Continued Care of Uranium Mill Sites: Some Economic Considerations

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by

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#### Introduction

The New Mexico State Legislature recently enacted legislation (12-9-1 to 12-9-12 NMSA 1953) expanding the authority of the State Environmental Improvement Agency to regulate uranium mills in the state. In particular, the Agency was authorized to require payment from all licensed mills in the state into a continued care fund. The purpose of this fund is to provide for the maintenance, in perpetuity, of the mill sites after decommissioning, especially the tailings pile. The Agency may require each mill to contribute up to 10 cents per pound of yellowcake  $(U_3O_8)$ , until a total of \$1,000,000 has been deposited by that mill.

In this paper it will be argued that even the maximum amount is not likely to generate an income stream which would support an adequate maintenance program. This is a parable of the futility of writing dollar amounts into legislation in the presence of persistent inflation.

In the first section I compute an estimated annual maintenance cost for a typical 5,000 ton per day (tpd) mill. In the second section I compute that mill's contribution to the continued care fund, and compare it to the minimum endowment necessary to generate revenues sufficient to cover those costs. In the third section we will discuss the calculations.

#### I. The Cost of Continued Maintenance

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A uranium mill processes are to remove uranium oxide  $(U_{3}O_{8})$ , which is then sent to the enrichment plant for further processing. Typical ore grades are on the order of 0.2 percent (as  $U_{3}O_{8}$ ), which means that on a tonnage basis virtually all (99.8 percent) the ore processed is waste, or tailings. The mills are quite efficient at removing uranium, as over 95 percent is captured. However, a significant amount of radioactive material remains in the tailings, principally due to radium and thorium. Inasmuch as radium has a half-life of 1,600 years, the piles will continue to emit radiation on a time scale which is for practical purposes an eternity. This radiation is a threat to the health of future generations; hence the need for continued care.

At each mill site the continued care fund must be sufficient to meet the following expenses:

- (i) <u>Fencing</u> It is assumed in this paper that a fence will surround the tailings pile. This fence must be replaced when it wears out. In addition, sections may have to be repaired or replaced annually, due to washouts, theft, etc.
- (ii) <u>Monitoring and Repair</u> On a regular basis, perhaps once or twice a year, the site will have to be inspected, and repairs made if the structural integrity of the pile has been compromised by erosion, animals, or other cause.

Groundwater quality will also be monitored. Although migration of radioactivity from the site into nearby aquifers is considered by most experts to be very unlikely, the possibility must not be discounted. Otherwise there would be no need to monitor groundwater. Correction of such a problem could be achieved at enormous expense, if at all.

- (iii) <u>Emergency</u> In case of natural disaster (earthquake, flood, etc.), repairs to the pile may be necessary.
- (iv) <u>Unanticipated problems</u> By definition, at this time it is impossible to say what those problems might be. Suffice it to say that at some existing stabilized sites (such as Monticello, Utah) problems have been encountered which were not anticipated at the time of stabilization.

The most extensive work on the problem of stabilization and maintenance of tailings piles has been done by Ford, Bacon, and Davis - Utah, Inc. FB&DU has completed a series of cost studies on the cost of stabilizing a number of abandoned tailings piles throughout the West. Reports have been published ("Phase II - Title I Engineering Assessment of Inactive Uranium Mill Tailings") for about fifteen such sites. In addition to estimating the investment cost of various stabilization alternatives, these reports also estimated the annual cost of maintenance. FB&DU maintenance cost estimates for four abandoned sites on the Navajo Reservation are given in Table I.

## Table I

### Estimated Maintenance Costs for Four Abandoned Tailings Piles on the Navajo Reservation

(Source: Ford, Bacon, and Davis Utah, Inc., "Phase II - Title I Engineering Assessment of Inaction Uranium Mill Tailings," Four Reports, USERDA, Grand Junction, Colorado, 1977.)

