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COMBINATION OF TRANSIENT RESPONSES BY SRSS METHOD

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BASES FOR CRITERIA FOR COMBINATION OF EARTHQUAKE AND OTHER TRANSIENT RESPONSES BY THE SQUARE-ROOT-SUM-OF-THE-SQUARES METHOD

R. P KENNEDY (EDAC)
N. M. NEWMARK (CES)

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BASES FOR CRITERIA FOR COMBINATION OF EARTHQUAKE AND OTHER TRANSIENT RESPONSES BY THE SQUARE-ROOT-SUM-OF-THE-SQUARES METHOD

R. P. Kennedy (Engineering Decision Analysis Company)
N. M. Newmark (Consulting Engineering Services)

(Prepared in Cooperation with Mark II Owner's Group)

Reviewed:
J. D. Gilman, Manager
Dynamic & Seismic Analysis

Approved:
W. F. English, Manager
Dynamics Loads Technology

NUCLEAR ENERGY PROJECTS DIVISION • GENERAL ELECTRIC COMPANY
SAN JOSE, CALIFORNIA 95125

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1. BACKGROUND

Structures and components of nuclear power facilities are designed for a large number of load combinations. These load combinations include both multiple dynamic loads and static loads. In most cases, peak responses from each of the dynamic loads are calculated elastically. These results are then combined to obtain a resultant peak combined dynamic response. Once the resultant peak combined dynamic response has been determined from a proper combination of the multiple peak dynamic responses, the resultant is added absolutely to the elastically calculated static response. This elastically calculated combined maximum response is then compared to code allowable stress levels with the acceptance criterion being that the combined response must be lower than the code allowable level.

The question of how to combine several multiple peak dynamic responses has been studied extensively for earthquake and blast response of structures. Appendix A summarizes some of these studies and contains an extensive reference list of papers which can be consulted for further details. In 1951, Rosenblueth[1] first proposed that peak dynamic seismic responses be combined using the square-root-sum-of-the-squares (SRSS) method. This method is founded on a statistical basis summarized in Section 5 such that peak* combined response is expected to have approximately the same nonexceedance probability as exists for each of the individual peak responses being combined. The method was first published in 1953.[2] Since that time, this method of response combination has been widely studied (see Appendix A) and, with a few well-defined exceptions, has been accepted as the preferred method for response combination in the field of earthquake response of structures.

As other transient loadings on nuclear power facility components have been identified, it has become necessary to combine peak responses from these transient loadings also. For such loadings, peak responses have been generally combined using SRSS based upon extensive experience in earthquake response analysis. However, questions were raised by the Nuclear Regulatory Commission (NRC)

*Throughout this report, the words "peak response" are used to represent the maximum peak response.

as to whether the SRSS method is an appropriate method for combining such responses. Several studies were initiate to demonstrate the adequacy of combining peak dynamic responses from other transient loadings using the SRSS method.[3,4,5]

Reference 3 documents a methodology for developing cumulative distribution function curves (called CDF curves) of the conditional probability of nonexceedance of any peak combined response as a result of multiple input response time histories having random relative time phasing. These nonexceedance probability curves are based upon a defined probability density function for relative time phasing between the multiple dynamic inputs. All of the multiple dynamic responses in the response combination are assumed to occur concurrently. Thus, the resultant nonexceedance probabilities are conservative in that they ignore the possibility that the events may not occur concurrently.

These CDF curves present exceedance probabilities resulting from the randomness of time phasing only and do not consider uncertainty of amplitude of the individual peak response or the nonexceedance probability at which the individual peak responses being combined are defined. The actual nonexceedance probability of the peak combined response is a function of both the nonexceedance probability of the amplitude of the individual peak responses and the nonexceedance probability obtained from the CDF curves resulting from random time phasing. So long as the individual peak responses are conservatively defined, the actual nonexceedance probability of the peak combined response will be greater than that defined by these time phasing only CDF curves. Throughout this report, the term CDF curves refers to plots of nonexceedance probabilities associated with random time phasing only, which is in accordance with how such curves were developed in Reference 3.

A total of 291 different load combination cases which included multiple dynamic response time histories generated from actual Mark II plant structures and components were studied. For the cases studied, it was shown in References 3 and 5 that the median probability of nonexceedance of the SRSS combined response associated with random time phasing (i.e., amplitude known) was about 86%, with about 98% confidence that the nonexceedance probability was greater than 50%. Furthermore, it was shown that the median probability of exceeding the SRSS combined response by more than 20% was only about 4% with about a 98% confidence that the probability of exceeding the SRSS combined response by more than 20% was less than 15%. For such transient response cases, one can conclude that there is a low conditional probability of exceedance of the SRSS combined response and a very low probability of significant exceedance due to random time phasing.

Reference 3 also reported reliability studies which compare the component reliability for components designed using SRSS combination of peak dynamic responses versus those designed using absolute summation combination when both were subject to dynamic loadings. So long as:

1. The design dynamic load events (such as earthquake ground acceleration) are defined with sufficient conservatism to cover reasonable uncertainty in their definition so that there is high confidence that the likelihood of occurrence of more severe load events is no greater than the likelihood of occurrence upon which the corresponding allowable stress criteria are based,

2. The dynamic loadings (such as design response spectrum anchored to the earthquake ground acceleration) for the given dynamic event are also defined with sufficient conservatism to cover reasonable uncertainty, and

3. The allowable stress criteria are designed with sufficient conservatism (low probability of component failure consistent with the likelihood of exceedance of the design load when stresses are held to allowable stress criteria),

then it can be concluded that use of absolute combination of peak dynamic responses does not result in significant increase in component reliability over that obtained from SRSS combination of peak dynamic responses. In other words, a low probability of structural failure can be achieved by the proper application of the sources of conservatism defined above. When this is done, very little added structural reliability is achieved by requiring a conservative response combination procedure (i.e., absolute summation of responses). In fact, requiring excessive conservatism in the response combination (i.e., absolute summation of responses) may lead to lesser reliability under normal expected loading conditions.

