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November 9, 1979

Mr. Franz P. Schauer, Chief
Structural Engineering Branch
Division of Systems Safety
U. S. Nuclear Regulatory Commission
Washington, D. C. 20555

Subject: NUREG CR-0793
"Buckling Criteria and Application of Criteria To Design of Steel Containment Shell", May 1979

Dear Mr. Schauer:

Chicago Bridge & Iron Company (CBI) has reviewed the subject document with a great deal of interest since we have a long history of involvement with design, construction, and experimental testing of shell structures subjected to compressive loads. We would like to offer a number of comments on NUREG CR-0793 which reflect our experience in design of containment vessels. Our detailed comments are attached as Enclosure No. 1. Enclosure No. 2 contains an elaboration of some of our concerns regarding the NUREG recommendation on the use of two-dimensional analyses.

We note that NUREG CR-0793 lists Mr. C. D. Miller of CBI as one of the individuals contributing information used in the preparation of the report. It should be clarified that Mr. Miller's contribution was limited to a verbal presentation to the consultants during the late stages of their study. That presentation resulted only in the consultants' referencing of some of Mr. Miller's papers in their report. Mr. Miller does not concur with some of the contents and recommendations of the consultants' report.

CBI has cooperated with a Task Force of the ASME Working Group on Containments on preparation of Code rules for buckling design of containment shells. The Task Force's report has been approved by the Working Group on Containment and forwarded to the Subgroup on Design of the ASME's Section III Code. A commentary providing the basis and the justification for the rules contained in the Task Force's report has been prepared and submitted to the Subgroup on Design. That commentary is still in a preliminary

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stage and, when completed, will provide detailed justification for the proposed Code rules. The NRC representatives on these groups, S. B. Kim and K. R. Wichman, have copies of these documents. We fully concur with the recommendations of the ASME Task Force report and suggest it be considered as an alternative to the recommendations of NUREG CR-0793.

We are hopeful that the enclosures will be helpful in NRC's consideration of proposed rules for buckling evaluation of containment vessels. We would be pleased to discuss this subject with you to provide clarification of our comments as you deem necessary.

Very truly yours,

W. R. Mikesell
Assistant Chief Engineer

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cc:
R.J. Bosnak-U.S. NRC
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Enclosure No. 1

CHICAGO BRIDGE & IRON COMPANY's
COMMENTS ON NUREG CR-0793
"BUCKLING CRITERIA AND APPLICATION
OF CRITERIA TO DESIGN OF STEEL
CONTAINMENT SHELLS", May 1979

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November, 1979

INTRODUCTION

NUREG CR-0793 contains a valuable survey of the design and analysis methods for buckling evaluation of containment shells and provides some very helpful references. However, we feel that certain areas of concern have not been adequately addressed and in some other areas we do not agree with the conclusions and recommendations of the report. In this enclosure, we have listed a number of comments which we feel would be helpful in evaluating the NUREG.

Our basic critism of the NUREG is that it proposes the use of complex two-dimensional finite element models for the stress analysis and buckling analysis of containment vessels (Section 4.6). The rationale for this recommendation is given in 3.2 by the use of arguments that we do not support. We feel that both of these analyses (stress and buckling) can be accomplished with simpler and more reliable approaches for the vast majority of containment vessel geometries and loadings. When the multiple load cases used in the design of containment vessels and the time and space varying nature of the dynamic responses (load cases often involve more than one dynamic component) are considered, the complexities of the two-dimensional analysis are magnified and its reliability further diminshed. The alternative of axisymmetric analyses permits the evaluation of all representative locations on the vessel and all specified load cases in a straight forward and tractable manner. In a complex two-dimensional analysis, the potential errors in modeling, in making the complex calculations, or in overlooking a governing load combination may produce the opposite result, a less accurate and less reliable analysis. (See Enclosure No. 2 for further treatment of this subject.) Other specific comments on 3.2 are mentioned later in this commentary.

The issue of acceptable and reliable knockdown factors is also critical. We are in basic agreement with the general approach proposed by the authors of the NUREG. We agree that the complete body of relevant test data should be used to determine reliable and conservative design values; and that for those cases where adequate data is not available, additional testing should be undertaken. The general accuracy of containment design and analysis procedures for buckling is governed directly by the precision with which the knockdown factors are defined. The use of the lower bound of the available data should be conservative. The large amount of scatter shown in buckling test data, which is used to arrive at knockdown factors, further reinforces our belief that the complex modeling and analyses procedures proposed by the NUREG would not significantly add to the accuracy and utility of the final results.

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The following are our specific comments on the contents of the NUREG (the paragraph numbers referenced are those of the NUREG):

1. In Paragraph 3.2, it is stated that "in the case of a nonlinear one-dimensional code, the load is axisymmetric only". We would like to point out that there are a number of nonlinear one-dimensional codes with non-axisymmetric loading capabilities.

2. We do not agree with the statement in Paragraph 3.2 implying that it may be more convenient to use a two-dimensional shell analysis to avoid using Fourier harmonics for describing non-axisymmetric loads in an axisymmetric shell analysis. We have extensive experience with both methods and have found the converse to be true.

3. We agree with the statement in Paragraph 3.2 that for large enough holes, the stress state in the entire shell will be affected. However, we believe that, typically, reinforced openings in containment vessels are not large enough to affect the overall state of stress to the extent that a two-dimensional analysis will be required. Paragraph 4.5.1.4, in the discussion of reinforcing openings per the ASME Code requirements, supports our contention.

4. We believe that the sample stress analysis calculations of Paragraph 3.3 are rather misleading. The mathematical model used is not nearly fine enough to provide an accurate estimate of the response of the vessel. With such coarse mesh, local discontinuity stresses will not be obtained. A mesh adequate for providing accurate stress results would have to be significantly finer than that of the report. The complexities of generating such a mesh and the costs of running such analysis have been grossly underestimated in Paragraph 3.3 (see Enclosure #2).

5. In seismic analysis of Paragraph 3.3.4, the use of the first 15 natural modes for determining the response of the containment vessel to dynamic loads will not be sufficient to calculate the local response near penetrations and attached masses. A separate analysis, similar to that commonly used in conjunction with a one-dimensional analysis of the vessel, is required.

6. We strongly agree with the proposals of Paragraph 4.4 on the use of the available body of test data to define knockdown factors.

7. We agree with the statement under Paragraph 4.5.1.1 that "the use of the critical uniform stress as a measure of the critical maximum axial stress is conservative".

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8. We agree with Paragraph 4.5.1.2 in that the critical equivalent uniform pressure is not the maximum pressure but the length average of the pressure distribution. Evidence of such behavior has been provided by a sample problem in the commentary on the proposed ASME rules.

9. We agree with the statement in Paragraph 4.5.1.5 that the reduction in a cylinder's load carrying capacity "can be disregarded safely if a dynamic stress analysis is used to determine the maximum axial stress, which is then applied as a static uniform stress in the structure". However, we believe that such application of maximum stresses, obtained from a dynamic analysis, as quasi-static stresses is always conservative for any shell structure under any kind of loading. We, therefore, do not see any need for the tentative recommendation of Paragraph 4.6, which requires that the dynamic axial stress be always greater than 140% of the axial stress obtained with a static load application.

10. Paragraph 4.6(a) implies that all penetrations should be included with a two-dimensional model. Penetrations on a typical containment vessel are numerous and mostly small. Accurate modeling of all penetrations, is impractical and unwarranted, regardless of whether one-dimensional or two-dimensional modeling is used.

11. Paragraph 4.6(c) requires that a linear bifurcation analysis be performed for the buckling evaluation. We believe that the theoretical critical stresses and the interaction relationships proposed in the proposed ASME rules would be a convenient and acceptable alternative to a computer analysis. The proposed interaction relationships are conservative estimates of theoretical relationships, which have been confirmed by test.

12. Paragraph 4.6 makes reference to NASA SP-8007 for values of capacity reduction factors for unstiffened cylinders. We believe that the values recommended by Paragraph 1511 of the proposed ASME rules are better estimates of these factors. The justification for values of those rules and a comparison of the proposed values with test results are included in the commentary document submitted with the proposed rules.

13. The statement at top of Page 4-40 implies that no specific recommendations for reduction factors of stiffened shells are available and the conservative recommendation is made that the reduction factors for stiffened shells be based on unstiffened shells having buckling load capacity the same as that of the stiffened cylinder. A great deal of work has been done in the area of developing capacity reduction factors for stiffened shells. The proposed ASME rules contain suggested values for these factors. The basis and justification for those factors are provided in the commentary document.

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We also disagree with the statement that for local buckling between closely spaced longitudinal stiffeners, the reduction factor may be taken as 1.0. As a minimum, "closely spaced" should be defined.

14. We agree with the Paragraph 4.6 recommendation that a safety factor of 2.0, in combination with proper capacity reduction factors, is sufficient to achieve a conservative design. However, to use a 0.1 factor on theoretical, to arrive at design values, for stiffened cylinders under axial compression could result in gross overdesigns. While a knockdown factor of 5.0 is realistic for long and thin unstiffened cylinders, the value of this factor for stiffened cylinders could be as low as 1.6 (see Figure 1511-2 of proposed ASME rules). For short (stiffened) or thick cylinders, the critical axial stress approaches the yield strength of the material. Obviously a factor of 0.1 applied to the failure stress, to account for capacity reduction and safety factor, is not realistic in such cases.

15. Under Section 5, it appears that references to (3a) and (3b) in subparagraphs (a) and (b) have been mistakenly interchanged. As indicated by the above comments, we don't agree with some of the conclusions of Section 5. However, we would like to strongly endorse the call for a rational method of combining various loadings, based on the use of probability statistics and risk analysis methods, to avoid the overconservatism of straight addition of worst possible conditions.

16. We would like to point out that NUREG CR-0793 does not address the question of inelastic buckling. The stiffener spacing on most of the recently designed containment vessels is such that buckling failure would occur at a stress above the proportional limit of the fabricated material. For such cases, a plasticity reduction factor will have to be applied. Furthermore, the failure behavior in the inelastic range will be different from that predicted by a linear bifurcation analysis.

17. The NUREG does not provide adequate guidelines for selection of knockdown factors, for either panel buckling or overall instability. Adequate rules for sizing of stiffeners are not provided either. Proposals in these areas are contained in the proposed ASME rules.

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Enclosure #2

An Evaluation of 1-D and 2-D Analysis Methods For The Solution of Thin Shell Containment Vessel Problems

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INTRODUCTION

In engineering work, the simplest method which can adequately solve a given problem should be used. Simple methods, when justified, are the most effective engineering tools. Simple analysis methods reduce the amount of engineering judgement that must be employed to assure safe, reliable designs. A 1-D analysis method can accurately predict the response of thin shell containment vessels in almost all cases of current practical interest. Guidelines can be established to isolate those few cases where 2-D analysis methods are required. This enclosure substantiates the technical acceptability and desirability of 1-D methods. Furthermore, it shows that using 2-D methods as a routine approach cannot be cost justified.

The remainder of this enclosure is organized in four major sections. The first section is a brief summary of important findings and conclusions. The second and third sections are technical discussions of 1-D and 2-D analysis methods, respectively. Important benefits and problems are presented. The final section is a cost study and cost-benefit evaluation. In developing the expected cost of a 2-D analysis, references are cited and examples are presented which form the basis of some good modeling rules. These rules are then used to develop a mesh which will give an accurate solution to a real containment problem. The cost of an analysis using this mesh is then estimated and evaluated.

SUMMARY

The primary conclusions supported by this enclosure are as follows.

1. 1-D analysis methods are accurate for most containment vessel problems of interest.

2. Since 1-D methods are accurate in most cases, the use of 2-D methods will not improve the solution. In actual practice, potential errors and misinterpretations due to the increased complexity of 2-D methods are likely to yield less accurate solutions.

3. 2-D methods are more expensive than 1-D methods by a factor of at least 100 for typical containment problems.

4. NUREG CR-0793 quotes a cost of $150 to perform a 2-D eigenvalue analysis of a "somewhat coarsely modeled" containment vessel. CBI feels that the model is not somewhat inadequate, but rather is grossly inadequate. As a result, the cost estimate is misleading. The cost of an adequate analysis is difficult to predict but will probably be in the range of $30,000 to $50,000. One commonly used and generally accurate rule of thumb is that the cost of a computer solution is proportional to the number of degrees of freedom squared. Based on this rule and an adequate mesh, the cost of an equivalent run could be as much as $350,000.

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TECHNICAL DISCUSSION OF 1-D METHODS

CBI believes that a one-dimensional analysis is an effective engineering tool for the analysis of thin shell containment vessels under static and dynamic loads. A one-dimensional analysis provides a reliable and accurate solution to the overall problem of determining the deformations and state of stress in an axisymmetric shell subjected to any arbitrary loading.

It is realized that a 1-D analysis will not predict the detailed state of stress in the containment vessel in the immediate vicinity of the larger penetrations (such as an equipment hatch or a personnel lock). However, these penetrations are not large enough to significantly affect the overall response of the vessel. Certainly any subtle changes in response would be much less significant than other uncertainties, such as the proper knockdown factors to be used in the buckling evaluation. The adequacy of local areas can be assured by simple design rules (e.g., area replacement) and, when required, verified by a local 2-D analysis.

