AN ANALYTICAL METHOD FOR DETERMINING THE PROBABILITY DISTRIBUTION OF THE MAXIMUM OF COMBINED RESPONSES

A Verification of the Newmark-Kennedy SRSS Criteria

C. Allin Cornell, Daniele Veneziano, and Robert Kilcup

C. Allin Cornell
Consultant
Box 289
MIT Branch P.O.
Cambridge, MA 02139

prepared for
N. M. Newmark Consultants
December, 1979

REPORT NO. 1C1979

8003270425

Disclaimer of Responsibility

As this document is submitted by the General Electric Company on behalf of the Mark II Owners Group, the following disclaimer is incorporated:

The only undertakings of the General Electric Company respecting information in this document are contained in the contracts for Mark II Containment Consulting Services between the General Electric Company and each of the members of the U. S. Mark II Owners Group, effective variously June 9, 1975, June 13, 1975, and July 29, 1975, and nothing contained in this document shall be construed as changing the contracts. The use of this information by anyone other than the members of the U. S. Mark II Owners Group either themselves or through their technical consultants, or for any purpose other than that for which it is intended under the contracts is not authorized; and with respect to any unauthorized use, the General Electric Company makes no representation or warranty, express or implied, and assumes no liability of any kind as to the completeness, accuracy, usefulness or non-infringing nature of the information contained in this document.

TABLE OF CONTENTS

1. INTRODUCTION AND SUMMARY OF CONCLUSIONS

When two histories of component response (e.g., stress time histories) are hypothesized to act simultaneously it is desirable to have straightforward rules for determining a satisfactorily conservative combined peak design response. The Square Root of the Sum of the Squares (SRSS) rule* has been proposed as being applicable, provided certain criteria are satisfied. The most recent statement of such criteria, called the Newmark-Kennedy criteria, is given in Reference 1.

The primary purpose of this study is to support the criteria confirmation and other conclusions drawn in Reference 1 by considering a broader range of variations in the sensitivity study reported there. This report is a companion to Reference 1 in that respect, and the reader is assumed to be familiar with it. Only a brief summary of the background is given in the next section.

A secondary purpose of this study has been to develop and confirm an analytical (or, when computer-implemented, a numerical) procedure to generate both time-phase-only (TPO) cumulative distribution functions (CDF's) and random-amplitude-plus-time-phase distribution functions of the peak combined response when time histories of the individual responses are given.** The new procedure can evidently replace the more costly simulation

---

* The rule, the method, and the examples here are interpreted to apply to the signed responses. The method can be adapted to absolute extremes by replacing the mean up-crossing rate by the sum of the up-crossing and down-crossing rates.

** The procedure also applies to cases in which, instead of explicit time histories, appropriate random process descriptions of the responses are specified. These might be developed in the future from random vibrations approaches to the structural/mechanical dynamic analyses.

analyses that have been used for this purpose in preceding studies (References 1, 2, and 3). Because it is new, somewhat more space is given in this report to the bases of the procedure; most of this material has been placed in Appendix A, however, not to detract from the main emphasis of this study.

> The primary conclusion of this study is simply a confirmation of Reference 1. If the modified preamble and Criterion 2 of the Newmark-Kennedy criteria are satisfied for response time histories with individual peaks defined at 84% NEP (non-exceedance probability), then the SRSS combination rule will produce a value with approximately the same (84%) NEP. This conclusion has been verified here for a large variety of amplitude distribution types (lognormal, uniform, normal, exponential, and a bimodal mixture of uniform distributions), for a range of preliminary relative amplitude scalings, and for a range of relative durations of the two response histories. In addition, a range of amplitude dispersions has been considered to investigate the dependence of the conclusions on this variable. As determined in Reference 1, the 84% fractile (or the design amplitude) should be at least 15% greater than the median to insure that the SRSS procedure is adequate.

2. A BRIEF BACKGROUND

In summary, the primary purpose of Reference 1 and of this study is to demonstrate the adequacy of Criterion 2, which states:

"When response time histories are available for all dynamic loadings being combined, SRSS methods may be used for peak combined response when CDF calculations, using appropriate assumptions on the range of possible time lags between the response time histories, show the following criteria are met:

1. There is estimated to be less than approximately a 50% conditional probability that the actual peak combined response from these conservatively defined loadings exceed approximately the SRSS calculated peak response, and

2. There is estimated to be less than approximately a 15% conditional probability that the actual peak combined response exceeds approximately 1.2 times the SRSS calculated response."

The conditional probabilities stated are conditional on knowing the peak amplitudes of the individual responses, that is, the only random quantity reflected in these CDF curves is the random time lag between the two responses. We refer to these conditonal CDF curves as time-phase only (TPO) curves or TPO CDF's.

Adequacy of the SRSS rule is defined to be that it produces a design peak combined response that has a non-exceedance probability (NEP) of approximately 84%, when the randomness in the peak amplitudes is recognized explicitly. It is presumed (see modified preamble, pg. 4-1, Reference 1) that the individual given time histories have been scaled, conservatively, such that their individual peaks have approximately 84% NEP's also (or that each peak is at least 1.2 times the median level, whichever is greater).

In Reference 1, the authors selected six cases of sets of time histories from the 291 which had been considered earlier (Reference 2 and 3). All non-trivial cases satisfied Criterion 2. To be conservative, the six selected cases were those that came closest to failing the requirements of Criterion 2, either because the SRSS value was close to 50% in the TPO CDF or because 1.2 SRSS was close to the 85% value. (In addition, cases were rejected in which the SRSS value was within 25% of the absolute sum of the peaks, the difference being insignificant and uninteresting.) The first five of these cases are also considered here.*

In Reference 1, simulations were conducted with both random amplitudes and random time lags. The former were introduced as random variable multipliers**, F_R, on the deterministic time histories, $A(t)$, producing random time histories $R(t) = F_R \cdot A(t)$. Various dispersions were assumed for the distribution of F_R but, consistent with intent of the study, in all cases the median was defined such that the unity value of the scale factor F_R corresponded to the 84% NEP. The dispersion factor δ was defined as the ratio of the 84% NEP value to the median, i.e., simply as 1/median. For the lognormal distribution (used in most of the Reference 1 study), δ corresponds to $\exp(\beta)$ in which β is the standard deviation of the (natural) log of F_R. (Approximately, $\delta \cong 1 + \beta$, and, approximately, β is the coefficient of variation of the amplitude distribution.) δ values of 1.0 (i.e., the TPO case), 1.1, 1.2, and 1.3 were considered in Reference 1 and in this

---

* The sixth case involving three rather than two loads, was not pursued simply because it would have required computer program extensions that were not believed to be justified in the light of the two-load case results. Recall that the combination of three loads can always be considered as the combination of one with the combination of the other two. Because the amplitude dispersion for the combination of any two loads involves both TPO dispersion and individual amplitude dispersions, it was never found to fall below the 15% level. Therefore, there is every reason to assume that the SRSS procedure will work for three as well as for two loads.

** This is consistent with seismic practice of scaling motions by their peaks.

study as well.

Random time lags were introduced in Reference 1 in a manner consistent with the preceding studies (References 2 and 3): the beginning of the shorter time history was assumed to lag behind the beginning of the longer time history by an amount uniformly distributed on the interval from the beginning of the longer history to the end of the longer history. In contrast, in this study, for reasons of analytical simplicity, the lag is uniformly distributed on an interval equal to the sum of the two histories durations (a somewhat unconservative assumption, relative to previous studies), but the motions are assumed to be periodic implying that, if it extends beyond the termination of the longer motion, the tail end of the shorter motion can "wrap around" and superpose on the beginning of the longer motion (a conservative assumption relative to previous studies). In addition to simplicity (it permits the adoption of stationary processes), the assumption here is more realistic in that it assumes, in effect, that the shorter motion may be the first of the two to occur. In any case, the net effect is believed to be small and comparisons between results here and in Reference 1 are very satisfactory.

In Reference 1 simulation was used to determine the CDF's. Here an approximate analytical method has been developed, based on random process theory and on previous theoretical study of load combination problems. The procedure permits determination of the CDF's by simply evaluating several convolution integrals. These integrals can be evaluated numerically by computer. (A convenient program has been developed for the specific purposes of response combination studies.). The basis of this method is presented in a subsequent section and in more analytical detail in Appendix A.

The information required for each response history .s

(1) the duration, T_i.

(2) the CDF of the random amplitude scale factor, F_{R_i}.

(3) the marginal CDF of the deterministic time history, $A_i(t)$. (This function gives the fraction of the duration, T_i, that the history spends below level x, as a function of x.)

(4) the mean upcrossing rate function of $A_i(t)$. (This function gives the ratio of the number of times $A_i(t)$ crosses--from below to above--level x to the duration T_i. It is a function of level x.)

The last two functions are easily obtained if the deterministic time history $A_i(t)$ is given.* For example, if $A_i(t)$ is given as a set of values at uniform time intervals, Δt, the determination of the marginal CDF requires simply counting the number of values in the list with $A_i \leq x$, dividing by $T_i/\Delta t$, and repeating for a set of x values. Similarly the upcrossing rate function involves observing values at the two ends of each time interval and counting the number of times that both the earlier value is below x and the later value is above x (i.e., an upcrossing of x occurs). This number is divided by T_i to obtain a rate per unit time. The process is repeated for a set of x values above and below the (zero) mean. If negative peaks are also to be considered, a mean downcrossing function is needed for all values of x. These functions are equal in value. Examples of these functions, provided by EDAC, are shown in Figures E-2, 4, 6, etc.

---

* As mentioned above, these functions might also be provided directly by random vibration theory without calculating response histories.

The following sections will summarize first the analysis method used and then the results. Those uninterested in the former can skip the next section. Those interested in more detail about the method are referred to Appendices A and B; details of results are given in Appendices C and D.

3. METHOD OF ANALYSIS

The analytical method used to evaluate the SRSS rule of 84% fractile combination is based on an approximate theory of extremes of stationary random functions (Reference 4 and 5).

A requirement for applicability of this analytical technique to the present problem is that the random response functions (random because of phasing and because of amplitude) correspond to stationary processes. Although each response, $A_i(t)$, may appear to be the realization of a nonstationary stochastic process, (see, for example, Fig. E-1 , in Appendix C), the introduction of random phasing between responses gives an opportunity for stationary representation of the uncertainty. For the i^{th} response, one such stationary representation can be obtained in three steps:

1. A deterministic and periodic function $A_{ip}(t)$ is obtained by shifting the i^{th} response, $A_i(t)$ to start at time $t = 0$ and by then repeating the same function at intervals of length $T = T_1 + T_2$, the combined duration of the two loads.
2. $A_{ip}(t)$ is translated randomly by an amount t_{0_i}. The random variables t_{0_1} and t_{0_2} are independent with an identical uniform distribution on the interval [0, T]. (In application, it is sufficient simply that the phase between the two responses has such a uniform distribution.)
3. In the final step, randomness of the amplitudes is accounted for. After

this is done, the i^{th} response is represented as a stationary periodic random process, $R_{ip}(t) = F_{R_i} A_{ip}(t)$, in which F_{R_i} = random amplitude of response i and $A_{ip}(t)$ = (random, periodic) response process for $F_{R_i} = 1$. These three steps need be done formally only; in practice (Appendix A) one deals numerically only with the marginal CDF and mean up-crossing rate functions of the given response histories. Application of the approximate theory of Poisson upcrossings to the stationary and periodic sum process $Z(t) = \sum_i A_{ip}(t)$ over the interval [0, T] yields an analytical approximation to the distribution of the maximum combined response. Details of the model and of the analysis that eventually leads to this distribution are given in Appendix A.

The method requires much less numerical effort than simulation. It is, however, an approximate method. In a variety of applications to load combination problems it has been found to be satisfactorily accurate (Ref. 4). For this response combination application, its accuracy must be verified through comparison with Monte Carlo simulation results. Specifically, one may anticipate that inaccuracies could originate from three features of the analytical model:

1. Representation of random phase through the artificial introduction of stationary, periodic response processes;
2. Approximation of the mean upcrossing rate function of the sum process $Z(t)$; and
3. Assumption of Poisson upcrossings.

From comparison with the simulation result in Reference 1, it was concluded that none of these features is critical in the case of random phase only (i.e., TPO CDF's), whereas the Poisson approximation can produce sizeable errors when amplitudes are random with large coefficients of variation (large δ). In this last case, a major improvement in accuracy followed from assuming that upcrossing events are <u>conditionally</u> Poisson, given the response amplitude factors. This improvement over the ordinary treatment of mean upcrossing properties has been incorporated into the analytical approach (Appendix A). Through it, one accounts for the fact that the mean upcrossing rate of Z(t) across level x, $\nu_Z(x)$, varies from realization to realization due to randomness of the response amplitude factors, F_{R_i}. On the contrary, the method neglects sample variation of $\nu_Z(x)$ due to random phase. This factor introduces conservatism into the analytical results for moderate to high fractiles. Another source of conservatism is the formula by which mean upcrossing rates of Z(t) are approximated (Eq. A.4). These conservative trends are anticipated on the basis of theoretical considerations; numerical comparison with simulation results has shown that biases of the (improved) analytical method are small. See, for example, Fig. C-1. Details are given in Appendices C and D. Note that for the sample sizes used in the Reference 1 simulations the simulated CDF's are statistically accurate only between about 0.0 and 0.9.

In the analytical treatment, considerable computation time savings result if the two random amplitude scaling factors, F_{R_1} and F_{R_2}, are the same (perfectly dependent) for both the responses*, as opposed to them being probabilistically independent for the different responses. Replacement of the independence assumption by the perfect dependence assumption (Appendix B) causes little modification in the median value of the peak combined response, $Z_{max} = \max_{0 \leq t < T} Z(t)$, but increases the variance of Z_{max},

* This assumption is analogous to that adopted in Chapter 3 of Reference 1; the method there applies only to lognormal distributions, but the conclusions are similar to those found here.

thickens the distribution tails, and therefore adds conservatism in the estimation of high fractiles (of low fractiles for $Z_{min} = \min_{0 \le t < T} Z(t)$). A typical comparison between results from the two assumptions is shown in Fig. C-1 and many more cases are presented in Appendix C.

Numerical values for the NEP's of SRSS values in Table 1 can be used to confirm quantitatively the dependent-amplitude analytical method a .pted. The figures in parentheses are analytical results for independent amplitude scale factors. They are, on average, only 1% higher than those determined assuming dependent amplitude scale factors. A typical <u>net</u> comparison bet een the dependent-amplitude analytical results and the simulation results is obtained by comparing the left-most columns of NEP values in Table 1. Analysis of these results suggests the true (simulation) values are (on average over the cases shown) 1.05 times the dependent-amplitude analytical results used in the bulk of our study. This factor is somewhat higher (1.09) for $\delta = 1.0$, i.e., TPO, and somewhat lower (1.03) for $\delta = 1.2$ and 1.3. These factors can be used to apply approximate corrections to the (conservative) analytical results discussed below.

In summary: for fractiles above the median (below the median for Z_{min}) the analytical procedure tends to err on the conservative side. Conservatism is more pronounced but still not large under the simplifying assumption that amplitude scaling factors are the same for all responses. Therefore, the SRSS rule is validated here under this last assumption; in this case there is no need for more sophisticated modeling and analysis. On the basis of this reasoning, most calculations have been made assuming identical, perfectly dependent random amplitude factors.

4. NUMERICAL RESULTS

Representative results from the analytical procedure (the complete set is in Appendices C and D) are shown in Fig. D-1. They refer to EDAC Case 1, identical amplitude factors ($F_{R_1} = F_{R_2} = F_R$) with lognormal distribution, and dispersion parameter* δ = 1, 1.1, 1.2, 1.3. Due to the fact that the distribution of amplitude is "anchored" at the 0.84 fractile, larger δ (larger variance) of F_R implies smaller mean value of the maximum combined response, Z_{max}. Clearly, it also implies larger variance of Z_{max}. The two effects combine to cause the cumulative distribution functions for different δ to cross at points above the median. Of special interest here is the 0.84 fractile of the distributions and, even more, the probability F(SRSS) of not exceeding the SRSS value, i.e., the NEP of the SRSS.

The NEP of the SRSS value depends on many parameters, including duration of the responses, distribution of the time phase, distribution of the amplitude factor, relative size of the loads (whether or not one response dominates the sum), whether interest is in the minimum or in the maximum of the sum ("positive" case versus "negative" case). Among these parameters, the more critical ones have been identified through sensitivity analysis of the NEP with respect to:

1. Dispersion of the response-amplitude distribution (δ between 1.0 and 1.3);
2. Type of response-amplitude distribution (uniform, lognormal, normal, exponential, bimodal mixture of uniform distribution);
3. Preliminary amplitude scaling of the response functions;

---

* In all studies here both scale factors have the same δ value. This case is the most critical. See Reference 1.

4. Relative duration of the responses;

5. Positive (maximum) versus negative (minimum) case.

All calculations were performed on the five response-combination problems previously identified through simulation as those critical to the application of SRSS. Therefore, one can expect present numerical values of the NEP of the SRSS value to be conservative (low) with respect to those for the entire spectrum of response-combination problems.

Tables 1 and 2 summarize the numerical results from some of the sensitivity analyses. With the exceptions shown for comparison, all results were obtained under the conservative assumption that the random amplitude factors of the two responses are the same. The following conclusions can be drawn:

1. As expected, the NEP of the SRSS value tends to increase with δ (see also Fig. 1). This fact is considered in the Newmark and Kennedy criteria by limiting the applicability of the SRSS rule to, in effect, $\delta > 1.15$. The reason is that the amplitude-factor distribution is anchored at the 0.84 fractile. For all but small values of δ (e.g., for $\delta \geq 1.15$), the NEP of the SRSS value has small dispersion about a value which is itself close to 0.84. Recall that the cases studied here are the near critical ones from the original 291 cases. The remaining cases would show typically equal or higher NEP values.

2. The difference between the NEP of the maximum combined response and the NEP of the minimum combined response* explains most of the dispersion of the results in Fig. 1 for given δ. This variability overshadows sensitivity of the NEP of the SRSS value to all parameters considered here, except δ. It is due

---

* More precisely 1 - NEP in the minimum case.

primarily simply to small asymmetries in some of the sample time histories.

3. Parameters to which the NEP of the SRSS value is <u>not</u> sensitive include:

(a) Distribution type (shape) of the amplitude factor (simple or bi-modal rectangular, lognormal, etc.). See Table 1. Within the present choice of distributions, sensitivity is very small and negligible.

(b) Relative duration of the loads. Relative duration is varied here by stretching or compressing the time axis of the response with shorter duration. Sensitivity to this parameter is displayed in the last four columns of Table 2, in which TF is the factor by which duration of the shorter response is multiplied (duration = TF x original duration).

(c) Preliminary deterministic scaling of the responses (first four columns of Table 2). One of the responses is multiplied by a constant before phase and amplitude are randomized. The parameter that is varied in the table is the ratio R between the peak of the function with longer duration and the peak of the function with shorter duration. (Positive peaks for "positive" case, negative peaks for "negative" case.) There is only a very small reduction in NEP for R close to 1, which may actually be due to the method of calculation (i.e., to the assumption that the amplitude factors F_{R_1} and F_{R_2} are perfectly dependent).

5. CONCLUSIONS

A set of five pairs of response histories, selected from 291 cases as the most critical tests of Criterion 2, have been used as a basis to study the sensitivity of the NEP value of the SRSS value. It is found to be insensitive to the shape of the distribution of the random scale factor, insensitive to the (median) <u>relative</u> amplitude of the peaks of the two histories, and insensitive to relative stretching (and hence relative durations) of the two histories. The NEP is sensitive to the dispersion factor δ, at least for values of δ between 1.0 (i.e., TPO) and about 1.15. These are unrealistically low values, but in any case they are excluded by the (modified) preamble of the Newmark-Kennedy criteria.

In absolute terms, for cases that satisfy the criteria, the NEP's of the SRSS values are estimated to lie above about 85% for all but a very few cases studied. This conclusion is demonstrated in Fig. 1. The right-hand scale has been adjusted to reflect the typical conservative bias (a factor of 1.05) observed to exist in the analytical procedure adopted for these studies. To meet the criteria, δ must exceed 1.15. At this δ value (by interpolation in Fig. 1) only three of the cases in Table 2, for example, fall below 80%, the lowest being about 75%. The results in Table 2 are typical for all th... (most critical) five cases.

A second, methodological conclusion is that an analytical theoretical procedure, although approximate, gives satisfactorily accurate results for such studies. The computation time is believed to be much shorter than that associated with simulation. The approximate NEP results are found by comparison to simulation to be typically 1% to 5% conservative in the range of interest depending upon which of two analytical models is used.

REFERENCES

1. Kennedy, R. P. and Tong, W. H., "Study to Demonstrate that Approximately the SRSS Combined Response has Greater than an 84-Percent Non-Exceedance Probability when the Newmark-Kennedy Acceptance Criteria are Satisfied," EDAC, Irvine, Calif., June, 1979, (prepared for General Electric Co., San Jose, Calif.; Also published as "Supplement 3 Additional Demonstration of Statistical Basis for the SRSS Method" GE Report NEDO-24010-03, August, 1979, and Submitted to NRC, November 6, 1979).

2. Singh, A. K., Tagart, S. W., and Subramanian, C. V., "Technical Basis for the Use of the Square-Root-of-the-Sum-of-Squares (SRSS) Method for Combining Dynamic Loads for Mark II Plants," GE Report NEDO-24010-2, December, 1978.

3. Kennedy, R. P., Campbell, R. D., Merz, K. L., and Narver, R. B., "Combining Multiple Dynamic Responses by the Square-Root-of-the-Squares Method", EDAC 134-240.7R, Engineering Decision Analysis Company, Irvine, Calif., May 1978.

4. Larrabee, R. D., "Approximate Stochastic Analysis of Combined Loading," Supervised by C. A. Cornell, MIT CE R78-28, MIT Dept. of Civil Engr., Cambridge, Mass., September, 1978.

5. Madsen, H., Kilcup, R., and Cornell, C. A., "Mean Upcrossing Rate for Sums of Pulse-Type Stochastic Load Processes," Proceedings of ASCE Specialty Conference on Probabilistic Mechanics and Structural Reliability, Tucson, Arizona, Published by ASCE, New York, January, 1979.

CASE NO.	DISPERSION FACTOR δ	MAX (POS) OR MIN (NEG)	LOGNORMAL SIM. NEP (SRSS)	LOGNORMAL ANALYTICAL NEP(SRSS)	UNIFORM NEP(SRSS)	GAUSSIAN NEP(SRSS)	EXPONENTIAL NEP(SRSS)	BIMODAL (MIXED) UNIFORM NEP(SRSS)
1	1.0	NEG	0.48	0.51 (0.51)				
	TPO	POS	0.40	0.38 (0.38)				
	1.1	NEG	0.77	0.77				
		POS	0.74	0.71				
	1.2	NEG	0.81	0.82				
		POS	0.81	0.76 (0.80)	0.79 (0.79)	0.76 (0.80)	0.79 (0.81)	0.73 (0.79)
	1.3	NEG	0.84	0.83 (0.81)				
		POS	0.82	0.76 (0.80)				
2	1.0	NEG	0.61	0.54 (0.54)				
	TPO	POS	0.66	0.5[illegible] (0.59)				
	1.1	NEG	0.78	0.77				
		POS	0.80	0.80				
	1.2	NEG	0.82	0.81				
		POS	0.85	0.83	0.85			
	1.3	NEG	0.83	0.83				
		POS	0.85	0.84				
3	1.0	NEG	0.74	0.66 (0.66)				
	TPO	POS	0.54	0.53 (0.53)				
	1.1	NEG	0.86	0.83				
		POS	0.79	0.75				
	1.2	NEG	0.86	0.85				
		POS	0.82	0.81	0.83			
	1.3	NEG	0.87	0.86				
		POS	0.87	0.83 (0.83)				
4	1.0	NEG	0.90	0.89 (0.89)				
	TPO	POS	0.70	0.56 (0.56)				
	1.1	NEG	0.94	0.93				
		POS	0.82	0.71 (0.71)				
	1.2	NEG	0.93	0.93				
		POS	0.86	0.79 (0.81)	0.81 (0.83)	0.80 (0.82)	0.82 (0.82)	0.79 (0.83)
	1.3	NEG	0.92	0.92				
		POS	0.89	0.82 (0.85)				
5	1.0	NEG	0.94	0.90 (0.90)				
	TPO	POS	0.74	0.65 (0.65)				
	1.1	NEG	0.96	0.94				
		POS	0.86	0.76 (0.77)				
	1.2	NEG	0.93	0.94				
		POS	0.88	0.83 (0.84)	0.84			
	1.3	NEG	0.93	0.93				
		POS	0.88	0.85 (0.87)				

TABLE 1 Sensitivity of NEP of the SRSS Value to Shape of the Amplitude Distribution. Amplitudes Assumed to be Perfectly Dependent. (Values in Parentheses are for Independent Amplitudes). No Prior Deterministic Scaling of the Response Functions. Cases 4 and 5 give Similar Results.

CASE NO.	DISPERSION FACTOR δ	MAX (POS) OR MIN (NEG)	R, RATIO OF SCALING FACTORS				TF, SCALING FACTOR ON DURATION OF SHORTER LOAD (1.0 IS ORIGINAL CASE, REF. 1)			
			1/3	2/3	1	3	0.5	0.25	4.0	10.0
			NEP(SRSS)	NEP(SRSS)	NEP(SRSS)	NEP(SRSS)	NEP(SRSS)	NEP(SRSS)	NEP(SRSS)	NEP(SRSS)
1	1.0 TPO	NEG	0.53	0.55	0.64	0.85	0.54			
		POS	0.40	0.51	0.60	0.89	0.42			
	1.1	NEG	0.78	0.75	0.79	0.91	0.79			
		POS	0.73	0.72	0.76	0.91	0.73			
	1.2	NEG	0.82	0.81	0.84	0.90	0.83	0.83	0.81	0.80
		POS	0.79	0.79	0.82	0.90	0.79	0.79	0.78	0.77
	1.3	NEG	0.83	0.83	0.85	0.90	0.83			
		POS	0.81	0.81	0.84	0.89	0.81			
2	1.0 TPO	NEG					0.56			
		POS	0.61	0.67	0.72	0.90	0.61			
	1.1	NEG					0.79			
		POS	0.82	0.81	0.84	0.92	0.82			
	1.2	NEG					0.83	0.83	0.81	
		POS	0.84	0.85	0.87	0.91	0.84	0.85	0.83	
	1.3	NEG					0.84			
		POS	0.85	0.86	0.88	0.90	0.85			
4	1.0 TPO	NEG	0.81	0.86	0.87	0.92	0.90			
		POS	0.58	0.55	0.57	0.74	0.58			
	1.1	NEG	0.89	0.91	0.92	0.93	0.94			
		POS	0.78	0.72	0.72	0.86	0.73			0.94
	1.2	NEG	0.90				0.94	0.94	0.93	
		POS	0.83	0.80	0.80	0.87	0.81	0.81	0.79	
	1.3	NEG					0.93			
		POS	0.84	0.82	0.83	0.87	0.83			

TABLE 2 Sensitivity of NEP of SRSS Value to Relative Scaling of the (Deterministic) Amplitudes and Time Scaling (Stretching) of Shorter Load and its Duration. Maxima and Minima (Postitive and Negative Preak Combined Responses) are Shown. Lognormal Distribution on (Perfectly Dependent) Amplitude Factors. Cases 3 and 5 give Similar Results.

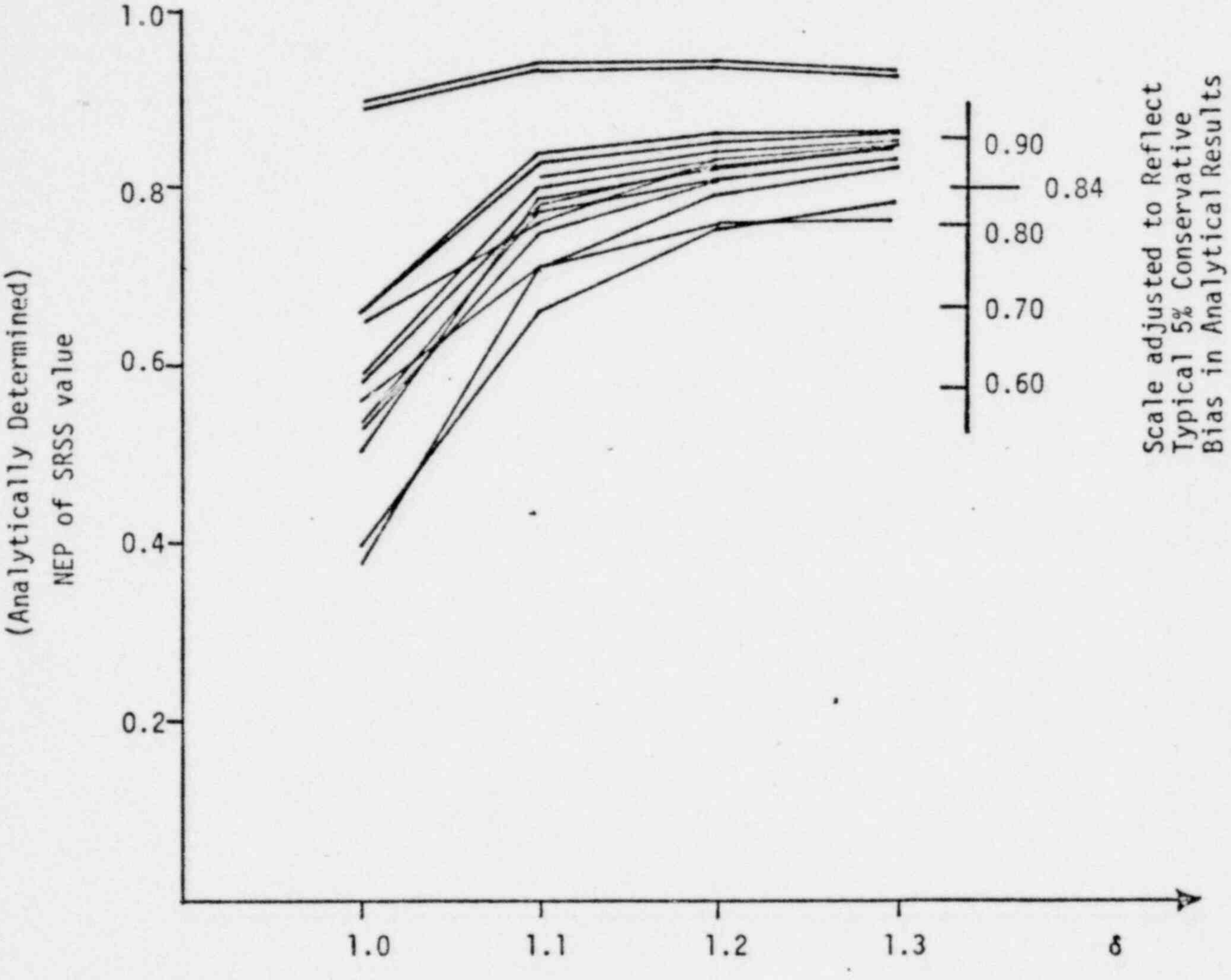


FIG. 1 - Dependence of (Analytically Determined) NEP of the SRSS value on δ. Same cases as in Table 2. Actual NEP values are about 5% higher (read right-hand scale).

