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Detection of Damage in Structures from Changes in Their Dynamic (Modal) Properties -- A Survey

Prepared by M. H. Richardson

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Structural Measurement Systems, Inc.

Prepared for
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ABSTRACT

The stated object of this study was to survey the technical literature and interview selected experts in the fields of dynamic testing and analysis to determine the state-of-the-art of the relationship between physical damage to a structure and changes in its dynamic (modal) properties.

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FOREWORD

This study is the result of discussions which began over a year ago with several individuals in the Engineering Mechanics Section of the Mechanical Engineering Department of Lawrence Livermore Laboratory, Livermore, California. Through our discussions with Dr. H. Joseph Weaver and his colleagues at the Laboratory, we determined that much of the newly developed technology, in vibration measurement, digital signal processing and parameter identification could be utilized for monitoring the integrity of a variety of structural components in nuclear power plants.

Nuclear plants are a complex assemblage of interconnected structures and machinery which must be maintained at a sufficient level of integrity in order to function properly. Because of the potential damage and lost production time which could occur due to structural failures in these plants, these problems deserve our best engineering efforts on a long term basis in order to develop better monitoring and diagnostic procedures for damage detection and assessment should it occur.

Although vibration levels in certain key components such as coolant pumps, turbo generators and the reactor core are currently monitored, simple checking of overall vibration levels is often not a sensitive enough indicator of impending failure in a structure. The purpose of this study, then, was to examine a more sophisticated approach to signal processing which involves the identification of parameters describing the so called "modes of vibration" of a vibrating structure.

The hypothesis of this study is that changes in the modes of vibration of a structure are sensitive indicators of change in its physical properties; namely its mass, stiffness and damping. Indeed, we found much substantiating evidence of this in the literature.

In Chapter 2 we present a tutorial review of modal analysis. We were not able to present a thorough discussion of this subject without the use of mathematics. However, we include a general discussion of dynamic testing and analysis at the beginning of the chapter for those not interested in reading through the mathematics.

This report contains a large number of quotations which are essentially the text taken from papers we reviewed and which contained comments we felt contributed to our survey. The risk in doing this is that the point being made by the author appears to be something different when taken out of context. It was certainly not our intention to misrepresent anyone's results. To avoid any misrepresentations, we encourage interested readers to obtain copies of the references quoted if any of our quotations appear to present only half of the story.

I want to thank the technical and contracts personnel at LLL and the NRC who supported us throughout the course of this contract. Lastly, I want to congratulate my colleagues Dave Formenti, Ken Ramsey, and Don Kientzy on a job well done.

Mark H. Richardson, PhD.
President
STRUCTURAL MEASUREMENT SYSTEMS, INC.

1.0 INTRODUCTION

This report is the result of a study effort performed over approximately a 3 1/2 month period between April 1979 and July 1979 by the personnel of Structural Measurement Systems, Inc. (SMS) under contract to the Mechanical Engineering Department of Lawrence Livermore Laboratory, Livermore, California, and at the request of the Research Branch of the Nuclear Regulatory Commission (NRC).

During the study approximately 800 man hours of effort were expended in searching computer data bases for references, rooting through libraries for articles, reading and discussing the articles among ourselves, and in talking to a few chosen experts in the field. Given the time constraints of this study, we tended to emphasize the methods, publications, and acquaintences most familiar to us in the dynamic testing and analysis field. There are, of course, numerous contributors to this field whom we did not interview or whose publications we did not examine in detail. However, there have been a number of other state-of-the-art surveys done recently of both dynamic testing and parameter identification methods. We have cited a number of them in this report in an effort to give as complete and up-to-date synopsis of the state-of-the-art as possible.

1.1 OBJECTIVES OF THE STUDY

The stated object of this study was to survey the technical literature and interview selected experts in the fields of dynamic testing and analysis to determine the state-of-the-art of the relationship between physical damage to a structure and changes in its dynamic (modal) properties.

During the course of this study we took a broad look at the problem of relating physical damage to changes in modes of vibration. We did not attempt to discover specific schemes for detecting damage. Instead we concentrated on three major areas of investigation related to this problem. They are:

1. A tutorial treatment of modal analysis.
2. Modal testing methods.
3. Analytical/experimental studies related to the verification of the relationship between physical damage in a structure and changes in its modes of vibration.

The first topic we chose, in agreement with our contract monitor, out of a need we felt to clearly state the modal analysis theory as it relates to vibration measurements. There is a multitude of terminology associated with the structural dynamics field, and structural engineers, depending upon the industry in which they work (e.g. aerospace, automotive, nuclear, heavy construction, or otherwise) and the methods they commonly use describe the same concepts in different terms. Furthermore, the majority of literature on modal theory is contributed by dynamicists who work with computer based dynamic models (finite element models) and have little or no contact with measurements. In short, we have included in this report a consistent presentation of the concept of a mode of vibration and explained many of the terms and assumptions surrounding the use of modal analysis to characterize the dynamics of structures.

The second topic of this report, namely,testing and data analysis methods, is an area where we collected a large amount of information. With the advent of mini and micro computers, and the implementation of efficient computational algorithms such as the FFT (Fast Fourier Transform) in these small machines, many new testing and data processing methods have sprung up during the past 5 to 10 year period. Emphasis is placed on these newer digitally based methods in this report because they appear to hold more promise for the future.

Lastly, the topic most germane to this study, that is, experimental or analytical evidence of the relationship between physical damage and changes in modal properties, yielded some very significant results. This material was further subdivided into four major areas because the vast majority of the literature surveyed fell into one of these categories. The four areas we chose are:

1. Nuclear Power Plants
2. Large Structures
3. Rotating Machinery
4. Offshore Platforms

Our findings are presented in this report primarily in the form of quotations of the results, conclusions and recommendations of the researchers whom we interviewed or whose literature we reviewed.

At the end of this report we include a bibliography of the literature which we reviewed. This is by no means an exhaustive list of literature on this subject. However, many of the references listed here contain excellent lists of additional references which will take the interested reader back through the history of experiences of the author(s) of that particular paper.

In 1973 D. R. Houser & M. J. Drosjack wrote a very comprehensive report (Ref. G.43) on vibration signal analysis techniques, which discusses integrity monitoring from a broader perspective than is covered in this report. They do not limit their discussion to the use of modal parameters as the primary discriminants for detecting structural damage as we do here

1.2 DESCRIPTION OF THE PROBLEM

During our interview with Dr. Sheldon Rubin of Aerospace Corporation, El Segundo, California, he stated the problem of structural integrity monitoring very succinctly; "The real question is, can we detect when a failure has gotten to the point where it is intolerable." This statement implies several things; first, that we have defined what we mean by an intolerable situation. In most cases this is probably the most difficult step of the entire process. Secondly, do we know where and how to monitor a structure in order to detect the intolerable failure? Thirdly, can we measure vibrations and process the data with instrumentation sufficiently accurate to detect the failure?

The hypothesis which motivated this study, namely that structural damage can be detected by measuring (or monitoring) a structure's dynamic (modal) properties is expanded upon here in order to give the reader a clearer overview of the problem and to prepare him for the more intensive reading which follows in later sections.

Stated simply, most structures operate in a dynamic environment, which means that they are subjected to a variety of external forces which can tend to damage them. Examples include the constant wave and wind motions which can contribute to the fatigue of offshore platforms; seismic loadings such as earthquakes which can affect the structural integrity of large structures; and unbalance forces and resonance conditions which can cause fatigue and excessive wearout in rotating machinery.

Continued loading (or an excessive single load) is known to have the potential to damage structures by promoting fractures (cracks) in certain load-carrying members. These cracks can have a significant influence on the structure's internal damping or stiffness properties, and consequently can cause the structure's dynamics to change.

If the stiffness of a structure changes, it will behave differently because its resonances (frequencies where the structure readily absorbs energy) are shifted to different values. As an example, consider a simple cantilever beam which is solidly mounted into a wall. If we deformed the beam at its free end and released it, it would vibrate at its fundamental frequency. Now, if we were to make a saw cut half way through the beam near its root, and repeat the test, we would observe that the beam vibrated at a lower frequency even though we have not changed the physical dimensions of the beam. This change in frequency is caused by a stiffness change in the beam due to the saw cut.

Using modern measurement and data processing techniques, it is possible to measure a structure's dynamic properties and express them conveniently in terms of its modes of vibration. Identifying the modes of vibration over a given frequency range completely quantifies the dynamics of the structure over that same frequency range. The modes of vibration are represented by the mode shapes and their corresponding frequencies and damping parameters.

Thus, if we can accurately measure modal parameters and detect changes in their values, we are incrementally closer to correlating these often subtle changes with damage in the structure.

The underlying assumption of this survey effort is that changes in modal parameters are a reliable (and sensitive) indicator of changes in structural integrity.

It is our contention, of course, based upon approximately 40 yrs. of combined modal testing experience, that this is indeed the case.

In one of our references (Ref. A.1), Savage & Hewlett describe a "non destructive testing" method for assessing structural integrity. Some of their comments are the following:

> "The method relies principally on the fact that if a change occurs in the elastic load path from one point on a structure to another, whether this change is due to plastic (failure) or elastic (temporary load) motion, such motion is accompanied by a change in natural frequencies and a shift in mode shape. The detection of such changes is easily accomplished, but the interpretation of the data requires expert knowledge."

In their paper, the authors give results of tests they performed on concrete beams. They tested the beams to distruction (by dropping a weight on them) and plotted the frequencies of their modes at various stages of failure. One of the typical curves is shown below.

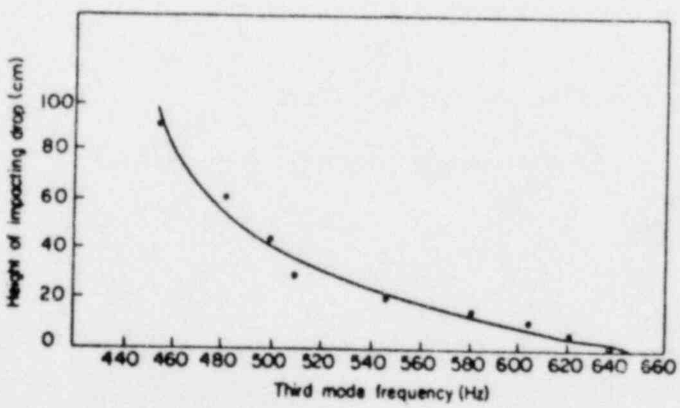


Fig. 10 Third mode frequency as a function of the height of impacting drop showing damage is occuring in the beam

Their observations from this test are the following:

> "Visible signs of minute cracking first appeared after the 610 mm drop test. Prior to this, no distress to the beam was visible, although evidently from the diminishing frequency response, some internal damage had been progressively occurring."

Many other results similar to these, but involving different structures and materials, were found in the literature. Many of them are reported in this report.

1.3 TYPES OF STRUCTURAL DAMAGE

As the reader reviews our findings, he should also keep in mind that damage is a very broad term and is assumed here to describe several different types of failure mechanisms.

Generally speaking, these failure mechanisms can be divided into the following categories:

A. Excessive elastic deformation

B. Yielding or excessive plastic deformation

C. Fracture

The following brief tutorial discussion is presented in order to acquaint the reader with a basic understanding of the definitions and terminology commonly encountered in literature describing structural damage.

In general, when damage is referred to in the literature it typically is equated to fracture. Fracture is defined as the formation of a crack which can result in degradation of the strength of a member.

Fractures can occur in all engineering materials having the range of brittle to ductile material properties. Concrete typically exhibits brittle behavior whereas a low carbon steel behaves in a ductile manner. Forces which cause fracture can be either "static" or "dynamic" in nature. Failures can occur as the result of a single overload or from repeated application of loads. Fatigue cracking and failure are the result of the latter type of loading. Additionally fatigue failure can result from either low or high levels of cyclic loading. Fatigue failure can occur in a relatively small number of high loading applications (as might be experiened during strong motion earthquakes) or a large number of light loading applications (as caused by operating machinery). Thermal loadings as well as mechanical loadings can result in fatigue failures.

Other factors contributing to lose of strength in materials are corrosion and irradiation..

1.4 DETECTING STRUCTURAL DAMAGE

Although the objective of this survey was to search for evidence (both experimental and analytical) that a predictable relationship exists between physical damage and changes in a structure's modes of vibration, the ulterior motive is to implement monitoring schemes which can effectively detect damage in various components of a nuclear power plant.

There are many types of mechanical components in a nuclear power plant, e.g., pressure vessels, containment structures, piping systems, pumps, motors, and compressors, which are made out of varied types of material. The task of monitoring the vibration of all these components is indeed a complex one.

The major steps of a damage assessment scheme using vibration measurements are shown in Figure 1.1.

The first step involves the sensing of the structure's vibrations with some type of motion sensitive transducer. These transducers typically convert motion (sensed as displacement, velocity, or acceleration) into an electronic signal. This signal can be stored directly on a magnetic tape, or it can be digitized, i.e., converted to a sequence of numbers, and stored in a digital memory.

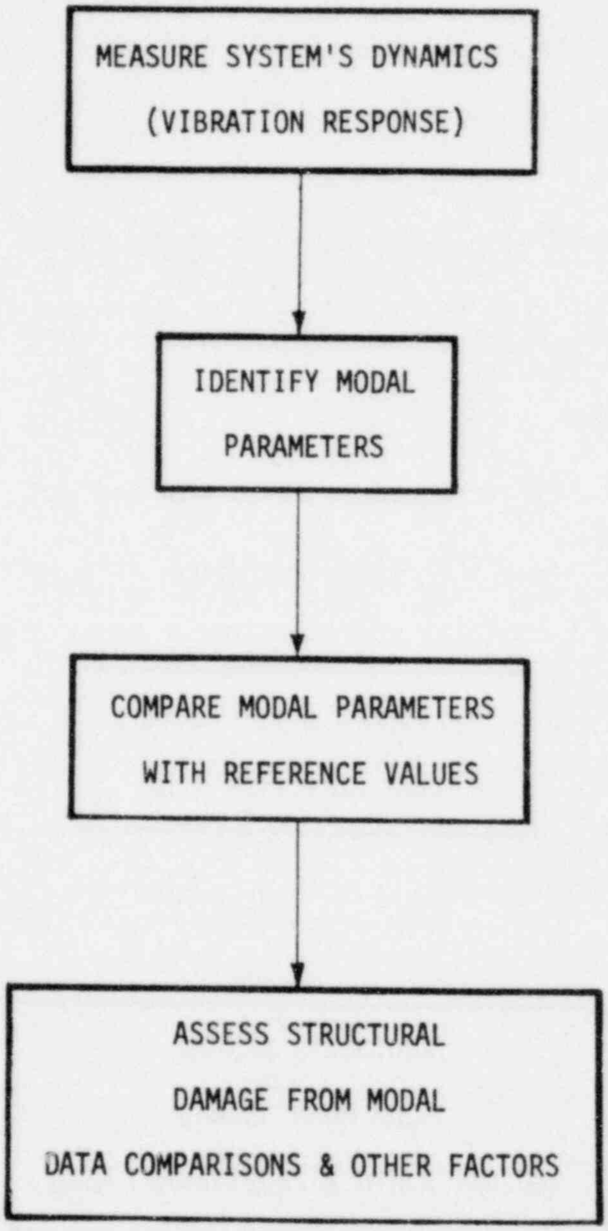


FIGURE 1.1 MAJOR STEPS OF DAMAGE ASSESSMENT

Once the signal has been stored in the form of digital data in a computer's memory, many different processing techniques can be used to identify the modal parameters of the structure. So, the second step of the process is basically a data processing task.

The third step, comparison of the modal parameters with reference values, can also be viewed as a data processing task, accomplished by the computer, or it could be done manually.

The final step, however, is basically a subjective decision making task, since the assessment of acceptable damage must take into account many other factors than simply a shift in the structure's modal parameters.

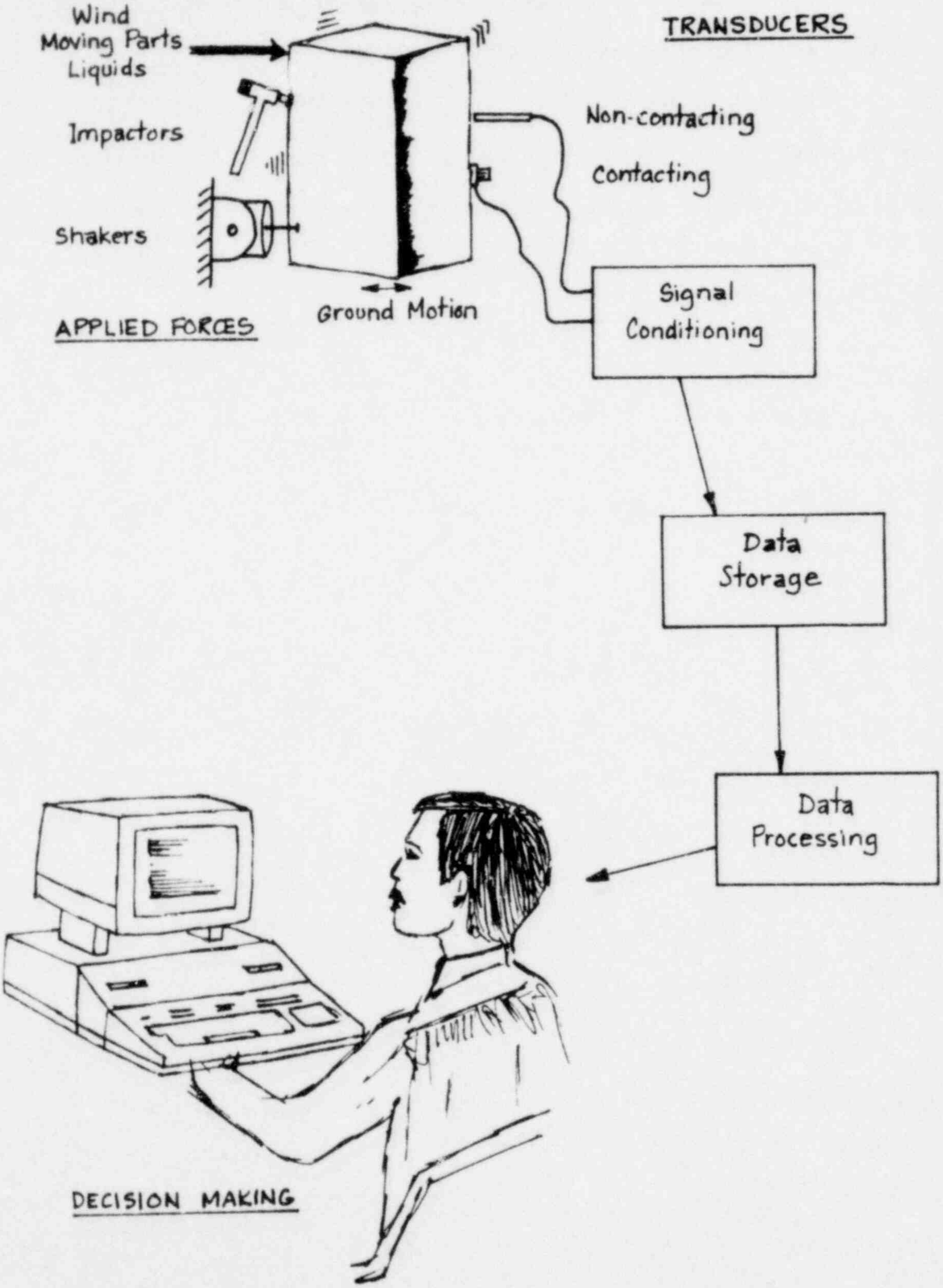
Another way of characterizing this process is to liken it to a medical doctor using a stethoscope to listen to a patient's heartbeat as an aid to determining whether he or she is healthy or not.

Just as the heartbeat does not give a complete picture of a person's health, vibration signals from a mechanical structure will probably never conclusively reveal whether or not it has undergone significant damage. However, it is generally agreed that vibration signals are a useful indicator of physical damage in many cases.

Implementation of this process involves the proper choice and location of motion sensitive transducers, electronic signal conditioning, data acquisition, storage, and processing using digital computers, and some type of strategic decision making process involving both machines and human beings. This last step will certainly involve some amount of qualitative judgement before a final decision is reached. This process is depicted in figure 1.2 .

Figure 1.2

Vibration Monitoring Process



1.5 SUMMARY AND RECOMMENDATIONS FOR FURTHER WORK

During the course of this survey we discovered that there has been a considerable amount of research directed at the broad problem of assessing structural integrity. However, we found few examples of cases where structures (especially nuclear power plant structures) were being monitored using modal parameters as the primary discriminants. The majority of the monitoring cases we discovered recognized the need for this type of monitoring but were generally still measuring only ambient vibration data with little or no data processing to detect vibrational changes. The most sophisticated work appears to have been done with offshore platforms, where efforts are directed specifically at monitoring structural integrity via modal parameter identification methods.

We found that a variety of testing techniques have been developed for exciting structures with a controlled force input, and that they are sufficiently accurate for identifying the modes of many types of nuclear power plant structures. It was also found that ambient vibration measurements (i.e. no artificially applied forces) can be used to identify modes.

While we felt that the greatest amount of the work done to date has been on the refinement of excitation methods, in our opinions, refinement of the associated data processing and parameter estimation methods were noticeably lacking. For example, many articles told of using either the "half-power point method" or the "log decrement method" to estimate the percent critical damping in the structure. Estimates using these methods had a large amount of variance and generally yielded only "ball park" estimates.

Although a number of more sophisticated modal parameter estimation (i.e. curve fitting) algorithms have been developed in recent years, and many of them are referenced here, there is still a need to substantiate their usefulness in real world monitoring situations.

Another area of investigation which has received little attention is determining the sensitivity of modal parameter changes to structural failures in certain classes of structures such as containment vessels, condensors, pumps, etc. in nuclear power plants. There are, however, some current research efforts being conducted on offshore platforms to determine the sensitivity of modal parameters to structural failure.

Dr. Paul Ibanex and co-workers at ANCO Engineers, Santa Monica, California are currently working on the combined use of finite element computer modeling, forced vibration testing and parameter identification techniques for monitoring the structural integrity of offshore platforms.

Dr. Sheldon Rubin and co-workers at Aerospace Corporation have recently completed a computer study of the sensitivity of modal parameter changes to failures in certain standard types of offshore platform trusses. These results will be available from the U.S. Geological Survey in the near future.

In conclusion, it has been made clear to us through this study that the feasibility of monitoring modal parameters for determining structural damage has been demonstrated in a number of isolated instances.

The application of this technology to nuclear power plant structures has not as yet been clearly demonstrated, however. In order to move closer to the accomplishment of this goal, further work is recommended in the following areas.

1) Controlled Tests: Systematically modify various types of nuclear power plant structures and monitor their modal parameters to determine if and when structural degradation can be detected.

2) Conduct experiments to monitor modal parameters from structures being excited by various types of ambient forces (e.g. wind, fluid, seismic, machinery) to determine the quality of the data that can be derived with these kinds of forcing functions.

3) Investigate various curve fitting algorithms for identifying modal parameter estimates from power spectrum, impulse response or transfer function data taken from nuclear plant structures.

4) Investigate new methods for simple and economical in-situ forced excitation of nuclear power plant structures.

5) Begin a continuous data collection and modal parameter monitoring program at several nuclear power plants to build up a base of historical data relating modal parameter changes to actual failures.

2.0 MODAL ANALYSIS - A TUTORIAL REVIEW

The theory of modal analysis has been well developed for many years. There are many text books on the subject, and most university courses on dynamic analysis nowadays include a treatment of the basic concepts of modal analysis.

In this section we review all of the basic concepts, and much of the terminology associated with modal analysis. There are a number of simplifying assumptions associated with the use of modal analysis. These assumptions can be satisfied in a large number of test situations, but they must be kept in mind during a modal test since they can be easily violated when testing complex structures.

Modal Analysis is the process of characterizing the dynamics of an elastic structure in terms of its modes of vibration. Physically speaking modes of vibration are the so called "natural" frequencies at which a structure's predominant motion is a well defined waveform, as shown in Figure 2.1.

Mathematically speaking modes of vibration are defined by certain parameters of a linear dynamic model. As shown in Figure 2.2 each mode of vibration is defined by a resonant frequency, a damping factor and a mode shape. It will be shown later that a dynamic model can be completely represented in terms of these parameters.

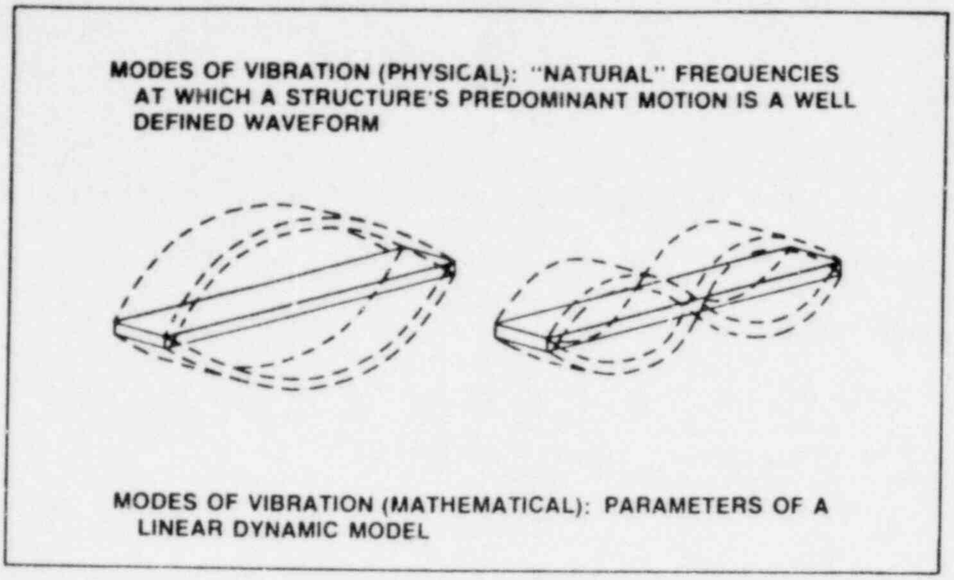


FIGURE 2.1

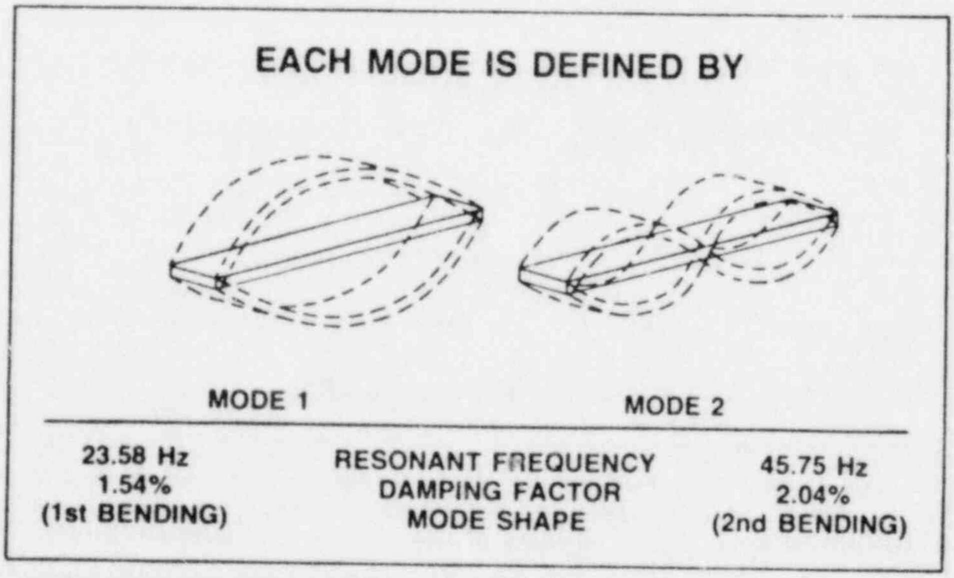


FIGURE 2.2

The purpose of modal testing, then, is to artifically excite a structure so that the frequencies, damping and mode shapes of its predominant modes of vibration can be identified.

2.0.1 MODES OF VIBRATION AND LINEAR DYNAMICS

One of the advantages, and apparent limitations, of modal analysis is that modes of vibration are only defined for linear dynamical systems. Hence, all of the developments of analysis and measurement techniques discussed here apply to structures undergoing linear dynamic motion only. Now, it is well known that most structures will distort, i.e., behave in a non-linear manner, if excited with sufficiently large forces. For instance, several of the references we reviewed (Refs. F.6, F.7 & F.15) are concerned with accurately modeling the non-linear dynamics of structures so that their response to earthquakes can be predicted.

Nevertheless, most structures will behave in an approximate linear manner if excited at low levels. Furthermore, using modern-day measurement and data processing methods, modal parameters can be accurately identified from low level vibration signals.

Hence, the application of modal analysis to low level vibration signals offers a sound measurement and analysis approach for detecting structural damage, even though the failure process itself may be highly non-linear.

2.1 MODELING STRUCTURAL DYNAMICS

For the purposes of modeling their dynamics, mechanical structures can be divided into two classes; RIGID bodies and ELASTIC bodies. Although most structures are elastic throughout, many times analysis of their dynamics can be greatly simplified without significant loss of accuracy by assuming that they behave as if they were an assemblage of rigid masses connected together by spring and damper elements. (A simple mass, spring, damper system is shown in Figure 2.3) Rigid body systems (also called lumped parameter systems) can be analyzed in a straightforward manner by applying Newton's Second Law to all masses in the system. By performing a "force balance" on the system, that is by setting all forces acting on each mass equal to its mass multiplied by its acceleration, a set of differential equations is derived which completely describes the dynamics of the system. These equations can then be solved analytically to yield structural responses to specified external forces and boundary conditions.

However many dynamics problems cannot be solved by using a rigid body approximation of the structure. These problems require that the distributed elastic behavior of the structure be modeled more accurately.

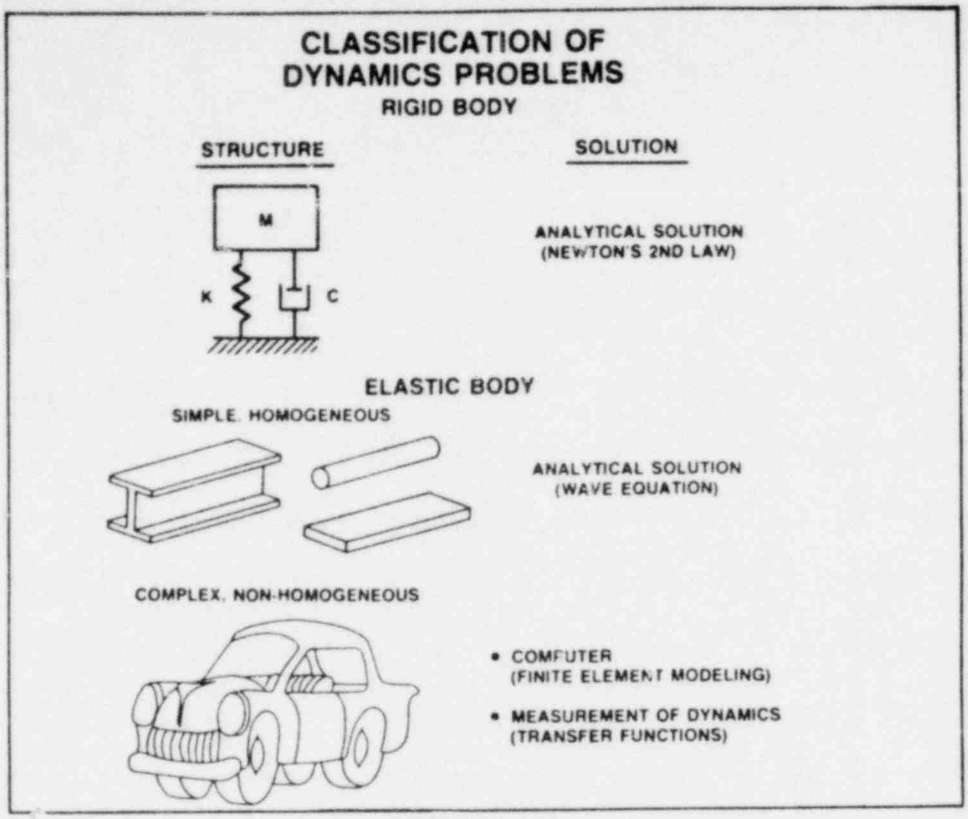


FIGURE 2.3

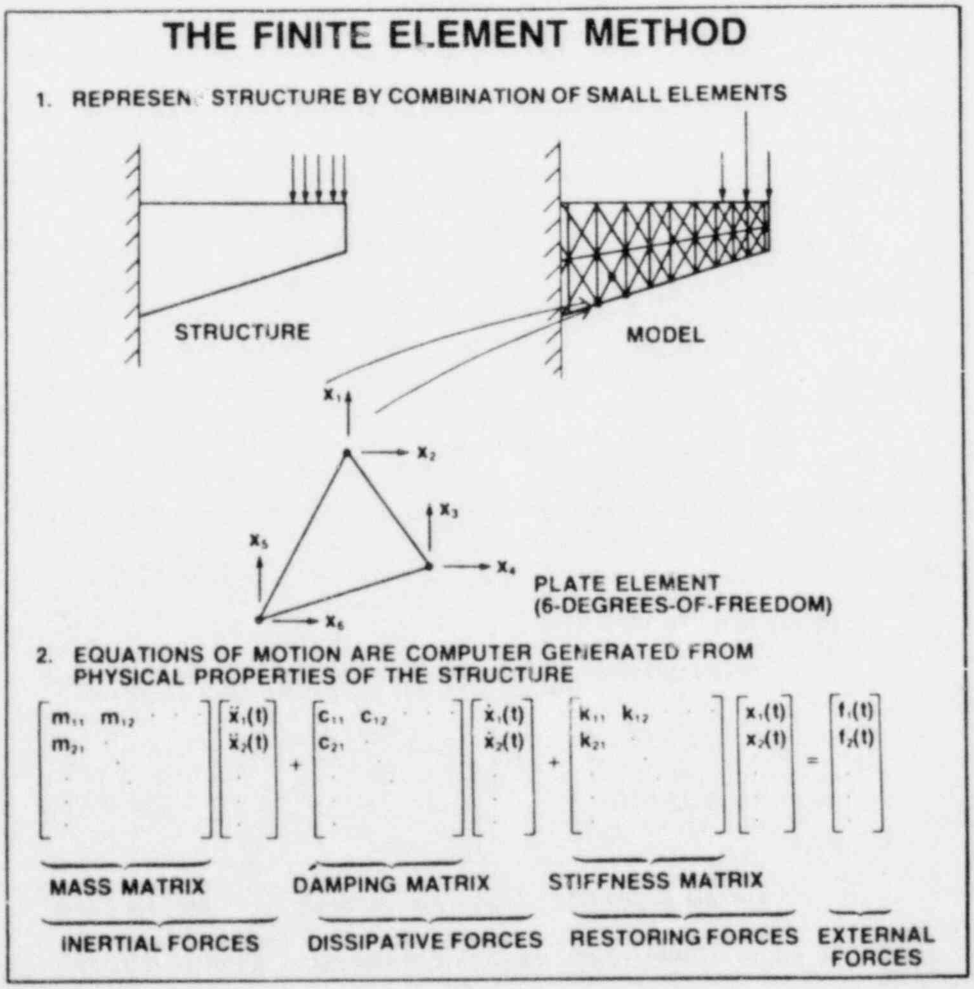


FIGURE 2.4

If the structure has a simple geometric shape and its physical properties (eg. density and elasticity) are more or less uniform throughout, then a partial differential equation of the form known as the "wave equation" can be used to describe its dynamics. There are well known solutions to the wave equation for many types of simple mechanical structures, such as beams, shafts and plates. However the approximations required in order to apply these analytical methods are often too restrictive to adequately describe the dynamics of complex structures.

The requirement for a more generalized method for modeling the dynamics of large, complex structures with non-homogeneous physical properties has brought about the development in recent years of the finite element modeling method.

2.1.1 THE FINITE ELEMENT METHOD

The objective of the finite element method is to sub-divide a structure into an assemblage of many smaller elements such as plates, beams, shafts, etc. Then the overall equations of motion of the structure are constructed from equations describing the motions of each of the individual elements, plus all the boundary conditions at the connection points between elements. (This process is depicted in Figure 2.4)

A primary advantage of this approach is that it has been computerized, and readily available programs such as the NASTRAN program, which was initially developed by NASA (The National Aeronautics and Space Administration of the United States) to assist in the design of space vehicles, can be used to build very large dynamic models of complex structures.

As shown in Figure 2.4, the differential equations of motion are a set of simultaneous, second order differential equations which represent a "force balance" among the inertial, dissipative, restoring and externally applied forces on the structure.

Note that the space variable x(t) has been "discretized" in this modeling process. That is, a finite number of degrees-of-freedom* have been chosen to represent the motion of the structure. This approximation removes all derivatives with respect to the space variable from the equations and reduces them to a "generalized" statement of Newton's Second Law.

Although these equations are of the same form as those obtained by using rigid body analysis, the mass, stiffness, and damping coefficient matrices do not necessarily contain mass values, spring constants and damping coefficients which are readily associated with lumped elements on the structure. Rather, these matrices can be viewed simply as coefficients which are necessary to satisfy force balances between the various finite elements of the structure.

* - a degree-of-freedom is motion at a point in a particular direction.

Once the mathematical model has been built, (i.e. the mass, stiffness and damping matrices have been synthesized), the equations of motion can be solved, again by using computer methods.

A popular method of solving the equations of motion is to "diagonalize" them. This is done by finding the "eigenvalues" and "eigenvectors" of the equations. A commonly used approach is to assume that the damping forces on the structure are negligible, and can therefore be ignored. The equations of motion, minus the damping terms, can be transformed to a new coordinate system, called "generalized" coordinates, and written in diagonal or uncoupled form as shown in Figure 2.5. The transformation relating the generalized coordinates to the actual degrees-of-freedom of the structure is a matrix, the columns of which are the eigenvectors of the system. Once the equations of motion are in diagonal form it is much easier to solve them to obtain the structure's response to externally applied forces. This model is known as a "normal mode" model.

2.1.2 USES OF THE MATHEMATICAL MODEL

A finite element model can be used to perform several different types of analysis. They include: LOAD ANALYSIS, MODAL ANALYSIS, "WHAT IF" INVESTIGATIONS and DYNAMIC SIMULATION.

THE FINITE ELEMENT METHOD

3. THE EQUATIONS OF MOTION ARE "DIAGONALIZED" SO THAT RESPONSES TO EXTERNAL FORCES CAN BE SIMULATED
   - DIAGONALIZATION INVOLVES FINDING THE "EIGENVALUES" AND "EIGENVECTORS" OF THE EQUATIONS OF MOTION
   - DISSIPATIVE (DAMPING) FORCES ARE USUALLY IGNORED

$$\begin{bmatrix} m_1 & 0 & \cdots \\ 0 & m_2 & \\ \vdots & & \ddots \end{bmatrix} \begin{bmatrix} \ddot{q}_1(t)\\ \ddot{q}_2(t)\\ \vdots\end{bmatrix} + \begin{bmatrix} k_1 & 0 & \cdots \\ 0 & k_2 & \\ \vdots & & \ddots\end{bmatrix}\begin{bmatrix} q_1(t)\\ q_2(t)\\ \vdots\end{bmatrix} = \begin{bmatrix} u_{11} & u_{21} & \cdots\\ u_{12} & \ddots & \\ \vdots & & \ddots\end{bmatrix}\begin{bmatrix} f_1(t)\\ f_2(t)\\ \vdots\end{bmatrix}$$

GENERALIZED MASSES — GENERALIZED COORDINATE ACCELERATION — GENERALIZED STIFFNESSES — GENERALIZED COORDINATES — EIGENVECTOR MATRIX

$$\begin{bmatrix} x_1(t)\\ x_2(t)\\ \vdots\end{bmatrix} = \begin{bmatrix} u_{11} & u_{12} & \cdots\\ u_{21} & \ddots & \\ \vdots & & \ddots\end{bmatrix}\begin{bmatrix} q_1(t)\\ q_2(t)\\ \vdots\end{bmatrix}$$

FIGURE 2.5

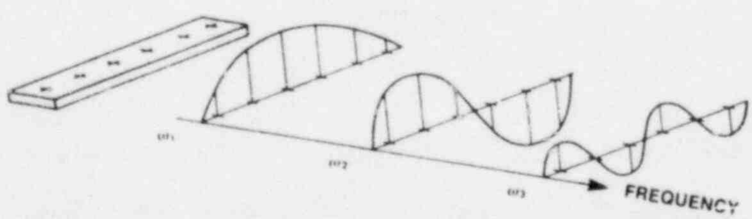
USES OF MATHEMATICAL MODEL

1. LOADS ANALYSIS: EXTERNAL FORCES ⇨ INTERNAL STRESSES, STRAINS

2. MODAL ANALYSIS: IDENTIFICATION OF RESONANT FREQUENCIES OF VIBRATION AND AMPLITUDES OF VIBRATION AT RESONANT FREQUENCIES

   EIGENVALUES ⇨ RESONANT FREQUENCIES

   EIGENVECTORS ⇨ AMPLITUDES OF VIBRATION (MODAL VECTORS, MODE SHAPES)



3. "WHAT IF" INVESTIGATIONS: WHAT HAPPENS IF MASS AND/OR STIFFNESS IS ADDED TO OR REMOVED FROM STRUCTURE (MODAL SYNTHESIS)

4. DYNAMIC SIMULATION: STRUCTURE RESPONSE TO REAL WORLD EXTERNAL FORCES (FATIGUE PREDICTION)

FIGURE 2.6

LOAD ANALYSIS is basically an analysis of the internal stresses and strains in a structure due to external loads. If the structure is in a static condition (i.e. all accelerations are zero) then the equations of motion reduce to a force balance between the internal restoring forces and externally applied forces. By performing analyses with these equations, areas of high static stress or strain can be located on the structure. In a similar manner dynamic stress and strain levels can be analyzed if the inertial terms are also included in the equations of motion.

MODAL ANALYSIS is defined as the process of characterizing the dynamics of a structure in terms of its modes of vibration. It turns out that the eigenvalues and eigenvectors of the previously defined normal mode mathematical model are also parameters which define the resonant frequencies and mode shapes of the modes of vibration of the structure. (This is shown in Figure 2.6) That is, the eigenvalues of the equations of motion correspond to frequencies at which the structure tends to vibrate with a predominant, well defined deformation. The amplitude of this wave motion on the structure is specified by the corresponding eigenvector. Each mode of vibration, then, is defined by an eigenvalue (resonant frequency) and corresponding eigenvector (mode shape). If the dynamic model has n-degrees-of-freedom then it also has n-eigenvalue-eigenvector pairs, or n-modes of vibration.

Knowing the modes of vibration of a structure is useful information in itself, for it tells at what frequencies the structure can be excited into resonant motion, and the predomonant wave-like motion it will assume at a resonant frequency. In many cases, this information is sufficient for modifying the structural design in order to reduce noise and vibration.

WHAT IF INVESTIGATIONS can be conducted using a finite element model to determine how changes in the mass or stiffness of the structure will affect its dynamic characteristics. These types of investigations can be made using the mathematical model long before the first prototype structure is even built. This way, any deficiencies in the design can be spotted early in the design cycle where changes are a lot less costly than in the later stages. This capability is perhaps the single most important advantage of finite element modeling.

A finite element model can also be used for SIMULATION of the dynamic response of the structure to real world external forces. These forces might be of short duration and high amplitude, i.e. impulsive in nature, in which case they could cause immediate damage to the structure. Or they may be of long duration and cyclic in nature, and could cause fatigue damage to the structure over a long period of time.

The simulated response of the mathematical model could be used as input to fatigue damage prediction algorithms, thus giving information about the fatigue life of a prototype design even before the first prototype is constructed.

Finite element modeling does however have two major disadvantages. Most finite element computer programs are very large in size, and require large computers with lots of memory in which to operate. Hence, it is not unusual to spend tens of thousands of dollars to develop a single finite element model. To obtain the required accuracy, models containing several thousand degrees-of-freedom are not uncommon. Models of this size require many man-hours of effort to develop, debug, and operate.

A second disadvantage of finite element modeling is that the dynamic response of the model can differ substantially from that of the actual structure. This can occur because of errors in entering model parameters, but can also occur when the finite elements do not approxi :e the real world situation well enough. Many times the model will turn out to be much stiffer than the actual structure. This can be due to the use of an inadequate number of elements or unrealistic boundary conditions between elements.

Both of these disadvantages point to a need for dynamic testing of the structure in order to confirm the validity of the model.

2.1.3 WHY DYNAMIC TESTING?

Not only is dynamic testing necessary in order to check a finite element model, but some other advantages can be gained from testing, as shown in Figure 2.7.

WHY DYNAMIC (MODAL) TESTING?