In order to substantiate the advantages of 1-D methods, Reference 1 is cited. This reference compares the computational efficiency and reliability of one-dimensional analysis methods to that of the two-dimensional finite element method.

Reference 1 presents a comparative analysis of a model motor casing for static loads. The casing is in the form of a cylindrical shell which is 304 cm. in diameter and 2121 cm. long with hemispherical heads at both ends. The loading is in the form of a pinching load as shown in Figure 1. The problem was run using the STARS, BOSOR4, NASTRAN, and MARC computer codes. The pertinent computer run times for the various programs and idealizations are shown in Table 1.

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A review of Table 1 shows that the agreement for the maximum deflection was within 1% for all programs except the MARC program. It is possible that the MARC 2-D program was used improperly. This is a serious danger with 2-D programs. Nevertheless, the excellent agreement between the two 1-D programs (STARS-2 and BOSOR4) and the 2-D NASTRAN program proves that a 1-D solution is accurate for this kind of problem. (Later discussion will show that a 1-D solution may be more accurate than 2-D for other problems.)

The particular problem in Reference 1 included gross geometry discontinuities (head to shell junctions) and a nonaxisymmetric loading. It can also be shown that 1-D methods will accurately solve thin shell problems which include ring stiffeners, vertical stiffeners, any arbitrary time varying nonaxisymmetric loading, and fluid-structure interaction. It is also possible to perform a coupled 1-D analysis which actually calculates the effect, if any, of a local mass on the overall response of the vessel.

Reference 1 also provides some information about costs. The STARS-2 one dimensional shell of revolution program (based on a numerical integration technique) solved the problem in just 2.5 minutes. The one-dimensional BOSOR4 program (based on a finite difference formulation) solved the problem in 3 minutes. However, it took NASTRAN 70 minutes to solve the problem. The NASTRAN 2-D finite element code used a fine mesh of one-quarter of the structure. Note that, in general, a quarter structure model could not be used for containment vessels if one were trying to determine the effect of nonaxisymmetric attachments or loading conditions. Based on these results, it is evident that when solving specialty structures, such as shells of revolution, the 1-D programs offer distinct advantages for static analysis.

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The same general comparison was made between STARS-2 and NASTRAN for an eigenvalue problem. The problem analyzed was the natural vibration analysis of a free cylinder; R = 10", L = 150" and t = .02". The STARS-2 program extracted 10 mode shapes in 6.7 minutes. However, the NASTRAN program took 21 minutes to extract one mode shape. This was only achieved by using the results of STARS to bracket the eigenvalue search ranges. Initially, when the run was made on NASTRAN without prior knowledge of the eigenvalues, the 30 minute cutoff time was exceeded without calculating even one eigenvalue.

Reference 1 also presents a practical comparison for a cylinder subjected to a blast loading using the same three methods. The analyses determine the linear transient response of a cylinder subjected to the harmonic dynamic loading shown in Figure 2. The results, also shown in Figure 2, are identical for the three techniques. However, the idealizations used serve to accent the significant differences. These idealizations are contrasted in Table 2. As can be seen, the numerical integration idealization is satisfactory using an order of magnitude fewer degrees of freedom. An extra benefit of the accuracy of the numerical integration method is an increase in the time integration step allowed before artifical damping becomes evident.

The standard approach used by CBI to analyze containment vessels for their various loadings is to use a CBI proprietary one-dimensional shell of revolution program which is based on linear classical shell theory. The method of solution is one used by Kalnins (Reference 2). A special version of CBI's program is used to extract eigenvalues (calculation of mode shapes and frequencies of the shell) and also to evaluate the dynamic response using the direct integration method. The static and dynamic versions of CBI's shell of revolution programs have been verified for their intended applications.

At this point, CBI would like to make a few comment about the use of Fourier series to model nonaxisymmetric loads. In order to limit the computational effort needed to obtain accurate stresses and displacements, a technique was developed for using a greatly abbreviated Fourier series to represent the circumferential variation of the loading. This technique is described in Reference 3. The truncation error in the variable (deformations, stress and moment resultants) at any given point on the shell can be evaluated from the shell solution using the last two harmonics of the Fourier series. This error can be reduced to any level desired. The important point is that the error is known. An engineer can decide that a 2% error in the load will not affect the validity of his results. On the other hand, for 2-D methods the analysis appears to be accurate. However, its accuracy is, in general, unknown. Convergence studies using several mesh sizes would have to be conducted to evaluate the accuracy of the numerical results for a particular mesh.

To date, the approach adopted by CBI with regard to containment vessel design has been to evaluate the overall behavior of the vessel using a shell of revolution type analysis and then subsequently to perform a detailed design in the regions where there are large openings (such as the locks and equipment hatch). All openings of any size are reinforced in accordance with ASME Code Section III rules. Section III of the Code specifies that the shell material cutout by the opening be replaced within a specified reinforcing zone. Application of these rules to openings in the size range usually encountered assures that the local area is just as safe from a buckling standpoint as the unpenetrated shell would have been.

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Technical Discussion of 2-D Methods

CBI recognizes the need for, and value of, 2-D analysis methods for certain structural problems. In fact, we have actively used 2-D methods for over 10 years in situations where simpler design rules and analysis methods were not adequate. Over that time, we have gained an appreciation for the power of general purpose finite element programs. We have also learned that, due to the increased complexity of these programs, there are very real dangers associated with their use. The complexity of a 2-D analysis will invariably lead to more errors and misinterpretations of results than a 1-D analysis. It is CBI's firm belief that, if 2-D methods were required on a routine basis (and especially in situations where their use should be optional), these errors and misinterpretations would result in a relative net loss of confidence and safety.

To understand CBI's positon, one must understand three important points. First, it should be recognized that it is CBI's policy to design safe structures, and that we would not knowingly use design or analysis methods which might yield inadequate structures. CBI designs its structures to meet all specified customer, Code and NRC requirements. Moreover, the company has additional internal requirements to further assure the safety of its designs.

The second point is that a 2-D analysis is far more complex than a 1-D analysis. A mesh must be generated and associated data prepared. This data would involve as many as 100,000 numbers for a simple static analysis of a properly modeled containment. The output generated would be even more voluminous. Furthermore, the output would not generally be in a usable form so that additional calculations and manipulations would have to be performed. To properly prepare and interpret all these numbers is a difficult assignment. To do so on a routine basis would increase the likelihood of errors, confusion and misinterpretations of results. It does not make sense to risk the occurence of these problems when a simpler method is available which yields an accurate solution.

The third important point is that the containment vessel design procedure is, in reality, an extremely complex iterative process. This fact further complicates the analysis required. The many trials normally needed to reach an acceptable design are further increased by common changes in specified loadings and other input information. Many different loads and load combinations must be studied. Design details must be adjusted and reanalyzed. The complete process is long, involved and difficult - even when 1-D methods are used. To prepare good designs, engineers should have a "feel" for the problem being solved and the behavior of the structure. Reliance on 2-D methods would make it more likely that engineers would become lost in the numbers and less able to make good engineering decisions.

COST STUDY & COST BENEFIT EVALUATION

In this section, the computer costs associated with performing a 1-D and 2-D dynamic analysis for a typical containment vessel will be evaluated. The higher engineering manhour costs associated with 2-D methods will not be included. The problem used for comparison is the containment vessel eigenvalue problem analyzed in NUREG CR-0793. The containment vessel is assumed to be a cylindrical shell 120 ft. in diameter and 150 ft. high with a 2:1 ellipsoidal top head. Six circumferential ring stiffeners are attached to the cylindrical shell. The spacing of these stiffeners, starting at the point of embedment, is 200", 200", 400", 200", 400", and 200".

The first step in the evaluation will be to confirm that the computer cost rate used in the NUREG examples is reasonable. Next the cost of a 1-D analysis will be presented. The third step will be to develop the 2-D analysis cost. CBI will show what kind of modeling detail is required for an accurate 2-D analysis of a real containment vessel. It will also show that the mesh proposed in the NUREG is grossly inadequate. Finally the cost of an adequate 2-D analysis will be estimated and compared with the 1-D costs.

COMPUTER COST RATE

CBI has recreated the coarse 2-D mesh used in the NUREG examples and has confirmed that the computer rates quoted are reasonable. The $30 static run used 0.93 minutes of CPU on CBI's computer, and a computer rate of about $30 per minute is reasonable. Note that the basis of this rate is the NUREG example. An eigenvalue run to extract 15 modes took 5.67 CPU minutes. Based on $30 per minute (and assuming cost proportional to CPU), the run would cost about $170. The NUREG quoted $150 for the analysis, including a modal superposition time history analysis.

The cost of a modal superposition time history solution varies widely depending on how many points in time and space and how many variables per point are evaluated. The cost of this part of the solution should be about the same for both 1-D and 2-D methods. For the amount of data reported the NUREG example, CBI would estimate the cost of evaluation at about $40. Thus, the $150 quote compares reasonably well with an estimate based on $170 + $40 = $210. It is possible that the NUREG did not include the cost of the time history solution.

ESTIMATE FOR 1-D ANALYSIS

Based on CBI's experience with similar problems, the computer cost for a complete 1-D modal time history dynamic analysis is estimated to be about $300. For the eigenvalue solution alone, the cost would be about $115. This is for a model with 150 segments (more than enough for a real containment) and 15 modes (the same number used in the NUREG example).

As discussed above, the time history solution would add about $40 for the number of points evaluated in the NUREG example. Assuming that more points in space and more variables are evaluated for a real containment analysis, the cost could be a few hundred dollars. For the purpose of comparing the 1-D and 2-D costs, we will use a figure of $185. Thus the total cost for the 1-D analysis would be $115 + $185 = $300.

ESTIMATE FOR 2-D ANALYSIS

The computer cost of a 2-D eigenvalue analysis is directly related to the size of the mesh. Hence, in a 2-D analysis it is important to select "a priori" the optimum mesh. An optimum mesh is defined as one which essentially provides converged results (to within some engineering accuracy) for any pertinent response variable of interest. The only way to prove convergence is to redo the analysis with a finer mesh and show that the results do not change. Since this must be done for each load case, this approach is generally not feasible and can be avoided by developing and following good modeling rules. Such rules are not easy to define since they depend on the structure, closeness to discontinuities and the loading. For this reason, no published and generally accepted rules are available for general use. However, for the purpose of this evaluation, some realistic rules can be developed for the particular problem of interest.

In the following, three examples are presented which give some guidance as to what kind of modeling is required for accurate solutions of different types of thin shell problems. These examples only tackle the initial problem of accurately calculating the static stress concentration factor at the shell to stiffener junction and around openings. The static response must be accurately predicted before one can expect the dynamic response to be valid. The examples also provide some useful information about the relationship between the modeling detail and the computer costs in a 2-D analysis.

EXAMPLE 1: PLANE STRESS ANALYSIS OF CIRCULAR PLATE WITH HOLE

The problem considered was that of a circular plate with a central circular hole subjected to an in-plane uniform tension load of 1.0 ksi. It shows what kind of modeling detail is required in the vicinity of an opening. The dimensions of the plate are shown in Figure 3. The geometric parameters are R = 10", a = 1.00", t = 1.00".

Symmetry of the loading, geometr. and material properties made the analysis of only one-quarter of the plate sufficient. Adequacy of the mesh size is determined by comparing two separate models which vary in grid size. The finite element idealizations of the quarter panel using a fine and coarse mesh are shown in Figures 4 and 5. For the fine mesh the radial length of the elements in the region extending around the hole was made equal to .25" (one-quarter the radius of the hole) for a radial distance of 1". In the outer regions, progressively larger elements were used (½" and 1" sizes). For the fine mesh there are 84 elements with 105 nodes, resulting in 180 degrees of freedom. For the coarse mesh the length of the elements was made twice as long as the corresponding elements in the fine mesh, resulting in 27 elements with 35 nodes and 54 degrees of freedom.

The problem was analyzed using the SAP4 program and plane stress membrane (SAP Type 3) elements. The tensile and compressive stress concentration factors predicted at the edge of the hole using the fine and coarse meshes are given in Table 3 together with the results of the solution using CBI's 1-D program. These results are compared to the elasticity solution of the same problem given in Theory of Elasticity by Timoshenko and Goodier.

The results from CBI's 1-D program are correct to three signifcant digits when compared to the exact elasticity solution. The results from CBI's program can also be considered "exact" because the program uses a numerical integration technique to solve the plate differential equations and uses a very small convergence criteria.

The SAP4 solution for the fine mesh shows an error of 3% for the tensile SCF and an error of 15% for the compressive SCF. The corresponding errors for the coarse mesh are 14% and 40% respectively. The agreement between the exact solution and the fine mesh finite element solution is relatively close; however, the results of the coarse mesh solution show considerable inaccuracy and indicate that the solution has not converged.

The questionable adequacy of the fine mesh to solve this problem shows that in order to obtain good engineering accuracy for stresses in the immediate vicinity of the hole, the size of the elements in the neighborhood of the cutout should not exceed one-fourth the radius of the hole, and, preferably, should be smaller.

