NRCExecSec Resource

From: Timothy Margulies <t.margulies@gmail.com>
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Subject: [External_Sender] Learning thru MAPLE models-beginners to advanced researchers
Attachments: MapleappsTiM.pdf

About the Report Collection

Growing up with the "Sputnik generation" that needed to catch-up in math and the sciences following the satellite orbiting demonstration, industry aiming for automation and digital systems, as well as, understanding environmental pollution has significantly influenced societies' education and jobs. The computer concept attributed to von Neumann of a *programmable calculator*, such as, an electronic numerical integrator has advanced an ambitious craze with accompanying technological innovations.

This purpose of this report is to provide a variety of applied math problems with computation on selected topics of physics, engineering, and anthropology. Readers are encouraged to get their start on their own journey of math explorations with computation with experiential learning.

Additional mostly analytical studies are published in,

Mathematical Science Explorations, Create Space Pub. ISBN-13: 978-1986909952 (2018).

Timothy S. Margulies

Math & Computations: MAPLE Explorations

T. Margulies

About the Report Collection

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This purpose of this report is to provide a variety of applied math problems with computation on selected topics of physics, engineering, and anthropology. Readers are encouraged to get their cravings and start on their own journey of math explorations with computation with experiential learning.

About The Author

Timothy S. Margulies enjoys presenting a collection of investigations on the computer for introducing and for teaching quantitative studies, to motivate students to conduct their own explorations on neoclassical engineering physics onto validations with data analysis. He enjoys applied mathematical recreations, as well as, photography, art, and bicycling. He earned both Masters and Doctorate degrees at The Johns Hopkins University in Baltimore, Maryland.

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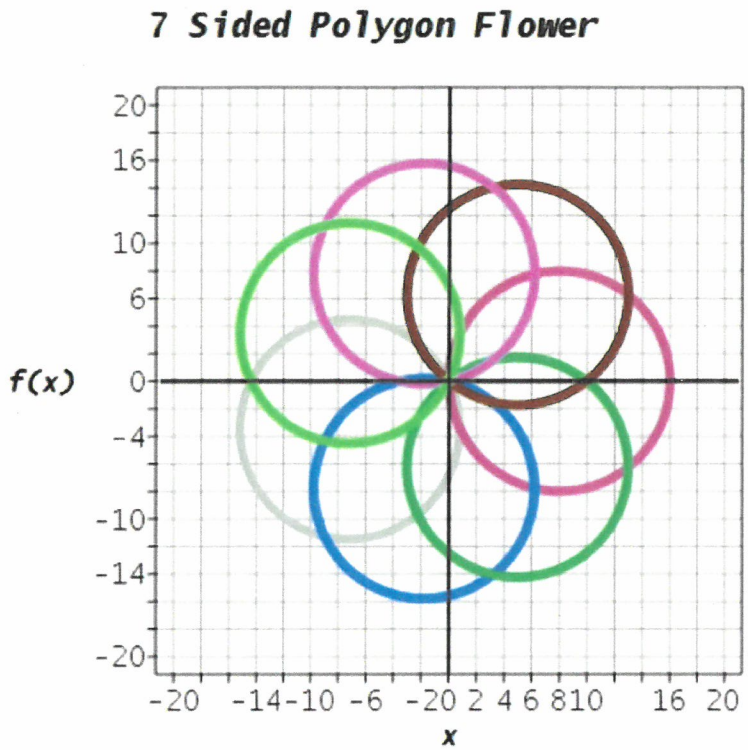
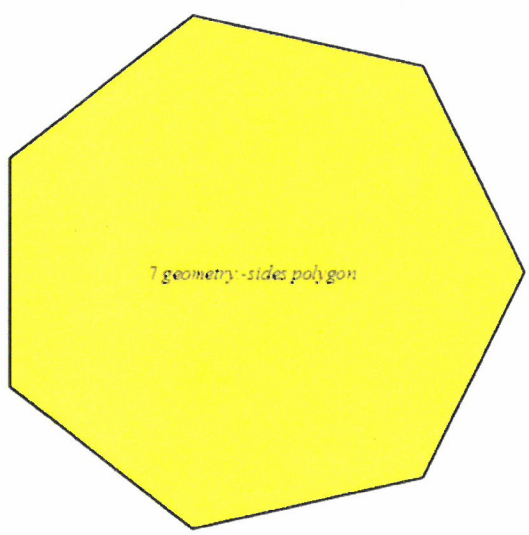
Dedicated To: Parents, Children, educators, and the MAPLE user community

1.1 Circle of Life & Polygons

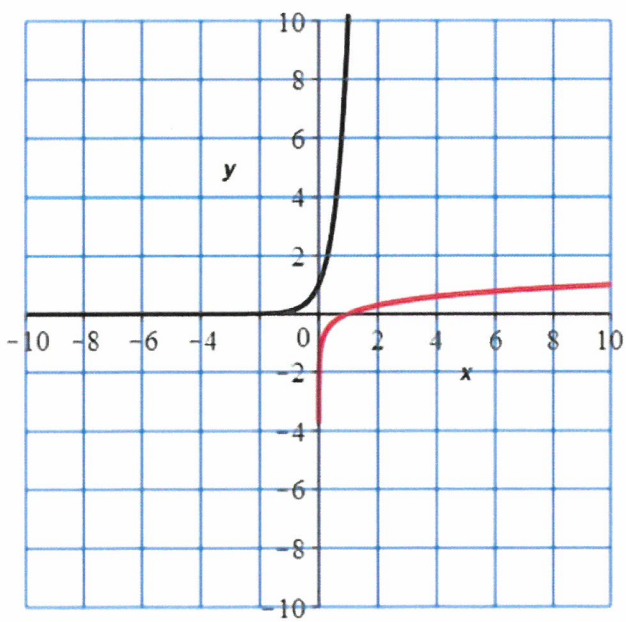




restart; with(plots); with(Student); with(RandomTools); with(geometry); n1 := 7; r1 := n1+1; x1 := 3*n1; col1 := seq(COLOR(RGB, rand()/10^12, rand()/10^12, rand()/10^12), j = 1 .. n1); theta1 :=

seq(i*(2*3.14159)/n1, I = 1 .. n1); `cosθ1` := seq(cos(theta1[i]), I = 1 .. n1); `sinθ1` := seq(sin(theta1[i]), I = 1 .. n1); for I to n1 do h1[i] := r1*`cosθ1`[i]; k1[i] := r1*`sinθ1`[i]; pl[i] := implicitplot((x-h1[i])^2+(y-k1[i])^2-r1^2, x = -x1 .. x1, y = -x1 .. x1, color = col1[i], thickness = 5, axes = boxed, title = 'n1*Sided*Polygon*Flower', titlefont = [COURIER, Bold, 14], labels = ["x", "f(x)"], labelfont = [COURIER, BOLDITALIC, 12], axesfont = [COURIER, 12], gridlines, tickmarks = [20, 20], scaling = constrained) end do; g := proc (x) options operator, arrow; 0*x end proc; h := proc (x) options operator, arrow; 10000000*x end proc; pl2 := plot(g(x), x = -x1 .. x1, y = -x1 .. x1, color = black, thickness = 2, axes = boxed, title = 'n1*Sided*Polygon*Flower', titlefont = [COURIER, Bold, 14], labels = ["x", "f(x)"], labelfont = [COURIER, BOLDITALIC, 12], axesfont = [COURIER, 12], gridlines, tickmarks = [10, 10], scaling = constrained); pl3 := plot(h(x), x = -x1 .. x1, y = -x1 .. x1, color = black, thickness = 1, axes = boxed, title = 'n1*Sided*Polygon*Flower', titlefont = [COURIER, Bold, 14], labels = ["x", "f(x)"], labelfont = [COURIER, BOLDITALIC, 12], axesfont = [COURIER, 12], gridlines, tickmarks = [20, 20], scaling = constrained); display({pl2, pl3, seq(pl[j], j = 1 .. n1)}); RegularPolygon(pgon, n1, point(o, 0, 0), 16); detail(pgon); pgon2 := proc (n1) options operator, arrow; [seq([cos(2*Pi*j/n1), sin(2*Pi*j/n1)], j = 1 .. n1)] end proc; display([polygonplot(pgon2(n1), color = yellow), textplot([0, 0, n1*sides*polygon])], axes = none); g := proc (x) options operator, arrow; 0*x end proc; h := proc (x) options operator, arrow; 10000000*x end proc; plt2 := plot(g(x), x = -x1 .. x1, y = -x1 .. x1, color = black, thickness = 2, axes = boxed, gridlines, tickmarks = [10, 10], scaling = constrained); plt3 := plot(h(x), x = -x1 .. x1, y = -x1 .. x1, color = black, thickness = 1, axes = boxed, gridlines, tickmarks = [20, 20], scaling = constrained); display({plt2, plt3});

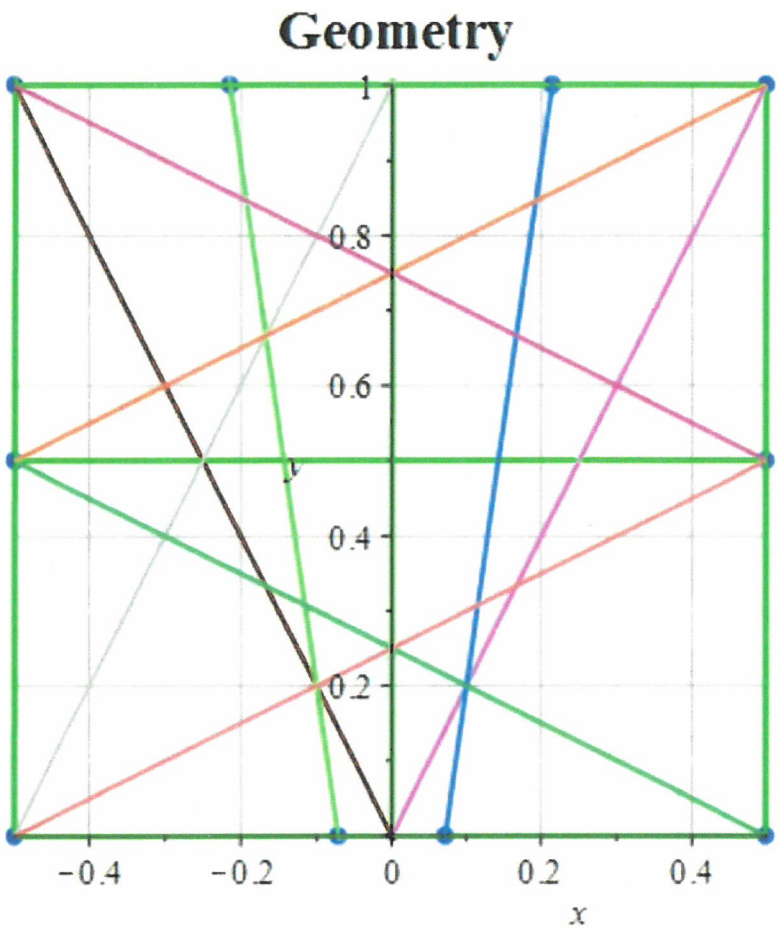
1.2 Logarithm & Exponential Functions



restart; with(Student); with(plots); p1 := plot(0*x, x = -10 .. 10, y = -10 .. 10, axis = [gridlines = [10, color = blue]], labelfont = [COURIER, BOLDITALIC, 12], axesfont = [BOLDCOURIER, 14], gridlines, tickmarks = [10, 10]); p2 := plot(x/10^9, x = -10 .. 10, y = -10 .. 10, axis = [gridlines = [10, color = blue]], labelfont =

[COURIER, BOLDITALIC, 12], axesfont = [BOLDCOURIER, 14], gridlines, tickmarks = [10, 10]); f := proc (x) options operator, arrow; 10^x end proc; p3 := plot(f(x), x = -10 .. 10, y = -10 .. 10, axis = [gridlines = [10, color = blue]], labelfont = [COURIER, BOLDITALIC, 12], axesfont = [BOLDCOURIER, 14], gridlines, tickmarks = [10, 10], color = black, thickness = 3); g := proc (x) options operator, arrow; log10(x) end proc; p4 := plot(g(x), x = -10 .. 10, y = -10 .. 10, axis = [gridlines = [10, color = blue]], labelfont = [COURIER, BOLDITALIC, 12], axesfont = [BOLDCOURIER, 14], gridlines, tickmarks = [10, 10], color = red, thickness = 3); display({p1, p2, p3, p4});

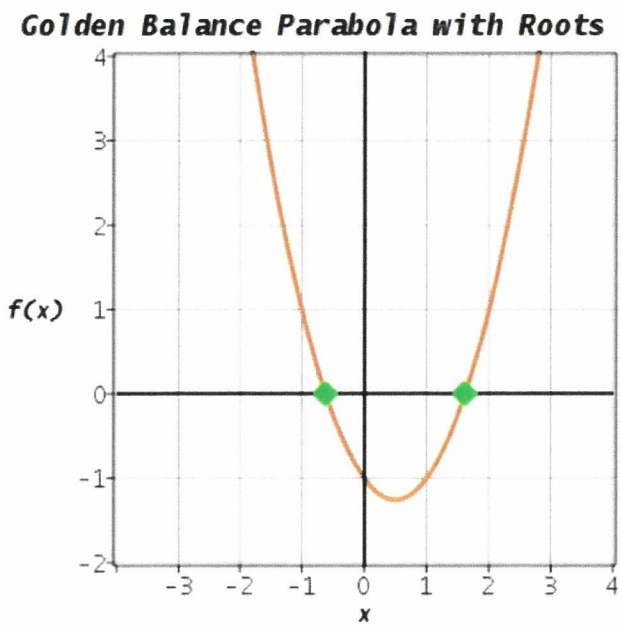
1.3 Geometry with Lines



restart; with(Student); with(plots); with(RandomTools); n := 10; col1 := [seq(COLOR(RGB, rand()/10^12, rand()/10^12, rand()/10^12), j = 1 .. n)]; a := 1; p0 := [0, a]; p1 := [-(1/2)*a, a]; p2 := [-3/14*a, a]; p3 := [3/14*a, a]; p4 := [(1/2)*a, a]; p5 := [(1/2)*a, (1/2)*a]; p6 := [(1/2)*a, 0]; p7 := [-(1/2)*a, 0]; p8 := [-(1/2)*a, (1/2)*a]; q1 := [-(1/14)*a, 0]; q2 := [(1/14)*a, 0]; L1 := y = -(1/2)*x+(1/4)*a; L2 := y = (1/2)*x+(1/4)*a; L3 := y = -2*x; L4 := y = 2*x; L5 := y = -7*x-(1/2)*a; L6 := y = 7*x-(1/2)*a; L7 := y = -(1/2)*x+3*a*(1/4); L8 := y = (1/2)*x+3*a*(1/4); L9 := y = 2*x+a; L10 := y = 2*x+a; L1x := proc (x) options operator, arrow; -(1/2)*x+(1/4)*a end proc; L2x := proc (x) options operator, arrow; (1/2)*x+(1/4)*a end

proc; L3x := proc (x) options operator, arrow; -2*x end proc; L4x := proc (x) options operator, arrow; 2*x end proc; L5x := proc (x) options operator, arrow; -7*x-(1/2)*a end proc; L6x := proc (x) options operator, arrow; 7*x-(1/2)*a end proc; L7x := proc (x) options operator, arrow; -(1/2)*x+(3/4)*a end proc; L8x := proc (x) options operator, arrow; (1/2)*x+(3/4)*a end proc; L9x := proc (x) options operator, arrow; 2*x+a end proc; L10x := proc (x) options operator, arrow; 2*x+a end proc; Lup := proc (x) options operator, arrow; 0*x+a end proc; Llow := proc (x) options operator, arrow; 0*x end proc; Lc := proc (x) options operator, arrow; 1000000*x end proc; LcL := proc (x) options operator, arrow; 10000000*x+5000000*a end proc; LcR := proc (x) options operator, arrow; Heaviside(x-(1/2)*a) end proc; Lh := proc (x) options operator, arrow; 0*x+(1/2)*a end proc; p17 := solve({L5, L7}, {x, y}); p16 := solve({L5, L8}, {x, y}); p10 := solve({L1, L5}, {x, y}); p51 := solve({L2, L7}, {x, y}); outer := [p1, p2, p3, p4, p5, p6, q1, q2, p7, p8]; plpts := pointplot(outer, style = point, symbol = solidcircle, symbolsize = 14, color = blue, axes = normal, gridlines, title = "Geometry", titlefont = ["ARIAL", bold, 18], scaling = constrained); x1 := -.5; x2 := .5; x21 := .500000000001; y1 := 0.; y2 := 1.; plinesxy := plot({Lc(x), LcL(x), LcR(x), Lh(x), Llow(x), Lup(x)}, x = x1 .. x21, y = y1 .. y2, axes = normal, gridlines = true, title = "Geometry ", titlefont = ["ARIAL", bold, 18], style = line, thickness = [2, 2], color = green); plines := plot({L10x(x), L1x(x), L2x(x), L3x(x), L4x(x), L5x(x), L6x(x), L7x(x), L8x(x), L9x(x)}, x = x1 .. x2, y = y1 .. y2, axes = normal, gridlines = true, title = "Geometry ", titlefont = ["ARIAL", bold, 18], style = line, thickness = [2, 2], color = col1); plots[display]({plines, plpts, plinesxy});

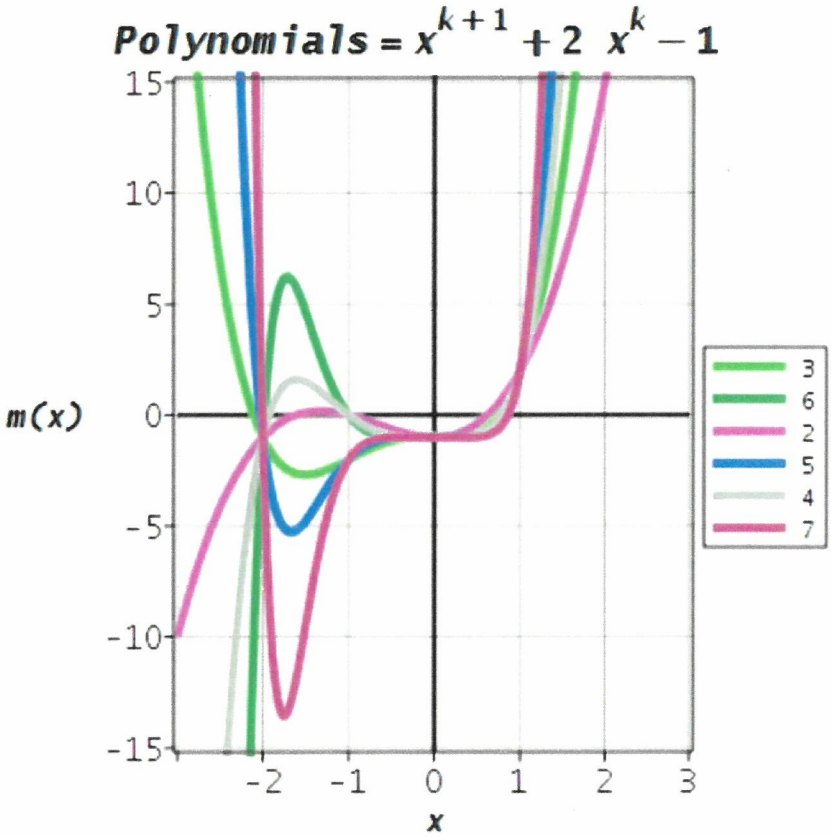
1.4 Golden Parabola with Roots



restart; with(plots); with(Student); with(RandomTools); f := proc (x) options operator, arrow; x^2-x-1 end proc; eq1 := f(x) = 0; sol := solve(eq1, x); r1 := evalf(sol[1]); r2 := evalf(sol[2]); g := proc (x) options operator, arrow; 0*x end proc; h := proc (x) options operator, arrow; 10000000*x end proc; p5 := plot([[r2, 0], [r1, 0]], x = -4 .. 4, y = -2 .. 4, style = point, symbol = soliddiamond, symbolsize = 30, color =

green, axes = boxed, title = 'Golden*Balance*Parabola*with*Roots', titlefont = [COURIER, Bold, 14], labels = ["x", "f(x)"], labelfont = [COURIER, BOLDITALIC, 12], axesfont = [COURIER, 12], gridlines, tickmarks = [10, 10]); p6 := plot(f(x), x = -4 .. 4, y = -2 .. 4, color = gold, thickness = 3, axes = boxed, title = 'Golden*Balance*Parabola*with*Roots', titlefont = [COURIER, Bold, 14], labels = ["x", "f(x)"], labelfont = [COURIER, BOLDITALIC, 12], axesfont = [COURIER, 12], gridlines, tickmarks = [10, 10]); p7 := plot(g(x), x = -4 .. 4, y = -2 .. 4, color = black, thickness = 2, axes = boxed, title = 'Golden*Balance*Parabola*with*Roots', titlefont = [COURIER, Bold, 14], labels = ["x", "f(x)"], labelfont = [COURIER, BOLDITALIC, 12], axesfont = [COURIER, 12], gridlines, tickmarks = [10, 10]); p8 := plot(h(x), x = -4 .. 4, y = -2 .. 4, color = black, thickness = 2, axes = boxed, title = 'Golden*Balance*Parabola*with*Roots', titlefont = [COURIER, Bold, 14], labels = ["x", "f(x)"], labelfont = [COURIER, BOLDITALIC, 12], axesfont = [COURIER, 12], gridlines, tickmarks = [10, 10]); display({p5, p6, p7, p8});

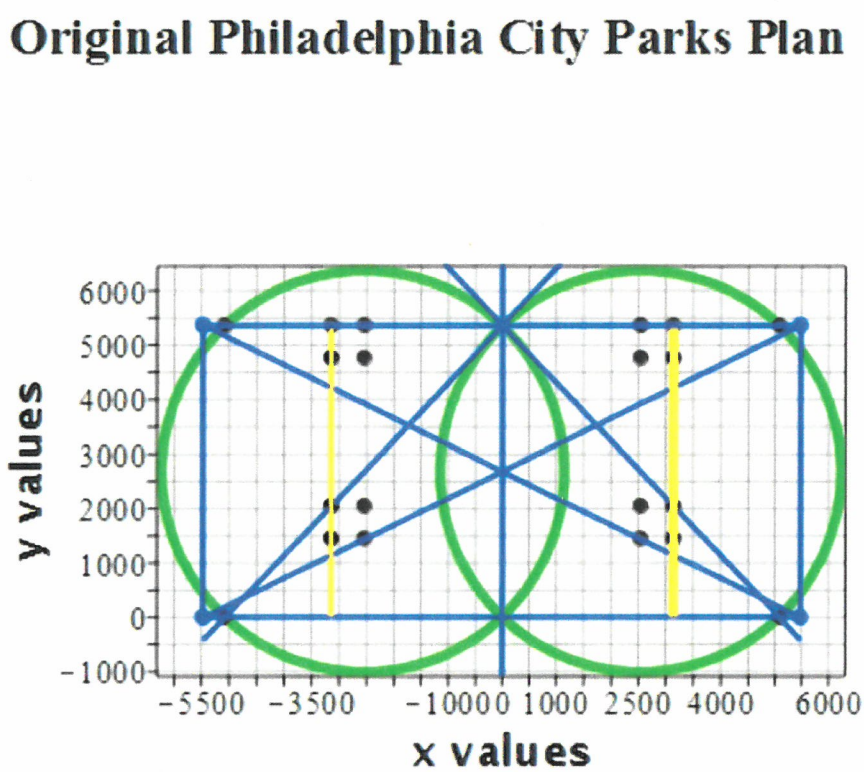
1.5 Polynomials



restart; with(plots); with(Student); with(RandomTools); n := 7; r00 := 3; r11 := 15; col1 := seq(COLOR(RGB, rand()/10^12, rand()/10^12, rand()/10^12), j = 1 .. n); for I from 2 to n do m := proc (x) options operator, arrow; x^(i+1)+2*x^i-1 end proc; eqn := m(x) = 0; sol := solve(eqn, x); rt :=

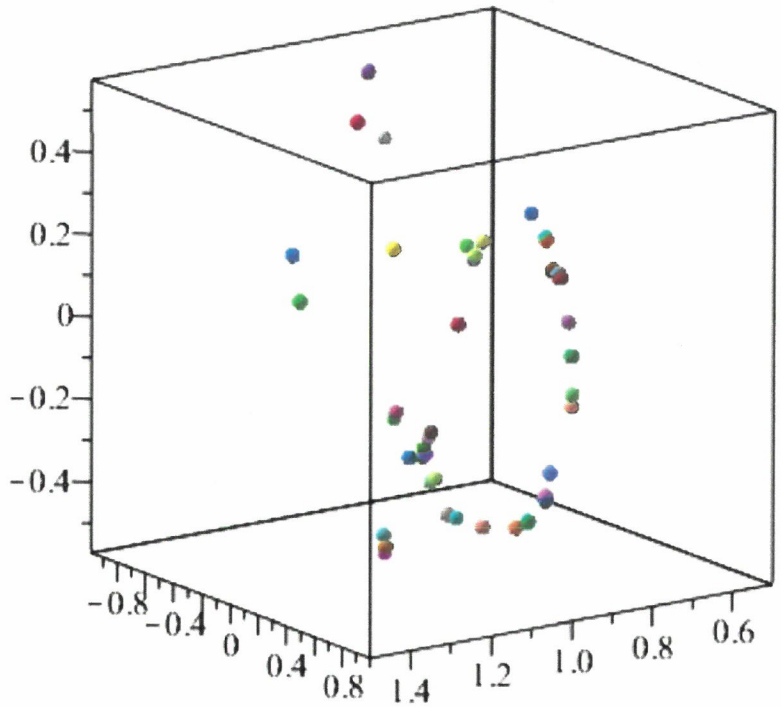
```
seq(evalf(Re(sol[j])), j = 1 .. i); g := proc (x) options operator, arrow; 0*x end proc; h := proc (x) options operator, arrow; 10000000*x end proc; plt[i] := plot(m(x), x = -r00 .. r00, y = -r11 .. r11, color = col1[i], thickness = 4, axes = boxed, title = 'Polynomials*x^(k+1)+2*x^k-1', titlefont = [COURIER, Bold, 14], labels = ["x", " m(x)"], labelfont = [COURIER, BOLDITALIC, 12], axesfont = [COURIER, 12], gridlines, tickmarks = [10, 10], legend = [i], legendstyle = [font = ["HELVETICA", 9], location = right]); pl1 := plot(g(x), x = -r00 .. r00, y = -r11 .. r11, color = black, thickness = 2, axes = boxed, title = 'Polynomials = x^(k+1)+2*x^k-1', titlefont = [COURIER, Bold, 14], labels = ["x", "m(x)"], labelfont = [COURIER, BOLDITALIC, 12], axesfont = [COURIER, 12], gridlines, tickmarks = [10, 10]); pl2 := plot(h(x), x = -r00 .. r00, y = -r11 .. r11, color = black, thickness = 2, axes = boxed, title = 'Polynomials = x^(k+1)+2*x^k-1', titlefont = [COURIER, Bold, 14], labels = ["x", "m(x)"], labelfont = [COURIER, BOLDITALIC, 12], axesfont = [COURIER, 12], gridlines, tickmarks = [10, 10]) end do; display({pl1, pl2, seq(plt[j], j = 2 .. n)})
```