1. TO CONFIRM ANALYTICALLY-DERIVED DYNAMIC MODELS
2. TO TROUBLESHOOT VIBRATION PROBLEMS
3. TO EVALUATE DESIGN "FIXES" TO STRUCTURES
4. TO CONSTRUCT A DYNAMIC MODEL FOR PARTS OF A STRUCTURE TOO DIFFICULT TO MODEL ANALYTICALLY

FIGURE 2.7

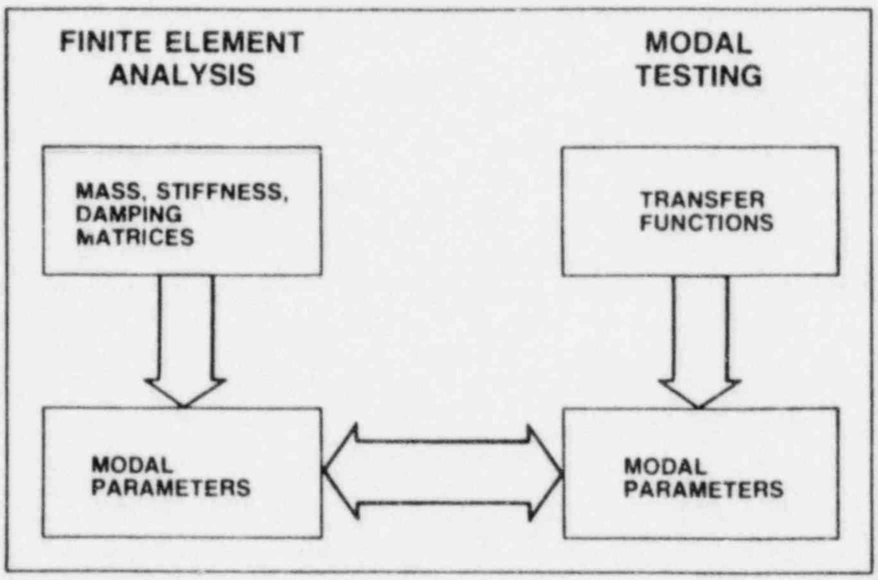


FIGURE 2.8

Dynamic testing can be used for troubleshooting noise and vibration problems in existing mechanical systems. These problems can occur because of errors in the design or construction of the system, or they could occur as a result or wearout, failure or malfunction in some of its components. Not only can testing be used to locate a problem, but it can also be used to evaluate fixes to the problem. Finally dynamic testing can be used to construct a dynamic model for components of a structure which are too difficult to model analytically.

In all the cases mentioned above, the objective of the dynamic testing procedure is to excite and identify the test specimen's modes of vibration. As shown in Figure 2.8 the common element between finite element modeling and dynamic, (or modal) testing is the modal parameters of the structure.

2.1.4 TIME DOMAIN MODEL

In a measurement situation the actual input forces and responses for a finite number of degrees-of-freedom of the structure are measured. So if a model were constructed from the measurements involving these specific degrees-of-freedom, the model would give an accurate description of the structural dynamics involving those points. This is a different situation than with a finite element model where the degrees-of-freedom and the size and shape of the elements are chosen so as to approximate the dynamics of the structure as closely as possible.

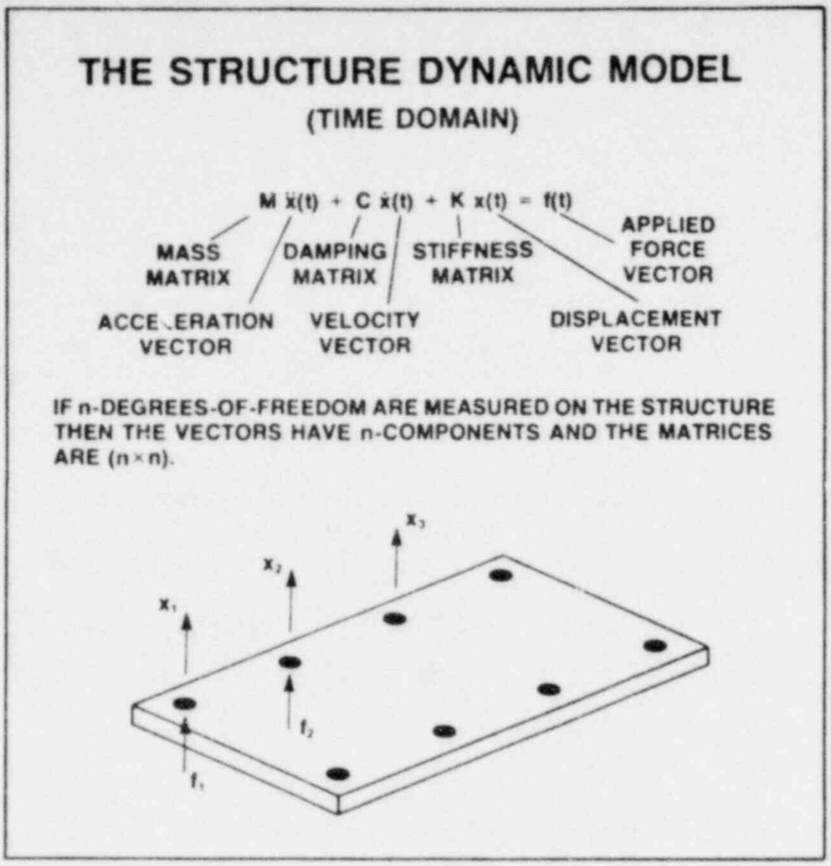


FIGURE 2.9

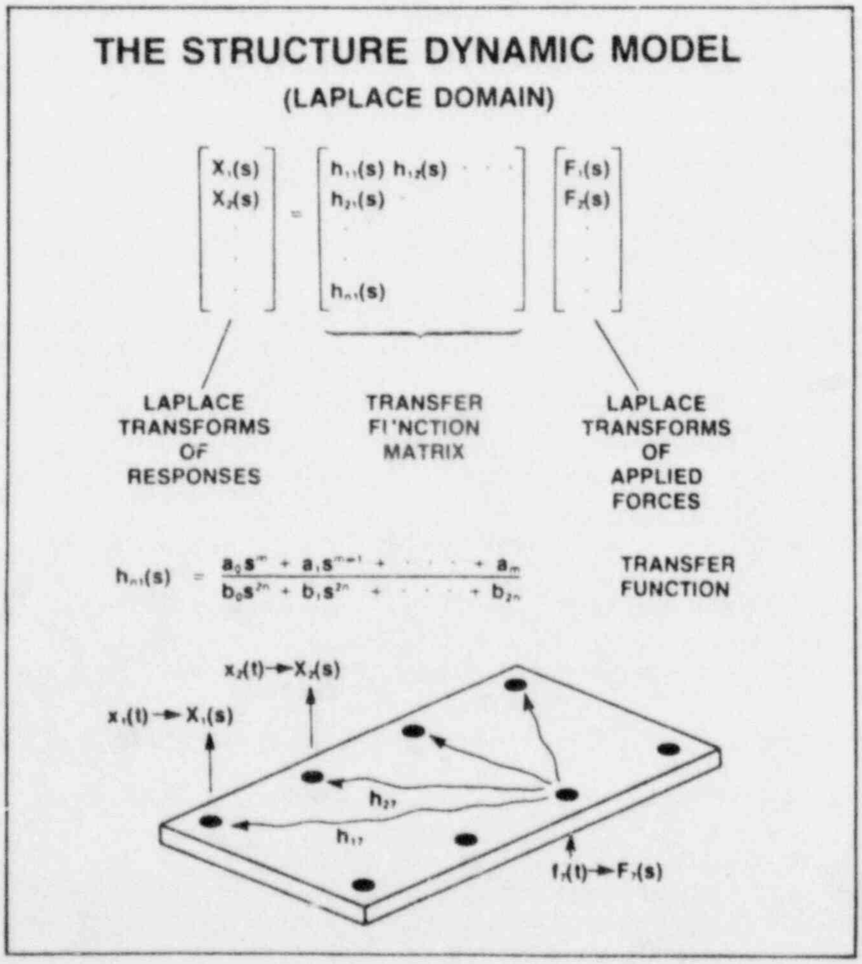


FIGURE 2.10

The time domain structural dynamic model, as shown in Figure 2.9 exhibits the same form as the finite element model but, at least in principle, is an exact model of the structural dynamics if obtained from measurements.

2.1.5 Laplace Domain Model

In modal testing we typically do not directly identify parameters of the time domain model of Figure 2.9, but rather parameters of its Laplace domain equivalent, shown in Figure 2.10.

In this model the inputs and responses of the structure are represented by their Laplace transforms. Time domain derivatives (i.e. velocity and acceleration) do not appear explicity in the Laplace domain model but are accounted for in the transfer functions. The transfer matrix contains transfer functions which describe the effect of an input at each degree-of-freedom (D.O.F.) upon the response at each D.O.F. Because the model is linear, the transformed total motion for any D.O.F. is the sum of each transformed input force multiplied by the transfer function between the input D.O.F. and the response D.O.F. For example the response of D.O.F. number 2 in Figure 2.10 is:

$$X_2(s) = h_{21}(s)F_1(s) + h_{22}(s)F_2(s) + \cdot \cdot \cdot \cdot + h_{2n}(s)F_n(s)$$

All of the concepts (mathematical and dynamical) associated with this seemingly complex multiple - D.O.F. (MDOF) model can be better understood by examining in detail a single - D.O.F. (SDOF) system.

2.2 SINGLE DEGREE OF FREEDOM SYSTEMS

A single degree-of-freedom (SDOF) system is any system whose motion can be described by a single coordinate.

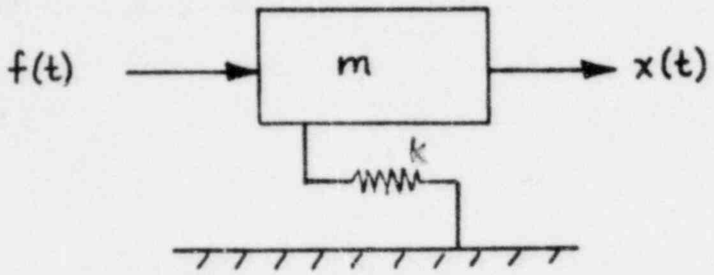


Figure 2.11

where: m = mass of the system

k = spring stiffness constant

x(t) = system response (displacement)

f(t) = applied force

The equation of motion of the SDOF system represented by Figure 2.11 is:

$$m\ddot{x}(t) + kx(t) = f(t) \qquad (2.1)$$

where $\ddot{x}(t)$ = response acceleration.

The equation of motion for free vibration of the system (i.e. f(t)=0) completely defines the system's dynamic characteristics.

$$m\ddot{x}(t) + kx(t) = 0 \qquad (2.2)$$

To solve this differential equation, we assume a solution of the form $x = Ae^{\alpha t}$ and substitute it into equation (2.2) yielding

$$A(m\alpha^2 + k)e^{\alpha t} = 0 \tag{2.3}$$

The unknown is then determined from the equation

$$m\alpha^2 + k = 0 \tag{2.4}$$

This equation is defined to be the system's <u>characteristic</u> <u>equation</u>. The roots of this equation are the system's resonant frequencies. For an undamped system such as this the solution for the free vibration response is:

$$x(t) = A_1 e^{\alpha_1 t} + A_2 e^{\alpha_2 t} = B_1 \cos(\Omega_0 t) + B_2 \cos(\Omega_0 t) \tag{2.5}$$

where:

$$\alpha_1 = j\sqrt{\frac{k}{m}} \quad , \quad \alpha_2 = -j\sqrt{\frac{k}{m}} \tag{2.6}$$

The <u>undamped natural frequency</u> (Ω_0) of this system is now defined as

$$\Omega_0 = \sqrt{\frac{k}{m}} \tag{2.7}$$

The response of the system in free vibration is said to be harmonic, and the constants B_1 and B_2 are determined from the initial displacement and velocity of the system. When equivalent viscous damping is added to the system the equations of motion become

$$m\ddot{x}(t) + c\dot{x}(t) + kx(t) = f(t) \tag{2.8}$$

where C = damping constant

$\dot{x}(t)$ = system velocity

The system can now be represented by the schematic diagram in Figure 2.12.

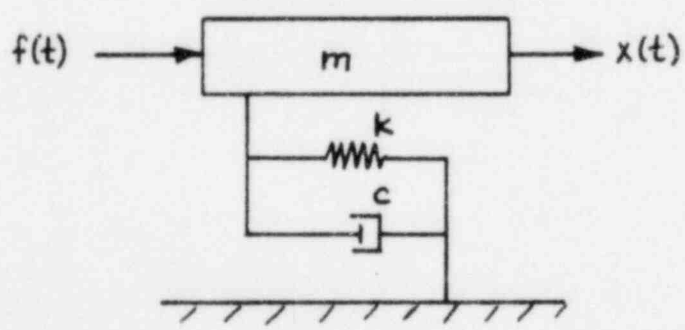


Figure 2.12

The system's characteristic equation now becomes

$$m\alpha^2 + c\alpha + k = 0 \tag{2.9}$$

The solutions to this equation are still referred to as the resonant frequencies of the system

$$\alpha_1 = \frac{-c + \sqrt{c^2 - 4mk}}{2m} \qquad \alpha_2 = \frac{-c - \sqrt{c^2 - 4mk}}{2m} \tag{2.10}$$

When the constants m, c and k are such that the roots are real and negative the response of the system will be that of a damped exponential. Those systems that exhibit this type of a response are defined as overdamped. The double roots of the characteristic equation reduce to one when the quantity under the radical in eqs. (2.10) becomes zero. The amount of damping required to cause this condition is defined as the critical damping (Cc) of the system.

$$C_c = 2\sqrt{mk} = 2m\Omega_0 \tag{2.11}$$

Systems that have critical damping are referred to as critically damped.

When the roots of the characteristic equation become complex (i.e. the quantity under the radical sign in eqs. 2.10 is negative) the response of the system is damped harmonic motion. This type of system is referred to as an underdamped system.

The damping ratio of a structure is used to classify it according to one of the above three categories. The damping ratio (ζ) is the amount of damping in the system divided by the critical damping of the system.

$$\zeta = \frac{c}{c_c} = \frac{c}{2m\Omega_0} \tag{2.12}$$

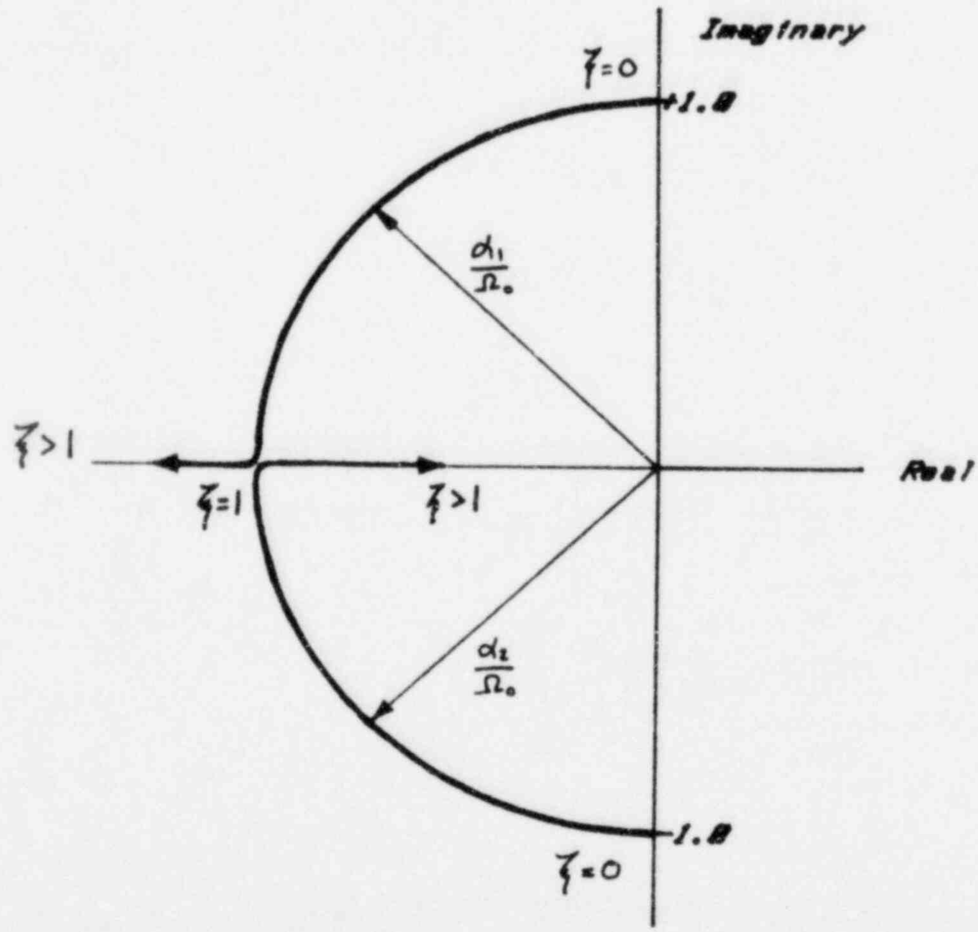
The roots of the characteristic equation can also be written in terms of the damping ratio as:

$$\alpha_1 = \left(-\zeta + \sqrt{\zeta^2 - 1}\right)\Omega_0 \quad , \quad \alpha_2 = \left(-\zeta - \sqrt{\zeta^2 - 1}\right)\Omega_0 \tag{2.13}$$

The damping ratio for the three different types of systems are

1) $\zeta > 1$ overdamped
2) $\zeta = 1$ critically damped
3) $\zeta < 1$ underdamped

If one plots the ratio of the roots of the characteristic equation to the undamped natural frequency (as shown in Figure 2.13) if the damping ratio equals zero the roots start out as totally imaginary; As the damping ratio becomes greater than zero the roots become complex, and when $\zeta = 1$ the two roots coalesce to one real root.



Imaginary
ζ=0
+1.0
$\frac{\alpha_1}{\Omega_0}$
ζ>1
Real
ζ=1
ζ>1
$\frac{\alpha_2}{\Omega_0}$
-1.0
ζ=0

Figure 2.13

The systems that are of primary importance in structural analysis, are those that are underdamped, i.e. $\zeta < 1$. The resonant frequencies of an underdamped system are always complex and can be rewritten as:

$$\alpha_1 = (-\zeta + j\sqrt{1-\zeta^2})\Omega_o \quad , \quad \alpha_2 = (-\zeta - j\sqrt{1-\zeta^2})\Omega_o \qquad (2.14)$$

The free response of the system in Figure 2.12 takes on the following form.

$$x(t) = Ae^{-\zeta\Omega_o t} \sin(\sqrt{1-\zeta^2}\,\Omega_o t + \phi) \qquad (2.15)$$

where the constants A and ϕ depend on the initial conditions of the system. From equation (2.15) we define the damped natural frequency (W_o) of a single degree of freedom system.

$$w_o = \sqrt{1-\zeta^2}\;\Omega_o \qquad (2.16)$$

2.2.1 Transfer Function of a Single Degree-of-Freedom

The Laplace variable is a complex number, normally denoted by $S = \sigma + jw$. Since the transfer function is a function of the S-variable, it too is complex valued.

The equation of motion of a damped SDOF system can be transformed to the S-Domain by using the Laplace transform.

$$L\left[m\ddot{x}(t) + c\dot{x}(t) + kx(t)\right] = L\left[f(t)\right] \tag{2.17}$$

or

$$(ms^2 + cs + k)\,X(s) = F(s) \tag{2.18}$$

where all initial conditions are assumed to be zero.

The transfer function(H(s)) is defined as the ratio the Laplace transform of the response (X(s)) divided by the Laplace transform of the forcing function (F(s)). Or

$$H(s) = \frac{X(s)}{F(s)} = \frac{1}{ms^2 + cs + k} \tag{2.19}$$

or

$$H(s) = \frac{1/m}{s^2 + 2\zeta\Omega_0 s + \Omega_0^2} \tag{2.20}$$

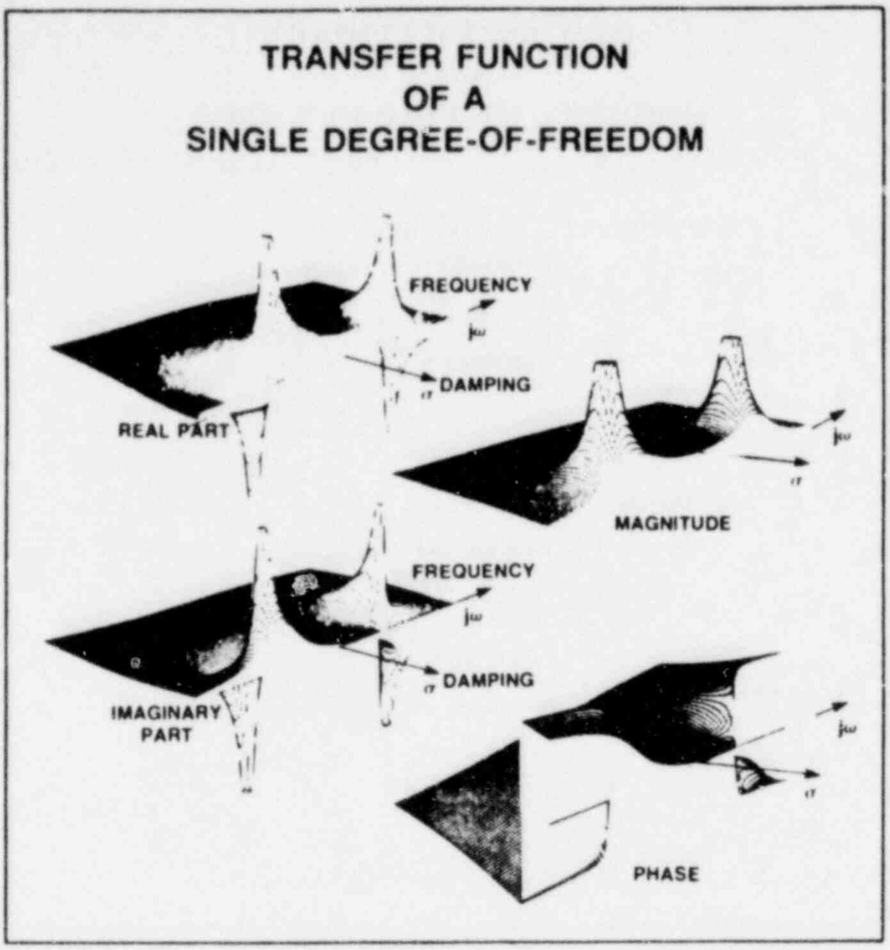


FIGURE 2.14

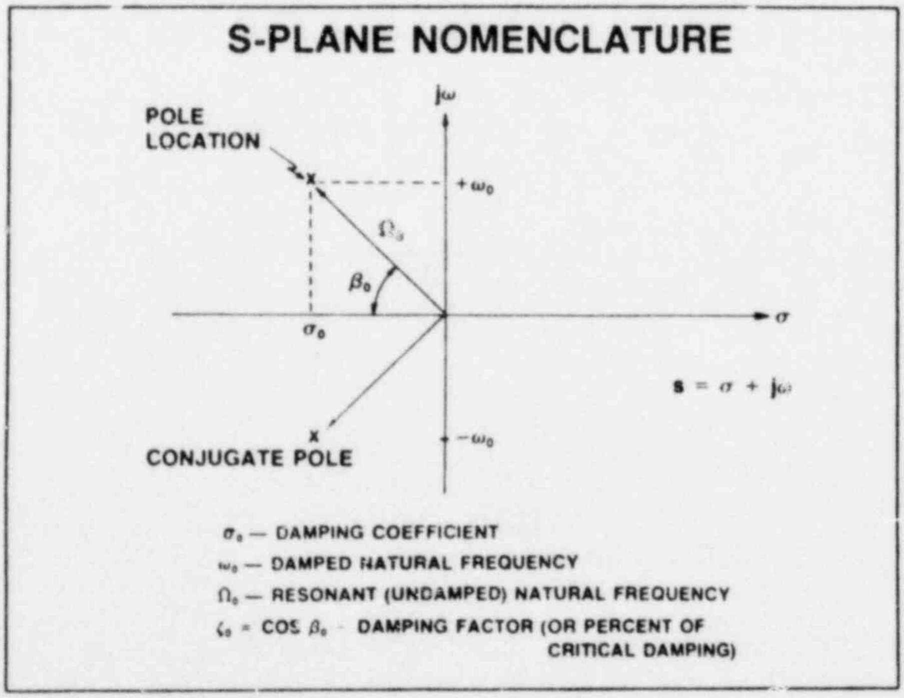


FIGURE 2.15

or:

$$H(s) = \frac{1/m}{(s-p_1)(s-p_2)} \tag{2.21}$$

where:

$$p_1 = (-\zeta + j\sqrt{1-\zeta^2})\Omega_0 = -\sigma_0 + j\omega_0 \tag{2.22}$$

$$p_2 = (-\zeta - j\sqrt{1-\zeta^2})\Omega_0 = -\sigma_0 - j\omega_0$$

Plots of a typical transfer function on the S-plane are shown in Figure 2.14. Because it is complex, the transfer function is represented by its REAL and IMAGINARY parts or equivalently by its MAGNITUDE and PHASE. Note that in this case, the transfer function is only plotted over half of the S-plane, i.e. it is not plotted for any positive values of σ . This was done to give a clear picture of the transfer function values along the $j\omega$-axis. These values will become important later in this development.

Note also that the magnitude of the transfer function goes to infinity at two points in the S-plane. These discontinuities are called the POLES of the transfer function. These poles define resonant conditions on the structure which will "amplify" an input force. The location of each of these poles in the S-plane is defined by a FREQUENCY and DAMPING value as shown in Figure 2.15, and expressed mathematically by equations (2.22). Hence the σ-axis and jw-axis of the S-plane have become known as the damping axis and the frequency axis respectively. The frequency and damping which define a pole in the S-plane are the frequency and damping of a mode of vibration of the structure.

The transfer function if typically expressed in its equivalent partial fraction form as:

$$H(s) = \frac{A_1}{s-p_1} + \frac{A_2}{s-p_2} \tag{2.23}$$

where:

$$A_1 = \frac{1}{2jm\omega_0}, \quad A_2 = A_1^* = \frac{-1}{2jm\omega_0}$$

* - denotes conjugate

The constants A_1 and A_2 are referred to as the residues of the poles p_1 and p_2. Equation (2.23) is conveniently written in standard form as:

$$H(s) = \frac{R}{2j(s-p)} - \frac{R^*}{2j(s-p^*)} \tag{2.24}$$

where: $R = R^* = 1/m\omega_0$

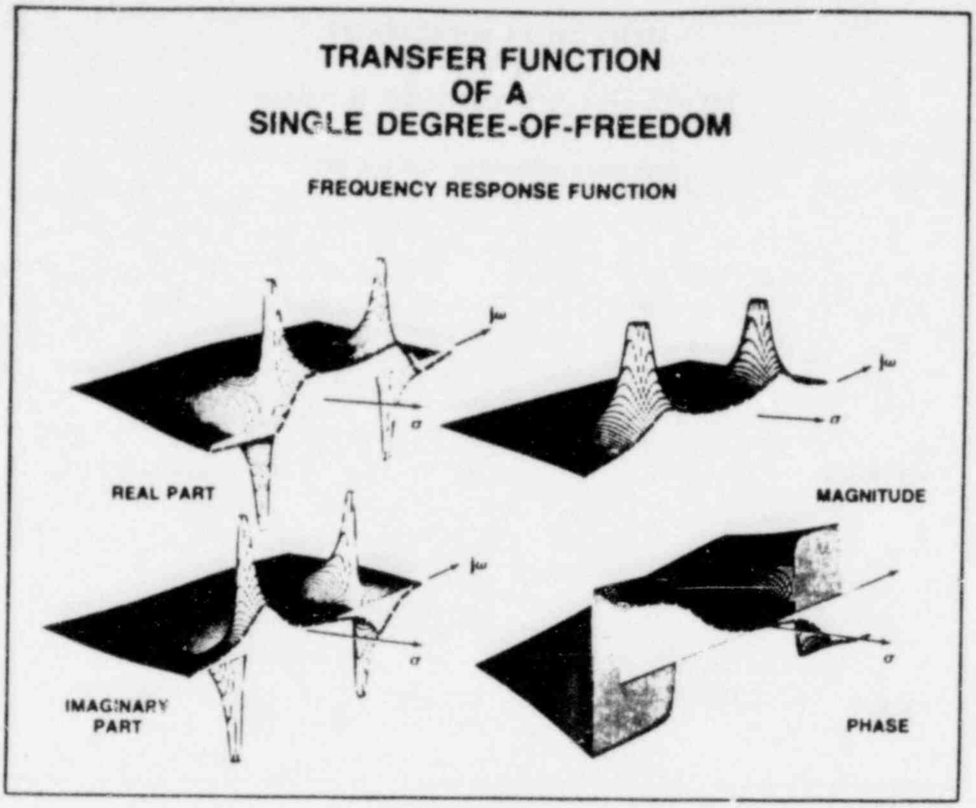


FIGURE 2.16

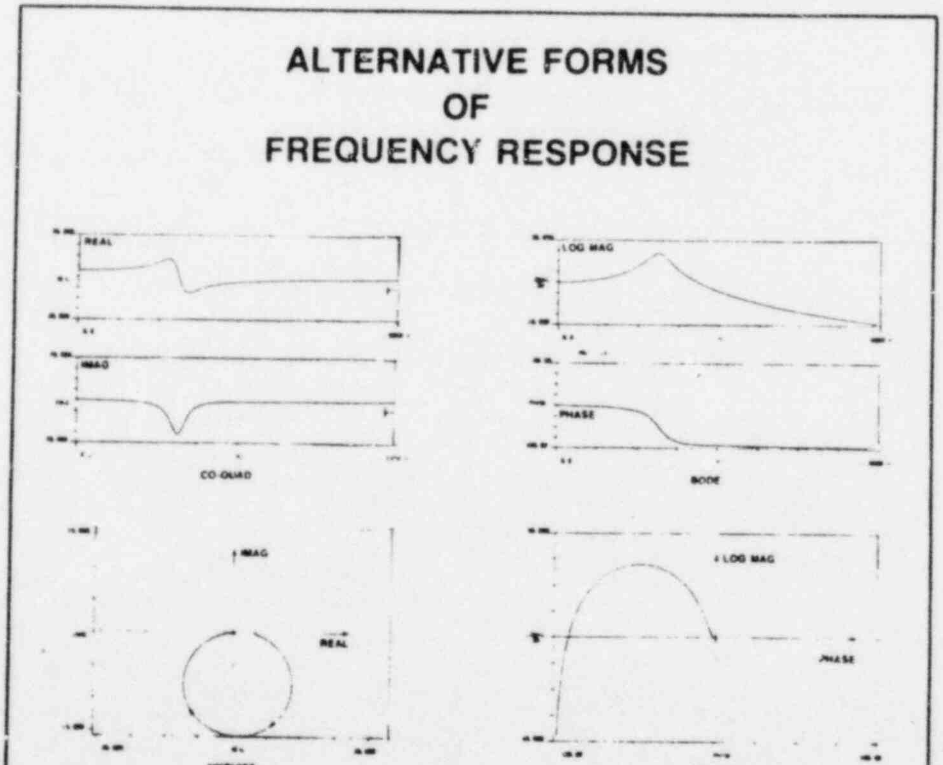


FIGURE 2.17

2.2.2 The Frequency Response Function

In a test situation we do not actually measure the transfer function over the entire S-plane, but rather its values along the jw (or frequency) axis. These values are known as the frequency response function as shown in Figure (2.16).

Since the transfer function is an "analytic" function its values throughout the S-plane can be inferred from its values along the jw-axis. More specifically, if we can identify the unknown modal parameters of a transfer function by "curve fitting" the analytical form (Eq. 2.24) to measured values of the function along the jw-axis, then we can synthesize the function throughout the S-plane.

The Fourier transform of a function is the Laplace transform of that function evaluated along the imaginary axis of the S-plane. The frequency response function of a structure, can be determined (measured) experimentally by measuring an input waveform or function and its corresponding response function. The ratio of the Fourier transform of the response to the input then yields the frequency response function between the input and response points of the structure. The frequency response function of a SDOF system takes the following form:

$$H(s)\Big|_{s=j\omega} = H(\omega) = \frac{X(\omega)}{F(\omega)} = \frac{1/m}{(\Omega_0^2-\omega^2)+j2\zeta\Omega_0\omega} \qquad (2.25)$$

2.2.3 Alternate Forms of the Frequency Response

The frequency response function, being complex valued, is represented by two numbers at each frequency. Figure 2.17 shows some of the alternative forms in which this function is commonly plotted. The so called CO-QUAD plot, or real and imaginary parts, derives its origin from the days of swept sine testing when the real part was referred to as the COincident waveform and the imaginary part as the QUADrature waveform. The Bode plot, or log magnitude and phase vs. frequency, is named after H.W. Bode who made many contributions to the analysis of frequency response functions. (Many of Bode's techniques involved plotting these functions along a log-frequency axis).

The Nyquist plot, or real vs. imaginary part, is named after the gentleman who popularized its use for determining the stability of linear systems. The Nichols plot, or log magnitude vs. phase angle, is named after N.B. Nichols who used such plots to analyze servo-mechanisms.

2.2.4 The Impulse Response

When a system (structure) is subjected to an impulsive type of input the response of the structure is a sum of decaying sinusoids. The impulse response, like the transfer function and frequency response function, contains information about the structure's dynamic characteristics.

To solve for the response of a SDOF system, subjected to an impulse, it is convenient to use equation (2.24). The Laplace transform of the forcing function (i.e. the impulse) is equal to unity in the S-domain. Thus the response of the system in the S-domain becomes

$$X(s) = \frac{R}{2j(s-p)} - \frac{R^*}{2j(s-p)} \tag{2.26}$$

The <u>impulse</u> <u>response</u> <u>function</u> is then determined by taking the inverse Laplace transform of equation (2.26). Hence the impulse response of a damped SDOF system is:

$$h(t) = L^{-1}[X(s)] = Re^{-\sigma_o t} \sin(\Omega_o t + \phi) \tag{2.27}$$

or alternatively:

$$h(t) = \left(\frac{1}{m\sqrt{1-\zeta^2}\,\Omega_o}\right) e^{-\zeta\Omega_o t} \sin\left(\sqrt{1-\zeta^2}\,\Omega_o t + \phi\right) \tag{2.28}$$

(since R is real valued in this case ϕ = 0 degrees)

In the case of a SDOF system (structure) the impulse response is a damped sinusoid. It will be shown later that for structures with multiple degrees of freedom, the impulse response will be a sum of damped sinusoids. The parameters that make up the impulse response contain all the information about the dynamic properties of the structure.

The impulse response of a structure can also be determined by taking the inverse Fourier Transform of the frequency response function.

Figure 2.18 shows a typical SDOF impulse response function.

Experimentally the impulse response of a structure can be determined in two different ways. First the structure could be excited by an "approximate" delta function ; for instance, a hammer blow or an explosive force. The measured response of the structure is then an "approximate" impulse response function. Or secondly, the frequency response of the structure could be measured, using a variety of excitation techniques, and then inverse Fourier transformed to obtain the impulse response.

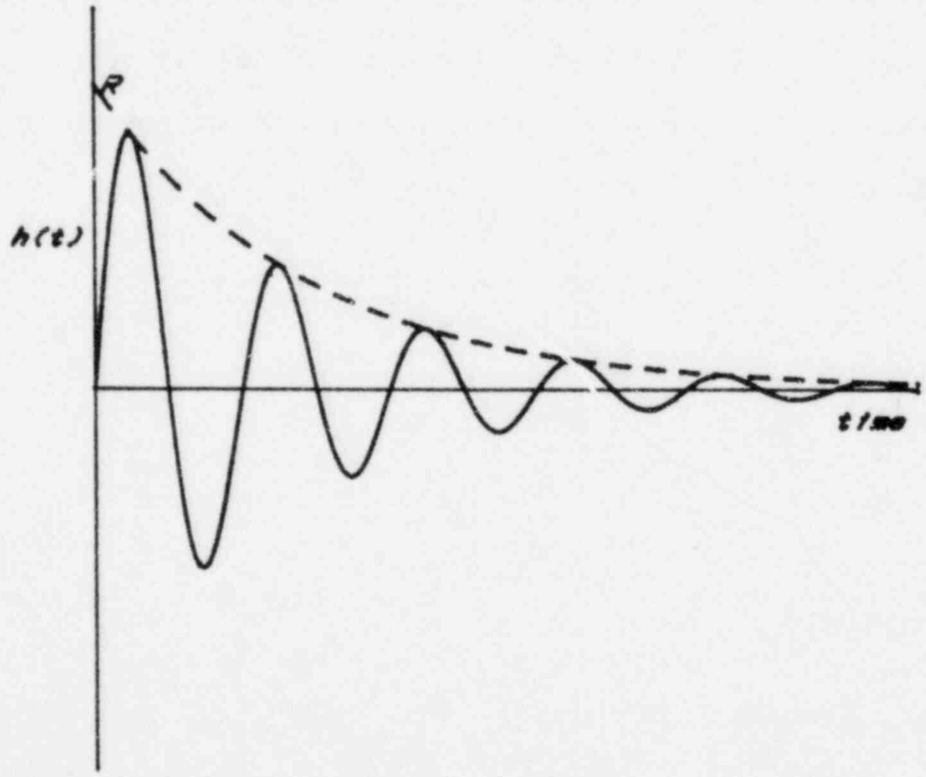


Figure 2.18

Impulse Response Function

2.3 MULTIPLE DEGREE OF FREEDOM (MDOF) SYSTEMS

Multiple degree-of-freedom systems (MDOF's), are those that require 2 or more independent coordinates to describe their motion (position). In general all physical systems (structures) have an infinite number of degrees-of-freedom. However because of their associated size and complexity, most structures are modeled (analytically or experimentally) using a finite number of degrees-of-freedom. The model is an "approximate" model because of the finite number of degrees-of-freedom used to describe the motion of the structure. Figure 2.19 is an idealized lumped mass model of a building where only the horizontal translational degrees of freedom of the floors have been retained.

The equations of motion for the free vibration of the structure in Figure 2.20 can be expressed in matrix form as:

$$[M][\ddot{x}(t)] + [C][\dot{x}(t)] + [K][x(t)] = [0] \qquad (2.29)$$

The solution of the undamped equations of motion([C] = [0])for free vibration can be found by assuming a solution of the form.

$$[x] = [u]e^{pt}$$

Substituting into equation (2.29) yields

$$[p^2[M] + [K]][u]e^{pt} = [0] \qquad (2.30)$$

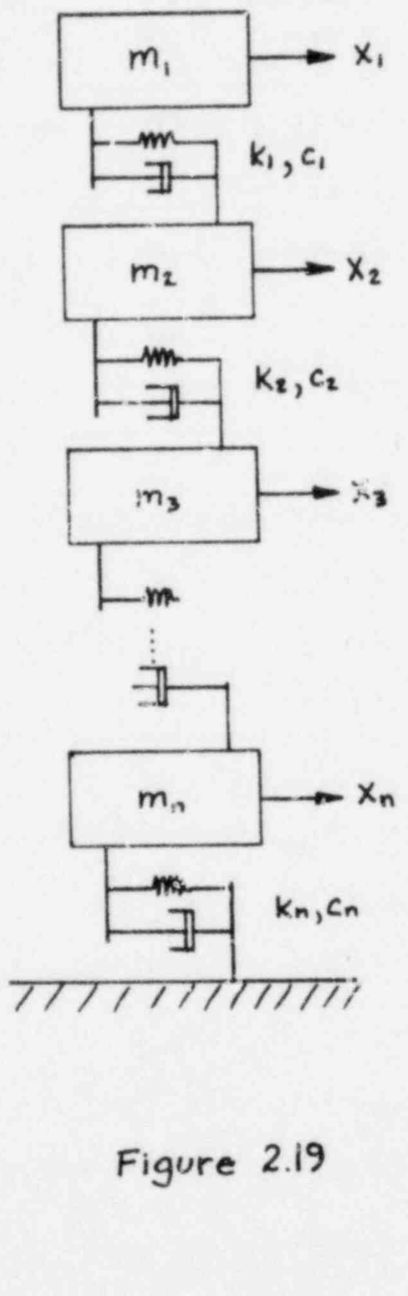


Figure 2.19

Equation (2.30) is referred to as an"eigenvalue problem", where the unknown p is defined as the eigenvalue or pole, and the solution vector [u] is the corresponding eigenvector. The eigenvalue is also called the modal frequency and the eigenvectors are called mode shapes or modal vectors.

The set of homogeneous equations (equ. 2.30) have a non-trival solution if the determinant of the coefficient matrix equals zero. That is:

$$\left| p^2[M] + [K] \right| = 0 \tag{2.31}$$

Equation (2.31) is a polynomial in p and is defined as the characteristic equation of the system. The roots of this characteristic equation are the modal frequencies (eigenvalues) of the system. Once the eigenvalues are known they can be substituted into equation (2.30) to determine the mode shapes. There is an unique mode shape for every distinct eigenvalue. Since equation (2.30) is a set of homogeneous equations, there is no unique solution in terms of the [u]. Therefore the solution vectors can only be determined to within an arbitrary constant. This fact emphasizes that the mode shapes of a structure are arbitrary in terms of amplitude and that only the ratio of the mode shape components is unique. As in the SDOF case, the roots of the characteristic equation (equ. 2.31) appear in complex conjugate pairs. The mode shapes of the system likewise occur in conjugate pairs.

2.3.1 Orthogonality of Modes

The modal frequencies p and mode shapes [u] satisfy equation 2.30. Therefore substituting the model frequency p_r along with its corresponding mode shape u_r into equ. 2.30 yields

$$-p_r^2[M][u_r] = [K][u_r] \tag{2.32}$$

Another solution to the eigenvalue problem is p_s with its corresponding mode shape $[u_s]$ yielding

$$-p_s^2[M][u_s] = [K][u_s] \tag{2.33}$$

Now, if we premultiply equation (2.32) by the mode shape transposed $[u_s]^t$ and equation (2.33) by $[u_r]^t$ gives

$$-p_r^2[u_s]^t[M][u_r] = [u_s]^t[K][u_r] \tag{2.34}$$

$$-p_s^2[u_r]^t[M][u_s] = [u_r]^t[K][u_s] \tag{2.35}$$

Takeing the transpose of equation (2.35) and subtracting the results from equation (2.34) gives

$$(p_r^2 - p_s^2)[u_s]^t[M][u_r] = 0 \tag{2.36}$$

Since in general the two modal frequencies are not the same value, we have the following conclusion:

$$[u_s]^t [M][u_r] = 0 \tag{2.37}$$

Similarly one can show that

$$[u_s]^t [K][u_r] = 0 \tag{2.38}$$

Equations (2.37) and (2.38) are statements of the orthogonality of the mode shapes with respect to the mass and stiffness matrices of the system. This orthogonality property is sometimes referred to as a weighted orthogonality property. If r = s the triple product of equation 2.37 equals a constant

$$[u_r]^t [M][u_r] = m_r \tag{2.39}$$

The constant m_r is defined as the modal mass for mode (r). Similarly the modal stiffness (k_r) is defined by:

$$[u_r]^t [K][u_r] = k_r \tag{2.40}$$

From equation (2.35) when r = s we have the relationship

$$p_r^2 = \frac{k_r}{m_r} \tag{2.41}$$

The mode shapes of a system can be grouped together in assending order with respect to their appropriate modal frequency. This matrix of mode shapes is referred to as the Modal Matrix.

$$[\phi] = \Big[[u_1][u_2] \cdots [u_n] \Big] \tag{2.42}$$

Similarly the eigenvalues or modal frequencies can be grouped together as a diagonal matrix. This diagonal matrix is the systems Spectral Matrix.

$$[\Omega] = \begin{bmatrix} p_1 & & 0 \\ & p_2 & \\ & & \ddots \\ 0 & & & p_n \end{bmatrix} \tag{2.43}$$

The unique orthogonality properties of the mode shapes, with respect to the systems mass and stiffness matrices, can be used to uncouple the original equations of motion. For an undamped system we have the following equations of motion.

$$[M][\ddot{x}(t)] + [K][x(t)] = [f(t)] \tag{2.44}$$

In general the mass and stiffness matrices of a structure will contain off diagonal elements. In other words, the equations of motion are coupled. That is, each equation involves two or more of the dependent variables. In order to solve equations (2.44) one must first uncouple them. One way to accomplish this is to transform these equations to a new coordinate space where the equations are uncoupled. If one used the modal matrix (2.42) as the coordinate transformation matrix we have the following relationship, Let

$$[x(t)] = [\phi][q(t)] \tag{2.45}$$

where the x's are the physical coordinates and the q's are another set of coordinates called the Modal or Generalized Coordinates. If one substitutes the above coordinate transformation into equation (2.44) and then pre-multiplies by the transpose of the modal matrix we have:

$$[\phi]^t[M][\phi]\ddot{q}(t) + [\phi]^t[K][\phi]q(t) = [\phi]^t f(t) \tag{2.46}$$

Now the orthogonality properties of the mode shapes are used in the above set of equations to yield.

$$[\,{}^\backslash M_\backslash\,][\ddot{q}(t)] + [\,{}^\backslash K_\backslash\,][q(t)] = [\phi]^t[f(t)] \tag{2.47}$$

Equation (2.47) is a set of uncoupled second order equations in terms of the modal coordinates q(t) where each equation is of the form of a single degree of freedom system. Once the solution is obtained in modal coordinates, the solution in terms of the physical coordinates can be found using the coordinate transformation (equ. 2.45).

