BAW-10120A

Topical Report
July 1979

COMPARISON OF CORE PHYSICS CALCULATIONS WITH MEASUREMENTS

Babcock & Wilcox

BAW-10120A

Topical Report
July 1979

COMPARISON OF CORE PHYSICS CALCULATIONS WITH MEASUREMENTS

by

J. J. Woods
T. L. Wilson
W. G. Pettus

BABCOCK & WILCOX
Power Generation Group
Nuclear Power Generation Division
P. O. Box 1260
Lynchburg, Virginia 24505

Babcock & Wilcox

UNITED STATES
NUCLEAR REGULATORY COMMISSION
WASHINGTON, D. C. 20555

JUN 14 1979

Mr. James H. Taylor
Manager, Licensing
Babcock & Wilcox Company
Nuclear Power Generation
P. O. Box 1260
Lynchburg, Virginia 24505

Dear Mr. Taylor:

SUBJECT: EVALUATION OF BAW-10120P

We have completed our evaluation of Babcock & Wilcox Topical Report BAW-10120P, "Comparison of Core Physics Calculations with Measurements." We have determined that BAW-10120P is acceptable for reference to describe the techniques used to measure core physics parameters. A summary of our evaluation is enclosed.

If our criteria or regulations change, such that our conclusions concerning BAW-10120P are invalidated, we will notify you and provide you with an opportunity to revise and, if you desire, resubmit this report for our review.

We request, that within three months, you issue a revised version of BAW-10120P and a non-proprietary version incorporating this letter and your responses to our requests for additional information.

In your letter of March 14, 1978, you requested that BAW-10120P be withheld from public disclosure pursuant to 10 CFR 2.790. In support of this request, you submitted an affidavit with your letter of July 14, 1978, which contained statements as to the reasons for withholding this information from public disclosure. We have reviewed your application and material based on the requirements and criteria of 10 CFR 2.790 and have determined that the above-mentioned document sought to be withheld contains trade secrets or confidential or privileged commercial or financial information. We also have found at this time that the right of the public to be fully apprised as to the bases for and effects of the proposed licensing action does not outweigh the demonstrated concern for protection of your competitive position. Accordingly, we have determined that the information should be withheld from public disclosure. We therefore approve your request for withholding pursuant to 10 CFR 2.790 and are withholding BAW-10120P from public inspection as proprietary.

Withholding from public inspection shall not affect the right, if any, of persons properly and directly concerned to inspect the documents. If the

904329

need arises, we may send copies of this information to our consultants working in this area. We will, of course, assure that the consultants have signed the appropriate agreements for handling proprietary data.

If the basis for withholding this information from public inspection should change in the future such that the information could then be made available for public inspection, you should promptly notify the NRC.

Sincerely,

Steven A. Varga, Chief
Light Water Reactors Branch No. 4
Division of Project Management

Enclosure:
As stated

cc: Mr. Robert B. Borsum
Babcock & Wilcox Company
7735 Old Georgetown Road
Bethesda, Maryland 20014

Review . BAW-10120P - "Comparison of C e Physics Calculations with Measurements."

Report Number:	BAW-10120P (Proprietary)
Report Title:	Comparison of Core Physics Calculations with Measurements
Report Date:	March, 1978
Originating Organization:	Babcock & Wilcox
Reviewed by:	Core Performance Branch/Walter L. Brooks

The Power Generation Group of Babcock & Wilcox has submitted licensing topical report BAW-10120P entitled "Comparison of Core Physics Calculations with Measurements." The report describes the techniques used to measure various core parameters, discusses the accuracy of the measurements, and presents a comparison of measured values with those calculated by methods which have been described in other topical reports. BAW-10120P is one of a series of topical reports which have been submitted by Babcock & Wilcox in order to provide the staff with generic information on the nuclear design of B&W reactors.

Our review of the subject topical report follows.

1. Summary of Report

This report documents the techniques currently used to measure core physics parameters in B&W reactors, provides estimates of the uncertainties in the measurements and compares measured and calculated values of the various parameters. The parameters discussed in the reports are: control rod worths - individual (ejected) rods, individual bank, regulating banks and total worth, the all rods out boron concentration as a function of core life, and the moderator and fuel temperature coefficients.

For each parameter discussed the measurement technique is described (including the data analysis method), the measurement uncertainty is assessed, and comparisons are made between measured and calculated results. As a result of the comparisons values for calculational biases and uncertainties are obtained. Certain calculated values (e.g., rod bank worths) used in safety analyses are corrected by the biases but conservative values are employed for the uncertainties. For other parameters (e.g., moderator coefficients) conservative values are used in the safety analyses and the measurements are used to confirm their conservatism

2. Summary of Evaluation

We reviewed the description of the measurement techniques, the evaluation of measurement uncertainties, the comparisons between measurement and calculation, and the conclusion drawn from the comparisons. The following discussion summarizes our findings.

Reactivity measurements are made with the Babcock & Wilcox Reactimeter which uses periodic samples of neutron flux as input to the mono-energetic, point-reactor kinetics equations with six delayed neutron groups to compute the overall core reactivity. The algorithm employed in the Reactimeter is described and an analysis of the errors in the reactivity measurement is presented. The Reactimeter is similar in nature to other reactivity meters employed in the industry and we conclude that it is state-of-the-art and therefore acceptable.

Control rod worths are measured by boron swap, rod swap or rod drop techniques. These techniques are described and analyses performed of the uncertainties in the measurements. These techniques are standard ones used throughout the industry and are acceptable

Reactivity coefficients are determined directly by making a change in the appropriate parameter (moderator temperature, power, etc.) and measuring the resultant reactivity change. Reactivity changes are measured with the Reactimeter or by making compensating control rod changes on previously calibrated rods. The techniques are described and analyses are presented of measurement uncertainties. The techniques are state-of-the-art and are acceptable.

Comparisons are presented between calculated and measured values of rod bank worths, single rod worths, critical boron concentration, and reactivity coefficients. The calculations were performed for the same reactor conditions at which the measurements were made. In some cases earlier experimental values were reanalyzed using current analysis techniques.

The comparisons show that rod bank worths may be calculated to within the measurement uncertainty (about 7 percent). Single (ejected) rod worths tend to be overpredicted with the largest difference between prediction and measurement being 0.17 percent reactivity change. It should be noted that conservative values are used in the safety analyses rather than adding an uncertainty to the calculated value.

Critical boron concentrations are predicted to within 20 parts per million for cycles which contain no lumped burnable poison and to within 30 parts per million for cycles that contain lumped burnable poison. Reactivity coefficients are also predicted to within approximately the measurement error.

It should be noted that the measurements and calculations were performed for several Babcock & Wilcox reactors for up to three cycles of operations. Thus the conclusions are applicable for both first and succeeding cycles.

3. Evaluation Procedure

The review of topical report BAW-10120P has been conducted within the guidelines provided by the Standard Review Plan, Section 4.3. Sufficient information is presented to permit a knowledgeable person to conclude that appropriate techniques have been employed to do the measurements, suitable analyses of the measurement uncertainty have been performed, and proper conclusions have been drawn with respect to calculational uncertainties and conservations.

4. Regulatory Position

Based on our review of licensing topical report BAW-10120P we conclude that it is acceptable for reference in licensing actions in regard to comparison of core physics calculations and measurements.

Babcock&Wilcox

Power Generation Group

P.O. Box 1260, Lynchburg, Va. 24505
Telephone: (804) 384-5111

December 7, 1978

Mr. S. A. Varga, Chief
Light Water Reactors Branch No. 4
Division of Project Management
U. S. Nuclear Regulatory Commission
Washington, D.C. 20555

Dear Mr. Varga:

Subject: Response to Questions on Topical Report BAW-10120.

The responses to the questions in your July 12, 1978 letter are attached. We hope this adequately answers your question, however, if you desire additional information please contact Mr. R. J. Finnin (Ext. 2892) of my staff.

Very truly yours

James H. Taylor
Manager, Licensing

JHT/fw
cc: R. B. Borsum - B&W

Attachment - As stated

Question #232.1:

Please provide a summary table of calculational uncertainties to be applied to the various quantities considered in this report. These should be in the form (absolute magnitude, percent of calculated value, etc.) most appropriate for each quantity.

Answer:

The mean differences and standard deviations for the control rod worths and reactivity coefficients reported in BAW-10120 are shown below.

	Mean Difference	Standard Deviation
Control Rod Worths		
Individual Bank Worths, %Δρ	.02	.06
Regulating Bank Worths, %Δρ	.04	.12
Total Pattern Worths, %Δρ	-.33	.46
Ejected Rod Worths, %Δρ	.04	.11
Reactivity Coefficients		
Temperature Coefficients at Zero Power, μρ/°F	- 6	8
Temperature Coefficients at Power, μρ/°F	12	13
Power Doppler Coefficients, μρ/%FP	-17	12
Differential Boron Worths, %Δρ/ppm	-.0004	.00076

Calculational uncertainties with assigned confidence limits have not been determined for the parameters addressed in BAW-10120 because it is B&W's opinion that there is not sufficient data available to perform a reliable statistical determination of calculational uncertainty. When calculational uncertainties are required for design analysis, a conservative estimate is used. For example, a 10% uncertainty is used with control rod worths when performing shutdown margin calculations as compared to a standard deviation which is less than 5% of the average total pattern worth.

Question #232.2 (Section 3.2.3):

For the value of the uncertainty in β_{eff} in mixed environments has the uncertainty in the ratio of fissions between uranium and plutonium been considered?

Answer:

Yes, the 4% uncertainty assigned in Section 3.2.3 to the absolute delayed neutron yield is derived as follows:

The average delayed neutron fraction can be written as neutron yield weighted average:

$$f\beta_{U^{235}} + (1-f)\beta_{Pu^{239}} = \bar{\beta}$$

where

f = fraction of fissions in U^{235}

$\beta_{Pu^{239}}$ $\beta_{U^{235}}$ = delayed neutron fractions for Pu^{239} and U^{235}

$\bar{\beta}$ = average delayed neutron fraction

The uncertainty can be written as:

$$\varepsilon^2(\bar{\beta}) = \left[\left(\frac{\partial\bar{\beta}}{\partial f}\right)\varepsilon(f)\right]^2 + \left[\left(\frac{\partial\bar{\beta}}{\partial\beta_{U^{235}}}\right)\varepsilon(\beta_{U^{235}})\right]^2 + \left[\left(\frac{\partial\bar{\beta}}{\partial\beta_{Pu^{239}}}\right)\varepsilon(\beta_{Pu^{239}})\right]^2$$

$$= \left[(\beta_{U^{235}} - \beta_{Pu^{239}})\,\varepsilon(f)\right]^2 + \left[f\varepsilon(\beta_{U^{235}})\right]^2 + \left[(1-f)\,\varepsilon(\beta_{Pu^{239}})\right]^2$$

This expression can be rewritten in terms of percentage error:

$$\frac{\varepsilon\bar{\beta}}{\bar{\beta}} \times 100\% = \left\{\left[(\beta_{U^{235}} - \beta_{Pu^{239}})\,\varepsilon(f)\right]^2 + \left[f\varepsilon(\beta_{U^{235}})\right]^2 + \left[(1-f)\,\varepsilon(\beta_{Pu^{239}})\right]^2\right\}^{1/2} \Big/ (f\beta_{U^{235}} + (1-f)\beta_{Pu^{239}})$$

$\beta_{U^{235}}$ and $\beta_{Pu^{239}}$ are .0065 and .00212 respectively,(1)

$\varepsilon(\beta_{U^{235}})$ and $\varepsilon(\beta_{Pu^{239}})$ are 3.1% and 4.8% respectively,(2)

f is approximately 0.8 at the beginning of a typical reload cycle, and $\varepsilon(f)$ is approximately 3%, based on measured versus predicted isotopics from post-irradiation examination.

(2)

Hence, the uncertainty in $\bar{\beta}$ is:

$$\frac{\epsilon(\bar{\beta})}{\bar{\beta}} \times 100\% = \left\{\left[(.0065 - .00212)(.03)\right]^2 + \left[(0.8)(.031)(.0065)\right]^2 + \left[(.2)(.048)(.00212)\right]^2\right\}^{\frac{1}{2}} \Big/ \left[(.8)(.0065) + (.2)(.00212)\right]$$

$$= 3.72\%$$

This has been rounded to 4% in section 3.2.3.

Reference 1 - Hetrick, Dynamics of Nuclear Reactors, p. 12.
Reference 2 - Keepin, Physics of Reactor Kinetics, Table 4-4.

Question #232.3 (Page 3-3):

It appears that the ΔT_{avg} in the denominator of the first and last equations on this page should be replaced by T_{avg}. Please confirm or present reasons for the correctness of the present form.

Answer:

The symbol ΔT_{avg} indicates the full span of the instrument, that is 520 to 620°F. Thus, the denominator is 100°F rather than the full scale temperature, 620°F, or the measured temperature, T_{avg}. This ambiguity could be eliminated by correcting a typographical omission.

$$\frac{\epsilon(T_{avg})}{\Delta T_{avg}} = \left[(0.36)^2 + (0.3)^2 + \left(\frac{5}{2}\right)^2 (0.21)^2 + (1.0)^2 + (0.1)^2\right]^{\frac{1}{2}}$$

$$= 1.2\% \text{ of span (520 to } 620^{\circ}\text{F)} =$$

and

$$\epsilon(T_{avg}) = 1.2^{\circ}\text{F}$$

Question #232.4 (Equation 3-15):

The term $\sqrt{\pi}$ in this equation apparently should be $\sqrt{\eta}$.
Please clarify.

Answer:

This was a typographical error; the term should be $\sqrt{\eta}$.

Question #232.5 (Section 4.2.1):

Calculational bias is taken into account for critical boron concentrations and other quantities. Please provide a summary table of such biases.

Answer:

Section 4.2.1

Calculational bias has not been included in the calculated k_{eff} at hot zero power in Table 4-5. However, based on the results shown in Table 4-5, a calculational bias of 30 ppmb is taken into account when the model is used to make AROCBC predictions at hot zero power.

Section 4.2.2.1

Standard reactivity corrections which account for reactivity effects due to such known phenomena as nonuniform axial burnup and provide an overall normalization of the model to operating plant data have been included in the predicted critical boron concentrations at power shown in Figures 4-2 thru 4-7.

Figures 4-2, 4-3 - never had LBP - no correction.
Figure 4-4 - LBP - non-rodded - See Table 1.
Figure 4-5 - LBP-rodded - See Table 1.
Figures 4-6, 4-7 - non-LBP after LBP - constant correction of +25 ppmb.

These corrections are re-evaluated periodically as additional operating plant data is received at B&W.

Section 4.2.2.2

The constant reactivity correction factor applied to the FLAME eigenvalue is $0.85 \times 10^{-3}\%\ \Delta k/k$.

Section 4.3

The biases discussed in this section have not been included in the predicted coefficients in Tables 4-6, 4-7, and 4-8. The biases refer to observations that the model tends to overpredict or underpredict particular coefficients.

Table 1

Burnup MWD/T	Correction Factor (ppm boron) LBP-Rodded	LBP - Non-Rodded
800	+66	+16
1600	+67	+17
2400	+68	+18
3200	+73	+23
4000	+74	+24
4800	+71	+21
5600	+68	+18
6400	+67	+17
7200	+61	+11
8000	+53	+ 3
8800	+50	0
9600	+43	- 7
10400	+35	-15
11200	+28	-22
12000	+20	-30
12800	+10	-40
13600	+ 1	-49

UNITED STATES
NUCLEAR REGULATORY COMMISSION
WASHINGTON, D. C. 20555

JUL 21 1978

Babcock & Wilcox Company
ATTN: Mr. James H. Taylor
Manager, Licensing
Nuclear Power Generation
P. O. Box 1260
Lynchburg, Virginia 24505

Gentlemen:

SUBJECT: REVIEW OF TOPICAL REPORT BAW-10120

In order to complete our review of the subject report, we require adequate responses to the enclosed requests for additional information. If you have any questions on this matter, please contact us.

Sincerely,

Steven A. Varga, Chief
Light Water Reactors Branch No. 4
Division of Project Management

Enclosure:
As stated

cc: Mr. Robert B. Borsum
7735 Old Georgetown Road
Bethesda, MD 20014

904341

232.1 Please provide a summary table of calculational uncertainties to be applied to the various quantities considered in this report. These should be in the form (absolute magnitude, percent of calculated value, etc.) most appropriate for each quantity.

232.2 (3.2.3) For the value of the uncertainty in β_{eff} in mixed environments has the uncertainty in the ratio of fissions between uranium and plutonium been considered?

232.3 (p. 3-3) It appears that the ΔT_{avg} in the denominator of the first and last equations on this page should be replaced by T_{avg}. Please confirm or present reasons for the correctness of the present form.

232.4 (Eq. 3-15) The term $\sqrt{\pi}$ in this equation apparently should be $\sqrt{\eta}$. Please clarify.

232.5 (4.2.1) Calculational bias is taken into account for critical boron concentrations and other quantities. Please provide a summary table of such biases.

Babcock & Wilcox
Power Generation Group
Nuclear Power Generation Division
Lynchburg, Virginia

Topical Report BAW-10120A

July 1979

Comparison of Core Physics Calculations With Measurements

J. J. Woods, T. L. Wilson, W. G. Pettus

Key Words: Control Rod Worth, Reactivity Coefficient, Reactivity Depletion, Reactor Physics

ABSTRACT

The techniques currently used to measure core physics parameters in Babcock & Wilcox's operating reactors are described. The uncertainties inherent in these measurements are estimated, and measured and calculated data are compared. These comparisons confirm the accuracy of B&W's standard calculational procedures and techniques.

CONTENTS

CONTENTS (Cont'd)

List of Tables

List of Tables (Cont'd)

List of Figures

1. INTRODUCTION AND SUMMARY

1.1. Introduction

This report documents the techniques currently used to measure core physics parameters in B&W's operating reactors. The uncertainties inherent in these measurements are estimated, and measured and calculated data are compared. These comparisons confirm the accuracy of B&W's standard calculational procedures and techniques. The core physics parameters included and the importance of each are summarized below.

Control rod worth is measured as a part of the zero power physics tests performed during cycle startups. The results are compared with predicted control rod worths from the Physics Test Manual and must fall within a certain acceptance criteria to ensure that adequate shutdown margins exist. In the case of the ejected rod, comparison verifies that the limits assigned in safety analysis calculations are met.

Reactivity depletion and boron worth calculations are used to construct the boron letdown curves used by the plant operator throughout the cycle to determine critical control rod positions and boron concentrations. Technical Specifications require that the measured reactor coolant boron concentration be periodically compared to the predicted concentration. If the difference between the observed and predicted concentrations reaches the equivalent of 1% in reactivity, the incident must be reported to the NRC as a reactivity anomaly. All-rods-out critical boron concentration (AROCBC) measurements are taken as a part of the zero power physics tests performed during every cycle startup. The results are compared with predicted AROCBCS from the Physics Test Manual, and must fall within a certain acceptance criteria to ensure that the actual excess reactivity of the core is consistent with design calculations.

Reactivity coefficients are measured at operating moderator temperatures and at several core power levels during cycle startup. Power Doppler coefficients are measured at BOC and at other times during the cycle. The results of the

measurements are compared with predictions from the Physics Test Manual and specified acceptance criteria must be met. Variations in the reactivity coefficients as a function of temperature, core power level, and core burnup must be accurately known, so that the plant operator can (1) determine critical rod positions and boron concentrations, and (2) anticipate plant response to control system changes. Comparisons of measured and calculated reactivity coefficients also verify that the limiting values used in various safety analysis calculations are met.

It is noted that power distributions are not included in this report. Topical Report BAW-10119[1] documents power distribution measurement and calculational techniques and their associated uncertainties.

1.2. Summary

The measurement techniques used for control rod worths, critical boron concentrations, and reactivity coefficients are described in section 2. Data analysis methods for each test are also described.

In section 3, measurement uncertainties are estimated by identifying sources of errors and probabalistically combining individual components to obtain the total uncertainty in the measurement. The measurement uncertainties obtained in this section are estimates of the standard deviation and are summarized in Table 3-1.

Measured and calculated data are compared in section 4. All calculations were performed using B&W's current standard calculational techniques and for the actual plant conditions at the time of the measurement. The comparisons demonstrate taht B&W's standard calculational models are accurate tools for predicting core physics parameters.

Very good agreement was obtained between measured and calculated control rod bank and total rod worths.

Measured and calculated ejected rod worths agree within $\Delta\rho$.

Measured critical boron concentrations at power generally lie within ppm of predicted valves for non-LBP cycles and within ppm of predicted for LBP cycles.

Measured and calculated temperature and power Doppler coefficients agree within

Measured and calculated differential boron worths agree very well, the difference being less than

Section 4 shows that B&W's standard calculational techniques and models accurately predict core behavior.

2. MEASUREMENT TECHNIQUES AND DATA ANALYSIS METHODS

Measurement techniques described in this section include previously used and current test techniques for B&W test programs. In some cases, several techniques have been used for the same measurement; this is the result of striving for improved accuracy and reduced test time.

Data analysis methods for each test are also described. Generally, data are analyzed in the field by evaluating data from both a B&W Reactimeter and a pen recorder. In certain cases, data recorded by the Reactimeter on magnetic tape have been analyzed in Lynchburg using the CDC 7600 computer. The B&W Reactimeter which is used in almost all the core physics measurements, is described in section 2.1. The techniques for measuring control rod worths, reactivity depletion, and reactivity coefficients are described in sections 2.2, 2.3, and 2.4, respectively.

2.1. B&W Reactimeter

The B&W Reactimeter is a reactivity computer and data logging system used primarily during zero power physics and power escalation testing. It is functionally described in section 2.1.1, the reactivity algorithm used is described in section 2.1.2, and uncertainties in the measured reactivities are discussed in section 3.2.

