Uncertainty and Uncertainties

Lecture 3-2

Key Topics

- Types of uncertainty (as treated in NPP PRA): aleatory and epistemic
- Epistemic uncertainty
  - Characterization (subjective probability)
  - Magnitude for typical parameters
  - Types: completeness, model, parameter
  - Characterization for overall system (“propagation”)

Resources

- G. Apostolakis, “Probability and risk assessment: the subjectivistic viewpoint and some suggestions,” *Nuclear Safety*, **9**, 305–315, 1978.
- G. Apostolakis, “The concept of probability in safety assessments of technological systems,” *Science*, **250**, 1359–1364, 1990.
- N. Siu, et al., “Probabilistic Risk Assessment and Regulatory Decisionmaking: Some Frequently Asked Questions,” *NUREG-2201*, U.S. Nuclear Regulatory Commission, September 2016.
- U.S. Nuclear Regulatory Commission, “Guidance on the Treatment of Uncertainties Associated with PRAs in Risk-Informed Decision Making,” *NUREG-1855, Revision 1*, March 2013.
- M. Granger Morgan, “Use (and abuse) of expert elicitation in support of decision making for public policy,” *National Academy of Sciences Proceedings (NASP)*, 111, No. 20, 7176-7184, May 20, 2014.

Other References

- A. Mosleh, et al., *Model Uncertainty: Its Characterization and Quantification*, Center for Reliability Engineering, University of Maryland, College Park, MD, 1995. (Also available as *NUREG/CP-0138*)
- N. Siu, D. Karydas, and J. Temple, "Bayesian Assessment of Modeling Uncertainty: Application to Fire Risk Assessment," in *Analysis and Management of Uncertainty: Theory and Application*, B.M. Ayyub, M.M. Gupta, and L.N. Kanal, eds., North-Holland, 1992, pp. 351-361.
- E. Droguett and A. Mosleh, “Bayesian methodology for model uncertainty using model performance data,” *Risk Analysis*, **28**, No. 5, 1457-1476, 2008.
- U.S. Nuclear Regulatory Commission, “Reliability and Availability Data System (RADS)” https://nrcoe.inl.gov/resultsdb/RADS/
- U.S. Nuclear Regulatory Commission, “Industry Average Parameter Estimates,” https://nrcoe.inl.gov/resultsdb/AvgPerf/

Other References (cont.)

- W.E. Vesely, et al., "Measures of Risk Importance and Their Applications," *NUREG/CR-3385*, July 1983.
- Siu, N. and D.L. Kelly, "On the Use of Importance Measures for Prioritizing Systems, Structures, and Components," *Proceedings 5th International Topical Meeting on Nuclear Thermal Hydraulics, Operations, and Safety (NUTHOS-5)*, Beijing, China, April 14-18, 1997, pp. L.4-1 through L.4-6.
- American Nuclear Society and the Institute of Electrical and Electronics Engineers, "PRA Procedures Guide," NUREG/CR-2300, January 1983.
- E. Zio and N. Pedroni, "How to effectively compute the reliability of a thermal-hydraulic passive system," *Nuclear Engineering and Design*, **241**, 310-327, 2011.

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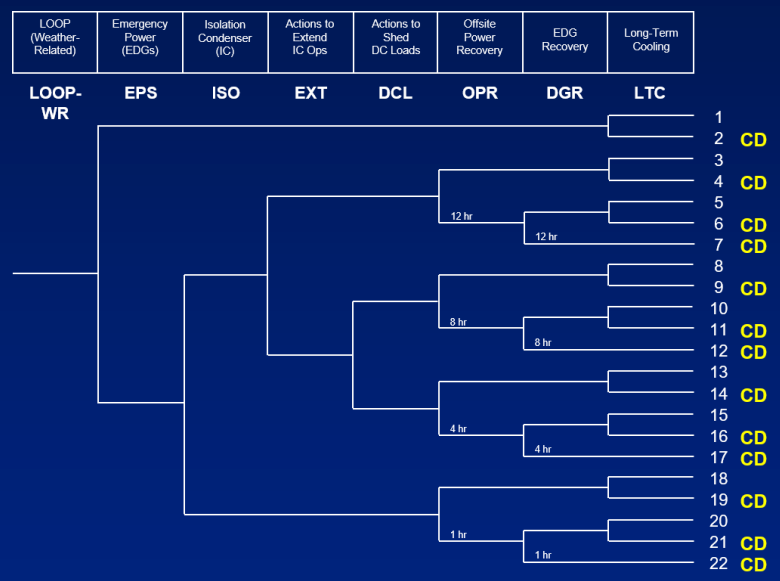
The Big Picture (Level 1)



$$CDF_{LOOP-WR} = \sum_{i:C_{si}=CD} CDF_{si}$$
$$= CDF_{s2} + CDF_{s4} + \cdots$$

$$CDF_{Total} = \sum_{all\ ET} CDF_{ET\,j}$$

- CDF is a measure of uncertainty
- Random variables:
  - number of core damage events
  - time to a core damage event

How well do we know CDF?

Types of Uncertainty Addressed by NPP PRA/RIDM

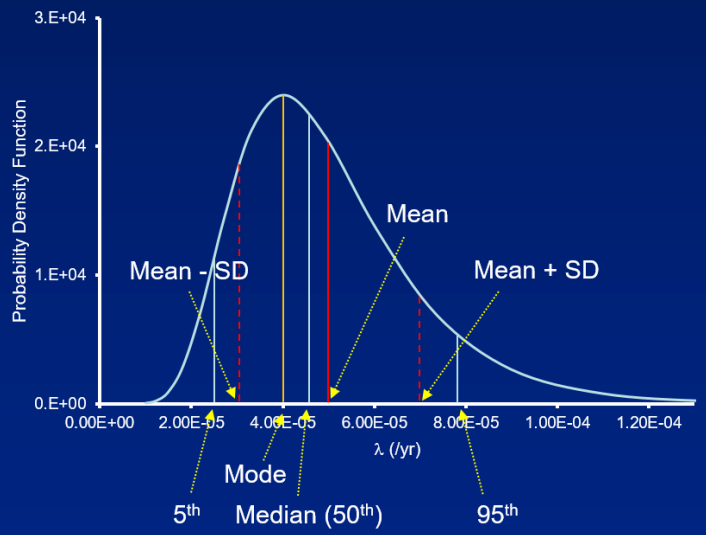
- Aleatory (random, stochastic)
  - Irreducible
  - Examples:
    - How many times a safety relief valve (SRV) operates before failing
    - How long it takes for operators to initiate a fire response procedure
  - Modeling addressed in Lecture 3-1
- Epistemic (state-of-knowledge)
  - Reducible
  - Examples:
    - “True value” of SRV failure rate
    - “Best” model for operator response