EXAMPLE 2: CYLINDRICAL SHELL WITH A CIRCULAR CUTOUT SUBJECTED TO AXIAL TENSION LOAD

This example has been taken from the paper "The Analysis of Thin Shells with Transverse Shear Strains by the Finite Element Method", by S. W. Key and Z. Beisinger. It was published in the Proceedings of the Second Conference on Matrix Methods in Structural Mechanics. This problem, illustrated in Figure 6, will provide additional guidance into modeling requirements near an opening.

Taking advantage of the two planes of symmetry, only one-quarter of the shell needs to be modeled. Figures 7 & 8 illustrate the fine mesh which has a 27x27 grid with 533 elements and 589 nodes. The size of the smallest elements in the immediate vicinity of the cutout for the fine mesh is of the order of r/10 (one-tenth the cutout radius) or about $1/10\sqrt{Rt}$. The coarse mesh, shown in Figures 9 & 10, uses a 14x14 finite element grid with 148 elements and 176 nodal points. For the coarse mesh the size of the smallest elements is of the order of r/3 (one-third the hole radius) or about $1/3\sqrt{Rt}$. Typical results are given in Table 4 along with the analytical solution obtained by Lekkerkerker. The stresses are compared at points A and B which correspond to the side and top edges of the hole respectively. The finite element solutions for the membrane and surface stresses are compared with the analytical solution.

For the meridional surface stresses at point A, the results for the fine and coarse meshes are within 1% and 8% respectively of the analytical solution. For the circumferential surface stresses at point B, the errors for the fine and coarse meshes are of the order of 10% and 20% respectively.

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The authors have given the execution time for the two meshes. For the coarse mesh the execution time was 0.3 hours, but for the fine mesh the time increased by a factor of nearly 12 to 3.5 hours. Note that the number of nodes (and degrees of freedom) in the fine mesh increased by 589/176 = 3.35 over the coarse mesh. Assuming that the run time is proportional to $(DOF)^2$, one would expect an increase in the run time from 0.3 hours to $0.3\ (3.35)^2 = 3.36$ hours. The fine mesh run time of 3.5 hours confirms that this rule is reasonable.

The conclusions to be drawn from this example problem are that:

1. For a cylindrical shell with a circular cutout subjected to a uniform loading, the coupled membrane - bending behavior near the hole appears to be characterised reasonably well if the size of the elements used in the immediate vicinity of the hole is of the order of $1/3\ \sqrt{Rt}$.

2. The CPU time is about proportional to $(DOF)^2$.

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EXAMPLE 3: STIFFENED CYLINDRICAL SHELL WITH PINCHING LOAD

This example was selected because it gives some indication of the mesh size necessary to give accurate results for a cylindrical shell of the same size as a containment vessel. The problem includes important real phenomenon like a stiffener and a nonaxisymmetric loading.

A pinching load is applied to the shell because it will cause a rapid variation of the displacement normal to the shell in the circumferential direction. This pinching load is analogous to the asymmetric nature of the Safety Relief Valve Loading for which some containment vessels have to be analyzed. The dimensions of the shell are R = 720", t = 1.5". The length of the cylindrical shell is taken as 100' and the boundary conditions at the ends are assumed as simple supports. The shell has been stiffened by a ring stiffener as shown in Figure 11.

A study was carried out using two mesh sizes and the SAP4 program. The fine mesh as shown in Figure 12 has 437 nodes spaced such that there are 18 elements in the circumferential direction. The size of the elements in the circumferential direction is 63". In the meridional direction, the size of the elements is made equal to about $\frac{1}{2}\sqrt{Rt}$ (16") for a distance of $2\sqrt{Rt}$ (64") from the shell-to-stiffener junction. The size of the elements in the axial direction away from the stiffener (distance greater than $2\sqrt{Rt}$) is made equal to $\sqrt{Rt}$ (32"). The maximum aspect ratio of the quadrilateral elements is limited to about (4:1). The mesh has a total of 378 quadilateral shell-plate elements (SAP Type 6) including 18 which are used to model the stiffener flange. In the web, there are an additional 18 membrane plane stress elements. This mesh idealization gives rise to 2321 unknowns with a maximum bandwith of 204.

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The coarse mesh (shown in Figure 13) has the same configuration except that the dimensions of all elements have been doubled. This mesh has 99 shell-plate elements and 9 plane stress elements. There are 130 nodes wnich give rise to 620 unknowns with a maximum bandwidth of 99.

The pinching load is imposed by means of a 20 psi pressure applied over the complete length of the shell in the meridional direction and encompassing an arc of 20° circumferentially.

Both meshes were analyzed using the SAP4 program. To provide a comparison of these two analyses, the stress results at and near the stiffener-to-shell connection are tabulated in Table 5 along with the results of a solution obtained using CBI's 1-D program. For this problem, the results from CBI's program are very nearly exact (error less than 1%) because of the method of solution (i.e: numerical integration, small convergence criteria) and the fact that the Fourier series used to describe the load was carried to the second zero point (as described in Reference 3).

Because the SAP finite element solution gives stresses at the centroid of the element, the stress comparisons are shown for point A on the fine mesh and point B for the coarse mesh. These points are slightly away from the stiffener junction, and corresponding points from the 1-D solution are used. The SAP centroidal stresses are also compared to the corresponding maximum discontinuity stresses from the 1-D analysis. The maximum stresses occur at the shell-to-stiffener junction along a longitudinal section which passes through the center of the loaded area.

For the fine mesh, the hoop stresses at point A on both the inside and outside surfaces are underpredicted by 19% The hoop membrane stress is also underpredicted by 19%. For the meridional

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stresses, SAP underpredicted the surface stress by 15%. However, the meridional membrane stress is overpredicted by 32%.

For the coarse mesh, the stresses are compared at point B which is 16" from the transverse plane of symmetry and 5° from the longitudinal plane of symmetry. In this case, the hoop stresses both on the inside and outside are underpredicted by about 45%. The membrane hoop stress is also underpredicted by 45%. For the meridional stresses, the stresses on the inside and outside surfaces are underpredicted by 18% and 39% respectively. However, the membrane stress is overpredicted by 52%.

The maximum surface discontinuity stresses predicted by the 1-D shell analysis are in the meridional direction. The fine mesh underpredicts these values by 60%. The coarse meshes underestimates the stress by 95%. The maximum value of the membrane discontinuity stress is in the hoop direction. The fine mesh underestimates this stress by 31% and the coarse mesh by 70%. This comparison shows that both meshes are inadequate to perform the analysis.

These comparisons illustrate some of the dangers of 2-D analysis. It is apparent that elements near a discontinuity in geometry or load should be smaller than $\frac{1}{2}\sqrt{Rt}$. Elements away from discontinuities may be larger. However, it is CBI's experience that real containment loadings do not result in any large areas of constant stress. In areas removed from obvious discontinuities, elements should be no larger than $1.0\sqrt{Rt}$.

The number of degrees of freedom increased by a factor of 2321/620 = 3.74 and the cost increased by a factor of 173.4/25.9 = 6.69. This implies that, for this problem, the cost is proportional to $(DOF)^{1.44}$.

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DEVELOPMENT OF AN ADEQUATE 2-D MESH AND COST ESTIMATE

The results of the previous three examples show that the maximum element length in the meridional direction should be about 1/3 $\sqrt{Rt}$ or 1/4 the hole radius in the vicinity of large openings, less than $\frac{1}{2}\sqrt{Rt}$ near stiffeners and $1\sqrt{Rt}$ in other shell regions. Elements can be longer in the circumferential direction. In no case should the maximum aspect ratio be more than 4:1. Using these rules, the size of the mesh necessary to model the containment vessel analyzed in NUREG CR-0793 can be determined in a manner which will approximate, but not overestimate, the engineering requirements.

For the normal stiffener spacing of 200", a typical 90° slice of the vessel would require (4+2+4)18 = 180 elements. This is based on 4 rows of $\frac{1}{2}\sqrt{Rt}$ elements above the first stiffener (or bottom) and also below the next stiffener with 2 rows of $1\sqrt{Rt}$ elements in between. Using a 4:1 maximum aspect ratio, 18 elements are needed per 90° segment per row. For a 400" stiffener spacing, the corresponding number of elements is (4+8+4)18 = 288. An additional 36 elements are required to model the web and flange of each stiffener.

The NUREG document recommends modeling all the major locks. This requires that all 360 degrees of the shell be modeled. It can be shown that each properly modeled lock or hatch (based on the 1/4 radius rule) would cause a net increase of about 500 elements. This number would be even larger if $1/10\sqrt{Rt}$ elements were used. Hence, for the subject vessel with six stiffeners (4 with 200" spacings and 2 at 400" spacings) and 3 locks (2 personnel locks and 1 equipment hatch), the number of elements required to model the cylindrical vessel will be 4*4*180+4*2*288+4*6*36+3*500 = 7548 elements. A slice of the resulting mesh is show in Figure 14.

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Assuming that about 600 elements would be needed to model the top head, the total number of elements required to model the complete vessel will be about 8200. Even with this many elements, the mesh does not account for any fine mesh layouts which might be required for other penetrations nor for any node points at locations where floor response spectra are needed. It also does not account for the modeling required for fluid-structure interaction problems.

The containment vessel sample problem in the NUREG document used only 170 elements. The size of each element was 200" which is about $6\sqrt{Rt}$. This mesh will not adequately describe the curved surface and will not yield an accurate solution for real containment loadings. CBI's more realistic estimate of about 8200 elements is about 48 times the number given in the NUREG document. The NUREG example is quite misleading.

Because the NUREG mesh is inadequate, the estimated computer cost of $150 is grossly underestimated. A generally accepted rule of thumb is that the computer cost of a 2-D analysis is proportional to the square of the number of degrees of freedom $(DOF)^2$. Example 2 confirmed this rule while Example 3 showed the cost was proportional to $(DOF)^{1.44}$ for that particular problem. Using the $(DOF)^2$ rule (and assuming that the ratio of the number of elements is about equal to the ratio of the number of degrees of freedom), the cost of a single eigenvalue analysis would increase by a factor of $(48)^2$ to $350,000. Based on a rule of $(DOF)^{1.44}$, the cost would increase to $40,000. It should be noted that this estimate is based on an extrapolation using the NUREG example's computer cost rate.

CBI recognizes the uncertainty in the cost formula and the fact that an extrapolation of this magnitude cannot be made with as great a confidence as one involving a smaller change in the number of degrees of freedom. Nevertheless, CBI believes a realistic estimate of the computer cost for one run would be in the range of $30,000 to $50,000. Using this figure, 2-D methods are more

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expensive than 1-D methods by a factor of at least 30,000/300 = 100. The cost of the modal superposition time history evaluation would still be about $200 and would be insignificant relative to the cost of the 2-D eigenvalue solution.

A cost of say $40,000 would cover only the cost for one dynamic analysis for seismic loads. Many containment vessels also have to be analyzed for several nonaxisymmetric time varying pressure load cases, in addition to several static load cases. Many load combinations exist. Hence, the actual cost of analyzing the vessel could approach or exceed $1,000,000. That kind of expenditure cannot be justified when there is no real benefit to be gained and the potential exists for increased confusion and errors.

ADDITIONAL COMMENTS

There are two additional problems with the 2-D approach which should be discussed at this time. Both relate to the accuracy of the solution. The first problem is that it appears that only 2 of the 15 modes found by SAP represent zero or first harmonic displacement patterns. These two harmonics are the only ones important in determining the overall vessel response to a seismic load. Therefore, the 1-D analysis could have used only 2 modes to obtain the same accuracy as the 2-D analysis. This would reduce the 1-D cost.

On the other hand, if 15 modes were required from the first two harmonics, SAP would probably have to extract well over 100 modes to find 15 in the first two harmonics. This would further increase the 2-D cost by an order of magnitude. A requirement to find 15 or more modes is not unusual. On some real containment contracts, CBI has been required to find all modes in the first two harmonics whose frequencies were less than 33 hertz. The number of such modes would be greater than 15.

The second problem with the 2-D approach is that in attempting to calculate the response near openings, a relatively inaccurate solution will be found in areas away from the openings. These areas away from openings account for about 96% of the shell surface. The critical region of the shell will likely be in a region away from the large penetrations.

