Uncertainty and sensitivity analysis for hydraulic models with dependent inputs

Lucie Pheulpin[1], Vito Bacchi[1] and Nathalie Bertrand[1]
[1]Institut de Radioprotection et de Sûreté Nucléaire, Fontenay-aux-Roses, France
Contact : lucie.pheulpin@irsn.fr

1. Introduction

Nowadays, **flooding hazard** is usually assessed through **numerical modelling,** generally affected by **uncertainties**. **Uncertainty Quantification** (UQ) and **Global Sensitivity Analysis** can be useful tools to improve the quantification of the flooding hazard. Traditionally, to perform these kinds of analyses, the input parameters are supposed to be independent, which is not always the case. In the framework of the **NARSIS** European Research-project, our objective is to develop a methodology to perform UQ and GSA by considering **dependent inputs.** This methodology will be applied to the **Loire River 2D hydraulic model**, currently under construction. However, before applying the general methodology presented here, we tested it on a very **simplified model of river flood inundation**.

2. Methodology

Step A: Problem specification

Input parameters:
- **Fixed**: Time step, grid resolution, *etc.*
- **Uncertain**:
  - Hydraulic parameters: hydrograph parameters, Strickler coefficient, *etc.*
  - Breach parameters: length, depth, time formation, *etc.*