APPENDIX A: METHODOLOGY

Let $X_1(\tau)$, $X_2(\tau)$ be stationary, independent random processes (normalized responses) and A_1, A_2 be independent positive random variables (response-amplification factors).* We are interested in the extremal properties of the stationary (combined response) process $Z(\tau)$; defined

$$Z(\tau) = A_1X_1(\tau) + A_2X_2(\tau) = Y_1(\tau) + Y_2(\tau) \tag{A.1}$$

in which $Y_i(\tau) = A_iX_i(\tau)$ is the i^{th} response process. Conditionally on $A_1 = a_1$, the upcrossing rate of $Y_1(\tau)$ is given by

$$\nu_{Y_1|A_1}(y, a_1) = \nu_{X_1}(\frac{y}{a_1}) \tag{A.2}$$

and the first-order (marginal) probability density function of $Y_1(\tau)$ is

$$f_{Y_1|A_1}(y, a_1) = \frac{1}{a_1} f_{X_1}(\frac{y}{a_1}) \tag{A.3}$$

similarly for $Y_2(\tau)$. The upcrossing rate of the sum process $Z(\tau)$ conditional on given values of A_1 and A_2 can be approximated by the following conservative formula (Reference 4 and 5):

$$\nu_{Z|A_1,A_2}(z,a_1,a_2) = \int_{-\infty}^{\infty} \nu_{X_1}(\frac{x}{a_1})\frac{1}{a_2} f_{X_2}(\frac{z-x}{a_2})dx + \int_{-\infty}^{\infty} \nu_{X_2}(\frac{x}{a_2})\frac{1}{a_1}f_{X_1} -(\frac{z-x}{a_1})dx \tag{A.4}$$

---

* A simpler notation is used here compared to main body.

This is a basic relationship in the approximate calculation of the probability distribution of $Z_{max} = \max_{0<\tau<T} Z(\tau)$ by the analytical method proposed here. Before completing the method, we observe that in order to apply equation 4 to the present problem, it is necessary to model the (finite length) response functions as stationary processes. This operation can be performed as follows:

Let T_1 be the duration of event $X_1(\tau)$, and T_2 be the duration of event $X_2(\tau)$. Define periodic functions $X_{1p}(\tau)$ and $X_{2p}(\tau)$, both with period $T = T_1 + T_2$, as follows: $X_{jp}(\tau)$ $(j = 1,2)$ is obtained by shifting $X_j(\tau)$ to start at time $\tau = 0$ and by then repeating the functions at intervals of length T. See Figure A.1.

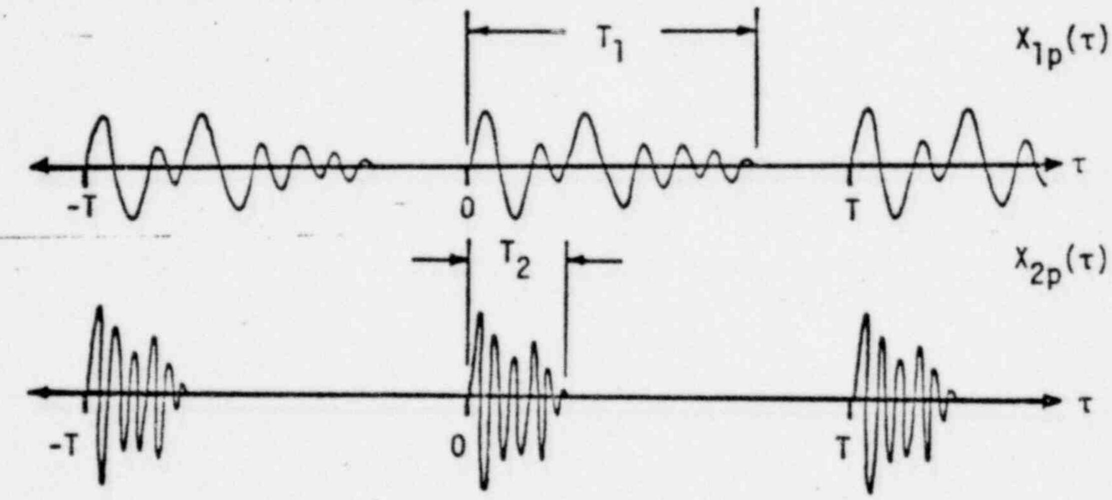


FIGURE A.1

Next, $X_{1p}(\tau)$ and $X_{2p}(\tau)$ are shifted randomly by amounts t_{01} and t_{02}, respectively. t_{0_1} and t_{0_2} are independent variables with identical uniform distribution in [0, T]. (Note that the phase $|t_{0_1} - t_{0_2}|$ is also uniformly sitributed on [0, T].) This operation produces random processes $X_{1p}(\tau)$ and $X_{2p}(\tau)$ which are stationary, periodic with period T, and independent (Fig. A.2). One replaces $X_i(t)$ in Eq. A.1 by $X_{ip}(\tau)$, and uses the results which follow to calculate the distribution of the maximum of $Z(\tau)$

over an interval of duration T.

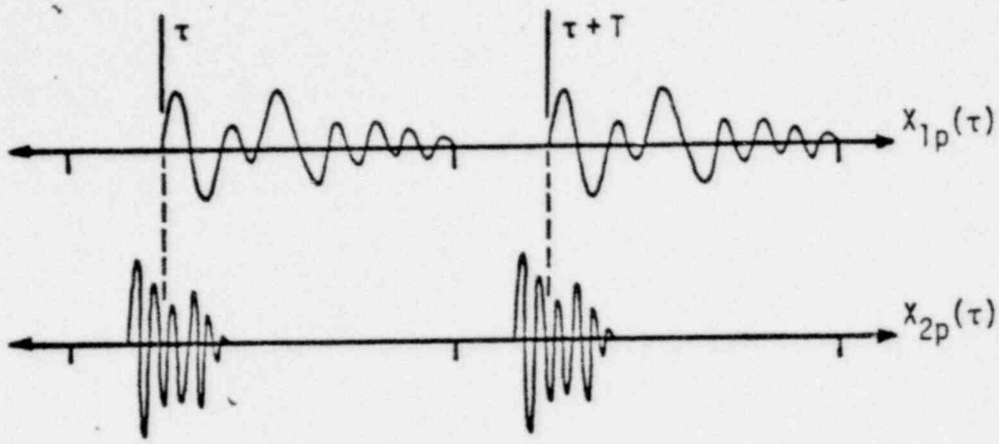


FIGURE A.2

If the upcrossing rate functions and first order probability density functions are obtained from the original time functions $X_j(\tau)$ $(j = 1,2)$, defined on the intervals $[0, T_j]$ $(j = 1,2)$, respectively, then the first two functions must be modified to reflect presence of intervals of zero amplitude in the stochastic model. With $\nu_{j,T}(x)$ and $f_{j,T}(x)$ the modified functions, and $\nu_{j,T_j}(x)$ and $f_{j,T_j}(x)$ the original functions, it is easy to show that

$$\nu_{j,T}(x) = \frac{T_j}{T} \nu_{j,T_j}(x) \qquad \text{(A.5)}$$

and

$$f_{j,T}(x) = \frac{T_j}{T} f_{j,T_j}(x) + (1 - \frac{T_j}{T}) \delta(x) \qquad \text{(A.6)}$$

where $\delta(x)$ is the Dirac delta function. These modified functions can be used in equation A.4 for $\nu_{Xj}(x)$ and $f_{Xj}(x)$.

It was originally thought that the best solution strategy was to calculate the upcrossing rate of the sum $\nu_Z(z)$, as

$$\nu_Z(z) = \int_0^\infty \int_0^\infty \nu_{Z|A_1,A_2}(z,a_1,a_2) f_{A_1}(a_1) f_{A_2}(a_2) da_1\, da_2 \qquad (A.7)$$

and then to calculate $F_Z(z)$ by the approximation

$$F_Z(z) \simeq \exp\{-\nu_Z(z) \cdot T\} \qquad (A.8)$$

However, when compared to simulation this approach proved to give unsatisfactory results in the middle portion of the distribution of Z_{max}. A different approach, which produced much better results, consists of calculating the conditional distribution of $Z_{max} = \max_{0 \le \tau \le T} Z(\tau)$,

$$F_{Z|A_1,A_2}(z,a_1,a_2) \simeq \exp\{-\nu_{Z|A_1,A_2}(z,a_1,a_2) \cdot T\} \qquad (A.9)$$

and then removing conditionality:

$$F_Z(z) = \int_0^\infty \int_0^\infty F_{Z|A_1,A_2}(z,a_1,a_2) f_{A_1}(a_1) f_{A_2}(a_2) da_1\, da_2 \qquad (A.10)$$

Results from Equations A.4, A.9, and A.10 are shown in Appendix C, where they are compared with simulation analyses.

APPENDIX B: SIMPLIFIED METHOD

The method of Appendix A, which involves a double integration in Equation* A.10, can be expensive to implement if a large number of cases are to be studied. A simplification, similar to the simplified method proposed by Kennedy and Tong (Ref. 1), is to assume that the two responses have identical random amplitude factor, $A = A_1 = A_2$. In this case Equation A.4 becomes

$$\nu_{Z|A}(z,a) = \int_{-\infty}^{\infty} \nu_{X_1}(\tfrac{x}{a}) \, \frac{1}{a} \, f_{X_2}(\tfrac{z-x}{a}) dx + \int_{-\infty}^{\infty} \nu_{X_2}(\tfrac{x}{a}) \, \frac{1}{a} \, f_{X_1}(\tfrac{z-x}{a}) \, dx \qquad (B.1)$$

and Equations A.9 and A.10 combine to give

$$F_Z(z) = \int_0^{\infty} \exp \{-\nu_{Z|A}(z,a) \cdot T\} \, f_A(a) \, da \qquad (B.2)$$

The two methods will be distinguished by calling the method in Appendix A the <u>independent amplitudes approach</u>, and the above method the <u>dependent amplitudes approach</u>.

Comparisons in Appendix C indicate that there is reasonable agreement between methods and between each method and simulation when the amplitude factors are identically distributed.** It can be observed that

---

* The original method, involving Eq. A.7, <u>is</u> accurate in the tail (NEP very close to 1.0). For applications in which only these NEP's are required that method may be used. Eq. A-7 (after substitution from Eq. A.4) reduces to single integrations over the A_i distributions.

** In the case when the random amplitude factors have different distribution, the dependent amplitudes method is generalized by assuming $A_i = c_i + d_i A$ with c_i and d_i given constants.

the dependent amplitudes method tends to be conservative with respect to the independent amplitudes method in the upper portion of the distribution of Z_{max}.

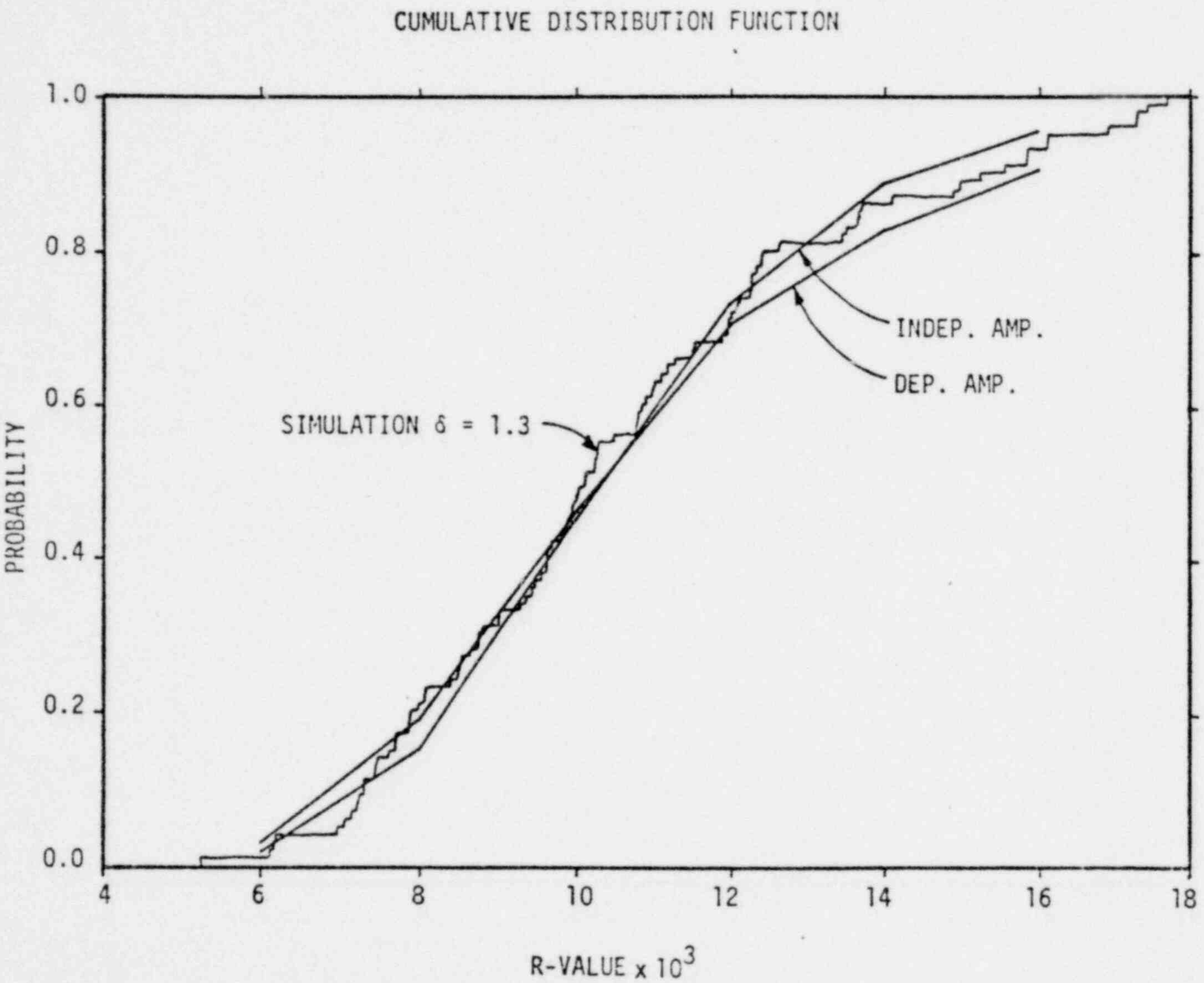
Since the SRSS values tend to occur in the upper half of the distribution, it is probably conservative to use the dependent amplitudes method to estimate the probability of non-exceedance of the SRSS value. Neither method seems to have a clear advantage over the other in matching the simulation results.

APPENDIX C COMPARISON OF RESULTS OF TWO ANALYTICAL METHODS

Results for the independent amplitudes method and the dependent amplitudes method are compared to simulation results in the following figures (see Appendices A and B for a description of the analytical methods). Figures C.1 through C.8 illustrate the case of lognormal random amplitude distributions and a dispersion factor of $\delta = 1.3$. Figures C.9 through C.11 illustrate the case of uniform random amplitude distributions and a dispersion factor of $\delta = 1.2$. Of those available (1.0, 1.1, 1.2, and 1.3) these are the δ-values admitted by the Newmark-Kennedy criteria.

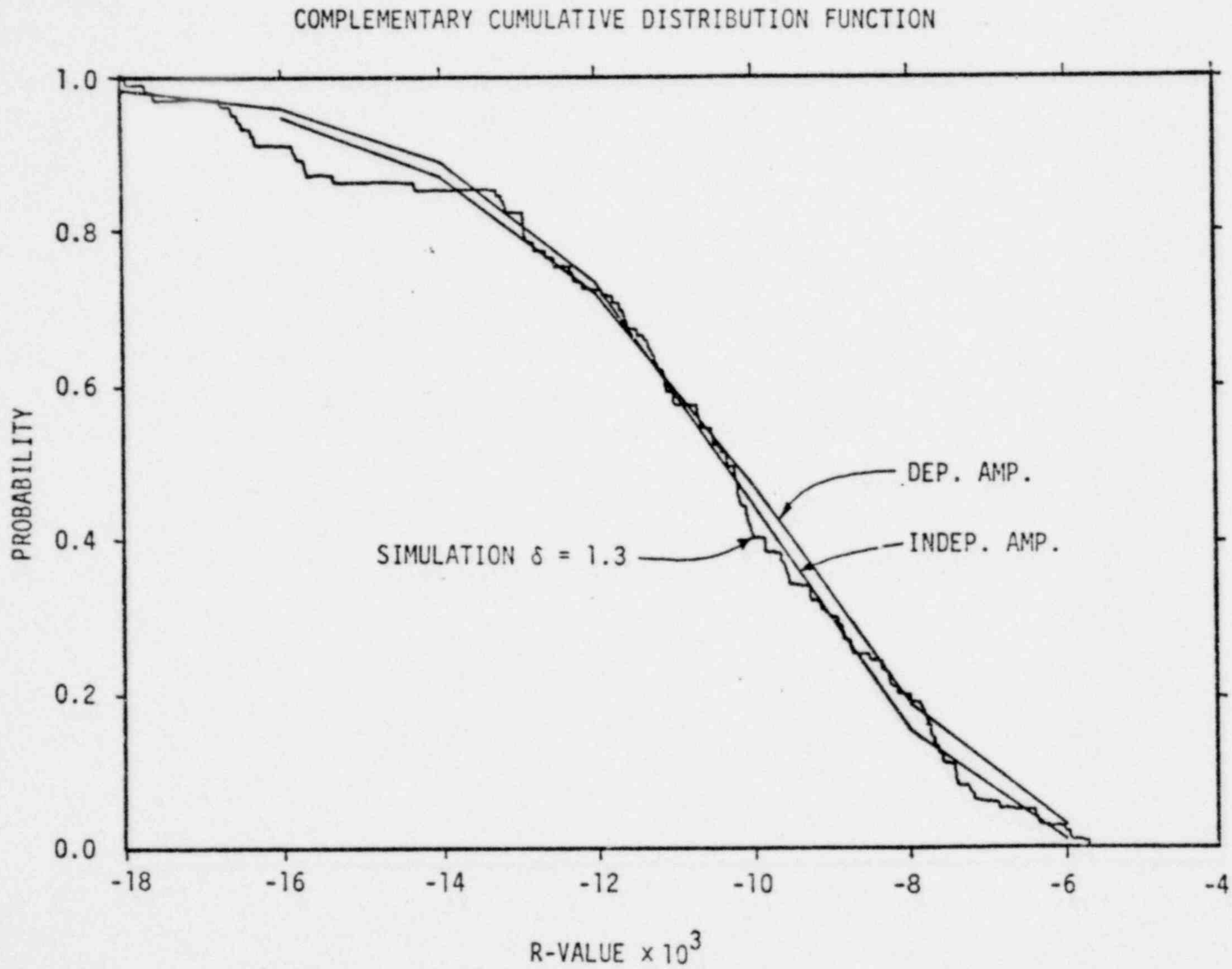
The independent amplitudes method and the dependent amplitudes method give essentially the same result around the median value of the distribution, but the dependent amplitudes method gives a larger variance and therefore thicker tails on the distribution. The dependent amplitudes method therefore tends to be conservative in the upper tail of the distribution, which is the region of greatest interest. Because of the approximation introduced by the assumption of Poisson crossings, the lower tails of the distributions tend to show the greatest errors, and it must be regarded as coincidental that the dependent amplitudes method sometimes gives better results in this region.

For the cases studied, the two analytical methods give similar results, and the dependent amplitudes method is generally conservative in the region of interest. Therefore the dependent amplitudes method was used in subsequent comparisons to simulations (Appendix D).



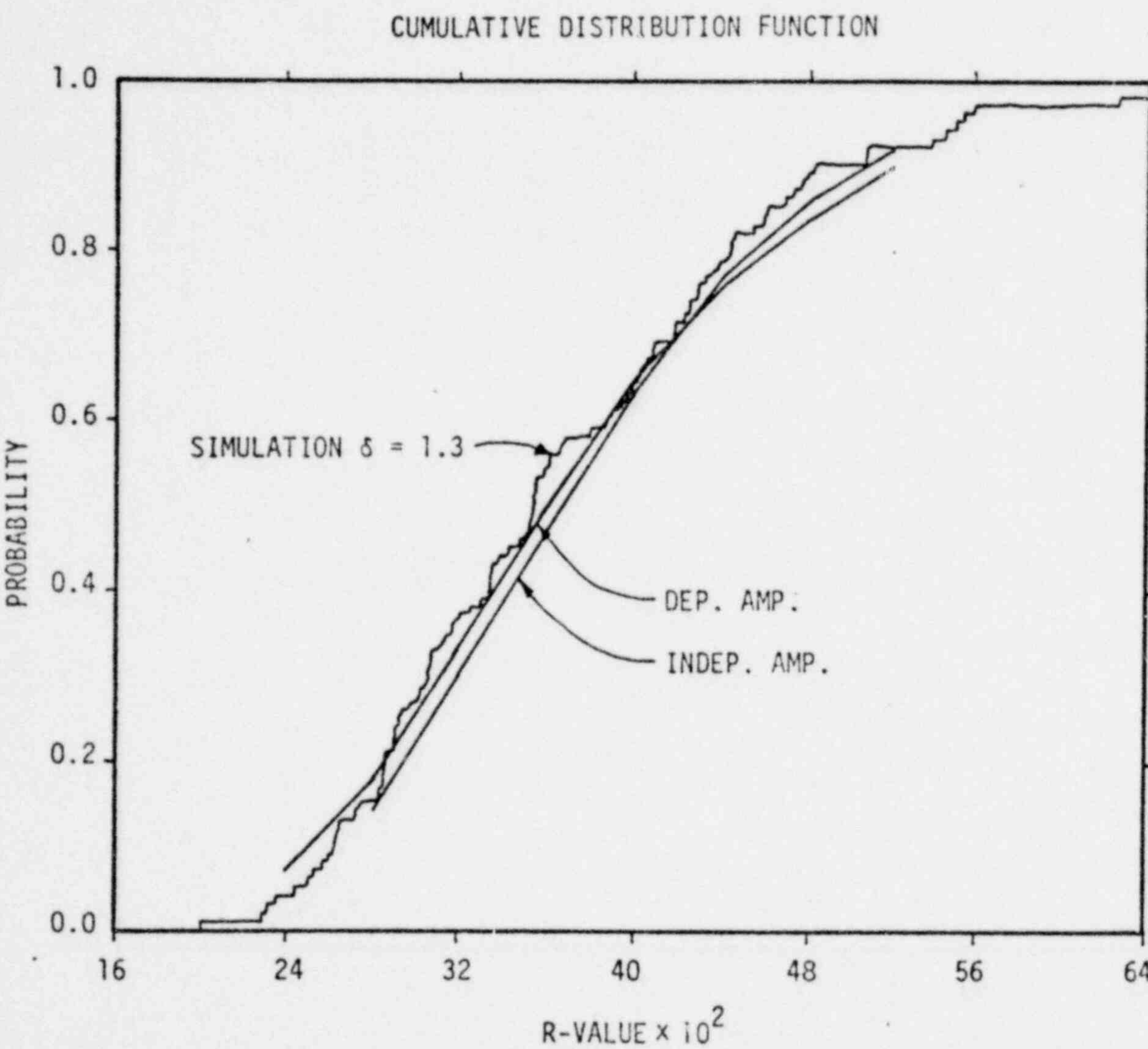
NOTE: SIMULATION TRACED FROM EDAC FIGURE 2-28. CASE 1: MS - OBE + SRV, M_a LOGNORMAL AMPLITUDE PDF-ALL DISPERSIONS EQUAL

FIGURE C.1 COMPARISON ANALYTICAL METHODS TO SIMULATION; CASE 1



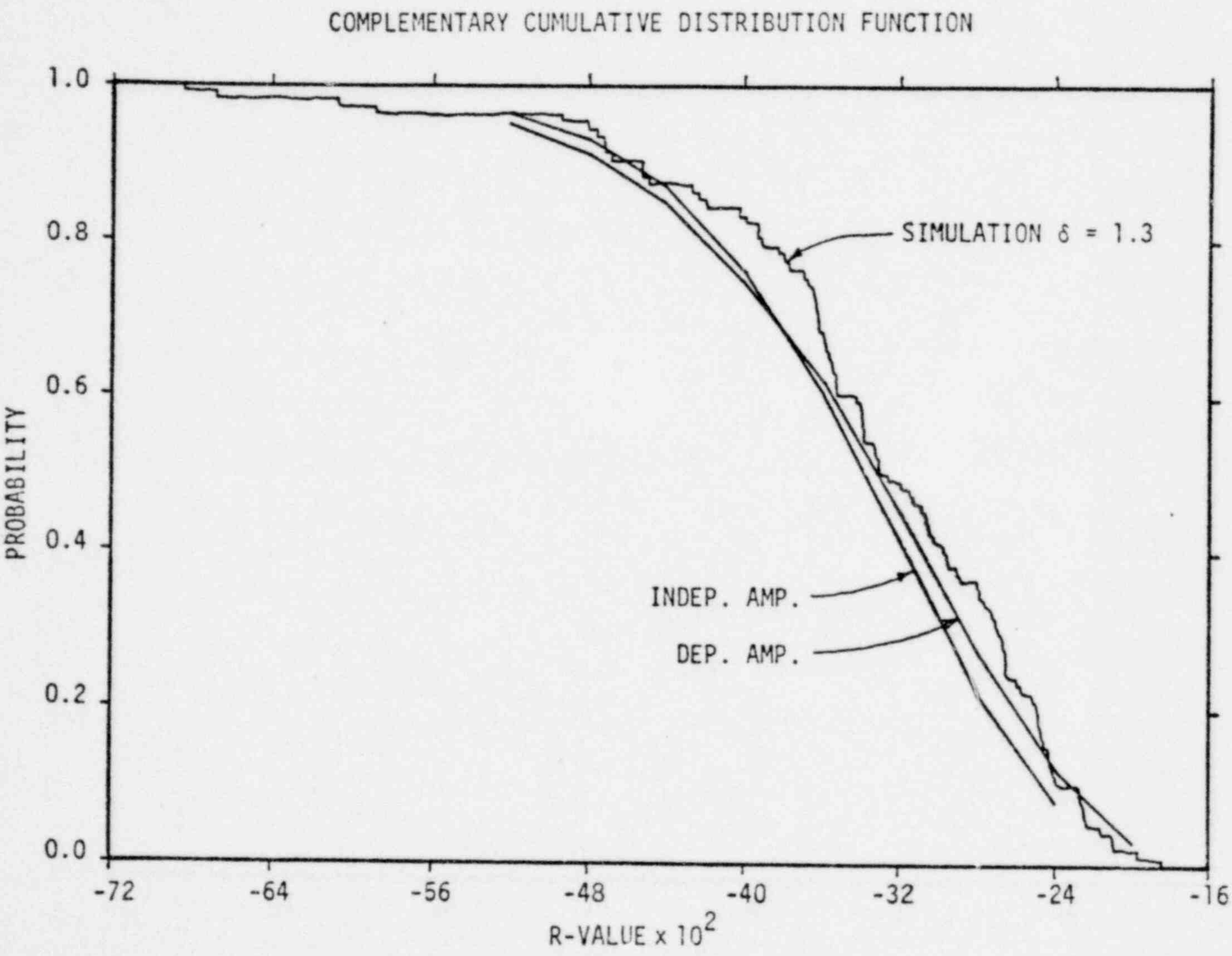
NOTE: SIMULATION TRACED FROM EDAC FIGURE 2-29. CASE 1: MS OBE + SRV M_a, LOGNORMAL AMPLITUDE PDF-ALL DISPERSIONS EQUAL

FIGURE C.2 COMPARISON ANALYTICAL METHODS TO SIMULATION; CASE 1



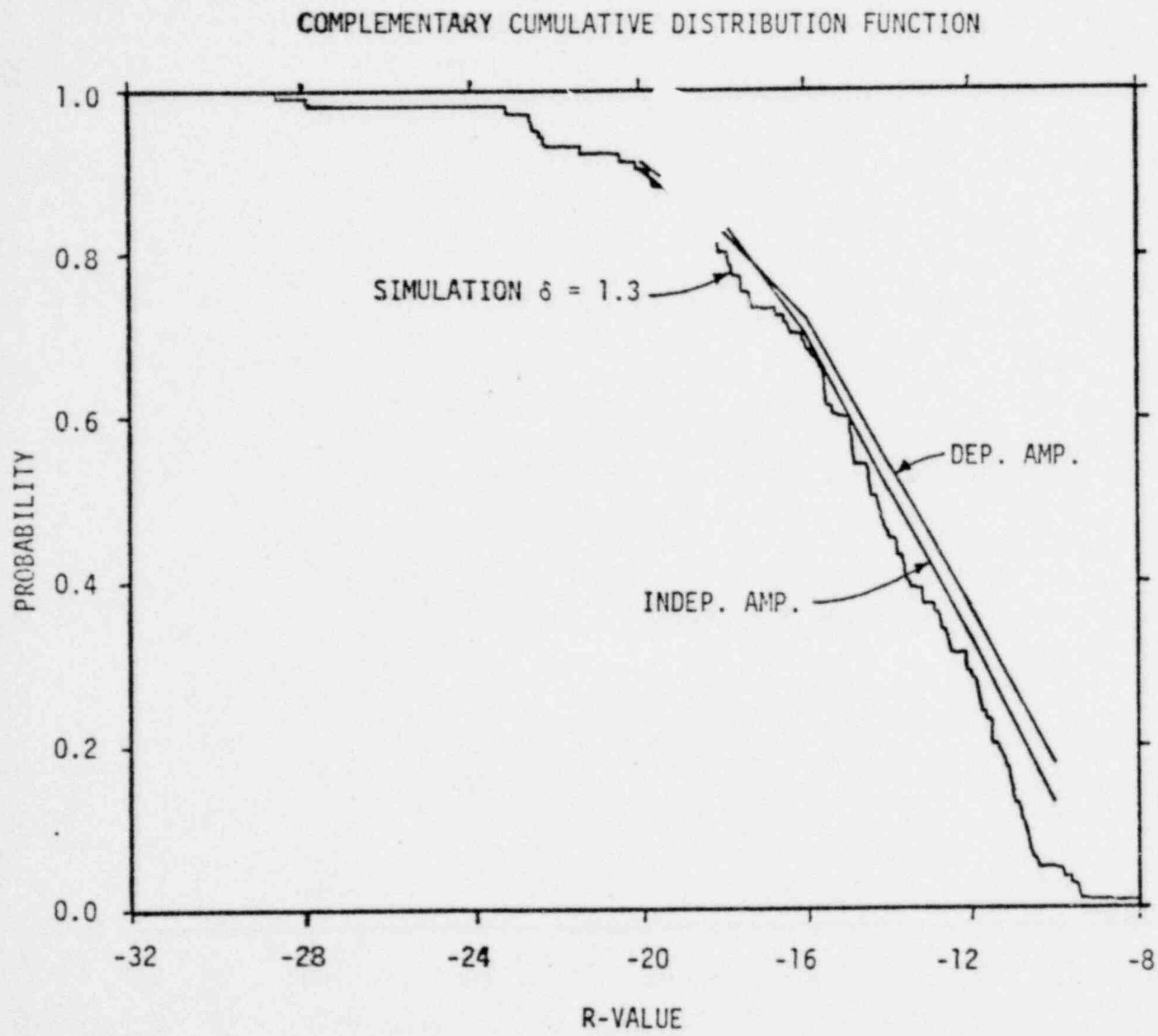
NOTE: SIMULATION TRACED FROM EDAC FIGURE 2-32. CASE 3: RHR-WETWALL OBE + SRV M_a
LOGNORMAL AMPLITUDE PDF-ALL DISPERSIONS EQUAL