1.6 City Grid Design



1.7 Ancient Monuments

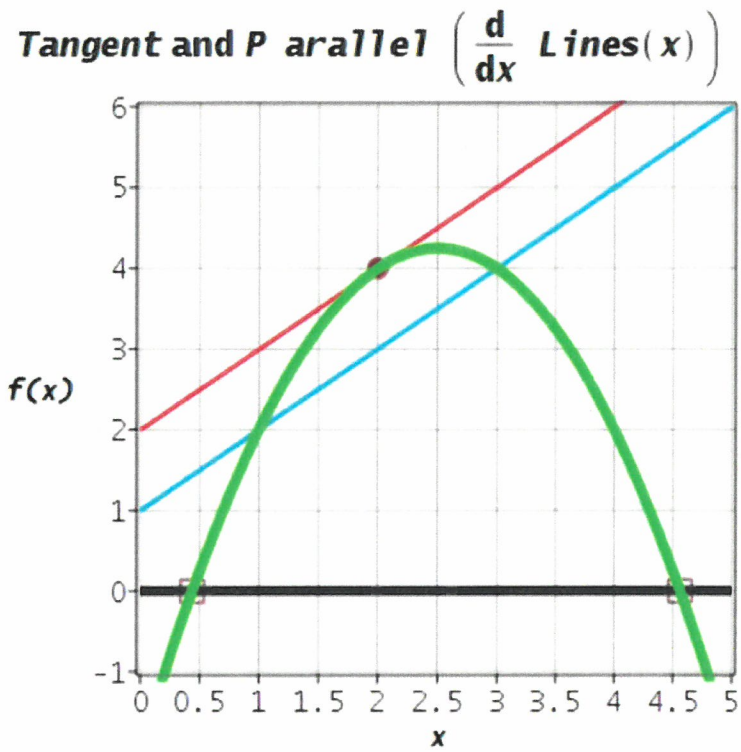
Ancient Monuments – Cities



restart; with(Student); with(plots); with(LinearAlgebra); with(RandomTools); n := 45; col1 := seq(COLOR(RGB, rand()/10^12, rand()/10^12, rand()/10^12), I = 1 .. n); lat1 := vector(45, Vector[column](45, [.258509265, .216261751, .240860488, .42927772, 0.61048595e-1, -.240858422, -.225801707, -.425376852, -.396607122, -.37067725, -.293687143, .265133166, -.233679239, -.42492514, -.30331112, -.533731764, -0.44926713e-1, .222267317, .218313309, 0.43072168e-1, -.241631982, -.255434957, -.289921942, -.301082779, -.308616469, -.559741263, -.572640075, .442351234, .167929381, -0.55265592e-1, 0.66997757e-1, .146921779, .245482199, .255309753, .326906347, .330393023, .376671487, -.350184702, -.427078035, -.361074684, -0.24683178e-1, .24718693, .254025434, .57429465, -.323089755])); long1 := vector(45, Vector[column](45, [.855816237, .299319453, .253195848, -.331610986, -.98419611, -.253198802, -.239949354, .370013005, .603668306, .693188859, .811231122, .363252345, .842606467, .122442689, .816974406, -.325571674, -.928162509, .309294748, .304588458, .193950308, 0.7855246e-2, -0.9306539e-2, -0.39258582e-1, -0.34316934e-1, -0.50881693e-1, -.308738709, -.317579431, -.521752781, -.260503018, .995312156, .993305742, .968074824, .905198443, .896744232, .812762244, .806788819, .70504736, 0.41004827e-1, .186525443, 0.6401283e-2, .999497375, .915662727, .903584836, -.444833703, -.144405029])); Radius1 := vector(45, Vector[column](45, [1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1])); pl1 := [seq(pointplot3d([Radius1[i], long1[i], lat1[i]], axes = boxed, symbol = solidcircle, symbolsize = 16, title =

'Ancient*Monuments-Cities', titlefont = [COURIER, Bold, 14], color = col1[i], coords = rectangular), I = 1 .. 45)]; display(pl1);

1.8 Parabola and Tangent Line at a Point



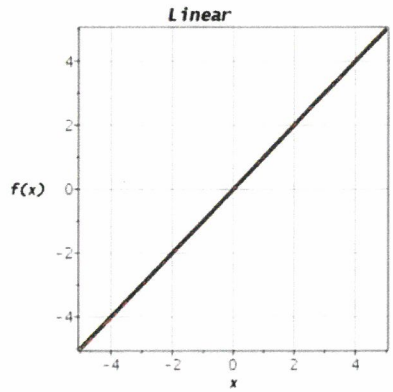
restart; with(plots); with(Student); with(RandomTools); f := proc (x) options operator, arrow; -x^2+5*x-2 end proc; g := proc (x) options operator, arrow; x+2 end proc; h := proc (x) options operator, arrow; x+1 end proc; k := proc (x) options operator, arrow; (1/10000000)*x end proc; eq1 := f(x) = 0; sol := solve(eq1, x); r1 := evalf(sol[1]); r2 := evalf(sol[2]); p0 := plot([[2, 4]], x = 0 .. 5, y = -1 .. 6, style = point, symbol = solidcircle, symbolsize = 20, axes = boxed, title = 'Tangent and P*arallel*Lines(x)', titlefont = [COURIER, Bold, 14], labels = ["x", "f(x)"], labelfont = [COURIER, BOLDITALIC, 12], axesfont = [COURIER, 12], gridlines, tickmarks = [10, 10]); p1 := plot([[r2, 0], [r1, 0]], x = 0 .. 5, y = -1 .. 6, style = point, symbol = box, symbolsize = 26, axes = boxed, title = 'Tangent and P*arallel*(diff(Lines(x), x))', titlefont = [COURIER, Bold, 14], labels = ["x", "f(x)"], labelfont = [COURIER, BOLDITALIC, 12], axesfont = [COURIER, 12], gridlines, tickmarks = [10, 10]); p2 := plot({f(x), k(x)}, x = 0 .. 5, y = -1 .. 6, color = [black, green], thickness = 5, axes = boxed, title = 'Tangent and P*arallel*(diff(Lines(x), x))', titlefont = [COURIER, Bold, 14], labels = ["x", "f(x)"], labelfont = [COURIER, BOLDITALIC, 12], axesfont = [COURIER, 12], gridlines, tickmarks = [10, 10]); p3 := plot(g(x), x = 0 .. 5, y = -1 .. 6, color = red, thickness = 2, axes = boxed, title = 'Tangent and P*arallel*(diff(Lines(x), x))', titlefont = [COURIER, Bold, 14], labels = ["x", "f(x)"], labelfont = [COURIER, BOLDITALIC, 12], axesfont = [COURIER, 12], gridlines, tickmarks = [10, 10]); p4 := plot(h(x), x = 0 .. 5, y = -1 .. 6, color = cyan, thickness = 2, axes = boxed, title = 'Tangent and P*arallel*(diff(Lines(x), x))', titlefont

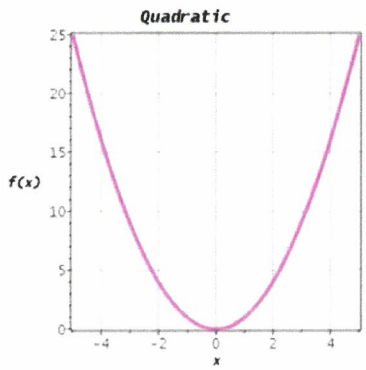
= [COURIER, Bold, 14], labels = ["x", "f(x)"], labelfont = [COURIER, BOLDITALIC, 12], axesfont = [COURIER, 12], gridlines, tickmarks = [10, 10]); display({p0, p1, p2, p3, p4})

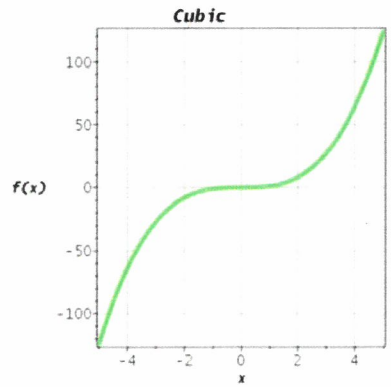
1.9 Functions, Differentials, Integrals, & Riemann Sums

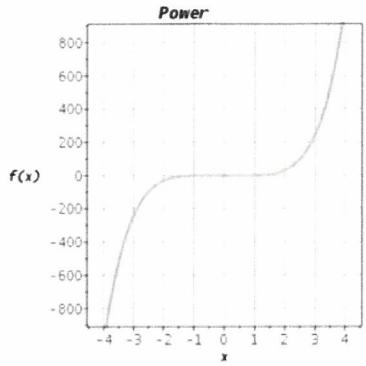
$$n := 5$$

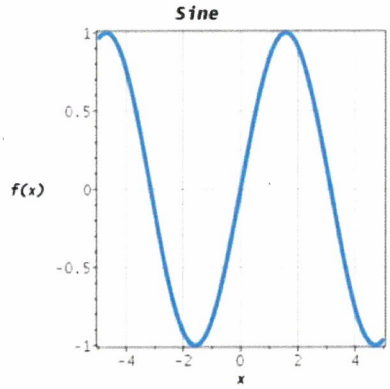
$$func1 := [x, x^2, x^3, x^5, \sin(x), \cos(x), \tan(x), e^x, \ln(x), \arcsin(x)]$$

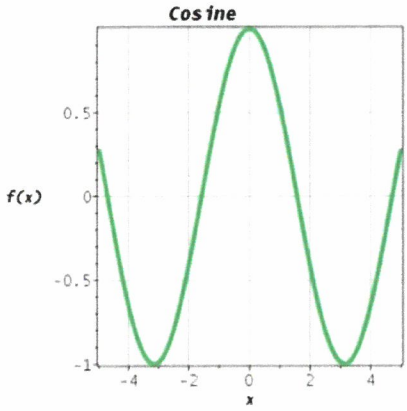


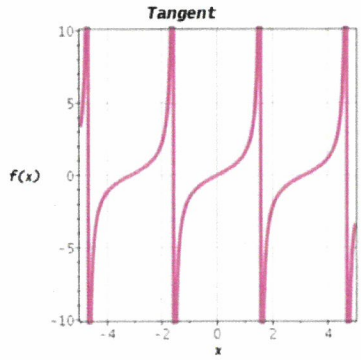


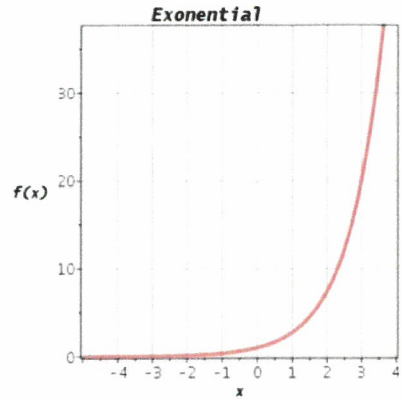


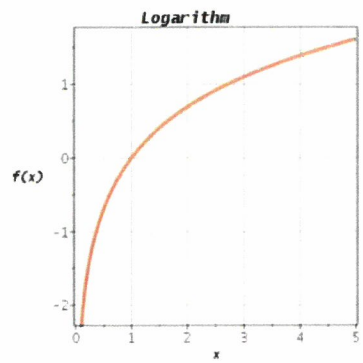

Power
f(x)
x

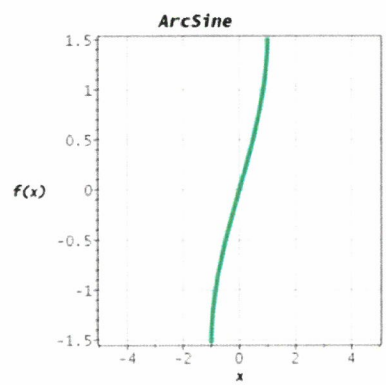

Sine
f(x)
x


Cosine
f(x)
x


Tangent
f(x)
x

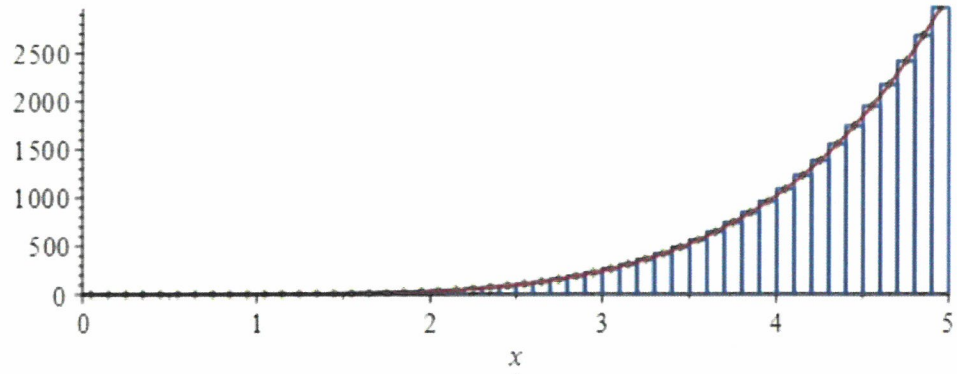




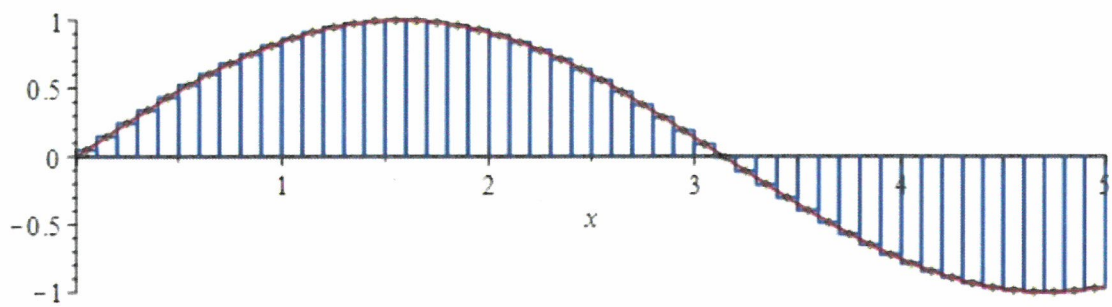


$$1,\ 2\,x,\ 3\,x^2,\ 5\,x^4,\ \cos(x),\ -\sin(x),\ 1+\tan(x)^2,\ e^x,\ \frac{1}{x},\ \frac{1}{\sqrt{-x^2+1}}$$

$$\frac{x^2}{2},\ \frac{x^3}{3},\ \frac{x^4}{4},\ \frac{x^6}{6},\ -\cos(x),\ \sin(x),\ -\ln(\cos(x)),\ e^x,\ x\ln(x)-x,\ x\arcsin(x)+\sqrt{-x^2+1}$$



A midpoint Riemann sum approximation of $\int_0^5 f(x)\,dx$, where $f(x) = x(x)^5$ and the partition is uniform. The approximate value of the integral is 2602.864766. Number of subintervals used: 50.



A midpoint Riemann sum approximation of $\int_0^5 f(x)\,dx$, where $f(x) = \sin(x)(x)$ and the partition is uniform. The approximate value of the integral is 0.7166363756. Number of subintervals used: 50.

$$f1 := x \mapsto \sin(x)$$

$$a := 0.$$

$$b0 := 1.570796325$$

$$b := 5.0$$

$$NT := 50$$

$$dh := 0.1000000000$$

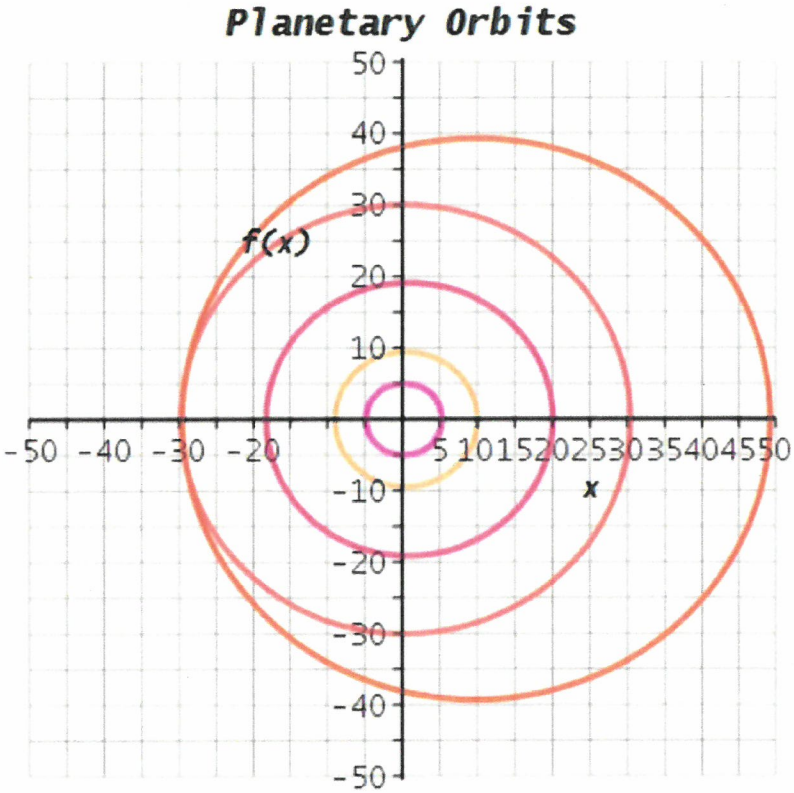
$$a1 := 0.$$

$$b1 := -0.9589242747$$

$$Intxtrap := 0.7157407668$$

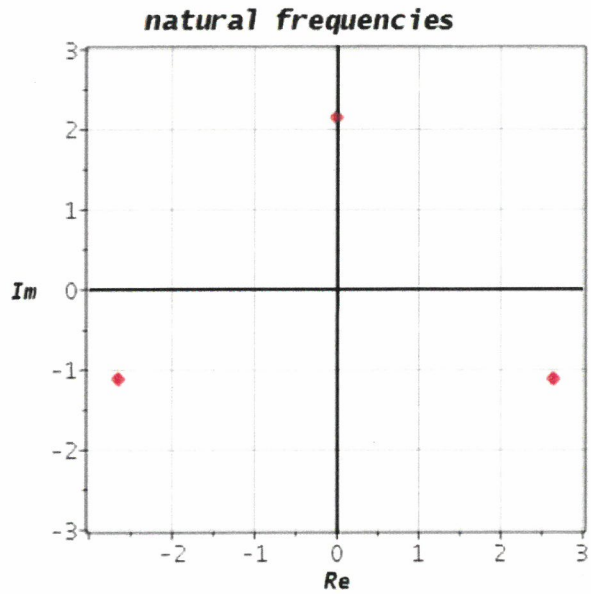
$$IntxRS := 0.7540977378$$

2.0 Planetary Orbits




```
restart; with(plots); with(Student); with(RandomTools); r1 := 1.; n1 := 9; col1 := seq(COLOR(RGB,
rand()/10^11, rand()/10^12, rand()/10^12), j = 1 .. n1); a := [.387, .723, 1., 1.52, 5.2, 9.54, 19.18, 30.07,
39.44]; b := [.264664, .719491, .991464, 1.472766, 5.175944, 9.511959, 19.15699, 30.065, 39.3158];
epsilon := [.206, 0.7e-2, 0.17e-1, 0.93e-1, 0.48e-1, 0.56e-1, 0.46e-1, 0.1e-1, .248]; c1 :=
seq(evalf(epsilon[i]*a[i]), I = 1 .. n1); x1 := 50; for I to n1 do pl[i] := implicitplot((x-
c1[i])^2/a[i]^2+y^2/b[i]^2-r1^2, x = -x1 .. x1, y = -x1 .. x1, color = col1[i], thickness = 3, axes = normal,
title = 'Planetary*Orbits', titlefont = [COURIER, Bold, 14], labels = ["x", "f(x)"], labelfont = [COURIER,
BOLDITALIC, 12], axesfont = [COURIER, 12], gridlines, tickmarks = [20, 20], gridrefine = 1) end do; g :=
proc (x) options operator, arrow; 0*x end proc; h := proc (x) options operator, arrow; 10000000*x end
proc; pl2 := plot(g(x), x = -x1 .. x1, y = -x1 .. x1, color = black, thickness = 2, axes = normal, title =
'Planetary*Orbits', titlefont = [COURIER, Bold, 14], labels = ["x", "f(x)"], labelfont = [COURIER,
BOLDITALIC, 12], axesfont = [COURIER, 12], gridlines, tickmarks = [20, 20]); pl3 := plot(h(x), x = -x1 .. x1, y
= -x1 .. x1, color = black, thickness = 1, axes = normal, title = 'Planetary*Orbits', titlefont = [COURIER,
Bold, 14], labels = ["x", "f(x)"], labelfont = [COURIER, BOLDITALIC, 12], axesfont = [COURIER, 12],
gridlines, tickmarks = [20, 20]); display({pl2, pl3, seq(pl[j], j = 1 .. n1)});
```

2.1 Bicycle Natural Frequencies




```
restart; with(Student); with(LinearAlgebra); with(DETools); with(plots); A0 := Matrix(2, 2, [[3., -0.5e-2], [-
0.2e-2, 0.5e-1]]); A0 := Matrix(2, 2, [[80.81722, 2.31941332208709], [2.31941332208709,
.29784188199686]]); g := 9.81; v := 3.5; omega1 := 10; A := v . Matrix(2, 2, [[0, 33.86641391492494], [-
.85035641456978, 1.68540397397560]]); K0 := Matrix(2, 2, [[-80.95, -2.59951685249872], [-
2.59951685249872, -.80329488458618]]); K2 := Matrix(2, 2, [[0, 76.59734589573222], [0,
2.65431523794604]]); B := evalf(K2*v^2+K0*g); q := Matrix(2, 1, [x(t), y(t)]); zer0 := Matrix(2, 1, [0, 0]);
zer1 := Matrix(2, 1, [exp(-t)*sin(omega1*t), 0]); Id1 := IdentityMatrix(2); ampl := Matrix(2, 2, [[a0, 0], [0,
b0]]); qs := Matrix(2, 1, [exp(a*t), exp(a*t)]); q2 := ampl . qs; q[1] := q2[1]; q[2] := q2[2]; de1 := A0 .
(diff(q, t, t))+A . (diff(q, t))+B . q; mult := Matrix(2, 2, [[exp(-a*t), 0], [0, exp(-a*t)]]); sys0 := evalf(mult .
de1); sys1 := simplify(sys0[1]); sys2 := simplify(sys0[2]); sys1f := collect(sys1, [a0, b0]); sys2f :=
collect(sys2, [a0, b0]); m11 := coeff(sys1f, a0); m21 := coeff(sys2f, a0); m12 := coeff(sys1f, b0); m22 :=
coeff(sys2f, b0); mat := Matrix(2, 2, [[m11, m12], [m21, m22]]); p := Determinant(%); poly := subs(a =
I*omega, p); natf := [solve(poly = 0, omega)]; natf[1]; for I to 4 do pl[i] := complexplot([natf[i]], -3 .. 3, -3
.. 3, style = point, symbol = soliddiamond, symbolsize = 16, color = red, gridlines, axes = boxed, title =
'natural*frequencies', titlefont = [COURIER, Bold, 14], labels = ["Re", "Im"], labelfont = [COURIER,
BOLDITALIC, 12], axesfont = [COURIER, 12]) end do; g := proc (x) options operator, arrow; 0*x end proc;
h := proc (x) options operator, arrow; 10000000*x end proc; plx := plot(g(x), x = -3 .. 3, y = -3 .. 3, style =
line, thickness = 2, color = black); ply := plot(h(x), x = -3 .. 3, y = -3 .. 3, style = line, thickness = 2, color =
black); plt := plots[display](seq(pl[i], I = 1 .. 4), plx, ply);
```

2.2 Linear Bicycle Dynamics

$$\underline{\underline{M}}\,\ddot{\underline{V}} + v\underline{\underline{A}}\dot{\underline{V}} + \left(g\underline{\underline{K}}_0 + v^2\underline{\underline{K}}_2\right)\underline{V} = \underline{f}$$

$$\underline{\underline{M}} = \begin{pmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{pmatrix}$$

$$\underline{\underline{A}} = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}$$

$$\underline{\underline{K}}_0 = \begin{pmatrix} k_{11} & k_{12} \\ k_{21} & k_{22} \end{pmatrix}$$

$$\underline{\underline{K}}_2 = \begin{pmatrix} K_{11} & K_{12} \\ K_{21} & K_{22} \end{pmatrix}$$

$$\underline{V} = \begin{pmatrix} \phi \\ \delta \end{pmatrix},\ \dot{\underline{V}} = \begin{pmatrix} \dot{\phi} \\ \dot{\delta} \end{pmatrix},\ \ddot{\underline{V}} = \begin{pmatrix} \ddot{\phi} \\ \ddot{\delta} \end{pmatrix},\ \underline{f} = \begin{pmatrix} f_1 \\ f_2 \end{pmatrix}$$

lean ϕ , steer δ

$$m_{11}\ddot{\phi} + m_{12}\ddot{\delta} + v\left(a_{11}\dot{\phi} + a_{12}\dot{\delta}\right) + g(k_{11}\phi + k_{12}\delta) + v^2(K_{11}\phi + K_{12}\delta) = f_1 \rightarrow e^{-\frac{t}{\tau}}\sin(\omega t + \theta)$$

$$m_{21}\ddot{\phi} + m_{22}\ddot{\delta} + v\left(a_{21}\dot{\phi} + a_{22}\dot{\delta}\right) + g(k_{21}\phi + k_{22}\delta) + v^2(K_{21}\phi + K_{22}\delta) = f_2 \rightarrow 0$$

$$\phi(0) = \phi_0,\ \delta(0) = \delta_0\ ,\ \dot{\phi}(0) = 0\ ,\ \dot{\delta}(0) = 0$$

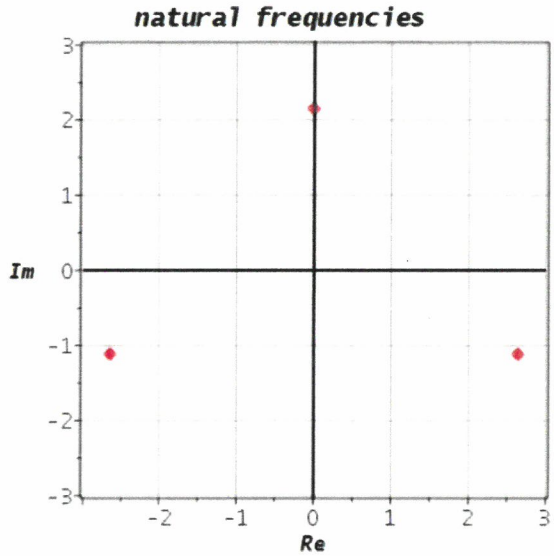
$$eq1 := \left[80.81722\frac{d^2}{dt^2}x(t) + 2.31941332208709\frac{d^2}{dt^2}y(t) + 33.8664139149249\frac{d}{dt}y(t) - 794.119500000000\,x(t) + 51.0960855727198\,y(t) - e^{-t}\sin(10\,t)\right]$$

$$eq2 := \left[2.31941332208709\frac{d^2}{dt^2}x(t) + 0.29784188199686\frac{d^2}{dt^2}y(t) - 0.850356414569780\frac{d}{dt}x(t) + 1.68540397397560\frac{d}{dt}y(t) - 25.5012603230124\,x(t) - 5.22600757984439\,y(t)\right]$$

x(0) := 5., y(0) := 1., D(x)(0) := 0, D(y)(0) := 0

Ref Meijaard, Jaap P., Jim M. Papadopoulos, Andy Ruina, and Arend L. Schwab. "Linearized dynamics equations for the balance and steer of a bicycle: a benchmark and review." In *Proceedings of the*

Royal Society of London A: Mathematical, Physical and Engineering Sciences, vol. 463, no. 2084, pp. 1955-1982. The Royal Society, 2007.