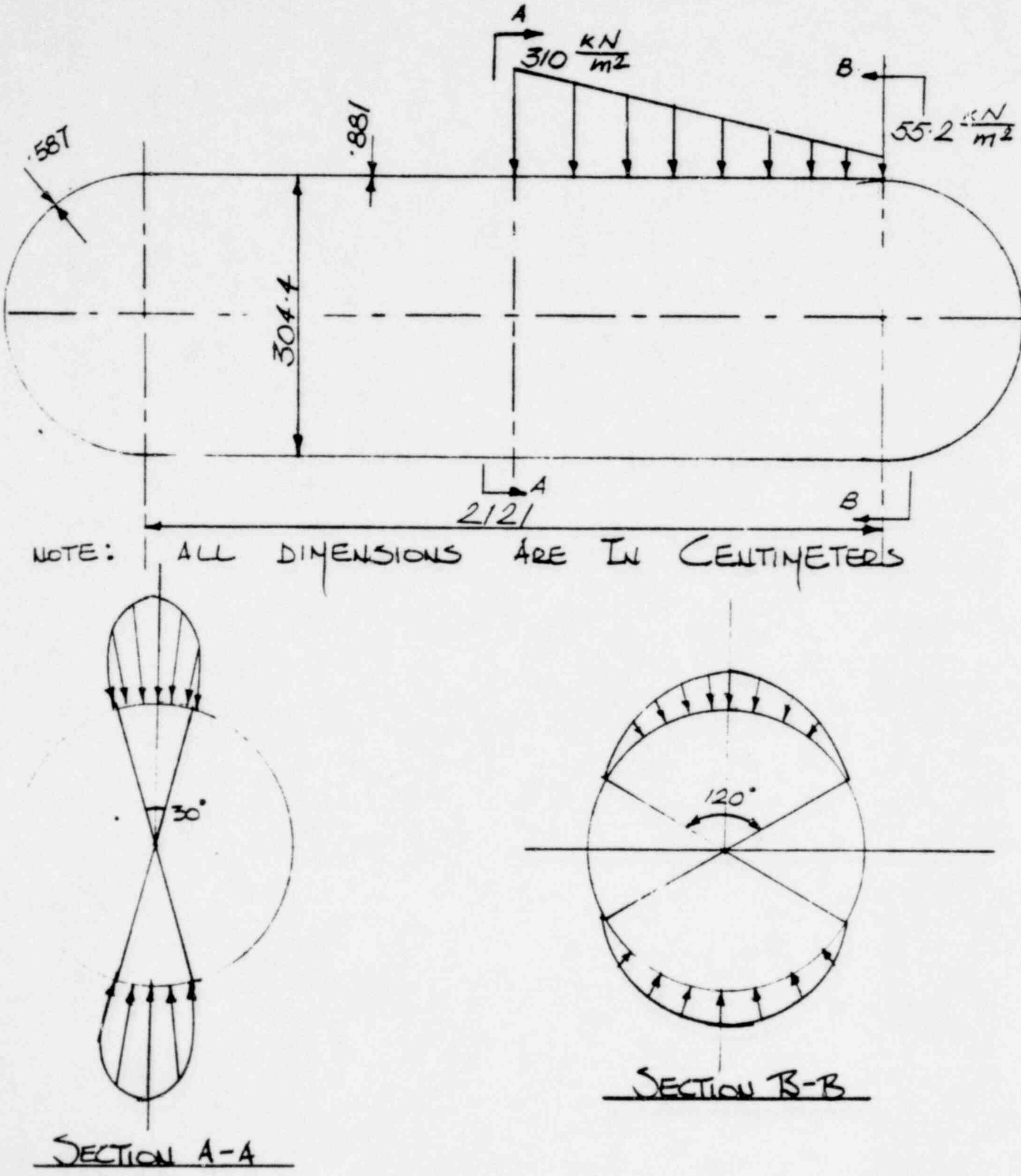
The main concern of the NUREG document is that the detailed state of stress in the vicinity of penetrations be accurately determined, on the basis that these stresses are needed to realistically evaluate the safe buckling loads. On the other hand, the document states that studies of the buckling of cylinders with reinforced holes have shown that the critical buckling stress for a cylindrical shell with a reinforced cutout is higher than the corresponding critical stress for the unpenetrated shell. All openings on a containment vessel are reinforced in accordance with ASME Code rules. These rules are intended to insure that the penetrated shell is at least as strong as the plain shell. It appears that it would serve no purpose to use a two-dimensional finite element analysis to study these local effects, especially one that modeled the entire shell.

REFERENCES

1. Svalbonas, V., and Key, J., "Static Stability, and Dynamic Analysis of Shells of Revolution by Numerical Integration - A Comparison", Nuclear Engineering and Design 27, 1974, pp. 30-45.

2. Kalnins, A., "Analysis of Shells of Revolution Subjected to Symmetric and Nonsymmetrical Loads", ASME Journal of Applied Mechanics, Vol. 31, 1964, pp 467-476.

3. Endicott, J. S., "Analysis of Piping Loads on Shells of Revolution," Transaction of the 4th International Conference on Structural Mechanics in Reactor Technology, Vol J(a), 1977.



NOTE: ALL DIMENSIONS ARE IN CENTIMETERS

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FIGURE 1
(2-24)

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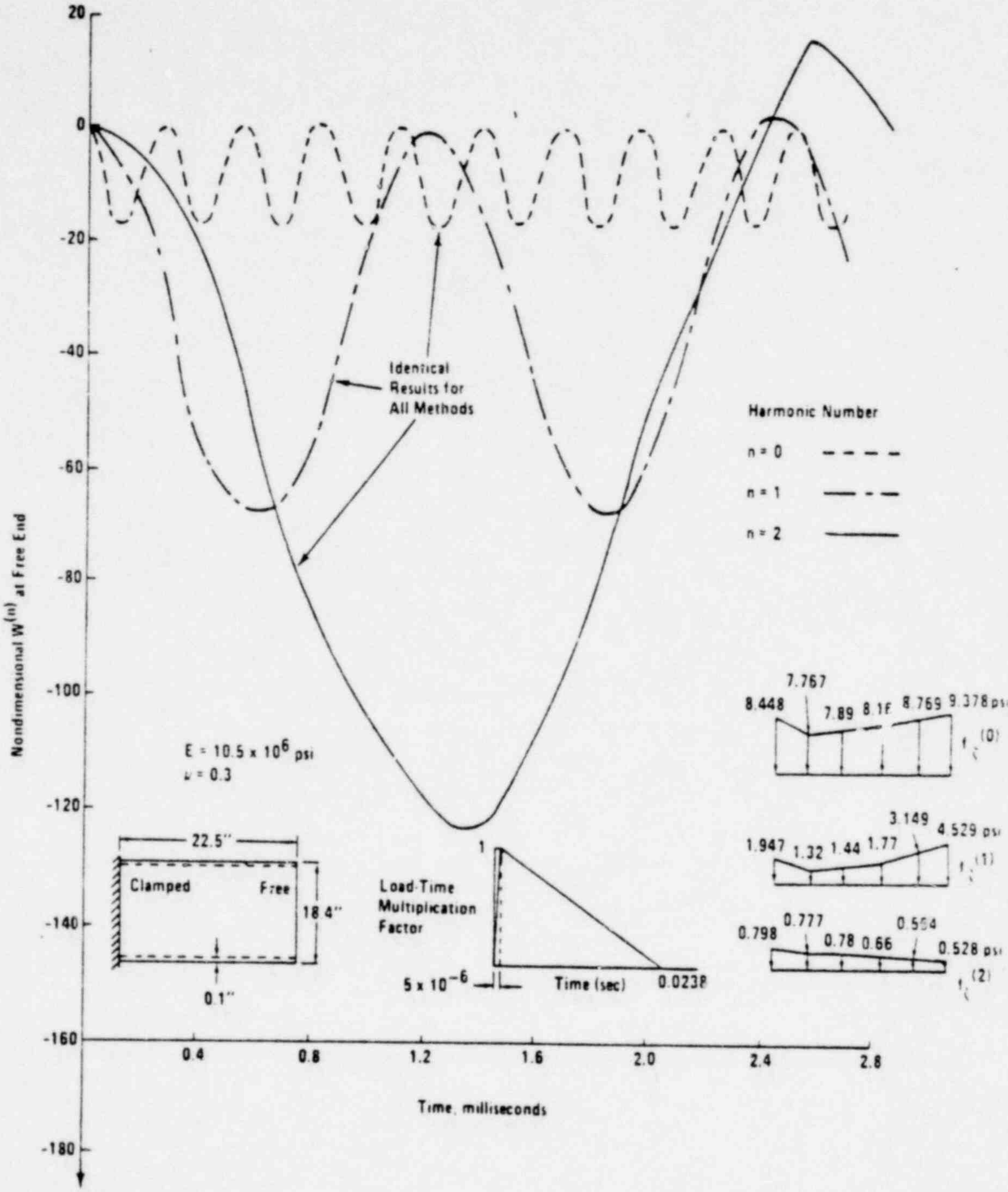

Cylinder subjected to blast load.

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FIGURE 2

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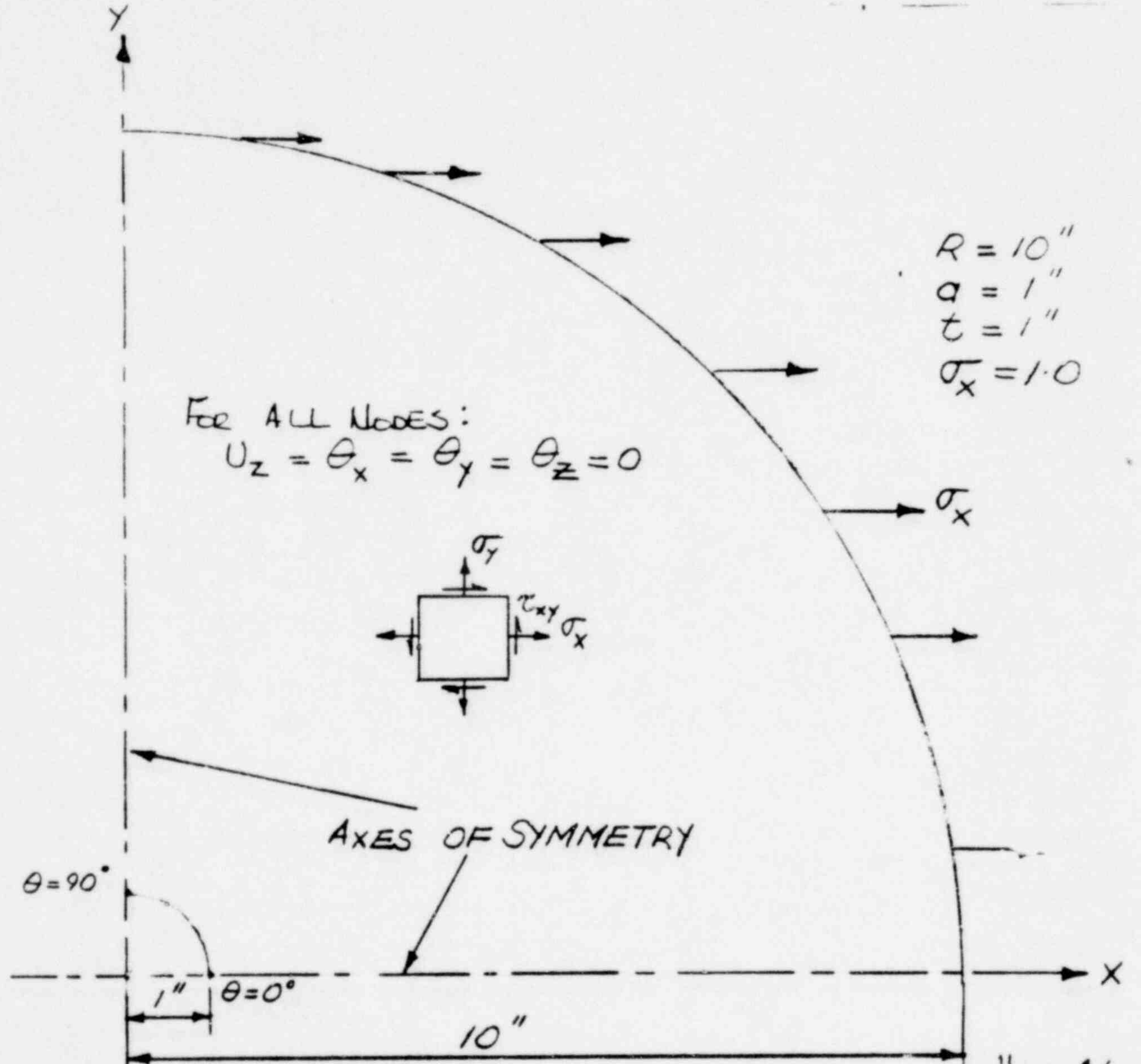
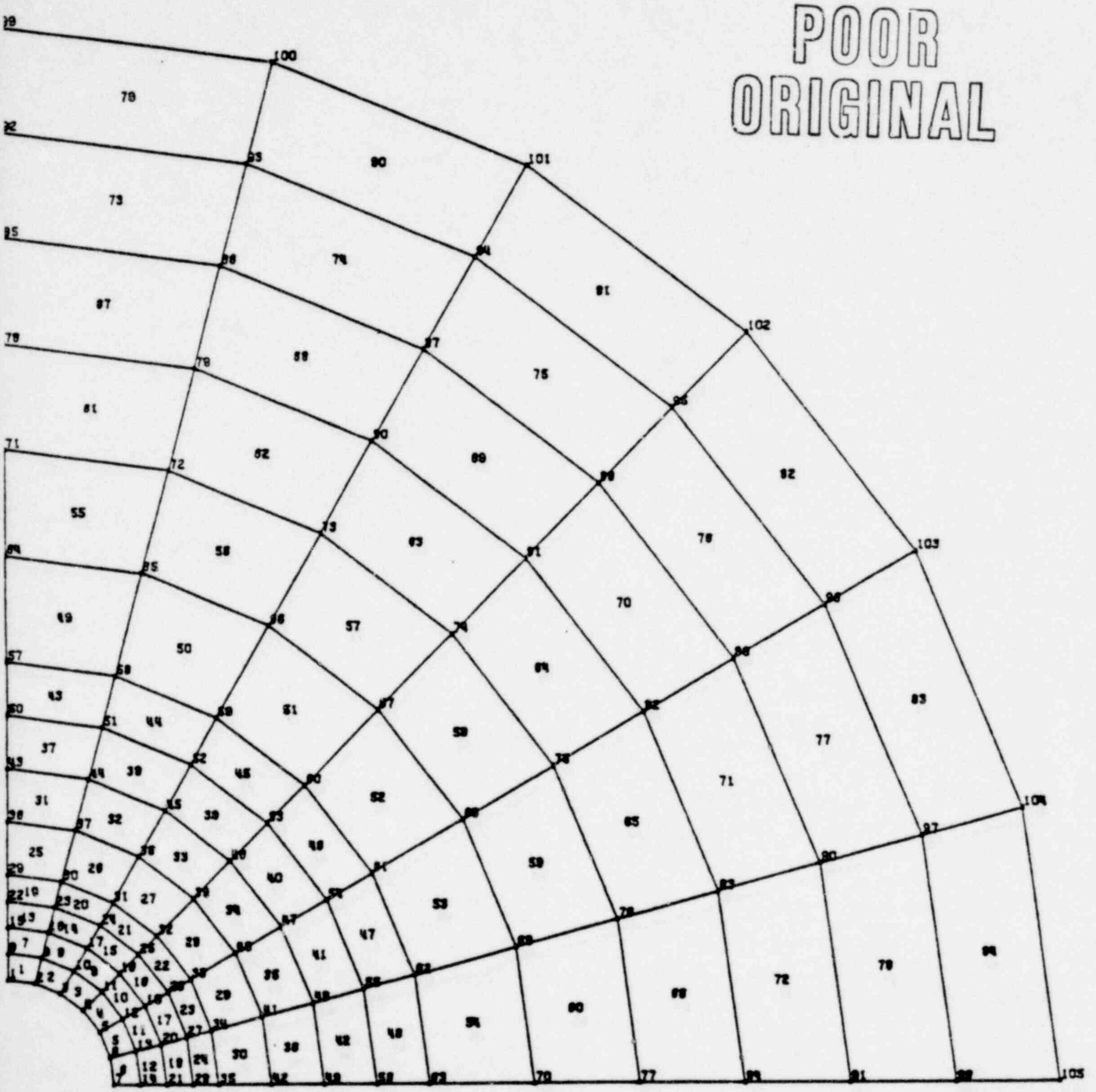


