{X}\right) v_x^o \, \hat{j} + \left(\frac{\Delta w}{X}\right) w_x^o \, \hat{k} \right] \\
& + (v^o \Delta v + v^i) \left[\left(\frac{\Delta u}{Y}\right) u_y^o \, \hat{i} + \left(\frac{\Delta v}{Y}\right) v_y^o \, \hat{j} + \left(\frac{\Delta w}{Y}\right) w_y^o \, \hat{k} \right] \\
& + (w^o \Delta w + w^i) \left[\left(\frac{\Delta u}{Z}\right) u_z^o \, \hat{i} + \left(\frac{\Delta v}{Z}\right) v_z^o \, \hat{j} + \left(\frac{\Delta w}{Z}\right) w_z^o \, \hat{k} \right] \Bigg\} \\
& + \mu \left[\left(\frac{\Delta u}{X^2}\right) u^o{}_{xx} + \left(\frac{\Delta u}{Y^2}\right) u_{yy}^o + \left(\frac{\Delta u}{Z^2}\right) u^o{}_{zz} \right] \hat{i} + \mu \left[\left(\frac{\Delta v}{X^2}\right) v_{xx}^o + \left(\frac{\Delta v}{Y^2}\right) v_{yy}^o + \left(\frac{\Delta v}{Z^2}\right) v^o{}_{zz} \right] \hat{j} \\
& + \mu \left[\left(\frac{\Delta w}{X^2}\right) w_{xx}^o + \left(\frac{\Delta w}{Y^2}\right) w_{yy}^o + \left(\frac{\Delta w}{Z^2}\right) w^o{}_{zz} \right] \hat{k}
\end{aligned} \tag{9}$$

Energy:

$$\rho\left[\left(\frac{\Delta e}{\tau}\right) e_t^o + (u^o \Delta u + u^i)\left(\frac{\Delta e}{X}\right) e_x^o + (v^o \Delta v + v^i)\left(\frac{\Delta e}{Y}\right) e_y^o + (w^o \Delta w + w^i)\left(\frac{\Delta e}{Z}\right) e_z^o\right]$$

$$= \quad \phi^o \Delta\phi + \phi^i + K\left[\left(\frac{\Delta T}{X^2}\right) T_{xx}^o + \left(\frac{\Delta T}{Y^2}\right) T_{yy}^o + \left(\frac{\Delta T}{Z^2}\right) T_{zz}^o\right] \qquad (10)$$

3.1 POOL MOTION — WATER CLEARING

The first item of interest is pool motion during water clearing from the discharge device. To estimate water clearing velocity and time, a simple geometry is devised and first principles used (Figure 3-2). From Newton's law and assuming constant acceleration:

$$F = ma \qquad \text{or} \qquad (P_{SH} - P_p) = \rho La \qquad (11)$$

$$x = 1/2\ at^2. \qquad (12)$$

Solving for clearing time τ_{wc} (i.e., when $x = L$):

$$\tau_{wc} = \left(\frac{2L^2\rho}{P_{SH} - P_P}\right)^{1/2} . \qquad (13)$$

If the water were accelerated out the end of the pipe, then the final velocity U would be:

$$U = a\tau_{wc} \quad \left(\frac{2(P_{SH} - P_P)}{\rho}\right)^{1/2} . \qquad (14)$$

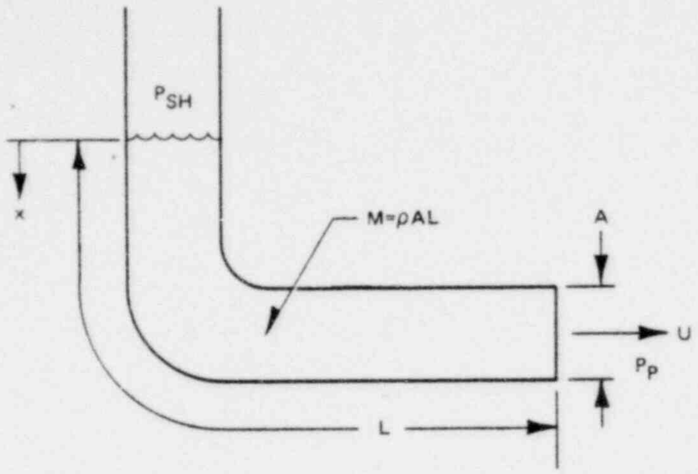

Figure 3-2. Water Clearing Model

The velocity out the pipe end U is of no interest but rather the velocity out the quencher holes. If the end of the pipe shown in Figure 3-2 is capped, then the water is forced out the mitigator holes (Figure 3-3). The approximate exiting velocity is determined from mass conservation:

$$V = U \frac{A_{PIPE}}{A_{HOLES}} = \left(\frac{2(P_{SH} - P_P)}{\rho} \right)^{1/2} \frac{A_{PIPE}}{A_{HOLES}} \tag{15}$$

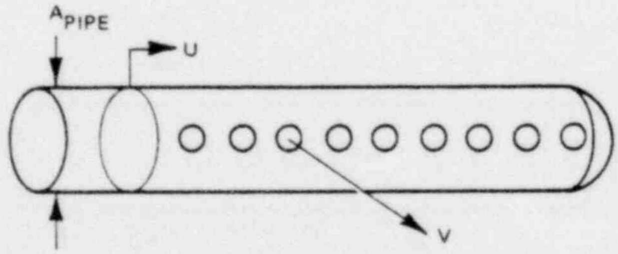

Figure 3-3. U to V Mass Conservation

Inserting typical values then:

$$\rho = 62.4 \left[\frac{lb_m}{f^3}\right] \quad \text{- Water density}$$

$$L = 10 \left[f\right] \quad \text{- Water leg length}$$

$$P_{SH} - P_P = 340 - 15 = 324 \text{ [psia]} \quad \text{(16)}$$

- Driving pressure difference, see Appendix A for shock pressure. Pool pressure is slightly above 1 atm

$$A_{PIPE} \approx A_{HOLES} \approx 100 \text{ [in.}^2\text{]} \quad \text{- Pipe and hole areas}$$

Then from (13) and (15):

$$\tau_{wc} = 0.09 \text{ [sec]}$$

$$V = 220 \text{ [f/s]}$$

The clearing velocities in the x and z directions are estimated from geometry. The value U_{MAX} is found according to the maximum drill angle of the quencher holes. In the z direction, W_{MAX} is due to the increase in pool volume from pipe water expulsion (Figure 3-4):

$$V_{MAX} = V = 220\text{[f/s]}$$

$$U_{MAX} = V \sin \alpha = 110\text{[f/s]} \quad \text{(18)}$$

$$W_{MAX} = \frac{Vol}{A_{PW}\tau_{wc}} = 0.25\text{[f/s]}$$

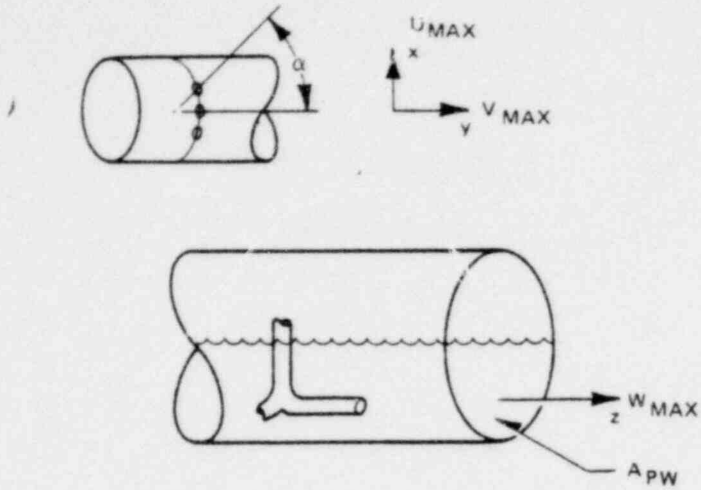

Figure 3-4. Water Velocities in X and Z Directions

where

$\alpha = 30°$ - Drill angle

$\tau_{wc} = 0.09$ [sec] - Water clearing time (19)

$A_{PW} = 308[f^2]$ - Pool water area

$Vol = 7[f^3]$ - Half the initial water volume of the quencher

Initially, the pool will have zero velocity and, in $\tau_{wc} = 0.09$ seconds, will have reached U_{MAX}, V_{MAX}, and W_{MAX} in the x, y, and z directions, respectively. Since there are no temperature differences in the pool, application of the energy equation is unnecessary during water clearing.

Reference values for water clearing are:

$X = 6$ [f]	- Quencher submergence	
$Y = 10$ [f]	- Quencher-to-wall length	
$Z = 7$ [f]	- Quencher arm length	
$\tau = 0.09$ [sec]	- Water clearing time	
$u^i = v^i = w^i = 0$	- Water is initially at rest	
$u^R = 110$ [f/s]	- Maximum velocity in x direction	
$v^R = 220$ [f/s]	- Maximum velocity in y direction	(20)
$w^R = 0.25$ [f/s]	- Maximum velocity in y direction	
$\Delta P = \rho g X = 2.6$ [psia]	- Static head	
$\rho = 62.4$ [lbm/f^3]	- Water density	
$\mu = 2.10^{-5}$ [lbf - s/f^2]	- Water viscosity at 70°F, 1 atm.	
$f^x = -\rho g = -62.4$ [lbf/f^3]	- Gravitational body force	
$f^y = f^z = 0$		

Inserting the reference values into the mass and momentum equations (8) and (9), the eliminating terms two orders of magnitude less than the greatest give the following differential equations and similitude parameters:

<u>Mass</u>

$$(P_1 P_3) u_x^o + v_y^o = 0 \qquad (21)$$

Momentum

$$(P_2P_3)u_t^o\,\hat{i} + (P_2)v_t^o\,\hat{j} + \left(P_1P_3^2\right)u^o u_x^o\,\hat{i} + (P_1P_3)u^o v_x^o\,\hat{j}$$
$$+ (P_3)v^o u_y^o\,\hat{i} + v^o v_y^o\,\hat{j} = 0 \qquad (22)$$

where

$$P_1 = (Y/X)$$
$$P_2 = (Y/\Delta v\tau) = (Y/v^R\tau) \qquad (23)$$
$$P_3 = (\Delta u/\Delta v) = (u^R/v^R)$$

Thus, for modeling pool motion during water clearing, pressure variations, viscous forces, and all "z" direction, similitude parameters are negligible. The three dimensionless parameters of equation (23) must be numerically equal in full and model scales; that is, the same equations govern the responses in both systems. Two controllable scaling parameters are defined, one for submergence and one for fluid density (in case the model has a fluid other than water):

$$\lambda_x \triangleq \frac{X_m}{X_F} \qquad (24)$$

$$\lambda_\rho \triangleq \frac{\rho_m}{\rho_F} \qquad (25)$$

It follows immediately from P_1 that:*

$$X \neq Y \neq \lambda_x \qquad (26)$$

---

*See the properties of the scaling operator presented in Appendix B.

Examining P_2 and from equation (26) then:

$$Y \neq v^R \tau \neq \lambda_x \tag{27}$$

Since $v^R = V_{MAX}$ (equation 15) and τ is given in equation (13), Equation (27) can be written as:

$$v^R \tau = L \frac{A_{PIPE}}{A_{HOLES}} \neq \lambda_x \tag{28}$$

Parameter P_3 indicates that velocities scale the same. From P_3 and equation (15) then:

$$u^R \neq v^R \neq \left(\frac{P_{SH} - P_P}{\rho}\right)^{1/2} \frac{A_{PIPE}}{A_{HOLES}} \tag{29}$$

Five nondimensional parameters resulted in the governing equations (21) and (22), namely, t^o, x^o, y^o, u^o, and v^o. Nondimensional parameters must be equal in both model and full scales to compare one scale with the other, thus:

$$(t^o)_m = (t^o)_F \quad \text{or} \quad (t/\tau)_m = (t/\tau)_F.$$

Rearranging gives an expression in τ which is known from equations (13) and (28):

$$\frac{t_m}{t_F} = \frac{\tau_m}{\tau_F} \neq L \left(\frac{\rho}{P_{SH} - P_P}\right)^{1/2} \tag{30}$$

Proceeding similarly for x^o and y^o results in:

$$x \neq X \neq \lambda_x$$

$$y \neq Y \neq \lambda_x \tag{31}$$

Likewise, velocities scale like ation (29) since initial velocities are zero:

$$v \neq v^R \neq u \neq u^R \neq \left(\frac{P_{SH} - P_P}{\rho}\right)^{1/2} \frac{A_{PIPE}}{A_{HOLES}} \tag{32}$$

One other parameter is determined from P_3 and equation (18):

$$P_3 = u^R/v^R = \sin \alpha \neq 1 \tag{33}$$

Thus, the angle α must be preserved between model and full scales.

Summarizing, for modeling water flow into the pool, lengths scale as:

$$x,\ y,\ X,\ Y,\ L\ .\ \frac{A_{PIPE}}{A_{HOLES}} \neq \lambda_x \tag{34}$$

Velocities scale according to the driving pressure difference and pool density:

$$u, v, u^R, v^R \neq \left(\frac{P_{SH} - P_P}{\rho}\right)^{1/2} \cdot \frac{A_{PIPE}}{A_{HOLES}} \tag{35}$$

Time scales like Y/v^R:

$$t,\tau \neq \left(\frac{\rho}{P_{SH} - P_P}\right)^{1/2} \frac{A_{HOLES}}{A_{PIPE}} \; x \tag{36}$$

and mitigator hole angles scale one to one:

$$\alpha \neq 1.0 \tag{37}$$

Later, after examining pool motion during air clearing, it will be found that:

$$\tau \neq \lambda_x^{0.5}$$

and

$$P_{SH} \neq P_P \neq \lambda_\rho \lambda_x$$

Consequently from (36); $A_{HOLES} \neq A_{PIPE}$ and from (35); $u \neq \lambda_x^{0.5}$ and from (34); $L \neq \lambda_x$.