Indicators of Uncertainty

- Percentiles

$$x_\alpha : F_X(x_\alpha) = P\{X \le x_\alpha\} = \int_{-\infty}^{x_\alpha} f_X(x)dx = \alpha$$

  - "Bounds" (e.g., 5th, 95th)



- Moments
  - General

$$E[X^n] \equiv \int_{-\infty}^{\infty} x^n f_X(x)\, dx$$

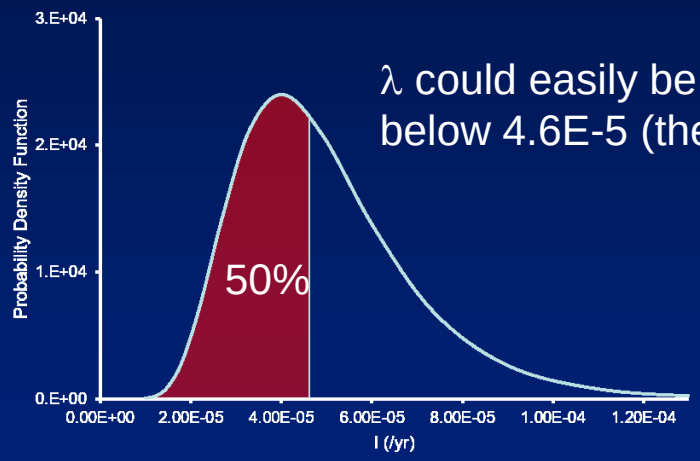
  - Mean

$$E[X] \equiv \int_{-\infty}^{\infty} x \cdot f_X(x)\, dx$$

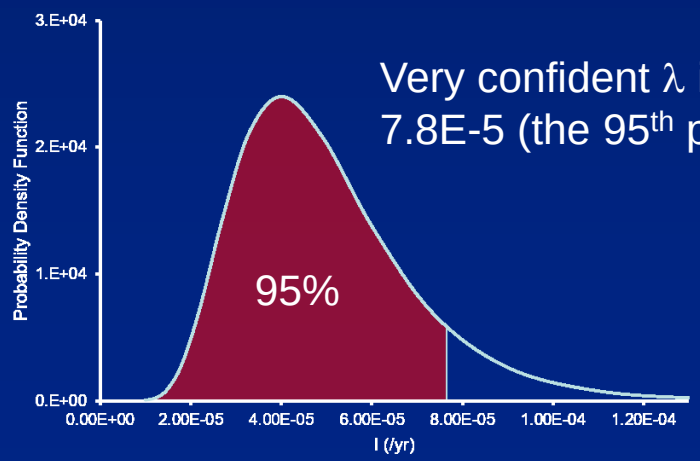
  - Variance

$$Var[X] \equiv E[(X - E[X])^2] \qquad SD[X] \equiv \sqrt{Var[X]}$$

Epistemic Uncertainty: The "Probability of Frequency" Model



λ could easily be above or below 4.6E-5 (the median)

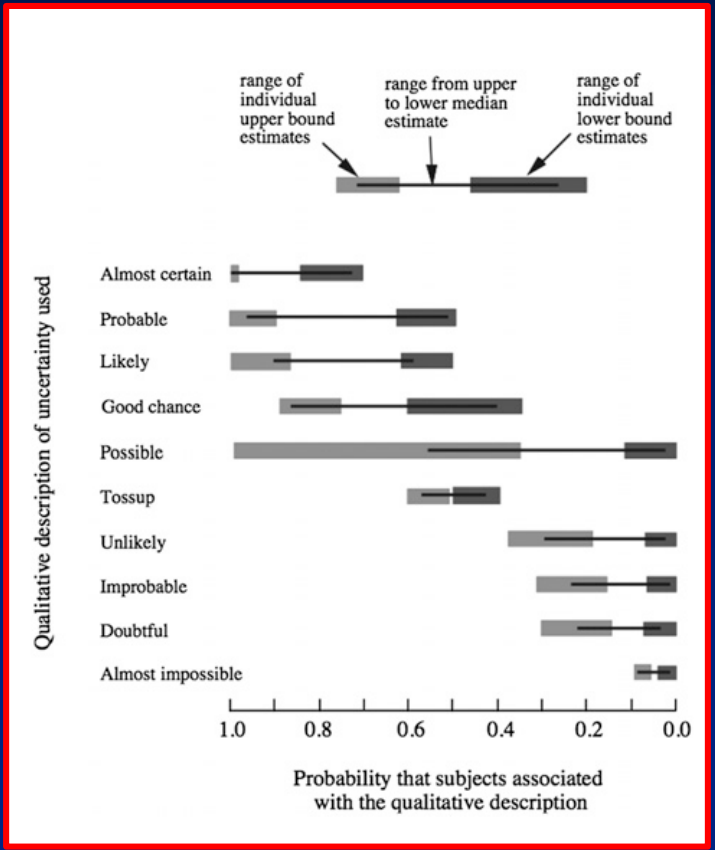


Very confident λ is below 7.8E-5 (the 95th percentile)

Subjective Probability

- Probability is an internal measure of degree of belief in a proposition
- To be useful, need:
  - Consistent scale for comparing judgments
  - Convergence with classical (relative frequency) probabilities given enough data
- "Coherent" analyst: probabilities conform to the laws of probability

Qualitative to Quantitative: Examples



M. Granger Morgan, "Use (and abuse) of expert elicitation in support of decision making for public policy," *National Academy of Sciences Proceedings (NASP)*, 111, No. 20, 7176-7184, May 20, 2014.

Table 4.1 Guidelines for Assigning Conditional Probabilities to Events with "State-of-Knowledge" Uncertainty (Ref. 7)