2.1.1. Description

The B&W Reactimeter uses periodic samples of neutron flux as input to the monoenergetic, point reactor kinetics equations with six delayed neutron groups to compute the overall core reactivity. The neutron flux is monitored with a standard ion chamber connected to a multi-range picoammeter with ranging controlled by the computer. In addition to the neutron flux, the Reactimeter also samples 24 other reactor and plant parameters once every 0.2 second. The signals are converted to digital information and may be recorded on magnetic tape. The logging rate on magnetic tape can be selected from one data set per 0.2

second to one per 12.6 seconds. One data set consists of reactivity and 24 other channels of information. The magnetic tape format is compatible with most large computer systems, making the data available for reanalysis at a later date. The reactivity and one other variable can be displayed online during a test. In addition, connection points are provided for recording the neutron flux and one other variable on an analog strip chart recorder. The Reactimeter system includes a teletype which may be used to print out data stored on magnetic tape whenever the Reactimeter is not being used to record data.

2.1.2. Reactivity Algorithm

The algorithm for computing reactivity from flux is based on the integral form of the point kinetics equations given in equation 1[2]:

$$n(t) = \sum_{i=0}^{6} \gamma_i \int_0^{\infty} n(t - \tau)k(t - \tau)\lambda_i e^{-\lambda_i \tau} d\tau \qquad (1)$$

where

$n(t)$ = neutron population,

$k(t)$ = multiplication factor,

$1/\lambda_0$ = neutron lifetime,

$$\gamma_0 = 1 - \beta - \sum_{i=1}^{6} \beta_i \lambda_i / (\lambda_0 - \lambda_i),$$

$$\gamma_i = \beta_i \lambda_0 / (\lambda_0 - \lambda_i) \text{ for } i = 1,6.$$

The prompt jump approximation is valid for the range of reactivities of interest. This approximation is equivalent to letting $1/\lambda_0$ approach zero. In this case, $\lambda_0 e^{-\lambda_0 \tau}$ becomes a delta-function at $\tau = 0$, $\gamma_0 = 1 - \beta$, and $\gamma_i = \beta_i$ for $i = 1,6$. Then we have

$$n(t) = (1 - \beta)n(t)k(t) + \sum_{i=1}^{6} \beta_i \int_0^{\infty} n(t - \tau)k(t - \tau)\lambda_i e^{-\lambda_i \tau} d\tau. \qquad (2)$$

Using the definition $\rho = (k - 1)/k$ and rearranging terms gives the following:

$$n(t) = \frac{1 - \rho}{\beta - \rho} \sum_{i=1}^{6} \beta_i \int_0^{\infty} n(t - \tau)k(t - \tau)\lambda_i e^{-\lambda_i \tau} d\tau. \qquad (3)$$

Since most of the contributions to the integrals in equation 3 come from near $\tau=0$ and since $k(t)$ is weakly varying in any case, we take $k(t)$ outside of the integral, giving

$$n(t) = \sum_{i=1}^{6} \frac{\beta_i}{\beta - \rho(t)} \int_0^{\infty} n(t-\tau)\lambda_i e^{-\lambda_i \tau} d\tau. \tag{4}$$

Solving for $\rho(t)$ in equation 4 and substituting $Y_i(t) = \int_0^{\infty} n(t-\tau)\lambda_i e^{-\lambda_i \tau} d\tau$ and $\beta_i = a_i \beta$,

$$\rho(t) = \beta\left[1 - \frac{\sum_{i=1}^{6} a_i Y_i(t)}{n(t)}\right]. \tag{5}$$

A recursion relation can be derived for $Y_i(t)$; let $Y_i(t)$ be represented by

$$Y_i(t+\Delta T) = \int_0^{\Delta t} n(t-\tau+\Delta t)\lambda_i e^{-\lambda_i \tau} d\tau + \int_{\Delta t}^{\infty} n(t-\tau+\Delta t)\lambda_i e^{-\lambda_i \tau} d\tau. \tag{6}$$

The first integral in equation 6 can be approximated by assuming that the variation of $n(t)$ between t and $t+\Delta t$ is small:

$$\int_0^{\Delta t} n(t-\tau+\Delta t)\lambda_i e^{-\lambda_i \tau} d\tau \approx n(t)(1 - e^{-\lambda_i \Delta t}) \tag{7}$$

The second integral in equation 6 can be simplified by making the substitution $u = \tau - \Delta t$:

$$\int_{\Delta t}^{\infty} n(t-\tau+\Delta t)\lambda_i e^{-\lambda_i \tau} d\tau = e^{-\lambda_i \Delta t} Y_i(t). \tag{8}$$

Thus, $Y_i(t+\Delta t)$ can be expressed as

$$Y_i(t+\Delta t) = n(t)(1 - e^{-\lambda_i \Delta t}) + e^{-\lambda_i \Delta t} \cdot Y_i(t), \tag{9}$$

or, using $Y_{ij} = Y_i(j \cdot \Delta t)$ and $n_j = n(j \cdot \Delta t)$,

$$Y_{i,j+1} = n_j(1 - e^{-\lambda_i \Delta t}) + e^{-\lambda_j \Delta t} \cdot Y_{ij}. \tag{10}$$

Equations 5 and 10 have been programmed for computation by the Reactimeter.

2.2. Control Rod Worths

B&W's current 177-fuel assembly cores use 69 control rod assemblies (CRAs), which are assigned to eight rod groups. Groups 1 through 4 are safety groups and during operation are withdrawn from the core. Groups 5, 6, and 7 are regulating banks. Group 8 contains partial-length rods used for axial power shaping.

Control rod group worths, total control rod worths, and ejected rod worths are measured with the reactor at hot zero power conditions. Differential rod worths are measured at power for use in the analysis of reactivity coefficient measurements.

2.2.1. Control Rod Group Worths

Two methods have generally been used to measure control rod group worths. These are the boron swap and rod drop techniques.

2.2.1.1. Boron Swap

The reactivity worths of the regulating groups are measured by the boron swap technique. In this technique, boron is continuously increased or decreased, while control rods are moved in small increments to keep reactivity near zero. The control rod movements are made so that the reactivity swings between approximately -300 and +300 μρ. The resultant reactivity trace is a sawtooth-like curve such as that shown in Figure 2-1.

B&W's Reactimeter is used to record control rod position, flux, and other plant parameters and to compute reactivity. These data are stored on magnetic tape. Reactivity and control rod position are also recorded by a two-pen chart recorder.

In the field, group worths are obtained as illustrated in Figure 2-1. Lines are drawn through the reactivity trace from the chart recorder before and after a rod movement. The reactivity difference determined from the intersections of these lines with a vertical line drawn through the midpoint of the rod movement represents the reactivity change due to rod movement alone. Group worths are obtained by summing the reactivity increments.

Selected data from reactimeter tapes have been reanalyzed using a computer program.[3] In this analytical method, linear least-squares fits are made to the boron change portions of the reactivity traces before and after a rod movement. Since the rod movement causes a flux redistribution, the time interval for the

latter linear fit is chosen well after the rod movement to allow time for any redistribution effects to "die out." The difference in reactivities at the midpoint of a rod movement divided by the corresponding change in rod position gives the differential rod worth. The integral worth of a group is obtained by fitting piece-wise, cubic splines to the differential rod worth and integrating under the fitted curve. The connecting points between splines are chosen to be the 0, 20, 40, 60, and 100% withdrawn positions. Figures 2-2 and 2-3 illustrate typical results of this method. Figure 2-2 shows reactivity versus time and the fitted straight lines about one rod movement. Figure 2-3 shows differential rod worth data and the spline fit through this data. Rod worths obtained by this technique generally agree with rod worths evaluated in the field to within the measurement uncertainty (see section 3.3.1).

2.2.1.2. Rod Drop

Because the Technical Specifications require that a shutdown margin of 1.0% $\Delta k/k$ be maintained at all times, safety bank worth has been measured using a rod drop rather than a boron swap technique. The measurement consists of dropping the rods to be measured (typically banks 1-4) into the core while the reactivity is logged on the B&W Reactimeter. By plotting the reactivity as a function of time, the reactivity at zero time (the time of the drop) can be determined.

Early in the development of the rod drop measurement technique, it was recognized that large negative reactivity insertions are not measured accurately by the Reactimeter. The inaccuracy is due in part to flux redistribution effects but also to the fact that a wide range of large negative reactivities give small changes in the stable period. For example, the period for a reactivity insertion of -$1.0 is -82.9 seconds, while the period for a reactivity insertion of -$10.0 is 80.3 seconds.

2.2.2. Ejected Rod Worths

Three measurement techniques have been used to measure ejected rod worth: boron swap, rod swap and rod drop.

2.2.2.1. Boron Swap

The boron swap technique for measuring ejected rod worth is essentially the same as the boron swap technique for group worth. Beginning with the ejected rod fully inserted, the boron concentration is slowly, continuously increased. The ejected rod is withdrawn in discrete steps to compensate for the changing

boron concentration. The analytical method described in section 2.2.1.1 is used to determine the ejected rod worth.

2.2.2.2. Rod Swap

In the rod swap method, a control rod configuration is established with the ejected rod 100% withdrawn and the remainder of the bank containing the ejected rod fully inserted (or at the rod insertion limit). The initial position of the controlling group is recorded. The ejected rod is then inserted into the core as the controlling group is withdrawn. The final controlling group position with the ejected rod 0% withdrawn is recorded. Integral rod worth curves are obtained by integrating the differential curves obtained as the control rods were deborated into the core in normal sequence (section 2.2.1.1). The ejected rod worth is computed from the integral worth curves by taking the difference between the initial and final controlling group reactivity worths.

2.2.2.3. Rod Drop

A rod configuration is established with the ejected rod 100% withdrawn. The ejected rod is tripped, and the reactivity is logged using a B&W Reactimeter. By plotting reactivity as a function of inverse time, 1/t, the reactivity can be extrapolated to infinite time. Figure 2-6 illustrates this procedure. Experiments have shown that the shapes of reactivity traces versus 1/t depend on the relative position of rod and detector (i.e., the line depends on flux shape effects), but the intercepts of the lines at $1/t = 0$ are relatively invariant with respect to the relative position of rods and detector. Thus, the spatial effects are effectively removed by extrapolation.

2.2.3. Differential Rod Worths at Power

The fast insert-withdraw method is used to measure differential rod worths at power. In this measurement, the regulating group is inserted for approximately 6 seconds, followed immediately by the reverse motion for approximately 6 seconds. Reactivity, rod position, and neutron flux are recorded by the B&W Reactimeter every 0.2 second. Since the duration of the measurement is less than the primary system loop time, the core inlet temperature remains constant.

The ΔT across the core varies slightly; however, this has been shown to have a negligible effect on the rod worth determination. The average or effective fuel temperature varies during the measurement and contributes to the reactivity change through the Doppler coefficient.

The differential rod worth can then be evaluated using the following linearized reactivity model:

$$\rho = \frac{\partial \rho}{\partial H} H + \alpha_D T_F + \rho_0$$

where

ρ = reactivity,

α_D = Doppler coefficient,

$\partial \rho / \partial H$ = differential rod worth,

ρ_0 = constant representing all remaining reactivity in the core,

H = rod position,

T_F = fuel temperature.

A hypothetical quick-insertion, quick withdrawal measurement is illustrated in Figure 2-7. Since the insertion and withdrawal times normally differ, the final rod position is slightly different from the starting position. The reactivity at t_2 and t_3 is given by

$$\rho_2 = \rho_1 + \frac{\partial \rho}{\partial H} (H_2 - H_1) + \alpha_D (T_{f2} - T_{f1})$$

$$\rho_3 = \rho_1 + \frac{\partial \rho}{\partial H} (H_3 - H_1) + \alpha_D (T_{f3} - T_{f1})$$

where α_D is the Doppler coefficient, and $\partial\rho/\partial H$ is the average differential rod worth over the range of the measurement. Defining

$$f = \frac{T_{f2} - T_{f1}}{T_{f3} - T_{f1}},$$

and solving the two equations above for $\partial\rho/\partial H$ gives

$$\frac{\partial\rho}{\partial H} = \frac{\rho_2 - (1 - f)\rho_1 - f\rho_3}{H_2 - (1 - f)H_1 - fH_3}$$

Because of random fluctuations in the data, it is desirable to use all the data points during an insert-withdrawal rather than three. A linear regression scheme is used to determine the best differential rod worth.

2.3. Reactivity Depletion

The AROCBC is a measure of the excess reactivity of the core in terms of the boron concentration required for criticality. The measurement is taken at BOL during zero power physics testing and at power throughout the life of the core. However, the test procedures differ somewhat for the two cases.

2.3.1. AROCBC at Zero Power

During zero power physics testing, criticality is established with the regulating rods near the all-out configuration, 80 to 90% withdrawn on the last

controlling group. The boron concentration at this critical configuration is measured. The remaining control rods in the core are then withdrawn fully and the resulting reactivity insertion is measured. This measured reactivity worth is converted to an equivalent boron concentration using previously determined differential boron worths. The sum of the reactor coolant boron concentration and the equivalent boron worth is the AROCBC.

2.3.2. AROCBC at Power

During power operation, the measurement is taken based on normal operating data. When the reactor is at equilibrium full-power conditions, the boron concentration is measured and the control rod positions are recorded. The worth of the control rods in the core is converted to equivalent boron concentration using measured or calculated control rod and soluble boron worths. The AROCBC is the sum of the measured boron concentration and the concentration equivalent to the inserted control rod worth. The boron concentration in the core with the transient rods fully inserted, the APSRs inserted, and all other rods fully withdrawn is also calculated for comparison with the results of two-dimensional calculations.

2.4. Reactivity Coefficients

Four reactivity coefficients are considered to be of primary importance in describing feedback effects in pressurized water reactors:

1. Temperature coefficient, defined as the change in reactivity per unit change in core average temperature while holding the power constant.
2. Moderator temperature coefficient, defined as the change in reactivity per unit change in moderator average temperature while holding the average fuel temperature constant.
3. Power Doppler coefficient, defined as the change in reactivity per unit change in core power while holding the moderator average temperature constant.
4. Doppler coefficient, defined as the change in reactivity per unit change in fuel temperature while holding the moderator average temperature constant.

Test and data analysis methods for reactivity coefficients are different for zero power and power escalation testing and are discussed separately.

2.4.1. Isothermal Temperature Coefficient at Zero Power

At zero power, all parts of the core are at the same temperature; thus, the isothermal temperature coefficient is measured. The isothermal temperature coefficient of reactivity, defined as the change in reactivity per unit change in core temperature ($\Delta\rho/°F$), is given by

$$\alpha_T = \alpha_M + \alpha_D$$

where

α_T = isothermal temperature coefficient,
α_M = moderator temperature coefficient,
α_D = Doppler (fuel temperature) coefficient.

To obtain the moderator coefficient, the Doppler coefficient is subtracted from the isothermal temperature coefficient. Calculated values of the Doppler coefficient are used for this purpose.

The test procedure for obtaining isothermal temperature coefficients begins with the core at equilibrium zero-power conditions. An increasing or decreasing temperature ramp of approximately 30°F/h is initiated by changing the turbine header pressure setpoint. When a change of 5-10°F is attained, the temperature is stabilized to allow all parts of the core to come to a uniform temperature. Temperature, reactivity, and other data are recorded by the B&W Reactimeter. The isothermal temperature coefficient is calculated using the following formula:

$$\alpha_T = \frac{\rho_2 - \rho_1}{T_2 - T_1}$$

where

T_1, T_2 = temperatures on stable plateaus,
$\rho_2 - \rho_1$ = measured reactivity change between temperature plateaus.

Control rods are not usually moved during the test. However, if neutron power or reactivity approach their limiting values, control rods can be moved in discrete steps to return them to their normal range. In this case, the temperature coefficient is

$$\alpha_T = \frac{\rho_2 - \rho_1 + \rho_{CR}}{T_2 - T_1}$$

where ρ_{CR} = reactivity insertion due to control rod movement.

A second determination of the isothermal temperature coefficient is obtained on the return to the original temperature. The average of the two results is the reported value.

2.4.2. Reactivity Coefficients at Power

The temperature and power Doppler coefficients are measured at power, as discussed below. The moderator coefficient cannot be directly measured in an operating reactor because a change in the moderator temperature also causes a similar change in the fuel temperature. However, the moderator coefficient is determined from the measured temperature coefficient and the calculated Doppler coefficient.

$$\alpha_M = \alpha_T - \alpha_D$$

where

α_M = moderator coefficient,

α_T = temperature coefficient,

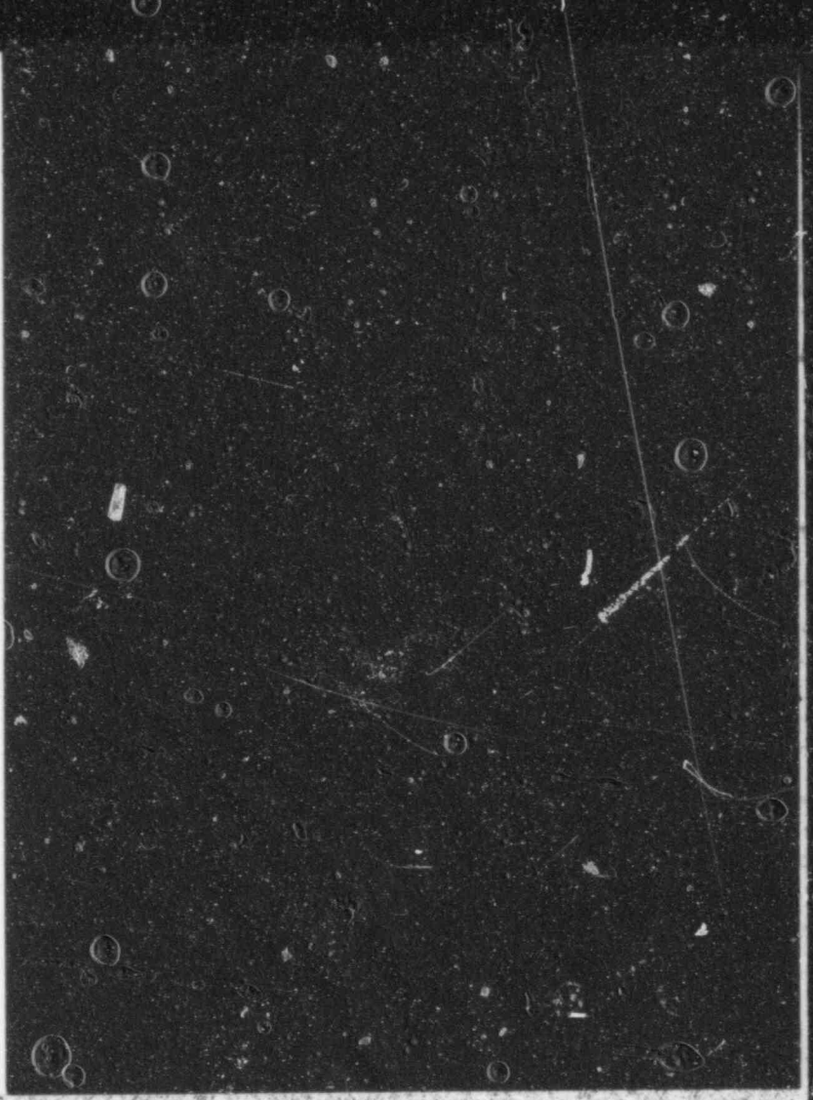
α_D = Doppler coefficient.

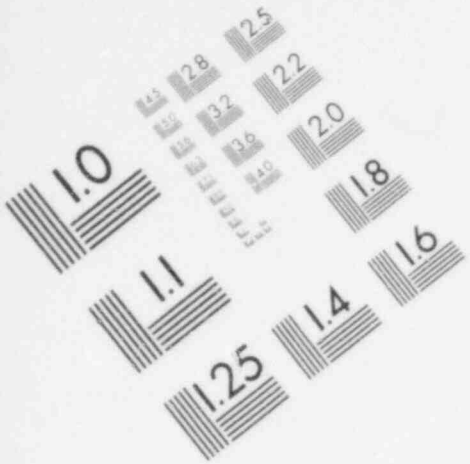
2.4.2.1. Temperature Coefficient

The temperature coefficient is measured at power with the integrated control system (ICS) in automatic. The basic sequence is to first establish steady-state conditions with standard operating temperature and pressure, equilibrium xenon, and uniform boron concentration in the reactor coolant system, pressurizer, and makeup tank. The differential reactivity worth of the controlling rod group is then measured by the fast insert-withdraw method (section 2.2.3). Steady-state conditions are re-established and data, such as rod position, average coolant temperature, power, and reactivity are recorded for 10 minutes by the B&W Reactimeter. The reactor coolant average temperature setpoint is then increased by 5F. A small movement of the controlling rod group compensates for the reactivity change caused by the temperature change. After 10 minutes of data recording at the new temperature level, the reactor is returned to the original average temperature by resetting the temperature setpoint.