Reference 4 documents the results of a study conducted to show that structures designed elastically to code allowable stress levels generally have much greater margin against failure when subjected to dynamic loadings than when subjected to static loadings. In this study, the dynamic margin, R_D, was defined as the ratio of the dynamic time history load amplitude which results in failure strains to the dynamic time history load amplitude corresponding to code allowable stress levels for elastic analysis. Similarly, the static margin, R_S, was defined as the static load of failure divided by the static load corresponding to the same code allowable stress level for elastic analysis. Then, the dynamic to static margin ratio (D/S Margin) is defined by:

$$\text{D/S Margin} = R_D/R_S$$

For structures with even very moderate ductility (inelastic energy absorption capability), it is shown that for earthquakes or pulsive dynamic loadings which result in dynamic structural response, this D/S Margin is greater than 1.3 (often much greater than 1.3). It is also shown by other studies[4] that this D/S Margin alone is sufficient to cover the possible exceedance of the SRSS combined response from multiple dynamic events. For moderate ductility structures subjected to dynamic responses, one can conclude that the combination of dynamic responses by SRSS generally results in greater reserve margin than is obtained for static responses when both responses are held to the same code allowable stress levels for elastic analyses.

The Nuclear Regulatory Commission Working Group on Methodology for Combining Dynamic Responses recently issued a report[6] recommending the approval, on a limited basis, of the SRSS response combination method for combining peak responses from transient loadings. That report also provided guidance for developing justification for a more generic acceptance of the SRSS method. The need for general criteria for determining when transient responses can be reasonably combined using the SRSS method was clearly demonstrated. Such criteria must provide reasonable assurance that the conditional probability of the combination of dynamic responses exceeding the SRSS value is acceptably low (given the condition of simultaneous occurrence of events).

Based upon this need for generic criteria against which to judge the applicability of the SRSS method of response combination, the authors developed and presented suggested criteria in August 1978. These criteria are presented in Section 2 of this report. They are based upon the conclusions summarized above, study of the references reported in Appendix A, and professional judgment concerning the important parameters influencing the applicability of the SRSS method.

Section 2 documents the recommended acceptance criteria and Section 3 is intended to clarify our intent on several potentially ambiguous points. Section 4 presents the philosophic basis for the overall criteria while Section 5 documents the basis for Criterion 1 and Section 6 does the same for Criterion 2.

The probability distribution for the responses to earthquake motions is based on the concepts underlying U.S. NRC Regulatory Guide 1.60, where the standard deviation is 30 to 40% of the median value.

It was proved some decades ago that modal responses to earthquake motions may be conservatively combined by SRSS methods with the same degree of conservatism as that of the motions. If each of such responses is considered to be at the level of mean plus one standard deviation, the SRSS value is also at this level. For the same reasons, responses from the three component directions of earthquake motions may also be conservatively combined by SRSS methods.

2.3 CRITERION 2

When response time histories are available for all multiple dynamic loadings being combined, SRSS methods may be used for peak combined response when CDF calculations, using appropriate assumptions on the range of possible time lags between the response time histories, show the following criteria are met:

1. There is estimated to be less than approximately a 50% conditional probability that the actual peak combined response from these conservatively defined loadings exceeds approximately the SRSS calculated peak response, and

2. There is estimated to be less than approximately a 15% conditional probability that the actual peak combined response exceeds approximately 1.2 times the SRSS calculated peak response.

2. CRITERIA FOR COMBINATIONS OF EARTHQUAKE AND/OR OTHER TRANSIENT RESPONSES

N. M. Newmark
R. P. Kennedy

2.1 PREAMBLE

The intent of the methods proposed for combinations of transient, dynamic responses is to achieve a nonexceedance probability of approximately 84% for the peak combined response of the system, component, or element considered. This goal is achieved by compliance with any one of the following criteria, or any alternative method that meets the intent stated above, provided that the intensity of loads or accelerations for each input are conservatively represented (approximately at the level of the 84th percentile, or the mean plus one standard deviation, of the expected input intensity).

2.2 CRITERION 1

Dynamic or transient responses of structures, components, and equipment arising from combinations of dynamic loading or motions may be combined by SRSS provided that each of the dynamic inputs or responses has characteristics similar to those of earthquake ground motions, and that the individual component inputs can be considered to be relatively uncorrelated; i.e., the individual dynamic inputs or responses considered are either from independent events or have random peak phasing. This similarity involves a limited number of peaks of force or acceleration (not more than 5 exceeding 75% of the maximum, or not more than 10 exceeding 60% of the maximum), with approximately zero mean and a total duration of strong motion (i.e., exceeding 50% of the maximum) of 10 seconds or less.

Explanation. Since earthquake motions in various directions produce responses which are combined conservatively by the use of SRSS, the descriptions of dynamic or transient inputs are based on those applicable to earthquake motions. The coefficient of correlation for these is less than 0.4, and the pattern of peaks is based on Table 2 of Circular 672 of the U.S. Geological Survey describing earthquake ground motions for use in the design of the Alaska oil pipeline.