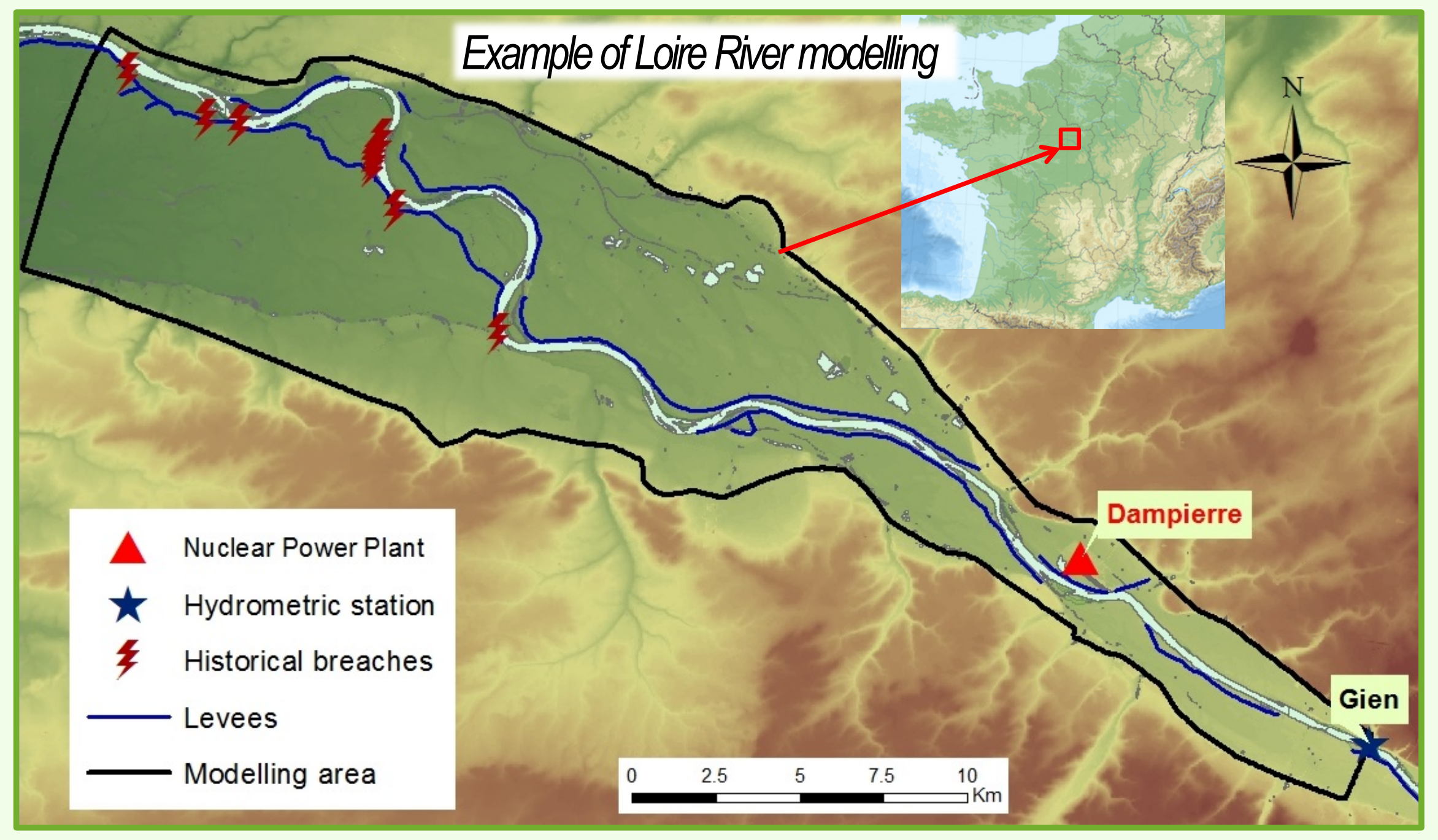
↓

Independent parameters or not?

Variables of interest
- Water levels at certain location in the flood plain (*e.g.* near the breaches)

Quantities of interest
- Probability, variance, *etc.*

Hydraulic and levee breach modelling: Example for the Loire River
- 50 km-long reach modelling, between Gien and Orléans
- 2D modelling with Telemac-2D
- Numerous levees along this reach with known historical breaches

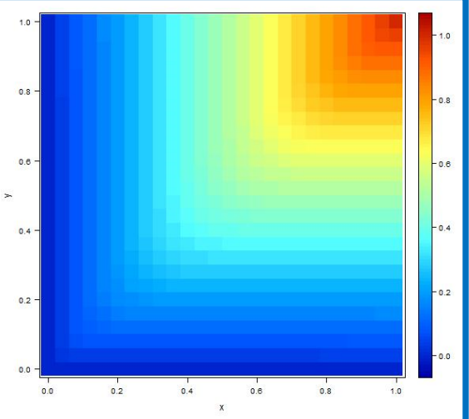

Example of Loire River modelling

Step B: Uncertainty sources quantification

For all parameters, definition of:
- Parameter bounds
- Parameter distribution laws

For dependent parameters:
- Groups of parameters identification
- Copula selection (*e.g.* normal copula) adapted to each group of parameter and definition of the correlation coefficients (*r*)
- Construction of multivariate distributions

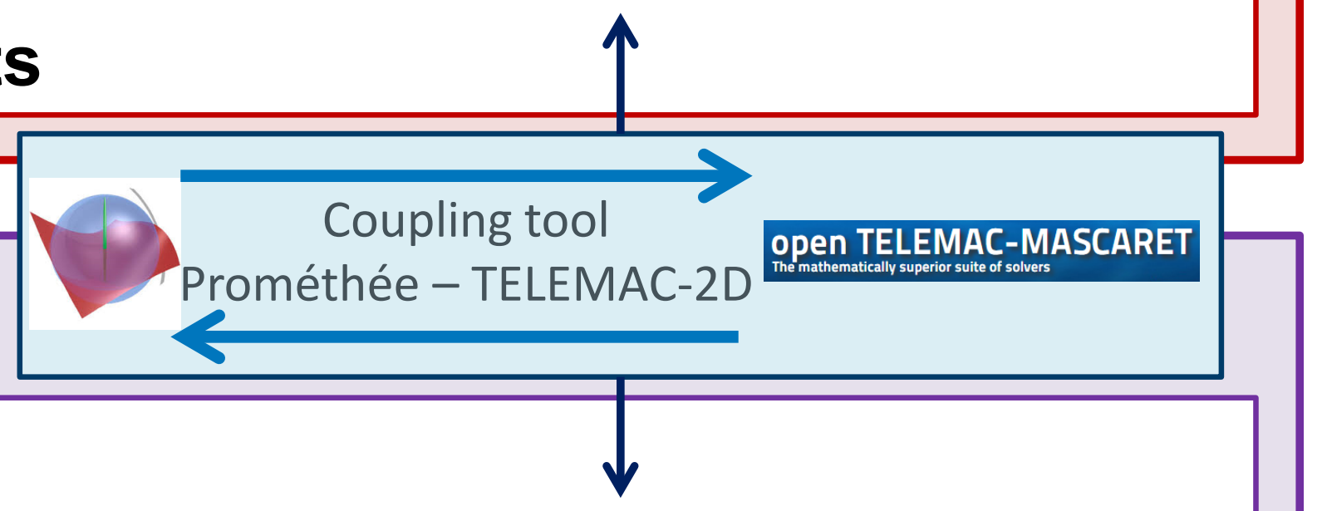
Example of a normal copula cumulative distribution function



