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A Comparison of Piping Models for Digital Power Plant Simulators

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Nuclear Engineering

Date

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ABSTRACT

Two piping models intended for use in a digital power plant simulator are compared. One is a finite difference approximation to the partial differential equation called PIPE, and the other is a function subroutine that acts as a delay operator called PDELAY. The two models are compared with respect to accuracy and execution time. In addition, the stability of the PIPE model is determined.

The PDELAY model is found to execute faster than the PIPE model with comparable accuracy.

CHAPTER 1

INTRODUCTION

A digital simulator (BRENDA) for the Clinch River Breeder Reactor Plant (CRBRP) has been written at The University of Arizona. The simulator is a low order simulator, of about sixty differential equations, that will execute quickly on the computer and provide a low cost tool for analysis of the CRBRP. This simulator would be appropriate for 1) Plant sensitivity studies, 2) Verification of higher order models, and 3) Identification of areas for further study.

The simulator consists of a system of coupled non-linear first order ordinary differential equations and some auxiliary algebraic equations. These differential equations are integrated, using the DARE P simulation language developed by Dr. John V. Wait at The University of Arizona, to provide the system response. The system of ordinary differential equations (ODEs) results from the application of the principles of conservation of mass, momentum, and energy to specific regions in the CRBRP.

There are two major factors which affect the execution time of a system of ODEs: the order of the system and the size of time step the integration routine is allowed to use. The execution time for

a model may be reduced by decreasing the order of the model or by increasing the size of the computational time step. The latter f these methods is probably the most difficult to accomplish.

Generally the size of the allowed time step is governed by the size of the largest eigenvalue for the system. In a non-linear system the eigenvalues are a function of time, and variable time step integration rules are used to keep the size of the time step as large as possible. Another problem called "stiffness" may cause further difficulties.

A stiff system of equations is one that has a large spread of eigenvalue moduli and at least one large negative eigenvalue. A stiff system of equations must be integrated with very small time steps when using most methods and may eventually yield an unstable solution. Thus a system such as this must be integrated with an implicit integration rule that will allow larger timesteps. However, implicit integration rules are much more complicated than the more familiar explicit rules and take roughly an order of magnitude more computation time. Thus, a significant increase in the size of a computational timestep is not easily accomplished. This leaves the first method, reducing the execution time by reducing the order of the system. This approach was used in BRENDA from the beginning.

In many power plant simulators a large number of differential equations is used to describe the flow of a working fluid through a pipe between two power plant components. Instead of describing this

flow with differential equations, a function subroutine, PDELAY, was used in BRENDA from the project's inception. This paper is intended to compare these two solution methods based on their accuracy and the savings in execution time.

In Chapter 2 the models used for the comparison are presented along with an analytic solution to the partial differential equation that describes the flow of a fluid through a pipe. Chapter 3 will compare the results from these models and Chapter 4 will contain the conclusions and summary.

CHAPTER 2

THE MODELS

The models used for comparison are presented in this chapter. The model used in BRENDA has been mentioned previously and is called PDELAY (Pipe DELAY). The differential equation model to which it is compared is called PIPE. Before these are presented, an analytic solution to the partial differential equation is derived. This solution will provide one form of comparison in Chapter 3.

2.1 Analytic Solution

The fluid in the pipe is being modeled as an incompressible fluid with inviscid flow in the axial direction only, known as slug flow. In addition, heat flow into the fluid is neglected. Thus the applicable equation is a reduced form of the energy equation:

$$\rho \frac{\partial u}{\partial t} = - \frac{\dot{m}}{A} \frac{\partial h}{\partial z} \qquad 2.1$$

In this equation

ρ = density, (lbm/ft^3)

u = internal energy, (BTU/lbm)

t = time, (sec)

$\dot{m}$ = mass flow rate, (lbm/sec)

A = flow cross section area, (ft^2)

h = enthalpy, (BTU/lbm)

z = axial coordinate, (ft).

Because the fluid is incompressible Cp equals Cv. Therefore,

$$u = C_p T$$

and

$$h = C_p T$$

where,

C_p = heat capacity,

T = temperature, (F).

In addition

$$\dot{m} = \rho v A$$

where,

v = fluid velocity.

Substituting these expressions into eq. (2.1) yields

$$\frac{\partial T}{\partial t} = -v \frac{\partial T}{\partial z} \qquad 2.2$$

which is the equation to be solved. The boundary condition is

$$T(0,t) = T_i\ (t) \qquad 2.3$$

and the initial condition is

$$T(z,0) = T_o. \qquad 2.4$$

To solve this equation it is first convenient to transform it using Laplace transforms. Throughout this derivation the transform of a variable will be indicated by writing that variable with a bar over it, i.e.,

$$\mathcal{L}[T(z,t)] = \bar{T}(z,s).$$

Let

$$v = 1/\tau,$$

and the transform of eq. (2.2) is

$$s\bar{T} - T(z,0) = -1/\tau \ (d\bar{T}/dz). \qquad 2.5$$

Note that the result is an ordinary differential equation in space, and the transform variable s is only a coefficient in this equation. Using the initial condition, eq. (2.4), eq. (2.5) may be written in the usual form

$$d\bar{T}/dz + s\tau T = \tau T_o \qquad 2.6$$

The particular solution is given by

$$\bar{T}_p = T_o/s \qquad 2.7$$

and the homogeneous solution is given by

$$\bar{T} = C(s)\exp(-S\tau z) \qquad 2.8$$

where C(s) is an arbitrary function of s. The general solution is the sum of the homogeneous solution and the particular solution,

$$\bar{T} = C(s)\exp(-s\tau t) + T_o/s. \qquad 2.9$$

To evaluate the function C(s), the boundary condition must be transformed to s-space. The result of this,

$$\bar{T}(0,s) = \bar{T}_i(s), \qquad 2.10$$

may be applied to the general solution, eq. (2.9), yielding

$$\bar{T}_i(s) = C(s) + T_o/s$$

or

$$C(s) = \bar{T}_i(s) - T_o/s. \qquad 2.11$$

The solution to eq. (2.6) is then given by

$$\bar{T} = [\bar{T}_i(s) - T_o/s]\exp(-s\tau z) + T_o/s \qquad 2.12$$

and the solution to the partial differential equation may be obtained by taking the inverse transform of this equation.

If eq. (2.12) is rewritten as

$$\bar{T} = \bar{T}_i(s)\exp(-s\tau z) - (T_o/s)\exp(-s\tau z) + T_o/s$$

the inverse transform, term by term, is

$$T(z,t) = T_i(t - \tau z) \cdot \begin{bmatrix} 1 \text{ if } t>\tau z \\ 0 \text{ if } t<\tau z \end{bmatrix}$$

$$- \begin{bmatrix} T_o \text{ if } t>\tau z \\ 0 \text{ if } t<\tau z \end{bmatrix} + T_o \qquad 2.13$$

This result may be interpreted by remembering

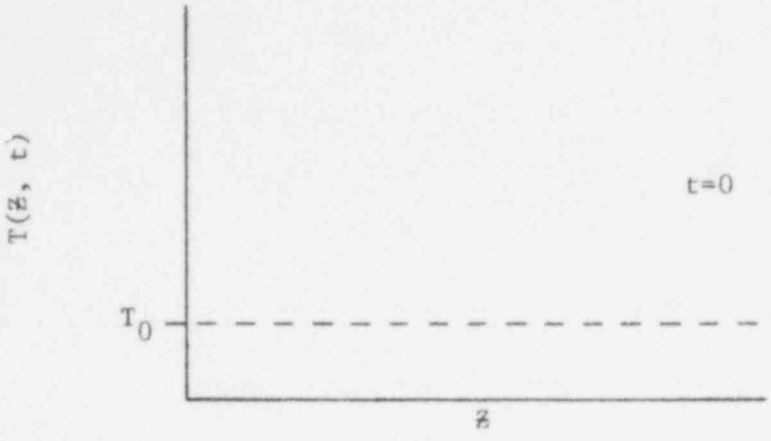
$$\tau = 1/v$$

and by reference to Figures 2.1a and 2.1b.