	Mexican Hat, Utah	Monument Valley, Utah	Tuba City, Arizona	Shiprock, New Mexico
Total site area (acres)	555	90	88	230
Tailings pile area (acres )	68	30	22	72
Tailings pile mass (10 <sup>6</sup> tons)	2.2	1.1	0.8	1.5
Tailings pile height (ft)	3-14	55 (max.)	16 (avg.)	14-40
Annual maintenance cost	\$12,000-15,000	\$8,000-10,500	\$20,000-24,000	\$14,000

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A telephone conversation with FB&DU engineers established that the major elements in the annual maintenance cost were the cost of fencing and the cost of repair of the pile itself (recontouring). It was estimated that fencing would deteriorate and require replacement every ten to fifteen years, at an installed cost of about \$10 per linear foot. In their opinion, the fencing cost would be the only significant cost at a new tailings pile. The relatively high estimated annual cost of recontouring at the abandoned sites was due to the fact that the piles had not been designed properly. Presumably, future piles will be designed in such a way as to be much more stable structurally. Based on their comments I concluded that the annual cost of monitoring and occasional repair of a site using state-of-the-art design methods might be as low as \$2,000 per year.

This is one reason to expect continued care expenses to be somewhat less at newly designed sites than at old ones. On the other hand, the new sites are likely to be much larger: about 33 million tons for a 5,000 tpd plant (see Table III) rather than one or two million tons typical of the sites in Table I. The new sites will therefore be much larger in area, and therefore will have a greater perimeter, which of course is directly proportional to the cost of fencing.

Table II gives data on the five uranium mills now in operation in New Mexico. The average capacity of these mills is approximately 5,000 tpd; apparently, future mills will also be approximately that size. As shown, the tailings piles are considerably larger than those examined by FB+DU. Moreover, with another decade to go before closing down, they will become larger still. 806 332

#### Table II

#### Some Characteristics of Uranium Mills Currently in Operation in New Mexico

(Source: New Mexico Environmental Improvement Agency)

<u>M111</u>	Capacity Expected		Tailin	gs Pile			
	<u>tpd</u> Remaining Life, Years	Mass (10 <sup>6</sup> tons)	Max.Height (ft)	Area (Acres)	Hypothetical Fenced Area*		
Anaconda	6,000	9	17	24	270	4000 X 3000 ft.	
Kerr-McGee	7,000	1	23	100	265	4200 X 3000 /t.	
United Nuclear- Nomestate	3,400	7	19	70	150	2000 X 4000 ft.	
Sohto	1,500	15	estimated	maximum area =	160 acres		
United Nuclear- Churchrock	4,000	17	estimated	maximum area =	160 acres		

The maximum length and width of the existing pile, estimated from maps.

\*

Let us turn now to the problem of estimating the annual continued care expense for a hypothetical 5,000 tpd mill. At a capacity factor of 90 percent, such a mill will produce 33 X 10<sup>6</sup> tons of tailings over the course of a 20-year lifetime. The problem is, how do we convert that mass of tailings into a perimeter, needed for the estimation of fencing costs? It can be done, but a few assumptions about the shape of the pile are required.

We begin with an observation from geometry: given any two similar solids, the ratio of the areas of corresponding faces is proportional to the two-thirds power of the ratio of volumes. In symbols, if A is area and V is volume,

$$A = b \nabla .^{2/3}$$
 (1)

Now assume that our representative tailings pile has an area-volume relationship equal to the average of the top three entries in Table II. For this average, A = 228 acres, and  $V = 20 \times 10^6$  tons. Using (1), we find that b = 31 (when A is measured in acres and V in millions of tons). Substituting a mass of 33 X 10<sup>6</sup> tons into (1) yields an area of 320 acres.

The relationship between area and perimeter again depends on shape. Loosely speaking, the more closely a figure approximates a circle, the smaller its perimeter. But tailings piles are desiccated impoundments, and as anyone will agree who has ever looked at an aerial photograph of an artificial lake, their shapes are very irregular indeed. Minimizing the amount of fence needed means that some additional area not covered by tailings must be contained. It is assumed to be 10 percent. Loosely, the perimeter depends mostly on the ratio of length to width; the exact shape of the figure is not so important. Here it is assumed that the area can be enclosed by a rectangle whose width is one half its length.

These calculations are presented in Table III, whereupon we find a perimeter for fenced area of 16,800 feet. Assuming fencing costs of \$10/linear foot (FB&DU typical value), a fence lifetime of 12 years, and an annual monitoring/repair cost of \$2,000, we have an annual continued care expense of \$16,000. This estimate does not include an allowance for surprises or emergencies, so it may well be a low estimate.