3.2 POOL MOTION - AIR CLEARING

Once the water has cleared the quencher, compressed air begins to enter the pool. The expelled bubbles coalesce into larger bubbles as they begin to rise to the surface. In considering air clearing down one arm of the quencher (Figure 3-5), half the air in the S/RV pipe will flow through each arm and exit at a roughly choked flow through the mitigator holes. Air clearing time τ_{ac} can then be estimated using:

$$\frac{M_a}{2} = \int_o^{\tau_{ac}} \dot{m}_a \, dt \tag{38}$$

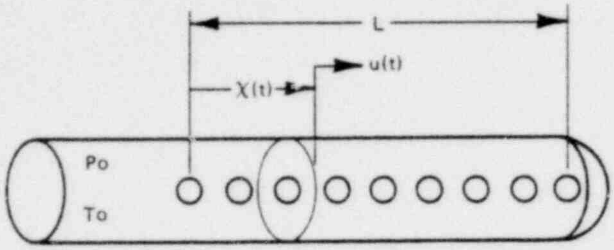

Figure 3-5. Hole Exposure Down Quencher Arm

where:

M_a = mass of air in pipe

$$\dot{m}_a = \frac{P_{SH}A(t)}{\sqrt{RT_{SH}}} \sqrt{\gamma\, g_c \left(\frac{2}{\gamma + 1}\right)^{\frac{\gamma+1}{\gamma-1}}} \quad \text{- Choked flow out uncovered holes} \tag{39}$$

$$A(t) = \frac{A_{HOLES}}{L}\, x(t) = \frac{A_{HOLES}}{L}\, Ut$$ - Air flow area assuming constant water velocity and uniform hole distribution

Inserting equation (39) into equation (38) and integrating gives an expression for air clearing time τ_{ac}:

$$\tau_{ac} = \left(\frac{M_a L \sqrt{RT_{SH}}}{P_{SH} \sqrt{\gamma\, g_c \left(\frac{2}{\gamma + 1}\right)^{\frac{\gamma+1}{\gamma-1}}}\; A_{HOLES}\, U} \right)^{1/2} \tag{40}$$

Using the typical values:

$$R = 53.3\left[\frac{lb_f - f}{lb_m - {}^oR}\right]$$ - Gas constant for air

$$\gamma = 1.4$$ - Ratio of specific heats for air (41)

$$L = 6\ [f]$$ - Quencher arm length of holes

$$P_{SH} = 340\ [psia]$$ - Driving air pressure

$$M_a = \frac{P_a^i}{RT_a^i} V_{PIPE} = 3.3\ [lbm]$$ - Mass of air in pipe, $P_a^i = 14.7$ [psia], $T_a^i = 595[°R]$, $V_{PIPE} = L \cdot A = (100)(0.499) = 49.9\ [f^3]$

(41)

$$T_{SH} = T_a^i \left(\frac{P_{SH}}{P_a^i}\right)^{(\gamma-1)/\gamma} = 1460[°R]$$ - Driving air temperature

$$A_{HOLES} \simeq 100\ [in.^2]$$ - Quencher arm hole area

$$U = 220\ [f/s]$$ - Equation (14)

then from (40):

$$\tau_{ac} \approx 0.01\ [sec] \quad (42)$$

Note that $x = U\tau_{ac} = 2.9$ [f]; thus, only about half the quencher arm is passing air by the time air clearing is over.

Pool velocities are influenced by bubble oscillations, which can be estimated from the Rayleigh bubble equation:

$$R\ddot{R} + \frac{3}{2}\dot{R}^2 = \frac{(P_B - P_P)}{\rho} \qquad (43)$$

The maximum R occurs when R = 0:

$$\dot{R}_{MAX} = \left(\frac{2}{3}\frac{P_B - P_P}{\rho}\right)^{1/2} \qquad (44)$$

The bubble pressure can be estimated assuming that the exiting air expands as a sphere (Figure 3-6):

$$\dot{R} = \frac{V^i}{4} = \frac{220}{4} = 55 \text{ [f/s]} \qquad (45)$$

Inserting equation (45) into equation (44) with P_P = 15 [psia] gives:

$$P_B = P_P + \frac{3}{2}\rho\dot{R}^2 = 76 \text{ [psia]} \qquad (46)$$

The exiting bubble will be higher in temperature due to compression. Assuming adiabatic compression:

$$T_B = T_a^i \left(\frac{P_R}{P_a^i}\right)^{(\gamma-1)/\gamma} = 951 \text{ [°R]} \qquad (47)$$

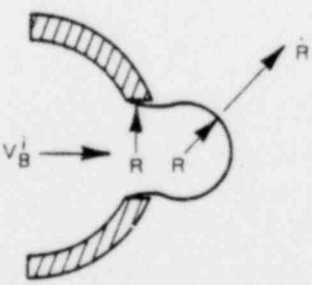

Figure 3-6. Initial Bubble Expansion

Physically, the pool will have initial velocities equal to the final water clearing velocities. As the bubbles enter, the velocity components increase due to bubble motion. Pool pressures vary from a high pressure at the bubble interface to roughly hydrostatic pressures at the pool boundaries. A temperature difference now exists between the compressed air bubble and the pool surface, so application of the energy equation is necessary during air clearing.

Reference values for pool motion during air clearing are:

$X = 6[f]$	- Quencher submergence	
$Y = 10\ [f]$	- Quencher-to-wall length	
$Z = 19\ [f]$	- Pool segment length	
$\tau = 0.01\ [sec]$	- Air clearing time	
$u^i = 110\ [f/s]$	- Final water clearing velocity x direction	
$v^i = 220\ [f/s]$	- Final water clearing velocity, y direction	
$w^i = 0\ [f/s]$	- Final water clearing velocity, z direction	
$\Delta u = \Delta v = \Delta w = \dot{R}_{MAX} = 55\ [f/s]$	- Bubble velocity	(48)
$P^i = P_\infty = 14.7\ [psia]$	- Pool surface pressure	
$\Delta P = \rho g X + P_B = 80\ [psia]$	- Static head plus bubble pressure	
$\rho = 62.4\ [lb_m/f^3]$	- Water density	
$\mu = 2.10^{-5}\left[lb_f - s/f^2\right]$	- Water viscosity at 70°, 1 atm	
$f^x = -\rho g = -62.4\left[lb_f/f^3\right]$	- Gravitational body force	
$f^y = f^z = 0$		
$T^i = T_p = 520\ [^\circ R]$	- Pool temperature	

$\Delta T = T_B - T_P = 431$ [°R]	- Temperature difference for heat Conduction	
$e^i = C_v T_P = 520$ [B/lb$_m$]	- Initial pool-specific internal energy	
$\Delta e = C_v \Delta T = 431$ [B/lb$_m$]	- Maximum change in internal energy	
$K = 1.10^{-4} \left[\frac{B}{s - f°F}\right]$	- Conduction coefficient for water	
$\phi^i = \Delta\phi_{\text{WATER CLEARING}} = 0\left(2\mu\left(\frac{\Delta v}{X}\right)^2\right)_{\text{WATER CLEARING}}$	- Initial energy dissipation function is found from the final water clearing velocities	(48 Cont)
$= 0(0.087)\left[\frac{lb_f}{f^2 - s}\right]$		
$\phi^R = 0\left(2\mu\left(\frac{\Delta u}{X}\right)^2\right) = 0(0.005)\left[\frac{lb_f}{f^2 - s}\right]$	- Energy dissipation for bubble expansion	

Inserting reference values equation (48) into equations (8), (9), and (10), and neglecting terms two orders of magnitude less than the greatest gives:

Mass

$$(P_1P_3)u^o_x + v^o_y + (P_4P_7)w^o_z = 0 \tag{49}$$

Momentum

$$\begin{aligned}
&(P_2P_3)u^o_t\,\hat{i} + (P_2)v^o_t\,\hat{j} + (P_2P_4)w^o_t\,\hat{k} + \left[\left(P_1P_3^2\right)u^o + (P_1P_3P_5)\right]u^o_x\,\hat{i} \\
&+ \left[(P_1P_3)u^o + (P_1P_5)\right]v^o_x\,\hat{j} + \left[(P_1P_3P_4)\,u^o + (P_1P_4P_5)\right]w^o_x\,\hat{k} \\
&+ \left[(P_3)v^o + (P_3P_6)\right]u^o_y\,\hat{i} + \left[v^o + (P_6)\right]v^o_y\,\hat{j} + \left[(P_4)v^o + (P_4P_6)\right]w^o_y\,\hat{k} \\
&+ \left[(P_3P_4P_7)\right]w^o u^o_z\,\hat{i} + \left[(P_4P_7)\right]w^o v^o_z\,\hat{j} + \left[\left(P_4^2P_7\right)\right]w^o w^o_z\,\hat{k} \\
&+ (P_1P_8)\,P^o_x\,\hat{i} + (P_8)P^o_y\,\hat{j} + (P_7P_8)P^o_z\,\hat{k} = 0
\end{aligned} \tag{50}$$

Energy

$$(P_2)e_t^o + \left[(P_1P_3)u^o + (P_1P_5)\right] e_x^o + \left[v^o + (P_6)\right] e_y^o + \left[(P_4P_7)\right] w^o e_z^o = 0 \qquad (51)$$

where the resulting similarity parameters are:

$$P_1 = Y/X$$
$$P_2 = Y/\Delta v\tau$$
$$P_3 = \Delta u/\Delta v$$
$$P_4 = \Delta w/\Delta v$$
$$P_5 = u^i/\Delta v \qquad (52)$$
$$P_6 = v^i/\Delta v$$
$$P_7 = Y/Z$$
$$P_8 = \Delta P/\rho \Delta v^2$$

From the momentum equation gravitational and viscous forces are found to be unimportant relative to other terms. The mass equation shows that modeling the "z" direction is important for bubble motion. The energy equation shows that temperature differences and viscous heating may be neglected.

Again, to insure that the model and full scale systems are governed by the same equations, each similitude parameter must scale as 1.0. From P_1 and P_7, all reference lengths must scale the same, that is:

$$X \neq Y \neq Z \neq \lambda_x \tag{53}$$

Examining P_6, the initial velocity v^i must scale like Δv:

$$v^i \neq \Delta v \neq \dot{R}_{MAX} - v^i \tag{54}$$

Using the comparative property and equation (44) for $\dot{R}$ allows equation (54) to be written:

$$\Delta v \neq \dot{R}_{MAX} \neq \left(\frac{P_B - P_P}{\rho}\right)^{1/2} \tag{55}$$

At this point let us examine P_8:

$$P_8 = \Delta P/\rho\Delta v^2$$

Parameter P_8 is the "pressure coefficient"* and is the ratio of pressure to inertia forces. Another parameter of physical significant is the Froude number:

$$F_r = \frac{\Delta v^2}{gX} = \frac{\text{INERTIA FORCE}}{\text{GRAVITATIONAL FORCE}}$$

The Froude number need not be scaled since it was found that gravitational forces are negligible compared with pressure forces. One characteristic of pressure coefficient scaling is that pool pressure differences and velocities are related; both are uncontrollable under normal conditions. For example, P^o must scale as 1.0 between model and full scales

$$P - P^i \neq \Delta P \neq \rho\Delta v^2 \quad \text{(Pressure coefficient scaling)} \tag{56}$$

*Sometimes written $\Delta P/\frac{1}{2}\rho V^2$.

If, on the other hand, Froude scaling were important, velocity Δv^2 is related to ρX, which is a controllable parameter and (56) could be written:

$$P - P^i \neq \Delta P \neq \rho g X \neq \lambda_\rho \lambda_x \quad (57)$$

where gravity is the same in model and full scales. Because Froude scaling results in controllability, it will be incorporated in the scaling. Imposing Froude scaling reduces the flexibility of the scale model since it defines how the pressure difference (ΔP) scales, but it does not violate scaling laws and eliminates the problem of measuring ΔP in the model and full scales.

From equations (55) and (57) and the reference values for P^i and ΔP:

$$P - P_\infty \neq P_B + \rho g X \neq \rho \Delta v^2 \neq \lambda_\rho \lambda_x \quad (58)$$

From the comparative property it follows that:

$$P_B \neq \lambda_\rho \lambda_x \quad (59)$$

and also from equations (58), (59), and (55):

$$(v^i)^2 \neq \Delta v^2 \neq \dot{R}^2 \neq \frac{P_B - P_P}{\rho} \neq \lambda_x \quad (60)$$

Applying the comparative property to equations (58) and (59):

$$P_P \neq \lambda_\rho \lambda_x \quad (61)$$

Since $P_P \triangleq P_\infty + \rho g X$:

$$P_\infty \neq \lambda_\rho \lambda_x \quad (62)$$

Parameters P_3, P_4, P_5, and P_6 indicate that all reference velocities scale like Δv or:

$$v^i \neq u^i \neq \Delta u \neq \Delta v \neq \Delta w \neq \lambda_x^{0.5} \qquad (63)$$

Parameter P_2 scales reference time τ:

$$\tau \neq Y/\Delta v \neq \lambda_x^{0.5} \qquad (64)$$

Looking at the nondimensional parameters t^o, x^o, y^o, and z^o shows that since each scales as 1.0:

$$t \neq \tau \neq \lambda_x^{0.5} \qquad (65)$$

$$x \neq y \neq z \neq X \neq Y \neq Z \neq \lambda_x \qquad (66)$$

Similarly for u^o, v^o, and w^o, from equation (62) and the comparative property:

$$u \neq v \neq w \neq \lambda_x^{0.5} \qquad (67)$$

Applying equations (57) and (61) to P^o gives:*

$$P \neq \lambda_\rho \lambda_x \qquad (68)$$

Summarizing for pool motion during air clearing, lengths scale the same:

$$x, y, z, X, Y, Z \neq \lambda_x \qquad (69)$$

*Note: Without Froude scaling, P would scale like $P - P_\infty = P_B + \rho g X$; consequently, the bubble pressure P_B would have to be measured in both model and full scale to get a reference pressure difference.