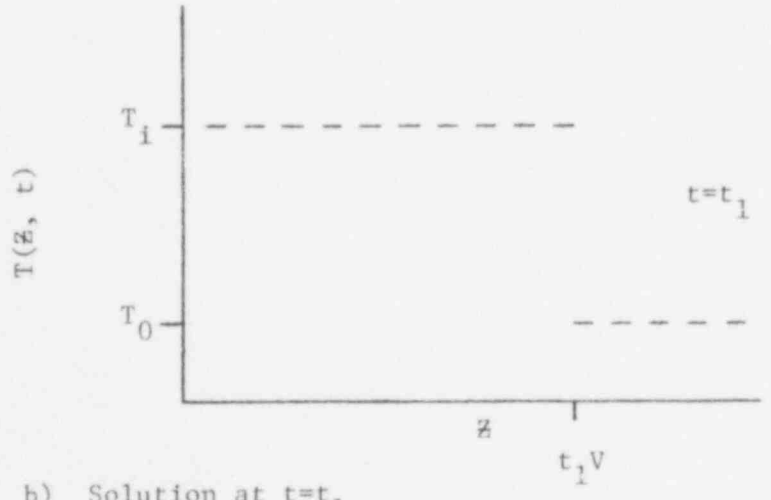
In Figure 2.1a, $t = 0$ and the initial temperature is T_o. $T_i(t)$ will be assumed to be a constant, T_i. In Figure 2.1b, $t = t_1$ and for all $z>t_1v$, $T = T_o$, while for $z<t_1v$, $T = T_i$. This means that the temperature at any position z is the same as it was at $z = 0$ a time $t = z/v$ in the past where the time t is simply the transit time for the fluid. This solution is valid for T_i equal to any function of time. In the following chapter this result will be compared with results from the models presented in the next two sections of this chapter.

2.2 Ordinary Differential Equation Model (PIPE)

This method of solution involves dividing the pipe into a finite number of regions and writing an ordinary differential equation to describe the transient behavior of each region. Physical properties are assumed to be uniform within a region and the model is called a lumped parameter model because of this. There are many methods of deriving the region equations and the reader is referred to Appendix A for comparisons.



a) Initial conditions



b) Solution at $t=t_1$

Figure 2.1 - Analytic Solution to a Step Increase at t=0

The method presented here is to start with the partial differential eq. (2.2) repeated below,

$$\frac{\partial T}{\partial t} = -v \frac{\partial T}{\partial z} \qquad 2.2$$

and discretize the spatial derivative using a general alpha finite difference approximation. This leads to an ordinary differential equation for the nth region,

$$\frac{dT_n}{dt} = -v \frac{[\alpha T_{n+1} + (1 - \alpha) T_n] - [\alpha T_n + (1 - \alpha) T_{n-1}]}{\Delta z} \qquad 2.14$$

In this equation alpha is a parameter assigned a value between zero and one. If alpha = 0 the finite difference approximation is known as a "backwards difference", alpha = 1 is a "forward difference", and alpha = 0.5 is a "central difference".

This equation is applied to each of the N regions in the pipe, and the resulting set of coupled differential equations is integrated to provide the system response. The temperature in the Nth region is the outlet temperature from the pipe. The two end regions are treated separately due to their special nature.

The first region, n = 1 is bounded on the left by a "ghost" region, n = 0. The temperature in this zone is considered to be the exit temperature of the component flowing into the pipe. The last region of the pipe, n = N, is treated in a similar manner. A "ghost" region, n = N + 1, has its temperature defined so the difference approximation for the last zone is reduced to a "backwards difference" approximation.

These equations are the core of the PIPE model. Other equations were used to calculate error terms used for choosing an optimum value

of alpha for use in the model. These equations are essentially energy balance equations for the pipe as a whole and are presented here.

$$E_p(t) = E_p(\mathcal{J}) + \int_o^t E_{in}(t')\,dt' - \int_o^t E_{out}(t')\,dt' \qquad 2.16$$

this is to say that the energy in the pipe is equal to the initial energy plus the total energy that has flowed in minus the total energy that has flowed out. The energy in the pipe may also be represented by

$$E_p(t) = \sum_{n=1}^{N} m_n C_{pn} T_n \qquad 2.17$$

where,

m_n = mass of the nth region

C_{pn} = specific heat of the nth region,

and a measure of the accuracy of a model is the ratio of these two different calculations. If this ratio is much different from one, the method is not conserving energy well.

2.3 PDELAY

This method of solution was based on the realization that the desired behavior of a pipe subroutine was similar to the benavior of a delay operator that already existed in the DARE P library. PDELAY was designed to operate in the same manner, with one exception. The delay time in PDELAY is designed to be a function of time to enable variable flow rates to be modeled.

PDELAY is a FUNCTION subroutine and is called with the expression

```
TOUT = PDELAY (TIN. TAU, I, TINI),
```

where

TOUT = outlet temperature

TIN = inlet temperature

TAU = transit time for the fluid

I = an index number

TINI = initial temperature of the fluid in the pipe.

PDELAY works by saving values in two linear arrays. The values are a state variable, such as temperature, and the time when that value of the state variable will exist at the exit of the pipe. Variable flow rates are modeled by adjusting the time the state variable is expected to exit the pipe. The FORTRAN expression that does this is

TEXIT = [(TEXIT - T)/TAUL]* TAU + T.

In this expression

T = simulator time

TEXIT = fluid exit time

TAUL = previous transit time

TAU = current transit time.

The expression in square brackets is the fraction of the pipe left to be traveled by a particular fluid element. This quantity is multiplied by the new transit time and then added to the system time to define the new exit time for the fluid element in question. This method of solution is essentially a way to reproduce the analytic solution by numeric means. For a constant flow rate the output of PDELAY is identical to the input delayed by a time TAU.

CHAPTER 3

RESULTS

In this chapter the results from the models are presented. In the first section the results from PDELAY and PIPE for three different input functions, a step, a ramp, and a sine wave are presented. Since the PDELAY function is the same as an analytic solution, only PDELAY and PIPE outputs are presented. In the second section, optimum values of alpha for the PIPE model are discussed. In the third section the models are incorporated in the CRBRP model, BRENDA, and results from BRENDA are presented.

It is worth noting that there are actually an infinite number of PIPE models. This is due to two factors; first, the parameter N, the number of regions, may be set to any positive integer value and second, the parameter alpha may vary between zero and one. For this reason multiple results for the PIPE model are presented.

3.1 PDELAY and PIPE Results

The results for the PDELAY and PIPE models are presented in Figures 3.1 through 3.4. Three inlet functions were used to drive the models, a step, a ramp, and a sine wave.

Figure 3.1 is the response of PDELAY and PIPE (alpha = 0.0, alpha = 0.5) to a step, as is expected. The two PIPE outputs share the common characteristic that they do not agree with the analytic solution.

