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INFORMAL REPORT

EFFECT OF DISPERSED CONDENSED PHASE ON THE RAYLEIGH-TAYLOR INSTABILITY OF A GAS-LIQUID INTERFACE

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ABSTRACT

The growth of Rayleigh-Taylor instabilities is studied in relation to liquid entrainment at the interface between accelerating fluids of unequal density. The upper fluid is pure liquid, and the lower fluid is a mixture of vapor and a heavy entrained droplet or particulate phase. Entrainment through this mechanism would occur when the liquid spikes grow into the lower fluid and, eventually, separate into droplets. This work estimates the effect of the presence of heavy droplets or particulates in the immediate vicinity of the interface on the early (linear) stages of instability growth.

The growth of the Taylor instability is computed using a porous medium model of the multi-phase lower fluid, which assumes that the entrained phase is characterized by infinite inertia. The vapor simply flows around the entrained phase. The model is described and calculation results are presented for the rate of instability growth during HCDA bubble expansion. Results are compared with the classical Taylor theory which neglects the presence of the entrained phase, and with a homogeneous model of the multi-phase bubble.

The models suggest that there are physical processes which limit the potential for sodium entrainment by growth of Taylor instabilities. In the case of the porous medium model the limiting factor is the drag imposed by the entrained phase. In the case of the homogeneous flow model the factor is the loss of the driving force (upper-to-lower fluid density difference) for growth of the instability. The results suggest the need for development of a theory of Taylor instabilities which applies to multi-phase expanding bubbles and which accounts for the finite inertial characteristics of the entrained particulate or droplet phase.

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NOMENCLATURE

a	acceleration
g	gravitational acceleration
k	permeability
n	growth constant
m_1	defined by Eq. (30)
p	pressure
p_o	pressure at undisturbed interface
t	time
u	velocity component in x-direction
v	velocity component in y-direction
V	interfacial velocity
x	coordinate perpendicular to undisturbed interface
y	coordinate parallel to undisturbed interface
β	defined by Eq. (40)
κ	disturbance wave number
κ_{CRIT}	critical wave number
κ_{CUTOFF}	cutoff wave number
μ	viscosity
ν	kinematic viscosity
ξ	radius of curvature
ρ	density
σ	surface tension
ϕ	potential function
ψ	stream function

Subscripts

1	upper fluid
2	lower fluid
s	interface
1,s	upper fluid at interface
2,s	lower fluid at interface

I. INTRODUCTION

Safety analyses of sodium-cooled fast breeder reactors include studies of hypothetical core disruptive accidents (HCDA's). A loss-of-flow accident with failure to scram would lead to coolant boiling and expulsion from the reactor core. Fuel disruption and melting would ensue. One possible accident sequence would, at this point, lead to a rapid deposition of energy in the fuel and concomitant vaporization of a portion of the fuel and structural steel. A vapor bubble would develop in the core region, containing substantial amounts of liquid and solid UO_2 and steel; liquid sodium could also be present. Figure 1 schematically represents the expanding HCDA bubble within the reactor vessel.

The bubble expands through the upper, undamaged, core region into the sodium pool and in the process entrains some of the liquid coolant. It has been argued [1] that the amount of work of which the expanding bubble is capable depends strongly on the quantity of entrained coolant. "Optimal" entrained amounts can double the work potential. More importantly, however, larger amounts of coolant entrainment can act as energy sinks and reduce the work done very substantially and with it the danger of structural damage to the reactor vessel.

Several mechanisms of entrainment have been suggested [2,3]. One of them is the Rayleigh-Taylor instability of the coolant-bubble interface due to the acceleration of the interface into the coolant. The instability leads to a growth of "spikes" of liquid into the bubble and penetration of vapor columns into the coolant. Another possible mechanism is jet-like entrainment by the gas jet or bubble penetrating into the coolant pool. At the walls of the

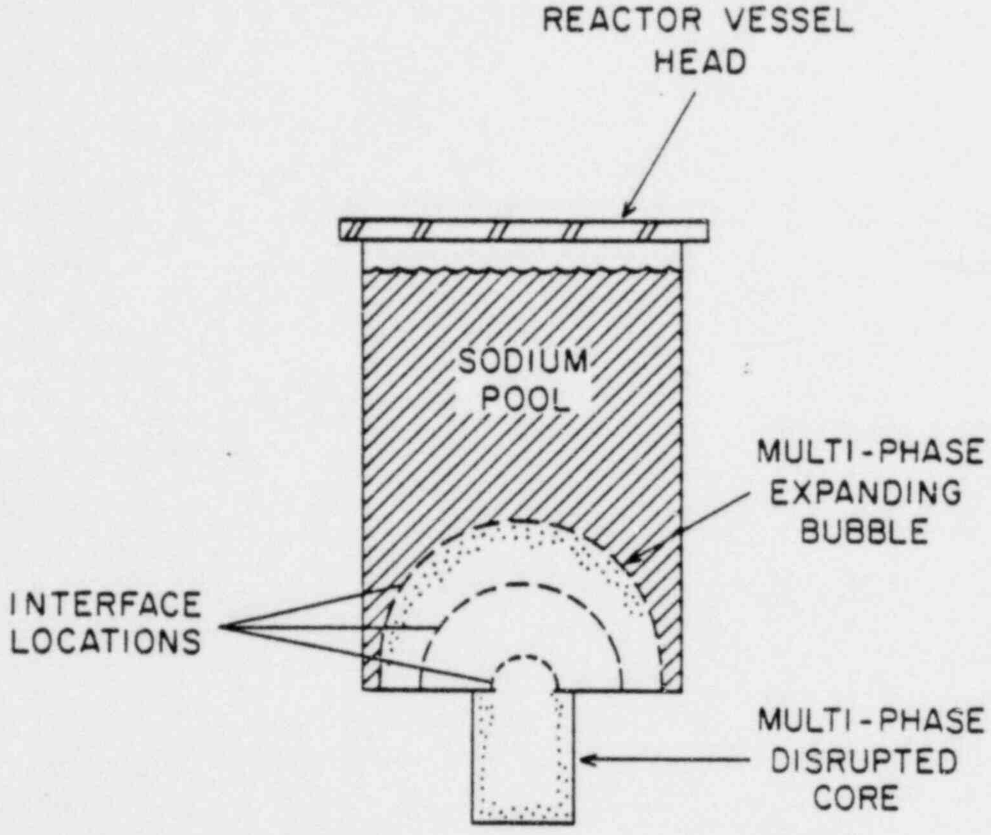


Figure 1. Schematic of HCDA Bubble in Reactor Vessel.
(BNL Neg. No. 3-1296-80)

passage between the core and the pool, and later on the walls of this coolant plenum and possibly any above-core structure, thin liquid films may be left behind and the gas flow parallel to the resulting interface may give rise to a Kelvin-Helmholtz instability. This last mechanism may also become involved if the gas develops a swirling flow within the expanding bubble. A less clearly defined mechanism involving sudden condensation in the proximity of the interface has also been proposed [4]. Experimental evidence [3] suggests that several, if not all, of these mechanisms may be involved in various stages of the expansion of the gas bubble and that possibly some even stronger mechanisms of a transient nature may be responsible for the unusually strong mixing during the early stages of the expansion.