Froude velocity scaling,

$$u, v, w, u^i, v^i, \dot{R} \neq \lambda_x^{0.5} \tag{70}$$

Pressure coefficient scaling of pressures,

$$P,\ P_\infty,\ P_B, P_P = \lambda_\rho \lambda_x \tag{71}$$

Time scales as $Y/\Delta v$,

$$t \neq \tau \neq \lambda_x^{0.5} \tag{72}$$

3.3 BOUNDARY CONDITIONS

There are two boundary conditions for pool motion: a nonflow condition at the torus wall and a flow condition at the pool surface. A fixed bounding surface can be described by the functional equation:

$$F(x_1, x_2, x_3) = 0 \tag{73}$$

The nonflow restriction permits no fluid velocity component normal to surface F so:

$$\vec{V} \cdot \frac{\nabla F}{|\nabla F|} = 0 \qquad \text{on} \quad F \tag{74}$$

where $\nabla F/|\nabla F|$ is the unit vector normal to the surface. Unit normal vectors are the same in any scale; therefore, equation (73) can be written in non-dimensional form as:

$$\left\{ \left[u^o \left(\frac{\Delta u}{\Delta v}\right) + \left(\frac{u^i}{\Delta v}\right) \right] \hat{i} + \left[v^o + \left(\frac{v^i}{\Delta v}\right) \right] \hat{j} + \left[w^o \left(\frac{\Delta w}{\Delta v}\right) + \left(\frac{w^i}{\Delta v}\right) \right] \hat{k} \right\} \cdot \frac{\nabla^o F^o}{|\nabla^o F^o|} = 0 \qquad \text{on } F \tag{75}$$

All coefficients of equation (75) appeare' f· ·e governing differential equations, so no new similarity parameters ... introduced.

The boundary between two moving fluids* ..ch as the pool surface) or a fluid and a nonrigid, flexible, or moving solid can be classified as a moving nonflow boundary. Such a boundary can be described by the functional equation:

$$G(x_1, x_2, x_3, t) = 0 \tag{76}$$

The normal and tangential velocity components at a point on the bounding surface G are given by:

$$V_n = \vec{V} \cdot \frac{\nabla G}{|\nabla G|} \qquad \text{on } G \tag{77}$$

$$V_t = \vec{V} \cdot \left[\frac{\nabla G}{|\nabla G|} \times \frac{\vec{V}}{|\vec{V}|} \times \frac{\nabla G}{|\nabla G|} \right] \qquad \text{on } G \tag{78}$$

The functional equation must also satisfy the condition that:

$$\frac{DG}{Dt} = 0 \tag{79}$$

To nondimensionalize V_n and V_t coefficients are introduced that function as direction cosines $V_n = \vec{V} \cdot \vec{\beta}$; $V_t = \vec{V} \cdot \vec{\zeta}$. Nondimensionalizing equations (77), (78), and (79) then:

$$\beta_1 \left[\left(\frac{\Delta u}{\Delta v}\right) u^o + \left(\frac{u^i}{\Delta v}\right) \right] + \beta_2 \left[v^o + \left(\frac{v^i}{\Delta v}\right) \right] + \beta_3 \left[\left(\frac{\Delta w}{\Delta v}\right) w^o + \left(\frac{w^i}{\Delta v}\right) \right]$$

$$= \left\{ \left[\left(\frac{\Delta u}{\Delta v}\right) u^o + \left(\frac{u^i}{\Delta v}\right) \right] \hat{i} + \left[v^o + \left(\frac{v^i}{\Delta v}\right) \right] \hat{j} + \left[\left(\frac{\Delta w}{\Delta v}\right) w^o + \left(\frac{w^i}{\Delta v}\right) \right] \hat{k} \right\} \cdot \frac{\nabla^o G^o}{|\nabla^o G^o|} \quad \text{on } G \tag{80}$$

---

*Surface tension has been ignored.

$$\zeta_1 \left[\left(\frac{\Delta u}{\Delta v} \quad u^o + \left(\frac{u^i}{\Delta v}\right)\right] + \zeta_2 \left[v^o + \left(\frac{v^i}{\Delta v}\right) \right] + \zeta_3 \left[\left(\frac{\Delta w}{\Delta v}\right) w^o + \left(\frac{w^i}{\Delta v}\right) \right]$$

$$= \left\{ \left[\left(\frac{\Delta u}{\Delta v}\right) u^o + \left(\frac{u^i}{\Delta v}\right) \right] \hat{i} + \left[v^o + \left(\frac{v^i}{\Delta v}\right) \right] \hat{j} + \left[\left(\frac{\Delta w}{\Delta v}\right) w^o + \left(\frac{w^i}{\Delta v}\right) \right] \hat{k} \right\} \tag{81}$$

$$\cdot \left[\frac{\nabla^o G^o}{|\nabla^o G^o|} X \frac{\vec{V}^o}{|\vec{V}^o|} X \frac{\nabla G^o}{|\nabla^o G^o|} \right] \qquad \text{on } G$$

$$\left(\frac{Y}{\Delta v \tau}\right) G_t^o + \left[\left(\frac{\Delta u}{\Delta v}\right) u^o + \left(\frac{u^i}{\Delta v}\right) \right] \left(\frac{Y}{X}\right) G_x^o + \left[v^o + \left(\frac{v^i}{\Delta v}\right) \right] G_y^o$$

$$+ \left[\left(\frac{\Delta w}{\Delta v}\right) w^o + \left(\frac{w^i}{\Delta v}\right) \right] \left(\frac{Y}{Z}\right) G_z^o = 0 \tag{82}$$

All parameters were previously derived from the governing equations except $\vec{\beta}$ and $\vec{\zeta}$. Since these act as direction cosines, modeling $\vec{\beta}$ and $\vec{\zeta}$ means angles between model and full scales must be preserved in the pool.

3.4 SUPPRESSION POOL AIR SPACE

As water and air enter the pool, the top air space becomes compressed. The reference time was determined for air and water clearing; 0.09 + 0.01 = 0.1 sec. Comparing the reference time with the time for an acoustic wave to travel through the top air space, using a length of 15 feet and an acoustic speed of 1200 fps, only 0.0125 sec would be required for disturbances to pass through the air space; thus, the top air can be treated as a node.

The governing equations for nodal systems exclude momentum but require a state equation:

Mass

$$\dot{m}_{IN} - \dot{m}_{OUT} = \frac{dM}{dt} \tag{83}$$

Energy

$$P\frac{dV}{dt} + q_{OUT} - q_{IN} + (\dot{m}h_o)_{OUT} - (\dot{m}h_o)_{IN} + \frac{dE}{dt} = 0 \tag{84}$$

State

$$P\forall = MRT \tag{85}$$

More complex equations of state might be used, but air behaves similarly to an ideal gas.

Introducing the nondimensional parameters:

$$\dot{m}^{o} = (\dot{m} - \dot{m}^{i})/\Delta\dot{m}$$

$$M^{o} = (M - M^{i})/\Delta M$$

$$P^{o} = (P - P^{i})/\Delta P$$

$$V^{o} = (V - V^{i})/\Delta V$$

$$E^{o} = (E - E^{i})/\Delta E \qquad (86)$$

$$T^{o} = (T - T^{i})/\Delta T$$

$$q^{o} = (q - q^{i})/\Delta q$$

$$(\dot{m}h_{o})^{o} = \left[\dot{m}h_{o} - (\dot{m}h_{o})^{i}\right]/\Delta(\dot{m}h_{o})$$

$$t^{o} = t/\tau$$

The resulting normalized equations are:

<u>Mass</u>

$$(\dot{m}^{o}\Delta\dot{m} + \dot{m}^{i})_{IN} - (\dot{m}^{o}\Delta\dot{m} + \dot{m}^{i})_{OUT} = \left(\frac{\Delta m}{\tau}\right) M_{t}^{o} \qquad (87)$$

<u>Energy</u>

$$(P^{o}\Delta P + P^{i}) \left(\frac{\Delta V}{\tau}\right) V_{t}^{o} + (q^{o}\Delta q + q^{i})_{OUT} - (q^{o}\Delta q + q^{i})_{IN}$$

$$+ \left[(\dot{m}h_{o})^{o}\Delta(\dot{m}h_{o}) + (\dot{m}h_{o})^{i}\right]_{OUT} - \left[(\dot{m}h_{o})^{o}\Delta(\dot{m}h_{o}) + (\dot{m}h_{o})^{i}\right]_{IN} \qquad (88)$$

$$+ \left(\frac{\Delta E}{\tau}\right) E_{t}^{o} = 0$$

State

$$(P^o \Delta P + P^i)(V^o \Delta V + V^i) = (M^o \Delta M + M^i)\ R(T^o \Delta T + T^i) \tag{89}$$

Reference values for top air are:

$R = 53.3 \left[\frac{lb_f - f}{lb_m - {}^\circ R}\right]$ - Gas constant - air

$\tau = 0.1$ [sec] - Air + water clearing time

$\gamma = 1.4$ - Ratio of specific heats

$P^i = P_\infty = 14.7$ [psia] - Initial air pressure

$V^i = 11700$ [f^3] - Initial air volume

$E^i = \frac{P^i \forall^i}{\gamma - 1} = 79584$ [B] - Ideal gas energy

$M^i = \frac{P^i \forall^i}{RT^i} = 860$ [lbm] - Ideal gas mass (90)

$T^i = 540$ [°R] - Initial air temperature

$\forall^R = \forall^i - \forall_{PIPE} = 11650$ [f^3] - Air volume reduces with S/RV actuation

$P^R = P^i \left(\frac{\forall^i}{\forall^R}\right)^\gamma = 14.8$ [psia] - Adiabatic compression of ideal gas

$T^R = T^i \left(\frac{\forall^i}{\forall^R}\right)^{\gamma - 1} = 541$ [°R] - Adiabatic compression of ideal gas

$E^R = \frac{P^R \forall^R}{\gamma - 1} = 79783$ [B] - Ideal gas energy

$$\left.\begin{aligned} \dot{m}^i_{IN} = \dot{m}^i_{OUT} = \Delta\dot{m}_{IN} = \Delta\dot{m}_{OUT} = 0 \\ (\dot{m}h_o)^i_{IN} = (\dot{m}h_o)^i_{OUT} = \Delta(\dot{m}h_o)_{IN} = \Delta(\dot{m}h_o)_{OUT} = 0 \end{aligned}\right\}$$ - No mass flow into top air

$q^i_{IN} = q^i_{OUT} = \Delta q_{IN} = 0$ - No heat conduction except out

$\Delta q_{OUT} = HA\Delta T = 6.1 \left[\frac{B}{S}\right]$ - Where $H = 10 \left[\frac{B}{hr \quad f^2 \quad {}^oF}\right]$ Natural convection (90 Cont)

$A = 2203 \; [f^2]$ Surface area of top air

$\Delta T = 541 - 540 = 1[°F]$

Inserting equation (90) into equations (87), (88), and (89) results in the following, after eliminating terms two orders of magnitude smaller than the greatest:

Mass

(No contribution)

Energy

$$(P_{12})V^o_t + E^o_t = 0 \qquad (91)$$

State

$$(P_{13}) = 1.0 \qquad (92)$$

To scale the top air, P_{12} and P_{13} must be scaled as 1.0:

$$P_{12} = \frac{P^i \Delta V}{\Delta E} = (\gamma - 1) \Big/ \left\{ \left(1 - \frac{V^i}{V_{PIPE}}\right)^{\gamma} - \left[\left(1 - \frac{V^i}{V_{PIPE}}\right)^{\gamma} - 1\right] \frac{V^i}{V_{PIPE}} \right. \qquad (93)$$

$$P_{13} = \frac{P^i V^i}{M^i R T^i} \triangleq 1.0 \qquad (94)$$

Thus, the γ in the model must be the same as for full scale, and the top air initial volume must scale as pipe volume. The bubble formation analysis will show that to give a large enough bubble, pipe volume scales geometrically (i.e., λ_x^3),

$$\forall^i \neq \forall_{PIPE} \neq \lambda_x^3 \tag{95}$$

Since top air $\forall°$ and $E°$ must be the same in both scales, it follows as a consequence of geometric volume scaling and $P_\infty \neq \lambda_\rho \lambda_x$ for pool motion that:

$$\forall \neq \Delta\forall \neq \forall^i \neq \forall_{PIPE} \neq \lambda_x^3 \tag{96}$$

$$E \neq \Delta E \neq E^i \neq P^i \forall_{PIPE} \neq \lambda_\rho \lambda_x^4 \tag{97}$$

3.5 BUBBLE FORMATION

Consider a typical quencher hole (Figure 3-7). The effects of adjacent bubbles and pressure superpositions are neglected, and the bubble is assumed to act as a node. A nodal analysis implies the momentum equation may be neglected. The momentum equation can be examined for a simple geometry to show that no similarity parameters are neglected.