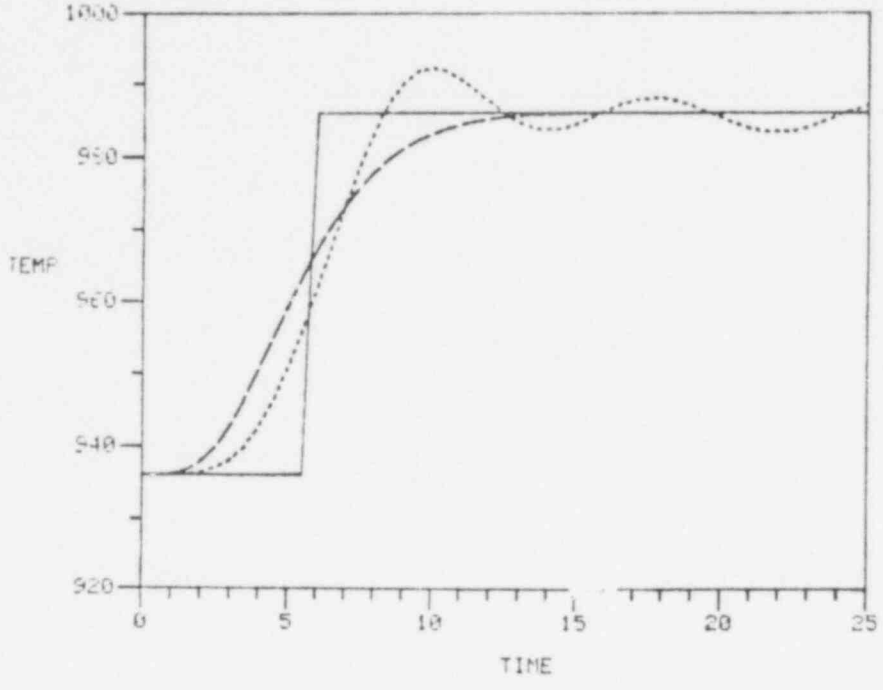


Figure 3.1 - Response of the Outlet Temperature to a 50° Step in the Inlet Temperature.

N=5, tau=1.16 sec, ΔZ=24.36 ft

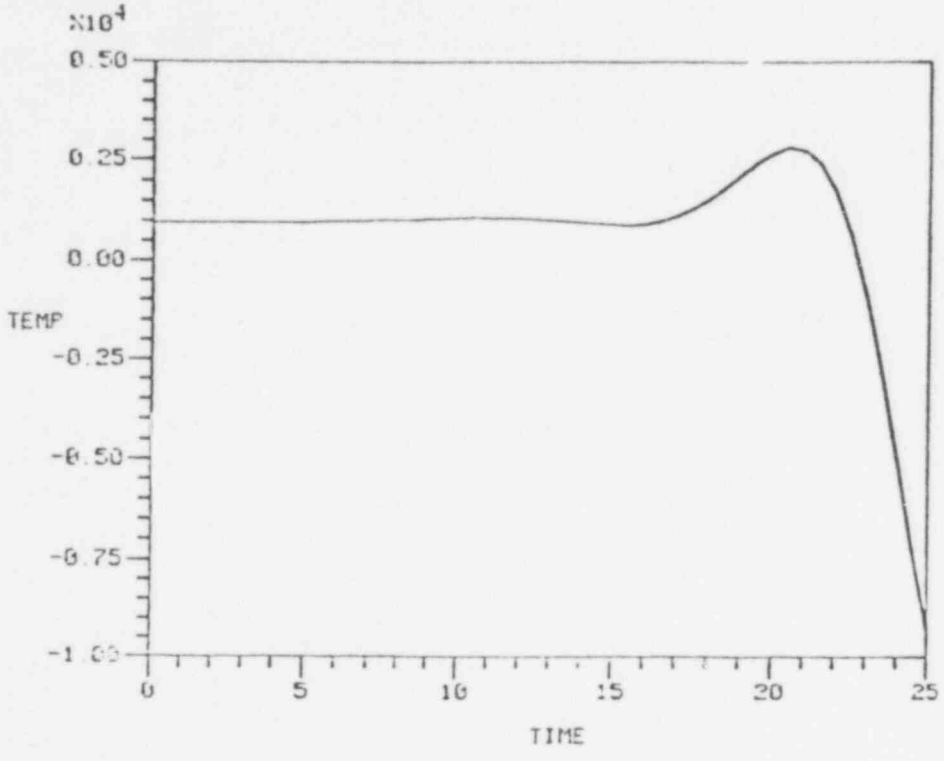


Figure 3.2 - Response of the Outlet Temperature to a 50° Step in the Inlet Temperature.

N=5, tau=1.16 sec, ΔZ=24.36 ft, alpha=0.75

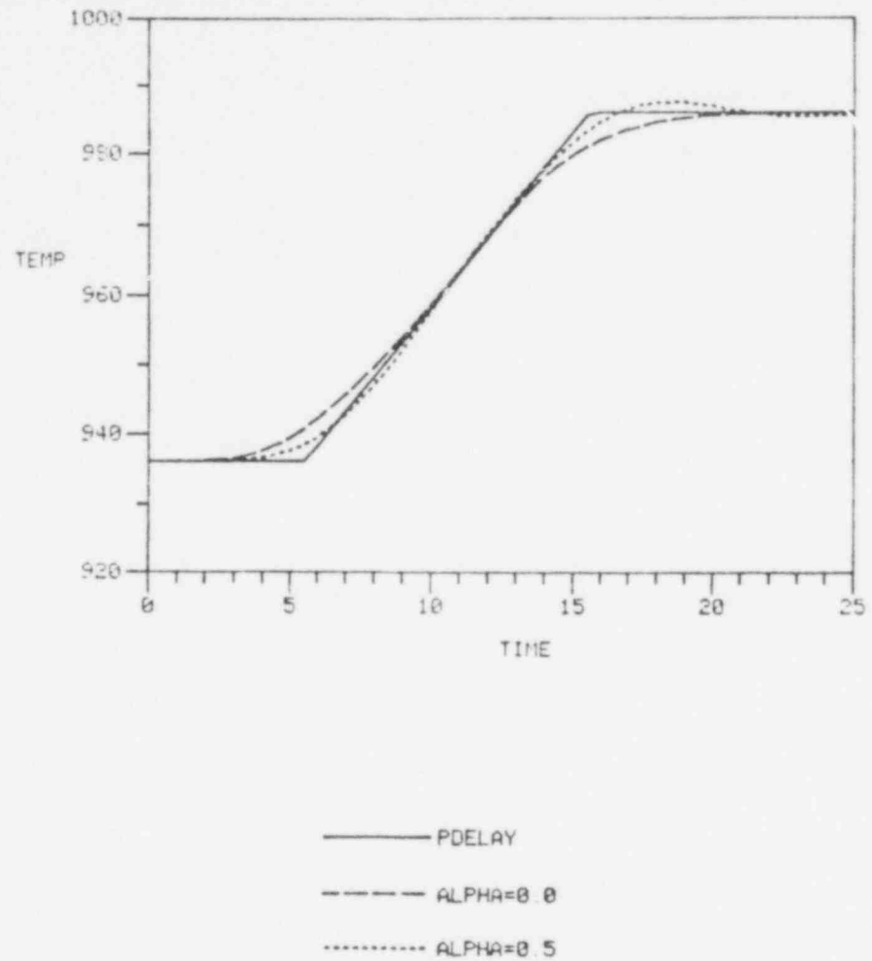


Figure 3.3 - Response of the Outlet Temperature to a 10°/sec Ramp in the Inlet Temperature for Ten Seconds.