The work presented in this report is concerned with the Rayleigh-Taylor instability mechanism. The objective of the work is to estimate the effect of the presence of droplets or solid particles of heavy condensed fuel or steel in the immediate vicinity of the interface on the instability growth mechanism.

Entrainment through the Rayleigh-Taylor mechanism occurs when the liquid spikes grow into the gas and can no longer be considered as "small" disturbances of the interface. The growth rates of the spikes and, therefore, the entrainment rate, cannot be expected to be governed by "small-disturbance" type linear theory, but rather by a non-linear theory appropriate to the later stages of development of the Rayleigh-Taylor instability. Unfortunately, analytical attempts at developing a nonlinear theory of the instability [5-7] have met with only limited success even under the most simple of circumstances, although considerable insight has been provided by numerical studies [8-9]. Under these circumstances, it is reasonable to initially seek the effect of the presence of droplets or solid particles in the gas phase on the initial, linear period of interfacial growth of disturbances. This is the approach adopted here. The results of the analysis provide the range of conditions (based upon model assumptions) for which linear growth is possible. These conditions also govern the circumstances for which liquid entrainment may occur by growth of Rayleigh-Taylor instabilities. The non-linear analysis governing the rate of entrainment is left to future research.

The analysis requires that some assumption be made regarding the relative magnitude of the drag forces between the vapor and the droplets (or particles) and the inertia forces. In one extreme, the drag forces may be assumed so large that the particles follow the continuous gas phase motion without slip. In this case, one could treat the mixture as a homogeneous one with an appropriate mean density. If this mixture density reached the density of the homogeneous liquid on the other side of the interface, the Rayleigh-Taylor instability would disappear [12]. In the other extreme, the droplets and particles could be considered so heavy that they remain stationary, forming a "porous" matrix through which the gas moves. The latter is the model adopted here in the hope that it will provide a bounding estimate of the effect of the particles. The analysis of the intermediate situations, with finite inertia and slip of the condensed phase will be presented in a companion report [10].

II. ANALYSIS

2.1 Classical Raleigh-Taylor Analysis of Homogeneous Fluid Systems

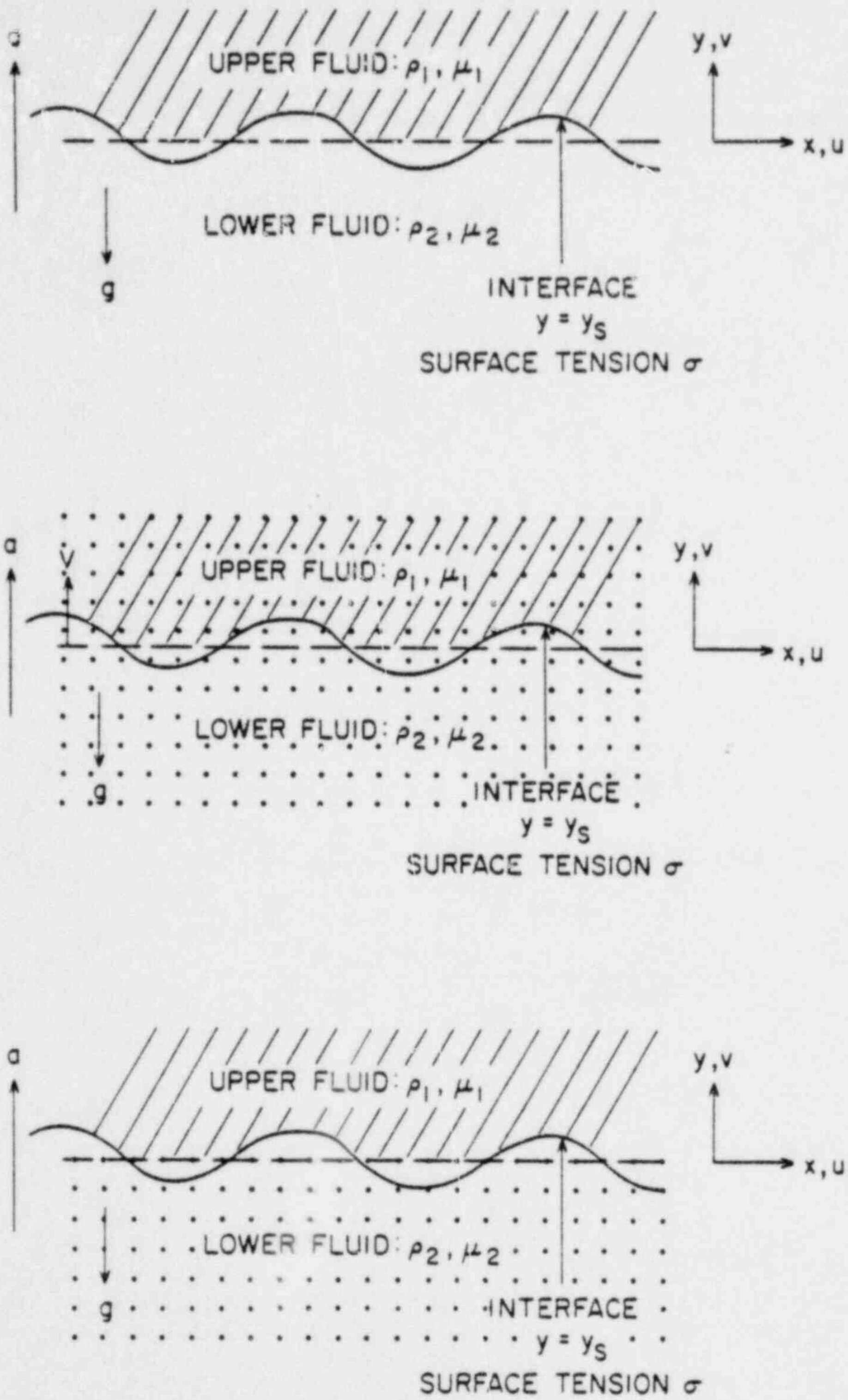
In the classical analysis of the Rayleigh-Taylor instability [11,12] the interface between two initially stagnant incompressible fluids is accelerated from the less dense to the more dense fluid. The interface, shown systematically in Fig. 2(a) is assumed to be subjected to an infinitisimal disturbance and the resulting flow is required to satisfy conditions of continuity of normal velocity component at the interface, and vanishing of the velocity far away, in both directions, from the interface. Continuity of pressure at the interface yields a dispersion relation for the disturbance, relating the growth rate of the disturbance amplitude to the wave number of the disturbance. Under the assumptions of no viscosity or surface tension, disturbance potentials of the form