Value	Description
1.	The indicated outcome is CERTAIN given the conditions defined by the case in question. Usually this is reserved for logical outcomes not requiring analysis to support them. Analysis or calculations that are needed to support a certain outcome use only methods appearing in textbooks or peer-reviewed journals. The results of the analysis demonstrate the indicated outcome to be appropriate considering all relevant uncertainties. Other analysis approaches have been considered, and these either yield the same result or are not applicable. No debate as to the outcome would be expected from individuals who are informed of the specifics of the case and the associated phenomena.
1. - 1.0E-3 (i.e., 0.999)	The indicated outcome is ALMOST CERTAIN. Detailed analysis has been performed that includes all phenomena identified as relevant and has been subjected to independent review. At least one other individual who has analyzed the situation [other than the analyst and reviewer(s)] agrees that the outcome is almost certain. Separate analysis exists that supports this outcome. Consideration of all identified uncertainties has been made, and none has been found to have a credible effect on the outcome.
(1. - 1.0E-2) i.e., 0.99	The indicated outcome is EXTREMELY LIKELY. Either detailed analysis has been performed and subjected to independent review or a significant body of directly applicable experimental data published in the technical literature supports this position. The indicated outcome is obtained for all credible assumptions as to the values of parameters in supporting analysis. Arguments against this position are not supported by either analysis or data.
(1. - 5.0E-2) i.e., 0.95	The indicated outcome is VERY LIKELY. Either detailed analysis has been performed and reviewed for completeness or a significant body of relevant experimental data supports this position. Arguments against this position are obviously flawed or data exist that contradict the arguments presented in some measure.
0.9	The indicated outcome is LIKELY. Either it is supported by analysis or the preponderance of experimental evidence points to this result. Arguments against this position are apparently flawed, and the technical basis for disagreement with the counter argument has been established. Alternatively, no analysis has been performed, but there is general agreement between two or more independent individuals knowledgeable of the situation that the indicated outcome is appropriate.
0.5	The indicated outcome is fully POSSIBLE. Either no analysis has been performed or existing analysis is inconclusive. Inconclusive analysis includes that for which no concurrence from an independent party can be gained. Experimental data do not clearly indicate this outcome to be more likely or experiments are obviously no directly pertinent.
0.1	The indicated outcome is UNLIKELY. It cannot be supported by incontrovertible analysis or a preponderance of data. However, it is a credible outcome when attendant uncertainties are considered.
5.0E-2	The indicated outcome is VERY UNLIKELY. Analysis cannot rule it out completely. However, arguments in favor of this outcome are not supported by the available data. At most, a few experiments suggest that this outcome could occur.
1.0E-2	The indicated outcome is EXTREMELY UNLIKELY. Uncertainties in the available analysis that show the outcome not to occur can be identified. Consideration of these uncertainties might lead to this outcome, but no analytical or experimental support can be found.
1.0E-3	The indicated outcome is ALMOST IMPOSSIBLE. It has credibility only if a number of unsupported (but not demonstrably incorrect) assumptions are made. No analysis is available to support this result even when relevant uncertainties in the parameters of the analysis are considered.
0.	The indicated outcome is IMPOSSIBLE. It is either ruled out by the physical situation or a large body of analysis and experiments support alternate outcomes.

NUREG/CR-6771

Aleatory vs. Epistemic – Relationship

- Subjective view: all uncertainties are epistemic. Aleatory model
  - brings in analyst's understanding of underlying processes
  - supports quantification
  - supports communication regarding model parameters
- Coin toss example
  - Uncertain proposition X: observation of *n* "heads" on next *m* tosses (trials)
  - P{X|C,H} can include consideration of conditions: fairness of coin, tossing, measurement; whether outcomes are truly binary
  - If conditions are "ideal," epistemic uncertainty (for a "coherent" analyst) is given by aleatory (binomial) distribution:

$$P\{N = n \text{ in } m \text{ trials} | \phi\} = \binom{m}{n} \phi^n (1-\phi)^{m-n}$$

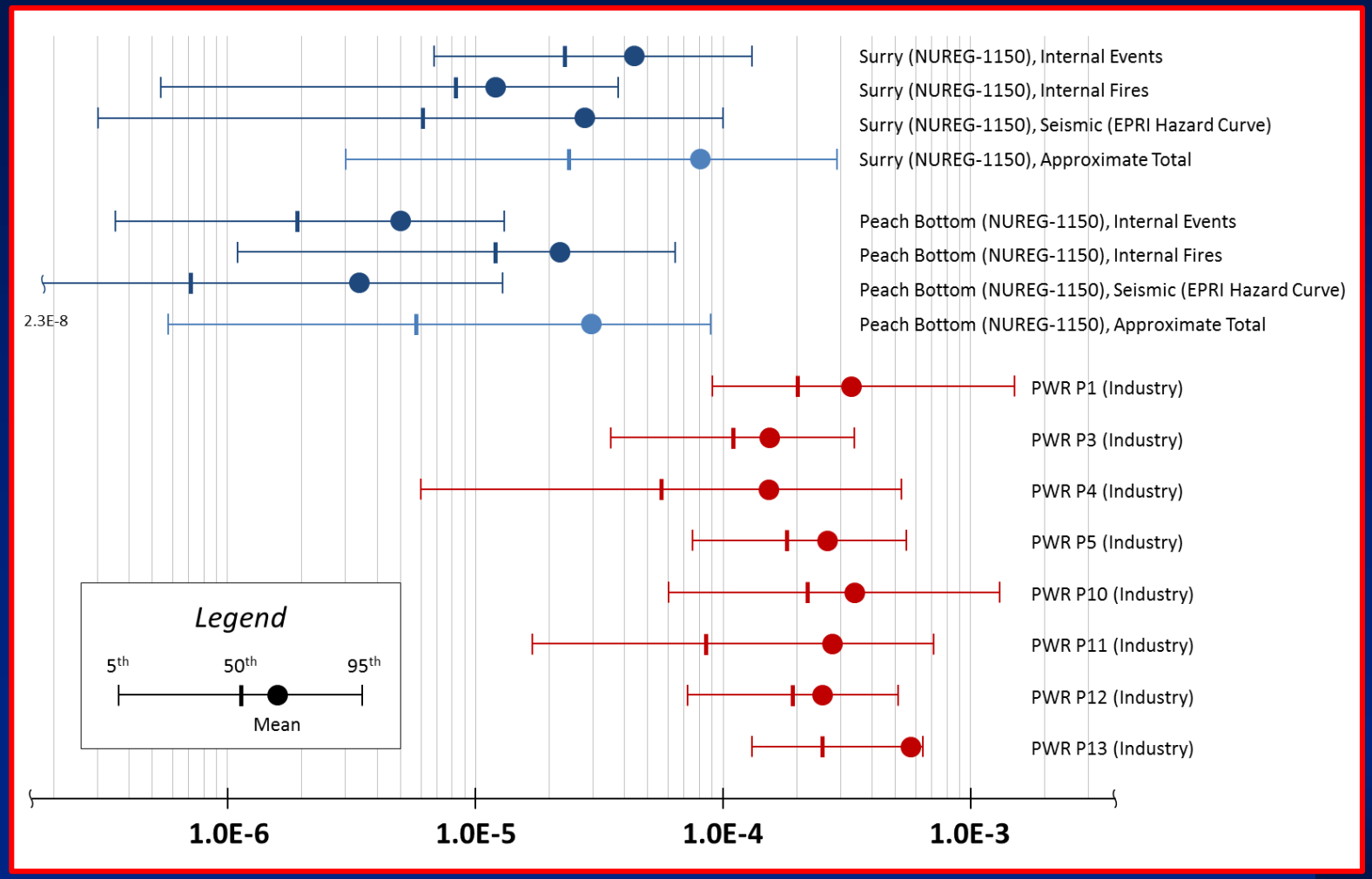
  - If there is uncertainty in the conditions, as reflected by a probability distribution for ϕ, the total probability is the expectation of the aleatory result:

$$P\{N = n \text{ in } m \text{ trials} | C, H\} = \int_0^1 \binom{m}{n} \phi^n (1-\phi)^{m-n} \underbrace{\pi_\Phi(\phi|H)}_{\text{epistemic distribution for } \phi} d\phi$$