N=5, tau=1.16 sec, ΔZ=24.36 ft

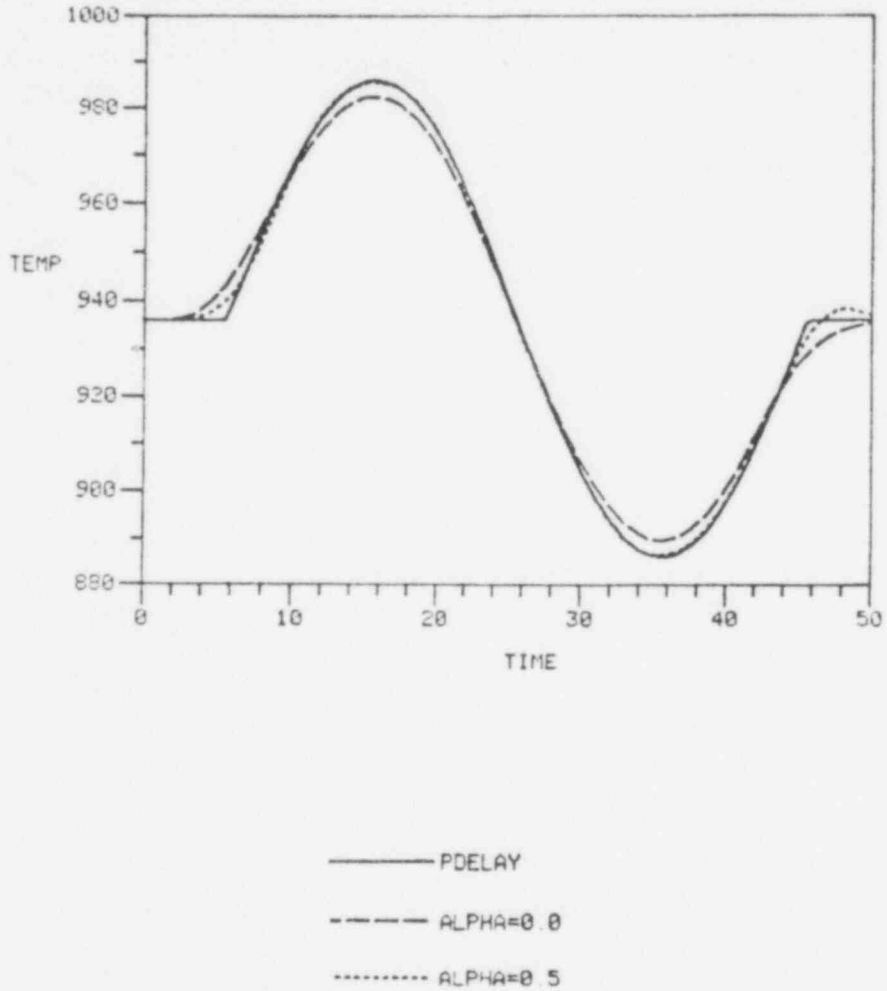


Figure 3.4 - Response of the Outlet Temperature to a Sine Wave Inlet Variation.

Amplitude=50°, period=40 sec, N=5, tau=1.16 sec, Z=24.36 ft

The solution for alpha = 0.5 has a further characteristic; it oscillates. This behavior is very nonphysical in that fluid flows out of the pipe at a higher temperature than it ever flows in.

Figure 3.2 is the response of PIPE to a step at time zero, with alpha = 0.75. This model is unstable. Since the system of equations in PIPE are linear, the stability of the system is determined by the eigenvalues of the system. Furthermore, the stability of the system is independent of the type of the inlet function. Thus, the PIPE model will always be unstable with alpha = 0.75, independent of the inlet function. The value of alpha that is the boundary between stability and instability is a practical limit for alpha. This value will be investigated in the next section.

Figure 3.3 is the response of PDELAY and PIPE (alpha = 0.0, alpha = 0.5) to a ramp inlet. Again PDELAY is essentially the analytic solution. The PIPE model (alpha = 0.5) exhibits oscillatory behavior, as the model did for the step inlet. This is expected because the oscillatory behavior results from the fact that some eigenvalues of the system are complex for alpha = 0.5.

Figure 3.4 is the response of PDELAY and PIPE (alpha = 0.0, alpha = 0.5) to a sine wave inlet. Once more PDELAY is the analytic solution. The PIPE model with alpha = 0.5 seems to be a much better representation than the PIPE model with alpha = 0.0. This topic will be pursued further in the next section.

These results point to the basic question addressed in the next section, the choice of alpha to be used in PIPE when it is incorporated in the CRBRP simulator, BRENDA.

3.2 Optimum Values of Alpha

There are two basic issues involved in choosing alpha: stability and accuracy. The question of stability will be discussed first, followed by a discussion of the accuracy of the model.

It was indicated in the previous section that the stability of the PIPE model was determined by the eigenvalues, and the eigenvalues are determined by the choice of alpha. For any value of alpha it is possible to write the coefficient matrix for the system of equations. The eigenvalues are the roots of the characteristic equation, formed by the operation

$$\det(\bar{A} - \lambda\bar{I}) = 0$$

where,

$\bar{A}$ = the coefficient matrix

λ = the eigenvalue vector

$\bar{I}$ = the identity matrix.

Stability of the system depends on the real parts of all eigenvalues being negative. Thus examining the signs of the real parts of the eigenvalues as a function of alpha determines the limit of stability. Except for the cases alpha = 0.0, alpha = 1.0 and the case alpha = 0.5, $N = \infty$, finding the eigenvalues is a difficult analytic problem, so a numerical approach was used.

Figure 3.5 is a root locus plot for the PIPE model, with N = 5 and alpha varying between zero and one. For alpha equal to zero there are five identical negative real roots. As alpha is increased the roots