$$\phi = Ae^{nt \pm \kappa y} \cos \kappa x \qquad (1)$$

satisfy the equations of motion and conditions at infinity. The well-known analysis yields the result that the interface is unstable to disturbances of all wavelengths and that the growth rate is given by

$$n = \sqrt{a \kappa \frac{\rho_1 - \rho_2}{\rho_1 + \rho_2}} \qquad (2)$$

As long as $\rho_1 > \rho_2$, n is real for $a > 0$ and all κ. In the presence of surface tension, the pressure at the disturbed interface becomes dis-



(a) Raleigh-Taylor

(b) Saffman-Taylor

(c) Present Work

Figure 2. Schematics of Interfacial Region and Definition of Coordinate System. (BNL Neg. No.s 3-1292/93/94-80)

continuous, the discontinuity being proportional to the curvature of the interface. This leads to the result that [11,13]

$$n = \left[a\kappa \frac{\rho_1 - \rho_2}{\rho_1 + \rho_2} - \frac{\kappa^3 \sigma}{\rho_1 + \rho_2} \right]^{1/2} \tag{3}$$

and the fact that n becomes imaginary for all $\kappa > \kappa_{MAX}$, where

$$\kappa_{MAX} = \sqrt{\frac{a\,(\rho_1 - \rho_2)}{\sigma}} \tag{4}$$

In other words for $\kappa > \kappa_{MAX}$, exponential growth is replaced by oscillatory behavior. As a corollary n has a maximum for

$$\kappa_{CRIT} = \sqrt{\frac{a\,(\rho_1 - \rho_2)}{3\sigma}} \tag{5}$$

which corresponds to the most unstable disturbance mode. If viscosity is taken into consideration [11,13], the velocity components normal and parallel to the interface must be continuous; continuity of the shear stress at the interface is also required. Solutions of the equations of motion, satisfying these conditions are constructed by appropriate linear combinations of a potential ϕ and a stream function ψ of the form

$$\begin{aligned} \phi &= A\, e^{nt \pm \kappa y} \cos \kappa x \\ \psi &= A\, e^{nt \pm my} \sin \kappa x \end{aligned} \tag{6}$$

This leads to a complex dispersion relation of higher order in n. The overall result is that viscosity reduces the growth rates, and that the reduction is more pronounced for higher wave numbers.

The classical theory can be applied to two-phase lighter fluids (denoted by subscript '2') by assuming that two-phase mixtures act as homogeneous fluids with effective density ρ_2. The theoretical result given by Eq. (2) then predicts, as discussed in the Introduction, that as the mixture density ρ_2 approaches ρ_1, the growth rate of disturbances vanishes.

2.2 Instability Growth in a Porous Matrix

Saffman and Taylor [14] considered the penetration of a less viscous fluid into a more viscous one in a porous medium under the action of gravity. Both fluids are assumed to be moving within the porous medium, as shown in Fig. 2(b). Here the velocity of each fluid is assumed to obey Darcy's law

$$\underset{\sim}{V} = -\frac{k}{\mu} \text{ grad } (p + \rho g y) \tag{7}$$

The interface between the fluids, in the Saffman-Taylor model has a velocity V and it is stable or unstable with respect to small disturbances according to whether

$$\left(\frac{\mu_2}{k_2} - \frac{\mu_1}{k_1} \right) V + (\rho_2 - \rho_1) g \begin{matrix} > 0 \\ < 0 \end{matrix} \tag{8}$$

respectively. Both the fluids are moving within the porous body but the permeability may be different with respect to each fluid.

2.3 Instability Growth in Two-Phase Systems: Porous Matrix Model of Lower Fluid Containing Heavy Entrained Phase

2.3.1. Neglecting Surface Tension and Viscosity of Upper Fluid.

In our analysis, we extend the Saffman and Taylor model in applying it to model the behavior of two-phase lower fluids (component 2). The system is represented schematically in Fig. 2(c). The upper fluid (component 1) is assumed to be incompressible and inviscid. Surface tension is neglected. (The effects of viscosity and surface tension are considered below.)

The lower fluid is assumed to be a two-phase medium containing a discrete heavy phase in droplet or particulate form, embedded in a continuous light phase. It is assumed that the discrete phase is infinitely heavy, such that it remains stationary as the light fluid flows around it. With this infinite inertial approximation, the continuous fluid-particle (or droplet) slip ratio is infinite. The effect of the particles is to exert a drag force on the flowing continuous fluid. It is further assumed that this drag force can be approximated by using a model which represents the stationary particulate or droplet phase as a proous matrix and considers the continuous phase as flowing through it.

The lower fluid is moving within the porous body in accordance with Darcy's law. The undisturbed interface between the fluids coincides with the upper boundary of the porous body and is accelerated towards the heavier fluid, as shown in Fig. 2(c). The linearized equations of motion for the upper fluid are

$$\frac{\partial u_1}{\partial x} + \frac{\partial v_1}{\partial y} = 0$$

$$\frac{\partial u_1}{\partial t} = -\frac{1}{\rho_1}\frac{\partial p_1}{\partial x} \qquad (9)$$

$$\frac{\partial v_1}{\partial t} = -\frac{1}{\rho_1}\frac{\partial p_1}{\partial y} - a$$

and their solution is assumed in the form

$$u_1 = -\frac{\partial\phi_1}{\partial x} \qquad v_1 = -\frac{\partial\phi_1}{\partial y} \tag{10}$$

$$\phi_1 = A\ e^{-\kappa y + nt} \cos \kappa x \tag{11}$$

$$p_1 = p_o - \rho_1\, a\, y + \rho_1 \frac{\partial\phi_1}{\partial t} \tag{12}$$

The form of ϕ_1 guarantees vanishing of the velocity far above the interface. The integration constant p_o represents the average pressure of the undisturbed interface.