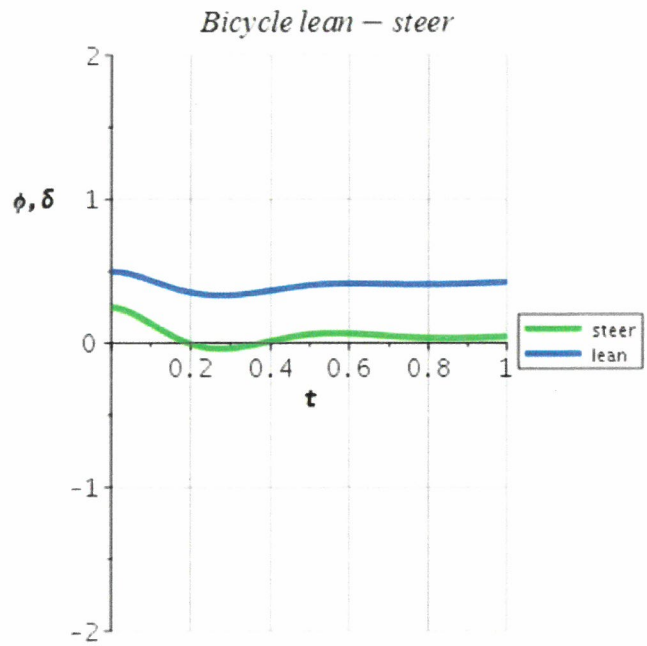


$$poly := 18.69107474\,\omega^4 - 208.7112305\,\mathrm{I}\,\omega^3 - 49.145052\,\omega^2 + 1055.051236\,\mathrm{I}\,\omega + 3714.79950$$

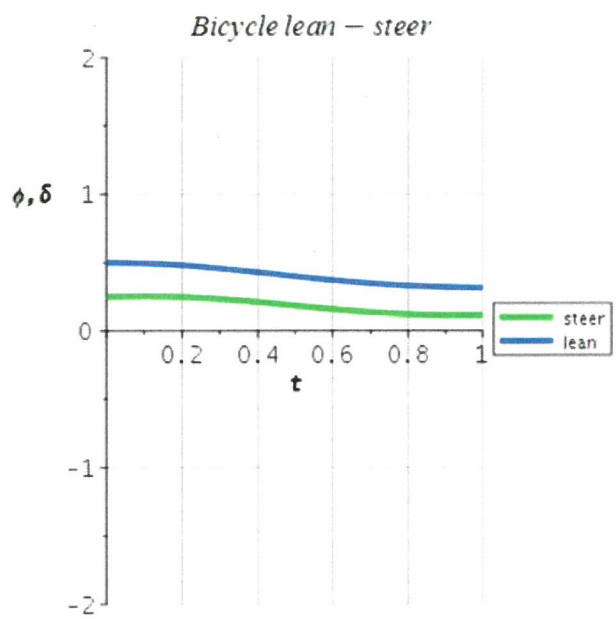
natf := [2.644144863 - 1.111251629 I, 2.149580430 I, 11.23928075 I, -2.644144863 - 1.111251629 I]

$$eq1 := \left[80.81722\,\frac{d^2}{dt^2}\,x(t) + 2.31941332208709\,\frac{d^2}{dt^2}\,y(t) + 338.664139149249\,\frac{d}{dt}\,y(t) - 794.119500000000\,x(t) + 7634.23332925021\,y(t) - 0.5\,e^{-t}\sin(1.5\,t)\right]$$

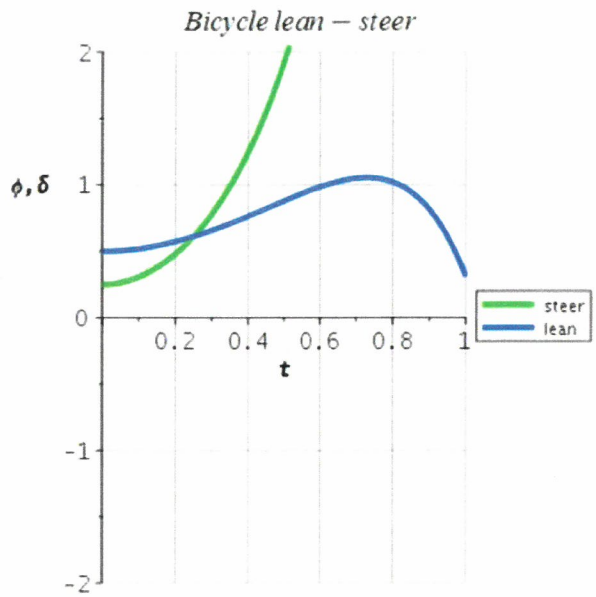
$$eq2 := \left[2.31941332208709\,\frac{d^2}{dt^2}\,x(t) + 0.29784188199686\,\frac{d^2}{dt^2}\,y(t) - 8.50356414569780\,\frac{d}{dt}\,x(t) + 16.8540397397560\,\frac{d}{dt}\,y(t) - 25.5012603230124\,x(t) + 257.551200976814\,y(t)\right]$$


Bicycle lean − steer
2
1
0
-1
-2
ϕ, δ
0.2
0.4
0.6
0.8
1
t
steer
lean

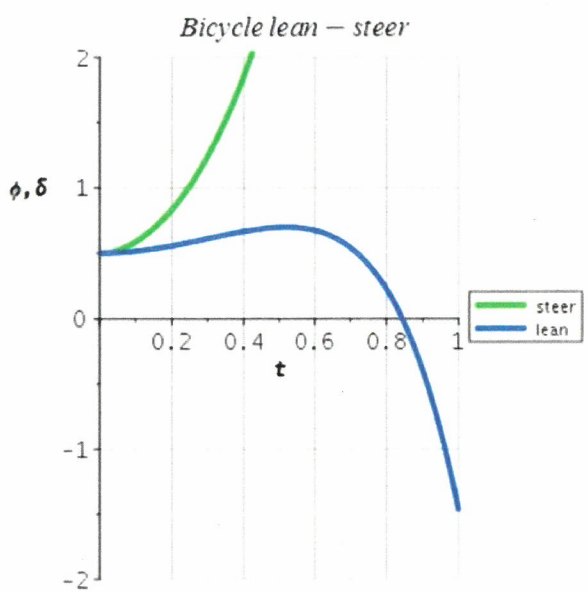
$v = 10.$


Bicycle lean − steer
2
1
0
-1
-2
ϕ, δ
0.2
0.4
0.6
0.8
1
t
steer
lean

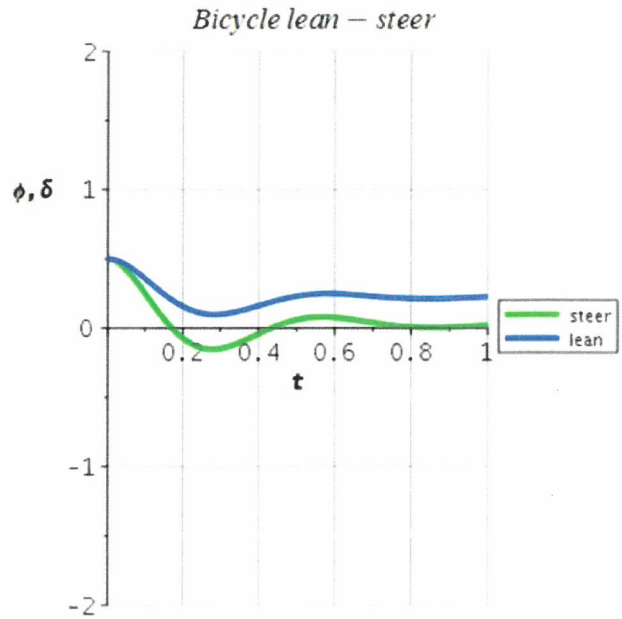
$v = 5..$



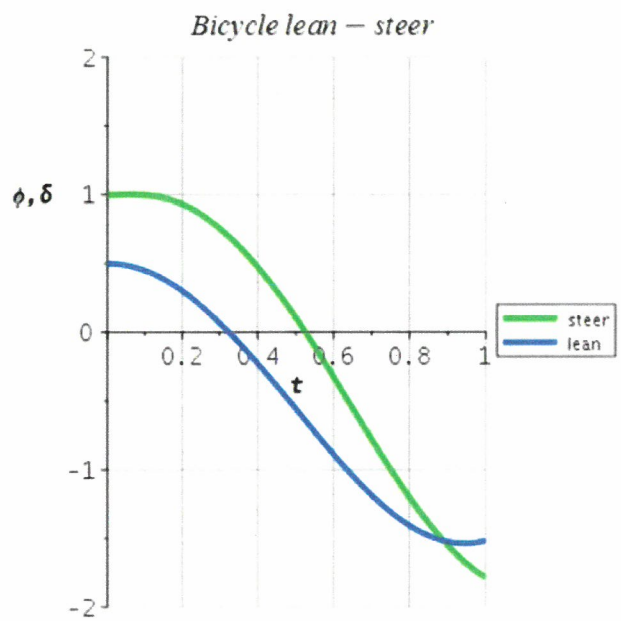
$v = 1.5.$



$v = 1.5., x(0) = 0.5 = y(0)$

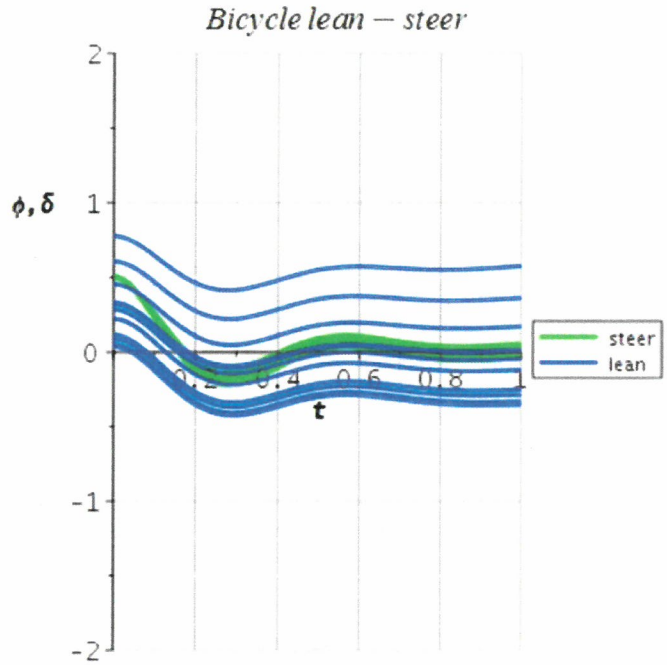


$v = 10.., x(0) = 0.5 = y(0), \omega = 1.5$



$v = 4.$

Random amplitude and phase



$$v = 10.., x(0) = 0.5 = y(0), \omega = 1.5$$

$$f_1 = \exp\left(\frac{-t}{\tau}\right)\left(1 - \frac{t}{\tau}\right) \mathrm{A(i)} \sin(\omega t + \theta(i)); \text{ sample i}$$

$$\mathrm{A(i)} \in [0.5,1.5],\ \theta(\mathrm{i}) \in [0.5,1.5]$$

2.3 Fractional Calculus Generalization

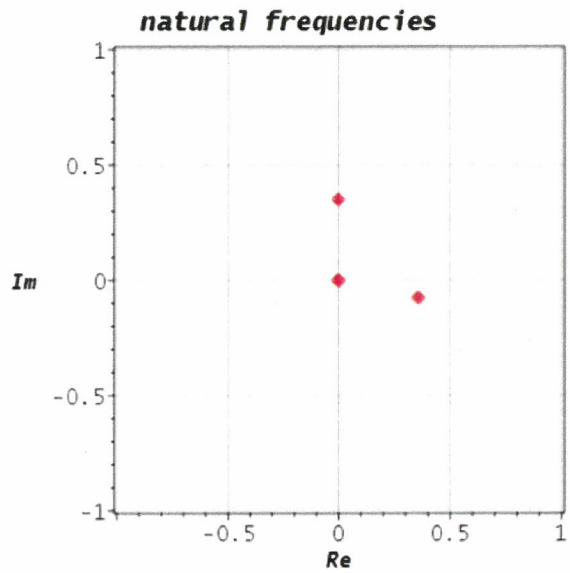
$$\underline{\underline{M}}\,\ddot{\underline{V}} + v\underline{\underline{A}}\,\frac{d^{0.5}}{dt^{0.5}}\underline{V} + \left(g\underline{\underline{K_0}} + v^2\underline{\underline{K_2}}\right)\underline{V} = \underline{f}$$

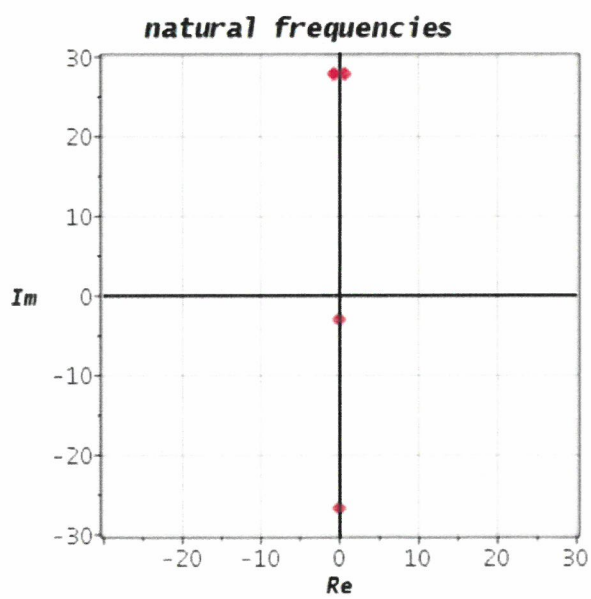
$$poly := 1.\omega^4 + 769.4844612\,\omega^2 - 5.898913909\,\omega^2\sqrt{\mathrm{I}\omega} + 3714.79950 + 1055.051236\sqrt{\mathrm{I}\omega} + 352.7818983\,\mathrm{I}\omega$$

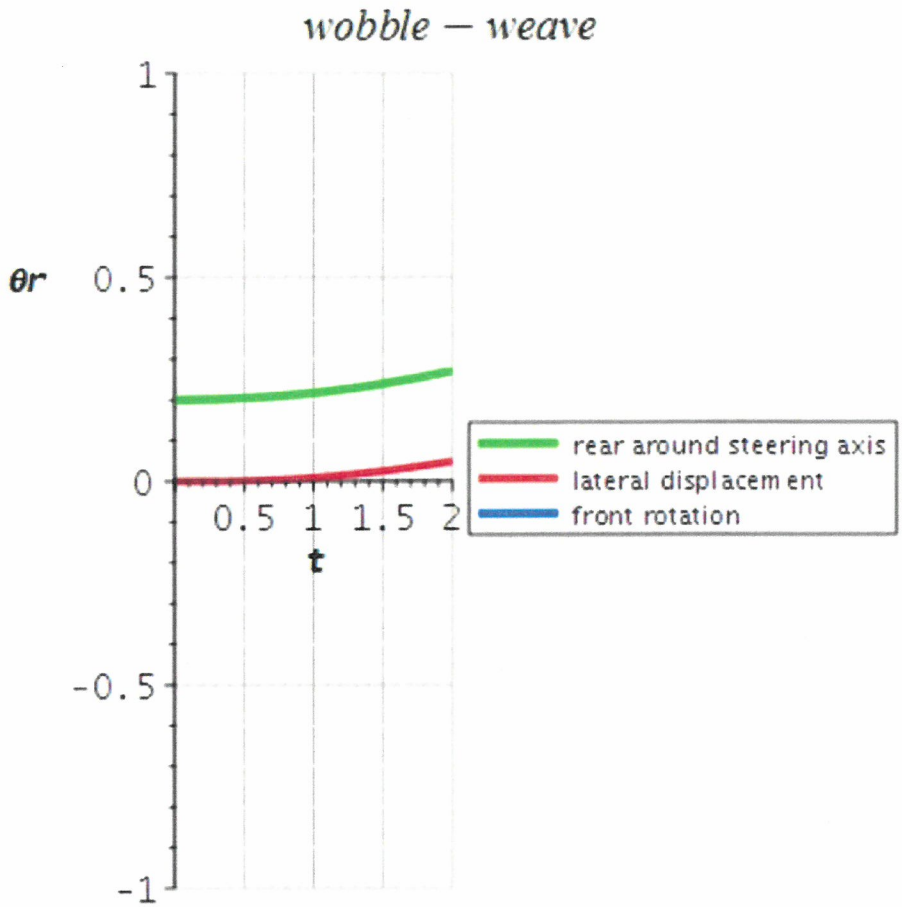
$$natff := -2.959457495\,\mathrm{I},\ -26.70080877\,\mathrm{I},\ 0.6854949836 + 27.88745367\,\mathrm{I},\ -0.6854949836 + 27.88745367\,\mathrm{I}$$

$$poly := -13554.35232\,\omega^6 + 73884.9139\,I\,\omega^5 + 15318.11212\,\omega^4 - 6418.56323\,I\,\omega^3 - 3344.379138\,\omega^2$$

$$natf := [0., 0., 0.3583029063 - 0.07537103514\,I, 0.3504834739\,I, 5.251268881\,I, -0.3583029063 - 0.07537103514\,I]$$







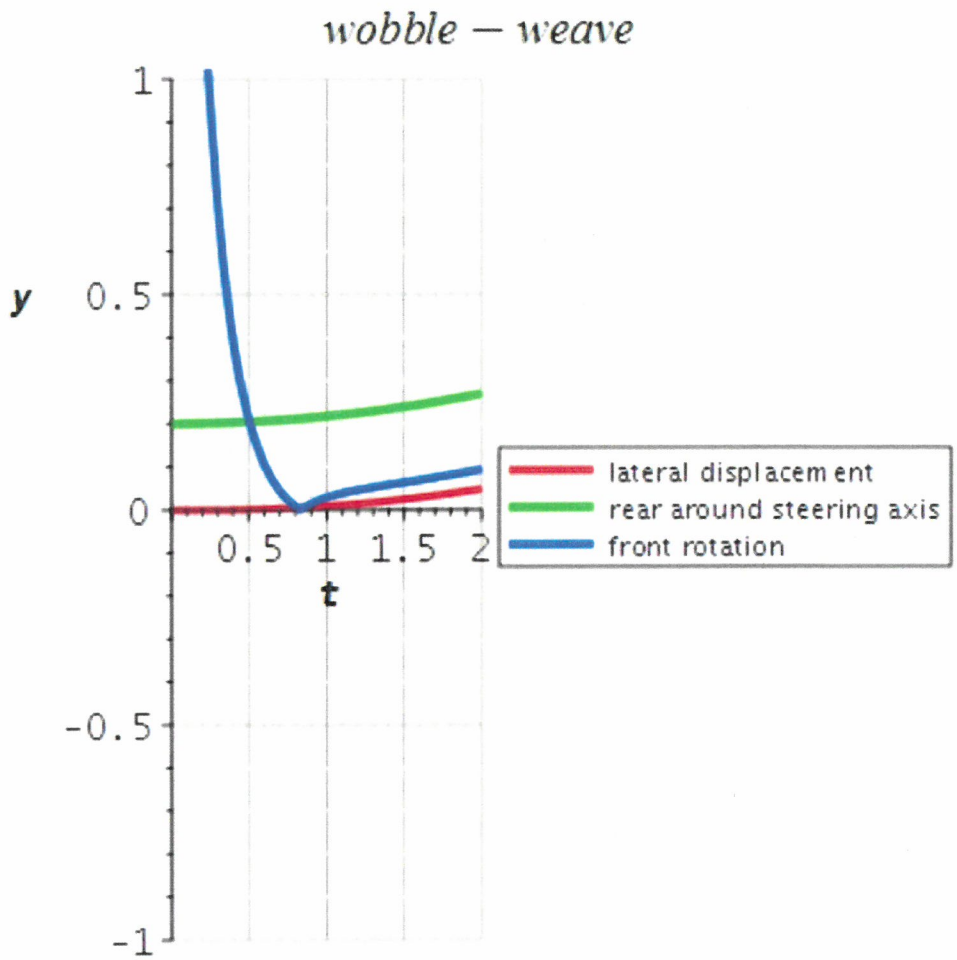
$$v = 30$$

$$\left[248.2 \frac{d^2}{dt^2} x(t) - 145.725 \frac{d^2}{dt^2} y(t) + 0.7368 \frac{d^2}{dt^2} z(t) + 0.899999999910000 \frac{d}{dt} x(t) - 0.726799999927320 \frac{d}{dt} y(t) - 0.0433066666623360 \frac{d}{dt} z(t) - 14.07790594 y(t) - 9.979275095 z(t)\right]$$

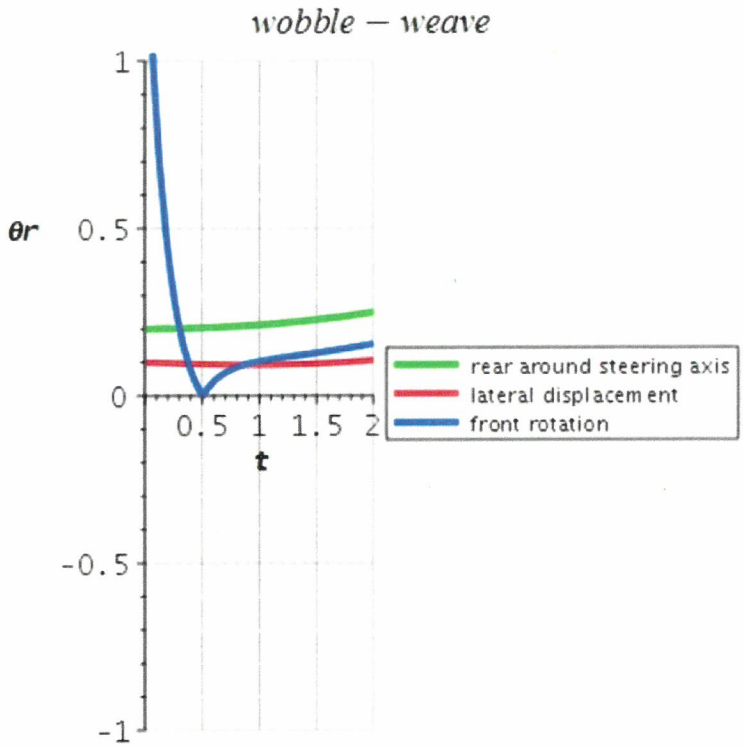
$$-145.725 \frac{d^2}{dt^2} x(t) + 21. \frac{d^2}{dt^2} y(t) - 0.726799999927320 \frac{d}{dt} x(t) - 6.79999999932000 \frac{d}{dt} y(t) + 6.80502357265283 \frac{d}{dt} z(t) + 19.42751019 y(t)$$

$$\left[0.7368 \frac{d^2}{dt^2} x(t) + 1.2176832 \frac{d^2}{dt^2} z(t) - 0.0433066666623360 \frac{d}{dt} x(t) - 6.79999999932000 \frac{d}{dt} y(t) + 6.80502357265283 \frac{d}{dt} z(t) + 1.157595911 z(t)\right]$$

$$x(0) := 0.1$$
$$y(0) := 0.2$$
$$z(0) := 0.$$
$$D(x)(0) := 0$$
$$D(y)(0) := 0$$
$$D(z)(0) := 0$$



$$v = 10$$



$$v = 15$$

x(0) := 0.1

y(0) := 0.2

z(0) := 0.1

D(x)(0) := 0

D(y)(0) := 0

D(z)(0) := 0

```
restart; with(Student); with(LinearAlgebra); with(DETools); with(plots); with(inttrans); A0 := Matrix(2, 2, [[80.81722, 2.31941332208709], [2.31941332208709, .29784188199686]]); g := 9.81; v := 4.; omega1 := 1.5; A := v . Matrix(2, 2, [[0, 33.86641391492494], [-.85035641456978, 1.68540397397560]]); K0 := Matrix(2, 2, [[-80.95, -2.59951685249872], [-2.59951685249872, -.80329488458618]]); K2 := Matrix(2, 2, [[0, 76.59734589573222], [0, 2.65431523794604]]); B := evalf(K2*v^2+K0*g); q := Matrix(2, 1, [x(t), y(t)]); zer0 := Matrix(2, 1, [0, 0]); ic1 := .5; ic2 := .5; zer1 := Matrix(2, 1, [ic1*exp(-t)*sin(omega1*t)*(Heaviside(t)-Heaviside(t-1)), 0]); Id1 := IdentityMatrix(2); diff(q[1], t, t); diff(q[1], t); de1 := A0 . (diff(q, t, t))+A . (diff(q, t))+B . q-Id1 . zer1; eq1 := de1[1]; eq2 := de1[2]; x(0) := ic1; y(0) := ic2; (D(x))(0) := 0; (D(y))(0) := 0; delp1 := laplace(eq1[1], t, s); delp1s := collect(delp1, [laplace(x(t), t, s), laplace(y(t), t, s)], recursive); delp2 := laplace(eq2[1], t, s); delp2s := collect(delp2, [laplace(x(t), t, s),
```

```
laplace(y(t), t, s)], recursive); sol1 := solve({delp1s, delp2s}, {laplace(x(t), t, s), laplace(y(t), t, s)}); xt :=
invlaplace(rhs(sol1[1]), s, t); yt := invlaplace(rhs(sol1[2]), s, t); tm := 1; y1 := 2; pl1 := plot(evalf(xt), t = 0 ..
tm, y = -y1 .. y1, axes = normal, gridlines, thickness = 3, color = blue, style = line, title =
'Bicycle*Pulsed*lean*sinusoid-steer', titlefont = [COURIER, Bold, 14], labels = ["t", "&phi;,&delta;"],
labelfont = [COURIER, BOLDITALIC, 12], axesfont = [COURIER, 12], legend = ["lean"], titlefont = ["ARIAL",
15], legendstyle = [font = ["HELVETICA", 9], location = right]); pl2 := plot(evalf(yt), t = 0 .. tm, y = -y1 .. y1,
axes = normal, gridlines, thickness = 3, color = green, style = line, title = 'Bicycle*lean-steer', titlefont =
[COURIER, Bold, 14], labels = ["t", "&phi;,&delta;"], labelfont = [COURIER, BOLDITALIC, 12], legend =
["steer"], titlefont = ["ARIAL", 15], legendstyle = [font = ["HELVETICA", 9], location = right]);
plots[display]({pl1, pl2})
```

2.3 Simple Tire Model Dimensionless Parameters

```
restart; with(Student); with(combinat, randcomb); with(linalg); with(Student[LinearAlgebra]);
with(plots); m := 6; n1 := 3; dd := m-n1; N := factorial(m)/(factorial(m-n1)*factorial(n1)); m0 := [delta, w,
f, F, mu, R]; for j to N do mc[j] := randcomb(m0, n1) end do; M := [seq(mc[k], k = 1 .. N)]; for k to N do
m0s[k] := convert(m0, set); Mc[k] := convert(M[k], set); A0[k] := m0s[k] minus Mc[k]; Cm[k] := [A0[k],
Mc[k]]; V1[k] := Vector(dd, [seq(A0[k][j], j = 1 .. dd)]); V2[k] := Vector(n1, [seq(Mc[k][j], j = 1 .. n1)]);
V1tr[k] := Transpose(V1[k]); V2t[k] := Transpose(V2[k]) end do; V1tr[1]; V1tr[1](1, 1); V1tr[1](1, 2);
V2t[1]; V2t[1](1, 1); V2t[1](1, 2); V2t[1](1, 3); Am2[1] := [seq(A0[1][i], I = 1 .. dd)]; delta := Vector(3, [0, 1,
0]); w := Vector(3, [0, 1, 0]); f := Vector(3, [1, 1, -2]); F := Vector(3, [1, 1, -2]); mu := Vector(3, [1, 0, 0]); R
:= Vector(3, [0, 0, -1]); for k to N do AM1[k] := Matrix(n1, dd, [seq(V1tr[k][i], I = 1 .. dd)]); Bm1[k] :=
Matrix(n1, n1, [seq(V2t[k][i], I = 1 .. n1)]); det1[k] := Determinant(Bm1[k]) end do; for n to N do if det1[n]
= 0 then next end if; Bm1i[n] := inverse(Bm1[n]); I01[n] := -1.*Matrix(n1, n1, shape = identity); A1[n] :=
I01[n] . AM1[n]; Dim[n] := Bm1i[n] . A1[n] end do; for n to N do if det1[n] = 0 then next end if; dimpar[n]
:= seq(V2t[n](1, 1)^convert(Dim[n](1, i), rational)*V2t[n](1, 2)^convert(Dim[n](2, i), rational)*V2t[n](1,
3)^convert(Dim[n](3, i), rational)*V1tr[n](1, i), I = 1 .. n1) end do; RowDimension(V1tr[1]);
ColumnDimension(V1tr[1]); RowDimension(V2t[1]); ColumnDimension(V2t[1]); M[1]; V1[1]; V1tr[1];
V2[1]; V2t[1]; Dim[1]; Dim[1][1]; Dim[1][2]; Dim[1][3];
```

$$dimpar_1 := \frac{F}{R^2\,\mu\,w}, \frac{\delta}{w}, \frac{f}{R^2\,\mu\,w}$$

$$dimpar_3 := \frac{R^2\,\mu\,\delta}{F}, \frac{f}{F}, \frac{R^2\,\mu\,w}{F}$$

$$dimpar_4 := \frac{F}{f}, \frac{\sqrt{\delta}\,\sqrt{\mu}\,R}{\sqrt{f}}, \frac{w}{\delta}$$

$$dimpar_7 := \frac{R^2\,\mu\,\delta}{F}, \frac{f}{F}, \frac{R^2\,\mu\,w}{F}$$

$$dimpar_8 := \frac{F}{f}, \frac{\delta}{w}, \frac{R^2\,w\,\mu}{f}$$

$$dimpar_9 := \frac{\sqrt{\delta}\,\sqrt{\mu}\,R}{\sqrt{F}}, \frac{f}{F}, \frac{w}{\delta}$$

$$dimpar_{10} := \frac{F}{f}, \frac{R^2 \delta \mu}{f}, \frac{w}{\delta}$$

$$dimpar_{11} := \frac{R^2 \mu \delta}{F}, \frac{f}{F}, \frac{R^2 \mu w}{F}$$

$$dimpar_{12} := \frac{F}{f}, \frac{R^2 \delta \mu}{f}, \frac{R^2 w \mu}{f}$$

$$dimpar_{15} := \frac{F}{f}, \frac{\sqrt{\delta}\sqrt{\mu}\, R}{\sqrt{f}}, \frac{w}{\delta}$$

$$dimpar_{16} := \frac{F}{f}, \frac{\sqrt{\mu}\sqrt{w}\, R}{\sqrt{f}}, \frac{\delta}{w}$$

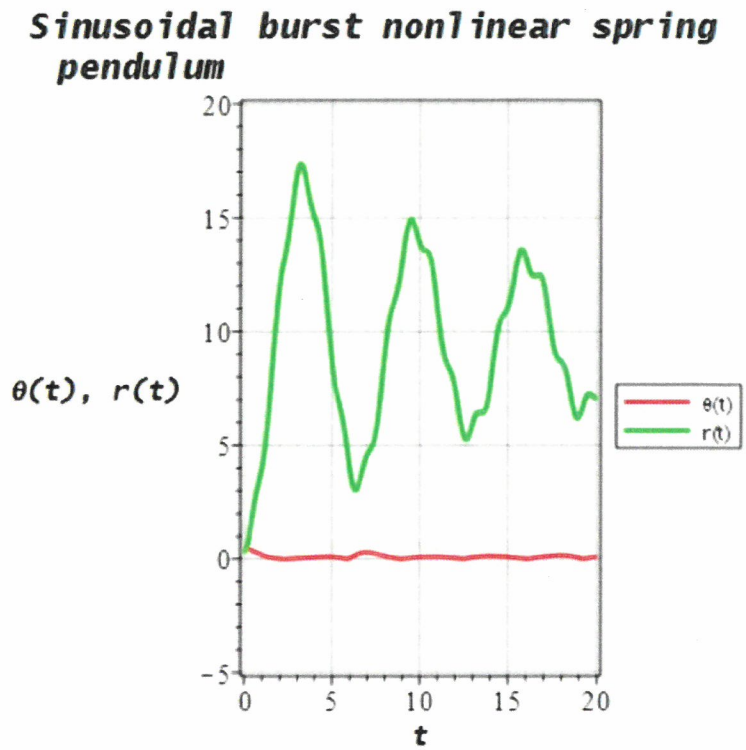
$$dimpar_{18} := \frac{\sqrt{\delta}\sqrt{\mu}\, R}{\sqrt{F}}, \frac{f}{F}, \frac{w}{\delta}$$