FIGURE C.3 COMPARISON ANALYTICAL METHODS TO SIMULATION; CASE 3



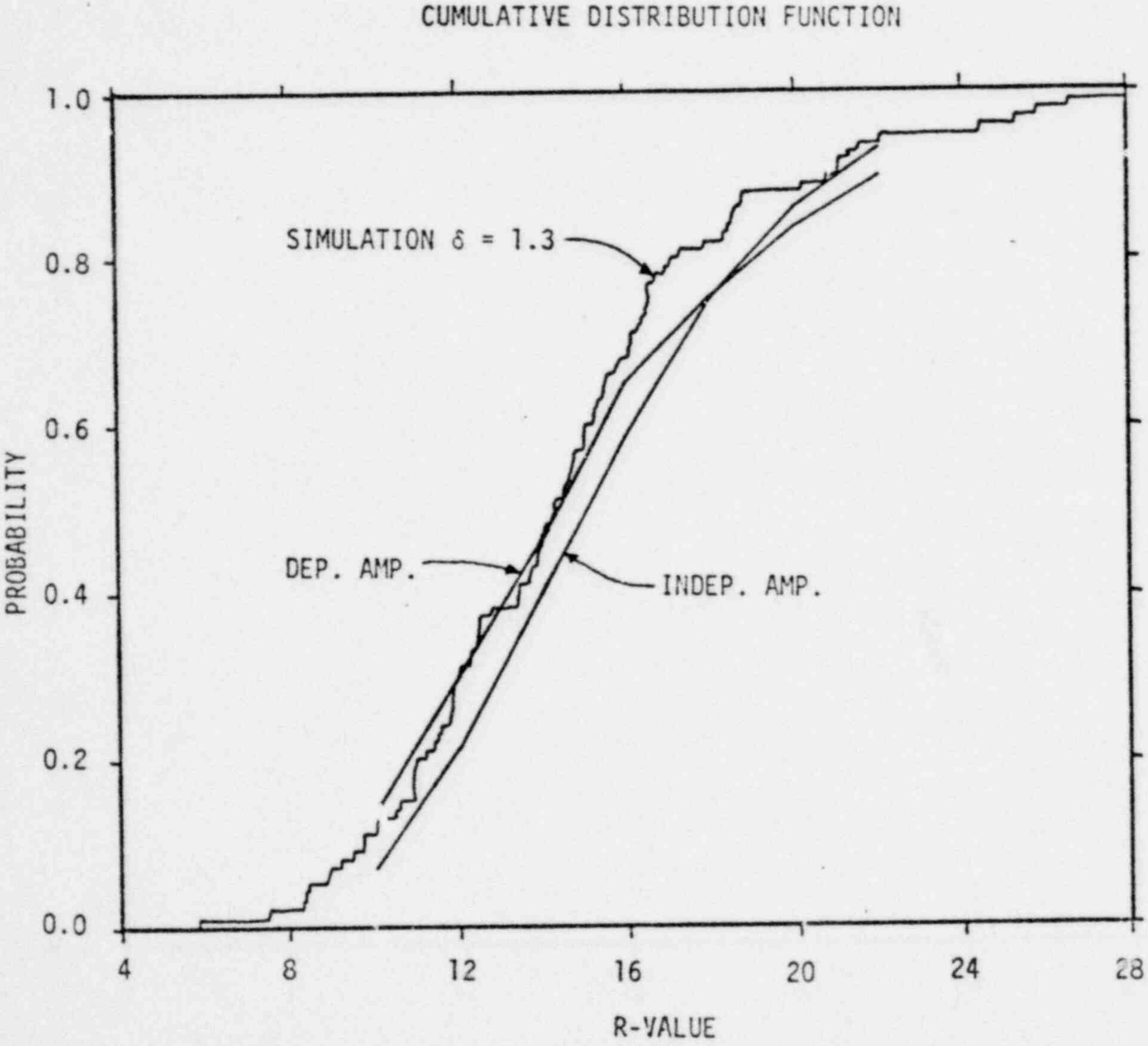
NOTE: SIMULATION TRACED FROM EDAC FIGURE 2-33. CASE 3: RHR-WETWALL OBE + SRV M_a
LOGNORMAL AMPLITUDE PDF-ALL DISPERSIONS EQUAL

FIGURE C.4 COMPARISON ANALYTICAL METHODS TO SIMULATION; CASE 3



NOTE: SIMULATION TRACED FROM EDAC FIGURE 2-35. CASE 4: ZIMMER OBE(NS) + SRV
LOGNORMAL AMPLITUDE PDF-ALL DISPERSIONS EQUAL

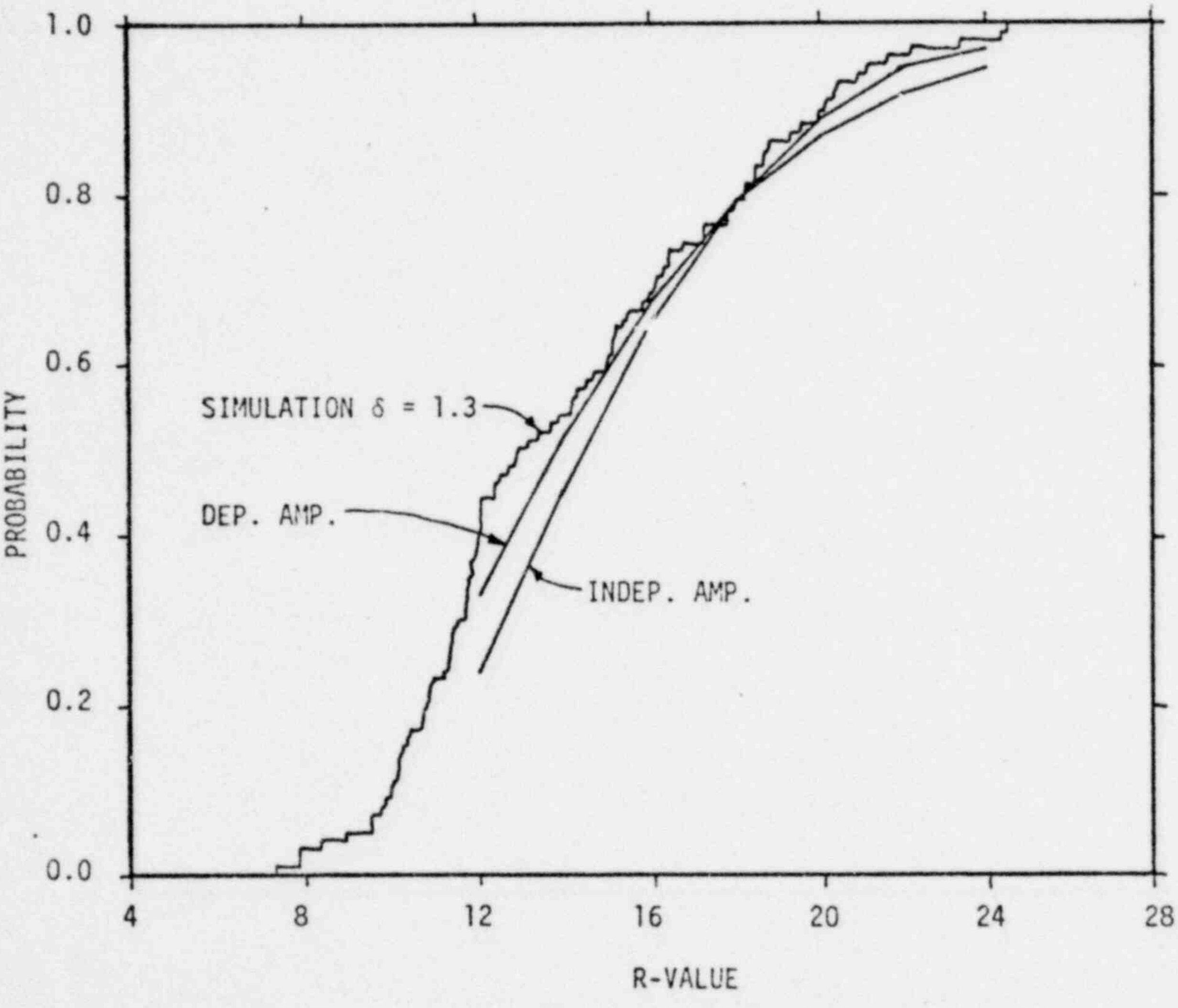
FIGURE C.5 COMPARISON ANALYTICAL METHODS TO SIMULATION; CASE 4



NOTE: SIMULATION TRACED FROM EDAC FIGURE 2-34. CASE 4: ZIMMER OBE(NS) + SRV.
LOGNORMAL AMPLITUDE PDF-ALL DISPERSIONS EQUAL

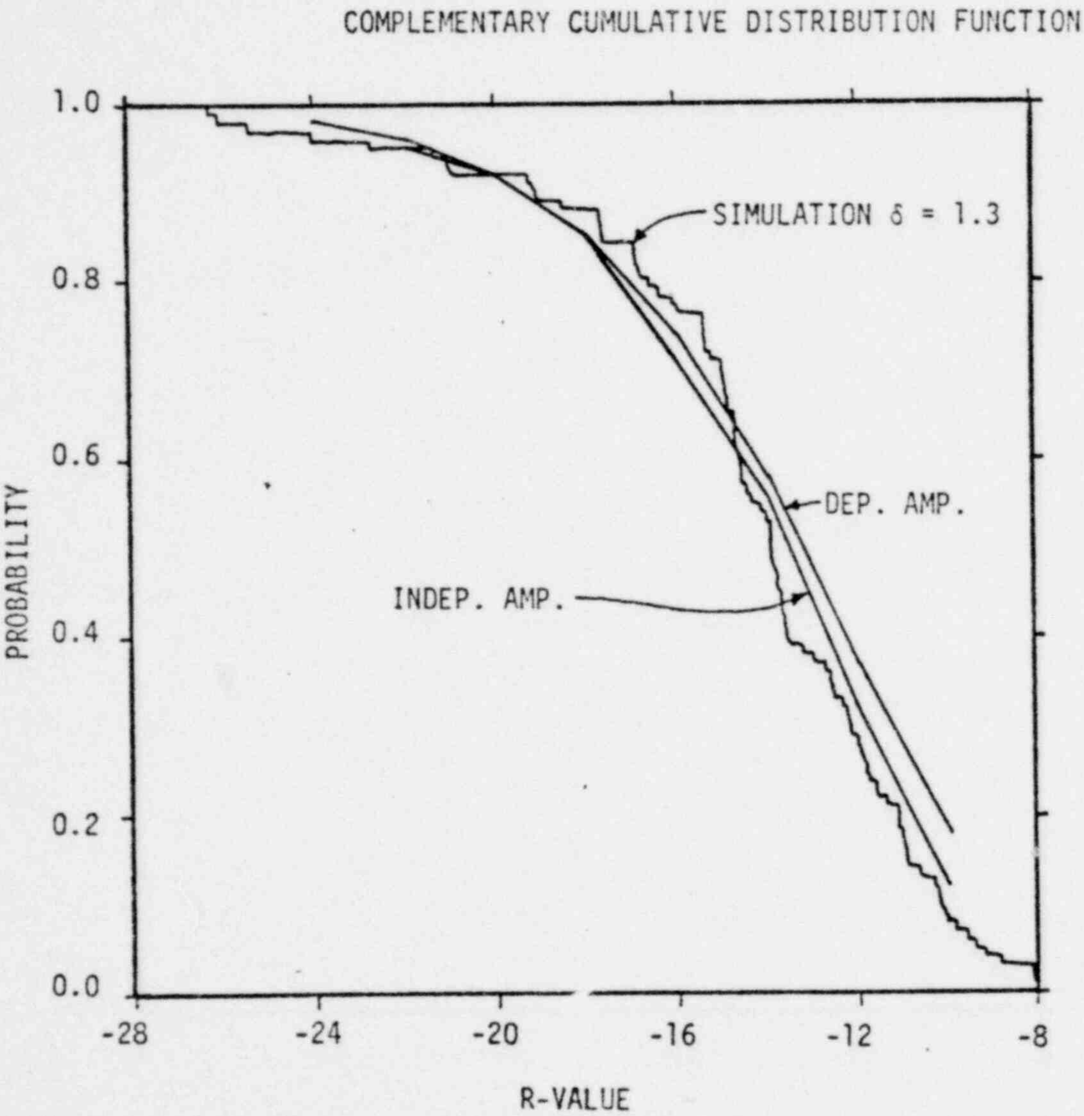
FIGURE C.6 COMPARISON ANALYTICAL METHODS TO SIMULATION; CASE 4

CUMULATIVE DISTRIBUTION FUNCTION



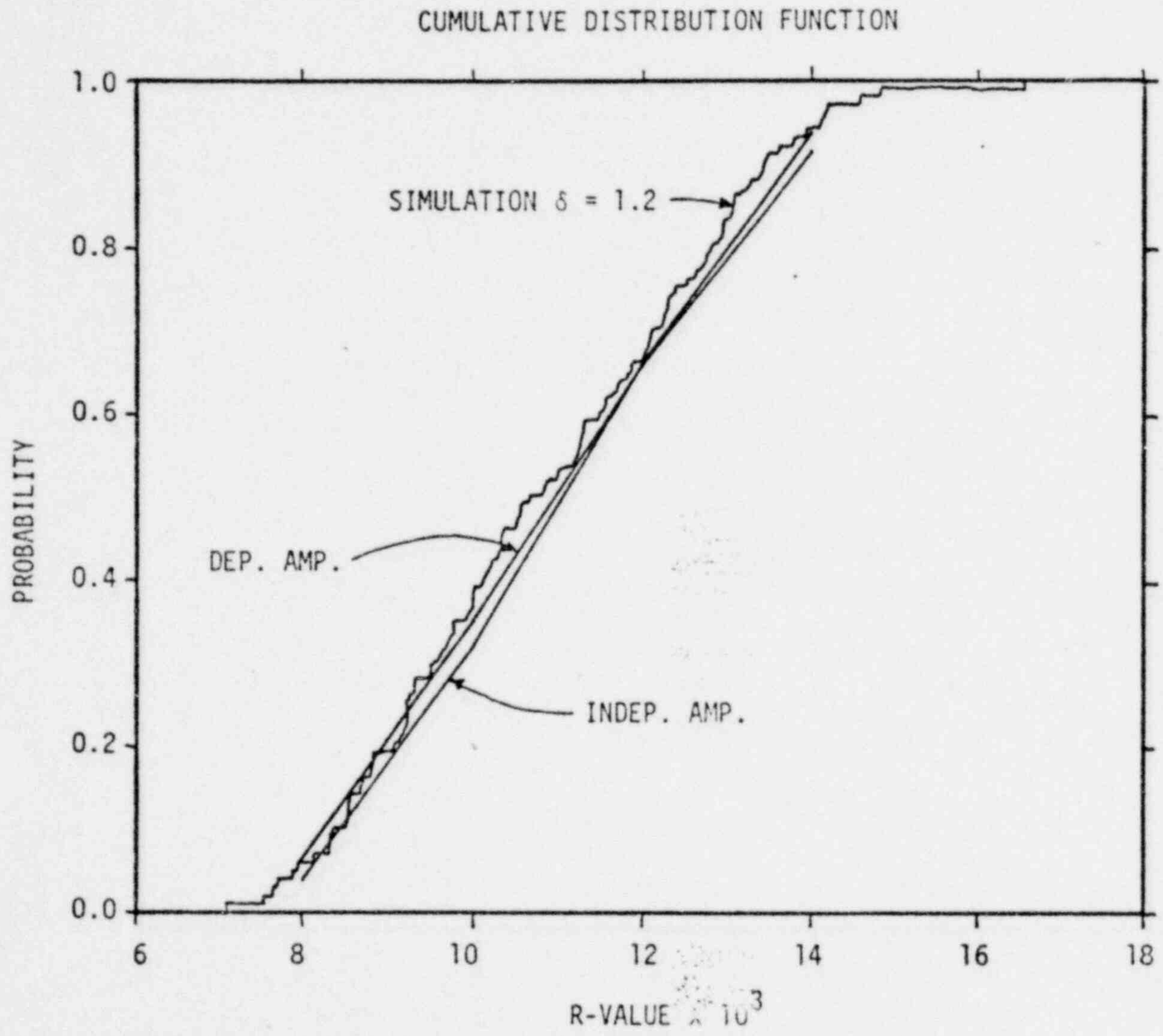
NOTE: SIMULATION TRACED FROM EDAC FIGURE 2-36. CASE 5: ZIMMER OBE(EW) + SRV
LOGNORMAL AMPLITUDE PDF-ALL DISPERSIONS EQUAL

FIGURE C.7 COMPARISON ANALYTICAL METHODS TO SIMULATION; CASE 5



NOTE: SIMULATION TRACED FROM EDAC FIGURE 2-37. CASE 5: ZIMMER OBE(EW) + SRV, LOGNORMAL AMPLITUDE PDF-ALL DISPERSIONS EQUAL

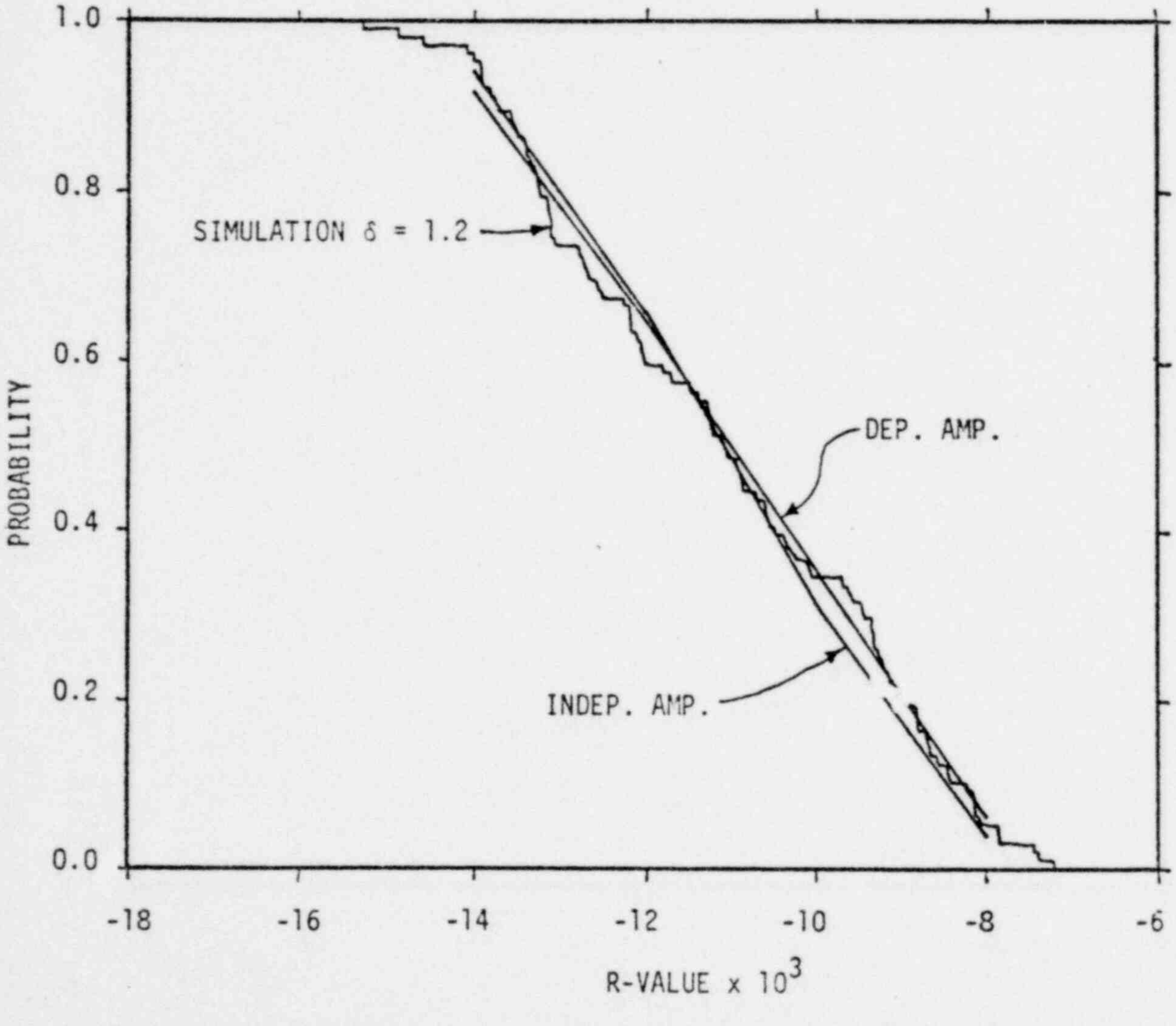
FIGURE C.8 COMPARISON ANALYTICAL METHODS TO SIMULATION; CASE 5



NOTE: SIMULATION TRACED FROM EDAC FIGURE 2-52. CASE 1: MS - OBE + SRV - M_a, UNIFORM AMPLITUDE PDF-ALL DISPERSIONS EQUAL

FIGURE C.9 COMPARISON ANALYTICAL METHODS TO SIMULATION; CASE 1

COMPLEMENTARY CUMULATIVE DISTRIBUTION FUNCTION

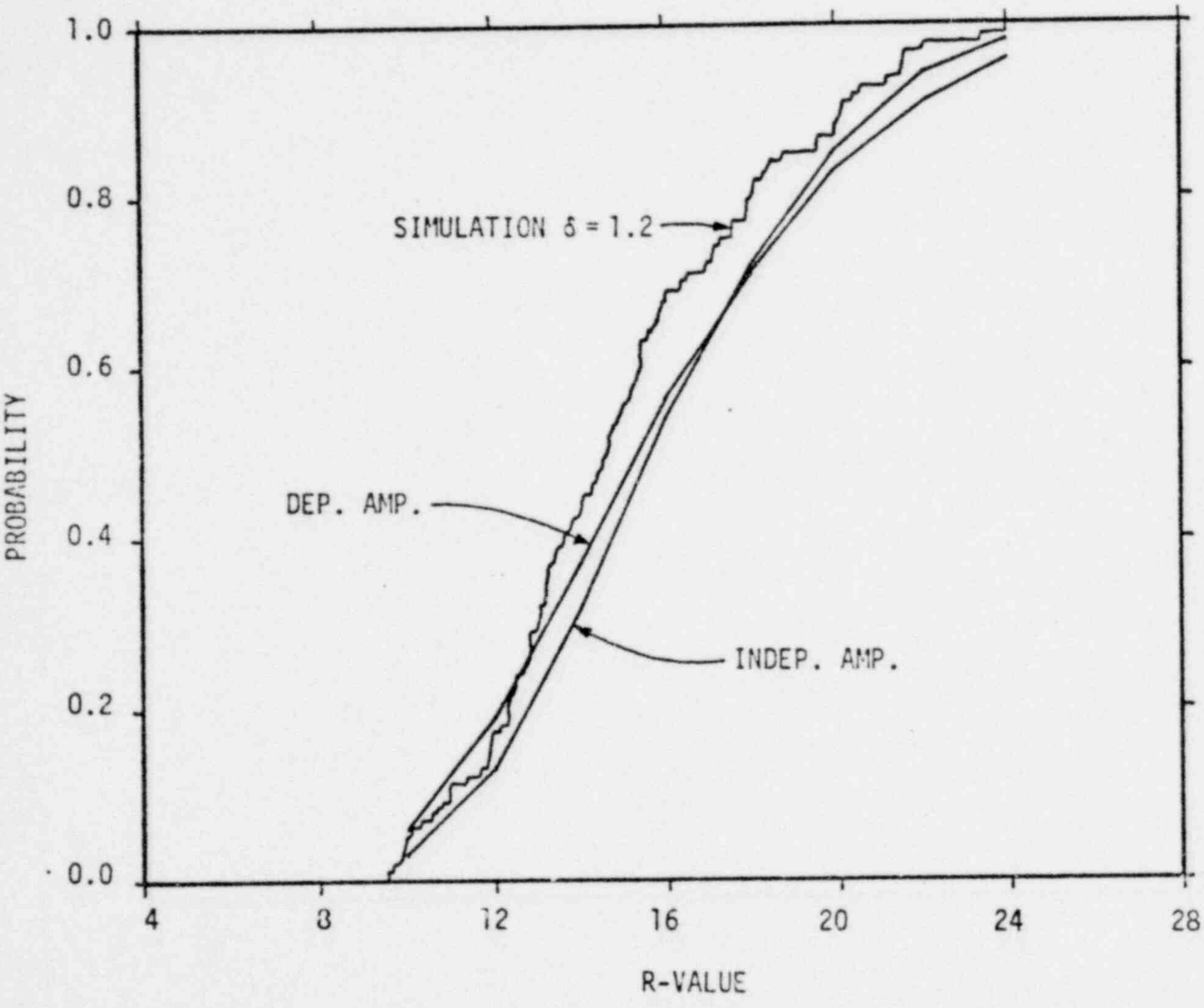


NOTE: SIMULATION TRACED FROM EDAC FIGURE 2-53. CASE 1: MS - OBE + SRV - M_a

UNIFORM AMPLITUDE PDF-ALL DISPERSIONS EQUAL

FIGURE C 10 COMPARISON ANALYTICAL METHODS TO SIMULATION; CASE 1

CUMULATIVE DISTRIBUTION FUNCTION



NOTE: SIMULATION TRACED FROM EDAC FIGURE 2-58. CASE 4: ZIMMER OBE(NS) + SRV UNIFORM AMPLITUDE PDF-ALL DISPERSIONS EQUAL

FIGURE C.11 COMPARISON ANALYTICAL METHODS TO SIMULATION; CASE 5

APPENDIX D: COMPARISON OF ANALYTICAL AND SIMULATION RESULTS

The dependent amplitudes analytical method (see Appendix B) is compared to simulation results in this appendix. Figures D.1 through D.10 show results for the five cases studied for a lognormal amplitude distribution. Figures D.11 through D.18 show results for four cases for a uniform amplitude distribution.

Figures D.1 and D.2 (Case 1) are typical of the first three cases studied. The analytical method compares very favorably to the simulation results. The lower tails of the distributions must be regarded as very approximate because of the approximation introduced by the assumption of Poisson crossings; it is the upper tail, however, which is of interest.

Figures D.7 through D.10 (Cases 4 and 5) are the least successful in matching the entire simulation results. This may be due in part to the unusual asymmetry in the SRV load (see Figure E.15) and to the near-vertical portion of the distribution of the time phase only (δ = 1.0) curves. This portion is caused by the high ratio of the durations (about 10 to 1) coupled with a long period of relatively low amplitudes in the longer response histories, (see Fig. E.13 and E.17), plus the presence of a single large peak in the longer of the combined responses (see Figures E.13 and E.17). Even so, the analytical method performs well in reproducing this behavior in Figures D.8 and D.10. In all these cases, of course, the analytical method gives a reasonable (and generally conservative) approximation for fractiles at and above the 0.84 level.