Thus the mode shapes of the system has provided a way tc uncouple the equations of motion, by transforming the equations to modal coordinates. Once uncoupled the solution is easily obtained.

2.3.2 Frequency Domain Analysis

The equations of motion for the undamped multiple degree-of-freedom system, as represented by equation (2.44), can be transformed to the S-domain (Laplace domain) via the Laplace transform.

$$\left[[M]s^2 + [K]\right]\left[X(s)\right] = [F(s)] \qquad (2.48)$$

or

$$[B(s)][X(s)] = [F(s)]$$

Where the initial conditions are assumed to be zero in the above equations the matrix [B (s)] is referred to as the system matrix. The response of the structure, in the S-domain, can be determined from equation (2.48) multiplying through by the inverse matrix $[B\ (s)]^{-1}$. So,

$$[X(s)] = [B(s)]^{-1}[F(s)] \qquad (2.49)$$

or

$$[X(s)] = [H(s)][F(s)]$$

The system's transfer function matrix, which relates the input forces to the response motion, is defined as the matrix [H (s)] and is the inverse of the system matrix. For a structure that has two response points of interest, one can write the following force-displacement relationships in the S-domain.

$$\begin{bmatrix} X_1(s) \\ X_2(s) \end{bmatrix} = \begin{bmatrix} h_{11}(s) & h_{12}(s) \\ h_{21}(s) & h_{22}(s) \end{bmatrix} \begin{bmatrix} F_1(s) \\ F_2(s) \end{bmatrix} \tag{2.50}$$

The first equation can be written as:

$$X_1(s) = h_{11}(s) F_1(s) + h_{12}(s) F(s) \tag{2.51}$$

As was pointed out previously when time-domain measurements are Fourier transformed this is equivalent to evaluating the Laplace transform of the measurement at s = jw. Thus the above equation becomes.

$$X_1(w) = h_{11}(w) F_1(w) + h_{12}(w) F_2(w) \tag{2.52}$$

Equation 2.52 relates the Fourier transform of the response X_1 to the Fourier transform of the two force inputs. This particular relationship is governed by the two frequency response functions h_{11} and h_{12}. From a condition Where $F_2 = 0$ while $F_1 \neq 0$

$$X_1(w) = h_{11}(w) F_1(w) + h_{12}(w) F_2(w) \tag{2.53}$$

Thus $h_{11}(w)$ can be determined as

$$h_{11}(w) = \frac{X_1(w)}{F_1(w)} \tag{2.54}$$

Similarly one can determine h_{12} by letting $F_1 = 0$ while $F_2 \neq 0$ thus,

$$h_{12}(w) = \frac{X_1(w)}{F_2(w)} \tag{2.55}$$

The force-displacement relationships in the frequency domain are expressed as

$$\begin{bmatrix} X_1(w) \\ X_2(w) \end{bmatrix} = \begin{bmatrix} h_{11}(w) & h_{12}(w) \\ h_{21}(w) & h_{22}(w) \end{bmatrix} \begin{bmatrix} F_1(w) \\ F_2(w) \end{bmatrix} \tag{2.56}$$

Any structure can be modeled in the frequency domain using frequency response measurements

$$[X(w)] = [H(w)][F(w)] \tag{2.57}$$

where $[H(w)]$ is the structures frequency response function matrix. Commonly, structures can be represented by a set of linear 2nd order differential equations with constant coefficients. Since the coefficient matrices [M], [C], [K] are generally symmetric matrices, the system matrix and therefore the frequency response matrix are also symmetric.

2.3.3 Mode Shape Determination

The mode shapes, frequency and damping of a structure can be obtained from experimentally measured frequency response functions. As in the SDOF case the frequency response function can be expressed in a partial fraction expansion representation. Unlike the SDOF case the MDOF frequency response function can be represented as a summation of SDOF frequency response function.

$$h_{rs}(w) = \sum_{k=1}^{n} \left[\frac{R_{rsk}}{2j(jw - p_k)} - \frac{R^*_{rsk}}{2j(jw - p^*_k)} \right] \tag{2.58}$$

The summation is over the number of modes of vibration in the frequency range of the measurement. Figure 2.20 illustrates a typical frequency response measurement over a frequency range with two modes being present.

The parameters, R and p in equation 2.58 are defined as <u>Modal Parameters</u>, where R is the complex residue for the pole p. The residues, for any given mode of vibration, vary depending on where the frequency response function was measured on the structure whereas the pole p remains the same all over the structure. The modal parameters of equation 2.58 are typically determined via a curve fitting algorithm.

The mode shape of a structure can be determined from the residues. In fact, the mode shapes are proportional to the residues. To show this

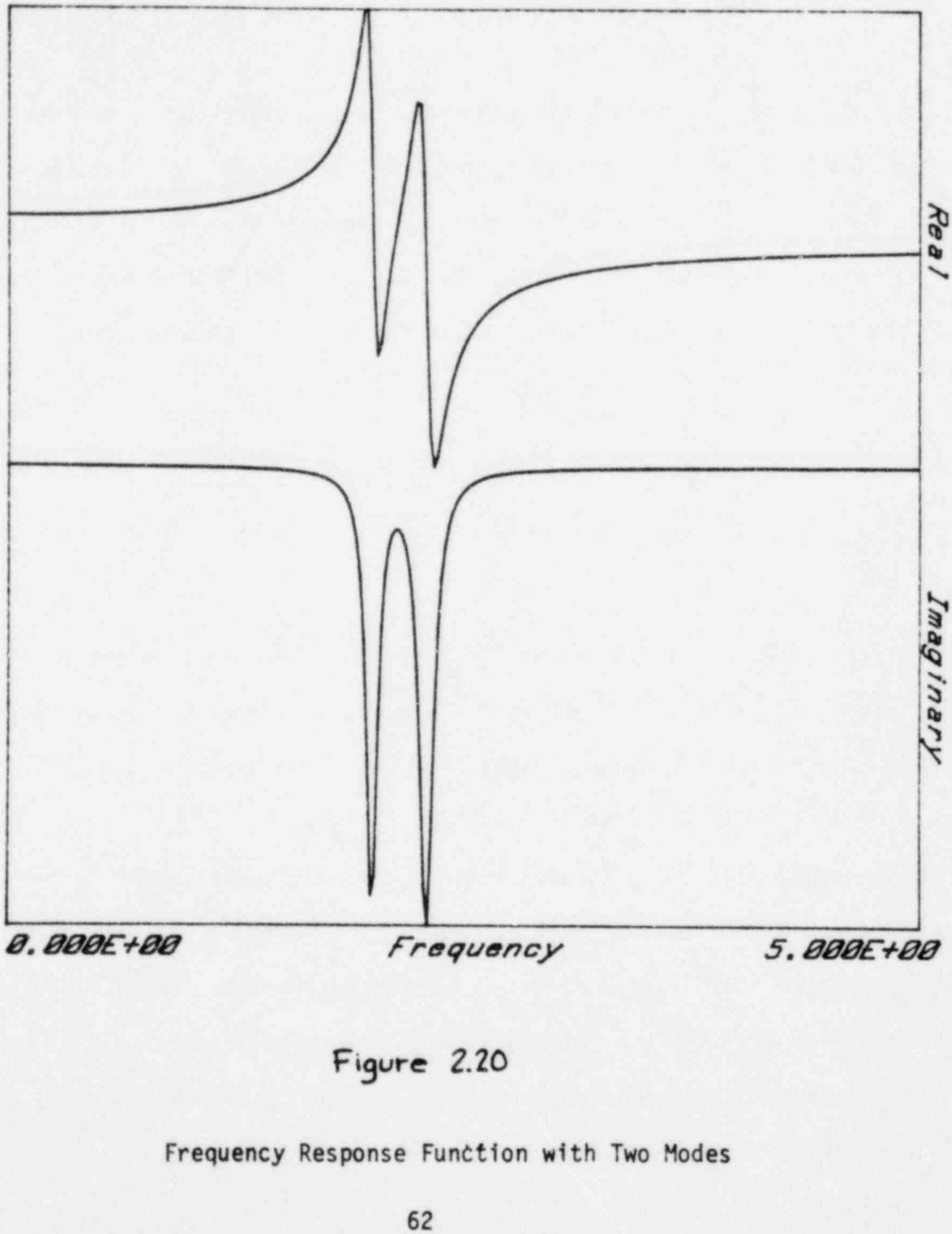


Figure 2.20

Frequency Response Function with Two Modes we first investigate the relationship of the system matrix to the transfer function matrix.

From the definition of the transfer function matrix we have,

$$[H(s)] = [B(s)]^{-1} \tag{2.59}$$

The following identity is true for the matrix inverse

$$[B(s)][B(s)]^{-1} = [I] \tag{2.60}$$

where [I] is the identity matrix. The definition of the matrix inverse is

$$[B(s)]^{-1} = \frac{[A(s)]}{|B(s)|} \tag{2.61}$$

where [A(s)] is the adjoint matrix of [B(s)] and [B(s)] is the determinant of [B(s)]. The determinant of [B(s)] is the systems characteristic equation (see equation 2.31). Substituting equation 2.61 into 2.60 yields

$$[B(s)][A(s)] = |B(s)|[I] \tag{2.62}$$

If we now evaluate the above equation at a pole $s = p_k$, we have

$$[B(p_k)][A(p_k)] = [0] \tag{2.63}$$

since the characteristic equation evaluated at a root (pole) is equal to zero. We can now rewrite the above set of equations using only the j - th column of $[A(p_k)]$

$$[B(p_k)][A(p_k)]_j = [0]_j \tag{2.64}$$

or substituting for $[B(p_k)]$

$$[p_k^2[M] + [K]][A(p_k)]_j = [0]_j \tag{2.65}$$

The above equation is a set of homogeneous equations in $A(p_k)$. The solution of this equation can only be determined to within a arbitrary constant. That is, if one of the $A(p_k)$'s are chosen the other $A(p_k)$'s can be determined in terms of it. The form of the above equation is exactly the same as the eigenvalue problem as shown in equation 2.30. In equation 2.30 the vector $[u]_k$ is the mode shape for eigenvalue p_k, thus the vector $[A(p_k)]j$ is proportional to the mode shape vector $[u]_k$ since the mode shape is arbitrary in amplitude.

$$[A(p_k)]_j = C_k [u]_k \tag{2.66}$$

Where C_k is a constant of proportionality. Thus, every column of the adjoint matrix of the system matrix evaluated at the pole p_k, is proportional to the mode shape $[u]_k$. Because the matrix $[B(p_k)]$ is symmetric, its adjoint matrix is also symmetric, hence if the j - th column of $[A(p_k)]$ is proportional to the mode shape for pole p_k so is the j - th row of $[A(p_k)]$. The form of the adjoint matrix must then be

$$[A(p_k)] = Q_k [u]_k [u]_k^t \tag{2.67}$$

where Q_k is an arbitrary constant. The mode shapes can now be related to the frequency response measurements as follows,

$$[B(s)]^{-1} = \frac{[A(s)]}{|B(s)|} = [H(s)] \tag{2.68}$$

Since $|B(s)|$ is the system characteristic equation, whose roots are the modal frequencies it can be represented as a product of its roots.

$$|B(s)| = (s-p_1)(s-p_2)\cdots(s-p_{2n}) \tag{2.69}$$

Where n is the number of modes in the frequency range of interest. Each root (pole) appears in complex conjugate pairs. Thus the systems transfer function matrix can be represented as:

$$[H(s)] = \frac{[A(s)]}{(s-p_1)(s-p_2)\cdots(s-p_{2n})} \tag{2.70}$$

The above equation can now be expanded in terms of its partial fractions.

$$\frac{[A(s)]}{(s-p_1)(s-p_2)\cdots(s-p_{2n})} = \frac{[R_1]}{s-p_1} + \frac{[R_2]}{s-p_2} + \cdots \frac{[R_{2n}]}{s-p_{2n}} \tag{2.71}$$

The constant matrix $[R_k]$ is called the residue matrix for the pole p_k. The k-th residue matrix can be determined in the following manner:

1. multiply both sides of equation 2.71 by the factor $(s-p_k)$
2. evaluate the resulting expressions at the pole, $s = p_k$

Since every term on the right of equation 2.71 is multiplied by the factor $(s-p_k)$, except the k-th residue matrix $[R_k]$, all the terms go to zero at $(s=p_k$ except $[R_k]$. Thus the k-th residue matrix is

$$[R_k] = \frac{[A(p_k)]}{(p_k-p_1)(p_k-p_2)\cdots(p_k-p_{2n})} = \frac{[A(p_k)]}{\prod_{\substack{l=1 \\ l\neq k}}^{2n}(p_k-p_l)} \qquad (2.72)$$

The denominator term is just the difference between every pole and the k-th pole, so it is nothing more than a constant. Let

$$E = \frac{1}{\prod_{\substack{l=1 \\ l\neq k}}^{2n}(p_k-p_l)} \qquad (2.73)$$

The residue matrix for the k-th mode is

$$[R_k] = E[A(p_k)] \qquad (2.74)$$

The matrix $[A(p_k)]$ is the adjoint matrix of $[B(s)]$ evaluated at the pole p_k, this has previously been shown to be proportional to the mode shape of the system for pole p_k (see equation 2.67). Since the constant of proportionality in equation 2.67 is arbitrary ,every row and column of the residue matrix for the pole p_k , is directly proportional to the mode shape for the pole p_k.

$$[R_k] = Q_k [u]_k [u]_k^t \qquad (2.75)$$

Because frequency response measurements are just transfer functions, evaluated at s = jw, the form of equation 2.71 remains the same. Thus the residues determined from frequency response measurements are the same as those for the corresponding transfer function.

Only one row or column of the systems residue matrix for a particular modal frequency need be determined to obtain the mode shape of the system. This is because of the form of equation 2.75, where every row and column of the residue matrix is proportional to the mode shape. For example, consider a structure where the frequency response information is known for three response points. That is:

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} & h_{13} \\ h_{21} & h_{22} & h_{23} \\ h_{31} & h_{32} & h_{33} \end{bmatrix} \begin{bmatrix} f_1 \\ f_2 \\ f_3 \end{bmatrix} \qquad (2.76)$$

The systems frequency response matrix is

$$[H(w)] = \begin{bmatrix} h_{11} & h_{12} & h_{13} \\ h_{21} & h_{22} & h_{23} \\ h_{31} & h_{32} & h_{33} \end{bmatrix} \qquad (2.77)$$

Each one of the frequency response measurements is assumed to have the following general form.

$$h_{rs}(\omega) = \sum_{k=1}^{n} \left[\frac{R_{rsk}}{2j(j\omega - p_k)} - \frac{R^*_{rsk}}{2j(j\omega - p^*_k)} \right] \tag{2.78}$$

where the residue R_{rsk} is the residue for the k-th mode involving response point r and input point s. One can express the residue matrix for the k-th mode as follows.

$$[R]_k = \begin{bmatrix} R_{11} & R_{12} & R_{13} \\ R_{21} & R_{22} & R_{23} \\ R_{31} & R_{32} & R_{33} \end{bmatrix}_k \tag{2.79}$$

From equation 2.75

$$\begin{bmatrix} R_{11} & R_{12} & R_{13} \\ R_{21} & R_{22} & R_{23} \\ R_{31} & R_{32} & R_{33} \end{bmatrix}_k = \begin{bmatrix} u_1^2 & u_1 u_2 & u_1 u_3 \\ u_2 u_1 & u_2^2 & u_2 u_3 \\ u_3 u_1 & u_3 u_2 & u_3^2 \end{bmatrix}_k \tag{2.80}$$

where the constant Q_k was arbitrarily set to unity because of the mode shapes of a structure are completely arbitrary in amplitude. From the above equation one can see that every row or column of the residue matrix corresponds to the mode shape $[u]_k$ multiplied by a component of itself. This is the essence of experimental modal testing, only having to measure one row or column of the frequency response matrix to determine the systems mode shapes.

In the above example of a three input-output system we can demonstrate the modal testing methology by assuming only the second column of frequency response measurements were made.Each of the frequency response measurements have the following form:

$$h_{12} = \sum_{k=1}^{n} \frac{R_{12k}}{2j(j\omega - p_k)} - \frac{R_{12k}^{*}}{2j(j\omega - p_k^{*})} \tag{2.81}$$

$$h_{22} = \sum_{k=1}^{n} \frac{R_{22k}}{2j(j\omega - p_k)} - \frac{R_{22k}^{*}}{2j(j\omega - p_k^{*})} \tag{2.82}$$

$$h_{32} = \sum_{k=1}^{n} \frac{R_{32k}}{2j(j\omega - p_k)} - \frac{R_{32k}^{*}}{2j(j\omega - p_k^{*})} \tag{2.83}$$

The three experimental frequency response measurements are then curve fit to their corresponding forms represented by the above equations. In the process, modal parameters, namely the residues and poles for each mode are thus determined. Instead of knowing the entire residue matrix we now only have determined the second column for each mode. The residue column for the k-th mode is:

$$[R]_k = \begin{bmatrix} R_{12} \\ R_{22} \\ R_{32} \end{bmatrix}_k \tag{2.84}$$

The relationship between the measured residues of the k-th mode and the k-th mode shape becomes.

$$\begin{bmatrix} R_{12} \\ R_{22} \\ R_{32} \end{bmatrix}_k = \begin{bmatrix} u_1 u_2 \\ u_2^2 \\ u_3 u_2 \end{bmatrix}_k \tag{2.85}$$

From the above relationship we see that the residue vector is equal to the mode shape multiplied by the second component of the mode shape U_2 . The actual mode can be determined by recognizing that the driving point residue is,

$$R_{22} = u_2^2 \qquad (2.86)$$

or

$$u_2 = \sqrt{R_{22}}$$

Thus the mode shape for mode k is

$$\begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix}_k = \frac{1}{\sqrt{R_{22}}} \begin{bmatrix} R_{12} \\ R_{22} \\ R_{32} \end{bmatrix}_k \qquad (2.87)$$

2.3.4 SYNTHESIS OF FREQUENCY RESPONSE FUNCTIONS

Frequency response functions can be synthesized from mode shape data utilizing the relationship between the residue matrix and mode shapes. (see equation 2.75)In the previous example the second column of the frequency response matrix was used to determine the second column of the residue matrices for each mode. From this set of residues the mode shapes were calculated. The entire residue matrix can now be re-constructed from the mode shape data. For mode k :

$$\begin{bmatrix} R_{11} & R_{12} & R_{13} \\ R_{21} & R_{22} & R_{23} \\ R_{31} & R_{32} & R_{33} \end{bmatrix}_k = \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix}_k \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix}_k^t \qquad (2.88)$$

Once the residue matrices for all the modes are known, any particular frequency response function can be synthesized using equation 2.78.

2.4 STEADY STATE RESPONSE - SINE TESTING

The SDOF equation of motion can be rewritten in terms of the damping ratio and undamped natural frequency

$$\ddot{x}(t) + 2\zeta\Omega_o\dot{x}(t) + \Omega_o^2 x(t) = \Omega_o^2 F(t) \quad \text{where} \quad F(t) = f(t)/k \tag{2.89}$$

Transforming equation 2.89 to the s-domain using the Laplace transform yields

$$X(s) = \frac{1}{B(s)}\Omega_o^2 F(s) + \frac{1}{B(s)}(s + 2\zeta\Omega_o)X(0) + \frac{1}{B(s)}\dot{x}(0) \tag{2.90}$$

where the last two terms are a result of any initial displacement or velocity. We assume the initial displacements and velocity to be zero ,giving

$$X(s) = \frac{\Omega_o^2 F(s)}{(s^2 + 2\zeta\Omega_o + \Omega_o^2)} \tag{2.91}$$

The forcing function is now assumed to be of sinusoidal nature

$$F(t) = e^{j\Omega t} \tag{2.92}$$

where Ω is the frequency of excitation. Thus,

$$F(s) = \frac{1}{s - j\Omega} \tag{2.93}$$

The response in the s-domain becomes,

$$X(s) = \frac{\Omega_0^2}{(s^2+2\zeta\Omega_0 s+\Omega_0^2)(s-j\Omega)} = \frac{\Omega_0^2}{(s-p_1)(s-p_1^*)(s-j\Omega)} \tag{2.94}$$

To facilitate the taking of the inverse Laplace transform, the above equation is expressed in its partial fraction form.

$$\frac{\Omega_0^2}{(s-p_1)(s-p_1^*)(s-j\Omega)} = \frac{A_1}{s-p_1} + \frac{A_2}{s-p_1^*} + \frac{A_3}{s-j\Omega} \tag{2.95}$$

The time domain response is obtained via the inverse Laplace transform

$$x(t) = A_1 e^{p_1 t} + A_2 e^{p_1^* t} + A_3 e^{j\Omega t} \tag{2.96}$$

The "pole" pairs p_1 have previously been shown to be

$$p_1 = -\zeta\Omega_0 \pm \Omega_0(1-\zeta^2)^{1/2} j$$

The first two terms of equation 2.96 are decaying sinusoids, these two terms represent the transient part of the response for X(t). This transient part is similar to the impulse response of the system. The transient part will eventually decay to zero amplitude, leaving only the steady state response due to the last term in equation 2.96. The time constant for this decay is

$$\tau = \frac{1}{\zeta\Omega_0} \tag{2.97}$$

The time constant is a function of both the damping and undamped natural frequency of the system. For systems that are lightly damped and have a low natural frequency the time constant can be very large. This fact is the primary reason that sine testing (exciting a structure with a sinusoidal forcing function) is very time consuming for structures with light damping and low natural frequency. This is typically the case we have with large structures. When a sinusoidal forcing function is used to measure the system response one must wait approximately 3 time constants, so that the transient response has decayed to 95% of its original response. Thus the response of the structure would be primarily the steady state response. For example if the structure that was being tested had a damping ratio and natural frequency:

$\zeta = .02$

$W_n = 2$ Hz

The total time required for the transient to decay to 95% of its original value would be approximately 12 seconds.

Sine testing has been historically the testing method used to measure a systems frequency response function. The frequency response function can be measured by exciting the structure at a single frequency at a time and then waiting for the transient response to decay so that the steady state response can be measured. As shown previously with lightly damped structures the time required at each different frequency can be of the order of 10 seconds or more. Sine testing also has important implications when used to excite non-linear structures.

The steady state response from equation 2.96 becomes

$$x(t) = A_3 e^{j\Omega t} = A_3 F(t) \tag{2.98}$$

where A_3 can be determined from the partial fraction expansion.

$$A_3 = \frac{1}{1 - \frac{\Omega^2}{\omega_n^2} + 2\zeta\frac{\Omega}{\omega_n} j} = H(\omega)\Big|_{\omega=\Omega} = H(\Omega) \tag{2.99}$$

Thus the constant A_3 is really the frequency response function, as previously defined, evaluated at the frequency $\omega = \Omega$. Therefore the frequency response function can be determined by measuring the steady state force and response of a structure simultaneously

$$H(\Omega) = \frac{X(t)}{F(t)} \quad \text{where} \quad F(t) = e^{j\Omega t} \tag{2.100}$$

at many different frequencies Ω.

The response to the harmonic motion F(t) can be expressed as:

$$x(t) = H(\Omega) F(t) \tag{2.101}$$

$$x(t) = |H(\Omega)| e^{-j\phi} F(t) = |H(\Omega)| e^{j(\Omega t - \phi)} \tag{2.102}$$

where the magnitude and phase of $H(\Omega)$ is

$$|H(\Omega)| = \left[\left(1-\left(\frac{\Omega}{\Omega_o}\right)^2\right) + \left(2\xi\frac{\Omega}{\Omega_o}\right)^2\right]^{-1/2} \tag{2.103}$$

$$\phi = \tan^{-1}\left[\frac{2\xi\frac{\Omega}{\Omega_o}}{1-\left(\frac{\Omega}{\Omega_o}\right)^2}\right]$$

The displacement as represented in equation 2.102 is lagging the harmonic excitation force by the angle ϕ .

The terms in equation 2.89 represent a force balance between inertial, damping and elastic forces with the imposed forcing function. The velocity and acceleration terms can be expressed as functions of the displacement and excitation frequency Ω .

$$\dot{x}(t) = j\Omega\,|H(\Omega)|\,e^{j(\Omega t-\phi)} = j\Omega x(t) \tag{2.104}$$

Similarly

$$\ddot{x}(t) = -\Omega^2 x(t)$$

From these equations we see that the velocity leads the displacement by $\frac{\pi}{2}$ (since $j = e^{j\pi/2}$) and the acceleration leads displacement by π . Equation 2.102 can now be represented in terms of a vector diagram as follows.

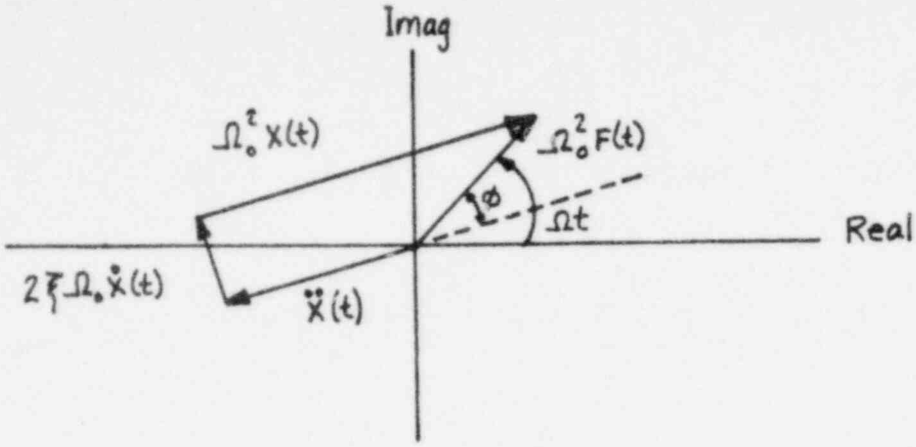


Figure 2.21

The phase of the displacement (and hence the velocity and acceleration) depend on the excitation frequency with respect to the systems undamped natural frequency as shown in Figure(2.22). At the undamped natural frequency the displacement lags the force by $\pi/2$. If the frequency response is measured with an accelerometer transducer, which is generally the case in modal testing, the phase of the acceleration w.r.t. the force will lead by $\pi/2$ at the undamped natural frequency. The phase plot of the acceleration frequency response is the same as that of the displacement (Figure 2.22) except displaced by π , since

$$\frac{\ddot{x}(t)}{F(t)} = -\Omega^2 \frac{x(t)}{F(t)} = -\Omega^2 \left| H(\Omega) \right| e^{j(\pi-\phi)} \tag{2.105}$$

2.5 MAGNITUDE OF FREQUENCY RESPONSE FUNCTION

The magnitude of the complex frequency response function $|H(w)|$ is determined from the square root of the magnitude squared, where the magnitude squared is

$$\left| H(w) \right|^2 = H(w) \cdot H^*(w) \tag{2.106}$$

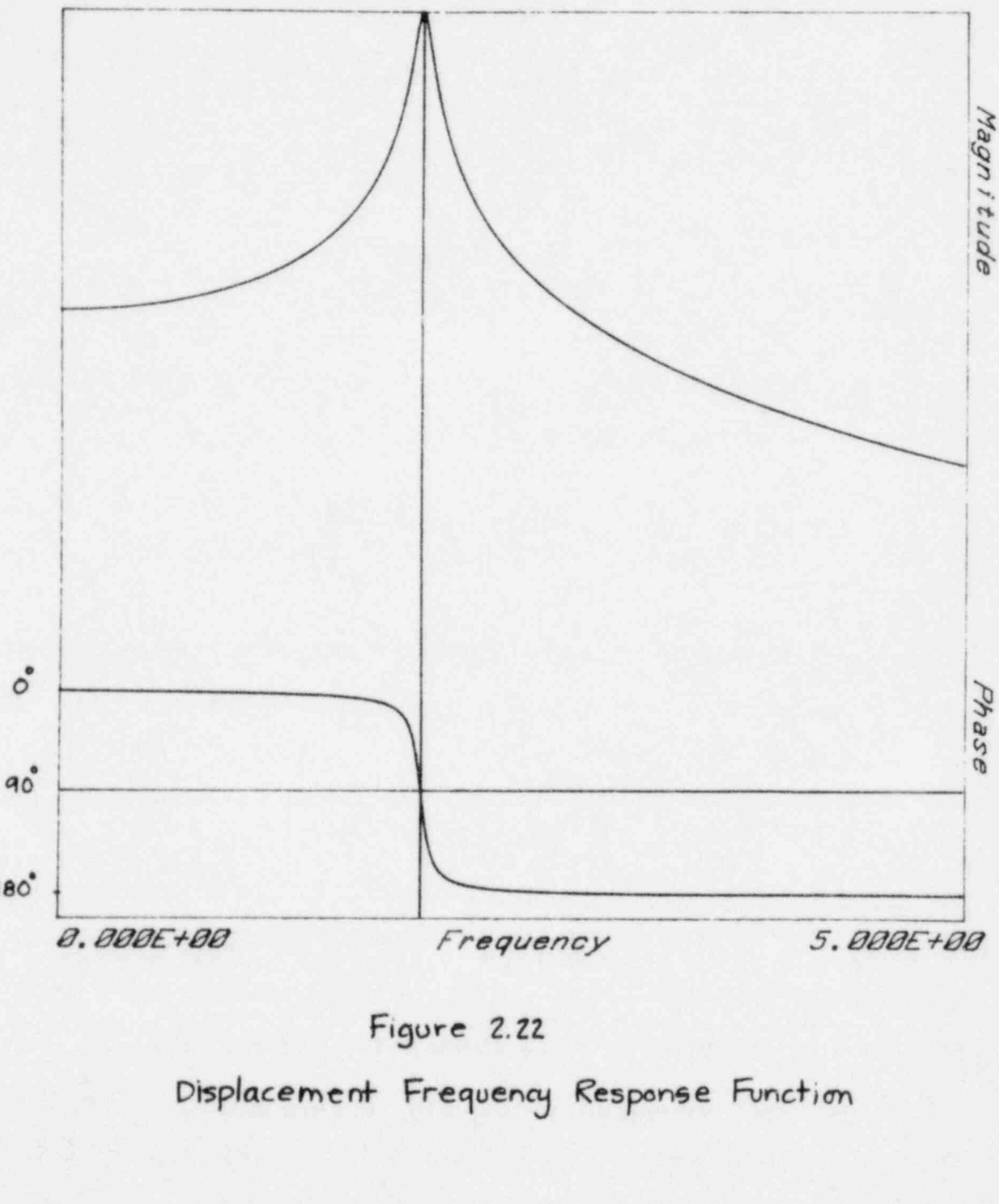


Figure 2.22

Displacement Frequency Response Function where $H^*(w)$ is the complex conjugate of the frequency response function.

$$H^*(w) = \frac{1/k}{1-\left(\frac{w}{\Omega_0}\right)^2 - j2\zeta\left(\frac{w}{\Omega_0}\right)} \qquad (2.107)$$

thus

$$\left|H(w)\right|^2 = \frac{1/k^2}{\left(1-\left(\frac{w}{\Omega}\right)^2\right)^2 + \left(2\zeta\left(\frac{w}{\Omega_0}\right)\right)^2} \qquad (2.108)$$

The plot of $|H(w)|$ is shown in Figure 2.23, where the constant k has been included in the system input, i.e.

$$F(t) = \frac{f(t)}{k} \qquad \text{thus} \qquad H(w) = \frac{X(w)}{F(w)} \qquad (2.109)$$

The peak amplitude of the frequency response function is obtained by differentiating equation 2.108 w.r.t. w and letting the result equal zero. The peak occurs when,

$$w = \Omega_0 \left(1-2\zeta^2\right)^{1/2} \qquad (2.110)$$

Thus, the peak of the magnitude of the frequency response function occurs at a frequency less than both w_0 and Ω_0. For very lightly damped systems the peak occurs approximately at the undamped natural frequency Ω_0.

The maximum value $|H(w)|$ for a SDOF system is referred to as the Q of the system. When a system has light damping

$$\left|H(\Omega_0)\right| \cong Q \cong \frac{1}{2\zeta} \qquad (2.111)$$

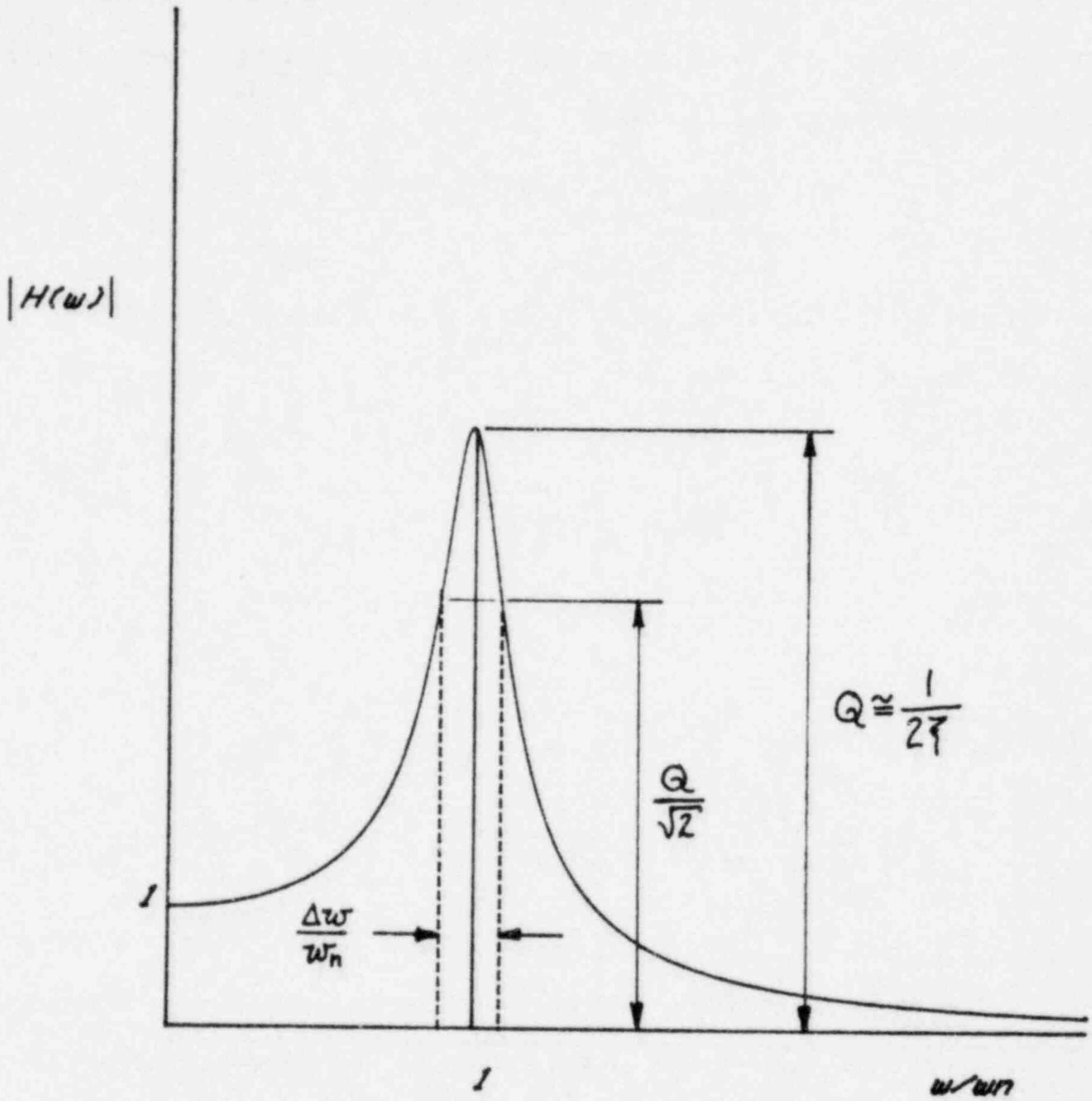


Figure 2.23

Magnitude of Frequency Response Function

An approximate value of damping for a SDOF can be computed by using the frequency difference between the systems half power points. The half power points are defined as the amplitude

$$\frac{|H(w)|^2_{max}}{2} \tag{2.112}$$

or in terms of the magnitude $|H(w)|$, when it is equal to $\frac{Q}{\sqrt{2}}$. The half power points thus occur when

$$\left(\frac{Q}{\sqrt{2}}\right)^2 = \frac{1}{2}\left(\frac{1}{2\xi}\right)^2 = \frac{1}{\left[1-\left(\frac{w}{\Omega_0}\right)^2\right]^2+\left[2\xi\frac{w}{\Omega_0}\right]^2} \tag{2.113}$$

assuming, light damping, $\xi \ll 1$

$$\left(\frac{w}{\Omega_0}\right)^2 = 1 \pm 2\xi \tag{2.114}$$

letting the roots of equation 2.13 equal to the frequencies w_1 and w_2, the difference of these two frequencies is sometimes referred to as the band width of the resonance.

$$w_2 - w_1 = \Delta w = \text{bandwidth}$$

For lightly damped systems

$$\Delta w = 2\xi\Omega_0 \tag{2.115}$$

Thus the approximate damping ratio of the system can be obtained from the magnitude of the frequency response function by,

$$\zeta = \frac{\Delta\omega}{2\Omega_0} \tag{2.116}$$

3.0 MODAL TESTING METHODS

The underlying assumption of this entire study is that structural damage can be detected by measuring changes in a structure's modes of vibration. Hence, a crucial step of this process is to accurately identify modal parameters from measured vibration data. Once the modal parameters have been identified further comparisons of these results with historical data (i.e. acceptable values determined from previous measurements) must be made. We do not discuss here any schemes for performing this last step, but merely limit the discussion to alternative methods for determining the modal parameters themselves.

Depending upon the type of mechanical structure involved, vibration monitoring may be done on a continuous basis or only periodically. For example, operating machinery such as pumps or compressors may typically be monitored on a continuous basis in an effort to detect damage which can often develop over relatively short periods of time. On the other hand, the containment structure or piping systems of a nuclear plant may only be monitored on a monthly, yearly, or less frequent basis, e.g. only after earthquakes.

During the literature survey a large amount of literature was found on the general subject of modal testing. Several recent survey papers were found which contain extensive bibliographies on dynamic testing and parameter identification. (See refs. D.6, D.1, D.27, D.17)

Ibanez et. al. (Ref. D.6) and Hart and Yao (Ref. D.17) have both conducted recent surveys of techniques, algorithims, and measurement systems specifically for modal testing.

In the discussion here we concentrate on what we consider to be the two fundamentally different modal testing methods in use today, the so called normal mode method and the transfer function method. Reference literature is organized in the Bibliography which is specific to both of those methods. All other literature, which didn't specifically pertain to one of these methods is organized under the heading of "General Measurement and Parameter Identification Methods."

Lastly we include in this chapter a discussion of ambient testing methods, and here discuss the Random Decrement Method. Although this method can be used for modal testing also, its real advantage lies in its use for processing ambient vibration signals when it is applicable.

3.0.1 Alternative Testing Methods

Two fundamentally different methods of modal testing have become popular today for testing structures. These methods are referred to as the NORMAL MODE method and the TRANSFER FUNCTION method. The NORMAL MODE method is the more traditional of the two, and has been used for the past 25 years by the aerospace industry to test large spacecraft and aircraft structures.

The TRANSFER FUNCTION method as become popular within the past 5 to 10 years and is being used by many manufacturing industries, such as the automotive and machine tool industries for prodúct improvement.

3.1 The Normal Mode Method

The primary objective of this method is to excite the undamped (or "normal") modes of a structure, one at a time. This is typically done by attaching several shakers to the structure, as shown in Figure 3.1, and driving them with a sinusoidal signal equal in frequency to the natural frequency of the mode to be excited. The amplitudes and polarity of the sinusoidal drive signals are adjusted so that the externally applied forces so that the predominant motion of the structure is due to the desired mode of vibration.

Resonant frequencies of modes are first located by performing so called WIDE BAND sweeps, usually with only one shaker active. Once a rise in the amplitude of response is detected (by watching the response signal on an oscilloscope), a NARROW BAND sweep is performed using smaller frequency changes to more accurately identify the frequency of the mode.

Once a resonant condition is located, multiple shakers are turned on in an effort to excite a single mode of vibration.

The process of adjusting the amplitude, polarity, and frequency of the shakers to excite a normal mode is called MODAL TUNING. Once a mode is

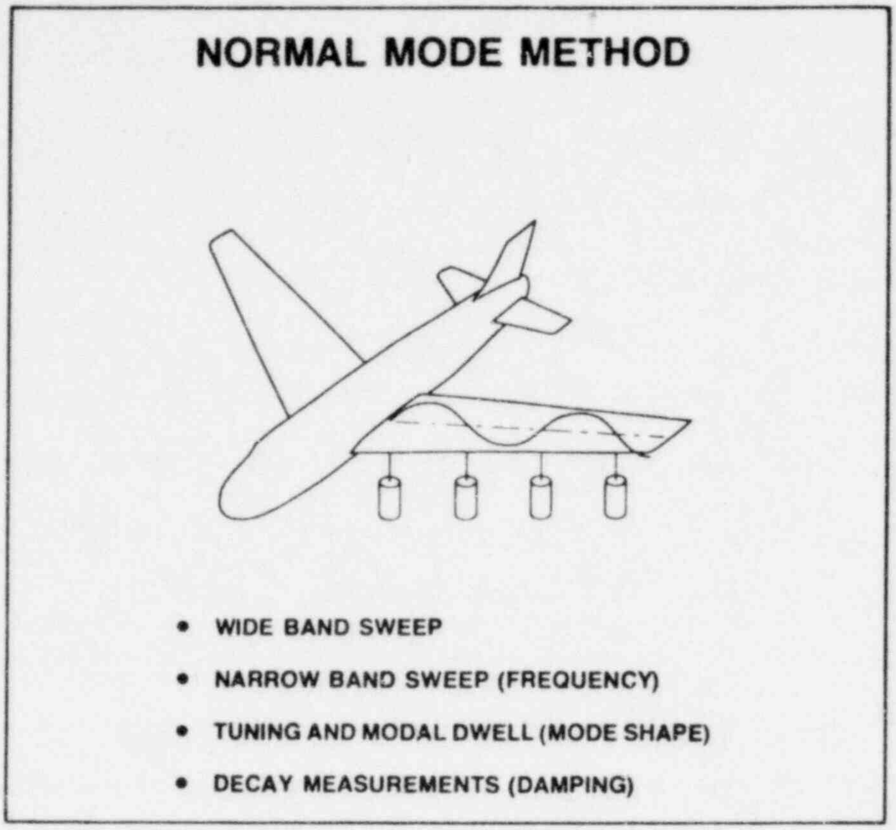


FIGURE 3.1

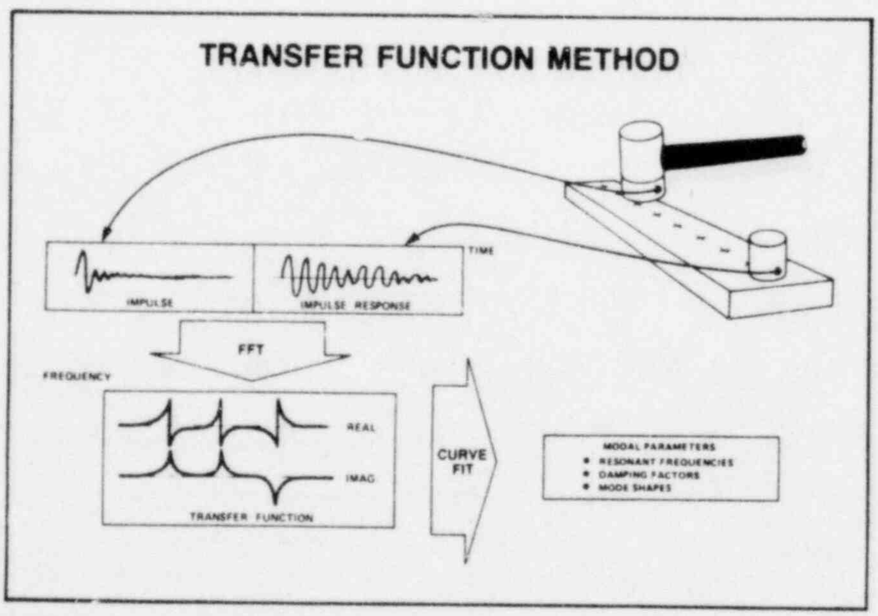


FIGURE 3.2

properly excited its amplitudes of vibration at many points on the structure are measured, and taken as the mode shape. This condition is referred to as MODAL DWELL.