FIGURE 3

1329 351

STRESS CONCENTRATION AT A HOLE IN A UNIFORM STRESS FIELD

PLOT 1 VIEW POINT 0.0 0.0 1.0 SCALE 1.250



FINE MESH

FIGURE 4

(2-27)

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STRESS CONCENTRATION IN A UNIFORM STRESS FIELD COARSE MESH

PLOT 1 VIEW POINT 0.0 0.0 1.0 SCALE 1.250

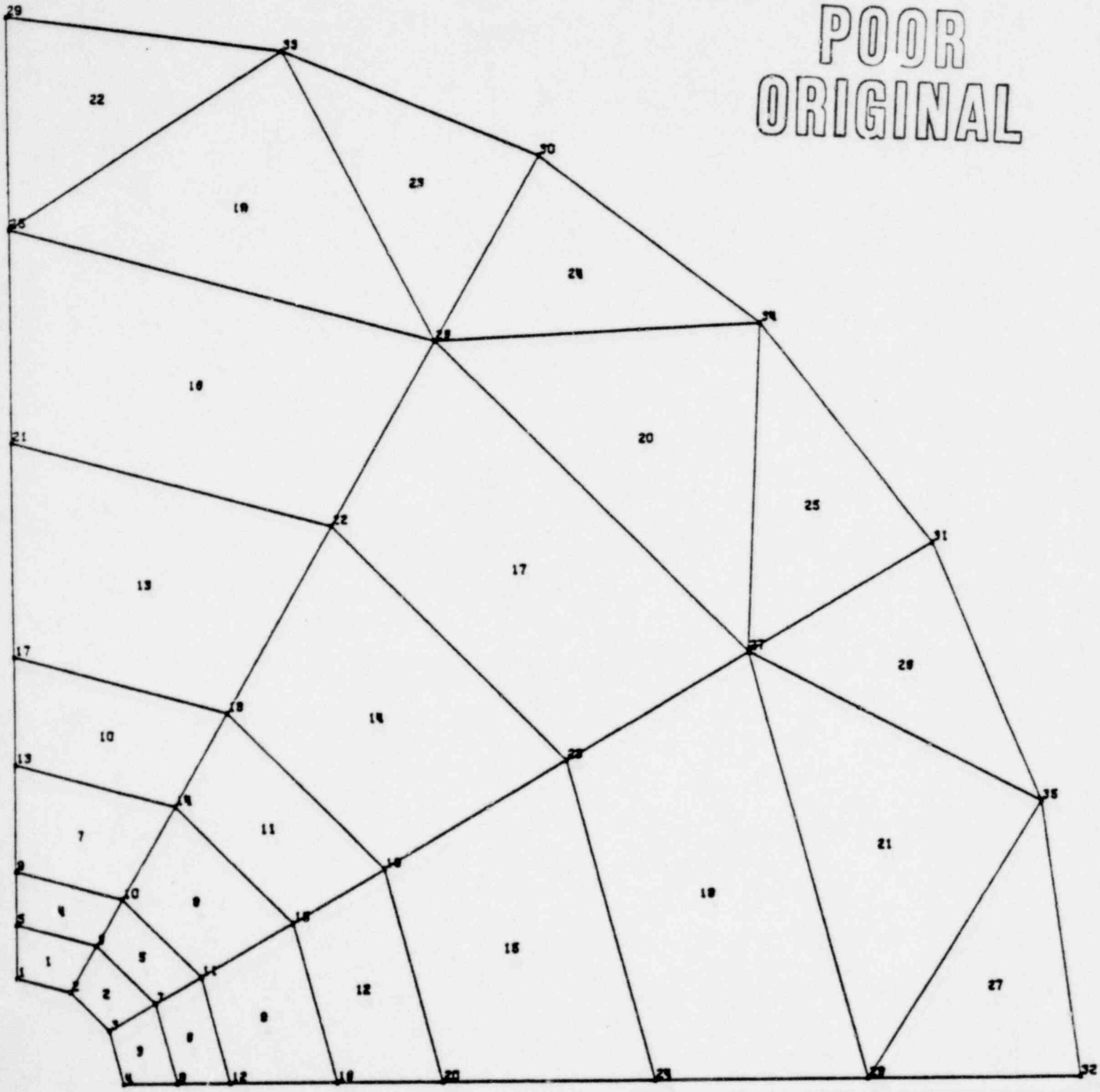

FIGURE 5

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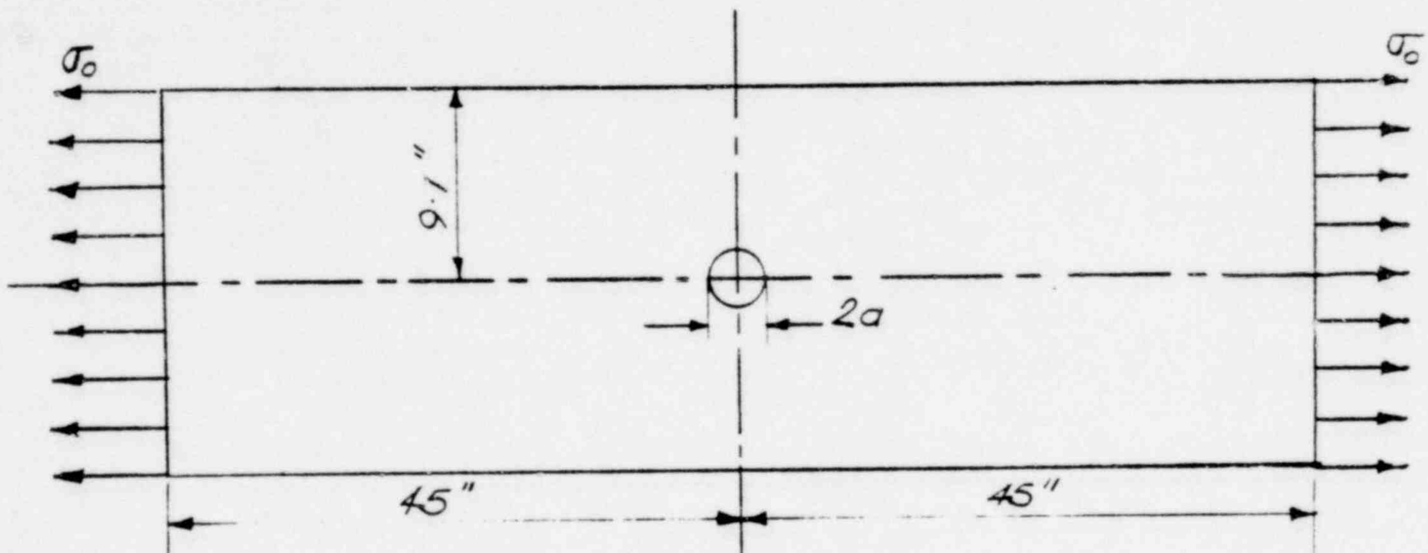

$R = 9{\cdot}1''$ $t = {\cdot}091''$ $L = 45''$ $a = 1{\cdot}00''$

FIG. 6

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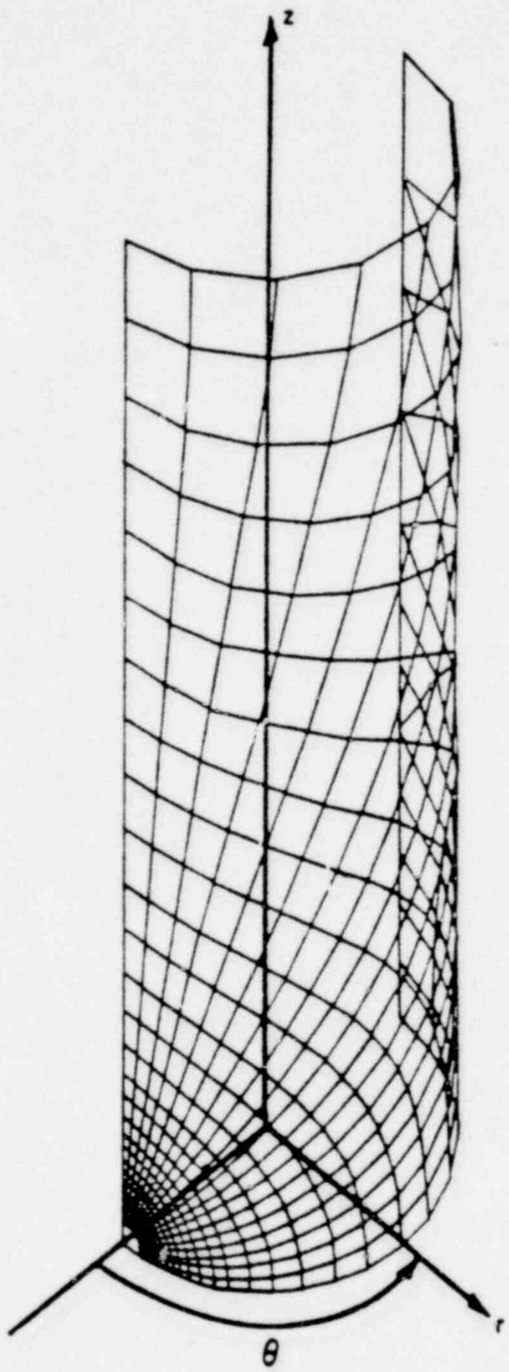