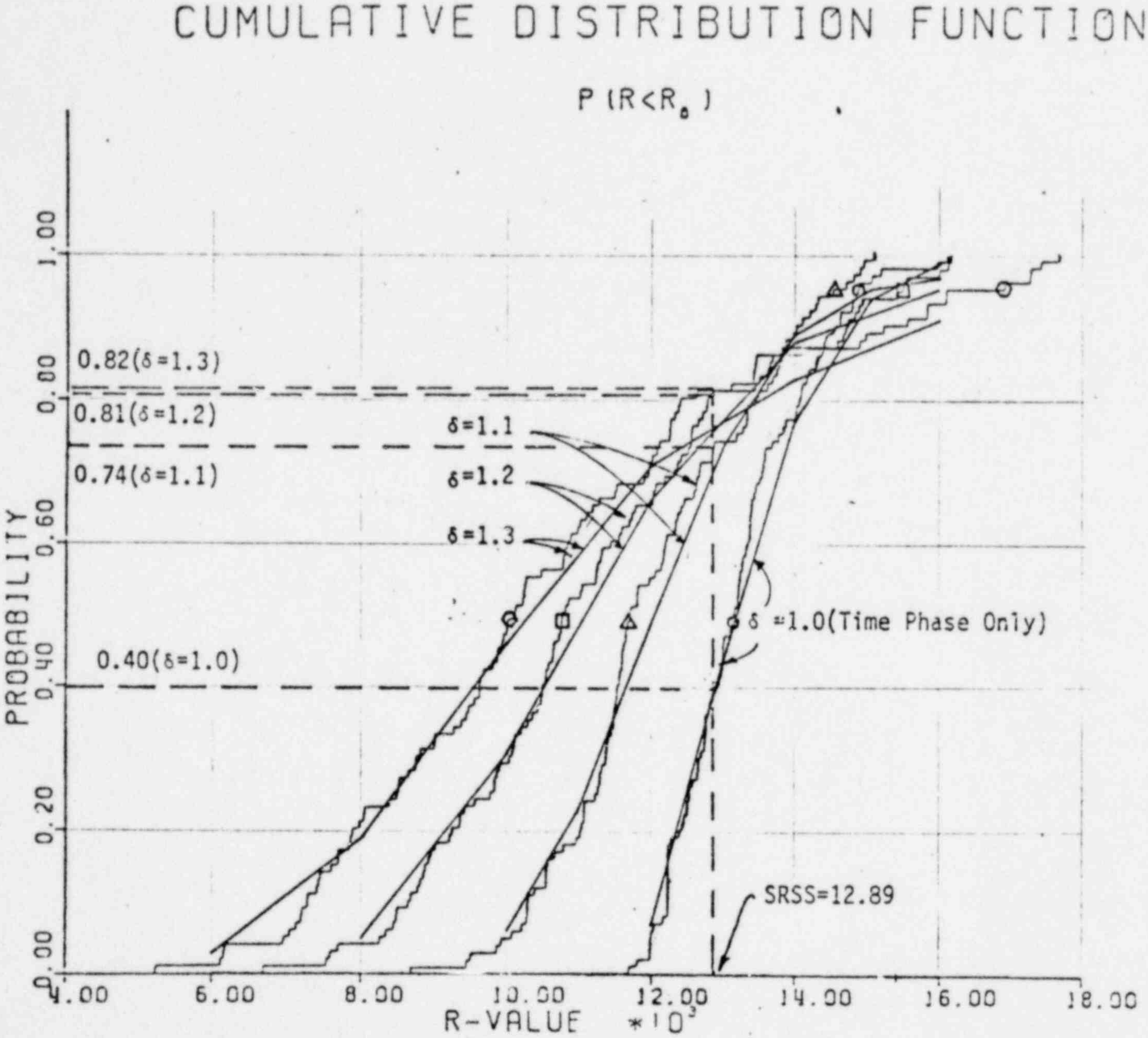


FIGURE D.1 COMPARISON DEPENDENT AMPLITUDES METHOD TO SIMULATION

EDAC FIGURE 2-28. CASE 1: MAIN STEAM-461 OBE + SRVBDG M_a (POSITIVE)
LOGNORMAL AMPLITUDE PDF-ALL DISPERSIONS EQUAL

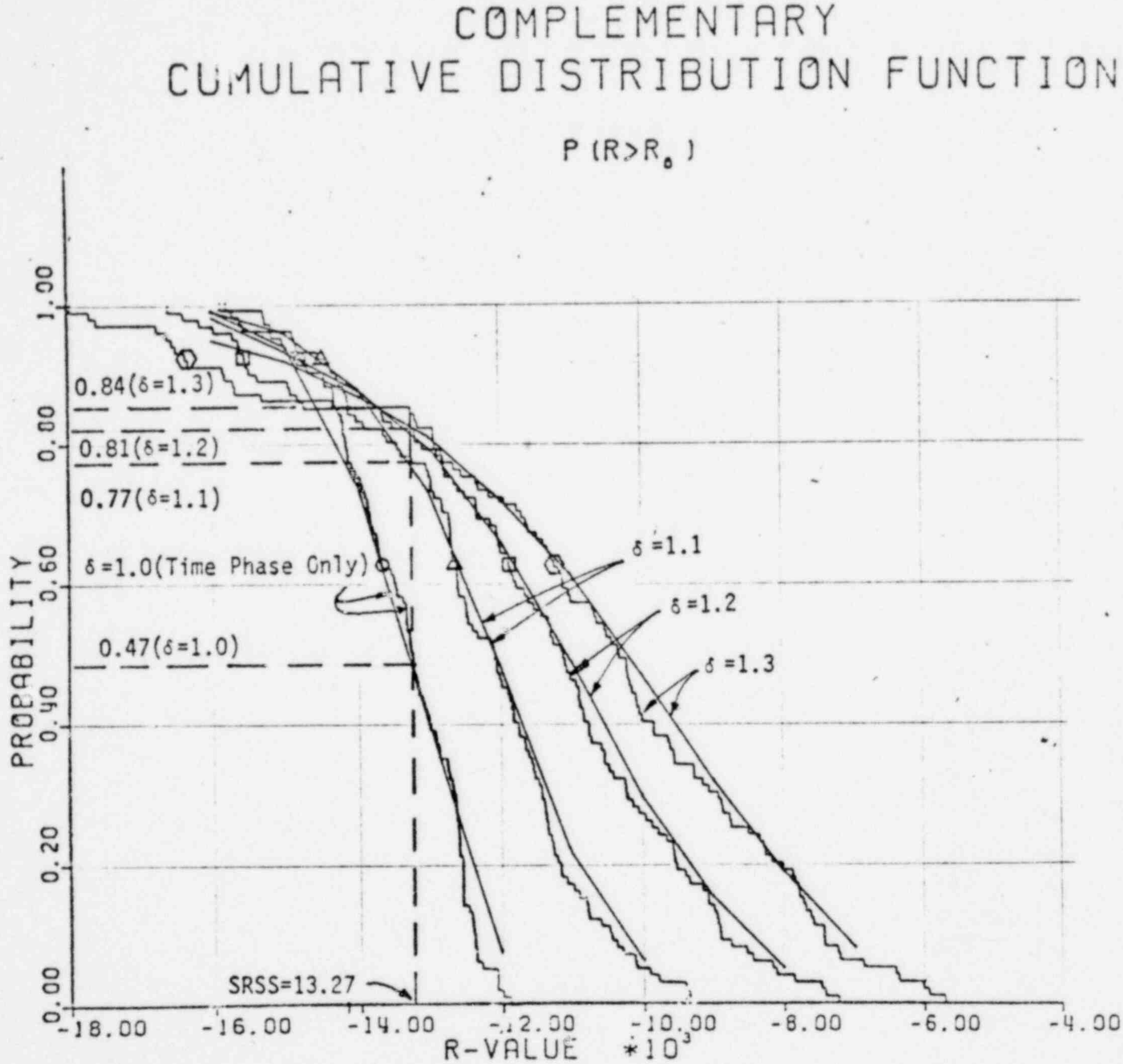


FIGURE D.2 COMPARISON DEPENDENT AMPLITUDES METHOD TO SIMULATION

EDAC FIGURE 2-29. CASE 1: MAIN STEAM-A | OBE + SRVBDG M_a (NEGATIVE)
LOGNORMAL AMPLITUDE PDF-ALL DISPERSIONS EQUAL

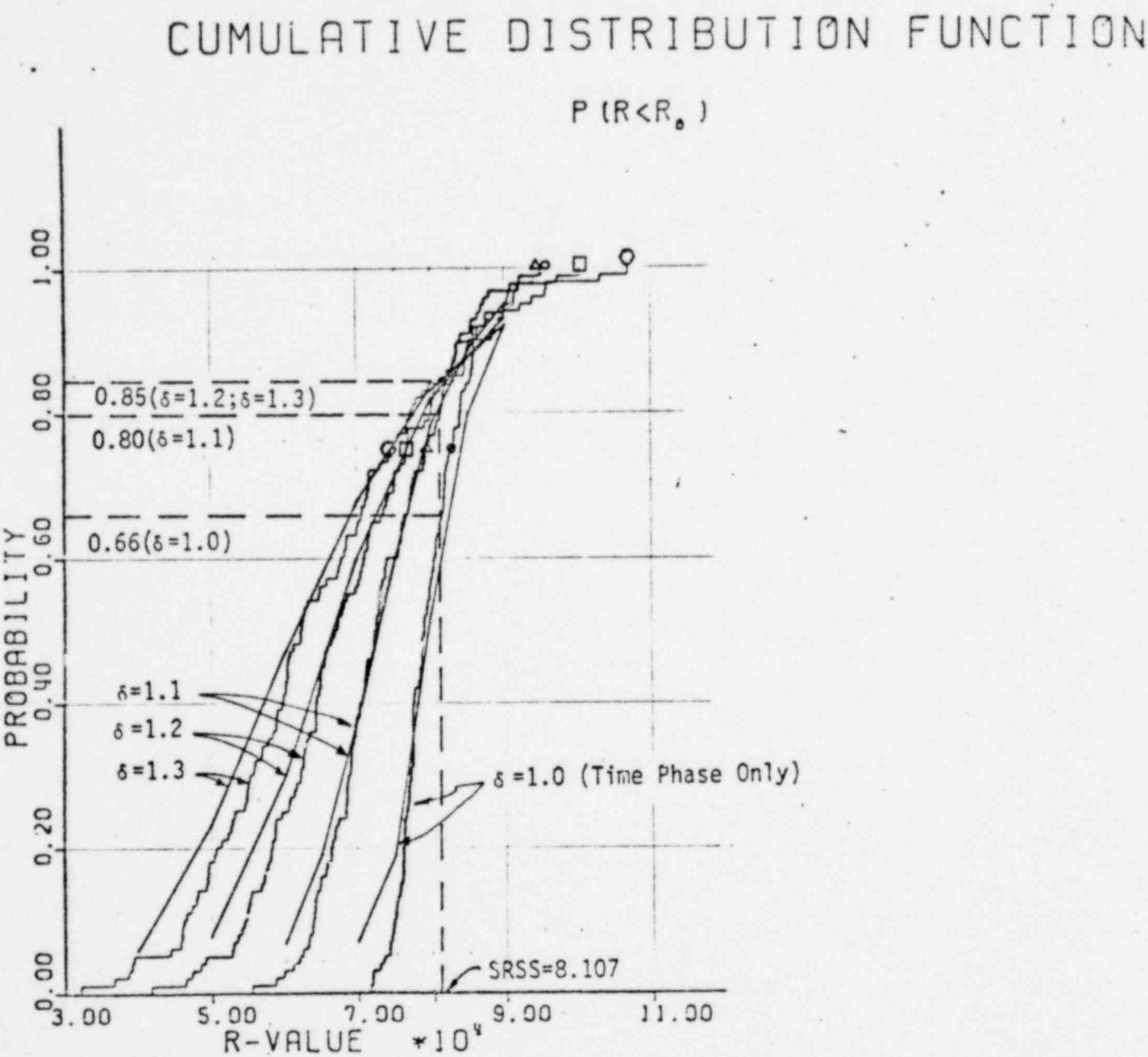


FIGURE D.3 COMPARISON DEPENDENT AMPLITUDES METHOD TO SIMULATION

EDAC FIGURE 2-30. CASE 2: MAIN STEAM-461 OBE + SRVBDG M_C (POSITIVE)
LOGNORMAL AMPLITUDE PDF-ALL DISPERSIONS EQUAL

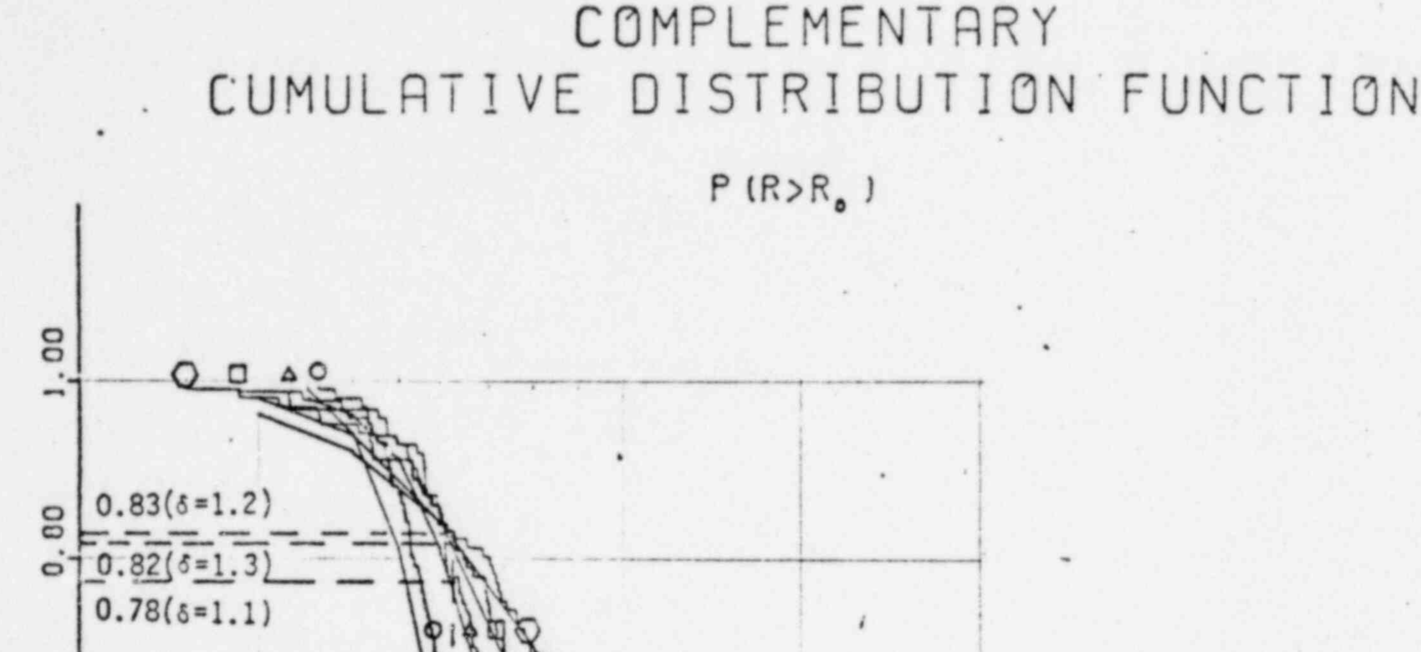


FIGURE D.4 COMPARISON DEPENDENT AMPLITUDES MEHTOD TO SIMULATION

EDAC FIGURE 2-31. CASE 2: MAIN STEAM-461 OBE + SRVBDG M_c (NEGATIVE)
LOGNORMAL AMPLITUDE PDF-ALL DISPERSIONS EQUAL

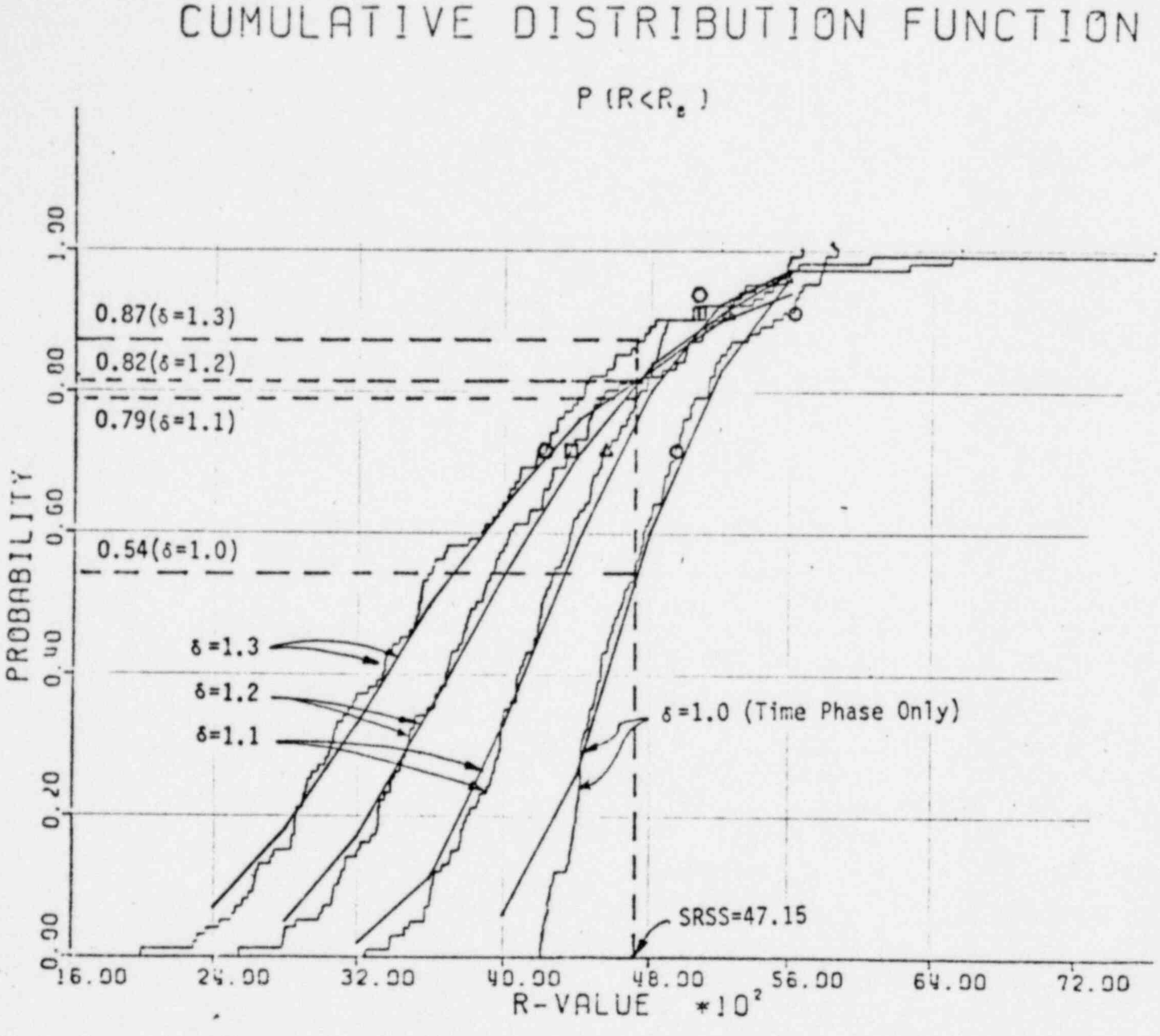


FIGURE D.5 COMPARISON DEPENDENT AMPLITUDES METHOD TO SIMULATION

EDAC FIGURE 2-32. CASE 3: RHR-WETWELL OBE + SRVBUB M_a (POSITIVE)
LOGNORMAL AMPLITUDE PDF-ALL DISPERSIONS EQUAL

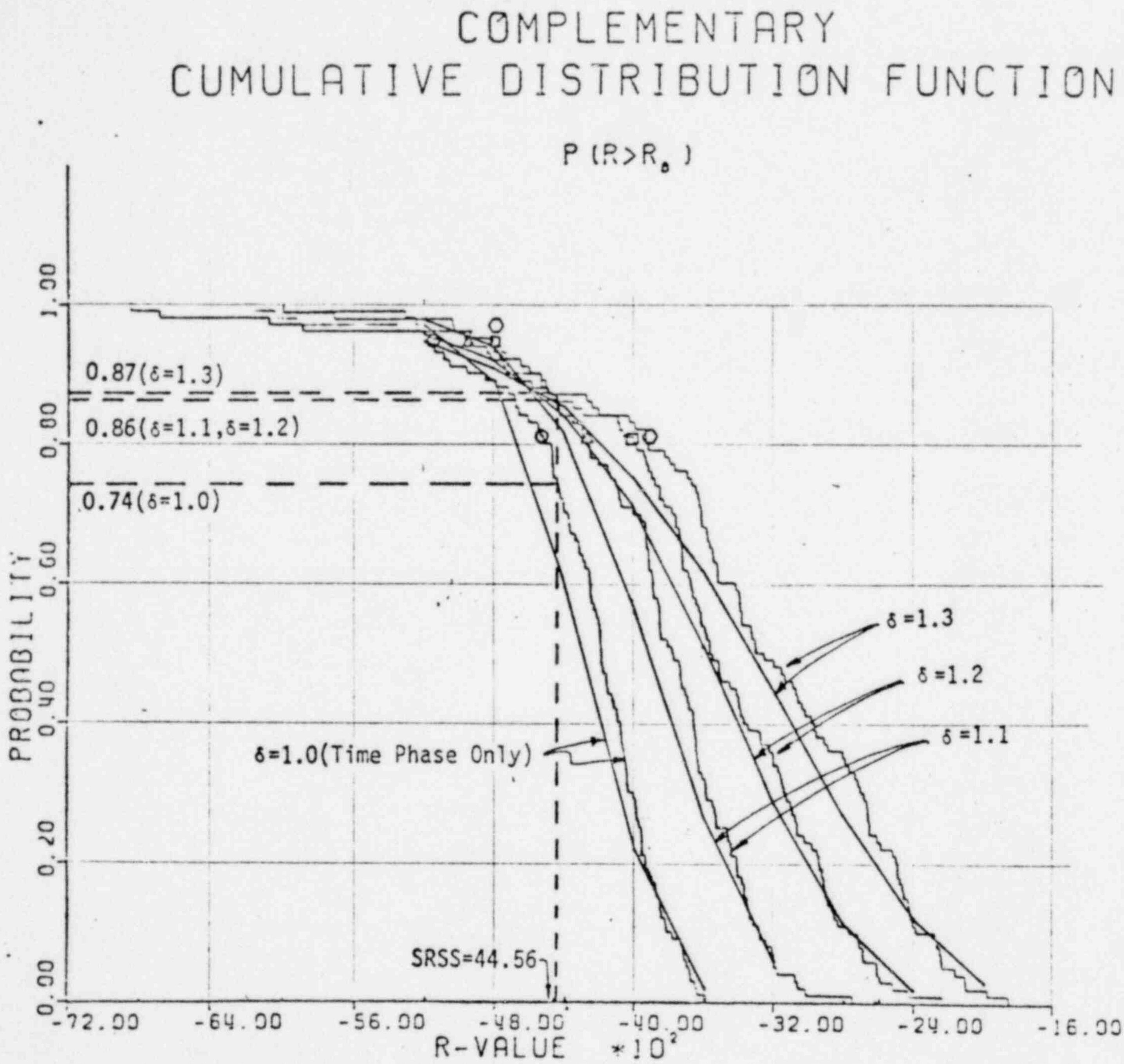


FIGURE D.6 COMPARISON DEPENDENT AMPLITUDES METHOD TO SIMULATION

EDAC FIGURE 2-33. CASE 3: RHR-WETWELL OBE + SRVBUB M_a (NEGATIVE)
LOGNORMAL AMPLITUDE PDF-ALL DISPERSIONS EQUAL

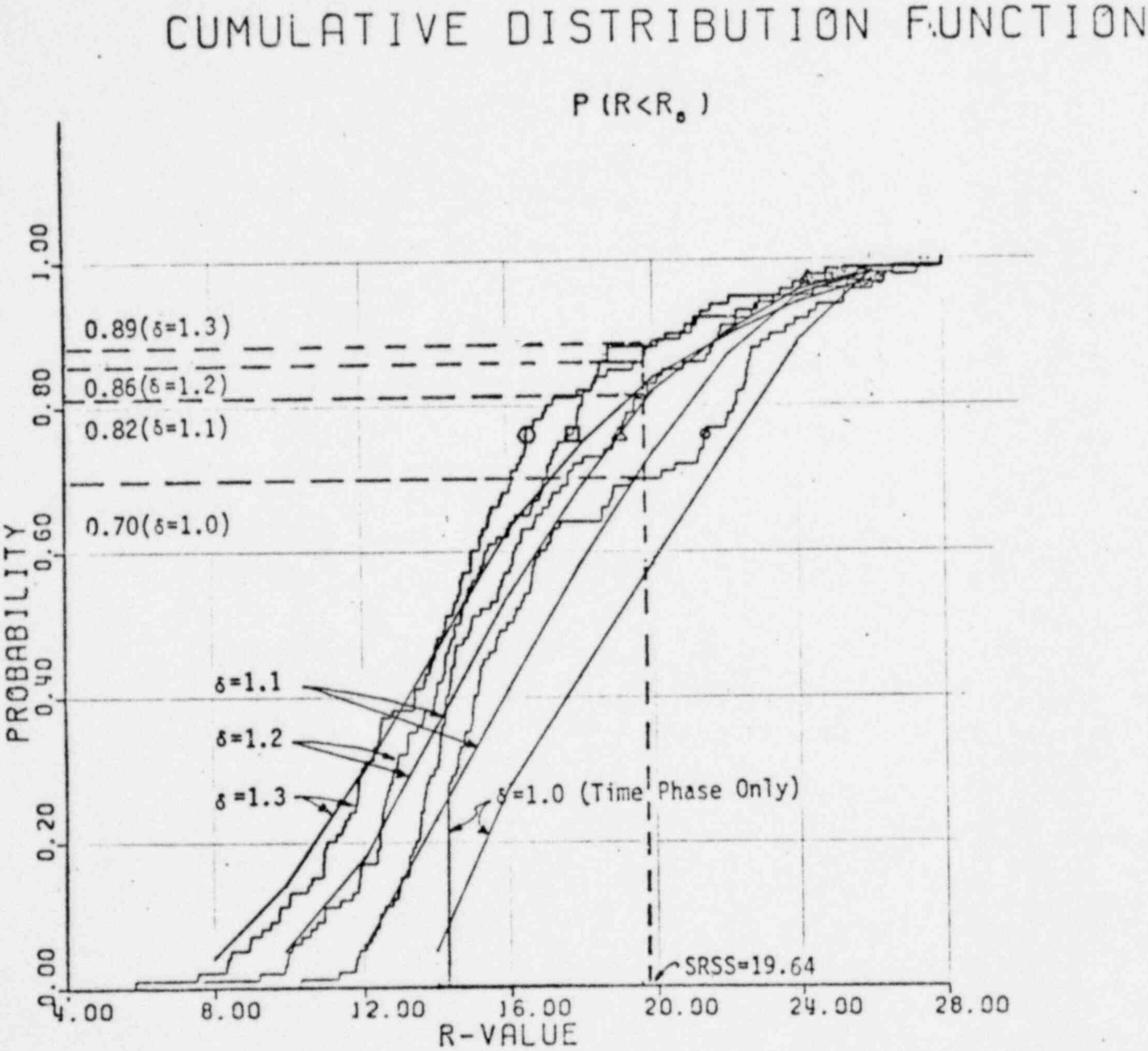


FIGURE D.7 COMPARISON DEPENDENT AMPLITUDES METHOD TO SIMULATION

EDAC FIGURE 2-34. CASE 4: ZIMMER PLANT OBE(NS) + SRV (ALL),
CONTAINMENT WALL AT DRYWELL FLOOR ELEVATION.
LOGNORMAL AMPLITUDE PDF-ALL DISPERSIONS EQUAL

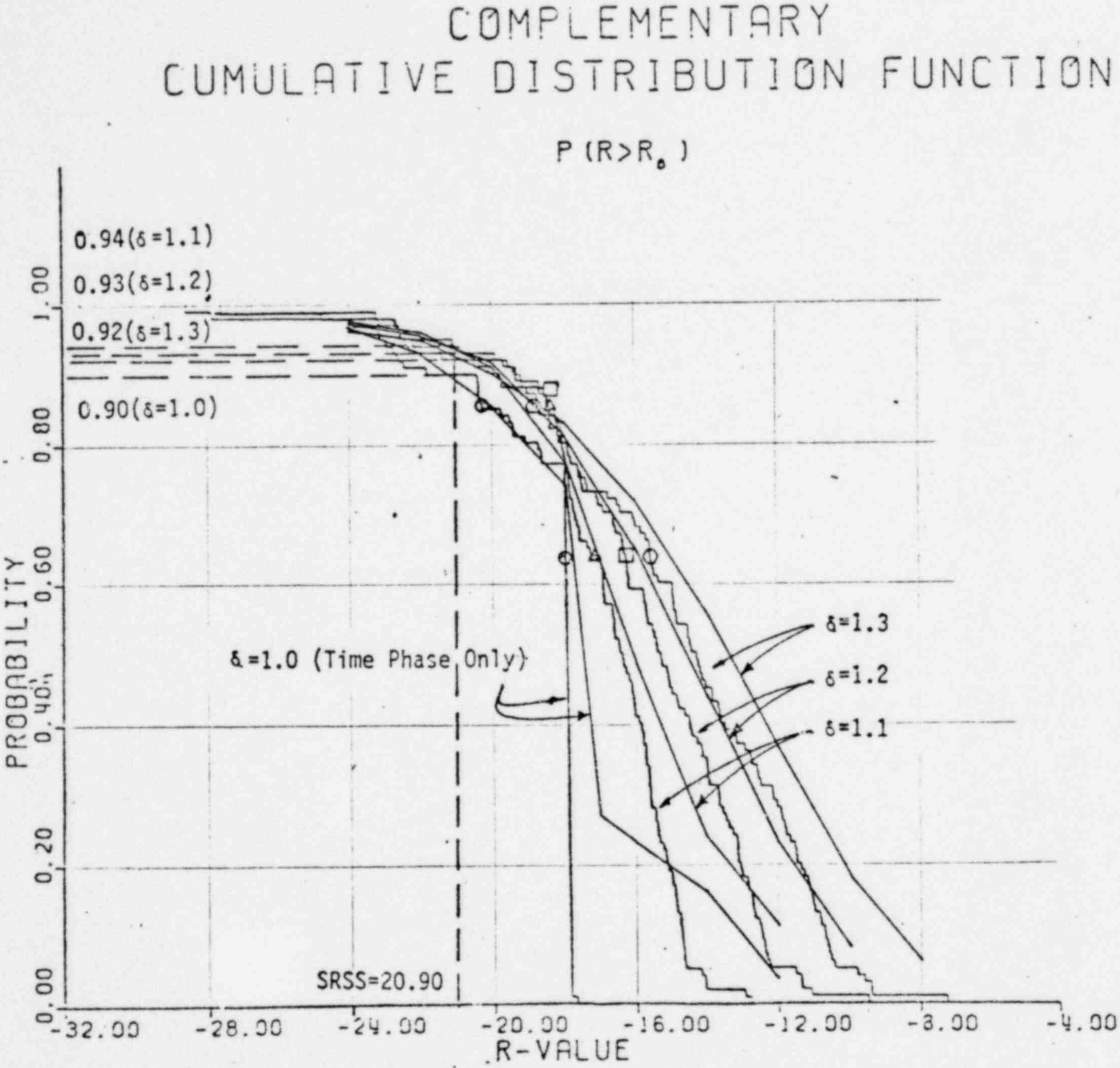


FIGURE D.8 COMPARISON DEPENDENT AMPLITUDES METHOD TO SIMULATION

EDAC FIGURE 2-35. CASE 4: ZIMMER PLANT OBE(NS) + SRV(ALL), CONTAINMENT WALL AT DRYWELL FLOOR ELEVATION. LOGNORMAL AMPLITUDE PDF-ALL DISPERSIONS EQUAL