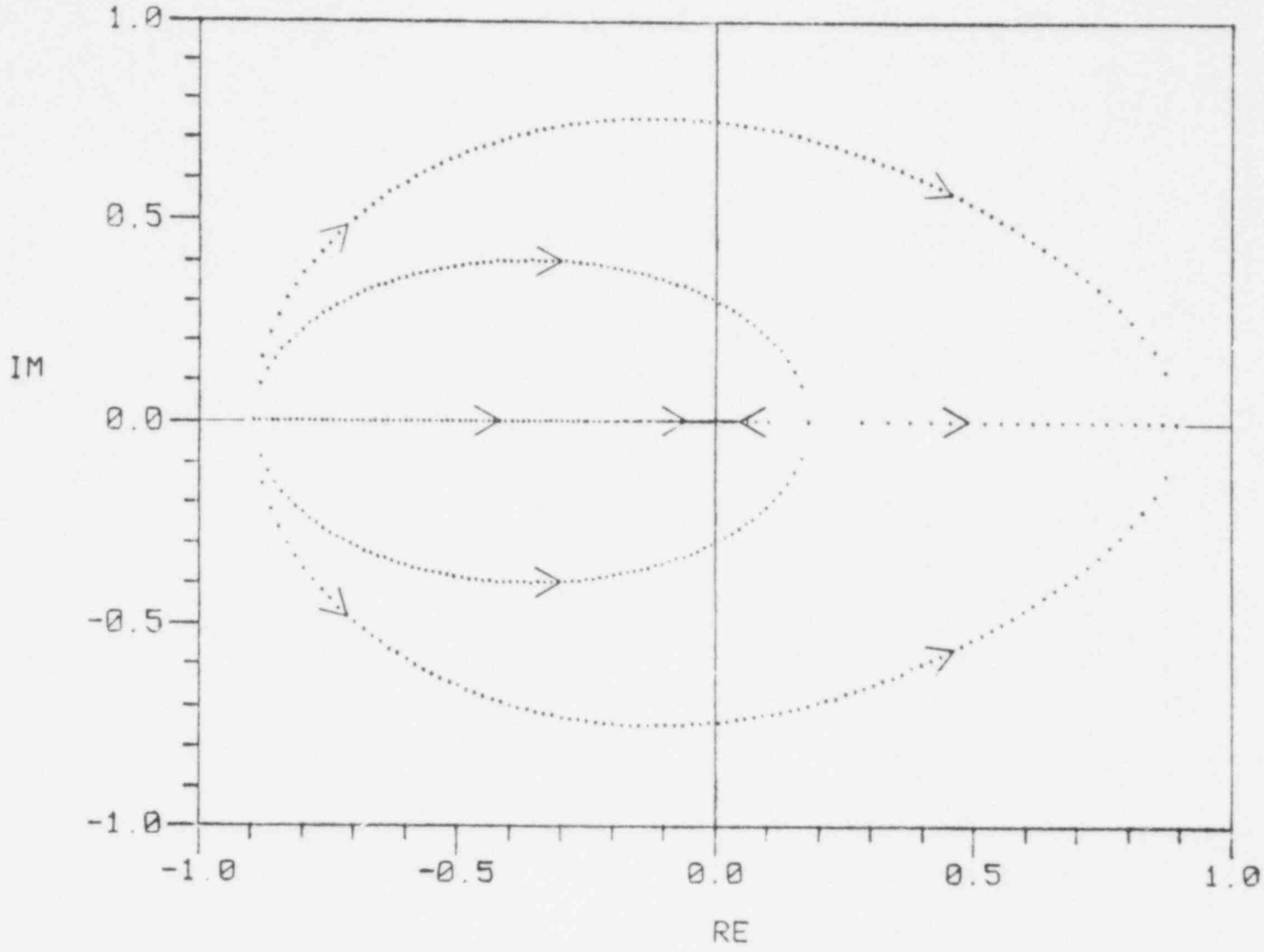


Figure 3.5 - Root Locus Plot for N=5, $0 \leq \text{alpha} \leq 1$

move toward the imaginary axis as one real root and two complex conjugate pairs. At the point where they cross the imaginary axis into the right half plane, the PIPE model becomes unstable.

Using an iterative procedure this value was found to be alpha equal to 0.537. This was then repeated for various values of N. Figure 3.6 is a plot of the "critical" alpha vs. N. For stability alpha must be less than this "critical" value. As N goes to infinity the Jacobian matrix for alpha equal to one-half is skew-symmetric. The eigenvalues for a skew-symmetric matrix are always purely imaginary. Thus as N goes to infinity the "critical" value of alpha approaches one-half. This defines the acceptable range of values on the basis of stability and it is this range of values that can be examined with respect to accuracy. An important consequence of this stability analysis has a direct application in power plant simulation. When flow reversal occurs in a flow path the spatial finite difference must be reversed to reflect this fact. Thus the difference approximation changes as the flow direction changes so that the system stability may be maintained.

Two criteria are used in this paper to judge the accuracy of a particular method. First a common measure of accuracy is applied to the outlet of the pipe. The total error is defined as

$$E_{rr}(t) = \left[\frac{1}{t}\int_o^t (T_{out} - T_{an})^2 dt'\right]^{\frac{1}{2}}$$

The error is equal to the square root of the average square of the difference between the outlet temperature and the analytic solution, at an arbitrary time t during the simulation.

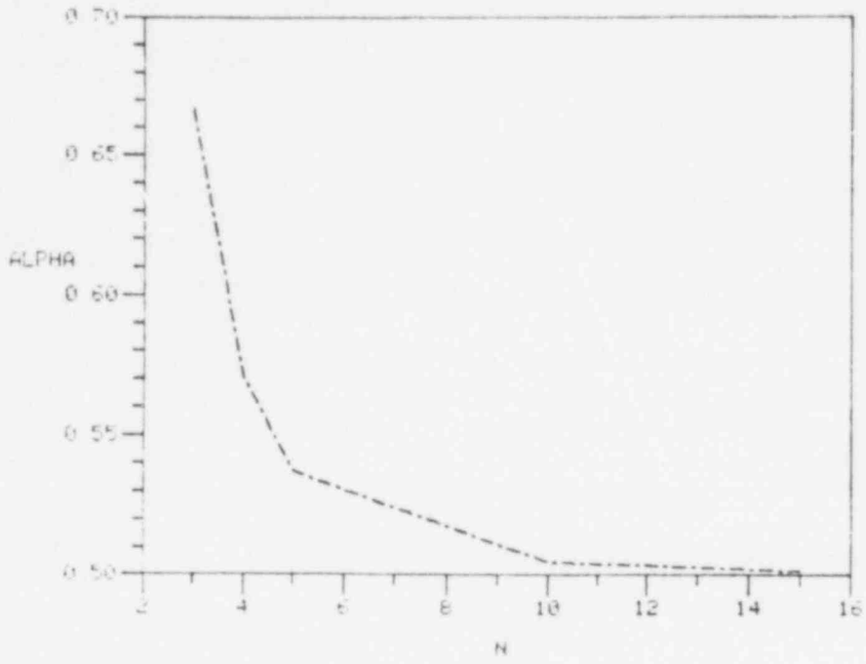


Figure 3.6 -Critical Value of Alpha vs. Total Number of Zones.

The Stable Region is below the Curve

For the purposes of comparison the inlet function is a sine wave one period in length, and the time of the simulation is constant for all values of alpha. The results of this error analysis are presented in Figure 3.7. It can be seen that the error increases as alpha decreases with the upper curve being for alpha equal to zero. The value of Err(t) can be interpreted as the time integrated error in temperature averaged over the length of the simulation.

The second measure of error employed is the degree to which a model conserves energy. Referring to eqs. (2.16) and (2.17), the energy conservation ability of a method may be judged by comparing values from these two equations. This ratio is plotted in Figure 3.8.

The model that conserves energy the best is the model with alpha equal to zero. As alpha increases, the energy conservation gets worse. This result is slightly surprising because it means the method that conserves energy the best does the worst job of predicting outlet temperatures. It should be noted that all of the methods conserve energy to a high degree.

The next section will present results obtained from BRENDA when these models were used.

3.3 BRENDA Results

Two models were implemented in BRENDA for comparison. The PDELAY model was used throughout in one simulator and in the second simulator the PIPE model was used in all of the sodium pipes. The BRENDA with PDELAY is a fifty-seventh order system. BRENDA with PIPE models in the sodium pipes is a hundred and forty-first order system. The

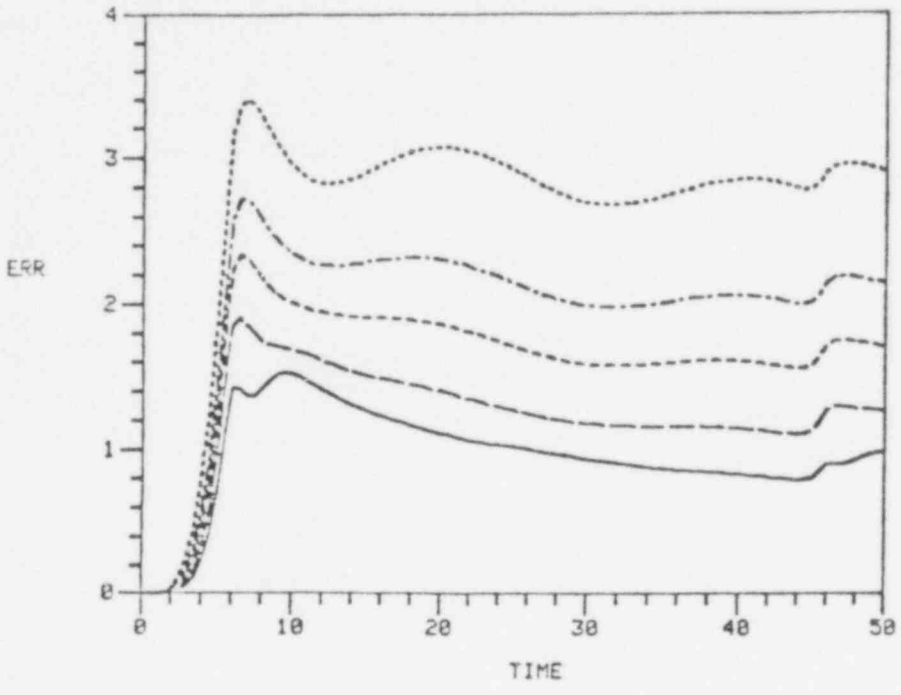


Figure 3.7 - Difference from Analytic Solution vs. Alpha, Err in F, Time in Seconds