589 Nodal Points

533 Elements

Figure 7 Fine Mesh Layout

1329 355

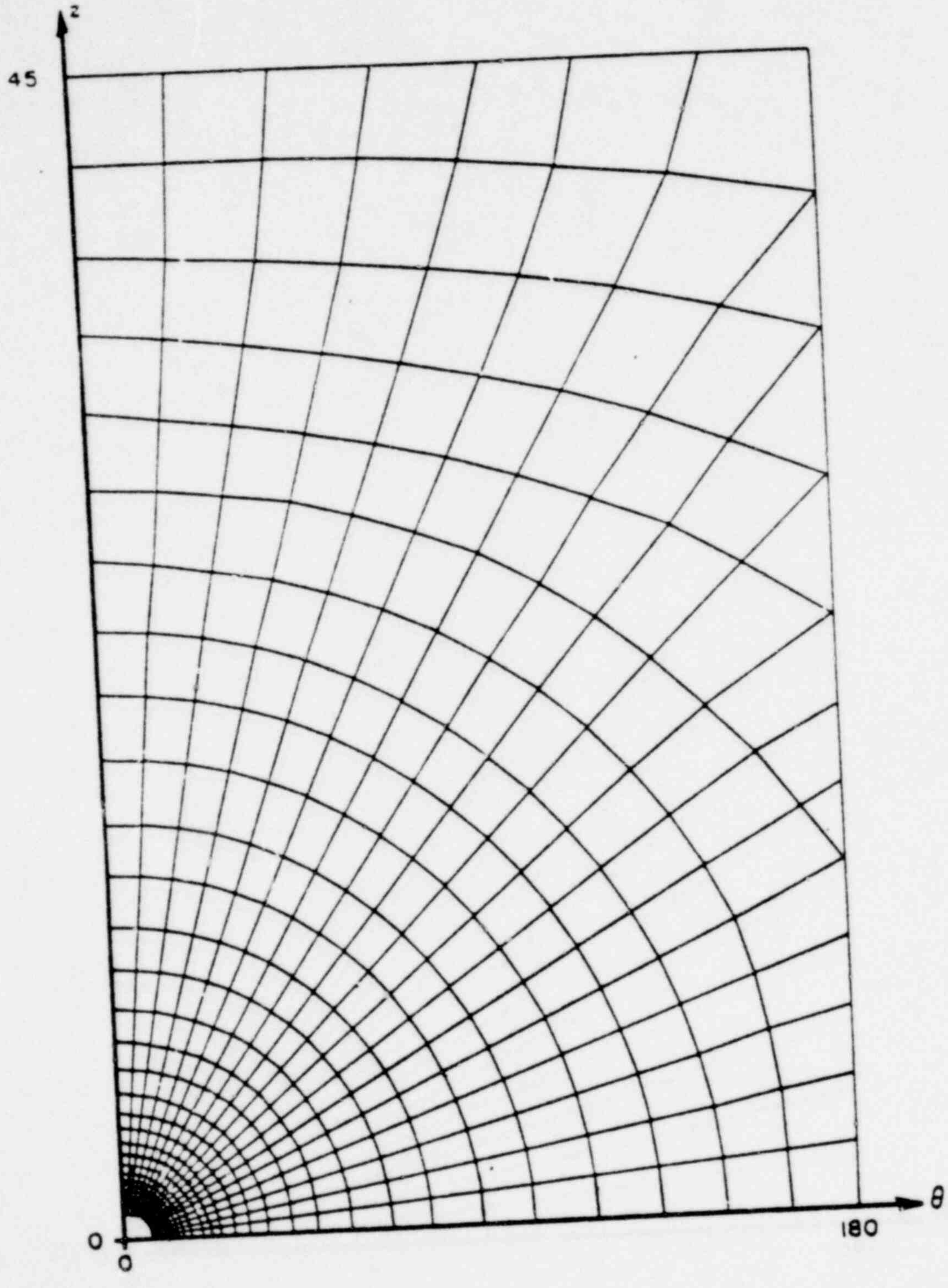

Figure 8 Developed Fine Mesh

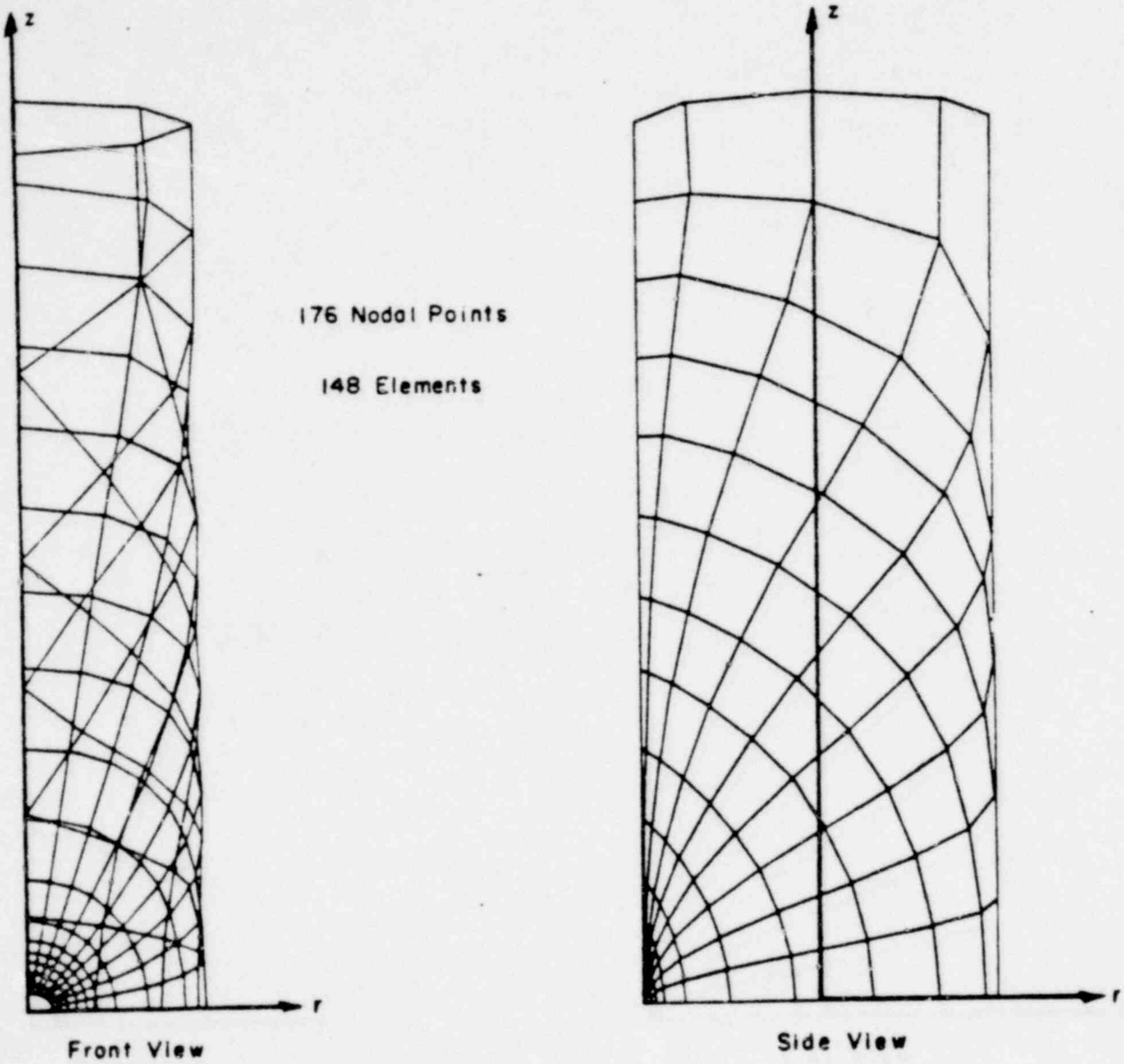

Figure 9 Coarse Mesh Layout

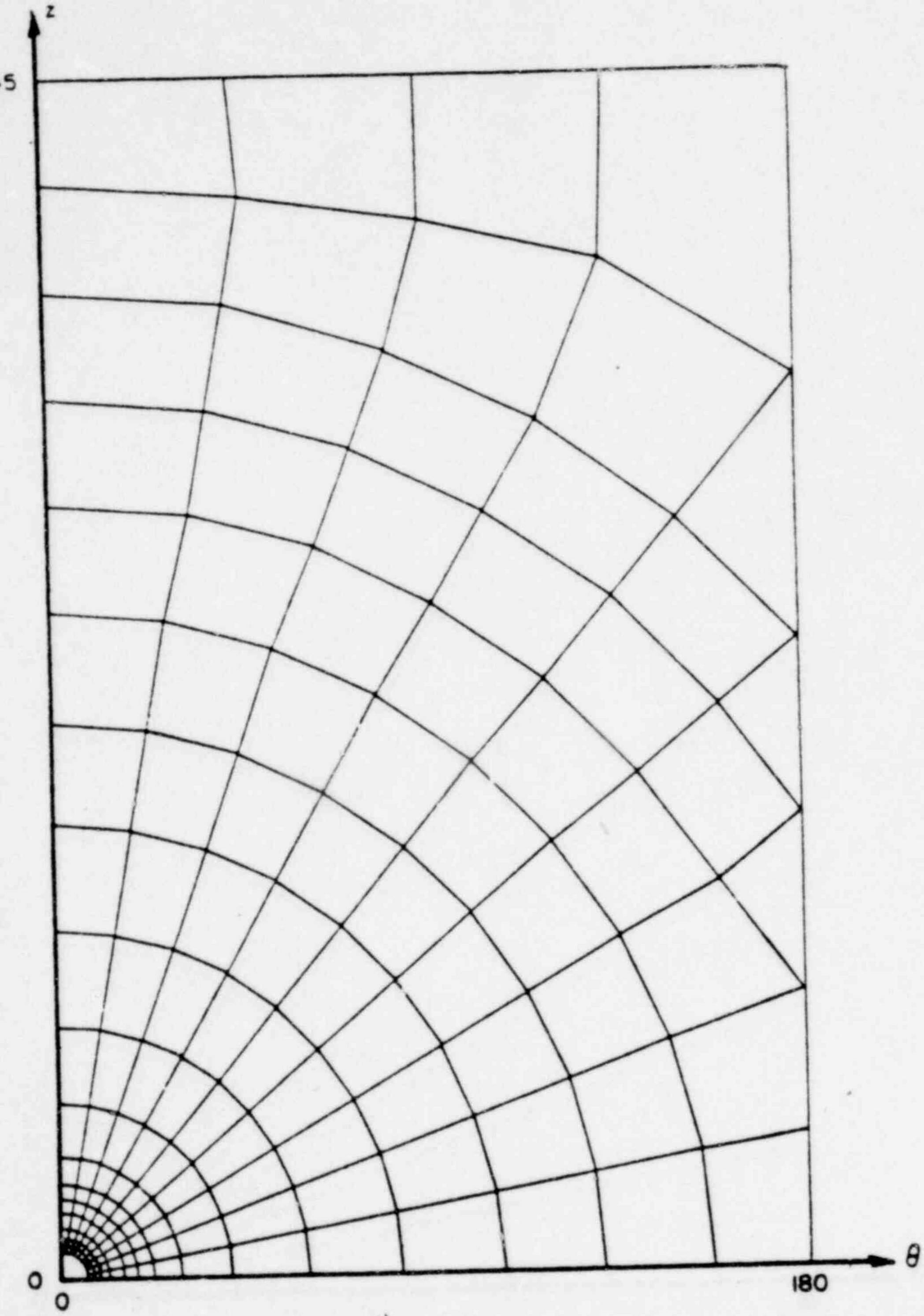

Figure 10 Developed Coarse Mesh

120 FT. CONT. VESSEL W/ONE STIFFENER 100 FT. SPACING COARSE MESH

PLOT 1 VIEW POINT 1.0 1.0 1.0 SCALE 170.031

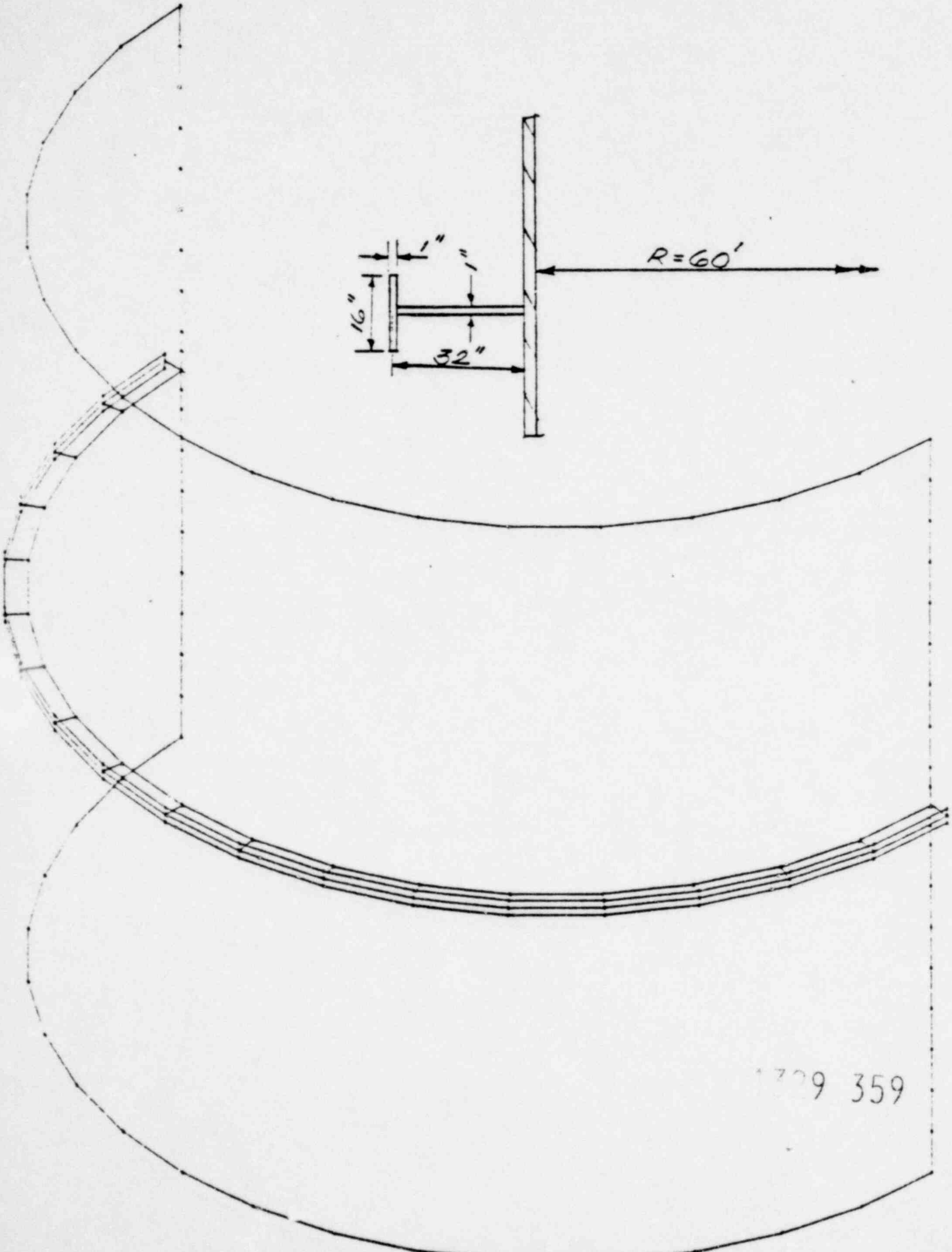

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FIG. 11

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FINE MESH MODEL