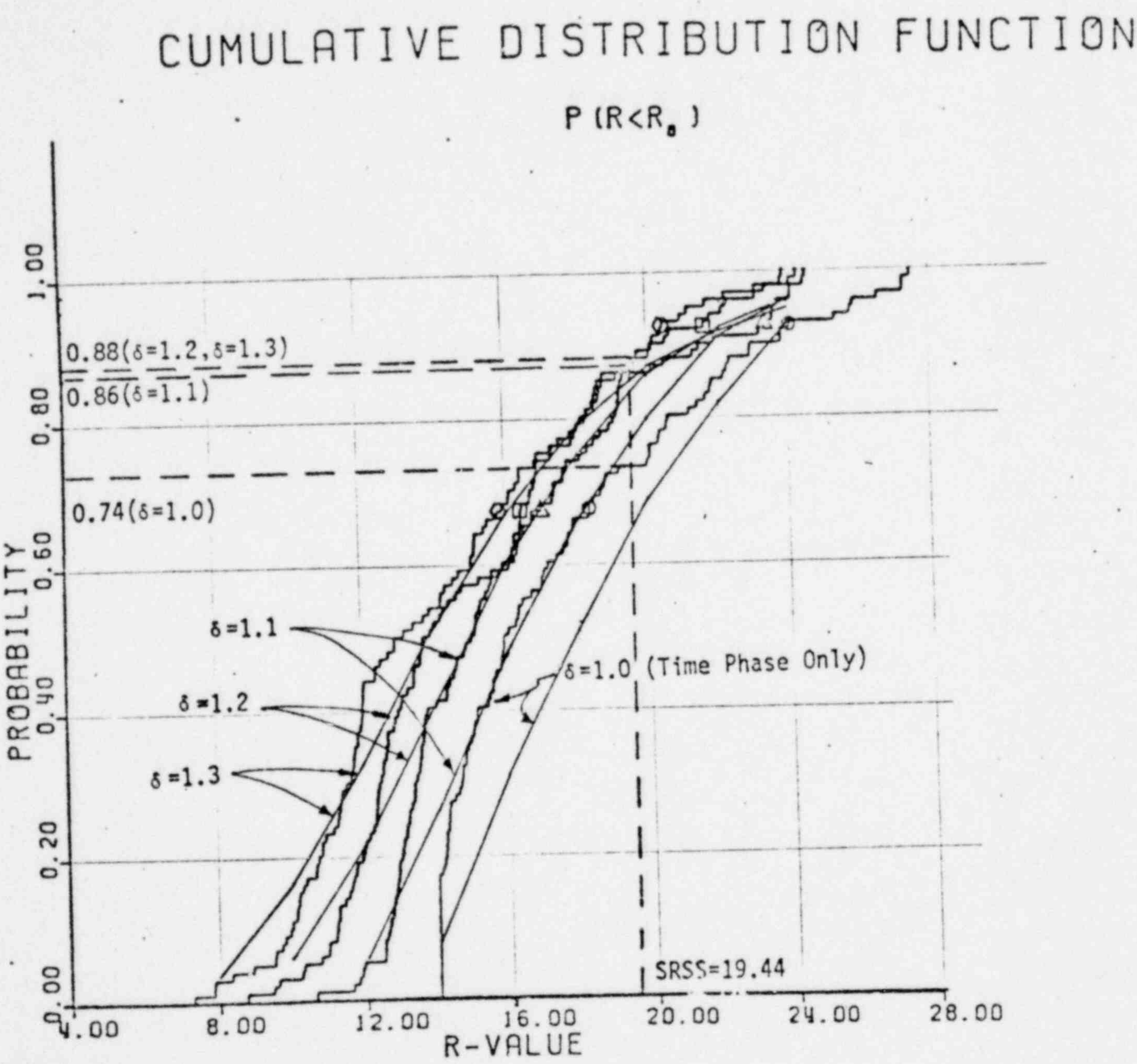


FIGURE D.9 COMPARISON DEPENDENT AMPLITUDES METHOD TO SIMULATION

EDAC FIGURE 2-36. CASE 5: ZIMMER PLANT OBE(EW) + SRV(ALL), CONTAINMENT WALL AT DRYWELL FLOOR ELEVATION. LOGNORMAL AMPLITUDE PDF-ALL DISPERSIONS EQUAL

COMPLEMENTARY
CUMULATIVE DISTRIBUTION FUNCTION

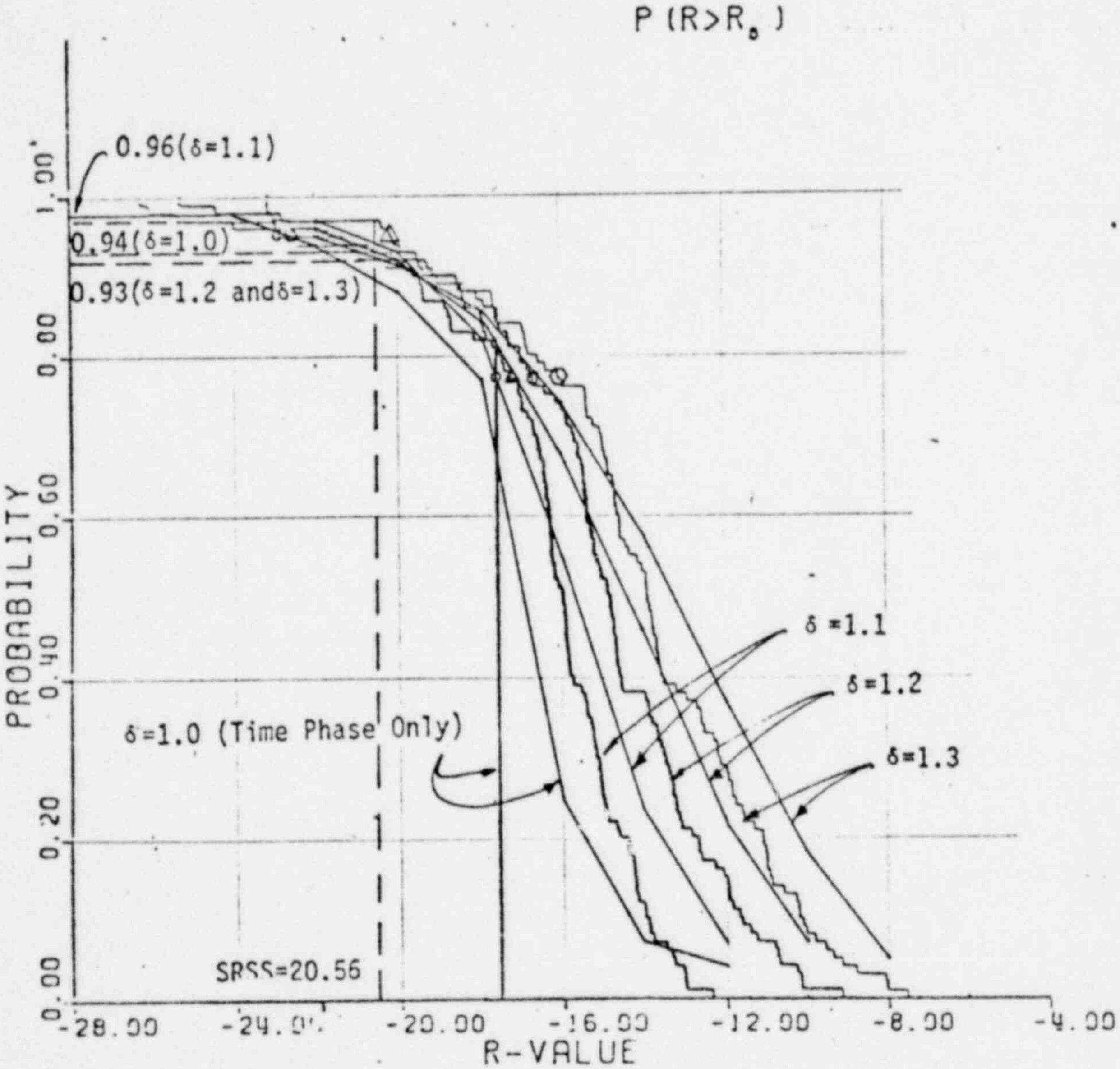


FIGURE D.10 COMPARISON DEPENDENT AMPLITUDES METHOD TO SIMULATION

EDAC FIGURE 2-37. CASE 5: ZIMMER PLANT OBE(EW) + SRV(ALL), CONTAINMENT WALL AT DRYWELL FLOOR ELEVATION.

LOGNORMAL AMPLITUDE PDF-ALL DISPERSIONS EQUAL

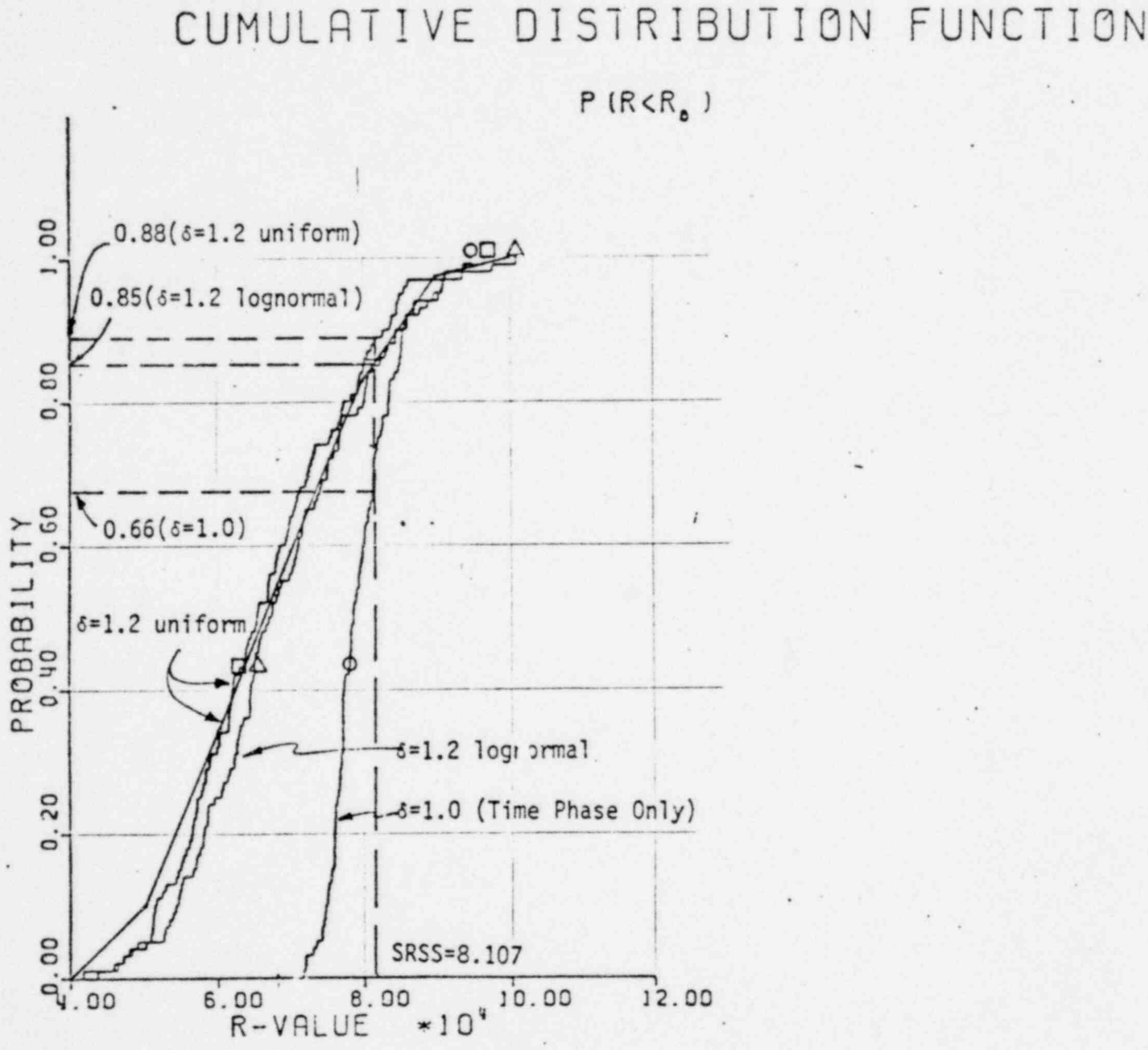


FIGURE D.11 COMPARISON DEPENDENT AMPLITUDES METHOD TO SIMULATION

EDAC FIGURE 2-54. CASE 2: MAIN STEAM-46I OBE + SRVBDG M_C (POSITIVE)
INFLUENCE OF SHAPE OF AMPLITUDE PDF

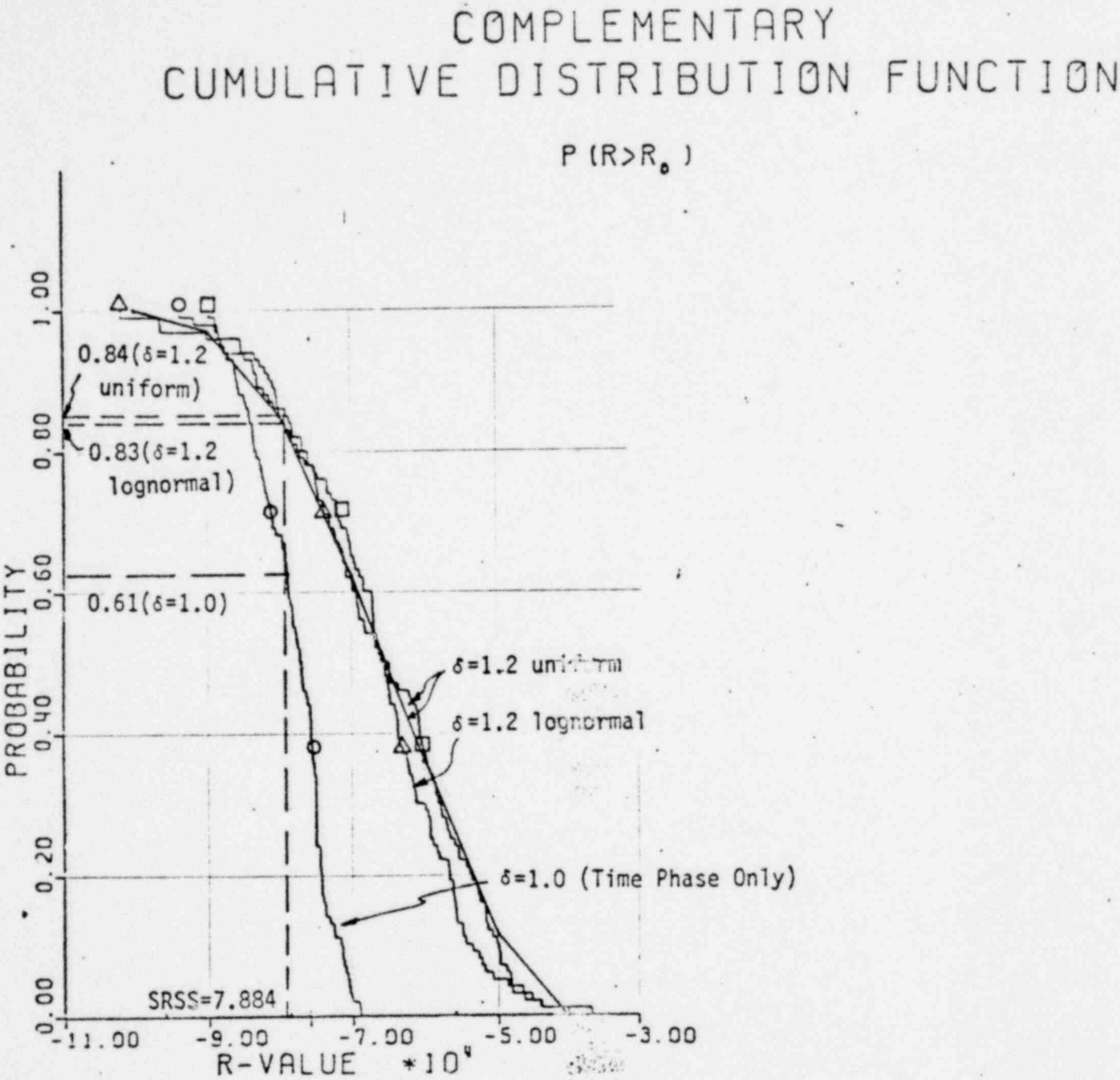


FIGURE D.12 COMPARISON DEPENDENT AMPLITUDES METHOD TO SIMULATION

EDAC FIGURE 2-55. CASE 2: MAIN STEAM-46I OBE + SRVBDG M_c (NEGATIVE)

INFLUENCE OF SHAPE OF AMPLITUDE PDF

CUMULATIVE DISTRIBUTION FUNCTION

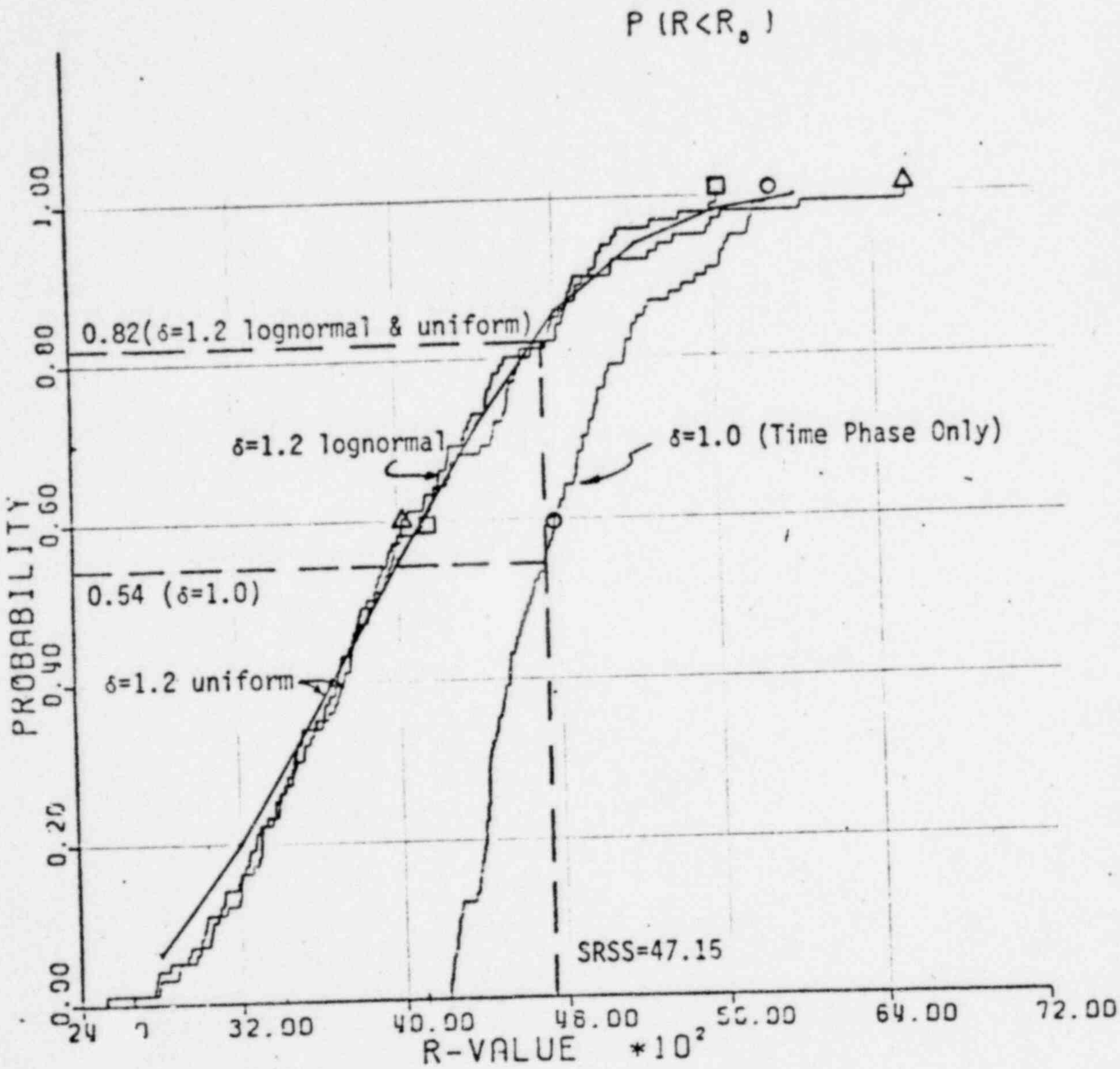


FIGURE D.13 COMPARISON DEPENDENT AMPLITUDES METHOD TO SIMULATION

EDAC FIGURE 2-56 CASE 3: RHR-WETWELL OBE + SRVBUB M_a (POSITIVE)
INFLUENCE OF SHAPE OF AMPLITUDE PDF

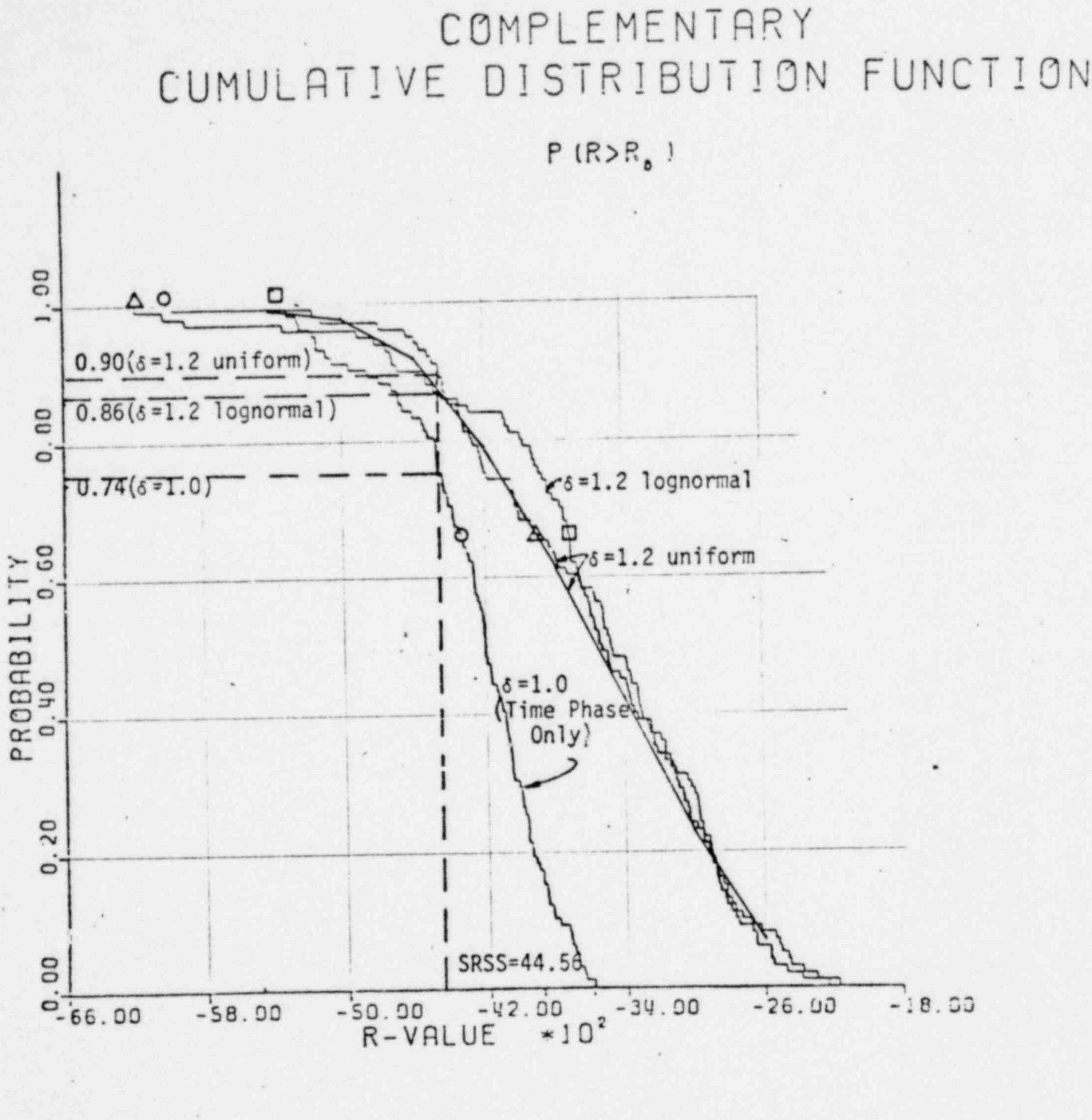


FIGURE D.14 COMPARISON DEPENDENT AMPLITUDES METHOD TO SIMULATION

EDAC FIGURE 2-57. CASE 3: RHR-WETWELL OBE + SRVBUB M_a (NEGATIVE)
INFLUENCE OF SHAPE OF AMPLITUDE PDF

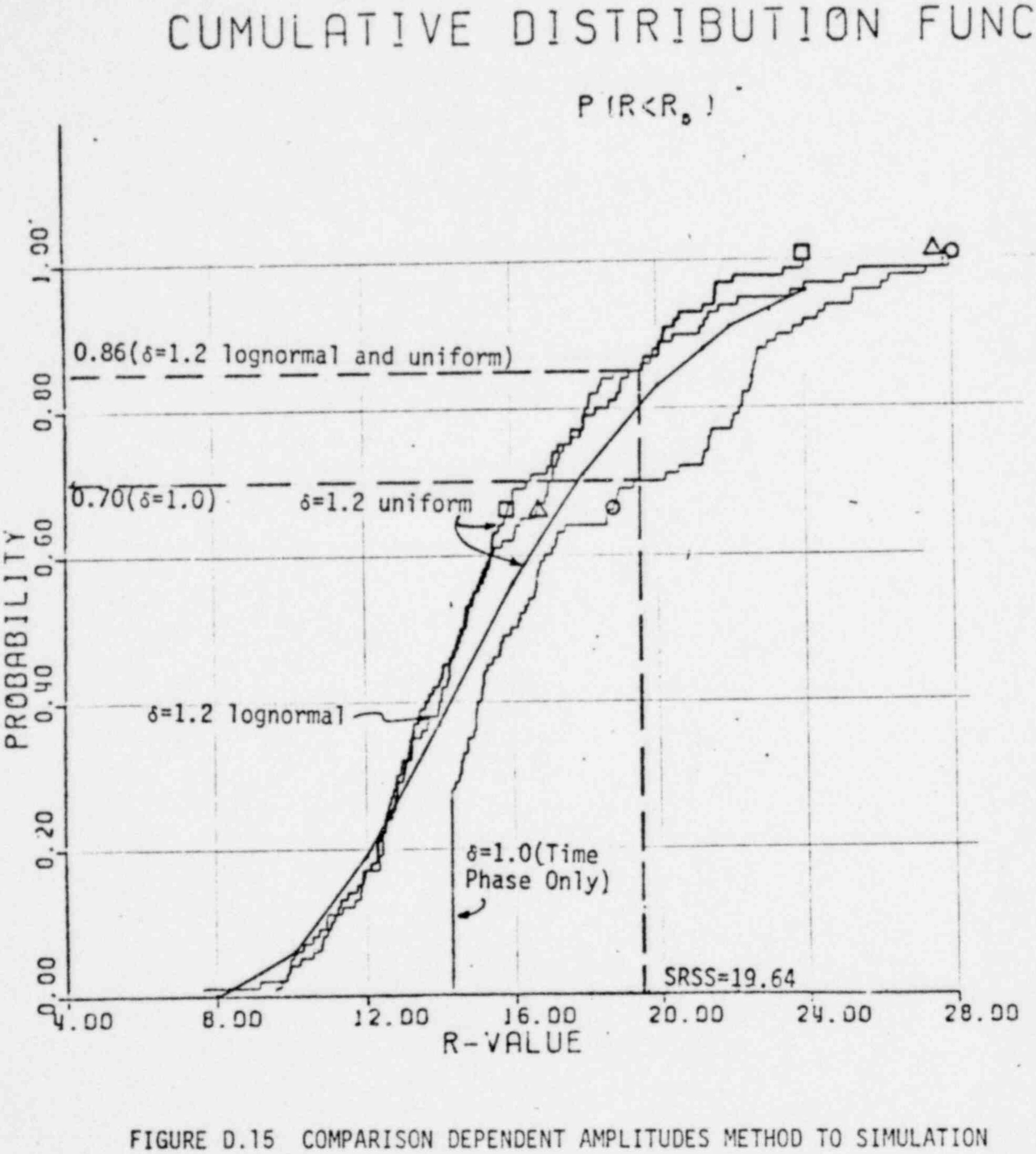


FIGURE D.15 COMPARISON DEPENDENT AMPLITUDES METHOD TO SIMULATION

EDAC FIGURE 2-58. CASE 4: ZIMMER PLANT OBE(NS) + SRV(ALL), CONTAINMENT WALL AT DRYWELL FLOOR ELEVATION. INFLUENCE OF SHAPE OF AMPLITUDE PDF

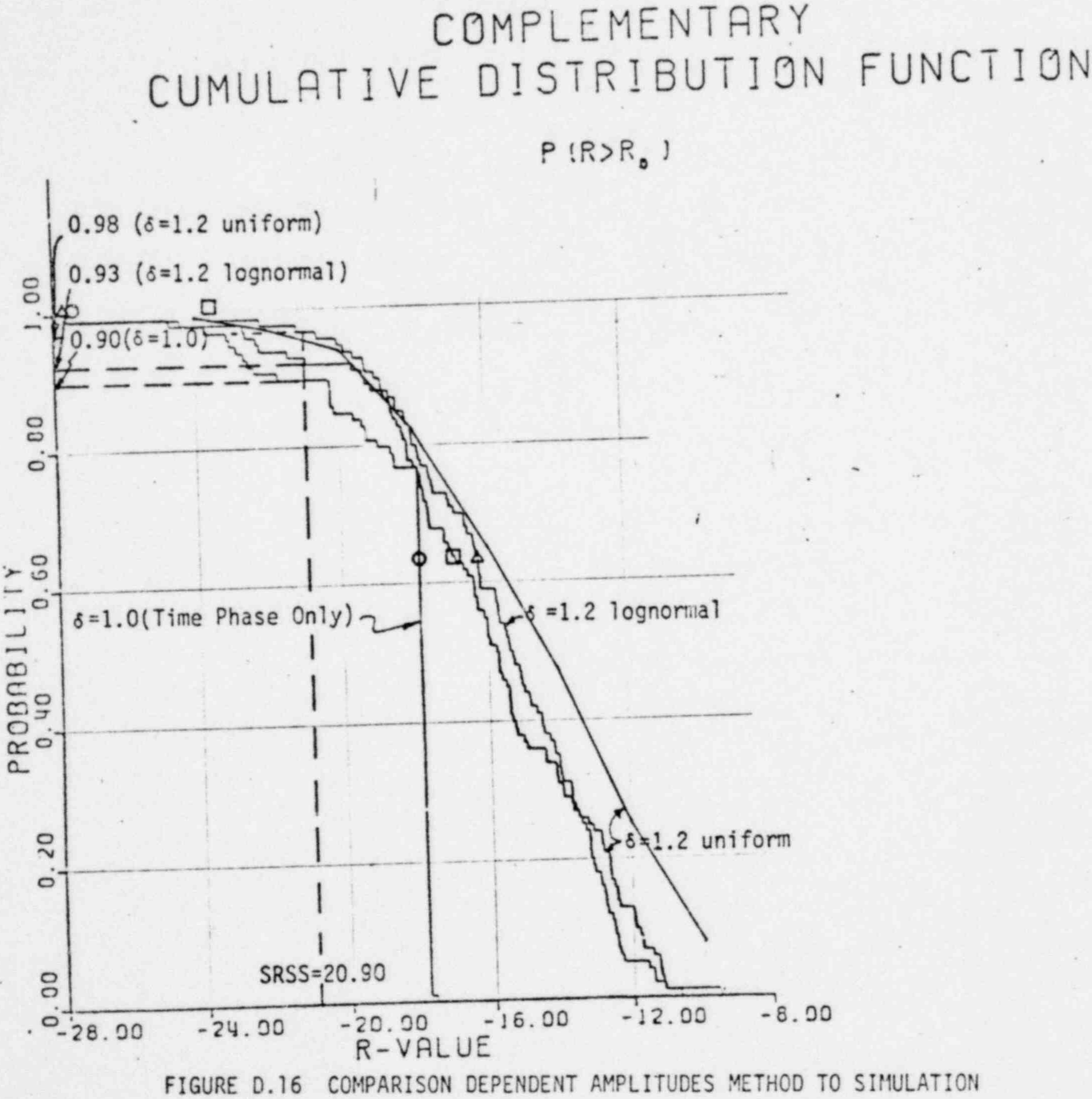


FIGURE D.16 COMPARISON DEPENDENT AMPLITUDES METHOD TO SIMULATION

EDAC FIGURE 2-59. CASE 4: ZIMMER PLANT OBE(NS) + SRV(ALL), CONTAINMENT WALL AT DRYWELL FLOOR ELEVATION.
INFLUENCE OF SHAPE OF AMPLITUDE PDF