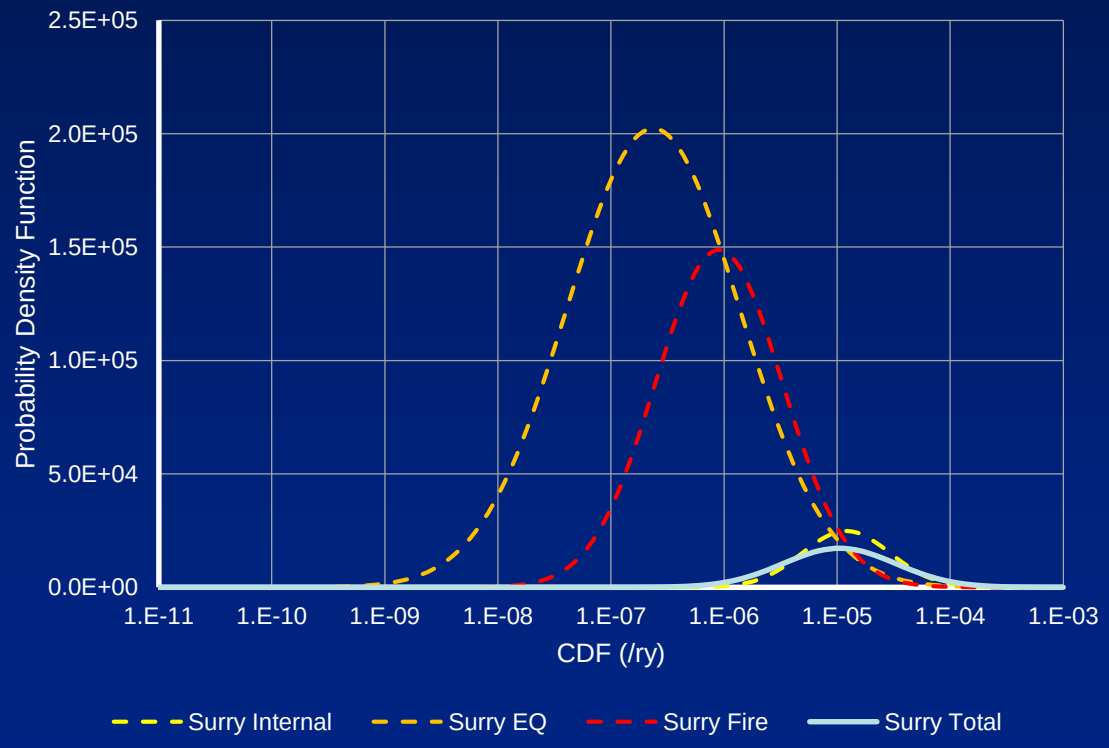
Aleatory vs. Epistemic Uncertainty – Commentary and Observations

- Distinction depends on frame of reference ("model of the world").
- Example: Weld crack propagation depends on local material conditions (e.g., Cu content, flaw geometry).
  - Aleatory model: conditions have a statistical distribution.
  - Epistemic model: local conditions are knowable (in principle).
  - "Appropriate" model depends on question being asked (e.g., probability of conditions at a randomly sampled location vs. conditions at a specific location).
- Practice of distinction is generally accepted, has proven useful in RIDM.
- NPP PRA community (particularly Level 1) uses "probability of frequency" modeling convention (used in these lectures).
- Terminology has been (and remains) a source of angst.
- Some arguments in the broader risk community for alternate measures of epistemic uncertainty (e.g., possibility, degree of membership)