2.4 Nonlinear Spring-Pendulum Model

```
restart; with(DEtools); with(Student); with(plots); -linear*vibration+non; l1 := 2.; k1 := 10.; m := 10.;
`&omega;y` := 5.; a0 := .2; ys := a0*sin(`&omega;y`*t); a0 := .5; b0 := .5; omega1 := 1.5; `d&theta;` := .2; g
:= 9.81; dr := 1.; tau := 100.; DEqp1 := l1*(diff(theta(t), [t$2]))+r1(t)*(diff(theta(t), [t$2]))+2*(diff(theta(t),
t))*(diff(r1(t), t))-ys*sin(theta(t))+g*sin(theta(t))+`d&theta;`*(diff(theta(t), t))/m = exp(-t/tau)*(1-
t/tau)*a0*sin(omega1*t)*Heaviside(t); DEqp2 := diff(r1(t), [t$2])+dr*(diff(r1(t), t))/m+k1*r1(t)/m-
g*cos(theta(t))-a0*`&omega;y`^2*cos(`&omega;y`*t)*cos(theta(t))+l1*(diff(theta(t),
t))^2+r1(t)*(diff(theta(t), t))^2 = 0; init := eval(theta(t), t = 0) = a0, (D(theta))(0) = 0, eval(r1(t), t = 0) = b0,
(D(r1))(0) = 0; sol3 := dsolve({DEqp1, DEqp2, init}, {r1(t), theta(t)}, numeric, output = listprocedure,
abserr = 0.1e-2); T10 := 0.; T2 := 20.; y1 := 5.; y2 := 20; pl01 := odeplot(sol3, [t, abs(theta(t))], t = T10 ..
T2, numpoints = 500, labels = ["t", "&theta;(t), r(t)"], thickness = 2, title = ` Sinusoidal burst nonlinear
spring pendulum`, titlefont = [COURIER, Bold, 14], labelfont = [COURIER, BOLDITALIC, 12], color = red,
axes = boxed, view = [T10 .. T2, -y1 .. y2], legend = ["&theta;(t)"], legendstyle = [font = ["HELVETICA", 9],
location = right], gridlines); pl02 := odeplot(sol3, [t, abs(r1(t))], t = T10 .. T2, numpoints = 500, labels =
["t", "&theta;(t), r(t)"], thickness = 2, title = ` Sinusoidal burst nonlinear spring pendulum`, titlefont =
[COURIER, Bold, 14], labelfont = [COURIER, BOLDITALIC, 12], color = green, axes = boxed, view = [T10 ..
T2, -y1 .. y2], legend = ["r(t)"], legendstyle = [font = ["HELVETICA", 9], location = right], gridlines);
plots[display]({pl01, pl02});
```

$$DEqp1 := 2.\frac{d^2}{dt^2}\theta(t) + r1(t)\left(\frac{d^2}{dt^2}\theta(t)\right) + 2\left(\frac{d}{dt}\theta(t)\right)\left(\frac{d}{dt}r1(t)\right)$$
$$- 0.2\sin(5.t)\sin(\theta(t)) + 9.81\sin(\theta(t)) + 0.1000000000\frac{d}{dt}\theta(t)$$
$$= 0.5\,e^{-0.01000000000\,t}(1 - 0.01000000000\,t)\sin(1.5\,t)\,\mathrm{Heaviside}(t)$$

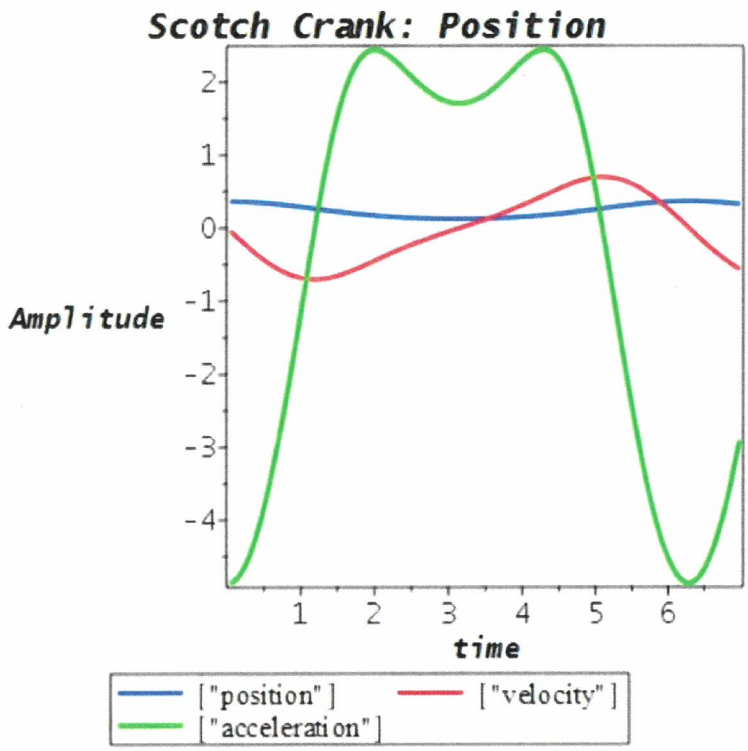
$$DEqp2 := \frac{d^2}{dt^2}\, r1(t) + 0.1000000000 \frac{d}{dt}\, r1(t) + 1.000000000\, r1(t) - 9.81 \cos(\theta(t)) - 12.5 \cos(5.\, t) \cos(\theta(t)) + 2. \left(\frac{d}{dt}\, \theta(t)\right)^2 + r1(t) \left(\frac{d}{dt}\, \theta(t)\right)^2 = 0$$



2.5 Scotch Crank

restart; with(RandomTools); THDrpm := 50; r = .12; c = .25; THD := evalf(2*THDrpm*Pi*(1/60)); tf := evalf(2*Pi/THD); t := seq(evalf(2*tf*i*(1/360)), I = 1 .. 360); TH := evalf(THD*t); d2s := seq(subs(c = .25, r = .12, c^2-r^2*sin(evalf(TH[i]))^2), I = 1 .. 200); x := seq(subs(r = .12, r*cos(TH[i])+sqrt(d2s[i])), I = 1 .. 200); xd := seq(subs(r = .12, evalf(-r*THD*sin(TH[i])-r^2*THD*sin(2*TH[i])/(2*sqrt(d2s[i])))), I = 1 .. 200); xdd := seq(subs(r = .12, evalf(-r*THD^2*cos(TH[i])-(4*r^2*THD^2*cos(2*TH[i])*d2s[i]+(r^2*THD*sin(2*TH[i]))^2)/(4*d2s[i]^1.5))), I = 1 .. 200); with(plots); pointplot({seq([TH[i], x[i]], I = 1 .. 200)}, axes = boxed, color = blue, style = line, thickness = 2, title = `Scotch Crank: Position`, titlefont = [COURIER, Bold, 14], labels = ["time", "Amplitude"], labelfont = [COURIER, BOLDITALIC, 12], axesfont = [COURIER, 12]); pointplot({seq([TH[i], xd[i]], I = 1 .. 200)}, axes = boxed, color = red, style = line, thickness = 2, title = `Scotch Crank: Velocity`, titlefont = [COURIER, Bold, 14], labels = ["time", "Amplitude"], labelfont = [COURIER, BOLDITALIC, 12], axesfont = [COURIER, 12]); pointplot({seq([TH[i], xdd[i]], I = 1 .. 200)}, axes = boxed, color = green, style = line, thickness = 2, title = `Scotch Crank: Acceleration`, titlefont = [COURIER, Bold, 14], labels = ["time", "Amplitude"], labelfont = [COURIER, BOLDITALIC, 12], axesfont = [COURIER, 12]); p1 := pointplot({seq([TH[i], x[i]], I = 1 .. 200)}, axes = boxed, color = blue, style = line, thickness = 2, title = `Scotch Crank: Position`, titlefont =

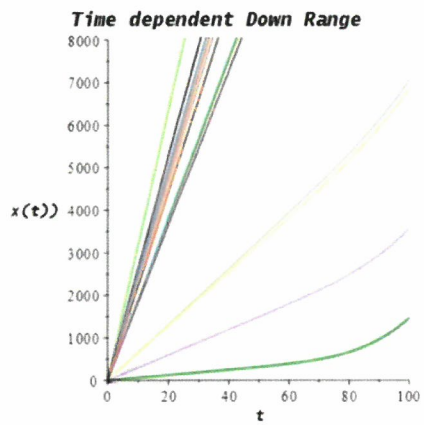
[COURIER, Bold, 14], labels = ["time", "Amplitude"], labelfont = [COURIER, BOLDITALIC, 12], axesfont = [COURIER, 12], legend = ["position"]); p2 := pointplot({seq([TH[i], xd[i]], I = 1 .. 200)}, axes = boxed, color = red, style = line, thickness = 2, title = `Scotch Crank: Velocity`, titlefont = [COURIER, Bold, 14], labels = ["time", "Amplitude"], labelfont = [COURIER, BOLDITALIC, 12], axesfont = [COURIER, 12], legend = ["velocity"]); p3 := pointplot({seq([TH[i], xdd[i]], I = 1 .. 200)}, axes = boxed, color = green, style = line, thickness = 2, title = `Scotch Crank: Acceleration`, titlefont = [COURIER, Bold, 14], labels = ["time", "Amplitude"], labelfont = [COURIER, BOLDITALIC, 12], axesfont = [COURIER, 12], legend = ["acceleration"]); plots[display]([p1, p2, p3]);

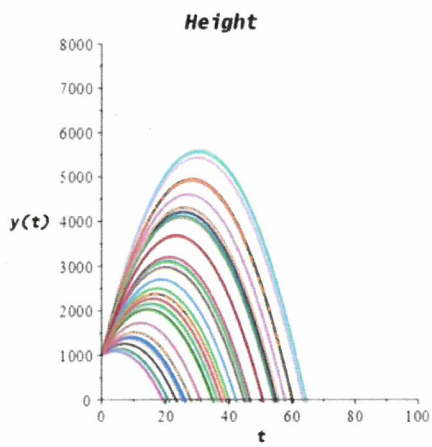


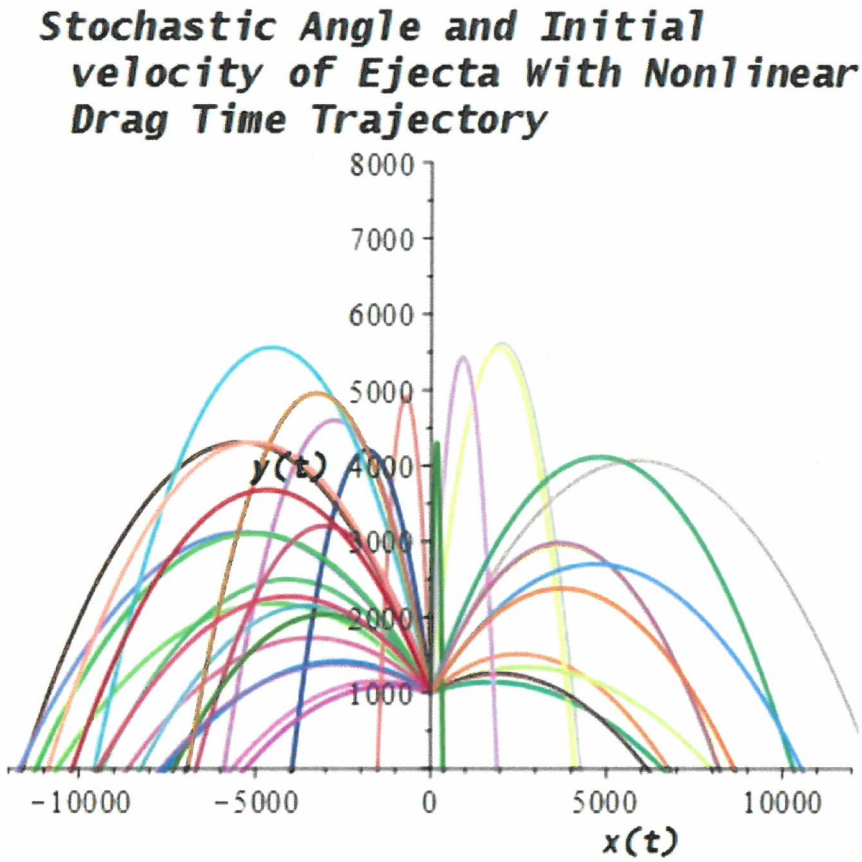
2.6 Volcanic Tephra

restart; with(DEtools); with(Student); with(plots); with(Statistics); with(RandomTools); R := 8.3143; T0 := 290; `ρd` := 1.6; rho0 := 1.225; r0 := 0.1e-2; phi1 := evalf((1/180)*(39*3.14159)); a1 := 0.1e-1; b1 := 1.; `gφ` := evalf(9.7803267714*(1+0.193185138639e-2*sin(phi1^2))/sqrt(1-0.6943779013e-2*sin(phi1^2))); g00 := evalf(`gφ`/(R*T0)); Cd := .4; Ch := .35; m0 := evalf((4/3*3.14159)*r0^3*`ρd`); A := evalf(3.14159*r0^2); n := 35; col1 := seq(COLOR(RGB, rand()/10^12, rand()/10^12, rand()/10^12), j = 1 .. n); theta0 := evalf(3.14159*(5*2)*(1/360)); theta11 := evalf(3.14159*(175*2)*(1/360)); `θr` := Generate(list(float(range = theta0 .. theta11, method = uniform), n)); v0 := Generate(list(float(range = 250 .. 350, method = uniform), n)); RE := 6378.137*10^3; g := 9.810; for I to n do de1 := diff(v(t), t)+.5*Cd*(1+a1*sqrt(v(t)^2+h(t)^2)^.637/Cd-b1/(1+1/sqrt(v(t)^2+h(t)^2)^1.09))*rho0*exp(-g00*h(t))*A*v(t)*sqrt(v(t)^2+h(t)^2)+.5*Ch*rho0*exp(-g00*h(t))*A*h(t)*sqrt(v(t)^2+h(t)^2) = 0; de2 := diff(h(t),

```
t)+`g&phi;`*RE/(RE+h(t))+.5*Cd*(1+a1*sqrt(v(t)^2+h(t)^2)^.637/Cd-
b1/(1+1/sqrt(v(t)^2+h(t)^2)^1.09))*rho0*exp(-g00*h(t))*A*h(t)*sqrt(v(t)^2+h(t)^2)-.5*Ch*rho0*exp(-
g00*h(t))*A*v(t)*sqrt(v(t)^2+h(t)^2) = 0; de3 := diff(x(t), t)-v(t) = 0; de4 := diff(y(t), t)-h(t) = 0; sol :=
dsolve({de1, de2, de3, de4, h(0) = v0[i]*sin(`&theta;r`[i]), v(0) = v0[i]*cos(`&theta;r`[i]), x(0) = 0, y(0) =
1000}, {h(t), v(t), x(t), y(t)}, numeric, output = listprocedure, abserr = 0.1e-2); T1 := 0.; T2 := 100; pl1[i] :=
odeplot(sol, [t, x(t)], t = T1 .. T2, numpoints = 1000, labels = ["t", "x(t))"], thickness = 2, title = ` Time
dependent Down Range `, titlefont = [COURIER, Bold, 14], titlefont = [COURIER, Bold, 14], labelfont =
[COURIER, BOLDITALIC, 12], color = col1[i], axes = normal, view = [T1 .. T2, 0 .. 8000]); pl2[i] :=
odeplot(sol, [[t, y(t)]], t = T1 .. T2, numpoints = 500, labels = ["t", " y(t)"], thickness = 2, title = ` Height`,
titlefont = [COURIER, Bold, 14], titlefont = [COURIER, Bold, 14], labelfont = [COURIER, BOLDITALIC, 12],
color = col1[i], axes = normal, view = [T1 .. T2, 0 .. 8000]); pl4[i] := odeplot(sol, [x(t), y(t)], t = T1 .. T2,
numpoints = 1000, labels = ["x(t)", "y(t)"], thickness = 2, title = ` Stochastic Angle and Initial velocity of
Ejecta With Nonlinear Drag Time Trajectory`, titlefont = [COURIER, Bold, 14], titlefont = [COURIER, Bold,
14], labelfont = [COURIER, BOLDITALIC, 12], color = col1[i], axes = normal, view = [-12000 .. 12000, 0 ..
8000]) end do; plots[display](seq(pl1[i], I = 1 .. n)); plots[display](seq(pl2[i], I = 1 .. n));
plots[display](seq(pl4[i], I = 1 .. n));
```

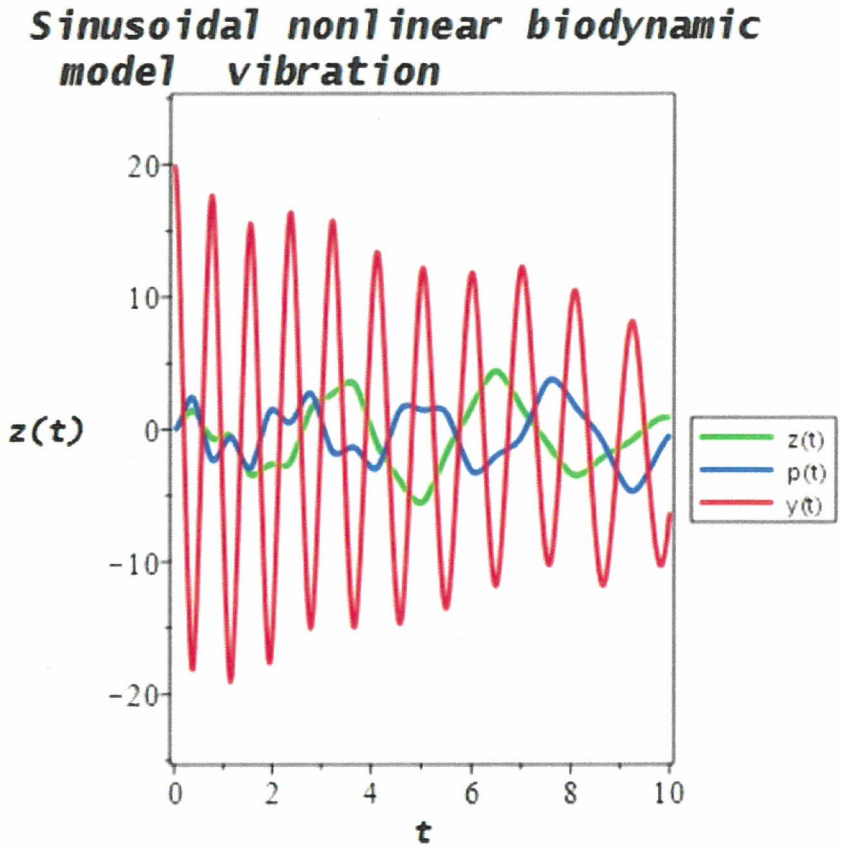






restart; with(plots); with(Student); with(RandomTools); r1 := 1.; n1 := 9; col1 := seq(COLOR(RGB, rand()/10^11, rand()/10^12, rand()/10^12), j = 1 .. n1); a := [.387, .723, 1., 1.52, 5.2, 9.54, 19.18, 30.07, 39.44]; b := [.264664, .719491, .991464, 1.472766, 5.175944, 9.511959, 19.15699, 30.065, 39.3158]; epsilon := [.206, 0.7e-2, 0.17e-1, 0.93e-1, 0.48e-1, 0.56e-1, 0.46e-1, 0.1e-1, .248]; c1 := seq(evalf(epsilon[i]*a[i]), I = 1 .. n1); x1 := 50; for I to n1 do pl[i] := implicitplot((x-c1[i])^2/a[i]^2+y^2/b[i]^2-r1^2, x = -x1 .. x1, y = -x1 .. x1, color = col1[i], thickness = 3, axes = normal, title = 'Planetary*Orbits', titlefont = [COURIER, Bold, 14], labels = ["x", "f(x)"], labelfont = [COURIER, BOLDITALIC, 12], axesfont = [COURIER, 12], gridlines, tickmarks = [20, 20], gridrefine = 1) end do; g := proc (x) options operator, arrow; 0*x end proc; h := proc (x) options operator, arrow; 10000000*x end proc; pl2 := plot(g(x), x = -x1 .. x1, y = -x1 .. x1, color = black, thickness = 2, axes = normal, title = 'Planetary*Orbits', titlefont = [COURIER, Bold, 14], labels = ["x", "f(x)"], labelfont = [COURIER, BOLDITALIC, 12], axesfont = [COURIER, 12], gridlines, tickmarks = [20, 20]); pl3 := plot(h(x), x = -x1 .. x1, y = -x1 .. x1, color = black, thickness = 1, axes = normal, title = 'Planetary*Orbits', titlefont = [COURIER, Bold, 14], labels = ["x", "f(x)"], labelfont = [COURIER, BOLDITALIC, 12], axesfont = [COURIER, 12], gridlines, tickmarks = [20, 20]); display({pl2, pl3, seq(pl[j], j = 1 .. n1)});

2.8 Biodynamic Model



$$DEq0 := 10\,\frac{d^2}{dt^2}\,y(t) + 2\,\zeta 1\,\omega\left(\frac{d}{dt}\,y(t)\right) + \omega^2\,y(t) + (y(t) - z(t))^2 + (y(t) - z(t))^3 + z(t)\,(y(t) - p(t))^2 + (y(t) - p(t))^3 = e^{-0.01000000000\,t}\,(1 - 0.01000000000\,t)\,F0\sin(\omega\,t)$$

$$DEq1 := 100\,\frac{d^2}{dt^2}\,z(t) + 2\,\frac{d}{dt}\,z(t) + \omega^2\,z(t) + (z(t) - y(t))^2 + (z(t) - y(t))^3 + y(t)\,(z(t) - p(t))^2 + (z(t) - p(t))^3 = 0$$

$$DEq2 := 60\,\frac{d^2}{dt^2}\,p(t) + 2\,\frac{d}{dt}\,p(t) + \omega^2\,p(t) + z(t)\,(p(t) - y(t))^2 + (p(t) - y(t))^3 + (p(t) - z(t))^2 + (p(t) - z(t))^3 = 0$$

$$\omega := 10.$$

$$F0 := 20$$

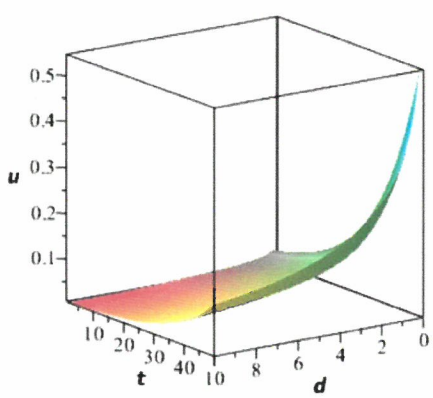
$$\zeta 1 := 0.1$$

$$ini := y(0) = 20, D(y)(0) = 0, z(0) = 0, D(z)(0) = 0, p(0) = 0, D(p)(0) = 0$$

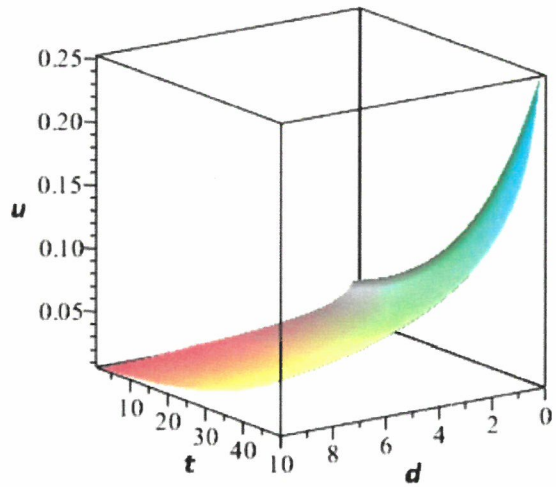
```
> restart : with(DEtools) : with(Student) : with(plots) : non-linear vibration :
>
τ := 100.; DEq0 := 10·diff(y(t), [t$2]) + 2·ζ1·ω·diff(y(t), t) + ω^2·y(t) + (y(t) − z(t))^2
    + (y(t) − z(t))^3 + z(t)·(y(t) − p(t))^2 + (y(t) − p(t))^3 = exp(−t/τ)·(1 − t/τ)·F0
    ·sin(ω·t); DEq1 := 100·diff(z(t), [t$2]) + 2·diff(z(t), t) + ω^2·z(t) + (z(t) − y(t))^2
    + (z(t) − y(t))^3 + y(t)·(z(t) − p(t))^2 + (z(t) − p(t))^3 = 0; DEq2 := 60·diff(p(t), [t
    $2]) + 2·diff(p(t), t) + ω^2·p(t) + z(t)·(p(t) − y(t))^2 + (p(t) − y(t))^3 + (p(t)
    − z(t))^2 + (p(t) − z(t))^3 = 0; ω := 10.; F0 := 20; ζ1 := 0.1; ini := y(t)|_{t=0} = F0,
    D(y)(0) = 0, z(t)|_{t=0} = 0, D(z)(0) = 0, p(t)|_{t=0} = 0, D(p)(0) = 0; F := dsolve({DEq0,
    DEq1, DEq2, ini}, {y(t), z(t), p(t)}, type = numeric); sol2 := dsolve({DEq0, DEq1,
    DEq2, ini}, {y(t), z(t), p(t)}, numeric, output = listprocedure, abserr = 0.001) : T10 :=
    0.0; T2 := 10.; y2 := 25.; pl11 := odeplot(sol2, [t, y(t)], t = T10 ..T2, numpoints = 500,
    labels = ["t", "y(t)"], thickness = 2, title
    = ` Sinusoidal nonlinear biodynamic model vibration`, titlefont = [COURIER, Bold, 14],
    labelfont = [COURIER, BOLDITALIC, 12], color = red, axes = boxed, view = [T10 ..T2,
    -y2 ..y2], legend = ["y(t)"], legendstyle = [font = ["HELVETICA", 9], location = right])
    : pl12 := odeplot(sol2, [t, z(t)], t = T10 ..T2, numpoints = 500, labels = ["t", "z(t)"],
    thickness = 2, title = ` Sinusoidal nonlinear biodynamic model  vibration`, titlefont
    = [COURIER, Bold, 14], labelfont = [COURIER, BOLDITALIC, 12], color = green, axes
    = boxed, view = [T10 ..T2, -y2 ..y2], legend = ["z(t)"], legendstyle = [font
    = ["HELVETICA", 9], location = right]) : pl13 := odeplot(sol2, [t, p(t)], t = T10 ..T2,
    numpoints = 500, labels = ["t", "p(t)"], thickness = 2, title
    = ` Sinusoidal nonlinear biodynamic model vibration`, titlefont = [COURIER, Bold, 14],
    labelfont = [COURIER, BOLDITALIC, 12], color = blue, axes = boxed, view = [T10 ..T2,
    -y2 ..y2], legend = ["p(t)"], legendstyle = [font = ["HELVETICA", 9], location = right])
    : plots[display]({pl11, pl12, pl13});
```

2.8 Flamant Bousinesq

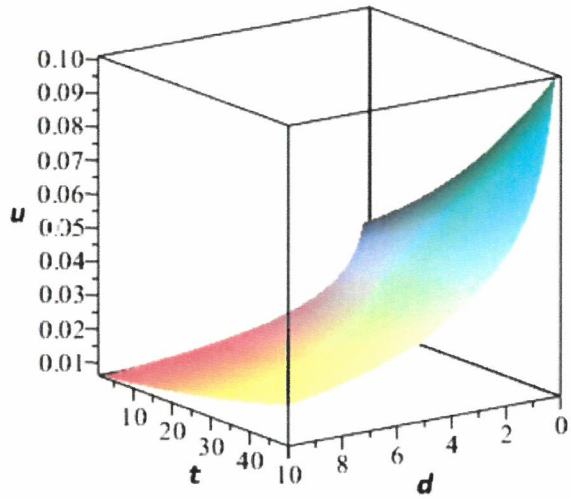
Time dependent Fractional Calculus Viscoelastic Settlement Magnitude