The lower fluid obeys Darcy's law

$$u_2 = -\frac{k}{\mu_2}\frac{\partial p}{\partial x} = -\frac{\partial\psi_2}{\partial x} \tag{13}$$

$$v_2 = -\frac{k}{\mu_2}\left(\frac{\partial p_2}{\partial y} + \rho_2 a\right) = -\frac{\partial\psi_2}{\partial y} \tag{14}$$

$$p_2 = p_o + \frac{\mu_2}{k}\psi_2 - \rho_2\, a\, y \tag{15}$$

$$\psi_2 = -Ae^{\kappa y + nt} \cos \kappa x \tag{16}$$

At the interface $y = y_s \approx 0$, $v_1 = v_2 = v_s$ and since

$$\frac{Dy_s}{Dt} = \frac{\partial y_s}{\partial t} + u\frac{\partial y_s}{\partial x} \approx \frac{\partial y_s}{\partial t} = v_s$$

then

$$y_s = \frac{A\,\kappa}{n} e^{nt} \cos \kappa x \tag{17}$$

Substituting Eq. (17) into (12) and (15) and equating pressures at the interface one obtains the following dispersion relation:

$$n^2 + \frac{\mu_2}{\rho_1 k} n + \frac{\rho_2 - \rho_1}{\rho_1} a\kappa = 0 \tag{18}$$

and

$$n = - \frac{\mu_2}{2\rho_1 k} \pm \sqrt{\frac{\mu_2{}^2}{4\rho_1{}^2 k^2} + \left(1 - \frac{\rho_2}{\rho_1}\right) a\kappa} \tag{19}$$

The result given by Eq. (19) indicates that the effect of the heavy entrained phase is to decrease the rate of growth of Taylor instabilities for all disturbance wave numbers with increasing values of the parameter $\mu_2/2\rho k$ the growth rate is progressively reduced in comparison with that predicted by the simple Taylor theory but remains finite for all wave numbers. It is apparent that in the limit of very large porosity we retrieve essentially the classical Taylor result as long as as $\rho_2 \ll \rho_1$, which is the case of interest.

2.3.2. Effect of Surface Tension.

If the surface tension is considered, then for a two-dimensional interface the condition of continuity of pressure at the interface must be replaced by

$$p_{1,s} - p_{2,s} = \frac{\sigma}{\xi} \simeq \sigma \frac{\partial^2 y_s}{\partial x^2} \tag{20}$$

With the aid of Eqs. (11), (12), and (15-17), one obtains the modified dispersion relation

$$n^2 + \frac{\mu_2}{k\rho_1} n + \frac{\rho_2 - \rho_1}{\rho_1} a \kappa + \frac{\sigma \kappa^3}{\rho_1} = 0 \tag{21}$$

and

$$n = - \frac{\mu_2}{2k\rho_1} \pm \sqrt{\left(\frac{\mu_2}{2k\rho_1}\right)^2 + \frac{\rho_1 - \rho_2}{\rho_1} a \kappa - \frac{\sigma \kappa^3}{\rho_1}} \tag{22}$$

For the growth rate to vanish

$$\frac{\rho_1 - \rho_2}{\rho_1} \kappa_{CUTOFF} - \frac{\sigma \kappa^3_{CUTOFF}}{\rho_1} = 0 \tag{23}$$

and, therefore,

$$\kappa_{CUTOFF} = \sqrt{\frac{(\rho_1 - \rho_2) a}{\sigma}} \tag{24}$$

so that the cut-off wave number is the same as that predicted for the simple Taylor model. For wave numbers below the maximum, surface tension aids the porous body resistance in reducing disturbance growth rates. The effect increases strongly with increasing wave number.

The most dangerous wave number is also the same as for the simple Taylor model as may be readily shown by differentiating Eq. (21).

2.3.3 Effect of Surface Tension and Viscosity of the Upper Fluid.

When the upper fluid has a finite viscosity, then the equations of motion (9) become

$$\frac{\partial u_1}{\partial x} + \frac{\partial v_1}{\partial y} = 0$$

$$\frac{\partial u_1}{\partial t} = \frac{1}{\rho_1}\frac{\partial p_1}{\partial x} + \nu_1\left(\frac{\partial^2 u_1}{\partial x^2} + \frac{\partial^2 u_1}{\partial y^2}\right) \quad (25)$$

$$\frac{\partial v_1}{\partial t} = -\frac{1}{\rho_1}\frac{\partial p_1}{\partial y} - a + \nu_1\left(\frac{\partial^2 v_1}{\partial x^2} + \frac{\partial^2 v_1}{\partial y^2}\right)$$

These equations can be satisfied by the introduction of a potential ϕ_1 and stream function ψ_1 such that

$$u_1 = -\frac{\partial \phi_1}{\partial x} - \frac{\partial \psi_1}{\partial y} \qquad v_1 = -\frac{\partial \phi_1}{\partial y} + \frac{\partial \psi_1}{\partial x} \quad (26)$$

$$\nabla^2\phi_1 = 0 \qquad \nu_1 \nabla^2 \psi_1 = \frac{\partial \psi_1}{\partial t} \quad (27)$$

$$p_1 = p_o - \rho_1 a y + \rho_1 \frac{\partial \phi_1}{\partial t} \quad (28)$$

If we put

$$\phi_1 = A e^{-\kappa y + nt} \cos \kappa x$$

$$\psi_1 = B e^{-m_1 y + nt} \cos \kappa x \quad (29)$$

the second of Eqs. (27) requires that

$$m_1^2 = \kappa^2 + \frac{n}{\nu_1} \tag{30}$$

The normal stress, which now replaces the pressure in the continuity requirement at the interface, is given by

$$p_1 - 2\,\mu_1 \frac{\partial v_1}{\partial y} \tag{31}$$

and the shear stress, which should also be continuous at the interface is given by

$$\mu_1 \left(\frac{\partial u_1}{\partial y} + \frac{\partial v_1}{\partial x} \right) \tag{32}$$

We treat the lower fluid (vapor-particle mixture) as before and assume that it obeys Darcy's law Eqs. (13) and (14). We put

$$\psi_2 = C\, e^{\kappa y + nt} \cos \kappa x \tag{33}$$

In comparison with the inviscid case, we can introduce two new conditions at the interface, namely the continuity of the u-component of the velocity and of the shear stress. Since we have introduced only one new constant, one of these conditions appear superfluous. If we regard the gas as having very low viscosity $\mu_2/\mu_1 \ll 1$ and the high resistance to its flow through the porous body to be due to the effect of the very large surface-to-volume ratio, it seems reasonable to neglect the shear stress in the gas at the interfac and at the same time to allow a discontinuity in the u-component of the velocity.