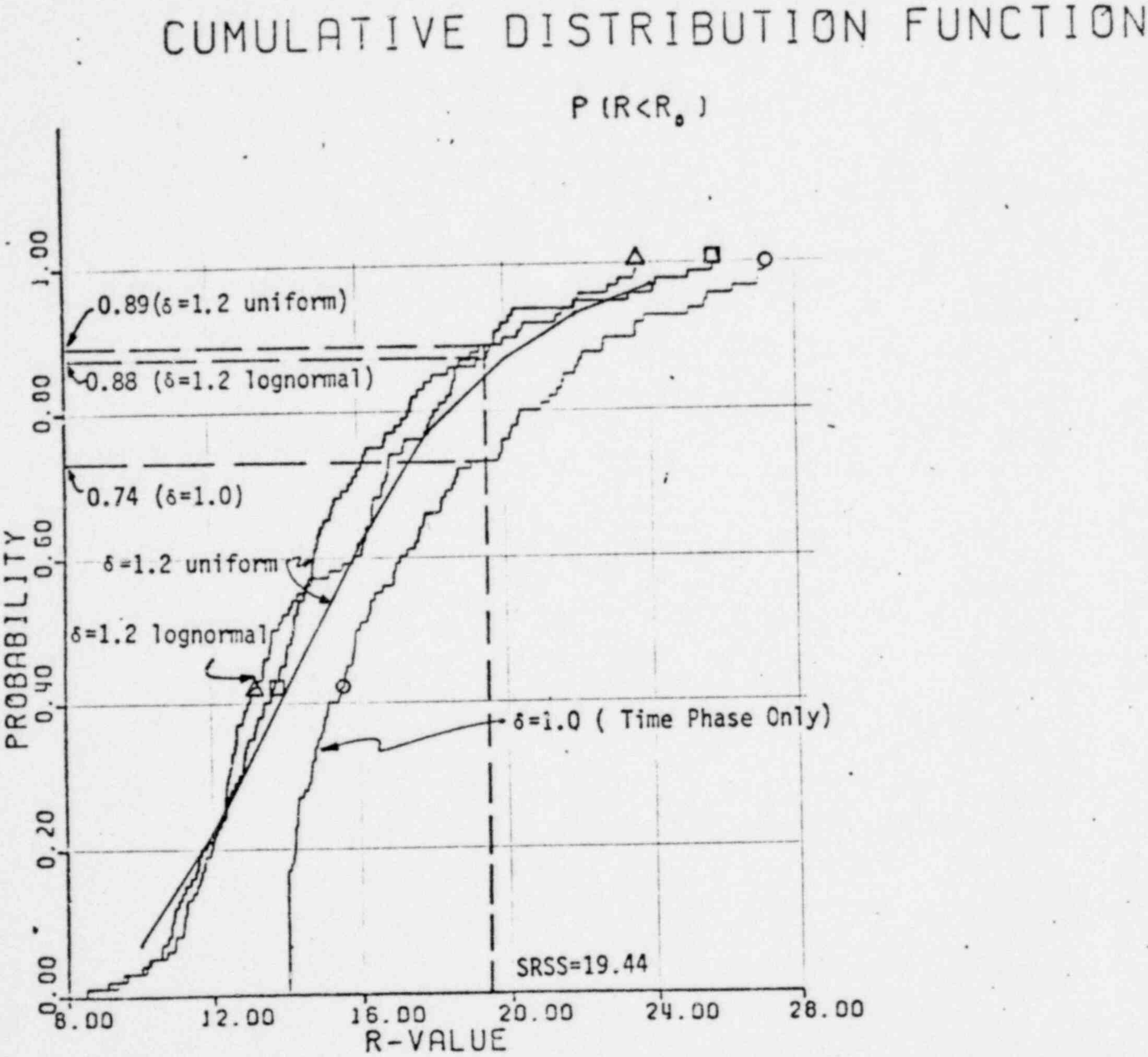


FIGURE D.17 COMPARISON DEPENDENT AMPLITUDES METHOD TO SIMULATION

EDAC FIGURE 2-60. CASE 5: ZIMMER PLANT OBE (EW) + SRV (ALL), CONTAINMENT WALL AT DRYWELL FLOOR ELEVATION.
INFLUENCE OF SHAPE OF AMPLITUDE PDF

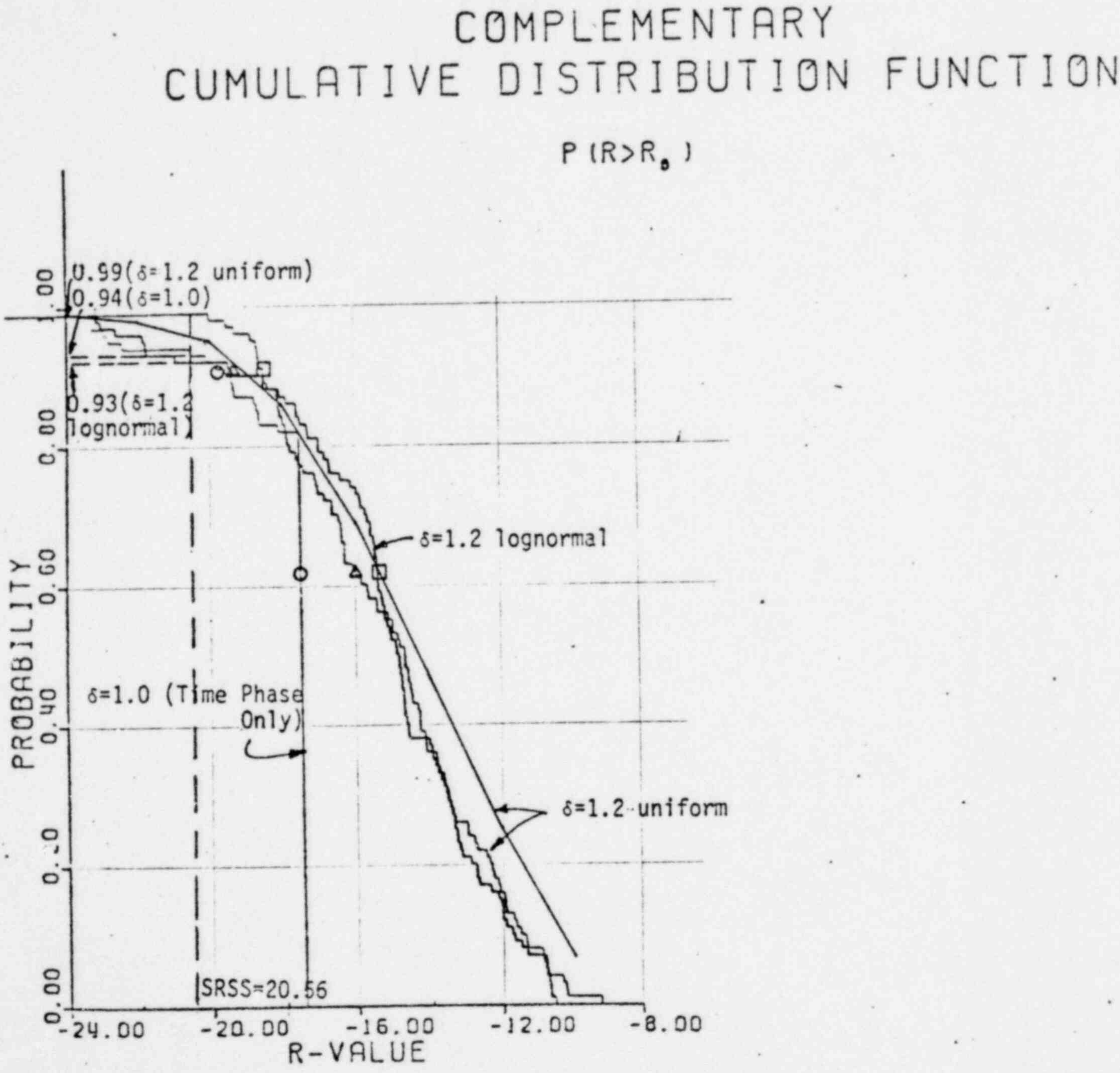


FIGURE D.18 COMPARISON DEPENDENT AMPLITUDES METHOD TO SIMULATION

EDAC FIGURE 2-61. CASE 5: ZIMMER PLANT OBE(EW) + SRV(ALL),
CONTAINMENT WALL AT DRYWELL FLOOR ELEVATION.
INFLUENCE OF SHAPE OF AMPLITUDE PDF

CASE NO.	DISPERSION FACTOR δ	MAX (POS) OR MIN (NEG)	LOGNORMAL SIM. NEP (SRSS)	LOGNORMAL ANALYTICAL NEP(SRSS)	UNIFORM NEP(SRSS)	GAUSSIAN NEP(SRSS)	EXPONENTIAL NEP(SRSS)	BIMODAL (MIXED) UNIFORM NEP(SRSS)
1	1.0	NEG	0.48	0.51 (0.51)				
	TPO	POS	0.40	0.38 (0.38)				
	1.1	NEG	0.77	0.77				
		POS	0.74	0.71				
	1.2	NEG	0.81	0.82				
		POS	0.81	0.76 (0.80)	0.79 (0.79)	0.76 (0.80)	0.79 (0.81)	0.73 (0.79)
	1.3	NEG	0.84	0.83 (0.81)				
		POS	0.82	0.76 (0.80)				
2	1.0	NEG	0.61	0.54 (0.54)				
	TPO	POS	0.66	0.59 (0.59)				
	1.1	NEG	0.78	0.77				
		POS	0.80	0.80				
	1.2	NEG	0.82	0.81				
		POS	0.85	0.83	0.85			
	1.3	NEG	0.83	0.83				
		POS	0.85	0.84				
3	1.0	NEG	0.74	0.66 (0.66)				
	TPO	POS	0.54	0.53 (0.53)				
	1.1	NEG	0.86	0.83				
		POS	0.79	0.75				
	1.2	NEG	0.86	0.85				
		POS	0.82	0.81	0.83			
	1.3	NEG	0.87	0.86				
		POS	0.87	0.83 (0.83)				
4	1.0	NEG	0.90	0.89 (0.89)				
	TPO	POS	0.70	0.56 (0.56)				
	1.1	NEG	0.94	0.93				
		POS	0.82	0.71 (0.71)				
	1.2	NEG	0.93	0.93				
		POS	0.86	0.79 (0.81)	0.81 (0.83)	0.80 (0.82)	0.82 (0.82)	0.79 (0.83)
	1.3	NEG	0.92	0.92				
		POS	0.89	0.82 (0.85)				
5	1.0	NEG	0.94	0.90 (0.90)				
	TPO	POS	0.74	0.65 (0.65)				
	1.1	NEG	0.96	0.94				
		POS	0.86	0.76 (0.77)				
	1.2	NEG	0.93	0.94				
		POS	0.88	0.83 (0.84)	0.84			
	1.3	NEG	0.93	0.93				
		POS	0.88	0.85 (0.87)				

TABLE 1 Sensitivity of NEP of the SRSS Value to Shape of the Amplitude Distrib ion. Amplitudes Assumed to be Perfectly Dependent. (Values in Parentheses for Independent Amplitudes). No Prior Deterministic Scaling of the Response Functions. Cases 4 and 5 give Similar Results.

CASE NO.	DISPERSION FACTOR δ	MAX (POS) OR MIN (NEG)	R, RATIO OF SCALING FACTORS				TF, SCALING FACTOR ON DURATION OF SHORTER LOAD (1.0 IS ORIGINAL CASE, REF. 1)			
			1/3	2/3	1	3	0.5	0.25	4.0	10.0
			NEP(SRSS)	NEP(SRSS)	NEP(SRSS)	NEP(SRSS)	NEP(SRSS)	NEP(SRSS)	NEP(SRSS)	NEP(SRSS)
1	1.0 TPO	NEG POS	0.53 0.40	0.55 0.51	0.64 0.60	0.85 0.89	0.54 0.42			
	1.1	NEG POS	0.78 0.73	0.75 0.72	0.79 0.76	0.91 0.91	0.79 0.73			
	1.2	NEG POS	0.82 0.79	0.81 0.79	0.84 0.82	0.90 0.90	0.83 0.79	0.83 0.79	0.81 0.78	0.80 0.77
	1.3	NEG POS	0.83 0.81	0.83 0.81	0.85 0.84	0.90 0.89	0.83 0.81			
2	1.0 TPO	NEG POS	0.61	0.67	0.72	0.90	0.56 0.61			
	1.1	NEG POS	0.82	0.81	0.84	0.92	0.79 0.82			
	1.2	NEG POS	0.84	0.85	0.87	0.91	0.83 0.84	0.83 0.85	0.81 0.83	
	1.3	NEG POS	0.85	0.86	0.88	0.90	0.84 0.85			
4	1.0 TPO	NEG POS	0.81 0.58	0.86 0.55	0.87 0.57	0.92 0.74	0.90 0.58			
	1.1	NEG POS	0.89 0.78	0.91 0.72	0.92 0.72	0.93 0.86	0.94 0.73			0.94
	1.2	NEG POS	0.90 0.83	0.80	0.80	0.87	0.94 0.81	0.94 0.81	0.93 0.79	
	1.3	NEG POS	0.84	0.82	0.83	0.87	0.93 0.83			

TABLE 2 Sensitivity of NEP of SRSS Value to Relative Scaling of the (Deterministic) Amplitudes and Time Scaling (Stretching) of Shorter Load and its Duration. Maxima and Minima (Postitive and Negative Preak Combined Responses) are Shown. Lognormal Distribution on (Perfectly Dependent) Amplitude Factors. Cases 3 and 5 give Similar Results.

APPENDIX E: TIME HISTORIES, CDF's, AND UP-CROSSING RATES

The figures in this appendix show the time histories and the corresponding upcrossing rate functions and CDF's for all of the load response events used in this study. The following events are included:

CASE 1: MAIN STEAM-46I OBE, M_a
MAIN STEAM-46I SRVBDG, M_a

CASE 2: MAIN STEAM-46I OBE, M_c
MAIN STEAM-46I SRVBDG, M_c

CASE 3: RHR WETWELL-1I OBE, M_a
RHR WETWELL-1I SRVBUB, M_a

CASE 4: ZIMMER NODE 4 OBE-NS
ZIMMER NODE 4 SRV-ALL

CASE 5: ZIMMER NODE 4 OBE-EW
ZIMMER NODE 4 SRV-ALL

The upcrossing rate function and the CDF contain important information about the process. Figures E.1 and E.2, for example, show a typical earthquake response. The CDF has a common "S-shaped" curve and the upcrossing rate function is "bell-shaped". By contrast, Figures E.3 and E.4 show an SRV response, in which the CDF is almost linear and the upcrossing rate function is almost uniform. Although these functions do not uniquely define the process, they present some of the most significant information needed for load combination purposes and in a form which is easier to interpret and generalize than the full time histories.

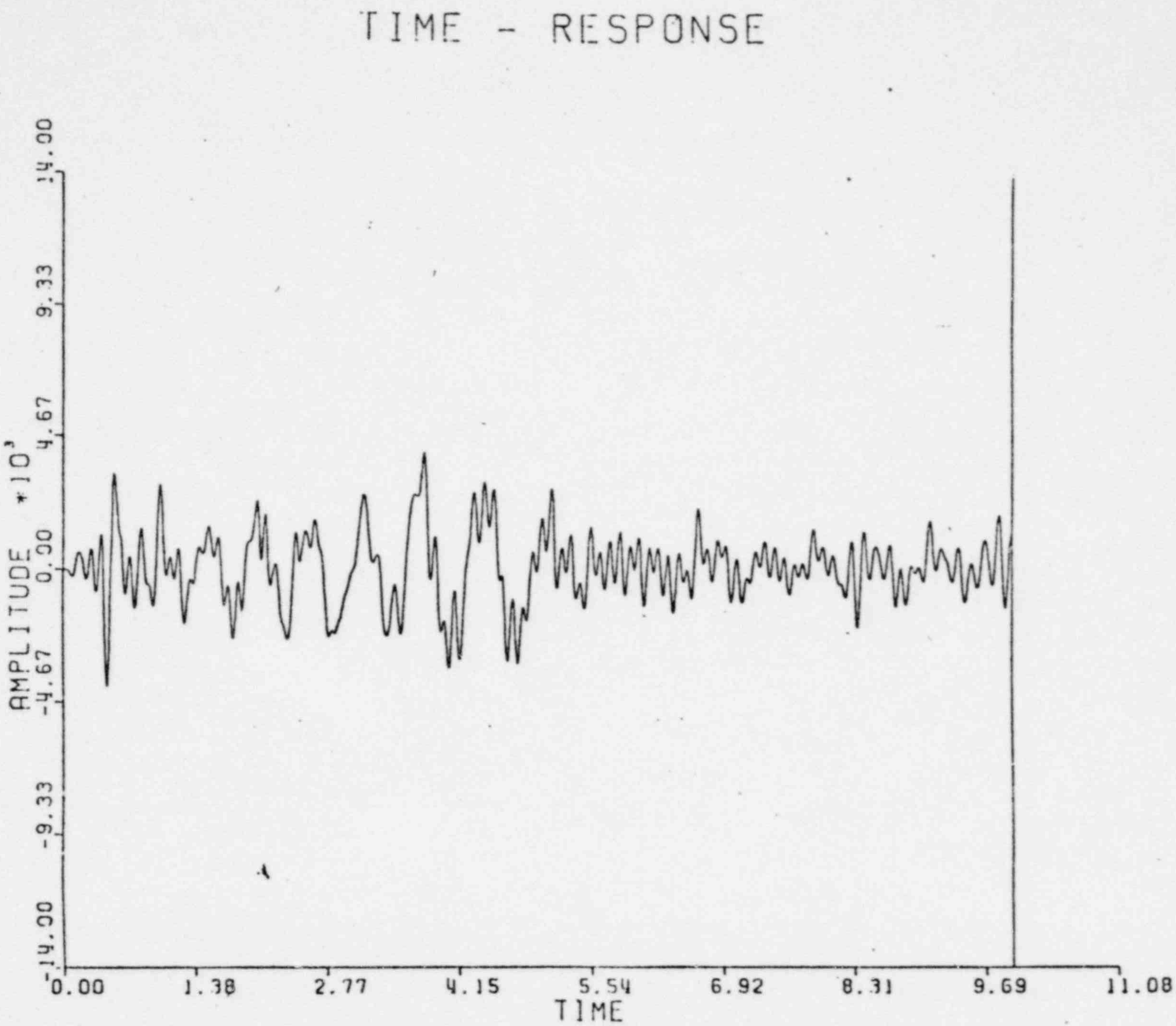


FIGURE E.1. MAIN STEAM-46I OBE, M_a, CASE 1

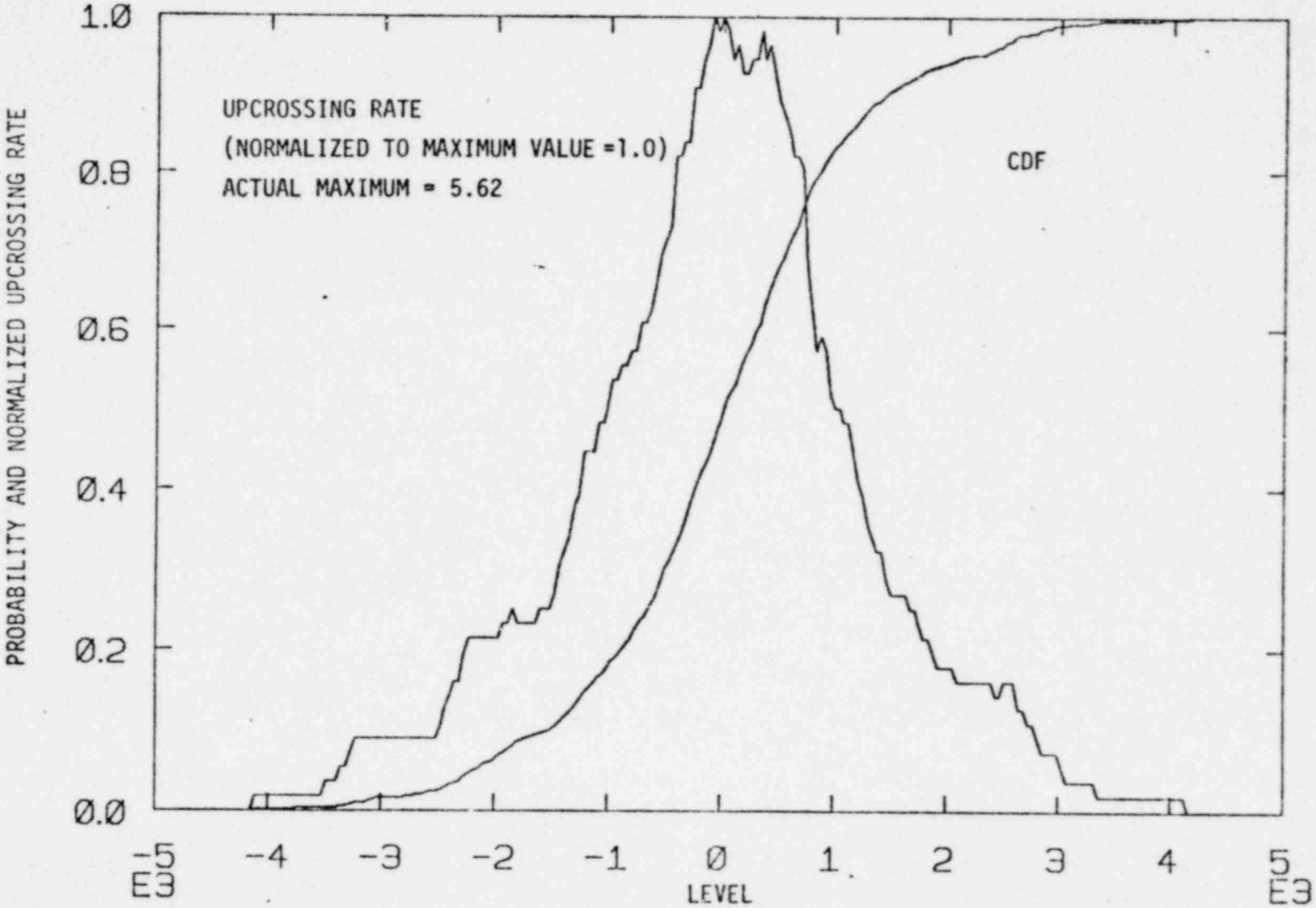


FIGURE E.2 UPCROSSING RATE FUNCTION AND CUMULATIVE DISTRIBUTION FUNCTION FOR EVENT MAIN STEAM-46I OBE, M_a, CASE 1

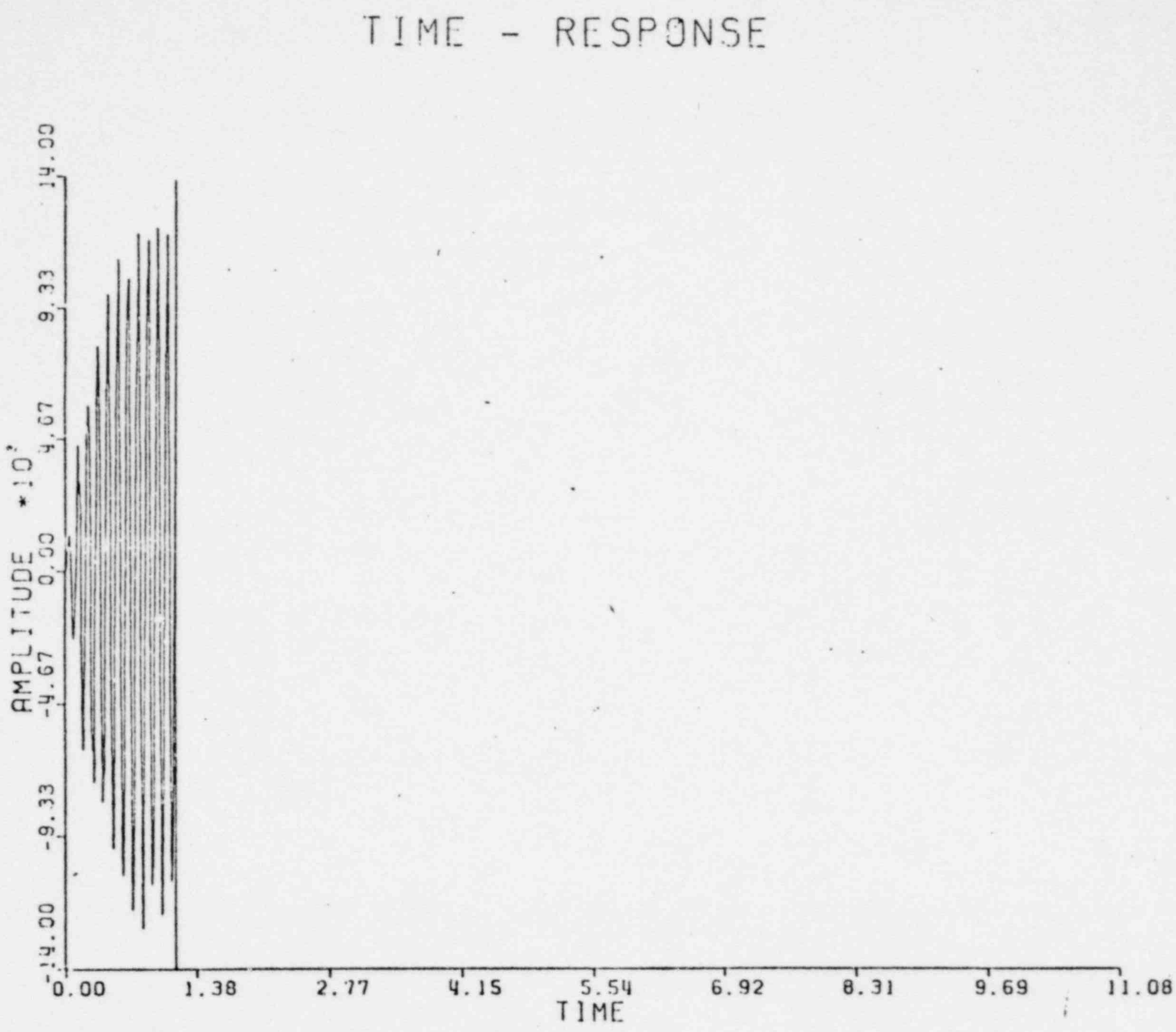


FIGURE E.3. MAIN STEAM-46I SRVBDC, M_a, CASE 1

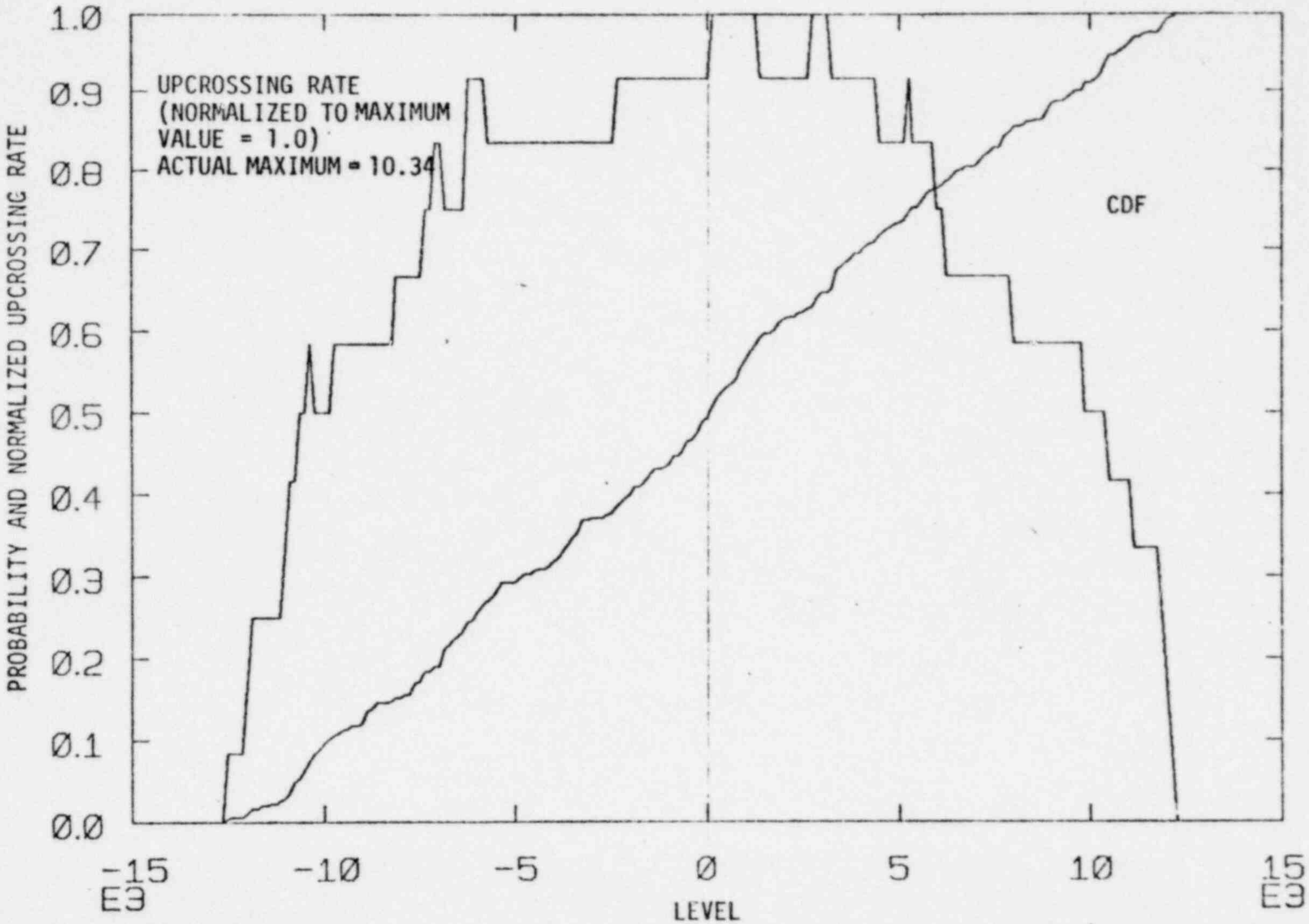


FIGURE E.4 UPCROSSING RATE FUNCTION AND CUMULATIVE DISTRIBUTION FUNCTION FOR EVENT MAIN STEAM-46I SRVBDG, M_a, CASE 1 .