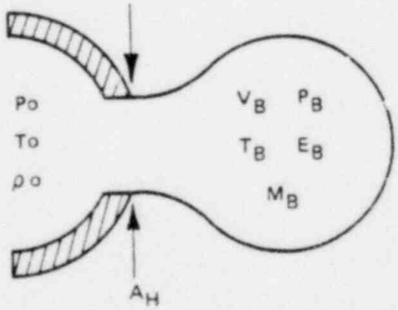

Figure 3-7. Bubble Formation From a Typical Quencher Hole

The mass, energy, and state equations are nondimensionalized as they are for top air. Reference values for bubble formation are:

$$R = 53.3 \left[\frac{lb_f - f}{lb_m - {}^\circ R}\right] \quad \text{- Air gas constant}$$

$$\gamma = 1.4 \quad \text{- Ratio of specific heats}$$

$$\tau = 0.01 \text{ [sec]} \quad \text{- Air clearing time}$$

$$P^i = P_{SH} = 340 \text{ [psia]} \quad \text{- Upstream stagnation pressure}$$

$$T^i = T_{SH} = T_a^i \left(\frac{P_{SH}}{P_a^i}\right)^{(\gamma-1)/\gamma} = 1460 \text{ [°R]} \quad \text{- Upstream stagnation temperature}$$

$$\dot{m}_{IN}^i = \frac{P_{SH} A_H}{RT_{SH}} V^i = 0.117 \text{ [lbm/s]} \quad \text{- Initial mass flow rate } (A_H = 0.0008454 [f^2]) \tag{98}$$

$$h_o = RT_{SH}\, \gamma/(\gamma-1) = 350.1 \text{ [B/lbm]} \quad \text{- Ideal stagnation enthalpy}$$

$$(\dot{m}h_o)_{IN}^i = m_{IN}^i h_o = 41 \text{ [B/s]} \quad \text{- Initial enthalpy flux}$$

$$M^i = V^i = E^i = \dot{m}_{OUT}^i = (\dot{m}h_o)_{OUT}^i = q_{IN}^i = q_{OUT}^i = 0$$

$$P^R = P_B = 76 \text{ [psia]} \quad \text{- Bubble pressure}$$

$$T^R = T_B = T_a^i \left(\frac{P_B}{P_a^i}\right)^{(\gamma-1)/\gamma} = 951 \text{ [°R]} \quad \text{- Bubble temperature}$$

$$\dot{m}_{IN}^R = \frac{P_{SH} A_H}{\sqrt{RT_{SH}}} \sqrt{g_c\, \gamma \left(\frac{2}{\gamma + 1}\right)^{\frac{\gamma+1}{\gamma-1}}}$$

$$= 0.5765 \text{ [lbm/s]} \quad \text{- Choked flow}$$

$$M^R = \dot{m}^R_{IN}\tau = 0.005765 \text{ [lbm]}$$ - Reference mass

$$(\dot{m}h_o)^R_{IN} = \dot{m}^R_{IN} \cdot h_o = 201.8 \text{ [B/S]}$$ - Reference enthalpy flux

$$\forall^R = \frac{M^R R T^R}{P^R} = 0.0267 \text{ [f}^3\text{]}$$ - Reference volume

$$E^R = \frac{P^R \forall^R}{\gamma - 1} = 0.94 \text{ [B]}$$ - Ideal energy

$$q^R_{OUT} = HA(T_B - T_P) = 5.16 \text{ [B/S]}$$ - Reference heat loss where (98 Cont)

$$H = 100\left[\frac{B}{hr\ f^2\ °F}\right]$$ Forced convection heat transfer coefficient

$$A = 4\pi\left(\frac{3}{4}\frac{\forall^R}{\pi}\right)^{2/3}$$ Surface area of sphere

$$T_B - T_P = 951 - 520 = 431\ °F$$ Temperature difference

$$\dot{m}^R_{OUT} = (\dot{m}h_o)^R_{OUT} = q^R_{IN} = 0$$

Inserting equation (98) into equations (87), (88), and (89) results in the following, after eliminating terms two orders of magnitude less than the greatest:

Mass

$$(P_{14})\dot{m}^o_{IN} + 1 = (P_{15})M^o_t \quad (99)$$

Energy

$$\left[(P_{16})P^o + (P_{17})\right] V^o_t - (P_{18})(\dot{m}h_o)^o - (P_{19}) + E^o_t = 0 \quad (100)$$

State

$$(P_{20})P^{o}\forall^{o} + (P_{21})V^{o} = (P_{22})M^{o}T^{o} + M^{o} \tag{101}$$

Again, to insure that the same differential equation governs both model and full scales, each parameter "P" must scale as 1.0:

$$P_{14} = \frac{\Delta\dot{m}_{IN}}{\dot{m}^{i}_{IN}} = \frac{\dot{m}^{R}_{IN}}{\dot{m}^{i}_{IN}} - 1$$

$$P_{15} = \frac{\Delta M}{\dot{m}^{i}_{IN}\tau} \triangleq 1.0$$

$$P_{16} = \frac{\Delta P \Delta V}{\Delta E} = (\gamma - 1)\left(1 - \frac{P_{SH}}{P_{B}}\right)$$

$$P_{17} = \frac{P^{i}\Delta V}{\Delta E} = (\gamma - 1)\left(\frac{P_{SH}}{P_{B}}\right)$$

$$P_{18} = \frac{\Delta(\dot{m}h_{o})_{IN}\tau}{\Delta E} = \left(1 - \frac{\dot{m}^{i}_{IN}}{\dot{m}^{R}_{IN}}\right)\gamma\left(\frac{P_{SH}}{P_{B}}\right)^{(\gamma-1)/\gamma} \tag{102}$$

$$P_{19} = \frac{(\dot{m}h_{o})^{i}_{IN}\tau}{\Delta E} = \left(\frac{\dot{m}^{i}_{IN}}{\dot{m}^{R}_{IN}}\right)\gamma\left(\frac{P_{SH}}{P_{B}}\right)^{(\gamma-1)/\gamma}$$

$$P_{20} = \frac{\Delta P \Delta \forall}{\Delta M R T^{i}} = \left(1 - \frac{P_{SH}}{P_{B}}\right)\left(\frac{P_{B}}{P_{SH}}\right)^{(\gamma-1)/\gamma}$$

$$P_{21} = \frac{P^{i}\Delta \forall}{\Delta M R T^{i}} = \left(\frac{P_{SH}}{P_{B}}\right)\left(\frac{P_{B}}{P_{SH}}\right)^{(\gamma-1)/\gamma}$$

$$P_{22} = \frac{\Delta T}{T^{i}} = \left(\frac{P_{B}}{P_{SH}}\right)^{(\gamma-1)/\gamma} - 1$$

Examining P_{22} first shows:

$$\left(\frac{P_B}{P_{SH}}\right)^{(\gamma-1)/\gamma} \neq 1.0 \qquad (103)$$

From (103) and P_{21}:

$$\left(\frac{P_{SH}}{P_B}\right) \neq 1.0 \qquad (104)$$

To satisfy both equations (103) and (104), the exponent $\lambda - 1/\lambda$ must scale as 1.0, or:

$$\gamma_m = \gamma_F \qquad (105)$$

Parameter P_{14} indicates that the reference to initial mass flow rate ratio must scale as 1.0, from equation (98):

$$\frac{\dot{m}^R_{IN}}{\dot{m}^i_{IN}} = \frac{\sqrt{RT_{SH}}}{v^i} \sqrt{g_c \, \gamma \left(\frac{2}{\gamma + 1}\right)^{\frac{\gamma+1}{\gamma-1}}} \neq 1.0 \qquad (106)$$

but from equation (105) $\gamma \neq 1$ and equation (106) is satisfied if:

$$\frac{\sqrt{RT_{SH}}}{v^i} \neq 1.0 \qquad (107)$$

The initial velocity was determined from pool motion for air clearing to scale as $\lambda_x^{0.5}$. To satisfy equation (107) and thereby scale P_{14}:

$$RT_{SH} \neq \lambda_x \qquad (108)$$

All remaining parameters are scaled by equations (104), (105), and (106).

Again, each nondimensionable variable must scale as 1.0. Conside. nondimensional volume $\forall^{o} = \forall/\forall^{R} \neq 1.0$:

$$\forall \neq \forall^{R} = \frac{P_{SH}A_{H}}{\sqrt{RT_{SH}}} \sqrt{g_c \gamma \left(\frac{2}{\gamma + 1}\right)^{\frac{\gamma+1}{\gamma-1}}} \frac{\tau RT_{B}}{P_{B}} \neq A_{H}\tau\sqrt{RT_{SH}} \tag{109}$$

As a consequence of geometric pool scaling, the bubble volume must scale geometrically; that is $\forall = \lambda_x^3$. Remembering time τ scales as $\lambda_x^{0.5}$ and from equation (108) $RT_{SH} \neq \lambda_x$, then solving equation (109) for A_H:

$$A_H \neq \frac{\forall^{R}}{\tau\sqrt{RT_{SH}}} \neq \lambda_x^2 \tag{110}$$

From P^{o}:

$$P - P_{SH} \neq P_B - P_{SH} \tag{111}$$

From equation (104) and pool motion for air clearing, $P_B \neq P_{SH} \neq \lambda_\rho \lambda_x$, then applying the comparative property to (111):

$$P \neq P_B \neq P_{SH} \neq \lambda_\rho \lambda_x \tag{112}$$

Examining $\dot{m}_{IN}^{o}$:

$$\dot{m}_{IN} - \dot{m}_{IN}^{i} \int \dot{m}_{IN}^{R} - \dot{m}_{IN}^{i} \tag{113}$$

But from P_{14} and equation (98) $\dot{m}_{IN}^{R} \neq \dot{m}_{IN}^{i} \neq P_{SH}A_{H}V^{i}/RT_{SH} \int \lambda_\rho \lambda_x^{2.5}$.

Again the comparative property applies to equation (113) giving:

$$\dot{m}^o_{IN} \neq \dot{m}^i_{IN} \neq \dot{m}^R_{IN} \neq \lambda_\rho \lambda_x^{2.5} \tag{114}$$

M^o and E^o follow directly from equation (98):

$$M \neq M^R \neq \frac{P_{SH} A_H \tau}{\sqrt{RT_{SH}}} \neq \lambda_\rho \lambda_x^3 \tag{115}$$

$$E \neq E^R \neq \frac{P_{SH} \forall^R}{\gamma - 1} \neq \lambda_\rho \lambda_x^4 \tag{116}$$

Finally, from P_{18} and P_{19}, $(\dot{m}h_o)^i_{IN}$ will scale like $\Delta(\dot{m}h_o)_{IN}$ and from equation (98):

$$(\dot{m}h_o)_{IN} \neq (\dot{m}h_o)^i_{IN} \neq (\dot{m}h_o)^R_{IN} \neq \frac{E^R}{\tau} \neq \frac{P^R \forall^R}{(\gamma - 1)\tau} \neq \frac{P_{SH} A_H V^i \gamma}{\gamma - 1}$$

$$\neq \frac{P_{SH} A_H \gamma \sqrt{RT_{SH}}}{\gamma - 1} \neq \lambda_\rho \lambda_x^{3.5} \tag{117}$$

Summarizing for bubble formation, pressures scale like bubble pressure for pool motion:

$$P \text{ , } P_{SH} \text{ , } P_B \neq \lambda_\rho \lambda_x \tag{118}$$

volume scales geometrically so that pool motion is modeled:

$$\forall, \forall^R \neq \lambda_x^3 \tag{119}$$

to model bubble pressue due to compression:

$$\gamma \neq 1.0 \tag{120}$$

to model mass flow into the bubble:

$$RT_{SH} \neq \lambda_x \tag{121}$$

As a result of equations (118) through (121), hole size, bubble mass, energy, and enthalpy flux model as:

$$
\begin{aligned}
&A_H \neq \lambda_x^2 \\
&M\ ,\ M^R \neq \lambda_\rho \lambda_x^3 \\
&E,\ E^R \neq \lambda_\rho \lambda_x^4 \\
&\dot{m}_{IN}\ ,\ \dot{m}_{IN}^i\ ,\ \dot{m}_{IN}^R \neq \lambda_\rho \lambda_x^{2.5} \\
&(\dot{m}h_o)_{IN},\ (\dot{m}h_o)_{IN}^R\ ,\ (\dot{m}h_o)_{IN}^i \neq \lambda_\rho \lambda_x^{3.5}
\end{aligned} \tag{122}
$$

Since the exiting air reaches sonic velocity, air momentum may be important. To examine this, consider the simplified one-dimensional model pictured in Figure 3-8. Writing the integral form of the momentum equation:

$$\sum \vec{F} g_c = \frac{d}{dt} \int_{C.V.} \rho \vec{V} \, dV + \oint_{C.S.} \rho \ (\vec{V} \cdot d\vec{A}) \ \vec{V} \tag{123}$$

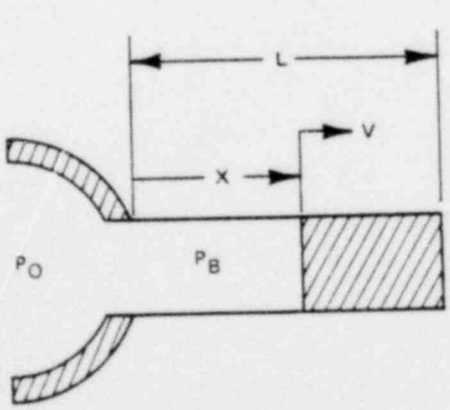

Figure 3-8. One-Dimensional Momentum Model

... applying equation (123) to the simple model shown in Figure 3-8:

$$(P_B - P_P)A_H g_c = \frac{d}{dt}\left[\rho_B \dot{x} A_H + \rho_P \dot{x} (L - x)A_H\right] + \rho_P \dot{x}^2 A_H - \rho_B \dot{x}^2 A_H$$

or

$$(P_B - P_P)g_c = \rho_B x\ddot{x} + \rho_P(L - x)\ddot{x} \qquad (124)$$

Nondimensionalizing using (assume constant ρ_P and P_P):

$$P_B^o = \frac{P_B - P_B^i}{\Delta P_B}$$

$$\rho_B^o = \frac{\rho_B - \rho_B^i}{\Delta\rho_B} \qquad (125)$$

$$x^o = x/L$$

$$t^o = t/\tau$$

Inserting equation (125) into equation (124) gives the following:

$$(P_{23})P_B^o + (P_{24}) = \left[(P_{25})\rho_B^o + (P_{26})\right] x^o x_{tt}^o + (1 - x^o)x_{tt}^o \qquad (126)$$

where:

$$P_{23} = \frac{\Delta P_B g_c}{\rho_P (L/\tau)^2} \qquad \text{- Pressure coefficient}$$

$$P_{24} = \frac{\left(P_B^i - P_P\right) g_c}{\rho_P (L/\tau)^2} \qquad \text{- Pressure coefficient} \qquad (127)$$

$$P_{25} = \Delta\rho_B/\rho_P \qquad \text{- Air momentum term}$$

$$P_{26} = \rho_B^i/\rho_P \qquad \text{- Air momentum term}$$

Reference values are:

$$
\begin{aligned}
& P_B^i = P_{SH} = 340 \text{ [psia]} && - \text{ Initial bubble pressure} \\
& T_B^i = T_{SH} = 1460 \text{ [°R]} && - \text{ Initial bubble temperature} \\
& \rho_B^i = P_B^i/RT_B^i = 0.629 \text{ [lbm/f}^3\text{]} && - \text{ Ideal density} \\
& P_B^R = 76 \text{ [psia]} && - \text{ Bubble reference pressure} \\
& T_B^R = 951 \text{ [°R]} && - \text{ Bubble reference temperature} \\
& \rho_B^R = P_B^R/RT_B^R = 0.216 \text{ [lbm/f}^3\text{]} && - \text{ Ideal density} \\
& \rho_P = 62.4 \text{ [lbm/f}^3\text{]} && - \text{ Pool density} \\
& P_P = 15 \text{ [psia]} && - \text{ Pool pressure} \\
& L/\tau = \begin{cases} V^i = 220 \text{ [f/s]} & - \text{ Initial velocity} \\ C = \sqrt{\gamma g_c R T_B^R} = 1512 \text{ [f/s]} & - \text{ Sonic velocity} \end{cases}
\end{aligned}
\qquad (128)
$$