Since the reactor is critical at both temperature setpoints, the reactivity due to temperature can be determined by a reactivity balance. Thus, the temperature coefficient is given by

$$\alpha_T = \frac{\Delta\rho_T}{\Delta T_M} = -\frac{\Delta\rho_{CR} + \Delta\rho_P + \Delta\rho_{Xe}}{\Delta T_M}$$





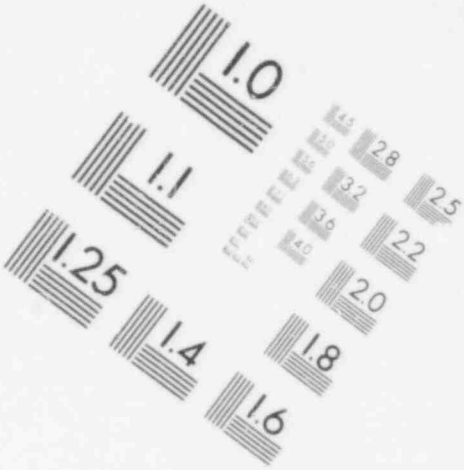
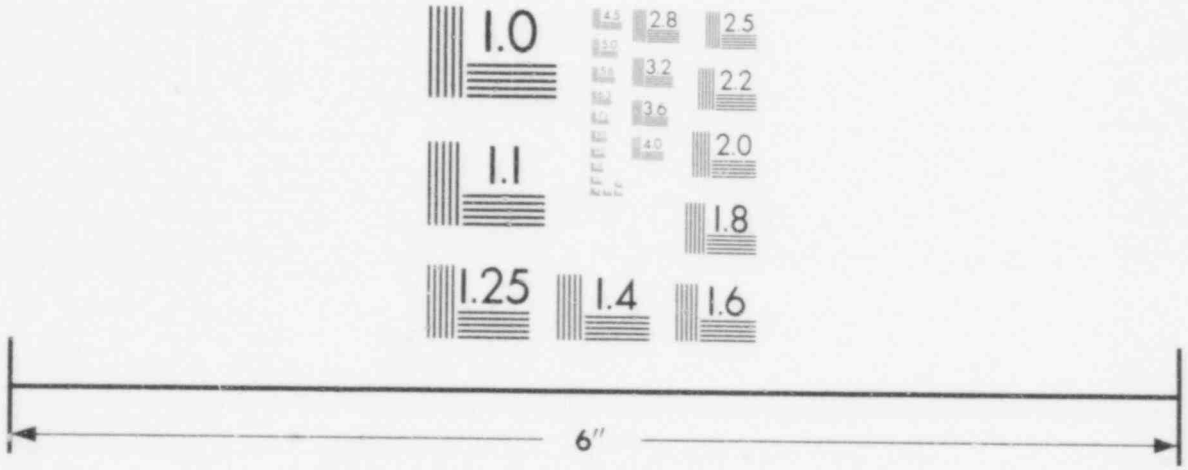
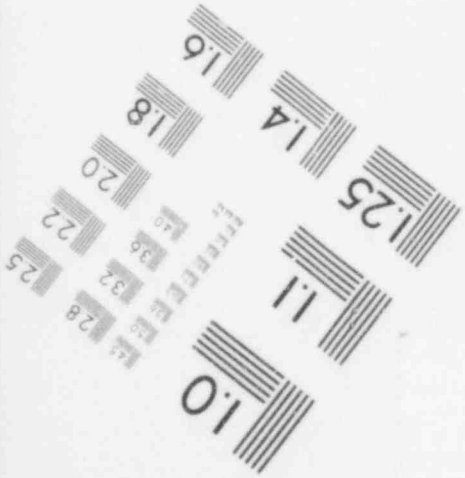
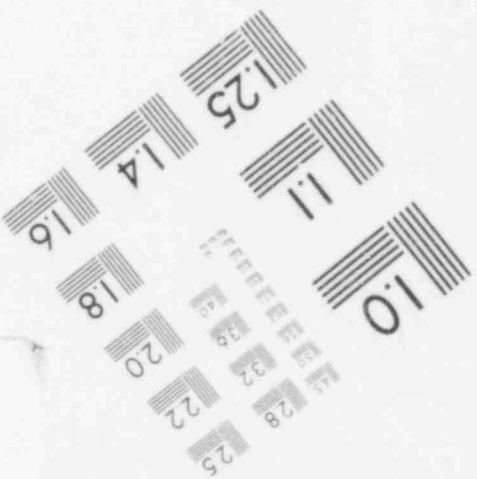
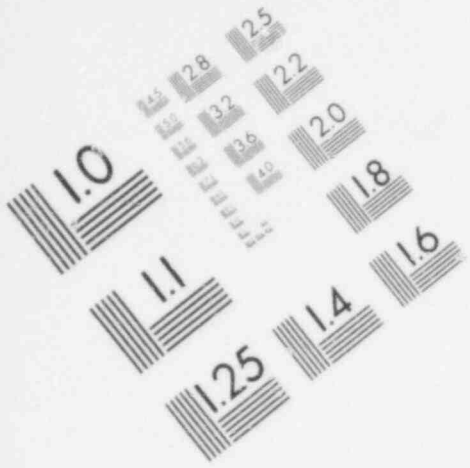


IMAGE EVALUATION
TEST TARGET (MT-3)









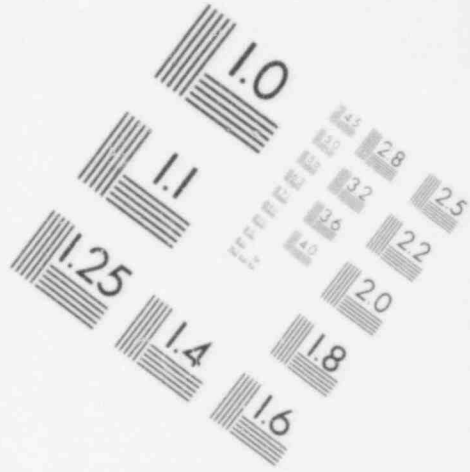
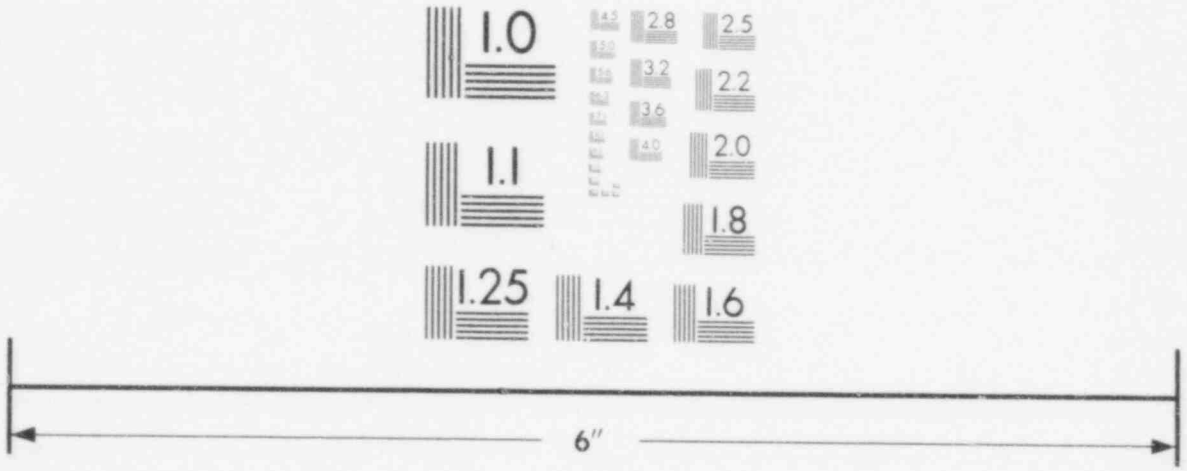
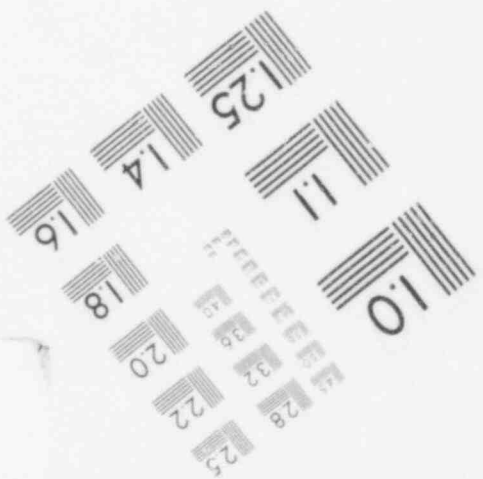


IMAGE EVALUATION TEST TARGET (MT-3)





or, assuming linear relationships between reactivity and control rod insertion and between reactivity and power,

$$\alpha_T = - \frac{\frac{\partial \rho}{\partial H} (H_2 - H_1) + \alpha_{PD}(P_2 - P_1) + \Delta\rho_{Xe}}{T_{M2} - T_{M1}}$$

where

$\Delta\rho_T$ = change in reactivity due to moderator temperature change, ΔT_M,

$\Delta\rho_{CR} = \frac{\partial \rho}{\partial H} (H_2 - H_1)$ = change in reactivity due to control rod movement between states 1 and 2,

$\Delta\rho_P = \alpha_{PD}(P_2 - P_1)$ = change in reactivity due to power level change between states 1 and 2,

$\Delta\rho_{Xe}$ = change in reactivity due to xenon.

Power is held nearly constant by the integrated control system (ICS) during the test. However, small changes do occur and must be accounted for in the reactivity balance. Calculated power Doppler coefficients are used for this purpose. The change in xenon worth during this measurement is usually small enough to be neglected. Measured differential rod worths are used to determining the change in control rod worth.

2.4.2.2. Power Doppler Coefficient

The power Doppler coefficient measurement is similar to the temperature coefficient measurement. Data are recorded for a period of 5 minutes at the initial steady-state conditions, and then a 5% power reduction is input to the unit master control station. The ICS reduces and levels out the reactor power at the new power level within approximately 5 minutes, while holding the reactor coolant average temperature constant. Data recording continues during this period of operation and also for a period of about 10 minutes at the new power level.

The power Doppler coefficient is determined by a reactivity balance:

$$\alpha_{PD} = \frac{\Delta\rho_P}{\Delta P} = - \frac{\Delta\rho_{CR} + \Delta\rho_T + \Delta\rho_{Xe}}{\Delta P}$$

or

$$\alpha_{PD} = - \frac{\frac{\partial \rho}{\partial H} (H_2 - H_1) + \alpha_T(T_{M_2} - T_{M_1}) + \Delta\rho_{Xe}}{P_2 - P_1}$$

where $\Delta\rho_P$ = change in reactivity due to power change ΔP,

$\Delta\rho_T = \alpha_T(T_{M_2} - T_{M_1})$ = change in reactivity due to moderator temperature change between states 1 and 2.

Small changes in reactor coolant average temperature are accounted for by using calculated temperature coefficients. The xenon worth is calculated as a function of time from the power history during the transient. Differential rod worths measured by the fast insert-withdrawal method are used to determine the change in control rod worth.

A second determination of the power Doppler coefficient is obtained on the return to the original power level. The average of the two results is the reported value.

2.4.3. Average Differential Boron Worth

The average differential boron worth is obtained from the control rod worth measurements by boron swap. The boron worth measurement makes use of equilibrium boron states, called boron endpoints, before and after a deboration. The reactivity worth of control rods between these states is obtained from step-like insertions (see section 2.2.1.1). Since the reactor is exactly critical before and after the deboration, the change in boron worth is equal in magnitude to the change in inserted control rod worth.

$$\Delta\rho_B = -\Delta\rho_{CR}.$$

Therefore, the average differential boron worth, $\overline{\partial\rho/\partial B}$ between the boron endpoints is given by

$$\overline{\frac{\partial\rho}{\partial B}} = \frac{\Delta\rho_B}{\Delta B} = -\frac{\rho_{CR_2} - \rho_{CR_1}}{B_2 - B_1}$$

where B_1, B_2 are the measured soluble boron concentrations at the boron endpoints.

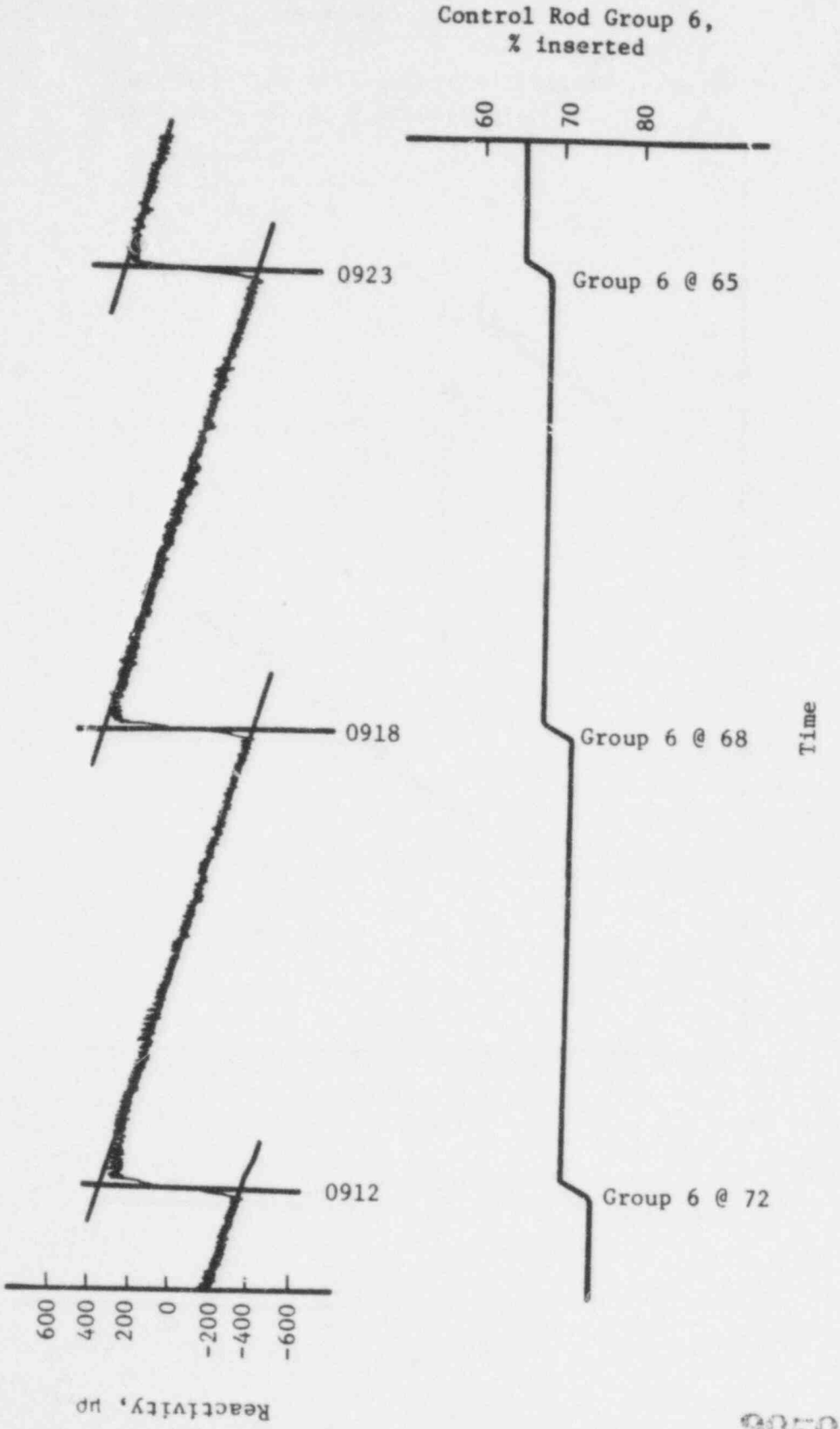


Figure 2-1. Reactivity Trace for Boron Swap Measurement

Figure 2-2. Typical Reactivity Signal and Straight Line Fits

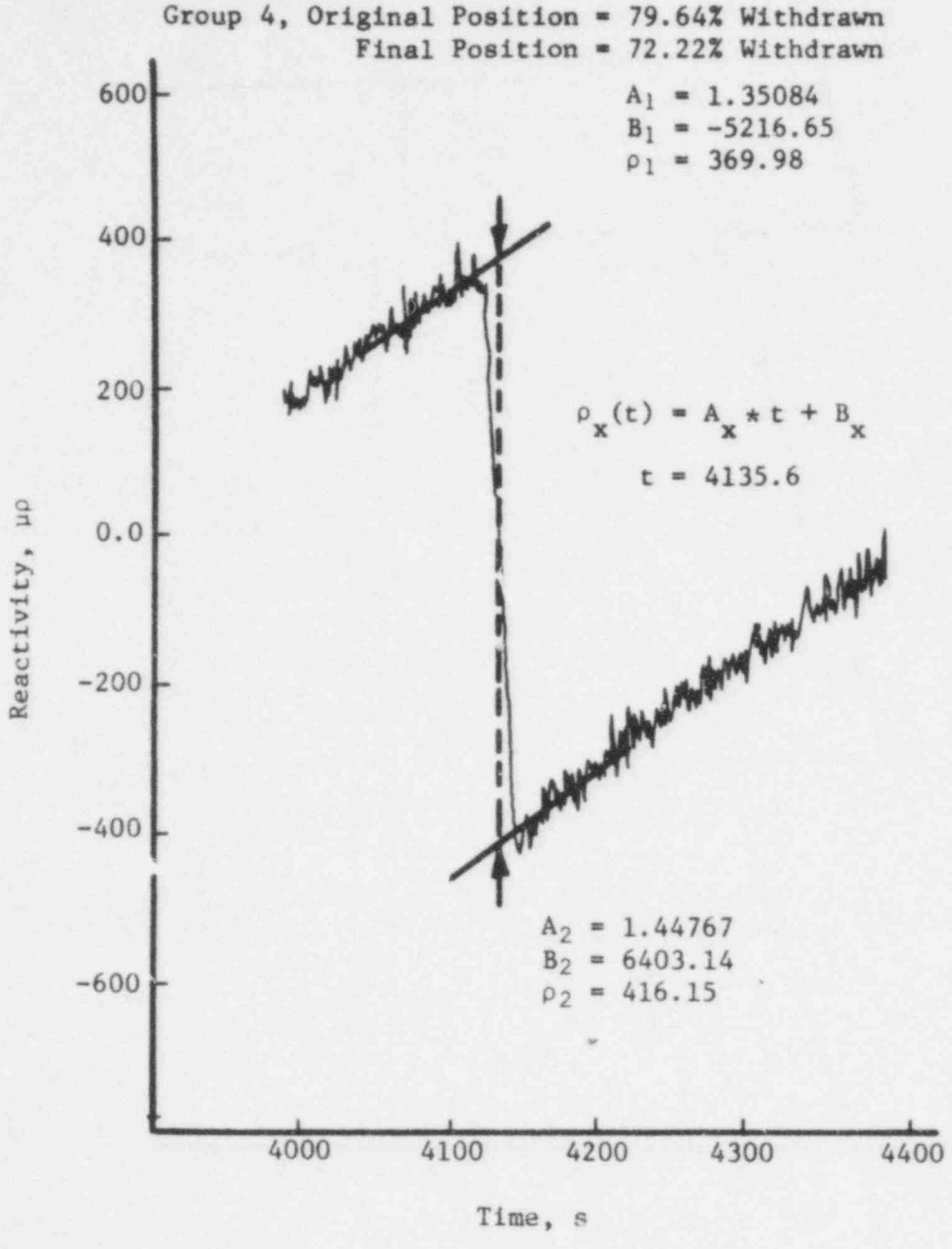


Figure 2-3. Typical Spline Fit Through Differential Rod Worth Data

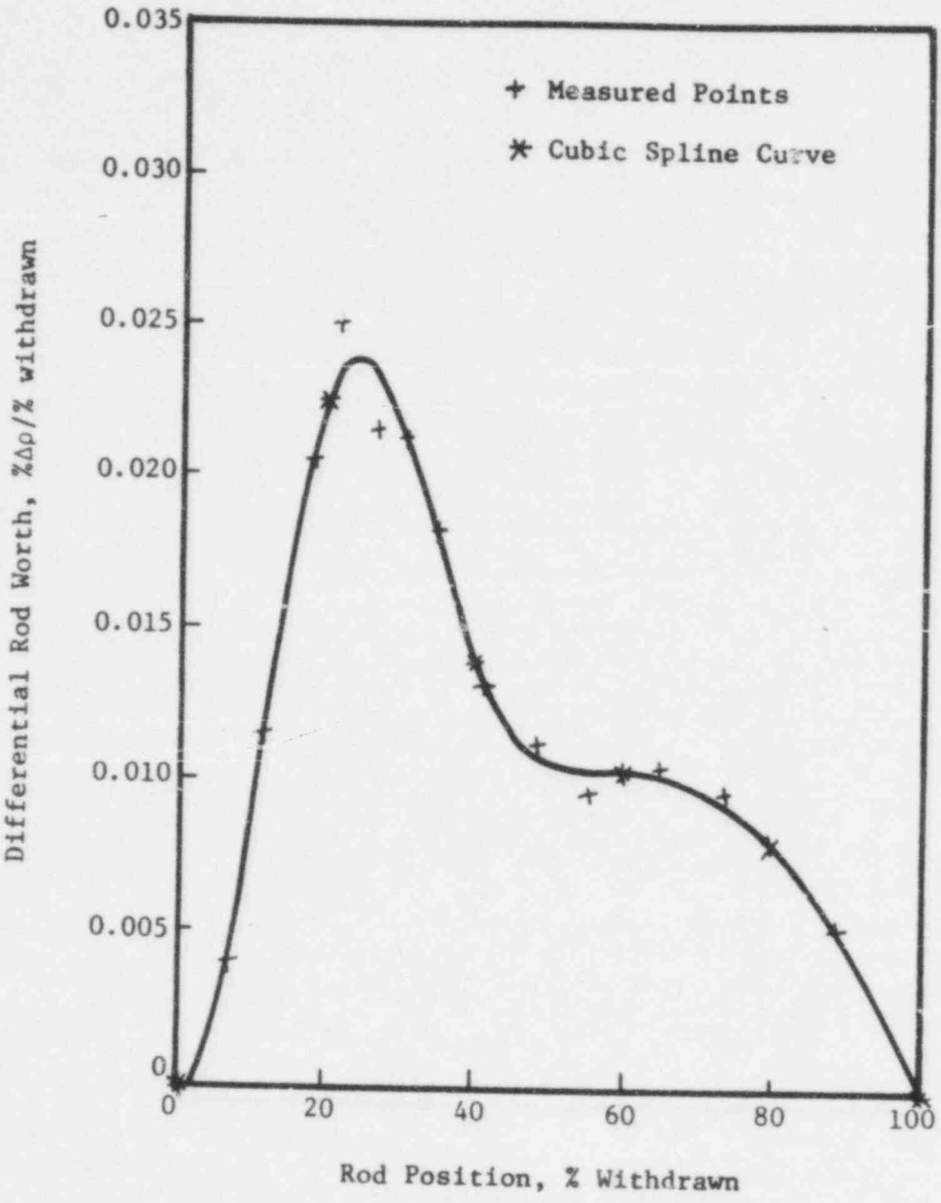


Figure 2-4. (Deleted)

Figure 2-5. (Deleted)