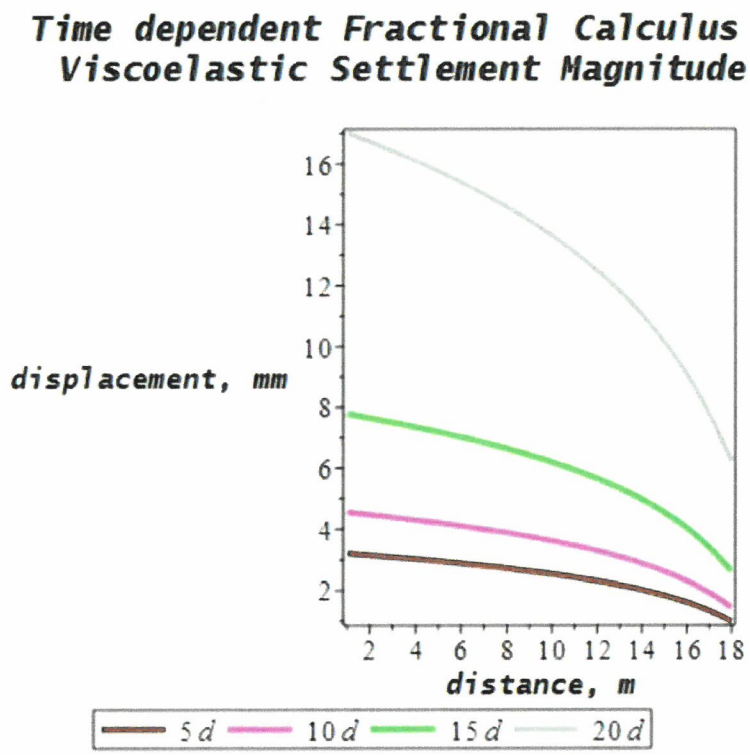
Time dependent Fractional Calculus Viscoelastic Settlement Magnitude

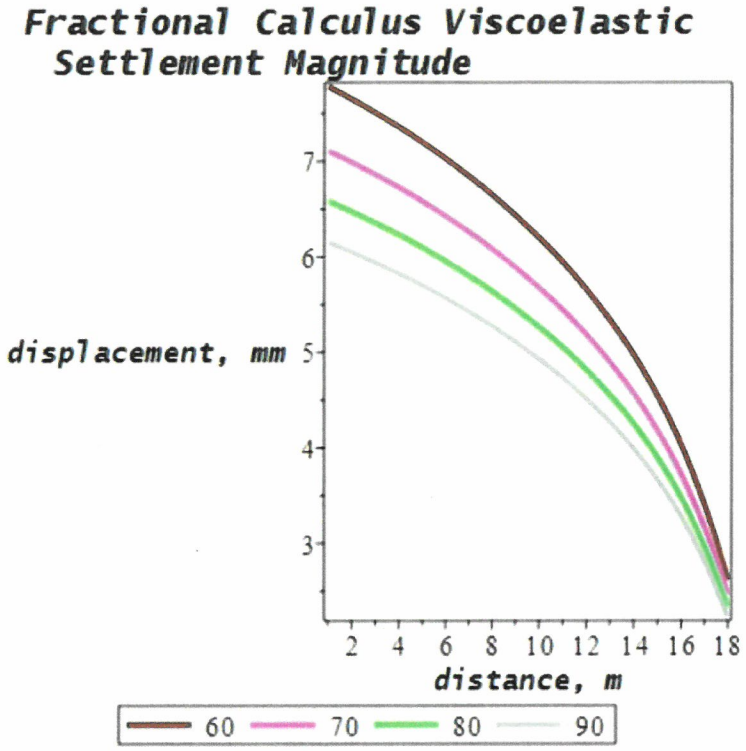


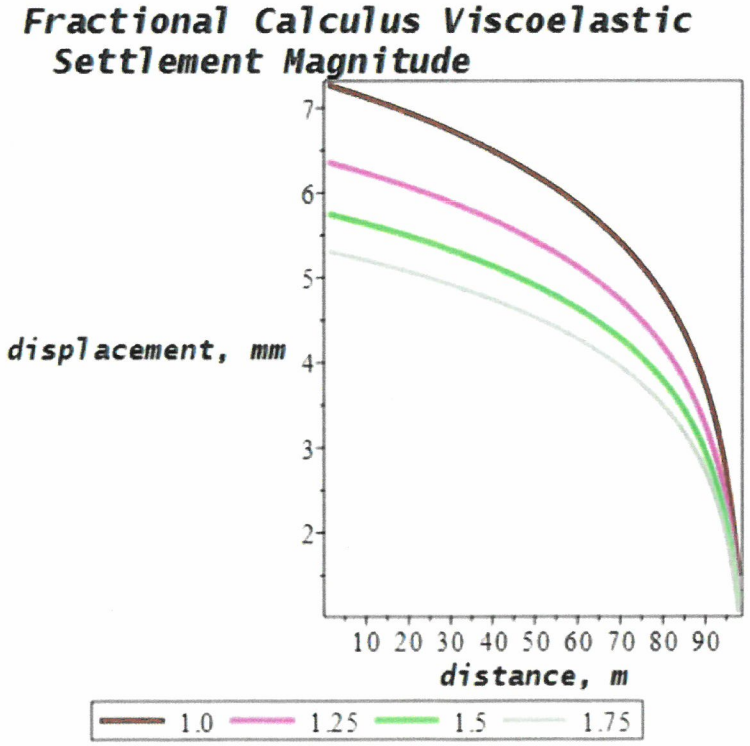
Time dependent Fractional Calculus Viscoelastic Settlement Magnitude



```
restart; with(Student); with(plots); N := 50; p0 := 1; K := 80; G := 60; E := 144; r0 := 15; eta := 1000; alpha := [1., 1.2, 1.7]; tau1 := evalf(4*eta/(3*K+G)); tau2 := evalf(eta/G); for I to 3 do ur[i] := proc (t) options operator, arrow; evalf(9*p0^2*(1-sum(-(t/tau1)^n/GAMMA(n*alpha[i]+1), n = 0 .. N))^2/(12*K+4*G)^2) end proc; `u&theta;`[i] := proc (r, t) options operator, arrow; evalf((1/4)*p0^2*ln(r0/r)^2*((3-3*(sum(-(t/tau1)^n/GAMMA(n*alpha[i]+1), n = 0 .. N)))/(3*K+G)+(1-sum(-(t/tau2)^n/GAMMA(n*alpha[i]+1), n = 0 .. N))/G)^2/3.14159^2) end proc; zz := proc (r, t) options operator, arrow; evalf((ur[i](t)+`u&theta;`[i](r, t))^.5) end proc; plot3d(zz(r, t), r = 0 .. 10, t = 1 .. 50, axes = boxed, title = ` Time dependent Fractional Calculus Viscoelastic Settlement Magnitude  `, titlefont = [COURIER, Bold, 14], labels = ["d", "t", "u"], labelfont = [COURIER, BOLDITALIC, 12]) end do;
```

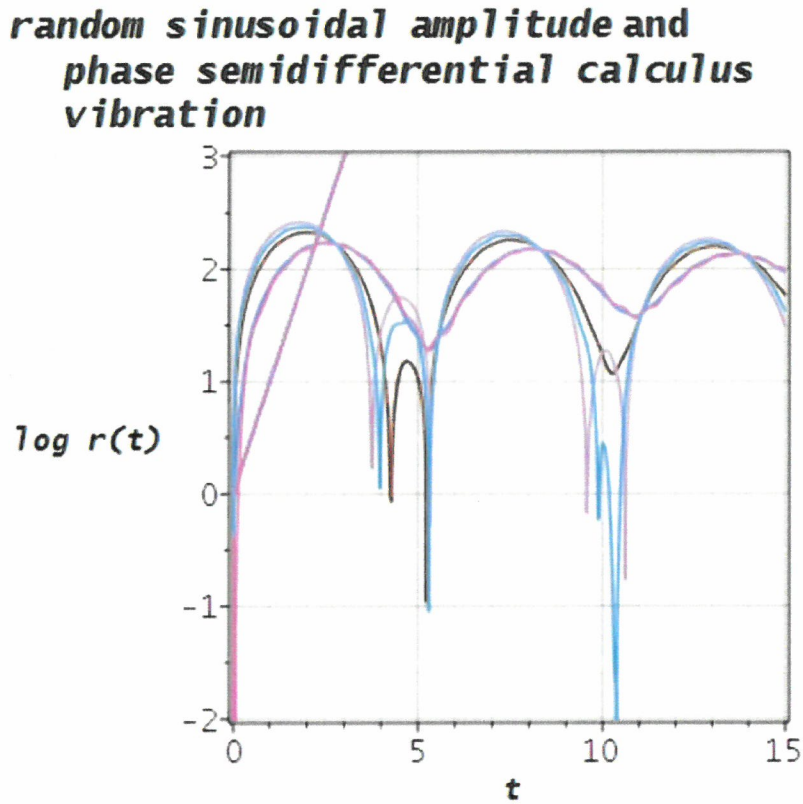



Fractional Calculus Viscoelastic
Settlement Magnitude
displacement, mm
7
6
5
4
3
2 4 6 8 10 12 14 16 18
distance, m
60 70 80 90

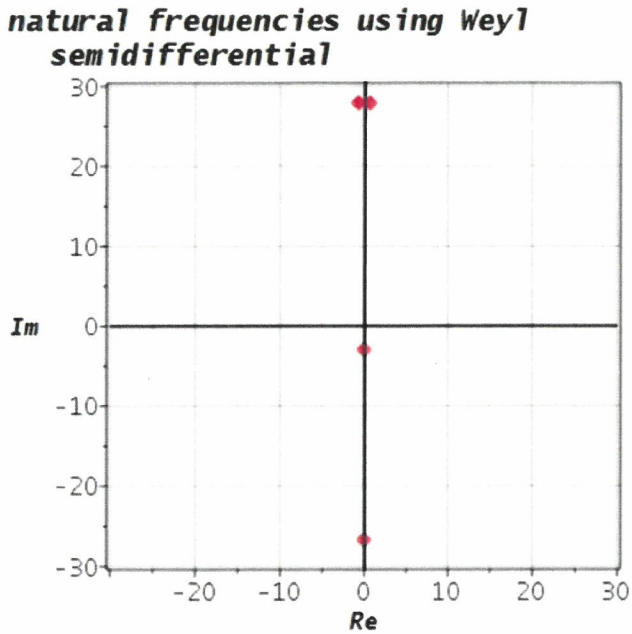

Fractional Calculus Viscoelastic
Settlement Magnitude
displacement, mm
7
6
5
4
3
2
10 20 30 40 50 60 70 80 90
distance, m
1.0 1.25 1.5 1.75

2.9 Fractional Calculus Bubble Oscillator

```
restart; with(Student); with(plots); with(RandomTools); N := 50; p0 := 10^2; K := 80; G := 60; E := 144; r0
:= 1; eta := 1000; alpha := .5; tau1 := evalf(4*eta/(3*K+G)); tau2 := evalf(eta/G); T := 4; col1 :=
seq(COLOR(RGB, rand()/10^11, rand()/10^12, rand()/10^12), j = 1 .. T); t5 := [5, 10, 15, 20]; for I to T do
ur[i] := subs(r0 = 1, p0 = 10^6, K = 80, G = 60, alpha = .6, t1 = t5[i], evalf((-3*p0*(1-sum(-
(t1/tau1)^n/GAMMA(alpha*n+1), n = 0 .. N))/(4*(3*K+G)))^2)); `u&theta;`[i] := proc I options operator,
arrow; subs(r0 = 1, p0 = 1000000, K = 80, G = 60, alpha = .6, t1 = t5[i], evalf((1/4)*p0^2*ln(r0/r)^2*((3-
3*(sum(-(t1/tau1)^n/GAMMA(alpha*n+1), n = 0 .. N)))/(3*K+G)+(1-sum(-
(t1/tau2)^n/GAMMA(alpha*n+1), n = 0 .. N))/G)^2/3.14159^2)) end proc; zz1[i] := proc I options
operator, arrow; evalf((ur[i]+`u&theta;`[i]I)^.5) end proc; R := 20; pl[i] := plot([zz1[i](R-r)], r = 1 .. R-2,
axes = boxed, title = ` Time dependent Fractional Calculus Viscoelastic Settlement Magnitude `, titlefont
= [COURIER, Bold, 14], labels = ["distance, m", "displacement, mm"], labelfont = [COURIER, BOLDITALIC,
12], color = col1[i], legend = [t5[i]*d], style = line, thickness = 3) end do; plots[display]([seq(pl[i], I = 1 ..
T)]); G1 := [60, 70, 80, 90]; for I to T do ur2[i] := subs(r0 = 1, p0 = 10^6, K = 80, alpha = .6, t1 = t5[3],
evalf((-3*p0*(1-sum(-(t1/tau1)^n/GAMMA(alpha*n+1), n = 0 .. N))/(4*(3*K+G1[i])))^2)); `u&theta;2`[i]
:= proc I options operator, arrow; subs(r0 = 1, p0 = 1000000, K = 80, alpha = .6, t1 = t5[3],
evalf((1/4)*p0^2*ln(r0/r)^2*((3-3*(sum(-(t1/tau1)^n/GAMMA(alpha*n+1), n = 0 .. N)))/(3*K+G1[i])+(1-
sum(-(t1/tau2)^n/GAMMA(alpha*n+1), n = 0 .. N))/G1[i])^2/3.14159^2)) end proc; zz2[i] := proc I
options operator, arrow; evalf((ur2[i]+`u&theta;2`[i]I)^.5) end proc; R := 20; pl2[i] := plot([zz2[i](R-r)], r =
1 .. R-2, axes = boxed, title = ` Fractional Calculus Viscoelastic Settlement Magnitude `, titlefont =
[COURIER, Bold, 14], labels = ["distance, m", "displacement, mm"], labelfont = [COURIER, BOLDITALIC,
12], color = col1[i], legend = [G1[i]], style = line, thickness = 3) end do; plots[display]([seq(pl2[i], I = 1 ..
T)]); r01 := [1, 5, 10, 15]; for I to T do ur3[i] := subs(r0 = r01[i], p0 = 10^6, K = 80, alpha = .6, t1 = t5[3],
evalf((-3*p0*(1-sum(-(t1/tau1)^n/GAMMA(alpha*n+1), n = 0 .. N))/(4*(3*K+G1[1])))^2)); `u&theta;3`[i]
:= proc I options operator, arrow; subs(r0 = r01[i], p0 = 1000000, K = 80, alpha = .6, t1 = t5[3],
evalf((1/4)*p0^2*ln(r0/r)^2*((3-3*(sum(-(t1/tau1)^n/GAMMA(alpha*n+1), n = 0 .. N)))/(3*K+G1[1])+(1-
sum(-(t1/tau2)^n/GAMMA(alpha*n+1), n = 0 .. N))/G1[1])^2/3.14159^2)) end proc; zz3[i] := proc I
options operator, arrow; evalf((ur3[i]+`u&theta;3`[i]I)^.5) end proc; R := 100; pl3[i] := plot([zz3[i](R-r)], r
= 1 .. R-2, axes = boxed, title = ` Fractional Calculus Viscoelastic Settlement Magnitude `, titlefont =
[COURIER, Bold, 14], labels = ["distance, m", "displacement, mm"], labelfont = [COURIER, BOLDITALIC,
12], color = col1[i], legend = [r01[i]], style = line, thickness = 3) end do; plots[display]([seq(pl3[i], I = 1 ..
T)]); alpha1 := [1.0, 1.25, 1.5, 1.75]; for I to T do ur4[i] := subs(r0 = r01[1], p0 = 10^6, K = 80, alpha =
alpha1[i], t1 = t5[3], evalf((-3*p0*(1-sum(-(t1/tau1)^n/GAMMA(n*alpha1[i]+1), n = 0 ..
N))/(4*(3*K+G1[1])))^2)); `u&theta;4`[i] := proc I options operator, arrow; subs(r0 = r01[1], p0 =
1000000, K = 80, alpha = alpha1[i], t1 = t5[3], evalf((1/4)*p0^2*ln(r0/r)^2*((3-3*(sum(-
(t1/tau1)^n/GAMMA(n*alpha1[i]+1), n = 0 .. N)))/(3*K+G1[1])+(1-sum(-
(t1/tau2)^n/GAMMA(n*alpha1[i]+1), n = 0 .. N))/G1[1])^2/3.14159^2)) end proc; zz4[i] := proc I options
operator, arrow; evalf((ur4[i]+`u&theta;4`[i]I)^.5) end proc; R := 100; pl4[i] := plot([zz4[i](R-r)], r = 1 .. R-
2, axes = boxed, title = ` Fractional Calculus Viscoelastic Settlement Magnitude `, titlefont = [COURIER,
Bold, 14], labels = ["distance, m", "displacement, mm"], labelfont = [COURIER, BOLDITALIC, 12], color =
col1[i], legend = [alpha1[i]], style = line, thickness = 3) end do; plots[display]([seq(pl4[i], I = 1 .. T)]);
```



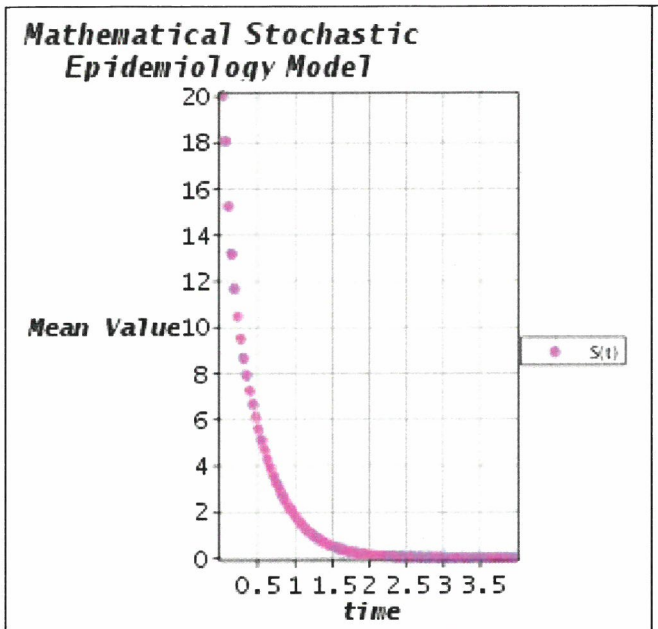
restart; with(Student); with(inttrans); with(RandomTools); n := 5; A := Generate(list(float(range = .1 .. 10., method = uniform), n)); Pie := evalf(Pi); phi := Generate(list(float(range = -Pie .. Pie, method = uniform), n)); col1 := seq(COLOR(RGB, rand()/10^12, rand()/10^12, rand()/10^12), I = 1 .. n); assume(0 < a); omega1 := 10.; alpha1 := .5; mu0 := 0.1e-2; rho0 := 1.0*10^3; R0 := 5.0*10^(-3); a0 := 10.; sigma1 := 72.8*10^(-3); p0 := 1.983*10^(-5); A1 := evalf(4*mu0/(rho0*R0^2)); B1 := evalf(4*sigma1/(rho0*omega1*R0^2)+2*p0*R0*(1/(rho0*omega1*R0^2))); tau := 100.; di1 := fracdiff(y(t), t, -.5); di0 := laplace(di1, t, s); ic1 := invlaplace(di0, s, 0); sol3 := solve(s^2+A1*s^alpha1+B1 = 0, s); y1 := 3.; for I to n do ic0[i] := ((a0*A[i]*sin(omega1*0+phi[i])*Heaviside(t)-p0)*R0-2*sigma1)/(rho0*omega1*R0^3); numerator := proc (s) options operator, arrow; laplace(((a0*A[i]*sin(omega1*t+phi[i])*Heaviside(t)-p0)*R0-2*sigma1)/(rho0*omega1*R0^3), t, s) end proc; paf := proc (s) options operator, arrow; convert(1/((s-sol3[1])*(s-sol3[2])), parfrac) end proc; fls[i] := invlaplace(paf(s)*(numerator(s)+ic1+ic0[i]), s, t); plt[i] := plot([t, log10(abs(fls[i](t)))], t = 0 .. 15, -2 .. y1, axes = boxed, style = line, color = col1[i], gridlines, title = 'random*sinusoidal*amplitude and phase*semidifferential*calculus*vibration', titlefont = [COURIER, Bold, 14], labels = ["t", "log r(t)"], labelfont = [COURIER, BOLDITALIC, 12], axesfont = [COURIER, 12]) end do; plots[display]({seq(plt[i], I = 1 .. n)});

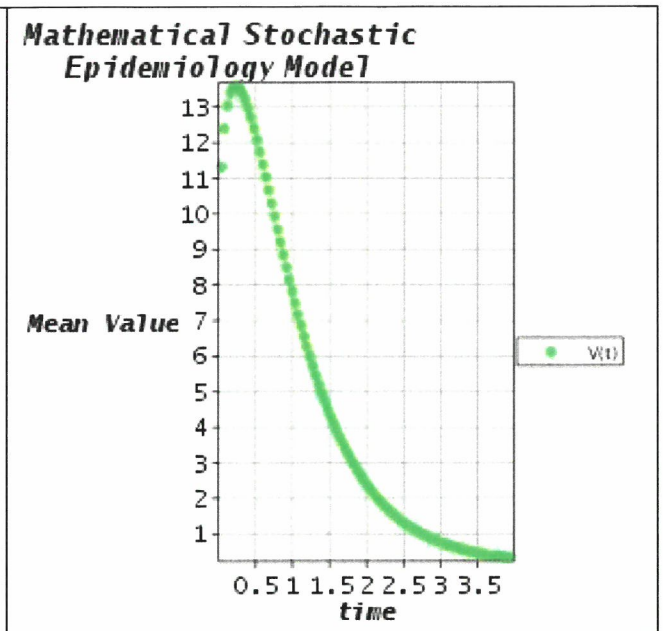


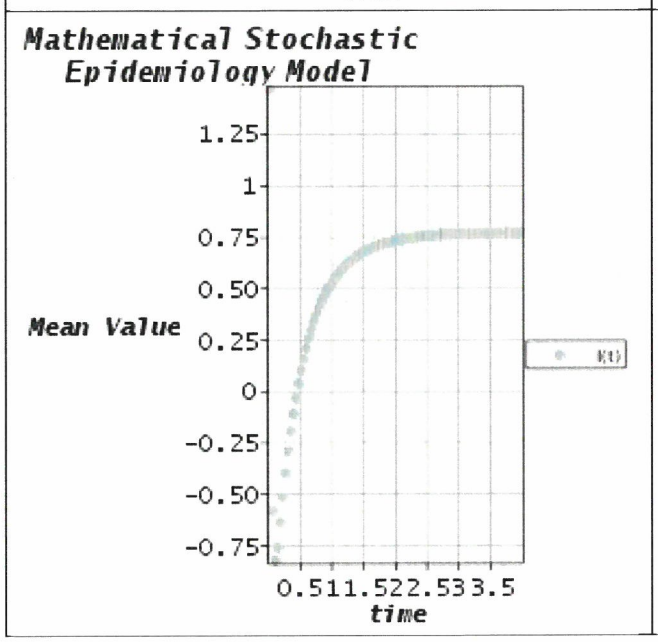

```
restart; with(Student); with(LinearAlgebra); with(DETools); with(plots); with(RandomTools); n := 2; col1
:= seq(COLOR(RGB, rand()/10^12, rand()/10^12, rand()/10^12), j = 1 .. n); A0 := Matrix(2, 2, [[80.81722,
2.31941332208709], [2.31941332208709, .29784188199686]]); g := 9.81; v := 3.5; omega1 := 10; A := v .
Matrix(2, 2, [[0, 33.86641391492494], [-.85035641456978, 1.68540397397560]]); K0 := Matrix(2, 2, [[-
80.95, -2.59951685249872], [-2.59951685249872, -.80329488458618]]); K2 := Matrix(2, 2, [[0,
76.59734589573222], [0, 2.65431523794604]]); B := evalf(K2*v^2+K0*g); q := Matrix(2, 1, [x(t), y(t)]);
zer0 := Matrix(2, 1, [0, 0]); Id1 := IdentityMatrix(2); alpha := .5; de0 := A0 . (diff(q, t, t))+A . (diff(q, t))+B .
q; ampl := Matrix(2, 2, [[a0, 0], [0, b0]]); ampl2 := Matrix(2, 2, [[a^alpha, 0], [0, b^alpha]]); qs := Matrix(2,
1, [exp(a*t), exp(b*t)]); q2 := ampl . qs; q2f := ampl2 . ampl . qs; diff(qs, t, t); de1 := diff(q2, t, t)+B . q2;
sys0 := de1+A . q2f; sys1 := simplify(sys0[1]); sys2 := simplify(sys0[2]); sys1f := collect(factor(sys1), [a0,
b0]); sys2f := collect(factor(sys2), [a0, b0]); m11 := coeff(sys1f, a0); m21 := coeff(sys2f, a0); m12 :=
coeff(sys1f, b0); m22 := coeff(sys2f, b0); t := 0.; mat := Matrix(2, 2, [[m11, m12], [m21, m22]]); a :=
I*omega; b := I*omega; poly := Determinant(mat); dl := 1.; natff := solve(poly = 0, omega); for I to 4 do
pl[i] := complexplot([natff[i]], -30 .. 30, -30 .. 30, style = point, symbol = soliddiamond, symbolsize = 16,
color = red, gridlines, axes = boxed, title = 'natural*frequencies*using*Weyl*semidifferential', titlefont =
[COURIER, Bold, 14], labels = ["Re", "Im"], labelfont = [COURIER, BOLDITALIC, 12], axesfont = [COURIER,
12]) end do; g := proc (x) options operator, arrow; 0*x end proc; h := proc (x) options operator, arrow;
10000000*x end proc; plx := plot(g(x), x = -30 .. 30, y = -30 .. 30, style = line, thickness = 2, color = black);
ply := plot(h(x), x = -30 .. 30, y = -30 .. 30, style = line, thickness = 2, color = black); plt :=
plots[display](seq(pl[i], I = 1 .. 4), plx, ply);
```