3. CLARIFICATION OF THE CRITERIA

In the criteria preamble (Section 2), it is stated that the intensity of loads or accelerations for each input must be conservatively represented (approximately at the level of the 84th percentile). This level of conservatism has been historically (and, in our opinion, properly) chosen for defining the earthquake input motion associated with a peak ground acceleration (load case) for use in design of nuclear power facilities. With the load case combinations conservatively defined, the 84th percentile level of conservatism in the loading definition is considered to provide a reasonable conservatism to cover loading definition uncertainties. No major significance should be placed on the 84th percentile number contained in the preamble. It is simply our intent to require a conservative loading definition and to provide some guidance as to a reasonable level of conservatism. It is not our intent to require statistical studies to be conducted to define all nonexceedance probabilities on loading amplitude. Loadings based upon limited test results or analyses in which the tests or analyses are known to conservatively bound the true event meet this intent.

It is intended that either Criterion 1 or Criterion 2 can be used to determine the adequacy of combining dynamic responses by the SRSS method. It is not intended that both criteria must be checked or that both criteria must be met. Criterion 1 represents a simple check which can be relatively easily performed but which may be overly restrictive and be subject to differing interpretations in some cases. Criterion 2 requires the development of CDF curves based on the assumption of random relative time phasing between the individual response time histories being combined. Development of such curves requires the availability of response time histories for each loading. Criterion 2 is intended to be used primarily for those cases which might not meet Criterion 1 because of its restrictiveness.

Criterion 1 may be met either at the input or response level. A substantial body of evidence exists, from the studies referenced in Appendix A, that earthquake ground motion induced responses can be rationally combined by the SRSS methods. For the reasons described in Section 5, it is considered to be equally applicable to combine all other transient responses which are similar to earthquake ground motion responses by the SRSS method. It is judged that, if the input function has similar characteristics to earthquake ground motion, then the responses will automatically be similar to responses from earthquake ground motion.

Even though it is the similarity in response which is of actual interest, this similarity can be justified at either the input or response level.

Criterion 1 requires an approximately zero mean response (or input). The intent is that during the strong motion of response, the ratio of mean to maximum response is near zero. If this ratio for the input is near zero, then the response ratio is automatically near zero for linear analysis. Ratios of mean to maximum response over the duration of strongest response of less than 0.1 meet the intent of this provision. Higher ratios (possibly as high as 0.2) might also meet the intent.

Criterion 2 is based upon the development of CDF curves similar to those in Reference 2. Such curves must be based upon the use of a conservative or reasonable probability density function for time phasing of the independent dynamic time history events. Bounds must be set on the range of relative times at which each time history response event may start. Such bounds must encompass the timing at which peak responses from all events in the combination are worst-case time-phased (absolute summed). So long as this time-phasing is encompassed, the narrower the time bounds on time phasing, the more conservative will be the resultant CDF curve. The time bounds selected on relative time-phasing must be justified. Use of a uniform probability density function for time-phasing within these time bounds is considered reasonable.

4. RATIONALE BEHIND CRITERIA

The criteria in Section 2 for acceptance of SRSS combination of multiple peak responses from dynamic loadings are based upon several key assumptions. These are:

1. Many sources of conservatism exist in each element of the design and evaluation process. These conservatisms are necessary to cover uncertainty and to ensure an acceptably low probability of unacceptable behavior of structures and components.

2. Additional conservatism does not have to be incorporated within the methodology for combining dynamic responses. In fact, conservatism should not be added at this step in an attempt to cover any possible unconservatism in Item 1. The response combination methodology cannot rationally or uniformly cover potential unconservatism inadvertently introduced elsewhere in the design process.

3. It is necessary only for the response combination methodology to preserve approximately the nonexceedance probability that exists for the individual responses. In other words, the combination methodology should reasonably assure that the peak combined response has no greater probability of exceedance than do the individual responses being combined. Conversely, combination of peaks by absolute sum will always result in probability of nonexceedance of the combination less than either of the individual peak responses. Hence, combination by absolute sum is more conservative than either of the individual events.

The goal of the acceptance criteria in Section 2 is to meet the statement under Item 3, above. The above three statements represent the key points upon which to judge whether dynamic responses should be combined by SRSS.

Sufficient conservatism can be provided in each of the following areas to cover uncertainty and to provide an adequately low probability of unacceptable behavior of structures or components:

1. The design dynamic load cases and combinations can be defined with sufficient conservatism so that the probability of exceedance of the defined load case combinations is acceptably low. Each load case and combination can be defined so that there is high confidence that the likelihood of the occurrence of more severe load cases is no greater than the likelihood of occurrence upon which the corresponding allowable stress criteria are based. As an example, the operating basis earthquake (OBE) is generally defined in terms of a peak ground acceleration. The allowable stress criteria for those load cases which include the OBE has been selected consistent with the assumption of one to five occurrences in the life of the plant. Uncertainty exists as to the level of peak ground acceleration which might occur one to five times in the life of the plant. Thus, the design OBE ground acceleration level is selected sufficiently high that there is high confidence that the design OBE is not likely to be exceeded more than one to five times in the life of the plant.