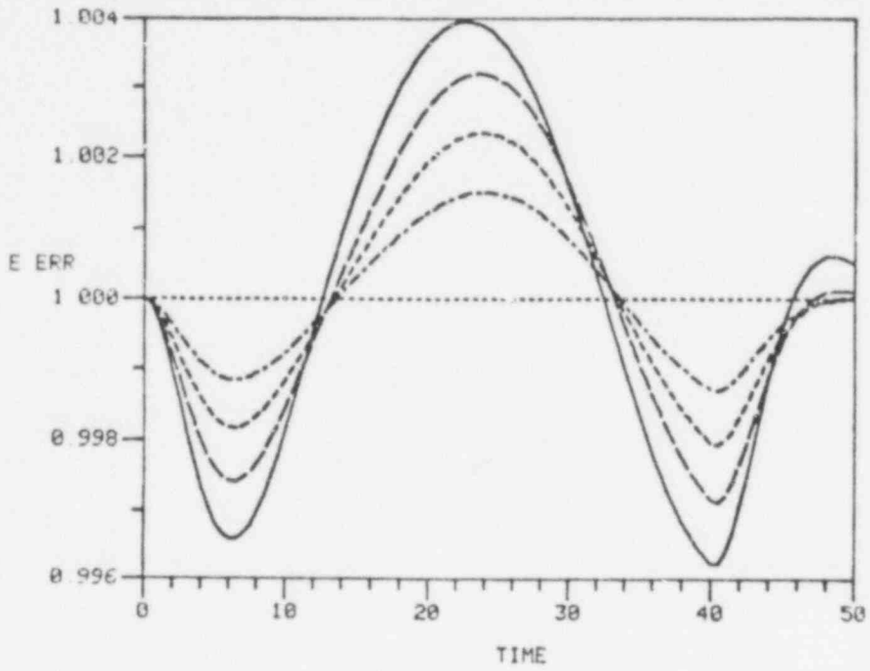


Figure 3.8 - Energy Conservation vs. Alpha

region sizes in the PIPE models were chosen such that the time constant, tau, for a region was approximately one second. The two models were then run for various transients. Results are shown in Figure 3.9. for a ten cent reactivity step at time zero. In these figures the dotted line is BRENDA with the PIPE models in the sodium loops and the solid line is BRENDA with the PDELAY model throughout. It can be seen that the differences in the results from the two models are very small. The most significant difference is seen in Figure 3.9a. In the graph of TS9 the PIPE models have a damping effect on the temperature. This is a result of the PIPE model that can be minimized by increasing the number of regions.

The two models were both integrated with a Runge-Kutta-Merson variable step method with a simulation time of one hundred seconds and with an implicit method, EPISODE, with a simulation time of twenty seconds. Execution times for the models on a CYBER 175 are summarized in Table 3.1. As is expected the difference in execution times is greater with the implicit method than with the explicit method.