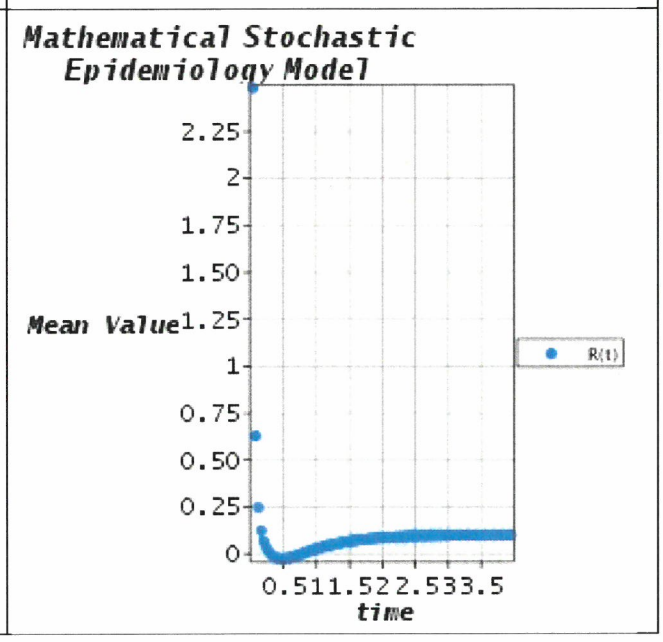
3.0 Epidemiological Model

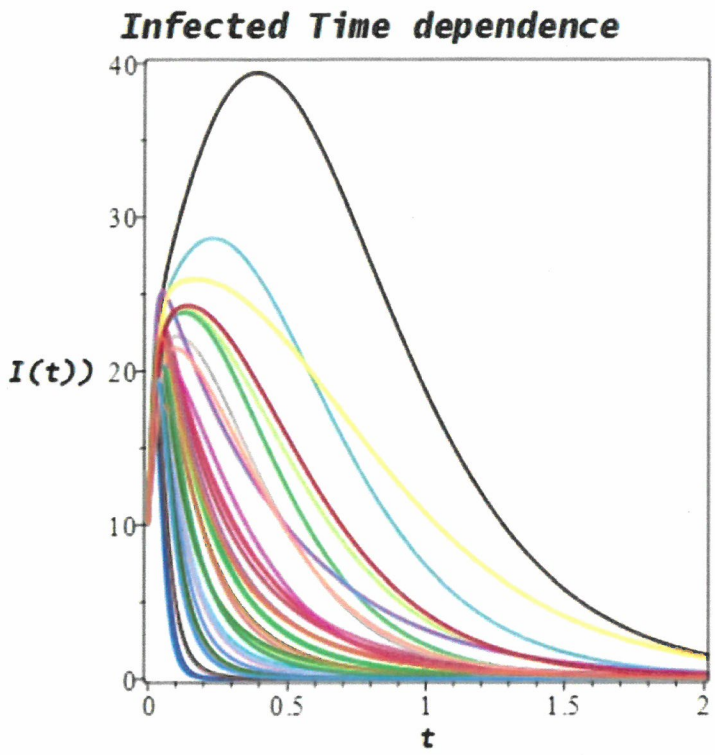
```
restart : with(DEtools) : with(Student) : with(plots) : with(Statistics) : with(RandomTools) :
with(Threads) : n := 35; col10 := seq(COLOR(RGB, rand()/10^12, rand()/10^12, rand()/10^12), j
= 1 ..n) : θr := Generate(list(float( range = 0.2 ..1.1, method = uniform), n)) : β1r :=
Generate(list(float( range = 1 ..3, method = uniform), n)) : γr :=
Generate(list(float( range = 2 ..3, method = uniform), n)); γ0r :=
Generate(list(float( range = 2 ..4, method = uniform), n)) : β0r :=
Generate(list(float( range = 1 ..4, method = uniform), n)) ; μr := Generate(list(float( range
= 1 ..3, method = uniform), n)) ;for i from 1 to n do ; de1 := d/dt s1(t) − μr[i] + μr[i]
·s1(t) + β0r[i]·v1(t)·I1(t) + θr[i]·s1(t) = 0; de2 := d/dt v1(t) − θr[i]·s1(t) + β1r[i]
·v1(t)·I1(t) + γr[i]·v1(t) + μr[i]·v1(t) = 0; de3 := d/dt I1(t) − β0r[i]·s1(t)·I1(t)
− β1r[i]·v1(t)·I1(t) + γr[i]·I1(t) + μr[i]·I1(t) = 0; de4 := d/dt R11(t) − γr[i]·v1(t)
− γr[i]·I1(t) + μr[i]·R11(t) = 0; sol1[i] := dsolve({de1, de2, de3, de4, s1(0) = 10,
v1(0) = 10, I1(0) = 10, R11(0) = 10}, {s1(t), v1(t), I1(t), R11(t)}, numeric, output
= listprocedure, abserr = 0.001) : T10 := 0.0; T2 := 4; pl11[i] := odeplot(sol1[i], [t,
I1(t)], t = T10 ..T2, numpoints = 500, labels = ["t", "I(t))"], thickness = 2, title
= ` Infected Time dependence`, titlefont = [COURIER, Bold, 14], labelfont = [COURIER,
BOLDITALIC, 12], color = col10[i], axes = boxed, view = [T10 ..2, 0 ..40]) : pl21[i] :=
odeplot(sol1[i], [[t, R11(t)]], t = T10 ..T2, numpoints = 500, labels = ["t", " R(t)"],
thickness = 2, title = `Recovered Time dependence `, titlefont = [COURIER, Bold, 14],
labelfont = [COURIER, BOLDITALIC, 12], color = col10[i], axes = boxed, view = [T10
..T2, 0 ..40]) : pl31[i] := odeplot(sol1[i], [[t, v1(t)]], t = T10 ..T2, numpoints = 500,
labels = ["t", " V(t)"], thickness = 2, title = `Vaccine density Time dependence `, titlefont
= [COURIER, Bold, 14], labelfont = [COURIER, BOLDITALIC, 12], color = col10[i],
axes = boxed, view = [T10 ...2, 0 ..10]) :end do : q[1] := plots[display](seq(pl11[i], i = 1
.. n)); q[2] := plots[display](seq(pl21[i], i = 1 ..n)); q[3] :=
plots[display](seq(pl31[i], i = 1 ..n)); nint := 100; nn := nint − 1; np := (T2 − T10)/nint;
for w from 1 to 5 do for j from 1 to n do sol2[j] := [seq(sol1[j][w](T10 + i·np), i = 1
..nn)]; end do;for m from 1 to n do v11[m] := [ seq(rhs(sol2[m][h]), h = 1 ..nn)] ;end
do; v10 := [seq(Add(v11[m][j], m = 1 ..n), j = 1 ..nn)]; T2 := [seq(T10 + i·np, i = 1
..nn)]; pts := seq([T2[i], v10[i]/n], i = 1 ..nn); Titles := ["Time", "S(t)", "V(t)", "I(t)",
"R(t)"];; pll[w] := pointplot([pts], axes = boxed, color = col10[w], symbol = solidcircle,
symbolsize = 13, title ='Mathematical Stochastic Epidemiology Model', titlefont
= [COURIER, Bold, 14], labels = ["time", "Mean Value"], labelfont = [COURIER,
BOLDITALIC, 12], axesfont = [COURIER, 12], gridlines, tickmarks = [10, 10], legend
= Titles[w], legendstyle = [font = ["HELVETICA", 9], location = right]);end do:
m1[1] := plots[display]([seq(pll[w], w = 2 ..5)]); display(Array(1 ..2, 1 ..2, [[pll[2],
pll[3]], [pll[4], pll[5]]])); display(Array(1 ..2, 1 ..2, [[q[1], q[2]], [q[3], m1[1]]]));
```

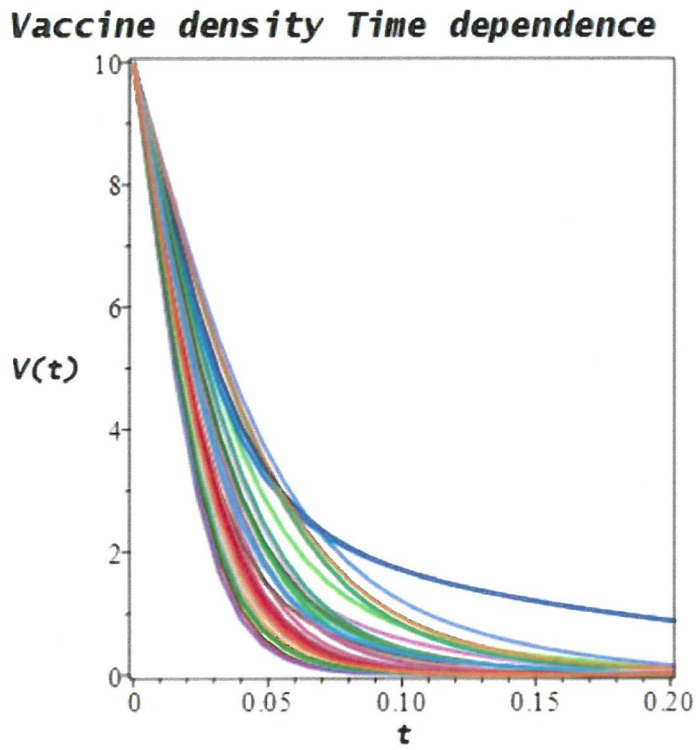

Mathematical Stochastic
Epidemiology Model
Mean Value
time
S(t)

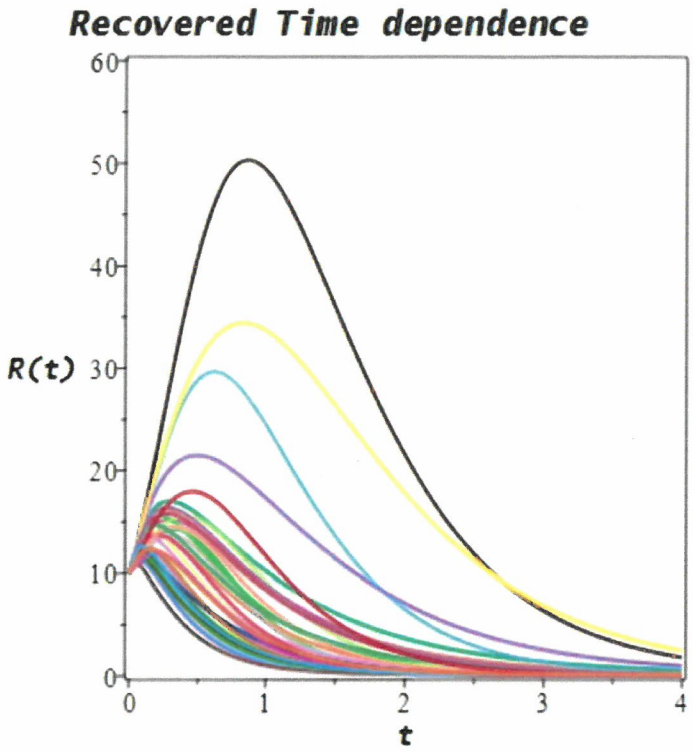

Mathematical Stochastic
Epidemiology Model
Mean Value
time
V(t)

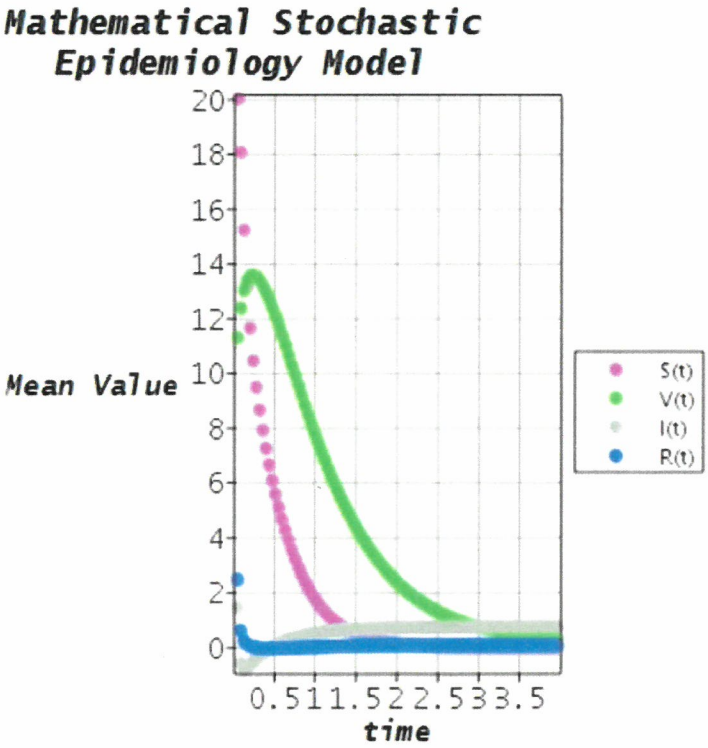

Mathematical Stochastic
Epidemiology Model
Mean Value
time
I(t)


Mathematical Stochastic
Epidemiology Model
Mean Value
time
R(t)


Infected Time dependence
I(t))
t
0
10
20
30
40
0.5
1
1.5
2


Vaccine density Time dependence
V(t)
t
0
2
4
6
8
10
0.05
0.10
0.15
0.20


Recovered Time dependence
R(t)
t
0
10
20
30
40
50
60
0
1
2
3
4


Mathematical Stochastic
Epidemiology Model
Mean Value
time
S(t)
V(t)
I(t)
R(t)
0
2
4
6
8
10
12
14
16
18
20
0.5
1
1.5
2
2.5
3
3.5

3.1 Pi Thereom Parameters: Falling Particle

```
restart; with(Student); with(combinat, randcomb); with(linalg); with(Student[LinearAlgebra]);
with(plots); m := 10; n1 := 3; dd := m-n1; N := factorial(m)/(factorial(m-n1)*factorial(n1)); m0 := [delta, L,
A, rho, U, c, p, mu, G, F]; for j to N do mc[j] := randcomb(m0, n1) end do; M := [seq(mc[k], k = 1 .. N)]; for
k to N do m0s[k] := convert(m0, set); Mc[k] := convert(M[k], set); A0[k] := m0s[k] minus Mc[k]; Cm[k] :=
[A0[k], Mc[k]]; V1[k] := Vector(dd, [seq(A0[k][j], j = 1 .. dd)]); V2[k] := Vector(n1, [seq(Mc[k][j], j = 1 ..
n1)]); V1tr[k] := Transpose(V1[k]); V2t[k] := Transpose(V2[k]) end do; Am2[1] := [seq(A0[1][i], I = 1 .. dd)];
delta := Vector(3, [0, 1, 0]); L := Vector(3, [0, 1, 0]); A := Vector(3, [0, 2, 0]); rho := Vector(3, [1, -3, 0]); U
:= Vector(3, [0, 1, -1]); c := Vector(3, [0, 1, -1]); p := Vector(3, [1, -1, -2]); mu := Vector(3, [1, -1, -1]); G :=
Vector(3, [1, -2, -2]); F := Vector(3, [1, 1, -2]); for k to N do AM1[k] := Matrix(n1, dd, [seq(V1tr[k][i], I = 1 ..
dd)]); Bm1[k] := Matrix(n1, n1, [seq(V2t[k][i], I = 1 .. n1)]); det1[k] := Determinant(Bm1[k]) end do; for n
to N do if det1[n] = 0 then next end if; Bm1i[n] := inverse(Bm1[n]); I01[n] := -1.*Matrix(n1, n1, shape =
identity); A1[n] := I01[n] . AM1[n]; Dim[n] := Bm1i[n] . A1[n] end do; for n to N do if det1[n] = 0 then next
end if; dimpar[n] := seq(V2t[n](1, 1)^convert(Dim[n](1, i), rational)*V2t[n](1, 2)^convert(Dim[n](2, i),
rational)*V2t[n](1, 3)^convert(Dim[n](3, i), rational)*V1tr[n](1, i), I = 1 .. dd) end do;
```

(boundary layer thickness, length, area, density, speed, sound speed, pressure, shear viscosity, pressure gradient, force-drag)

$$m0 := [\delta, L, A, \rho, U, c, p, \mu, G, F]$$

$$dimpar_1 := \frac{pA}{F}, \frac{\sqrt{F}\,G}{p^{3/2}}, \frac{\sqrt{p}\,L}{\sqrt{F}}, \frac{U}{c}, \frac{\sqrt{p}\,\delta}{\sqrt{F}}, \frac{c\mu}{\sqrt{F}\sqrt{p}}, \frac{c^2\rho}{p}$$

$$dimpar_2 := \frac{A}{L^2}, \frac{F}{L^2 c^2 \rho}, \frac{LG}{c^2\rho}, \frac{U}{c}, \frac{\delta}{L}, \frac{\mu}{Lc\rho}, \frac{p}{c^2\rho}$$

$$dimpar_3 := \frac{A}{\delta^2}, \frac{F}{U\delta\mu}, \frac{\delta^2 G}{U\mu}, \frac{L}{\delta}, \frac{c}{U}, \frac{\delta p}{U\mu}, \frac{U\delta\rho}{\mu}$$

$$dimpar_4 := \frac{A}{L^2}, \frac{L^3 G}{F}, \frac{c}{U}, \frac{\delta}{L}, \frac{LU\mu}{F}, \frac{L^2 p}{F}, \frac{L^2 U^2 \rho}{F}$$

$$dimpar_5 := \frac{A}{L^2}, \frac{\rho F}{\mu^2}, \frac{L^3 \rho G}{\mu^2}, \frac{L\rho U}{\mu}, \frac{L\rho c}{\mu}, \frac{\delta}{L}, \frac{L^2 \rho p}{\mu^2}$$

$$dimpar_7 := \frac{A}{L^2}, \frac{L^3 G}{F}, \frac{LU\mu}{F}, \frac{L\mu c}{F}, \frac{\delta}{L}, \frac{L^2 p}{F}, \frac{\rho F}{\mu^2}$$

$$dimpar_8 := \frac{A}{\delta^2}, \frac{F}{G\delta^3}, \frac{L}{\delta}, \frac{U}{c}, \frac{c\mu}{G\delta^2}, \frac{p}{G\delta}, \frac{c^2\rho}{G\delta}$$

$$dimpar_9 := \frac{F}{Ap}, \frac{\sqrt{A}\,G}{p}, \frac{L}{\sqrt{A}}, \frac{c}{U}, \frac{\delta}{\sqrt{A}}, \frac{U\mu}{\sqrt{A}\,p}, \frac{U^2\rho}{p}$$

$$dimpar_{10} := \frac{A^{3/2} G}{F}, \frac{L}{\sqrt{A}}, \frac{\sqrt{A}\,\mu U}{F}, \frac{\sqrt{A}\,\mu c}{F}, \frac{\delta}{\sqrt{A}}, \frac{pA}{F}, \frac{\rho F}{\mu^2}$$

$$dimpar_{12} := \frac{A}{L^2}, \frac{L^3 G}{F}, \frac{c}{U}, \frac{\delta}{L}, \frac{L U \mu}{F}, \frac{L^2 p}{F}, \frac{L^2 U^2 \rho}{F}$$

$$dimpar_{13} := \frac{A}{L^2}, \frac{\rho F}{\mu^2}, \frac{L^3 \rho G}{\mu^2}, \frac{L \rho U}{\mu}, \frac{L \rho c}{\mu}, \frac{\delta}{L}, \frac{L^2 \rho p}{\mu^2}$$

$$dimpar_{14} := \frac{G^2 A}{p^2}, \frac{G^2 F}{p^3}, \frac{G L}{p}, \frac{c}{U}, \frac{G \delta}{p}, \frac{G U \mu}{p^2}, \frac{U^2 \rho}{p}$$

$$dimpar_{15} := \frac{A^{3/2} G}{F}, \frac{L}{\sqrt{A}}, \frac{\sqrt{A}\sqrt{\rho}\, U}{\sqrt{F}}, \frac{\sqrt{A}\sqrt{\rho}\, c}{\sqrt{F}}, \frac{\delta}{\sqrt{A}}, \frac{\mu}{\sqrt{F}\sqrt{\rho}}, \frac{p A}{F}$$

$$dimpar_{16} := \frac{G^{2/3} A}{F^{2/3}}, \frac{G^{1/3} L}{F^{1/3}}, \frac{\sqrt{\rho}\, U}{F^{1/6} G^{1/3}}, \frac{\sqrt{\rho}\, c}{F^{1/6} G^{1/3}}, \frac{G^{1/3} \delta}{F^{1/3}}, \frac{G^{\frac{1}{9007199254740992}} \mu}{\sqrt{F}\sqrt{\rho}},$$

$$\frac{p}{F^{1/3} G^{2/3}}$$

$$dimpar_{17} := \frac{G^2 A}{p^2}, \frac{G^2 F}{p^3}, \frac{G L}{p}, \frac{G U \mu}{p^2}, \frac{G \mu c}{p^2}, \frac{G \delta}{p}, \frac{p^3 \rho}{G^2 \mu^2}$$

$$dimpar_{19} := \frac{A}{L^2}, \frac{F}{L^2 p}, \frac{G L}{p}, \frac{\sqrt{\rho}\, U}{\sqrt{p}}, \frac{\sqrt{\rho}\, c}{\sqrt{p}}, \frac{\delta}{L}, \frac{\mu}{L\sqrt{p}\sqrt{\rho}}$$

$$dimpar_{20} := \frac{A}{\delta^2}, \frac{F}{c \delta \mu}, \frac{\delta^2 G}{c \mu}, \frac{L}{\delta}, \frac{U}{c}, \frac{\delta p}{c \mu}, \frac{c \delta \rho}{\mu}$$

$$dimpar_{21} := \frac{A}{\delta^2}, \frac{F}{\delta^2 p}, \frac{G \delta}{p}, \frac{L}{\delta}, \frac{\sqrt{\rho}\, U}{\sqrt{p}}, \frac{\sqrt{\rho}\, c}{\sqrt{p}}, \frac{\mu}{\delta\sqrt{p}\sqrt{\rho}}$$

$$dimpar_{22} := \frac{A}{L^2}, \frac{F}{G L^3}, \frac{c}{U}, \frac{\delta}{L}, \frac{U \mu}{G L^2}, \frac{p}{G L}, \frac{U^2 \rho}{G L}$$

$$dimpar_{23} := \frac{F}{A^{3/2} G}, \frac{L}{\sqrt{A}}, \frac{\sqrt{\rho}\, U}{A^{1/4}\sqrt{G}}, \frac{\sqrt{\rho}\, c}{A^{1/4}\sqrt{G}}, \frac{\delta}{\sqrt{A}}, \frac{\mu}{A^{3/4}\sqrt{G}\sqrt{\rho}}, \frac{p}{\sqrt{A}\, G}$$

$$dimpar_{24} := \frac{c^2 \rho A}{F}, \frac{\sqrt{F}\, G}{c^3 \rho^{3/2}}, \frac{c\sqrt{\rho}\, L}{\sqrt{F}}, \frac{U}{c}, \frac{c\sqrt{\rho}\, \delta}{\sqrt{F}}, \frac{\mu}{\sqrt{F}\sqrt{\rho}}, \frac{p}{c^2 \rho}$$

$$dimpar_{25} := \frac{A}{L^2}, \frac{F}{L^2 p}, \frac{G L}{p}, \frac{\mu U}{L p}, \frac{\mu c}{L p}, \frac{\delta}{L}, \frac{L^2 \rho p}{\mu^2}$$

$$dimpar_{28} := \frac{c^2 \mu^2 A}{F^2}, \frac{F^2 G}{c^3 \mu^3}, \frac{L \mu c}{F}, \frac{U}{c}, \frac{c \mu \delta}{F}, \frac{F p}{c^2 \mu^2}, \frac{\rho F}{\mu^2}$$

$$dimpar_{29} := \frac{A}{L^2}, \frac{F}{L c \mu}, \frac{L^2 G}{c \mu}, \frac{U}{c}, \frac{\delta}{L}, \frac{L p}{c \mu}, \frac{L \rho c}{\mu}$$

$$dimpar_{30} := \frac{p \rho A}{\mu^2}, \frac{\rho F}{\mu^2}, \frac{\mu G}{p^{3/2}\sqrt{\rho}}, \frac{\sqrt{p}\sqrt{\rho}\, L}{\mu}, \frac{\sqrt{\rho}\, U}{\sqrt{p}}, \frac{\sqrt{\rho}\, c}{\sqrt{p}}, \frac{\sqrt{p}\sqrt{\rho}\, \delta}{\mu}$$

$$dimpar_{31} := \frac{G^{2/3} A}{F^{2/3}}, \frac{G^{1/3} L}{F^{1/3}}, \frac{\sqrt{\rho}\, U}{F^{1/6} G^{1/3}}, \frac{\sqrt{\rho}\, c}{F^{1/6} G^{1/3}}, \frac{G^{1/3} \delta}{F^{1/3}}, \frac{G^{\frac{1}{9007199254740992}} \mu}{\sqrt{F}\sqrt{\rho}},$$

$$\frac{p}{F^{1/3} G^{2/3}}$$

$$dimpar_{34} := \frac{A}{\delta^2}, \frac{F}{c^2 \delta^2 \rho}, \frac{\delta G}{c^2 \rho}, \frac{L}{\delta}, \frac{U}{c}, \frac{\mu}{c \delta \rho}, \frac{p}{c^2 \rho}$$

$$dimpar_{37} := \frac{A^{3/2} G}{F}, \frac{L}{\sqrt{A}}, \frac{\sqrt{A}\sqrt{\rho}\, U}{\sqrt{F}}, \frac{\sqrt{A}\sqrt{\rho}\, c}{\sqrt{F}}, \frac{\delta}{\sqrt{A}}, \frac{\mu}{\sqrt{F}\sqrt{\rho}}, \frac{pA}{F}$$

$$dimpar_{38} := \frac{p^2 A}{c^2 \mu^2}, \frac{Fp}{c^2 \mu^2}, \frac{G \mu c}{p^2}, \frac{Lp}{c \mu}, \frac{U}{c}, \frac{\delta p}{c \mu}, \frac{c^2 \rho}{p}$$

$$dimpar_{39} := \frac{A}{\delta^2}, \frac{F}{G \delta^3}, \frac{L}{\delta}, \frac{U}{c}, \frac{c \mu}{G \delta^2}, \frac{p}{G \delta}, \frac{c^2 \rho}{G \delta}$$

$$dimpar_{40} := \frac{A}{L^2}, \frac{F}{G L^3}, \frac{c}{U}, \frac{\delta}{L}, \frac{U \mu}{G L^2}, \frac{p}{G L}, \frac{U^2 \rho}{G L}$$

$$dimpar_{42} := \frac{F}{A^{3/2} G}, \frac{L}{\sqrt{A}}, \frac{\mu U}{A G}, \frac{\mu c}{A G}, \frac{\delta}{\sqrt{A}}, \frac{p}{\sqrt{A}\, G}, \frac{A^{3/2} G \rho}{\mu^2}$$

$$dimpar_{43} := \frac{F}{A U^2 \rho}, \frac{\sqrt{A}\, G}{U^2 \rho}, \frac{L}{\sqrt{A}}, \frac{c}{U}, \frac{\delta}{\sqrt{A}}, \frac{\mu}{\sqrt{A}\, U \rho}, \frac{p}{U^2 \rho}$$

$$dimpar_{44} := \frac{F}{A^{3/2} G}, \frac{L}{\sqrt{A}}, \frac{\mu U}{A G}, \frac{\mu c}{A G}, \frac{\delta}{\sqrt{A}}, \frac{p}{\sqrt{A}\, G}, \frac{A^{3/2} G \rho}{\mu^2}$$

$$dimpar_{46} := \frac{A}{\delta^2}, \frac{F}{G \delta^3}, \frac{L}{\delta}, \frac{c}{U}, \frac{U \mu}{G \delta^2}, \frac{p}{G \delta}, \frac{U^2 \rho}{G \delta}$$

$$dimpar_{48} := \frac{A}{L^2}, \frac{F}{G L^3}, \frac{\sqrt{\rho}\, U}{\sqrt{G}\sqrt{L}}, \frac{\sqrt{\rho}\, c}{\sqrt{G}\sqrt{L}}, \frac{\delta}{L}, \frac{\mu}{\sqrt{G}\, L^{3/2} \sqrt{\rho}}, \frac{p}{G L}$$

$$dimpar_{49} := \frac{A}{\delta^2}, \frac{F}{U \delta \mu}, \frac{\delta^2 G}{U \mu}, \frac{L}{\delta}, \frac{c}{U}, \frac{\delta p}{U \mu}, \frac{U \delta \rho}{\mu}$$

$$dimpar_{50} := \frac{A}{L^2}, \frac{F}{G L^3}, \frac{c}{U}, \frac{\delta}{L}, \frac{U \mu}{G L^2}, \frac{p}{G L}, \frac{U^2 \rho}{G L}$$

$$dimpar_{53} := \frac{A}{\delta^2}, \frac{F}{G \delta^3}, \frac{L}{\delta}, \frac{c}{U}, \frac{U \mu}{G \delta^2}, \frac{p}{G \delta}, \frac{U^2 \rho}{G \delta}$$

$$dimpar_{54} := \frac{U^2 \rho^2 A}{\mu^2}, \frac{\rho F}{\mu^2}, \frac{\mu G}{U^3 \rho^2}, \frac{L \rho U}{\mu}, \frac{c}{U}, \frac{U \delta \rho}{\mu}, \frac{p}{U^2 \rho}$$

$$dimpar_{55} := \frac{A}{\delta^2}, \frac{\delta^3 G}{F}, \frac{L}{\delta}, \frac{c}{U}, \frac{U \delta \mu}{F}, \frac{\delta^2 p}{F}, \frac{U^2 \delta^2 \rho}{F}$$

$$dimpar_{56} := \frac{A^{3/2}G}{F}, \frac{L}{\sqrt{A}}, \frac{c}{U}, \frac{\delta}{\sqrt{A}}, \frac{\sqrt{A}\,\mu U}{F}, \frac{pA}{F}, \frac{AU^2\rho}{F}$$

$$dimpar_{57} := \frac{A}{\delta^2}, \frac{F}{\delta^2 p}, \frac{G\delta}{p}, \frac{L}{\delta}, \frac{U}{c}, \frac{c\mu}{\delta p}, \frac{c^2\rho}{p}$$

$$dimpar_{58} := \frac{A}{L^2}, \frac{\rho F}{\mu^2}, \frac{L^3\rho G}{\mu^2}, \frac{L\rho U}{\mu}, \frac{L\rho c}{\mu}, \frac{\delta}{L}, \frac{L^2\rho p}{\mu^2}$$

$$dimpar_{61} := \frac{A}{\delta^2}, \frac{\delta^3 G}{F}, \frac{L}{\delta}, \frac{c}{U}, \frac{U\delta\mu}{F}, \frac{\delta^2 p}{F}, \frac{U^2\delta^2\rho}{F}$$

$$dimpar_{62} := \frac{A}{L^2}, \frac{L^3 G}{F}, \frac{LU\mu}{F}, \frac{L\mu c}{F}, \frac{\delta}{L}, \frac{L^2 p}{F}, \frac{\rho F}{\mu^2}$$

$$dimpar_{66} := \frac{U^2\rho^2 A}{\mu^2}, \frac{\rho F}{\mu^2}, \frac{\mu G}{U^3\rho^2}, \frac{L\rho U}{\mu}, \frac{c}{U}, \frac{U\delta\rho}{\mu}, \frac{p}{U^2\rho}$$

$$dimpar_{69} := \frac{A}{L^2}, \frac{F}{L^2c^2\rho}, \frac{LG}{c^2\rho}, \frac{U}{c}, \frac{\delta}{L}, \frac{\mu}{Lc\rho}, \frac{p}{c^2\rho}$$

$$dimpar_{70} := \frac{pA}{F}, \frac{\sqrt{F}\,G}{p^{3/2}}, \frac{\sqrt{p}\,L}{\sqrt{F}}, \frac{\sqrt{\rho}\,U}{\sqrt{p}}, \frac{\sqrt{\rho}\,c}{\sqrt{p}}, \frac{\sqrt{p}\,\delta}{\sqrt{F}}, \frac{\mu}{\sqrt{F}\sqrt{\rho}}$$

$$dimpar_{72} := \frac{F}{A^{3/2}G}, \frac{L}{\sqrt{A}}, \frac{\mu U}{AG}, \frac{\mu c}{AG}, \frac{\delta}{\sqrt{A}}, \frac{p}{\sqrt{A}\,G}, \frac{A^{3/2}G\rho}{\mu^2}$$