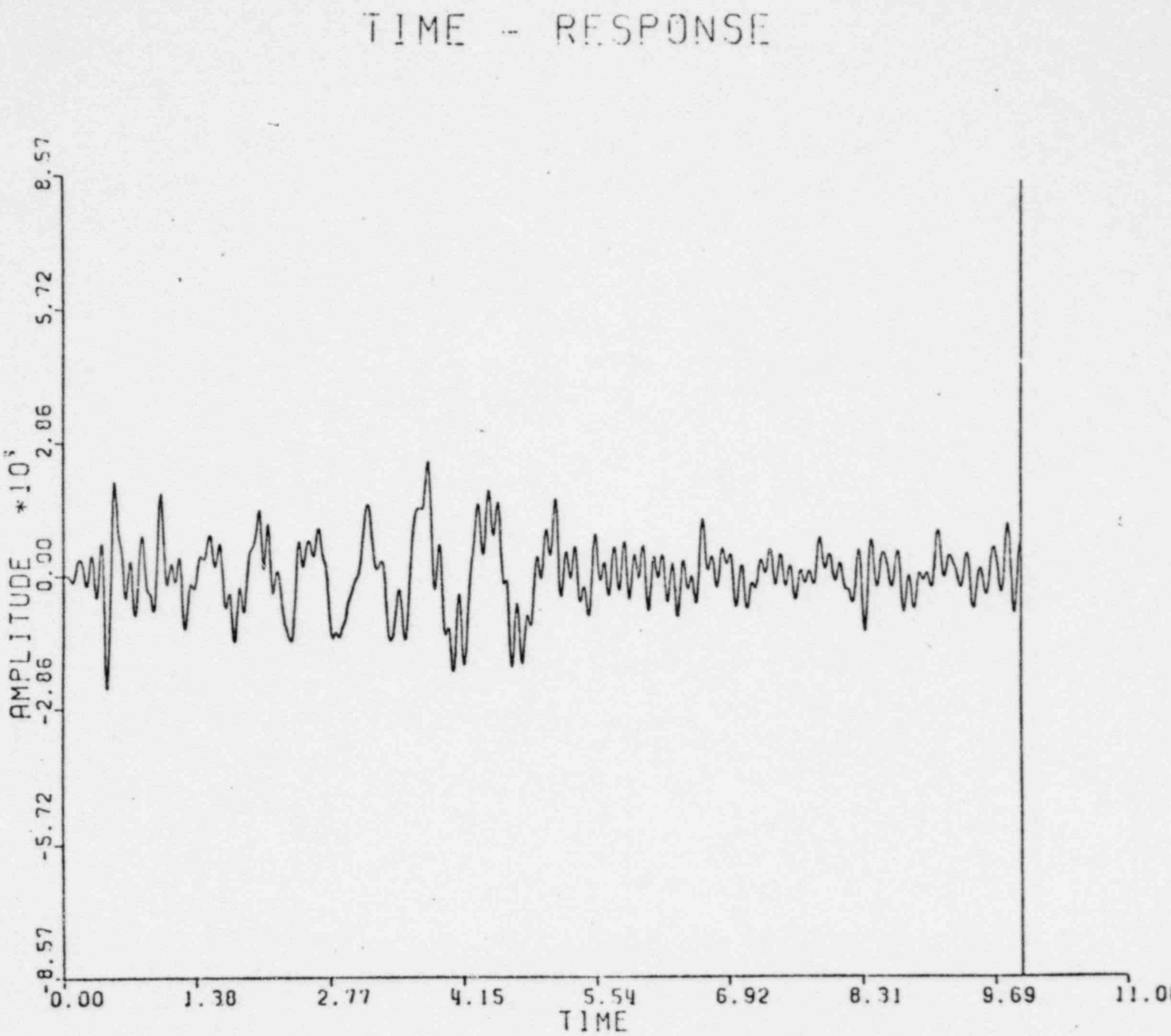


FIGURE E.5 . MAIN STEAM-46I OBE, M_c , CASE 2

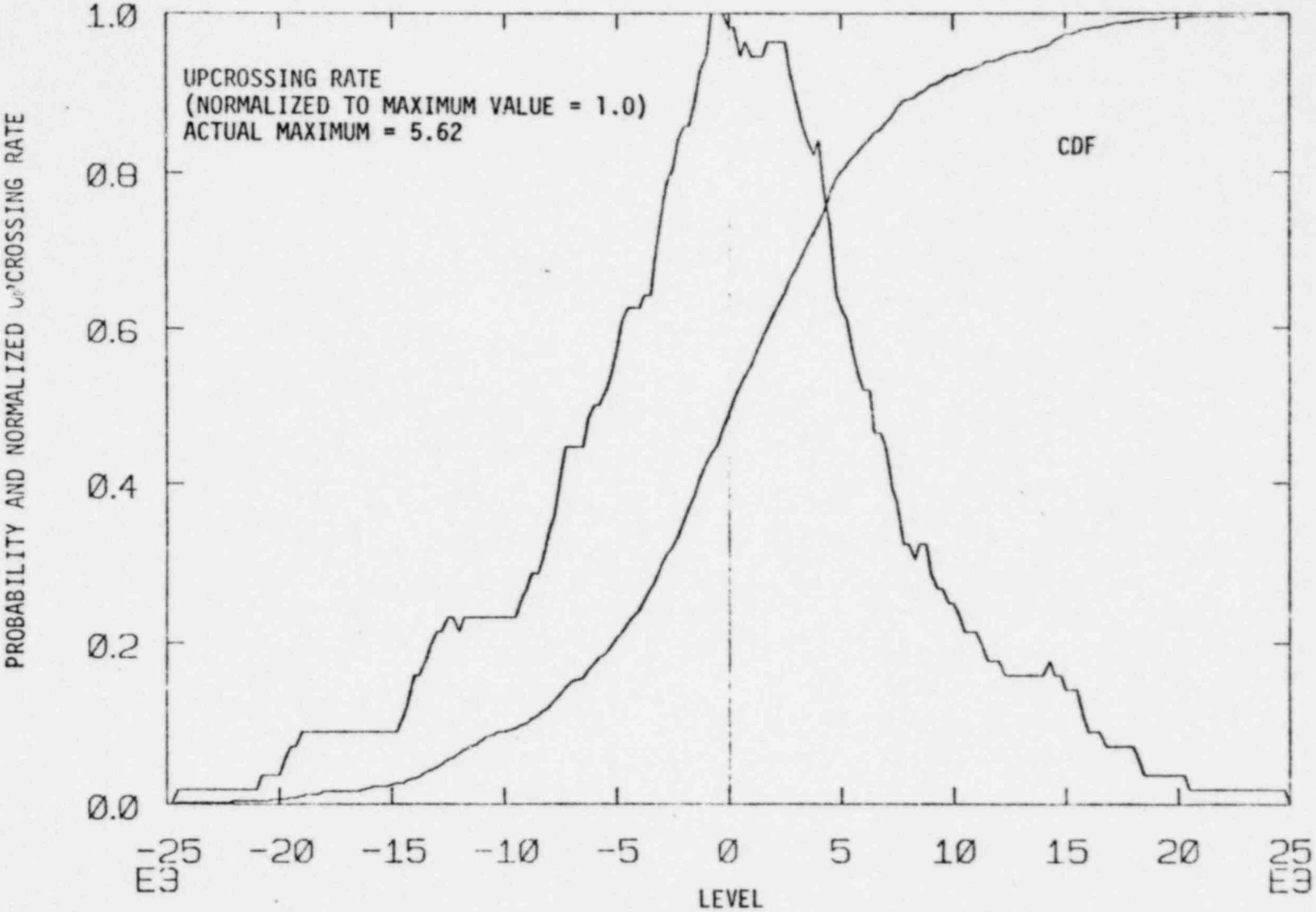


FIGURE E.6 UPCROSSING RATE FUNCTION AND CUMULATIVE DISTRIBUTION FUNCTION FOR EVENT MAIN STEAM-46I OBE, M_c, CASE 2

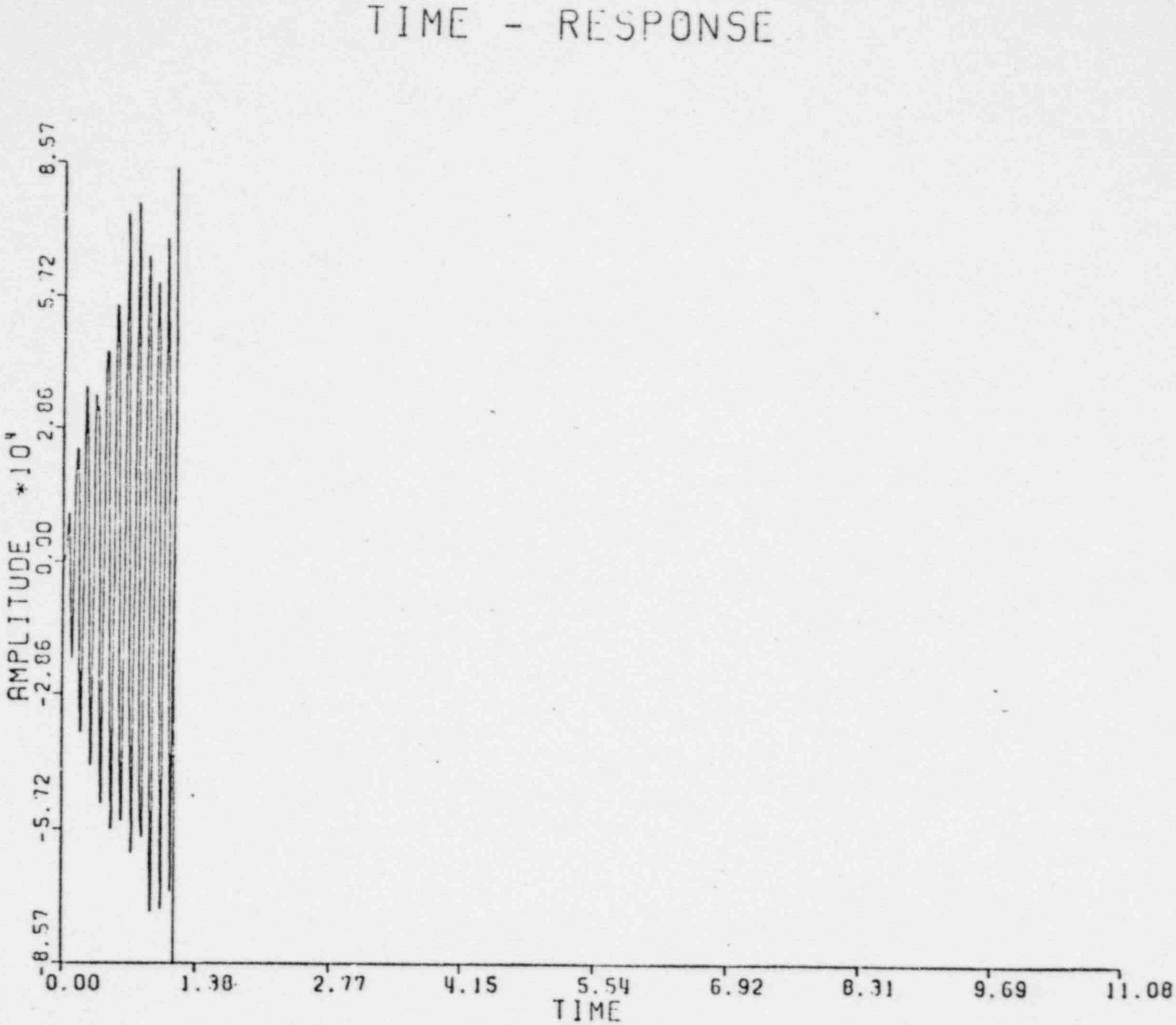


FIGURE E.7. MAIN STEAM-46I SRVBDG, M_c, CASE 2

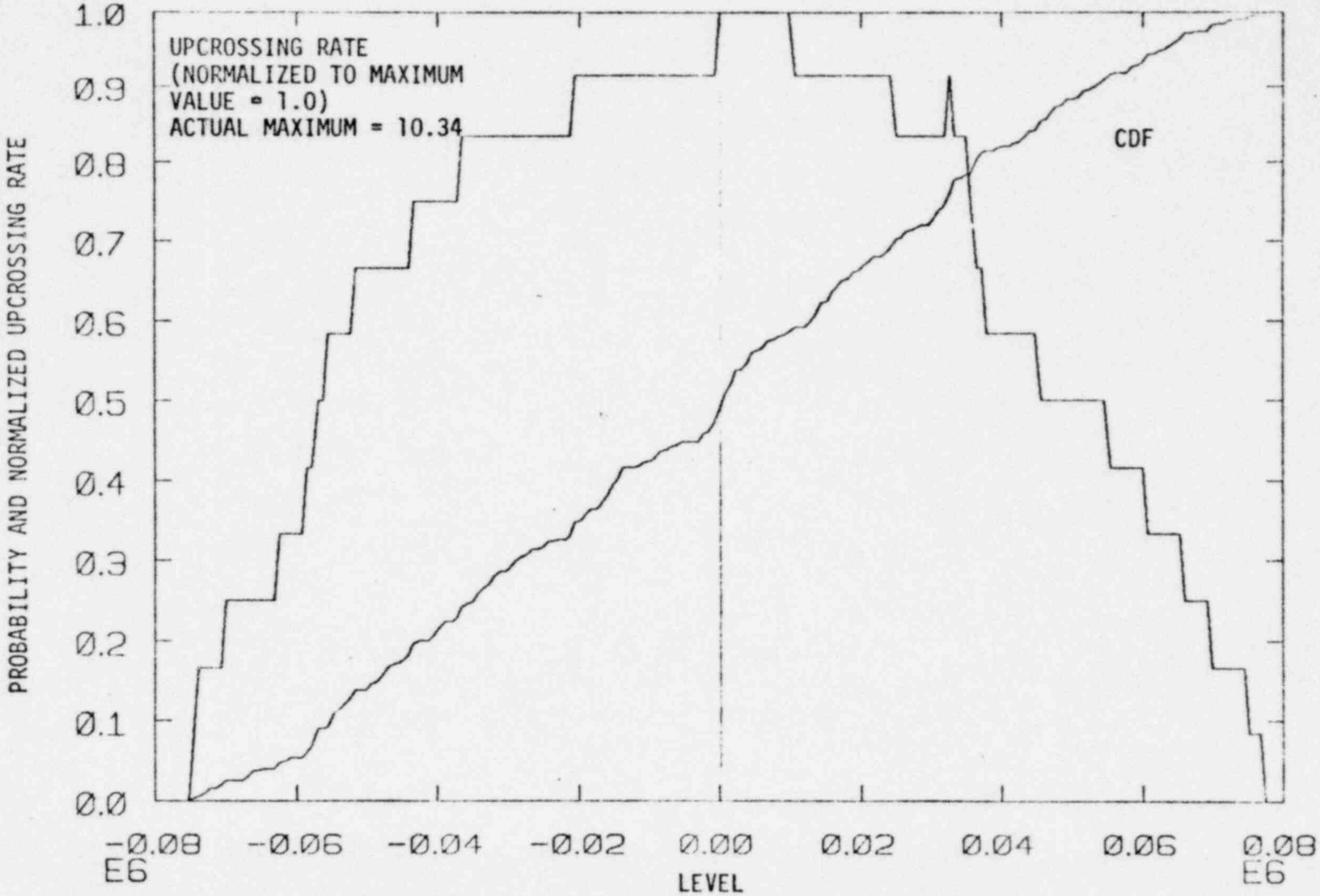


FIGURE E.8 UPCROSSING RATE FUNCTION AND CUMULATIVE DISTRIBUTION FUNCTION FOR EVENT MAIN STEAM-46I SRVBDG, M_c, CASE 2

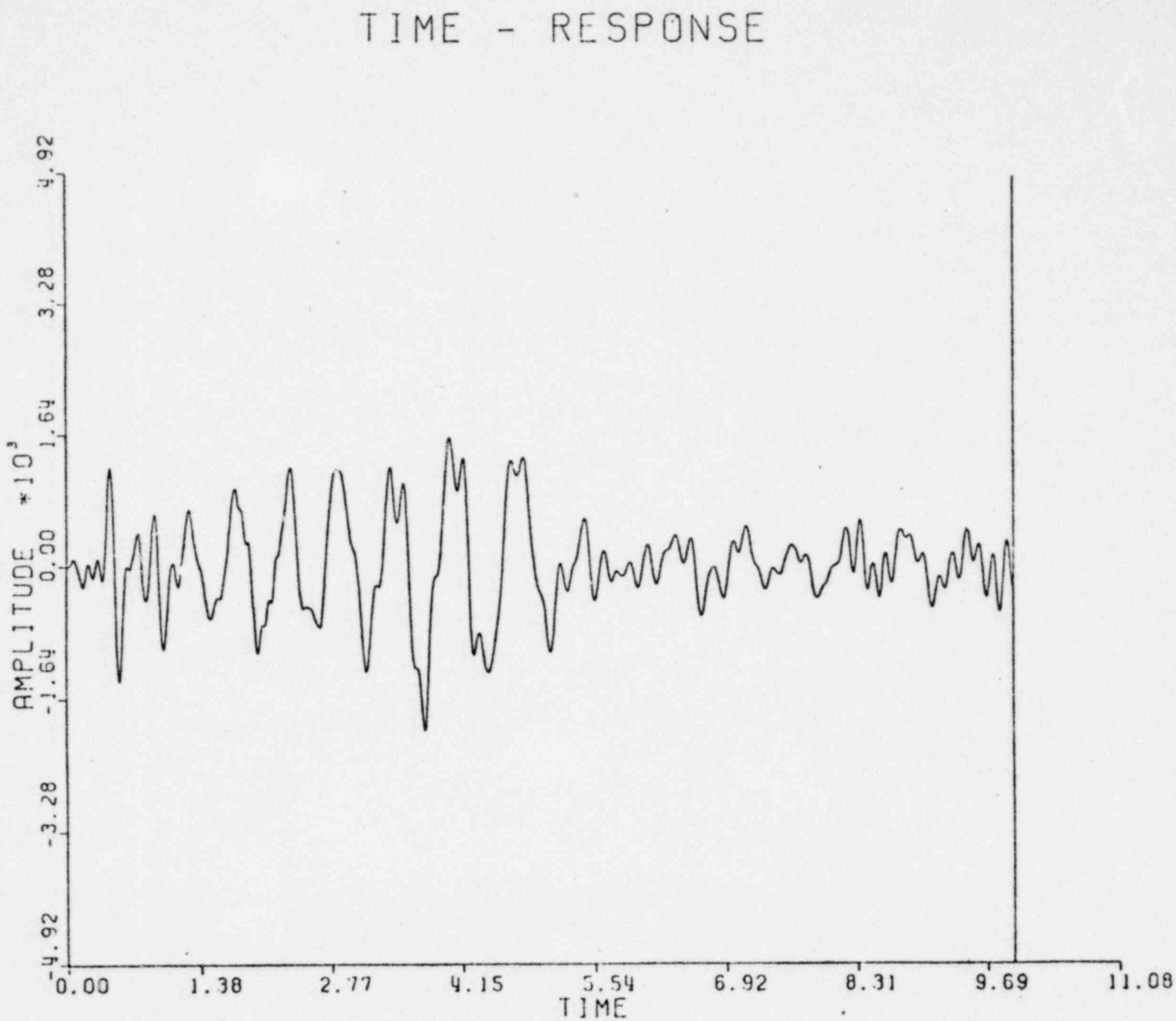


FIGURE E.9 . RHR-WETWELL-1I OBE, M_a, CASE 3

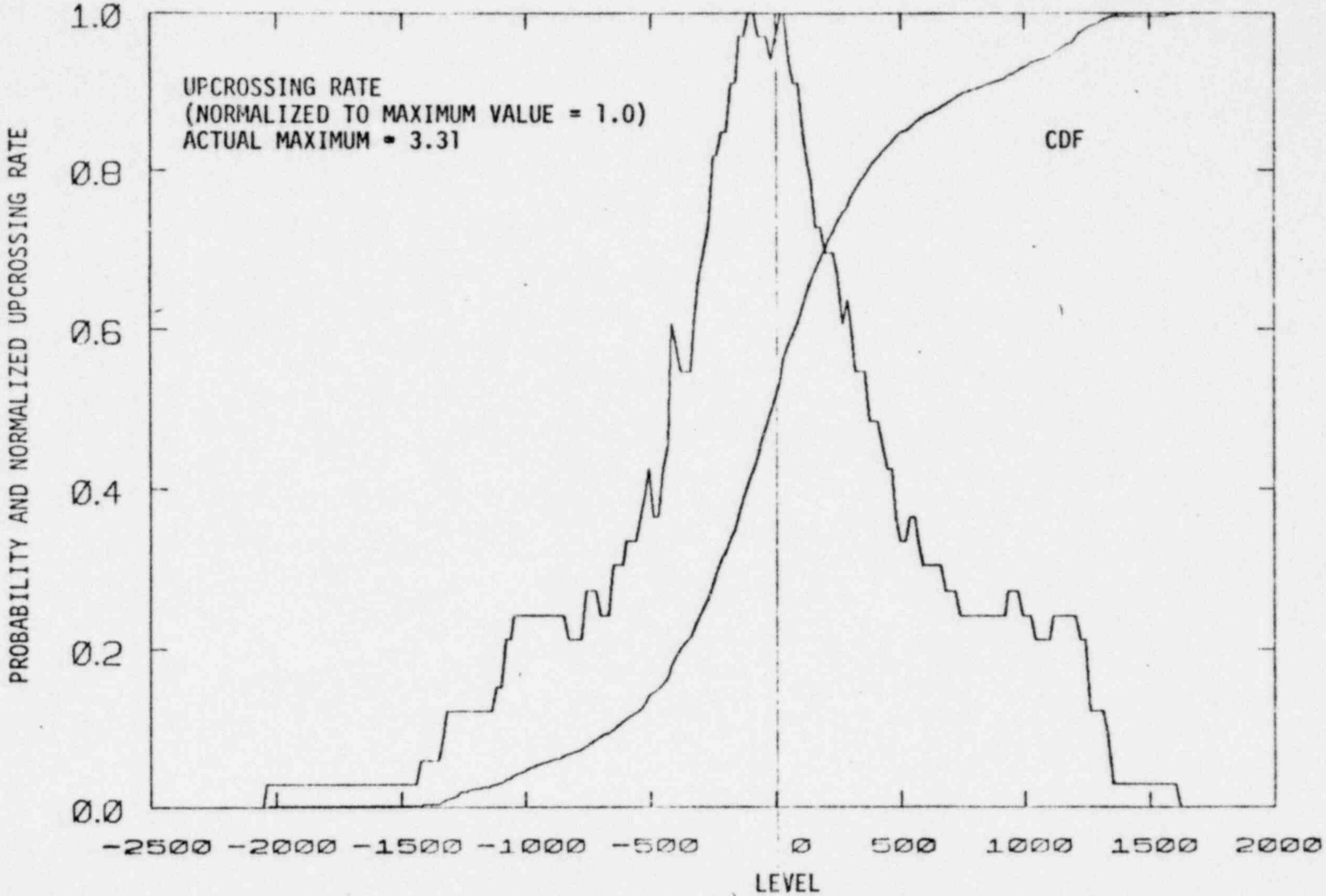


FIGURE E.10 UPCROSSING RATE FUNCTION AND CUMULATIVE DISTRIBUTION FUNCTION FOR EVENT RHR-WETWELL-11 OBE, M_a, CASE 3

TIME - RESPONSE

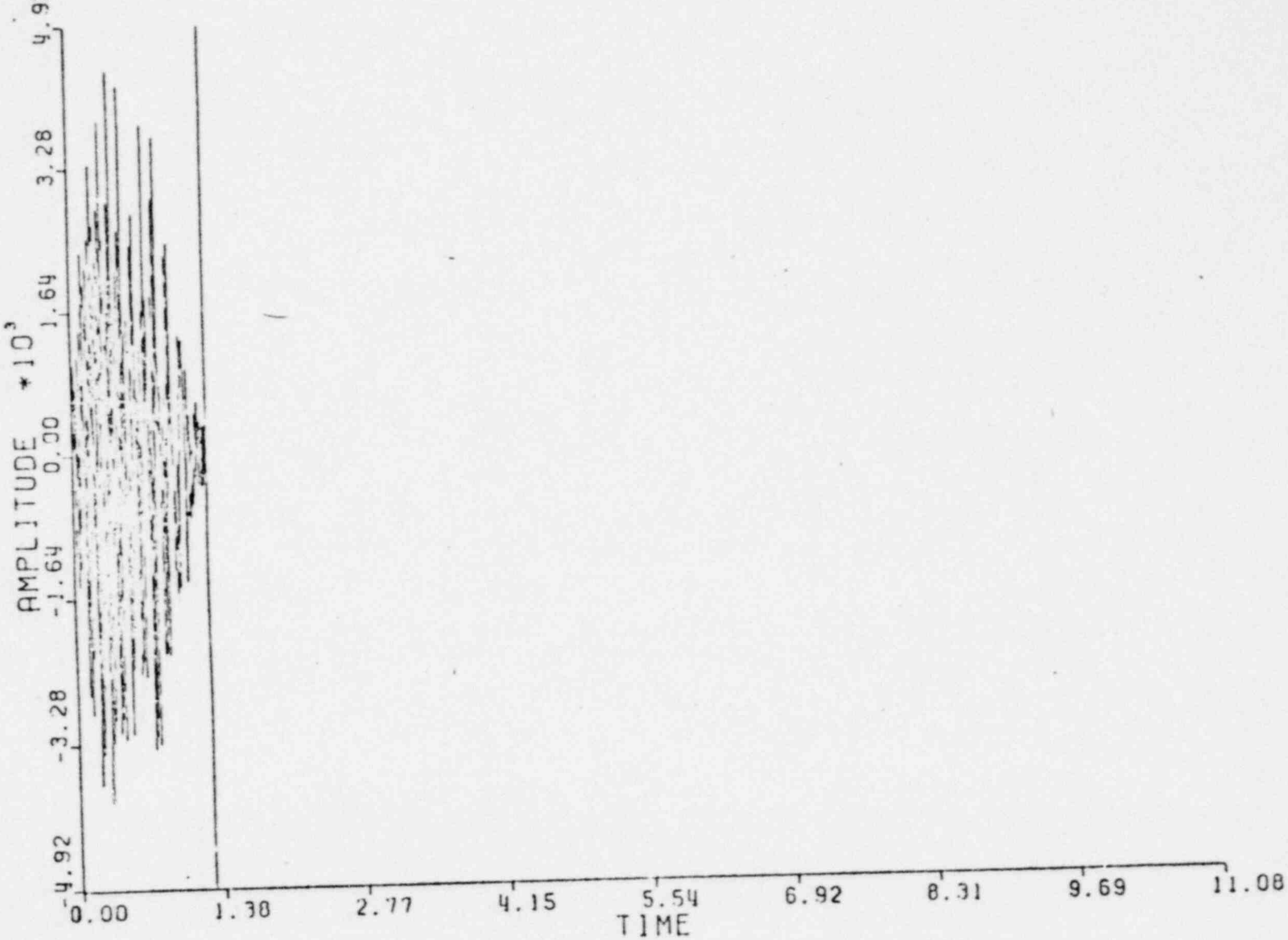


FIGURE E.11 . RHR-WETWELL-1I SRVBUB, M_a, CASE 3

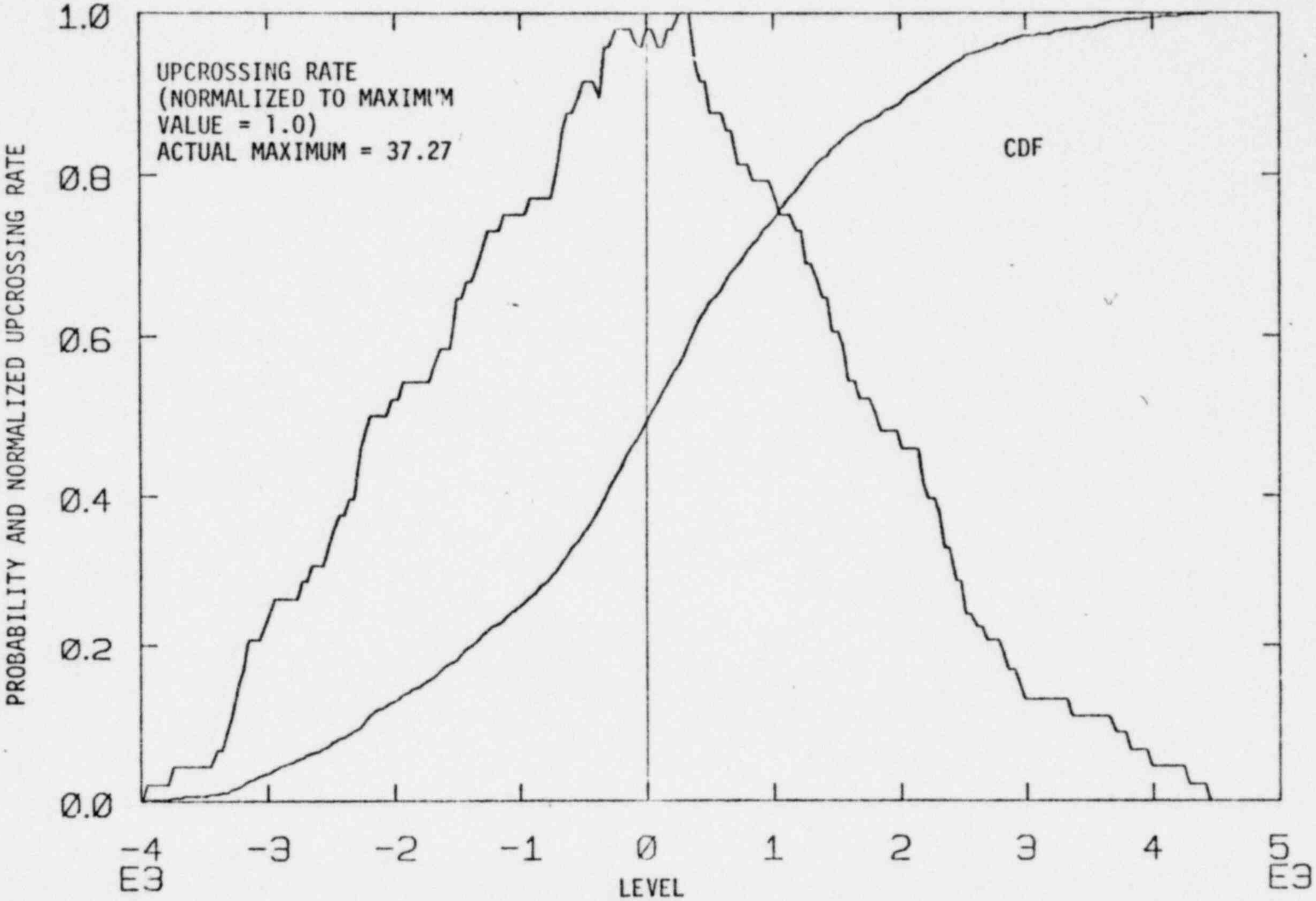


FIGURE E.12 UPCROSSING RATE FUNCTION AND CUMULATIVE DISTRIBUTION FUNCTION FOR EVENT RHR-WETWELL-1I SRVBUB, M_a, CASE 3