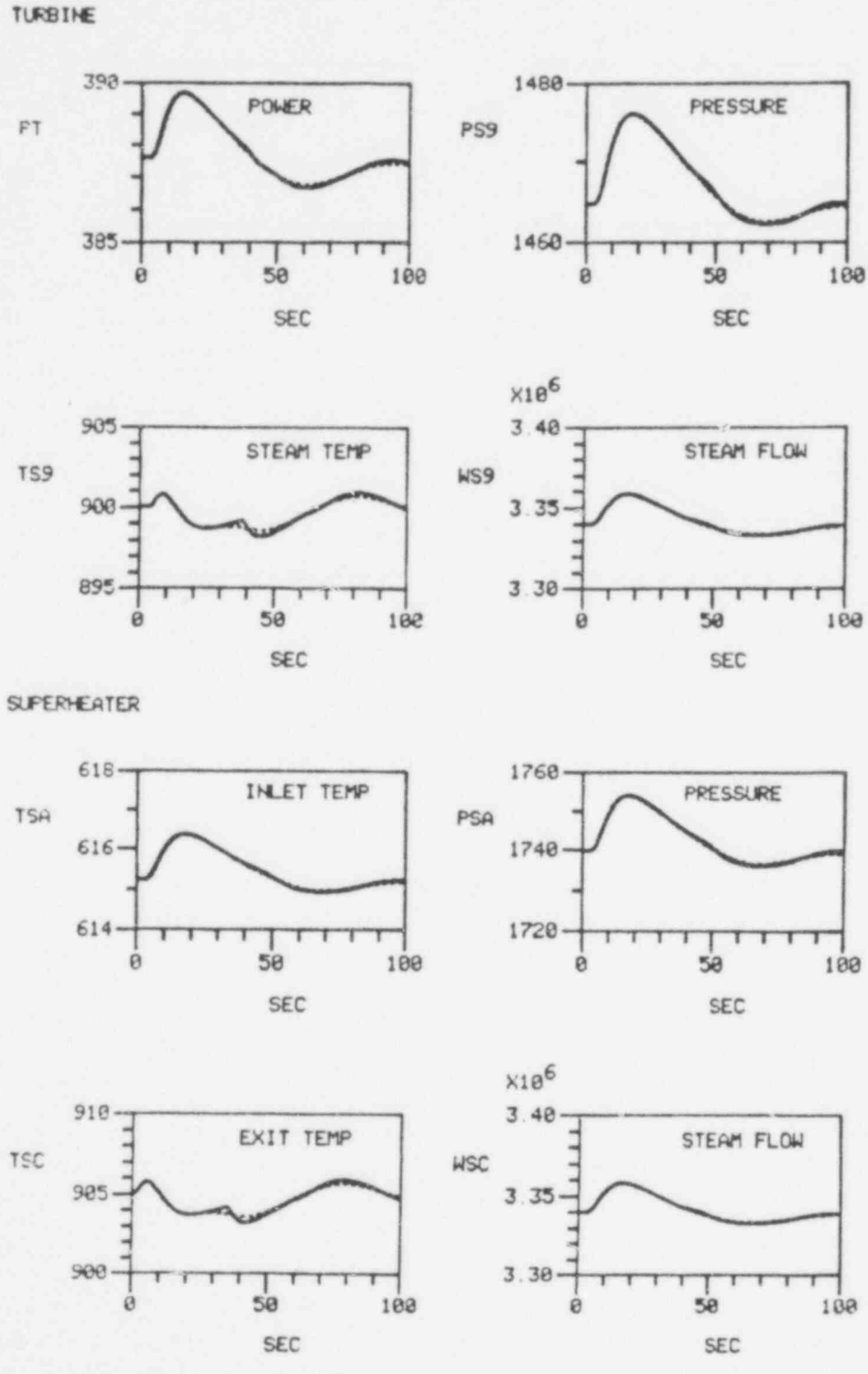


Figure 3.9 - Response of BRENDA to a Ten Cent Reactivity Step

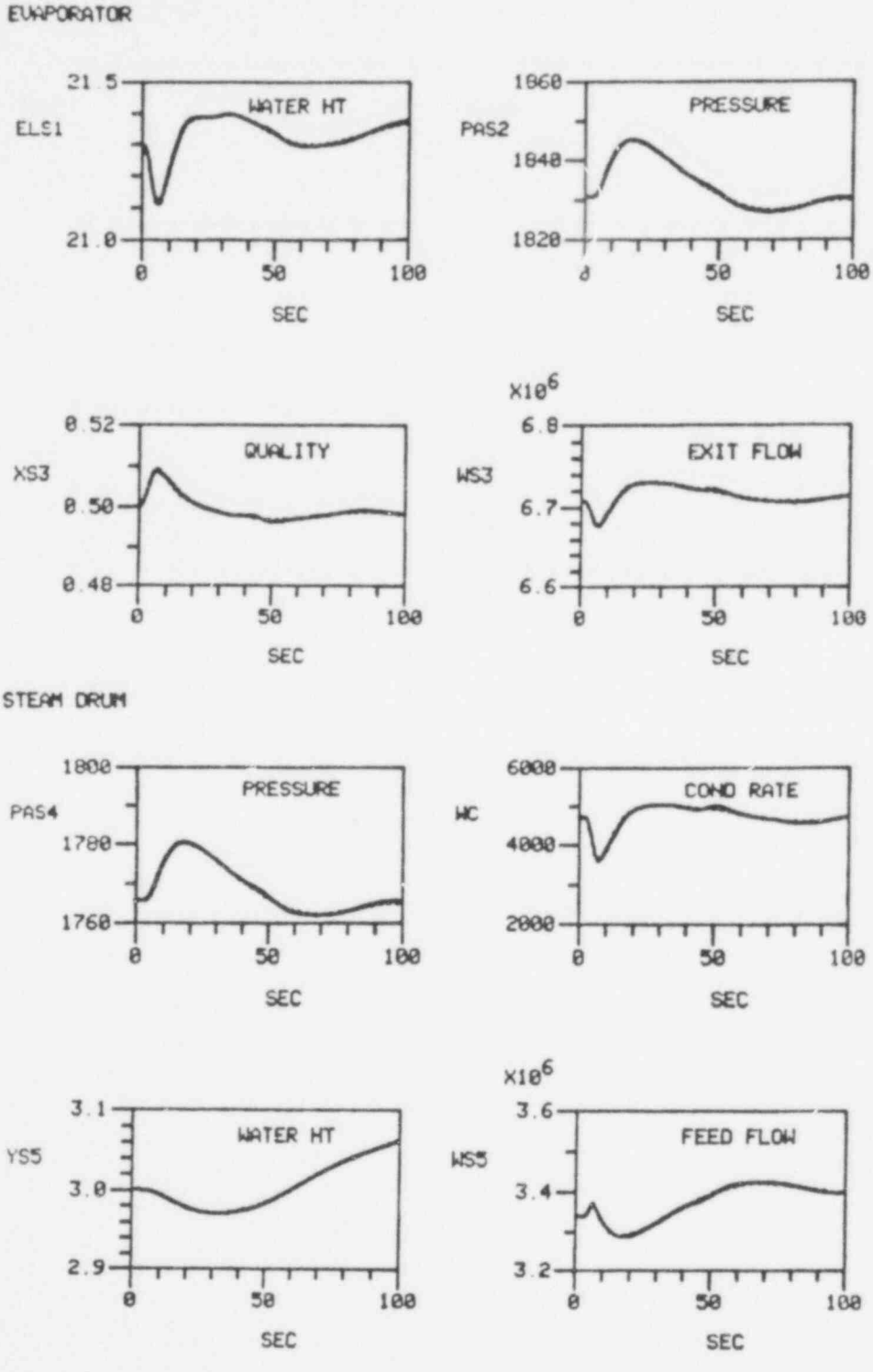


Figure 3.9, Continued - Response of BRENDA

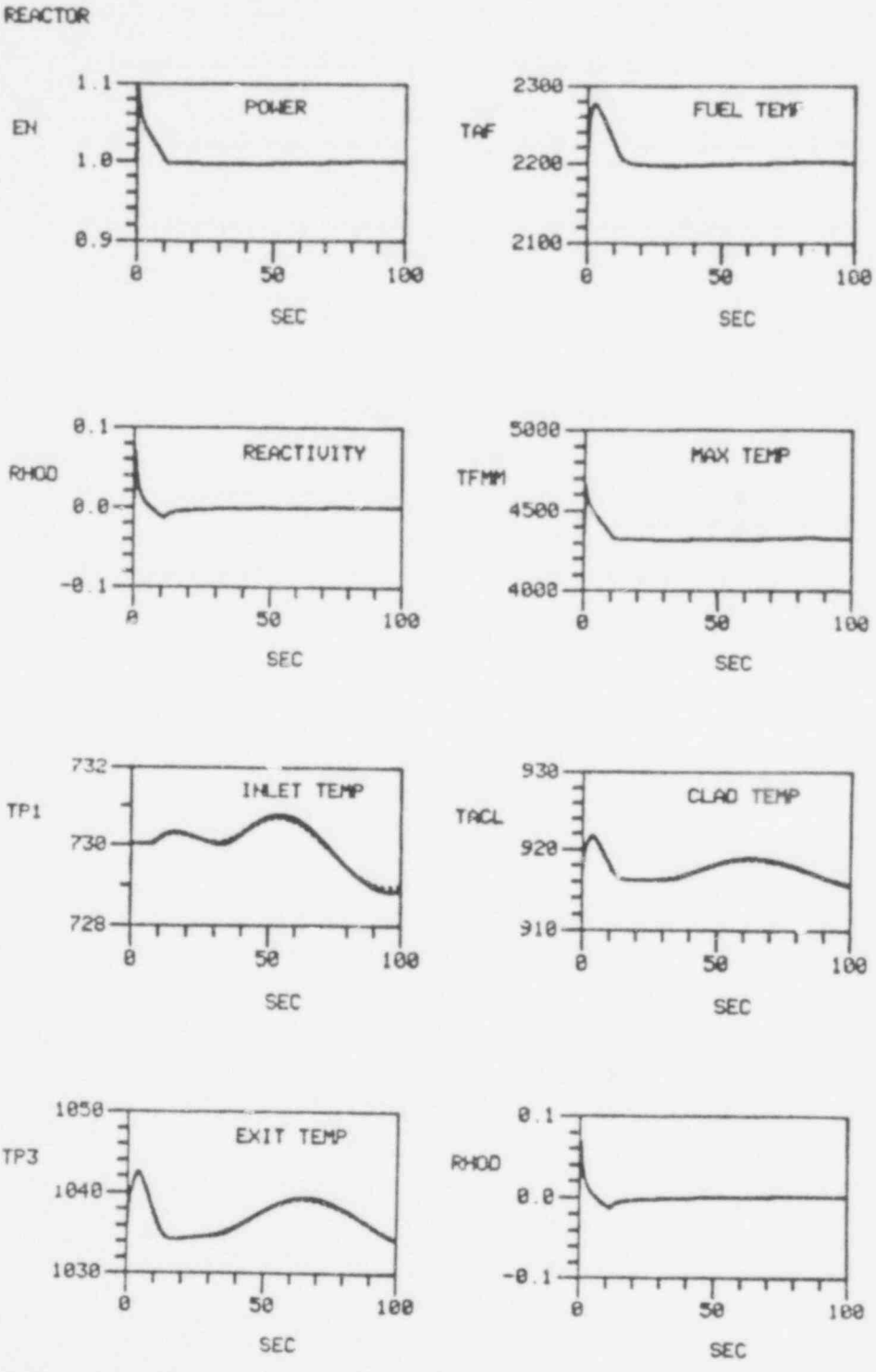


Figure 3.9, Continued - Response of BRENDA

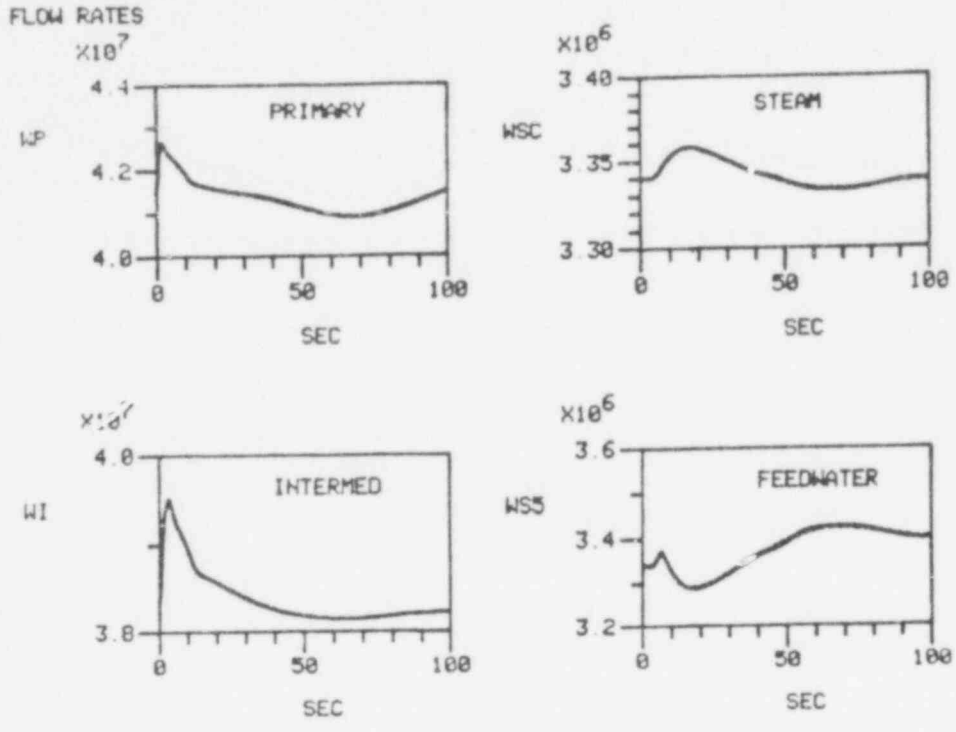


Figure 3.9, Continued - Response of BRENDA

Table 3.1 Comparative Execution Times on the CYBER 175, in Seconds

Integration Method (Model)	RUNGE-KUTTA MERSON TMAX=100	EPISODE TMAX=20
BRENDA-PDELAY	76.3	138.6
BRENDA-PIPE	86.0	353.5

CHAPTER 4

CONCLUSIONS AND SUMMARY

The intent of this paper was to show that PDELAY could be implemented in a low order power plant simulator with little change in accuracy, and a significant savings in execution time. Results showing that this was the case were presented in Chapter 3. It should be emphasized that the savings in execution time is a conservative estimate. If all of the PDELAY operators had been replaced with PIPE models instead of only those for the sodium pipes, a greater difference would have been seen. Another advantage of using PDELAY in a simulator is the savings in core storage. The PDELAY simulator required three quarters of the core that the PIPE simulator required.

These are the major conclusions of this paper and they reflect the primary goals. However, during the course of this research other conclusions were reached that affected the method of solution used in the PIPE model.

The finite difference method used for the pipes in BRENDA was the method with alpha equal to zero, or a backwards difference approximation. There is one major reason why this approximation was used: the solution does not oscillate. There are three reasons why this behavior is undesirable. First, it is not physically reasonable to expect the exit temperature to be higher than the inlet temperature. Second, this

behavior would be interpreted by the controllers as real and they would act accordingly. Thus, the model would initiate a controller action which would be entirely unreal. Third, to introduce a nonphysical behavior into a simulator may cause unpredictable results.

These facts, plus the fact that the average increase in error is on the order of only 4% when alpha is zero as opposed to one-half, make the backwards difference model the most desirable.

APPENDIX A

ALTERNATE DERIVATIONS

There are, of course, many ways to derive the equations used in PIPE. There are so many, in fact, that equating the many approaches taken in the literature is sometimes a non-trivial task. This appendix is intended to illustrate how some derivations are special cases of the general alpha difference presented in Chapter 2, and how others are equivalent to it.

The most common derivation is one based on a control volume with an inlet and an exit. The rate of change of internal energy is equal to the energy flow in minus the energy flow out. Referring to Figure A.1 this can be written as