Thus, the new condition to be imposed is

$$\mu_1 \left(\frac{\partial u_1}{\partial y} + \frac{\partial v_1}{\partial x} \right)_{y = y_s} = 0 \tag{34}$$

The condition $v_{1,s} = v_{2,s}$ yields

$$A + B = - C \tag{35}$$

and

$$y_s = \frac{\kappa}{n} \left(A + B \right) e^{nt} \cos \kappa x = - \frac{\kappa}{n} C e^{nt} \cos \kappa x \tag{36}$$

From Eq. (34)

$$2 A \kappa^2 + B \kappa^2 + B m_1^2 = 0$$

or

$$2 A \kappa^2 + B \left(2 \kappa^2 + \frac{n}{\nu_1} \right) = 0 \tag{37}$$

Finally, continuity of normal stress, with surface tension accounted for yields

$$\left(\rho_1 n^2 + 2\mu_1 \kappa^2 n \right) A + 2 \mu_1 \kappa mn B +$$
$$\left[a \left(\rho_1 - \rho_2 \right) \kappa - \frac{\mu_2}{k} n - \sigma \kappa^3 \right] C = 0 \tag{38}$$

The three homogeneous Eqs. (36), (37), and (38) yield a nontrivial solution if the determinant of the coefficient vanishes. This condition results in the required dispersion relation:

$$\rho_1 \beta - \left(\rho_1 n + 2 \mu_1 \kappa^2 \right)^2 + 4 \mu_1^2 \kappa^3 m = 0 \tag{39}$$

where

$$\beta = a \left(\rho_1 - \rho_2 \right) \kappa - \frac{\mu_2}{k} n - \sigma \kappa^3 \tag{40}$$

The resulting equation is of fourth order in n.

III. APPLICATION OF TWO-PHASE BUBBLE (LOWER FLUID) INSTABILITY THEORY TO HCDA PHENOMENA

In the prototypic system of interest in HCDA analysis, a multi-component, multi-phase bubble expands and accelerates a pool of liquid sodium. This system is represented schematically in Fig. 1. The bubble would consist of (i) a vapor phase containing UO_2, steel and sodium vapor, (ii) an entrained liquid phase composed of the same constituents and, perhaps, (iii) some solid constituents.

Entrainment of liquid sodium from the sodium pool into the bubble may occur as a result of growth of Taylor instabilities at the bubble-pool interface. The linear theory is used to compute the early stages of instability growth. Calculations are presented below, based upon

(i) the classical Taylor theory, ignoring the presence of the heavy entrained phase,

(ii) the porous medium model of the multi-phase, multi-component bubble developed in this report,

(iii) the homogeneous model of the multi-phase bubble.

Instability growth calculations were performed using the models which incorporate the effect of interfacial surface tension, but neglect the effect of viscosity of the upper fluid. Equation (3) is the classical Taylor result for the magnitude of instability growth constant including surface tension, but neglecting the presence of the heavy entrained phase. Equation (22) is the result using the porous medium model, presented in this report, to describe the multi-phase characteristics of the HCDA bubble. The property values listed in Table 1 were used in the calculations.

Table 1

PROPERTY VALUES

Density Upper Liquid (Sodium), ρ_1 = 823.3 kg/m^3

Density Vapor in Bubble (UO_2), ρ_2 = 1.1 kg/m^3

Surface Tension (Sodium), σ = 0.1529 N/m

Viscosity Vapor in Bubble (UO_2), $\mu_2 = 8.4 \times 10^{-5}$ N /m^2

Density Entrained Heavy Phase in Bubble (UO_2), ρ_E = 8700 kg/m^3

To assess the magnitude of the permeability of the porous medium we shall use the result of Brinkman's theory [15] which relates the permeability to the mean particle radius and the void fraction

$$k = \frac{r^2}{18} \left[3 + \frac{4}{1 - \alpha} - 3 \sqrt{\frac{8}{1 - \alpha} - 3} \right] \tag{41}$$

Table 2 presents the permeabilities for a void fraction of 0.9 and radii of 1, 10 and 100 μm, respectively. These numbers are used in the calculations.

Figure 3 presents the growth constant results for both the Taylor and porous medium models, for accelerations of 10, 10^2, 10^3 and 10^4 m/s^2. Note that both theories predict a cutoff wave number, beyond which disturbances do not grow. This is due to the surface tension effect. A strong influence of permeability is observed, especially for small accelerations. The smaller the permeability, the greater is the influence of the entrained heavy phase on the instability growth. The drag between the particles (or droplets) and the accelerating vapor provides a stabilizing effect on the interface. For large permeability, the porous medium model results approaches that of the Taylor model. This is especially true for the larger accelerations, where the influence of the permeability is not as strong as for the smaller accelerations.

Both the Taylor and porous medium theories predict that at some critical wave number, κ_{CRIT}, the growth constant is a maximum. This is apparent from Fig. 3, and also from Table 3. These constants represent the growth rate of the fastest growing instability for a given set of conditions (permeability

Table 2

POROUS MEDIUM PERMEABILITIES

$\bar{\alpha}$	r (μm)	k (m^2)
0.9	1	0.93×10^{-12}
0.9	10	0.93×10^{-10}
0.9	100	0.93×10^{-8}

Table 3

CUTOFF AND CRITICAL WAVE NUMBERS:

PROTOTYPE SYSTEM

Acceleration (m/s^2) a	Cutoff Wave Number (m^{-1}) κ_{MAX}	Critical Taylor Growth Rate (s^{-1}) n^{TAYLOR}_{CRIT}	Critical Wave Number (m^{-1}) κ_{CRIT}
10	231.9	29.8	133.9
10^2	733.3	167.8	423.4
10^3	2319.0	943.5	1339.0
10^4	7333.0	5305.6	4234.0

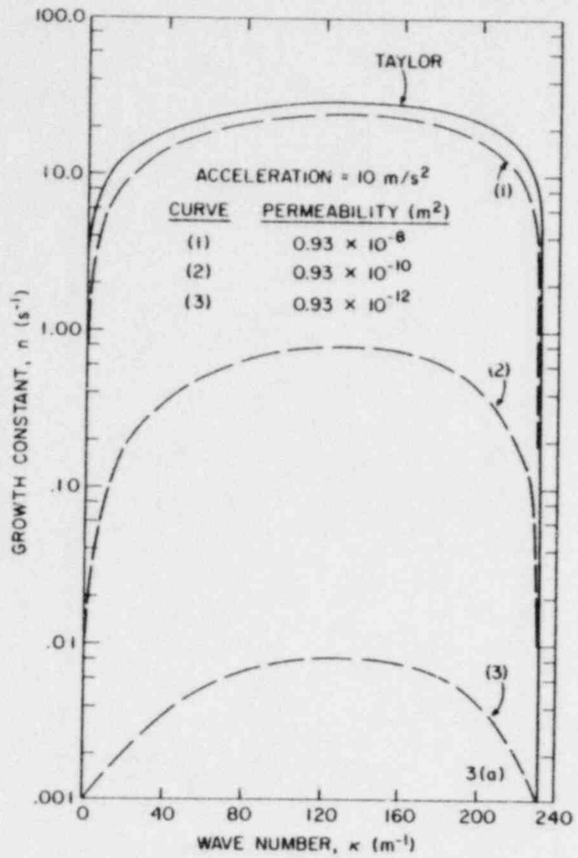


Figure 3(a). Growth Constants from Taylor and Porous Medium Instability Analyses. (BNL Neg. No. 3-1291-80)