Then, to measure the damping of the mode, the shakers are simultaneously shut off to simulate an impulse response of the structure at the frequency of the mode. Ideally the structure should exhibit a damped sinusoidal response at all points, with a single frequency of vibration being the frequency of the excited mode. Typically if more than one mode was excited during modal tuning and dwell, the impulse responses will show a "beating" of several modal frequencies. If, however, a so-called "pure" mode was excited, the damping of the structure at the modal frequency can be measured from the envelope of the damped sinusoidal response.

There are a number of problems which make this testing method difficult, time consuming, and expensive to implement. First of all, it is difficult to know where to locate shakers on the structure without some forehand knowledge of its modes of vibration. Secondly, it is often extremely difficult to excite closely coupled modes (i.e. close in frequency with heavy damping) one at a time. Thirdly, since all the mode shape data is collected during modal dwell, the structure must be completely instrumented with enough transducers and signal conditioning equipment so that amplitudes for all the desired degrees-of-freedom can be measured at once.

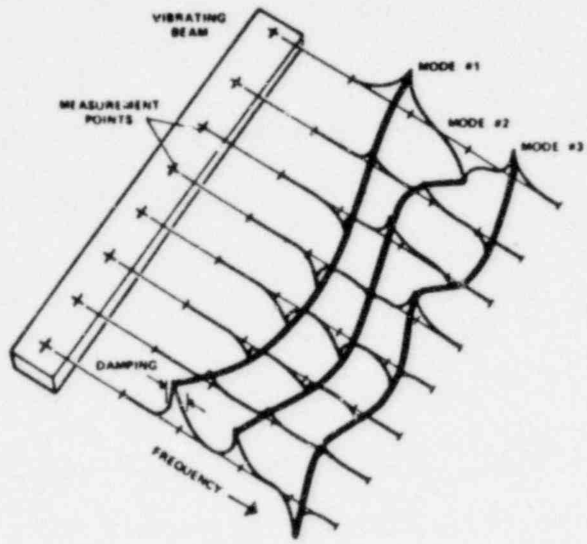
3.2 The Transfer Function Method

This method as gained much popularity in recent years because it is faster and easier to perform, and is much cheaper to implement than the normal mode testing method.

The major steps of the transfer function method are depicted in Figure 3.2. It is based upon the use of digital signal processing techniques and the FFT (Fast Fourier Transform) algorithm to measure transfer functions between various points on the structure. For example, on the simple beam in Figure 3.2 a set of transfer functions is measured between each of the X's marked on the beam, and a single response point. A single transfer function measurement is obtained by exciting the beam with a hammer at one of the X's, simultaneously measuring the input force and corresponding response motion signals, and then dividing the Fourier transform of the response by the transform of the input.

Modal parameters are identified by performing further computations (i.e. "curve fitting") on this set of transfer function measurements. Figure 3.3 shows how modal parameters can be obtained from transfer function measurements. Although much more sophisticated curve fitting algorithms are often used to identify modal parameters, this figure shows fundamentally how the parameters are obtained. The figure shows the imaginary part of each transfer function made between an impact point and the reference point.

MODAL DATA FROM TRANSFER FUNCTIONS



DAMPING & FREQUENCY —
SAME AT EACH MEASUREMENT POINT

MODE SHAPE—
OBTAINED AT SAME FREQUENCY FROM ALL MEASUREMENT POINTS

FIGURE 3.3

MODAL TESTING METHODS

NORMAL MODE METHOD	TRANSFER FUNCTION METHOD
MULTI-SHAKER	SINGLE POINT EXCITATION
SINUSOIDAL	BROADBAND
ONE MODE AT A TIME	MANY MODES AT A TIME
ANALOG INSTRUMENTATION	FFT-BASED DIGITAL INSTRUMENTATION

FIGURE 3.4

Modal frequencies correspond to peaks in the imaginary part of the transfer functions. A peak should exist at the same frequency in all measurements, except those measured at "node" points where the modal amplitude is zero. The width of the modal peak is related to the damping of the mode. That is, the wider the peak, the higher the modal damping. The mode shape is obtained by assembling the peak values at the same frequency from all measurements. As shown in Figure 3.3, as modal frequency increases, the complexity of the mode shape also increases.

Some major differences between the two modal testing methods are shown in Figure 3.4. A fundamental difference is that one method attempts to excite one mode at a time using a narrow band signal, while the other attempts to excite many modes simultaneously, using a broadband signal. A distinct advantage of the transfer function method is that any type of broadband excitation method can be used since the measurement being made is a response signal divided by the input which caused it. This type of "normalized" response measurement is independent of the type of input signal used, as long as it can be measured and has sufficient energy to excite the structure over the frequency range of interest. Hence, a simple excitation device such as a hammer can many times be used to measure transfer functions at a great savings of time and money compared to attaching shakers to the structure.

Another cost advantage on the transfer function method is that the measurements can be made one at a time. This means that a large set of measurements can be made using only one accelerometer (motion transducer) and one load cell (force transducer), and the corresponding signal conditioning equipment. If a shaker is used for excitation, the accelerometer is moved to each new measurement point on the structure. If a hammer is used, the structure is impacted at each new measurement point.

In general, it is easier to make transfer function measurements on a structure than to isolate one of its modes of vibration. In addition, once the measurement signals have been digitized and stored in the computer's memory, further processing of the data can be performed to reduce the effects of noise and distortion. Large amounts of data can also be stored on a mass memory device such as a magnetic disc or tape, and later recalled for further processing. Statistical estimation algorithms which use large amounts of measurement data, can also be used to estimate modal parameters with more accuracy.

3.2.1 Digital Fourier Analyzers

An analog tracking filter is a device which gives the spectral content of a time domain signal, one frequency at a time. By comparison, the digital Fourier transform (DFT) can be viewed as a parallel process which yields an entire frequency spectrum of a time signal in a selected frequency range.

Since the digital Fourier Analyzer provides a broad band frequency spectrum very quickly (e.g. 100 ms to give 512 spectral lines), it can be used for obtaining broad band response spectrums from a structure which is excited by a broad band input signal.

Furthermore, if the input and response time signals are measured simultaneously, Fourier transformed, and the transform of the response is divided by the transform of the input, a transfer function between the input and response points on the structure is measured. (Division of one frequency domain function by another is straightforward when the data is in digital form.) Hence, a digital Fourier Analyzer which can simultaneously measure two (or more) signals is an ideal tool for measuring transfer functions quickly and accurately. Furthermore, since the modes of vibration of an elastic structure can be identified from transfer function measurements, a multi-channel Fourier Analyzer with additional processing capability can also be used for identifying modal parameters from test data.

3.3 Transfer Function Measurement

In a test situation we do not actually measure the transfer function over the entire S-plane, but rather its values along the jw-axis.

These values are known as the FREQUENCY RESPONSE FUNCTION, as shown in Figure 3.5. Since the transfer function is an "analytic" function, its values throughout the S-plane can be inferred from its values along the jw-axis. More specifically, if we can identify the unknown modal parameters of a transfer function by "curve fitting" an analytical form of it to

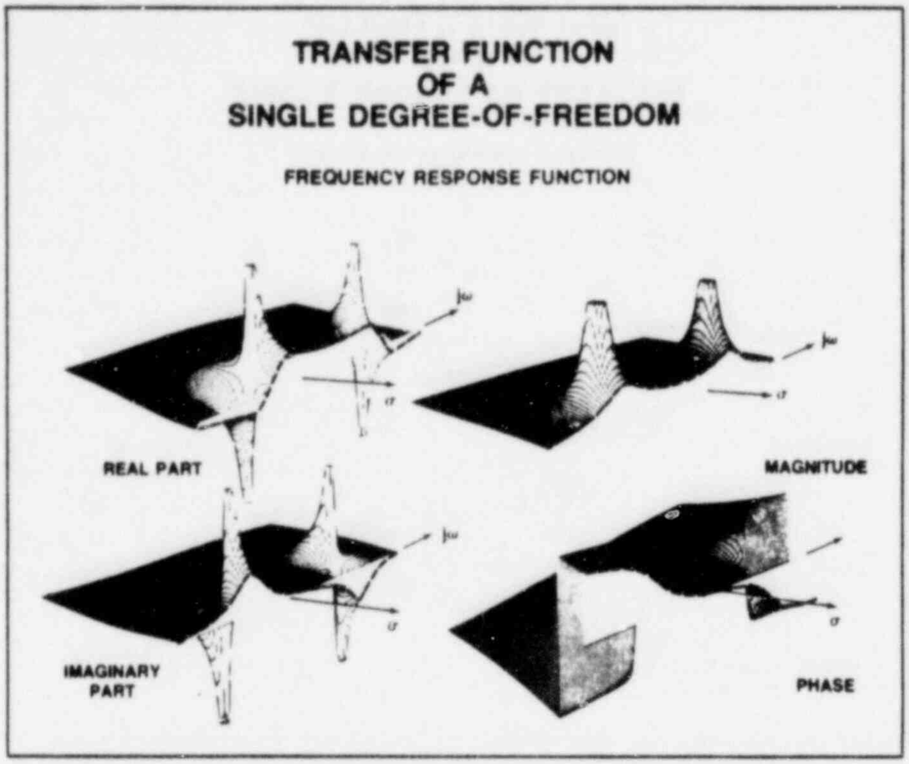


FIGURE 3.5

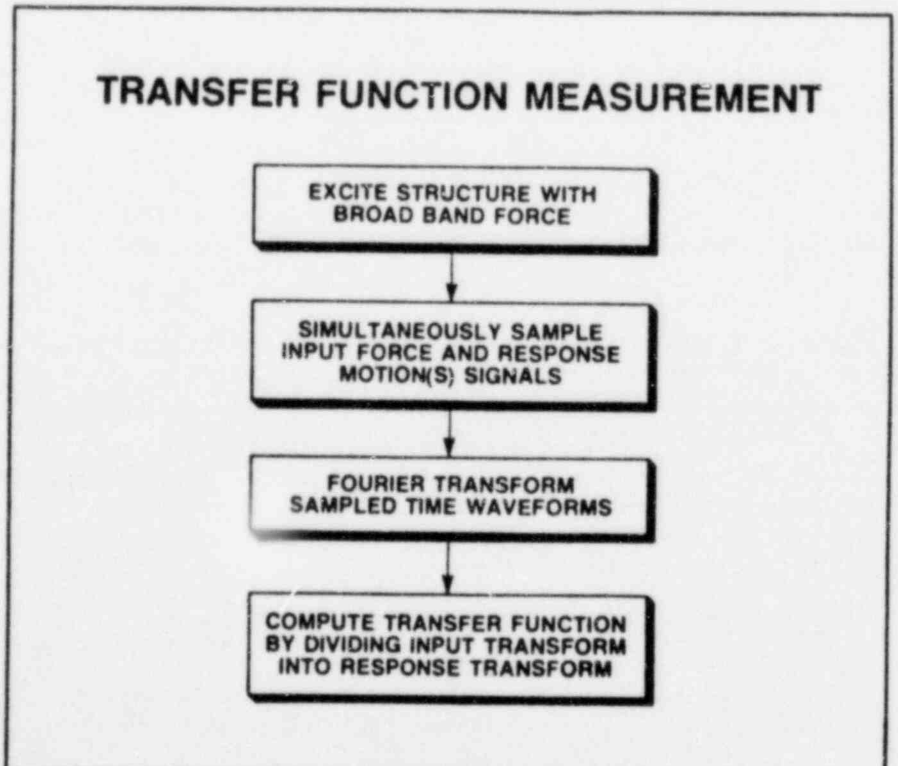


FIGURE 3.6

measured values of the function along the jw-axis, then we can synthesize the function throughout the S-plane.

3.3.1 The Measurement Process

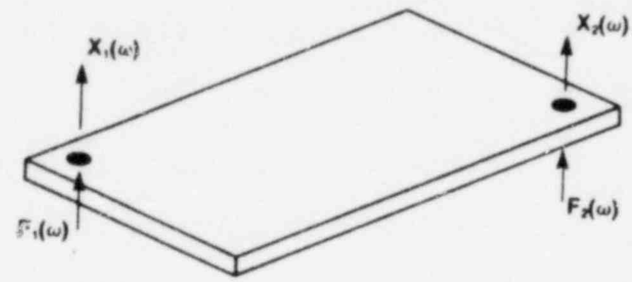
The transfer function measurement process is depicted in Figure 3.6. A key step in the measurement of the transfer function is that two signals from the structure, an input force signal and a response motion signal, are simultaneously digitized (sampled) and stored in the computer memory. From that point on, the measurement consists of performing mathematical operations on the digital data. The diagram in Figure 3.6 is actually a much simplified explanation of the signal processing that is typically used during transfer function measurements. This signal processing process is explained in more detail later on.

3.3.2 Measuring Elements of the Transfer Matrix

The simplest way of measuring elements of the transfer matrix (i.e. frequency response functions) is to measure them one at a time, as shown in Figure 3.7. In this simple 2-dimensional case the frequency response function (h_{11} (w)) is measured by exciting the structure at pt. #1 and measuring response at pt. #1. Then the function is formed by dividing the Fourier transform of the measured response motion (X_1 (w)) by the Fourier transform of the measured input force (F_1 (w)). Likewise the second element in the first row (h_{12}(w)

MEASURING ELEMENTS OF THE TRANSFER MATRIX

$$\begin{bmatrix} X_1(\omega) \\ X_2(\omega) \end{bmatrix} = \begin{bmatrix} h_{11}(\omega) & h_{12}(\omega) \\ h_{21}(\omega) & h_{22}(\omega) \end{bmatrix} \begin{bmatrix} F_1(\omega) \\ F_2(\omega) \end{bmatrix}$$



FIRST ROW

$$X_1(\omega) = h_{11}(\omega) F_1(\omega) + h_{12}(\omega) \cancelto{0}{F_2(\omega)} \qquad h_{11}(\omega) = X_1(\omega)/F_1(\omega)$$

$$X_1(\omega) = h_{11}(\omega) \cancelto{0}{F_1(\omega)} + h_{12}(\omega) F_2(\omega) \qquad h_{12}(\omega) = X_1(\omega)/F_2(\omega)$$

SECOND ROW

$$X_2(\omega) = h_{21}(\omega) F_1(\omega) + h_{22}(\omega) \cancelto{0}{F_2(\omega)} \qquad h_{21}(\omega) = X_2(\omega)/F_1(\omega)$$

$$X_2(\omega) = h_{21}(\omega) \cancelto{0}{F_1(\omega)} + h_{22}(\omega) F_2(\omega) \qquad h_{22}(\omega) = X_2(\omega)/F_2(\omega)$$

FIGURE 3.7

is measured by exciting the structure at pt. #2 and then dividing the Fourier transform of the response motion ($X_1(w)$) by the Fourier transform of the input force ($F_2(w)$). The second row of elements can be measured in a similar manner.

More sophisticated measurement methods involving multiple inputs and responses could be implemented, but this simplified single input-single output approach is most commonly used.

If time savings is a significant test objective, as it often is in larger modal tests, than test time can be significantly reduced by measuring a single input force and several response motions simultaneously. From this data, several transfer functions in a single column of the transfer matrix can be computed.

3.3.3 Typical Measurement Setups

A typical measurement setup using a shaker driven by a broadband random signal is shown in Figure 3.8. This figure shows the measurement being made with a Fourier analyzer system. After the transducer signals are amplified, via charge amplifiers, they are passed through low pass anti-aliasing filters before being digitized by the analog to digital converter (ADC) of the Fourier analyzer. Some transfer function analyzers contain built-in anti-aliasing filters, and some modern day transducers contain built-in charge amplifiers, which make them both more convenient to use.

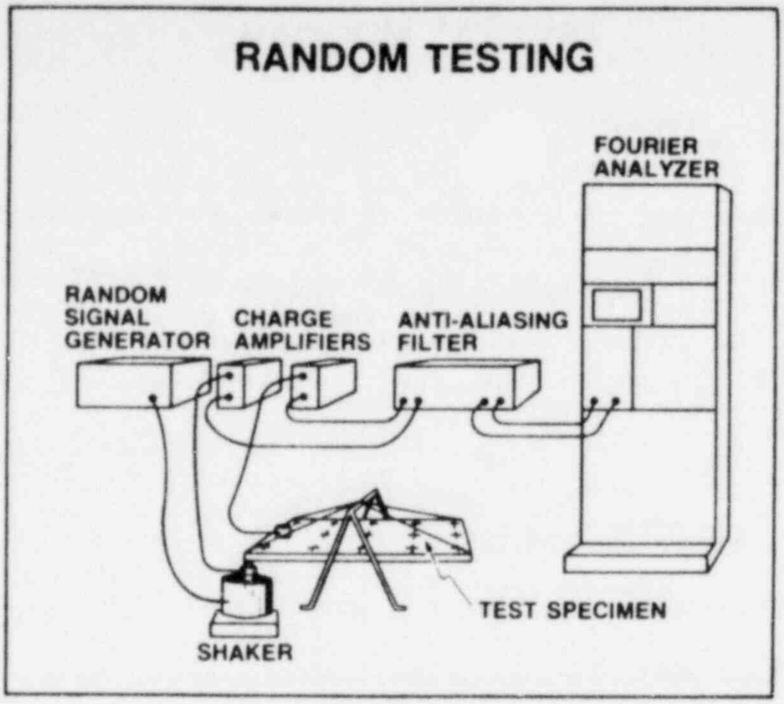


FIGURE 3.8

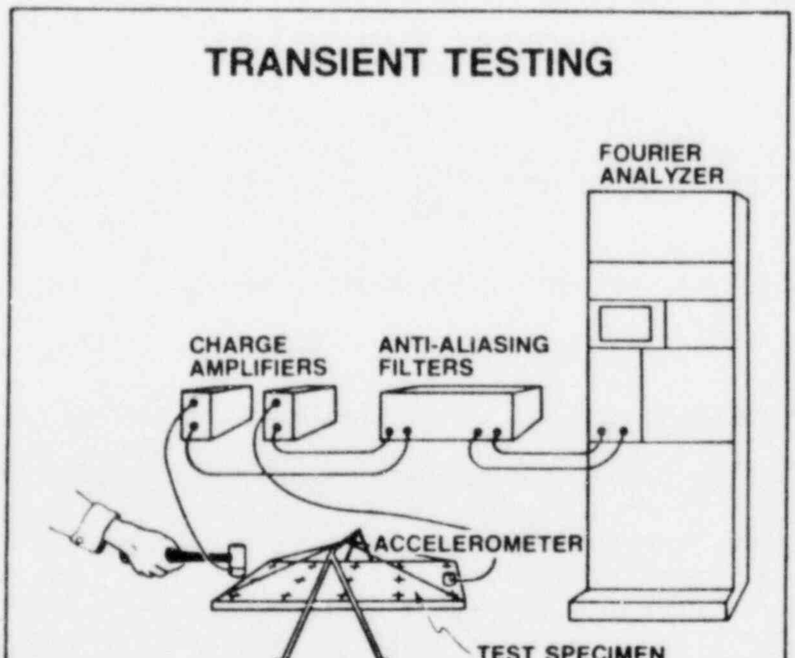


FIGURE 3.9

The test specimen is normally mounted in a manner which allows it to vibrate freely (called a free-free condition), or in a manner which allows it to vibrate the way it would in an actual operating environment.

Figure 3.9 shows a typical test setup for transient testing. When a small hammer is used to excite the structure as shown in the example, then a set of measurements (i.e. a row of the transfix matrix) is normally obtained by impacting the structure at various points while measuring its response at a single stationary point. However, in other situations where the impactor cannot be easily moved, measurements would be made in a manner similar to a shaker test.

3.4 Digital Signal Processing Methods

Figure 3.10 lists some of the signal processing methods that are commonly used to improve the quality of transfer function measurements.

3.4.1 Removing Noise From Measurements

Power spectrum averaging is usually done during transfer function measurement to remove extraneous noise from the measurement. The measurement algorithm depicted in Figure 3.11 shows how the three power spectrums; input auto power, output auto power, and the cross power spectrum are estimated by way of an

DIGITAL SIGNAL PROCESSING METHODS

- *POWER SPECTRUM AVERAGING* REDUCES EXTRANEOUS NOISE
- *COHERENCE FUNCTION* INSURES VALIDITY OF MEASUREMENT
- *RANDOM EXCITATION* REDUCES DISTORTION
- *PERIODIC SIGNALS* AND *TIME RECORD AVERAGING* REDUCE "LEAKAGE" (SMEARING OF SPECTRUM)
- *DIGITAL FILTERING* (ZOOM TRANSFORM) IMPROVES FREQUENCY RESOLUTION

FIGURE 3.10

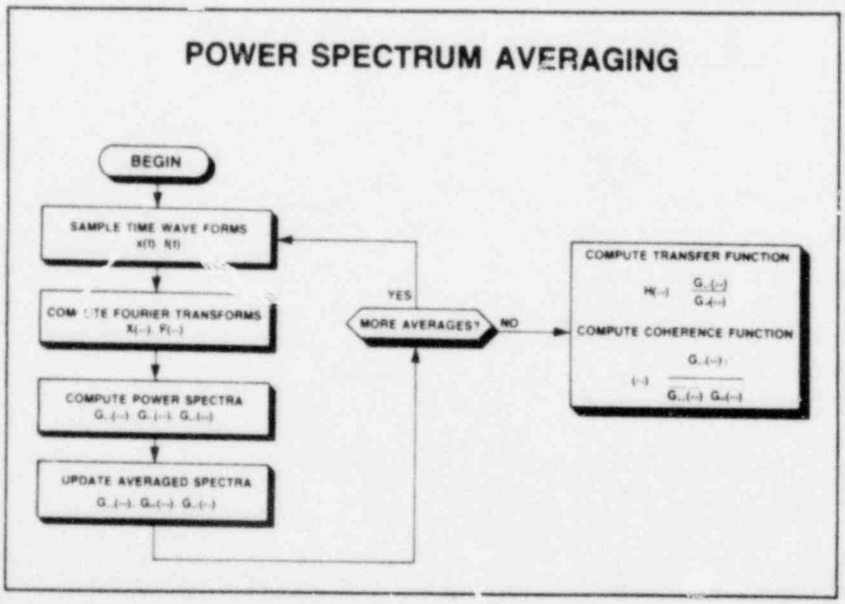


FIGURE 3.11

averaging scheme. Once enough records of data have been averaged together, the transfer function is computed by dividing the cross power spectrum estimate by the input auto power spectrum estimate. It can be shown that this method yields an unbiased estimate of the transfer function in the presence of noise. Figure 3.12 shows how power spectrum averaging can improve a measurement.

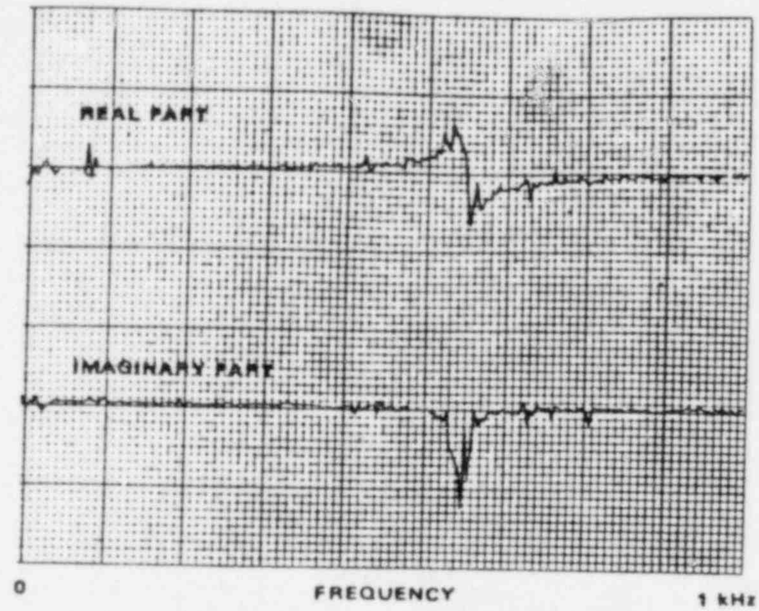
3.4.2 The Coherence Function

Whenever transfer functions are measured on a digital Fourier Analyzer in the manner just discussed, the coherence function can also be easily computed. The coherence function denoted $\gamma^2(w)$ is defined as the ratio

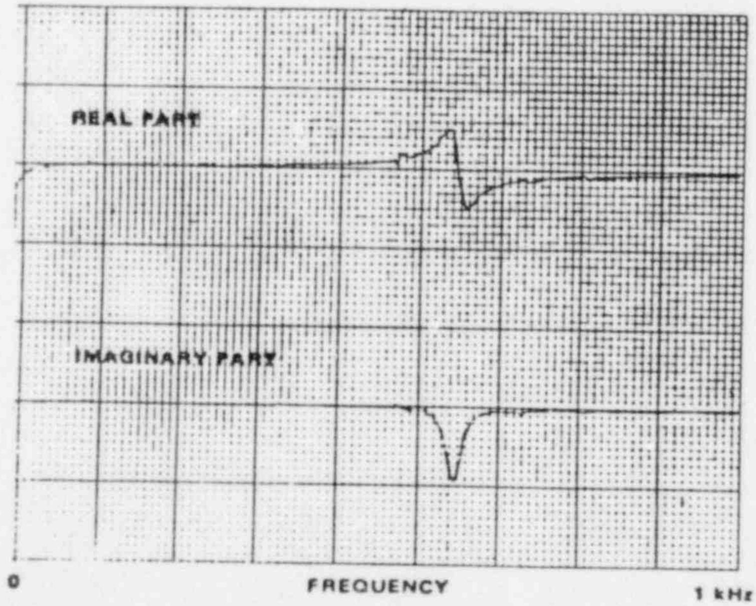
$$\gamma^2(w) = \frac{\text{(response power caused by applied input)}}{\text{(measured response power)}}$$

The coherence function is also dependent upon frequency. The measured response power spectrum contains the power caused by the input and the power due to extraneous noise sources.

The coherence function is normally always calculated with the transfer function. As shown in Figure 3.11, it is computed by dividing the input and response auto power spectrum into the magnitude squared of the cross power spectrum between input and response.

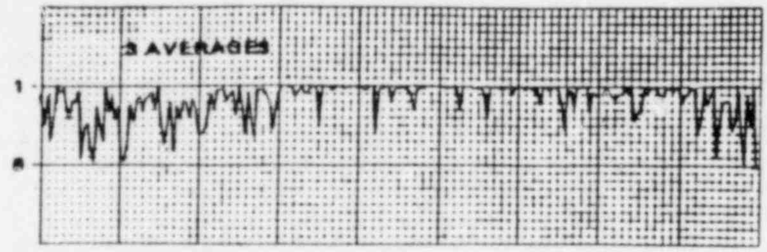


Measurement with 3 Averages

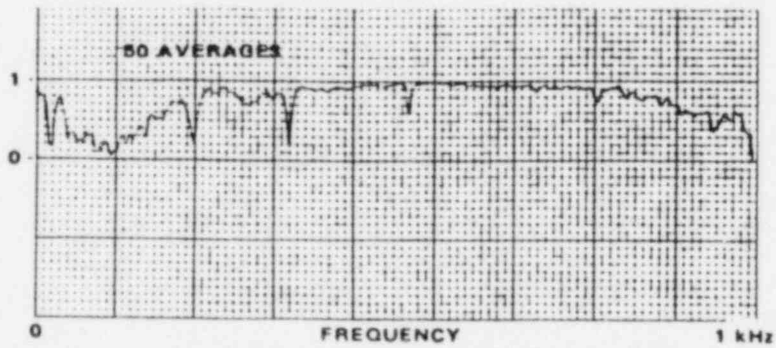


Measurement with 50 Averages

FIGURE 3.12



Coherence of Measurement with 3 Averages



Coherence of Measurement with 50 Averages

FIGURE 3.13

From the definition, it is clear that when the measured response power is in fact caused by the measured input power, the coherence value is one, for all frequencies. But when the measured response power is greater than the measured input power because some extraneous noise source is contributing to the output power, then the coherence value will be less than one (but greater then zero) for those frequencies where the noise source adds power to the response signal.

Hence, the coherence function is used to indicate the degree of noise contamination in a transfer function measurement and it can be used in a qualitative way to determine how much averaging is necessary to remove noise from the measurement. Figure 3.13 shows coherence functions corresponding to the transfer function measurement in Figure 3.12. Note that with more averaging the estimate of coherence contains less variance, thus giving a better estimate of the noise energy in the measured signal.

3.4.3 Exponential Smoothing

If after a reasonable amount of averaging, the transfer function measurements are still relatively noisy, the noise can be further removed by means of an exponential smoothing process. Many different types of schemes could be used to smooth data, but the exponential function is advantageous in this case because its effect on the data is known.

The inverse Fourier transform of a transfer function is the sum of the impulse responses of the modes in the measurement bandwidth. An example of this sum of decaying sinusoids is given in Figure 3.14. Measurement noise adds uniformly to this impulse response and therefore gives the result shown in Figure 3.15 or alternatively 3.17.

Since the noise is distributed evenly throughout the block and the signal decays exponentially, the signal-to-noise ratio of the combined signal also decays exponentially.

Now, if the measurement plus noise is multiplied by a decreasing exponential function (called an exponential window), as shown in Figure 3.16, the noise at the right hand end of the block is truncated, while the signal toward the left hand end is preserved. The overall signal to noise ratio of the data is increased since the combined signal plus noise is weighted more heavily in favor of the signal and less in favor of the noise. When the resulting waveform is transformed back to the frequency domain, the corresponding transfer function has been smoothed, as illustrated in Figure 3.18.

Furthermore, the width of the modal resonances, which are governed by the amount of damping in each mode have increased by a known amount. If the exponential window is represented by

$$W(t) = e^{-bt}$$

b = a known constant, then a known amount of damping (b) is added to each mode with each multiplication of the impulse response by the exponential

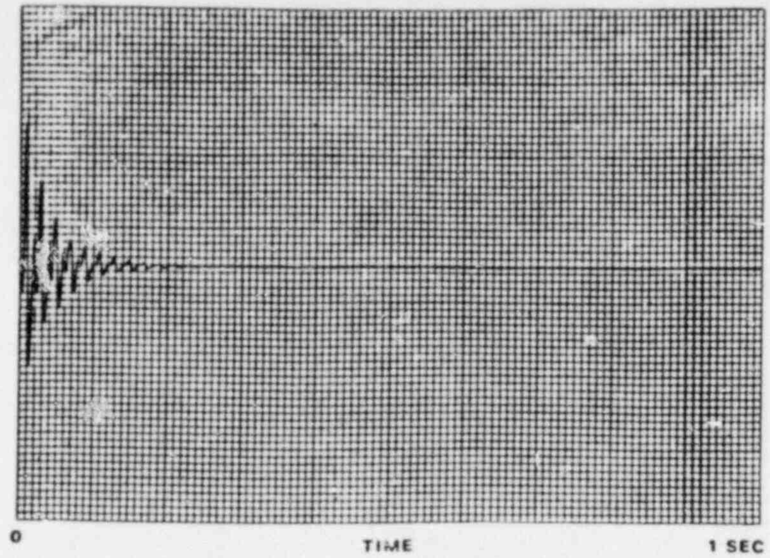


Figure 3.14 Impulse Response

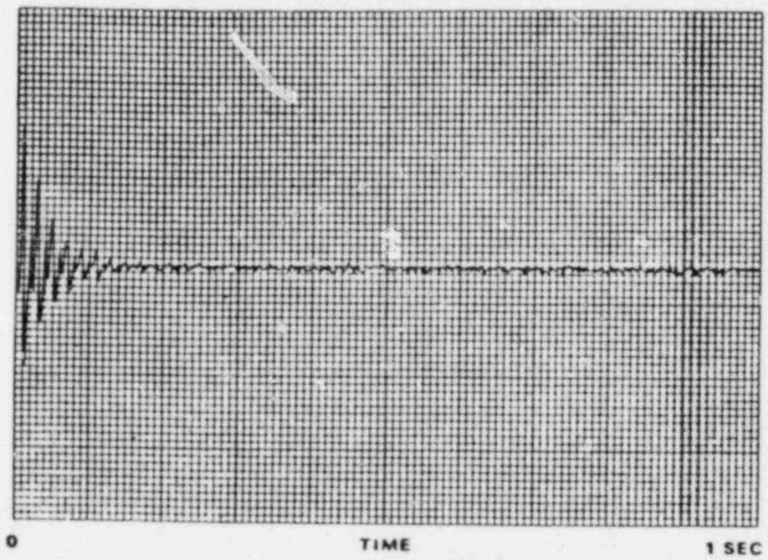


Figure 3.15 Impulse Response plus Random Noise

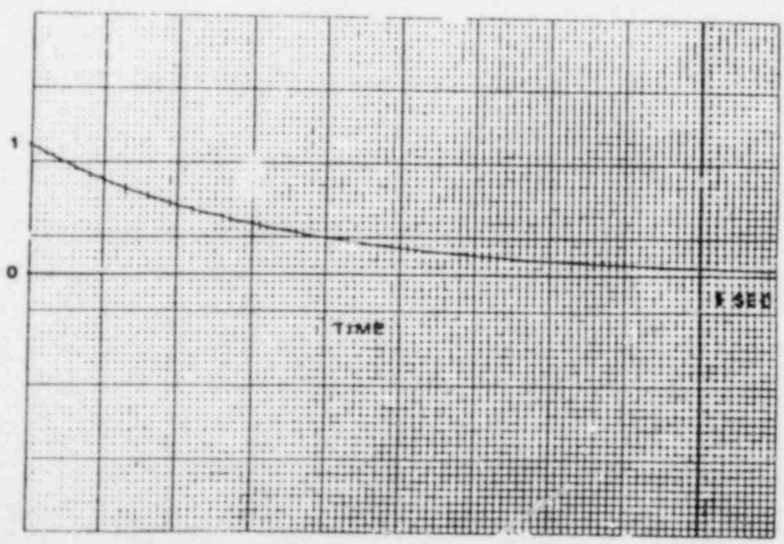


Figure 3.16 Exponential Window

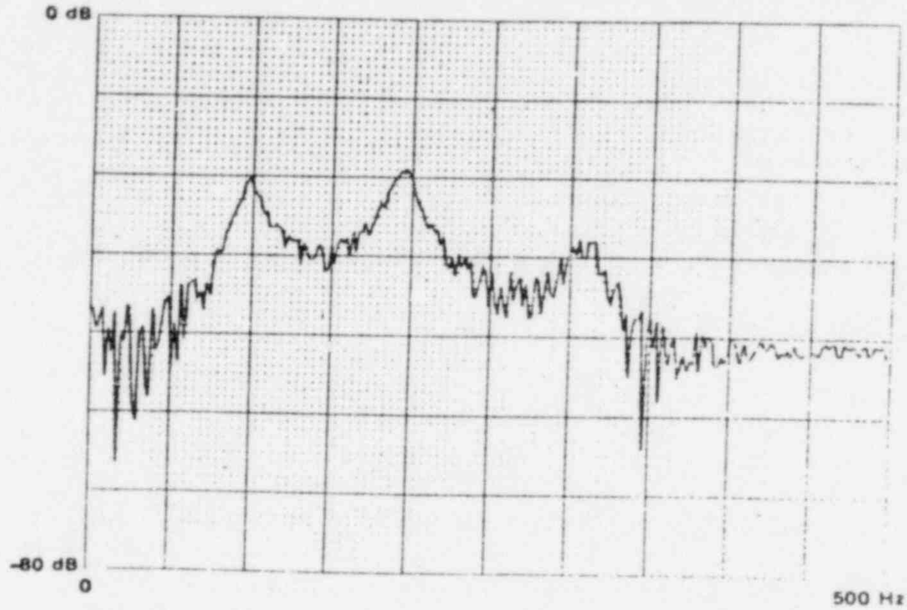


Figure 3.17 **Transfer Function Before Smoothing**

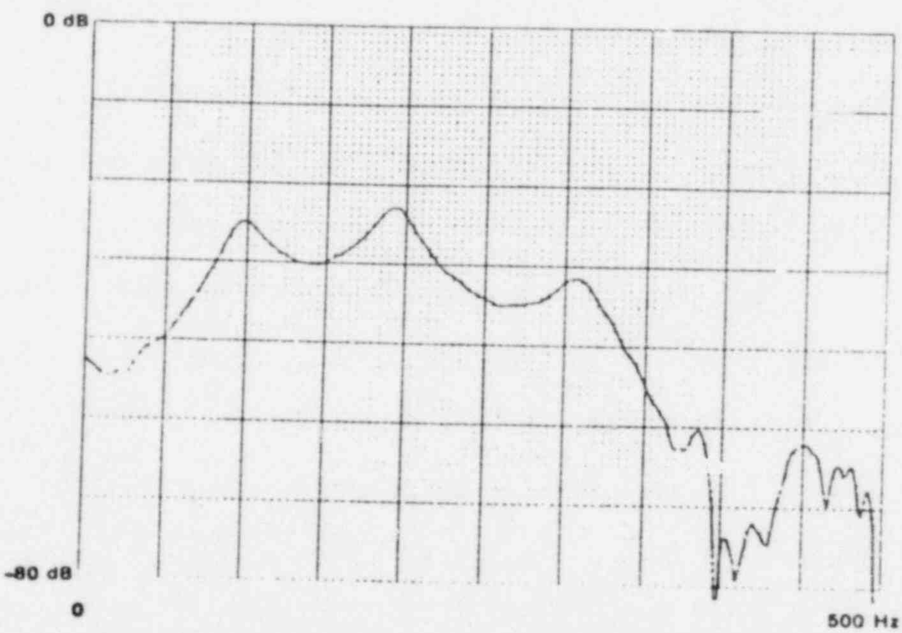


Figure 3.18 **Transfer Function After Smoothing**

window.

The smoothed transfer function data can therefore be used to identify modal parameters and the correct damping coefficient can be recovered by subtracting the amount of damping due to the smoothing process from the identified damping value.

The drawback of this approach to noise removal is that if modes are closely spaced in frequency, too much exponential smoothing will smear them together so that they are no longer discernable as two modes.

3.4.4 Windowing To Reduce Leakage

A key assumption of digital Fourier analysis is that the time waveforms be exactly periodic in the observation window (see reference C.22 for an explanation). If this condition is not met, the corresponding frequency spectrum will contain so-called "leakage" due to the nature of the discrete Fourier transform; that is, energy from the non-periodic parts of the signal will "leak" into the periodic parts of the spectrum, thus giving a less accurate result.

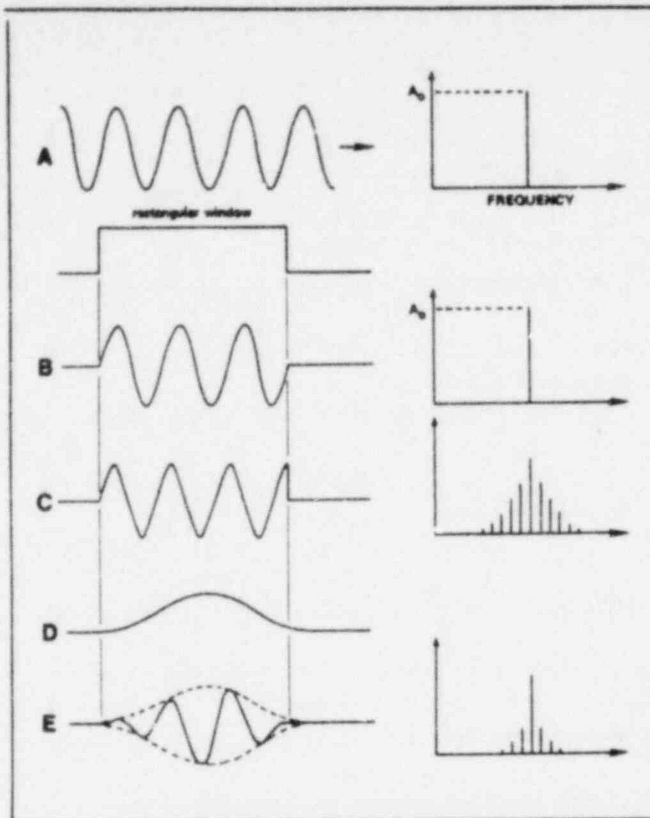
In digital signal analyzers, non-periodic time domain data is typically multiplied by a weighting function such as a Hanning window to help reduce the leakage caused by non-periodic data and a standard rectangular window.

When a non-periodic time waveform is multiplied by this window, the values of the signal in the measurement window more closely satisfy the requirements of a periodic signal. The result is that leakage in the spectrum of a signal which has been multiplied by a Hanning window is greatly reduced.

However, multiplication of two time waveforms, i.e. the non-periodic signal and the Hanning window, is equivalent to the convolution of their respective Fourier transforms. Hence, although multiplication of a non-periodic signal by a Hanning window reduces leakage, the spectrum of the signal is still distorted due to the convolution with the Fourier transform of the Hanning window. Figure 3.19 illustrates these points for a simple sinewave.

3.4.5 Increasing Frequency Resolution

Certainly the single most important factor affecting the accuracy of modal parameters is the accuracy of the transfer function measurements. And, in general, frequency resolution is the most important parameter in the measurement process. From a practical viewpoint, in many complex structures, modal density is so great, and modal coupling (or overlap) so strong, that increased frequency resolution over that obtainable with baseband techniques is an absolute necessity for achieving reliable results. In other words, it is simply not possible to extract the correct values of the modal parameters when there is inadequate information available to process.



(A) A sinewave is continuous throughout time and is represented by a single line in the frequency domain; (B) when observed with a standard rectangular window, it is still a single spectral line, if it is exactly periodic in the window; (C) if it is not periodic in the measurement window, leakage occurs and energy "leaks" into adjacent frequency channels; (D) the Hanning window is one of many types of windows which are useful for reducing the effects of leakage; and (E) multiplying the time domain data by the Hanning window causes it to more closely meet the requirement of a periodic signal, thus reducing the leakage effect.

FIGURE 3.19

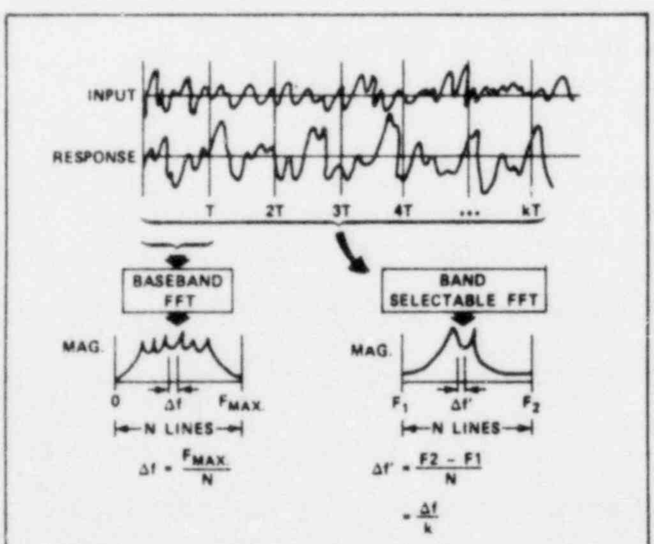


FIGURE 3.20

Modern curve fitting algorithms are highly dependent on adequate resolution in order to give correct modal parameter estimates.

In the past, many digital Fourier Analyzers have been limited to baseband spectral analysis; that is, the frequency band under analysis always extends from dc to some maximum frequency (FMAX). With the Fourier transform, the available number of discrete frequency lines (typically 1024 or 512) are equally spaced over the analysis band. This, in turn, causes the available frequency resolution to be, Δf = (FMAX) (N/2), where N is the Fourier transform block size, i.e. the number of samples describing the real-time function. There are N/2 complex (magnitude and phase) samples in the frequency domain. Thus, FMAX and the block size, N, determine the maximum obtainable frequency resolution.

The problem with baseband Fourier analysis is that, to increase the frequency resolution for a given value of FMAX, the number of lines in the spectrum (i.e. the block size) must increase. There are two important reasons why this is an inefficient way to increase the frequency resolution:

1) As the block size increases, the processing time required to perform the Fourier transform increases.
2) Because of available computer memory sizes, the block size is limited to a relatively small number of samples (typically a maximum of 4096).

More recently, however, the implementation of the so called "Zoom" transform, has made it possible to perform Fourier analysis over a frequency band whose upper and lower frequency limits are independently selectable. Zoom provides this increase frequency resolution without increasing the number of spectral lines in the computer.