Step C: Uncertainty Quantification (UQ)

Random sample of input parameters with the computational environment Prométhée (*e.g.* with a Monte-Carlo method)
- For independent parameters: inside their distributions laws
- For dependent parameters: inside the multivariate distributions coming from copulas

→ Construction of histograms, boxplots, *etc.* of outputs


Coupling tool
Prométhée – TELEMAC-2D

Step D: Global Sensitivity Analysis (GSA)

Variance based method: computation of Sobol indices (1^{st} and total order)
- With a FAST (Fourier Analysis Sensitivity Test) method for independent parameters
- Calculation of multidimensional sensitivity indices for dependent parameters *(Jacques, Lavergne, et al. 2006)*

Screening method: computation of sensitivity indices (elementary mean and standard deviation) with Morris method
- With a classic Morris method for independent parameters
- With an extension of the Morris method which integrates dependency through copulas for dependent parameters *(Ţene et al.,2018)*

→ Parameter ranking, uncertainty reduction, model siplification, *etc.*

3. Test case: simplified model of river flood inundation

Model description

- Based on simplified 1D hydro-dynamical equations of Saint-Venant, considering uniform and constant flowrate and large rectangular sections (used in *Iooss and Lemaître, 2015*)
- Simulation of river water level (h) and comparison with levee height (H_d)
- 8 input parameters: 3 groups of 2 inputs (Q/K_s, Z_v/Z_m, L/B) and 2 independent ones: (C_b and H_d)

Model equations

$$h = \left(\frac{Q}{BK_s\sqrt{\frac{Z_m - Z_v}{L}}}\right)^{0.6} \quad \text{with } S = Z_v + h - H_d - C_b$$

- Random sampling of 10,000 parameter combinations in the univariate (independent inputs) or multivariate (dependent inputs) distributions

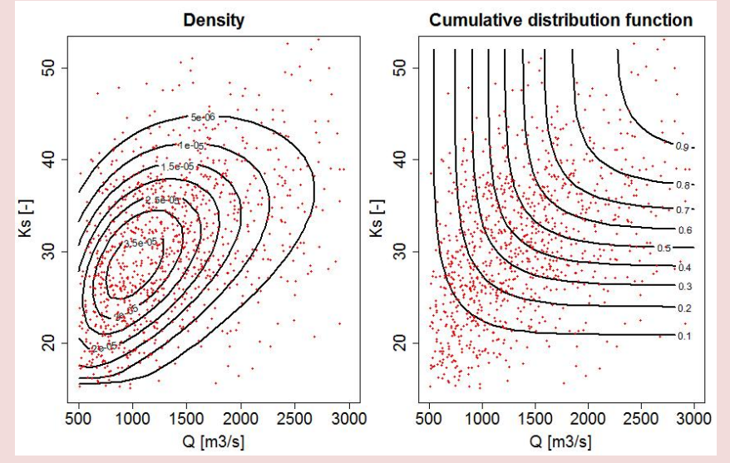
Uncertainty sources quantification

Inputs	Symbols	Units	PDF
Maximal annual flow rate	Q	m^3/s	Truncated Gumbel
Strickler coefficient	K_s	-	Truncated Normal
River downstream level	Z_v	m	Triangle
River upstream level	Z_m	m	Triangle
Levee height	H_d	m	Uniform
Bank level	C_b	m	Triangle
Length of the river stretch	L	m	Triangle
River width	B	m	Triangle