$$dimpar_{73} := \frac{F}{A^{3/2}G}, \frac{L}{\sqrt{A}}, \frac{c}{U}, \frac{\delta}{\sqrt{A}}, \frac{\mu U}{AG}, \frac{p}{\sqrt{A}\,G}, \frac{U^2\rho}{\sqrt{A}\,G}$$

$$dimpar_{74} := \frac{A}{\delta^2}, \frac{F}{G\delta^3}, \frac{L}{\delta}, \frac{U\mu}{G\delta^2}, \frac{c\mu}{G\delta^2}, \frac{p}{G\delta}, \frac{G\delta^3\rho}{\mu^2}$$

$$dimpar_{76} := \frac{F}{Ap}, \frac{\sqrt{A}\,G}{p}, \frac{L}{\sqrt{A}}, \frac{U\mu}{\sqrt{A}\,p}, \frac{\mu c}{\sqrt{A}\,p}, \frac{\delta}{\sqrt{A}}, \frac{p\rho A}{\mu^2}$$

$$dimpar_{77} := \frac{A}{\delta^2}, \frac{F}{\delta^2 p}, \frac{G\delta}{p}, \frac{L}{\delta}, \frac{\sqrt{\rho}\,U}{\sqrt{p}}, \frac{\sqrt{\rho}\,c}{\sqrt{p}}, \frac{\mu}{\delta\sqrt{p}\sqrt{\rho}}$$

$$dimpar_{78} := \frac{A}{\delta^2}, \frac{F}{c\delta\mu}, \frac{\delta^2 G}{c\mu}, \frac{L}{\delta}, \frac{U}{c}, \frac{\delta p}{c\mu}, \frac{c\delta\rho}{\mu}$$

$$dimpar_{79} := \frac{A}{\delta^2}, \frac{\delta^3 G}{F}, \frac{L}{\delta}, \frac{\delta\sqrt{\rho}\,U}{\sqrt{F}}, \frac{c\sqrt{\rho}\,\delta}{\sqrt{F}}, \frac{\mu}{\sqrt{F}\sqrt{\rho}}, \frac{\delta^2 p}{F}$$

$$dimpar_{81} := \frac{A}{\delta^2}, \frac{\delta^3 G}{F}, \frac{L}{\delta}, \frac{U}{c}, \frac{c\mu\delta}{F}, \frac{\delta^2 p}{F}, \frac{c^2\delta^2\rho}{F}$$

$$dimpar_{82} := \frac{A}{\delta^2}, \frac{F}{\delta^2 p}, \frac{G\delta}{p}, \frac{L}{\delta}, \frac{U}{c}, \frac{c\mu}{\delta p}, \frac{c^2\rho}{p}$$

$$dimpar_{83} := \frac{A}{L^2}, \frac{F}{G L^3}, \frac{U \mu}{G L^2}, \frac{\mu c}{G L^2}, \frac{\delta}{L}, \frac{p}{G L}, \frac{L^3 \rho G}{\mu^2}$$

$$dimpar_{85} := \frac{p A}{F}, \frac{\sqrt{F}\, G}{p^{3/2}}, \frac{\sqrt{p}\, L}{\sqrt{F}}, \frac{c}{U}, \frac{\sqrt{p}\, \delta}{\sqrt{F}}, \frac{U \mu}{\sqrt{F}\, \sqrt{p}}, \frac{U^2 \rho}{p}$$

$$dimpar_{86} := \frac{F}{A U^2 \rho}, \frac{\sqrt{A}\, G}{U^2 \rho}, \frac{L}{\sqrt{A}}, \frac{c}{U}, \frac{\delta}{\sqrt{A}}, \frac{\mu}{\sqrt{A}\, U \rho}, \frac{p}{U^2 \rho}$$

$$dimpar_{87} := \frac{G^2 A}{p^2}, \frac{G^2 F}{p^3}, \frac{G L}{p}, \frac{c}{U}, \frac{G \delta}{p}, \frac{G U \mu}{p^2}, \frac{U^2 \rho}{p}$$

$$dimpar_{90} := \frac{A^{3/2} G}{F}, \frac{L}{\sqrt{A}}, \frac{c}{U}, \frac{\delta}{\sqrt{A}}, \frac{\sqrt{A}\, \mu U}{F}, \frac{p A}{F}, \frac{A U^2 \rho}{F}$$

$$dimpar_{91} := \frac{F}{A^{3/2} G}, \frac{L}{\sqrt{A}}, \frac{c}{U}, \frac{\delta}{\sqrt{A}}, \frac{\mu U}{A G}, \frac{p}{\sqrt{A}\, G}, \frac{U^2 \rho}{\sqrt{A}\, G}$$

$$dimpar_{93} := \frac{A}{L^2}, \frac{F}{L c \mu}, \frac{L^2 G}{c \mu}, \frac{U}{c}, \frac{\delta}{L}, \frac{L p}{c \mu}, \frac{L \rho c}{\mu}$$

$$dimpar_{94} := \frac{A}{L^2}, \frac{F}{L U \mu}, \frac{L^2 G}{U \mu}, \frac{c}{U}, \frac{\delta}{L}, \frac{L p}{U \mu}, \frac{L \rho U}{\mu}$$

$$dimpar_{96} := \frac{G^{2/3} A}{F^{2/3}}, \frac{G^{1/3} L}{F^{1/3}}, \frac{U}{c}, \frac{G^{1/3} \delta}{F^{1/3}}, \frac{c \mu}{F^{2/3} G^{1/3}}, \frac{p}{F^{1/3} G^{2/3}}, \frac{c^2 \rho}{F^{1/3} G^{2/3}}$$

$$dimpar_{97} := \frac{G^{2/3} A}{F^{2/3}}, \frac{G^{1/3} L}{F^{1/3}}, \frac{U}{c}, \frac{G^{1/3} \delta}{F^{1/3}}, \frac{c \mu}{F^{2/3} G^{1/3}}, \frac{p}{F^{1/3} G^{2/3}}, \frac{c^2 \rho}{F^{1/3} G^{2/3}}$$

$$dimpar_{98} := \frac{G^{2/3} \rho^{2/3} A}{\mu^{4/3}}, \frac{\rho F}{\mu^2}, \frac{G^{1/3} \rho^{1/3} L}{\mu^{2/3}}, \frac{\rho^{2/3} U}{G^{1/3} \mu^{1/3}}, \frac{\rho^{2/3} c}{G^{1/3} \mu^{1/3}}, \frac{G^{1/3} \rho^{1/3} \delta}{\mu^{2/3}},$$
$$\frac{\rho^{1/3} p}{G^{2/3} \mu^{2/3}}$$

$$dimpar_{99} := \frac{p^2 A}{c^2 \mu^2}, \frac{F p}{c^2 \mu^2}, \frac{G \mu c}{p^2}, \frac{L p}{c \mu}, \frac{U}{c}, \frac{\delta p}{c \mu}, \frac{c^2 \rho}{p}$$

$$dimpar_{100} := \frac{A}{\delta^2}, \frac{F}{G \delta^3}, \frac{L}{\delta}, \frac{U \mu}{G \delta^2}, \frac{c \mu}{G \delta^2}, \frac{p}{G \delta}, \frac{G \delta^3 \rho}{\mu^2}$$

$$dimpar_{101} := \frac{A}{\delta^2}, \frac{\rho F}{\mu^2}, \frac{G \delta^3 \rho}{\mu^2}, \frac{L}{\delta}, \frac{U \delta \rho}{\mu}, \frac{c \delta \rho}{\mu}, \frac{\delta^2 \rho p}{\mu^2}$$

$$dimpar_{102} := \frac{A^{3/2} G}{F}, \frac{L}{\sqrt{A}}, \frac{U}{c}, \frac{\delta}{\sqrt{A}}, \frac{\sqrt{A}\, \mu c}{F}, \frac{p A}{F}, \frac{c^2 \rho A}{F}$$

$$dimpar_{103} := \frac{A}{L^2}, \frac{F}{G L^3}, \frac{\sqrt{\rho}\, U}{\sqrt{G}\, \sqrt{L}}, \frac{\sqrt{\rho}\, c}{\sqrt{G}\, \sqrt{L}}, \frac{\delta}{L}, \frac{\mu}{\sqrt{G}\, L^{3/2} \sqrt{\rho}}, \frac{p}{G L}$$

$$dimpar_{104} := \frac{A}{L^2}, \frac{F}{L^2 p}, \frac{G L}{p}, \frac{\mu U}{L p}, \frac{\mu c}{L p}, \frac{\delta}{L}, \frac{L^2 \rho p}{\mu^2}$$

$$dimpar_{105} := \frac{G^2 A}{p^2}, \frac{G^2 F}{p^3}, \frac{G L}{p}, \frac{U}{c}, \frac{G \delta}{p}, \frac{G \mu c}{p^2}, \frac{c^2 \rho}{p}$$

$$dimpar_{106} := \frac{A}{L^2}, \frac{F}{L^2 p}, \frac{G L}{p}, \frac{\mu U}{L p}, \frac{\mu c}{L p}, \frac{\delta}{L}, \frac{L^2 \rho p}{\mu^2}$$

$$dimpar_{108} := \frac{G^{2/3} A}{F^{2/3}}, \frac{G^{1/3} L}{F^{1/3}}, \frac{U}{c}, \frac{G^{1/3} \delta}{F^{1/3}}, \frac{c \mu}{F^{2/3} G^{1/3}}, \frac{p}{F^{1/3} G^{2/3}}, \frac{c^2 \rho}{F^{1/3} G^{2/3}}$$

$$dimpar_{109} := \frac{F}{A^{3/2} G}, \frac{L}{\sqrt{A}}, \frac{\mu U}{A G}, \frac{\mu c}{A G}, \frac{\delta}{\sqrt{A}}, \frac{p}{\sqrt{A}\, G}, \frac{A^{3/2} G \rho}{\mu^2}$$

$$dimpar_{110} := \frac{A^{3/2} G}{F}, \frac{L}{\sqrt{A}}, \frac{\sqrt{A}\sqrt{\rho}\, U}{\sqrt{F}}, \frac{\sqrt{A}\sqrt{\rho}\, c}{\sqrt{F}}, \frac{\delta}{\sqrt{A}}, \frac{\mu}{\sqrt{F}\sqrt{\rho}}, \frac{p A}{F}$$

$$dimpar_{112} := \frac{A}{L^2}, \frac{F}{G L^3}, \frac{\sqrt{\rho}\, U}{\sqrt{G}\sqrt{L}}, \frac{\sqrt{\rho}\, c}{\sqrt{G}\sqrt{L}}, \frac{\delta}{L}, \frac{\mu}{\sqrt{G}\, L^{3/2}\sqrt{\rho}}, \frac{p}{G L}$$

$$dimpar_{113} := \frac{\rho F}{\mu^2}, \frac{A^{3/2} G \rho}{\mu^2}, \frac{L}{\sqrt{A}}, \frac{\sqrt{A}\, \rho U}{\mu}, \frac{\sqrt{A}\, \rho c}{\mu}, \frac{\delta}{\sqrt{A}}, \frac{p \rho A}{\mu^2}$$

$$dimpar_{114} := \frac{G^2 A}{U^4 \rho^2}, \frac{G^2 F}{U^6 \rho^3}, \frac{G L}{U^2 \rho}, \frac{c}{U}, \frac{G \delta}{U^2 \rho}, \frac{\mu G}{U^3 \rho^2}, \frac{p}{U^2 \rho}$$

$$dimpar_{116} := \frac{A}{L^2}, \frac{F}{G L^3}, \frac{\sqrt{\rho}\, U}{\sqrt{G}\sqrt{L}}, \frac{\sqrt{\rho}\, c}{\sqrt{G}\sqrt{L}}, \frac{\delta}{L}, \frac{\mu}{\sqrt{G}\, L^{3/2}\sqrt{\rho}}, \frac{p}{G L}$$

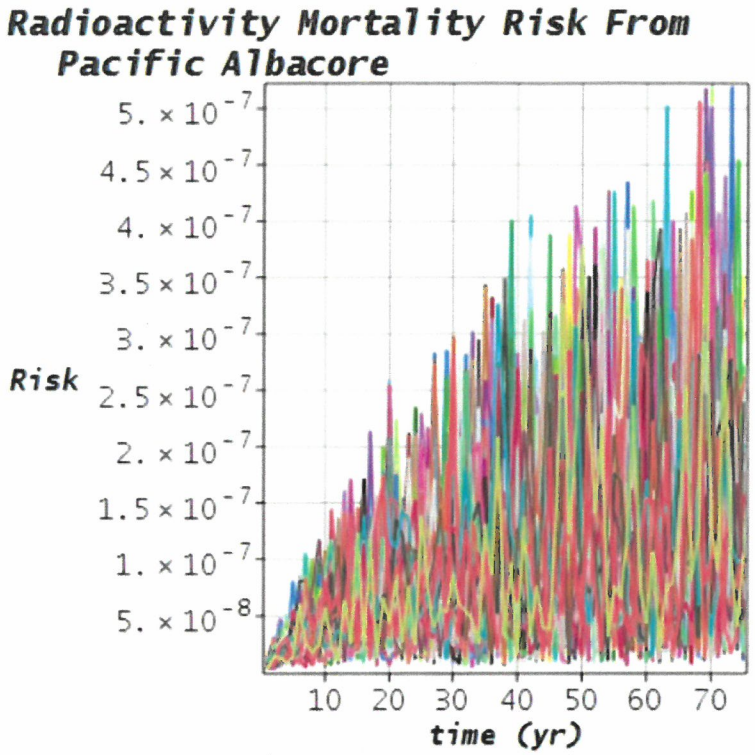
$$dimpar_{118} := \frac{A}{L^2}, \frac{F}{G L^3}, \frac{\sqrt{\rho}\, U}{\sqrt{G}\sqrt{L}}, \frac{\sqrt{\rho}\, c}{\sqrt{G}\sqrt{L}}, \frac{\delta}{L}, \frac{\mu}{\sqrt{G}\, L^{3/2}\sqrt{\rho}}, \frac{p}{G L}$$

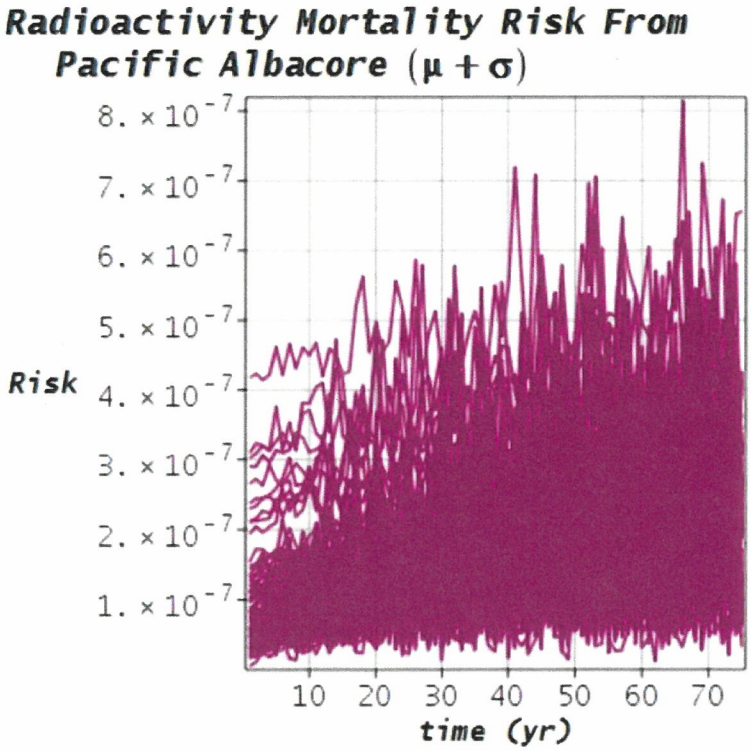
$$dimpar_{119} := \frac{G^{2/3} A}{F^{2/3}}, \frac{G^{1/3} L}{F^{1/3}}, \frac{c}{U}, \frac{G^{1/3} \delta}{F^{1/3}}, \frac{U \mu}{F^{2/3} G^{1/3}}, \frac{p}{F^{1/3} G^{2/3}}, \frac{U^2 \rho}{F^{1/3} G^{2/3}}$$

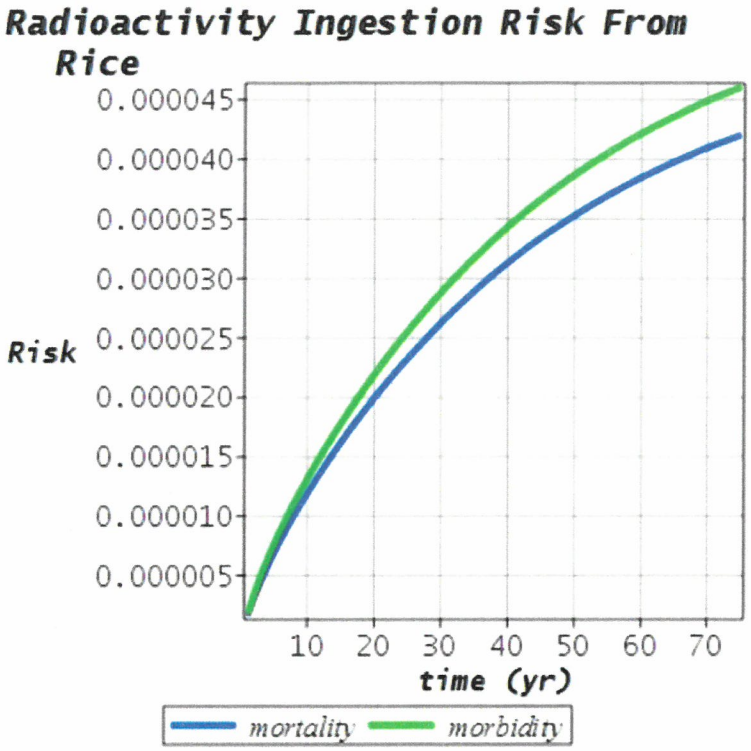
>

3.2 Radiological Accident Contaminated Tuna Off West Coast

```
restart : with(Student) : with(plots) : with(RandomTools) : with(Statistics) : wth(ListTools) :
N := 75; col1 := seq(COLOR(RGB, rand()/10^12, rand()/10^12, rand()/10^12), i = 1 ..N) : mort :=
[(9.57·10^-10, 6.88·10^-10)]; morb := [(1.39·10^-9, 1.01·10^-9)]; HLsec := [(65007360,
9.50·10^8)]; T := 75; cf := evalf(3600·24·364.2); ingest := 24.1; cic := evalf(3.7·10^10
·10^-12); c1 := evalf(200·cic); c2 := evalf(200·cic); contam := [(c1, c2)] : dos134f :=
t→ingest·contam[1]· mort[1]·e^(-ln(2)/HLsec[1]·cf·t); dos137f := t→ingest·contam[2]
· mort[2]·e^(-ln(2)/HLsec[2]·cf·t); dos134b := t→ingest·contam[1]· morb[1]
·e^(-ln(2)/HLsec[1]·cf·t); dos137b := t→ingest·contam[2]· morb[2]·exp(-ln(2)/HLsec[2]·cf·t);
xx := [seq(i, i = 1 ..T)]; yy0 := [seq(evalf(ingest·contam[1]· mort[1]
·(1 - exp(-ln(2)/HLsec[1]·cf·i))/(ln(2)/HLsec[1]·cf)), i = 1 ..T)] : yy1 :=
[seq(evalf((ingest·contam[2]· mort[2]·(1 - exp(-ln(2)/HLsec[2]·cf·i)))/(ln(2)/HLsec[2]·cf)), i = 1 ..T)] :
yy := [seq(yy1[j] + yy0[j], j = 1 ..T)]; zz0 :=
[seq(evalf((ingest·contam[1]· morb[1]·(1 - exp(-ln(2)/HLsec[1]·cf·i)))/(ln(2)/HLsec[1]·cf)), i = 1 ..T)] :
zz1 := [seq(evalf((ingest·contam[2]· morb[2]·(1 - exp(-ln(2)/HLsec[2]·cf·i)))/(ln(2)/HLsec[2]·cf)), i = 1
..T)] : zz := [seq(zz1[j] + zz0[j], j = 1 ..T)];for j from 1 to N do n := 75; un1 :=
Generate(list(float( range = 1 ..2, method = uniform), N)) : un11 :=
Generate(list(float( range = 0 ..10, method = uniform), N)) : b137 :=
Generate(list(float( range = 0.2 ..0.8, method = uniform), N)); b134 :=
Generate(list(float( range = 1 ..2, method = uniform), N)) : ingest :=
Generate(list(float( range = 1 ..3, method = uniform), N)) : dos134f := t→ingest
·b134[1]· mort[1]·e^(-ln(2)/HLsec[1]·cf·t) : dos137f := t
→ingest·b137[2]· mort[2]·(1-e^(-ln(2)/HLsec[2]·cf·t))/(ln(2)/HLsec[2]·cf) : xx2 := [seq(i, i = 1 ..T)]; yy0 :=
[seq(evalf((ingest[i]·b134[i]· mort[1]·un1[i]·(1 - exp(-ln(2)/HLsec[1]·cf·i)))/(ln(2)/HLsec[1]·cf)), i = 1
..T)] : yy1 :=
[ ( ( ingest[i]·b137[i]· mort[2]·un11[i]·(1 - exp(-ln(2)/·cf·i))) )
```

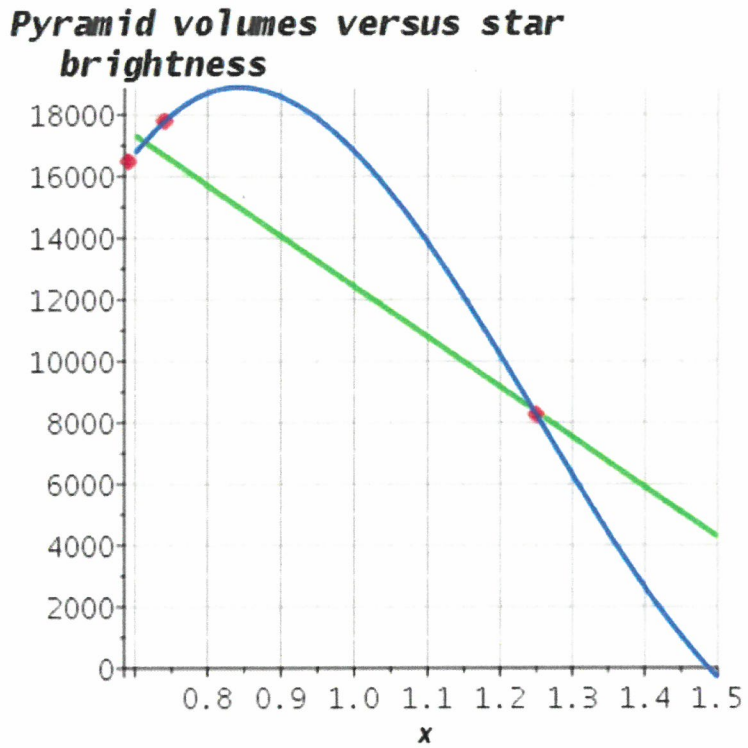

Radioactivity Mortality Risk From
Pacific Albacore
Risk
5. × 10^-7
4.5 × 10^-7
4. × 10^-7
3.5 × 10^-7
3. × 10^-7
2.5 × 10^-7
2. × 10^-7
1.5 × 10^-7
1. × 10^-7
5. × 10^-8
10 20 30 40 50 60 70
time (yr)

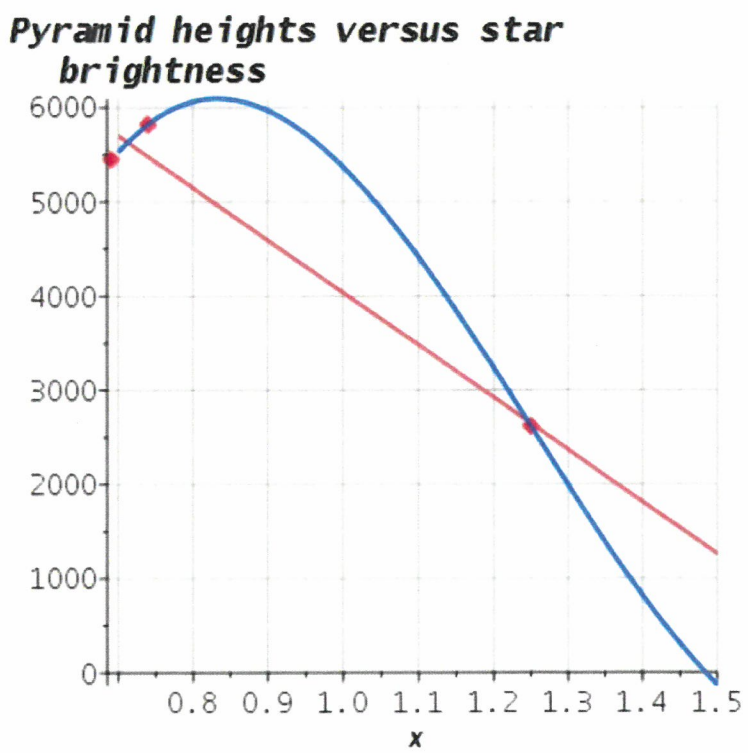

Radioactivity Mortality Risk From
Pacific Albacore (μ + σ)
Risk
8. × 10^-7
7. × 10^-7
6. × 10^-7
5. × 10^-7
4. × 10^-7
3. × 10^-7
2. × 10^-7
1. × 10^-7
10 20 30 40 50 60 70
time (yr)



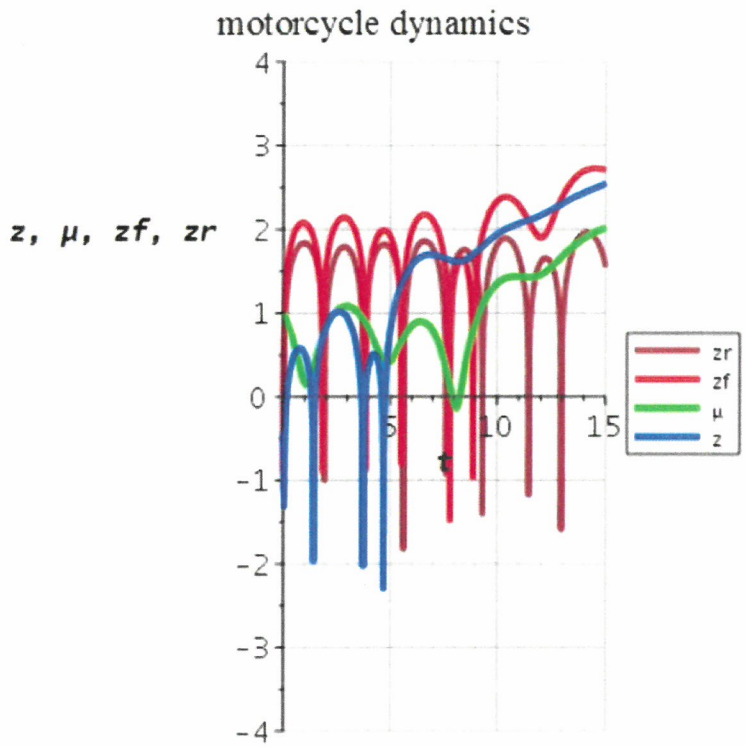
r*estart; with(Student); with(plots); mort := [1.66*10^(-9), 2.77*10^(-9)]; morb := [1.89*10^(-9), 3.03*10^(-9)]; HLsec := [65007360, 9.50*10^8]; T := 75; cf := evalf(364.2*(3600*24)); ingest := 54.6375; cic := evalf(3.7*10^10*10^(-12)); c1 := evalf(200*cic); c2 := evalf(200*cic); contam := [c1, c2]; dos134f := proc (t) options operator, arrow; ingest*contam[1]*mort[1]*e^(-ln(2)*cf*t/HLsec[1]) end proc; dos137f := proc (t) options operator, arrow; ingest*contam[2]*mort[2]*e^(-ln(2)*cf*t/HLsec[2]) end proc; dos134b := proc (t) options operator, arrow; ingest*contam[1]*morb[1]*e^(-ln(2)*cf*t/HLsec[1]) end proc; dos137b := proc (t) options operator, arrow; ingest*contam[2]*morb[2]*exp(-ln(2)*cf*t/HLsec[2]) end proc; xx := [seq(I, I = 1 .. T)]; yy0 := [seq(evalf(ingest*contam[1]*mort[1]*(1-exp(-ln(2)*cf*i/HLsec[1]))*HLsec[1]/(ln(2)*cf)), I = 1 .. T)]; yy1 := [seq(evalf(ingest*contam[2]*mort[2]*(1-exp(-ln(2)*cf*i/HLsec[2]))*HLsec[2]/(ln(2)*cf)), I = 1 .. T)]; yy := [seq(yy1[j]+yy0[j], j = 1 .. T)]; zz0 := [seq(evalf(ingest*contam[1]*morb[1]*(1-exp(-ln(2)*cf*i/HLsec[1]))*HLsec[1]/(ln(2)*cf)), I = 1 .. T)]; zz1 := [seq(evalf(ingest*contam[2]*morb[2]*(1-exp(-ln(2)*cf*i/HLsec[2]))*HLsec[2]/(ln(2)*cf)), I = 1 .. T)]; zz := [seq(zz1[j]+zz0[j], j = 1 .. T)]; pl1 := pointplot({seq([xx[i], yy[i]], I = 1 .. T)}, axes = boxed, color = blue, style = line, thickness = 3, title = 'Radioactivity*Ingestion*Risk*From*Rice', titlefont = [COURIER, Bold, 14], labels = ["time (yr)", "Risk"], labelfont = [COURIER, BOLDITALIC, 12], axesfont = [COURIER, 12], legend = mortality, gridlines, tickmarks = [10, 10]); pl2 := pointplot({seq([xx[i], zz[i]], I = 1 .. T)}, axes = boxed, color = green, style = line, thickness = 3, title = `Radioactivity Ingestion Risk From Rice  `, titlefont = [COURIER, Bold, 14], labels = ["time (yr)", "Risk"], labelfont = [COURIER, BOLDITALIC, 12], axesfont = [COURIER, 12], legend = morbidity, gridlines, tickmarks = [10, 10]); plots[display]([pl1, pl2])