Depending on what reference velocity is used, the momentum of air (terms P_{25} and P_{26}) may or may not be important when compared with pressure.

$$P_{23} = -0.638 \text{ (Initial Flow)} \qquad -0.014 \text{ (Sonic flow)}$$

$$P_{24} = 0.789 \text{ (Initial Flow)} \qquad 0.016 \text{ (Sonic flow)} \tag{129}$$

$$\left.\begin{array}{l} P_{25} = -0.009 \\ \\ P_{26} = 0.014 \end{array}\right\} \text{Momentum Terms}$$

Momentum terms are small initially. During the first part of bubble formation, momentum is unimportant, but as the bubble grows and velocity becomes sonic, air momentum becomes important. Nonetheless, using the already determined scale parameters confirms that momentum is scaled:

$$P_{23} = \frac{\Delta P_B g_c}{\rho_P (L/\tau)^2} \quad \neq \frac{\lambda_\rho \lambda_x}{\lambda_\rho \lambda_x} = 1.0$$

$$P_{24} = \frac{\left(P_B^i - P_P\right) g_c}{\rho_P (L/\tau)^2} \neq \frac{\lambda_\rho \lambda_x}{\lambda_\rho \lambda_x} = 1.0 \tag{130}$$

$$P_{25} = \frac{\Delta \rho_B}{\rho_P} = \left(\frac{P_B}{RT_B} - \frac{P_{SH}}{RT_{SH}}\right) \cdot \frac{1}{\rho_P} \neq \frac{\lambda_\rho \lambda_x}{\lambda_\rho \lambda_x} = 1.0$$

$$P_{26} = \frac{\rho_B^i}{\rho_P} = \frac{P_{SH}}{RT_{SH} \rho_P} \neq \frac{\lambda_\rho \lambda_x}{\lambda_\rho \lambda_x} = 1.0$$

3.6 QUENCHER MODEL

At this point the quencher hole size and diameter have been scaled. It is important to see how the quencher arm length and hole spacing should scale so that the air bubble enters the pool in a geometrically scaled fashion. Consider Figure 3-5, which shows the water exiting the quencher as (equation 15):

$$V = \left(\frac{2(P_{SH} - P_P)}{\rho}\right)^{1/2} \cdot \left(\frac{A_{PIPE}}{A_{HOLES}}\right) \tag{131}$$

Mass conservation in the pipe gives the centerline velocity:

$$u(t) = \left(\frac{A_{HOLES}}{A_{PIPE}}\right) \left(1 - \frac{A(t)}{A_{HOLES}}\right) \cdot V \tag{132}$$

but for full scale the air flow area A(t) and length of quencher exposed to air flow x(t) is:

$$A(t) = A_{HOLES} \cdot \frac{x(t)}{L} \tag{133}$$

and integrating (132):

$$x(t) = \int_0^t u(t)\, dt \tag{134}$$

Combining equations (131) through (134) gives the following relation for x(t):

$$x(t) = \int_0^t \left(1 - \frac{x(t)}{L}\right) \left(\frac{2}{\rho} (P_{SH} - P_P)\right)^{1/2} dt \tag{135}$$

Pressure and time scaling have already been determined (namely $P_{SH} \neq P_P \neq \lambda_\rho \lambda_x$; $t \neq \lambda_x^{0.5}$). To model pool motion x(t) must scale geometrically. The only solution to equation (134) is:

$$L \neq \lambda_x \tag{136}$$

which is consistent with geometric pool scaling.

Pressure losses across an orifice, such as a quencher hole, are obtained from Figure 3-9:

$$P_{SH} - P_P = \frac{K}{2g_c} \rho V^2 \tag{137}$$

For water flow:

$$V \neq \lambda_x^{0.5}$$

$$\rho \neq \lambda_\rho \tag{138}$$

$$P \neq \lambda_\rho \lambda_x$$

thus, from equation (137) it follows that $K \neq 1.0$.

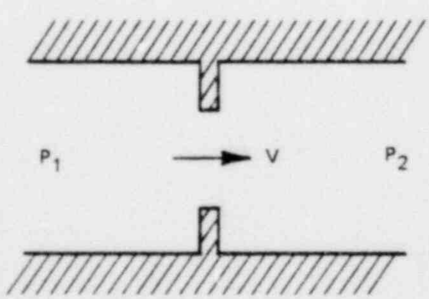

Figure 3-9. Orifice Pressure Loss

For air flow:

$$V = \sqrt{RT} = \lambda_x^{0.5}$$

$$\rho \neq \frac{P}{RT} \neq \lambda_\rho \tag{139}$$

$$P \neq \lambda_\rho \lambda_x$$

Thus, from equation (137) it again follows that $K \neq 1.0$.

Since K is a function of orifice inside edge sharpness, it is important to keep the same sharpness between model and full scales so that $K_{MODEL} = K_{FULL}$.

3.7 PIPE FLOW

To give the correct driving pressure at the quencher, the acoustic phenomena (shock waves) and the compressible flow of air and steam through the S/RV line must be modeled. The pipe is assumed to be of constant area and the flow to be one-dimensional.

3.7.1 Moving Normal Shock

Whenever a moving normal shock occurs in a fluid, a moving boundary condition is introduced. Properties can be related across this moving boundary condition. Letting subscripts y and x refer to the shocked and undisturbed regions respectively, then mass, momentum and energy conservation equations for a moving normal shock are written as:

Mass

$$\rho_x(S - V_x) = \rho_y(S - V_y) \tag{140}$$

Momentum

$$\rho_x(S - V_x)^2 - \rho_y(S - V_y)^2 = P_y - P_x \tag{141}$$

Energy

$$\rho_x\rho_y(h_x - h_y) + \frac{1}{2}(P_y - P_x)(\rho_y - \rho_x) = 0 \tag{142}$$

where S is the shock speed.

Nondimensionalizing equations (140), (141), and (142) using:

$$\rho^o = \frac{\rho - \rho^i}{\Delta\rho}$$

$$V^o = \frac{V - V^i}{\Delta V}$$

$$P^o = \frac{P - P^i}{\Delta P} \qquad (143)$$

$$h^o = \frac{h - h^i}{\Delta h}$$

$$S^o = S/S^R$$

gives:

Mass

$$\Delta\rho \left\{ \left(\rho^o_x - \rho^o_y\right)\left(S^R S^o - V^i\right) - \Delta V\left(\rho^o_x V^o_x - \rho^o_y V^o_y\right) \right\} + \rho^i \Delta V \left[V^o_y - V^o_x \right] = 0 \qquad (144)$$

Momentum

$$\left(\rho^o_x \Delta\rho + \rho^i\right) \left[S^R S^o - \Delta V V^o_x - V^i \right]^2 - \left(\rho^o_y \Delta\rho + \rho^i\right) \left[S^R S^o - V^o_y \Delta V - V^i \right]^2$$

$$= \left(P^o_y - P^o_x\right) \Delta P \qquad (145)$$

Energy

$$\left(\rho^o_y \Delta\rho + \rho^i\right)\left(\rho^o_x \Delta\rho + \rho^i\right)\left[\left(h^o_x - h^o_y\right) \Delta h \right] + \frac{1}{2}\Delta P \Delta\rho \left(P^o_y - P^o_x\right)\left(\rho^o_y - \rho^o_x\right) = 0$$

$$(146)$$

Initial values are those before the shock (that is the x side), and reference values are those just behind the shock (the y side). Typical values for shock motion are:

$P^i = P^i_a = 14.7$ [psia] - Initial air pressure

$T^i = T^i_a = 595$ [°R] - Initial air temperature

$\rho^i = P^i/RT^i = 0.0667$ [lb_m/f^3] - Initial air density

$V^i = 0$ - Initial air velocity

$h^i = \frac{\gamma}{\gamma - 1} RT^i = 143$ [B/lb_m] - Ideal enthalpy

$P^R = 88.37$ [psia] - Shocked air pressure (Appendix A)

$$T^R = T^i_a \left(\frac{P^R}{P^i_a}\right)^{(\gamma-1)/\gamma} = 993 \text{ [°R]} \tag{147}$$

- Shocked air temperature (adabiatic) .

$\rho^R = P^R/RT^R = 0.2404$ [lb_m/f^3] - Shocked air density

$$V^R = V^i + \frac{\sqrt{\gamma g_c RT^i}\left(P^R/P^i - 1\right)}{\left(\frac{\gamma(\gamma - 1)}{2}\left[\left(\frac{\gamma + 1}{\gamma - 1}\right)\frac{P^R}{P^i} + 1\right]\right)^{1/2}}$$

- Shocked air velocity - Ideal gas Rankine - Hugoniot relationship, equals steam velocity V_S

$= 1860$ [f/s]

$h^R = \frac{\gamma}{\gamma - 1} RT^R = 238$ [B/lb_m] - Ideal enthalpy

$$S^R = V^i + \left(\frac{g_c}{2\rho^i}\left[(\gamma + 1)P^R + (\gamma - 1)P^i\right]\right)^{1/2}$$

- Ideal gas shock speed - Rankine-Hugoniot relationship

$= 2751$ [f/s]

Inserting values and eliminating terms two orders of magnitude less than the greatest gives:

Mass

$$\left(\rho_x^o - \rho_y^o\right)\left[S^o(P_{27}P_{28})\right] - (P_{27})\left[\rho_x^o V_x^o - \rho_y^o V_y^o\right] + \left[V_y^o - V_x^o\right] = 0 \qquad (148)$$

Momentum

$$\left(\rho_x^o(P_{27}) + 1\right)\left[S^o(P_{28}) - V_x^o\right]^2 - \left(\rho_y^o(P_{27}) + 1\right)\left[S^o(P_{28}) - V_y^o\right]^2 \qquad (149)$$

$$= (P_{29})\left(P_y^o - P_x^o\right)$$

Energy

$$\left(\rho_y^o + (1/P_{27})\right)\left(\rho_x^o(P_{27}) + 1\right)\left(h_x^o - h_y^o\right) + (P_{30})\left[P_y^o - P_x^o\right]\left[\rho_y^o - \rho_y^o\right] = 0 \qquad (150)$$

where each parameter must scale as one:

$$P_{27} = \frac{\Delta\rho}{\rho^i} = \frac{P_{SH-a}}{P_a^i}\left(\frac{P_a^i}{P_{SH-a}}\right)^{(\gamma-1)/\gamma} - 1$$

$$P_{28} = S^R/\Delta V = S^R/V_{SH-a} \qquad (151)$$

$$P_{29} = \frac{\Delta P}{\rho^i \Delta V^2} = \left(\frac{P_{SH-a}}{P_i^a} - 1\right)\frac{RT_a^i}{V_{SH-a}^2}$$

$$P_{30} = \frac{\Delta P}{2\rho^{i}\Delta h} = \left(\frac{P_{SH-a}}{P_a^i} - 1\right)\left(\frac{\gamma - 1}{\gamma}\right) \frac{1}{\left(\frac{P_{SH-a}}{P_a^i}\right)^{(\gamma-1)/\gamma-1}} \quad (151\ \text{Cont})$$

From P_{27} the reference pressure and the initial air pressure must scale the same (remembering $\gamma \neq 1$):

$$P_{SH-a} \neq P_a^i \quad (152)$$

Parameters P_{28} and P_{29} are automatically satisfied since from equation (147):

$$S^R \neq V_{SH-a} \neq \sqrt{RT_a^i} \quad (153)$$

3.7.2 Air Flow

The conservation equations for compressible one-dimensional flow in a straight rigid passage where wall shear is expressed by $\frac{f}{4}\rho\frac{V^2}{2}$ are given by:

Mass

$$\frac{\partial\rho}{\partial t} + u\frac{\partial\rho}{\partial x} + \rho\frac{\partial u}{\partial x} = 0 \quad (154)$$

Momentum

$$\rho\left(\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x}\right) + \frac{\partial P}{\partial x} + \rho\frac{f}{D_H}\frac{u^2}{2} = f^x \quad (155)$$

Energy

$$\rho\left(\frac{\partial e}{\partial t} + u\frac{\partial e}{\partial x}\right) + P\frac{\partial u}{\partial x} = q'''_{IN} - q'''_{OUT} + \rho\frac{f}{D_H}\frac{u^3}{2} + K\frac{\partial^2 T}{\partial x^2} \tag{156}$$

Nondimensionalizing using:

$$\rho^o = \frac{\rho - \rho^i}{\Delta\rho}$$

$$P^o = \frac{P - P^i}{\Delta P}$$

$$u^o = \frac{u - u^i}{\Delta u}$$

$$e^o = \frac{e - e^i}{\Delta e} \tag{157}$$

$$q'''^o = \frac{q''' - q'''^i}{\Delta q'''}$$

$$T^o = \frac{T - T^i}{\Delta T}$$

$$x^o = x/L$$

$$t^o = t/\tau$$

Inserting equation (157) into equations (154), (155), and (156) gives the normalized conservation equations:

Mass

$$\left(\frac{\Delta\rho}{\tau}\right)\rho^o_t + (u^o\Delta u + u^i)\left(\frac{\Delta\rho}{L}\right)\rho^o_x + (\rho^o\Delta\rho + \rho^i)\left(\frac{\Delta u}{L}\right)u^o_x = 0 \tag{158}$$

Momentum

$$\left(\rho^{o}\Delta\rho + \rho^{i}\right)\left[\left(\frac{\Delta u}{\tau}\right)u^{o}_{t} + (u^{o}\Delta u + u^{i})\left(\frac{\Delta u}{L}\right)u^{o}_{x}\right]$$

$$+ \left(\frac{\Delta P}{L}\right)P^{o}_{x} + (\rho^{o}\Delta\rho + \rho^{i})\left(\frac{f}{2D_{H}}\right)\left(u^{o}\Delta u + u^{i}\right)^{2} = f^{x} \tag{159}$$