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Calculation of Annual Maintenance Expense for Tailing File at Expothetical 5,000 tpd Plant, 20 Year Lifetime

1. Total annual tailings production = 5000 tons I 330 day = 1.65 X 10<sup>6</sup> tons/yr. day year

(90 percent capacity factor)

2. Lifetime tailings production = 20 X 1.65 X 10° = 33 X 10° tons.

3. Estimated area of tailings pile =

32 X (33) 2/3 acres = 330 acres = 14.3 X 10<sup>6</sup> sqft. (see text) Estimated quantity of fence required:

Lasumptions:	(1)	Fenced	Area :	- 1102	of pr	ile area	
	(2)	Fenced	Area a	a rec:	angle	s I 2s	
fenced Area =		14.3 X	1.1 =	15.7	X 10 <sup>6</sup>	sqft.	

2800 ft.

Perimeter = 6 s = 16,800 ft.

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5. Fencing cost at \$10/linear foot = \$158,000 Lifetime of fence = 10-15 years; assume 12. Hence on an average basis the cost of fencing is \$14,000. Fencing cost and life span assumptions were made after telephone conversations with Ford Bacon and Davis engineers.

 Annual monitoring and maintenance cost (exclusive of fance) was taken to be \$2,000/year.

7. Total annual cost: \$14,000 + 2,000 = \$15,000.

Note: It is explicitly assumed that revegatation will not be required. Revegetation may raise costs to \$50,000 per year (G.J. Boyle, " A Proposal for Financing the Stabilization and Maintenance of Uranium Mill Tailings," Department of Economics, University of New Mexico, Albuquerque, May 1976.

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#### II. Is the Continued Care Fund Adequate?

If the revenues generated by the continued care fund are successfully to cover its requirements, then <u>at a minimum</u> the contribution of each mill must generate revenues sufficient to meet the annual routine maintenance expenses of that mill's tailings pile.

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At the maximum levy of 10 cents pc. pound of  $U_3O_8$ , our representative 5,000 tpd mill will contribute \$660,000 to the fund in its initial year of operation and \$340,000 its second year. At that point the statutory limit of \$1,000,000 will be reached, and the mill will make no further contributions. This fund is then invested by the state treasurer as he invests other state funds.

At present, state monies are invested in state banks at interest rates equal to that offered by Federal securities.  $\frac{1}{2}$ Thus, short-term investments are made at the rate of U.S. Treasury bills, while longterm investments are made at the U.S. Treasury Note rate. Since the continued care fund is not to be used for twenty years, when the mill is abandoned, presumably the funds collected will be considered long-term investments. Recently, the average yield of Treasury Notes has been about 7 percent.<sup>2/</sup> If the continued care fund contribution of the representative mill is invested at this interest rate, its balance in twenty years will be \$3.5 million, which at 7 percent interest generates an annual income of \$247,000.

Personal communication, Mr. Davidson, New Mexico State Treasurer's Office, January 4, 1978.

<sup>2/</sup> U.S. Department of Commerce, Survey of Current Business, p. S-44, October 1977.

It would appear at first glance that this sum is enough to maintain the pile in fine style, but, due to inflation, appearances can be deceiving. How large an endowment is required to be able to pay one mill's annual maintenance expenses without eating into the principal?

Suppose

Kt is the size of the endowment at the end of year t, Ct the maintenance expense during year t, r the rate of interest, and

i the rate of inflation.

We have the following relationships:

- (1)  $C_t = C_{t-1} (1+i) = C_0 (1+i)^t$
- (2)  $K_t = K_{t-1} (1 + r) C_r$

If the purchasing power of the endowment is to remain constant, then we must have  $K_t / C_t$  a constant for all t. If so, then we have a third condition:

$$K_t = K_{t-1} (1 + i),$$

and therefore

$$K_{t-1} (1 + i) = K_{t-1} (1 + r) - C_{t-1} (1 + i),$$

so that

$$\frac{C_{t-1}}{K_{t-1}} = \frac{r-i}{1+i}, \text{ or } K_t = C_t \left(\frac{1+i}{r-i}\right) \text{ for all } t.$$

In other words, the endowment must be at least (1 + i) / (r - i)times as large as the annual expense: otherwise the purchasing power of the endowment will eventually be driven to 0. (This fact is discussed further in the Appendix.)