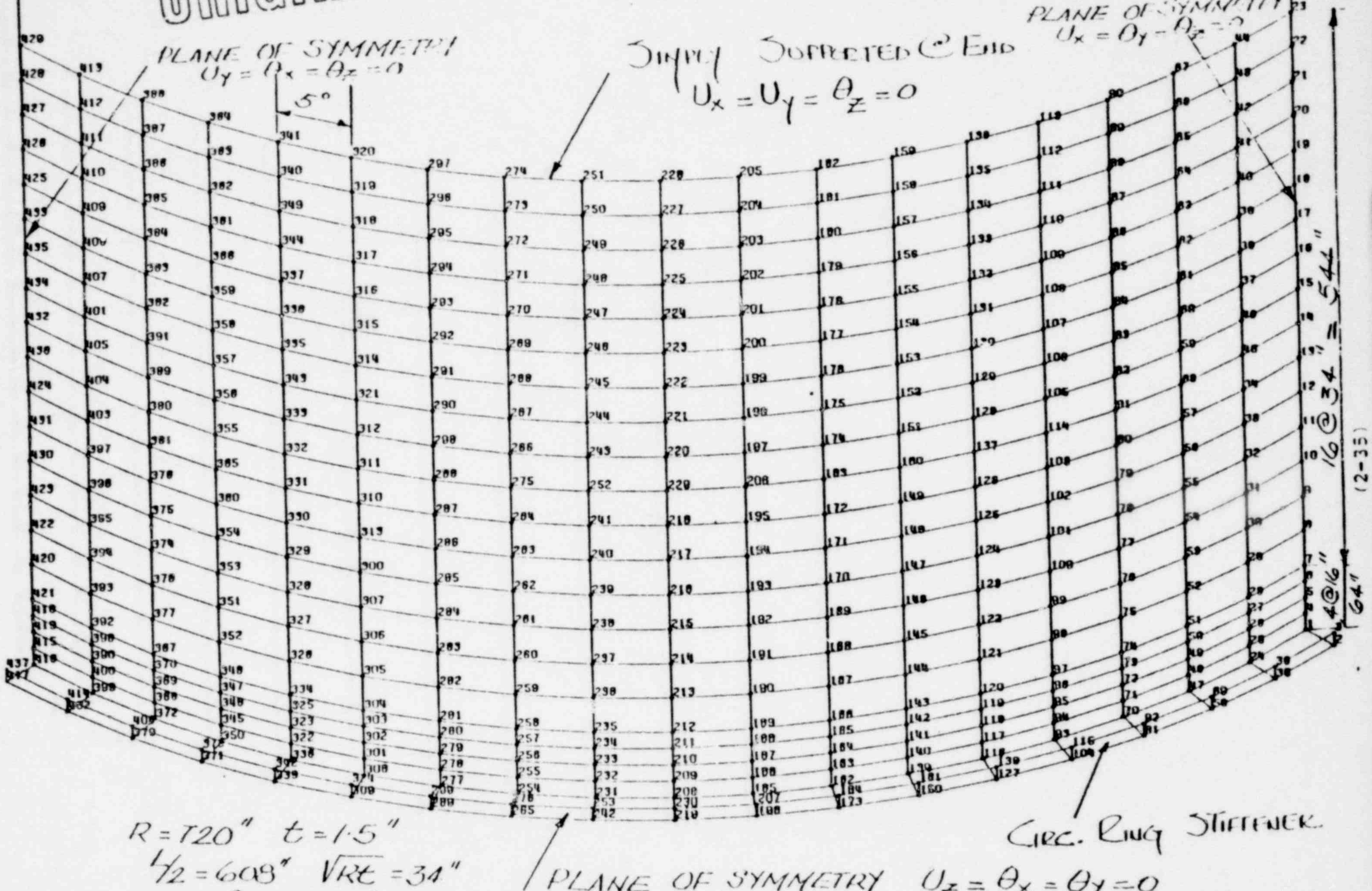
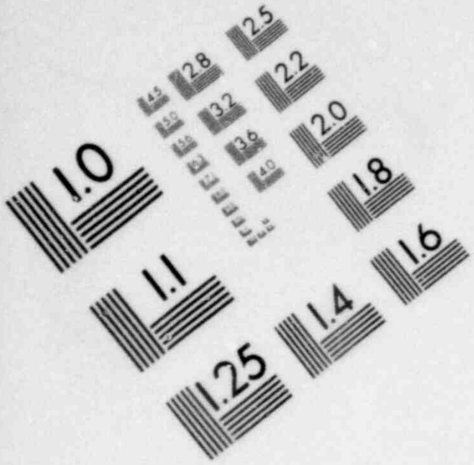


FIGURE 12



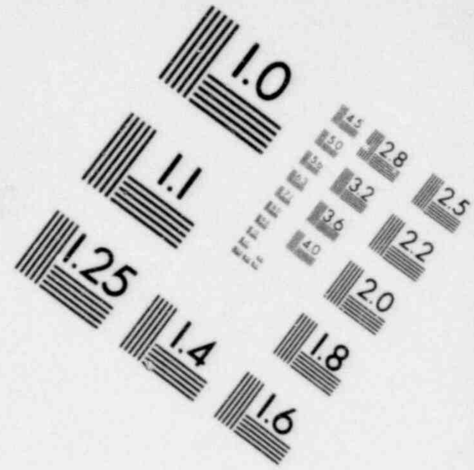
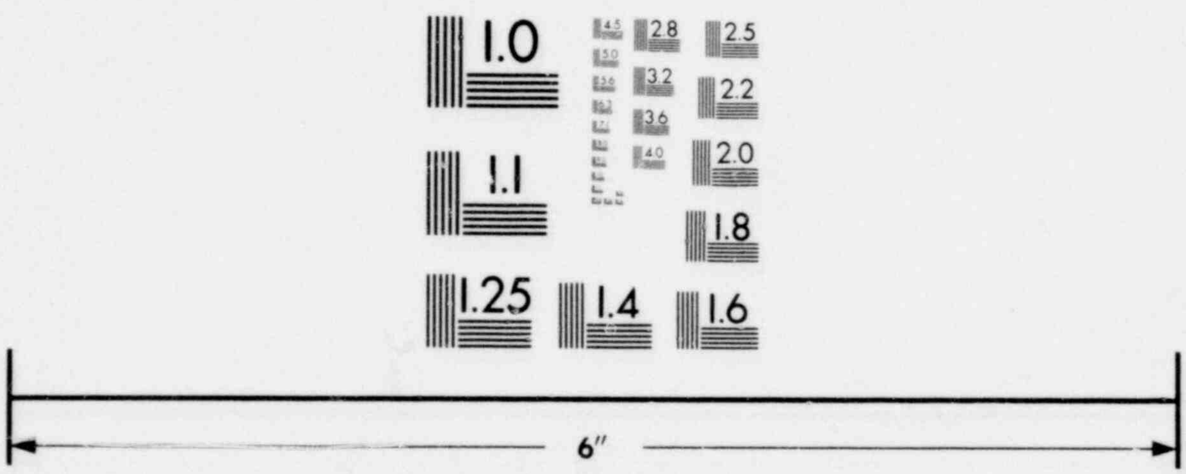
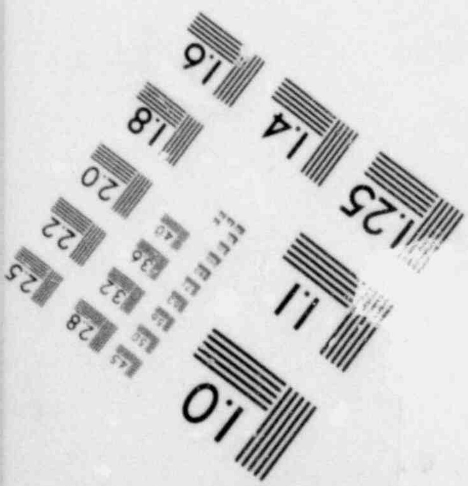


IMAGE EVALUATION TEST TARGET (MT-3)





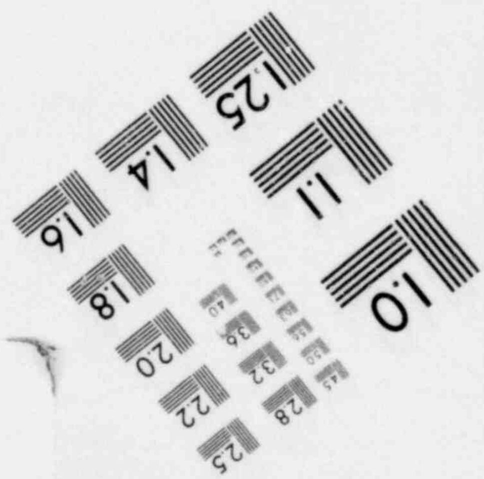
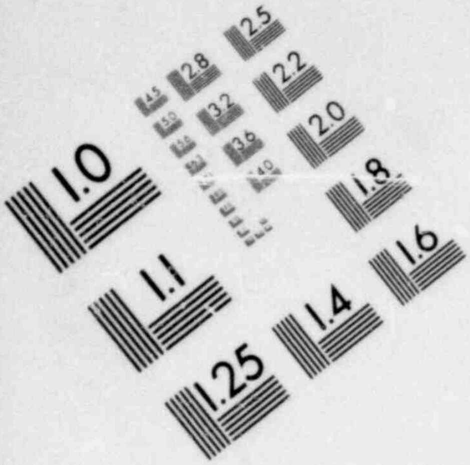


IMAGE EVALUATION
TEST TARGET (MT-3)

6"



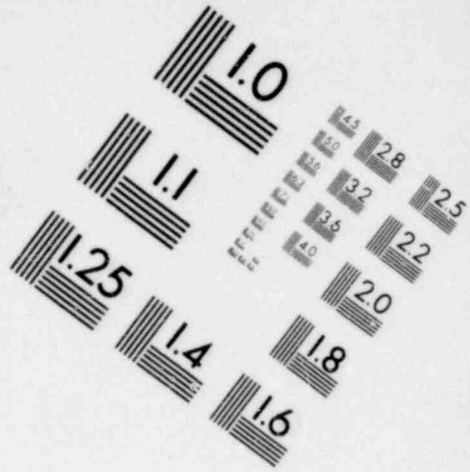
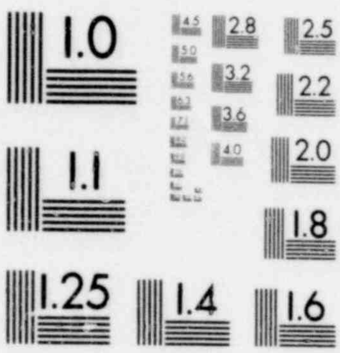
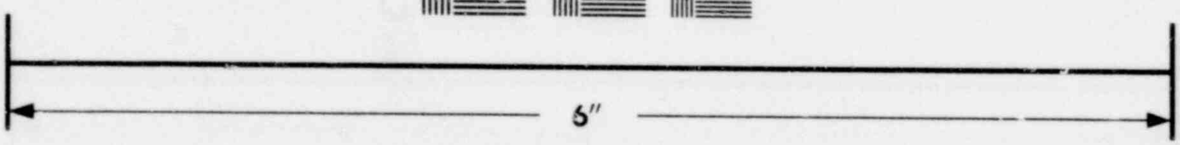
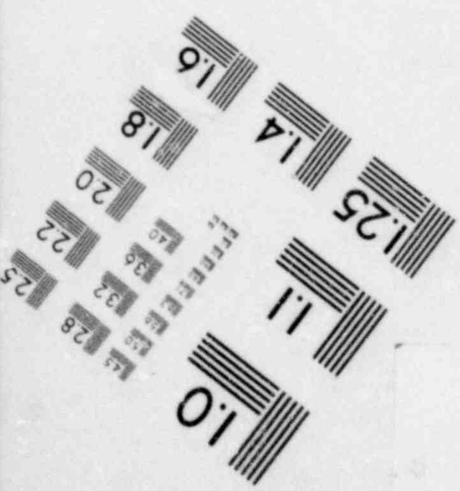
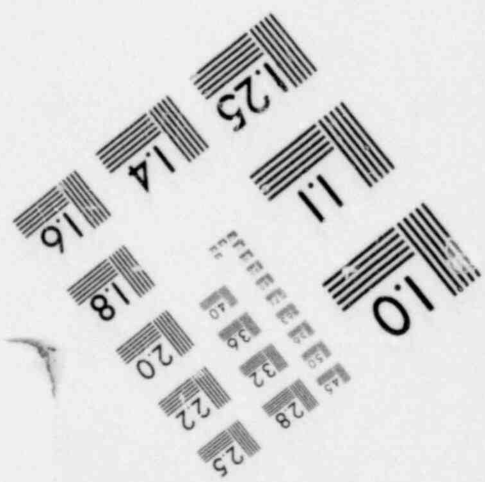