Zoom digitally filters the time domain data and stores only the filtered data in memory. The filtered data corresponds to the frequency band of interest as specified by the user. The procedure is completed by performing a Fourier transform on the filtered data.

Of fundamental importance is the fact that the laws of nature and digital signal processing also apply to the Zoom situation. Since the frequency resolution is always equal to the reciprocal of the observation time of the measurement, $\Delta f = 1/T$, the digital filters must process T seconds of data to obtain a frequency resolution of 1/T in the analysis band. Whereas in baseband Fourier analysis the maximum resolution is always $\Delta f = FMAX\ (N/2)$, the resolution with Zoom is $\Delta f = BW\ (N/2)$ where BW is the independently selectable bandwidth of the Zoom measurement. Therefore, by restricting our attention to a narrow region of interest below FMAX and concentrating the entire power of the Fourier transform in this interval, an increase in frequency resolution equal to FMAX/BW can be obtained (Figure 3.20).

The other significant advantage of Zoom is its ability to increase the dynamic range of the measurement to 90 dB or more in many cases. The increased dynamic range of Zoom is a direct result of the extremely sharp roll-off and out-of-band rejection of the pre-processing digital filters and of the increased frequency resolution which reduces the effect of the white quantizing noise of the analyzer's analog-to-digital converter.

Certain types of Zoom filters can provide more than 90 dB of out-of-band rejection relative to a full scale in-band spectral line, a characteristic which is not matched by more traditional analog range translators.

Illustrative Example #1: To illustrate the importance of Zoom, a mechanical structure was tested and modes in the area of 1225HZ to 1525HZ were to be investigated. Figure 3.21 is a typical baseband (dc-FMAX) transfer function measurement. It was taken with the following parameters:

Block size. 1024

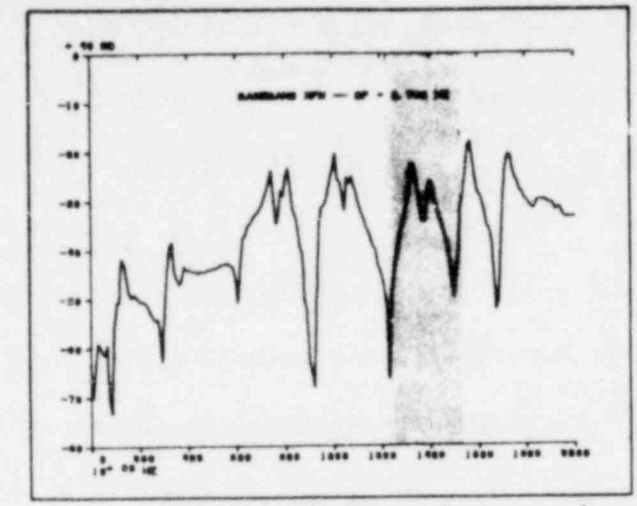
FMAX. . . . . 5000 Hz

Filter cutoff . . . . 2500 HZ

Δf . . . . .9.765 Hz

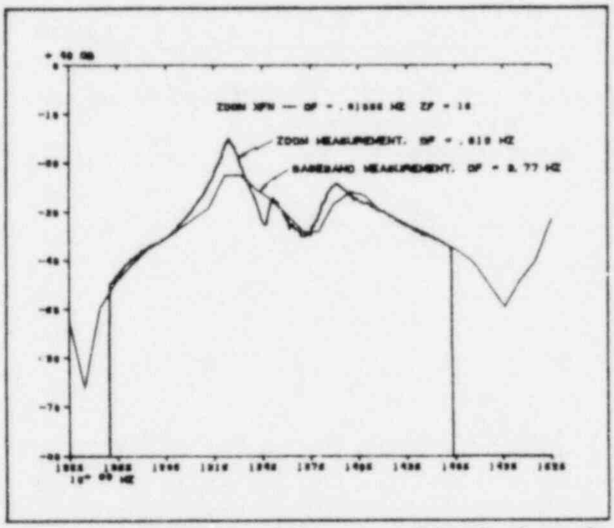
Random noise was used to excite the structure through an electro-dynamic shaker.

The Inadequacy of the Baseband Measurement: Note that two modes are clearly visible between 1225 Hz and 1525 Hz. In Figure 3.22, a partial display of the region between 1225 and 1525 Hz was made. The expanded quadrature display is shown in Figure 3.23. Realize that this represents no increase in frequency resolution, only an expansion of the plot. Clearly, only two modes were found. However, there is also a slight inflection in the response between these two modes which indicates that yet a third mode may be present. But there is insufficient frequency resolution to adequately define the mode.



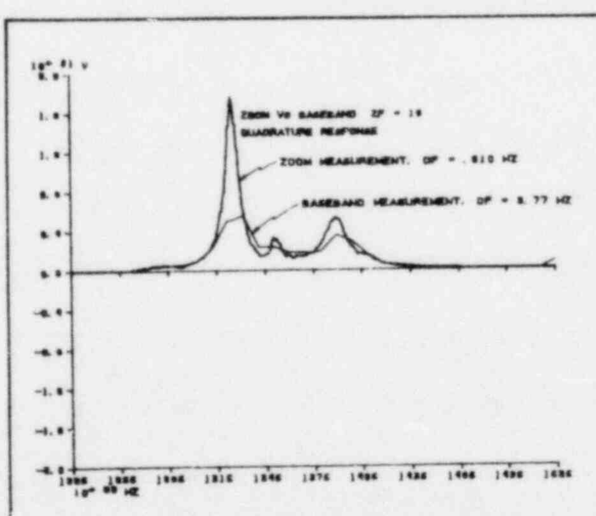
Baseband transfer function shows two modes at approximately 1320 Hz and 1400 Hz.

FIGURE 3.21



Comparison of the Zoom and baseband transfer functions between 1225 and 1525 Hz. In the BSFA result, three modes are clearly visible and well defined. The baseband data would have led to considerable error in estimates of frequency and damping.

FIGURE 3.22



Comparison of quadrature response of the baseband and Zoom result. The Zoom measurement clearly shows the small third mode and the poor result of the baseband measurement for the other two modes.

FIGURE 3.23

Accurate Measurements with Zoom: In order to accurately define the modes in this region, the structure was re-tested using the Zoom transform. All 512 lines of spectral resolution were placed in a band from 1225 to 1525 Hz, resulting in a resolution of 0.610 Hz instead of 9.76 Hz, as in the base-band measurement. The quadrature response attained with the Zoom is also shown in Figure 3.23 for comparitive purposes. Note that three modes are now clearly visible. The small (third) mode of approximately 1350 Hz is now well defined, whereas it was not even apparent before. In addition, the magnitude of the first mode of 1320 Hz is seen to be at least three times greater in value than the result indicated by the baseband measurement. The corresponding results in log form are shown in Figure 3.22. This Zoom result was obtained by using only a 16:1 resolution enhancement. Enhancements of more than 100:1 are possible.

Implications of Frequency Resolution in Determining Modal Parameters: Referring again to Figure 3.23, we can clearly see the necessity of using adequate frequency resolution for making a particular measurement. In addition, it is important to understand how the baseband result would lead to an incorrect answer in terms of modal parameters.

A) Modal Frequency and Damping: If the baseband result is compared to the Zoom result for the 1320 Hz mode it is obvious that the base-band result would indicate that the mode is much more highly damped than it actually is. The second small mode (1350 Hz) would not even be found, and the 1400 Hz mode would also have the wrong damping. Close inspection also shows that the estimate of the resonant.

frequency for the 1320 Hz mode would have significant error.

b) Mode Shape: Any technique for estimating the mode shape coefficients (e.g. quadrature response, circle fitting, differencing, least squares, etc.) would clearly be in error since it is apparent that the Zoom result shows a quadrature response at least three times greater than the baseband result.

Although the preceeding example presented a case where the use of Zoom was a necessity, it is very easy for the engineer to be misled into believing he has made a measurement of adequate resolution when in fact he has not. The following concluding example illustrates this point and presents the estimates of the modal parameters for each case.

Illustrative Example #2: A disc brake rotor was tested using an electro-dynamic shaker and random noise as a stimulus. A load cell was used to measure the input force and an accelerometer mounted near the driving point was used to measure the response. The baseband measurement had a resolution of 9.76 Hz. As can be seen in Figure 3.24, the two major modes at about 1360 Hz and 1500 Hz appear to be well defined. An expanded display (no increased resolution) from 1275 Hz to 1625 Hz clearly shows the two large modes and a much smaller mode at about 1580 Hz.

The rotor was re-tested using Zoom and the two sets of data are compared in Figure 3.24. This data clearly shows tha value of Zoom. The Zoom data provides increased definition of the modal resonances, as can be seen by

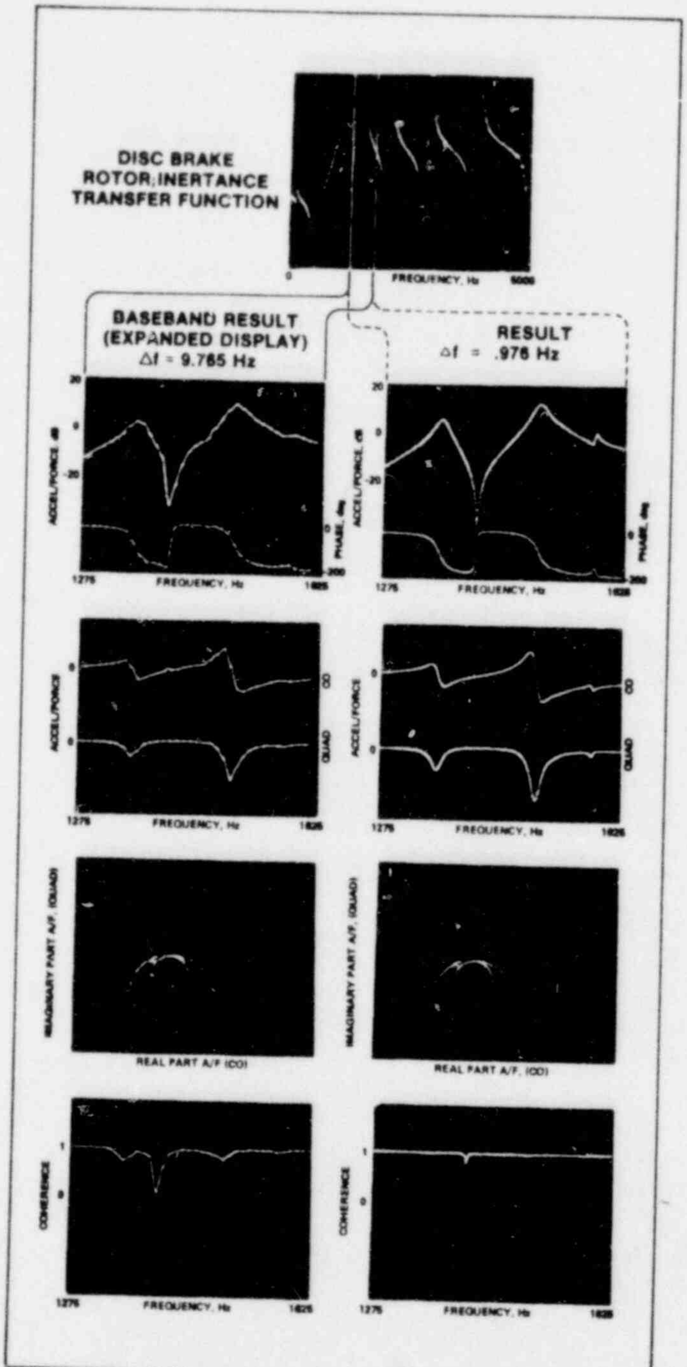

DISC BRAKE
ROTOR INERTANCE
TRANSFER FUNCTION
FREQUENCY, Hz
5000
BASEBAND RESULT
(EXPANDED DISPLAY)
Δf = 9.765 Hz
RESULT
Δf = .976 Hz
ACCEL/FORCE, dB
PHASE, deg
FREQUENCY, Hz
ACCEL/FORCE
QUAD
IMAGINARY PART A/F, (QUAD)
REAL PART A/F, (CO)
COHERENCE

FIGURE 3.24

comparing the baseband and Zoom results. The validity of each result is reflected in the respective coherence functions. The baseband transfer function contains inaccuracies due to the Hanning effect, as well as inadequate resolution. The coherence for the Zoom measurement is unity in the vicinity of all three modal resonances, indicating the quality of the transfer function measurement. Further proof of the increased modal definition is shown in the Zoom Nyquist plot (co versus quad). Here, all three modes are clearly discernible and form almost perfect circles, indicating an excellent measurement, almost totally free of distortion. In the baseband result, only three or four data points were available in the vicinity of each resonance, whereas in the Zoom data many more points are used.

The modal parameters for all three modes were identified from the baseband and Zoom data and the results are shown in Table 3.1. Comparison of results emphasizes the need for Zoom when accurate modal parameters are desired.

In summary, no parameter identification techniques are capable of accurately identifying modal parameters when the frequency resolution of the measurement is not adequate.

TABLE 3.1 -- COMPARISON OF MODAL PARAMETER TEST RESULTS.

Baseband Results, f=9.765 Hz

Mode	Frequency, Hz	Damping, %	Amplitude	Phase
1	1359.99	0.775	193.51	350.3
2	1503.92	0.763	483.30	11.1
3	1584.33	0.273	9.49	336.1

ZOOM Results, f=0.976 Hz

Mode	Frequency, Hz	Damping, %	Amplitude	Phase
1	1359.13	0.669	211.99	352.7
2	1502.65	0.652	509.52	9.4
3	1583.50	0.131	11.65	340.8

Error, %, Versus Baseband

Mode	Frequency, Hz	Damping	Amplitude
1	0	16%	8%
2	0	17%	5%
3	0	108%	19%

3.5 Broadband Excitation Signals

Three basic types of excitation signals are used for transfer function testing. They are transients, swept (or stepped) sine, and random signals. Transient testing typically requires less equipment (shakers and instrumentation) than does sine or random testing. The force level and spectral content of sine and random signals is generally more controllable than with transient signals though, giving them an advantage in situationswhere more control over these variables is needed.

3.5.1 Transient Testing

The transfer function of a system can be determined using virtually any physically realizable input, the only criteria being that some input signal energy exists at all frequencies of interest. However, before the advent of mini-computer-based Fourier analyzers, it was not practical to determine the Fourier transform of experimentally generated input and response signals unless they were purely sinusoidal.

These digital analyzers, by virtue of the fast Fourier transform, have allowed transient testing techniques to become widely used. There are two basic types of transient tests: (1) Impact, and (2) Step Relaxation.

3.5.2 Impact Testing

A very fast method of performing transient tests is to use a hand-held hammer with a load cell mounted to it to impact the structure. The load cell measures the input force and an accelerometer mounted on the structure measures the response. The process of measuring a set of transfer functions by mounting a stationary response transducer (accelerometer) and moving the input force around is equivalent to attaching a mechanical exciter to the structure and moving the response transducer from point to point. In the former case, we are measuring a row of the transfer matrix whereas in the latter we are measuring a column.

In general, impact testing enjoys several important advantages:

1) No elaborate fixturing is required to hold the structure under test.
2) No electro-mechanical exciters are required.
3) The method is extremely fast - often as much as 100 times as fast as an analog swept-sine test.

However, this method also has drawbacks. The most serious is that the power spectrum of the input force is not as easily controlled as it is when a mechanical shaker is used. This causes non-linearities to be excited and can result in some variablity between successive measurements. This is a direct consequence of the shape and amplitude of the input force signal.

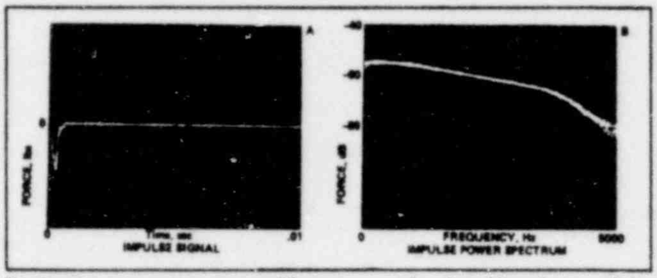
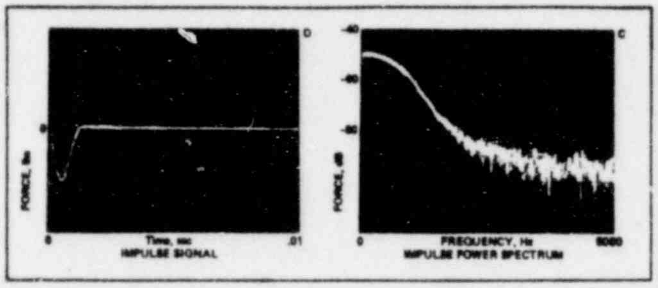


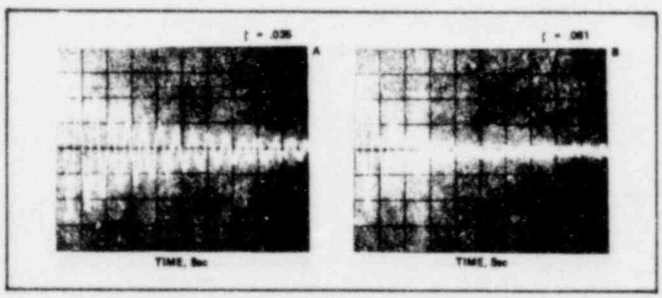
Figure 6 — An instrumented hammer with a hard head is used for exciting higher frequency modes but with reduced energy density.

FIGURE 3.25



An instrumented hammer with a soft head can be used for concentrating more energy at lower frequencies, however, higher frequencies are not excited.

FIGURE 3.26



The impulse responses for two single-degree-of-freedom systems with different amounts of damping. Each measurement contains exactly the same amount of noise. The Fourier transform of the heavily damped system will have more uncertainty because of the poor signal-to-noise ratio in the last half of the data record.

FIGURE 3.27

The impact force can be altered by using a softer or harder hammer head. This, in turn, alters the corresponding power spectrum. In general, the greater the width of the force impulse, the lower the frequency range of excitation. Therefore, impulse testing is a matter of trade-offs. A hammer with a hard head can be used to excite higher frequency modes, whereas a softer head can be used to concentrate more energy at lower frequencies. These two cases are illustrated in Figures 3.25 and 3.26.

Since the total energy supplied by an impulse is distributed over a broad frequency range, the actual excitation energy density is often quite small. This presents a problem when testing large, heavily damped structures, because the transfer function estimate will suffer due to the poor signal-to-noise ratio of the measurement. Ensemble averaging, which can be used with this method, will greatly help the problem of poor signal-to-noise ratios.

Another major problem is that of frequency resolution. Adequate frequency resolution is an absolute necessity in making good structural transfer function measurements. The fundamental nature of a transient response signal places a practical limitation on the resolution obtainable. In order to obtain good frequency resolution for quantifying very lightly damped resonances, a large number of digital data points must be used to represent the signal. This is another way of saying that the Fourier transform size must be large, since

$$\Delta f = \frac{\text{Maximum frequency of interest}}{\text{1/2 Fourier transform size}} = \frac{1}{T}$$

Thus, as the response signal decays to zero, its signal-to-noise ratio becomes smaller and smaller. If it has decayed to a small value before a data record is completely filled, the Fourier transform will be operating mostly on noise, therefore causing uncertainties in the transfer function measurement. Obviously, the problem becomes more acute as higher frequency resolutions are needed and as more heavily damped structures are tested. Figure 3.27 illustrates this case for a simple single-degree-of-freedom system. In essence, frequency resolution and damping form the practical limitations for impulse testing with baseband (dc to FMAX) Fourier analysis.

Since a transient signal may or may not decay to zero within the measurement window, windowing can be a serious problem in many cases, especially when the damping is light and the structure tends to vibrate for a long time. In these instances, the standard rectangular window is unsatisfactory because of the severe leakage. Some Digital Fourier analyzers allow the user to employ a variety of different windows which will alleviate the problem. Typically, a Hanning window would be unsuitable because it destroys data at the beginning of the record - the most important part of a transient signal. The exponential window can be used to preserve the important information in the waveform while at the same time forcing the signal to become periodic. It must, however, be applied with care, especially when modes are closely spaced, for exponential smoothing can smear modes together so that they are no longer discernible as separate modes. Reference C.9 explains this in more detail.

The Force Window

The particular characteristic of an impulsive force signal and the resulting structural response signal make the impulse technique especially susceptible to two problems: noise and truncation errors. While these problems occur to some extent with other frequency response testing techniques, their unique importance in the impulse technique requires special signal processing methods. The noise problem is further aggravated when employing the zoom transform, which yields increased resolution in a given frequency band by effectively increasing the sample length.

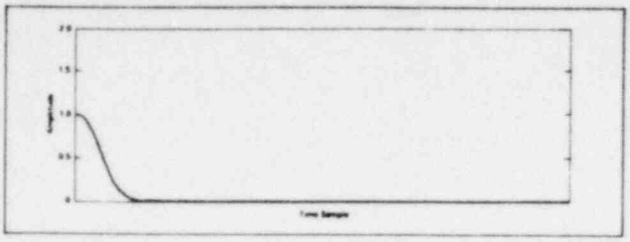
With other techniques, the effects of noise are reduced by averaging the power spectrum and cross-spectrum functions prior to the computation of the frequency response function. However, only a few averages are usually used in the impulse technique. Otherwise, the time advantage of the technique is lost. Therefore, special time-sample windows have been developed for the impulse technique.

At first thought it might seem appropriate to just set all time-sample values beyond the impulse to zero, since it is known that the true signal value after the impulse is zero. However, this would be equivalent to multiplying the signal by a narrow rectangular window. In applying any type of window, it is important to keep in mind that multiplication by a window in one domain is equivalent to convolution of the Fourier transforms of the window and the data in the other domain, resulting in distortion of the transformed signal.

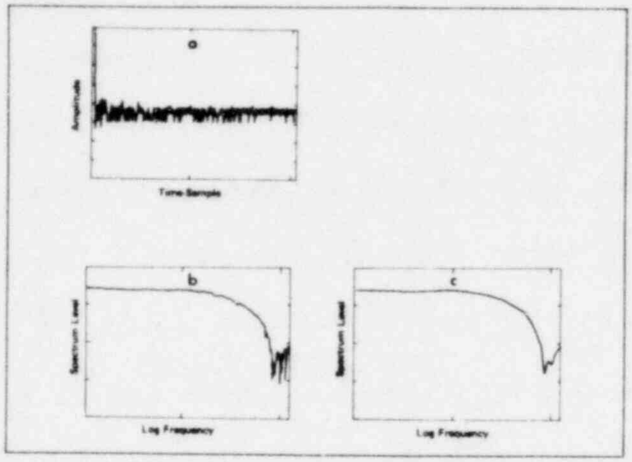
A good compromise has been arrived at in practice in the form of a window with unity amplitude for the duration of the impulse and a cosine taper, with a duration of 1/16 of the sample time, from unity to zero. This window is shown in Figure 3.28. Figure 3.28 shows the results of applying the force window to an impulse signal with significant measurement noise. Comparison of the computed frequency response functions with and without the window applied shows that the window substantially improves the frequency response estimate.

In spite of these problems, the value of impact testing for modal analysis cannot be overstressed. It provides a quick means for troubleshooting vibration problems. For a great many structures an impact can suitably excite the structure such that excellent transfer function measurements can be made. The secret of its success rests with the user and his understanding of the physics of the situation and the basics of digital signal processing.

Numerous ingenious methods have been applied in recent years to excite the transient response of structures. The ones discussed here, all of which are applicable to testing nuclear power plants, are the use of hammer, a dropped weight, snapback or step relaxation, and explosive.

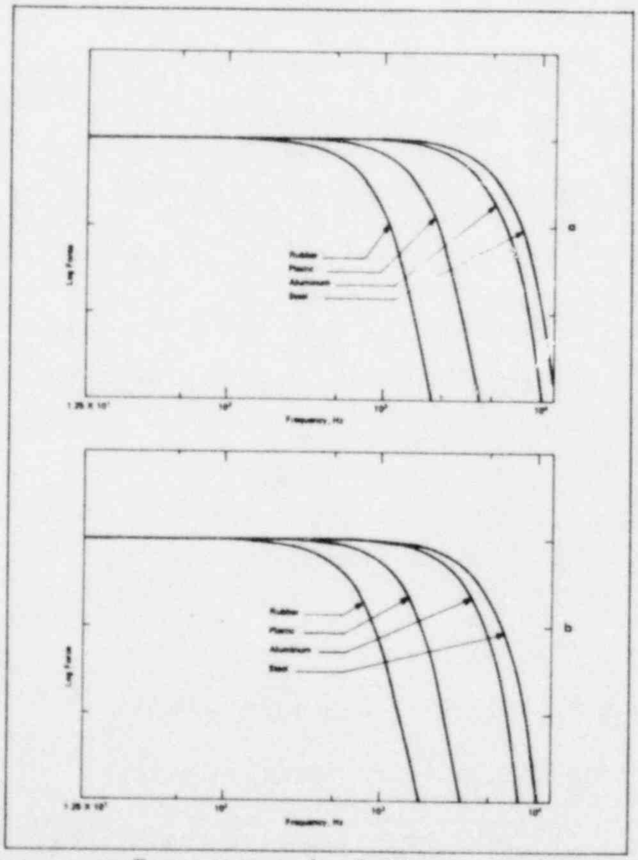


Force window.



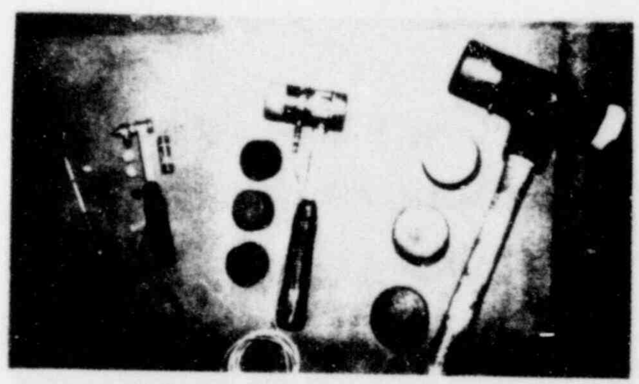
Effect of force window applied to force signal with noise added. a. Time-sample of force signal. b. Spectrum of force signal without window. c. Spectrum of force signal with window applied.

FIGURE 3.28



Force spectra produced with various combinations of hammer weights and tip materials. a. Hammer with no added mass. b. Hammer with added mass.

FIGURE 3.30



Collection of typical hammers used for impact testing.

FIGURE 3.29

Hammer Testing

A simple diagram of a typical hammer test setup is shown in Figure 3.9. The force imparted to the structure by the hammer can be measured by placing a load cell (which measures compressive force) on the head of the hammer. Hammers can vary in size from the small ones shown in Figure 3.29 to a large pendulum hammer supported by a crane.

The physical characteristics of the hammer determine the magnitude and duration of the force pulse. The two most important characteristics are weight and tip hardness. The width of the frequency spectrum is inversely proportional to the weight of the hammer and directly proportional to the hardness of the tip. The force magnitude and duration also depend on the dynamic characteristics of the structure at the point of impact. For instance, it is impossible to excite a weak structure with an impulse of long duration and at the same time impart a large magnitude force.

Figures 3.30 (a) and (b) show the effects of tip hardness and hammer weight upon the impulse power spectrum.

Dropped Weight

Various types of impactors based upon the concept of a dropped weight impacting against a stop and transmitting an impulsive force into the structure have been invented for structural testing. Agbabian Associates, under the direction of Dr. Fred Safford, have developed an impactor suitable for exciting large buildings. They are able to shape the power spectrum of the input force by placing various metal coupons in the machine, which are sheared off by a falling weight. (References C.23, C.24 and D.29 contain details on this machine.)

3.5.3 Step Relaxation Testing

Step relaxation is another form of transient testing which utilizes the same type of signal processing techniques as the impact test. In this method, an inextensible, light-weight cable is attached to the structure and used to pre-load the structure to some acceptable force level. The structure "relaxes" when the cable is severed, and the transient response of the structure, as well as the transient force input, are recorded.

Although this type of excitation is not nearly as convenient to use as the impulse method, it is capable of putting a great deal more energy into the structure in the low frequency range. It is also adaptable to structures which are too fragile or too heavy to be tested with the hand-held hammer described earlier. Obviously, step relaxation testing will also require a more complicated test setup than the impulse method but the actual data acquisition time is the same.

Step relaxation testing, also called snapback testing, has been used successfully to test a number of large structures. Its one advantage over the other transient methods is that it can be controlled somewhat more carefully than the application of an impulsive force.

Ibanez and co-workers (Reference D.6) say the following about snapback testing. "Snapback testing is subject to certain limitations in practice. First, the method tends to excite certain modes and not others. This disadvantage can be overcome by repeated tests with the excitation applied at various locations. Another limitation is that snapback tests often excite more than one mode, sometimes leading to difficulty in identifying damping coefficients. A method of applying the force and a method of quick release are required for snapback testing. Winches, cables, cranes, or hydraulic rams can be used to apply the force. Quick release is obtained by high-speed hydraulic valves, unlatching mechanisms, or frangible links which fail at a predetermined applied force."

3.5.4 Explosive Testing

Explosive testing has been used successfully on power plant structures to simulate motions caused by an earthquake. References F.5, F.16, E.12 and E.14 document some of these results.

3.5.5 Sinusoidal Testing

Until the advent of the Fourier analyzer, the measurement of transfer functions was accomplished almost exclusively through the use of swept-sine testing. With this method, a controlled sinusoidal force is input to the structure, and the ratio of output response to the input force versus frequency is plotted. Although sine testing was initially done using analog instrumentation, it is certainly not limited to that any more. Sinusoidal data can be digitized and processed with the Fourier analyzer to form transfer functions.

In general, swept sinusoidal excitation with analog instrumentation has several disadvantages which severely limit its effectiveness:

1) Using analog techniques, the low frequency range is often limited to a bandwidth of several Hertz.
2) The data acquisition time can be long.
3) The dynamic range of the analog instrumentation limits the accuracy range of the transfer function measurements.
4) Accuracy and repeatability are often difficult to achieve.
5) Non-linearities and distortion are not easily coped with.

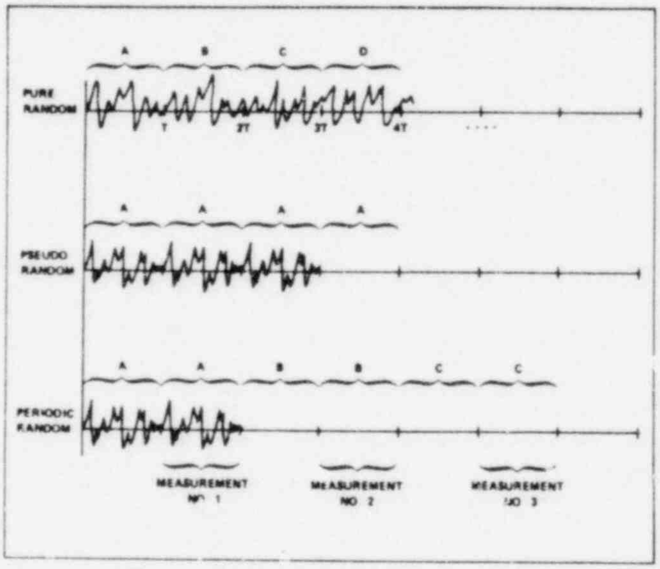
Nevertheless, swept sine testing does offer some advantages over other testing forms:

1) Large amounts of energy can be input to the structure at desired frequencies.
2) The excitation force level can be controlled accurately.

Being able to excite a structure with large amounts of energy provides at least two benefits. First, it results in relatively high signal-to-noise ratios which aid in determining transfer function accuracy and, secondly, it allows the study of structural non-linearities provided the sweep frequency can be controlled.

Sine testing can be very slow, depending upon the frequency range of interest and the sweep rate required to adequately define modal resonances. However, a sinusoidal stimulus can be utilized in conjunction with a digital Fourier Analyzer in many different ways. However, the fastest and most popular method utilizes a type of signal referred to as a "chirp". A chirp is a logarithmically swept sinewave that is periodic in the analyzer's measurement window T (assume that the window is T seconds wide). The swept sine is generated in the computer and output through a DAC (Digital-to-Analog Converter) every T seconds. Figure 3.32 shows a chirp signal. The important advantage of this type of signal is that it is sinusoidal and has a good peak-to-rms ratio. This is an important consideration in obtaining the maximum accuracy and dynamic range from the signal conditioning electronics which comprise part of the test setup. Since the signal is periodic, leakage is not a problem. However, the chirp suffers the same disadvantage as any sine si nal; it can easily cause distortion.

Any number of alternate schemes for using sinusoidal excitation can be implemented on a Fourier analyzer. However, they will not be discussed here because they offer few, if any, advantages over the chrip and, in fact, generally serve to make the measurement process more tedious and lengt .



Comparison of pure random, pseudo random, and periodic random noise. Pure random is never periodic. Pseudo random is exactly periodic every T seconds. Periodic random is a combination of both; i.e., a pseudo random signal that is changed for every ensemble average.

FIGURE 3.31

3.5.6 Random Excitation Techniques

In this section, three types of broadband random excitation which can be used for making frequency response measurements are discussed. Each one possesses a distinct set of characteristics which should be understood in order to use them effectively. The three types are (1) pure random, (2) pseudo random, and (3) periodic random.

Typically, pure random signals are generated by an external signal generator, whereas pseudo random and periodic random are generated by the analyzer's processor and output to the structure via a digital-to-analog converter (DAC). Figure 3.31 illustrates each type of random signal.

Pure Random

Pure random excitation typically has a Gaussian distribution and is characterized by the fact that it is in no way periodic, i.e. does not repeat after any time period. Typically this signal is generated by an independent signal generator and may be passed through a bandpass filter in order to concentrate energy in a frequency band of interest. Generally, the signal spectrum will be flat except for the filter roll-off and, hence, only the overall level is easily controlled.

One disadvantage of this approach is that, although the shaker is driven with a signal having a flat input spectrum, the structure is excited by a force with a different spectrum due to the impedance mismatch between the structure and shaker head. This means that the force spectrum is not easily controlled and the structure may not be excited in the optimum manner. Since it is difficult to shape the spectrum beforehand, some form of closed-loop force control system could ideally be used to modify the spectrum of the drive signal on line to compensate for the impedance mismatch. Fortunately, in most cases, the problem is not important enough to warrant this effort.

A more serious drawback of pure random excitation is that the measured input and response signals are not periodic in the measurement time window of the analyzer.

With a pure random signal, each sampled record of data T seconds long is different from the preceeding and following records (Figure 3.31). This gives rise to the single most important advantage of using a pure random signal for transfer function measurement. Successive records of frequency domain data can be ensemble averaged together to remove non-linear effects, noise and distortion from the measurement. As more and more averages are taken, all of these higher order components of a structure's motion will average toward an expected value of zero in the frequency domain data. Thus, a least squares estimate of the linear response of the structure is obtained.

This is especially important because modal parameter estimation schemes are all based on linear models and the premise that the structure behaves in a linear manner. Measurements that contain distortion are difficult to handle if the modal parameter identification techniques used are based upon a linear model of the structure's motion.

Pseudo Random

In order to avoid the leakage effects of a non-periodic pure random signal, a waveform known as pseudo random is commonly used. This type of excitation is easy to implement with a digital Fourier analyzer and its digital-to-analog (DAC) converter. The most commonly used pseudo random signal is referred to as "zero-variance random noise". It has uniform spectral density and random phase. The signal is generated in the computer and repeatedly output to the shaker through the DAC every T seconds (Figure 3.31). The length of the pseudo random record is thus exactly the same as the analyzer's measurement record length (T seconds), and is thus exactly periodic in the measurement window.

Because the signal generation process is controlled by the analyzer's computer, any signal which can be described digitally can be output through the DAC. The desired output signal is generated by specifying the amplitude spectrum in the frequency domain; the phase spectrum is normally random. The spectrum is then Fourier transformed to the time domain and output through the DAC. In this case, it is relatively easy to alter the stimulus spectrum to account for the exciter/system impedance characteristics.

In general, besides providing leakage-free measurements, a technique using pseudo random noise can often provide the fastest means for making a statistically accurate transfer function measurement when using a random stimulus. This proves to be the case when the measurement is relatively free of extraneous noise and the system behaves linearly.

The most serious disadvantage of this type of signal is that because it always repeats with every measurement record taken, non-linearities, distortion and periodicities due to rattling or loose components on the structure cannot be removed from the measurement by ensemble averaging, since the structure is excited in the same manner every time the pseudo random signal is output.

Periodic Random

Periodic random signals combine the best features of pure random and pseudo random, but without the disadvantages. That is, a periodic random signal satisfies the conditions for a periodic signal, yet changes with time so that it excites the structure in a truly random manner.

The process begins by outputting a pseudo random signal from the DAC to the exciter. After the transient part of the response has died out and the structure is vibrating in its steady-state condition, a measurement is taken; i.e. input, output, and cross power spectrums are formed. Then, instead of continuing to output the same signal again, a different uncorrelated pseudo random signal is synthesized and output (refer again to Figure 3.31). This new signal excites the structure in a new steady-state manner and another measurement is made.

When the power spectrums of these and many other records are averaged together, non-linearities and distortion components are removed from the transfer function estimate. Thus, the ability to use a periodic random signal eliminates leakage problems and ensemble averaging is now useful for removing distortion because the structure is subjected to a different excitation before each measurement.

The only drawback to this approach is that it is not as fast as pseudo random or pure random testing, since the transient part of the structure's response must be allowed to die out before a new ensemble average can be made. The time required to obtain a comparable number of averages may be anywhere from 2 to 3 times as long. Still, in many practical cases where a baseband measurement is appropriate, periodic random provides the best solution, in spite of the increased measurement time.

3.5.7 Removing Distortion From Measurements - A Case Study

Distortion, or non-linear motion, is an unwanted contaminant of vibration measurements when modal parameters are to be identified from them. Since the dynamic model upon which modal testing is based is linear, the measurements must accurately describe linear motion of the structure, and not reflect any non-linear motion.

In this case study a single-degree-of-freedom system was tested with each type of excitation method previously discussed. Besides measuring the linear characteristics of the system with each excitation type, gross non-

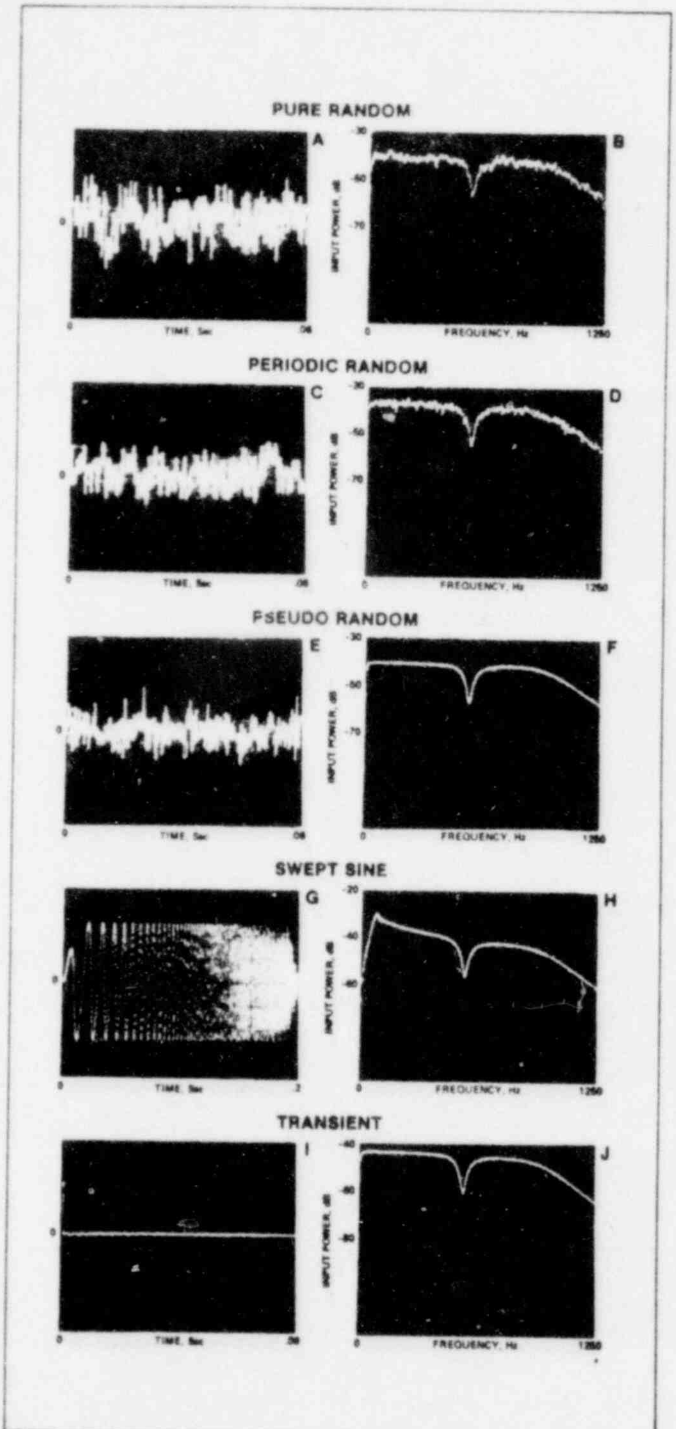


Figure 9 — Different excitation types and their power spectrums. Each type was used to test a single-degree-of-freedom system. Fifteen ensemble averages were used.

FIGURE 3.32

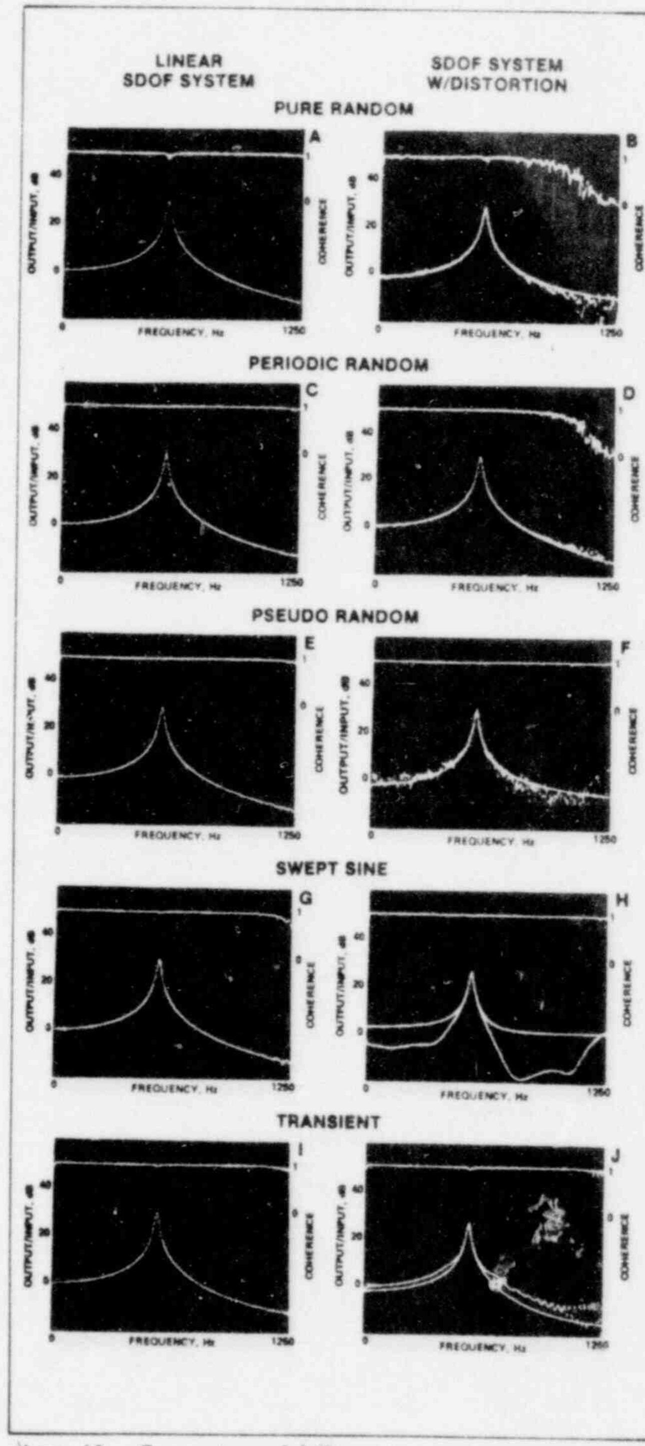


Figure 10 — Comparison of different excitation types for testing the same single-degree-of-freedom system with and without distortion.

FIGURE 3.33

linear response was simulated by clipping approximately one-third of the output signal. This condition simulates a "hard stop" in an otherwise unconstrained system. The intent of these tests was to show how certain forms of excitation can be used to measure the linear characteristics of a system even in the presence of a large amount of distortion. This is extremely important to the engineer who is interested in identifying modal parameters.