Note that when i = 0, we have C/K = r, the rate of interest. This suggests that we may consider (r - i) / (l + i) the "real" rate of interest, measuring the return to the endowment not in current dollars but in purchasing power. This value is not very sensitive to the absolute value of r or i, but to the difference between them.

In order to determine the minimum acceptable value for X, we need to know what future rates of interest and inflation will be. That, of course, is impossible, but by looking at the past we might be able to make an intelligent guess. In Table IV are twenty-year histories of the Treasury Bond rate, the overall rate of inflation, as measured by the GNP deflator, and the rate of increase in construction costs, as measured by the ENR Building Index. There is a palpable break in these series between the first decade, characterized by low interest and inflation rates and relative tranquillity, and the most recent decade, characterized by high and very volatile inflation rates, and higher interest rates.

Occasionally it is said that the interest rate for a security is made up of three elements: time preference, plus the rate of inflacion, plus a risk premium (which is zero for the securities we are speaking of, U.S. Treasury Notes). This is of course simplistic and can be misleading.

## Table IV

Year	(1) Treasury Bond Yield	(2) % Increase in GNP Deflator	(3) Z Increase in ENR Building Index	(4) (1)-(2)	(5) (3)-(2)	(6) (1)-(3)
77	6.94	4.96	6.78	1.98	1.82	0.16
76	6.78	5.27	9.10	1.51	3.83	-2.32
75	6.98	9.62	8.41	-2.64	-1.21	-1.43
74	6.99	9.54	5.87	-2.55	-3.67	1.12
73	6.30	5.92	8.53	0.38	2.61	-2.23
72	5.63	4.14	10.46	1.49	6.32	-4.83
71	5.82	5.10	12.96	0.72	7.86	-7.14
70	6.59	5.11	5.70	1.48	0.59	0.89
69	6.10	5.27	9.57	0.83	4.30	-3.47
68	5.25	4.49	7.38	0.76	2.89	-2.13
67	4.85	2.94	3.24	1.91	0.30	1.61
66	4.66	3.28	3.80	1.38	0.52	0.86
65	4.21	2.78	2.43	1.43	-0.35	1.78
64	4.15	1.56	2.97	2.59	1.41	1.18
63	4.00	1.47	2.42	2.53	0.95	1.58
62	3.95	1.83	2.09	2.12	0.25	1.86
61	3.90	0.89	1.56	3.01	0.67	2.34
60	4.01	1.70	2.12	2.31	0.42	1.89
59	4.07	2.21	4.14	1.86	1.93	-0.07
58	3.43	1.60	3.14	1.83	1.54	0.29
57	3.47	3.37	3.65	0.10	0.28	-0.13

Comparison of the Treasury Bond Yields the Rate of Inflation and the ENR Construction Cost Index

Average Difference

67-77	0.53	2.33	-1.80
57-66	1.92	0.76	1.15

For one thing, it is a concept which considers only the supply of investment funds; an investor would certainly like to get a return equal to the above sum, but the demand for funds may not allow it. Besides, an investor is interested not in the current inflation rate but in his expectations of the future. The insight as well as limitation of this conception are suggested in Table IV. As shown, the rate of interest exceeded the overall rate of inflation by one to three points except for a period in the early seventies, when the rate of inflation we very high (col. (4)). It is plausible that during the early seventies long-run inflation expectations were sufficiently below actual inflation for it to exceed the interest rate for a couple of years. In the earlier decade, the expectations of inflation were much more closely in line with then current values, and the difference between interest and inflation rates was much closer to true time preference. Today, it is widely anticipated that inflation will continue at an annual rate of 5 or 6 percent for at least another decade. If so, perhaps the earlier, more stable relationship between interest and inflation will reassert itself. That is the assumption here. If inflation 5 percent per year, the rate of interest will continue to is be 7 percent per year.