IMAGE EVALUATION TEST TARGET (MT-3)









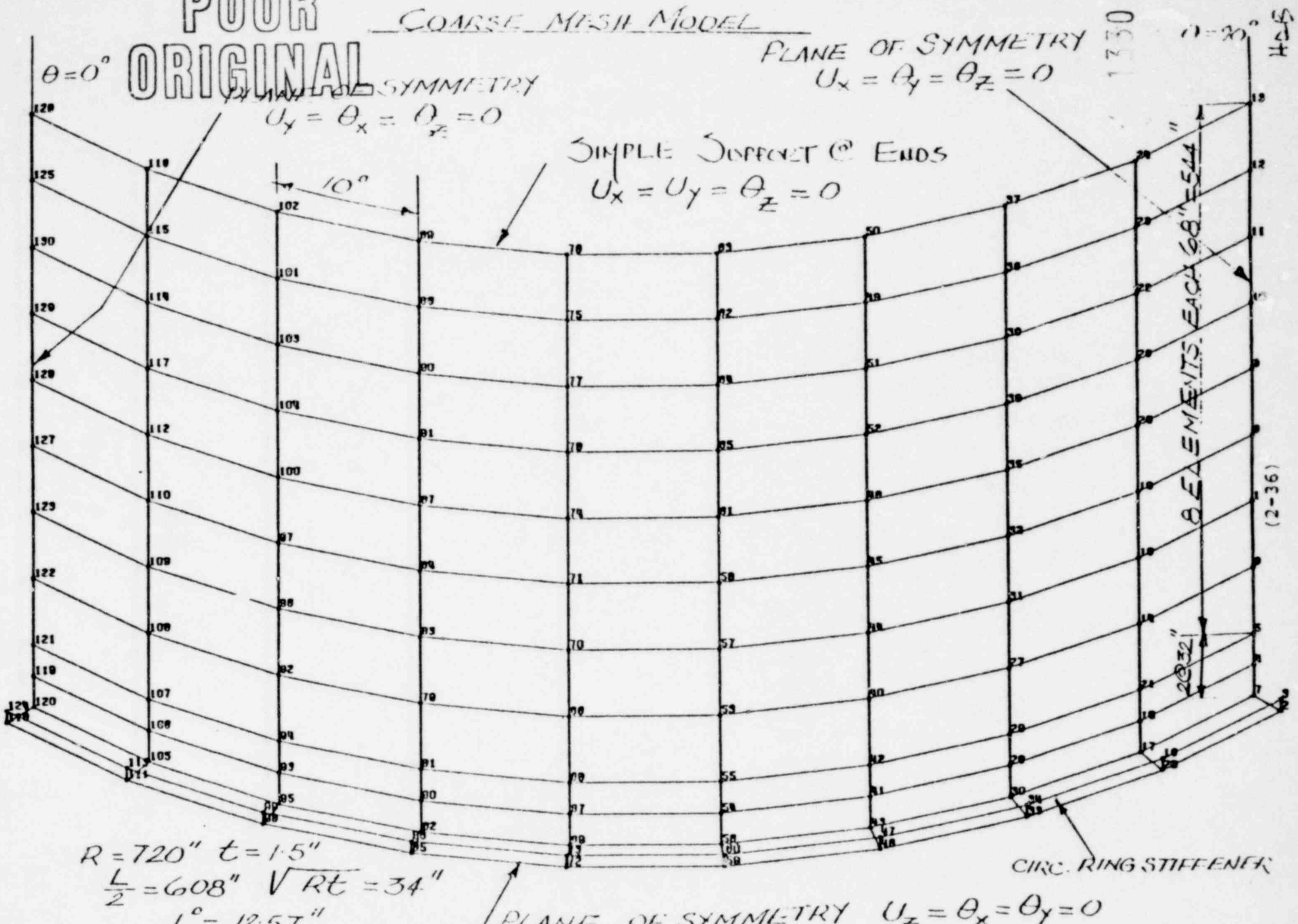

COARSE MESH MODEL
θ=0°
PLANE OF SYMMETRY
Uy = θx = θz = 0
θ=30°
PLANE OF SYMMETRY
Ux = θy = θz = 0
SIMPLE SUPPORT @ ENDS
Ux = Uy = θz = 0
10°
8 ELEMENTS EACH 68" = 544"
2@32"
R = 720" t = 1.5"
L/2 = 608" √Rt = 34"
1° = 12.57"
PLANE OF SYMMETRY Uz = θx = θy = 0
CIRC. RING STIFFENER

FIG. 13

WEINGARTEN CON.AINMENT EXAMPLE, MESH

PLOT 8 VIEW POINT 1.0 1.0 1.0 SCALE 220.000

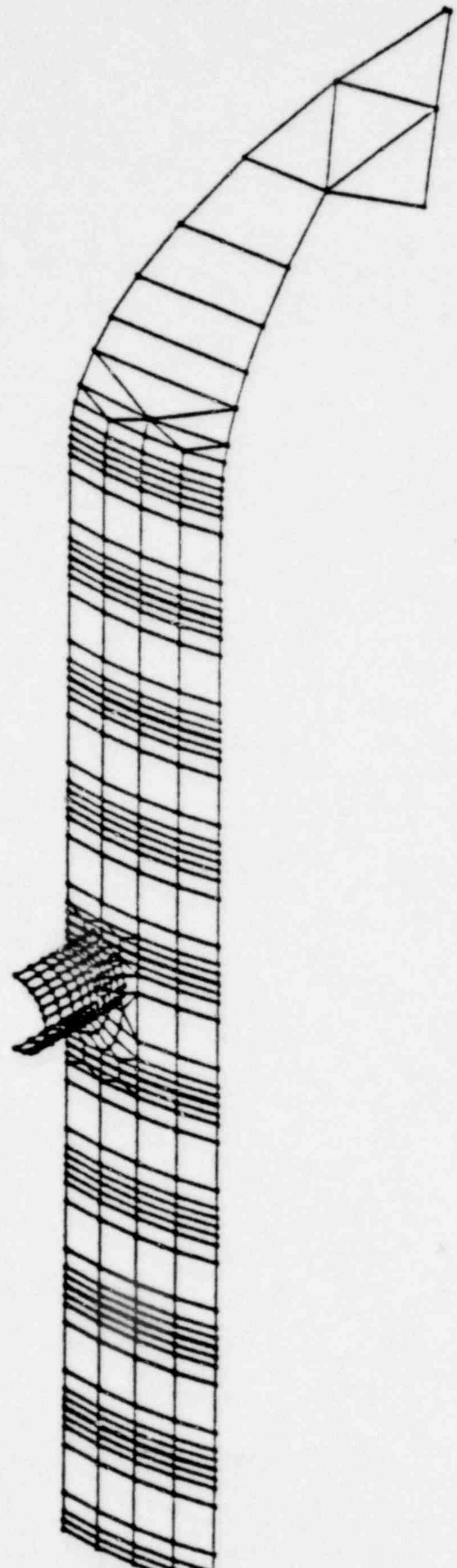


FIG. 14

PROGRAM	METHOD OF IDEALIZATION	RUN TIME	MAXIMUM DEFLECTION
NASTRAN VERSION 15	1900 NODES, 6 DOF PER NODE ONE QUARTER OF STRUCTURE RECTANGULAR GRID	70 MIN. UNIVAC 1108	~33 cm
BOSOR 4	346 MESH POINTS, 4 DOF PER POINT, 20 HARMONIC RUNS	3 MIN. UNIVAC 1108	32.77 cm
MARC	80 NODES, 9 DOF PER NODE HALF STRUCTURE MODELED CURVED TRIANGULAR ELEMENTS USED	360 MIN CDC	8.61 cm
STARS-2SF SPECIAL FAST VERSION	65 SEGMENTS/4 DOF PER POINT 5 INTEGRATION POINTS PER SEGMENT 20 HARMONIC RUNS	2.5 MIN IBM 360	32.9 cm

1330 003

TABLE 1

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STARS-2D	FINITE ELEMENTS	FINITE DIFFERENCE
SPATIAL IDEALIZATION 6 SEGMENTS 20 DOF	SPATIAL IDEALIZATION 50 FINITE ELEMENTS = 196 NET DOF	SPATIAL IDEALIZATION 90 FINITE DIFF. STATIONS = 356 NET DOF
TIME INTEGRATION $\Delta T = 1 \times 10^{-5}$ (N=0) $\Delta T = 2 \times 10^{-5}$ (N=1,2)	TIME INTEGRATION $\Delta T = 5 \times 10^{-6}$ SEC. FOR N = 0 TO 2	TIME INTEGRATION $\Delta T = 5 \times 10^{-6}$ SEC. FOR N = 0 TO 2

THE FINITE ELEMENT SOLUTION IS PRESENTED IN PAPER "NONLINEAR DYNAMIC ANALYSIS OF SHELLS OF REVOLUTION BY MATRIX DISPLACEMENT METHOD" J. STRICKLIN ET AL AIAA/ASME 11TH STRUCTURES, STRUCTURAL DYNAMICS + MATERIALS CONFERENCE 1970.

THE FINITE DIFFERENCE SOLUTION IS GIVEN IN "DYNAMIC RESPONSE OF A CYLINDRICAL SHELL: TWO NUMERICAL METHODS" D JOHNSON + R GRIEF AIAA JOURNAL 4(3)

1330 004

TABLE 2

HDB 9/79

POOR ORIGINAL

TABLE 3

	NUMBER OF EQUATIONS	STRESS CONCENTRATION FACTOR AT	
		$\theta = 0°$	$\theta = 90°$
FE RESULTS FINE MESH	180	−.906	2.999
FE RESULTS COARSE MESH	54	−.636	2.675
RESULTS FROM CBI PROGRAM 781	40	−1.071	3.091
ELASTICITY SOLUTION EXACT		−1.072	3.093

1330 005

POOR ORIGINAL

NATURE OF STRESS	LEKKERKERKER'S SOLUTION	FINITE ELEMENT SOL. COARSE MESH	FINITE ELEMENT SOL. FINE MESH
σ_ϕ^+ @ A	3350	3072 (-8%)	3338 (-.4%)
σ_ϕ^o @ A	3955	3637 (-8%)	3922 (-.8%)
σ_ϕ^- @ A	4560	4202 (-8%)	4506 (-1%)
σ_θ^+ @ B	-550	-397 (-28%)	-469 (-15%)
σ_θ^o @ B	-1375	-1148 (-16%)	-1285 (-7%)
σ_θ^- @ B	-2200	-1900 (-14%)	-2100 (-5%)

σ_ϕ AND σ_θ CORRESPOND TO MERIDIONAL + HOOP STRESSES IN PSI.

SUPERSCRIPTS +, O, - DENOTE OUTSIDE, MIDSURFACE AND INSIDE SURFACE STRESSES RESPECTIVELY

"A" AND "B" CORRESPOND TO SIDE AND TOP OF HOLE

TABLE 4

HDB

1330 006

TABLE 5

METHOD OF SOLUTION	σ_ϕ^+	σ_ϕ^o	σ_ϕ^-	σ_θ^+	σ_θ^o	σ_θ^-
STRESSES @ PT. A FINITE ELEMENT SOLUTION USING FINE MESH WITH 378 QUAD. SHELL/PLATE ELEMENTS + 18 MEMBRANE PLANE STRESS ELEMENTS	72.60	2.19	-68.26	74.86	59.00	43.14
STRESSES @ PT. A USING CBI SHELL ANALYSIS PROGRAM 781 "EXACT SOLUTION"	83.50	1.65	-80.20	92.34	72.57	52.80
STRESS @ PT. B FINITE ELEMENT SOLUTION USING COARSE MESH WITH 99 SHELL PLATE ELEMENTS & 9 PLANE STRESS ELEMENTS	10.53	-3.65	-17.83	24.90	24.70	24.50
STRESSES @ PT. B USING CBI SHELL ANALYSIS PROGRAM 781	17.20	-2.40	-21.75	46.40	46.10	45.70
"PEAK STRESSES" USING CBI SHELL ANALYSIS PROGRAM 781	180.0	2.65	-174.7	134.0	85.35	36.70

NOTE:-

σ_ϕ AND σ_θ CORRESPOND TO MERIDIONAL + HOOP STRESSES IN KSI.

SUPERSCRIPTS +, O, – DENOTE OUTSIDE, MEMBRANE AND INSIDE STRESSES RESPECTIVELY.

POINT "A" FOR FINE MESH IS AT LOCATION $\theta = 2.5°$ $x = 8"$

POINT "B" FOR COARSE MESH IS AT LOCATION $\theta = 5.0°$ $x = 16"$

FINITE ELEMENT PROGRAM USED IN ANALYSIS IS SAP4. HDB

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