Figure 3.32 illustrates the form of each type of stimulus and its power spectrum after fifteen ensemble averages. Notice that the input power spectrums for both the pure random and periodic random cases have more variance than the others. This is because each ensemble average consisted of a new and uncorrelated signal for these two stimuli. The pseudo random and swept sine (chirp) signals were controlled by the analyzer's digital-to-analog converter and each ensemble average was in fact the same signal, thus resulting in zero variance. In this test, the transient signal was also controlled by the DAC to obtain record-to-record repeatability and resulting zero variance. In all cases, the notching in the power spectrums is due to the impedance mismatch between the structure and the shaker. A final interesting note is that all spectrums except the swept sine are flat out to the cut-off frequency. The roll-off of the swept sine spectrum is due to the logarithmic sweep rate. Thus, the spectrum has reduced energy density as the frequency is increased.

In Figure 3.33, the results obtained from testing the single-degree-of-freedom system with and without distortion are shown. In Figures 3.33 (a) and (b), the cases for pure random excitation, notice that the coherence is noticeably different from unity in the vicinity of the resonance. This is due to the non-periodicity of the signals and the fact that Hanning windowing was used to reduce what would have otherwise been even more severe leakage. The leakage effect is much more sensitive here, due to the sharpness of the resonance, i.e. the rate of change of the function. Although the effect is certainly present throughout the rest of the band, the relatively small changes in response level between data points away from the resonance will obviously tend to minimize the leakage from adjacent channels. Although any number of different windowing functions could have been used, the phenomenon would still exist.

In all figures 3.33 showing the distorted case, the best fit of a linear model to the measured data is also shown. Note also that the coherence is almost exactly unity for the linear case. This is because all except the pure random case are ideally leakage-free measurements because they are periodic in the analyzer's measurement window. For the cases with distortion, the latter three show very good coherence even though the system output was highly distorted. This apparently good value of coherence is due to the nature of the zero-variance periodic signals used as stimuli. In cases B and D, the measurements are truly random from average to average and the coherence is more indicative of the quality of the measurement. The low coherence values at the higher frequencies are primarily a result of the poor signal energy. The conclusion is that the coherence function can be misleading if one does not understand the measurement situation.

It is apparent from Figure 3.33 that the pure random and periodic random stimuli provided the best means for transfer function measurements in the presence of distortion. Again, this is due to the effective use of ensemble averaging to remove the distortion components from the measurement in these cases. The distortion cannot be removed using the other types of periodic stimuli. The results obtained from fitting a linear model to the measured data are given in Table 3.2.

In all cases where the linear motion was measured, each type of excitation gave excellent results, as indeed they should. The one item worthy of mention is the estimate of damping with the pure random result. In this case, the value is about 7% higher than the correct value ($\approx$ 53 RAD/sec). This error is due to the windowing effect on the data. In this test, a Hanning window was used. However, any number of other windows could have been used and error would still be present. Further evidence of the Hanning effect on the data is shown by the error between the linear model and the measured data.

Of considerable importance is the data for the simulated distortion. The primary conclusion that can be drawn from this data is that the periodic random stimulus provides a good means for measuring the linear response of a linear system and is clearly superior to a pure random stimulus. It is also the best possible excitation for measuring the linear response of a system with distortion. Evidence of this is seen in the quality of the parameter estimates in Table 3.2 and the relative error (the error index between the ideal linear model and the measured data). The principal characteristics of each type of excitation are summarized in Table 3.3

Table 3.2 - Comparison of Single-Degree-of-Freedom System Modal Parameters From Measurements Using Various Excitation and Analysis Techniques.

Test Condition	Frequency (Hz)	Damping Coefficient (rad/sec)	Magnitude	Phase (deg)	Relative Error
Pure Random	549.44	56.83	3429.12	0.5	23.1
Pure Random w/Distortion	550.10	56.22	2963.28	357.1	21.7
Periodic Random	549.46	52.76	3442.18	0.6	3.6
Periodic Random w/Distortion	549.50	53.44	3272.00	0.4	4.2
Pseudo Random	549.55	52.76	3450.54	0.6	1.8
Pseudo Random w/Distortion	550.09	51.75	2766.06	359.3	32.4
Swept Sine	549.49	53.24	3444.01	0.6	2.2
Swept Sine w/Distortion	549.77	53.76	2411.52	4.5	21.5
Transient	549.63	53.75	3453.26	0.7	5.7
Transient w/Distortion	549.68	53.13	2200.84	359.4	102.9
Zoom with Pure Random	549.44	53.12	3446.84	0.7	3.2

Table - Principal Characteristics of Five Excitation Methods.

Characteristics	Pure Random	Pseudo Random	Periodic Random	Impact	Swept Sine (Chirp)
Force level is easily controlled	Yes	Yes	Yes	No	Yes
Force spectrum can be easily shaped	No	Yes	Yes	No	Yes
Peak-to-rms energy level	Good	Good	Good	Poor	Best
Requires a well-designed fixture and exciter system	Yes	Yes	Yes	No	Yes
Ensemble averaging can be applied to remove extraneous noise	Yes	Yes	Yes	Yes	Yes
Non-linearities and distortion effects can be removed by ensemble averaging	Yes	No	Yes	Somewhat	No
Leakage Error	Yes	No	No	Sometimes	No

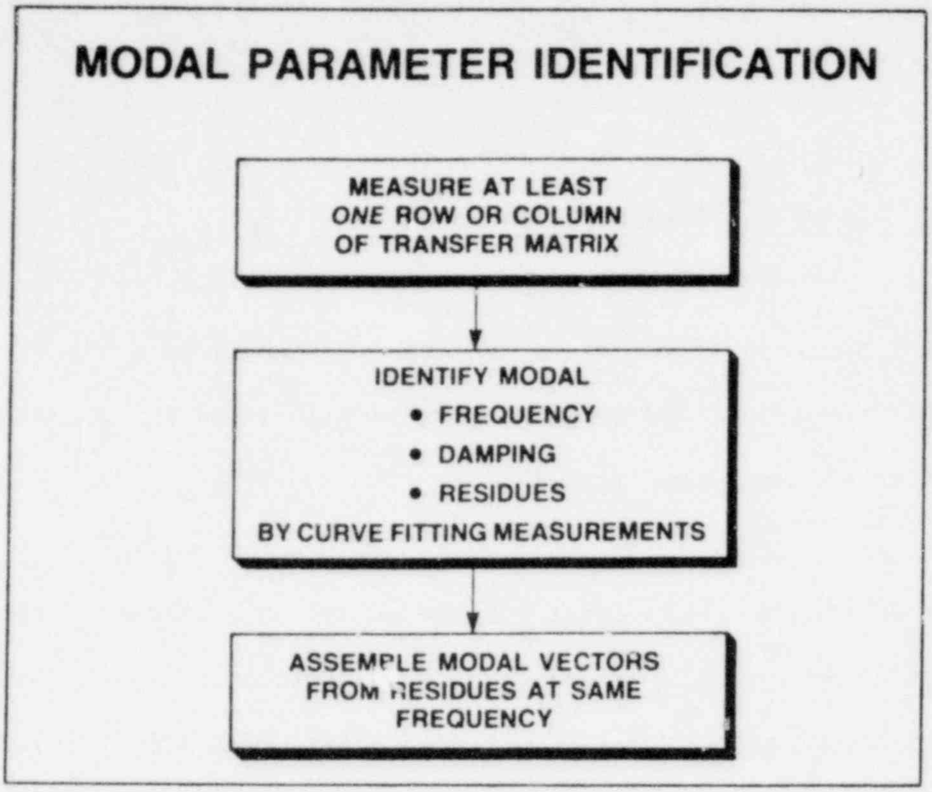


FIGURE 3.34

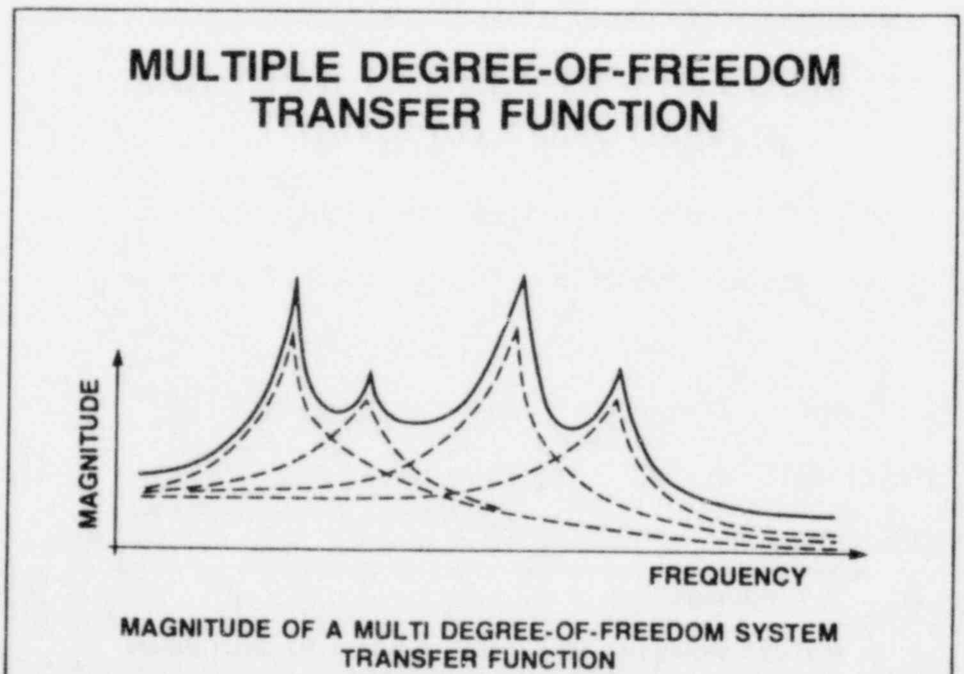


FIGURE 3.35

3.6 MODAL PARAMETER IDENTIFICATION BY CURVE FITTING

Once a set of transfer functions has been measured from a structure, modal parameters are identified by "curve fitting" an ideal form of the transfer function to the measurement data.

As shown in Figure 3.34, at least one row or column of transfer functions from the transfer matrix must be measured in order to identify the mode shapes. The mode shapes, or mode vectors, are then assembled from the identified residues from each measurement at the same modal frequency. This process is depicted in Figure 3.34.

Figure 3.35 shows a breakdown of a measurement into a summation of the contributions due to each of the modes of vibration. That is, the magnitude of the transfer function shown by the solid line in the figure, is really the summation of a number of magnitude functions shown by the dashed lines in the figure, each one due to a different mode of vibration.

The modal parameters (frequency, damping and residue), of a single mode can be identified by curve fitting the dashed line function corresponding to that mode. However, since only the solid line function was measured, it is clear that to identify modal parameters accurately, all the parameters of all the modes must be identified simultaneously by some method which fits a multiple mode form of the transfer function to the data. This method is called a "multiple mode" curve fitting method.

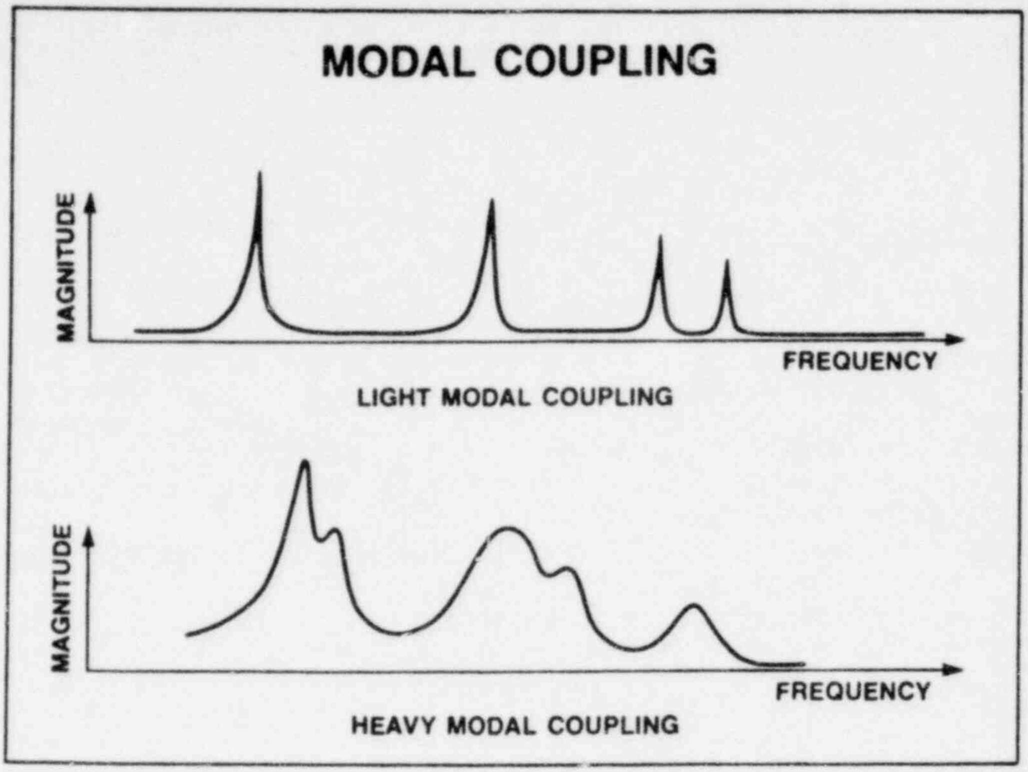


FIGURE 3.36

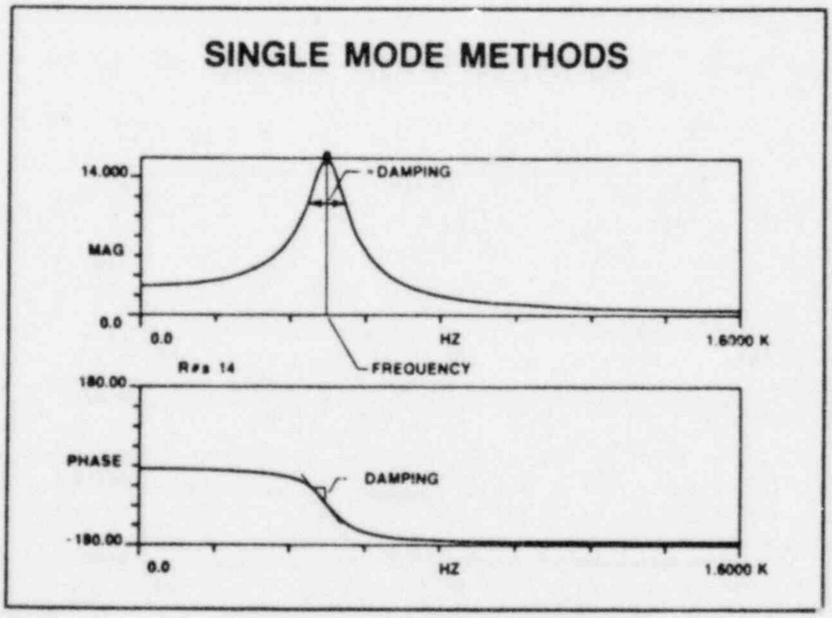


FIGURE 3.38

Many times, the accuracy of a multiple mode method is not required, so simple, easier-to-use methods known as "single mode" methods are used to identify the unknown parameters. A single mode method treats the data in the vicinity of a modal resonance peak as if it is due solely to a single mode of vibration. In other words the "tails" of the resonance curves from other modes are considered to have negligible contribution to the data in the vicinity of the resonance peak in question.

3.6.1 Modal Coupling

The amount of error incurred with the use of single mode methods is dictated by the amount of "modal coupling" in the measurements. Figure 3.36 shows cases of light and heavy modal coupling.

In a light modal coupling case the measurement data in the vicinity of a modal resonance peak is predominantly due to the mode, and the influence of the other modes is minimal. In this case a single mode curve fitting method can give accurate results.

In a heavy modal coupling case the influence of the tails of other modes is not negligible, so a single mode method will incorrectly identify modal parameters.

CURVE FITTING METHODS

1. SINGLE MODE METHODS
   - FAST
   - EASY TO USE
   - LARGE ERRORS CAN OCCUR WITH HEAVY NOISE OR MODAL COUPLING
2. MULTIPLE MODE METHODS
   - HANDLE NOISE AND MODAL COUPLING WELL IF FREQUENCY RESOLUTION IS SUFFICIENT
   - REQUIRE MORE OPERATOR SKILL
   - CAN GIVE ERRONEOUS RESULTS
3. MULTIPLE ROW/COLUMN RESIDUE SORTING
   - ALLOWS MORE MEASUREMENT FLEXIBILITY
   - IMPROVED ACCURACY OVER SINGLE ROW/COLUMN RESULTS
   - REQUIRES MORE MEASUREMENTS
4. MULTIPLE MEASUREMENT CURVE FITTING
   - MORE ACCURATE FREQ. AND DAMP. ESTIMATES
   - LARGE AMOUNT OF DATA PROCESSING

FIGURE 3.37

3.6.2 Curve Fitting Methods

Four different approaches to curve fitting measurements can be taken, as shown in Figure 3.37. The first two (single mode, and multiple mode) are applied to a single measurement at a time. The second two methods can be used on measurements from multiple rows and columns of the transfer matrix, to obtain better estimates of the modal parameters.

3.6.3 Single Mode Methods

Because of their speed and ease of use, single mode methods should be used whenever sufficiently accurate results can be obtained.

The simplest single mode methods are shown in Figures 3.38 and 3.39. Figure 3.38 shows that modal frequency is simply taken as the frequency of the peak of the transfer function magnitude. Damping can be obtained by measuring the width of the modal peak at 70.7% of its peak value or by computing the slope of the phase function at resonance. The first method is known as the "half power point" method since 70.7% of the magnitude is the same as 50%, or half, of the magnitude squared. The half power point is also referred to as the 3 db point (3 db = 10 Log (2)). Finally the residue can be estimated by using the peak value of the imaginary part of transfer function at resonance. This is known as "quadrature picking" or simply the quadrature method.

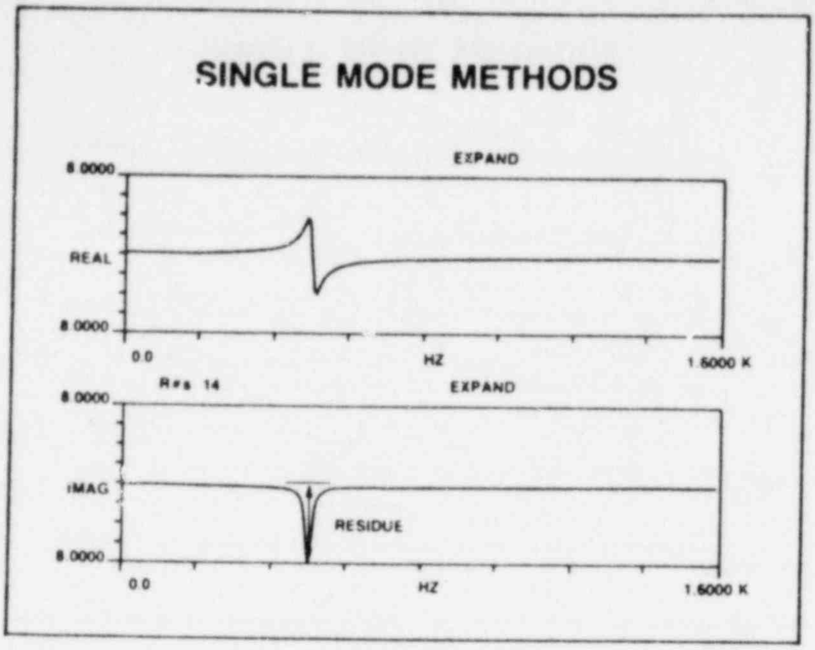


FIGURE 3.39

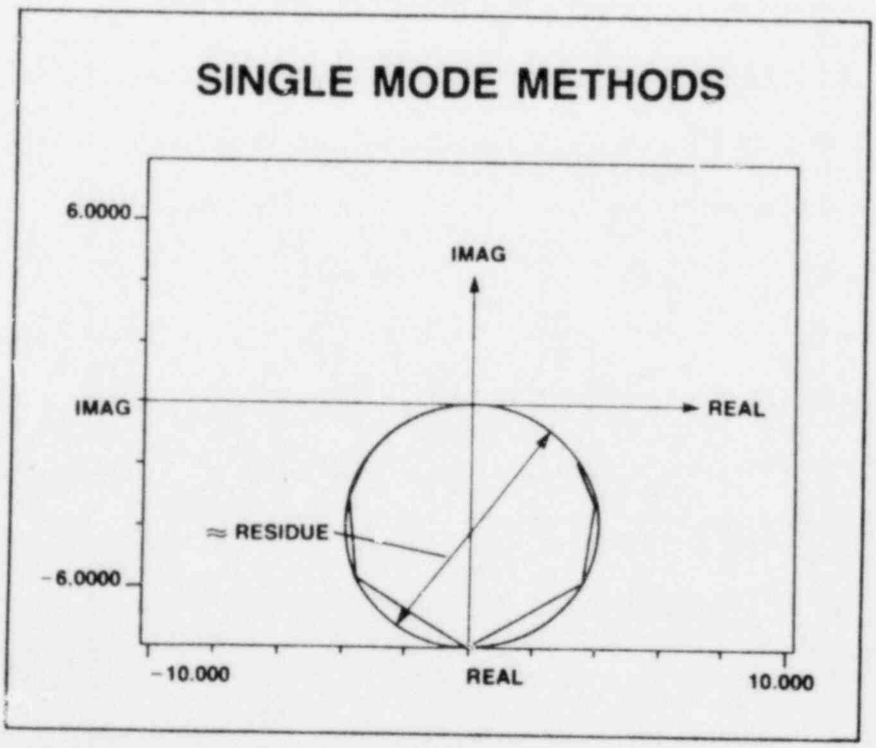


FIGURE 3.40

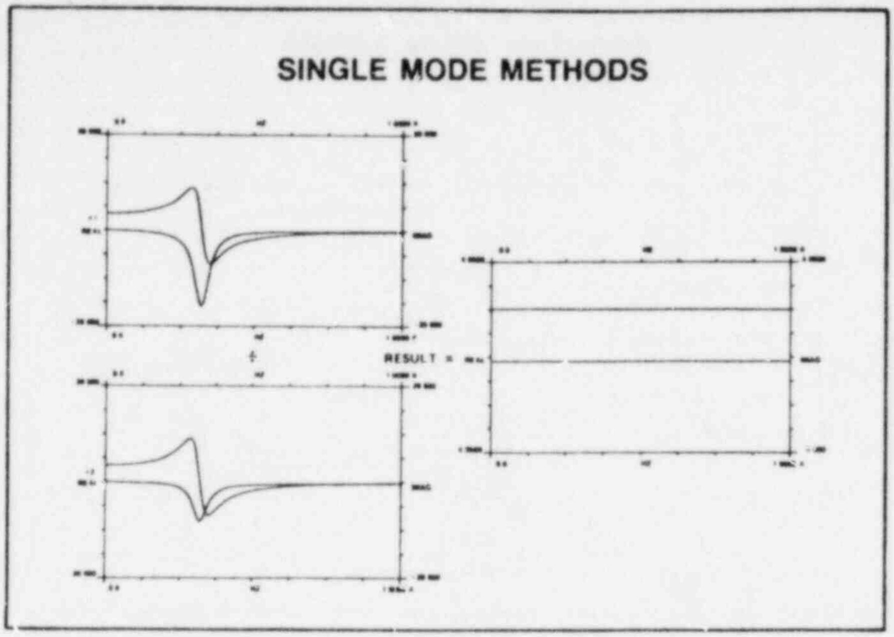


FIGURE 3.41

MULTIPLE MODE METHODS

LEAST SQUARED ERROR FIT OF *PARTIAL FRACTION* FORM

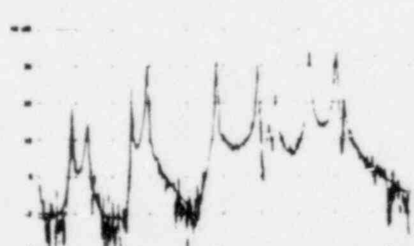
$$H(s) = \sum_{k=1}^{n} \left[\frac{r_k}{s - p_k} + \frac{r_k^*}{s - p_k^*} \right] \Bigg|_{s = j\omega}$$

OR *POLYNOMIAL* FORM

$$H(s) = \frac{a_0 s^m + a_1 s^{m-1} + \cdots + a_m}{b_0 s^{2n} + b_1 s^{2n-1} + \cdots + b_{2n}} \Bigg|_{s = j\omega}$$

MEAS. 1

MODE	FREQ(HZ)	DAMP(%)	RESIDUE AMPL	RESIDUE PHS
1	45.5497	.2475	87.1859	186.1
2	66.9754	1.5672	118.0868	2.5
3	126.1113	.1931	129.5194	163.1
4	148.4356	.1823	446.5868	349.8
5	248.5847	.1553	685.3258	354.4
6	258.5986	.2320	97.7994	159.4
7	303.5528	.1824	475.4058	181.0
8	319.2407	.7852	229.3786	182.8
9	328.2283	.2582	288.7471	169.2
10	373.5362	.2579	898.5685	358.1
11	413.4484	.1136	981.4277	169.2
12	422.8821	.3129	548.5385	186.6



Magnitude Curve — Multi Degree-of-Freedom Curve Fit

FIGURE 3.42

If the measurements are noisy or if the frequency resolution is not good, all of the above methods can yield largely incorrect results since they use only one or two data points from the measurement.

The methods shown in Figures 3.40 and 3.41 will yield better results in general, since they use more measurement data. The method shown in Figure 3.40, the so called "circle fitting" method, is a way of estimating the residue by least squared error fitting the parametric form of a circle to the measurement data in Nyquist form. This method was first pointed out by Kennedy and Pancu (Reference B.1). The method shown in Figure 3.41 is a simple complex division of one measurement into all the other measurements in the set. The result of each divide is a complex constant in the vicinity of a resonance. Several values over an interval of frequencies around the resonance can then be averaged together to obtain an estimate of the residue. These methods and others are explained in more detail in Reference C.3.

3.6.4 Multiple Mode Methods

Multiple mode methods involve curve fitting a multiple mode form of the transfer function to a frequency interval of measurement data containing several modal resonance peaks. In the process, all the modal parameters (frequency, damping and residue) for each mode are simultaneously identified.

Figure 3.42 shows two different forms of the transfer function which can be used for curve fitting. If the partial fraction form is used, the residues and poles (frequency and damping) can be identified directly from the curve fitting process. Normally, a least squared error curve fitting procedure is used. This yields a set of linear equations which must be solved for the residues, and a set of non-linear equations which must be solved for the poles. An iterative procedure is normally used to solve these equations.

If the polynomial form of the transfer function is used, the coefficients of the numerator and denominator polynomials are identified during curve fitting, and a root finding routine must then be employed to find the poles and residues. The advantage of the polynominal form, however, is that the least squared error equations are linear and thus can be solved for the unknown coefficients by non-iterative methods.

3.6.5 Complex Exponential Curve Fitting

A third method of fitting a parametric form of the impulse response function to impulse data, (obtained by inverse Fourier transforming the transfer function data into the time domain) is also commonly used. This method is called the Complex Exponential method or the Prony algorithm.

The impulse response can be written in terms of modal parameters as shown in Chapter 2.

This form, which can also be written as the sum of complex exponential functions, is curve fit to the time domain data using a very efficient algorithm. (Reference C.25)

All of these multiple mode methods will yield the same result, as shown in Figure 3.42, namely a list of modal parameters identified from a particular measurement. The solid line in the transfer function plot of Figure 3.42 is the ideal fit function superimposed on the data.

3.6.6 Residue Sorting

The accuracy of the curve fitting results obviously depends heavily upon the quality of the transfer functions measurements made. One of the key factors influencing the results is the proper choice of the reference D.O.F., i.e. single excitation point or reference response point. If the reference D.O.F. is chosen to correspond to a node point for a particular mode, then the response of that mode will not appear in the measurements and may be overlooked completely in a modal survey. On the other hand, it may be impossible to adequately excite certain modes from a single shaker location, especially on large structures.

The residue sorting algorithm described in Reference C.4 allows sorting of modal residue data identified from transfer function measurements made with different reference D.O.F.'s to obtain a modal vector which can give a much better estimate of the true behavior of the test structure. This algorithm allows modal testing in a more arbitrary manner than has been possible before. Use of this technique in transfer function testing offers several distinct and beneficial advantages.

(1) Much more test data in the form of rows or columns of the transfer matrix can be combined to permit superior estimates of modal parameters. This technique can be especially useful to reduce the effect of bad measurements by utilizing redundant sets of measurements and eliminating or reducing the contribution of inconsistent residue values.

(2) Judicious and selective excitation can be used on a structure to reduce modal density to an acceptable level. Unwanted responses can be minimized and desired modes enhanced by proper selection and subsequent combination of appropriate measurement sets using differ it reference D.O.F.'s.

(3) Redundant sets of data from different rows and/or columns of the residue matrix can be used to verify that modes have been properly identified, e.g. the same mode identified in all measurements.

(4) Signal-to-noise problems inherent in the testing of large structures with transfer function techniques can be overcome by application of multi-excitation point testing, i.e. the use of several reference D.O.F.'s appropriately located on the structure.

3.6.7 Multiple Row/Column Fitting

Many times modal coupling and/or noise on the measurements may be so great that it is difficult to determine how many modes there are, or to correctly identify their parameters from any single measurement. In these cases a curve fitting procedure that identifies modal parameters from the entire set of measurements should be used.

The last two "multiple measurement" methods are not commonly used today, but will probably be employed more in the future as curve fitting methods continue to improve.

3.7 Monitoring Ambient Vibrations

There are many sources of ambient vibration in or around a nuclear power plant. In many cases the vibration levels on certain machinery or structures may be of sufficient amplitude to identify the structure's modal properties without applying further excitation.

Sources such as rotating machinery, wind, moving fluids or seismic activity are common ambient sources of vibration excitation. These sources typically cause random vibrations to occur in the plant's structural components. (Rotating equipment will also cause sinusoidal vibrations at frequencies directly related to the rotational speeds of the machine.) Vibrations caused by operating machinery in the plant can, of course, be monitored whenever the plant is operational, whereas vibrations caused by seismic or wind effects could only be monitored when these forces cause vibrations of sufficient levels for identifying modal properties.

The primary difficulty however, with using ambient vibration signals to identify modal parameters is determining whether or not the structure is being excited sufficiently over the frequency range of the modes of interest. For example, if we are monitoring at 30 Hz mode from a particular measurement point and the ambient vibration signal from that point contains insufficient energy in the vicinity of 30 Hz, the parameters of the 30 Hz mode cannot be identified.

Nevertheless, in situations where the excitation force cannot be measured, none of the previously discussed modal testing methods which involve some kind of controlled and measured force input can be used anyway, which leaves the ambient techniques discussed here as the only possible means of identifying modal parameters.

Two different methods are discussed here for processing ambient vibration signals to identify modal parameters. They involve the curve fitting of power spectrums (i.e. PSD's) or impulse responses. The PSD's can be computed via the FFT and spectrum averaging techniques discussed earlier for transfer functions, and the impulse responses are computed via the so-called Random Decrement averaging process.

3.7.1 Modal Parameters From Power Spectrums

In any of the modern day FFT analyzers available on the market today the power spectrum or power spectral density is a common measurement function.

We will not discuss the details of the computation of this function here (Reference C.6 covers this subject), but merely define the power spectrum in terms of the Fourier transform as

$$G_{xx} = F_x F_x^*$$

where F_x = Fourier transform of a time waveform.

* denotes conjugate

Curve fitting techniques like those discussed in the previous section can be used to identify modal parameters from power spectrum data if the following assumption is satisfied.

Assumption: The power spectrum of the excitation force on the structure is flat in the vicinity of all modal resonance peaks of interest.

This is similar to assuming that the noise is "white" but is not quite as severe. The requirement for this assumption comes from the following simple analysis:

$$|H|^2 = HH^* = \frac{F_y}{F_x}\left(\frac{F_y^*}{F_x^*}\right) = \frac{G_{yy}}{G_{xx}}$$

where F_x, F_y = Fourier transforms of input and output signals respectively

G_{yy} = output power spectrum

G_{xx} = input power spectrum

H = transfer function

The above equation shows that the transfer function magnitude squared is proportional to the output (or ambient response) power spectrum when the input (or ambient excitation) power spectrum is assumed to be flat, i.e.

equal to a constant, for all frequencies. This is the least biased assumption possible when the input to the structure is unknown or not measured. Since we are only interested in the power spectrum data in the vicinity of a modal resonance peak we can relax the flatness assumption so that it is true only in the vicinity of a resonance peak.

For cases of light damping some of the single mode curve fitting methods used on transfer functions can also be used on power spectrums. For instance, the resonance peak frequency can be used as the modal natural frequency and modal damping can be gotten from the 3 db width of the resonance peak. More sophisticated curve fittings using a polynomial representation of the transfer function magnitude squared can also be used on power spectrums.

3.7.2 The Random Decrement Method

In May 1971, Henry Cole Jr. reported on a novel time domain signal averaging technique that had been used at Ames Research Center, Moffett Field, California to identify modal damping during wind tunnel tests of a Space Shuttle model. (Reference D.30) He called the method Random Decrement presumably because he applied it to random signals and used the logarithmic decrement method to identify damping from the resulting impulse response data.

Following are some of his introductory comments on the method.

"It is well known that a structure may be characterized by its impulse or frequency response and that these curves may be obtained, respectively, by taking cross correlations or cross spectra of the input and output when the input is a stationary random force. However, there are many practical situations in which the input occurs at so many points that it cannot be measured (i.e. the present case of a wing driven by a turbulent airflow), and the above methods cannot be employed. Fortunately, the forms of the spectral density of inputs due to turbulent flow do not vary greatly, and it was shown in reference D.41 that the form of the autocorrelation of the output is not very sensitive to the practical variations in input spectral densities. Thus, the structure may be characterized by the autocorrelation of the output even though the exact form of the input is unknown. The curve obtained is representative of the free vibration curve of the structure following a step displacement, and as suggested in reference D.41, may possibly be used as a means of failure detection.

The main problem with the use of autocorrelation signatures is that the level of the curve is dependent on the intensity of the random input, and in a natural environment this cannot be controlled. If the structure is a linear system, the level changes can be compensated for by normalizing the curves, but if the structure is nonlinear as is often the case, a different signature will be obtained with each level of excitation. This problem can be overcome by amplitude filtering one channel as shown in reference D.41, but the computational difficulties are considerable.

A third method for obtaining signatures is to average segments of the time history which start at a given constant amplitude. This method, called 'random decrement', is suggested by Section II of reference D.41, in which it is pointed out that the multiplications performed in autocorrelation merely serve to prevent the summation from tending to zero. Furthermore, the nonlinear systems part of that paper indicates that the curve obtained by autocorrelation is acutally a linearly dimensioned curve which represents the free vibration curve of the system with an initial amplitude equal to the root mean square of the response. If follows that such a curve may be more easily obtained by averaging the time history directly rather than multiplying and averaging as in autocorrelation. The averaging process has the further advantage that the results obtained do not vary with the intensity of the input and consistent results are obtained for nonlinear systems. For these reasons the 'random decrement' method was used in the present report to explore possibilities of its use as a failure detector."

The basic concept behind the random decrement method is that the response of a linear system to random excitation is made up of the summation of two responses (or solutions); a particular response and a homogeneous response (which is the system impulse response). If the particular response can be assumed to be zero-mean, Gaussian white noise, then if a large number of time records are properly averaged together, the random response will sum to zero leaving the impulse response.

As Cole claims above, this averaging process also "averages out" distortion (or nonlinear motion) from the response leaving a root mean square linear approximation of the response. This is equivalent to the frequency domain averaging with use of a pure random signal described earlier, which removes distortion from transfer function measurements.

Depending upon the frequency bandwidth of the excitation force and/or the bandpass filtering done on the signal before averaging, the resulting impulse response will be the summation of impulse responses of one or more modes of vibration.

Initially, Cole and co-workers bandpass filtered their data so that the resulting impulse response was the response of only one mode. They then used the logarithmic decrement method to obtain mode damping from the envelope of the impulse response.

Since then, other people (e.g. C. S. Chang (Ref. D.32) and S. R. Ibrahim (Ref. D.33) have applied multiple mode curve fitting techniques to Random Decrement signatures (impulse responses) to simultaneously identify parameters for several modes. In fact, Ibrahim et. al. (Ref. D.5, D.10, D.11) have combined Random Decrement with their own time domain curve fitting to perform complete modal surveys in a test laboratory environment where they controlled the excitation force on the test specimen.

The primary difficulty with the Random Decrement method as a modal testing method is that since the excitation force is not measured, it has to be assumed that it is <u>white</u>, e.g. its spectrum is <u>flat</u> over the entire analysis bandwidth of interest. This is a more severe assumption than with the use of power spectrums, since the response power spectrum only needs to be uncorrupted by the excitation force in the vicinity of the modal resonance peaks of interest. The impulse response function on the other hand, must remain totally uncorrupted by the excitation if the modal parameters are to be properly identified.

Aircraft Flutter Testing

Aircraft flutter testing involves the identification of modal parameters from ambient vibration measurements. Flutter is a nonlinear process whereby, as the speed of an aircraft increases, two of the modes of vibration of the wings begin to move toward one another in frequency, until they reach the point where they both coalesce, with the same frequency and damping values. Then as the speed continues to increase, one mode increases in damping while the other decreases, eventually taking on a negative (unstable) value and the wings begin to shake off. Hence, the objective of flutter testing is to monitor the frequency and damping of the flutter modes as the aircraft speed increases, and then use these values to extrapolate ahead to the onset of flutter.

Several references on flutter testing are given in the bibliography (Ref. D.34 through D.37)

4.0 EXPERIMENTAL/ANALYTICAL STUDIES

The majority of the literature we surveyed which contained specific investigative results relating to structural integrity monitoring was categorized as follows:

1) Nuclear Plants
2) Large Structures
3) Rotating Machinery
4) Offshore Platforms

By far the largest amount of literature found was on rotating machinery monitoring. Monitoring of overall vibration levels has become very commonplace but attempts to relate structural damage to measured model changes is still in its primitive stages.

The majority of the literature on testing of nuclear plants and large structures has been done primarily to develop dynamic models for predecting responses to earthquakes. This work is motivated by the need to predict and assess damage after an earthquake.

Offshore platforms are continually subjected to a very active dynamic environment where the threat of excessive external excitation, i.e. storms, is ever present. As a result a majority of the literature we found on offshore platforms was directly concerned with detecting structural damage via changes in modal properties.

Following are key excerpts from many of the papers we reviewed.

4.1 NUCLEAR POWER PLANTS

In our search of the literature on integrity monitoring of nuclear plants, we found several references specifically on the subject along with a large number of papers on dynamic testing and seismic qualification of nuclear plants.

In a recent paper Gopal and Ciaranitaro (Ref. E.3) detail several different types of diagnostic systems for nuclear plants which they have evaluated. They are:

> "(1) Vibration Monitoring System for detection of changes in vibrational characteristics of the major components of Nuclear Steam Supply System (NSSS) and Balance of Plant (BOP); (2) Acoustic Monitoring System for detection and location of leaks in the primary system pressure boundary and other piping systems in PWRs; (3) Metal Impact Monitoring for detection of loose debris in the reactor vessel and steam generators; (4) Nuclear Noise Monitoring System for monitoring core barrel vibration; (5) Sensor Response Time Measurement System for detecting any degradtion of process sensors; and (6) Transit Time Flow Meter for determining primary coolant flow rate."

The author's comments about the benefits of plant monitoring are quite appropriate;

> "Substantial economic benefits are relizable in nuclear power plants by increasing availability of these plants. A significant part of non-availability of plants is due to equipment failure causing forced outages. Benefits in improved availability are realizable through on-line surveillance systems by two ways. The first one is to reduce unscheduled downtime through the early detection of abnormalities and the subsequent prevention of major malfunctions. The second one is through improved maintenance scheduling. Prior knowledge of equipment condition will enable planned maintenance during a scheduled shutdown rather than be forced into an unscheduled outage or perform unnecessary maintenance before it is actually needed."

The authors have developed and tested a "Vibration Surveillance System" which monitors the following components of a plant.

> "1. Reactor System: Vibration monitoring establishes the data base or vibration signature for the reactor coolant system and supports to permit trend analysis for changes in the amplitude of the frequency spectra.
>
> 2. Rotating Equipment: Vibration monitoring of critical pumps and motors to provide an early warning of malfunctions.
>
> 3. Valves: Vibration monitoring of valves provides early detection of abnormal behavior of the valves.
>
> The Vibration Monitoring System is designed to:
>
> 1. Characterize and quantify vibration levels from external sensors on major components.
>
> 2. To determine significance of vibration level to an operator by indicators such as normal (green), caution (yellow), and alarm (red) and audible levels.
>
> 3. To determine trends at various frequencies."

One of the problems they discuss in detail is the monitoring of flow-induced vibration of secondary system piping. Some of their conclusions are:

> "The results of the combined experimental and analyses program indicated:
>
> In situ, dynamic structural monitoring of high pressure steam and feedwater piping can be used to obtain piping frequencies and modal displacements using a limited number of dynamic transducers. For the piping runs monitored, the approach taken provides frequencies and modal displacements for the first five to eight modes. Typically, the frequency range for these modes ranges from two to ten Hz."

The authors also discuss their so called "Nuclear Noise Monitor" system which monitors vibrations of the reactor core barrel. Their decription of the system is as follows:

> "The system detects changes in lateral core barrel vibration through analysis of signals from the excore power range neutron detectors of the Nuclear Instrumentation System (NIS). Lateral core oscillation causes a change of the neutron attenuation between the core and the excore neutron detectors and thus a fluctuation of the detector signal occurs. By appropriate signal conditioning procedures, lateral core barrel movement can be discriminated from noise sources and displayed on meters and/or recording devices."

They claim to have gotten excellent agreement between frequency spectra obtained with the NNM system and those measured with strain gauges mounted directly on the core barrel. A clear benefit of this is that nuclear noise monitoring may be able to replace more expensive instrumentation as a means of monitoring core barrel behavior.

Two papers by Fry, Kryter and co-workers at Oak Ridge National Laboratory also contain detailed discussions on nuclear noise monitoring. (Refs. E.4 and E.5)

In one reference (Ref. E.1), Ibanez and co-workers tested some electrical distribution equipment in order to collect modal data for modelling gross seismic responses. In their words:

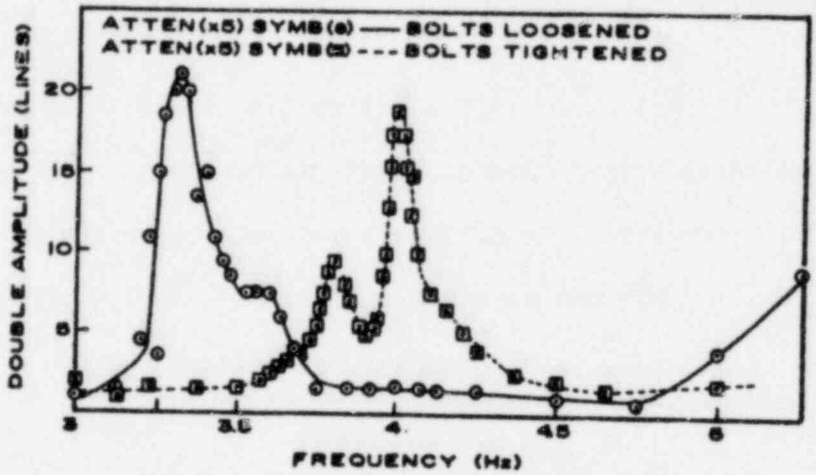
> "Forced vibration testing was carried out by mounting a sinusoidal steady-state eccentric mass vibration exciter (shaker) on the structure to be tested. The response of various points on the structure was measured by accelerometers as the frequency of excitation slowly changed in incremental steps. The object of the forced vibration testing and subsequent analysis was to identify the seismically important modes of vibration; their mode shapes, eigenfrequencies, and dampings. Once identified, these parameters can be used to predict the response of the tested system to a variety of earthquake inputs."
>
> "The equipment tested included:
>
> 1) a high voltage d.c. current divider
> 2) a 500 kV reactor disconnect
> 3) a 230 kV air circuit breaker
> 4) a free-standing 500 kV lightning arrester
> 5) a 500 kV lightning arrester connected to a transformer
> 6) a 500 kV transformer and bushing
> 7) a 230 kV SF-6 circuit breaker
>
> Four of these components were tested in detail using forced vibration methods (the two 500 kV lightning arresters, the 500 kV transformer and the 230 kV SF-6 circuit breaker)."