But obviously, goods do not change prices at the same rate, which means that the changes in purchasing power depend considerably on the good to be purchased. In the present instance we are interested in the changing price of construction services -- earth

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moving, fence installation, and the like. Of the many construction price indices available we choose the ENR Building Cost Index. A virtue of this index is its availability -- it is reported monthly in the Commerce Department's <u>Survey of Current Business</u>. It has one serious flaw which apparently it shares with other construction indices: it measures increases in the price of inputs (steel, lumber, cement, skilled labor), instead of outputs. Thus, it does not register technical innovations which allow fewer inputs to be used, and neither does it record the extra construction costs which may be imposed, say, for environmental reasons. Nonetheless, for lack of anything better the ENR Building Cost index will be used, though it is recognized that it may impart a slight upward bias to construction cost increases.

From Table IV it is apparent that over the past twenty years the cost of construction has increased faster than the overall rate of inflation, especially in the past decade. Why construction cost increases should have accelerated in this period is not clear. In the 1957-1966 decade, when economic relationships were apparently more stable, construction costs increased about one point faster than the overall price level.

Consider now two cases, one in which the rate of inflation is 6 percent and the other in which the rate is 5 percent (for both the interest rate-is 7 percent). How big an endowment is necessary to generate an income sufficient to cover the costs estimated in section I?

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#### Case I. 1 = 0.06

The cost in 1977 dollars of maintenance is assumed to be \$16,000. In twenty years, the cost will be

$$C_{20} = \$16,000 (1.06)^{20} = \$51,300 (1997 dollars).$$
  
 $K = C\left(\frac{1+i}{r-i}\right) = \$51,300 (106) = \$5,400,000.$ 

But the cash balance in 1997 of the mills' contribution to the continued care fund is \$3,500,000. The fund would be exhausted after paying expenses for about 110 years (see Appendix). For it to be 5,400,000, in 1977 about \$1,400,000 (in 1977 dollars) would be required.

#### Case II. 1 = 0.05

Again, annual costs are \$51,300 in 1997 dollars. The endowment required at that time is

$$K = C\left(\frac{1+1}{r-1}\right) = \$51,300 (53) = \$2,7000,000, (1997 dollars)$$

which is easily covered.

However, consider a mill opening in 1985. The size of the continued care fund contribution remains the same in current dollars: \$1,000,000 held for twenty years at 7 percent. This time, however, the mill is not abandoned until 2005. The annual maintenance cost in that year is \$81,800, in 2005 dollars. The endowment required at that time is

(\$81,800) (53) = \$4,300,000 (2005 dollars).

Any mill opening after about 1982 will not contribute funds sufficient to provide maintenance expenses in perpetuity. This is no accident. The way the law is written, as long as the inflation rate is positive, there will come a year when the continued care funds collected from a mill opening in that year will not cover the cost of maintenance. Such is the power of the exponential function.

#### III. Conclusions

Two rather obvious points emerge from the analysis of the preceding two sections. In the first place, our information ase could under no circumstances be described as ample; accordingly, the estimates made in the preceding two sections are quite prome to error. For the estimate of annual maintenance cost, two assumptions appear to be especiall crucial. It was assumed that the tailings' pile would have to be fenced off. If this is not a necessity, then apparently the annual maintenance cost can be cut to a fraction of the value estimated here. On the other hand, since there was no way of estimating the annual cost of emergencies or unanticipated expenses, the possibility was conveniently ignored. Conceivably these expenses could turn out to be an enormous burden, in which case the annual maintenance cost has been grossly underestimated.

Likewise, two assumptions are of special importance to the calculation of the real value of the continued care fund contribution from one mill. The first is the difference between the rate of interest and the rate of inflation, the "real" rate of interest, if you will. Examples in Section IT indicate that when the difference is only one percentage point the continued care fund would not be sufficient to neet the estimated expenses, while a two-point difference would easily provide enough funds. An equally critical variable is the rate of inflation itself, which gives the rate at which the contribution of future mills declines in real terms.

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The second point is as obvious so hardly to require mention. The only we to eliminate completely the possibility that the state at some future time will face the expense of tailings pile maintenance is to eliminate the piles altogether. In other words, a continued care fund guaranteed to protect the states' interest for all time would necessarily shut down the industry. This extreme is clearly beyond the intent of the legislation, but short of it no fund is sure to be large enough. This is of course due to the uncertainties mentioned above, which loom unusually large here because "all time" is such a long time. However, it would be invalid to conclude that nothing can -- or should -be done. Policy-makers are nearly always faced with uncertainty; this happens to be an extreme case.