2. The dynamic load time histories associated with a given dynamic load case can be conservatively biased so as to reasonably cover possible uncertainties. For earthquake load cases, the earthquake loading is defined in terms of design response spectra anchored to the peak ground acceleration (load case). Given the peak ground acceleration, the amplitude of peak responses at various damping and frequency levels as defined by response spectra are uncertain. The design response spectra have been defined with sufficient conservatism to cover such uncertainty. For a given peak ground acceleration (load case), the probability of nonexceedance of the design response value (loading) is estimated to be at about the 84th percentile level.

3. Dynamic analyses can be performed so as to reasonably cover uncertainties in the analysis parameter values.

4. The allowable stress criteria can be conservatively selected so as to cover possible uncertainties in structural capacity and to provide an acceptably low unacceptable structural behavior probability consistent with the probability of exceedance of the event combination.

2. One must judge the applicability and conservatism of the dynamic loadings independently of the method of response combination. If the loadings are not sufficiently conservative to justify SRSS response combinations, then they are equally inappropriate for absolute summation combination of responses.

3. The goal of a uniform probability of nonacceptable behavior between the components in different plants, or different components in the same plant, cannot be achieved unless the method of response combination simply preserves the nonexceedance probability that exists for the individual responses.

Thus, one has four places in the design process where conservatism can be added to cover uncertainty and to provide for an acceptably low probability of unacceptable performance.

If one were to hypothesize that sufficient conservatism did not exist at one or more steps in the evaluation process, then the methodology for combining dynamic responses would still be an appropriate place to make up for this potential deficiency. It is not possible to cover such a deficiency in any consistent or universal fashion through arbitrarily introducing conservatism in the method of response combination. Consider an example of a loading condition in which responses from dynamic loading A are to be combined with those from dynamic loading B. For one component in one plant the relative magnitude of peak responses from these two loadings will be different from the relative magnitude of these peak responses for a different component in the same or different plant. Assume the following conditions:

	Ratios of Peak Responses		
Component	A/B	AS/A	SRSS/A
1	1.0	2.0	1.414
2	10.0	1.10	1.005

In the first case, the peak responses from A and B are the same while in the second case the peak response from A is predominant over that from B. In the first case, the use of absolute sum (AS) combination of peak responses introduces considerable conservatism beyond that obtained from SRSS combinations, while in the second case use of AS does not introduce such added conservatism. Now let us hypothesize that A has been underestimated by 20% such that its true value is A' = 1.2A. This potential underestimation of A is covered by the conservatism introduced by absolute summation of peak responses in the first case but is not so covered in the second case. Thus, conservatism in the response combination methodologies has not been able to consistently protect against a potential unconservatism in the input. The following points should be emphasized:

1. Conservatism in the method of dynamic response combination cannot be used to cover potential unconservatism at some other point in the evaluation process.

5. BASIS FOR CRITERION 1

Under certain circumstances, a heuristic proof exists that the SRSS method of response combination does preserve the same nonexceedance probability for the combined response as exists for the individual peak dynamic responses being so combined.[9] The following conditions are necessary:

1. The response components being combined are statistically uncorrelated and each is the output of a linear dynamic narrowband system subjected to independent stochastic input forcing functions.

2. The mean response is zero. For linear systems this condition is automatically satisfied if the mean input is zero.

From an engineering viewpoint, response components can be considered statistically uncorrelated if their input forcing functions result from independent events (i.e., random time-phasing within a time band at least as long as the natural period of the structure) or have random Fourier phase spectra. Linear structural systems with low damping (on the order of 20% or less equivalent viscous damping) meet the requirement for linear dynamic narrowband systems. The mean response can be treated as zero so long as the ratio of mean to maximum response during the time period of strong response is low (less than about 0.1 to 0.2).

Under the above conditions, the variance of the total combined response σ_T^2 equals the sum of the variances of each individual response component σ_i^2. Thus,

$$\sigma_T^2 = \sum_{i=1}^{N} \sigma_i^2 \qquad (1)$$

Note that the standard deviations of response are equal to the square root of the variances. According to Reference 9, the extreme-values R_i of response (corresponding to a given nonexceedance probability) for each response component of a narrowband system are proportional to their respective standard deviations (i.e., $R_i = c_i \sigma_i$). Assuming the same extreme-value distribution and same nonexceedance probabilities for each response component, the c_i values are identical for all response components. Thus,

$$R_T = \sqrt{\sum_{i=1}^{N} R_i^2} \qquad (2)$$

where R_T is the combined extreme-value of response with the same nonexceedance probability as exists for each individual response component, R_i. Equation 2 is the SRSS rule for combining peak response components. Thus, within the limitations of the assumptions, a proof exists that the SRSS combined response does preserve the same nonexceedance probability for combined responses as exists for each peak individual response component in the combination.

It is recognized that the above proof contains several simplifying assumptions and is rigorous only for the case of independent stochastic input forcing functions. The validity of the conclusion must be empirically confirmed for other input motion time histories similar to those for which responses are being combined SRSS. The many studies referenced in Appendix A present results which tend to validate the above conclusion for the case of earthquake ground motion time histories. With a few well-defined exceptions, such validation exists for combination of modal responses. Based upon a study of the references in Appendix A it can be concluded that a similar validation exists for the combination of responses from multiple components of ground motion. This validation is briefly discussed below.