3.3 Egypt's Great Pyramid's Heights versus Orion Belt Star's Brightness





3.5 Four Degrees of Freedom Motorcycle Dynamics Model

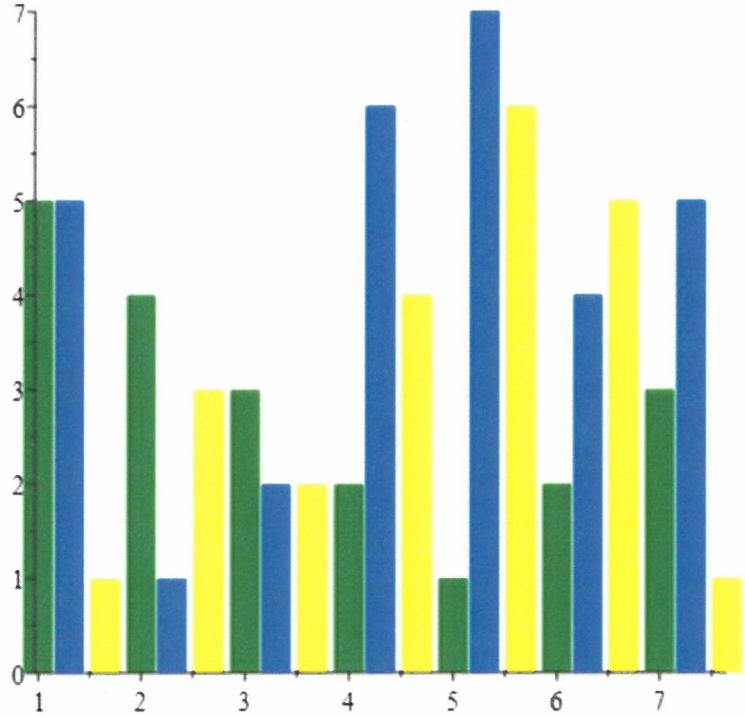



```
restart; with(Student); with(plots); with(LinearAlgebra); with(DETools); with(diffalg); with(inttrans); mf
:= 15.; mr := 18.; m := 200.; p := 1.414; b := .48; iyg := 38.; kf := 15.; kr := 24.; kp := 180; M1 := Matrix(4, 4,
[[m, 0, 0, 0], [0, iyg, 0, 0], [0, 0, mf, 0], [0, 0, 0, mr]]); M2 := Matrix(4, 4, [[kf+kr, (p-b)*kf-b*kr, -kf, -kr],
[(p-b)*kf-b*kr, (p-b)^2*kf+b^2*kf, -(p-b)*kf, b*kr], [-kf, -(p-b)*kf, b*kr], [-kr, b*kr, 0, kr+kr]]); q :=
Vector(4, [z(t), mu(t), zf(t), zr(t)]); A0 := 10.; omega1 := 1.5; tau := 100.; zer1 := Vector(4, [A0*exp(-
t/tau)*(1-t/tau)*Heaviside(t), 0, 0, 0]); eq0 := M1 . (diff(q, t, t))+M2 . q-zer1; de1 := eq0[1]; de2 := eq0[2];
de3 := eq0[3]; de4 := eq0[4]; ic1 := A0; ic2 := 0.5e-1; ic3 := 0.1e-1; ic4 := 0; z(0) := ic1; mu(0) := ic2; zf(0) :=
ic3; zr(0) := ic4; (D(z))(0) := 0; (D(mu))(0) := 0; (D(zf))(0) := 0; (D(zr))(0) := 0; delp1 := laplace(de1, t, s);
delp1s := collect(delp1, [laplace(z(t), t, s), laplace(mu(t), t, s), laplace(zf(t), t, s), laplace(zr(t), t, s)],
recursive); delp2 := laplace(de2, t, s); delp2s := collect(delp2, [laplace(z(t), t, s), laplace(mu(t), t, s),
laplace(zf(t), t, s), laplace(zr(t), t, s)], recursive); delp3 := laplace(de3, t, s); delp3s := collect(delp3,
[laplace(z(t), t, s), laplace(mu(t), t, s), laplace(zf(t), t, s), laplace(zr(t), t, s)], recursive); delp4 :=
laplace(de4, t, s); delp4s := collect(delp4, [laplace(z(t), t, s), laplace(mu(t), t, s), laplace(zf(t), t, s),
laplace(zr(t), t, s)], recursive); sol1m := solve({delp1s, delp2s, delp3s, delp4s}, {laplace(mu(t), t, s),
laplace(z(t), t, s), laplace(zf(t), t, s), laplace(zr(t), t, s)}); xt := invlaplace(rhs(sol1m[1]), s, t); yt :=
invlaplace(rhs(sol1m[2]), s, t); zt := invlaplace(rhs(sol1m[3]), s, t); wt := invlaplace(rhs(sol1m[4]), s, t); tm
:= 15; y1 := 4.; pl1 := plot(evalf(log10(abs(xt))), t = 0 .. tm, y = -y1 .. y1, axes = normal, gridlines, thickness
= 3, color = blue, style = line, title = " motorcycle dynamics ", titlefont = [COURIER, Bold, 14], labels = ["t",
"z, &mu;, zf, zr"], labelfont = [COURIER, BOLDITALIC, 12], axesfont = [COURIER, 12], legend = ["z"],
titlefont = ["ARIAL", 15], legendstyle = [font = ["HELVETICA", 9], location = right]); pl2 :=
```

```
plot(evalf(log10(abs(yt))), t = 0 .. tm, y = -y1 .. y1, axes = normal, gridlines, thickness = 3, color = green, style = line, title = "motorcycle dynamics", titlefont = [COURIER, Bold, 14], labels = ["t", "z, &mu;, zf, zr"], labelfont = [COURIER, BOLDITALIC, 12], legend = ["&mu;"], titlefont = ["ARIAL", 15], legendstyle = [font = ["HELVETICA", 9], location = right]); pl3 := plot(evalf(log10(abs(zt))), t = 0 .. tm, y = -y1 .. y1, axes = normal, gridlines, thickness = 3, color = red, style = line, title = " ", titlefont = [COURIER, Bold, 14], labels = ["t", "z, &mu;, zf, zr"], labelfont = [COURIER, BOLDITALIC, 12], legend = ["zf"], titlefont = ["ARIAL", 15], legendstyle = [font = ["HELVETICA", 9], location = right]); pl4 := plot(evalf(log10(abs(wt))), t = 0 .. tm, y = -y1 .. y1, axes = normal, gridlines, thickness = 3, color = brown, style = line, title = "motorcycle dynamics", titlefont = [COURIER, Bold, 14], labels = ["t", "z, &mu;, zf, zr"], labelfont = [COURIER, BOLDITALIC, 12], legend = ["zr"], titlefont = ["ARIAL", 15], legendstyle = [font = ["HELVETICA", 9], location = right]); plots[display]({pl1, pl2, pl3, pl4});
```

3.6 Interest Compounding, Present Value, Capital Recovery Factor calculations

3.7 Bar Chart




```
> restart : with(Student) : with(plots) : with(ColorTools) : with(plottools) :
  n := 7; x1 := seq(1.0·i, i = 1 ..n); wd := 0.3; x2 := seq(i + wd, i = 1 ..n);
  we := 0.65; x3 := seq(i + we, i = 1 ..n) ; y1 := [5, 4, 3, 2, 1, 2, 3]; y2 :=
  [5, 1, 2, 6, 7, 4, 5]; y3 := [1, 3, 2, 4, 6, 5, 1];for i from 1 to n
  do plx[i] := display(line([x1[i], 0], [x1[i], y1[i]]), linestyle = solid,
  color = "Green", thickness = 15); plx3[i] := display(line([x3[i], 0],
  [x3[i], y3[i]]), linestyle = solid, color = "yellow", thickness = 15);
  plx2[i] := display(line([x2[i], 0], [x2[i], y2[i]]), linestyle = solid,
  color = "blue", thickness = 15);end do: plty := plots[display]({seq(plx[i],
  i = 1 ..n), seq(plx2[i], i = 1 ..n), seq(plx3[i], i = 1 ..n)});
```

$$n := 7$$

$$x1 := 1.0, 2.0, 3.0, 4.0, 5.0, 6.0, 7.0$$

$$wd := 0.3$$

$$x2 := 1.3, 2.3, 3.3, 4.3, 5.3, 6.3, 7.3$$

$$we := 0.65$$

$$x3 := 1.65, 2.65, 3.65, 4.65, 5.65, 6.65, 7.65$$

$$y1 := [5, 4, 3, 2, 1, 2, 3]$$

$$y2 := [5, 1, 2, 6, 7, 4, 5]$$

$$y3 := [1, 3, 2, 4, 6, 5, 1]$$

3.8 Eigenvalues, Eigenvector Diagonalization

```
restart; with(Student); with(plots); with(LinearAlgebra); with(inttrans); with(DETools); A := Matrix(3, 3, [[0, 0, 3], [-1, -2, 2], [2, 0, 2]]); Determinant(A); Id3 := IdentityMatrix(3); x1 := Vector(3, [x, y, z]); lam := Matrix(3, 3, [[lambda, 0, 0], [0, lambda, 0], [0, 0, lambda]]); Alam := A-lam . Id3; eq := Determinant(Alam); sol := solve(eq = 0, lambda); sol1 := evalf(sol[1]); sol2 := evalf(sol[2]); sol3 := evalf(sol[3]); pts := [sol1, sol2, sol3]; complexplot(pts, style = point, symbol = solidbox, symbolsize = 16, color = red, axes = normal, gridlines); Eigenvalues(A); eig := evalf(%); eigv := evalf(Eigenvectors(A)); Av := Matrix(3, 3, eigv[2]); dtm := Determinant(Av); dtmr := evalf(1/dtm); Vinv := Adjoint(Av) . dtmr; Vinv . A . eigv[2];
```

$$A := \begin{bmatrix} 0 & 0 & 3 \\ -1 & -2 & 2 \\ 2 & 0 & 2 \end{bmatrix}$$

$$12$$

$$Id3 := \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$x1 := \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

$$lam := \begin{bmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{bmatrix}$$

$$Alam := \begin{bmatrix} -\lambda & 0 & 3 \\ -1 & -2-\lambda & 2 \\ 2 & 0 & 2-\lambda \end{bmatrix}$$

$$eq := -\lambda^3 + 10\lambda + 12$$

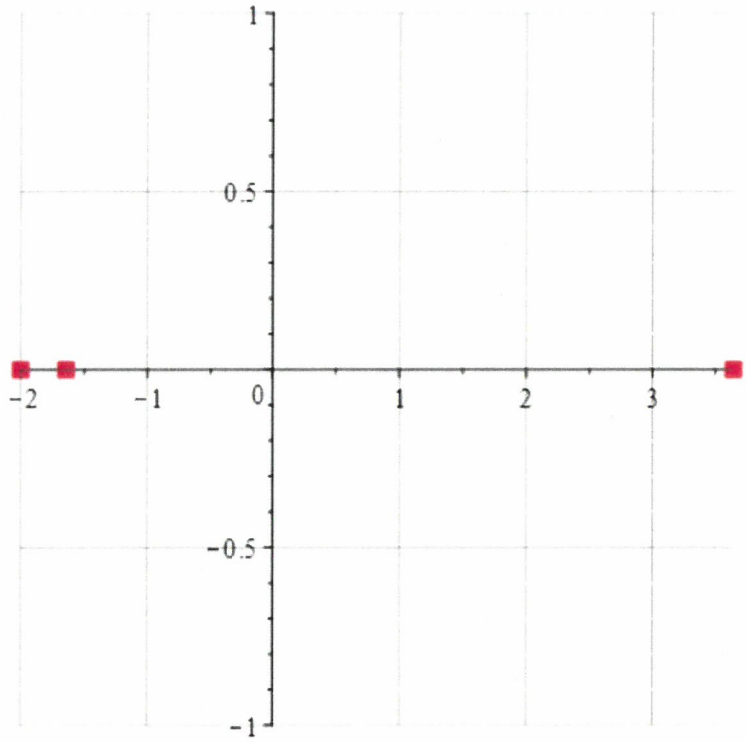
$$sol := -2, 1-\sqrt{7}, 1+\sqrt{7}$$

$$sol1 := -2.$$

$$sol2 := -1.645751311$$

$$sol3 := 3.645751311$$

$$pts := [-2., -1.645751311, 3.645751311]$$



$$eig := \begin{bmatrix} -2. \\ -1.645751311 \\ 3.645751311 \end{bmatrix}$$

$$eigv := \begin{bmatrix} 3.645751311 \\ -1.645751311 \\ -2. \end{bmatrix}, \begin{bmatrix} 0.8228756556 & -1.822875656 & 0. \\ 0.2084973779 & 10.79150262 & 1. \\ 1. & 1. & 0. \end{bmatrix}$$

$$Av := \begin{bmatrix} 0.8228756556 & -1.822875656 & 0. \\ 0.2084973779 & 10.79150262 & 1. \\ 1. & 1. & 0. \end{bmatrix}$$

$$dtm := -2.645751312$$

$$dtmr := -0.3779644729$$

$$Vinv := \begin{bmatrix} 0.377964472900000 & -0. & 0.688982236482282 \\ -0.377964472900000 & -0. & 0.311017763431096 \\ 3.99999999723454 & 1.00000000006456 & -3.49999999871411 \end{bmatrix}$$

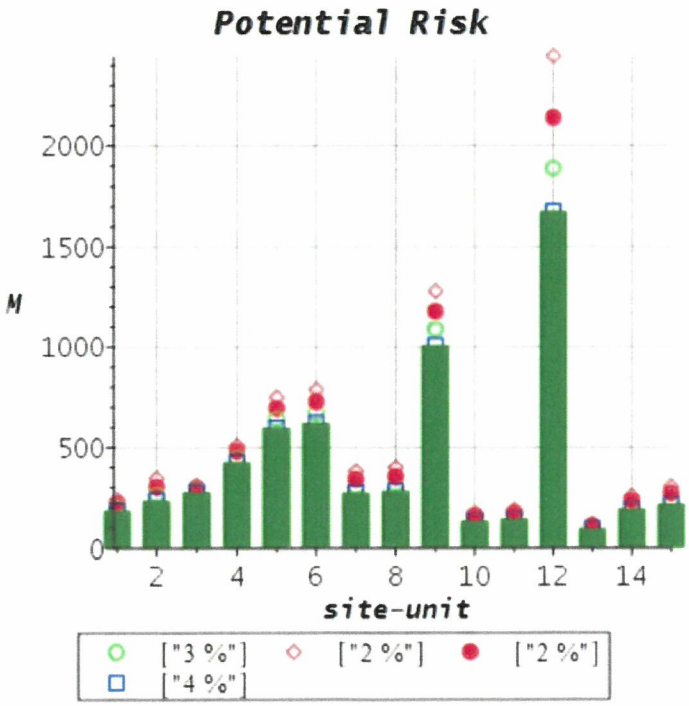
$$\begin{bmatrix} 3.64575131074879 & -9.35409083524519\,10^{-10} & 0. \\ 1.35409683466037\,10^{-10} & -1.64575131092203 & 0. \\ -4.15929068964260\,10^{-9} & -3.55928886364154\,10^{-9} & -2.00000000012913 \end{bmatrix}$$

>

3.9 Regional Nuclear Generating Risk Allocations

Study Region

Site/Unit	z pcm	Years	prem
Beaver Valley 1	-0.53	19.00	1.19
Beaver Valley 2	-0.53	30.00	1.19
Limerick 1	-0.49	7.00	9.16
Limerick 2	-0.49	12.00	9.16
Peach Bottom 2	-0.47	16.00	9.16
Peach Bottom 3	-0.49	17.00	9.16
Susquehanna 1	-0.60	25.00	1.67
Susquehanna 2	-0.61	27.00	1.63
Three Mile Island 1	-0.59	17.00	32.61
Calvert Cliffs 1	-0.52	17.00	0.41
Calvert Cliffs 2	-0.52	19.00	0.41
Hope Creek	0.96	29.00	28.80
Oyster Creek	-0.60	12.00	0.49
Salem 1	0.56	19.00	1.21
Salem 2	0.56	23.00	1.21



```
restart; with(Student); with(Finance); with(plots); with(ColorTools); ir := [0.2e-1, 0.3e-1, 0.4e-1, 0.5e-1];
cmp := 12; `&delta;oca` := 3300.; `&delta;occ` := 2.*10^4; Ccd := 1.5*10^9; Crp := evalf(1.2*10^8); Lvalue
:= 5000.; sec5 := evalf(16/3); epsilon := evalf(0.322e-4/0.641e-5); Onst := 3.*10^9; CV1 := 2.3; yrsr :=
Vector(15, Vector[column](15, [19, 30, 7, 12, 16, 17, 25, 27, 17, 17, 19, 29, 12, 19, 23])); repl :=
Vector(15, Vector[column](15, [10, 10, 7, 10, 10, 10, 10, 10, 10, 10, 10, 10, 10, 10, 10])); pwr1 :=
Vector(15, Vector[column](15, [967., 967., 1170., 1170., 1320., 1320., 1320., 1320., 856., 912., 912.,
1280., 643., 1170., 1130.])); PL := [seq(evalf((1/910)*pwr1[i]), i = 1 .. 15)]; offprem := Vector(15,
Vector[column](15, [1.191, 1.191, 9.16, 9.16, 9.16, 9.16, 1.67, 1.63, 32.61, .409, .409, 28.8, .486, 1.214,
1.214])); pcm := Vector(15, Vector[column](15, [0.3980000000e-5, 0.3980000000e-5, 0.5030000000e-5,
0.5030000000e-5, 0.5420000000e-5, 0.5030000000e-5, 0.1860000000e-5, 0.1830000000e-5,
0.2370000000e-5, 0.4080000000e-5, 0.4080000000e-5, 0.4440000000e-4, 0.2050000000e-5,
0.3350000000e-4, 0.3350000000e-4])); premgeo := Vector(15, Vector[column](15, [45.3, 45.3, 143.,
143., 164., 164., 57.9, 57.2, 267., 35., 35., 322., 32.1, 42.6, 42.6])); for j to 4 do crf0[j] :=
[seq(evalf(ir[j]*(1.+ir[j])^yrsr[i]/((1.+ir[j])^yrsr[i]-1.)), i = 1 .. 15)]; Ct[j] := [seq(evalf(1./crf0[j][i]), i = 1 ..
15)]; Ctc[j] := [seq(evalf((1-exp(-ir[j]*yrsr[i]))/ir[j]), i = 1 .. 15)]; Sm[j] := [seq(evalf((1-exp(-
ir[j]*repl[i]))/(repl[i]*ir[j])), i = 1 .. 15)] end do; for j to 4 do Offex[j] := [seq(evalf((Lvalue .
premgeo[i])*Ct[j][i]*sec5), i = 1 .. 15)]; OnLT[j] := [seq(evalf((pcm[i] . `&delta;occ` . Lvalue)*Ct[j][i] .
Sm[j][i]), i = 1 .. 15)]; Onst[j] := [seq(pcm[i] . Lvalue . `&delta;oca` . Ct[j][i], i = 1 .. 15)]; Ontot[j] :=
[seq(Onst[j][i]+Offex[j][i]+OnLT[j][i], i = 1 .. 15)]; Ocd[j] := [seq(evalf((pcm[i] . Crp)*Ct[j][i]^2 . PL[i]), i = 1
.. 15)]; Rp[j] := [seq(evalf(pcm[i]*Crp*Ct[j][i]^2*PL[i]), i = 1 .. 15)]; TotInt[j] :=
[seq(evalf(Ontot[j][i]+Ocd[j][i]+Rp[j][i]), i = 1 .. 15)] end do; for j to 4 do Totext[j] :=
[seq(evalf(epsilon*TotInt[j][i]), i = 1 .. 15)]; Total[j] := [seq(evalf((CV1*Totext[j][i]+TotInt[j][i])*10^(-6)), i
= 1 .. 15)] end do;

with(plottools); "use Enter twice"; pl4 := seq(pointplot([pts[4][i]], symbol = box, color = blue, symbolsize
= 16, axes = normal, gridlines, title = 'Potential*Risk', titlefont = [COURIER, Bold, 14], labels = ["site-unit",
"M"], labelfont = [COURIER, BOLDITALIC, 12], axesfont = [COURIER, 12], legend = ["4 %"]), i = 1 .. 15);
plots[display]({pl4}); pl3 := seq(pointplot([pts[3][i]], symbol = circle, color = green, symbolsize = 16, axes
= normal, gridlines, title = 'Potential*Risk', titlefont = [COURIER, Bold, 14], labels = ["site-unit", "M"],
labelfont = [COURIER, BOLDITALIC, 12], axesfont = [COURIER, 12], legend = ["3 %"]), i = 1 .. 15);
plots[display]({pl3}); pl2 := seq(pointplot([pts[2][i]], symbol = solidcircle, color = red, symbolsize = 16,
axes = normal, gridlines, title = 'Potential*Risk', titlefont = [COURIER, Bold, 14], labels = ["site-unit",
"M"], labelfont = [COURIER, BOLDITALIC, 12], axesfont = [COURIER, 12], legend = ["2 %"]), i = 1 .. 15);
plots[display]({pl2}); pl1 := seq(pointplot([pts[1][i]], symbol = diamond, color = orange, symbolsize = 16,
axes = normal, gridlines, title = 'Potential*Risk', titlefont = [COURIER, Bold, 14], labels = ["site-unit",
"M"], labelfont = [COURIER, BOLDITALIC, 12], axesfont = [COURIER, 12], legend = ["2 %"]), i = 1 .. 15);
plots[display]({pl1}); n := 15; wd := 1.; x1 := seq(wd*i, i = 1 .. n); for i to n do plx[i] := display(line([x1[i], 0],
[x1[i], Total[4][i]]), linestyle = solid, color = "Green", thickness = 15, axes = normal, gridlines, title =
'Potential*Risk', titlefont = [COURIER, Bold, 14], labels = ["site-unit", "M"], labelfont = [COURIER,
BOLDITALIC, 12], axesfont = [COURIER, 12]) end do; pltyy := plots[display]({seq(plx[i], i = 1 .. n)}); pts[1]
:= seq([i, Total[1][i]], i = 1 .. 15); pts[2] := seq([i, Total[2][i]], i = 1 .. 15); pts[3] := seq([i, Total[3][i]], i = 1 ..
15); pts[4] := seq([i, Total[4][i]], i = 1 .. 15); plots[display]({pl1, pl2, pl3, pl4, pltyy}); nmes := Vector(15,
```

["1=Beaver Valley 1", "2=Beaver Valley 2", "3=Limerick 1", "4=Limerick 2", "5=Peach Bottom 2", "6=Peach Bottom 3", "7=Susquehanna 1", "8=Susquehanna 2", "9=Three Mile Island 1", "10=Calvert Cliffs 1", "11=Calvert Cliffs 2", "12=Hope Creek", "13=Oyster Creek", "14=Salem 1", "15=Salem 2"])

Regional Optimization

Allocating resources to a subset of the region's power plants by choosing variable, $N \leq 15$, of the sites-units while maximizing the potential risk reduction obtains, for example, using MAPLE software.

N	Sites/Units
2	7, 8
4	7, 8, 9, 13
6	1, 2, 7, 8, 9, 13
8	1, 2, 7, 8, 9, 10, 11, 13

The weights on the binary decision variable to select a site/unit are the reciprocals of the IPE probabilities of core-melt (x 10^6), s well as, offsite person-rem . Prevention of core melt is 75% and offsite risk 25%.

```
restart; with(Student); with(Optimization); offprem0 := Vector(15, Vector[column](15, [1.191, 1.191, 9.16, 9.16, 9.16, 9.16, 1.67, 1.63, 32.61, .409, .409, 28.8, .486, 1.214, 1.214])); pcm0 := Vector(15, Vector[column](15, [0.3980000000e-5, 0.3980000000e-5, 0.5030000000e-5, 0.5030000000e-5, 0.5420000000e-5, 0.5030000000e-5, 0.1860000000e-5, 0.1830000000e-5, 0.2370000000e-5, 0.4080000000e-5, 0.4080000000e-5, 0.4440000000e-4, 0.2050000000e-5, 0.3350000000e-4, 0.3350000000e-4])); offprem := [seq(evalf(1/offprem0[i]), i = 1 .. 15)]; pcm := [seq(evalf(10^6/pcm0[i]), i = 1 .. 15)]; N := 15; for k to N do NTot := k; prev := .75; mit := .25; sol1[k] := LPSolve(sum((mit*offprem[j]+prev . pcm[j])*x[j], j = 1 .. N), {seq(x[i] >= 0, i = 1 .. N), sum(x[i], i = 1 .. N) <= NTot}, maximize, assume = binary) end do; nme := vector(15, ["1=Beaver Valley 1", "2=Beaver Valley 2", " 3=Limerick 1", "4=Limerick 2", "5=Peach Bottom 2", "6=Peach Bottom 3", "7=Susquehanna 1", "8=Susquehanna 2", "9=Three Mile Island 1", "10=Calvert Cliffs 1", "11=Calvert Cliffs 2", "12=Hope Creek", "13=Oyster Creek", "14=Salem 1", "15=Salem 2"]);
```

4.0 Spherical Trigonometry Haversine Distance Calculation

```
restart; with(Student); with(plots); with(LinearAlgebra); with(MTM); nmes := ["PHL", "NPoint", "JRS", "Mecca"]; M := Matrix(4, 2, [[39.95258, -75.1652], [90., -75.1652], [31.7683, 35.2137], [21.42664, 39.82563]]); f := 1.; M1 := evalf(f . M); RE := 3962.; Lat := [seq(M1[i, 1], i = 1 .. 4)]; Long := [seq(M1[k, 2], k = 1 .. 4)]; for j to 4 do for i to 4 do A[i, j] := evalf(2*RE*arcsin(sin(.5*(Lat[i]-Lat[j]))^2+cos(Lat[j])*cos(Lat[i])*sin(.5*(Long[i]-Long[j]))^2)) end do end do; dist := Matrix(4, 4, [[A[1, 1], A[1, 2], A[1, 3], A[1, 4]], [A[2, 1], A[2, 2], A[2, 3], A[2, 4]], [A[3, 1], A[3, 2], A[3, 3], A[3, 4]], [A[4, 1], A[4, 2], A[4, 3], A[4, 4]]]);
```

$$nmes := [\text{"PHL"}, \text{"NPoint"}, \text{"JRS"}, \text{"Mecca"}]$$

$$M := \begin{bmatrix} 39.95258 & -75.1652 \\ 90. & -75.1652 \\ 31.7683 & 35.2137 \\ 21.42664 & 39.82563 \end{bmatrix}$$

$$f := 1.$$

$$M1 := \begin{bmatrix} 39.9525800000000 & -75.1652000000000 \\ 90. & -75.1652000000000 \\ 31.7683000000000 & 35.2137000000000 \\ 21.4266400000000 & 39.8256300000000 \end{bmatrix}$$

$$RE := 3962.$$

$$Lat := [39.9525800000000, 90., 31.7683000000000, 21.4266400000000]$$

$$Long := [-75.1652000000000, -75.1652000000000, 35.2137000000000, 39.8256300000000]$$

$$dist := \begin{bmatrix} 0. & 93.8285077516663 & 763.230523645900 & 3062.73664150311 \\ 93.8285077516663 & 0. & 1225.20748931032 & 2588.44694381529 \\ 763.230523645900 & 1225.20748931032 & 0. & 2985.50470742029 \\ 3062.73664150311 & 2588.44694381529 & 2985.50470742029 & 0. \end{bmatrix}$$

>

Hobby On !

Disclaimer

The applications and any opinions expressed within are the author's only and do not reflect the views of the U. S. Nuclear Regulatory Commission, the U. S. Environmental Protection Agency or other government agencies.