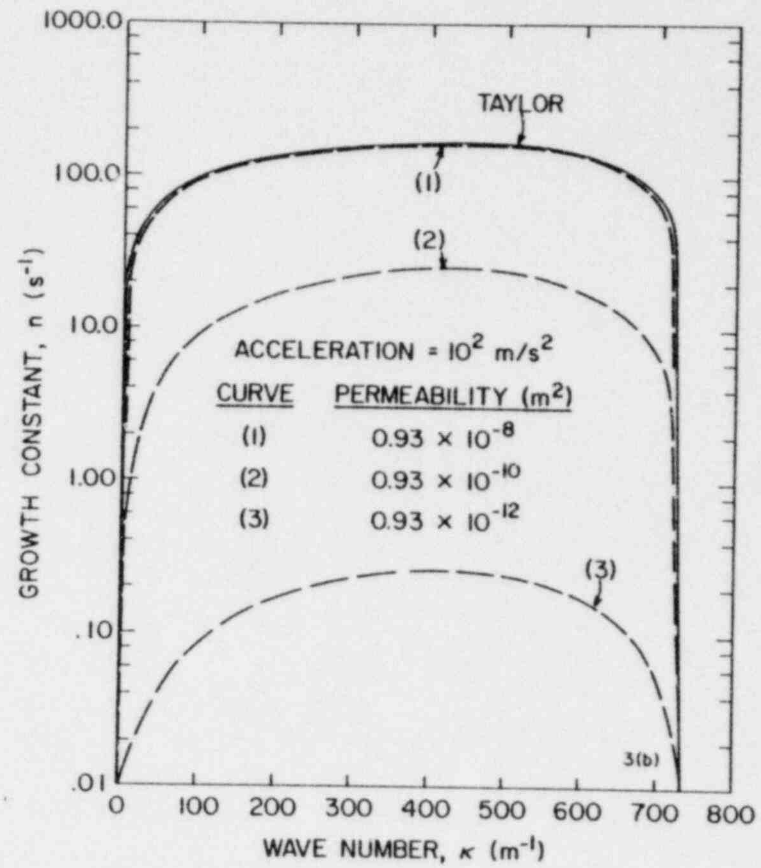


Figure 3(b). Growth Constants from Taylor and Porous Medium Instability Analyses. (BNL Neg. No. 3-1289-80)

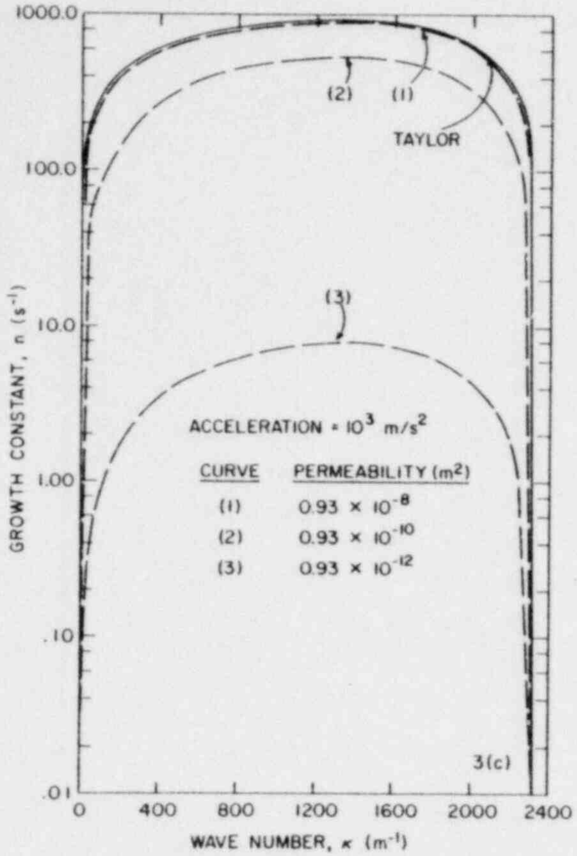


Figure 3(c). Growth Constants from Taylor and Porous Medium Instability Analyses. (BNL Neg. No. 3-1290-80)

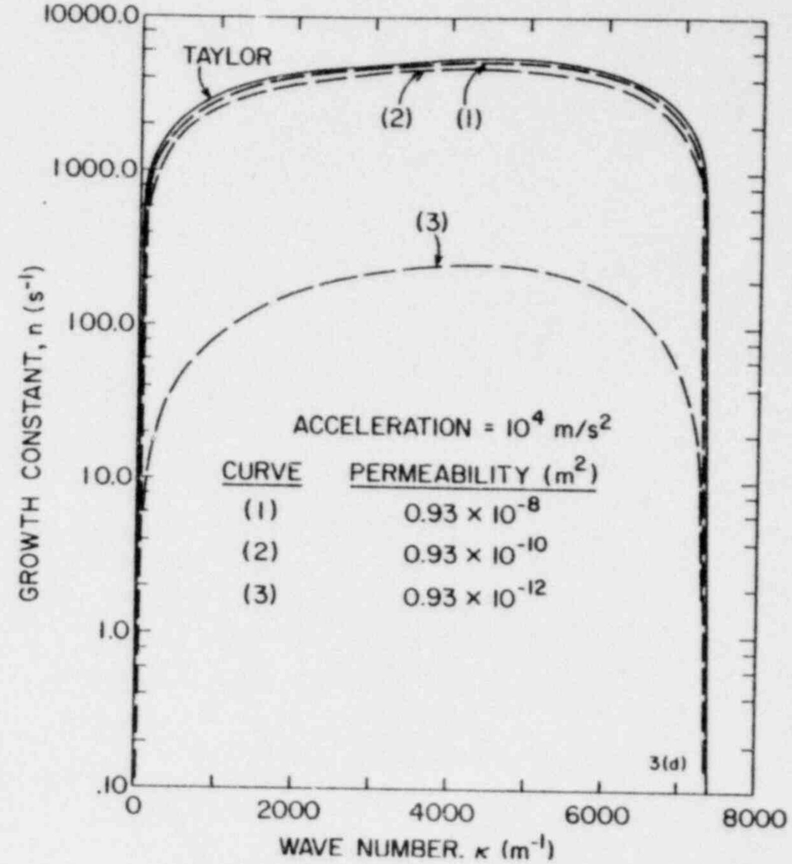


Figure 3(d). Growth Constants from Taylor and Porous Medium Instability Analyses. (BNL Neg. No. 3-1288-80) and acceleration). The magnitude of κ_{CRIT} is, for a given acceleration, independent of the permeability. The growth constants corresponding to the critical wave numbers are presented in Fig. 4 as a function of permeability. They are presented in dimensionless form, using the Taylor result as the scale factor.