From a study of the results presented in the references in Appendix A, for component responses from real earthquake ground motion time histories:

$$P\left[R_T \gtrsim R_{T_{SRSS}}\right] \quad \lesssim 50\% \qquad (3)$$

$$P\left[R_T \gtrsim 1.2\, R_{T_{SRSS}}\right] \quad \lesssim 15\% \qquad (4)$$

Equation 3 states that the probability that the actual peak combined response from time history combined response analyses exceeds or is approximately equal to the SRSS combined peak response is approximately 50%. Equation 4 states that the probability that the actual time history peak combined response exceeds the SRSS combined peak response by more than about 20% is about 15%. These probability statements are applicable for comparison of time history combined response with SRSS combination of individual peak components of response from real earthquake response time histories. They incorporate the influence of random time phasing between peak component responses but do not incorporate uncertainty about the peak amplitude of individual component responses. The peak amplitudes of response are known since actual time history records are used. The SRSS combined responses are based upon the same individual component peak amplitude as are used for the time history combined responses. Thus, these statements on probability of exceedance of SRSS combined responses due to random time phasing are consistent in interpretation with exceedance probabilities obtained from cumulative distribution function curves from Reference 3 since such curves also account only for randomness of relative time phasing. The amplitude of each individual peak component response of an earthquake is also uncertain. For earthquake loadings, each individual peak component response amplitude used for design is intended to be defined at about the 84th percentile nonexceedance probability which tends to be 30 to 40% above the median peak component response amplitude for each response component. With the individual peak component response amplitudes defined at the 84th percentile nonexceedance probability, the probability results presented by Equations 3 and 4 for random time phasing are sufficient to provide reasonable confidence that appproximately the SRSS combined response is also at about the 84th percentile nonexceedance probability. Thus, for the case of earthquake type response, approximately the SRSS combined response will be at about the 84th percentile nonexceedance probability if the individual response components are defined at about this nonexceedance probability.

Similar extensive empirical studies of the probability that the time history peak combined responses exceed the SRSS combined response are not available in the literature for responses from other transient loadings. However, it is our judgment that the results of the extensive studies using earthquake ground motion time histories can be extrapolated to other input time histories so long as such time histories are similar to earthquake ground motion time histories in those parameters which appear to influence the results presented by Equations 3 and 4.

In our opinion, the important earthquake ground motion characteristics influencing these results are:

1. Random peak phasing of responses. Responses from independent events with random phasing of start times over a time interval at least as long as the structure's natural period are considered to be equivalent.

2. A limited number of excursions to near peak response for each of the response time histories being combined. A review of actual earthquake time histories

and Reference 7 reveals that an earthquake input motion time history is generally characterized by a short duration of strong motion and by either no more than 5 acceleration pulses exceeding 75% of the maximum, or no more than 10 acceleration pulses exceeding 60% of the maximum. Such input motion characteristics will result in only a limited number of excursions to near peak response for each individual response component. Such characteristics are important to provide only a limited number of opportunities for the peak combined response to approach a maximum value and thus only limited opportunities for the peak combined response to potentially exceed the SRSS computed combined response.

3. An approximately zero mean response during the time of strong response. This condition is necessary for the derivation of the SRSS method for combining responses. An approximately zero mean response will automatically result for a linear system subject to input motion with an approximately zero mean during the time of strong motion. Such a condition is met for earthquake ground motion time histories. In order for the results obtained from earthquake ground motion time histories to be extrapolated to other loading conditions, they must also reasonably meet this condition.

Equations 3 and 4 represent statements on the conditional probability that time history calculated peak combined responses exceed, or significantly exceed, the SRSS combined response, respectively. These equations express the results of empirical studies conducted using earthquake ground motion time history inputs. They represent statements about what one can expect to achieve for input conditions which essentially meet the basic assumptions upon which the SRSS method of combining responses was developed. These probability statements associated with random time phasing (ignoring uncertainty on peak amplitude of component responses) are sufficient to reasonably assure that approximately the SRSS peak combined response maintains about an 84 percentile nonexceedance probability when a similar nonexceedance probability exists for the individual peak responses in the response combination. Reference 8 compares either the load or response time histories used in the studies previously described from Reference 3. The vast majority of these response time histories meet the time history characteristics defined by Criterion 1. For every case in which the loadings or responses meet Criterion 1, the CDF curves generated in the Reference 3 studies at least meet the probabilistic statements expressed by both Equations 3 and 4. This provides meet Criterion 1, there is reasonable assurance that Equations 3 and 4 are at least met. Thus, the intent of the criteria in Section 2 as expressed by its preamble is met.

6. BASIS FOR CRITERION 2

The basis for Criterion 2 follows directly from the previous discussion for Criterion 1. In the discussion on Criterion 1, it was stated that the probability statements accounting only for random time phasing defined by Equations 3 and 4 were considered sufficient to meet the intent of the dynamic response combination methodology which is to approximately preserve for the SRSS combined response a nonexceedance probability of about 84 percentile when each individual response in the combination is also defined at about this level. Empirical evidence has shown that both statements can be reasonably met for responses from multiple components of real earthquake ground motions. The greatest experience exists in the use of SRSS combination of responses from earthquake ground motions. It was considered prudent that other transient responses being combined SRSS should also meet the intent of Equations 3 and 4. Thus, Criterion 2 allows SRSS combination of peak responses when properly performed CDF calculations based upon appropriate assumptions (see Section 3) on the range of possible time-phasings between response time-histories show that both of the following are met:

$$P\left[R_T \gtrsim R_{T_{SRSS}}\right] \lesssim 50\% \qquad (5)$$

$$P\left[R_T \gtrsim 1.2\, R_{T_{SRSS}}\right] \lesssim 15\% \qquad (6)$$

These requirements are consistent with those being achieved for earthquake responses.