During the course of their test they discovered several conditions involving variations in structural physical characteristics which were detected through measurement of vibrations:

> "These tests and other tests which the authors have recently conducted point out the important effect on dynamic response caused by structural details. Three examples will be cited: the effect of lightning arrester conductors, the effect of mounting bolt tension, and the effect of variations in soils and foundation designs.
>
> The lightning arrester conductors were found to significantly modify its response, compared to a similar unit not having conductors.
>
> In another test, a slight reduction in mounting bolt tension caused dramatic changes in dynamic response, including changes in modal frequencies, modal dampings, and even mode shapes. Soil conditions and foundation designs have been observed to vary from site to site.
>
> The implication of these observations is that dynamic modeling, whether based on tests or analysis, must give adequate consideration to the actual conditions to be encountered in the field. Our concern here is that laboratory tests or a dynamic analysis may demonstrate the adequacy of a given design, only to have it fail in the field because as installed it magically becomes another structure."

One of their more graphic results showed the effect of mounting bolt tightness on one of the modal frequencies.

> "The dynamic response of the capacitor bank was highly dependent on the tightness of the bolts holding the lower insulators to the foundation. A slight loosening of these bolts caused the EW resonant frequency to drop from 4.00 to 3.3 Hz, and damping to increase from 1.5 to about 3.5%. Figure 4.1 shows the effect of loo ening the mounting bolts."



Effect of loose bolts.

FIGURE 4.1

As further evidence that modal properties (and hence physical characteristics) can vary widely between seemingly identical structures, Ibanez and co-workers examined four utility poles which were constructed of the same material and had the same physical dimensions. They included the following table in one of their recent papers (Ref. E.7):

SUMMARY OF POLE TEST RESULTS

Pole	Spring Constant, (k, lbf/in) Transverse	Parallel	1st Mode Frequency/Damping Transverse	Parallel
A	39.5	454.5	0.90 Hz/ 6%	8.0 Hz/4.4%
B	32.2	277.8	0.78 Hz/?	6.7 Hz/5.5%
C	34.6	262.5	0.85 Hz/ 5.5%	5.4 Hz/4.5%
D	40.0	375.0	0.83 Hz/2.5%	7.3 Hz/5%

These results indicate two things; firstly that slight variations in physical properties do cause perceptable variations in modal parameter values; and, secondly that historical modal data from a particular structure will be necessary in any successful integrity monitoring scheme.

Another recent paper by P.J. Pekrul at Rockwell International describes an on-line Reactor Noise, Vibration, and Loose Parts Monitoring system. (Ref. E.6).

> "This unit consists of piezoelectric sensors, preamplifiers, signal processor units, computers and other peripheral equipment, and provides on-line measurement without intereference to normal plant operation.
>
> An automatic scanning system (Spectra-Scan) device places all of the channels under computer control. The computer selects the channels sequentially for analysis on the spectrum analyzer. The output of the analyzer is compared with the spectra stored in the memory of the computer, and any significant deviations are annunciated. These deviations may be either in amplitude or in the characteristic frequency of a mechanical resonance. The graphical results are displayed on the screen of a storage-type cathode ray tube, showing both the reference and the newly obtained spectra. If no problems are found, the system continues to scan all of the channels in sequence. A status tabulation is maintained on the display, giving identification on each of the channels being monitored."
>
> "In the event of an alert or alarm condition, the operator is called and the component or location of the malfunction is indicated. The computer also prints out the appropriate action to be taken by the operator."

Another recent paper by German author B.J. Olma (Ref. E.18) discusses what he calls "incipient failure" monitoring. He makes the distinction between this and loose parts monitoring with the following comments:

"The objective of an incipient failure detection within the wider frame of structural analysis lies in the surveillance of the mechanical integrity and the protection of security relevant components of the primary circuit of a nuclear power plant during its operation."

"The essential parts of an incipient failure detection system for primary circuit components are vibration monitoring and monitoring of loose parts. Whereas loose parts monitoring indicates the occurrence of damage, vibration monitoring can give an indication of damage in the early developing stage of such a failure. Both parts are supplementary and offer with the necessary background a very good insight in the mechanical state of the primary circuit."

The authors describe the potential for integrity monitoring as follows:

"During the normal operation of a plant the pressure-, mass flow-, neutron flux-, acceleration- and displacement signals, which can be obtained from the process instrumentation or additional excore or incore instrumentation are stationary, random and partly superposed by deterministic spectral parts. These signals are basically of statistic nature which suggests the application of correlation analysis methods. The adequate respresentation of these signals in the frequency domain by means of power spectral densities gives a practicable method for the interpretation of the underlying structural dynamic processes.

With present possibilities of Fast-Fourier transformation VPSD's (vibration power spectral densities), coherences, phase relations and transfer functions can be determined either by on-line hardware processors or off-line with adequate digital programs."

The paper discusses a method for computing vibrational power spectral densities (VPSD's) of primary circuit components based on a finite-element representation of the primary circuit.

The author compares computed responses with some measurement data and concludes with the following comment:

> "The described method gives a mathematical representation of the real vibrative behaviour of primary circuit components with random and deterministic excitation, as could be shown by the comparison of measured and computed VPSD's. On the basis of these results it is possible to simulate the random structural dynamic processes which occur in the primary circuit, by means of a theoretical model. This offers an excellent possibility for a continuous vibration monitoring system, which is able to survey and protect the integrity of security relevant components in the pirmary circuit."

In a recent paper Kryter, Ricker and Jones of the Oak Ridge National Laboratory report on a survey they did of commercially available loose parts monitoring systems (LPMS) that were currently being used on light-water-cooled reactors. Details are contained in reference E.2.

They characterized the the current situation with the following comments:

> "The less-than-enthusiastic acceptance given LPMS by today's utility industry is not unique; other surveillance and diagnostic systems for which there is no presently established, clearly justifiable, self-recognized need or about which plant designers, architect-engineers (A-Es), and operating personnel are ill-informed have been similarly received. Utilities have been confused by conflicting information

> from designers, nuclear steam supply system vendors, the NRC, and LPMS vendors regarding the need for and the performance capabilities of LPMS, and this confusion has been compounded by a lack of practical guidance from the Commission staff regarding their expectations for LPMS performance and for action plans to be implemented when and if indications of a loose parts are received."

Some of the deficiencies they found are:

> "Our survey revealed that, with few exceptions, LPMS presently in operation produce too many false alarms. This is most unfortunate, because in many cases utility response to such malperforming, nonsafety-grade surveillance equipment is not to identify the cause and correct it but is rather to reduce the LPMS sensitivity to the point where the false alarms disappear. This action, of course, defeats the purpose of the instrumentation.
>
> Moreover, present-day LPMS are operated, in the main, as qualitative (not quantitative) devices; i.e., even though the several information channels of a given installation may be balanced with respect to each other, no attempt is ordinarily made to establish an absolute system calibration that is meaningful to other installations. This all but precludes the use of loose-parts data from one plant in diagnosing similar problems in another, but does not seriously compromise detection capability within a single plant. Also, such practice often leads to the use of completely arbitrary alarm level settings.
>
> Since loose-part characterization (i.e., sizing, locating and estimating point of loose part origin) is still in its developmental infancy, present LPMS have only limited built-in capability for estimating the nature of loose parts. This deficiency diminishes the usefulness of the LPMS because, in the event a loose-part is detected, no further information relevant to the subsequent assessment of the safety significance of the part is available."

"Finally, we judged that many present-day LPMS are marginally conceived and engineered, in the sense that signal amplitude information serves as the prinicpal indicator, whereas other signal characteristics (risetime, duration, frequency content) that could prove valuable in minimizing false alarms are underutilized. Also, the total number of sensors employed, their mounting positions, and mounting methods used have often been specified more on a basis of convenience than optimality (in many cases this is attributable to the retrofitted nature of convenience than optimality (in many cases this is attributable to the retrofitted nature of the LPMS). Moreover, at their present state of development, many LPMS are simply ill-suited to the typical utility environment from a human engineering point of view, owing to a requirement for considerable interpretation and judgment on the part of the LPMS operator in distinguishing genuine loose-part signals from spurious signals."

Their conclusions recommend both technical improvements to the systems and improvements in the regulation of their use. Some of their concluding remarks are:

"The technical improvements that we have suggested may be the easier of the two to achieve because, even though LPMS having both adequte sensitivty and reliability have not been, in our opinion, fully demonstrated at present, it is our best technical judgment that such systems are not only feasible but could, in fact, be implemented today using state-of-the-art technology.

The regulatory improvements that we have suggested, on the other hand, are likely to be more difficult to achieve because, in our opinion, a complete re-assessment of the basic safety justifications underlying the current regulatory requirement for loose-parts monitoring programs as well as a clear and concise policy statement of NRC staff's expectations for future performance of LPMS are required before any more detailed plans for industry implementation of better loose-parts programs can be made."

4.1.1 VIBRATION TESTING FOR SEISMIC ANALYSIS

A large number of technical papers appear in the literature on the general subject of vibration testing of nuclear plants. This testing is typically done to verify, improve, and synthesize analytical dynamic models for predicting plant responses during earthquakes.

A single source of ten different papers on the subject is a special issue in 1973 of the Nuclear Engineering and Design Journal (Ref. E.12). These papers were written by a multitude of nuclear engineers and structural dynamists who did their research over a number of years at University of California at Los Angeles. (UCLA). Their research was guided by several objectives as stated by Prof. C. B. Smith in the introduction. They wanted to acheive,

> "(a) a better idea of the response of nuclear power plant structures and equipment during earthquakes; this involves: (1) improved understandings of physical parameters such as modal frequencies, dampings, and mode shapes; (2) improved methods for testing large structures; and (3) the need to improve analytical models and develop more accurate methods of analysis;
> (b) better methods for designing nuclear power plants to resist earthquake forces;
> (c) rational methods for evaluating power plant sites;
> (d) the examination of several special problems, including non-linear effects, the effects of equipment supports, equipment performance during earthquakes, methods for seismic proof testing, evaluating the effectiveness of seismic design features, and improved seismic instrumentation."

Although none of this work was directly concerned with relating change in model parameters to structural damage, their efforts to develop better testing methods and to better understand the dynamics of nuclear tractor systems are certainly pertinent to the subject of this survey.

One of their articles (pages 51-93) contains results of modal tests conducted on nuclear power plants.

> "Tests have been conducted on three nuclear power plants; the experimental gas-cooled reactor (EGCR) at Oak Ridge, Tennessee; the Carolinas-Virginia tube reactor (CVTR) at Parr, South Carolina; and the San Onefre Nuclear Generating Station (SONGS), San Onofre, California. Analyses in varying detail have been performed; the most extensive work has been done at San Onofre. This article summarizes test results, dynamic models, and the results of seismic response calculations for each plant."

This article contains test results and analyses of containment buildings and structures as well as primary coolant loop equipment.

Some of their conclusions are the following:

> "Forced vibration experimentation can significantly aid in developing system models of the dynamic response of nuclear power plants. Direct experimental results are the natural frequencies of structures and major equipment as constructed, and other dynamic response characteristics are a basis for finding damping values and other model parameters. Experimental results thus provide evidence to permit the use of simple models to predict overall responses to earthquakes of more complicated systems if the response can be assumed linear, and can also be used in evaluating or checking detailed analytical models. Earthquake responses predicted by the linear methods used in this study provide a conservative upper bound for responses to strong motion earthquake inputs. If a stress analysis based on responses indicated by this method indicates that non-linear responses would occur, both engineering judgment and additional analysis would need to be applied to make a safety evaluation."

In another article (pages 112-125) the investigators discuss their results of attempts to identify modal damping from nuclear power plant structures. The paper contains a summary of damping values:

> "A summary of damping values for structures and equipment obtained in several full-scale dynamic tests performed on a research reactor, three experimental power reactor plants, and a commercial power reactor plant is given. The testing techniques used include steady state shakers, dynamite blasting, and snapback."

The authors used the "half power point" method to estimate damping, and include a discussion of the difficulties with its use. Some of their conclusions are:

> "In all cases discussed, it is reasonable to assume that the damping values reflect a lower bound to the damping that would be expected at elastic but near yield response levels."
>
> "The estimation of accurate modal damping values for use in reactor design is still not yet possible. However, as the previous sections indicate, it is possible to select, for a particular type of component and expected response level, damping values which represent 'in the ball park estimates' for design considerations."

Finally the special issue contains a bibliography of papers and reports by UCLA and ANCO Engineers, Santa Monica, California, on the subjects of vibration testing and seismic analysis.

More recently, Howard, Ibanez and Smith have completed an assessment of the seismic design of nuclear power plants for the Electric Power Research Institute, and have reported the results in two places. (Refs. E.16 and

E.17). Although the general topic is seismic design, the authors include a lot of information and references having to do with modal testing and data analysis techniques.

With respect to predicting the amount of damage in nuclear plants following an earthquake, the authors make some relevant comments:

> "Expected response nonlinearities arise from both large displacements affecting structural system geometry and from inelastic and plastic material behavior. While prediction of damage thresholds may be based in principle on linear elastic models (depending upon the level of geometric distortion under dynamic load), prediction of the degree of system damage and remaining margin to failure must be based on nonlinear analytical procedures."

> "Such numerical techniques as finite element and finite difference methods have been developed and employed in special structual problems, in part as a result of weapons effects studies. Such approaches are generally not employed in the analysis of nuclear power systems for reasons of economics, familiarity, confidence, and current acceptance of less difficult analytical procedures, even though nonlinear techniques are generally available or becoming so."

> "It is vital that parallel experimental programs be supported to ver fy the accuracy of analytical Such verified analytical capabilities could be extremely useful in determining remaining capacity of a nuclear power plant subjected to damaging seismic excitation, minimizing the time that will almost certainly be required for plant recertification following a significant earthquake."

References E.14, E.13, E.8 and E.11 all contain results of modal tests on nuclear power plant structures. References E.8 and E.11 contain comparative results between different testing methods.

4.2 LARGE STRUCTURES

The dynamic properties of large structures such as buildings and bridges have been investigated intensively during the past fifteen years in hopes that these structures can be designed or modified to survive the earthquake environment. To date the experimental testing of buildings, and bridges has been aimed primarily at the following:

1. Verification of analytical models.
2. Prediction of structural response to earthquakes and nuclear explosions.
3. Studies of structural non-linear behavior as a function of excitation level.
4. Studies of the effects of soil and foundation interactions upon structural response.

Many studies have been conducted to determine the best techniques for testing large structures to accurately determine their dynamic properties. The most commonly used measurement techniques involve the use of either a forced input from a shaker system, or ambient input such as the wind. The most popular forced excitation method is the stepped sine method, using rotating eccentric weights.

Ambient vibration measurements have been used successfully in recent years to determine modal frequency, damping and mode shapes of large structures. Typical vibration levels with ambient excitation are 10^{-8}g's. This is several orders of magnitude less than is typical with forced vibrations, but nevertheless is sufficient to identify modal parameters using modern day digital signal processing equipment.

One of the most common results of forced vibration testing on large concrete and steel structures is that they behave like a softening spring as the excitation level is increased. That is, their modal frequencies shift to lower values as the force level is increased.

A couple "rules of thumb" have been discovered for large framed buildings. One is that the fundamental modal frequency is approximately ten (10) divided by the number of stories above ground. For example, the fundamental mode of a 5 story building is 2 Hertz while that of a 10 story building is 1 Hertz. The other rule is that the ratios of modal frequencies to the fundamental approximately follows the sequence of odd numbers 1, 3, 5, 7, . . . this sequence is expected when the vibratory motion is primarily due to sheer deformations.

One reference produced some interesting results which are pertinent to the monitoring of structural integrity. (Reference F.1)

> "A study of a nine story building showed the pre-earthquake fundamental frequency was 1.54 Hz, during an earthquake the frequency shifted to 1.0 Hz, and then after the earthquake fundamental frequency was 1.27 Hz, but no "apparent" damage occurred to the structure. Tests since the earthquake indicate that the frequency is working its way back to 1.54 Hz."

Now quite clearly some physical change took place in the building or its surroundings as a result of the earthquake excitation, or the modes would not have shifted. The subsequent migration of modal frequencies back to their pre-earthquake values indicates that some material property (perhaps the stiffness of the foundation plus surrounding soil) has returned to its original state.

This example points out the fact that vibration monitoring can detect subtle physical changes but interpreting these changes and then determining whether the damage is acceptable or not remain as the difficult parts of the problem.

Donald E. Hudson has recently written his second survey paper on "Dynamic Tests of Full-Scale Structures " (Ref. F.1). His previous survey was done in 1970. Some of his pertinent comments are:

> "The main types of dynamic test suitable for full-scale structures are:
>
> 1. Free Vibration Tests: (a) Initial displacement--pull-back and quick release; and (b) Initial velocity--impacts, rockets.
> 2. Forced Vibration Tests: (a) Steady-state resonance, including mechanical oscillators and man-excited; (b) Variable frequency excitation, including sweep frequency--rundown, random, and pulse sequences; and (c) Transient excitations, including natural earthquakes, explosions, microtremors, and wind.
> 3. Vibration Table Tests."
>
> "Sinusoidal steady-state resonant testing with a rotating or reciprocating mechanical oscillator continues to be the most widely used technique for detailed dynamic investigations. The system of synchronized vibration generators with electronic speed control, developed by the Earthquake Engineering Research Institute (EERI) and the California State Division of Architecture in the early 1960's, continues to be a general purpose workhorse."

"It is becoming increasingly customary to provide important civil engineering structures such as dams and bridges with permanently installed instrumentation systems. Such systems not only facilitate many special tests of the structure during and after construction but remain as permanent monitoring systems for earthquake and wind loads."

Petroski, Stephan et.al. (Refs. F.2 and F.8) have extensively documented the results of both forced and ambient tests on a multi-story steel building. The building was 44 stories in height, with 6 parking levels below ground and triangular in shape at each level.

Their comments about the forced vibration test results are the following:

"In the forced vibration tests, the first six E-W and the first five N-S translational modes were excited, as well as, the first six torsional modes. Frequency response curves, in the region of the resonant frequencies were obtained for each of these modes.

The curves are plotted in the form of normalized displacement amplitude versus exciting frequency. The ordinates were obtained by dividing the measured acceleration by the square of the exciting frequency (cps) and then by the square of the circular frequency (rad/sec). Hence, the displacement amplitudes reflect the effect of a force amplitude that would be generated by the eccentric masses rotating at 1 cps. Damping capacities were obtained from the normalized frequency response curves by the formula: $\zeta = (\Delta f)/(2f)$, where ζ = damping factor (1% of critical damping), f = resonant frequency and Δf = difference in frequency of the two points on the resonance curve with amplitudes of 1/ 2 times the resonant amplitude."

In comparing the results of the two test methods they conclude:

"The resonant frequencies from both studies are in very good agreement in all separated modes of vibration with the maximum difference smaller than 2%. The ratios of the observed higher mode frequencies with respect to the fundamental one from both dynamic studies of the building indicate that the overall structural response is predominantly of the shear type.

Comparison of the forced and ambient vibration experiments demonstrate that it is possible to determine with adequate accuracy the natural frequencies and mode shapes of typical modern buildings using the ambient vibration method. Difficulties in evaluation of equivalent viscous damping factors from ambient vibrations studies are present and probably it will be more realistic from this type of study to expect assessment of the range of damping factors, rather than damping values associated with each mode of vibration."

In comparing experimental and analytical results they found:

"A comparison of the translational analytical results show very good agreement with the experimental studies, the maximum differences ranging from about 3% at the first mode to about 14% at the higher modes. It would appear from the first translational mode shape that the actual building is slightly more flexible than what the analysis indicates. In comparing the torsional analytical results with the experimental the differences are in the range of 10%."

Bouwkamp and Stephen performed similar tests on the 60 story Transamerica Building in San Francisco and obtained similar results (Ref F.10).

Ambient vibration monitoring has been done repeatedly on large structures recently. Some of the comments of Petrovski and Stephan (Ref. F.2) explain why.

> "In recent years a method for testing of full-scale structures based on wind and microtremor-induced vibrations has been developed. The ambient vibration study of the dynamic properties of the structures is a fast and relatively simple method of field measurements. It does not interfere with normal building function, and the measuring instruments and equipment can be installed and operated by a small crew."

In a very well written paper, Milhailo Trifunac (Ref. F.17) compares the results of several tests conducted on two different multi-story buildings. He compares estimates of modal parameters obtained from both ambient and forced vibration testing of the buildings. The paper contains numerous tables of frequency and damping values and plots of mode shapes comparing the ambient and shaker driven results. Some of his comments and conclusions are:

> "Although two case studies are not sufficient to justify general conclusions, the results summarized in this paper strongly suggest that, in the linear range of excitation, tests based on microtremor-and wind-induced vibrations give essentially the same results as would be obtained from the forced vibration experiments."

> "Except for the fifth and sixth NS modes and the fifth torsional mode, frequencies measured from the wind-excited motions are always the same or higher than those determined by the forced vibration experiment. In this respect, it is interesting that in nearly all forced vibration tests there have been observed monotonic increases of resonant frequency for decreasing amplitude of vibration."

"In measuring a single mode amplitude, the expected average discrepancies between the ambient and forced vibration tests are estimated to be within several per cent for the fundamental modes, and they increase for higher modes.

The accuracy of estimating the equivalent viscous damping from ambient vibration tests depends on the degree to which translational and torsional response curves can be separated, because the spectral overlap leads to the broadening of peaks and overstimated damping values. This difficulty may be encountered for a particular building, but apparently does not occur as a rule."

C. K. Chen et.al. of URS/John A. Blume and Associates (Ref.F.13) tested a concrete building beyond its elastic limit using an eccentric shaker, and recorded its fundamental modal frequency and damping values as it began to fail. Their description of the test is as follows:

"As part of this program, two identical full-scale 4-story reinforced concrete structures were built in 1965-1966 at the Nevada Test Site to investigate their dynamic response behavior. For eight years following their construction, the structures were the subject of a continuing program of vibration testing, and substantial data had been collected on the elastic response of these structures. In 1974 it was decided to conduct a high-amplitude vibration test that would cause the south structure (free of partitions) to deform beyond its elastic limit and cause major structural damage. This paper summarizes results of the 1974 testing program."

"A reciprocating mass vibration generator specifically for use in these high-amplitude test was designed and assembled by Sandia Laboratories of Albuquerque, New Mexico. This device had a frequency range up to 50 Hz and was capable of producing a maximum force of 12,000 lb at frequencies greater than 1.6 Hz."

"At all amplitudes of response below the elastic limit, the period was amplitude-dependent, which is characteristic of a system with nonlinear softening. The change in period below the design level was more pronounced than between the design and yield capacities. The period became nearly constant (0.54 sec) at amplitudes above the design capacity of the structure but below the elastic limit. However, as the input force was increased beyond the yield capacity, the structure's response was markedly nonlinear, resulting in a substantial change in the stiffness and period of the structure. After the structure sustained major damage, the period remained nearly constant at about 0.9 sec. Like the variation of period in the elastic range, the damping was also found to increase with an increase of amplitude of vibration but remained nearly constant (about 2.1% of critical) at amplitudes above the design capacity of the structure. Beyond the elastic limit, the structure experienced a marked change in damping, up to 4.1%, when the input reached its maximum level and the structure suffered major cracking."

Smith and Matthiesen (Ref.F.10) report briefly on several modal testing methods they have used on large structures (like containment vessels) around nuclear power plants. Their comments about the methods are:

"In theory, any method of applying energy to the structure being tested could be an appropriate means of excitation. In our experience, we have found certain techniques to be advantageous, and shall discuss them here. Undoubtedly, other methods or variations on these methods are possible. The following techniques are listed in order of increasing force capability:

Ambient vibration (random noise)
Structural vibrators (harmonic excitation)
Snapback tests (step function or initial displacement)
Explosive blasts (impulse)."

"The principle advantage of the forced vibration technique are the capability to excite specific frequencies (so individual modes can be emphasized) and the fact that the applied force is a known quantity. Also, the forced vibration technique permits levels of response which are 10^2 or 10^4

times greater than ambient vibrations, although generally still 10^1 ir 10^2 tunes less than strong motion earthquakes."

The authors site several other references containing more detailed test results.

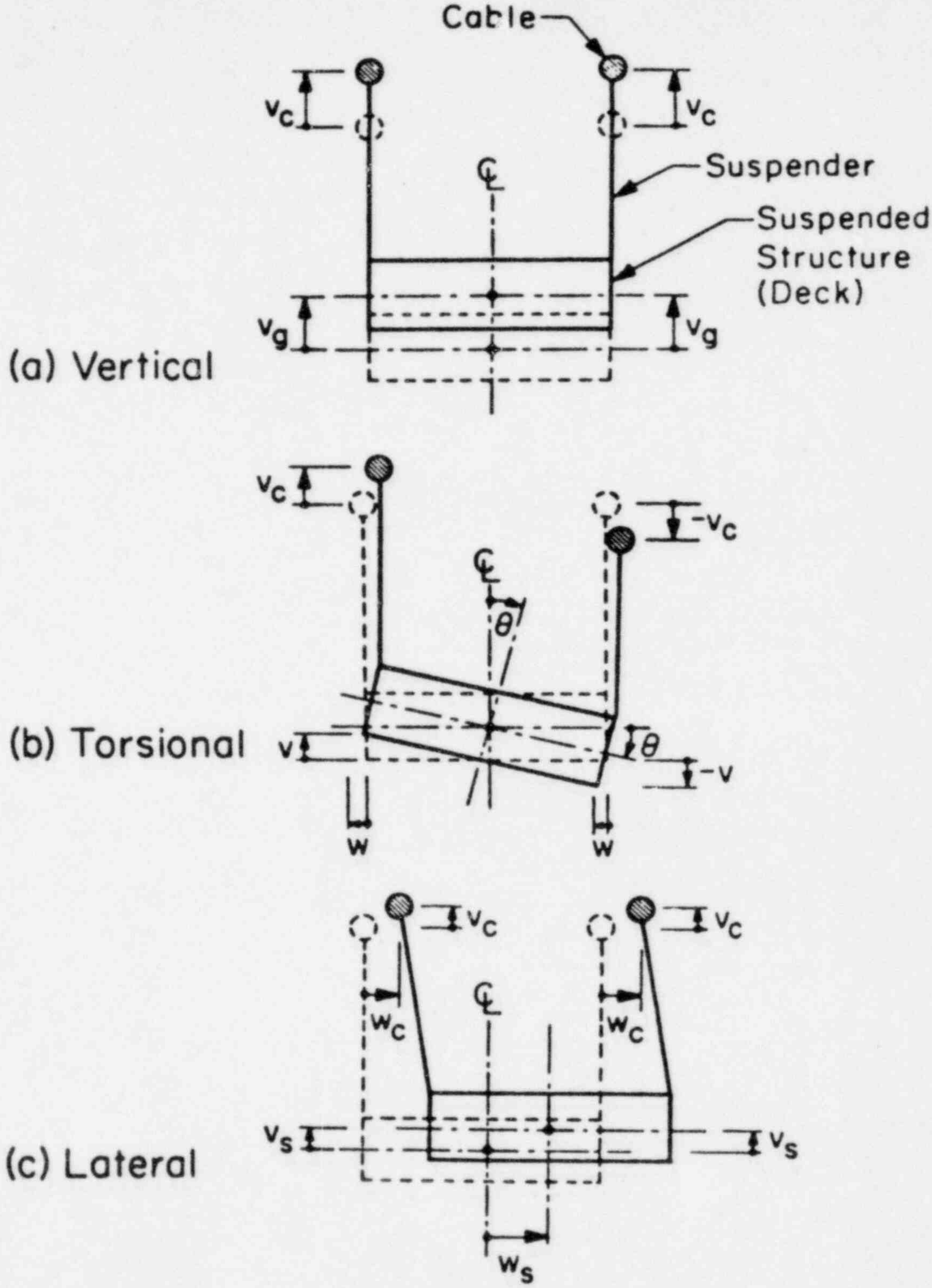
4.2.1 SUSPENSION BRIDGES

Modal testing has been done on a number of suspension bridges in the U.S., Canada and Japan. These bridges have been tested primarily to verify analytical dynamic models but the overall objective is to develop a reliable model for predicting the bridge's response to earthquakes.

We found one excellent report on the testing of the Vincent-Thomas Suspension Bridge in Los Angeles (Ref.F.9). The report documents the results of what must be considered as a very successful effort to identify a large number of the modes of vibration of a large structure from ambient vibration signals.

The modes of a suspension bridge can be classified as 3 different types, as explained by the authors:

> "The uncoupled vibrational modes of a suspension bridge may be classified as vertical, torsional and lateral, as shown in Fig. 4.2. In pure vertical modes, all points on a given cross section of the bridge move the same amount in only the vertical direction, and they remain in phase (Fig. 4.2-a). In pure torsional modes, each cross section of the bridge rotates about an axis which is parallel to the longitudinal axis of the bridge and which is in the same vertical plane as the centerline of the bridge. Corresponding points on opposite sides of the centerline of the roadway attain equal displacements, but in opposite directions (Fig.4.2-b).In pure lateral motion, each cross section swings in a pendular fashion in its own vertical plane, and therefore there is upward movement of the cables and of the suspended structure incidental to their lateral movements, as shown in Fig. 4.2-c."



TYPES OF VIBRATIONAL MOTION IN SUSPENSION BRIDGES

FIGURE 4.2

The report contains numerous plots of measured mode shapes vs. analytical results. The conclusions of the authors are as follows:

"The measurements identified sixteen vertical modes, eleven torsional modes and ten lateral modes and their natural frequencies, in the frequency range 0.0 Hz to 3.0 Hz. These frequencies and mode shapes were determined for small amplitude vibrations and, hence, indicate the structural behavior in the range of linear response. However, the measurements also demonstrated an interaction between side and center spans in the higher as well as the lower modes of vibration which indicate non-linear behavior.

Good modal identification was achieved by special deployment and orientation of the motion-sensing instruments and by summing and subtracting records to enhance vertical motions and torsional motions. Relatively long time intervals were recorded which contributed to higher resolution of the Fourier spectra. Consequently, better determination of the closely spaced modes of vibration was obtained. The characteristics of torsional modes were well-determined from recorded lateral motion. The vertical component of the torsional motion was not so well obtained and only six out of eleven torsional modes were recovered from the recorded vertical motions; the other five torsional modes were very close to vertical modes which dominated the vertical motion.

It was not possible to determine reliable damping values adequately by use of the half-power method due to closely spaced peaks and to spectral overlap which resulted in widening of the Fourier spectrum peaks; however, a rough estimation of the damping ratios was presented.

Finally, this comparison between measured and computed natural frequencies and mode shapes suggests for future earthquake-resistance analyses that computed values can be representative of the real structure."

4.3 ROTATING MACHINERY

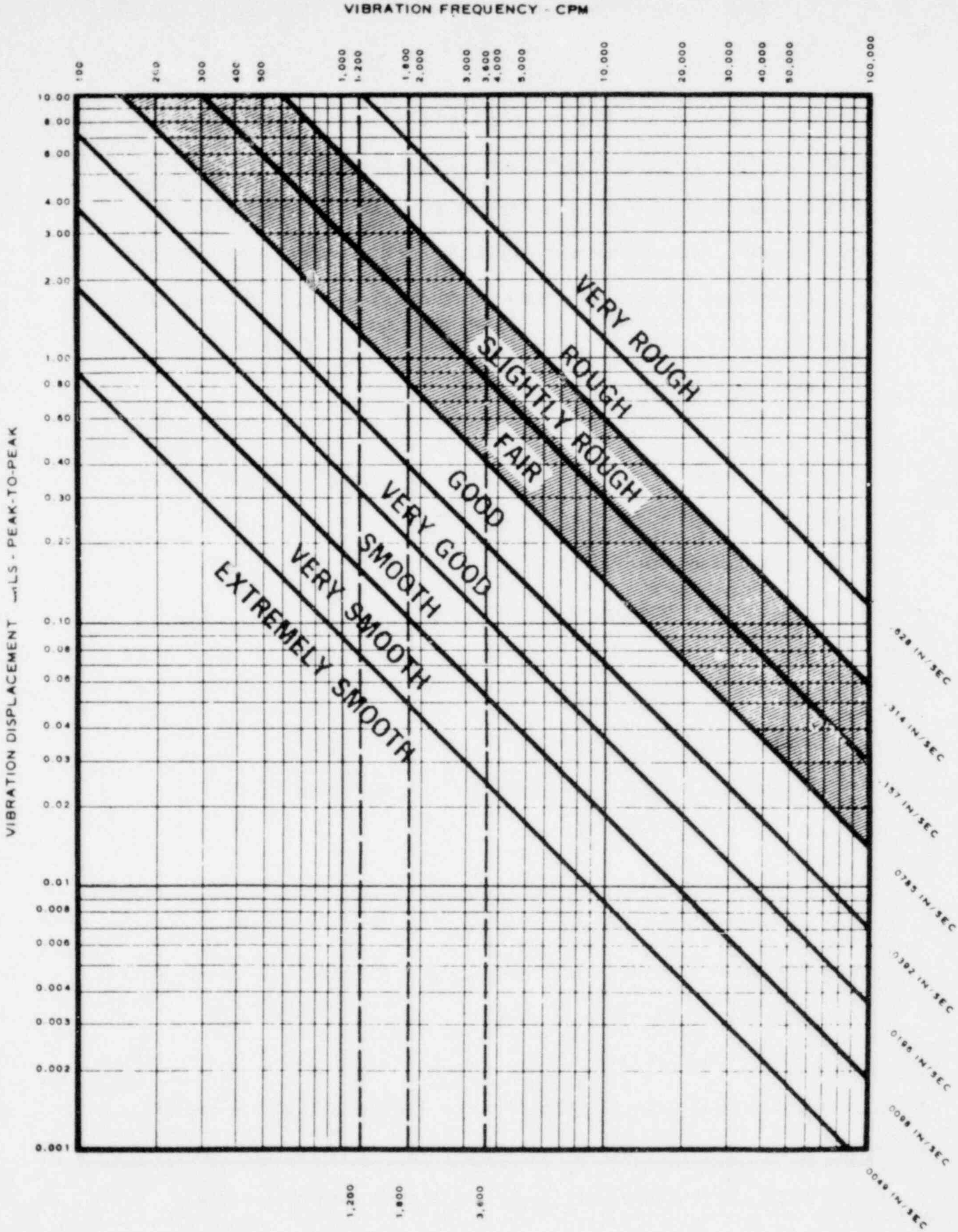
By far the largest number of integrity monitoring systems in use today have been applied to rotating machinery. The largest number of installations is clearly been done in the petrochemical industry, and there are a large number of equipment manufacturers who make monitoring equipment specifically for use with rotating machinery.

The majority of the literature we reviewed an integrity monitoring of machines can be divided into two categories:

1) Articles written by manufacturers and users of equipment describing general usage and some specific applications of the equipment. These articles are for the most part non-technical in nature.

2) Technical articles describing experimental/analytical studies of a particular diagnostic procedure.

Both types of papers serve a useful prupose in a state-of-the-art survey such as this, for they give a good picture of present day practice in the only really established application of integrity monitoring to date, and also indicate some of the uses of modal parameters to detect failures in rotating machines.



General machinery vibration severity chart

FIGURE 4.3

Vibration signals are the primary types of signals used to monitor the "health" of rotating machines. These signals are typically measured with accelerometers mounted directly on the machine, or more commonly with "proximity probes" which are typically mounted in the bearing blocks and measure dynamic displacement of the bearing journals. Acoustic emissions are also used to diagnose certain types of bearing malfunctions.

Vibration signals are used to detect a variety of different problems in rotating machines. They are:

1. Rotor unbalance and runout
2. Shaft misalignment
3. Bent shafts
4. Oil whirl/bearing whip
5. Bearing wear
6. Broken or worn gears or other internal parts

Traditionally the majority of the diagnosis methods have involved only the raw time domain signals. Peak or RMS signal levels are compared to a vibration severity chart such as that in Fig. 4.3. Shock pulse analysis (Ref. G.43) is also used on time domain signals.

With the availability of more and more low cost frequency analysis equipment, both narrowband and proportional band (Octave band) frequency spectrum analysis is becoming more commonplace for machinery monitoring.

Identification of the cause of the problem is based as much on a historical build up of experience with particular types of failures as it is upon any kind of analysis.

Vibration identification charts such as those shown in Figures 4.4 and 4.5 are typically used to identify causes of problems.

Table 1 Vibration Identification Chart

CAUSE	AMPLITUDE	FREQUENCY	PHASE	REMARKS
Unbalance	Proportional to unbalance. Largest in radial direction.	1 × RPM	Single reference mark.	Most common cause of vibration.
Misalignment couplings or bearings and bent shaft	Large in axial direction 50% or more of radial vibration	1 × RPM usual 2 & 3 × RPM sometimes	Single double or triple	Best found by appearance of large axial vibration. Use dial indicators or other method for positive diagnosis. If sleeve bearing machine and no coupling misalignment balance the rotor.
Bad bearings anti-friction type	Unsteady - use velocity measurement if possible	Very high several times RPM	Erratic	Bearing responsible most likely the one nearest point of largest high-frequency vibration.
Eccentric journals	Usually not large	1 × RPM	Single mark	If on gears largest vibration in line with gear centers. If on motor or generator vibration disappears when power is turned off. If on pump or blower attempt to balance.
Bad gears or gear noise	Low - use velocity measure if possible	Very high gear teeth times RPM	Erratic	
Mechanical looseness		2 × RPM	Two reference marks. Slightly erratic.	Usually accompanied by unbalance and/or misalignment.
Bad drive belts	Erratic or pulsing	1, 2, 3 & 4 × RPM of belts	One or two depending on frequency. Usually unsteady.	Strob light best tool to freeze faulty belt.
Electrical	Disappears when power is turned off.	1 × RPM or 1 or 2 × synchronous frequency.	Single or rotating double mark.	If vibration amplitude drops off instantly when power is turned off cause is electrical.
Aerodynamic hydraulic forces		1 × RPM or number of blades on fan or impeller × RPM		Rare as a cause of trouble except in cases of resonance.
Reciprocating forces		1, 2 & higher orders × RPM		Inherent in reciprocating machines can only be reduced by design changes or isolation.

FIGURE 4.4 (Ref. G.16)

Likely Causes of Vibration

Predominant Frequency*	Most-Likely Causes	Other Possible Causes; Remarks
R	Imbalance	Misalignment, eccentric journals, bent shaft, bad belts.
R	Misalignment	When axial reading is greater than half of either the vertical or horizontal reading.
$2R$	Mechanical looseness	Misalignment, rubbing, reciprocating forces as in auto engines, bad belts.
$3R$		Rare. Usually a combination of misalignment and looseness; sometimes bad antifriction bearings.
$0.5R$ or less	Oil whip or whirl	Occurs only on high-pressure lubricated machines with plain bearings.
Many times R	Bad antifriction bearings	Predominant frequency may correspond to operating speed multiplied by the number of balls or rollers.
Many times R	Gear noise	Frequency corresponds to number of gear teeth multiplied by R.
Many times R	Aerodynamic forces	Frequency corresponds to number of fan blades multiplied by R.
Many times R	Hydraulic forces	Impeller blades or lobes times R.

* Expressed in terms of operating speed, R rpm.

FIGURE 4.5 (Ref. G.42)

As examples of many of the papers we found, some of the experience of a couple of authors is included in the following.

R. H. Nittinger (Ref. G.42) says the following:

> "High amplitudes that occur at rotating-speed frequency are usually caused by imbalance. Imbalance can involve the following:
>
> 1. Corrosion or erosion of parts: Impellers and rotors of pumps and blowers, for example.
>
> 2. Static imbalance: We normally think of a static balance as one made by placing a rotating member on a shaft or mandrel, and then rolling it on a set of level knife edges to determine where the heavy spot may occur. We then add additional weights on the light side, or remove weight from the heavy side, so that the "wheel" will not stop at the same spot each time but will follow a random stopping pattern.
>
> 3. Dynamic imbalance: This is more likely to occur in multi-stage blowers, and also centrifugal pumps. These units can be statically balanced; but in most cases a true correction requires that a rather complex couple be fabricated into the unit."
>
> "4. Electrical imbalance: While rare, this type of imbalance does occasionally come up. It may be caused by removed or cut out field coils, or a broken slat in a rotor, which in turn creates unbalanced fields or currents in the motor.
>
> 5. Stray currents or static electricity: This type of imbalance is not easy to analyze and cure, but it must be considered. Electrical charges or cells can be produced in many ways; they may cause arcing or pitting of bearings, impellers and rotors, thus causing heavy vibrations and high frequencies due to excessive wear of bearings. We recently analyzed a fan that had an aluminum rotor in a cast iron housing. Under certain atmospheric or chemical conditions, the dissimilar metals apparently developed low voltage and high amperage (one measurement was 3 v. and 14 amp.). In a case like this, each time a charge builds up and arcs to ground, a small quantity of metal is disintegrated, which in turn can cause imbalance or loss of bearings."

"Misalignment is a second cause of excessive vibrations and can involve couplings, pulleys, belts and bearings. It is interesting that weather can be a temporary factor here; some of our case histories show that a sudden change in weather conditions--such as a cold rain that causes uneven cooling of the operating unit--can create misalignment and rough vibration readings until the complete unit has become stabilized.

The third cause of excessive vibration involves worn parts, such as:

Couplings: Worn gear teeth, bushings, pins, blocks or containers.

Pulleys: Worn grooves, chipped or broken flanges in belt area, hammered or flattened hubs and loose keys.

Belts: Worn or improperly tensioned.

Gears and clutches: Worn, slipping or damaged.

Shafts: Rough, improperly sized, or bent.

Friction and anti-friction bearings: Premature wear can be caused by improper alignment and fit, or by lubrication of the improper type, pressure or temperature.

Other causes of excessive vibrations include:

1. Stresses--e.g. expansion or contraction in piping.

2. Improper foundations or mountings--e.g., improper foundation mass, grout or bracings.

3. Unbalanced loads--including not only imbalance in the load itself, as might occur in a loaded centrifuge or wringer, but also such things as cavitation in pumps, and incorrect oil levels in variable-speed clutches."

C. A. Bowes (Ref.G.17) answers a number of key questions about vibration monitoring which reflect a wealth of experience. Though lengthy, the following excerpt from his paper gives a good picture of current day machinery monitoring practice.

"The selection of the location and direction of measurement is the most critical factor in machinery vibration monitoring and analysis. If the raw signal does not contain the components that are representative of machinery condition, no amount of analysis will reveal that condition. To borrow a phrase from the computer industry with respect to data input, "garbage in-garbage out."

What are the best locations for measuring machinery vibration?

The bearings are where the action is, where the basic, dynamic loads and forces of the machine are present, and they in themselves are a critical component with regard to machinery condition. Vibration measurements should be made on the bearing cap of each bearing in a machine. If this is not feasible, the measurements should be made at a point as close as possible to the bearing with the minimum possible mechanical impedance between that point and the bearing. It should be noted here that this is a discussion of vibration as a maintenance tool, not as a design or an experiment tool. There are many other locations for vibration measurements besides the bearings which can provide useful information to the machine designer or the person interested in an aspect of the machine other than its condition, such as structure borne noise."

"In what directions should the vibration be measured?

For a complete vibration signature of a machine, triaxial measurements should be made at each location. However, for rotating machinery, sufficient information can usually be obtained from an axial and a radial measurement at each location.

The different components of a machine vibrate at one or more discrete frequencies and different malfunctions in a given component can cause vibrations at different discrete frequencies. It is the combination of these discrete frequency vibrations that results in the complex vibration waveform at the measurement point. Therefore, a common and useful method of analyzing the measured signal is to reduce it to its discrete frequency components. The results of this type of analysis, usually presented as a plot of amplitude versus frequency, is what is commonly referred to as the vibration signature of a machine."

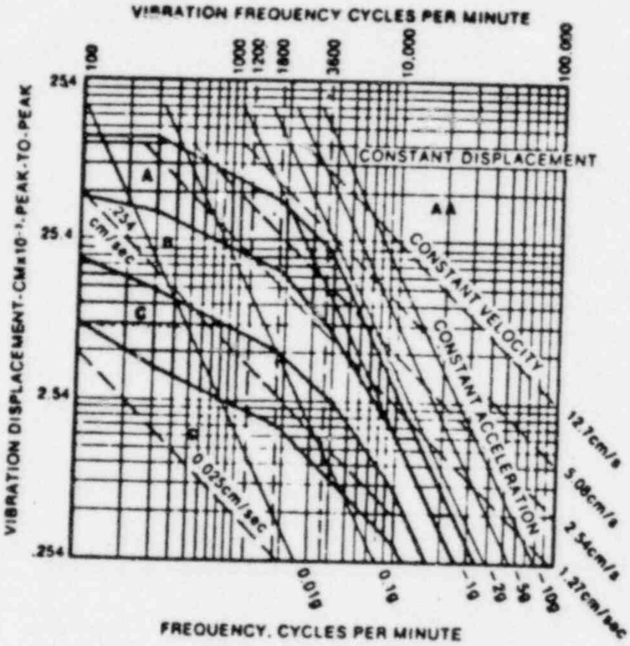
"Knowing which frequencies in a machine's vibration signature are caused by what components and what types of malfunction requires an understanding of the mechanical dynamics of the machine and the effects of the linear and nonlinear combination of discrete frequency signals. However, some generalizations can be made which are useful in a basic interpretation of a vibration signature:

(a) Unbalance causes radial vibration at the machine's rotational frequency.
(b) Misalignment, bent shafts, and bad coupling cause axial vibration at the machine's rotational frequency with large second and third harmonic components.
(c) Oil swirl in journal bearings causes radial vibration at subharmonics of the rotational frequency.
(d) Aerodynamic forces in fans or impellers cause radial vibration at a frequency equal to the product of the rotational frequency and the number of blades or vanes.
(e) Gears cause vibrations at the shaft rotational frequency and at frequencies equal to the product of the rotational frequency and the number of gear teeth.
(f) Anti-friction bearings cause vibrations at frequencies that are a function of the bearing geometry, the number of rolling elements, and the shaft rotational frequency. Bearing vibration is generally indicated by very high frequency radial components measured on the housing."