The results display the strong stabilizing effect of the permeability on the disturbance growth rate, especially for small and intermediate accelerations. For bubble void fractions of approximately 0.9, if the heavy entrained phase particle radius is 100 μm (corresponding to $k \simeq 10^{-8}$ m^2), then the stabilizing influence of the porous medium drag disappears. The presence of particles does not influence the interfacial disturbance growth rate. On the other hand, if the particle size is 1 μm, the growth constant is affected by at least one order of magnitude for all accellerations up to 10^4 m/s^2. This effectively means that the disturbance growth rate would become vanishingly small.

The third model to be considered is that of a homogeneous/no-slip model of the multi-phase bubble mixture. A specific case is considered where the entrained phase has the density listed in Table 1. The bubble mixture density is, then, approximately

$$\rho_2 = \rho_{MIXTURE} = (1-\bar{\alpha})\, \rho_E$$

where $\bar{\alpha}$ is the mixture void fraction. The Taylor theory result, given by Eq. (3), predicts that when the upper fluid density ρ_1 equals the lower fluid density ρ_2, then the disturbance vanishes. This would imply that if $\bar{\alpha} \leq 0.91$, then Taylor instabilities would not be capable of growing. This result is not

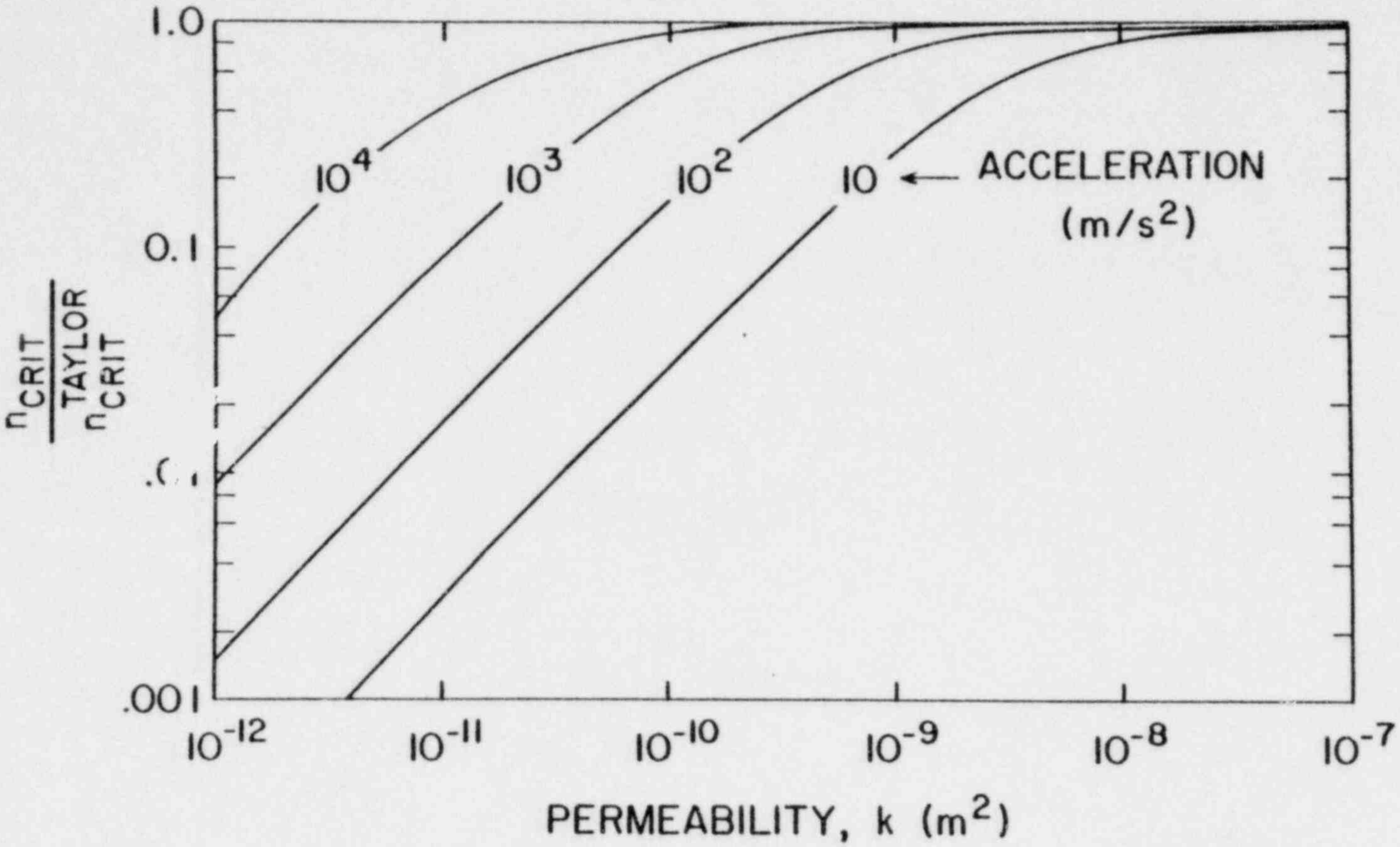


Figure 4. Variation of Critical Growth Constants with Permeability and Acceleration. (BNL Neg. No. 3-1295-80)

predicted by the porous medium model, in which the entrained phase density is not a parameter.

The homogeneous and porous medium (infinite particle/droplet inertia) models of multi-phase bubble dynamics represent two limits of particle-vapor interaction under acceleration. Both models predict that some multi-phase flow parameter does indeed limit instability growth and, hence, entrainment. The specific parameter, however, is model-dependent. In one case it is mixture void fraction, in the other case it is medium permeability. This disparity in results points to the need for development of a unified theory of Taylor instabilities applied to multi-phase expanding bubbles, which accounts in a more precise fashion for particle/vapor slip dynamics.