7. REFERENCES

1. E. Rosenblueth, "A Basis for Aseismic Design," Ph.D. Thesis, University of Illinois, Urbana, Illinois, 1951.
2. L. E. Goodman, E. Rosenblueth, and N. M. Newmark, "Aseismic Design of Firmly Founded Elastic Studies," Proceedings - Separate No. 349, ASCE, November 1953 (also Transactions, Vol. 120, ASCE, 1955, pp. 782-802).
3. A. K. Singh, S. W. Tagart, and C. V. Subramanian, Technical Basis for the Use of the Square-Root-of-the-Sum-of-Squares (SRSS) Method for Combining Dynamic Loads for Mark II Plants, General Electric Company, San Jose, California, July 1977 (NEDE-24010-P).
4. R. P. Kennedy, R. D. Campbell, K. L. Merz, and R. B. Narver, Combining Multiple Dynamic Responses by the Square-Root-of-the-Sum-of-the-Squares Method, Engineering Decision Analysis Company, Irvine, California, May 1978 (EDAC 134-240.7R).
5. C. V. Subramanian and R. P. Kennedy, Executive Summary Report and Supplement, Technical Basis for the SRSS Method of Combining Dynamic Responses, General Electric Company, San Jose, California, April 1978.
6. R. Cudlin, S. Hosford, R. Mattu, and K. Wickman, Methodology for Combining Dynamic Responses, Nuclear Regulatory Commission, September 1978 (NUREG-0484).
7. R. A. Page, D. M. Boore, W. B. Joyner, and H. W. Coulter, Ground Motion Values for Use in the Seismic Design of the Trans-Alaska Pipeline System, United States Geological Survey, 1972 (Circular No. 672).
8. A. K. Singh and C. V. Subramanian, SRSS Application Criteria as Applied to Mark II Load Combination Cases, General Electric Company, San Jose, California, October 1978 (NEDO-24010-1).
9. R. W. Clough and J. Penzien, Dynamics of Structures, McGraw-Hill, New York, 1975.

APPENDIX A

HISTORICAL APPLICATION OF SRSS METHODS IN STRUCTURAL DYNAMICS

K. L. Merz
Robert P. Kennedy

Several methods of dynamic analysis require the superposition of response components to obtain the peak response of a given system (i.e., acceleration, velocity, displacement, strain, stress, etc.) to dynamic loading. When the phase relationships between the response components are unknown, an estimate of the peak response is often obtained using the Square Root Sum of Squares (SRSS) "rule" for combining the maxima of each component response. The basis of the SRSS rule is the assumption of statistically uncorrelated response components viewed as the output of a linear dynamic system to stochastic (random) input forcing functions. If the response components are statistically uncorrelated, then the variance of the total response is simply the sum of the variance of each component. Noting that the standard deviation, σ, is defined as the square root of the variance, the standard deviation of the total response is given by the square root of the sum of the squares of the statistically uncorrelated response component standard deviations, or

$$\sigma = \sqrt{\sum_{i=1}^{n} \sigma_i^2} \qquad \text{(A-1)}$$

where the summation is over all n response components considered. Since the mean extreme-values, R_i, of response for each component of a narrowband system (i.e., a structural system with low damping) are proportional to their respective standard deviations (e.g., Reference A-1), say $R_i = c_i\sigma_i$, it may be inferred that an estimate of the mean peak response is given by,

$$R = \sqrt{\sum_{i=1}^{n} R_i^2} \qquad \text{(A-2)}$$

which, of course, is the functional statement of the SRSS rule. In general, the factors c_i are not the same for a given exceedance probability and, in general, are functions of system damping and system natural frequency. Thus, the heuristic development of the SRSS rule has prompted several empirical studies (Monte Carlo simulation) to assess the validity of the SRSS rule as an estimator. Originally, the SRSS rule was used (Reference A-2) as a means for combining modal maxima, but the SRSS rule has had other applications such as the combination of spatial response of a system subjected to independent excitation in three mutually orthogonal directions (Reference A-3) and the combination of the effects of wave passage due to multiple support excitation of long structures (Reference A-4). Most of the applications of the SRSS rule deal with the dynamic analysis of structures and equipment for earthquake ground motion. Other applications include the dynamic analysis of structures and equipment for nuclear weapons effects and shock loading for military and transportation environments. The following is a brief historical development of the use of the SRSS rule as an estimator of peak response when the phase relationships of response components are unknown.

Rosenblueth (Reference A-2) first suggested the widely used SRSS rule for combining modal maxima in an unpublished Ph.D thesis. The SRSS rule first appeared in the published literature in 1953 (Reference A-5). The justification was based on arguments stemming from a statistical analysis of earthquake ground motion viewed as a sequence of random velocity pulses. A more detailed presentation by Rosenblueth (Reference A-6) followed. The use of SRSS rule for combination of modal responses was noted by Housner (References A-7 and A-8) for earthquake response spectrum analysis of multi-degree-of-freedom structures. The SRSS rule was recommended for use in the modal analysis of structures subjected to nuclear weapons effects (Reference A-9). Since 1960, the SRSS rule has been the preferred method for combining modal responses in the earthquake design analysis of building structures using the response spectrum method (References A-10 through A-15).