PRA Uncertainties – Various Studies

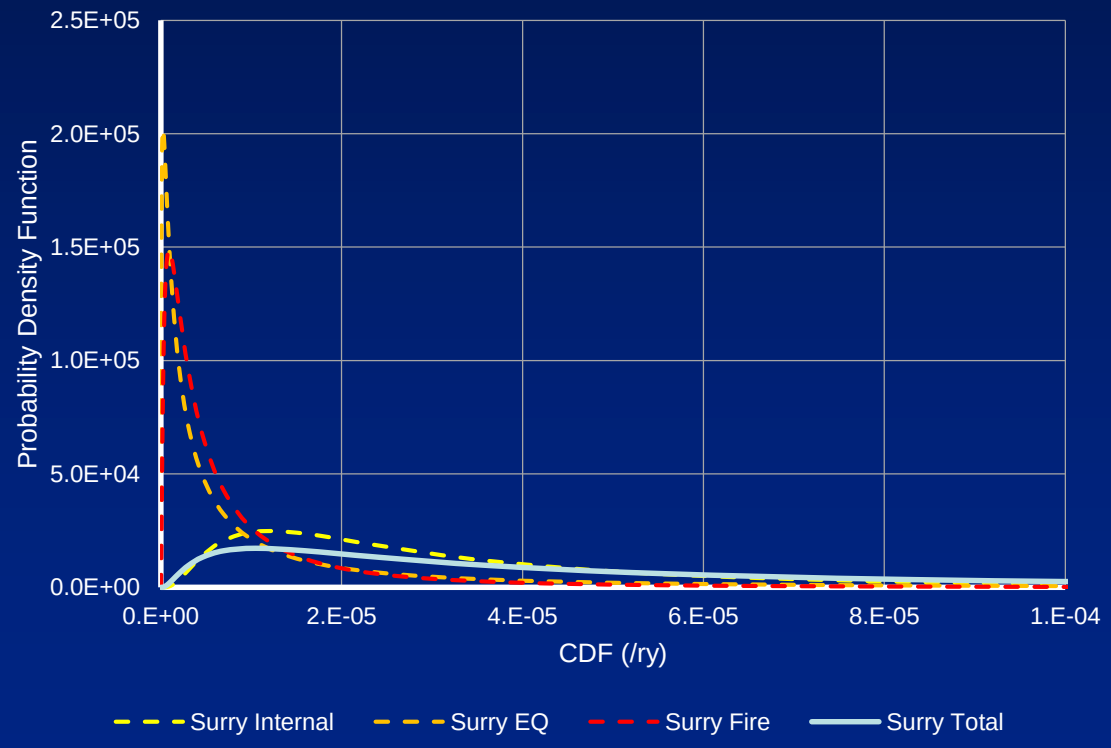


NUREG-1150 Uncertainties – Another View



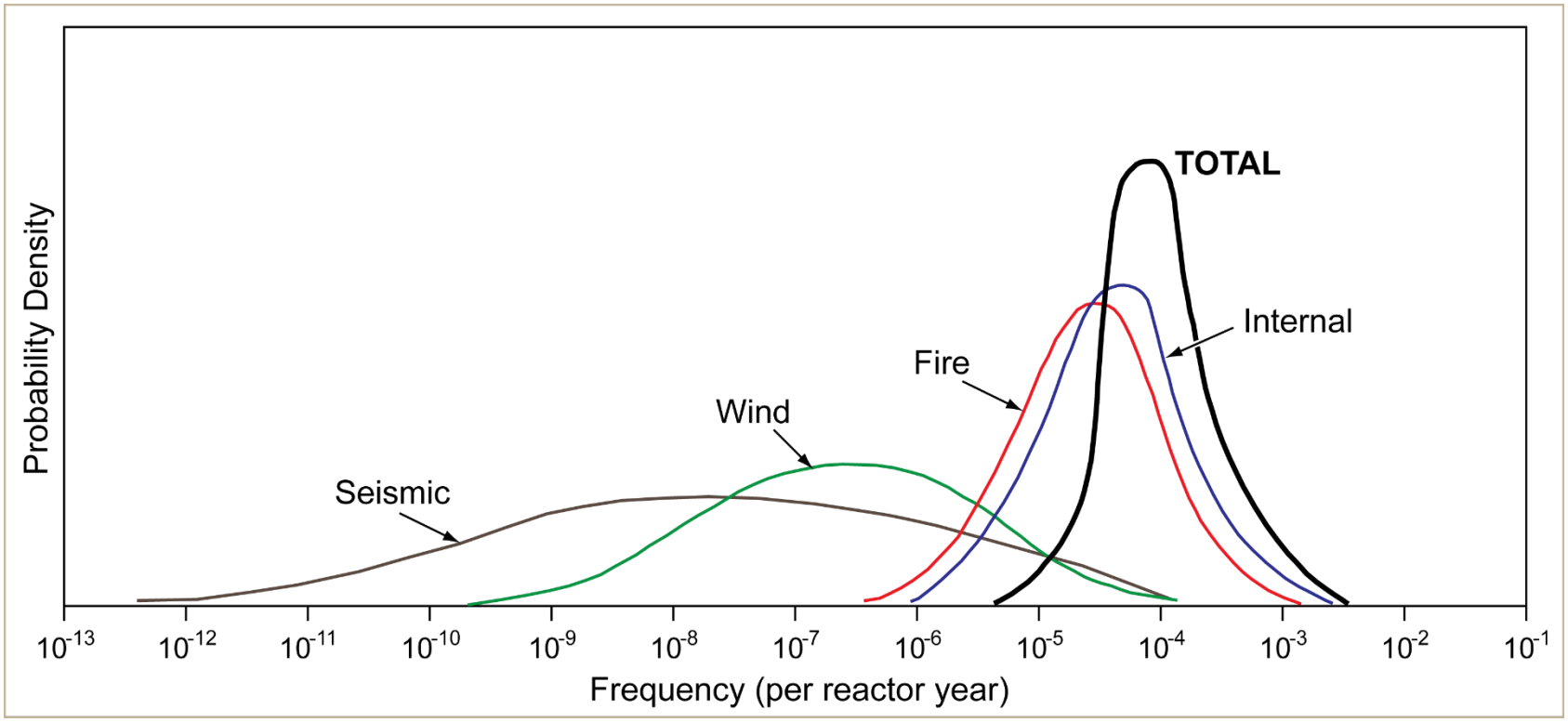
Estimated

NUREG-1150 Uncertainties – Another View



Estimated

PRA Uncertainties – Indian Point (1980s)



Knowledge Check

- Why do the NUREG-1150 curves get progressively taller with smaller values of CDF?
- What does this tell you about the Indian Point graph?

Types of Epistemic Uncertainty

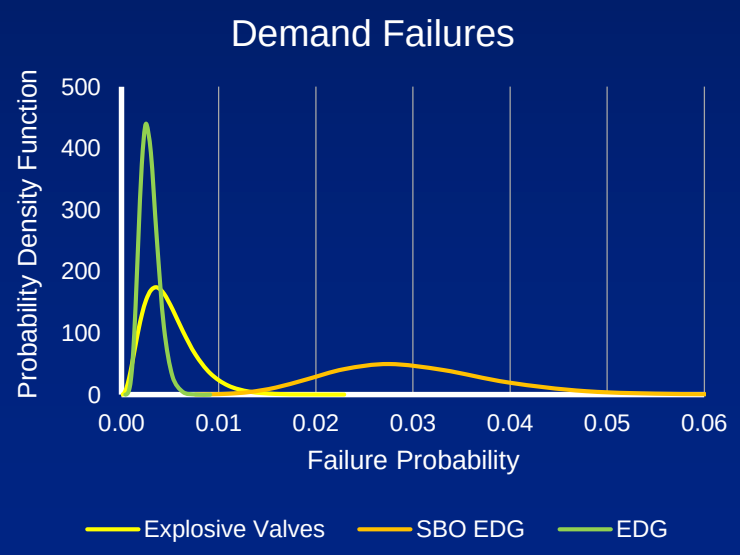
- Completeness
  - Recognized ("known unknowns")
  - Unrecognized ("unknown unknowns")
- Model
  - Competing models
  - Difference between predictions and "reality"
- Parameter
  - Conditioned on model
  - Estimated ("quantified") based on available evidence