Energy

$$(\rho^{o}\Delta\rho + \rho^{i})\left[\left(\frac{\Delta e}{\tau}\right) e^{o}_{t} + (u^{o}\Delta u + u^{i})\left(\frac{\Delta e}{L}\right) e^{o}_{x}\right] + (P^{o}\Delta P + P^{i})\left(\frac{\Delta u}{L}\right)u^{o}_{x}$$

$$= (q'''^{o}\Delta q''' + q'''^{i})_{IN} - (q'''^{o}\Delta q''' + q'''^{i})_{OUT} \tag{160}$$

$$+ (\rho^{o}\Delta\rho + \rho^{i})\left(\frac{f}{2D_{H}}\right)(u^{o}\Delta u + u^{i})^{3} + \left(\frac{K\Delta T}{L^{2}}\right) T^{o}_{xx}$$

Reference values for air flow are:

$\tau = 0.1$ [sec]	- Air plus water clearing times
$L = 100$ [f]	- Pipe length
$K = 0.0337 \left[\frac{B}{hr\text{-}f\text{-}^{\circ}F}\right]$	- Air conduction coefficient at 1000 °R.
$f/D_{H} = 0.03$ [1/f]	- Friction factor
$P^{i} = 14.7$ [psia]	- Initial air pressure
$T^{i} = 595$ [°R]	- Initial temperature

(161)

$\rho^i = P^i/RT^i = 0.0667\ [lb_m/f^3]$ - Ideal air density

$e^i = RT^i/(\gamma - 1) = 101.9\ [B/lb_m]$ - Ideal air energy

$P^R = P_{SH} = 340\ [psia]$ - Shock pressure (Appendix A)

$T^R = T^i\left(\frac{P_{SH}}{P^i}\right)^{(\gamma-1)/\gamma} = 1460\ [°R]$ - Adiabatic compression

$\rho^R = P^R/RT^R = 0.629\ [lb_m/f^3]$ - Ideal Air density

$e^R = RT^R/(\gamma - 1) = 250\ [B/lb_m]$ - Ideal air energy (161 Cont)

$u^R = 1860\ [f/s]$ - Shocked air velocity see equation (146)

$f^x = -\rho^R g = -27.9\left[\frac{lb_m}{f^2\text{-}s^2}\right]$ - Gravitational body force

$$q'''^R_{OUT} = \frac{HA\left(T^R - T_{PIPE}\right)}{V_{PIPE}} = \frac{H\ \ 4(T^R - T^i)}{D_H}$$ - Heat transfer to pipe wall

$$= 12.0\left[\frac{B}{s - f^3}\right]$$

$H = 10\ [B/hrf^2{}°F[$

$D_H = 9.564\ [in.]$

$u^i = q'''^i_{IN} = q'''^i_{OUT} = q'''^R_{IN} = 0$

Inserting reference values and eliminating terms two orders of magnitude less than the greatest gives:

Mass

$$(P_{31}P_{32})\rho^o_t + (P_{31})u^o\rho^o_x + \left[(P_{31})\rho^o + 1\right]u^o_x = 0 \qquad (162)$$

Momentum

$$\left[(P_{31}P_{32})\rho^{o} + (P_{32})\right] u_{t}^{o} + \left[(P_{31})\rho^{o} + 1\right] u^{o}u^{o}{}_{x} + (P_{33})\ P_{x}^{o} + \left[(P_{31}P_{35})\rho^{o} + (P_{35})\right] u^{o2} = 0 \tag{163}$$

Energy

$$\left[(P_{31}P_{32}P_{36})\rho^{o} + (P_{32}P_{36})\right] e_{t}^{o} + \left[(P_{31}P_{36})\rho^{o} + (P_{36})\right] u^{o}e_{x}^{o} + \left[(P_{33})\ P^{o} + (P_{34})\right] u_{x}^{o} = \left[(P_{31}P_{35})\rho^{o} + (P_{35})\right] u^{o3} \tag{164}$$

where six similarity parameters result that must scale as 1.0:

$$P_{31} = \frac{\Delta\rho}{\rho^{i}} = \left(\frac{P_{SH}}{P_{a}^{i}}\right)\left(\frac{P_{a}^{i}}{P_{SH}}\right)^{(\gamma-1)/\gamma} - 1$$

$$P_{32} = \frac{L}{\Delta u\tau} = \frac{L}{u^{R}\tau} \tag{165}$$

$$P_{33} = \frac{\Delta P}{\rho^{i}\Delta u^{2}} = \left(\frac{P_{SH}}{P_{a}^{i}} - 1\right)\left(\frac{RT_{a}^{i}}{\left(u^{R}\right)^{2}}\right)$$

$$P_{34} = \frac{P^i}{\rho^i \Delta u^2} = \frac{RT_a^i}{\left(u^R\right)^2}$$

$$P_{35} = \frac{fL}{2D_H} \qquad \text{(165 Cont)}$$

$$P_{36} = \frac{\Delta e}{\Delta u^2} = \frac{\left(RT_{SH} - RT_a^i\right)}{(\gamma - 1)\left(u^R\right)^2}$$

Heat conduction and gravitational forces are negligible in air flow modeling. To scale P_{31} (remembering $\gamma \neq 1.0$), it follows from shock scaling and bubble formation that:

$$P_{SH} \neq P_a^i \neq P_{SH - a} \neq \lambda_\rho \lambda_x \qquad (166)$$

Parameter P_{32} indicates how to "tune" the pipe length to make time τ scale consistently as $\lambda_x^{0.5}$, from equations (161) and (166), and remembering $RT \neq \lambda_x$:

$$L \neq \tau u^R \neq \lambda_x^{0.5} \sqrt{RT_a^i} \neq \lambda_x \qquad (167)$$

The pipe length scales geometrically.

Parameters P_{33}, P_{34}, and P_{36} are satisfied by previously determined scaling relationships. Parameter P_{35} is the friction scaling parameter. Remembering that volume scales geometrically, the cross sectional area is adjusted according to: $V = L \cdot A \neq \lambda_x^3$ it follows that $A \neq \lambda_x^2$ since $L \neq \lambda_x$ in equation (167) and from P_{35}:

$$f \neq \frac{D_H}{L} \neq 1.0 \tag{168}$$

Thus, the same friction factor is to be used in both scales.

3.7.3 Steam Flow

The mass, momentum, and energy equations for homogeneous steam-liquid flow are the same as given for air flow, which are equations (158), (159), and (160). Care must be taken in choosing reference values, however, since changes in phase can occur.

Reference values for steam flow are:

$f/D_H = 0.03$ [1/f]	- Pipe friction factor	
$K = 0.02$ [B/hr-f-°F]	- Steam conductivity at 100°F	
$L = 100$ [f]	- Pipe length	
$\tau = 0.1$ [sec]	- Water plus air cleaning time	
$P^i = 14.7$ [psia]	- Initial pressure in pipe	(169)
$T^i = 595$ [°R]	- Initial temperature in pipe	
$\rho^i = P^i/RT^i = 0.0667$ [lb_m/f^3]	- Initial density in pipe	
$e^i = RT^i/(\gamma-1) = 101.9$ [B/lb_m]	- Initial energy in pipe	
$P^R = P_s = 88.37$ [psia]	- Steam pressure (Appendix A)	
$T^R = T_s = 779$ [°R]	- Steam saturation temperature at P = 88.37 psia	
$\rho^R = 0.2184$ [lb_m/f^3]	- Homogeneous steam density x = 0.93155 (Appendix A)	

$u^R = 1860\ [f/s]$ = Steam velocity (Appendix A)

$e^R = 713.7\ [B/lb_m]$ = Steam energy at x = 0.93155, P = 88.37 psia

$f^x = -\rho^i g = -72.45 \left[\frac{lb_m}{f^2 - s^2}\right]$ - Gravitational body force

$$q'''^R_{OUT} = \frac{H \cdot 4\,(T^R - T_{PIPE})}{D_H} = 256.5 \left[\frac{B}{s - f^3}\right] \quad \text{Heat loss to pipe} \tag{169 Cont)}$$

$H = 1000 \left[\frac{B}{hr\ f^2\ °F}\right]$ Steam condensation

$T_{PIPE} = 595\ [°R]$

$D_H = 9.564\ [in.]$

$$u^i = q'''^i_{IN} = q'''^i_{OUT} = q'''^R_{IN} = 0$$

Inserting equation (169) into equations (158), (159), and (160), then eliminating terms two orders of magnitude less than the greatest give:

<u>Mass</u>

$$(P_{37}P_{38})\rho^o_t + (P_{37})u^o\rho^o_x + \left[(P_{37})\rho^o + 1\right] u^o_x = 0 \tag{170}$$

<u>Momentum</u>

$$\left[(P_{37}P_{38})\rho^o + (P_{38})\right] u^o_t + \left[(P_{37})\rho^o + 1\right] u^o u^o_x + (P_{39})P^o_x + \left[(P_{37}P_{41})\rho^o + (P_{41})\right] u^{o2} = 0 \tag{171}$$

Energy

$$\left[(P_{37}P_{38}P_{42})\rho^{o} + (P_{38}P_{42})\right] e_{t}^{o} + \left[(P_{37}P_{42})\rho^{o} + (P_{42})\right] u^{o}\, e_{x}^{o}$$
$$+ \left[(P_{39})P^{o} + (P_{40})\right] u_{x}^{o} = (P_{43})q_{OUT}^{'''o} + \left[(P_{37}P_{41})\rho^{o} + (P_{41})\right] u^{o3} \qquad (172)$$

where the resulting similarity parameters for steam flow are:

$$P_{37} = \Delta\rho/\rho^{i} = \frac{\rho^{R}}{\rho^{i}} - 1$$

$$P_{38} = L/\Delta u\tau = L/u^{R}\tau$$

$$P_{39} = \Delta P/\rho^{i}\Delta u^{2} = \frac{P^{R} - P^{i}}{\rho^{i}(u^{R})^{2}}$$

$$P_{40} = P^{i}/\rho^{i}\Delta u^{2} = \frac{P^{i}}{\rho^{i}(u^{R})^{2}} \qquad (173)$$

$$P_{41} = fL/2D_{H}$$

$$P_{42} = \Delta e/\Delta u^{2} = \frac{e^{R} - e^{i}}{(u^{R})^{2}}$$

$$P_{43} = \frac{q_{OUT}^{'''R}\, L}{\rho^{i}\Delta u^{3}} = \frac{4HL\left(T^{R} - T_{PIPE}\right)}{\rho^{i} D_{H}\left(u^{R}\right)^{3}}$$

Gravitational forces are negligible, but heat loss is important for steam flow modeling. Parameters P_{38} and P_{41} are identical to P_{32} and P_{35} for air flow and have been scaled. Similarly, parameters P_{37}, P_{39}, P_{40}, and P_{42}

depend upon the steam properties in the pipe. Since steam does not behave as an ideal gas, these parameters must be checked by tabular or graphical techniques. Parameters P_{39} and P_{40} define how steam pressure and, consequently, air density scale (remember $P_a \neq P_{SH-a} \neq \lambda_\rho \lambda_x$; $(u^R)^2 \neq RT_a^i \neq \lambda_x$):

$$P_a^i \neq P_s \neq \rho^i (u^R)^2 \neq \lambda_\rho \lambda_x \tag{174}$$

thus

$$\rho_a^i \neq \lambda_\rho \tag{175}$$

and from P_{37}:

$$\rho_a^i \neq \rho_s \neq \lambda_\rho \tag{176}$$

from P_{42}:

$$e_a^i - e_s \neq RT \neq \lambda_x \tag{177}$$

From P_{43} and the scale factors already determined:

$$q'''^R_{OUT} \neq \lambda_\rho \lambda_x^{0.5} \tag{178}$$

Relating equation (178) to temperature difference from equation (169):

$$H(T_s - T_{PIPE}) \neq \lambda_\rho \lambda_x^{1.5} \tag{179}$$

where T_s is the saturated steam temperature in the pipe. Parameter P_{43} defines the necessary heat loss relation, that is, it defines what the pipe wall temperature should be given the heat transfer coefficient H.

4. SUMMARY OF MODELING RESULTS

S/RV system analysis shows that two scaling parameters determine every aspect of the scale model - namely, quencher submergence and pool density:

$$\lambda_x \triangleq \frac{\text{QUENCHER SUBMERGENCE - MODEL SCALE}}{\text{QUENCHER SUBMERGENCE - FULL SCALE}}$$

$$\lambda_\rho \triangleq \frac{\text{POOL FLUID DENSITY - MODEL SCALE}}{\text{POOL FLUID DENSITY - FULL SCALE}} \quad (180)$$

POOL MODEL: The suppression pool models geometrically; that is, all lengths scale as λ_x and all angles are preserved. Froude* scaling of velocities ($V \neq \lambda_u^{0.5}$) and pressure coefficient scaling result in absolute pool pressures scaling as $\lambda_\rho \lambda_x$ and a time scale scaling of $\lambda_x^{0.5}$.

TOP AIR: As a result of geometric pool modeling and Froude-pressure coefficient scaling in the pool, the top air volume must also scale geometrically ($V \neq \lambda_x^3$) and compressibility must be preserved ($\gamma \neq 1.0$).

BUBBLE FORMATION: As a result of geometric pool modeling and Froude-pressure coefficient scaling in the pool, the bubble volume must scale geometrically ($V \neq \lambda_x^3$) and the bubble pressure must scale as $\lambda_\rho \lambda_x$. To accomplish this, the mitigator hole area must be scaled according to the initial air temperature in the pipe:

$$A_H \neq \lambda_x^{2.5} / \lambda_{RT}^{0.5} \quad (181)$$

*Froude scaling is not necessary, but is imposed to improve the controllability of the scale model.

The driving pressure in the pipe scales as $\lambda_\rho \lambda_x$. To scale initial air mass flow rate and later sonic flow rates and momentum effects $\lambda_{RT} = \lambda_x$ is necessary; consequently, hole area must scale geometrically ($A_H \neq \lambda_x^2$) from equation (181).