Figure 2-6. Determination of Ejected Rod Worth by Rod Drop Technique

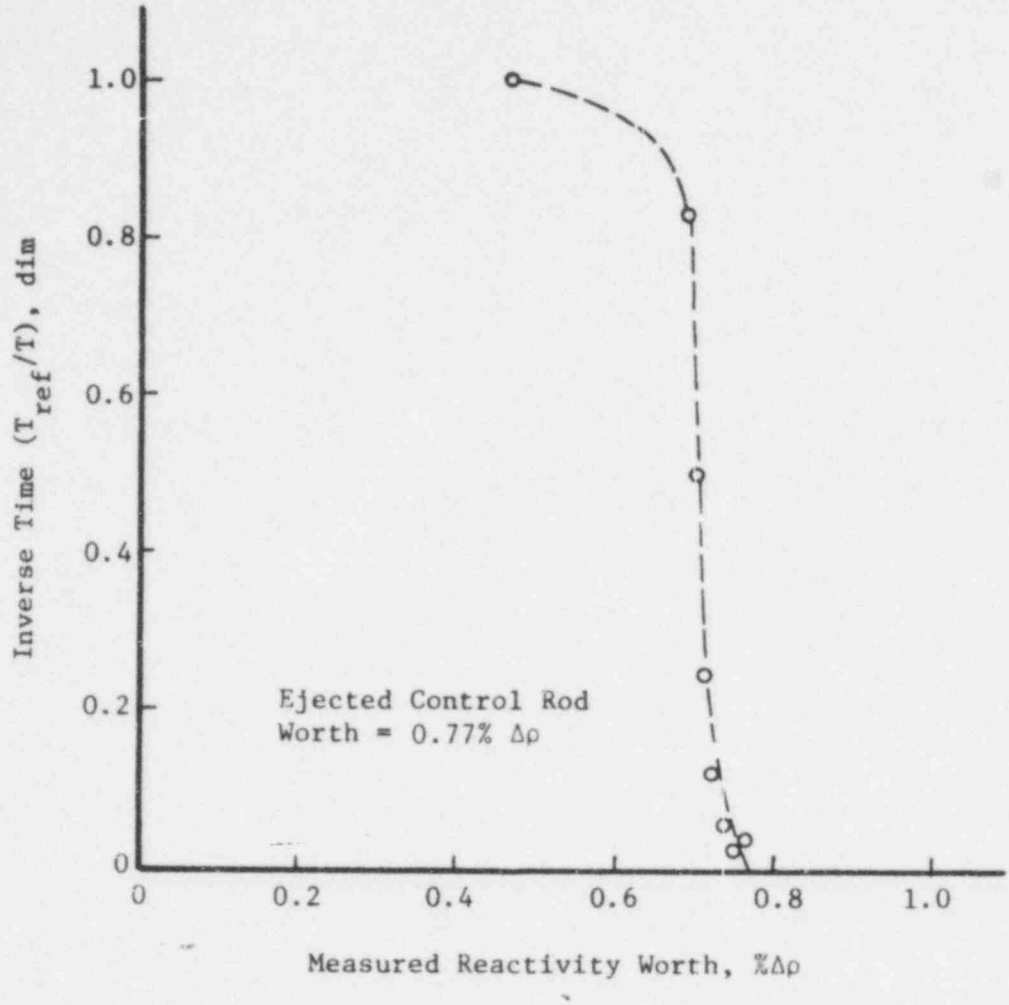
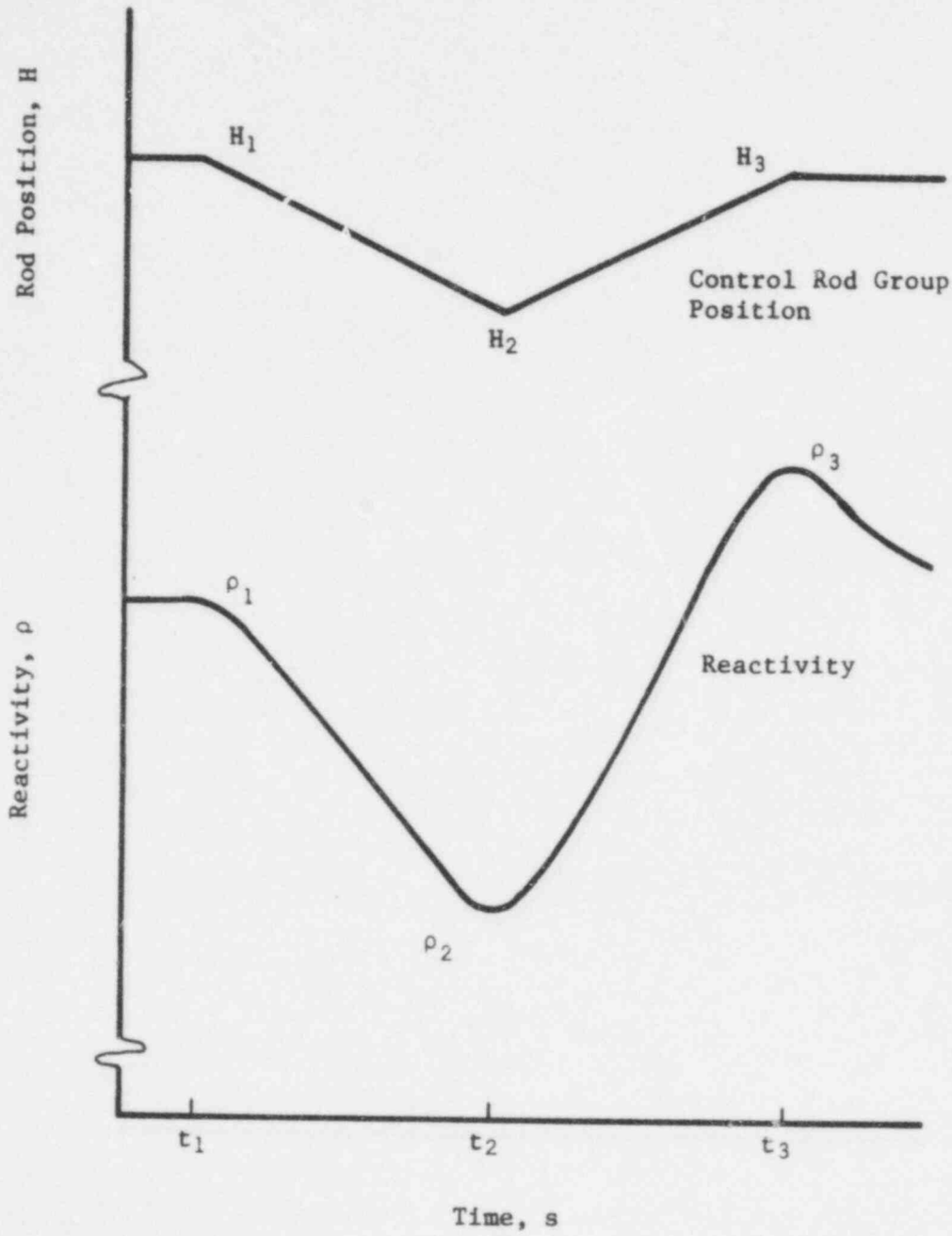


Figure 2-7. Quick-Insertion, Quick-Withdrawal Differential Rod Worth Measurement



3. MEASUREMENT UNCERTAINTIES

The measurement uncertainties obtained in this section are estimates of the standard deviation. They are derived by identifying sources of errors and probabilistically combining individual components to obtain the total uncertainty in the measurement. Considering the component sources to be random variables gives the following first-order approximation of the variance of a general function $T(X_1, X_2, \ldots, X_n)$ in terms of the component sources:

$$\sigma_T^2 = \sum_{i=1}^{n} \left(\frac{\partial T}{\partial X_i}\right)^2 \sigma_i^2 + 2 \sum_{i=1}^{n} \sum_{j=i+1}^{n} \frac{\partial T}{\partial X_i} \frac{\partial T}{\partial X_j} \sigma_{ij}^2$$

where

σ_T^2 = estimate of variance of T,

σ_i^2 = estimate of variance of component X_i,

σ_{ij}^2 = covariance of components X_i, X_j. (The partial derivatives are evaluated at mean values of the variables.)

The estimates above can be put on a relative basis by dividing both sides of the equation by $\bar{T}(X_1, X_2, \ldots, X_n)$, where

$$\bar{T}(X_1, X_2, \ldots, X_n) \approx T(\bar{X}_1, \bar{X}_2, \ldots, \bar{X}_n)$$

The covariance terms in the expression above are not usually directly estimable. For purposes of the analyses herein, it is assumed that the sources of error are independent.

The following sections describe the uncertainty analyses for the measurement procedures given in section 2. Table 3-1 summarizes the results of the measurement uncertainty analyses.

3.1. Instrument Uncertainties

The accuracy with which the directly observable parameters, such as flux, rod position, temperature, boron concentration, and time, can be measured

contributes to the overall uncertainty in core physics parameters. The following section summarizes the instrument uncertainty analyses.

3.1.1. Flux

The flux signal is taken from the nuclear instrumentation system intermediate range neutron detector. The detector is a compensated ion chamber. The equipment contributing to the signal, which is recorded by the B&W Reactimeter, and the equipment accuracies are given in Table 3-2.

The equipment accuracies combined as the sums of squares give a total uncertainty of 4.2% of the measured value. The major component of this uncertainty is the compensated ion chamber accuracy. This uncertainty is due largely to random noise in the flux signal and is observable in traces of flux versus time. The flux noise is propagated through the reactivity calculator. The magnitude of the resultant noise in reactivity is observable in reactivity traces (such as Figure 2-1). The magnitude of the noise is approximately 50 μρ or about 0.7¢ peak-to-peak. The effect of noise on the measurements is decreased by averaging a large number of reactivity samples.

3.1.2. Temperature

The reactor coolant average temperature is the average of six resistance temperature detectors (RTDs), four measuring inlet and two measuring outlet temperatures. The average is taken electronically using five summing amplifiers. The reactor coolant average temperature, T_{avg}, can be represented algebraically as

$$T_{avg} = G_2 G_3 S_1 \left\{ S_2 (T_{M_{H1}} + T_{M_{H2}}) + S_3 [S_4 (T_{M_{C1}} + T_{M_{C2}}) + S_5 (T_{M_{C3}} + T_{M_{C4}})] \right\}$$

where

G_2 = 24-channel amplifier and multiplier response,

G_3 = a/d converter response,

S = summing amplifier gain (five amplifiers used) = 0.5,

T_{M_H} = outlet temperature (two measurements), F,

T_{M_C} = inlet temperature (four measurements), F;

and

$$T_M = RTD \cdot G_1,$$

where

RTD = RTD and linear bridge response,

G_1 = signal converter response.

The uncertainty associated with the T_{avg} measurement can be estimated from the accuracies of the equipment contributing to the signal, which are listed in Table 3-3.

$$\frac{\varepsilon(T_{avg})}{\Delta T_{avg}} = \left[(0.36)^2 + (0.3)^2 + \left(\frac{5}{2}\right)^2 (0.21)^2 + (1.0)^2 + (0.1)^2\right]^{\frac{1}{2}}$$

1.2% of span (520 to 620F)

and

$$\varepsilon(T_{avg}) = 1.2°F.$$

In such measurements as the temperature coefficient, the difference between two temperatures is the quantity of interest. Examining traces of temperature signals reveals that the random noise component is negligible (less than 0.1%). Consequently, the error of 1.2F must be primarily systematic. Over a 5 to 10F range in temperature, the temperature can be represented as

$$T_i = T_{iM} + (\bar{\varepsilon} + \delta_i)$$

where

T_i = true temperature,

T_{iM} = measured temperature,

$\bar{\varepsilon}$ = average error over temperature range,

δ_i = difference between actual error at T_i and average error $\bar{\varepsilon}$.

Figure 3-1 is a hypothetical example illustrating this situation. The errors in the temperature measurements at T_1 and T_2 are of approximately the same magnitude and in the same direction. Assuming δ_1 and δ_2 to be independent, the error in $\Delta T = T_1 - T_2$ is

$$\frac{\varepsilon(\Delta T)}{\Delta T} = \sqrt{\left(\frac{\delta_1}{\Delta T}\right)^2 + \left(\frac{\delta_2}{\Delta T}\right)^2}.$$

Further assuming (based on engineering judgment) that relative errors over a part of the range are proportional to relative errors over the full scale,

$$\frac{\delta_i}{\Delta T} = \frac{\varepsilon(T_{avg})}{\Delta T_{avg}} = 1.2\%,$$

and the uncertainty on the temperature difference is

$$\frac{\epsilon(\Delta T)}{\Delta T} = \sqrt{(1.2)^2 + (1.2)^2} = 1.7\%.$$

3.1.3. Control Rod Group Position

The control rod group relative position signal represents the average relative position of all rods in the group. Table 3-4 lists the equipment contributing to the signal and the equipment accuracies. The uncertainty in the group average position signal includes components due to individual detector accuracy and the accuracy of the summing amplifier. The total uncertainty in control rod group position, obtained by summing the squares, is of instrumentation span (0 to 100% withdrawn), or approximately inches. The uncertainty in the difference between two control rod group positions, obtained in a manner similar to the uncertainty in the difference between two reactor coolant temperatures, is

3.1.4. Soluble Boron Concentration

Boron concentration is measured by an acid-base titration of a reactor coolant sample. The error in the measured value can be attributed to two sources: the error in the titration and the error due to the sample having a different boron concentration from the core average. These errors are conservatively estimated to be ppmB each when the reactor coolant (RC) system is in equilibrium. Hence, the total accuracy of a boron concentration measurement is ppmB. This estimate is supported by the fluctuations in measured boron concentration observed during physics testing.

3.1.5. Time

Time is used in the reactivity computation and is generated internally by the Reactimeter using a crystal-controlled real-time clock. The clock is sufficiently accurate, compared to other measured variables, that its contribution to error in core physics measurements is negligible.

3.2. Reactivity Measurement Uncertainties

The reactivity algorithm can be evaluated with regard to two sources of uncertainty: the accuracy of the approximations used to develop the algorithm and the uncertainty in the delayed neutron parameters used in the computation. The accuracy of the algorithm can be determined by computing flux due to an arbitrary reactivity input function using a very accurate kinetics calculation.

Then, using this computed flux as input to the reactivity algorithm, the difference between the Reactimeter computation and the input reactivity is a measure of the accuracy of the reactivity algorithm. The kinetics program used to generate flux is described in section 3.2.1, where the accuracy of the reactivity algorithm is also evaluated.

The uncertainty contributed by the delayed neutron parameters is evaluated by the method of propagation of errors from individual parameters in section 3.2.2.

The uncertainties in sections 3.2.1 and 3.2.2 are expressed in terms of dollars. When reactivity is converted into absolute units, the uncertainty in the effective delayed neutron fraction must also be included. This uncertainty is discussed separately in section 3.2.3.

3.2.1. Evaluation of the Reactivity Algorithm

Point reactor kinetics can be formulated from basic physical considerations in either a differential equation representation[5] or an integral representation.[2] Neglecting source terms, the conventional form of the differential equation representation is

$$\dot{n} = (\rho - \beta)\frac{n}{\Lambda} + \sum_{i=1}^{6} \lambda_i C_i,$$

$$\dot{C}_i = \beta_i \frac{n}{\Lambda} - \lambda_i C_i, \qquad (3\text{-}1)$$

and the integral representation is

$$n(t) = \sum_{i=0}^{6} \gamma_i \int_0^{\infty} n(t - \tau)k(t - \tau)\lambda_i e^{-\lambda_i \tau} d\tau \qquad (3\text{-}2)$$

where

Λ = neutron generation time,

λ_0^{-1} = neutron lifetime ℓ,

$\gamma_i = \beta_i\lambda_0/(\lambda_0 - \lambda_i)$, $i = 1, 6$,

$$\gamma_0 = 1 - \sum_{i=1}^{6} \gamma_i = 1 - \beta - \sum_{i=1}^{6} \beta_i\lambda_i/(\lambda_0 - \lambda_i).$$

There is a subtle difference between these formulations in that Λ is assumed to be constant in equation 3-1, whereas, $\lambda_0 = 1/\ell$ is assumed to be constant

in 3-2. Ordinarily, this distinction is insignificant because the uncertainty in either the neutron generation time or the neutron lifetime is far greater than the difference in their magnitudes. However, when the reactivity change is caused by control rod movement, as it is for measurements at operating reactors, it is more accurate to assume a constant neutron generation time.

These relations and the Reactimeter equations from section 2.1 have been programmed for the LRC Modcomp. Double precision was used for equations 3-5 and 3-6 to reduce error accumulation when short time steps are used. Time steps of 1 microsecond to 1 millisecond have been used in the present simulation work. Reactivity traces simulating various physics tests were input. Table 3-5 summarizes the results of the comparison for various types of measurements.

3.2.2. Uncertainties in Delayed Neutron Precursor Parameters

The errors in the delayed-neutron precursor parameters propagate through the Reactimeter algorithm to contribute to the overall error in the reactivity measurement. This contribution is difficult to determine because it is dependent on the nature of the perturbation of the system from equilibrium and on the time at which the measurement is made. In this treatment, only an ideal step insertion of reactivity is considered (since it is the only case that is even approximately analytically tractable). For a measurement taken immediately after the step, the reactivity is related to the flux jump by $\rho_{\$} = 1 - n(0-)/n(0+)$, and there is no error due to precursor parameters. In practice, however, Reactimeter measurements are taken over an extended period

after the initiation of the transient, and precursor parameter errors do contribute significantly. Estimates of this contribution are given in the following sections for the near and very long terms. Over the near term, which is some tens of seconds, the Reactimeter substantially retains its memory of the initial flux changes, and this information largely determines the reactivity. Over the long term, after a few hundred seconds, the initial change has faded from the memory of the Reactimeter as a consequence of the exponential decay terms in equation 3-2, and the reactivity is entirely determined by the slope of flux versus time. This is inherently a less accurate determination and, for large negative reactivities in particular, a small change in the slope implies a large change in the estimated reactivity. Therefore, the errors in the parameters of the long-lived precursors become very significant for long-term measurements. The intermediate term is difficult to deal with analytically, and some interpolation must be made between the errors calculated for the near term and those calculated for the asymptotic case.

3.2.2.1. Near-Term Measurements

The Reactimeter algorithm is based on the equation

$$\rho_{\$} = 1 - \frac{1}{n(t)} \sum_{i=1}^{6} a_i \int_0^{\infty} n(t - \tau)\lambda_i e^{-\lambda_i \tau} d\tau \qquad (3\text{-}7)$$

where $\rho_{\$} = \rho/\beta$, and $a_i = \beta_i/\beta$. For a step change in reactivity, certain simplifications of equation 3-7 are possible, and this case is used here to estimate the effect of errors in a_i and λ_i. In this case,

The values of $n_0/n(t)$ for these evaluations were obtained from the Reactimeter simulation program described in section 3.2.1. The typical BOL values of a_i and λ_i given in Table 3-7 were used. The errors in a_i and λ_i were taken from reference 6 and are for uranium-235.

3.2.2.2. Asymptotic Measurements

While most reactivity measurements are taken within one minute after a perturbation, some measurements, such as temperature coefficient and boron swap, involve tracking very slow changes over long time periods. The errors due to uncertainties in delayed-neutron parameters for these cases are probably more characteristic of the asymptotic state than of the near term considered in the preceding section.

3.2.3. Uncertainty in Effective Delayed Neutron Fraction

When reactivities are expressed in absolute units, the additional uncertainty in the effective delayed neutron fraction, β_{eff}, must be considered.

$$\beta_{eff} = \gamma\beta$$

where γ is an effectiveness factor, and β is the absolute delayed neutron yield fraction. The uncertainty in β is 3.1% for uranium-235 and 4.8% for plutonium-239.[7] An uncertainty of 4% has been assigned to this parameter for all measurements. The error in γ is difficult to estimate accurately; it has been assigned a value of here, based on engineering judgment. This gives an error in β_{eff} of .

3.3. Uncertainties in Control Rod Worths

The following sections summarize the uncertainties in control rod worth measurements.

3.3.1. Control Rod Group Worths by Boron Swap

In a typical boron swap measurement, the boron concentration is either increased or decreased in a slow and continuous manner, and rods are moved in discrete steps to counter the reactivity change. The total reactivity change at each rod movement is on the order of 10¢ or less, and the steps are made so as to maintain the reactor within about ±5¢ of critical with minimal drift.

The reactivity change due to each rod movement is determined from the Reactimeter readings just before and just after the movement. The errors in the reactivity determinations before and after the rod movement are somewhat different because of their dependence on previous operating conditions and because of the necessity to extrapolate through the short time interval during which the rod movement occurred. Table 3-9 gives a breakdown of the estimated errors at each reactivity step.

The sources of error given in Table 3-9 have been estimated by various means.

9005021

For all measurements where the boron-swap procedure has been used, this gives an error less than of the measured rod worth in dollars. Including the error in β_{eff} (section 3.2.3), this gives an error of in boron swap absolute rod worth measurements.

When the group worths are determined by integrating under a smooth curve through the measured differential worths the propagation of the random errors is less straight-forward. The random error contribution can be estimated, however, by calculating the random error in the area of a series of trapezoids through the differential worth points.

A simple analysis shows that if the fractional error, f, in the individual data points is constant, then the fractional error in the total area of the trapezoids is

$$\frac{\varepsilon(A)}{A} = f\sqrt{\sum_i A_i^2 / (\sum_i A_i)^2}$$

where A_i is the area of the ith trapezoid.

3.3.2. Total Rod Worths

Rod drop reactivity measurements used to determine total rod worths are affected by spatial dynamics effects which are not included in the kinetics model of the reactimeter and which are difficult to account for in the data analysis.

3.3.3. Ejected Rod Worths

The uncertainties in the three ejected rod worth measurement techniques are discussed below.

3.3.3.1. Boron Swap

The uncertainty in ejected rod worths measured by the boron-swap technique is the same as the uncertainty in control rod group worths measured by the boron swap technique, (see section 3.3.1).