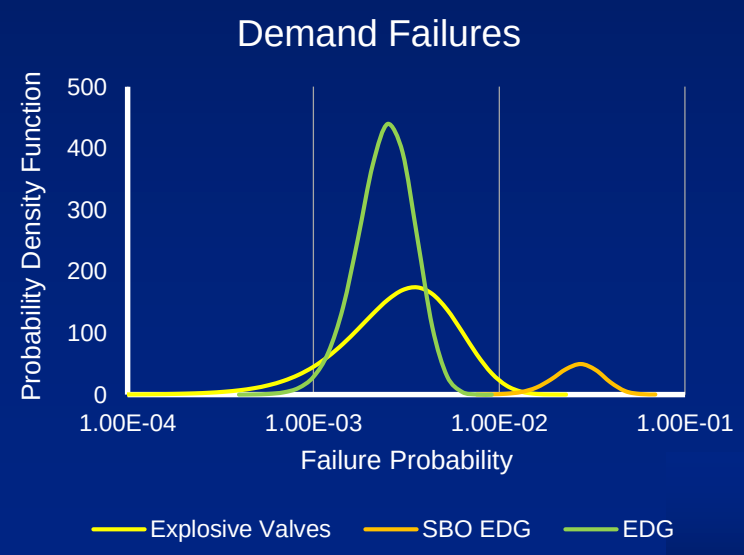
Parameter Uncertainty

- Typically characterized using parametric distributions (e.g., beta, gamma, lognormal)
- Parameters estimated using Bayes Theorem (see Lecture 5-1)
  - Statistical evidence (operational experience, test results); engineering judgment in processing
  - Large amounts of data => subjective probabilities converge with evidence
  - Small amounts of data => large uncertainties
- Note: pooling data increases confidence in estimates for population, but not necessarily for NPP studied.

Generic Demand Failure Probabilities

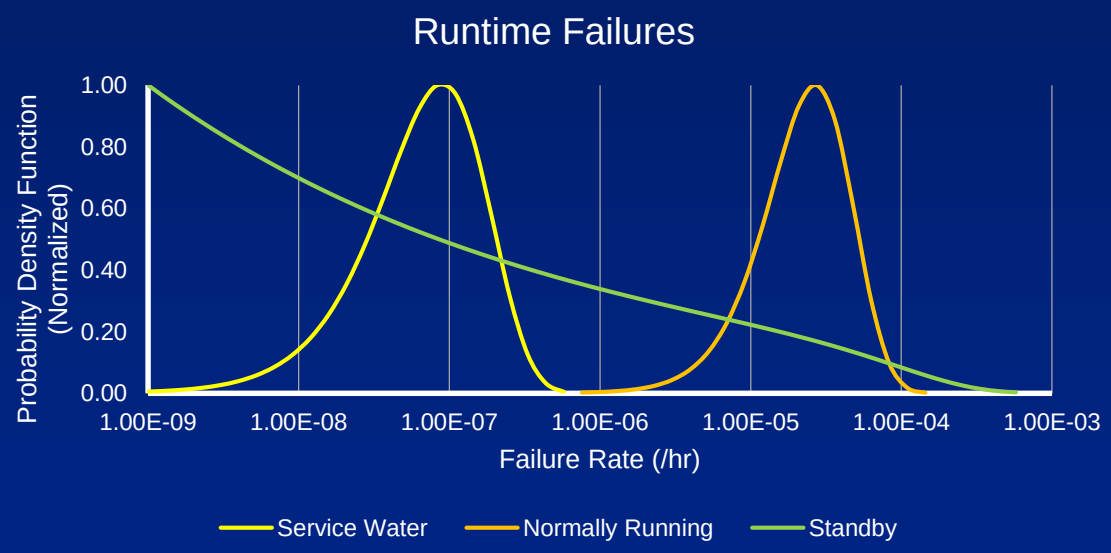
- 2015 Industry-wide estimates from: https://nrcoe.inl.gov/resultsdb/AvgPerf/
- Explosive valves: 3 failures in 713 trials
- SBO EDGs: 12 failures in 419 trials
- EDGs: 214 failures in 75,452 trials





Generic Runtime Failure Rates

- 2015 Industry-wide estimates from: https://nrcoe.inl.gov/resultsdb/AvgPerf/
- Service Water Pumps: 2 failures in 16,292,670 hours
- Normally Running Pumps: 225 failures in 59,582,350 hours
- Standby Pumps (1st hour operation): 48 failures in 437,647 hours



Note: point values won't alert user to potential irregularities

Model Uncertainty

- Some arbitrariness in distinction from “parameter uncertainty”
- Currently only requirements for characterization (not quantification)
- Typically addressed through “consensus models,” sensitivity studies
- If appropriately defined, uncertainties can be quantified and brought into the PRA
  - Focus on observables, difference between prediction and reality (vs. what is “best” or “correct” model)
  - Limited applications in practice

Thought Exercise

Under what conditions could a flawed consensus model be “good enough” for RIDM?

Example Quantification of Model Uncertainty

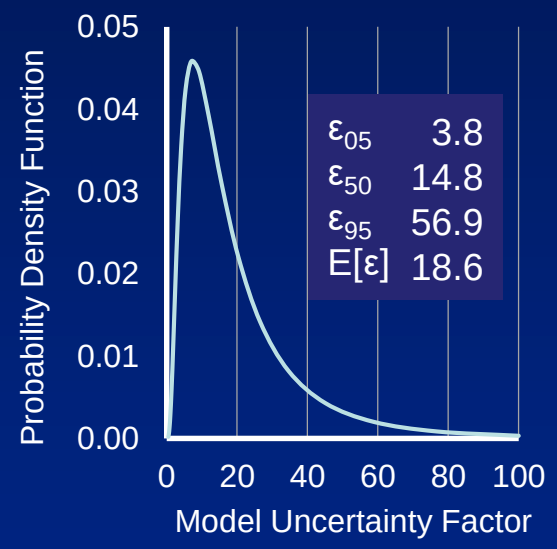
- Estimating uncertainty factor in "Deterministic Reference Model" (DRM) estimates based on comparisons with experiments
- DRM for cable fire propagation velocity

$$v = \frac{v_{air}(k\rho c)_{air}\left(T_{flame} - T_{ignition}\right)^2}{(k\rho c)_{cable}\left(T_{ignition} - T_{surface}\right)^2}$$

- Uncertainty factor (a random variable)

$$\varepsilon_i = \frac{v_i}{\langle v_{DRM,i} \rangle} \qquad f(\varepsilon|\mu,\sigma) = \frac{1}{\sqrt{2\pi}\sigma\varepsilon} e^{-\frac{1}{2}\left(\frac{ln\varepsilon-\mu}{\sigma}\right)^2}$$

- Note: bias in results indicates need for better DRM



N. Siu, D. Karydas, and J. Temple, "Bayesian Assessment of Modeling Uncertainty: Application to Fire Risk Assessment," in *Analysis and Management of Uncertainty: Theory and Application*, B.M. Ayyub, M.M. Gupta, and L.N. Kanal, eds., North-Holland, 1992, pp. 351-361.