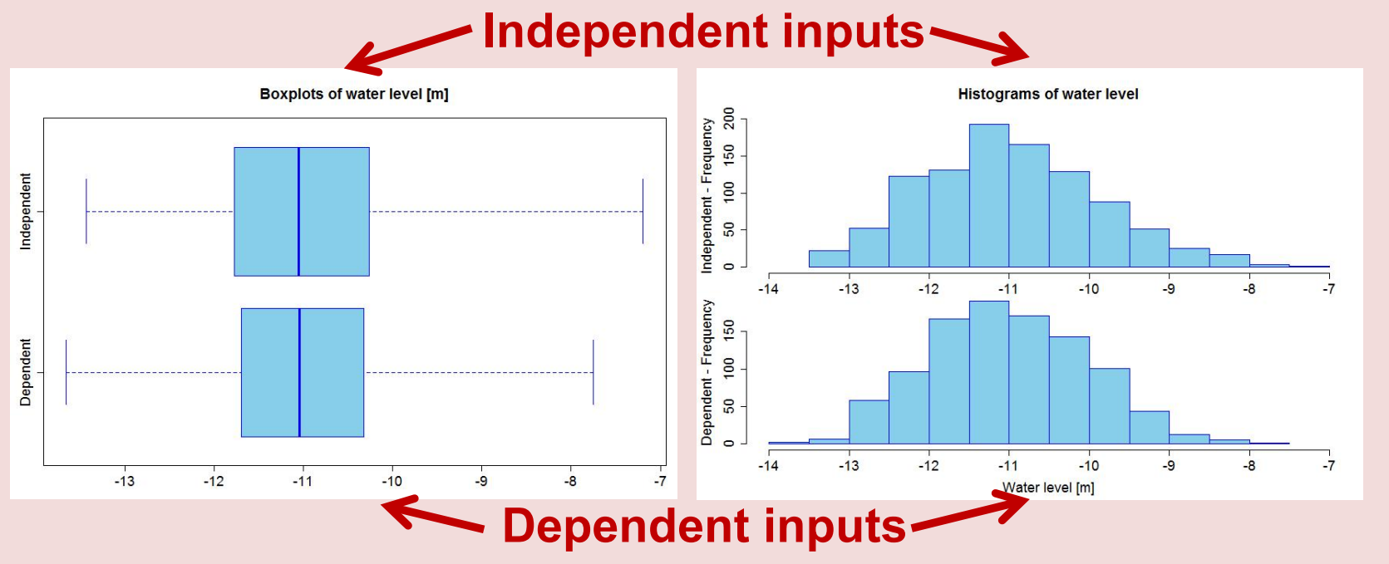
UQ for independent and dependent parameters

Dependent inputs → 3 normal copulas: Q/K_s ($r = 0.5$) ; Z_v/Z_m ($r = 0.3$) ; L/B ($r = 0.3$)