A number of numerical studies have been conducted, beginning with Hudson (Reference A-16) and Jennings and Newmark (Reference A-17), which compare the effectiveness of the modal SRSS method as an estimator of peak response of multi-degree-of-freedom structures. Notable additional studies were conducted by Merchant and Hudson (Reference A-18), Newmark, et al. (Reference A-19), Butzel and Merchant (Reference A-20), and Merchant and Golden (Reference A-21). Review of these studies indicates that the SRSS estimator tends to be a mean centered estimate of peak response for a variety of different types of input (ranging from single pulses to earthquake ground motion) to multi-degree-of-freedom structures. A general trend of the SRSS method toward more conservative estimates (i.e., above the mean) in lightly damped systems subjected to earthquake motion has also been noted (Reference A-21).

Random vibration theory has been used by a number of investigators to better define the technical basis and limitations on the use of the SRSS rule for model response combination in modal analysis (References A-22 through A-28). The use of the SRSS rule for modal analysis of light secondary systems has also been studied (References A-29 through A-31).

Additionally, consideration of random vibration theory indicates that the responses of a linear system to separate statistically independent input forcing functions are also statistically independent (e.g., Reference A-1). Thus, the peak responses of separately applied dynamic loads may be combined by the SRSS rule to obtain an estimate of the peak response to the simultaneous applied loading if the loading conditions are statistically uncorrelated. Newmark and Rosenblueth (Reference A-24) indicate that the SRSS rule is applicable, even when there is considerable correlation between inputs, for response components of systems that have negligible coupling between dynamic degrees-of-freedom. Studies (Reference A-15) have indicated that the three orthogonal components of earthquake ground motion are essentially uncorrelated (i.e., statistically independent). The SRSS combination of modal response in independent directions using the response spectrum method has been demonstrated (References A-3 and A-32 through A-34) for earthquake loading. The SRSS combination of peak earthquake response in each of the three component directions is a recommended method for design analysis (Reference A-35) and is reflected in the current NRC guidelines (Reference A-36) as the preferred method for combination of spatial components of response for structures and equipment for earthquake effects. It should be noted that SRSS combination of peak responses determined from any separate dynamic loading conditions should be a valid procedure as long as the forcing functions are essentially uncorrelated.

Thus, the SRSS method of combining response components which are essentially uncorrelated has substantial historical precedence and a firm technical basis stemming from random vibration theory. The combination of multiple spatial dynamic effects due to earthquake by SRSS is the current regulatory position of the NRC on the basis that the earthquake component motions (forcing functions) are essentially uncorrelated. From a technical viewpoint, the SRSS rule must be equally applicable for the combination of any peak response components which are uncorrelated.

REFERENCES

A-1. R. W. Clough and J. Penzien, Dynamics of Structures, McGraw-Hill, New York, 1975.

A-2. E. Rosenblueth, A Basis for Aseismic Design, Ph.D. Thesis, University of Illinois, Urbana, Illinois, 1951.

A-3. S. L. Chu, M. Amin, and A. K. Singh, Spectral Treatment of Actions of Three Earthquake Components on Structures, Nuclear Engineering and Design, Vol. 21, pp. 126-136, 1972.

A-4. D. E. Shaw and P. C. Rizzo, Seismic Structural Response Analysis Considering the Effects of Wave Passage, Second Specialty Conference on Structural Design of Nuclear Plant Facilities, ASCE, Vol. 1-A, pp. 15-29, New Orleans, Louisiana, 1975.

A-5. L. E. Goodman, E. Rosenblueth, and N. M. Newmark, A Seismic Design of Firmly Founded Elastic Structures, ASCE, Proceedings - Separate No. 349, November 1953. Also, Transactions, ASCE, Vol. 120, pp. 782-802, 1955.

A-6. E. Rosenblueth, Some Applications of Probability Theory in Aseismic Design, Proceedings of the 1st World Conference on Earthquake Engineering, pp. 8/1-18, Berkeley, California, 1956.

A-7. G. W. Housner, Behavior of Structures During Earthquakes, ASCE, of the Engineering Mechanics Division, EM4, pp. 109-129, October 1959.

A-8. G. W. Housner, Vibration of Structures Induced by Seismic Waves - Part I: Earthquakes, in Shock and Vibration Handbook, Vol. III, Edited by C. M. Harris and C. E. Crede, McGraw-Hill, New York, 1961.

A-9. Design of Structures to Resist Nuclear Weapons Effects, ASCE Manual 42, New York, 1961.

A-10. N. M. Newmark, Current Trends in the Seismic Analysis and Design of High-Rise Structures, in Earthquake Engineering, Edited by R. L. Wiegel, Prentice-Hall, Englewood Cliffs, New Jersey, 1970.

A-11. D. E. Hudson, M. D. Trifunac, and A. G. Brady, Analyses of Strong Motion Earthquake Accelerograms, Vol. III: Response Spectra, Part A, Earthquake Engineering Research Laboratory, California Institute of Technology, Pasadena, California, 1972 (EERL 72-80).

A-12. N. M. Newmark and W. J. Hall, Procedures and Criteria for Earthquake Resistant Design, Building Practices for Disaster Mitigation, National Bureau of Standards, Washington, D.C., Building Science Series 46, pp. 209-236, 1973.

A-13. An Evaluation of a Response Spectrum Approach to Seismic Design of Buildings, Applied Technology Council, San Francisco, California, 1974 (ATC-2).

A-14. W. K. Stockdale, Modal Analysis Methods in Seismic Design for Buildings, U.S. Army Corps of Engineers, Construction Engineering Research Laboratory, Champaign, Illinois, 1975 (M-132).