QUENCHER MODEL: The quencher cross-sectional area and mitigator hole area must scale the same in order to scale clearing time correctly,* and hole area is defined from bubble formation to scale geometrically ($A_p \neq A_H \neq \lambda_x^2$). To have the correct hole uncovering sequence and pool velocity directions, the quencher arm length and hole spacing must scale geometrically, and the drill angles of the holes must be preserved between model and full scales. To model pressure drop across the mitigator holes, the loss coefficients between model and full scales must be preserved ($K \neq 1.0$).

PIPE FLOW: Flow in the S/RV piping is characterized by shock waves and sonic velocities that scale like $\lambda_{RT}^{0.5}$. To match time scales between pipe and pool, pipe length must scale like $L \neq \tau \cdot V \neq \lambda_x^{0.5} \lambda_{RT}^{0.5}$ but from bubble considerations $\lambda_{RT} = \lambda_x$ and $L \neq \lambda_x$ (geometric). Pipe volume must scale geometrically to give geometric bubble volume; consequently, pipe area scales as $A \neq V/L \neq \lambda_x^2$. To get the correct pressure drop down the pipe, friction factor scales as $f \neq \sqrt{A}/L \neq 1.0$. Steam density in the pipe must scale like air density to model mass and momentum conservation $\rho_S \neq \rho_a^i = \lambda_\rho \lambda_x / \lambda_{RT} = \lambda_\rho$.

Scaling parameters found are summarized in Table 4-1.

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*Sometimes known as "segment scaling".

Table 4-1
SCALING RESULTS FOR S/RV SYSTEM

Pool Model

Parameter		Scaling
X*	(Submergence)	$\triangleq \lambda_x$
Z*	(Pool Length)	λ_x
R*	(Pool Radius)	λ_x
α*	(All Angles)	1.0
ρ*	(Pool Density)	$\triangleq \lambda_\rho$
V*	(Pool Velocities)	$\lambda_x^{0.5}$
P*	(Pool Pressures)	$\lambda_\rho \lambda_x$
t*	(All Time)	$\lambda_x^{0.5}$
u,v,w	(Velocities)	$\lambda_x^{0.5}$
x,y,z	(Distances)	λ_x

Quencher Model

Parameter		Scaling
L	(Arm Length)	λ_x
P	(Pressure)	$\lambda_\rho \lambda_x$
V	(Velocities)	$\lambda_x^{0.5}$

Quencher Model (Cont)

Parameter		Scaling
ρ	(Density)	λ_ρ
K	(Orifice Loss)	1.0
A	(Area)	λ_x^2

Top Air Model

Parameter		Scaling
V	(Volume)	λ_x^3
P	(Pressure)	$\lambda_\rho \lambda_x$
E	(Energy)	$\lambda_\rho \lambda_x^4$
γ	(Sp. Heat Ratio)	1.0
M	(Mass)	$\lambda_\rho \lambda_x^3$

Bubble Model

Parameter		Scaling
V	(Volume)	λ_x^3
P	(Pressure)	$\lambda_\rho \lambda_x$
E	(Energy)	$\lambda_\rho \lambda_x^4$
γ	(Sp. Heat Ratio)	1.0
M	(Mass)	$\lambda_\rho \lambda_x^3$

Table 4-1 (Cont)

SCALING RESULTS FOR S/RV SYSTEM

Bubble Model (Cont)

Parameters		Scaling
$\dot{m}$	(Mass Flow)	$\lambda_\rho \lambda_x^{2.5}$
$\dot{m}h_o$	(Enthalpy Flux)	$\lambda_\rho \lambda_x^{3.5}$
V	(Velocity)	$\lambda_x^{0.5}$

Pipe Model

Parameters		Scaling
P	(Pressure)	$\lambda_\rho \lambda_x$
ρ	(Density)	λ_ρ

Pipe Model (Cont)

Parameters		Scaling
V	(Velocity)	$\lambda_x^{0.5}$
L	(Length)	λ_x
e	(Energy)	λ_x
h	(Enthalpy)	λ_x
q'''	(Heat Loss)	$\lambda_\rho \lambda_x^{0.5}$
f	(Friction Factor)	1.0

*Note: These parameters are "control parameters"; all other parameter scalings are consequences of these scalings.

5. TEST APPLICATION DISCUSSION

As is often the case in a complex system, physical limitations dictate to what degree the scale can be altered. In the S/RV application; bubble modeling is $RT \neq \lambda_x$ and $\gamma \neq 1$. To meet this criterion, another gas with the same γ but smaller R might be found, for instance $R_m = 1/2\ R_F$ (chlorine, for example, has $R \approx 1/2\ R_{air}$), and use $T_m = T_F$. Thus, a half scale model is possible. Temperature might be reduced as $T_m = 1/2\ T_F$ and the same gas kept in both scales, but this would mean an air temperature of $1/2 \cdot 595 \approx 298°R$ or -162°F. Clearly reducing air temperature is not an effective way to reduce scale.

Because of steam's condensation properties, pool densities are limited to $\lambda_\rho \leq 1.0$. For example, if a gas is chosen such that $\lambda_{RT} = 1/2 = \lambda_x$ and a pool density $\lambda_\rho = 2$, then the model pressures scale as $P \neq \lambda_\rho \lambda_x = 1.0$. Steam in the modeled pipe must be at the same pressure as full scale but twice the density, which implies a steam-vapor mixture.

Another physical limitation is the friction factor f. Assuming $\lambda_\rho = \lambda_{RT} = 1$ and a very smooth pipe in the model (for example, drawn tubing), then the smallest the scale can reduce is $\lambda_x = 1/3$, as shown in Table 5-1.

Table 5-1

DARCY FRICTION FACTOR FOR $\lambda_\rho = \lambda_{RT} = 1$, $\epsilon_m = 0.000005$

Full Scale: $\mu = 1.10^{-5} \left[\frac{lbm}{f\text{-}S}\right]$, $\dot{m} = 200 \left[\frac{lbm}{S}\right]$, f = 0.024, D = 0.797[f],

A = 0.499 [f]

λ_x	Actual f	Desired f	Relative % Difference	
1/4	0.0115	0.0085	35%	Too Much Friction
1/3	0.0105	0.0105	0%	
1/2	0.0095	0.0143	33%	Too Little Friction

Although the friction factor is important for steady state flow, during the initial pipe charging until subsequent discharge, the velocities in the pipe are low and friction is not an important concern.

The smallest scale model that still meets all constraints is half scale. Using a different gas (chlorine, for example) and the same density fluid in both, then $\lambda_\rho \neq 1.0$, $\lambda_x \neq \lambda_{RT} \neq 1/2$. Steam charging rate $\dot{m}_s$ is modeled by scaling enthalpy flux $\dot{m}_s h_{os} = \lambda_x^{3.5}$. Bubble motion is directly dependent on the downstream pressure (P_{SH}), which is determined from steam pressure in the pipe. Neglecting heat loss q''' results in a higher bubble pressure and consequently higher torus wall loads in the model, so to ignore heat loss is conservative.

Reducing the scale beyond the scaling constraints will introduce model distortions.* In this instance we define λ_ρ, λ_x and λ_{RT}. The quencher system must be distorted to give as nearly correct results as possible.

The correct pool motion during air clearing is maintained as long as bubble volume and pressure are scaled correctly; i.e., $\forall_B \neq \lambda_x^3$ and $P_B \neq \lambda_\rho \lambda_x$. Time must scale correctly as well, $t \neq \lambda_x^{0.5}$. From the ideal gas law:

$$P_B \forall_B = M_B R T_B \neq \lambda_\rho \lambda_{(x^4)} \tag{182}$$

Thus, if the model bubble temperature T_B is too high, reducing the bubble mass M_B maintains the corre . pressure and volume scaling. From bubble formation equation (110), the quencher hole area is modified to give the correct bubble reference volume:

$$A_H \neq \lambda_x^{2.5} / \lambda_{RT}^{0.5} \tag{183}$$

*Meaningful results will still follow if model adjustments are made to scale the predominant phenomena.

To scale water clearing time and water velocities, the quencher cross-sectional area must scale as the hole area:

$$A_{PIPE} \neq A_H \neq \lambda_x^{2.5}/\lambda_{RT}^{0.5} \tag{184}$$

Equation (167) is used to time the pipe length acoustically to model sonic velocities.

$$L \neq \lambda_x^{0.5}\,\lambda_{RT}^{0.5} \tag{185}$$

To geometrically scale air volume, the cross-sectional pipe area is:

$$A = \frac{V}{L} \neq \lambda_x^{2.5}/\lambda_{RT}^{0.5} \tag{186}$$

Since quencher area and pipe area scale the same, there is no area discontinuity in the scaled model. In choosing this model, initial bubble pressure and volume will be distorted as will momentum effects. To illustrate this, consider pressure, volume, and momentum. Water clearing velocities will scale as before, $V \neq \lambda_x^{0.5}$. Initial air clearing velocities act like water velocity $V^i \neq \lambda_x^{0.5}$, but quickly reach sonic speed where $V^R \neq \lambda_{RT}^{0.5}$, then:

<u>Volume</u>:

$$\forall \neq Av\tau = \lambda_x^3 \qquad \text{Ideal}$$

$$\forall^i \neq Av^i\tau \neq \lambda_x^{3.5}/\lambda_{RT}^{0.5} \quad \text{- Initial volume distortion error} \left(\sqrt{\frac{\lambda_x}{\lambda_{RT}}} - 1\right)$$

$$\forall^R \neq Av^R\tau \neq \lambda_x^3 \qquad \text{- O.K.} \tag{187}$$

Pressure Difference:

$$(P_{SH} - P_B) \neq \rho_a v^2 \neq \frac{P}{RT} v^2 \neq \lambda_\rho \lambda_x \quad \text{Ideal}$$

$$(P_{SH} - P_B)^i \neq \frac{P}{RT} v^{i2} \neq \frac{\lambda_\rho \lambda_x^2}{\lambda_{RT}} \quad \text{- Initial pressure distortion error} \left(\frac{\lambda_x}{\lambda_{RT}} - 1\right) \tag{188}$$

$$(P_{SH} - P_B)^R \neq \frac{P}{RT} v^{R2} \neq \lambda_\rho \lambda_x \text{ - O. K.}$$

Air Momentum:

$$F_a \neq \rho_a V v \neq \frac{P}{RT} \forall v \neq \lambda_\rho \lambda_x^{3.5} \quad \text{Ideal}$$

$$F_a^i \neq \frac{P}{RT} \forall^i v^i \neq \lambda_\rho \lambda_x^{5.0} / \lambda_{RT}^{1.5} \quad \text{- Distortion error} \left(\frac{\lambda_x^{1.5}}{\lambda_{RT}^{1.5}} - 1\right) \tag{189}$$

$$F_a^R \neq \frac{P}{RT} \forall^R v^R \neq \lambda_\rho \lambda_x^4 / \lambda_{RT}^{0.5} \quad \text{- Distortion error} \left(\frac{\lambda_x^{0.5}}{\lambda_{RT}^{0.5}} - 1\right)$$

Water Momentum:

$$F_W \neq \rho \forall v \neq \lambda_\rho \lambda_x^{3.5} \quad \text{(Ideal)}$$

$$F_W \neq \rho A L v^i = \lambda_\rho \lambda_x^{4.0} / \lambda_{RT}^{0.5} \quad = \text{Distortion error} \left(\frac{\lambda_x^{0.5}}{\lambda_{RT}^{0.5}} - 1\right) \tag{190}$$

To illustrate the distortions, assume $\lambda_{RT} = 1$ and $\lambda_x < 1$. The modified model will then have bubble distortions of equation (a) - too small an initial volume, and equation (b) - too great an initial pressure. These distortions are eliminated as soon as exiting air velocity approaches sonic. Water momentum is too low, which translates into less measured water impact pressure and less "churning" of water far from the quencher. Air momentum is greatly distorted initially, but this is not a concern since initial air momentum is small compared with pressure effects.* Air momentum is too low even at sonic velocities, which translates into a more rounded bubble shape in the model. Even though some initial distortions occur, good measurements are expected to result since the predominant phenomena have been preserved.

One important question is how much is gained by using another gas. It can be qualitatively answered by examining the amount of bubble pressure distortion (equation 188) for various values of λ_x and λ_{RT}. Consider the following table for two different geometric sizes and two different gases:

Table 5-2

COMPARISON OF SCALING

	$\lambda_{RT} = 1$	$\lambda_{RT} = 1/2$	
$\lambda_x = 1/10$	$\Delta P_{err} = -90\%$	$\Delta P_{err} = -80\%$	→ Improve 10%
$\lambda_x = 1/4$	$\Delta P_{err} = -75\%$	$\Delta P_{err} = -50\%$	→ Improve 25%
	↓ Improve 15%	↓ Improve 30%	

*Refer to momentum effects portion of subsection 3.5.

Table 5-2 indicates the following: (a) given a very small scale test run with $\lambda_x = 1/10$, expending effort, expense, use of a different gas, gains very little, (b) given a relatively large scale test to be run ($\lambda_x = 1/4$), significant improvements can be made by using a different gas, and (c) given the same gas used in both full and small scales, $\lambda_{RT} = 1$, little improvement is achieved by going to the greatly increased scale, that is, from $\lambda_x = 1/10$ to $\lambda_x = 1/4$. The extra cost is probably unwarranted.

A simple quencher model with a uniform hole distribution was used to determine scaling laws. The same scaling laws apply to modeling the quencher actually installed (Figure 5-1); that is, arm lengths and hole distribution down the arm scale geometrically, drill angles are preserved, hole and cross-sectional areas scale as $\lambda_x^{2.5} \lambda_{RT}^{0.5}$.

Friction effects tend to be important at very small scales, especially with the longer, narrower S/RV line in the model. The friction parameter fL/D ideally scales as 1.0, but with $L \neq \sqrt{\lambda_x \lambda_{RT}}$ and $D \neq \lambda_x^{1.25}/\lambda_{RT}^{0.25}$, it follows that

$$f \frac{L}{D} \neq \left(\frac{\lambda_{RT}}{\lambda_x} \right)^{0.75} f$$

Moderately reduced scales do not appreciably affect friction scaling, which is not an extremely important parameter, but at greatly reduced scale (1/12 scale, for example), the pipe friction tends to weaken shock impact pressure and alter downstream pressurization.