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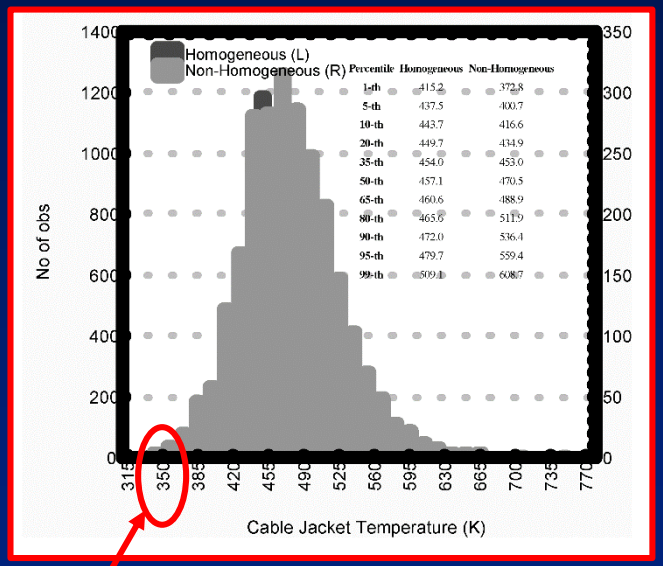
Another Example Quantification of Model Uncertainty

Time (s)	Experiment (K)	DRM (K)
180	400	450
360	465	510
720	530	560
840	550	565



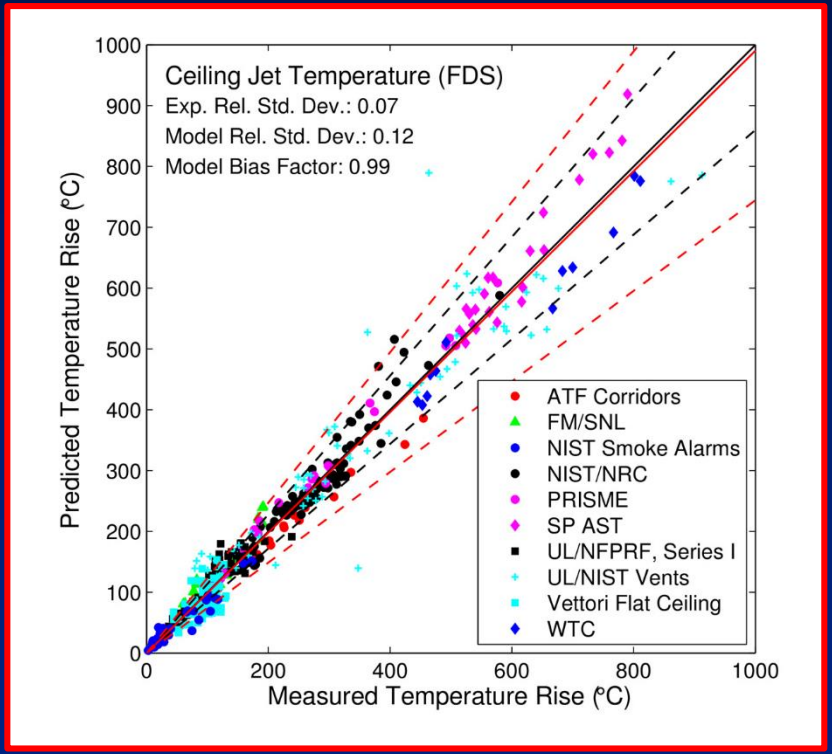
Notes:

1) Bayesian methodology accounts for possibility of inhomogeneous data.
2) Very large uncertainty bands might be unrealistic.

E. Droguett and Ali Mosleh, "Bayesian methodology for model uncertainty using model Performance data," *Risk Analysis*, **28**, No. 5, 1457-1476, 2008.

Model Uncertainty Quantification – Other Considerations

- Perspective matters
  - Model developer – ability of model to address phenomena
  - PRA user – include user effects
- Uncertainties in unmeasured parameters
- Sub-model limits of applicability



M.H. Salley and A. Lindeman, "Verification and Validation of Selected Fire Models for Nuclear Power Plant Applications," NUREG-1824 Supplement 1/EPRI 3002002182, November 2016.

Completeness Uncertainty – Example Known Sources

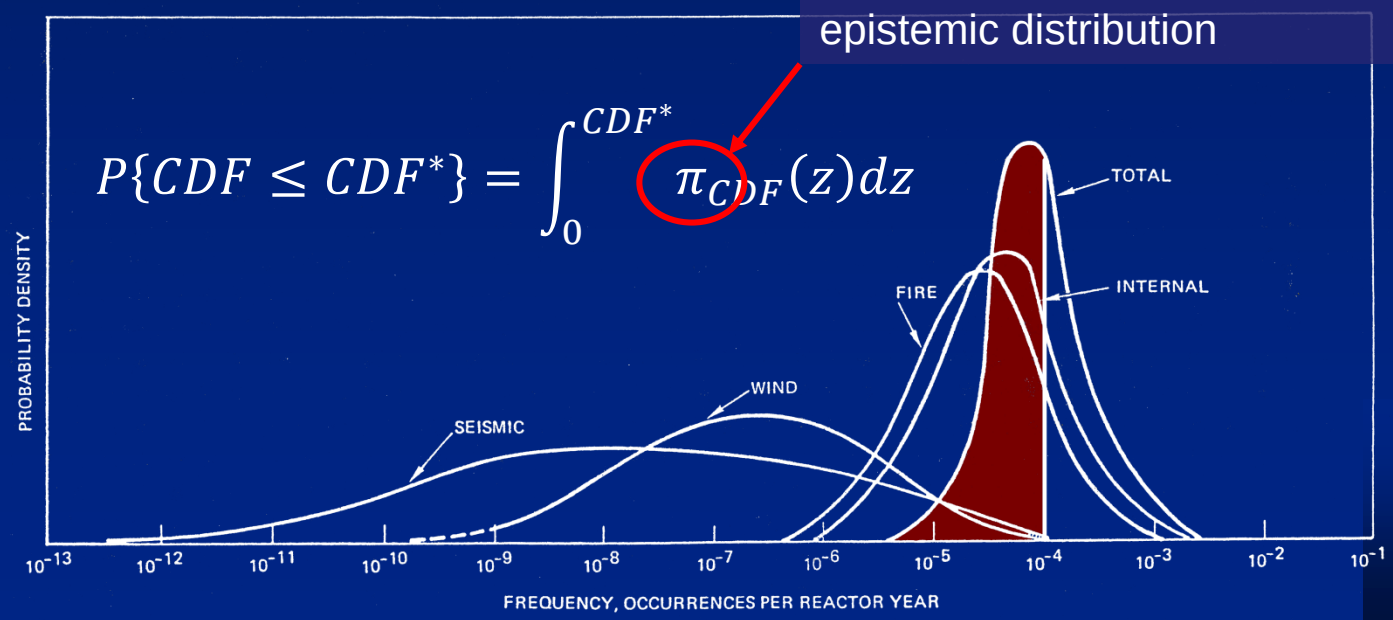
- Intentional acts (sabotage/terrorism)
- Operator errors of commission
- Organizational influences
  - Operation outside approved conditions (the “licensing basis envelope”)
  - Safety culture
  - External influences during an accident
- Design errors
- Phenomenological complexities
  - Combinations of severe hazards
  - Newly recognized hazards