3.3.3.2. Rod Swap

For those ejected rod worths measured by rod swap the error is essentially the same as the uncertainty in the reactivity increment ($\Delta\rho$) of the integral worth curve, for the bank being swapped, over the total movement (ΔH) of the bank required in the measurement. The total increment is composed of several step-like insertions or withdrawals from the previously measured integral worth

curve. But, since the endpoints of the ejected rod worth measurement movement do not necessarily correspond to increments measured previously, some interpolation at each end of the rod swap is necessary:

$$\Delta\rho = \frac{\partial\rho}{\partial H}\,\Delta H_I + \Delta\rho_1 + \Delta\rho_2 + \ldots + \Delta\rho_n + \frac{\partial\rho}{\partial H}\,\Delta H_F.$$

The terms $\Delta\rho_i$ are reactivity insertions from the integral worth measurement which count fully toward the ejected rod worth. The terms $(d\rho/dH)\Delta H_I$ and $(d\rho/dH)\Delta H_F$ are terms obtained by interpolation. The component errors in the $\Delta\rho_i$ terms are given in Table 3-9. The error in the sum of $\Delta\rho_i$ terms is given in section 3.3.1,

$$\varepsilon\left(\sum_{i=1}^{n}\Delta\rho_i\right) = \qquad (3\text{-}15)$$

The terms obtained by interpolation can be expressed as

$$\frac{\partial\rho}{\partial H}\,\Delta H = \frac{\Delta\rho_x}{\Delta H_x}\,\Delta H = \Delta\rho_x\,\frac{\Delta H}{\Delta H_x}$$

where $\Delta\rho_x$ is the previously measured reactivity over the interval ΔH_x. The error in the interpolation terms is obtained using the normal rules for propagation of errors. Assuming that the relative errors in ΔH and ΔH_x are equal,

$$\frac{\varepsilon\left(\frac{\partial\rho}{\partial H}\,\Delta H\right)}{\frac{\partial\rho}{\partial H}\,\Delta H} = \sqrt{\left[\frac{\varepsilon(\Delta\rho)}{\Delta\rho}\right]^2 + 2\left[\frac{\varepsilon(\Delta H)}{\Delta H}\right]^2}.$$

The uncertainty in $\Delta\rho$ can be obtained from section 3.3.1 by setting n = 1, and the uncertainty in ΔH is given by section 3.1.3:

$$\frac{\varepsilon\left(\frac{\partial\rho}{\partial H}\,\Delta H\right)}{\frac{\partial\rho}{\partial H}\,\Delta H} =$$

Including an error of in β_{eff} gives an error of of the measured value in absolute reactivity units.

3.3.3.3. Rod Drop

The reactivity recorded by the Reactimeter immediately following rod drops has been found to depend on the relative locations of detector and rod and on the elapsed time after the rod drop. However, after about 20 seconds, the flux redistribution effects diminish, and the measured reactivities begin to converge to a unique value (within about 10%) which is independent of the detector used or the particular rod (of a symmetric set) dropped. The standard procedure for determining the asymptotic value is described in section 2.2.2.3.

3.3.4. Differential Rod Worths at Power

The systematic errors associated with the differential rod worth reactivity ($\partial\rho/\partial H$) measurements are due to uncertainties in the Reactimeter algorithm, the delayed neutron precursor parameters, and the control rod position indication. For reactivities not exceeding about as in the $\partial\rho/\partial H$ measurements, the Reactimeter error is found by comparison with exact calculations to be less than (see Table 3-5). The error due to uncertainty in delayed neutron precursor parameters is discussed in section 3.2.2 and for reactivities of this

magnitude is found not to exceed about (see Table 3-6). Where the reactivity is expressed in absolute units, there is an additional systematic error of about due to uncertainties in β_{eff}.

There are small systematic errors in rod position, H, due to sensors, amplifiers, etc. The uncertainty in the measured change in rod position, ΔH, is estimated (section 3.1.3) to be

The spatial dynamics effects may be significant for this measurement.

Combining these errors gives a total systematic error of in differential rod worth.

The differential rod worth is obtained using the linear regression scheme discussed in section 2.2.3. The random error is evaluated based on the statistics of linear regression analysis. The equation for random error[8] is as follows:

$$S_{(\partial\rho/\partial H)} = \frac{1}{N-1}\frac{S^2}{S_H^2} r_{HH}^{-1},$$

where

$$S^2 = \frac{1}{N-3}\Sigma\left(\rho_i - \frac{\partial\rho}{\partial H} H - \alpha_D T_{Fi} - \rho_0\right)^2,$$

$$S_H^2 = \frac{1}{N-1}\left[\Sigma H_i^2 - \frac{1}{N}(\Sigma H_i)^2\right],$$

$$S_{HT}^2 = \frac{1}{N-1}\left(\Sigma H_i T_{Fi} - \frac{1}{N}\Sigma H_i \Sigma T_{Fi}\right),$$

$$S_T^2 = \frac{1}{N-1}\left[\Sigma T_{Fi}^2 - \frac{1}{N}(\Sigma T_{Fi})^2\right],$$

$$r_{HH}^{-1} = \text{inverse matrix element} = \left[1 - \left(\frac{S_{HT}^2}{S_H S_T}\right)^2\right]^{-1},$$

$$\Sigma = \sum_{i=1}^{N},$$

N = number of data points used.

90-027

This method accounts for the noise and goodness of fit in reactivity, fuel temperature, and control rod position. Errors due to the fuel temperature calculations and moderator temperature fluctuations are also taken into account.

3.4. Uncertainties in Critical Boron Concentrations

The all rods out critical boron concentration (AROCBC) is a measure of the reactivity depletion of the core. The measurement is taken at BOL during zero power physics testing and at power throughout the life of the core.

3.4.1. AROCBC at Zero Power

The uncertainty in the AROCBC comprises the uncertainty in the actual measured boron concentration plus the uncertainty due to the adjustment from critical near all-rods-out to critical with all rods completely out. This adjustment is typically The error in this term is certainly or smaller. The uncertainty in the actual measured boron concentration is estimated to be ppmB (section 3.1.4). Combining these components gives a total uncertainty of approximately ppmB.

3.4.2. Critical Boron Concentrations at Power

The uncertainty in the AROCBC at power contains components for the measured boron concentration and the adjustment to all rods out, as in the preceding section. However, the adjustment to all rods out is much larger at full power than at zero power. The equation that expresses the adjustment from the measured concentration to the all-rods-out concentration is

$$\text{AROCBC} = \text{MCBC} + \frac{\rho_{CR}}{(\overline{\partial\rho/\partial B})}$$

where

MCBC = measured critical boron concentration,

ρ_{CR} = worth of control rods inserted in core at time of measurement,

$\overline{\partial\rho/\partial B}$ = avg differential boron worth between MCBC and AROCBC.

The uncertainty in the AROCBC is

$$\varepsilon(\text{AROCBC}) = \sqrt{[\varepsilon(\text{MCBC})]^2 + \left(\frac{\rho_{CR}}{\partial\rho/\partial B}\right)^2 \left\{\left[\frac{\varepsilon(\rho_{CR})}{\rho_{CR}}\right]^2 + \left[\frac{\varepsilon(\overline{\partial\rho/\partial B})}{\partial\rho/\partial B}\right]^2\right\}}.$$

The adjusted measured critical boron concentration (AMCBC) is defined as the boron concentration with the transient rods fully inserted, the APSRs positioned for zero imbalance, and all other rods fully withdrawn. The uncertainty in the AMCBC is obtained in a manner similar to the uncertainty in the AROCBC. Since the adjustment to this core configuration is typically only ppmB, the uncertainty in the boron concentration is approximately ppmB.

3.5. Uncertainties in Reactivity Coefficients

3.5.1. Isothermal Temperature Coefficients at Zero Power

The temperature coefficient, α_T, is determined at zero power by changing the temperature uniformly about 5-10F and measuring the resultant change in reactivity, $\Delta\rho$. The change in reactivity is usually measured directly on the Reactimeter. The components of the uncertainty in $\Delta\rho$ are summarized in Table 3-10 for a typical -5¢ change in reactivity.

Since $\alpha_T = \Delta\rho/\Delta T$ (see section 2.4.1), the error in α_T is given by

$$\varepsilon(\alpha_T) = \sqrt{\left[\frac{\partial \alpha_T}{\partial(\Delta\rho)}\,\varepsilon(\Delta\rho)\right]^2 + \left[\frac{\partial \alpha_T}{\partial(\Delta T)}\,\varepsilon(\Delta T)\right]^2}$$

$$= \sqrt{\left[\frac{\varepsilon(\Delta\rho)}{\Delta T}\right]^2 + \left[\alpha_T\,\frac{\varepsilon(\Delta T)}{\Delta T}\right]^2} \quad .$$

The results are shown graphically in Figure 3-2.

3.5.2. Reactivity Coefficients at Power

3.5.2.1. Temperature Coefficients

The temperature coefficient at power is defined as $\alpha_T = \left.\frac{\partial \rho}{\partial T}\right|_P$ where the independent variable is the average core temperature T, and the reactor power P is held constant. Unfortunately, T cannot be varied independently, nor can P be held exactly constant. In practice, the moderator temperature is changed about 5F and the control rods are moved to compensate for the induced reactivity and to hold the power approximately constant.

Therefore, the net measured reactivity change is given by

$$\Delta\rho = \alpha_T \Delta T + \frac{\partial \rho}{\partial H} \Delta H + e$$

where $\partial\rho/\partial H$ is the differential rod worth and e is a second-order reactivity change due to the unavoidable power perturbation. Errors introduced by small changes in power and xenon worth are neglected since they are small compared to the rod worth error. Hence,

$$\alpha_T \approx \frac{\Delta\rho}{\Delta T} - \frac{\partial \rho}{\partial H} \frac{\Delta H}{\Delta T} .$$

The error in the temperature coefficient is given by

$$\varepsilon(\alpha_T) = \sqrt{\left[\varepsilon\left(\frac{\Delta\rho}{\Delta T}\right)\right]^2 + \left[\frac{\partial \rho}{\partial H} \frac{\Delta H}{\Delta T}\right]^2 \left\{ \left(\frac{\varepsilon\left(\frac{\partial \rho}{\partial H}\right)}{\frac{\partial \rho}{\partial H}} \right)^2 + \left[\frac{\varepsilon(\Delta T)}{\Delta T}\right]^2 + \left[\frac{\varepsilon(\Delta H)}{\Delta H}\right]^2 \right\}}$$

Since the experiment is performed so that the change in net reactivity, $\Delta\rho$, is near zero, and most of the reactivity change due to temperature is compensated by the movement of control rods,

$$\alpha_T \approx - \frac{\partial \rho}{\partial H} \frac{\Delta H}{\Delta T}$$

and

$$\varepsilon(\alpha_T) = \sqrt{\left(\varepsilon \frac{\Delta\rho}{\Delta T}\right)^2 + \alpha_T^2 \left\{ \left(\frac{\varepsilon\left(\frac{\partial \rho}{\partial H}\right)}{\frac{\partial \rho}{\partial H}} \right)^2 + \left[\frac{\varepsilon(\Delta T)}{\Delta T}\right] - \left[\frac{\varepsilon(\Delta H)}{\Delta H}\right]^2 \right\}} .$$

The uncertainty in the temperature coefficient at power is shown as a function of the coefficient in Figure 3-3.

3.5.2.2. Power Doppler Coefficients

The power Doppler coefficient is defined by $\alpha_{PD} = \partial\rho/\partial P\big|_{T_M}$ where P is the core power and T_M is the average moderator temperature.

It is measured by decreasing P by about 5% FP with T_M held approximately constant. Control rods are moved to compensate for the reactivity change and to hold the power constant at the decreased level. The total reactivity change is given by

$$\Delta\rho = \alpha_{PD}\Delta P + \frac{\partial\rho}{\partial H}\,\Delta H + \Delta\rho_{Xe} + \alpha_T\Delta T_M$$

where $\partial\rho/\partial H$ is the differential rod worth, $\Delta\rho_{Xe}$ is the xenon reactivity worth, and α_T is the temperature coefficient at constant power. Solving for α_{PD} gives

$$\alpha_{PD} = \frac{1}{\Delta P}\,[\Delta\rho - (\partial\rho/\partial H)\Delta H - \Delta\rho_{Xe} - \alpha_T\Delta T_M].$$

In this measurement, the change in xenon worth is significant. Furthermore, temperature cannot be held as nearly constant as power was for the temperature coefficient measurement. Using the expression above, the error in α_{PD} is

$$\varepsilon(\alpha_{PD}) = \left\{\frac{1}{\Delta P^2}\left[\left[\varepsilon(\Delta\rho)\right]^2 + \left[\varepsilon(\Delta Xe)\right]^2 + \left[\varepsilon(\alpha_T\Delta T_M)\right]^2\right] + \left(\frac{\partial\rho}{\partial H}\,\frac{\Delta H}{\Delta P}\right)^2\left[\left[\frac{\varepsilon\left(\frac{\partial\rho}{\partial H}\right)}{\frac{\partial\rho}{\partial H}}\right]^2 + \left[\frac{\varepsilon(\Delta\rho)}{\Delta\rho}\right]^2 + \left[\frac{\varepsilon(\Delta H)}{\Delta H}\right]^2\right]\right\}^{\frac{1}{2}}$$

Assuming

$$\alpha_{PD} \simeq -\frac{\partial\rho}{\partial H}\,\frac{\Delta H}{\Delta P},$$

$$\varepsilon(\alpha_{PD}) = \left\{ \frac{1}{\Delta P^2} \left[\left[\varepsilon(\Delta\rho) \right]^2 + \left[\varepsilon(\Delta Xe) \right]^2 + \left[\varepsilon(\alpha_T \Delta T_M) \right]^2 \right] \right.$$

$$\left. + \alpha_{PD}^2 \left[\left[\frac{\varepsilon\left[\frac{\partial\rho}{\partial H}\right]}{\frac{\partial\rho}{\partial H}} \right]^2 + \left[\frac{\varepsilon(\Delta\rho)}{\Delta\rho} \right]^2 + \left[\frac{\varepsilon(\Delta H)}{\Delta H} \right]^2 \right] \right\}^{\frac{1}{2}}$$

The uncertainty in the power Doppler coefficient is shown as a function of the coefficient in Figure 3-4.

3.5.3. Average Differential Boron Worths

Average differential boron worths are obtained from control rod worth measurements by boron swap, as described in section 2.5.

$$\frac{\partial\rho}{\partial B} = - \frac{\Delta\rho_{CR}}{\Delta B} .$$

The uncertainty in differential boron worth is obtained from the following formula:

$$\frac{\varepsilon(\overline{\partial\rho/\partial B})}{\overline{\partial\rho/\partial B}} = \sqrt{\left[\frac{\varepsilon(\Delta\rho_{CR})}{\Delta\rho_{CR}} \right]^2 + 2\left[\frac{\varepsilon(B)}{\Delta B} \right]^2} .$$

90-033

The errors in reactivity $\epsilon(\Delta\rho_{CR})/\Delta\rho_{CR}$ and in boron concentration $\epsilon(B)$ are obtained from sections 3.3.1 and 3.4.1, respectively. The boron reactivity worth is equal to the total worth of those control rod groups measured by boron swap; therefore, their errors are the same, . The error in boron concentration at each endpoint is ppmB. Thus,

The uncertainty in the measured differential boron is shown as a function of ΔB in Figure 3-5.

Differential boron worth is usually obtained using the deboration of control rod groups 5, 6, and 7 into the core. In this case, ΔB is in the range of 300 to 400 ppmB,

Table 3-1. Summary of Measurement Uncertainties

(1 standard deviation)

Instrument	
Flux, % measured value	4.2
Temperature, F	1.2
Control rod group position, in.	
Boron concentration, ppmB	
Control Rod Worths	
Group worth, % meas. value	
Boron swap (standard method)	
Boron swap (computer method)	
Total worth, % meas. value	
Ejected rod worth, % meas. value	
Boron swap	
Rod swap	
Rod drop	
Differential rod worth at power, % meas. value	
Critical Boron Concentrations	
AROCBC (at zero power), ppmB	
AROCBC at power, ppmB	
AMCBC at power, ppmB	
Reactivity Coefficients	
Temp coeff at zero power, μρ/°F	
Temp coeff at power, μρ/°F	
Power doppler coeff, μρ/%FP	
Differential boron worth, % meas. value	

Table 3-2. Accuracies of Equipment Contributing to Flux Signal

Equipment	Accuracy	Reference
Compensated ion chamber	±4.0% measured value	9
Ranging current amplifier	±1.0% full scale of each range	10
24-channel preamp, multiplexer	±1.0% full scale	11
Analog-to-digital converter	±0.1% full scale	12

Table 3-3. Accuracy of Equipment Contributing to Temperature Signal

Equipment	Accuracy	Reference
RTD and linear bridge	±0.36% full scale (520 to 620F)	9
Signal converter	±0.3%	9
Summing amplifier	±0.21%	13
24-channel preamp, multiplexer	±1.0%	11
Analog-to-digital converter	±0.1%	12

Table 3-4. Accuracy of Equipment Contributing to Control Rod Group Average Relative Position Indication

Equipment	Accuracy	Reference
Individual sensor output		14
Group average position signal		14
24-channel preamp, multiplexer	1.0%	11
Analog-to-digital converter	0.1%	12

90-036

Table 3-5. Reactivity Algorithm Computational Accuracy

Test method	Accuracy
Boron swap: Before step, ¢ After step, ¢	
Rod drop, % drop worth	
Temp coefficient at zero power, % meas. $\Delta\rho$	
Differential rod worth at power, % meas. $\Delta\rho$	

Table 3-6. Uncertainty in Reactivity Due to Delayed Neutron Precursor Parameters (Near-Term Measurements)

	Uncertainty, %						
t(s)/$^{\rho}$$	0.01	-0.01	0.1	-0.1	-1.0	-10.0	-15.0
1							
10							
30							

Table 3-7. Delayed-Neutron Precursor Parameters[6]

i	a_i	$\varepsilon(a_i)$	λ_i, s^{-1}	$\varepsilon(\lambda_i)$, s^{-1}
1	0.0303	±0.004	0.0125	±0.0003
2	0.2052	±0.007	0.0308	±0.0012
3	0.1897	±0.024	0.114	±0.004
4	0.3952	±0.010	0.307	±0.012
5	0.1341	±0.012	1.19	±0.12
6	0.0455	±0.004	3.19	±0.55

Table 3 . Uncertainty in Reactivity Due to Delayed Neutron Precursor Parameters (Asymptotic Measurements)

$\rho_\$$	$\varepsilon(\rho_\$)/\rho_\$$, %
-0.0758	
-0.0500	
-0.0262	
0.0	
0.0280	
0.0500	
0.0927	

*Limit as $\rho_\$ \rightarrow 0.0$.

Table 3-9. Sources of Uncertainties in Control Rod Worths Measured by Boron-Swap Technique

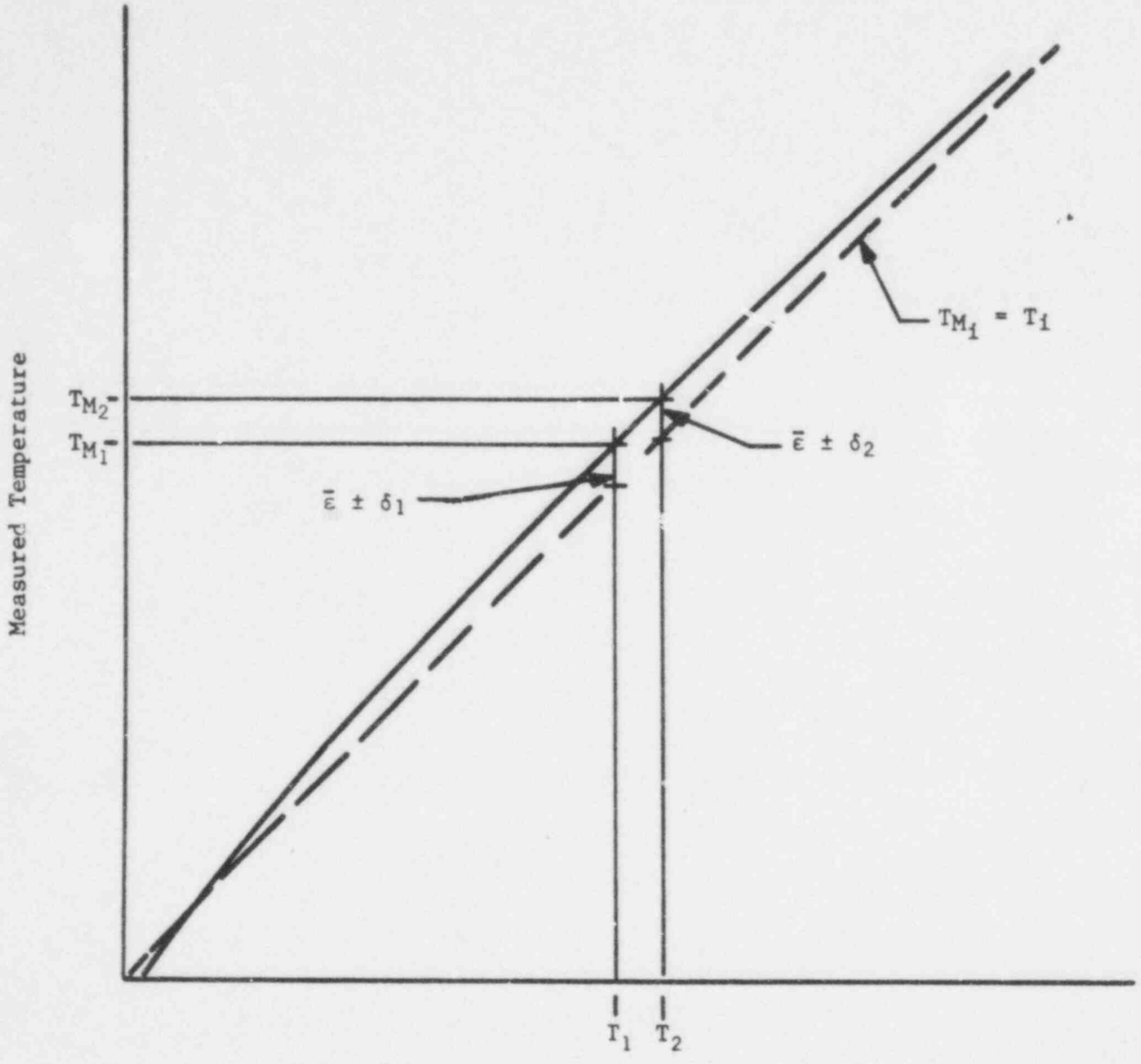
	Worth, ¢		
	Before step	After step	Net
Random Error			
Noise			
Curve extrapolation			
Total random error per step			
Systematic Error			
Delayed neutron parameters			
Reactimeter error			
Flux shape change			
Range changer			
Allowance for unidentified errors			
Total systematic error per step			