A-15. N. M. Newmark, Inelastic Design of Nuclear Reactor Structures and Its Implications on Design of Critical Equipment, Transactions of the 4th International Conference on Structural Mechanics in Reactor Technology, Vol. K(a), Paper K 4/1, San Francisco, California, 1977.

A-16. D. E. Hudson, A Comparison of Theoretical and Experimental Determinations of Building Response to Earthquakes, Proceedings of the 2nd World Conference on Earthquake Engineering, Vol. II, pp. 1105-1119, Tokyo, Japan, 1960.

A-17. R. L. Jennings and N. M. Newmark, Elastic Response of Multi-Story Shear Beam Type Structures Subjected to Strong Ground Motion, Proceedings of the 2nd World Conference on Earthquake Engineering, Vol. II, Tokyo, Japan, 1960.

A-18. H. C. Merchant and D. E. Hudson, Mode Superposition in Multi-Degree of Freedom Systems Using Earthquake Response Spectrum Data, Bulletin of the Seismological Society of America, Vol. 52, No. 2, pp. 405-416, 1962.

A-19. N. M. Newmark, W. H. Walker, and R. J. Mosborg, Design Procedures for Shock Isolation Systems of Underground Protective Structures - Response Spectra of Multi-Degree-of-Freedom Elastic Systems, U.S. Air Force Weapons Laboratory, Kirtland Air Force Base, New Mexico, 1965 (RDT-TDR-63-3096, Vol. 5).

A-20. L. M. Butzel and H. C. Merchant, The Use of Evaluation of Shock Spectra in the Dynamic Analysis of Structures, J. of Applied Mechanics, Vol. 40, pp. 433-438, 1973.

A-21. H. C. Merchant and T. C. Golden, Investigation of Bounds for the Maximum Response of Earthquake Excited Systems, Bulletin of the Seismological Society of America, Vol. 64, No. 4, pp. 1239-1249, 1974.

A-22. E. Rosenblueth and J. Elorduy, Responses of Linear Systems to Certain Transient Disturbances, Proceedings of the 4th World Conference on Earthquake Engineering, pp. A-1/185-196, Santiago, Chile, 1969.

A-23. A. Amin and A. H. S. Ang, Statistical Treatment of Seismic Loading on Reactor Buildings and Equipments, Transactions of the 1st International Conference on Structural Mechanics in Reactor Technology, Vol. 5, Part K, Paper K 3/1, Berlin, Germany, 1971.

A-24. N. M. Newmark and E. Rosenblueth, Fundamentals of Earthquake Engineering, Prentice-Hall, Englewood Cliffs, New Jersey, 1971.

A-25. A. K. Singh, S. L. Chu, and S. Singh, Influence of Closely Spaced Modes in Response Spectrum Method of Analysis, Specialty Conference on Structural Design of Nuclear Plant Facilities, ASCE, Vol. II, pp. 479-498, Chicago, Illinois, 1973.

A-26. A. H. S. Ang, Probability Concepts in Earthquake Engineering, in Applied Mechanics in Earthquake Engineering, Edited by W. D. Iwan, ASME, New York, AMD-Vol. 8, 1974.

A-27. F. C. Nelson, The Role of Closely Space Modes in the Seismic Response of Equipment and Structures, Proceedings of the 1st U.S. National Conference on Earthquake Engineering, Ann Arbor, Michigan, 1975.

A-28. E. H. Vanmarcke, Structural Response to Earthquakes, in Seismic Risk and Engineering Decisions, Edited by C. Lomnitz and E. Rosenblueth, Elsevier, New York, 1976.

A-29. A. Amin, W. J. Hall, N. M. Newmark, and R. P. Kassawara, Earthquake Response of Multiply Connected Light Secondary Systems by Spectrum Methods, in Seismic Analysis of Pressure Vessel and Piping Components, ASME, New York, 1971.

A-30. Z. D. Jankov and C. F. Reeves, Seismic Analysis of Lightweight Elastic Systems in Nuclear Power Plants, Proceedings of the 5th World Conference on Earthquake Engineering, Paper 253a, Rome, Italy, 1973.

A-31. A. K. Singh and A. H. S. Ang, Stochastic Prediction of Maximum Seismic Response of Light Secondary Systems, Nuclear Engineering and Design, Vol. 29, pp. 218-230, 1974.

A-32. C. W. Lin, On the Simultaneous Application of the Vertical and Horizontal Response Spectra in the Seismic Analysis, Nuclear Engineering and Design, Vol. 24, pp. 232-238, 1973.

A-33. A. P. Gelman, A Multidirectional Approach to Seismic Analysis, Journal of Pressure Vessel Technology, Trans. ASME, Series J, Vol. 96, No. 4, pp. 246-250, 1974.

A-34. J. A. M. Boulet and T. G. Carley, Response Spectrum Analysis of Coupled Structural Response to a Three Component Seismic Disturbance, Transactions of the 4th International Conference on Structural Mechanics in Reactor Technology, Vol. K(a), Paper K 3/5, San Francisco, California, 1977.

A-35. W. J. Hall, B. Mohraz, and N. M. Newmark, **Statistical Analysis of Earthquake Response Spectra**, Transactions of the 3rd International Conference on Structural Mechanics in Reactor Technology, Vol. 4, Park K, Paper K 1/6, London, England, 1975.

A-36. **Combining Modal Responses and Spatial Components in Seismic Response Analysis**, U.S. Nuclear Regulatory Commission, Washington, D.C., 1976 (USNRC Regulatory Guide 1.92, Revision 1).