Propagation of Uncertainties

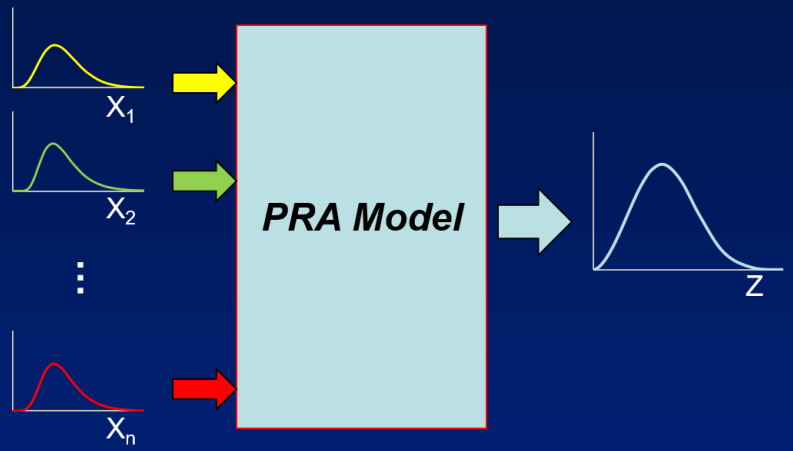
- Eventual aim: state uncertainty in risk metric estimates
- Quantitative aspects determined by "propagating" uncertainty in contributing parameters through PRA model

π notation used to indicate epistemic distribution

$$P\{CDF \leq CDF^*\} = \int_0^{CDF^*} \pi_{CDF}(z)dz$$



Propagation of Uncertainties - Methods



- Method of moments
- Sampling
  - Direct Monte Carlo
  - Latin Hypercube
- Advanced methods for computationally challenging situations
  - Importance sampling (line sampling, subset sampling, …)
  - Surrogate models (response surface, Gaussian process, neural nets, …)

Sensitivity Analysis

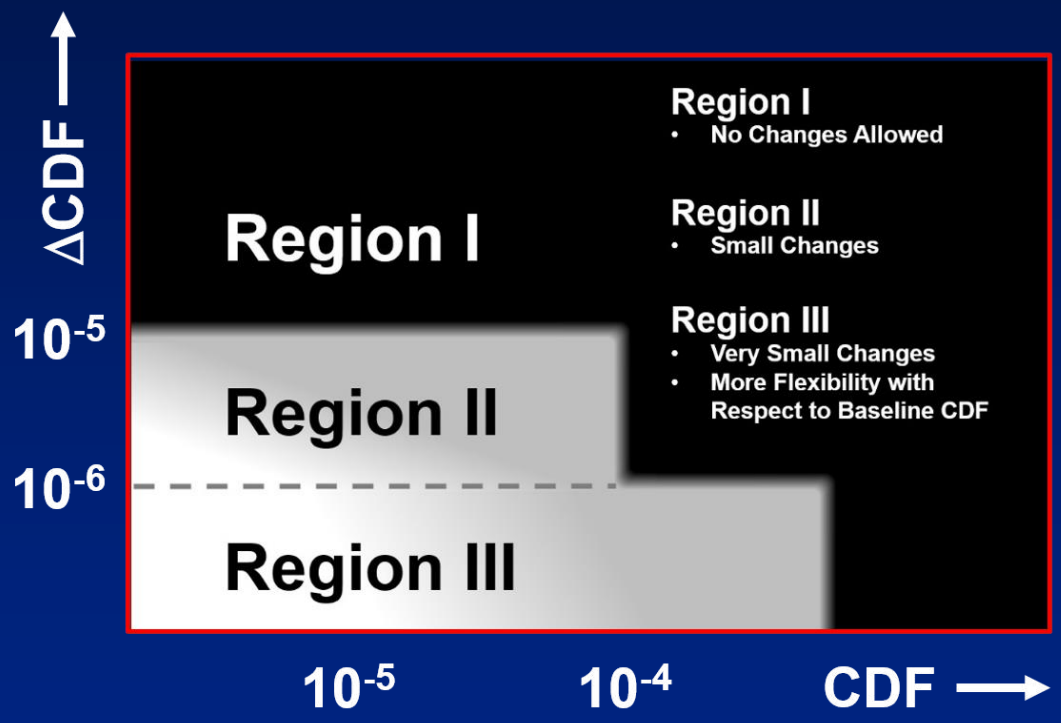
- Typical approach for treating model uncertainties
- In PRA/RIDM context, need to ensure analysis provides useful information
  - "Plausible" variations
  - Specific insights supporting decision problem (e.g., whether variation could lead to different decision)
- Importance measures, routinely available from current PRA software tools, provide results from "one-at-a-time" sensitivity calculations
- Advanced tools (e.g., global sensitivity analysis) available but not routinely used

Importance Measures

- Mathematical measures depend on definition of "importance"
- Some notation: R = risk metric of interest, P_i = probability of event i
- Common measures:

Measure	Definition	Alternate	Notes
Fussell-Vesely (F-V)	$FV_i \equiv \frac{P\left\{\bigcup_{j:i\in MCS_j} MCS_j\right\}}{R}$	$FV_i = \frac{P_i}{R}\frac{\partial R}{\partial P_i}$	Same rankings as RRW and RRR
Birnbaum	$B_i \equiv R(P_i = 1) - R(P_i = 0)$	$B_i = \frac{\partial R}{\partial P_i}$	Nearly same rankings as RAW and RIR
Risk Achievement Worth (RAW)	$RAW_i \equiv R(P_i = 1) - R(P_i)$		Nearly same rankings as Birnbaum
Risk Increase Ratio (RIR)	$RIR_i \equiv \frac{R(P_i = 1)}{R(P_i)}$		Nearly same rankings as Birnbaum
Risk Reduction Worth (RRW)	$RRW_i \equiv R(P_i) - R(P_i = 0)$		Same rankings as F-V
Risk Reduction Ratio (RRR)	$RRR_i \equiv \frac{R(P_i)}{R(P_i = 0)}$		Same rankings as F-V

Once uncertainties are characterized, then what?



- See Lecture 8-1