90-038

Table 3-10. Components of Uncertainty in Zero Power Temperature Coefficient Reactivity Measurements

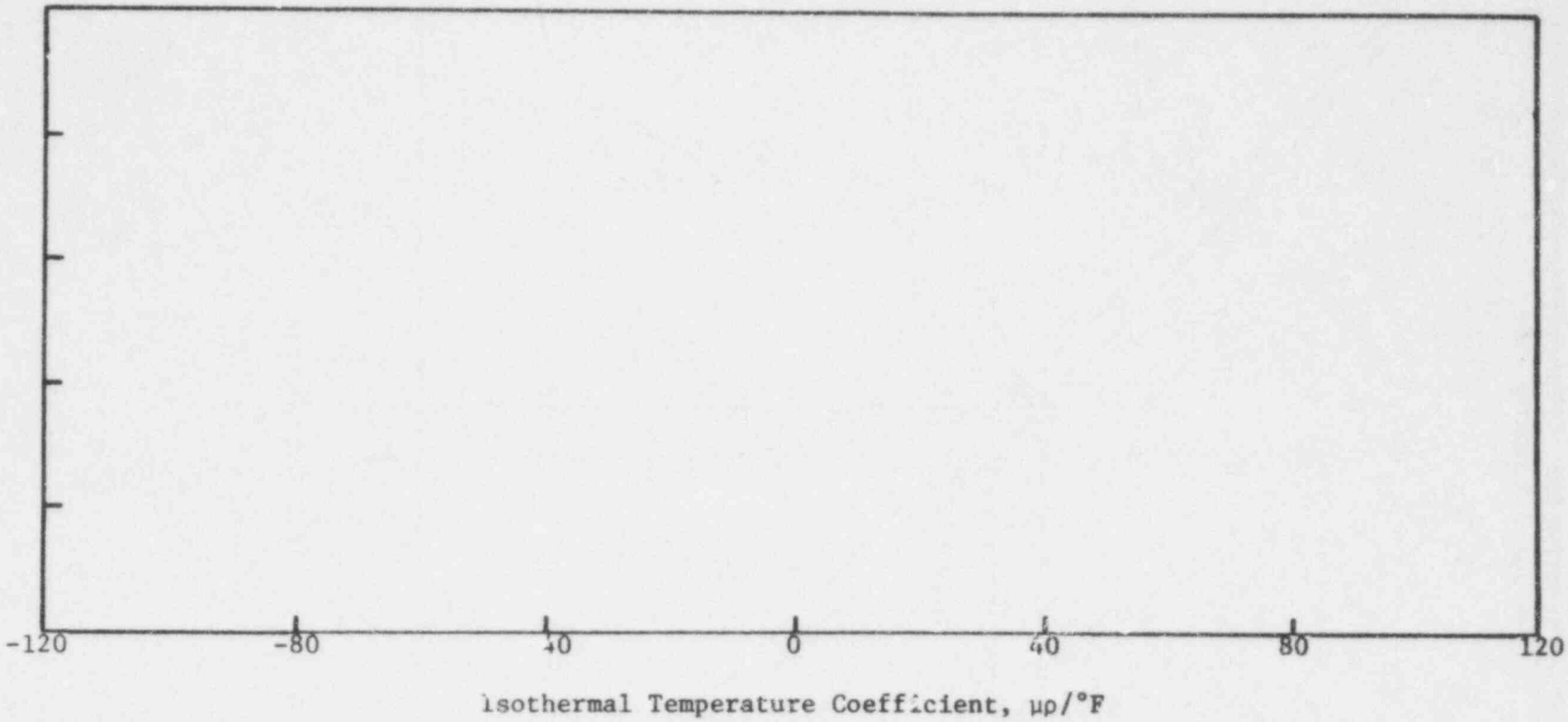
Noise in initial reactivity, ¢	
Noise in final reactivity, ¢	
Reactimeter algorithm error, ¢	
Delayed neutron precursor parameters, ¢	
Unidentified errors, ¢	
Total, ¢	

Figure 3-1. Hypothetical Illustration of True Vs Measured Temperature



90-010

Figure 3-2. Uncertainty in Measured Isothermal Temperature Coefficients



80-044

Figure 3-3. Uncertainty in Measured Temperature Coefficients at Power

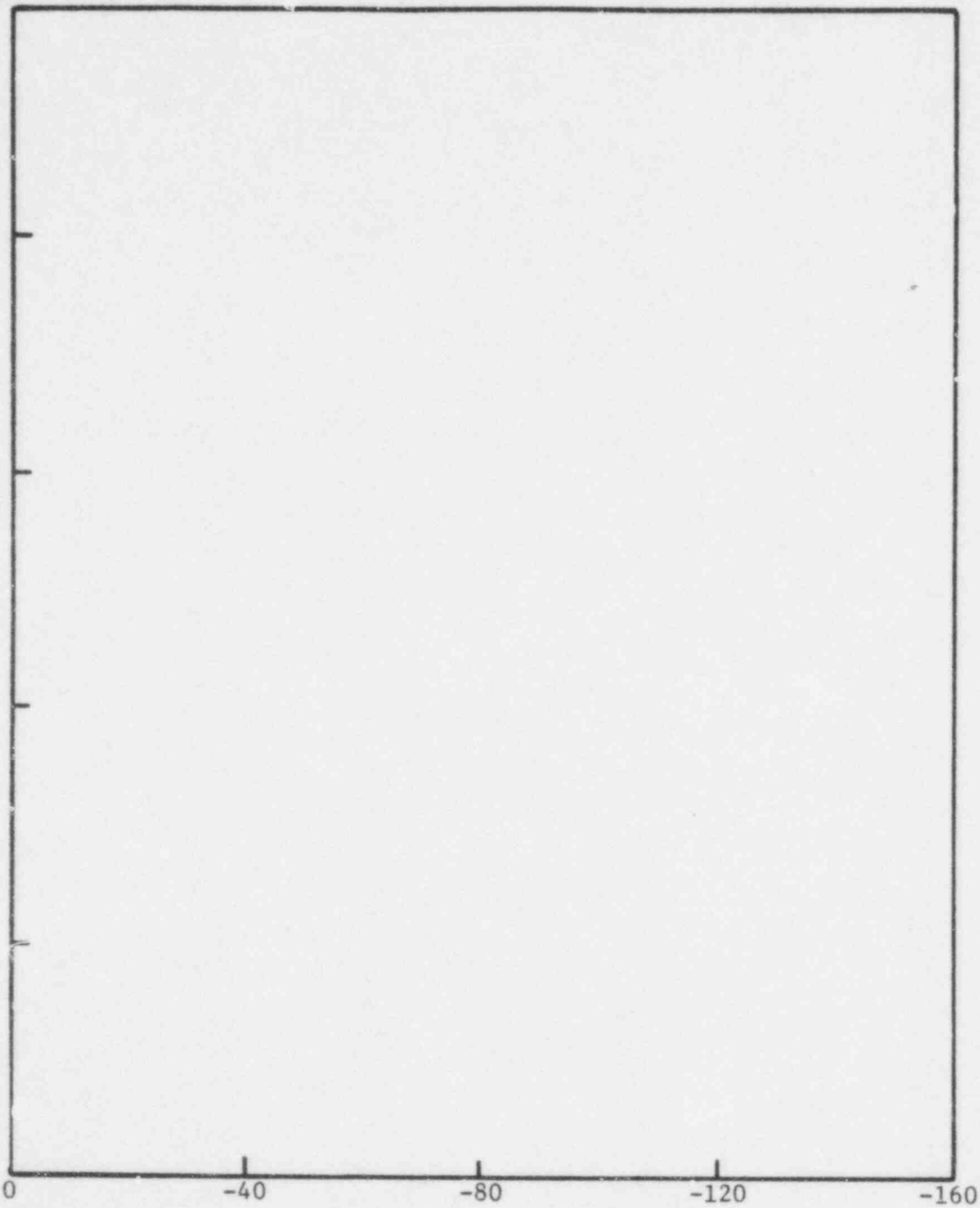


Figure 3-4. Uncertainty in Measured Power Doppler Coefficients

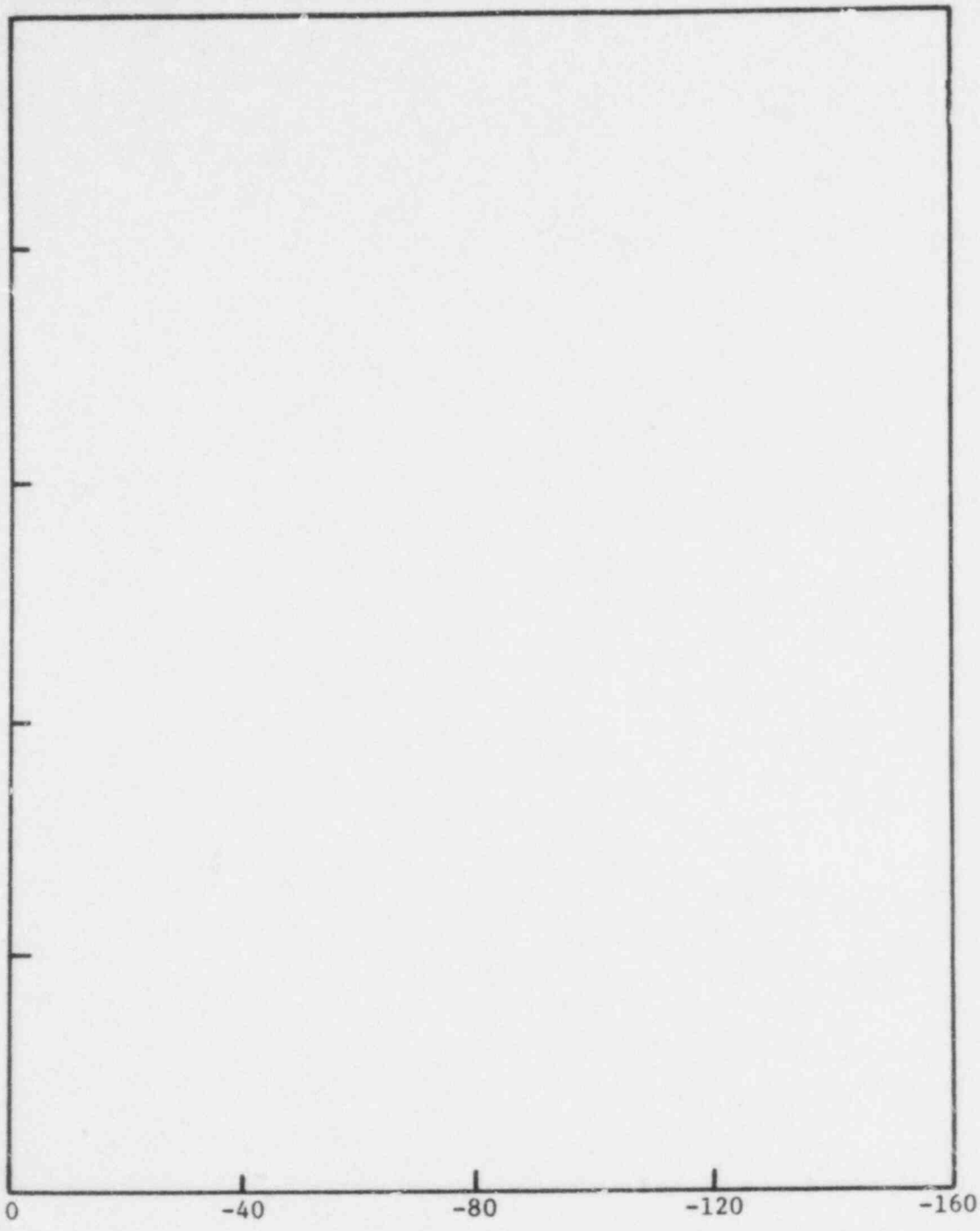
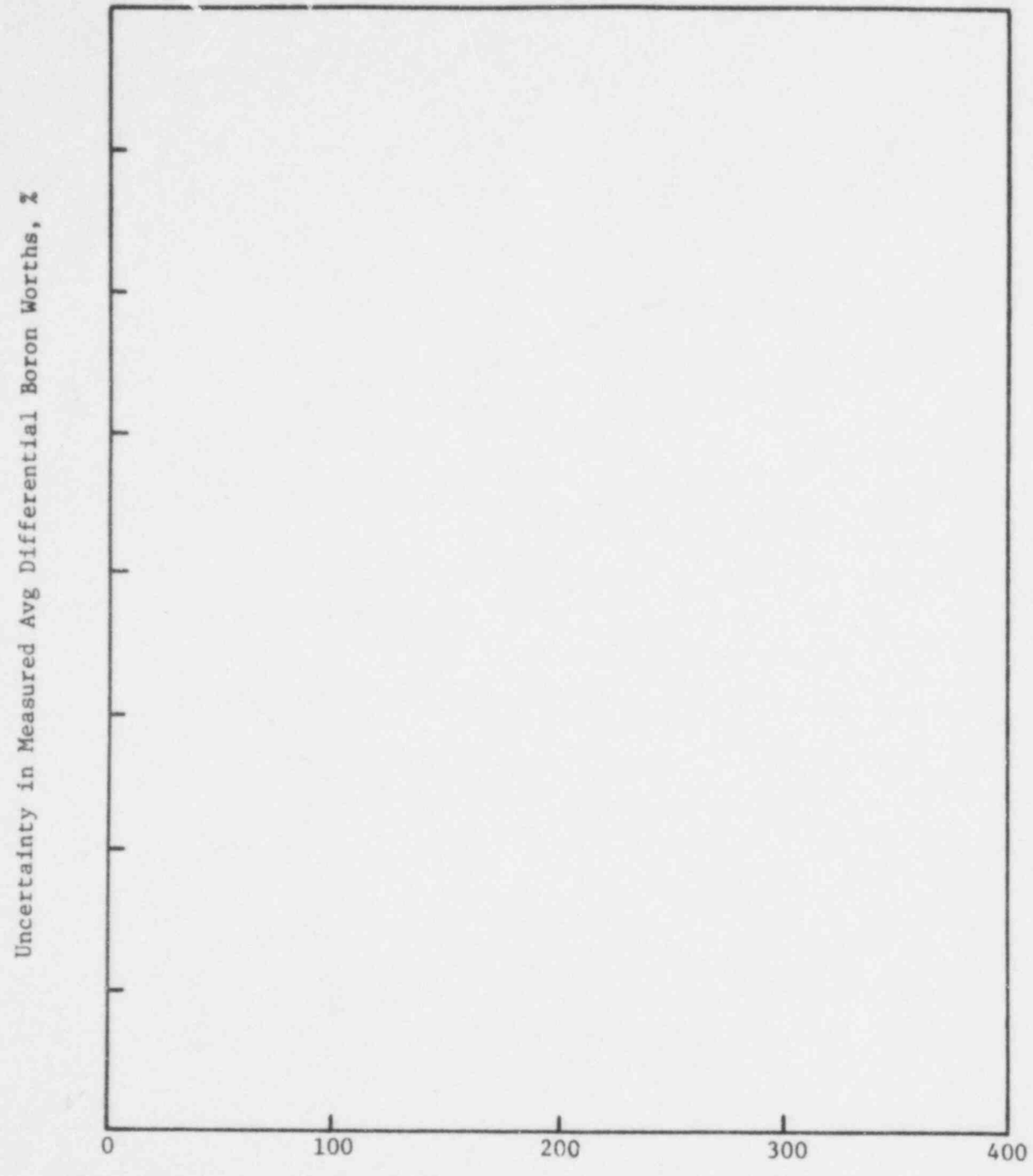


Figure 3-5. Uncertainty in Measured Average Differential Boron Worths



4. COMPARISONS OF CALCULATED AND MEASURED DATA

In this section, predicted control rod worths, boron concentrations, and reactivity coefficients are compared to data measured at B&W's operating reactors. The calculations were performed with PDQ07[15] and FLAME[16] using current standard calculational procedures and techniques. The procedures and techniques are discussed in detail in reference 17; the PDQ07 and FLAME models used are described in references 18 and 19. The calculated values presented in this section differ from those used in comparisons made during zero power physics and power escalation testing in two respects:

1. Predictions used during startup testing are obtained from calculations made prior to the measurements, and the measurements are seldom taken under plant conditions identical to those assumed in the calculations. Although the resultant comparisons are adequate to demonstrate compliance with acceptance criteria, they are not representative of the accuracy attainable with the calculational model. Therefore, the calculations reported in this section were made for the actual plant conditions at the time of the measurement.

2. In many cases, the predictions available during startup had been made using earlier calculational procedures and techniques.

It is also noted that the measured data reported in this section differs in some instances from previously reported data. All data were processed in accordance with B&W's current recommended measurement techniques, described in section 2. Measured data were re-evaluated where required to ensure a consistent data base with which to assess the accuracy of B&W's standard calculational procedures and techniques.

4.1. Control Rod Worths

Total control rod pattern and individual regulating bank worths are calculated to establish the ability of the reactor to provide adequate shutdown capability during normal and accident conditions and to maintain criticality during plant startup and power maneuvers. Ejected rod worths are calculated to verify that

ejected rod worths will not exceed the value used in safety analysis calculations to evaluate the consequences of an ejected rod accident.

During zero power physics testing, the total control rod pattern worth and the worths of individual regulating banks and ejected rods are measured with the reactor at hot zero power (HZP) conditions. Measured and predicted regulating bank and total rod worths are compared in section 4.1.1; measured and predicted ejected rod worths are compared in section 4.1.2. These comparisons show the accuracy with which B&W's standard calculational models predict control rod worths.

4.1.1. Regulating Bank and Total Rod Worths

Individual regulating bank and total-pattern control rod worths were calculated in quarter-core geometry with the two-dimensional discrete (one mesh interval per pin) PDQ model. Measured and predicted worths for individual regulating banks are compared in Table 4-1.

Since, as stated in section 3, the measurement uncertainty represents an estimate of the standard deviation, the agreement between measured and predicted individual bank worths is very good.

Measured and predicted worths for the regulating banks are compared in Table 4-2.

Table 4-3 compares measured and predicted total control rod pattern worths.

Since the measurement uncertainty represents the standard deviation, the agreement between measured and predicted regulating bank and total rod worths is very good.

4.1.2. Ejected Rod Worths

Ejected rod worths are calculated in full-core geometry with the two-dimensional discrete PDQ model. Normally, all transient control rods (banks 5 through 7) are inserted in the two-dimensional calculation. Since ejected rod worths (ERW) are usually measured with the controlling rod group (bank 5)

partially inserted, the two-dimensional ERWs were adjusted as follows using three-dimensional FLAME ERW calculations:

$$\frac{\text{FLAME 3D ERW Bank 5 at Measured Location}}{\text{FLAME 3D ERW Bank 5 Fully Inserted}} \times \text{PDQ 2D Discrete ERW} = \text{Adjusted PDQ Discrete ERW}$$

Measured and predicted (adjusted PDQ discrete) ERWs are compared in Table 4-4. Figure 4-1 is a map showing the core locations referred to in the table. The agreement between measured and predicted ejected rod worths is quite good;

4.2. Critical Boron Concentrations

4.2.1. AROCBCs at Zero Power

The all-rods-out critical boron concentration (AROCBC) is a measure in ppm boron of the excess reactivity in the core. As part of the zero power physics testing program performed during cycle startup, all rods out critical boron concentrations are measured. The measurement technique is described in section 2.3.1.

Two-dimensional discrete PDQ calculations have been performed using the measured AROCBCs. The PDQ calculations were made for the all-rods-out condition at HZP. However, measurements at Oconee 2 cycle 2, Oconee 3 cycle 2, and TMI-1 cycle 2 were taken with the axial power shaping rods (APSRs) inserted. Integral APSR worth curves generated from 3-D FLAME calculations and normalized to APSR worths from discrete PDQ calculations were used to account for the inserted worth of the APSRs. Table 4-5 shows the calculated critical eigenvalues at hot zero power. The average value of k_{eff} is with a standard deviation of Assuming a typical differential boron worth of 1% Δk/k/100 ppmB, the 2-D discrete PDQ model underpredicts the AROCBC by approximately

ppmB. This bias is taken into account when the model is used to make AROCBC predictions.

4.2.2. Critical Boron Concentrations at Power

Core reactivity depletion is compensated for by diluting the soluble boron concentration. Technical Specifications require that the reactor coolant boron concentration be routinely measured and compared to the predicted critical boron concentration to ensure that no unexpected reactivity anomalies have occurred. In this section, measured critical boron concentrations are compared to critical boron concentrations predicted from 2-D discrete PDQ fuel cycle calculations. Only boron concentrations measured while the plant was operating at full-power, steady-state conditions are included. Also the results of 3-D FLAME claculations made at the measured critical boron concentrations and control rod insertions are discussed. These comparisons show that B&W's calculational models accurately predict critical boron concentrations during full-power operation.

4.2.2.1. Discrete PDQ Calculations

Fuel cycle calculations for rodded cores using the two-dimensional discrete PDQ model are performed with rod groups 1 through 6 fully withdrawn and group 7 fully inserted. APSRs are simulated by flux- and volume-weighting the control rod channel cross sections using a calculated axial flux profile. The flux profile is determined from the case in which the APSRs are in a position maintaining approximately zero axial power imbalance. In addition, a modification to the buckling for APSR assemblies has been developed based on measured data.

Since the measured rod configuration is usually different from that assumed in the PDQ calculation, the measured critical boron concentrations must be adjusted to the rod configuration used in the PDQ calculation. These adjusted measured critical boron concentrations (AMCBCs) were computed using FLAME 3-D integral rod worth curves normalized to PDQ 2-D control rod bank worths and differential boron worths obtained from 2-D discrete PDQ calculations for the plants under consideration.

Since B&W's rodded plants operate with group 6 nearly all the way out and group 7 only slightly withdrawn, the FLAME integral rod worth curve was normalized to the PDQ 2-D bank 7 worth as follows:

$$\begin{pmatrix}\text{Normalized integral}\\ \text{worth as function}\\ \text{or rod index}\end{pmatrix} = \frac{\begin{pmatrix}\text{FLAME group 6 and 7}\\ \text{integral worth as a}\\ \text{function of rod index}\end{pmatrix}}{(\text{FLAME group 7 worth})} * \begin{pmatrix}\text{PDQ 2-D}\\ \text{group 7}\\ \text{worth}\end{pmatrix}$$

where rod index = % group 5 withdrawal + % group 6 withdrawal + % group 7 withdrawal, (giving a total maximum rod index of 300). For SMUD, a feed-and-bleed plant where banks 6 and 7 are withdrawn together, the FLAME integral rod worth curve was normalized to the total worth of banks 6 and 7.