Normal copula Q/K_s



Outputs Distribution



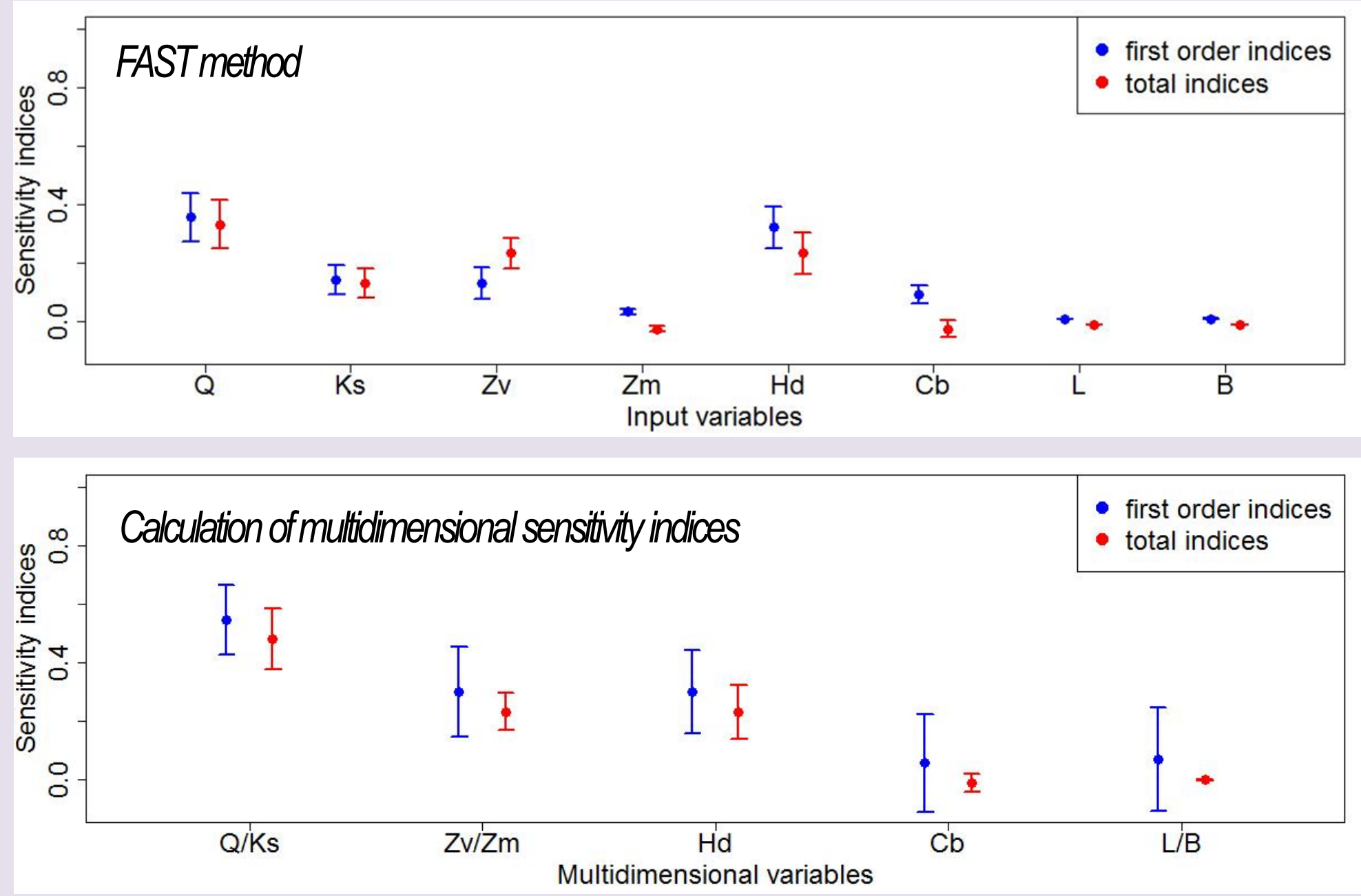
GSA

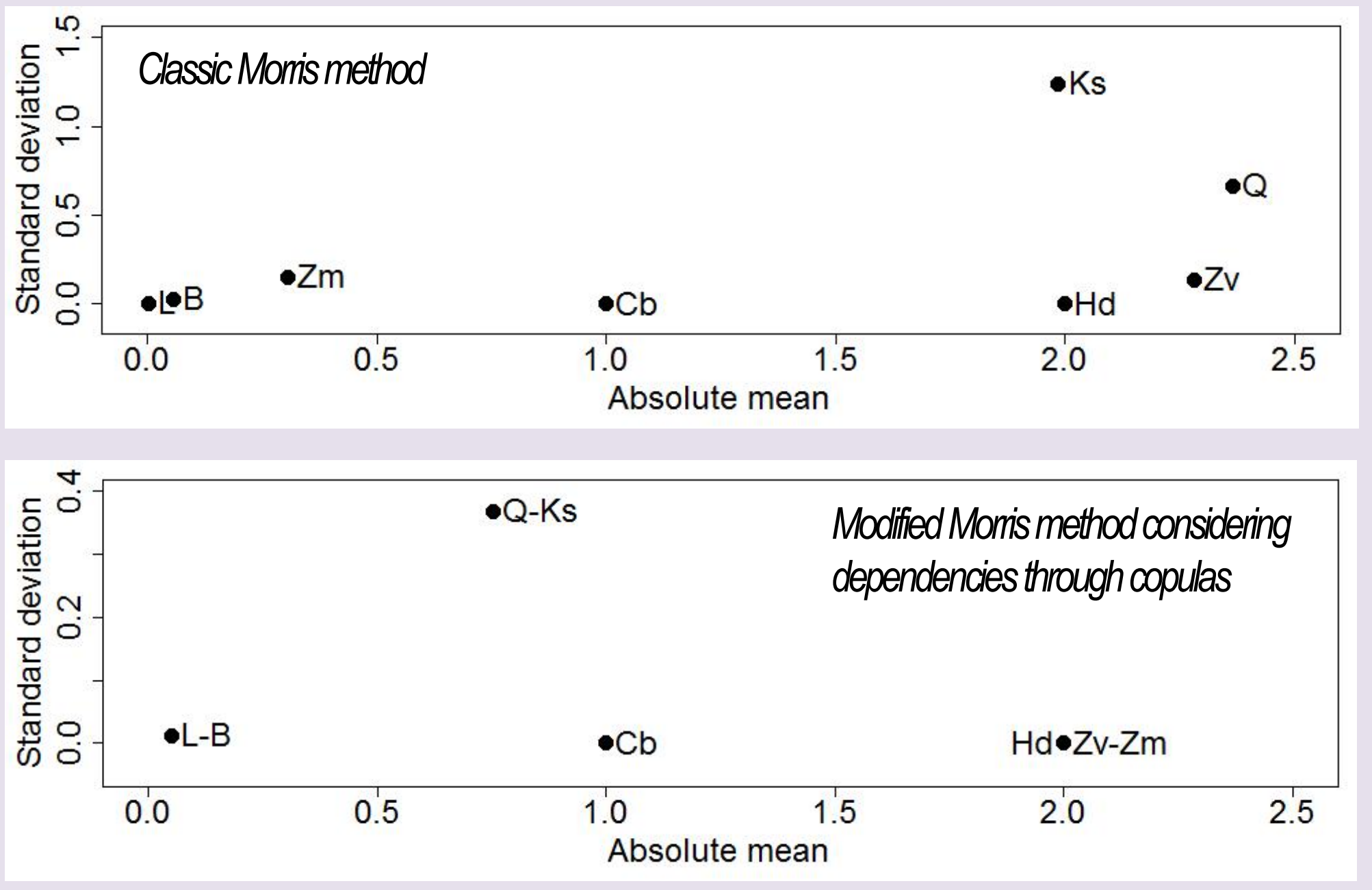
Variance-based methods (Sobol indices)

Screening methods (Morris)

Independent parameters

Dependent parameters





- In this example, the choice of the copula has very few impact on the outputs and there is almost no difference between the distribution of outputs by considering certain inputs dependent or not.
- The GSA methods show that some parameters (*e.g.* Z_m) can have more influence once included in a group than considered independent.

4. Conclusion and perspectives

- In the test case, the copulas and their correlation coefficients are defined arbitrarily. In the reality (*i.e.* in hydraulic models), it is necessary to test different types of copulas and different groups of parameters inside copulas, on observed data, and to validate them with a Cramer-von-Mises test for example.
- The UQ and GSA tools used for the test case were coded with **R** and now they must be included in the computational environment Prométhée.
- Once the Telemac-2D Loire model achieved, it will be coupled with Prométhée to process UQ and GSA on hydraulic parameter and on levee breach parameters. Finally, the whole point of our research is to better estimate the flooding hazard.

Références

J. Jacques, C. Lavergne, and N. Devictor, "Sensitivity analysis in presence of model uncertainty and correlated inputs", Reliability Engineering & System Safety, vol. 91, nº 10-11, p. 1126-1134, oct. 2006.
M. Ţene, D. E. Stuparu, D. Kurowicka, and G. Y. El Serafy, "A copula-based sensitivity analysis method and its application to a North Sea sediment transport model", Environmental Modelling & Software, vol. 104, p. 1-12, june 2018.
B. Iooss and P. Lemaître, "A Review on Global Sensitivity Analysis Methods", in Uncertainty Management in Simulation-Optimization of Complex Systems, vol. 59, G. Dellino et C. Meloni, Éd. Boston, MA: Springer US, 2015, p. 101-122.

Work carried out within the European project **NARSIS**
(**N**ew **A**pproach to **R**eactor **S**afety **I**mprovement**S**)

Photography encouraged