The problem is to give operational meaning to "perpetual care," a definition which weighs the needs of the state and the resources of the industry. To that end I would like to suggest two criteria which a perpetual care fund must satisfy:

- Based on currently available information on costs and interest and inflation rates, the fund should generate an income stream sufficient to meet all maintenance costs.
   We may not know what the future will bring, but the fund should at least be adequate according to our best knowledge.
- (ii) The terms of the continued care contribution should be readily alterable as new information becomes available. Thus, the perpetual care contribution from a mill opening, say, ten years from now should reflect ton addit teal years' data about inflation and maintenance costs. 806 346

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It is apparent that for the proposed New Mexico regulation the second of these conditions is not satisfied, and, based on the information used in this paper, at least, neither is the first. Perhaps a more effective policy would be to remove the upper bound now given in the statute and delegate to the New Mexico Environmental Improvement Agency the responsibility of setting the continued care fund at new mills. It would then be up to the Agency to review periodically maintenance responsibilities and their costs. In addition, it might be appropriate to review the contribution of a mill just prior to its decommissioning. This would allow adjustments to be made if conditions had changed substantially over the life of the mill.

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#### Mathematical Appendix

In this section we demonstrate somewhat more rigorously that the ratio of an endowment to the expenses paid out of it must exceed (1 + 1) / (r - 1) to avoid erosion of the principal. Recall

K. = balance of endowment, end of year t,

- C. = expenses during year t,
- r = interest rate,
- i = inflation rate.

Expenses grow at a constant rate, so that

$$C_{t} = C_{t-1} (1+i) = \dots = C_{0} (1+i)^{t}.$$

The basic accounting equation is

 $R_t = R_{t-1} (1+r) - C_t$ .

Expansion of this accounting equation yields

$$K_{t} = K_{t} - 1 (1 + t) - C_{t}$$

$$= K_{t} - 2 (1 + t)^{2} - C_{t-1} (1 + t) - C_{t}$$

$$= \dots = K_{0} (1 + t)^{t} - \sum_{i=0}^{t-1} (1 + t)^{i} C_{t-i}$$

$$= K_{0} (1 + t)^{t} - C_{0} \sum_{i=0}^{t-1} (1 + t)^{i} (1 + i)^{t-i}.$$

Therefore,

$$\frac{\mathbf{x}_{t}}{\mathbf{C}_{t}} = \frac{\mathbf{x}_{t}}{\mathbf{C}_{0}} \left[ \frac{1+\mathbf{r}}{1+\mathbf{i}} \right]^{t} = \frac{\mathbf{x}_{0}}{\mathbf{C}_{0}} \left[ \frac{1+\mathbf{r}}{1+\mathbf{i}} \right]^{t} = \frac{t-1}{\sum_{\mathbf{i}=0}^{t} \left[ \frac{1+\mathbf{r}}{1+\mathbf{i}} \right]^{t}}$$

The second term on the right hand side equals

$$\frac{1-\left(\frac{1+r}{1+i}\right)}{1-\frac{1+r}{1+i}} = \frac{1+i}{r-i} \left[ \frac{1+r}{1+i} - 1 \right].$$

Hence,

$$\frac{K_{t}}{C_{t}} = \begin{bmatrix} \frac{K_{0}}{C_{0}} & -\frac{1+i}{r-i} \end{bmatrix} \begin{bmatrix} \frac{1+r}{1+i} \\ 1+i \end{bmatrix}^{t} & +\frac{1+i}{r-i} & (2)$$
Since  $r > i$ ,  $\frac{(1+r)}{(1+i)}^{t}$  is unbounded above.
If  $\frac{K_{0}}{C_{0}} & <\frac{1+i}{r+i}$ , then  $\frac{K_{t}}{C_{0}}$  eventually becomes negative

In fact, we can use (2) to estimate how long an endowment will last if  $K \prec C_0$   $(\frac{1+i}{r-i})$ . Set the right-hand side of (2) equal to zero and solve for t.

This yields

$$\ln \left[ \frac{\left(\frac{1+i}{r-i}\right)}{\frac{1+i}{r-i} - \frac{\kappa_o}{c_o}} \right]$$

$$r = \frac{\ln \left(\frac{1+r}{k+i}\right)}{\frac{1+r}{k+i} - \frac{\kappa_o}{c_o}}$$