APSR (Bank 8) worth curves were also generated using FLAME and are normalized to PDQ 2-D bank 8 worths as follows:

$$\begin{pmatrix}\text{Normalized group 8}\\ \text{worth as function}\\ \text{of rod index}\end{pmatrix} = \frac{\begin{pmatrix}\text{FLAME group 8 worth as}\\ \text{function of rod index}\end{pmatrix}}{\begin{pmatrix}\text{FLAME group 8 worth}\\ \text{at 0 imbalance}\end{pmatrix}} * \begin{pmatrix}\text{PDQ 2-D}\\ \text{group 8}\\ \text{worth}\end{pmatrix}$$

The predicted boron concentrations include standard burnup-dependent reactivity corrections, which account for reactivity effects due to such known phenomena as nonuniform axial burnup and provide an overall normalization of the model to operating plant data.

Figures 4-2 through 4-7 compare the critical boron concentrations predicted from two-dimensional discrete PDQ calculations to the AMCBCs for selected cycles. The agreement between measured and predicted boron concentrations is very good. For non-LBP cycles, the AMCBCs generally lie within ppm of the predicted. For LBP cycles, the AMCBCs generally lie within ppm of the predicted.

4.2.2.2. Three-Dimensional FLAME Calculations

Three-dimensional FLAME core-follow calculations are routinely made for all B&W operating reactors. Since FLAME is a three-dimensional nodal code the measured rod configurations for a plant can be duplicated within limitations imposed by the node spacing. Therefore, the FLAME calculations are made for the measured control rod configuration and boron concentrations, and the calculated eigenvalues are compared to the known value of 1.0. A constant

reactivity correction factor is applied to the FLAME eigenvalues to account for the known average reactivity difference. The resultant distribution of FLAME eigenvalues lies within a band of around 1.0. This represents an equivalent boron uncertainty of approximately ppm boron.

4.3. Reactivity Coefficients

Reactivity coefficients quantify the changes in reactivity that result from incremental changes in such core parameters as moderator temperature, power, and soluble boron concentration. Reactivity coefficients are calculated to ensure that the reactor is capable of safely responding to possible perturbations to core conditions during normal plant operations. In the following sections, measured temperature coefficients, power Doppler coefficients, and soluble boron worths are compared to coefficients predicted from one-zone, two-dimensional, quarter-core PDQ calculations using six mesh intervals per assembly. The nuclide concentrations used in these calculations were obtained from two-dimensional discrete PDQ fuel cycle calculations. Thermal-hydraulic feedback is used when calculating reactivity coefficients at power.

In two-dimensional calculations, control rod banks must be represented as either fully inserted or fully withdrawn. Most of the measurements, however, were taken with partially inserted rod banks. The PDQ calculations were made with the two-dimensional rod configuration closest to the actual plant configuration at the time of the measurement. In selecting the measurements for which calculations were performed, preference was given to those measurements taken with rod configurations that could most closely be duplicated in the two-dimensional calculations.

4.3.1. Isothermal Temperature Coefficients at Zero Power

During zero power physics testing isothermal temperature coefficients are measured at HZP conditions for several different soluble boron concentrations and control rod configurations. The measured and predicted temperature coefficients are compared in Table 4-3. The agreement is good;

4.3.2. Reactivity Coefficients at Power

During beginning-of-cycle power escalation testing, temperature and power Doppler coefficient measurements are taken at each of the major test plateaus.

The measurement techniques are discussed in section 2.4.2 Measured differential control rod worths make a major contribution to both reactivity coefficient measurements. Selected measurements have been analyzed using the linear regression analysis described in section 2.2.3 to evaluate the differential rod worths.

4.3.2.1. Temperature Coefficients

Table 4-7 compares predicted and measured temperature coefficients at power. The agreement is good;

It is noted that the average core moderator temperature obtained from PDQ07 calculations is volumetrically averaged over the entire core. However, the measured average core moderator temperature is equal to the average of the inlet and outlet temperatures:

$$\bar{T}_{mod} = \frac{T_{inlet} + T_{outlet}}{2}$$

The difference between these two values is small and should have only minor effects on the comparisons of measured and calculated temperature coefficients.

4.3.2.2. Power Doppler Coefficients

The standard procedure for determining power Doppler coefficients involves both two- and three-dimensional PDQ07 calculations. The coefficients calculated with the two-dimensional, one-zone model with thermal-hydraulic feedback are corrected for three-dimensional effects using g[illegible]ic correction factors which have been obtained from three-dimensional, one-zone calculations with thermal-hydraulic feedback. These correction factors account for flux redistribution caused by variations in burnup and isotopics along the lengths of the fuel assemblies, and, to a lesser extent, nonuniform fuel temperature distributions. The correction factors are discussed in detail in reference 17.

Table 4-8 compares the measured and predicted power Doppler coefficients of reactivity. The agreement is good;

90-051

4.3.3. Average Differential Boron Worths

During zero power physics testing, average differential boron worths are calculated from data obtained during the control rod worth measurements by boron swap. Measured average differential boron worths are compared to predicted worths in Table 4-9. The agreement is very good;

4.4. Conclusion

The comparisons of predicted and measured core physics parameters presented in this section demonstrate that B&W's standard calculational models are accurate tools for predicting core behavior. Very good agreement was obtained between measured and calculated control rod bank and total rod worths.

Measured critical boron concentrations at power generally lie within ppm of the predicted for non-LBP cycles and within ppm of the predicted for LBP cycles. At zero power, the AROCBC is

Measured and calculated temperature and power Doppler coefficients agree within

Measured and calculated differential boron worths agree very well,

Finally, it is reiterated that the comparisons presented in this section are not necessarily typical of comparisons made during zero power physics and power escalation tests. Startup tests are intended to demonstrate compliance with the acceptance criteria. Since the purpose of this section is to demonstrate the accuracy attainable with B&W's calculational models, the measured data were re-evaluated and calculations were performed for the actual plant conditions at the time of the measurement.

Table 4-1. Control Rod Worths — Individual Banks

Cycle	Bank	Worth, % Δρ Predicted	Worth, % Δρ Measured	Diff., % Δρ
Oconee 1				
1[(a)]	7	1.10	1.09	0.01
	6	0.96	0.93	0.03
	5	0.72	0.68	0.04
	4	0.67	0.64	0.03
	3	2.78	2.75	0.03
1	7	1.12	1.09	0.03
	6	1.14	1.10	0.04
	5	0.67	0.68	-0.01
	4	0.51	0.59	0.02
	3	2.97	2.82	0.15
2	7	1.02	1.08	-0.06
	6	1.07	0.90	0.17
	5	1.42	1.42	0
3	7	1.21	1.32	-0.11
	6	1.14	1.07	0.07
	5	1.21	1.33	-0.12
Oconee 2				
1	7	1.17	1.14	0.03
	6	1.18	1.09	0.09
	5	1.06	1.04	0.02
2	7	0.77	0.76	0.01
	6	1.12	1.02	0.10
	5	0.79	0.71	0.08
Oconee 3				
1	7	1.28	1.24	0.04
	6	1.13	1.15	-0.02
	5	1.01	1.06	-0.05
2	7	0.81	0.79	0.02
	6	1.08	1.09	-0.01
	5	0.74	0.69	0.05
TMI-1				
1	7	1.17	1.21	-0.04
	6	1.18	1.23	-0.05
	5	1.06	1.10	-0.04
2	7	0.80	0.81	-0.01
	6	1.09	1.02	0.07
	5	0.77	0.69	0.08

Table 4-1. (Cont'd)

Cycle	Bank	Worth, % Δρ Predicted	Worth, % Δρ Measured	Diff., % Δρ
SMUD				
1[(a)]	7	1.43	1.42	0.01
	6	1.74	1.82	-0.08

(a) APSRs withdrawn; APSRs inserted in all other cases.

90-054

Table 4-2. Control Rod Worths — Regulating Banks

Cycle	Regulating banks	Worth, % Δρ		Diff., % Δρ
		Predicted	Measured	
Oconee 1				
1[(a)]	5-7	2.78	2.70	0.08
1	5-7	2.93	2.87	0.06
2	5-7	3.51	3.40	0.11
3	5-7	3.56	3.72	-0.16
Oconee 2				
1	5-7	3.41	3.27	0.14
2	5-7	2.68	2.49	0.19
Oconee 3				
1	5-7	3.42	3.45	-0.03
2	5-7	2.63	2.57	0.06
TMI-1				
1	5-7	3.41	3.54	-0.13
2	5-7	2.66	2.52	0.14
SMUD				
1[(a)]	6-7	3.17	3.24	-0.07

(a) APSRs withdrawn; APSRs inserted in all other cases.

96-055

Table 4-3. Control Rod Worths — Total Pattern

Cycle	Worth, % Δρ Predicted	Worth, % Δρ Measured	Diff., % Δρ
Oconee 1			
1	11.54	11.21	0.33
2	8.24	9.05	-0.81
3	8.98	10.32	-1.34
Oconee 2			
1	9.72	9.69	0.03
2	9.29	9.64	-0.35
Oconee 3			
1	9.68	9.86	-0.18
2	9.28	9.49	-0.21
TMI-1			
1	9.72	10.04	-0.32
2	9.23	9.57	-0.34
SMUD			
1	9.73	9.79	-0.06

90-056

Table 4-4. Ejected Rod Worths[a]

Cycle	ER loc'n	Controlling group, % wd	Measurement technique	Worth, % Δρ		Diff,. % Δρ
				Predicted	Measured	
Oconee 1						
2	H8	60	Boron swap	0.67	0.69	-0.02
2	H8	9	Rod swap	1.11	1.16	-0.05
2	L14	51	Rod swap	0.27	0.20	0.07
2	L14	8	Rod swap	0.62	0.46	0.16
3	K13	10	Boron swap	0.56	0.58	-0.02
Oconee 2						
1	P10	24	Rod drop	0.75	0.72	0.03
1	H4	32	Rod drop	0.47	0.34	0.13
1[b]	L14	6	Boron swap	0.77	0.54	0.23
2	N12	4	Boron swap	0.39	0.49	-0.10
2	N12	4	Rod swap	0.39	0.56	-0.17
Oconee 3						
1	F2	8	Rod drop	0.88	0.77	0.11
TMI-1						
1	F2	6	Boron swap	0.76	0.69	0.07
1	F2	13	Rod drop	0.75	0.66	0.09
2	N12	0	Boron swap	0.41	0.51	-0.10
ANO						
2	N12	9	Rod swap	0.41	0.54	-0.13
SMUD						
1	H2	50	Boron swap	0.81	0.68	0.13
1	H2	54	Rod drop	0.81	0.79	0.02
1[c]	H2	52	Boron swap	0.47	0.34	0.13
1[c]	H2	47	Rod swap	0.47	0.37	0.10
1[c]	H2	49	Rod drop	0.47	0.44	0.03

[a] Beginning of cycle unless otherwise noted.

[b] 127 EFPD.

[c] 134 EFPD.

90-057

Table 4-5. Calculated Critical Eigenvalues at HZP for All-Rods-Out Measured Critical Boron

Cycle	Meas. boron conc, ppmB	k_{eff}	$\frac{1 - k_{eff}}{k_{eff}} \times 100$
Oconee 1			
1			
2			
3			
Oconee 2			
1			
2			
Oconee 3			
1			
2			
TMI-1			
1			
2			
SMUD			
1			

$$\bar{k}_{eff} =$$

$$\sigma_{k_{eff}} = \sqrt{\sum_{i=1}^{m} \frac{(X_i - \bar{X})^2}{m - 1}} =$$

90-058

Table 4-6. Isothermal Temperature Coefficients at Zero Power

Cycle	Soluble boron conc, ppmB	Temp coeff, μρ/°F		Diff., μρ/°F
		Predicted	Measured	
Oconee 1				
1	952	-58	-39	-19
1	1364	58	75	-17
2	1013	-11	-14	3
2	1295	15	17	-2
3	1018	-81	-67	-14
3	1330	4	5	-1
Oconee 2				
1	1490	30	30	0
1	1630	43	38	5
Oconee 3				
1	1546	20	31	-11
2	1286	-7	-8	1
TMI-1				
1	1269	-66	-53	-13
1	1461	17	30	-13
1	1601	40	45	-5
2	1154	-51	-53	2
2	1375	8	9	-1

Table 4-7. Temperature Coefficients at Power

Cycle	Power level, % FP	Soluble boron conc, ppmB	Temp coeff, μρ/°F		Diff., μρ/°F
			Predicted	Measured	
Oconee 1					
2	98	782	-90	-103	13
Oconee 2					
1	75	1080	-30	-44	14
1	75	1216	-2	-18	16
Oconee 3					
2	99	650	-153	-155	2
TMI-1					
1	97	1110	-30	-59	29
2	98	835	-99	-119	20
SMUD					
1	75	1155	-50	-40	10

90-060

Table 4-8. Power Doppler Coefficients

Cycle	Power level, % FP	Soluble boron conc, ppmB	Power Doppler coeff, μρ/% FP		Diff., μρ/% FP
			Predicted	Measured	
Oconee 1					
2	96	782	-148	-108	-40
Oconee 2					
1	71	1216	-124	-111	-13
1	73	1080	-132	-104	-28
1	96	1095	-122	-115	-7
Oconee 3					
2	96	650	-84	-70	-14
TMI-1					
1	96	1090	-120	-114	-6
2	93	835	-129	-117	-12

Table 4-9. Differential Boron Worths

Cycle	Soluble boron conc, ppmB		Worth, % Δρ/ppm		Diff., % Δρ/ppm
	Initial	Final	Predicted	Measured	
Oconee 1					
1	1400	1200	0.0124	0.0120	0.0004
2	1301	1031	0.0106	0.0117	-0.0011
3	1350	1025	0.0100	0.0111	-0.0011
Oconee 2					
1	1630	1234	0.0102	0.0107	-0.0005
Oconee 3					
1	1554	1236	0.0107	0.0100	0.0007
2	1288	987	0.0101	0.0113	-0.0012
TMI-1					
2	1377	1136	0.0097	0.0097	0

90-062

Figure 4-1. 177-Fuel Assembly Core
Assembly Designations

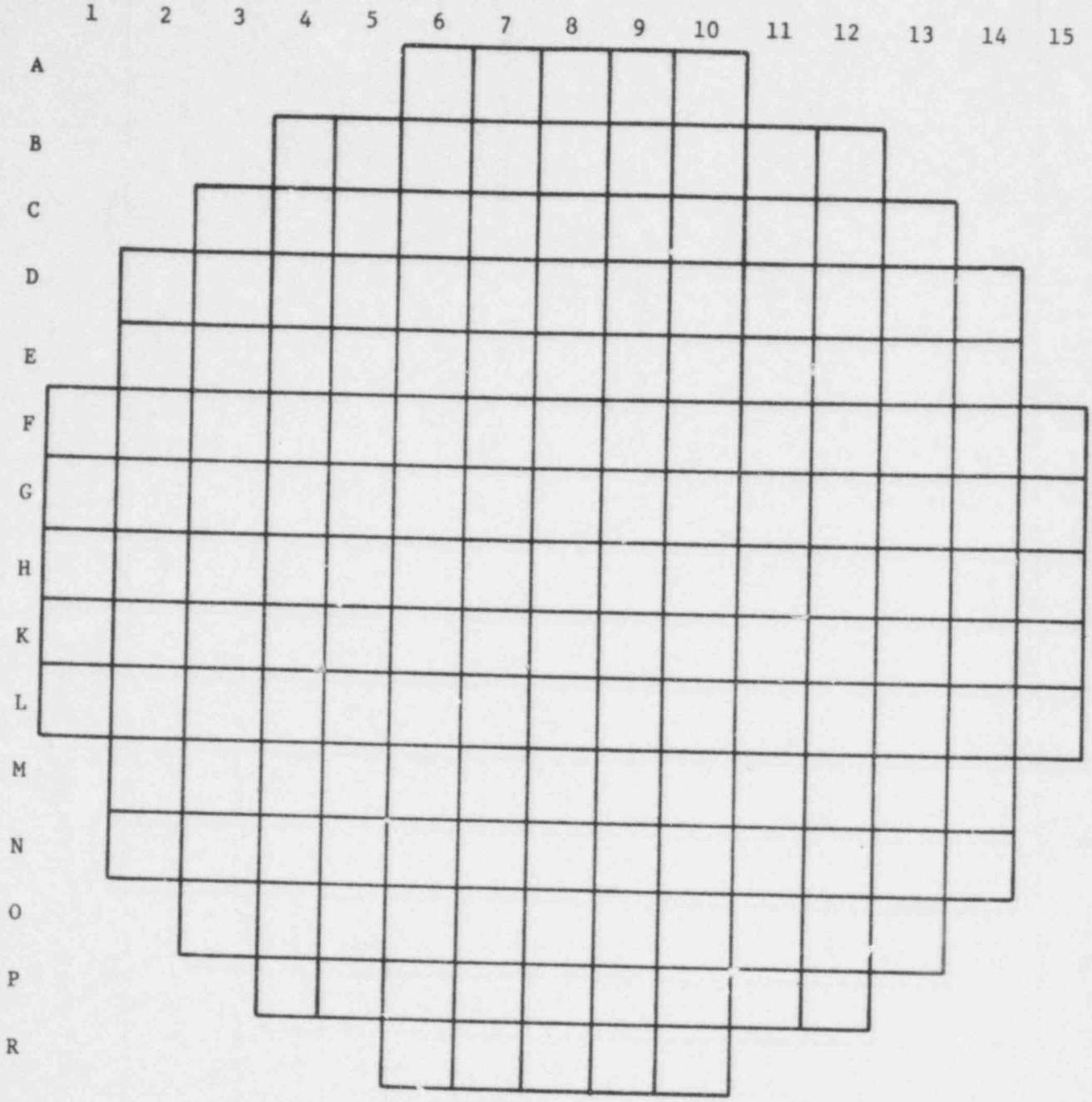
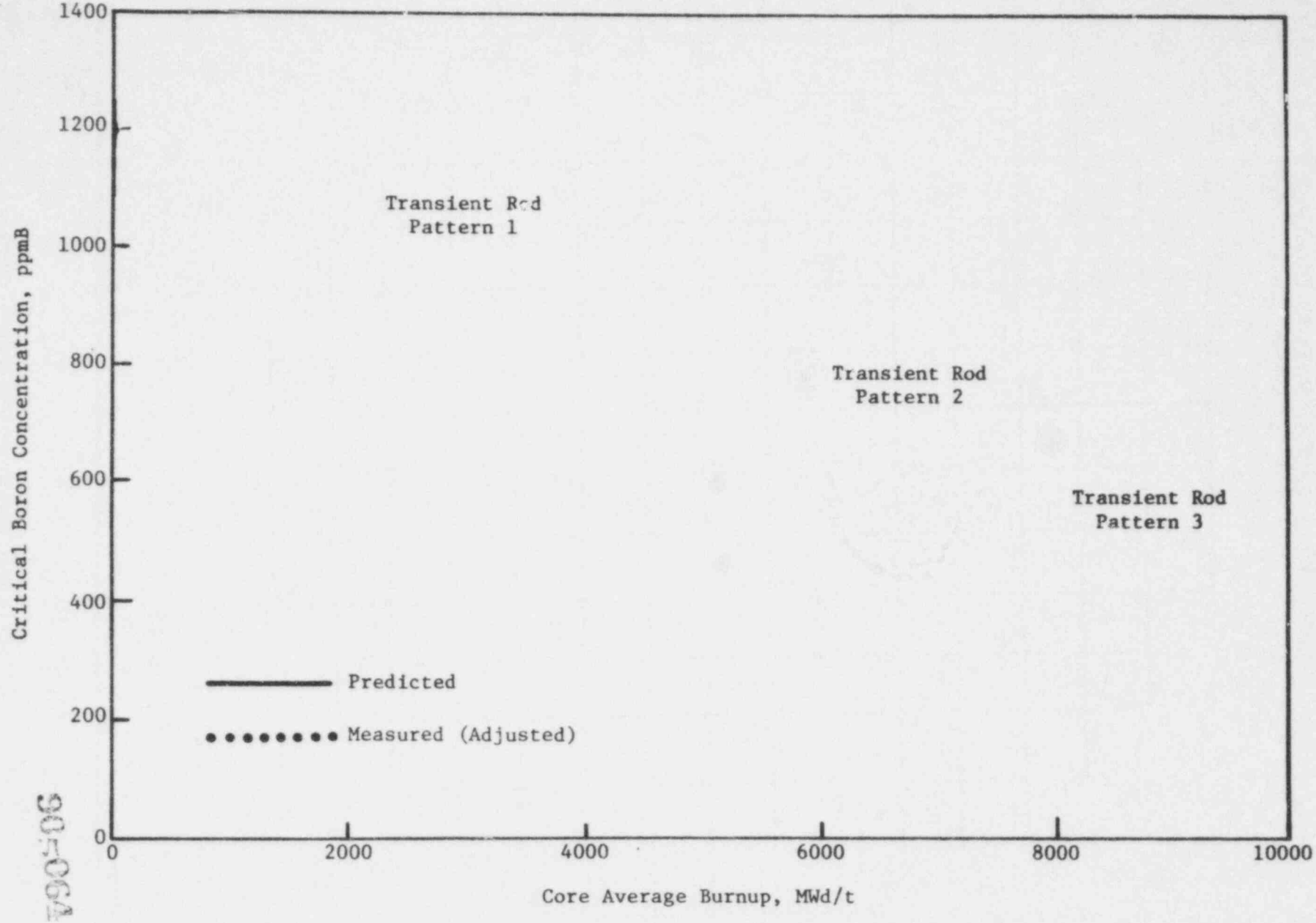