Another concern is choosing steam charging rate into the pipe. Two types of scaling exist depending on the elapsed time.

Initially, the steam charging rate behaves as a node, that is, the steam occupies a small section of the S/RV line and is uniform in pressure and temperature. Modeling nodal charging rate is achieved by scaling enthalpy flux

Company Proprietary

Figure 5-1. Quencher Arm Hole Pattern Detail (GE Company Proprietary)

($\dot{m}h_o \neq \lambda_\rho \lambda_x^{3.5}$) similar to bubble charging. As the steam proceeds down the S/RV line, it tends to behave more one-dimensionally, and steam charging is chosen to model pipe pressure and velocity as described in Appendix A.

6. CONCLUSIONS

Not all parameters can be modeled. Some phenomena of minor importance will be distorted to model predominant phenomena. The important nondimensional groupings to be maintained between model and prototype are:

POOL

Pressure Coefficient	$\frac{\Delta P}{\rho V^2}$	=	$\frac{\text{pressure forces}}{\text{inertial forces}}$
Froude Number	$\frac{V^2}{gX}$	=	$\frac{\text{inertia forces}}{\text{gravitational forces}}$

PIPE

Pipe Acoustics	$\frac{c\tau}{L}$	=	$\frac{\text{acoustic length}}{\text{pipe length}}$
Flow Path Losses	K	=	(orifice loss)
Gas Compressibility	γ	=	Ratio of Specific Heats

The recommended system scaling is sketched in Figure 6-1 and is based on the assumption that: (a) water is used in both systems, (b) air is used in both systems, and (c) the initial air temperature in the S/RV line is different between model and full scale (define $\lambda_T = T_{model}/T_{full}$). Let the scale factor be λ_x (e.g., for a half scale model $\lambda_x = 1/2$).

Following the figure is a summary of the scaling factors for the various parameters.

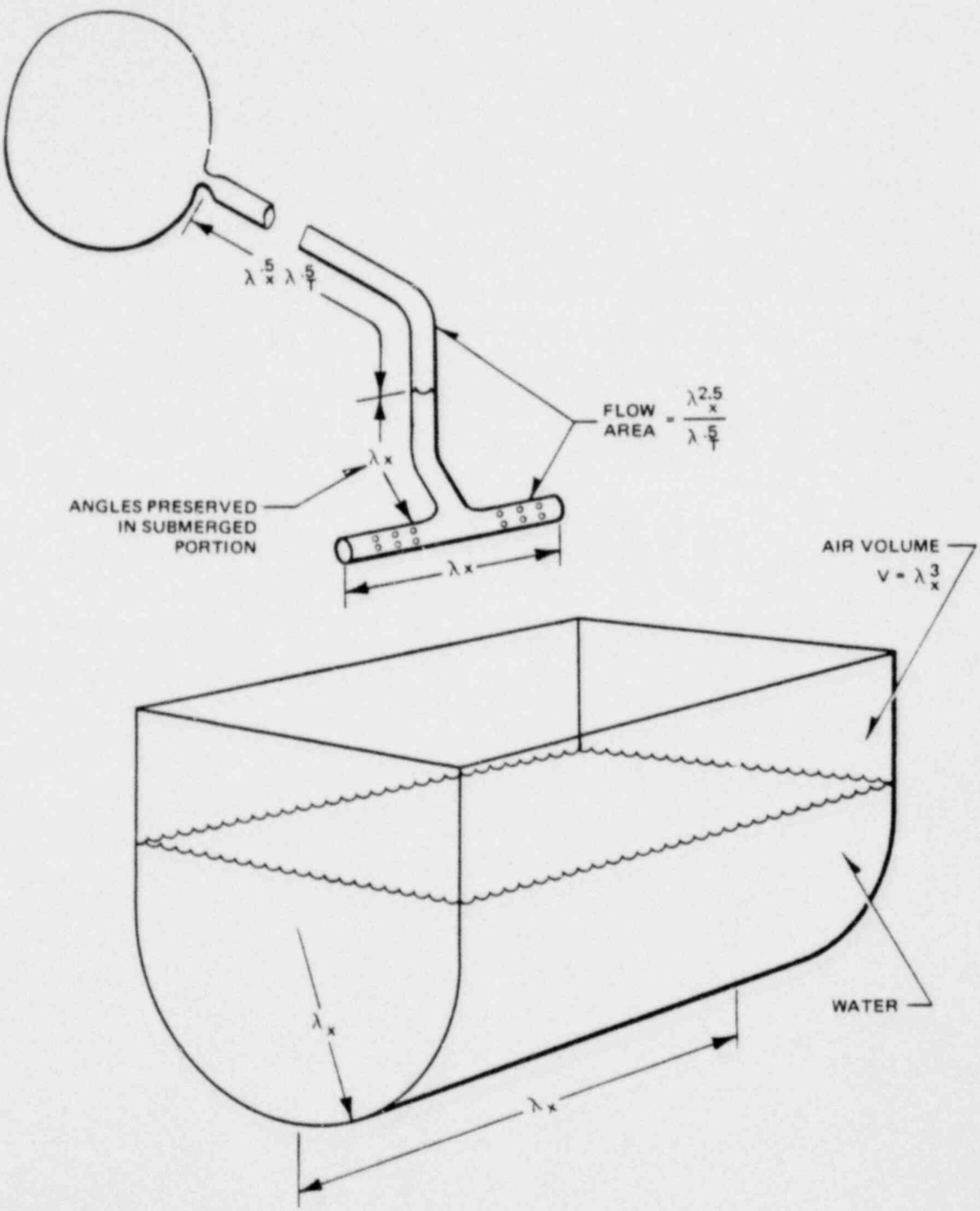

Figure 6-1. Proposed Model Scaling

SCALING FACTORS

TOTAL SYSTEM:

- Time scales as $\lambda_x^{0.5}$
  (things happen faster)

- Pressure scales as λ_x
  (reduced pressure)

TORUS AREA:

- All lengths scale as λ_x
  And all angles are preserved
  (geometric scaling)

- Air space volume scales as λ_x^3
  (geometric scaling)

QUENCHER:

- Quencher hole area scales as $\lambda_x^{2.5}/\lambda_T^{0.5}$
  (to maintain bubble pressure
  and volume scalings)

- Quencher cross-sectional area scales as $\lambda_x^{2.5}/\lambda_T^{0.5}$
  (to maintain scaled water clearing time)

- Quencher arm length scales as λ_x
  (to maintain geometric pool modeling)

- Quencher submergence scales as λ_x
  (to maintain geometric pool modeling)

SCALING FACTORS (cont)

QUENCHER: (cont)

- Quencher angles are preserved in submerged portion (to maintain water length leg scaling λ_x)

S/RV PIPING:

- S/RV line air length scales as $\lambda_x^{0.5}\lambda_T^{0.5}$ (to maintain acoustic scaling)
- S/RV line water leg length scales as λ_x (to maintain scaled water clearing time)
- S/RV line crosssectional area scales as $\lambda_x^{2.5}/\lambda_T^{0.5}$ (to maintain scaled bubble volume)

7. REFERENCES

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APPENDIX A

REFERENCE VALUES FOR PIPE FLOW

Consider the S/RV pipe shown in Figure A-1. Assuming h_{os} and $\dot{m}_s$ are known, all pipe properties can be determined based on steam properties for steam and Rankine-Hugoniot ideal gas relationships for air. Assuming a steady steam flow:

$$h_{os} = h_s + \frac{V_s^2}{2g_c} \qquad \text{(A-1)}$$

$$\dot{m}_s = \frac{AV_s}{\nu_s} \qquad \text{(A-2)}$$

where:

$$\begin{cases} h_s = h_s(P_s, \nu_s) \;;\quad \nu_s = \nu_s(P_s\,,\,h_s) & \text{- Saturated or superheated region} \\ h_s = h_s(P_s, x_s) \;;\quad \nu_s = \nu_s(P_s\,,\,x_s) & \text{- Steam-vapor mixture region} \end{cases} \qquad \text{(A-3)}$$

$$V_s = \frac{C_a\left(\frac{P_s}{P_a} - 1\right)}{\sqrt{\gamma\frac{(\gamma - 1)}{2}\left\{\left(\frac{\gamma + 1}{\gamma - 1}\right)\frac{P_s}{P_a} + 1\right\}}} \text{ - Rankine-Hugoniot velocity} \qquad \text{(A-4)}$$

$$C_a = \sqrt{\gamma g_c R T_a} \qquad \text{- Sonic speed in air}$$

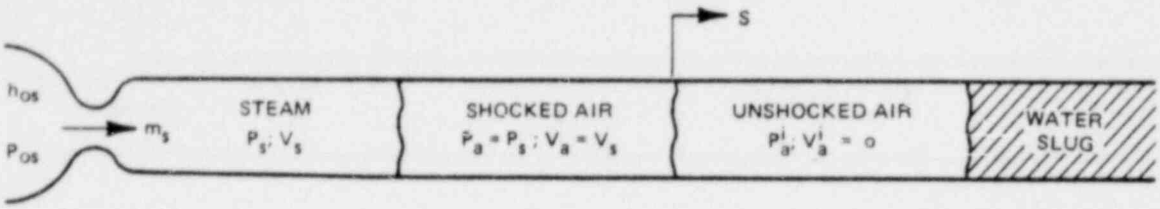

Figure A-1. S/RV Line Model

Since only one value of P_s will satisfy (A-1) through (A-4), one must iterate to find a solution. Once P_s is found, the impact shock-to-water pressure may be calculated from the Rankine-Hugoniot relationship:

$$P_{SH} = P_s \left(1 + \frac{\gamma(\gamma + 1)}{4} \left(\frac{V_S}{C_{SH - a}} \right)^2 + \frac{\gamma V_S}{C_{SH - a}} \sqrt{1 + \frac{1}{4} \left(\frac{\gamma + 1}{2} \right)^2 \left(\frac{V_s}{C_{SH - a}} \right)^2} \right) \quad \text{(A-5)}$$

where:

$$C_{SH - a} = C_a \left\{ \frac{\frac{P_s}{P_a} \left(\frac{\gamma + 1}{\gamma - 1} + \frac{P_s}{P_a} \right)}{\left(\frac{\gamma + 1}{\gamma - 1} \right) \frac{P_s}{P_a} + 1} \right\}^{1/2} \quad \text{(A-6)}$$

Iterating as described with reference values:

$\dot{m}_s$ = 200 [lbm/S]	- Steam mass flow rate	
h_{os} = 1192.8 [B/lbm]	- Steam stagnation enthalpy	saturated steam at P_o = 1000 psia
A = 0.499 [f^2]	- Pipe area	
γ = 1.4	- Ratio of specific heats	
R = 53.5 [lbf-f/lbm-°R]	- Air gas constant	
T_a = 595 [°R]	- Air initial temperature	
P_a = 14.7 [psia]	- Air initial pressure	(A-7)

Then:

P_s = 88.37 [psia]	- Steam pressure in pipe
x_s = 0.93155	- A small amount of quality results in pipe
V_s = 1860 [f/S]	- Steam and shocked air velocity
P_{SH} = 340 [psia]	- Shock impact pressure

APPENDIX B

PROPERTIES OF THE SCALING OPERATOR

For ease in manipulating scaling operations the "scale operator" is introduced and designated by $\neq$. The operator relates scaling to parameters; if some system parameter, for example, length L, scales as λ, then:

$$\frac{L_m}{L_F} = \lambda \qquad \text{(B-1)}$$

using the $\neq$ operator (B-1) is written

$$L \neq \lambda \qquad \text{(B-2)}$$

Some useful properties of the scaling operator are:

(1) Comparative property: If $A \neq \lambda$ and $A \pm B \neq \lambda$ then $B \neq \lambda$. Or conversely if $A \neq \lambda$ and $B \neq \lambda$, then $A \pm B \neq \lambda$.

Multiplication property: If two properties scale the same, $A \neq B$ for example, then introducing a different property "C," one can say that $A \cdot C \neq B \cdot C$ or conversely $A/C \neq B/C$.

(3) Elimination of constants property: Constant multipliers may be dropped with the scaling operator, for example, if $2 \cdot A \neq \lambda$ then $A \neq \lambda$.

APPENDIX C

NOMENCLATURE

Symbol	Description
α	Quencher hole drill angle
γ	ratio of specific heats
Δ	indicates change in
λ_x, λ_ρ, λ_{RT}	scale factor geometric, pool density, air temperature
μ	Viscosity
(v) $\forall$	Volume (specific)
ρ	Density
τ	Time
Φ	Energy dissipation function
ϕ	Some general parameter
A	Area
a	Acceleration
ac	Air clearing
c	Sonic speed
D	Diameter
D_H	Hydraulic diameter
(e) E	Internal energy (specific)
F	Force
$\overline{F}$	Body force vector
f	Friction factor
f^x, f^y, f^z	Body force components in the x, y, z directions
g	Acceleration of gravity
g_c	Gravitational constant

Symbol	Description
H	Heat convection coefficient
h	enthalpy
$\hat{i}$, $\hat{j}$, $\hat{k}$	Vector components in the x, y, z directions
K	Conductivity coefficient or orifice loss coefficient
L	Length
M	Mass
$\dot{m}$	Mass flow
m	Mass and water slug
P	Pressure
q'''	Heat flow per unit volume
q	Heat flow rate
R	Gas constant or bubble radius
S	Shock speed
T	Temperature
t	Time
U, V, W	Velocities
u, v, w	Velocity components in the x, y, z directions
wc	Water clearing
X, Y, Z	Reference lengths in the x, y, z directions
x, y, z	Position coordinates

Subscript	Description
a	Air
ac	Air clearing
B	Bubble
H	Single hole
HOLES	Total holes
"IN"	Into system
M, F	Model and full
o	Stagnation
"OUT"	Out of system
P	Pipe or pool
PW	Pool water
s	Steam
SH	Shock or impact
t	Derivative with respect to t^o
wc	Water clearing
x, y, z	Derivative with respect to x^o, y^o, z^o

Superscript	Description
i	Initial conditions
K	Indicates some general parameter
o	Nondimensional parameter
R	Reference condition