IV. SUMMARY AND CONCLUSIONS

4.1 Instability Model Development and Results

The growth of Rayleigh-Taylor instabilities is studied in relation to liquid entrainment at the interface between accelerating fluids of unequal density. The upper fluid is pure liquid, and the lower fluid is a mixture of vapor and a heavy entrained droplet or particulate phase. Entrainment through this mechanism would occur when the liquid spikes grow into the lower fluid and, eventually, separate into droplets. This work estimates the effect of the presence of heavy droplets or particulates in the immediate vicinity of the interface on the early (linear) stages of instability growth.

The growth of the Taylor instability is computed using a porous medium model of the multi-phase lower fluid, which assumes that the entrained phase is characterized by infinite inertia. The vapor simply flows around the entrained phase. The model is described and calculation results are presented for the rate of instability growth during HCDA bubble expansion. Results are compared with the classical Taylor theory which neglects the presence of the entrained phase, and with a homogeneous model of the multi-phase bubble.

The major results of the present analysis of instability growth at the HCDA bubble interface are:

- The effect of stationary particles on the linear growth of disturbances depends very strongly on the particle size, ranging from strong damping with small particles to negligible for particles of the order of 100 μm. If the effect is confirmed through the analysis of particles with finite inertia, the question of particle size distribution may become quite important. Further, it is important to note that with particles of the order of 10 μm, substantial reduction in the entrainment rate may be expected, except at very high wave numbers and accelerations.

- Application of a homogeneous model of multi-phase HCDA bubble characteristics to Taylor instability growth leads to the result that if the bubble void fraction is less than 0.9, the disturbance growth rate would vanish.

4.2 Implications with Respect to Sodium Entrainment

The basic issue to which this work is addressed is that of the potential for entrainment of liquid sodium into an expanding HCDA bubble. Whether the entrained sodium acts to amplify the work potential of the bubble, or whether it acts as a heat sink, depends upon the rate and quantity of sodium entrained. The work presented in this paper is directed specifically to evaluation of the potential for entrainment by the mechanism of growth of Rayleigh-Taylor instabilities.

The porous medium instability growth model described here suggests

- If the HCDA bubble is characterized by a void fraction of approximately 0.9, and if the heavy entrained phase droplet or particle radius is approximately 1 μm or less, then the potential for entrainment by Taylor instabilities is small for accelerations up to 10^4 m/s^2. Since the potential for entrainment is small, then sodium entrainment by Taylor instability growth would have little influence on the work potential of the HCDA bubble.

- If the heavy particle radius is approximately 100 μm or greater, then the presence of the heavy phase has no influence on the development of Taylor instabilities, and entrainment by the growth of these instabilities is possible. The rate of early (linear) growth of the disturbances may be computed using the Taylor theory, neglecting the presence of the heavy phase. The rate of entrainment cannot be predicted using the linear theory. The influence of sodium entrainment, by growth of Taylor instabilities, on the work potential of the HCDA bubble cannot be predicted using the methods described here.

Application of the classical Taylor theory to the multi-phase HCDA bubble expansion, using a homogeneous (no-slip) mixture model for the multi-phase dynamics suggests

- prediction of an instability cutoff at a void fraction of approximately 0.9. The implication would be, if the model is applicable, that entrainment by Taylor instabilities would not be possible for bubble void fractions less than 0.9. The porous medium model predicts no such cutoff. For conditions of void fraction less than 0.9 sodium entrainment by growth of Taylor instabilities would have no influence on the work potential of HCDA bubbles.

Both of the models discussed above suggest that there are physical processes which limit the potential for sodium entrainment by growth of Taylor instabilities. In the case of the porous medium model the limiting factor is the drag imposed by the entrained phase. In the case of the homogeneous flow model the factor is the loss of the driving force (upper-to-lower fluid density difference) for growth of the instability.

The results presented here suggest the need for development of a unified theory of Taylor instabilities which applies to multi-phase expanding bubbles and which accounts for the finite inertial characteristics of the entrained particulate or droplet phase. A confirmatory experimental effort should also be considered.

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VI. REFERENCES

1. Cho, D. H., M. Epstein, and H. K. Fauske, "Work Potential Resulting from a Voided Core Disassembly," Trans Am. Nucl. Soc., 18 p. 220 (1974).

2. Corradini, M. L., "Heat Transfer and Fluid Flow Aspects of Fuel-Coolant Interactions," MIT Nucl. Eng. Dept., COO-2781-12TR (September 1978).

3. Moszynski, J. R., "Liquid Entrainment by an Expanding Gas or Vapor Bubble - A Review of Experiments and Models," BNL Report to be published.

4. Saito, M. and T. G. Theofanous, "The Termination Phase of Core Disruptive Accidents in LMFBR's," Specialists' Workshop on Predictive Analysis of Material Dynamics in LMFBR Safety Experiments (March 1979).

5. Emmons, M. W., C. T. Chang, and B. C. Watson, "Taylor Instability of Finite Surface Waves," Journal of Fluids Mechanics 7, p. 177 (1960).

6. Nayfeh, A. H., "On the Nonlinear Lamb-Taylor Instability," J. Fluid Mech. 32, p. 619 (1969).

7. Dienes, J. K., "Method of Generalized Coordinates and an Application to Rayleigh-Taylor Instability," Physics of Fluids, 21, p. 736 (1978).

8. Daly, B. J., "Numerical Study of Two-Fluid Rayleigh-Taylor Instability," Physics of Fluids 10, p. 297 (1967).

9. Harlow, R. M. and J. E. Welch, "Numerical Study of Large-Amplitude Free-Surface Motions," Physics of Fluids 9, p. 842 (1966).

10. Moszynski, J. R., "Effect of Condensed Particles of Finite Inertia on the Rayleigh-Taylor Instability," BNL Report to be published.

11. Chandrasekhar, S., "Hydrodynamic and Hydromagnetic Stability," Chapter X, 2nd Ed., Oxford, England (1968).

12. Taylor, Sir G. I., "The Instability of Liquid Surfaces When Accelerated in a Direction Perpendicular to Their Planes I," Proc. Roy. Soc. (London) A, 201, p. 192 (1950).

13. Bellman, R. and Pennington, R. H., "Effects of Surface Tension and Viscosity on Taylor Instability," Quart. J. Appl. Math, 12, p. 151 (1954).

14. Saffman, P. G. and Sir G. I. Taylor, "The Penetration of a Fluid into a Porous Medium or Hele-Shaw Cell Containing a More Viscous Liquid," Proc. Roy. Soc. (London) 245A, p. 312 (1958).

15. Scheidegger, A. E., "The Physics of Flow Through Porous Media," The MacMillan Co., New York, Part VI (1957).

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