Figure 4-2. Critical Boron Concentrations — Oconee 1, Cycle 1



90-064

Figure 4-3. Critical Boron Concentrations — Oconee 1, Cycle 3

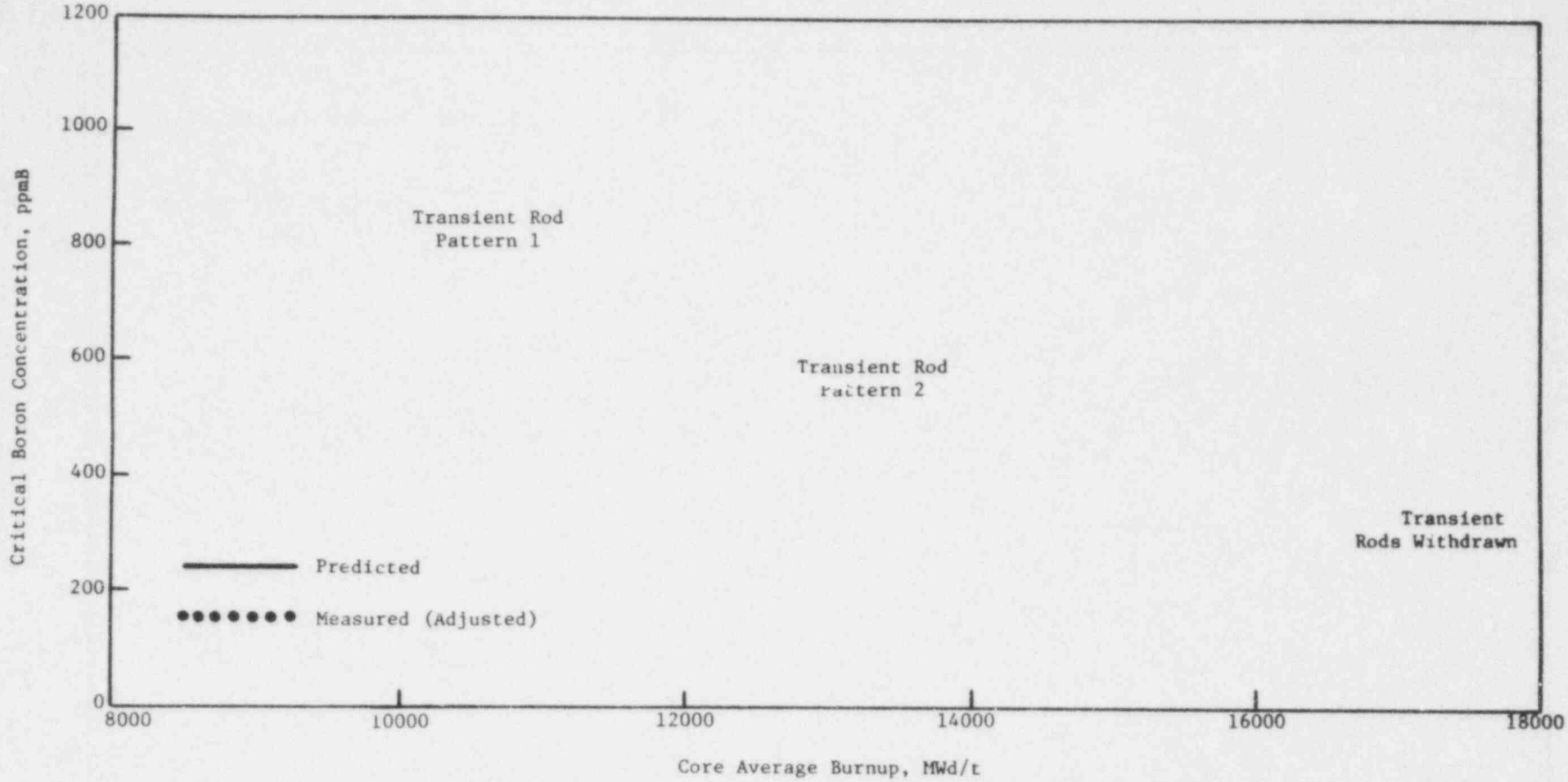


Figure 4-4. Critical Boron Concentrations — SMUD, Cycle 1

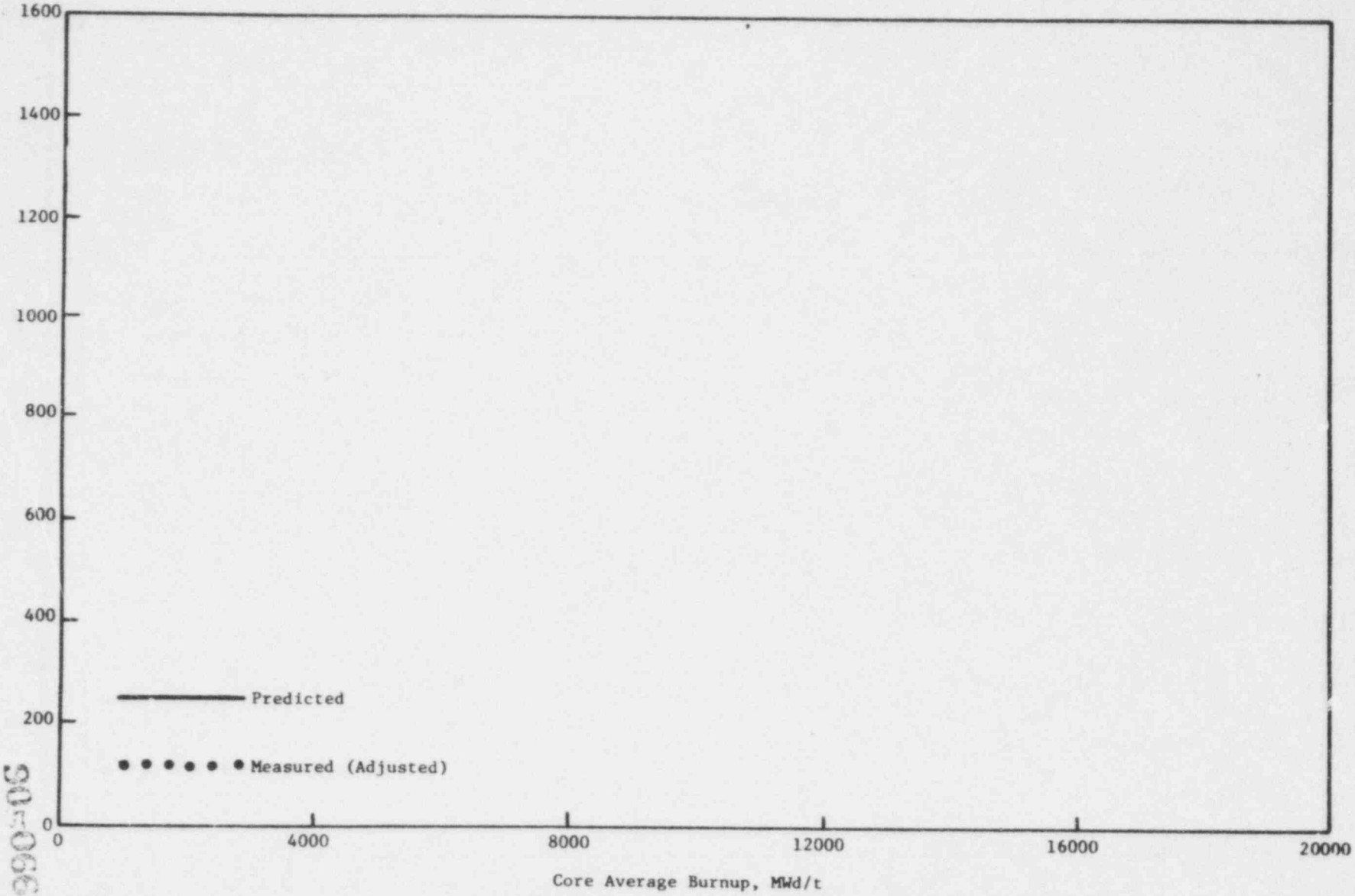


Figure 4-5. Critical Boron Concentrations — Oconee 2, Cycle 1

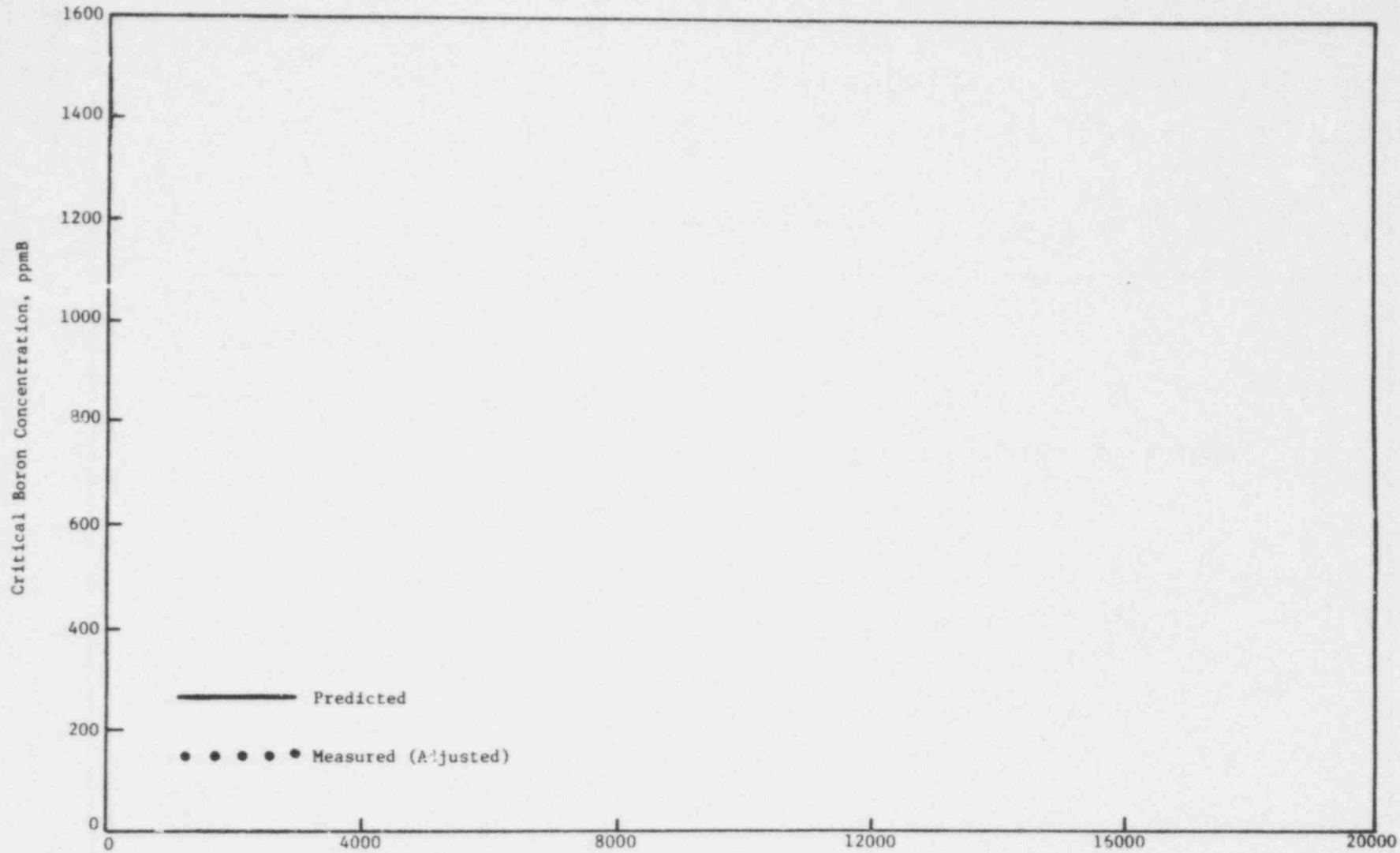


Figure 4-6. Critical Boron Concentrations — Oconee 3, Cycle 2

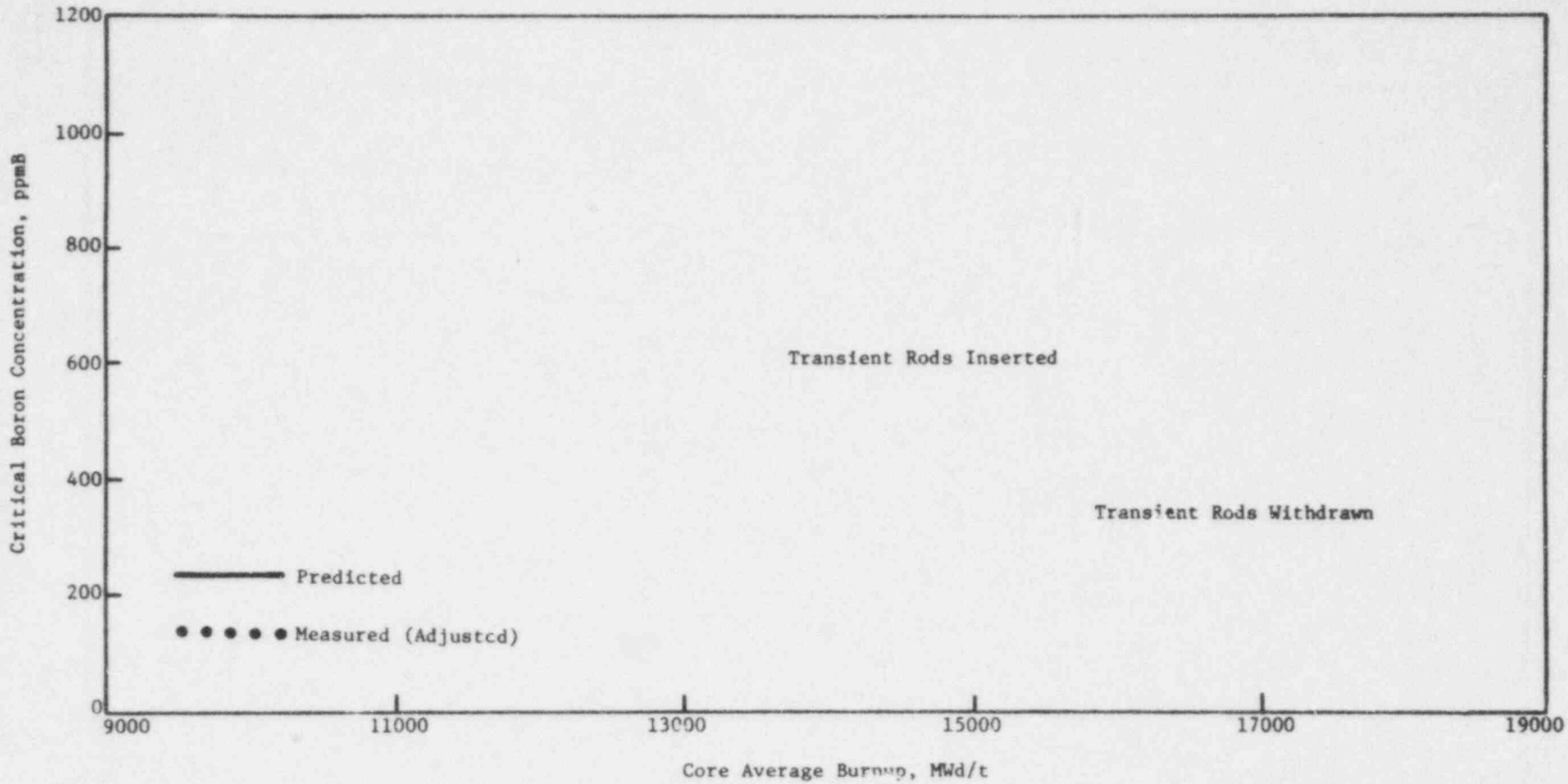
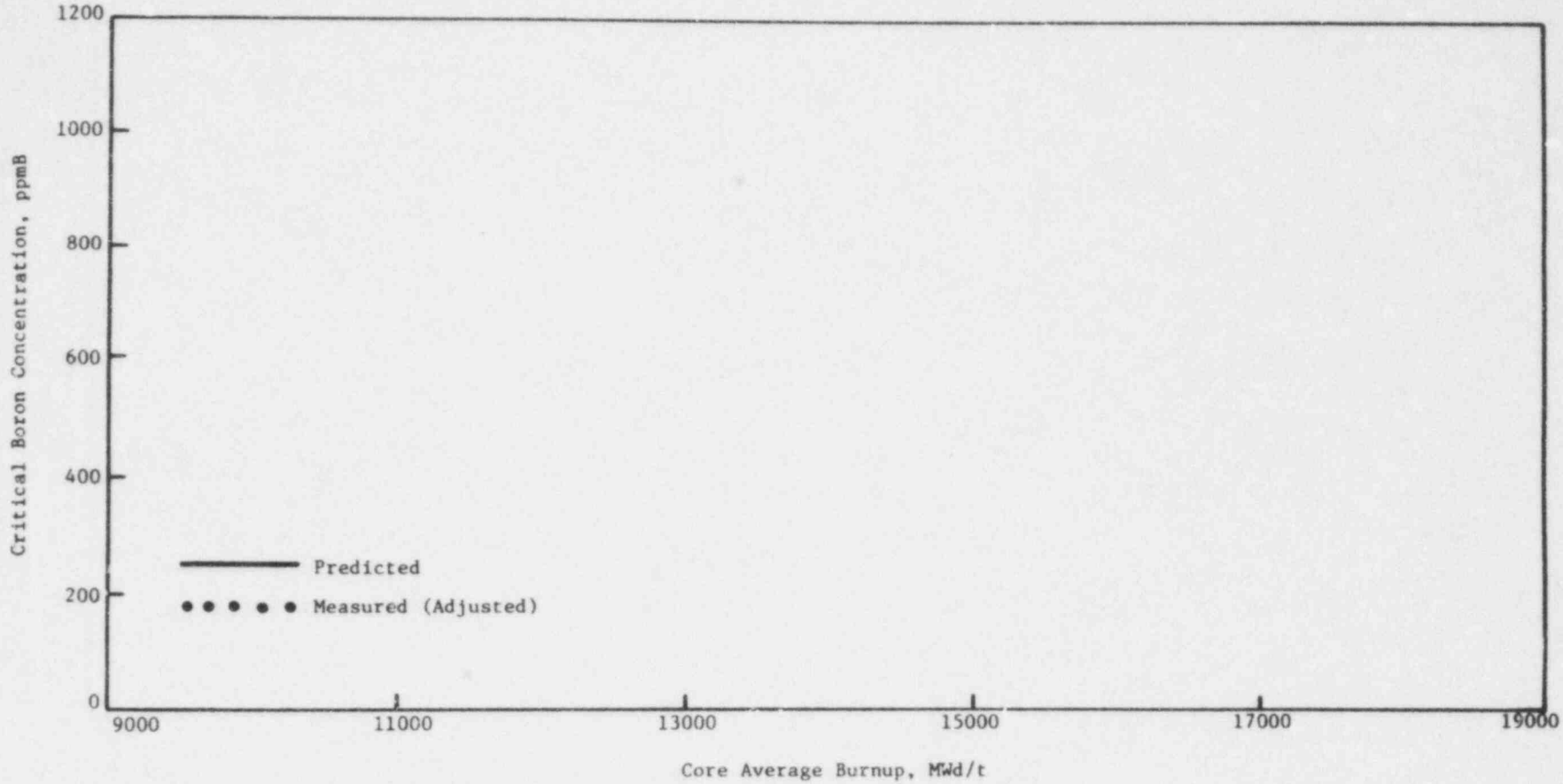


Figure 4-7. Critical Boron Concentrations — TMI 1, Cycle 2



REFERENCES

1 H. A. Hassan, et al., Power Peaking Nuclear Reliability Factors, BAW-10119P, Babcock & Wilcox, Lynchburg, Virginia, June 1977.

2 W. G. Pettus and N. L. Snidow, "A Direct Integral Formulation of Point Reactor Kinetics," J. Nuc. Energy, 26 (1972), p. 489.

3 W. M. Kelly, Reactivity Determination of Control Rod Groups, M. S. Thesis, University of Virginia, August 1976.

4 P. R. Bevington, Data Reduction and Error Analysis for the Physical Sciences, McGraw-Hill (1969), pp. 134-137.

5 D. L. Hetrick, Dynamics of Nuclear Reactors, University of Chicago Press (1971).

6 R. J. Tuttle, "Delayed-Neutron Data for Reactor Physics Analysis," Nuc. Sci. & Eng., 56 (1975), p. 37 (Table VII).

7 G. R. Keppin, Physics of Reactor Kinetics, Addison-Wesley (1965), Table 4-4.

8 Reference 4, p. 170.

9 Testing of Protection System Instrumentation, BAW-10003A, Rev. 4, Babcock & Wilcox, Lynchburg, Virginia, January 1976.

10 Keithley Instruments, Inc., Model 18010 High Speed Current Amplifier, Serial 606 through 610, Sales Literature.

11 Tidewater Technological, Inc., Model 7103, 24-Channel Data Preamplifier for Reactivity Meter, Instruction Manual.

12 Analogic, High-Speed 15-Bit A/D Converter, Technical Data, AN 2715.

13 Bailey Engineering Spec. Performance Data Sheet, Proportional and Summer Action Units for 721 Analog Control Systems, PD-E92-6-2, Rev. A.

14 Equipment Spec. 08-1086000002-01, Control Drive System, Contract 620-0073, Appendix C, p. 3, Babcock & Wilcox, Lynchburg, Virginia.

15 H. A. Hassan, et al., B&W Version of PDQ07 — User's Manual, BAW-10117, Babcock & Wilcox, Lynchburg, Virginia, October 1975.

16 C. W. Mays and M. Furtney, FLAME3 — Three-Dimensional Nodal Code for Calculating Core Reactivity and Power Distributions, BAW-10124, Babcock & Wilcox, Lynchburg, Virginia, January 1976.

17 J. J. Romano, Core Calculational Techniques and Procedures, BAW-10118, Babcock & Wilcox, Lynchburg, Virginia, September 1977.

18 M. R. Gudorf, G. E. Hanson, and J. R. Lojek, Assembly Calculations and Fitted Nuclear Data, BAW-10116, Babcock & Wilcox, December 1976.

19 C. W. Mays, Verification of the Three-Dimensional FLAME Code, BAW-10125, Babcock & Wilcox, Lynchburg, Virginia, August 1976.