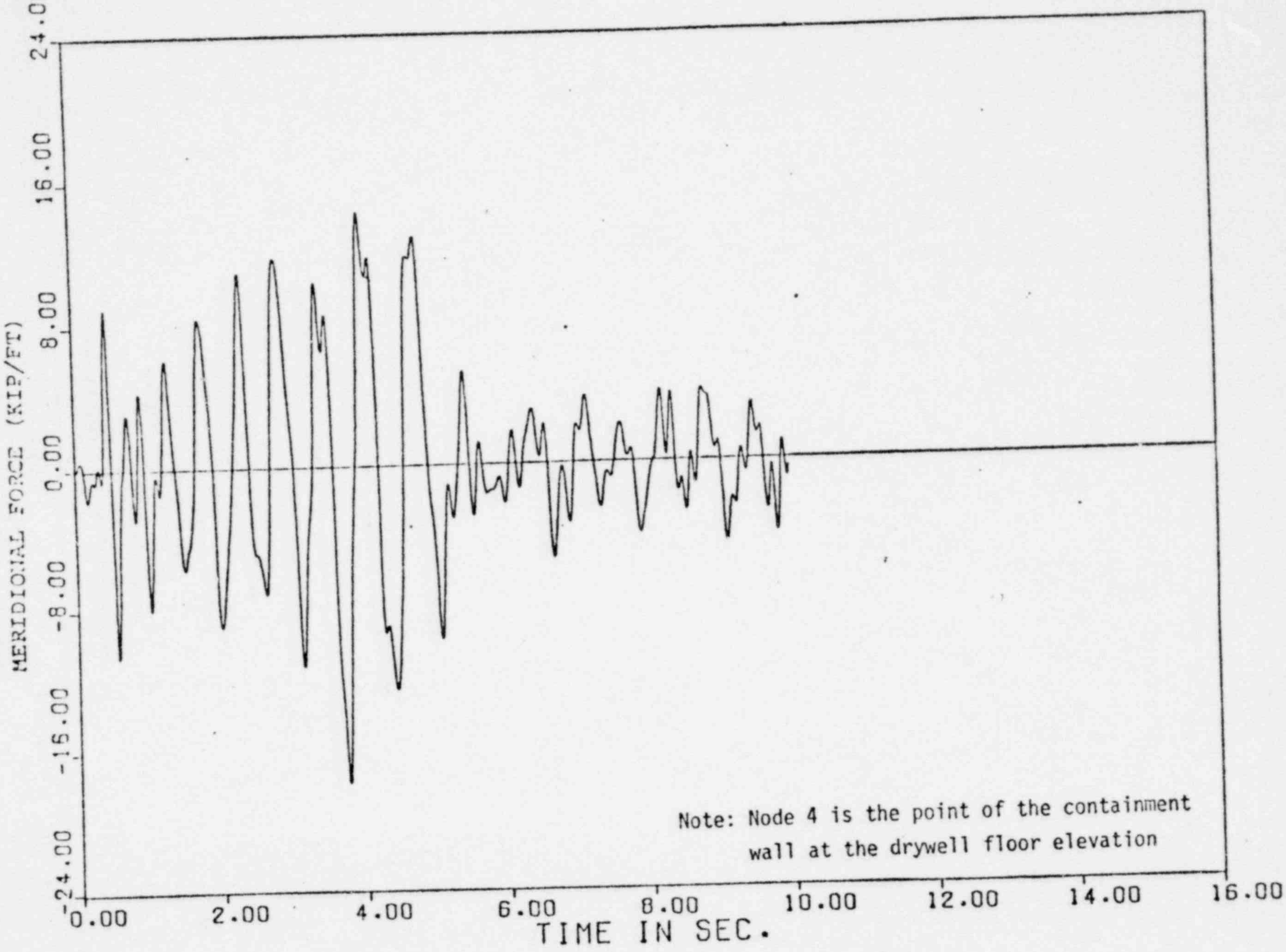


FIGURE E.13. ZIMMER NODE 4 OBE-NS, CASE 4

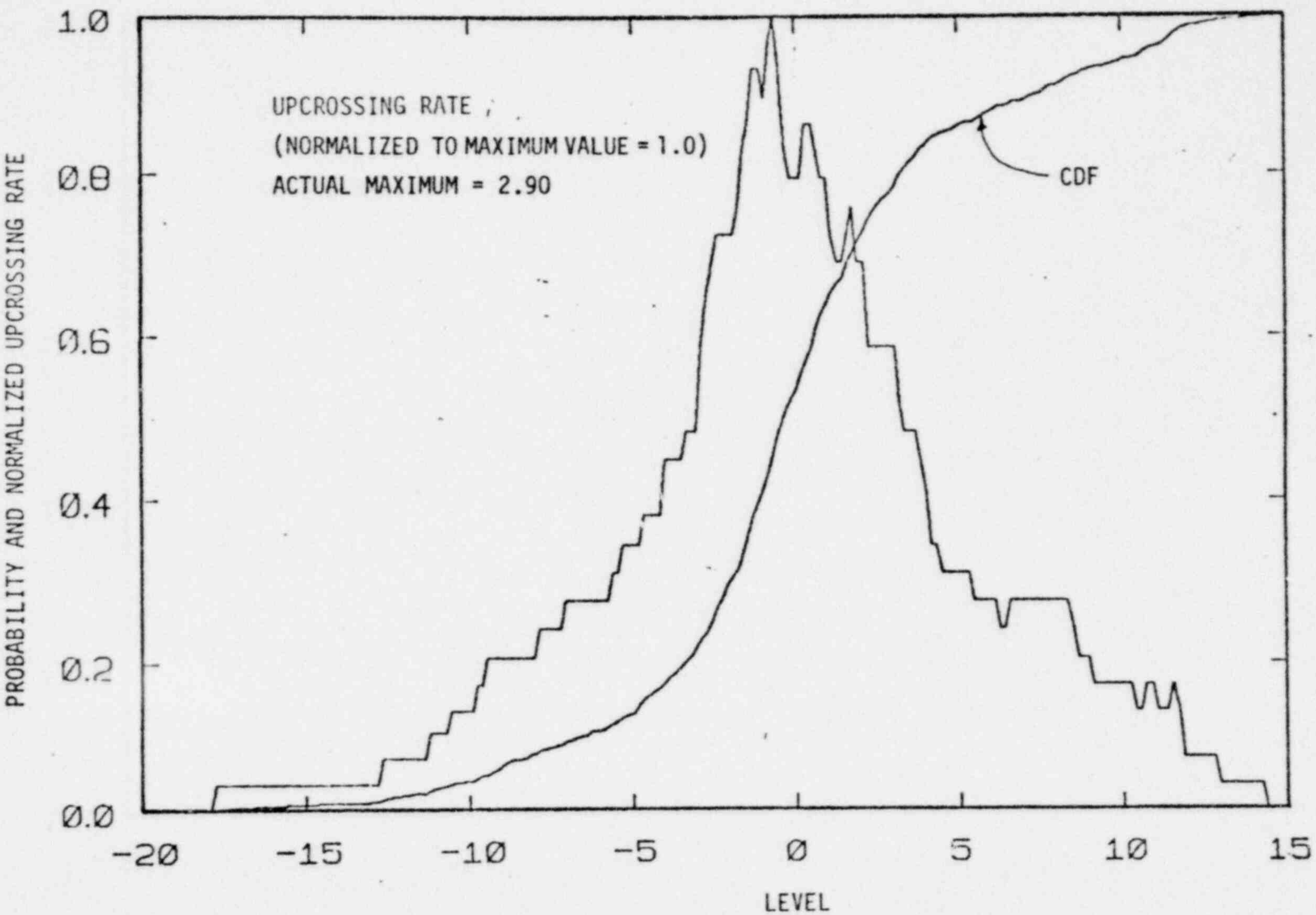


FIGURE E.14 UPCROSSING RATE FUNCTION AND CUMULATIVE DISTRIBUTION FUNCTION FOR EVENT ZIMMER NODE 4 OBE-NS, CASE 4

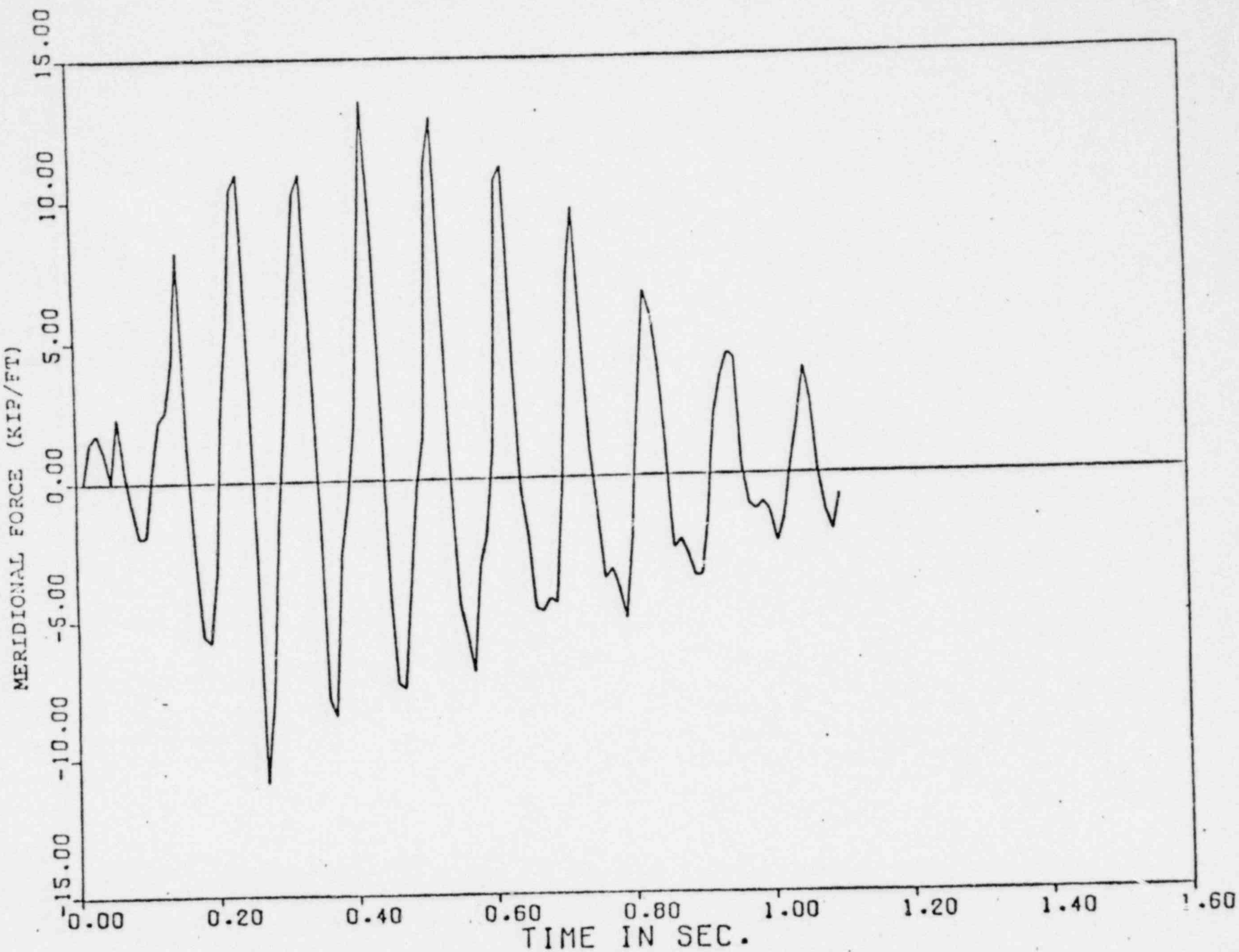


FIGURE E.15 ZIMMER NODE 4 SRV-ALL, CASE 4

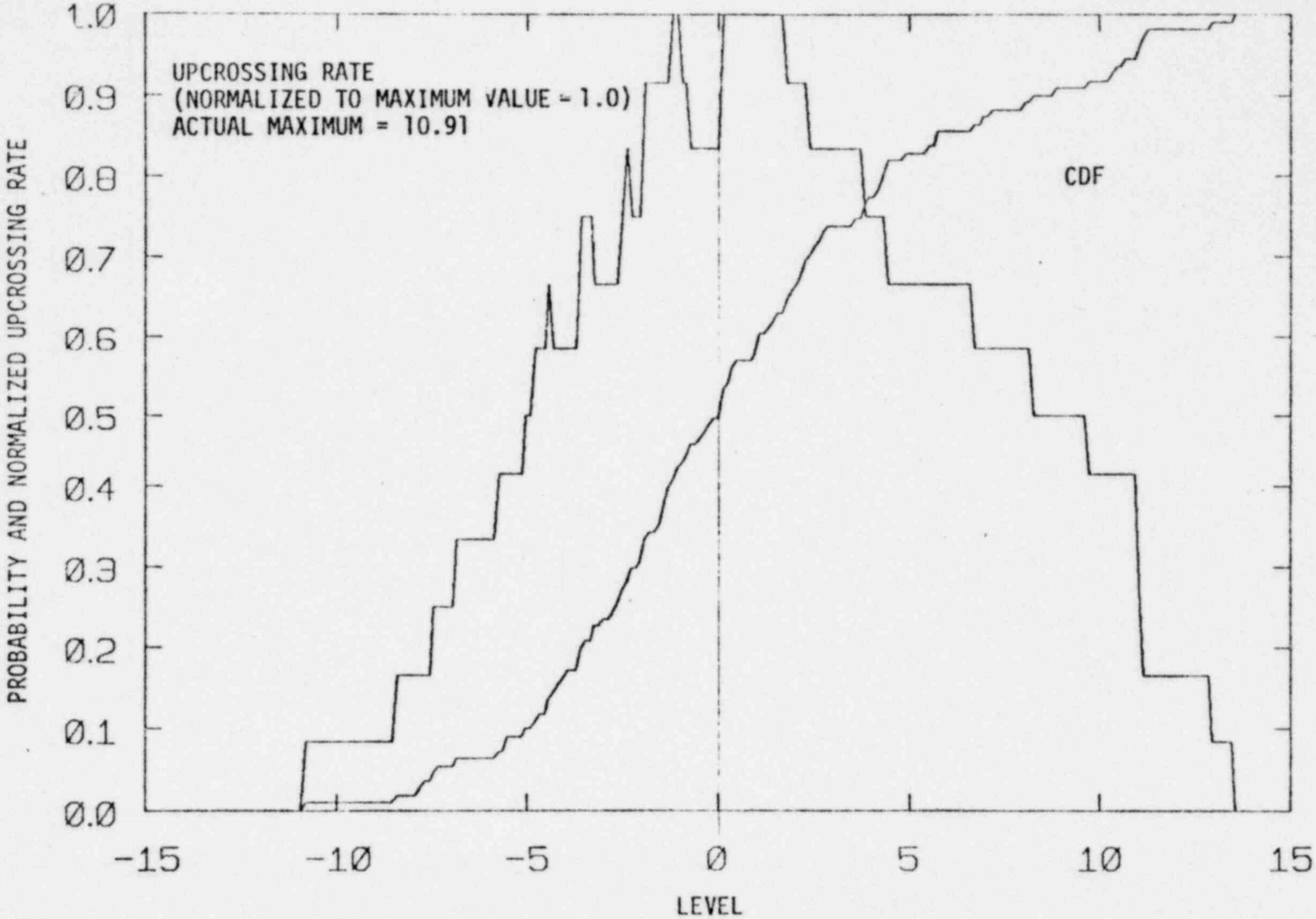


FIGURE E.16 UPCROSSING RATE FUNCTION AND CUMULATIVE DISTRIBUTION FUNCTION FOR EVENT ZIMMER NODE 4 SRV-ALL, CASE 4

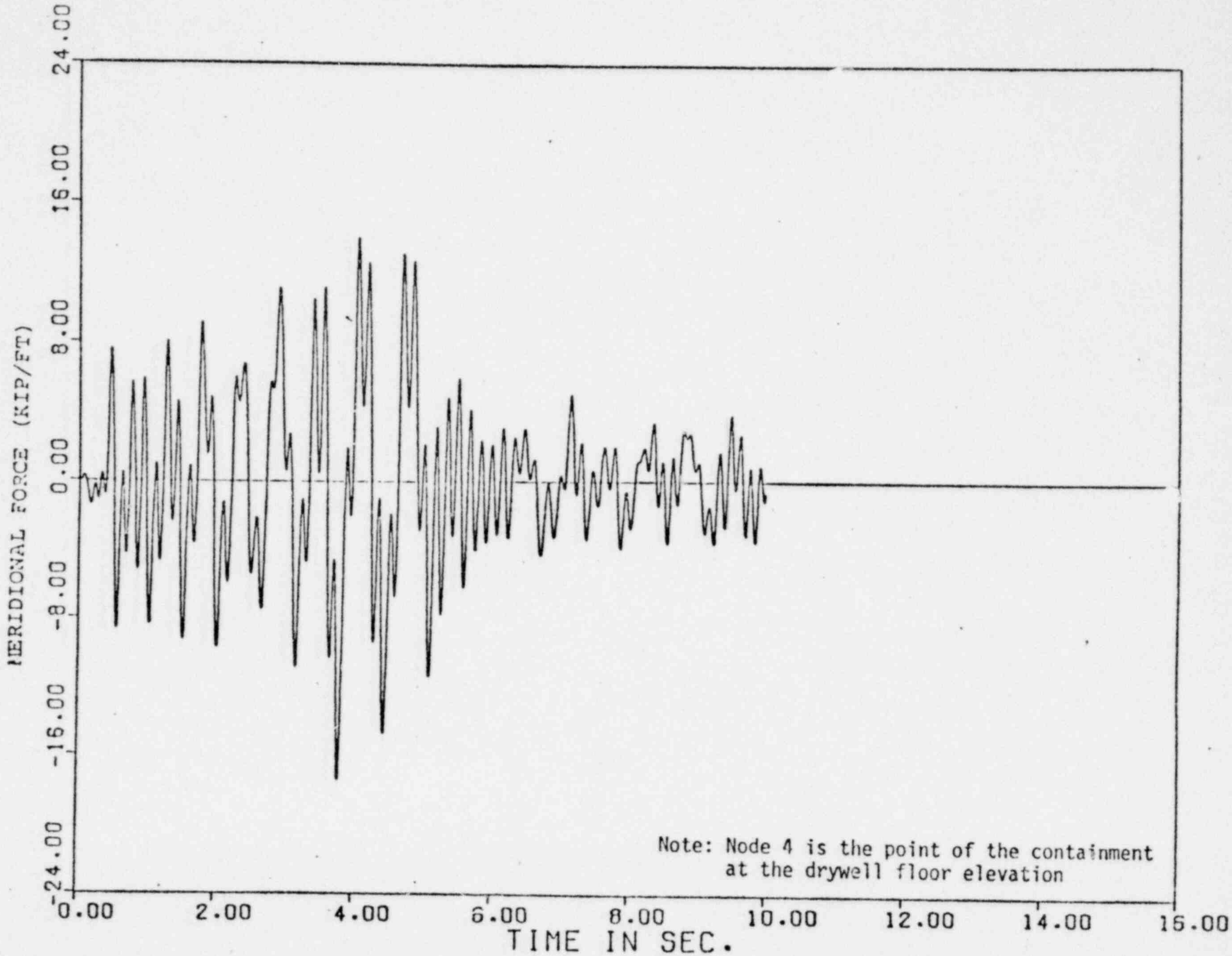


FIGURE E.17. ZIMMER NODE 4 OBE-EW, CASE 5

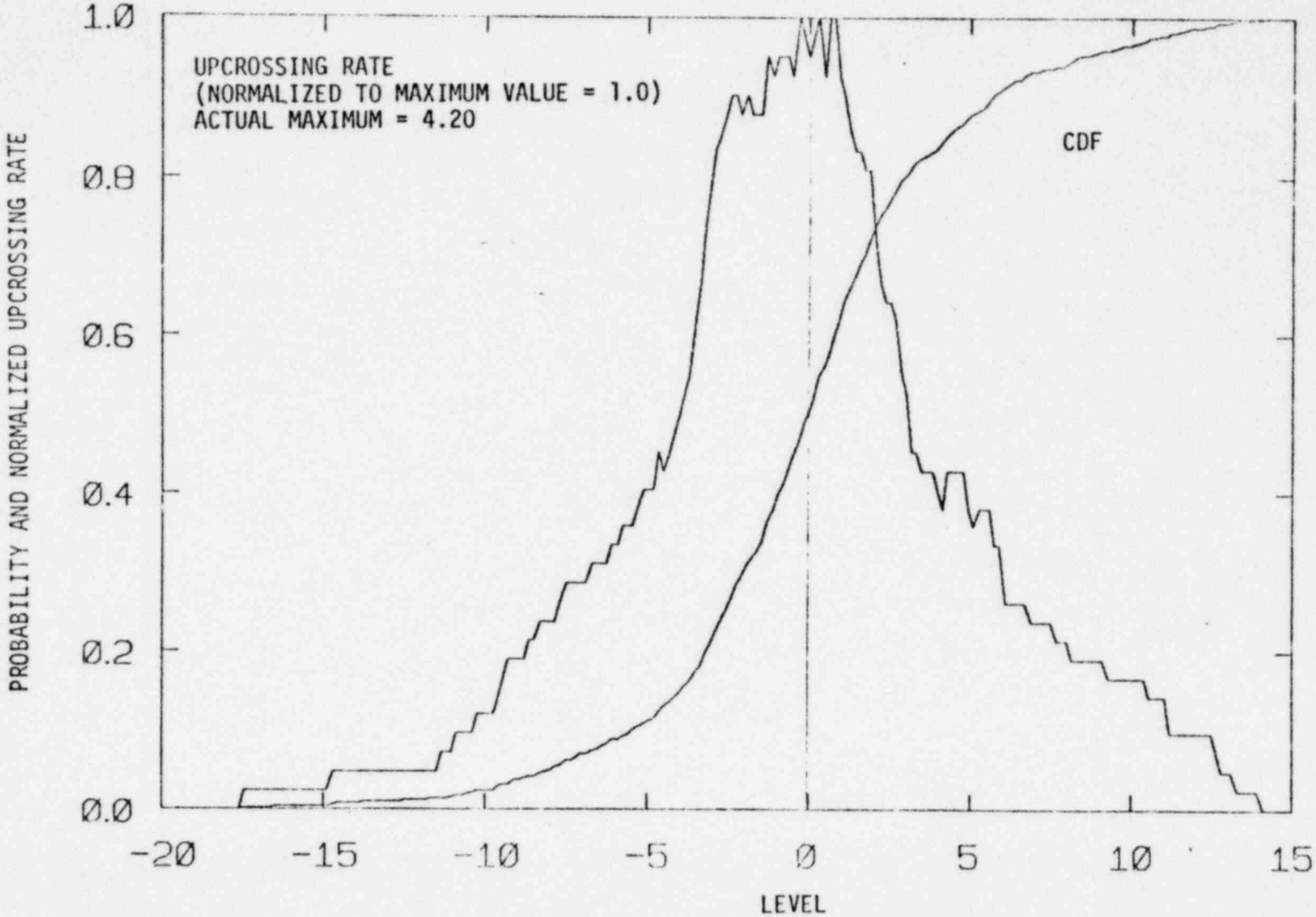


FIGURE E.18 UPCROSSING RATE FUNCTION AND CUMULATIVE DISTRIBUTION FUNCTION FOR EVENT ZIMMER NODE 4 OBE-EW, CASE 5 .

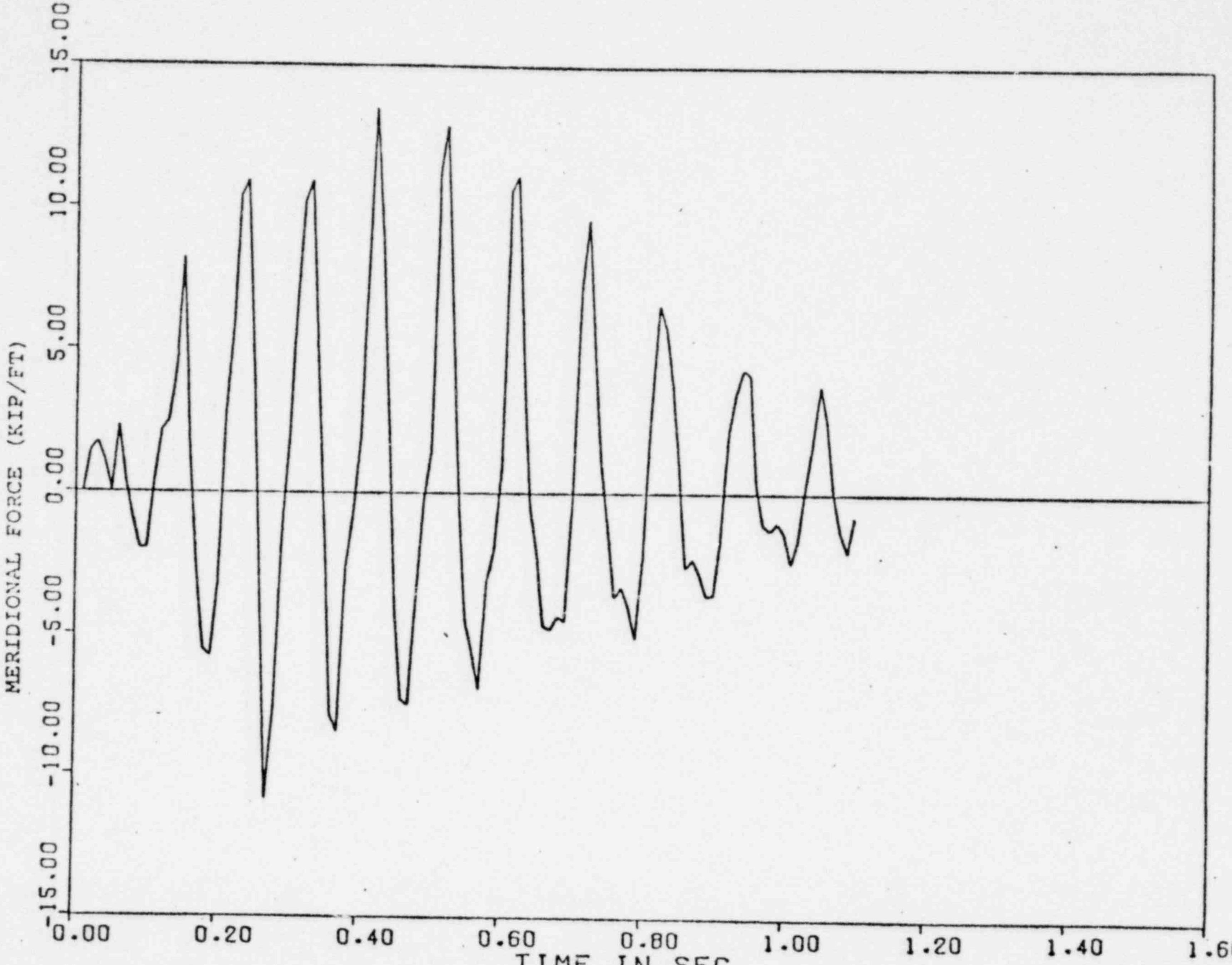


FIGURE E.19 ZIMMER NODE 4 SRV-ALL, CASE 5

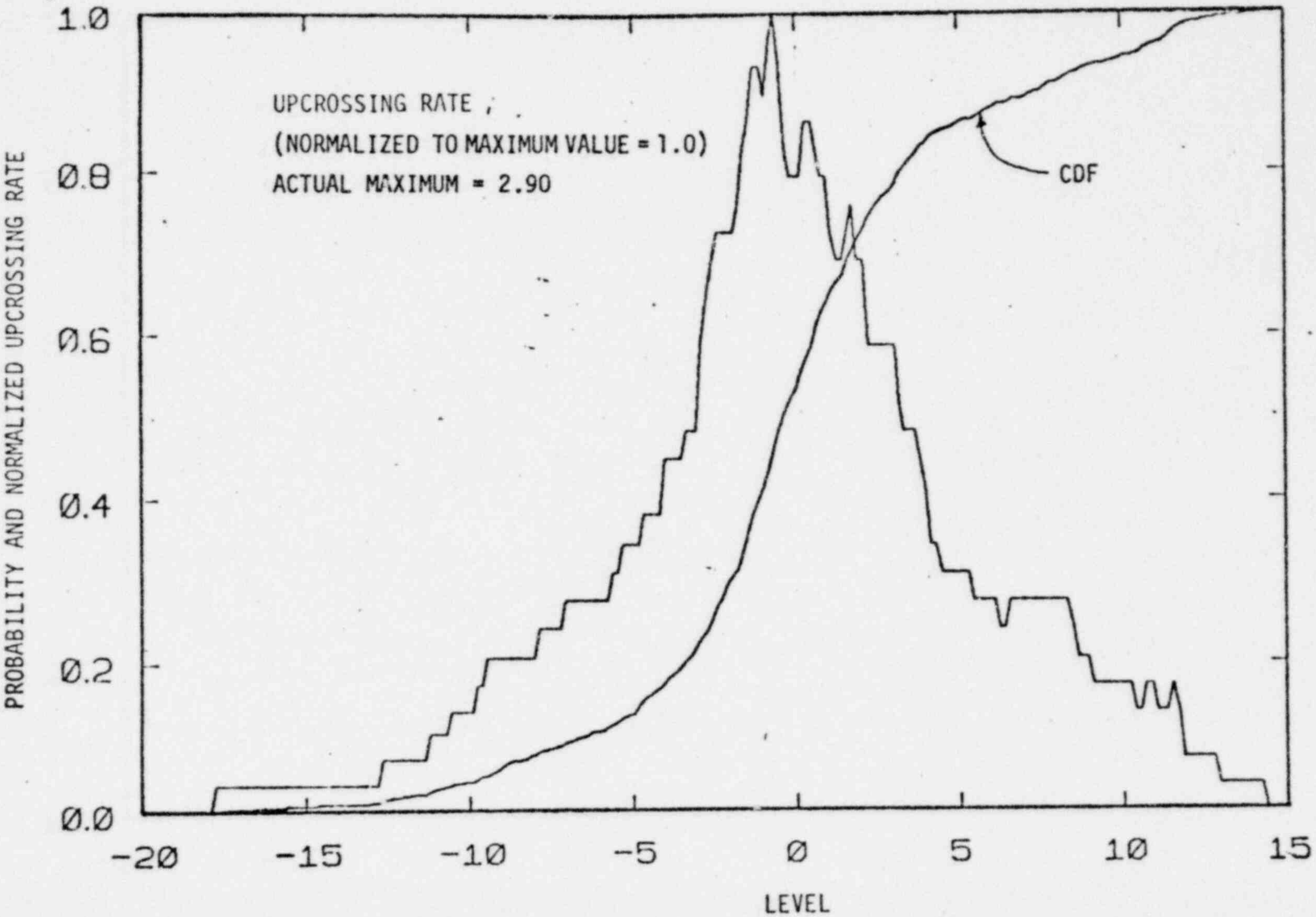


FIGURE E.20 UPCROSSING RATE FUNCTION AND CUMULATIVE DISTRIBUTION FUNCTION FOR EVENT ZIMMER NODE 4 OBE-NS, CASE 5