"What are the levels of vibration that represent a machine in good condition versus one in bad condition?

There are no hard and fast rules to answer this question, and there are definitely no rules based on scientific analysis and prediction. However, years of industrial experience in correlating machinery vibration with machinery health on thousands of machines have resulted in some empirically derived vibration standards. Figure 4.6 is one of the most widely used standards in the United States.

The standards are very general and should only be considered as rules of thumb for judging vibration severity. In actual practice, some machines can operate satisfactorily at vibration levels that would be considered unacceptable for other machines. A better method for judging vibration severity is to establish baseline signatures



Vibration severity criteria. AA dangerous; A failure is near; B faulty; C minor faults; D no faults.

FIGURE 4.6

for a machine known to be in good operating condition (this is not necessarily a new machine as some machines "wear in" to their normal operating levels) and to monitor changes in these signatures with time.

What magnitude of change in a signature component is considered significant?

After years of experience, the Canadian Navy has determined that an increase in vibration level is not significant unless it doubles. As important as the absolute level of change is the rate of change. The Canadian Navy has data which indicates that the mean level of a signature component as a function of time is a straight line with a slight positive slope for 75% of a machine's useful life, at which point it starts an exponential rise to the point of failure. Therefore, trend monitoring of vibration signatures is a more useful maintenance tool than a one-time survey of absolute magnitudes."

Michael H. Price of ARCO Chemical recently reported (Ref. G.10) some operating experience with a computer based system for monitoring a large compressor train. Some of his comments are:

"Use of a computer assisted vibration monitoring system has proven very valuable in two new, large single-train ethylene plants.

The over-all objective was to provide a system that would, (1) continuously monitor vibration levels of critical rotating equipment, (2) automatically analyze, store and update vibration information for troubleshooting purposes and (3) generate alarms that would indicate the source of trouble if problems develop.

The main benefit comes from knowing that most vibration analysis information on all critical compressor trains is being monitored and updated on a 24-hour-per-day basis. If conditions change, the compressor train operators are notified within minutes. This early warning system is useful in reducing the time to determine the cause of vibration increase (mechanical or operational). If a failure occurred in the middle of the night, detailed frequency spectrum information would be available immediately prior to failure time."

Some of his comments about the results obtained with the system are the following:

"Since system start up, many changes have been noted in the vibration signatures of monitored machines. Alert levels were set at fairly low values during the first few months of operation to determine how much a frequency spectrum would change during a 24-hour period. Changes of 200 percent in the level of a 2x or 3x running speed component were not uncommon. At first, some concern was generated over these seemingly large changes in vibration levels. As experience was gained with the system, it became apparent that for certain machines this was their "normal" vibration pattern. Such items as day vs. night, cold fronts, rain, sunny vs. cloudy days all affected the vibration signatures of the machines. Process changes also affected the frequency spectrums."

The British refer to the study of machinery as "tribology". In a three part paper (Ref. G.57), which appeared in a journal of the same name, they make the following comments:

"In recent years there has been considerable activity in the field of tribological failure investigations and it is widely recognized that important lessons can be learned from studies of failed components. The short-term benefit derived from these studies is an understanding of the nature and causes of the tribological failures which are responsible for the majority of machinery breakdown."

"The subject has assumed great significance as a result of the trend towards a smaller number of large machines in many modern technological applications. Evidence of this trend can be seen in the large process units in the chemical industry, steel-making plant, liquifaction equipment, electrical power-generating units and the power systems for air transport. This equipment is often complex and almost inevitably expensive and the consequences of unnecessary shut-down time can be serious and even disastrous in both the economic and technological sense."

"The successful introduction o. monitoring systems into modern machinery calls fo· a sound knowledge of the fundamental nature of failure mechanisms in tribological components, the operating characteristics of healthy equipment and the optimum type of monitoring device for a given situation."

For rotating machines they outline the following causes of incipient failure:

"Inadequate lubrication:

The volume of lubricant to the component might be inadequate.

The initial properties of the lubricant might not match up to the operating conditions in the machine.

The physical and chemical properties of the lubricant might change in service until the lubricant becomes inadequate for the task in hand.

The lubricant might be incompatible with other materials in the machine.

Dimension changes:

Movements of shafts, thrust plates etc. during service might indicate undesirable thermal or elastic distortions or excessive wear.

Changes in running clearances might adversely affect the performance of a machine and lead to premature failure.

Excessive heat:

The overall component temperature or lubricant temperature rise might lead to inadequate performance, lubricant degradation or accelerated failure of the component.

The mechanical strength of the machine components might be adversely affected by heat from the bearings, gears, brakes, etc."

"Thermal distortion and thermal stresses might influence the performance and safety of the machine.

Unsteady running:

Excessive amplitude of vibration or even a change in frequency is one of the best known indicators of tribological faults.

Changes in the noise spectrum associated with particular components operating under normal conditions in known environments can similarly indicate incipient failure.

Frictional changes:

An increase or decrease in friction in bearings, gears, etc. can be a point towards decreased performance and incipient failure.

Excessive wear:

Progressive wear of components resulting from chemical attack or mechanical action takes place in many tribological components. It often leads to a gradual deterioration in performance and once a certain amount of wear has taken place the component can be deemed to have failed. The failure might be gradual but it is often sudden and maybe catastrophic in its later stages.

Mechanical fatigue in which pits or cracks appear in the surfaces of tribological components is usually a powerful indicator of incipient failure, particularly in rolling elements and gears."

With respect to implementing monitoring schemes to detect failures, they make the following observations:

"The widespread introduction of monitoring systems must be accompanied by the development of records of average performance of tribological components in healthy machines. The present-day qualitative and subjective understanding of this aspect of the subject, based upon extensive oeprating experience and machinery development and the expertise of a small number of people, must be supplemented by careful scientific study."

As another indication of the state-of-the-art C. A. W. Glew (Ref. G.14) illustrates how frequency spectrum analysis is used to monitor rotating equipment in the Canadian Military ship and aircraft fleets. Some of his comments about the monitoring program are:

> "The Canadian Forces have developed a machinery health monitoring programme in ships and on aircraft, based upon the use of the portable octave band noise and vibration analyser as a vibration monitoring tool. In ships, this non-destructive testing technique is in general use prior to refit to help determine the machinery overhaul requirements and, after refit, to help determine the accepability of repair work. As a result, repair costs and ships' outage costs because of breakdown at sea have been almost halved."

> "Velocity measurements are taken near each principal bearing on a machine, on an overall and octave band basis. These readings enable the operator to diagnose rapidly any serious deterioration due to unbalance, misalignment, bent shafts, defective gears or bearings in the machine, because the overall velocity level defines whether a machine is in good condition, or not, and the octave band analysis enables the nature of the fault to be determined.
>
> To take the machinery health measurements, monitoring points are welded at standardized positions on each machine and vibration measurements are taken periodically at specified operating conditions. The state of the machine is determined by comparing these measurements with the machinery norms and with the previous readings at each point. These operations are computerized."

On the use of octave band analysis the author gives the following example:

"The Octave Band Vibration Analyzer has a Definite Diagnostic Capability

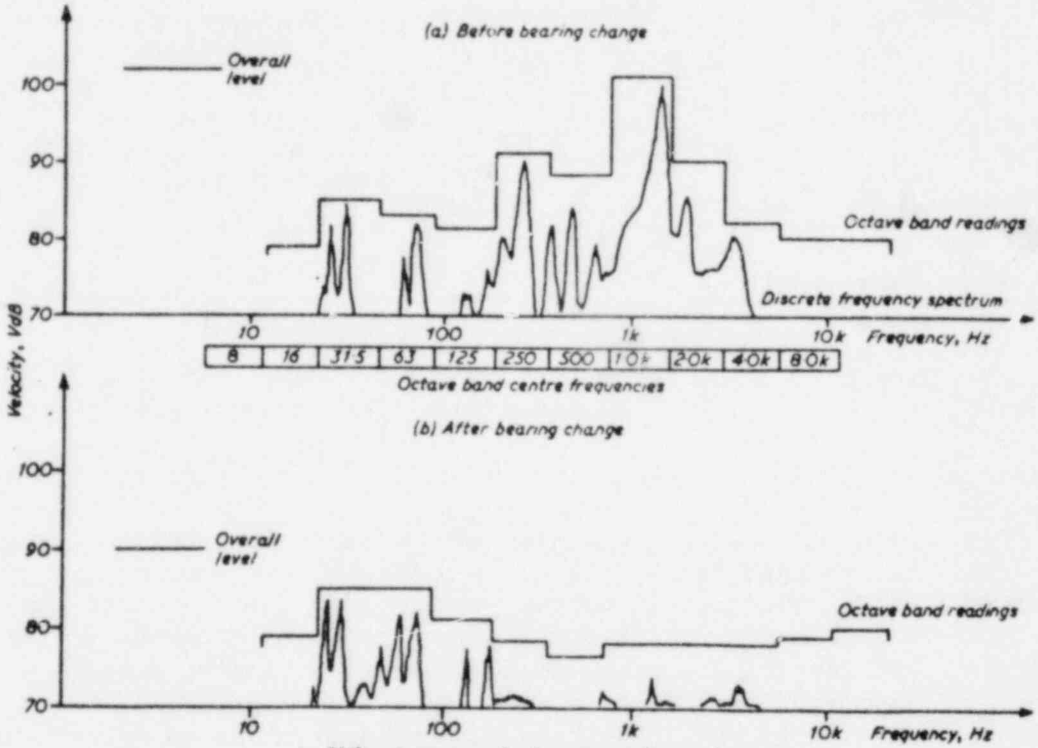
When a high overall vibration level is encountered, it has been found that an octave band analysis of the vibration is a convenient way of defining the nature of the problem which has arisen on the machine.

This is because most faults lie within the following simple grouping:

Type of fault	Axis of dominant vibration	Frequency of vibration
Unbalance	Radial	Shaft frequency
Misalignment	Axial	Shaft frequency and low multiples
Bent shaft	Axial	Shaft frequency and low multiples
Gears	Can be either axis	Gear teeth shaft frequency and multiples thereof
Bearings	Can be either axis	High frequency 250 Hz to 5 KHz

As an example Fig.4.7 shows the relationship between the octave band readings and the discrete frequency readings taken on a typical marine pump. It can be seen that the octave band reading is generally only a little higher than the largest discrete frequency reading in that octave band and, in this case, the relatively high octave band reading in the 1 KHz band gave an indication of bearing problems.

When using the octave band analyser on a complex machine, it is not always possible to define which gear or bearing in an assembly is giving trouble, but the indication is generally sufficient to enable the correct preventive action to be taken."



Vibration record of pump at lower bearing

FIGURE 4.7

Some of the author's concluding remarks are the following:

> "There are very considerable gains still to be made by more efficient utilization of the present machinery health information system.
>
> Other areas of development, which the author expects to see implemented, are the combined use of vibration analysis and spectrometric oil analysis or filter particle examination, together with the measurements of the flow, pressure, temperature and work parameters of the concerned machines to give a complete record of machinery condition on a periodic or real time basis. Maintenance routines will be based on the analysis of these records."

A recent very interesting paper by Nagy, Dousis and Finch (Ref. G.41) on the detection of mechanical flaws in railroad wheels clearly demonstrates how monitoring of modal properties can be used to detect structural failures. Some of the author's introductory comments are:

> "Well developed nondestructive test methods are in use for monitoring product quality during the manufacture of railroad wheels, including ultrasonic, magnetic particle and dye penetrant techniques. Locomotive wheels are inspected regularly in service, but there is a need for a fast, automatic and continuous testing system for freight car wheels in service."

The most prevalent types of failures are described in the following:

> "At present the most frequently occurring defects are thermal cracks, due to extended periods of brake shoe application. In this case both wheels on a given axle may be cracked. Sometimes a single brake shoe may be misplaced and then the cracks are found on only one wheel of the pair. The cracks appear as hair lines

on the tread or flange and there are frequently numerous cracks on the same wheel. Despite the barely visible exterior manifestation of this type of crack, below the surface it may occupy a sizeable fraction of the rim cross section. When a wheel with thermal cracks is withdrawn from service and subject to metal-lurgical examination, the cracks are typically found to extend over about a square inch in area perpendicular to the surface. This area is usually blackened, suggesting that the cracks may persist for long periods before progressing rapidly through the plate to cause a catstrophic failure. This final stage of the failure is surmised to be a result of changes in the internal state of stress of the wheel."

"Plate cracks are usually found as extensive bow shape fractures running round the hub and extending into the plate. Their occurrence a commonly ascribed to poor manufacture (either improper heat treatment or excessive force in fitting the wheel to the axle) followed by growth under stress cycling. This type of defect is not found as frequently as thermal cracking or overheating but some of the most catastrophic wheel failures have been ascribed to plate cracks."

"Another common problem is the occurrence of flat spots on the tread, due to sliding along the rail when the brake locks the wheels. Flat spots cause repeated impacting which results in further damage to both the rail and the wheels."

The investigators applied so called "acoustic signature analysis" to detect the structural faults. Basically, they measured vibration signals emminating from the wheels with a microphone and looked for shifts in modal frequencies in the power spectrum of the signal. They present results of computer analyses, laboratory tests and field tests which they performed in an effort to learn more about detection of these failures.

Some of their conclusions are the following:

"It is concluded that it is feasible to use acoustic signature inspection for detection of thermal and plate cracks in railroad wheels. It appears that the best form of excitation for detecting cracks is by impact and the best sensor is a microphone. The cracks cause shifts in resonance frequencies and these shifts can be observed best in narrow band analysis but are also manifest in third octave band analysis. Grease layers cause damping of resonance lines above about 4 kHz. Recognition of defective wheels can be carried out by comparing sound spectra with a standard or by comparing the sound spectra of wheels on either end of an axle. For finding flat spots a rail mounted accelerometer detecting both the spectral characteristics of impact on the rail and the repetitiveness of impact may be used."

It should be noted that the modal analysis techniques presented in the Chapter II do not apply to rotating structures. An article by R. M. Laurenson (Ref. G.38) details the differences. As he explains in the introduction:

"Conventional modal analysis techniques are not applicable in the case of an elastic structure spinning at a constant angular velocity. This is of interest because numerous structural configurations such as spinning satellites, rotating shafts, and rotating linkages fall into this category. The analysis of these spinning structures differs from that of stationary structures due to the complexity of the accelerations which act throughout the system. In addition to the accelerations resulting from elastic structural deformations, contributions due to Coriolis and centripetal acceleration may be of significance. Also, the stiffness characteristics of the structure may be modified by the steady state internal loads induced by the centrifugal forces."

Laurenson prefaces his own work with a discussion of the previous work of others, which he includes as references in his paper.

Gunter, Choy, and Allaire (Ref. G.50) also discuss modal analysis methods for rotating machines.

Wilson and Frarey (Ref. G.7) describe some preliminary operating experience they have obtained with a newly developed system for monitoring pumps in a nuclear power plant. Some of their introductory remarks are the following:

> "The number of critical machines, where monitoring is considered desirable, typically results in systems comprising between 100 and 200 individual sensors. The complex vibration information of each sensor must be analyzed periodically, frequencies identified, and amplitude trends recorded in order to provide the diagnostic information necessary to predict machinery health. This type of analysis requires expertise in the areas of machinery dynamics and instrumentation to provide meaningful results. Considering the number of sensors involved, coupled with the type of analyses required for each sensor, the need for a computer based automated type system becomes evident."
>
> "In order to demonstrate both feasibility and inherent advantages of such an approach, a demonstration system is being developed for installation in the Northeast Millstone II PWR nuclear power plant. Five pumps were selected utilizing a total of 27 sensors for system evaluation."

The authors illustrate how a preliminary modal analysis is key to a successful monitoring scheme. In their words:

> "In order to select the final instrumentation and its location on the pump, it is necessary to conduct a critical speed analysis of the pump. This information is necessary not only for location of the sensors, but is a necessary part of any diagnostic system in order to understand the vibrational characteristics of the system.
>
> In conducting a critical speed analysis, the rotating assembly is modeled as a beam composed of cylinders and discs. The natural frequencies of the rotor are

calculated as a function of bearing stiffness. This permits plotting system critical speeds as a function of rotor support stiffness. Figure 4.8 illustrates a typical plot of the rotor natural frequencies as a function of bearing stiffness. This analysis provides an indication of the rigidity of the rotating system, the mode shape through criticals, and the stability margin of the rotor bearing system.

Figure 4.8 is the critical speed analysis of the main feed-water pump. This is a single stage double inlet (back-to-back wheel) type centrifugal pump driven by a steam turbine. Cross plotted on this figure is the calculated bearing stiffness indicating the first critical speed of 4600 rpm. Operating speed of the turbine varies from 3000 to 5200 rpm, indicating that the first critical speed is in the operating speed range."

The mode shape is depicted on Fig. 4.9 and illustrates that maximum shaft deflections at rotor center is approximately twice the amplitude at the bearings verifying shaft rigidity. Bearing damping approaches 0.5 of critical damping, minimizing any amplification at the critical. This resonance, therefore, is well controlled; but its occurrence in the operating speed range is important in the monitoring and diagnostic functions of the system. Changes in amplitude at the critical may be indicative of changes in bearing lubricant viscosity and damping properties or occurrence of destabilizing functions between impellers and feedwater."

When any rotating machine is started or stopped the rotational unbalance of the machine acts as a swept sine excitation source covering the frequency range from D.C. to full speed of the machine. The author's comments about collecting data during these periods are pertinent:

"If resonances exist in the structural or rotor components of the system in this range, they will be excited during the start-stop operation and may be evaluated in terms of frequency and amplitude magnification. With sensors located at both ends of the rotating assembly, phase relationships can be used to determine the type of rotor unbalance (static or dynamic) and the mode shapes of the rotor.

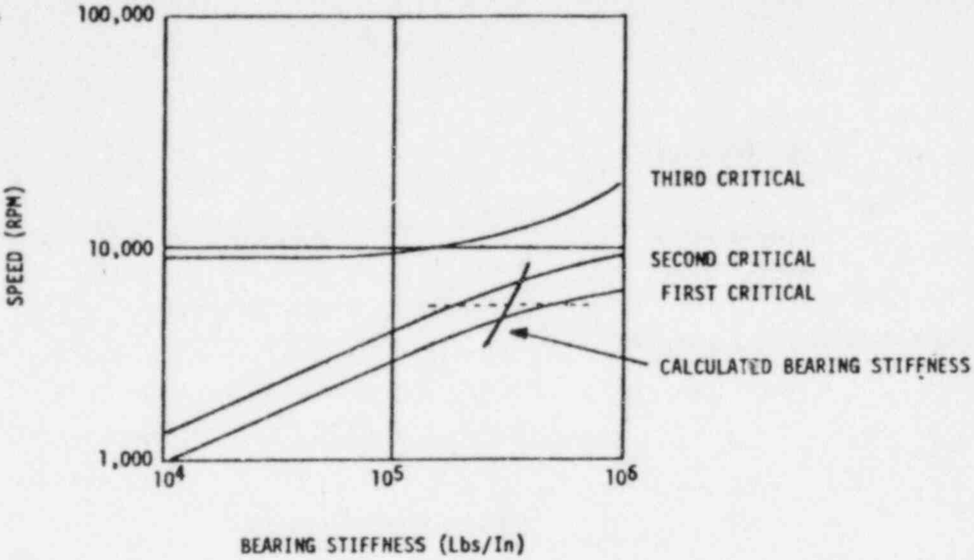
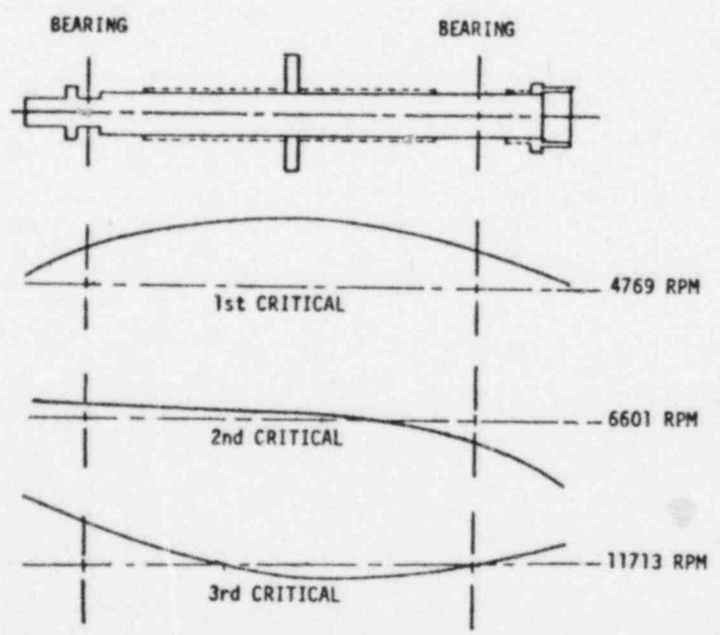


FIGURE 4.8



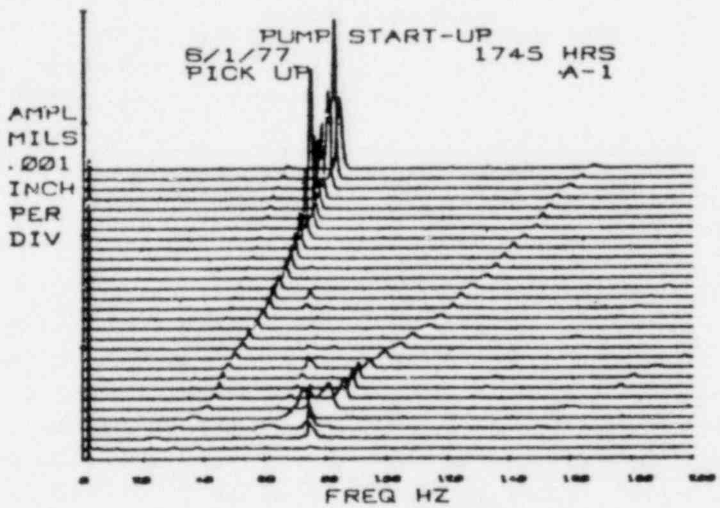
Shaft mode shapes of feedwater pump

FIGURE 4.9

During startup or shutdown, speed transients of motor driven pumps can be very rapid. Typically, a reactor coolant pump can start in a period less than 10 seconds. Three aspects rule out the traditional methods of analyzing such rapidly changing transient data; i.e.: the need for tracking discrete frequencies during startup, the need to remove shaft runout from shaft displacement measurements, and the need to obtain frequency spectra during startup. The tracking and frequency analysis requirement must be accomplished on as many as six probes simultaneously. The use of conventional tracking filters for each probe would be very costly, but more importantly, none are available with adequate response time. For frequency spectral analysis, conventional analyzers are also limited in response time for rapid transients that occur during start and stop. The solution to these problems is to digitize the raw data from the sensors during startup and store it in a computer for analysis after attaining speed."

Waterfall spectra are developed from accelerometer sensors to detect structural resonances during start or stop. Figure 4.10 is a typical waterfall spectra of a pump during startup. Each spectra in the figure represents the frequency content of the vibrational characteristics of the pump at an increasing speed condition. The data has been converted from acceleration to displacement prior to plotting. A structural resonance can be seen at 75 Hz (4500 rpm) which is excited by the second harmonic of running as well as the fundamental or running frequency. The second harmonic excitation can be seen in the third frequency spectra plotted.

In addition to the structural resonance, the first rotor critical is evidenced at 80 Hz (4800 rpm). Without previous knowledge from analysis, this resonance might be difficult to identify as a rotor critical. The waterfall presents a clear visual picture of the system resonances during a transient speed condition."



Waterfall vibrational spectra during startup

FIGURE 4.10

4.4 Offshore Platforms

A significant amount of research work has been done in the past 5 years to investigate the problem of detecting failures on off-shore platforms via measured vibration data.

The single largest source of literature we found on the subject came from papers given at the Offshore Technology Conference, an annual conference which is normally held in May of each year in Houston, Texas. The other large source of literature comes from the United Kingdom and Scotland reporting on study efforts conducted on platforms in the North Sea.

All of this activity on offshore platforms is perhaps best explained by the following quote from one of the references (Ref. H.1):

> "As the search for offshore petroleum moves into deeper water the problem of ensuring the structural integrity of the operating platforms becomes almost exponentially more severe. Because of the very rapid increase of diving cost, time and danger with increasing depth, pressure is strong to develop techniques for monitoring platforms without the use of divers or at least to make more efficient use of diving time and effort."

Dr. Begg explains vibrations on platforms and their relationship to modes as follows:

> "Such structures vibrate over a wide range of frequency, the vibration amplitude being especially pronounced at the structural resonances or natural frequencies (1).
>
> At the lower natural frequencies the vibration consists of the entire platform bending like a cantilever in

fundamental, second, third modes etc. Under such bending conditions the individual members are subjected principally to axial distortions, so that the frequencies of the motions are governed by the axial stiffnesses of the members. At each of the higher natural frequencies only a part of the structure, sometimes only one member, vibrates perceptibly and the motion involves bending of the members or member so that the frequency at the upper resonances is a function of the individual bending stiffnesses."

"It must be noted that, since the lower frequency vibration modes involve only axial deformation and since cracking has little effect on axial stiffness, the effect of cracking on the low frequency spectrum will always be extremely small, and it is not thought that it will ever be possible to detect cracks from the lower frequencies of the spectrum, measured above the water line."

"However, as has been stated, the higher vibration frequencies are governed principally by bending stiffness, which is affected by cracking, so that cracks may be detected by measuring changes in the upper parts of the spectrum. Unfortunately these vibration modes are very localised and it has not been found possible to identify spectrum peaks, measured above the water line, with individual small scale bending modes below the water.

Tests made on a 5 m high model platform have indicated that such measurements are possible under ideal laboratory conditions but it has been found that the additional damping due to the water ambient along with the non-linear interaction of the jacket with the piles have made such measurements extremely difficult offshore. Thus for crack detection it is necessary to make measurements of the spectrum on or very near to the member under inspection, which involves underwater work."

Begg's company seems to have had a considerable amount of success in the implementation of an onboard monitoring system for platforms. Some of his conclusions are:

"The overall, above water monitoring system has been fairly extensively tested on North Sea platforms over an eighteen month period, both on steel and concrete structures. Spectra, measured using accelerometers bonded to the structure by epoxy cement at various locations, have proved in general to be stable and consistent to within 2% over an extended period."

"Early tests offshore of the underwater accelerated diving inspection method temporarily, (mounting transducers underwater) have been very encouraging but as yet there is no indication of the smallest size of defect which such a system can detect. Already it is obvious that any crack large enough to permit ingress of water will register a frequency shift or around 10% which will be very evident."

In another presentation at the OTC in 1976 Begg et.al. (Ref. H.4) reported on some results obtained with laboratory models and from measurements on a North Sea platform:

"Integrity monitoring is based on the assumption that each load-carrying member is involved in the overall vibration modes. Since these overall modes are wholly dependent on the stiffness of the platform it follows that the more efficient the design the greater will be the effect on the spectrum of the failure of a member. The position of the member in the structural topology governs which of the overall modes is most affected by the failure. In general, the more heavily loaded is the member, the bigger will be the effect of its failure on the spectrum. The fact that the more serious the damage suffered the bigger the indication given is a decided advantage in a monitoring arrangement.

The second important consideration is the magnitude of the changes caused by a failure. Reference 2 tabulates the effects of the failure of a series of members on the first three natural frequencies of a typical 4 legged North Sea platform. This indicates changes of up to 30% in some of the frequencies and these figures have been confirmed experimentally in tests on the 15 ft model, which was based on that platform. (Reference 3) Finally, measurements made in the North Sea on a 16

> legged, highly redundant platform showed a 10% change in a fundamental mode after an extra bracing member was fixed in position."
>
> "It is to be noted that, at the lower frequencies, the mode shapes are such that the individual members deform primarily in tension and compression and not in bending. Since the introduction of a deep crack in a long member does not change the axial stiffness appreciably, the effect of cracks on the lower structural modes will be small."

On the use of digital processing (FFT) systems to process the vibration data, they note:

> "On such systems it is possible to detect frequency changes down to 0.1% of the frequency in question, which allows very fine changes in spectra to be detected. Since changes associated with structural damage are of the order of 5-30%, the accuracy of the data processing is more than adequate."

More recently Begg & Mackenzie have again reported on their experiences with platform monitoring (Ref. H.6). Some of their comments are the following:

> "In most platforms, the first and second groups of natural frequencies are easily obtained from measurements made with only natural excitation. Failure of a particular member produces larger changes in some of these frequencies than in others, and the location of the failed member can be deduced approximately from the pattern of these changes. In some structures, the frequencies of the third mode group can also be obtained and provide further confirmation of a failure and increased confidence in its location. Changes in mode shapes also assist in failure location."

In steel platforms, which they claim are generally very lightly damped, frequency changes can be accurately measured and hence detected:

> "Thus, in this instance, it is the natural frequencies which contain the information about structural integrity. The amplitudes give additional information about changes in the vibration mode shapes which assists the location of a stiffness change. In such lightly damped structures, the accuracy with which the frequencies can be measured is very high, so that considerable sensitivity can be achieved."

The author's experience with concrete structures has been less successful than with steel, though. Some of their comments are:

> "Recent measurements made on a multileg platform have shown that many vibration modes associated with overall sway and with vibration of individual legs can be measured and identified by adapting the instrumentation developed for steel jackets. Further tests are planned to determine how stable the measurements are, and to assess their sensitivity to structural change."

One of the earlier investigators of the relationship between vibration response and the structural integrity of offshore platforms is Professor J. Kim Vandiver of M.I.T. He wrote a Ph.D. thesis on the subject in 1975 and summerized many of his findings in a presentation at the Offshore Technology Conference in 1975. His work is reprinted in a more recent document (Ref H.8) also. His assessment of the state of the art in 1975 was as follows:

> "The detection of structural failure by measurement of a related change in natural frequency is not without

precedent. There is continuing industrial research in the field of expensive rotating machinery such as generators and jet engines. More closely related work has been performed by civil engineers interested in the seismic response properties of large buildings. For several years civil engineers have been able to measure the natural frequencies of large buildings using sensitive accelerometers. Wind and seismic forces have sufficient broad-band random excitation that most buildings respond at one or more of the natural frequencies included in the band. Measurements made before and after earthquakes have revealed damage-related frequency reductions as large as 50 percent. In many cases visual inspection had revealed no damage. For example, in steel-reinforced concrete buildings, microcracks that developed in the concrete were undetected in visual inspections, and yet caused a substantial reduction in the structural stiffness and, therefore, the natural frequency.

Force-balance-type accelerometers that have been developed for seismic work can be applied directly to measuring dynamic response of offshore towers to wind and wave forces. These devices are capable of resolving 10^{-6} g's, one-millionth of the acceleration of gravity. In extremely calm weather conditions the Buzzards Bay tower responds at 10^{-5} to 10^{-4} g's at its natural frequencies. Much of the instrumentation that works on buildings is readily adaptable to offshore towers.

Fast Fourier Transform (FFT) techniques have been used to analyze the dynamic response of offshore structures. FFT spectrum analysis was used in this work to obtain estimates of natural frequency and damping."

Vandiver conducted computer simulations as well as experimental tests on an operating platform at Buzzards Bay. On the uses of computer simulation he wrote:

"Concurrent with the instrumentation program, a computer model was formulated to predict the natural frequencies

> of the Buzzards Bay tower. The ultimate purpose of this model was to conduct a parametric study of the effect of simulated structural damage on natural frequency. Since it was impossible to actually conduct a systematic survey on a full-scale structure in which members would actually be removed or broken, it was reasoned that a computer simulation would be the next best thing.
>
> Once the results of the full-scale test were in and the accuracy of the natural frequency determinations was known, a comparison with the computer simulation would specify the minimum detectable level of damage that this inspection technique could resolve."

With the computer model he was able to correlate detected frequency shifts in certain directions with structural failures affecting stiffness in the given direction. He also simulated the effect of .050 inches of rust on the members below the water line, and showed that it greatly affected the stiffness of the tower, which caused significant frequency shifts in its modes.

He also makes a strong case for being able to detect mass changes in a structure as part of a successful integrity monitoring scheme:

> "The ability to identify structural damage is limited by the ability to estimate the change in mass of the structure from the time of the last inspection. On an active drill platform, the amount of mud, drillpipe, water, etc., must all be considered. Marine fouling and underwater flooding of structural members are also potential sources of error that must be detected and eliminated from structural-failure considerations.
>
> The computer simulation can reveal the percentage of change in natural frequency as a function of member damage. The severity of damage that the inspection can potentially detect is determined by comparing the

> computer results with the in-practice ability to detect the long-term changes in structure mass. This will vary from one structure to the next. Unmanned producing wells have rather constant masses and, hence, will have a very sensitive detection threshold. Exploratory drilling rigs will have much less sensitive detection limits."

Loland & Dodds (Ref. H.2), also from Structural Monitoring Limited, and Begg & Bendat (Ref. H.3) have reported on their experiences with instrumentation and data processing systems for detecting damage on platforms.

Some of the comments of Loland & Dodds are pertinent to any long term monitoring scheme:

> "The conclusions from the laboratory work carried out by Loland et.al. (4) made it abundantly clear that a successful integrity monitoring system for fixed steel offshore platforms had to fulfill certain requirements. If the laboratory results were to be extended and used in field case studies then the following requirements had to be met:
>
> 1. It must be possible to measure the vibration response of the platform under environmental excitation and from this response extract the natural frequencies with sufficient accuracy and reliability. Only transducers located above the water line could be used for this purpose and at least 9 vibrational modes (i.e. the fundamental group plus 2nd and 3rd harmonics) would be required to enable the possible identification of the failure area.
>
> 2. The vibration spectra, containing these frequencies, must remain stable in frequency over long periods in time and be independent, in the frequency axis, of tide, sea and wind conditions. The long term frequency stability will be affected by changes in platform deck mass, build up of marine growth on the structure and such like occurrences.

3. The instrumentation package must withstand the rigours of the environment and ensure accurate recording of information with signal-to-noise ratios compatible with the response spectrum (i.e. signal-to-noise ratios greater than 50 dB).

4. It must be possible to infer the mode shapes for the complete structure from the measurements taken from transducers located above the water line. Sufficient information must be obtained from transducers in this region to remove any ambiguity from mode shape identification.

5. The installation and operating costs of the monitoring system must enable obvious savings to be made over existing diver orientated inspection methods and be competitive with other monitoring techniques which are available such as underwater television."

Loland & Dodds also reported the following experimental results:

"Throughout the period of measurement (from April 1975 to date) the spectra on all platforms were found to be stable in frequency to within 3%."

"During the period of the investigation platform 1 underwent minor structural modifications and figure 3 shows the spectra obtained from this platform before and after this modification. The effects of this modification which can be taken as typical of a splash-zone failure, resulted in frequency changes of 10 to 15%. Tracking of these frequencies was made possible by identifying the mode shapes."

They also make some very optimistic conclusions:

"1. Spectra can be obtained with sufficient information for monitoring purposes.

2. Spectra are stable enough for monitoring purposes.

3. Instrumentation techniques have been developed which are capable of measuring the platform response spectra with sufficient accuracy and reliability for monitoring purposes.

4. Analysis techniques are available and have been developed which enable the determination of the platform mode shape above the water line, the accuracy limitation being on the measurement side. From these measurements the overall mode shape can be successfully inferred.

5. Changes in this response spectrum due to minor structural modifications are clearly observed and confirm the validity of this technique as a means of platform inspection for primary structural damage."

Lock & Jones (Ref H.5) carried out a short feasibility study in 1976 into the use of vibration monitoring to assess structural integrity, which they interpreted as maintaining structural stiffness.

They tested a triangular space frame, similar to one of the sections of an oil platform, in a laboratory both before and after one of its cross members had been cut in half.

They also modelled the space frame with a computer model and computed its model properties with and without the cross member. Their paper contains a table which shows the frequency shifts of 20 of the frames modes caused by the severed member. The percentage changes in the frequencies range from 1% to 18%.

Some of their conclusions are stated below:

"On the work which has been done it is not possible to definitely identify any given resonance in the response spectrum of the undamaged space frame with its equivalent in that of the damaged structure. However the spectrum of the undamaged frame, Figure , contains five energetic resonances in the range 125 to 171 Hz

> while a similar group appears between 105 and 154 Hz in the spectrum of the cut frame. This suggests that an overall decrease in frequency for these five modes of 12.5% when bar 5-9 was cut. Comparison of individual frequencies on this tentative identification give decreases from 10 to 16%. Apparent shifts at the low frequency end of the spectrum are much smaller though amplitudes do show some variation.
>
> The computer model predicts a modal density close to that found practically. It also predicts the existence of closely spaced modes. The model predictions have been included in Figures 2 and 3 for the purpose of comparison. In most cases insufficient work has been done to allow a theoretically predicted mode to be uniquely identified with one found in experiment. However Figures 2 and 3 show reasonable agreement in that fourteen theoretically calculated natural frequencies appear to correspond to within about 5% with measured resonances. In two cases theoretical points appear with no corresponding practical points though this may reasonably be explained in terms of sensors being unfavourably aligned to detect them or low energies in the practical modes."

At a conference of the Society for Earthquake Engineering and Civil Engineering Dynamics held in 1977, L. R. Wootton made the following claims:

> "Monitoring the dynamic response of steel offshore platforms is now being used to detect changes in the structural integrity. The work involves comparing accelerometer measurements taken on the deck or on the structure in the splash zone with calculated and previously measured response modes. The analysis falls into two parts; discerning that a change has occurred and then diagnosing the form of the failure and its position."

In his presentation he also stated:

"The accurate dynamic analysis of the jacket structure is very important in determining firstly the most suitable positions for the accelerometers on the jacket and then the likely cause of any particular measured change in the vibration response."

"The regular dynamic loads due to the on-board machinery (turbines, pumps, cranes etc.) tends to be at frequencies that are significantly higher than those used for monitoring but in any case their strong periodicity makes them readily identifiable in the measured spectra."

"One potential problem is that the same order of change in the spectrum could be caused by factors other than a change in a structural member. For example, marine growth increases the weight of a structure by increasing the added mass but this effect developes very slowly. Changes of top deck mass distribution can be much more sudden, the loading of pipes from a supply vessel for example."

All of these results would apply as well to monitoring of large structures or piping systems in a nuclear power plant. Some of the author's conclusions are:

"The purpose of this initial condition monitoring is to discern any change in the structure; if there is change then the cause must be diagnosed. Very careful analysis of the data already acquired will probably locate the damage to within a few members on a certain area. From this point, traditional non-destructive testing techniques can take over."

"There is no doubt that monitoring of the dynamic response of an offshore structure can provide an efficient system for the monitoring of its structural integrity. The techniques that are currently being evolved may have much wider application."

Sheldon Rubin and his co-workers at the Aerospace Corp. have recently completed some experimental and analytical work related to assessing the structural health of offshore platforms.

Rubin collected some data from a platform in the Gulf of Mexico and processed it using FFT techniques to identify the first few fundamental modes of vibration of the platform.

Some of the pertinent comments in Ref. H.11 are:

> "Fixed offshore platforms are subject to structural failures from storms, earthquakes, ice, ship collisions, and fatigue. Current diver inspection operations can be extremely costly and hazardous. Industry and governmental agencies have been studying the incorporation of other inspection and monitoring techniques to improve the cost effectiveness of the overall monitoring effort."
>
> "There has been a growing interest in the concept of using instrumentation to establish an initial indication of the structural health of these platforms. If the instrumentation can be shown to give reasonably reliable indications of damage, it could be used as a trigger for an inspection, which would then identify the specific failures. This capability would reduce the number of inspection operations needed to assure a platform's structural integrity. Another benefit, when an underwater inspection cannot be carried out immediately, is that precautionary actions can be instituted to reduce the risks of possible structural failure until such time that underwater inspection provides a more definitive assessment."
>
> "Autospectral analysis reveals narrow-band peaks at the natural frequencies; cross-spectral analysis can be employed to determine, for each natural mode, the amplitude and phase relationships among the measurement positions (11). The lower modes of vibration, hopefully up to at least the third in each lateral direction and in torsion, are identified in this manner. Changes in the frequencies and shapes of these modes relative to baseline data are then evaluated to establish whether significant damage has

> occurred, guided by failure sensitivity studies on a mathematical model of the platform. The approach can only be expected to detect significant failures, which we define as total severance of one or more members important to the platform's load-carrying capability."

Some of the findings of the study are the following:

> "The successful acquisition of data from the SP-62C platform and the quality of the quick-look spectral analyses lend support to the contention that ambient vibration testing is a promising concept for monitoring the structural health of offshore platforms. It is clear that available technology in the areas of instrumentation and data analysis are adequate for implementation of the concept.
>
> The quick-look analyses suggest that platform vibration induced by a calm sea provides good quality data, as does also that induced by a rough sea. Tentatively, nonlinearity in platform stiffness characteristics does not appear to significantly affect the frequencies of the modes. The net result appears to be that sea state is not a significant factor in the conduct of an ambient vibration test, at least for the type of platform tested."

Several papers were found which treat modelling of the dynamics of offshore platforms in detail. (Refs H.15 through H.18). Ruhl (Ref H.16) and Earle (Ref H.15) both report results of vibration measurements made on platforms and their correlation with analytical predictions. Ruhl made the following conclusions and recomendations:

> "The data presented in this paper is specific to the tested platforms, measured sea states, and levels of response. Although the platforms are typical of platforms for the indicated water depths, the data

should not be arbitrarily applied to other conditions without considering differences in all pertinent parameters. The following observations can be made from this work:

1. Measured natural periods are seen to agree closely with periods predicted on the basis of three-dimensional models with lumped nodal masses. Deck mass can significantly effect natural periods, but axial and lateral foundation stiffnesses are important only if the foundation is relatively soft. The condition of flooding and added mass coefficient are of secondary importance.

2. Estimates of damping values from measured motions ranged from 1.0 to 5.1 percent of critical depending on the methods of testing and data analysis. These are not consistant values, and it is recommended that the methods of data analysis be reviewed, including the assumed form of structural behavior, in an effort to improve these estimates. Measured results under low response levels are not directly applicable to higher levels of response, nor are values measured on short structures directly applicable to tall structures. The soil/structure system must be considered.

3. For one platform the measured significant platform displacements were seen to vary with the measured significant wave heights, and the major peak in the platform motion response spectrum matched the single large peak in the wave spectrum.

4. The largest measured wave (13.6 feet) caused a platform motion of 0.16 inches; the predicted motion based on regular Airy wave theory due to this wave was 0.20 inches. In another case, a measured motion of 0.16 inches was caused by a 9.1 foot wave, and the corresponding predicted motion was 0.15 inches. This is considered good agreement but represents only two data points.

5. Comparisons were made of the average response over a four hour period. A comparison using histograms indicated the average predicted response was 13 to 25 percent greater than the average measured response. Computation of a response spectrum from the measured wave spectrum and a theoretically derived transfer function showed the predicted significant response was 25 percent greater than the

> measured significant response. Similar comparisons should be made for different sea states on the basis of models that include the spreading of energy."

Earle made the following conclusions:

> "The most important conclusion to be drawn from this work is that the amount of damping in offshore platforms is less than 4 percent of critical. The data presented here suggest that this value depends very little on the details of the structure or the type or magnitude of the loading on the structure. Frequently, in the past, higher values have been assumed for damping. This would result in unconservative predictions of vibrations, since an increase in damping will result in smaller-amplitude vibrations.
>
> In the case of Cook Inlet platforms, the frequencies are independent of the magnitude of the loading, at least up to and including loading that was fairly severe."

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16. ABSTRACT (200 words or less)

The stated object of this study was to survey the technical literature and interview selected experts in the fields of dynamic testing and analysis to determine the state-of-the-art of the relationship between physical damage to a structure and changes in its dynamic (modal) properties.

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