$$mC_v \frac{d\bar{T}n}{dt} = \dot{m}C_p T_{n-1} - \dot{m}C_p T_n \qquad \text{A.1}$$

where the variables are defined as usual except

$\bar{T}_n$ = the average temperature in region n.

T_{n-1} = the inlet temperature to region n.

T_n = the exit temperature from region n.

Again assuming invompressibility, and defining

$$T_n = \emptyset.5(\bar{T}_n + \bar{T}_n)$$

eq. (A.1) may be written

$$\frac{d\bar{T}_n}{dt} = \frac{-v}{\Delta z}[\emptyset.5\bar{T}_{n+1} - \emptyset.5\bar{T}_{n-1}] \qquad \text{A.2}$$

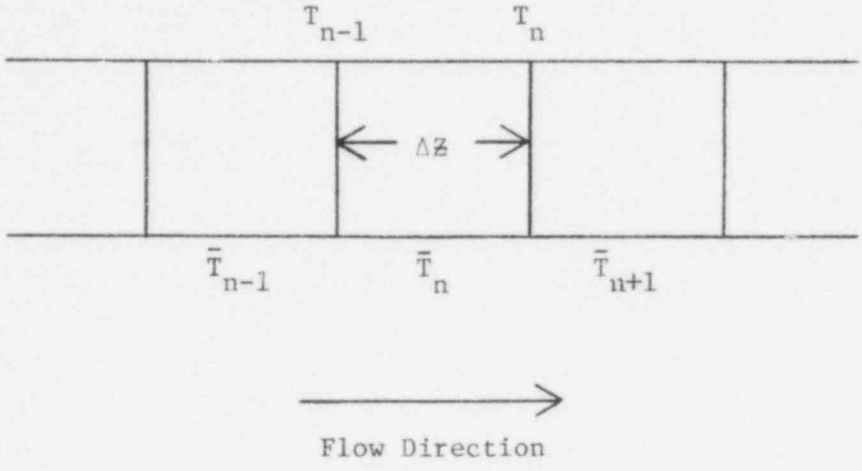


Figure A.1 - Finite Difference Notation

This is exactly the same result that can be obtained from eq. (2.14) by using alpha = 0.5, remembering that in eq. (2.14) the T's are region temperatures, not boundary temperatures. This is the central difference approximation. This type of analysis is sometimes carried one step further by assuming the rate of change of the outlet temperature is equal to the rate of change of the average temperature, or

$$\frac{dT_n}{dt} = \frac{d\bar{T}_n}{dt} \qquad \text{A.5}$$

Realizing that n is simply a reference variable and using eq. (A.5), eq. (A.1) may be rewritten as

$$\frac{dT_n}{dt} = \frac{V}{\Delta z}[T_n - T_n - 1]. \qquad \text{A.6}$$

This result can be obtained from eq. (2.15) by using alpha = 0, and is the backward difference approximation.

Another approach (Agrawal et al., 1977, p. 488) which leads to a result similar to that in Chapter 2, is to define an average temperature.

$$T_n = \frac{1}{\Delta z} \int_{n-1}^{n} T(z)dz \qquad \text{A.7}$$

The average temperature is then the state variable in the equation

$$\frac{d\,T_n}{dt} = \frac{-V}{\Delta}[T_n - T_{n-1}]. \qquad \text{A.8}$$

They then define the average temperature to be a linear combination of the end point values,

$$T_n = \alpha T_{n-1} + (1 - \alpha)T_n \qquad \text{A.9}$$

This approach is also equivalent to that presented in Chapter 2, differing only in point of view. In Chapter 2 the position of the time derivative is fixed, and the position of the space derivative is varied. Here, the space derivative is fixed, and the position of the time derivative is varied.

There are other ways to derive the differential equations, but these three illustrate how some common approaches are related.

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16. ABSTRACT (200 words or less)

Two piping models intended for use in a digital power plant simulator are compared. One is a finite difference approximation to the partial differential equation called PIPE, and the other is a function subroutine that acts as a delay operator called PDELAY. The two models are compared with respect to accuracy and execution time. In addition, the stability of the PIPE model is determined.

The PDELAY model is found to execute faster than the PIPE model with comparable accuracy.

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